



# Digital Communication Assignment

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## I. MAXIMUM LIKELIHOOD DETECTION: BPSK

- 1) Generate equiprobable  $X \in \{1, -1\}$ .
- 2) Generate

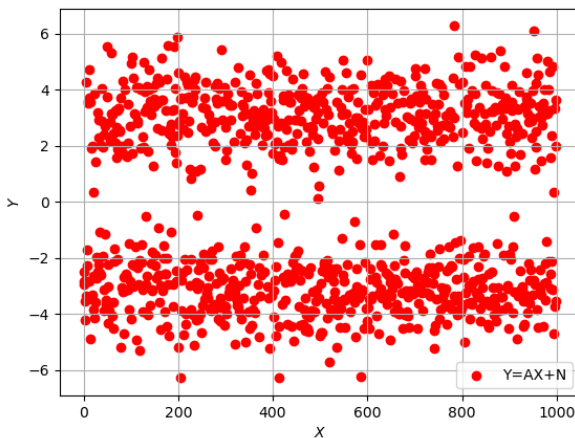
$$Y = AX + N,$$

where  $A = 5$  dB, and  $N \sim \mathcal{N}(0, 1)$ .

- 3) Plot  $Y$  using a scatter plot.

The following code provides the solution to (1), (2) and (3).

[https://github.com/nikhilnair90/FWC/tree/main/Module-II/Digital\\_Comm/7.1/Code/7\\_1.py](https://github.com/nikhilnair90/FWC/tree/main/Module-II/Digital_Comm/7.1/Code/7_1.py)



- 4) Guess how to estimate  $X$  from  $Y$ .  
In the question binary symbol '0' is represented as 1 and '1' as -1

$$\begin{aligned} 0 &\longrightarrow 1 \\ 1 &\longrightarrow -1 \end{aligned}$$

Comparing the conditional pdfs of  $Y$  for  $X=1$  and  $X=-1$  by MAP decision rule,

$$\Pr(Y|X=1) \underset{X=-1}{\overset{X=1}{\gtrless}} \Pr(Y|X=-1) \quad (1)$$

i.e.,

$$\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y-A)^2}{2}\right) \underset{-1}{\overset{1}{\gtrless}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y+A)^2}{2}\right) \quad (2)$$

Taking log of both sides eq.(2) can be simplified as,

$$(y-A)^2 \underset{-1}{\overset{1}{\gtrless}} (y+A)^2 \quad (3)$$

which gives the estimate of  $X$  from  $Y$  as,

$$y \underset{X=-1}{\overset{X=1}{\gtrless}} 0 \quad (4)$$

- 5) Find

$$P_{e|0} = \Pr(\hat{X} = -1|X = 1)$$

and

$$P_{e|1} = \Pr(\hat{X} = 1|X = -1)$$

From eq.(4)

$$\begin{aligned} P_{e|1} &= \Pr(\hat{X} = 1|X = -1) \\ &= \Pr(Y > 0|X = -1) \end{aligned} \quad (5)$$

$$= \Pr(AX + N > 0|X = -1) \quad (6)$$

$$= \Pr(N > A) \quad (7)$$

Q function is defined as,

$$Q(z) = \Pr(X > z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{x^2}{2}} dx. \quad (8)$$

where  $X$  is a normal random variable and  $\Pr(X > z)$  is the probability that  $X$  is greater than  $z$ .

Using eq.(8), eq.(7) can be re-written as

$$P_{e|1} = Q(A) \quad (9)$$

Similarly,

$$\begin{aligned} P_{e|0} &= \Pr(\hat{X} = -1|X = 1) \\ &= \Pr(N < -A) \end{aligned} \quad (10)$$

$$= 1 - Q(A) \quad (11)$$

$$= Q(-A) \quad (12)$$

- 6) Find  $P_e$  assuming that  $X$  has equiprobable symbols.

$$P_e = P_{e|0} + P_{e|1} \quad (13)$$

Since  $X$  has equiprobable symbols.

$$P_{e|0} = P_{e|1} \quad (14)$$

Now,

$$P_e = P_{e|0} + P_{e|1} \quad (15)$$

$$= 2P_{e|1} = 2P_{e|0} \quad (16)$$

From eq.(9) and eq.(12)

$$P_e = 2Q(A) = 2Q(-A) \quad (17)$$

- 7) Verify by plotting the theoretical  $P_e$  with respect to  $A$  from 0 to 10 dB.

