Matrix Assignment - Circle

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Contents

I. Problem

If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is .

II. Solution

The equation of a circle is given as,

$$\mathbf{x}^\top \mathbf{V_1}\mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0$$

The circle given in the question can be written in the above form as follows

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f_1 = 0$$

where,

$$\mathbf{V_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \mathbf{u_1}^\top = \begin{pmatrix} 0 & 0 \end{pmatrix}; f_1 = -4$$

Given that the two circles are orthogonal, therefore

$$r_1^2 + r_2^2 = \|\mathbf{u_1} - \mathbf{u_2}\|^2$$

$$4 + r_2^2 = \|\mathbf{u_1}\|^2 - \|\mathbf{u_2}\|^2 - 2\mathbf{u_1}^\top \mathbf{u_2} \tag{1}$$

$r_1 = 2$ and r_2 are the radius of the given circles

Now, we know

$$r_2 = \sqrt{\|\mathbf{u_2}\|^2 - f_2} \tag{2}$$

Solving eq1 and eq2 we get,

$f_2 = 4$

The equation of circle with centre at $u_2 = \begin{pmatrix} -h \\ -k \end{pmatrix}$ can be given by

$$\mathbf{x}^\top \mathbf{V_2}\mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f_2 = 0$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} -h & -k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 4 = 0$$

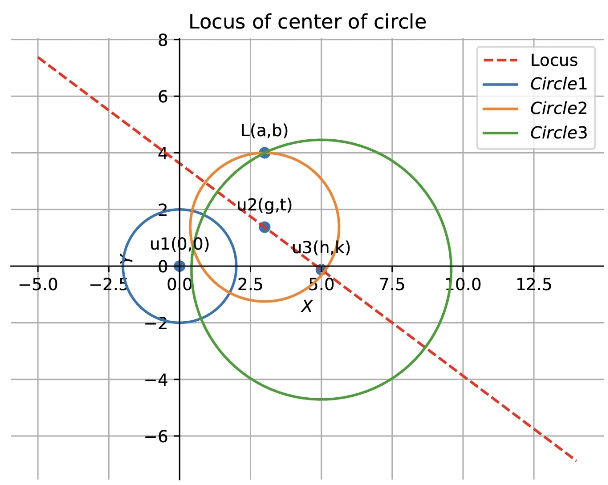
Since the circle passes through the point (a,b), the above equation can be simplified as

$$a^2 + b^2 - 2ha - 2kb + 4 = 0$$

Therefore the locus of the center of circle is

$$2ha + 2kb - (a^2 + b^2 + 4) = 0$$

III. Figure



IV. Code Link

https://github.com/nikhilnair90/FWC−2/blob/main/Matrix/Circle/circle.py

Execute the code by using the command
python3 circle.py