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2015010290

1. $x[n] = \begin{cases} 1 & \text{if } 0 \le n \le 9 \\ 0 & \text{elsewhere} \end{cases}$

$h[n] = \begin{cases} 1 & \text{if } 0 \le n \le N \\ 0 & \text{elsewhere} \end{cases}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=0}^{9} x[k]\, h[n-k] \quad (0 \text{ elsewhere})$$

$$= \sum_{k=0}^{9} h[n-k]$$

given $y[4] = 5$

$$\therefore y[4] = h[4] + h[3] + h[2] + h[1] + h[0] + 0 + 0 + 0 + 0 + 0$$

$$\therefore y[4] - 1 = 4 = h[4] + h[3] + h[2] + h[1]$$

Since $h[n]$ is either 0 or 1

$$h[4] = h[3] = h[2] = h[1] = 1$$

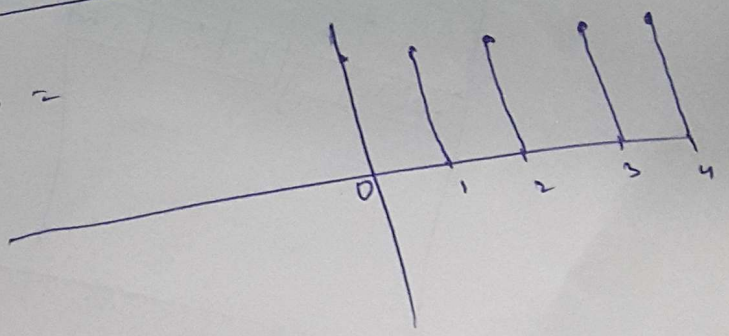
given $y[14] = 0$

$$\Rightarrow y[14] = 0 = h[14] + h[13] + \cdots h[5]$$

$\therefore$ All must be 0

$\boxed{N = 4}$

$h[n] =$



(2) Initially the input image has the first 250 rows — Black (0)

rows 251 to 500 — White (255)

Finally the output is

Rows 1 to 250 — black (0)

Rows 251 — white (255)

Rows 252 to 500 — black (0)

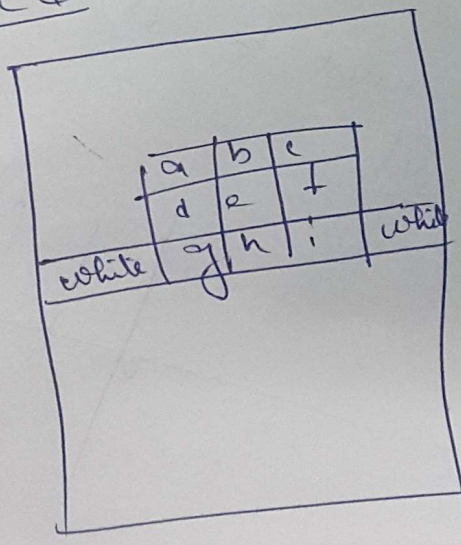
let the 3×3 correlation matrix be

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

(from co-relation matrix, we can get convolution matrix by flipping)

Here co-relation matrix means, the matrix obtained from convolution matrix by flipping twice

case ① :-



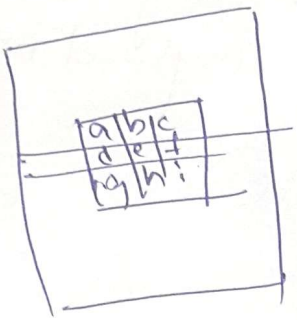
① (a+b+c+d+e+f)+
255 (g+h+i) = 0

(∵ portion of e is replaced with ~~portion~~ black

finally

g+h+i → ①

case ①



$0(a+b+c) + 255(d+e+f) + 255(g+h+i)$

$= 255$

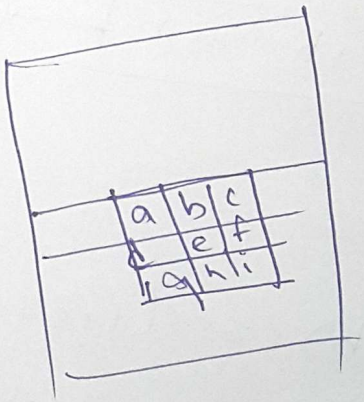
(∵ portion of e is finally replaced with white)

$\rightarrow d+e+f+g+h+i = 1$

from ①, $g+h+i = 0$

$\boxed{d+e+f = 1} \rightarrow ②$

case ③



$255(a+b+c+d+e+f+g+h+i) = 0$

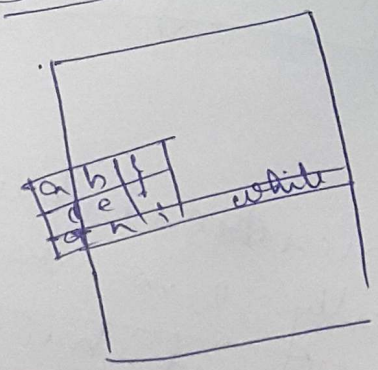
(∵ portion of e is finally replaced with 0)

from ①, ②; $d+e+f+g+h+i = 1$

$\Rightarrow a+b+c = 0-1 = -1$

$\boxed{a+b+c = -1} \rightarrow ③$

case ④

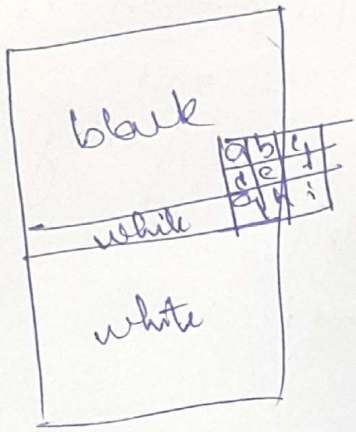


$0(b+c+e+f) + 255(h+i) = 0$

(∵ finally e is replaced with 0)

$\boxed{h+i = 0} \rightarrow ④$

case ⑤

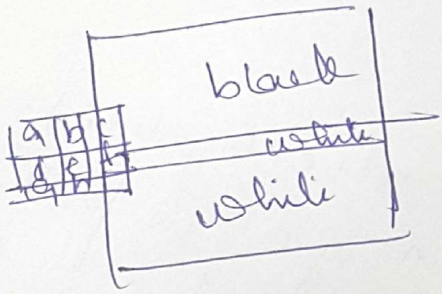


$0(a+b+d+e)+255(g+h)=0$

(∵ finally e is replaced with 0)

$\Rightarrow \boxed{g+h=0} \rightarrow (5)$

case ⑥ -

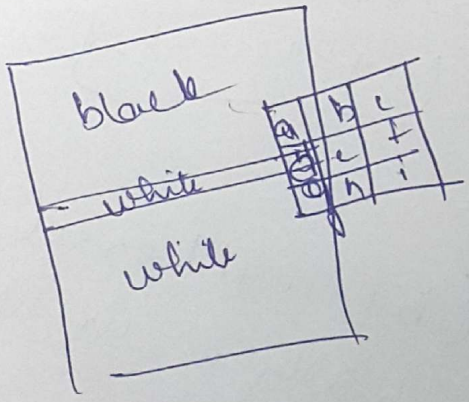


$0(b+c)+255(e+f+h+i)=255$

(∵ finally e is replaced with 255)

$\Rightarrow \boxed{e+f+h+i=1} \rightarrow (6)$

case ⑦ -

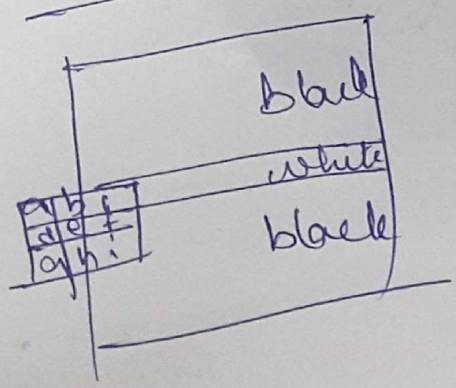


$0(a+b)+255(d+e+g+h)=255$

(∵ finally e is replaced with 255)

$\Rightarrow \boxed{d+e+g+h=1} \rightarrow (7)$

case ⑧ . . -

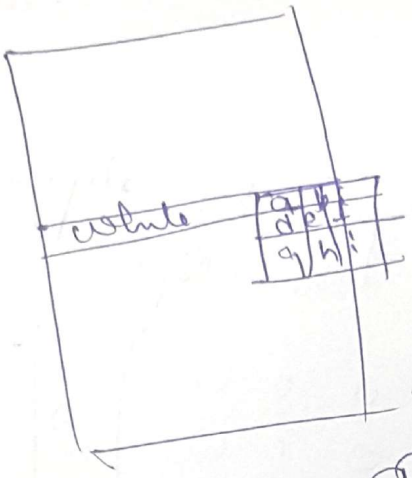


$255(b+c+d+e+f+h+i)=0$

(∵ finally e is replaced with 0)

$\Rightarrow \boxed{b+c+d+e+f+h+i=0} \rightarrow (8)$

case (9):-



$2s(a+b+d+e+g+h)=0$

C: finally e is replacing with 0

$a+b+d+e+g+h=0$ (9)

from (1), (4), (5)

$g+h+i=0$
$h+i=0$
$g+h=0$

$\Rightarrow$ $g=0, h=0, i=0$

from (2), (6), (7)

$d+e+f=1$
$e+f+0h+i=1$
$d+e+g+h=1$

$\Rightarrow$ $d+e+f=1$
$e+f=1$
$d+e=1$

since $g=0$, $h=0$, $i=0$

$\rightarrow d=0$; $e=1$ & $d=1$; $f=1-e=1-1=0$

$\Rightarrow$ $d=0, e=1, f=0$

from (3), (8), (9)

$a+b+c+0=-1$
$b+c+e+f+h+i=0$ $\rightarrow$ $b+c+1+0+0+0=0$ (subst values)

$b+c=-1$

$a+b+d+e+g+h=0$

$\Rightarrow a+b+c+1+0+0+0+0=0$

$\rightarrow a+b=-1$

$a=0$
$b=-1-a=-1$
$c=-1-b=-1+1=0$

$\Rightarrow$ $a=0, b=-1, c=0$

hence correlation matrix is

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

impulse response is got by flipping it upwards and to the left

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

∴ filter (impulse response) is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

problem 3:-

given $x[n] = [\ldots 0\ 0\ 1\ 1\ 1\ 0\ 0 \ldots]$

$y[n] = [\ldots 0\ 0\ 1\ 2\ 3\ 2\ 1\ 0 \ldots]$

of these two, let $y[n]$ be the impulse response

let output of convolution of $x[n]$, $y[n]$ be $z[n]$

$$\therefore z[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[n-k]$$

$$= x[-1]\, y[n+1] + x[0]\, y[n] + x[1]\, y[n-1]$$

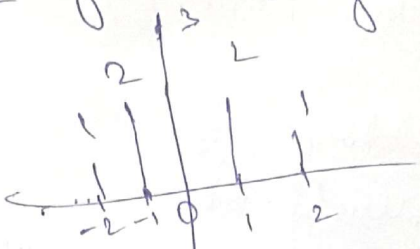
$(\because x[n] = 0;\ n \neq -1$ and $n \neq 0$ & $n \neq 1)$

$$= y[n-1] + y[n] + y[n+1]$$

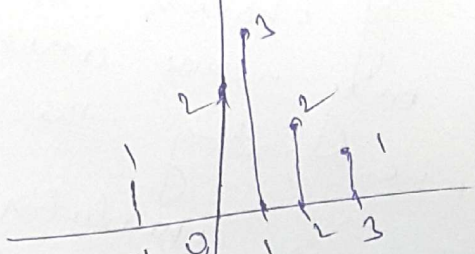
$y[n-1]$ obtained by shifting $y[n]$ to the right by 1 unit.

y[n+1] obtained by shifting y[n] to the left by 1 unit

y[n] :-



y[n-1]

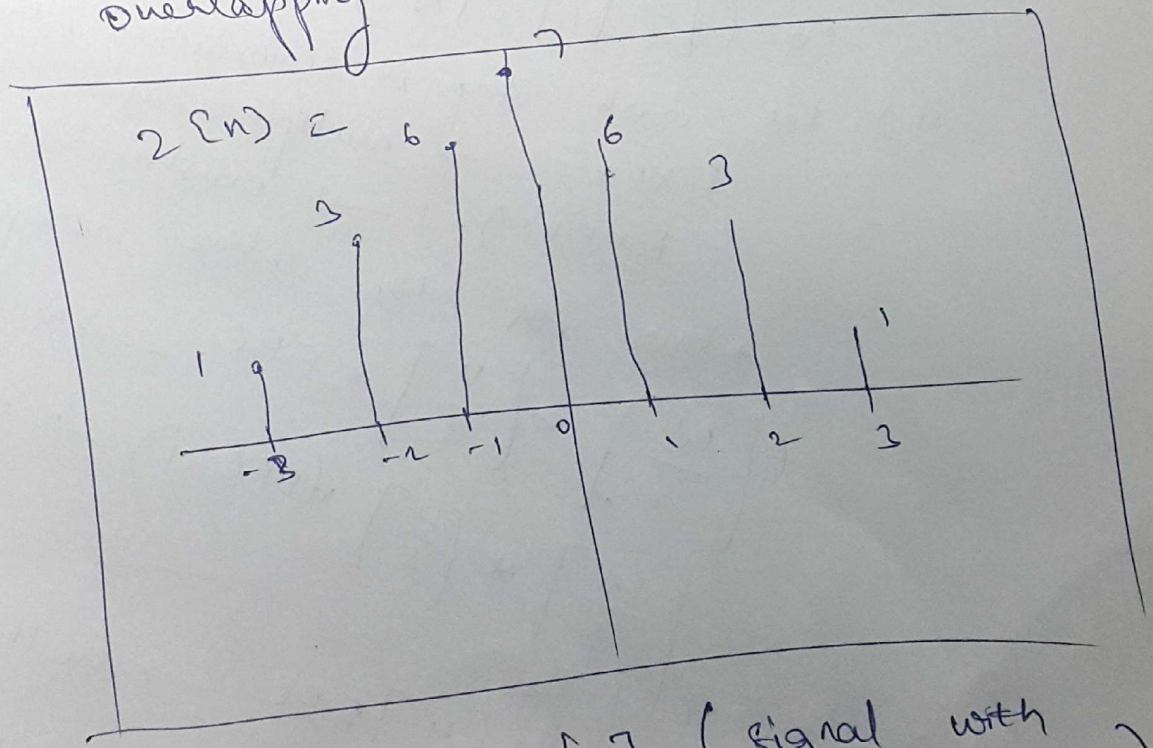


y[n+1]

$z[n] = y[n] + y[n-1] + y[n+1]$

overlapping above 3 graphs we get

z[n] =



Impulse response = y[n] (signal with unique naming)

(5) Size of image: (W, H, C)
Size of filter: (F, F, C)
step size: S
zero padding: Z
no. of filters: N

Number of channels are equal & so the filter only moves across the images in 2D, i.e along the width and height.

i) each step in convolution gives rise to 1 entry in output. so if step size = 1, each cell of image gives rise to 1 entry in output.

so if step size = S, each cell at a distance of S horizontally and vertically becomes the center (KE+1)

so $\left(\left\lceil \frac{W}{S} \right\rceil, \left\lceil \frac{H}{S} \right\rceil, 1\right)$ will be the dimension of

ii) for each step in convolution, there will be

$F \times F \times C$ → multiplication

$(F \times F \times C) - 1$ → substraction

now for the entire image

$\left\lceil \frac{W}{S} \right\rceil \times \left\lceil \frac{H}{S} \right\rceil \times FFC$ → multiplication

$\left\lceil \frac{W}{S} \right\rceil * \left\lceil \frac{H}{S} \right\rceil \times (FFC-1)$ → addition

so for N filters

$N \times \left\lceil \frac{W}{S} \right\rceil \left\lceil \frac{H}{S} \right\rceil FFC$ → multiplication

$N \times \left\lceil \frac{W}{S} \right\rceil \left\lceil \frac{H}{S} \right\rceil (FFC-1)$ → addition

(4) $y[n] = x[n] - y^2[n-1] + y[n-1]$

$x[n] = y[n] + y^2[n-1] - y[n-1]$

$\therefore y[n] \xrightarrow{T} x[n]$

if we proved that T is non-linear, so it will be $x[n] \xrightarrow{T'} y[n]$

replacing $\alpha y[n]$ in place of $y[n]$

$RHS = \alpha y[n] + \alpha^2 y^2[n-1] - \alpha y[n-1]$

Operating LHS,

$= \alpha x[n]$

$= \alpha (y[n] + y^2[n-1] - y[n-1])$

$= \alpha y[n] + \alpha y^2[n-1] - \alpha y[n-1]$

$\therefore LHS \neq RHS$

$\therefore$ system is Nonlinear.

Lts put $z[n] = y[n-k]$ instead of $y[n]$, we get $x[n-k]$ (if time invariant)

$LHS = x[n-k] = x[t] \quad t = n-k$

$= y[t] + y^2[t-1] - y[t-1]$

$= y[n-k] + y^2[n-k-1] - y[n-k-1]$

$R.H.S = z[n] + z^2[n-1] - z[n-1]$

$= y[n-k] + y^2[n-k-1] - y[n-k-1]$

$L.H.S = R.H.S \therefore$ time invariant.

(7) Observations :-

* After resampling and hearing the sound it was found that quality of sound decrease with decrease in sampling frequency.
* when played at default frames per second, it played faster with decreasing sampling rate.

simulations were done at :-

1) church
2) park
3) railway

* sounds were also recorded and sampled at 44kHz, 10kHz, 8kHz, 4kHz.
sampling rates digitalized at 24 bits.
played at default rate of matlab, at higher sampling freq, was distorted and playing slower than actual time
* For correctly playing sound, it should be played at same rate it was recorded.
* quality of sound increases with increase in digitization bits.
* for 3rd part, recorded sound is convolved with 3 earplay responses. Impulse response added to original rendered sound.

(6) we correlate 'shyam.jpg' with 'thumbnail.jpg' because cross-correlation helps to find exact matching in both.

matchTemplate() function is used for this.

From correlated graph, we find x and y coordinate and draw rectangle using rectangle.