# Matrix Assignment - Conic

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## I. PROBLEM

The minimum area of the triangle formed by the tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

## II. SOLUTION

The equation of a conic is given as,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$$

where,

$$\mathbf{V} = \begin{pmatrix} b^2 & 0 \\ 0 & a^2 \end{pmatrix} \text{ and } \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for the given conic

Let the point where the tangent touches the conic be $Q(acos\theta, bsin\theta)$

Equation of tangent is given by,

$$(\mathbf{V}\mathbf{Q} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{Q} + f = 0$$

$$(\mathbf{V}\mathbf{Q})^\top \mathbf{x} = -f$$

which can be written as

$$\mathbf{n}^\top \mathbf{x} = C$$

$$n = \mathbf{V}\mathbf{Q} = \begin{pmatrix} ab^2cos\theta \\ a^2bsin\theta \end{pmatrix} \text{ and } C = -f = (ab)^2$$

Since r and p are the intercepts on the x and y axis respectively

$$\mathbf{r} = \begin{pmatrix} r \\ 0 \end{pmatrix} \text{and } \mathbf{p} = \begin{pmatrix} 0 \\ p \end{pmatrix}$$

Now,

$$\therefore r = \frac{C}{\mathbf{n}^\top \mathbf{e_x}} \text{ and } p = \frac{C}{\mathbf{n}^\top \mathbf{e_y}}$$

where

$$\mathbf{e_x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{e_y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

are the units vectors on the x and y axis respectively

Area of triangle is given by

$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{o} & \mathbf{r} & \mathbf{p} \end{vmatrix}$$

Area of triangle

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & r & 0 \\ 0 & 0 & p \end{vmatrix}$$

$$\therefore \text{Area} = \frac{1}{2}\frac{ab}{cos\theta sin\theta} = \frac{ab}{sin2\theta}$$

Area will be minimum when $sin2\theta$ is maximum

Max. value of $sin2\theta = 1$

$\therefore$ Minimum area = ab

The minimum area of the traingle formed by the tangent to the conic is $ab$

## III. Verification using Gradient Descent

Using gradient descent method we can find minimum area of triangle

$$x_{n+1} = x_n - \alpha \nabla f(x_n)$$

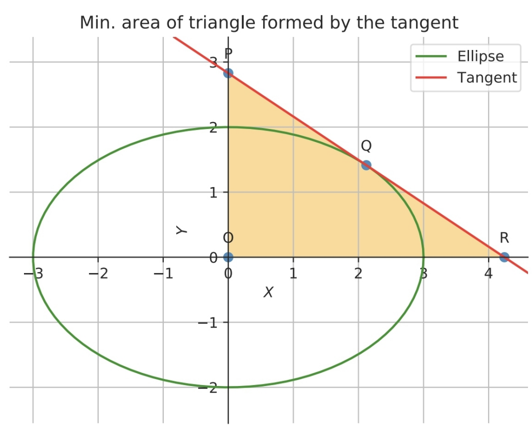
$$x_{n+1} = x_n + \alpha(2ab\frac{cos2\theta}{sin^2(2\theta)})$$

Taking a=3, b=2, $x_0$=1,$\alpha$=0.001 and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima} = 6} \quad (1)$$

$$\boxed{\text{Minima Point} = 0.78} \quad (2)$$

## IV. Figure




## V. Code Link

https://github.com/nikhilnair90/FWC−2/blob/main/Matrix/Conic/conic.py

Execute the code by using the command **python3 conic.py**