CS-350, Quiz-1, Nikhil Mehta, 190549

Q-1

(1) Small-step Semantics : $t_1 \otimes t_2$

$$\frac{\text{iszero } t_1 \rightarrow \text{false}}{t_1 \otimes t_2 \rightarrow t_1}$$

$$\frac{\text{iszero } t_1 \rightarrow \text{true}}{t_1 \otimes t_2 \rightarrow t_2}$$

(t_2 will be evaluated only when necessary)

(2)

$$\frac{t_1 : \text{Nat}}{\text{iszero } t_1 : \text{Bool}}$$

$$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T}$$

$\Gamma \vdash$ if (iszero t_1) then t_2 else t_3

$$\frac{t_1 : \text{Nat} \quad t_2 : \text{Nat}}{t_1 \otimes t_2 : \text{Nat}}$$

(3) $t_1 = 0$ & $t_2 =$ false is not stuck but reject.

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Q-2

Let us define $\overset{exp}{g}$ as follows: for (m^n)

$~~g=~~$ exp = $\lambda m n. (if (iszero\ n) (one) (mult\ m (exp\ m\ (pred\ n))))$

We want to use Y-combinator, ~~so~~ and exp should be a fixed point of some function as gexp.

$exp = (\lambda f m n. (if (iszero\ n) (one) (mult\ m (f m (pred\ n))))) exp$

or

$exp = Y(\lambda f m n. (if (iszero\ n) (one) (mult\ m (f m (pred\ n)))))$

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Q-3

~~$= (\lambda x.x(\lambda zpw.(\lambda xy.y))(\lambda xy.x))((\lambda xyz.zxy)b(\lambda xy.y))$~~

~~$= (\lambda x.x(\lambda zpw.(\lambda xy.y))(\lambda xy.x))((\lambda yz.zby)(\lambda xy.y))$~~

~~$= (\lambda x.x(\lambda zpw.(\lambda xy.y))(\lambda xy.x))(\lambda z.$~~

$= (\lambda x.x(\lambda zpw.(\lambda xy.y))(\lambda xy.x))((\lambda xyz.zxy)b(\lambda xy.y))$

$\rightarrow (\lambda xyz.zxy)b(\lambda xy.y)(\lambda zpw.(\lambda xy.y))(\lambda xy.x)$

$\rightarrow (\lambda yz.zby)(\lambda xy.y)(\lambda zpw.(\lambda xy.y))(\lambda xy.x)$

$\rightarrow (\lambda z.zb(\lambda xy.y))(\lambda zpw.(\lambda xy.y))(\lambda xy.x)$

$\rightarrow (\lambda z.zb(\lambda xy.y))(\lambda pw.(\lambda xy.y))$

$\rightarrow \lambda pw.(\lambda xy.y)\, b(\lambda xy.y)$

$\rightarrow (\lambda w.(\lambda xy.y))b(\lambda xy.y)$

$\rightarrow \lambda xy.y$ Ans

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