Assignment 1

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Prior probability: chance to encounter each class

$P(\omega_1) = 0.6$, $P(\omega_2) = 0.4$

For all problems, we use a single prior probability. Usually the prior of all classes is generic.

Class 1 generic $\mu_1 = 0.5$, $\sigma_1 = 0.5$

Class 2 generic $\mu_2 = 0.5$, $\sigma_2 = 0.5$

Bayes Rule

$$P(\omega_i|x) = \frac{P(x|\omega_i)\,P(\omega_i)}{P(x)} \propto P(x|\omega_i)$$

$P(x)$ is probability of drawing image with given pixel value x.

Since $P(x)$ is same across the classes we might not necessarily need to compute it.

For generic μ_1, σ_1 & μ_2, σ_2 generic classes

$$\text{class 1: } P(x|\omega_1) = \frac{1}{\sigma_1\sqrt{2\pi}}\, e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}$$

$$\text{class 2: } P(x|\omega_2) = \frac{1}{\sigma_2\sqrt{2\pi}}\, e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

Bias in the training and test probability

We will use training bias to derive the test conditional probability and apply Bayes theorem

$$P(\omega_1|x) = \frac{P(x|\omega_1)\,P(\omega_1)}{P(x)}$$

$$P(x) = \int_{-\infty}^{\infty} P(x|\omega_1)\,P(\omega_1) + P(x|\omega_2)\,P(\omega_2)\,dx$$

$$P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)$$

$$P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)$$

$$P(\omega_1|x) = \frac{P(x|\omega_1)P(\omega_1)}{P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)}$$

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$P(\omega_1|x) = \frac{0.6\,P(x|\omega_1)}{0.6\,P(x|\omega_1) + 0.4\,P(x|\omega_2)}$ = test probability that an item with given pixel value belongs to class 1

$P(x|\omega_1)$ = conditional probability of pixel value given class 1

$P(x|\omega_2)$ = conditional probability of pixel value given class 2

$P(\omega_1)$ = prior probability of class 1

$P(\omega_2)$ = prior probability of class 2

$P(x)$ = probability of pixel value

$P(\omega_1|x)$ = probability of class 1 given pixel value

$P(\omega_2|x)$ = probability of class 2 given pixel value

training test probability

$$P(\omega_1|x) = \frac{P(x|\omega_1)\,P(\omega_1)}{P(x)}$$

$$P(x|\omega_1) = \frac{P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)}{P(x)} = \frac{0.6\,P(x|\omega_1) + 0.4\,P(x|\omega_2)}{P(x)}$$

$$P(x|\omega_1) = \frac{P(x|\omega_1)P(\omega_1)}{P(x)}$$

$$P(x|\omega_1) = \frac{P(x|\omega_1)P(\omega_1)}{P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)}$$

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$$P(\omega_1|x) = \frac{0.6\,P(x|\omega_1)}{0.6\,P(x|\omega_1) + 0.4\,P(x|