Consider the data set shown in Table 5. **[10]**

(a) Estimate the conditional probabilities for P(A =1|+), P(B =1|+), P(C =1|+), P(A =1|-), P(B =1|-), and P(C =1|-)

Table 5

Instances	A	B	C	Class
1	0	0	1	-
2	1	0	1	+
3	0	1	0	-
4	1	0	0	-
5	1	0	1	+
6	0	0	1	+
7	1	1	0	-
8	0	0	0	-
9	0	1	0	+
10	1	1	1	+

(b) Use the conditional probabilities in part (a) to predict the class label for a test sample (A =1, B =1,C =1) using the Naive Bayes approach.

(c) Compare P(A = 1), P(B = 1), and P(A =1,B = 1). State the relationships between A and B

(d) Repeat the analysis in part (c) using P(A = 1), P(B = 0), and P(A =1,B = 0)

(e) Compare P(A =1,B =1|Class = +) against P(A =1|Class =+) and P(B =1|Class = +). Are the variables conditionally independent given the class?

Estimate the conditional probabilities for $P(A = 1|+)$, $P(B = 1|+)$, $P(C = 1|+)$, $P(A = 1|-)$, $P(B = 1|-)$, and $P(C = 1|-)$

Answer:
$P(A = 1|+) = 0.6$, $P(B = 1|+) = 0.4$, $P(C = 1|+) = 0.8$, $P(A = 1|-) = 0.4$, $P(B = 1|-) = 0.4$, and $P(C = 1|-) = 0.2$

(b) Use the conditional probabilities in part (a) to predict the class label for a test sample ($A = 1$, $B = 1$, $C = 1$) using the naıve Bayes approach.

Answer:
Let $R : (A = 1, B = 1, C = 1)$ be the test record. To determine its class, we need to compute $P(+|R)$ and $P(-|R)$. Using Bayes Theorem
$P(+|R) = P(R|+)P(+)/P(R)$ and $P(-|R) = P(R|-)P(-)/P(R)$. Since $P(+) = P(-) = 0.5$ and $P(R)$ is constant, R can be classified by comparing $P(+|R)$ and $P(-|R)$.

For this question,
$P(R|+) = P(A = 1|+) \times P(B = 1|+) \times P(C = 1|+) = 0.192$
$P(R|-) = P(A = 1|-) \times P(B = 1|-) \times P(C = 1|-) = 0.032$

Since P (R|+) is larger, the record is assigned to (+) class.

(c) Compare P (A = 1), P (B = 1), and P (A = 1, B = 1). State the relationships between A and B.
Answer:
P (A = 1) = 0.5, P (B = 1) = 0.4 and P (A = 1, B = 1) = P (A) × P (B) = 0.2. Therefore, A and B are independent.

(d) Repeat the analysis in part (c) using P (A = 1), P (B = 0), and P (A = 1, B = 0).
Answer:
P (A = 1) = 0.5, P (B = 0) = 0.6, and P (A = 1, B = 0) = P (A = 1) × P (B = 0) = 0.3. A and B are still independent.

(e) Compare P (A = 1, B = 1|Class = +) against P (A = 1|Class = +) and P (B = 1|Class = +). Are the variables conditionally independent given the class?
Answer:
Compare P (A = 1, B = 1|+) = 0.2 against P (A = 1|+) = 0.6 and P (B = 1|Class = +) = 0.4. Since the product of P (A = 1|+) and P (A = 1|−) is not the same as P (A = 1, B = 1|+), A and B are not conditionally independent given the class.