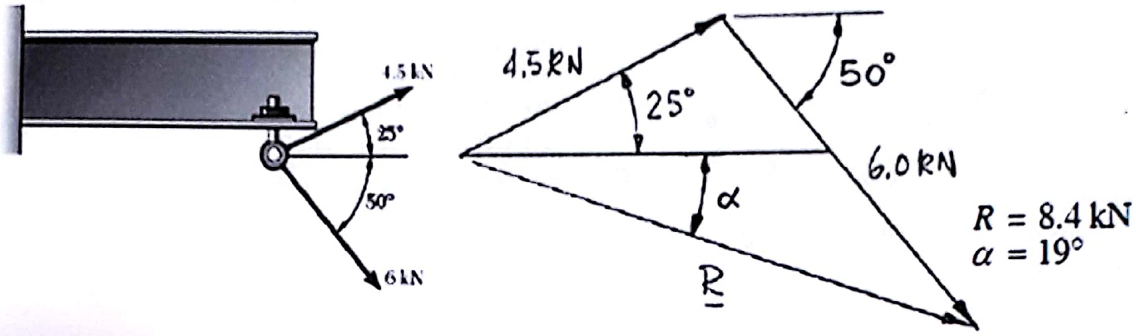
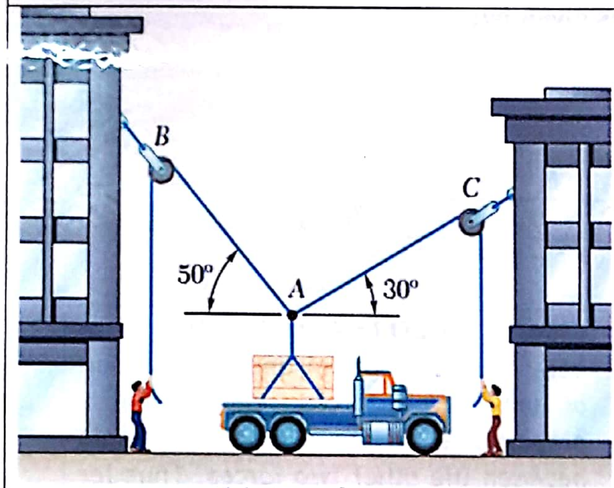
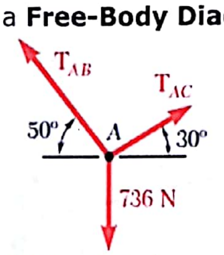
TRIANGLE LAW OF FORCES: If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, their resultant is represented by the closing side of the triangle taken in the opposite order.



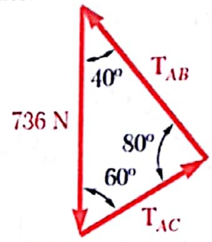


(*a*) Space diagram

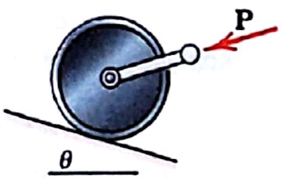
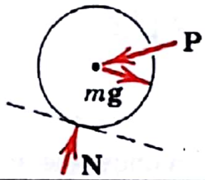
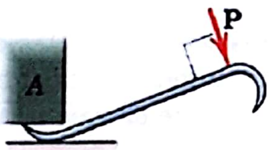
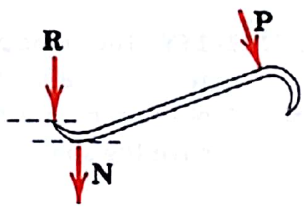
FREE-BODY DIAGRAMS: A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particle. This is done by choosing a significant particle and drawing a separate diagram showing this particle and all the forces acting on it. Such a diagram is called a **Free-Body Diagram**



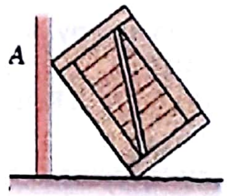
(*b*) Free-body diagram

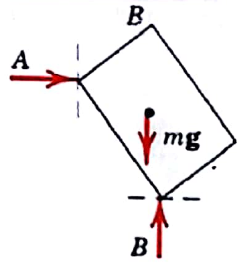


(*c*) Force triangle

	Body	Wrong or Incomplete *FBD*
Lawn roller of mass *m* being pushed up incline *θ*.		
Prybar lifting body *A* having smooth horizontal surface. Bar rests on horizontal rough surface.		

Uniform crate of mass *m* leaning against smooth vertical wall and supported on a rough horizontal surface.





Wrong FBD

Drawing a free-body diagram is the first step in the solution of a problem involving the equilibrium of a particle. This diagram shows the particle and all the forces acting on it. Indicate in your free-body diagram the magnitudes of known forces, as well as any angle or dimensions that define the direction of a force. Any unknown magnitude or angle should be denoted by an appropriate symbol. Nothing else should be included in the free-body diagram. *Drawing a clear and accurate free-body diagram is a must in the solution of any equilibrium problem. Skipping this step might save you pencil and paper, but is very likely to lead you to a wrong solution.*

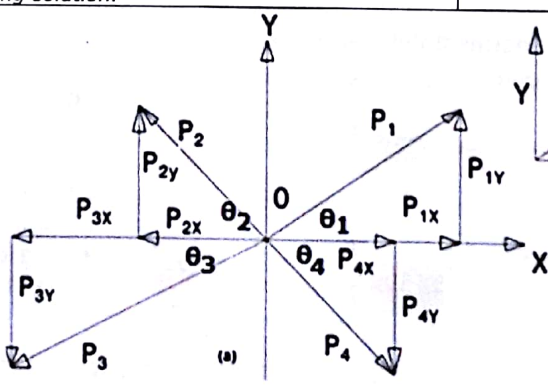
COMPOSITION OF FORCES BY RESOLUTION

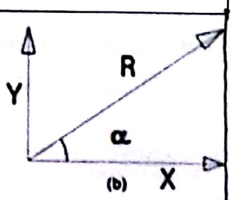
$\sum F_x = P_{1x} + P_{2x} + P_{3x} + P_{4x}$

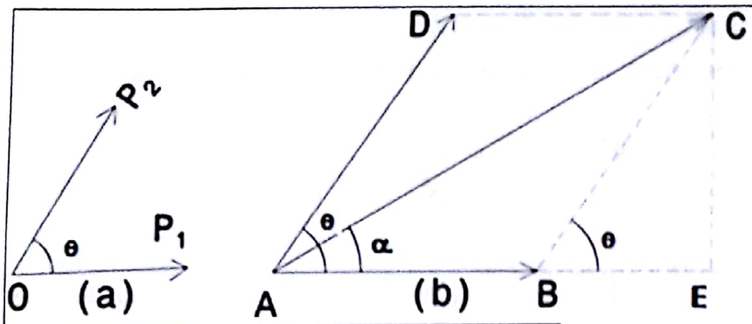
$\sum F_y = P_{1y} + P_{2y} + P_{3y} + P_{4y}$

Note: In the above case $\mathbf{P_{2x}}$, $\mathbf{P_{3x}}$, $\mathbf{P_{3y}}$ and $\mathbf{P_{4y}}$ have negative values

$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}}$, $\alpha = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$







ANALYTICAL METHOD

Now the resultant R of P_1 and P_2 is given by:

R=AC

$=\sqrt{AE^2+CE^2}$

$=\sqrt{(AB+BE)^2+CE^2}$

But AB= P_1

BE=BC$\cos\theta$= $P_2\cos\theta$

CE=BC$\sin\theta$= $P_2\sin\theta$

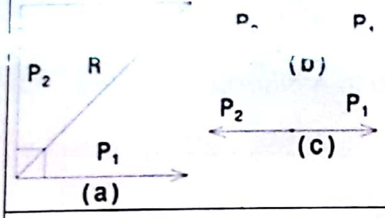
$\therefore\ R=\sqrt{(P_1+P_2\cos\theta)^2+(P_2\sin\theta)^2}$

$$=\sqrt{P_1^{\,2}+2P_1P_2\cos\theta+P_2^{\,2}Cos^2\theta+P_2^{\,2}sin^2\theta}$$

$$=\sqrt{P_1^{\,2}+2P_1P_2\cos\theta+P_2^{\,2}}$$

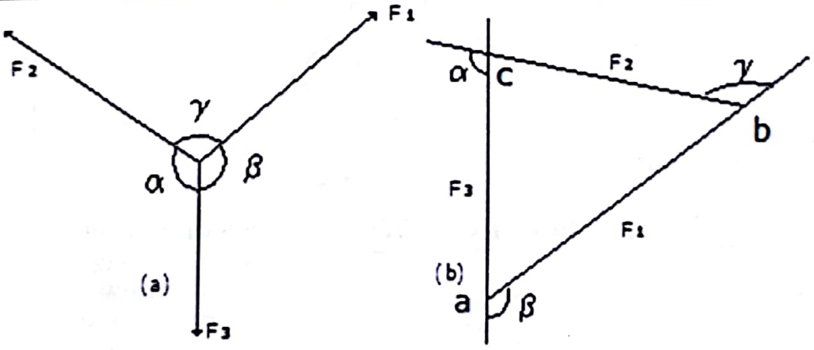
The inclination of the resultant to the direction of the force **P** is given by:

$$\tan\alpha=\frac{CE}{AE}=\frac{P_2\sin\theta}{P_1+P_2\cos\theta},\quad \alpha=\tan^{-1}\frac{P_2\sin\theta}{P_1+P_2\cos\theta}$$



Particular cases:

1. When $\theta=90^0$ [Fig.(a)] $R=\sqrt{P_1^{\,2}+P_2^{\,2}}$
2. When $\theta=0^0$ [Fig.(b)] $R=\sqrt{P_1^{\,2}+2P_1P_2+P_2^{\,2}}\ =\ P_1+P_2$
3. When $\theta=180^0$ [Fig.(c)] $R=\sqrt{P_1^{\,2}-2P_1P_2+P_2^{\,2}}\ =\ P_1-P_2$



LAMI'S THEOREM states that if a body is in equilibrium under the acting of only three forces, each force is proportional to the sine of the angle between the other two forces. Thus for the system of forces shown in figure.

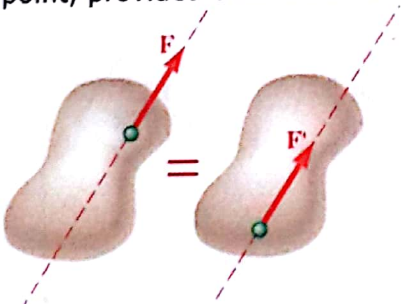
$$\frac{F_1}{\sin\alpha}=\frac{F_2}{\sin\beta}=\frac{F_3}{\sin\gamma}$$

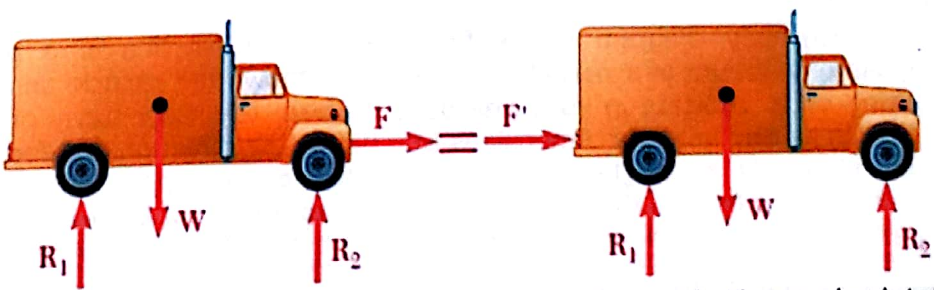
Applying sine rule for the triangle abc,

$$\frac{ab}{\sin(180^0-\alpha)}=\frac{bc}{\sin(180^0-\beta)}=\frac{ca}{\sin(180^0-\gamma)},$$

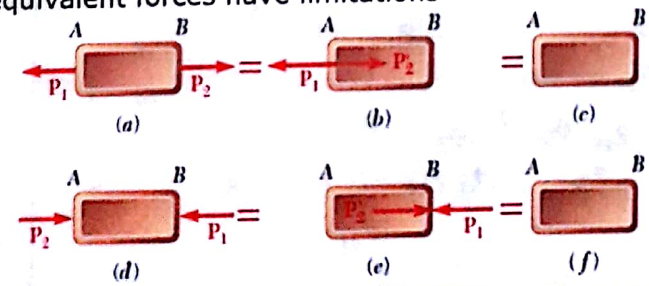
Note: when a body is in equilibrium under the action of only three forces, these three forces must be **concurrent.**

PRINCIPLE OF TRANSMISSIBILITY: The principle of transmissibility states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force **F** acting at a given point of the rigid body is replaced by a force **F**$'$ of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action





The principle of transmissibility and the concept of equivalent forces have limitations



The bar **(a)** is in tension and, if not absolutely rigid, will increase in length slightly; the bar of Fig **(d)** is in compression and, if not absolutely rigid, will decrease in length slightly. Thus, while the principle of transmissibility may be used freely to determine the conditions of motion or equilibrium of rigid bodies and to compute the external forces acting on these bodies, it should be avoided, or at least used with care, in determining internal forces and deformations.

Pruthviraj U, NITK Surathkal