Indian Institute of Technology Hyderabad

Matrix Assignment - Circle

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CONTENTS

I. PROBLEM

If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is .

II. SOLUTION

The equation of a circle is given as,

$$\mathbf{x}^\top \mathbf{V_1} \mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f_1 = 0$$

The circle given in the question can be written in the above form as follows

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f_1 = 0$$

where,

$$\mathbf{V_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \mathbf{u_1}^\top = \begin{pmatrix} 0 & 0 \end{pmatrix}; f_1 = -4$$

Given that the two circles are orthogonal, therefore

$$r_1^2 + r_2^2 = \|\mathbf{u_1} - \mathbf{u_2}\|^2$$

$$r_1^2 + r_2^2 = \|\mathbf{u_1}\|^2 + \|\mathbf{u_2}\|^2 - 2\mathbf{u_1}^\top \mathbf{u_2} \quad (1)$$

$r_1 = 2$ and r_2 are the radius of the given circles

Now, we know

$$r_2 = \|\mathbf{A} + \mathbf{u_2}\| \quad (2)$$

where A is a point (a,b) on the circle 2

$-\mathbf{u_2}$ i.e centre of 2nd circle

$$r_2^2 = \|\mathbf{u_2}\|^2 + \|\mathbf{A}\|^2 + 2\mathbf{u_2}^\top \mathbf{A} \quad (3)$$

Solving (1) and (3) we get,

$$r_1^2 + \|\mathbf{A}\|^2 - 2\mathbf{u_2}^\top \mathbf{A} = 0$$

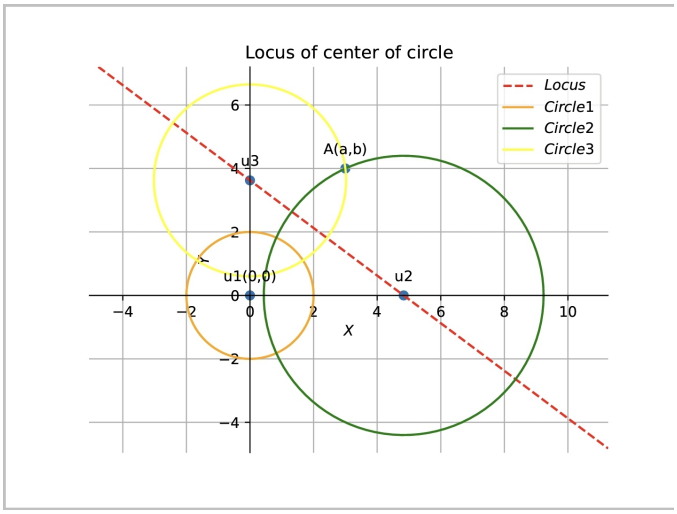
$$2\mathbf{u_2}^\top \mathbf{A} = r_1^2 + \|\mathbf{A}\|^2$$

Therefore the locus of the center of circle2 is

$$\mathbf{u_2}^\top \mathbf{A} = 2 + \tfrac{1}{2}\|\mathbf{A}\|^2$$

Which is a line equation in the form $\mathbf{n}^\top \mathbf{x} = C$

III. FIGURE



IV. CODE LINK

https://github.com/nikhilnair90/FWC−2/blob/main/Matrix/Circle/circle.py

Execute the code by using the command
python3 circle.py