CS 374 **Spring 2015**

Homework 1

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3. (a)

$$Q := Q_1 \times Q_2 \times Q_3$$
$$\Sigma := \Sigma$$
$$\delta((p,q,r),a) := (\delta_1(p,a), \delta_2(q,a), \delta_3(r,a)), p \in Q_1, q \in Q_2, r \in Q_3, a \in \Sigma$$
$$q_0 := (q_0^{(1)}, q_0^{(2)}, q_0^{(3)})$$
$$F := (F_1 \times F_2 \times Q_3) \cup (F_1 \times Q_2 \times F_3) \cup (Q_1 \times F_2 \times F_3)$$

Essentially, the accepting states are if you accept any two DFAs. So I just took all $\binom{3}{2}$ ways to succeed. The case where all 3 DFAs are fulfilled is included implicitly.

(b) First we have to prove that the extended δ^* function is accurate for all words with induction. [1]
So we're trying to prove that $\delta^*((q_0^{(1)}, q_0^{(2)}, q_0^{(3)}), w) = (\delta_1^*(q_0^{(1)}, w), \delta_2^*(q_0^{(2)}, w), \delta_3^*(q_0^{(3)}, w)), w \in \Sigma^*$ with induction on the length of the input:

Base Case:
The length of the string is 0, or is ϵ.

$$\delta_1^*(q_0^{(1)}, \epsilon) = q_0^{(1)}, \delta_2^*(q_0^{(2)}, \epsilon) = q_0^{(2)}, \delta_3^*(q_0^{(3)}, \epsilon) = q_0^{(3)}$$

ϵ as the input for our new δ^* function just returns the start state. So we get:

$$\delta^*((q_0^{(1)}, q_0^{(2)}, q_0^{(3)}), \epsilon) = (q_0^{(1)}, q_0^{(2)}, q_0^{(3)}) = (\delta_1^*(q_0^{(1)}, \epsilon), \delta_2^*(q_0^{(2)}, \epsilon), \delta_3^*(q_0^{(3)}, \epsilon))$$

Inductive Hypothesis:
Assume that for all words of length k, such that $0 \leq k < n$, that $\delta^*((p,q,r), w_k) = (\delta_1^*(p, w_k), \delta_2^*(q, w_k), \delta_3^*(r, w_k))$

Inductive Step:
Let $w_n = wa$ be a word of length n, such that $\|a\| = 1, \|w\| = n-1$. Then let the state of Q after reading w be (l, m, n).
This also means that $\delta_1^*(q_0^{(1)}, w) = l, \delta_2^*(q_0^{(2)}, w) = m, \delta_3^*(q_0^{(3)}, w) = n$ by the inductive hypothesis.
Now let $\delta((l,m,n),a) = (x,y,z)$. From the original definition of our delta function, we know that $\delta_1(l,a) = x, \delta_2(m,a) = y, \delta_1(n,a) = z$.

So by the inductive assumption, we know that $\delta^*((q_0^{(1)}, q_0^{(2)}, q_0^{(3)}), w) = (\delta_1^*(q_0^{(1)}, w), \delta_2^*(q_0^{(2)}, w), \delta_3^*(q_0^{(3)}, w))$

We now want to show that $L(Q) = (L(Q_1) \cap L(Q_2)) \cup (L(Q_1) \cap L(Q_3)) \cup (L(Q_2) \cap L(Q_3))$
Let : $w \in L(Q) = (L(Q_1) \cap L(Q_2)) \cup (L(Q_1) \cap L(Q_3)) \cup (L(Q_2) \cap L(Q_3))$.
Let there be some final states x,y,z, s.t. $x = \delta_1^*(q_0^{(1)}, w), y = \delta_2^*(q_0^{(2)}, w), z = \delta_3^*(q_0^{(3)}, w)$, such that at least 2 of x,y, or z are in F_1, F_2, F_3 respectively.
From the definition of our new F, w is in the language $L(Q)$, so $(L(Q_1) \cap L(Q_2)) \cup (L(Q_1) \cap L(Q_3)) \cup (L(Q_2) \cap L(Q_3)) \in L(Q)$.

[1] Proof outline inspired by past lectures : https://courses.engr.illinois.edu/cs373/fa2011/lectures/lect_04.pdf

Similarly, we can show that if w is an element of $L(Q)$, it must be a member of the set $(L(Q_1) \cap L(Q_2)) \cup (L(Q_1) \cap L(Q_3)) \cup (L(Q_2) \cap L(Q_3))$, showing that the two sets must be equal.