Digital Communication Assignment

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I. Maximum Likelihood Detection:BPSK

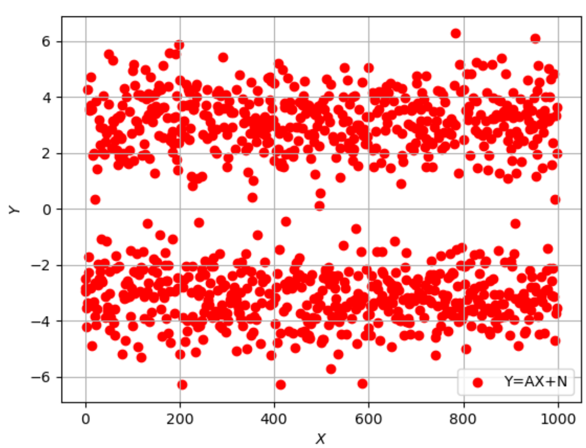
1) Generate equiprobable $X \in \{1, -1\}$.
2) Generate

$$Y = AX + N,$$

where $A = 5$ dB, and $N \sim \mathcal{N}(0,1)$.
3) Plot Y using a scatter plot.

The following code provides the solution to (1),(2) and (3) .

https://github.com/nikhilnair90/FWC/tree/main/Module–II/Digital_Comm/7.1/Code/7_1.py



4) Guess how to estimate X from Y.
In the question binary symbol '0' is represented as 1 and '1' as -1

$$0 \longrightarrow 1$$
$$1 \longrightarrow -1$$

Comparing the conditional pdfs of Y for X=1 and X=-1 by MAP decision rule,

$$\Pr(Y|X=1) \underset{X=-1}{\overset{X=1}{\gtrless}} \Pr(Y|X=-1) \tag{1}$$

i.e,

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(y-A)^2}{2}\right) \underset{-1}{\overset{1}{\gtrless}} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(y+A)^2}{2}\right) \tag{2}$$

Taking log of both sides eq.(2) can be simplified as,

$$(y-A)^2 \underset{-1}{\overset{1}{\gtrless}} (y+A)^2 \tag{3}$$

which gives the estimate of X from Y as,

$$y \underset{X=-1}{\overset{X=1}{\gtrless}} 0 \tag{4}$$

5) Find

$$P_{e|0} = \Pr\left(\hat{X} = -1|X = 1\right)$$

and

$$P_{e|1} = \Pr\left(\hat{X} = 1|X = -1\right)$$

From eq.(4)

$$\begin{aligned} P_{e|1} &= \Pr\left(\hat{X} = 1|X = -1\right) \\ &= \Pr(Y > 0|X = -1) \quad (5) \\ &= \Pr(AX + N > 0|X = -1) \quad (6) \\ &= \Pr(N > A) \quad (7) \end{aligned}$$

Q function is defined as,

$$Q(z) = \Pr(X > z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty e^{\frac{-x^2}{2}}\, dx. \tag{8}$$

where X is a normal random variable and $\Pr(X > z)$ is the probability that X is greater than z.
Using eq.(8), eq.(7) can be re-written as

$$P_{e|1} = Q(A) \tag{9}$$

Similarly,

$$\begin{aligned} P_{e|0} &= \Pr\left(\hat{X} = -1|X = 1\right) \\ &= \Pr(N < -A) \quad (10) \\ &= 1 - Q(A) \quad (11) \\ &= Q(-A) \quad (12) \end{aligned}$$

6) Find P_e assuming that X has equiprobable symbols.

$$P_e = P(0)P_{e|0} + P(1)P_{e|1} \tag{13}$$

Since X has equiprobable symbols.

$$P_{e|0} = P_{e|1} = Q(A) \tag{14}$$

Now,

$$P_e = \frac{1}{2}P_{e|0} + \frac{1}{2}P_{e|1} \tag{15}$$
$$= P_{e|1} = P_{e|0} \tag{16}$$

From eq.(9)

$$P_e = Q(A) \tag{17}$$

7) Verify by plotting the theoretical P_e with respect to A from 0 to 10 dB.