



# Probability Assignment-III

Nikhil Nair

## I. PROBLEM

Find the mean number of heads in three tosses of a fair coin.

## II. SOLUTION

Consider each trial results in success (i.e Heads) or failure (i.e Tails).

Let  $p$  and  $q = (1 - p)$  be the probability of success and failure respectively.

$$p = \frac{1}{2} \quad (1)$$

$$q = 1 - p = \frac{1}{2} \quad (2)$$

In  $n$  Bernoulli trials with  $x$  success and  $(n - x)$  failures, the probability of  $x$  success in  $n$ - Bernoulli trials can be given as

$${}^nC_x p^x q^{n-x} \quad (3)$$

Now the distribution of number of successes using (1), (2)&(3) can be given as,

X	0	1	2	3
P(X)	${}^3C_0(\frac{1}{2})^3$	${}^3C_1(\frac{1}{2})^3$	${}^3C_2(\frac{1}{2})^3$	${}^3C_3(\frac{1}{2})^3$

$$\text{Mean of X} = \mu = \sum_{i=1}^{n=3} x_i P(x_i) \quad (4)$$

$$\begin{aligned} \mu = 0 \times {}^3C_0(\frac{1}{2})^3 + 1 \times {}^3C_1(\frac{1}{2})^3 + 2 \times {}^3C_2(\frac{1}{2})^3 \\ + 3 \times {}^3C_3(\frac{1}{2})^3 \end{aligned} \quad (5)$$

$$\therefore \text{Mean of X} = \mu = 1.5 \quad (6)$$