



Digital Communication Assignment

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I. PROBLEM

Given

$$V = -2 \ln(1 - U)$$

Find a theoretical expression for $F_V(x)$.

II. SOLUTION

Let U be a uniform random variable between 0 and 1.

Now, CDF of U is

$$F_U(x) = \Pr(U \leq x) \quad (1)$$

$$F_U(x) = \begin{cases} 0 & 0 < x \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2)$$

Similarly, CDF of V is given as

$$F_V(x) = \Pr(V \leq x) \quad (3)$$

$$= \Pr(-2 \ln(1 - U) \leq x) \quad (4)$$

$$= \Pr\left(\ln(1 - U) \geq \frac{-x}{2}\right) \quad (5)$$

$$= \Pr\left(1 - U \geq e^{\frac{-x}{2}}\right) \quad (6)$$

$$= \Pr\left(U \leq 1 - e^{\frac{-x}{2}}\right) \quad (7)$$

From (1) and (7) $F_V(x)$ can be simplified as,

$$F_V(x) = F_U(1 - e^{\frac{-x}{2}}) \quad (8)$$

From (2) and (8)

$$F_U(x) = \begin{cases} 0 & 0 < x \\ 1 - e^{\frac{-x}{2}} & 0 \leq x < \infty \\ 1 & x = \infty \end{cases} \quad (9)$$