POPL Quiz 1

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1. Consider the language of arithmetic expressions with an addition of $\otimes$ binary operator:

`t` :=	– *terms*
`true`	– *constant true*
`false`	– *constant false*
`if t then t else t`	– *conditional*
0	– *constant zero*
`succ t`	– *successor*
`pred t`	– *predecessor*
`iszero t`	– *zero test*
`t` $\otimes$ `t`	– *a special operator*

Evaluating $\mathtt{t}_1 \otimes \mathtt{t}_2$: If $\mathtt{t}_1$ evaluates to a non 0 value, then the value of $\mathtt{t}_1 \otimes \mathtt{t}_2$ is same as the value of $\mathtt{t}_1$. Otherwise, it is same as the value of $\mathtt{t}_2$.

The set of values remains the same:

`v` :=	– *values*
`true`	– *value true*
`false`	– *value false*
0	– *value zero*
`succ v`	– *successor value*

   1. Give small step semantics for the evaluation of $\otimes$. Make sure that $\mathtt{t}_2$ is evaluated only if necessary.
   2. Provide typing rules for $\otimes$ that can be used for static type checking/inferencing. Typing rules for the other constructs remain the same as defined in the lectures.
   3. Are there any executable programs using $\otimes$ that are not "stuck", but are rejected by the typing rule you added? If yes, give an example. If no, provide a proof.

   Clearly list any assumptions you make. (3*5)

2. In λ-calculus, define the recursive version of power function (power $m\ n = m^n$)for church numerals, where the recursive definition (for Natural numbers) is:

   $$\mathsf{power}\ \mathrm{m}\ \ \mathrm{n} = \mathtt{if}\ \mathrm{n}{=}{=}0\ \mathtt{then}\ 1\ \mathtt{else}\ \mathrm{m} * \mathsf{power}\ \mathrm{m}\ (\mathrm{n}\text{-}1)$$

   You can use the λ-calculus terms defined in the class without redefining them, such as, *isZero, mult, add, sub, succ, pred, Y* etc. (5)

3. Show the steps in the evaluation of the below λ-term to its normal form. Reduce only one λ binding in a step.

   $(\lambda\ x\ \cdot\ x\ (\lambda\ z\ p\ w \cdot\ (\lambda\ x\ y \cdot\ y))\ (\lambda\ x\ y \cdot\ x))\ \ ((\lambda\ x\ y\ z\ \cdot\ z\ x\ y)\ b\ (\lambda\ x\ y \cdot\ y))$ (5)