

## Homework 2

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2. First, to do this problem we must define a new  $\delta$  function, which we'll call  $\delta_2$ . It is to represent the state transitions for  $L^R$ , and it's defined as :  $\delta_2(p, a) = \{q | \delta(q, a) = p, q \in Q\}$ . Then our new NFA is defined as follows:

$$Q^N = Q \times Q \cup q_0^N$$

$$\Sigma^N = \Sigma$$

$$\delta^N(q_0^N, \epsilon) = \{(q_0, r), r \in F\}$$

$$\delta^N((p, q), a) = (\delta(p, a), \delta_2(q, a))$$

$$q_0^N$$

$$F^N = \{(p, q) | \delta(p, a) = q, \delta_2(q, a) = p, p \in Q, q \in Q \text{ for some } a \in \Sigma\}$$

Essentially all the NFA is doing is that it has several “starting” points, where it marks two indices: one is the index of  $q_0$ , and the other is an element from  $F$ , and the idea is that the two will walk towards each other.

To actually implement multiple possible starting points <sup>1</sup>, I just have an artificial  $q_0^N$  that  $\epsilon$  transitions to all of the starting points that I want.

The two walk towards each other, with the first index incrementing with the forwards  $\delta$ , and the second index decrementing with the backward  $\delta_2$ . Eventually they reach a point where they're one transition away from one another, at the last character of  $w$ . At this point, we see that the set of final states has this case included, and we can return successfully.

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<sup>1</sup><https://piazza.com/class/i4mrddxr0h3sd?cid=265>