CS 374 **Spring 2015**

Homework 1

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1. I'll be inducting on the number of operations to construct a regular language. [1]

   Base Case:
   The base case is when the number of operations to construct the regular language is $n = 1$. The only way to get a regular language of a single operation is through the base case of the inductive definition of regular languages:

   - $\emptyset$ is represented by a top-plus regular expression, of the form $(\alpha_1 + \ldots + \alpha_k)$, where k = 1, because $\alpha_1 = \emptyset$ contains no '+'
   - $\{\epsilon\}$, is also represented by a top-plus expression, again where k = 1, where $\alpha_1 = \epsilon$ contains no '+'.
   - $\{a\}$, for any a in any arbitrary alphabet, Σ, is represented by a top-plus expression, where k = 1, where $\alpha_1 = a$ contains no '+'

   Inductive Hypothesis:
   Now assume all r_k and r_l where $1 \leq k < n$, $1 \leq l < m$, where r_k and r_l are regular expressions that represent regular languages, can be represented by a top-plus regular expression.

   Inductive Step:
   Without loss of generality, let r_{n-1} be any regular expression (representing a regular language) that was constructed in $(n-1)$ steps, and let r_{m-1} be any regular expression constructed in $(m-1)$ steps. Since they are both top-plus expressions, say that r_{n-1} is a top-plus expression with $k = x$ terms, and let r_{m-1} be a top-plus expression with $k = y$ terms.
   We can get a top-plus regular expression of construction size n by constructing on r_{n-1}:

   - $(r_{n-1} + r_{m-1})$ is a top-plus expression because it is the union of x and y terms all without any '+'. The result is a top-plus expression of $k = x + y$.
   - $(r_{n-1}r_{m-1})$ is a top-plus expression as well. Through the distributive law of concatenation over union, we have $(r^1_{n-1} + r^2_{n-1} + \ldots + r^x_{n-1})(r^1_{m-1} + r^2_{m-1} + \ldots + r^y_{m-1}) = (r^1_{n-1}r^1_{m-1} + r^1_{n-1}r^2_{m-1} + \ldots r^x_{n-1}r^y_{m-1})$, which represents a top-plus expression of $k = xy$. (Here the "exponents" represent indices.)
   - $(r_{n-1})^*$ is also a top-plus expression. Through the theorem included with the pset, we know that $(r^1_{n-1} + r^2_{n-1} + \ldots + r^x_{n-1})^* = ({r^1_{n-1}}^* {r^2_{n-1}}^* \ldots {r^x_{n-1}}^*)$, which is a top-plus expression of $k = 1$.

   Thus, every regular language can be represented by a top-plus regular expression.

[1] Creds to Prof. Pitt : https://piazza.com/class/i4mrvddxr0h3sd?cid=185