Indian Institute of Technology Hyderabad

Matrix Assignment - Lines

Nikhil Nair

CONTENTS

I. PROBLEM

Find the orthocenter of triangle with vertices (0,0), (3,4) and (4,0).

II. SOLUTION

Orthocenter of a triangle is the point where perpendiculars drawn to the opposite side from each vertex of the triangle intersect.

To find the orthocenter first we find the coordinates of point Q i.e the foot of the perpendicular drawn from point B as follows

$$\mathbf{OA} = O + \lambda.\mathbf{m}$$

where O is the origin and $\mathbf{m} = A - O$

$$\mathbf{OA} = \lambda. \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{1}$$

Now,

$$\mathbf{m}^\top(\mathbf{OA} - B) = 0$$

$$\mathbf{m}^\top[O + \lambda.\mathbf{m} - B] = 0$$

$$\lambda.\|m\|^2 = \mathbf{m}^\top(O - B)$$

$$\lambda = \frac{\mathbf{m}^\top(O - B)}{\|m\|^2} \tag{2}$$

Coordinates of Point Q can be found by replacing value of λ in eq 2

$$Q = \frac{\mathbf{m}^\top(O-B)}{\|m\|^2}. \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$Q = \frac{1}{\sqrt{(3^2+4^2)}} \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1.44 \\ 1.92 \end{pmatrix}$$

The orthocenter of the triangle can be calculated as follows

From the points,

A(3, 4) ; P(3, 0)
B(4, 0) ; Q(1.44, 1.92)

$$\mathbf{AP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda 1. \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{3}$$

$$\mathbf{BQ} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \lambda 2. \begin{pmatrix} -2.56 \\ 1.92 \end{pmatrix} \tag{4}$$

where $\lambda 1$ and $\lambda 2$ are scalars

Solving eq3 and eq4 we get the value of

$$\lambda 2 = 0.39$$

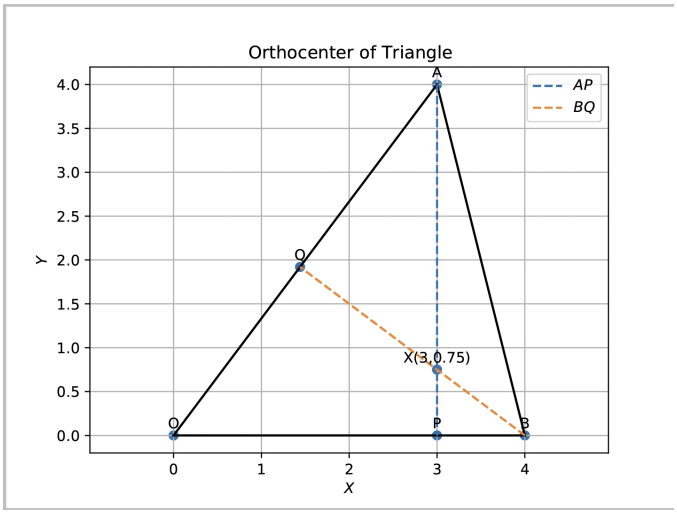
Replace the value of $\lambda 2$ in eq4 to find the intersection of $\mathbf{AP}$ and $\mathbf{BQ}$, ie point X.

$$\mathrm{X} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 0.39. \begin{pmatrix} -2.56 \\ 1.92 \end{pmatrix}$$

Therefore the orthocenter of the triangle is

$$\mathrm{X} = \begin{pmatrix} 3 \\ 0.75 \end{pmatrix}$$

III. Figure



IV. Code Link

https://github.com/nikhilnair90/FWC−2/blob/main/Matrix/Line/line.py

Execute the code by using the command
python3 line.py