# SIKSHA O ANUSANDHAN UNIVERSITY

**Department Of Comp. Sc. & Engg., ITER, BBSR-30**

**B.Tech.($5^{th}$ Sem., CSE &CSIT)**

# Theory of Computation

*Sample Questions* -

1. Answer the followings.

(a) Arrange DFA, NFA, and NFA-$\varepsilon$ in terms of their expressive power?

(b) Let $L_1$={10,1}, and $L_2$={011,11}. Find $L_1L_2$, $\{10,01\}^\star$, and $L_1^4$.

(c) L={All strings of 0's and 1's with at least two consecutives 0's}. Find the regular expression for the language.

(d) Determine the state sequences for the input string 01001 in the given NFA shown in Table:1

| $\delta$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $\{q_0,q_3\}$ | $\{q_0,q_1\}$ |
| $q_1$ | $\emptyset$ | $\{q_2\}$ |
| $q_2$ | $\{q_2\}$ | $\{q_2\}$ |
| $q_3$ | $\{q_4\}$ | $\emptyset$ |
| $q_4$ | $\{q_4\}$ | $\{q_4\}$ |

Table 1: Mapping function for NFA

(e) Find the regular expression for the language L={ All strings of 0's and 1's whose last two symbols are the same?

(g) Prove or disprove $(\epsilon + 00)^\star = 0^\star$, and $0(00)^\star(\epsilon + 0)1 + 1 = 0^\star 1$

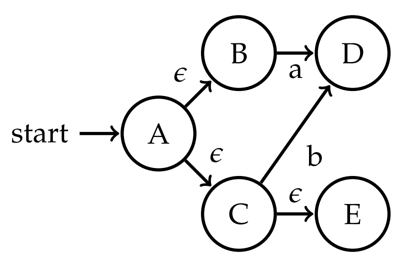
(h) Let $L_1$={10,11}, and $L_2$={011,101}. Find $L_1L_2L_2L_2$, and $L_1^5$.

(i) Find the regular expression for the language L={ All strings of 0's and 1's whose last two symbols are the different?

(j) State true or false $(00)^\star(\epsilon + 0) = 0^\star$

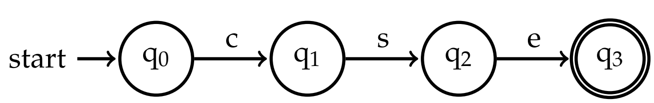
(k) Draw the NFA for the given regular expression $(a \mid b)^* ab$.

(l) Determine $\epsilon$-closure of state A in the following diagram.




(m) Let us consider an NFA as M =$(\{q_0, q_1, q_2, q_3, q_4, q_5\}, \Sigma, \delta, q_0, \{q_3\})$. Determine the maximum number of states possible in converting the given NFA to DFA using subset construction.

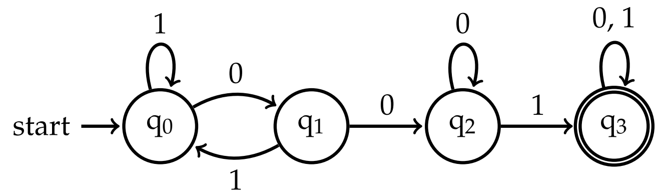
(m) Draw an equivalent DFA for the NFA as shown in the transition diagram over the alphabet $\Sigma = \{c, s, e\}$;



(p) Let a language L ={ The set of all strings over {0, 1}, where the fourth symbol from the left end is 1}. Design a deterministic finite automaton.

(q) Give the regular expression for the language L = { All strings of a's and b's, where the length of the string is at least 5}.

(s) Find the number of strings accepted by the following finite automaton, where the length of the string is exactly 7 and the $1^{st}$, $4^{th}$, and $7^{th}$ bits are 1 from the left end .




2. Convert to a DFA the following NFA:

| $\delta$ | 0 | 1 |
|---|---|---|
| →p | {p,q} | {p} |
| q | ∅ | {r} |
| Ⓡ | {p,r} | {q} |

3. Convert to a DFA the following NFA:

| $\delta$ | 0 | 1 |
|---|---|---|
| →p | {p,r} | {q} |
| q | {r, s} | {p} |
| Ⓡ | {p,s} | {r} |
| Ⓢ | {q, r} | ∅ |

4. Consider the following NFA-$\epsilon$ in Table-2:

| $\delta$ | $\epsilon$ | a | b |
|---|---|---|---|
| →p | {r} | {q} | {p,r} |
| q | ∅ | {p} | ∅ |
| Ⓡ | {p,q} | {r} | {p} |

Table 2: NFA-$\epsilon$

a. Compute the $\epsilon$-closure of each state.
b. Give the set of all string of length 3 or less accepted by the automaton
c. Convert the automaton to a DFA.

5. Design NFA-$\epsilon$ for the following languages. try to use $\epsilon$-transition to simplify your design

a. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.
b. The set of strings over $\{0,1\}^\star$ ending with 011.
c. The set of strings that consist of either 01 repeated one or more times or 010 repeated one of more times.

6. Give DFA's accepting the following strings over the alphabet{0, 1}:

a. The set of all strings beginning with 101.
b. The set of all string containing 1101 as a substring.
c. The set of all srtings conatining with exactly three consecutive 0's.
d. The set of all strings not containing 110 as a substring.
e. the set of all strings that begin with 01 and end with 11.

7. Design NFA's to recognize the following set of strings

    a. lab, cab, and dab. assume the alphabet is { a, b, c, d, l }
    b. 1101, 101, and 111. Assume the alphabet is { 0, 1 }
    c. Convert each of your NFA's from question 7(a) and 7(b) to DFA's.

8. Prove that if $D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ is the DFA constructed from NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ by subset construction, then L(D)= L(N).

9. Design a finite automata which accepts binary strings consisting of odd number of 0's and even number of 1's.

10. Language accepted by finite automata is
    (a) Context-free (b) Regular
    (c) Non-regular (d) None...
    Ans.

11. In some programming language an identifier is permitted to be a letter following by any number of letters or digits. If L & D denotes the sets of letters and digits respectively. Find the regular expression for the identifier.
    Ans.

12. Let $\sum = \{0, 1\}$, $L = \sum^\star$ and $R = \{0^n 1^n | n > 0\}$then the language $L \cup R$ & R are respectively
    (a) Regular, Regular (b) Not regular, Regular
    (c) Regular, Not Regular (d) Not regular, Not Regular
    Ans.

13. Give a regular expression over the alphabet {0, 1} to denote the set of *non-null* substring of the string 0110.
    Ans.

14. Which of the following regular expressions are equivalent
    (a) $(00)^\star(\epsilon + 0)$ (b) $(00)^\star$ (c) $0^\star$ (d) $0(00)^\star$
    Ans.

15. $L = \{w \in \{0, 1\}^\star |$w is interpreted as binary number divisible by 5(ignoring leading zeros)$\}$. What is the minimum number of DFA states required to recognize the language?
    Ans.

16. Let S & T are languages over $\sum = \{a, b\}$ represented by regular expression $(a + b^\star)^\star$ and $(a + b)^\star$ . Which of the follow is *True*?
    (a) S⊂T (b) T⊂S (c) S = T (d) S∩T=∅
    Ans.

17. Consider the following languages
    (a) $L_1 = \{ww | w \in \{a, b\}^\star\}$ (b) $L_2 = \{ww^R | w \in \{a, b\}^\star, w^R$ - Reverse of w$\}$
    (c) $L_3 = \{0^{2i} | i \geq 1\}$ (d) $L_4 = \{0^{i^2} | i \geq\}$
    which of the languages are regular?
    Ans.

18. Find the minimum number of states required for the regular expression $(0 + 1)(0 + 1) \cdots$ n times.
    .
    Ans.

19. $L = \{a^n | n \geq 1\}$
    (a) Regular (b) Not regular
    (c) Context-free Language (d) None
    Ans.

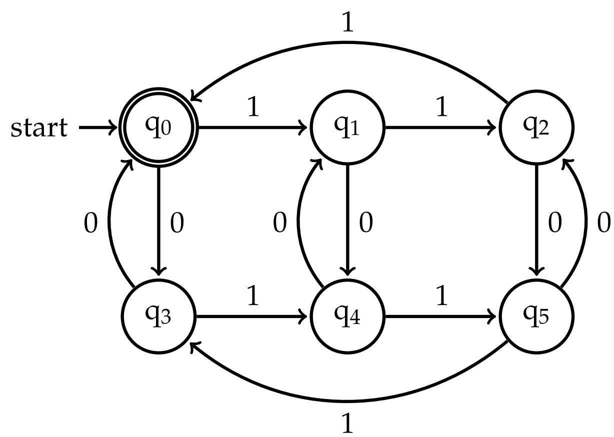
20. Regular languages are closed under
(a) Union (b) Intersection
(c) Reverse (d) All

Ans.

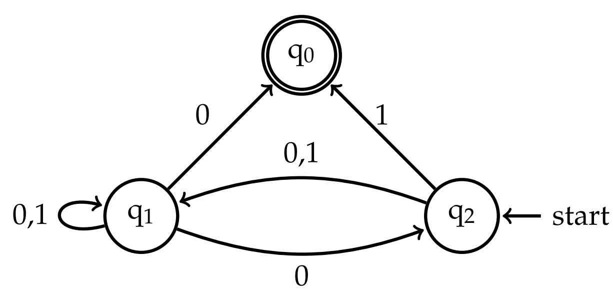
21. Let $M_1$, and $M_2$ be the finite automata(FA) recognizing the languages $L_1$ and $L_2$ respectively. Let $L_1$={All strings over {0, 1} are divisible by 2}, and $L_2$={All strings over {0, 1} are divisible by 3}. Draw the FAs recognizing the following languages due to the closure properties of regular sets over union and intersection operations.

(i) $L_{L_1 \cup L_2}$={All strings over {0, 1} are divisible by 2 or 3}
(ii) $L_{L_1 \cap L_2}$={All strings over {0, 1} are divisible by 2 and 3}
(iii) $L_{L_1 - L_2}$={All strings over {0, 1} are divisible by 2 and not divisible by 3}
(iv) $L_{L_2 - L_1}$={All strings over {0, 1} are divisible by 3 and not divisible by 2}

22. Specify the language accepted by the following finite state automaton $M$. Consider $\Sigma = \{0, 1\}$.




23. Consider the following non-deterministic finite automaton (NFA) $M$.




Say, the language accepted by $M$ be $L$. Say, $L_1$ be the language accepted by the NFA $M_1$, obtained after changing the accepting state of $M$ to a non-accepting state and by changing the non-accepting states of $M$ to accepting states without changing any direction of arrows. Derive the relationship between $L$ and $L_1$.

24. In a certain programming language, comments appear between delimiters such as /# and #/. Let $L$ be the language of all valid delimited comment strings. A member of $L$ must begin /# and end with #/ but should have no intervening #/. Design the DFA that recognizes $L$. Consider $\Sigma = \{a, b, /, \#\}$. [Hint: String like /#*aab*#/ is accepted but /#*aba*#/*aabab*#/ is not accepted.]

25. Say, $L$ be the language of all strings over $\{0, 1\}$ that do not contain a pair of 1s that are separated by an odd number of symbols. Design the NFA that recognizes $L$.

26. Is the class of languages recognized by NFAs closed under complement? Explain your answer mathematically.

27. Say, $A$ be a DFA and $q$ a particular state of A, such that $\delta(q, a) = q$ for all input symbols $a$. Show by induction on the length of the input that for all input strings $w$, $\hat{\delta}(q, w) = q$. Please note that $\hat{\delta}$ is the extended transition function.

28. Say, there be a language $L = \{ab, aa, baa\}$. Which of the following strings are in $L^*$ (Kleene Closure of $L$). Justify your answer.

    1. *abaabaaabaa*
    2. *aaaabaaaa*
    3. *baaaaabaaaab*
    4. *baaaabaa*

29. Design a DFA that recognizes a string that begins with a 01 and ends with 11. Consider $\Sigma = \{0, 1\}$.

30. Design an NFA which can recognize a language with exactly two occurrences of 01. Consider $\Sigma = \{0, 1\}$.

31. Proof by induction on the length of the string that the language accepted by the DFA with the following transition table will always contain even number of 0s.

| $\delta$ | 0 | 1 |
|---|---|---|
| A | B | A |
| B | A | B |

where A is the initial state and $F = \{A\}$ is the set of final states.

32. $L = \{\ w \in \{4, 8, 1\}^* \ | w \ contains \ '481'\}$

    (a) Draw a diagram of DFA for $L$.

    (b) Give a formal definiton of the same DFA, draw the transition table.

33. $L = \{\ w \in \{0, 1\}^* \mid \#0(w) \ is\ even\ and\ \#1(w)\ is\ a\ multiple\ of\ three\ \}$. Where $\#a(x)$ means total number of symbol $a$ present in the string $x$, $a \in \Sigma$ and $x \in \Sigma^*$.

    (a) Draw a diagram of DFA for $L$.

    (b) Give a formal definiton of the same DFA, draw the transition table.

34. Let $\Sigma = \{a, b, c\}$ be the alphabet set and $L = \{w \mid w \in \Sigma\ and\ w\ starts\ and\ ends\ with\ same\ symbol\}$. Construct a DFA for the given language.

35. Draw an automaton for the set of strings over $\{a, b\}$ containing at least three occurances of three consecutive $b$'s, overlapping permitted (e.g., the string *bbbbb* should be accepted).

36. Consider the alphabet $\Sigma = \{a, b\}$. Is there any language $L$ over this alphabet for which $(\overline{L})^* = \overline{L^*}$ ? If yes, give an example of such a language; if no, explain why.

37. Let us define a language $A_{k,p} = \{w \in \{0, 1, ..., p-1\}^* \mid x\ is\ a\ p-ary\ representation\ of\ a\ multiple\ of\ k\}$. For example, consider the following language $L_1 = \{\ w \in \{0, 1\}^* \mid w\ reprersents\ a\ multiple\ of\ three\ in\ binary\}$. If we choice $k = 3$ and $p = 2$ then $A_{3,2}$ is exactly $L_1$.

    Is $A_{k,p}$ regular? Justify your answer.

38. Consider language set as $A_{3,2}$.

(a) Can you draw a DFA diagram for this language? If yes, then draw the diagram. Define $F$, the set of final states for this DFA. Draw the transition table.

(b) Prove mathematically that this DFA accepts exactly the given language set.
**Hint:** Use the definition of extended transition function and use induction on the length of any string from $\{0,1\}^*$. So you have to show that whenever you have a string from the given set it will end up in one of the accepted states starting from the start state and if the string does not belong to the given language set then it will end up in some non-accepted state starting from the start state.

39. Let $A$ be a regular language. Consider the following operations on $A$:

$$\begin{aligned} 2A &= \{xy \mid x, y \in A and x = y\} \\ A^2 &= \{xy \mid x, y \in A\} \end{aligned}$$

(a) Are the sets regular or not?

(b) If regular then give a proof.

(c) If not regular provide a counter example.

40. Let $L_1$ and $L_2$ be languages over an alphabet $\Sigma$ such that $L_1 \subseteq L_2$. Which of the following is true:

(a) If $L_2$ is regular then $L_1$ must also be regular.

(b) If $L_1$ is regular then $L_2$ must also be regular.

(c) Either both $L_1$ and $L_2$ are regular, or both are not regular.

(d) None of the above.

**Hint:** think with counter examples.

41. Let $B$ be a set of strings over a fixed finite alphabet. We say $B$ is:

$$\begin{aligned} &transitive\ if\ BB \subseteq B \\ &reflexive\ if\ \epsilon \in B \end{aligned}$$

Prove that for any set of strings $A$, $A^*$ is the smallest reflexive and transitive set containing $A$.
**Hint:**

(a) Show that $A^*$ is reflexive and transitive set containing $A$.

(b) Show that if $B$ is any other reflexive and transitive set containing $A$, then $A^* \subseteq B$.

42. Consider the following two languages over the alphabet $\Sigma = \{a, b\}$.

$$L_1 = \{a^n : n \geq 1\}$$

$$L_2 = \{b^n : n \geq 1\}$$

Describe the following languages as per the set notations (similar to the above examples) and give precise definitions as well (like $L_1$ can be defined as the set of all strings that have one or more a's but no b's).

(a) $L_3 = L_1^*$

(b) $L_4 = \bar{L}_1$

(c) $L_5 = L_1 \cup L_2$

(d) $L_6 = L_1 L_2$

(e) $L_7 = (L_1 \cup L_2)^*$

$(f)$ $L_8 = (L_1L_2)^+$

43. Say, $\Sigma = \{a, b\}$, $L = w \in \Sigma^+$. Design a finite automaton that accepts the language $L$.

44. A language $L$ is called **regular** if a DFA accepting that language can be constructed. Consider the language $L_1$ over $\Sigma = \{a, b\}$, defined as

$$L_1 = \{w \in \Sigma^* | \, w = (ab)^n, \; n \geq 1\}$$

Is the language $L_1$ regular? Justify your answer.

45. Say, $\Sigma = \{a, b\}$. Design DFA(s) accepting the following languages:

 $(i)$ The set of all strings containing 1101 as substring.
 $(ii)$ The set of all strings beginning with 101.
 $(iii)$ The set of all strings that begin with 01 and end with 11.
 $(iv)$ The empty set.
 $(v)$ All strings except the empty string.
 $(vi)$ $\{w \,|\, every\, odd\, position\, of\, w\, is\, a\, 1\}$

46. Construct DFA(s) for the following strings:

 $(a)$ Starts with 'a' and $|w| \equiv 1(mod\ \ 4)$.
 $(b)$ Containing substring 'ab' but $|w|$ is not divisible by 2.
 $(c)$ Strings over $\Sigma^*$ which contain even number of a's and odd number of b's.

Consider $\Sigma = \{a, b\}$ and, $w \in \Sigma^*$.

47. Construct DFA(s) over the alphabet $\{0, 1\}$ that accept the following:

 $(i)$ Set of all strings beginning with a '1', such that when interpreted as an integer is a multiple of 5 (E.g. strings like 101, 1010, 1111 etc. are in the language but 0001, 00100 are not).
 $(ii)$ Set of all strings such that every 00 is immediately followed by a 1. (for instance, the strings 1001, 0010, 0010011001 are in the language but 0001, 00100 are not).

48. Design a DFA accepting the language $L = \{w \in \Sigma^* | \, w\, has\, the\, property\, p\}$ over $\Sigma = \{a, b, c\}$, where $p \equiv$ number of a's, b's and c's are even.

49. Construct an NFA with three states that accepts the language L=$\{ab, abc\}^*$.

50. Find an NFA with four states for L=$\{a^n \mid n \geq 0\} \cup \{b^n a \mid n \geq 1\}$

51. Draw Deterministic Finite Automata (DFA) to accept the following sets of strings over the alphabet $\{0, 1\}$:

 a All strings that contain exactly 4 "0"s.
 b All strings ending in "1101".
 c All strings containing exactly 4 "0"s and at least 2 "1"s.
 d All strings whose decimal interpretation is divisible by 5.
 e All strings that contain the substring 0101.
 f All strings that start with 0 and have odd length or start with 1 and have even length.
 g All strings that dont contain the substring 110.
 h All strings of length at most five.

i All strings where every odd position is a 1.

52. For the alphabet $\Sigma = \{0, 1, 2\}$, construct a DFA for the language L = { w ∈ $\Sigma^*$ | w contains exactly two 2s }.

- For the alphabet $\Sigma = \{0, 1\}$, construct a DFA for the language **L** = { w ∈ $\Sigma^*$ | w contains the same number of instances of the substring 01 and the substring 10 }. Note that substrings are allowed to overlap, so 010 ∈ **L** and 10101 ∈ **L**.

53. For the alphabet Σ = { a, b, c, ..., z}, construct a DFA for the language **L** = { $w \in \Sigma^*$| w contains the word "**cocoa**" as a substring }.

54. Suppose that you are taking a walk with your dog along a straight-line path. Your dog is on a leash that has length two, meaning that the distance between you and your dog can be at most two units. You and your dog start at the same position. Consider the alphabet, Σ = {**Y, D**}. A string in $\Sigma^*$ can be thought of as a series of events in which either you or your dog moves forward one unit. For example, the string **YYDD** means that you take two steps forward, then your dog takes two steps forward. Let **L** = {$w \in \Sigma^*$ | $w$ describes a series of steps that ensures that you and your dog are never more than two units apart }. Construct a DFA for **L**.