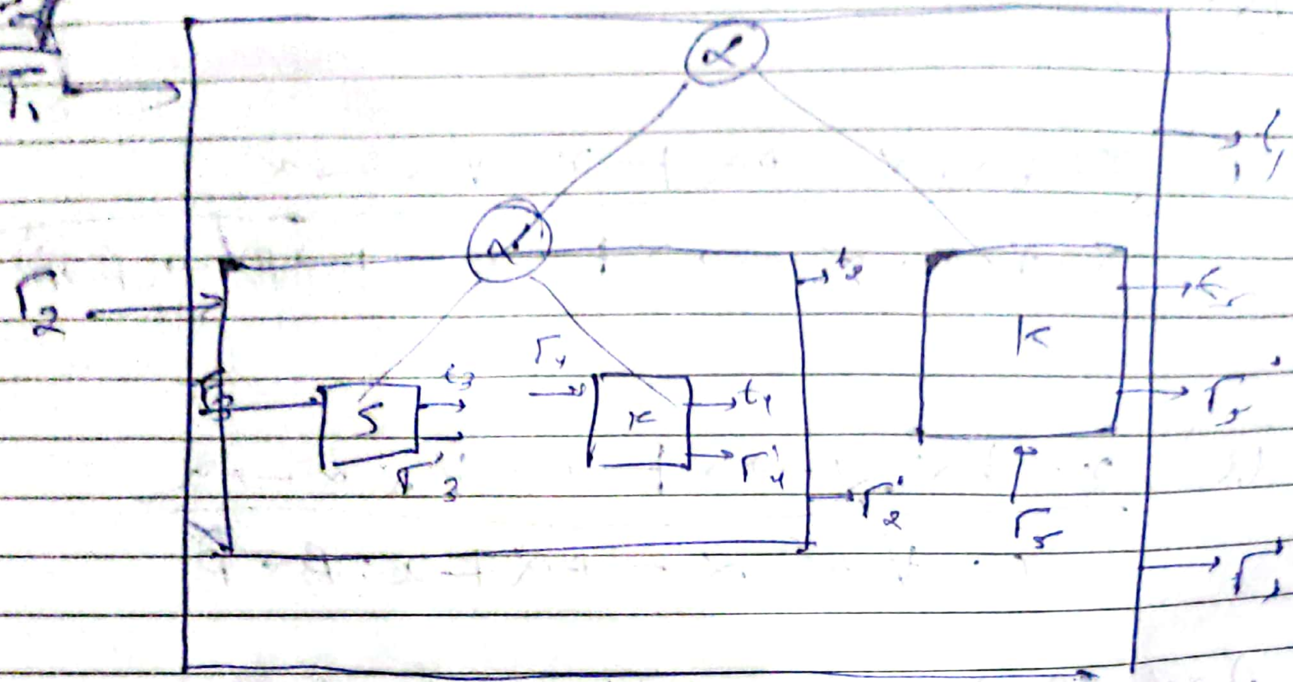
190549

Q:1



Given let

$k = \lambda x\, y \to x$

in SKK

$s :: \forall \alpha\beta\gamma . (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$

- type of k (observe box of K).

$\Gamma_2 = \Gamma_1 \cup (x :: \alpha)$

$\Gamma_4 = \Gamma_1 \cup (x :: \alpha) \cup (y :: \beta)$

$K = t \to (\beta \to \alpha)$

$K \Rightarrow \forall \alpha \beta . \ \alpha \to (\beta \to \alpha)$.

- Now observing the box of s & also type of s is given, we get

$\Gamma_1 = \{ s :: \forall \alpha\beta\gamma . (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$

$k :: \forall \alpha \beta . \ (\alpha \to (\beta \to \alpha)$

- ~~$\Gamma_1 + \Gamma$~~: $\Gamma_1 = \Gamma_2$ & $\Gamma_3 = \Gamma_2$ [same environment]

So $\Gamma_1 = \Gamma_2 = \underbrace{\Gamma_3 = \Gamma_3'}$ as ~~$t_3 = (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$~~

- $\Gamma_4 = \theta_3 \Gamma_3 = \Gamma_3$ [$\Gamma_3' = \Gamma_3 \Rightarrow \theta_3 = \theta_{id}$]
- $\Gamma_4 = \Gamma_3$ (observing box for K), $t_4 = \alpha \to (\beta \to \alpha)$
  - $\Gamma_4' = \theta_4 \Gamma_4 = \Gamma_4$ [$\theta_4 = \theta_{id}$]
- $\theta_3 = \theta_{id} = \theta_4$, $\theta_2 = \theta_{id}$, $\Gamma_2' = \theta_2 \Gamma_2$

Concluding from above line $\Gamma_2' = \Gamma_2$.

$t_2 = (\alpha \to \beta) \to \alpha \to \alpha$.

~~we have calculated Γ_1 in starting~~

~~$\theta_1 = \theta_1 \cdot \theta_2$, $t_1 = \alpha \to \alpha$~~

- Same environment follows for Γ_5 & Γ_2 ($\Gamma_5 = \Gamma_2$).

$\Gamma_5 = \theta_2 \Gamma_2 = \Gamma_2$

By box of K, we have

$t_5 = \alpha \to (\beta \to \alpha)$

$\Gamma_5' = \Gamma_5 \Rightarrow (\theta_5 = \theta_{id})$

- We have calculated Γ_1.

$\theta_1 = \theta_1 \cdot \theta_2$, $t_1 \to \alpha \to \alpha$

Concluding $\theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_{id}$

190549.

Q-3

(i) $$\frac{x : \forall\alpha.\alpha \to \alpha \vdash x : \forall\alpha \to \alpha}{x : \forall\alpha.\alpha \to \alpha \vdash x : (\beta \to \beta) \to (\beta \to \beta)}$$

(ii) $$\frac{x : \forall\alpha.\alpha \to \alpha \vdash x : \forall\alpha.\alpha \to \alpha}{x : \forall\alpha.\alpha \to \alpha \vdash x : \beta \to \beta}$$

(iii) Apply P-App on both (i) & (ii)

$$\frac{x : \forall\alpha.\alpha \to \alpha \vdash x : (\beta \to \beta) \to (\beta \to \beta) \quad x : \forall\alpha.\alpha \to \alpha \vdash x : \beta \to \beta}{x : \forall\alpha.\alpha \to \alpha \vdash xx : \beta \to \beta}$$

• P-ABS on (iii)

$$\frac{\dfrac{x : \forall\alpha.\alpha \to \alpha \vdash xx : \beta \to \beta}{x : \forall\alpha.\alpha \to \alpha \vdash \lambda x.xx : (\forall\alpha.\alpha \to \alpha) \to (\beta \to \beta)}}{\vdash \lambda x.xx : \forall\beta.(\forall\alpha.\alpha \to \alpha) \to (\beta \to \beta)}$$

$\sigma \Rightarrow \forall\beta.(\forall\alpha.\alpha \to \alpha) \to (\beta \to \beta)$.

which proves $\boxed{M : \sigma}$

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190549

Q-4

(a) i) α is not a unifier

$\alpha C_1 = p(a, f(g(v)), b)$

$\alpha C_2 = q(a, f(f(v)), b)$

ii) Outer functions of C_1 & C_2 are different, so MGU can't exist thus α is also not MGU.

(b) α is unifier as well as MGU

$\alpha = \{x := g(g(v)), y := h(a, z), w := x\}$

~~$W_5 = \{\langle q(x, h(a, z)), f(x)\rangle, \langle q(g(g(v)), y, f(w))$~~

$W_5 = \{\langle q(x, h(a, z), f(x)\rangle, \langle q(g(g(v)), y, f(w))\}$

$W_5 = \{q(\langle x, g(g(v))\rangle, \langle h(a z), y\rangle, f(\langle x, w\rangle)\}$

$MGU = \{x := g(g(v)), y := h(a, z), w := x\}$

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(c) $\alpha C_1 = q(g(g(v)), h(a,z), f(g(g(v))))$

$\alpha C_2 = q(g(g(v)), h(a,z), f(g(g(v))))$

α is unifier & MGU.

MGU is same as we defined in part (b) because on substituting once we get same

(d) α is a unifier but not MGU

$W_5 = \{ \langle r, (f(x), g(y)), r(z, g(v)) \rangle \}$

$W_5 = \{ r(\langle f(x), z \rangle, \langle g(y), g(v) \rangle) \}$

$W_5 = \{ r(\langle f(x), z \rangle, g(\langle y, v \rangle)) \}$

$MGU = \{ z := f(x), y := v \}$

$\alpha = \{ x := a, z := f(a), y := v \}$

$z := f(a) := f(x)$

Not MGU but unifier.

190599

Q-2

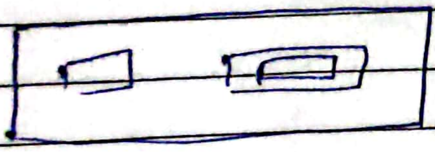
concat [ ] = [ ]

concat (x : xs) = x ++ concat xs.

Initial environment

(++) :: $\forall \alpha$. $[\alpha] \to [\alpha] \to [\alpha]$

Topic). Γ, θ, t



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