Nikhil Mehta, 190549

Question: 1

(a)

$L_1 = \{x \in \{0,1\}^* \mid x$ does not contain the substring $101\}$

- When 1 we get, next two characters cannot be 01

$RE = 0^*(1^*000^*)1^*0^*$

(b) $L_2 = \{x = \{0,1\}^* \mid x$ has atmost two 0's & at most three 1's$\}$.

RE for L_2 by observing each:

$(0+\varepsilon)(0+\varepsilon)(1+\varepsilon)(1+\varepsilon)(1+\varepsilon) +$

$(0+\varepsilon)(1+\varepsilon)(0+\varepsilon)(1+\varepsilon)(1+\varepsilon) +$

$(0+\varepsilon)(1+\varepsilon)(1+\varepsilon)(0+\varepsilon)(1+\varepsilon) +$

$\cancel{(0+\varepsilon)(1+\varepsilon)(1+\varepsilon)(1+\varepsilon)(0+\varepsilon)} +$

$(1+\varepsilon)(0+\varepsilon)(1+\varepsilon)(1+\varepsilon)(0+\varepsilon) +$

$(1+\varepsilon)(1+\varepsilon)(0+\varepsilon)(1+\varepsilon)(0+\varepsilon) +$

$\cancel{(1+\varepsilon)(1+\varepsilon)(1+\varepsilon)(0+\varepsilon)(0+\varepsilon)} +$

$(1+\varepsilon)(0+\varepsilon)(0+\varepsilon)(1+\varepsilon)(1+\varepsilon) +$

$(1+\varepsilon)(0+\varepsilon)(1+\varepsilon)(0+\varepsilon)(1+\varepsilon) +$

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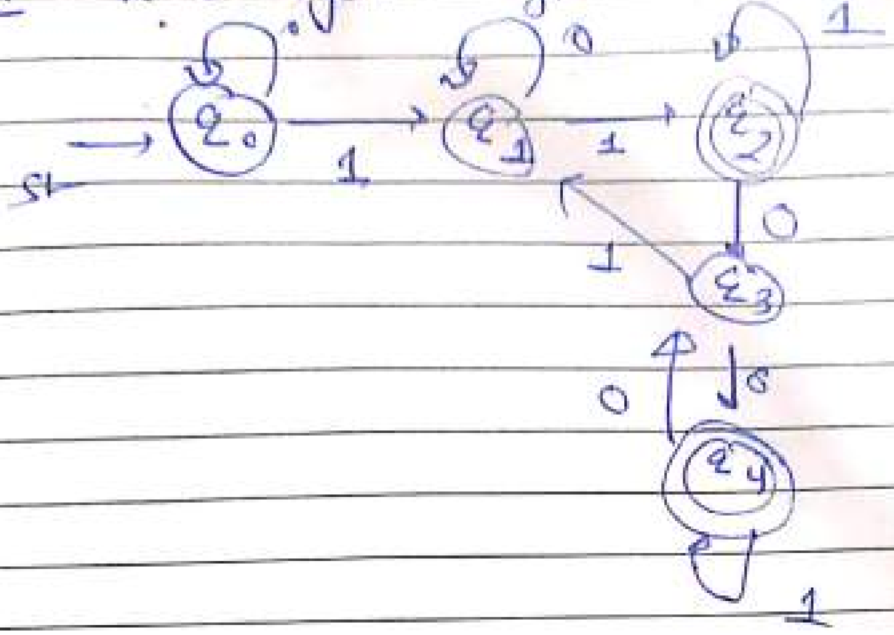
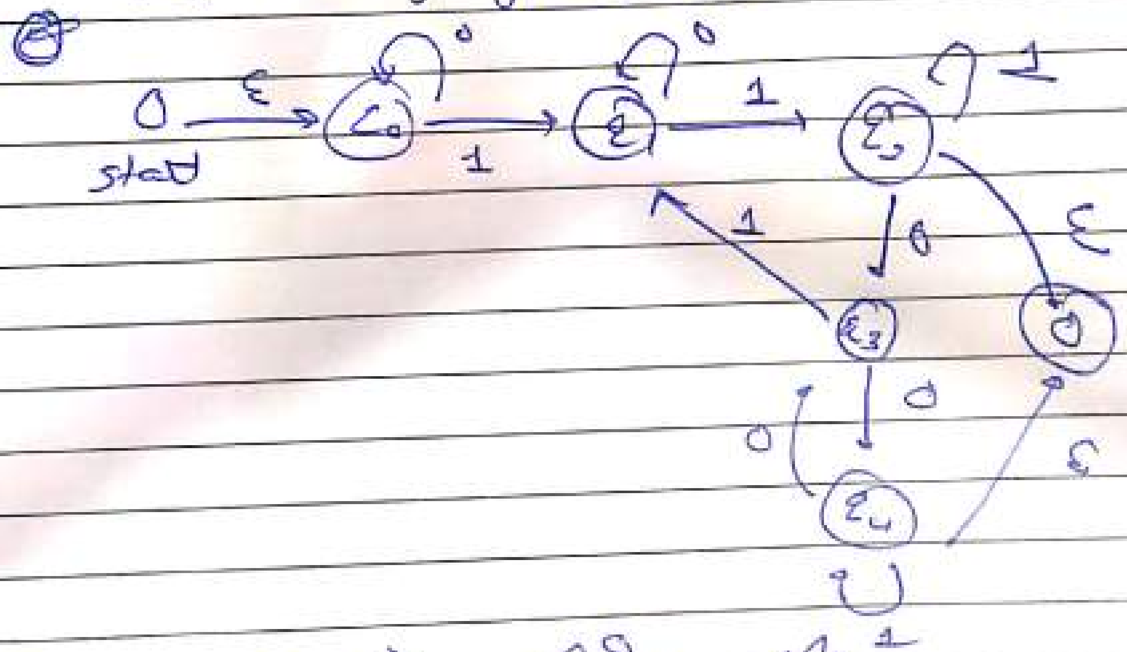
$(1+\varepsilon)(1+\varepsilon)(0+\varepsilon)(0+\varepsilon)(1+\varepsilon).$

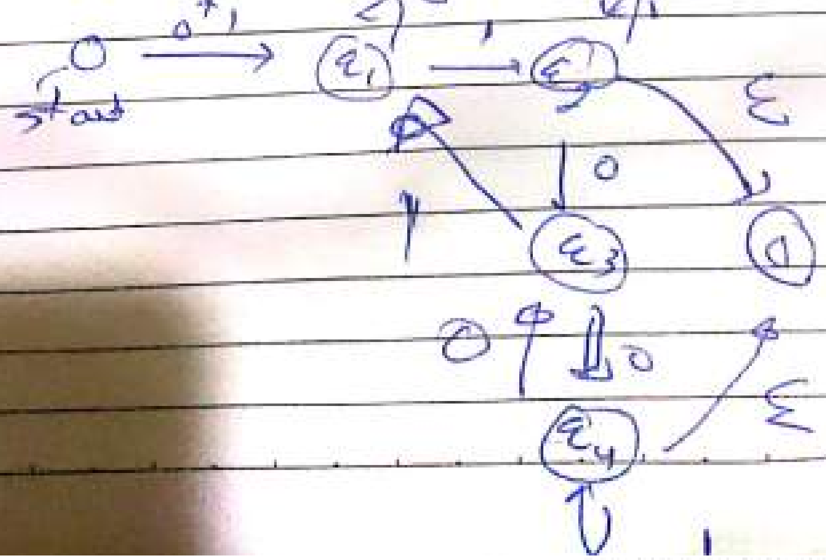
Q-2 R.E for given DFA



step-1 Adding a state for final state & removing ~~initial~~ edge from start



Step-2 :



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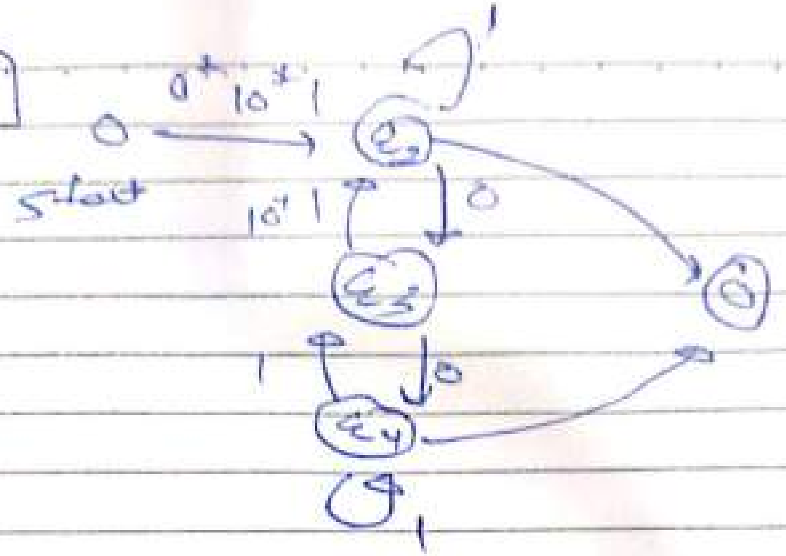
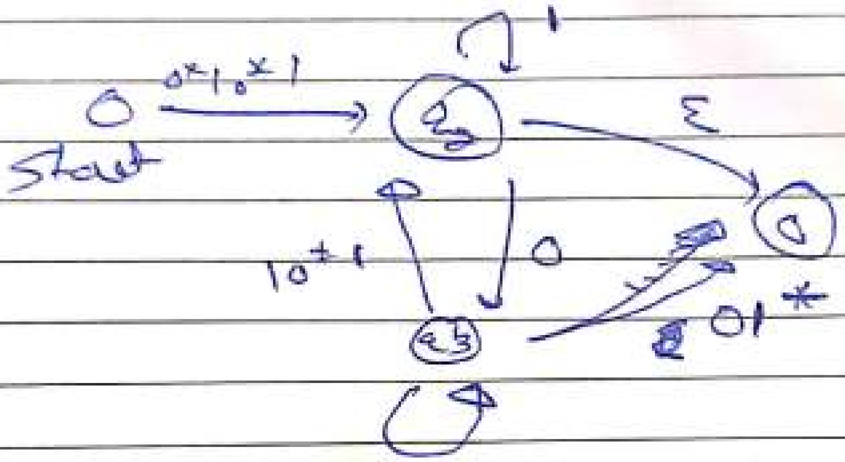
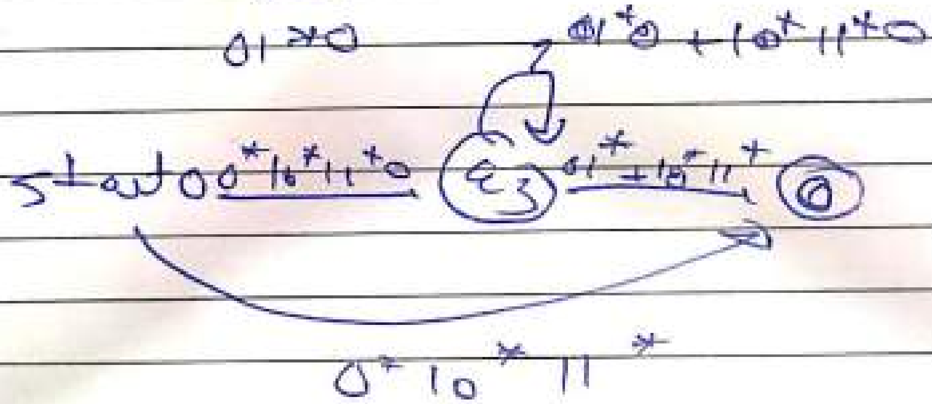
Step-3



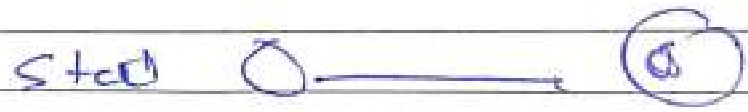
Step-4



Step-5



Step-6



$0^*10^*11^* + 0^*10^*1(^*0)($

$(01^*11^*0)^*(01^* + 10^*11^*)$

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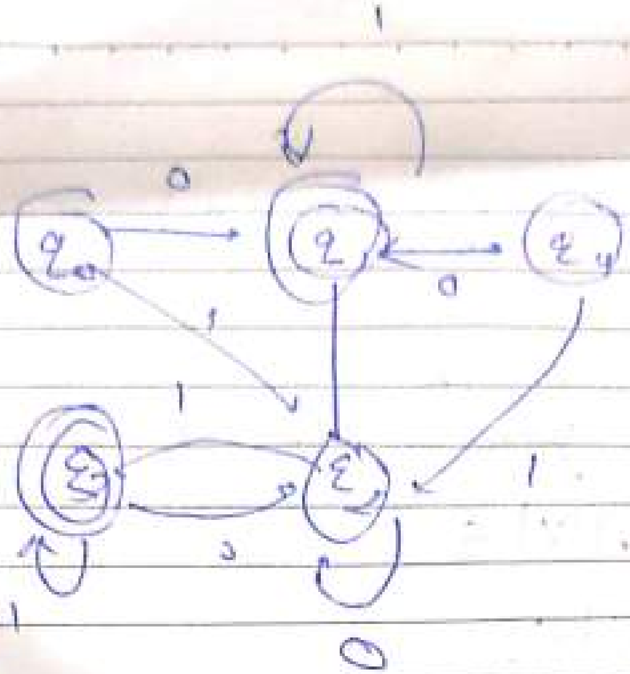
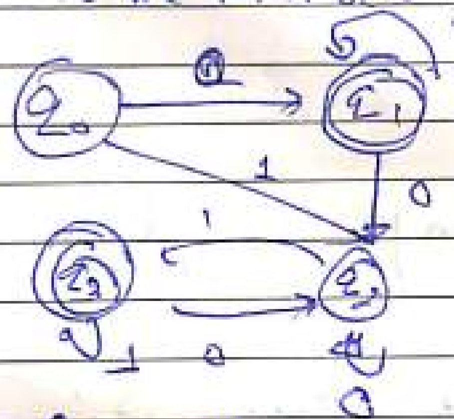
Question - 7

Step 1



As we observe q_4 is unreachable, we can remove & then apply the algorithm.
So the DFA to be minimized is:



- Creating ~~table~~ table & marking pairs $\langle p, q \rangle$, when $p \in F$ & $q \notin F$

q_0			
X	q_1		
	X	q_2	
X		X	q_3

- Now we iterate until no ~~ent~~ entry can be marked.

$\langle q_0, q_2 \rangle, a = 0$ $\langle \delta(q_1, 0), \delta(q_2, 0) \rangle$
$= \langle q_1, q_2 \rangle$

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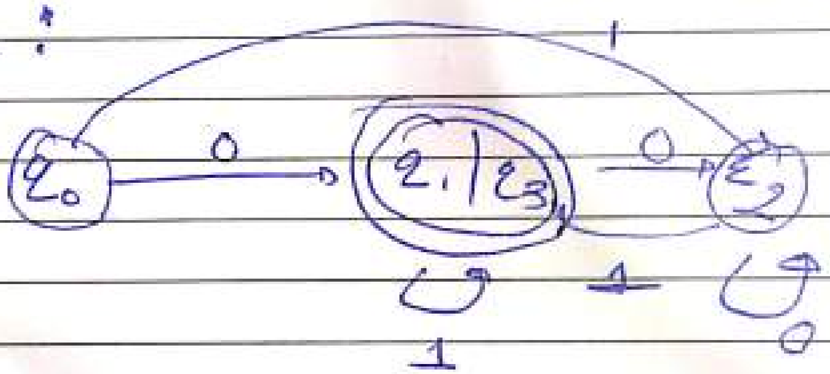
Q3 CTD.

~~(q_1, q_3)~~ $\langle q_1, q_3 \rangle$ is marked so we mark $\langle q_0, q_3 \rangle$

We get table as :

	q_0	q_1	q_2
q_1	x		
q_2	x	x	
q_3	x		x

So we observe that $q_1 \simeq q_3$, min DFA will be :



Question 4

(a) $L_1 = \{a^m b^n c^n d^{2m} \mid n, m \geq 0\}$

Context Free Grammar for the above

$S \rightarrow ASD \mid X \mid \epsilon$

$X \rightarrow BXC \mid \epsilon$

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$

$D \rightarrow dd$

(b) $L_2 = \{a^n b^m \mid n \neq m\}$

~~Cont~~ CFG for the above is :

$S \rightarrow AX \mid XB$

$A \rightarrow aA \mid a$

$B \rightarrow Bb \mid b$

$\cancel{X \rightarrow aX}$

$X \rightarrow aXb \mid \epsilon$

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(Q-5)

Show that following is not regular

$L = \{a^n b^m \mid \cancel{gcd(n,m)}\ gcd(n,m) = 1 \quad n,m \geq 0$

We will try to use pumping lemma to show that it is not regular.

- Let us assume L i.e. $\{a^n b^m \mid gcd(n,m) = 1 \quad n,m \geq 0\}$ is regular.
- Also, consider $U = a^* b^*$. Note that U is regular.
- By closure properties: $\bar{L}$ is regular.

  Let us consider $L_2 = \bar{L} \cap \bar{U}$.
  L_2 must be regular then.
- Now, we apply pumping lemma on L_2

  → Given $P > 0$
  Let us choose $w = a^q b^q$ ~~q is prime~~
  q is prime, $q > p$ $\cancel{q \geq p}$

For any partition of w as xyz, $|xy| \leq p$, $|y| > 0$, xyz must be of the form

$x = a^{u-v}$, $y = a^v$, $z = a^{q-u} b^v$

where $u \leq p$ & $v > 0$

~~[illegible]~~

P.T.O

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$$xy^iz = a^{q+v(i-1)}b^q$$

Take $i=0$, we get $xy^iz = a^{q-v}b^q$

~~We know gcd(q, q-v) = 1 as q is prime~~

We know $\gcd(q, q-v) = 1$ as q is prime & $v>0$.

So we observe:

$a^{q-v}b^q$ does not belong to L_2.

So L_2 can't be regular as by contradiction

$\boxed{\text{So } L \text{ is irregular}}$

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