

Fuzzy logic

Fuzzy Inference

Contents

- Mamdani Fuzzy Inference
 - Fuzzification of the input variables
 - Rule evaluation
 - Aggregation of the rule outputs
 - Defuzzification
- Sugeno Fuzzy Inference

Mamdani Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.
- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.
- The Mamdani-style fuzzy inference process is performed in four steps:
 1. Fuzzification of the input variables
 2. Rule evaluation (inference)
 3. Aggregation of the rule outputs (composition)
 4. Defuzzification.

Mamdani Fuzzy Inference

We examine a simple two-input one-output problem that includes three rules:

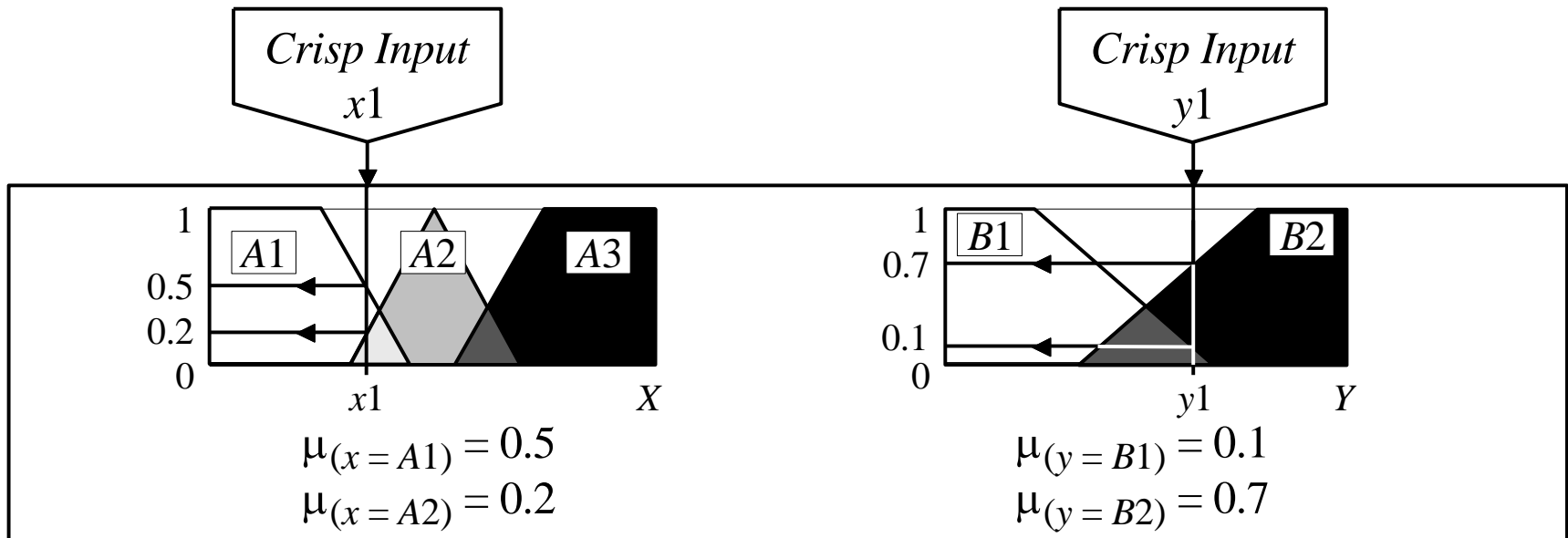
<u>Rule: 1</u>	IF x is A3	OR	y is B1	THEN	z is C1
<u>Rule: 2</u>	IF x is A2	AND	y is B2	THEN	z is C2
<u>Rule: 3</u>	IF x is A1			THEN	z is C3

Real-life example for these kinds of rules:

<u>Rule: 1</u>	IF project_funding is adequate	OR	project_staffing is small	THEN	risk is low
<u>Rule: 2</u>	IF project_funding is marginal	AND	project_staffing is large	THEN	risk is normal
<u>Rule: 3</u>	IF project_funding is inadequate			THEN	risk is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x_1 and y_1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

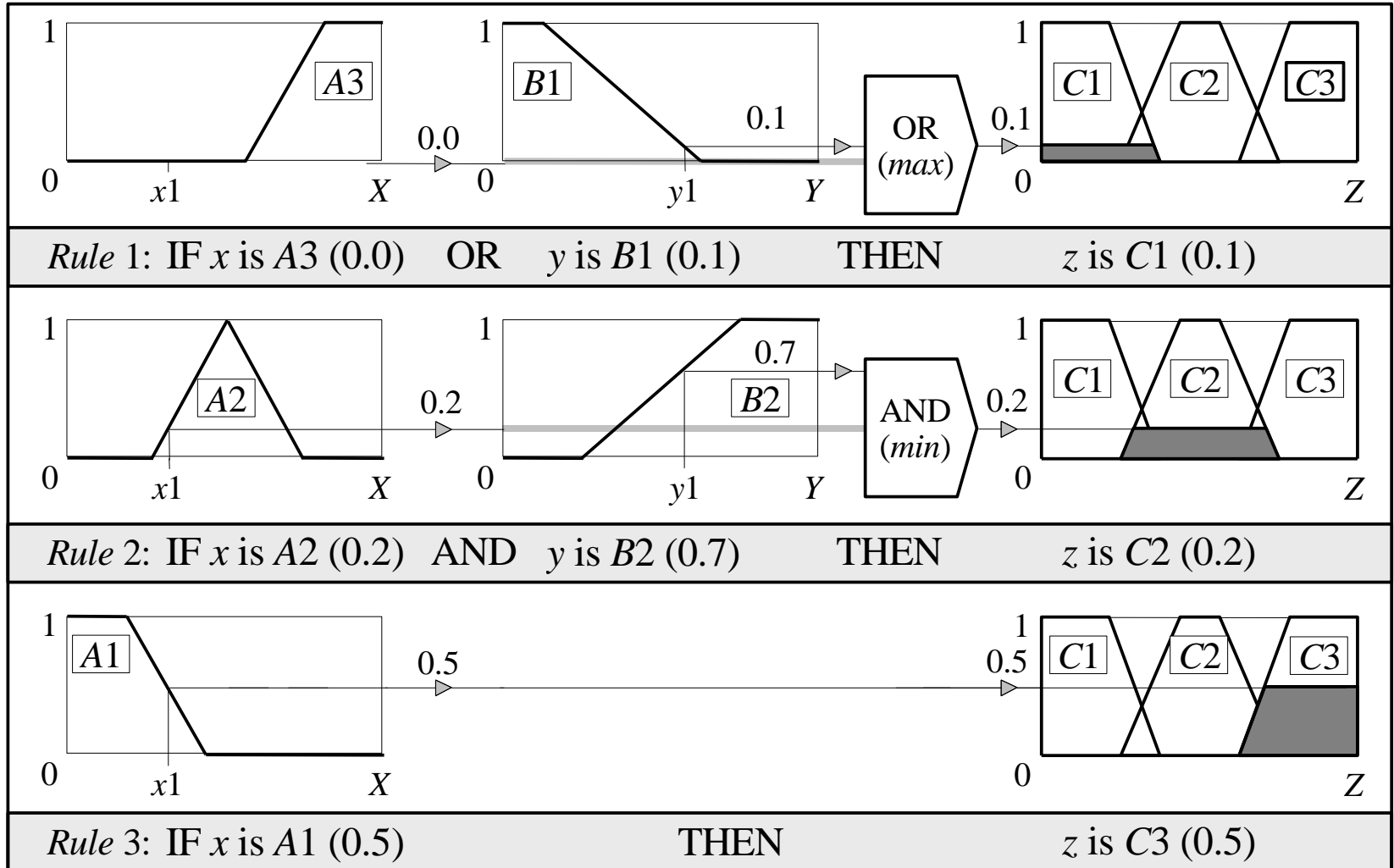
RECALL: To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

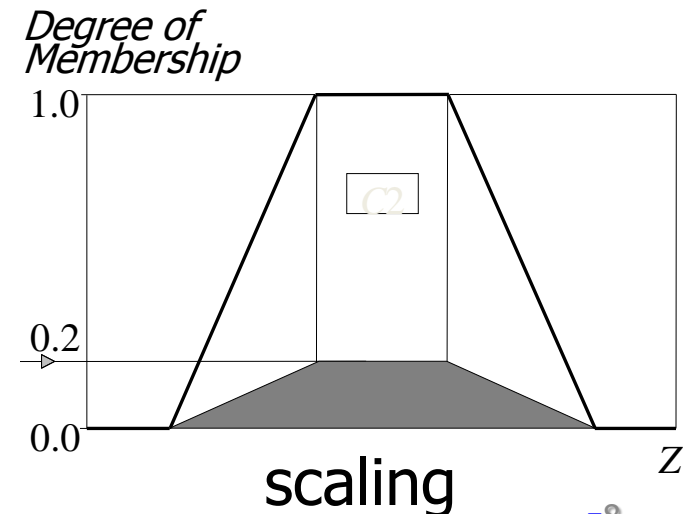
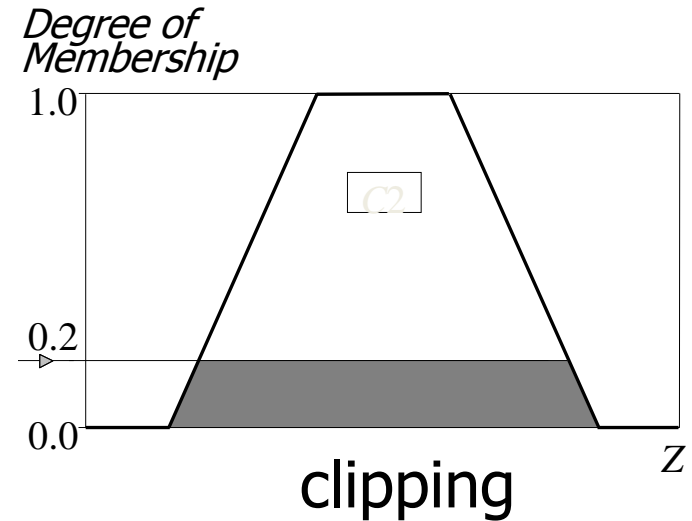
$$\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)]$$

Step 2: Rule Evaluation



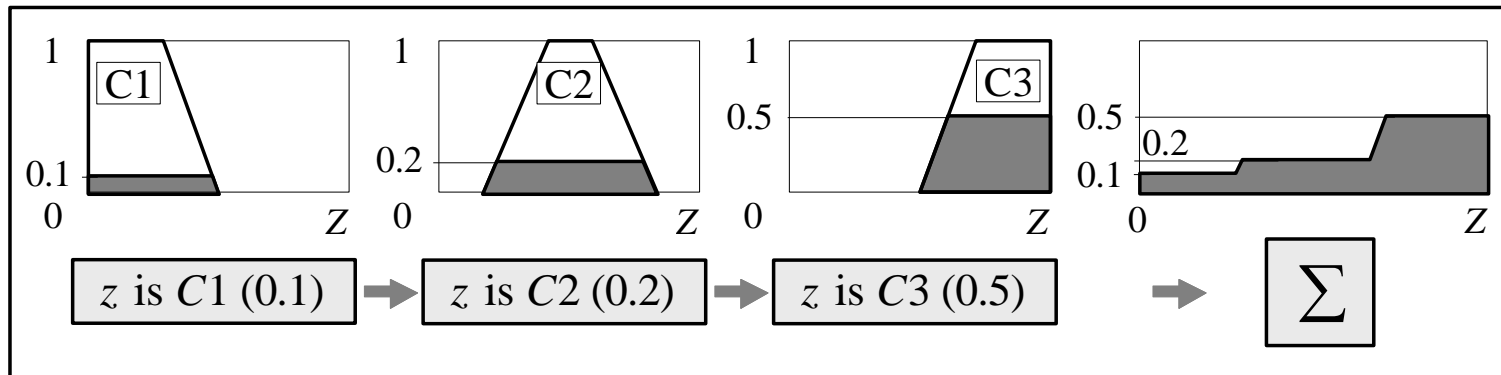
Step 2: Rule Evaluation

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.
- The most common method is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).
- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
 - The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
 - This method, which generally loses less information, can be very useful in fuzzy expert systems.



Step 3: Aggregation of the Rule Outputs

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.



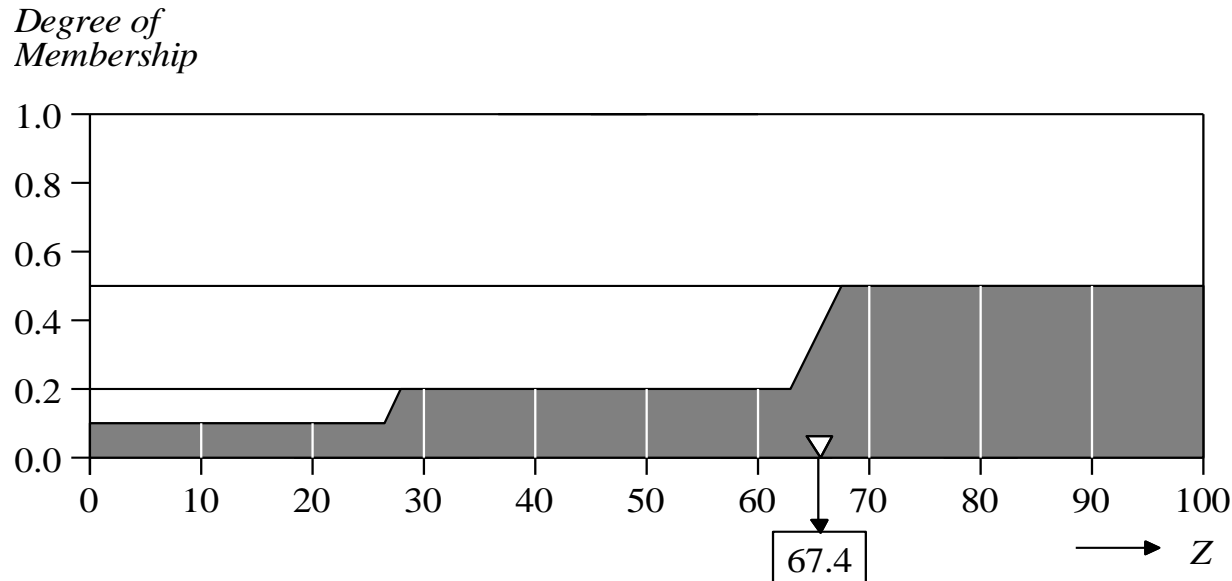
Step 4: Defuzzification

- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity (COG)** can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x) x dx}{\int_a^b \mu_A(x) dx}$$

Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the aggregated fuzzy set A , on the interval $[a, b]$.
- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20) \times 0.1 + (30+40+50+60) \times 0.2 + (70+80+90+100) \times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.
- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Sugeno Fuzzy Inference

- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent: instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

IF x is A AND y is B THEN z is $f(x, y)$

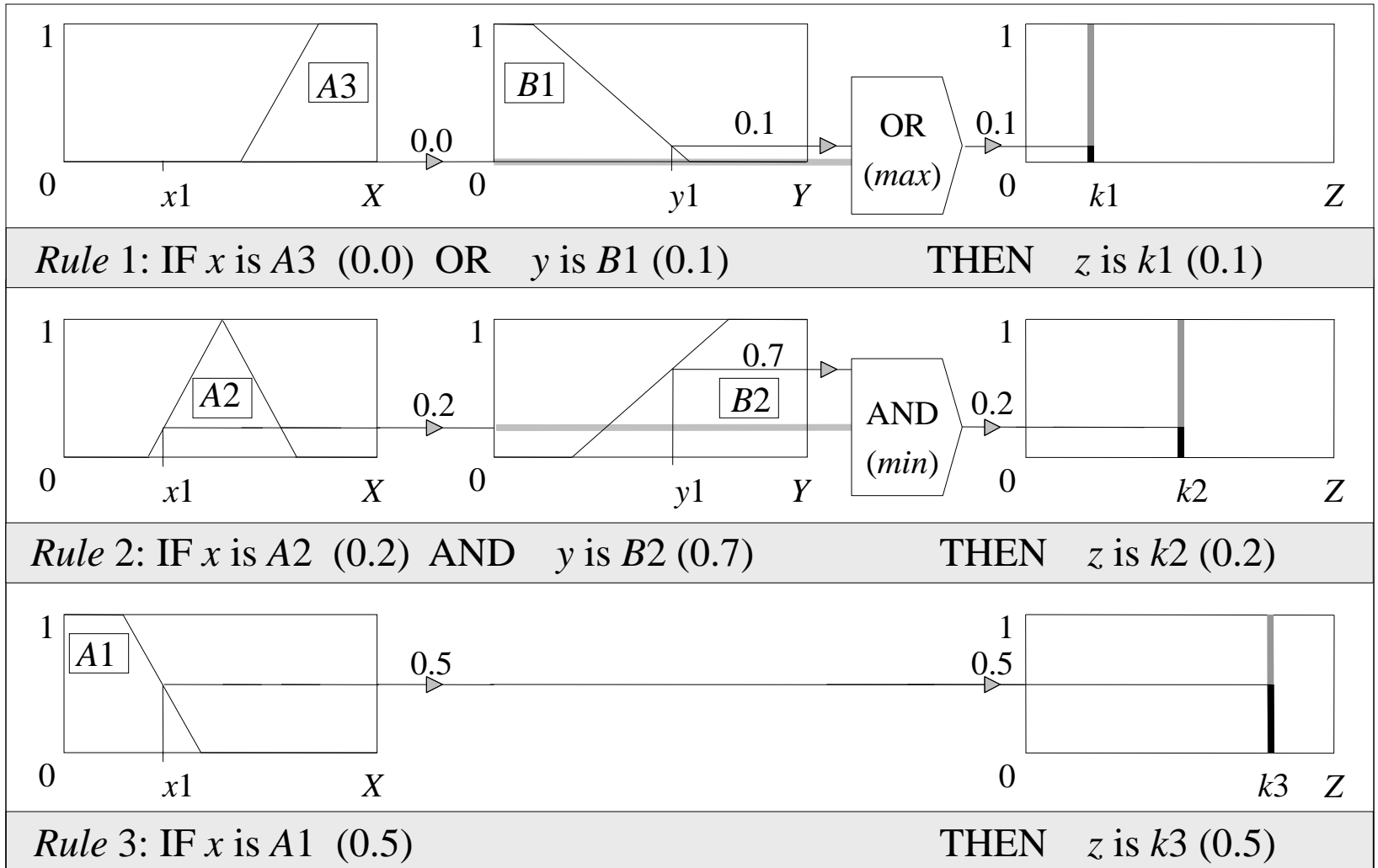
where:

- x, y and z are linguistic variables;
 - A and B are fuzzy sets on universe of discourses X and Y , respectively;
 - $f(x, y)$ is a mathematical function.
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- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

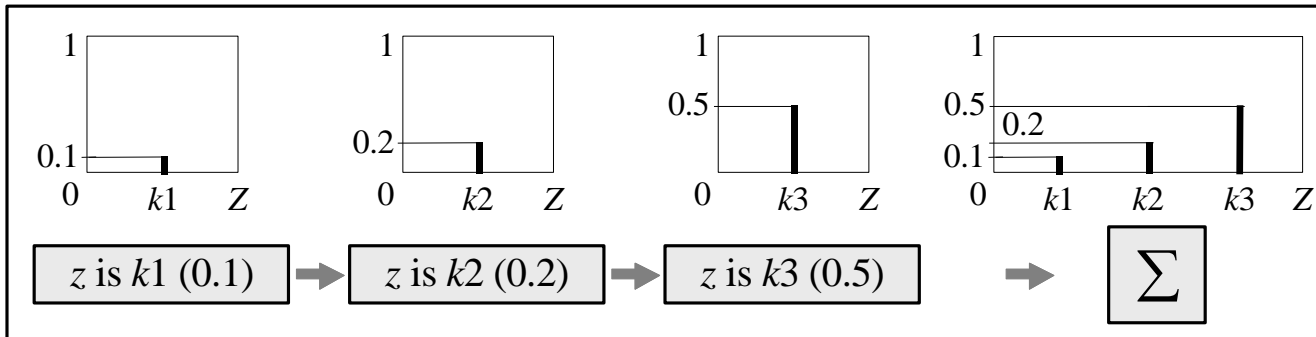
IF x is A AND y is B THEN z is k

- where k is a constant.
- In this case, the output of each fuzzy rule is constant and all consequent **membership functions** are represented by **singleton spikes**.

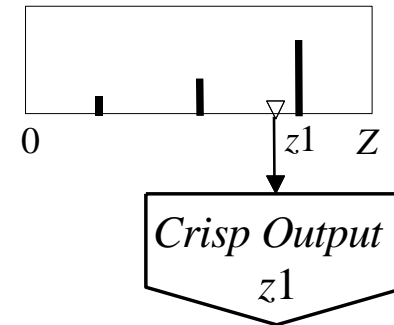
Sugeno Rule Evaluation



Sugeno Aggregation and Defuzzification



COG becomes Weighted Average (WA)



$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$

Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.