



Probability Assignment-III

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I. PROBLEM

Find the mean number of heads in three tosses of a fair coin.

II. SOLUTION

Consider each trial results in success (i.e Heads) or failure (i.e Tails).

Let p and $q = (1 - p)$ be the probability of success and failure respectively.

$$p = \frac{1}{2} \quad (1)$$

$$q = 1 - p = \frac{1}{2} \quad (2)$$

In n Bernoulli trials with x success and $(n - x)$ failures, the probability of x success in n - Bernoulli trials can be given as

$${}^nC_x p^x q^{n-x} \quad (3)$$

Now the distribution of number of successes using (1), (2)&(3) can be given as,

| X | 0 | 1 | 2 | 3 |
|------|--------------------------|--------------------------|--------------------------|--------------------------|
| P(X) | ${}^3C_0(\frac{1}{2})^3$ | ${}^3C_1(\frac{1}{2})^3$ | ${}^3C_2(\frac{1}{2})^3$ | ${}^3C_3(\frac{1}{2})^3$ |

$$\text{Mean of X} = \mu = \sum_{i=1}^{n=3} x_i P(x_i) \quad (4)$$

$$\begin{aligned} \mu &= 0 \times {}^3C_0(\frac{1}{2})^3 + 1 \times {}^3C_1(\frac{1}{2})^3 + 2 \times {}^3C_2(\frac{1}{2})^3 \\ &\quad + 3 \times {}^3C_3(\frac{1}{2})^3 \end{aligned} \quad (5)$$

$$\therefore \text{Mean of X} = \mu = 1.5 \quad (6)$$