Matrix Assignment - Circle

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Contents

I. Problem

If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is .

II. Solution

The equation of a circle is given as,

$$\mathbf{x}^\top \mathbf{V_1} \mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0$$

The circle given in the question can be written in the above form as follows

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f_1 = 0$$

where,

$$\mathbf{V_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \mathbf{u_1}^\top = \begin{pmatrix} 0 & 0 \end{pmatrix}; f_1 = -4$$

Given that the two circles are orthogonal, therefore

$$r_1^2 + r_2^2 = \|\mathbf{u_1} - \mathbf{u_2}\|^2$$

$$r_1^2 + r_2^2 = \|\mathbf{u_1}\|^2 - \|\mathbf{u_2}\|^2 - 2\mathbf{u_1}^\top \mathbf{u_2} \tag{1}$$

$r_1 = 2$ and r_2 are the radius of the given circles

Now, we know

$$r_2 = \|\mathbf{u_2} - \mathbf{L}\| \tag{2}$$

where L is a point (a,b) on the circle 2

$\mathbf{u_2}$ i.e centre of 2nd circle is taken as $\begin{pmatrix} g & t \end{pmatrix}$

$$r_2^2 = \|\mathbf{u_2}\|^2 + \|\mathbf{L}\|^2 - 2\mathbf{u_2}^\top \mathbf{L} \tag{3}$$

Solving 1 and 3 we get,

$$r_1^2 + \|\mathbf{L}\|^2 - 2\mathbf{u_2}^\top \mathbf{L} = 0$$

$$r_1^2 + \mathbf{L}^\top \mathbf{L} - 2\mathbf{u_2}^\top \mathbf{L} = 0$$

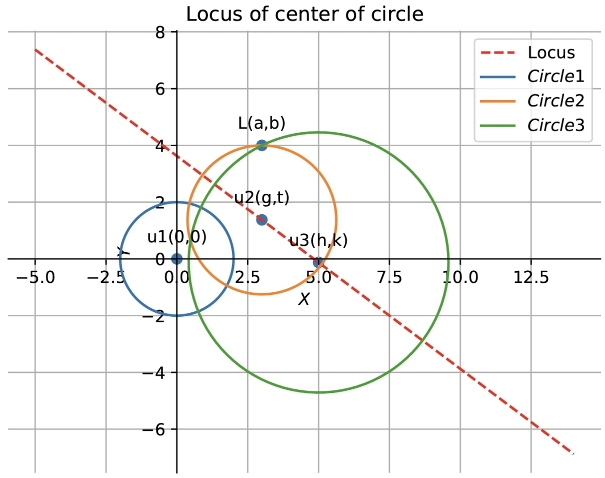
$$r_1^2 + \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} - 2 \begin{pmatrix} g \\ t \end{pmatrix} \begin{pmatrix} a & b \end{pmatrix} = 0$$

$$r_1^2 + a^2 + b^2 - 2ga - 2tb = 0$$

Therefore the locus of the center of circle2 is

$$2ga + 2tb - (a^2 + b^2 + 4) = 0$$

III. Figure



IV. Code Link

https://github.com/nikhilnair90/FWC−2/blob/main/Matrix/Circle/circle.py

Execute the code by using the command
python3 circle.py