Fuzzy Logic

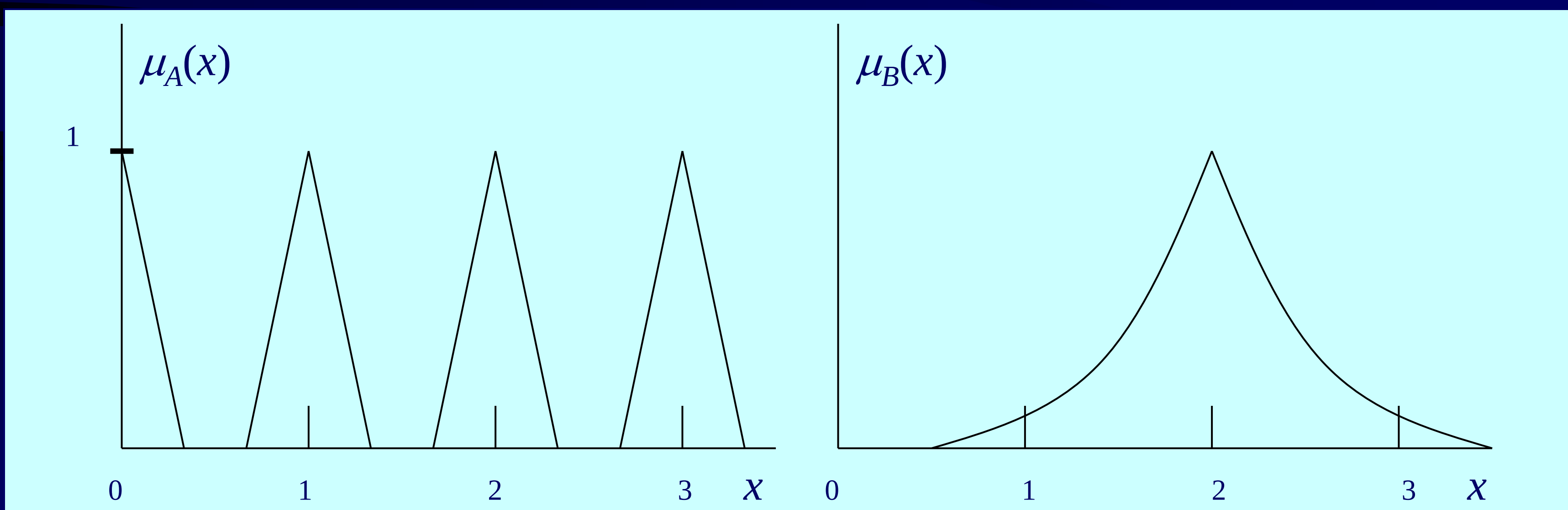
Criteria for fuzzy "and", "or", and "complement"

- Must meet crisp boundary conditions
- Commutative
- Associative
- Idempotent
- Monotonic

Example Fuzzy Sets to Aggregate...

$$A = \{\, x \mid x \text{ is } \textit{near} \text{ an integer} \}$$

$$B = \{\, x \mid x \text{ is } \textit{close} \text{ to } 2 \}$$



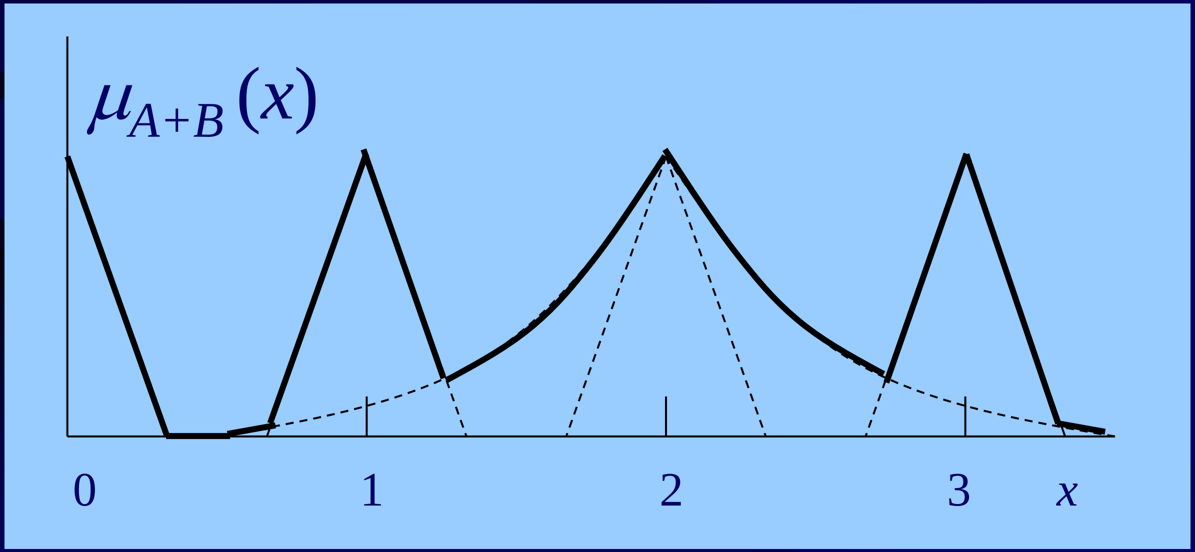
- Fuzzy Union (logic “or”)

$$\mu_{A+B}(x) = \max\left[\mu_{\mathrm{A}}(x), \mu_{B}(x)\right]$$

Fuzzy Union

A OR B = A+B ={ x | (x is *near* an integer) OR (x is *close* to 2)}

$$= \text{MAX}\left[\mu_A(x), \mu_B(x)\right]$$



Fuzzy Intersection

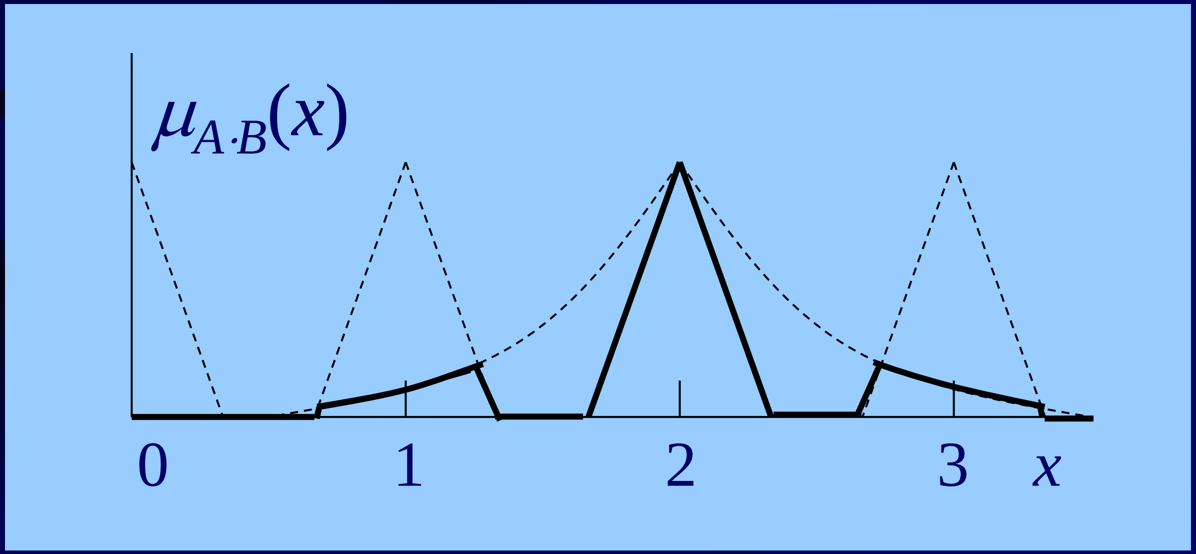
- Fuzzy Intersection (logic “and”)

$$\mu_{A \bullet B}(x) = \min\,[\mu_{\mathrm{A}}(x), \mu_{B}(x)]$$

Fuzzy Intersection

A AND B = $A \cdot B$ = $\{ x \mid$ (x is *near* an integer) AND (x is *close* to 2)$\}$

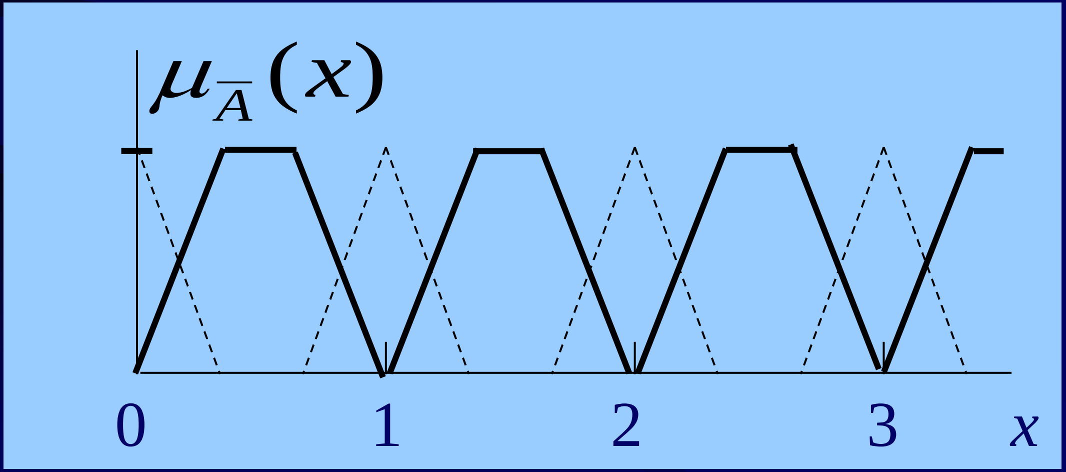
$$= \text{MIN}\left[\mu_A(x), \mu_B(x)\right]$$



Fuzzy Complement

The complement of a fuzzy set has a membership function...

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$



Associativity

Min-Max fuzzy logic has intersection distributive over union...

$$\mu_{A\bullet(B+C)}(x) = \mu_{(A+B)\bullet(A+C)}(x)$$

since

min[A,max(B,C)]=min[max(A,B), max(A,C)]

Associativity

Min-Max fuzzy logic has union distributive over intersection...

$$\mu_{A+(B\bullet C)}(x) = \mu_{(A\bullet B)+(A\bullet C)}(x)$$

since

$$\max[\ A, \min(B,C)\ ] = \max[\ \min(A,B), \min(A,C)\ ]$$

DeMorgan's Laws

Min-Max fuzzy logic obeys DeMorgans Law #1...

$$\mu_{\overline{B \bullet C}}(x) = \mu_{\overline{B}+\overline{C}}(x)$$

since

$$1 - \min(B,C) = \max[\,(1-B),\,(1-C)]$$

DeMorgan's Laws

Min-Max fuzzy logic obeys DeMorgans Law #2...

$$\mu_{\overline{B+C}}(x) = \mu_{\overline{B}\bullet\overline{C}}(x)$$

since

1 - max(B,C)= min[(1-B), (1-C)]

Excluded Middle

Min-Max fuzzy logic fails ***The Law of Excluded Middle.***

$$\boxed{A \cdot \overline{A} \neq \phi}$$

since

$$\min(\mu_A, 1-\mu_A) \neq 0$$

Thus, (the set of numbers *close* to 2) AND (the set of numbers <u>not</u> *close* to 2) $\neq$ null set

Contradiction

Min-Max fuzzy logic fails the ***The Law of Contradiction*.**

$$A + \overline{A} \neq U$$

since

$$\max(\mu_A, 1-\mu_A) \neq 1$$

Thus, (the set of numbers *close* to 2) OR (the set of numbers <u>not</u> *close* to 2) ≠ universal set