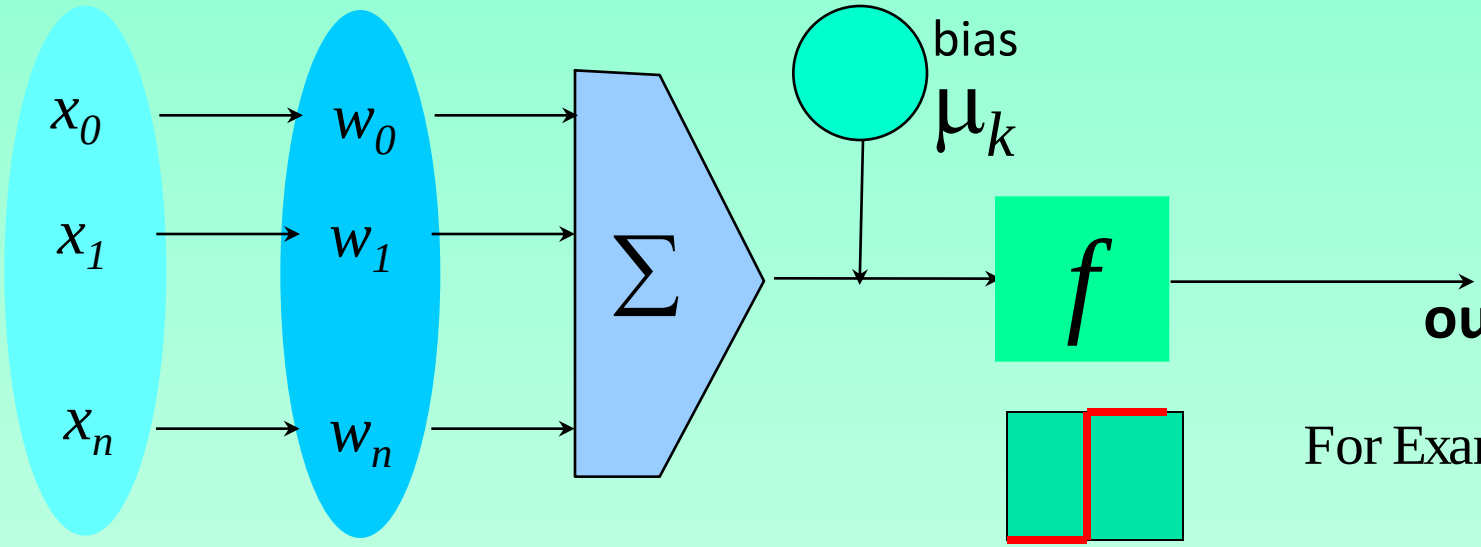
Classification by Backpropagation

- Backpropagation: A **neural network** learning algorithm
- Started by psychologists and neurobiologists to develop and test computational analogues of neurons
- A neural network: A set of connected input/output units where each connection has a **weight** associated with it
- During the learning phase, the **network learns by adjusting the weights** so as to be able to predict the correct class label of the input tuples
- Also referred to as **connectionist learning** due to the connections between units

Backpropagation

- Iteratively process a set of training tuples & compare the network's prediction with the actual known target value
- For each training tuple, the weights are modified to **minimize the mean squared error** between the network's prediction and the actual target value
- Modifications are made in the "**backwards**" direction: from the output layer, through each hidden layer down to the first hidden layer, hence "**backpropagation**"
- Steps
  - Initialize weights to small random numbers, associated with biases
  - Propagate the inputs forward (by applying activation function)
  - Backpropagate the error (by updating weights and biases)
  - Terminating condition (when error is very small, etc.)

Neuron: A Hidden/Output Layer Unit



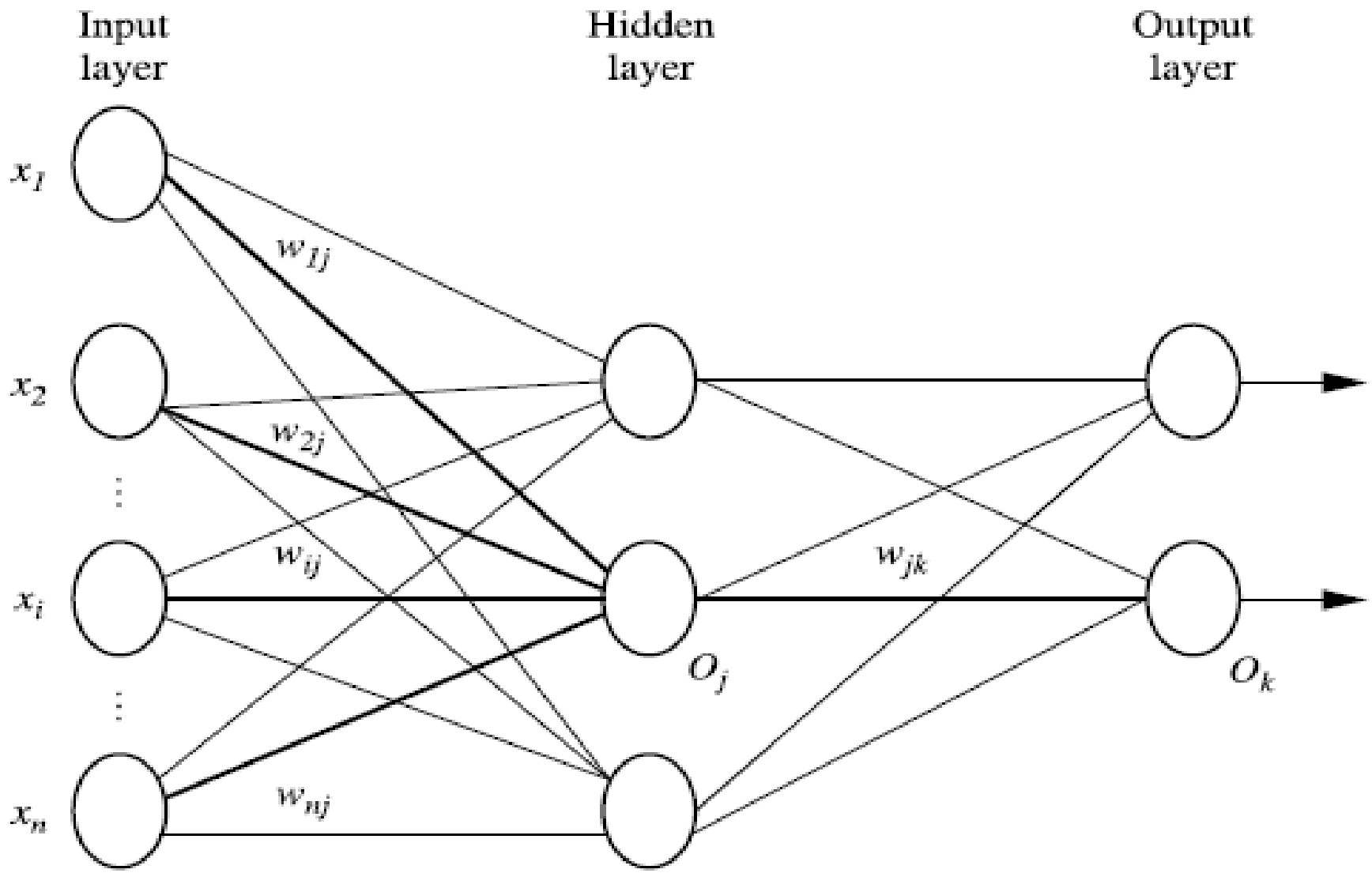
For Example

$$y = \text{sign}\left(\sum_{i=0}^{n} w_i x_i - \mu_k\right)$$

Input vector x **weight vector w** **weighted sum** **Activation function**

- An *n*-dimensional input vector **x** is mapped into variable y by means of the scalar product and a nonlinear function mapping
- The inputs to unit are outputs from the previous layer. They are multiplied by their corresponding weights to form a weighted sum, which is added to the bias associated with unit. Then a nonlinear activation function is applied to it.

A Multi-Layer Feed-Forward Neural Network



Backpropagation Algorithm

Input:

- D, a data set consisting of the training tuples and their associated target values;
- l, the learning rate;
- *network*, a multilayer feed-forward network.

Output: A trained neural network.

Method:

(1) Initialize all weights and biases in *network*;
(2) while terminating condition is not satisfied {
(3) for each training tuple X in D {
(4) // Propagate the inputs forward:
(5) for each input layer unit j {
(6) $O_j = I_j$; // output of an input unit is its actual input value
(7) for each hidden or output layer unit j {
(8) $I_j = \sum_i w_{ij} O_i + \theta_j$; //compute the net input of unit j with respect to the previous layer, i
(9) $O_j = \frac{1}{1+e^{-I_j}}$; } // compute the output of each unit j

Backpropagation Algorithm

(10) // Backpropagate the errors:
(11) for each unit j in the output layer
(12) $Err_j = O_j(1 - O_j)(T_j - O_j)$; // compute the error
(13) for each unit j in the hidden layers, from the last to the first hidden layer
(14) $Err_j = O_j(1 - O_j)\sum_k Err_k w_{jk}$; // compute the error with respect to the next higher layer, k
(15) for each weight w_{ij} in *network* {
(16) $\Delta w_{ij} = (l) Err_j O_i$; // weight increment
(17) $w_{ij} = w_{ij} + \Delta w_{ij}$; } // weight update
(18) for each bias θ_j in *network* {
(19) $\Delta\theta_j = (l) Err_j$; // bias increment
(20) $\theta_j = \theta_j + \Delta\theta_j$; } // bias update
(21) } }

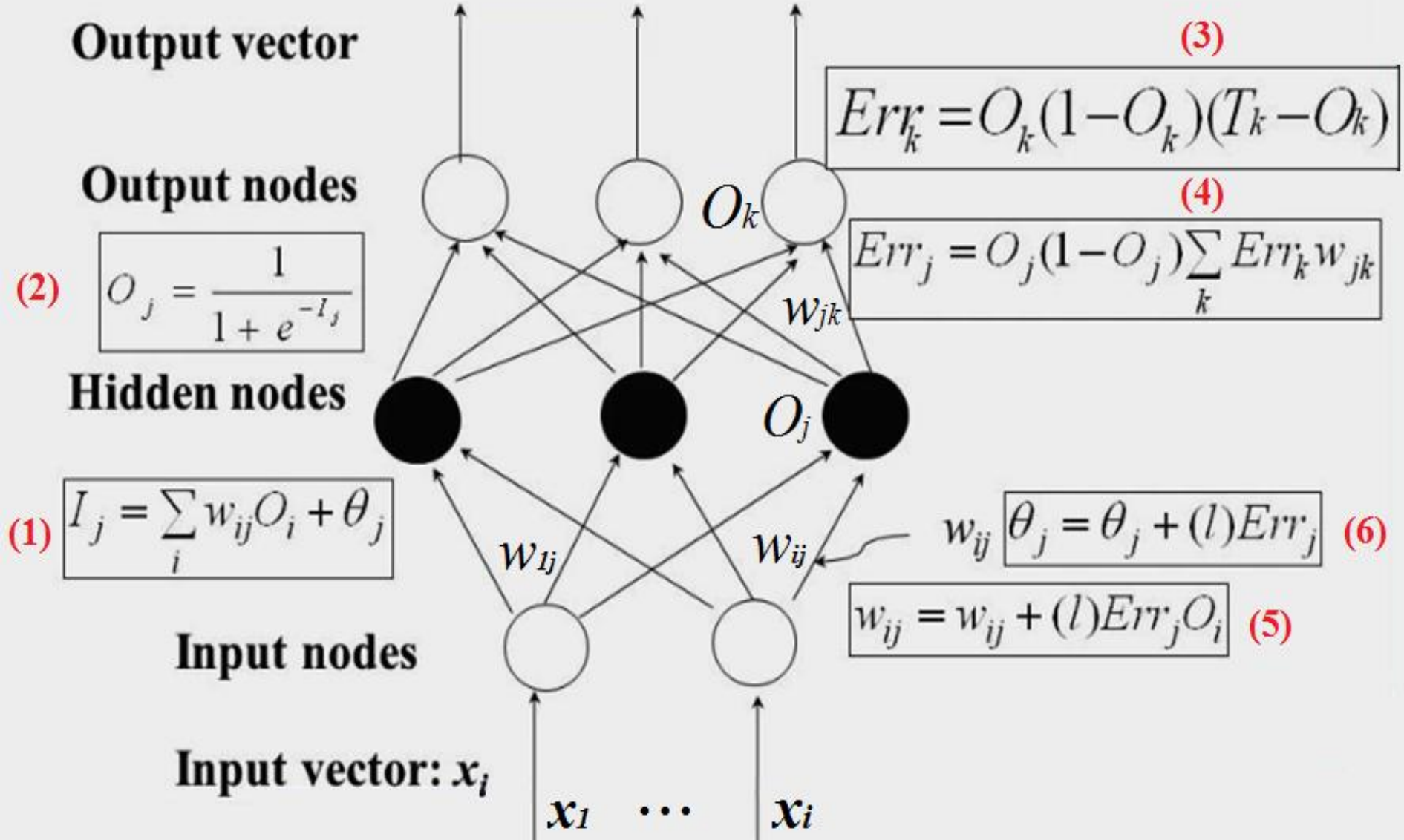
Backpropagation Termination

Terminating condition: Training stops when

- All Δw_{ij} in the previous epoch were so small as to be below some specified threshold, or
- The percentage of tuples misclassified in the previous epoch is below some threshold, or
- A prespecified number of epochs has expired.

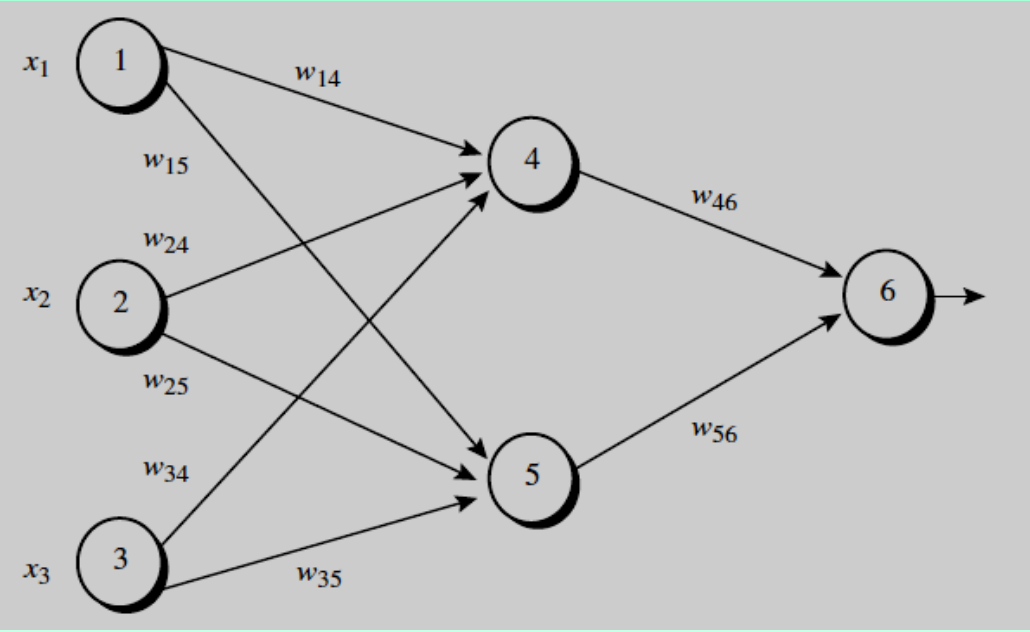
In practice, several hundreds of thousands of epochs may be required before the weights will converge.

Back Propagation Formulas



Example

Sample calculations for learning by the backpropagation algorithm. Figure shows a multilayer feed-forward neural network. Let the learning rate be 0.9. The initial weight and bias values of the network are given in Table , along with the first training tuple, ***X = (1, 0, 1), whose class label is 1.***



Initial input, weight, and bias values.

x_1	x_2	x_3	w_{14}	w_{15}	w_{24}	w_{25}	w_{34}	w_{35}	w_{46}	w_{56}	θ_4	θ_5	θ_6
1	0	1	0.2	−0.3	0.4	0.1	−0.5	0.2	−0.3	−0.2	−0.4	0.2	0.1

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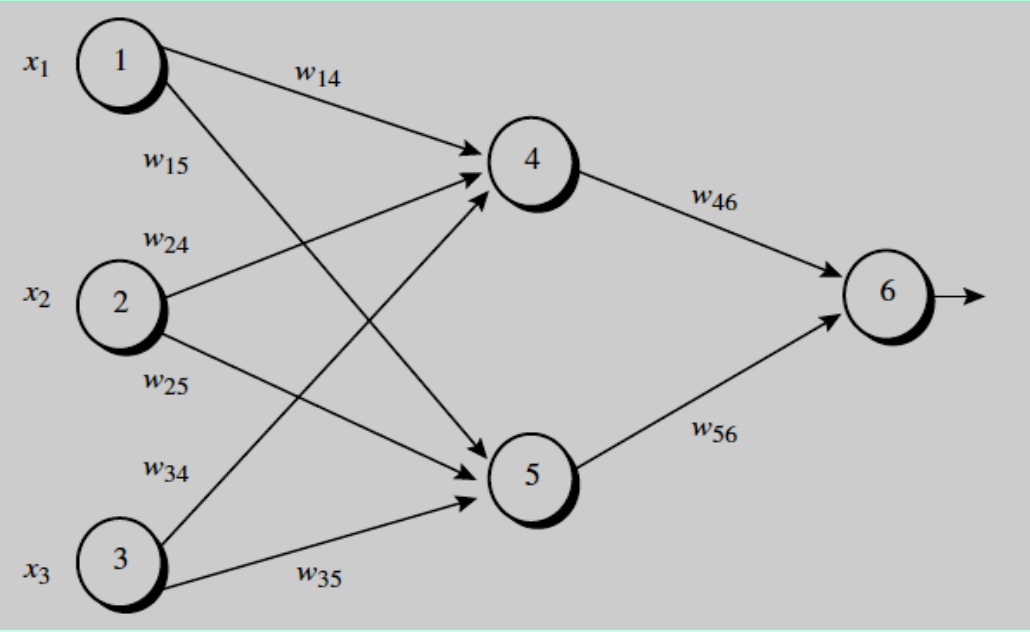

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Net Input and Output Calculation

The net input and output calculations.

Unit j	Net input, I_j	Output, O_j
4	$0.2+0-0.5-0.4=-0.7$	$1/(1+e^{0.7})=0.332$
5	$-0.3+0+0.2+0.2=0.1$	$1/(1+e^{-0.1})=0.525$
6	$(-0.3)(0.332)-(0.2)(0.525)+0.1=-0.105$	$1/(1+e^{0.105})=0.474$

$$I_j = \sum_i w_{ij} O_i + \theta_j,$$

Net Input

$$O_j = \frac{1}{1+e^{-I_j}}.$$

Net Output

Calculation of error at each node

Calculation of the error at each node.

Unit j	Err_j
6	$(0.474)(1-0.474)(1-0.474) = 0.1311$
5	$(0.525)(1-0.525)(0.1311)(-0.2) = -0.0065$
4	$(0.332)(1-0.332)(0.1311)(-0.3) = -0.0087$

$$Err_j = O_j(1-O_j)(T_j - O_j),$$

Output Node

$$Err_j = O_j(1-O_j)\sum_k Err_k w_{jk},$$

Hidden Node

Calculation of Weights and Biases Updating

Weight or bias	New value
w_{46}	$-0.3+(0.9)(0.1311)(0.332)=-0.261$
w_{56}	$-0.2+(0.9)(0.1311)(0.525)=-0.138$
w_{14}	$0.2+(0.9)(-0.0087)(1)=0.192$
w_{15}	$-0.3+(0.9)(-0.0065)(1)=-0.306$
w_{24}	$0.4+(0.9)(-0.0087)(0)=0.4$
w_{25}	$0.1+(0.9)(-0.0065)(0)=0.1$
w_{34}	$-0.5+(0.9)(-0.0087)(1)=-0.508$
w_{35}	$0.2+(0.9)(-0.0065)(1)=0.194$
θ_6	$0.1+(0.9)(0.1311)=0.218$
θ_5	$0.2+(0.9)(-0.0065)=0.194$
θ_4	$-0.4+(0.9)(-0.0087)=-0.408$

Weight Update

$$\Delta w_{ij} = (l) Err_j O_i$$

$$w_{ij} = w_{ij} + \Delta w_{ij}$$

Bias Update

$$\Delta \theta_j = (l) Err_j$$

$$\theta_j = \theta_j + \Delta \theta_j$$

Neural Network as a Classifier

- **Weakness**
  - Long training time
  - Require a number of parameters typically best determined empirically, e.g., the network topology or "structure."
  - Poor interpretability: Difficult to interpret the symbolic meaning behind the learned weights and of "hidden units" in the network
- **Strength**
  - High tolerance to noisy data
  - Ability to classify untrained patterns
  - Successful on an array of real-world data, e.g., hand-written letters
  - Techniques have recently been developed for the extraction of rules from trained neural networks