# DS-GA 1008 HW2 - Part 1

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## 1 Convolutions

a. 3 x 3

b. $\frac{I+2P-K}{S}+1$

c. $C = \begin{pmatrix} 113 & 87 & 75 \\ 114 & 80 & 75 \\ 112 & 73 & 81 \end{pmatrix}$

d. Let $a_{i,j}$ denote the elements of the input image.
Let $b_{i,j}$ denote the elements of the filter of size 3 x 3.
Let $c_{i,j}$ denote the elements of the output.

$$c_{ij} = \sum_{n=1}^{3}\sum_{m=1}^{3} a_{i+m-1,j+m-1} b_{m,n}$$

$$\frac{\partial c_{pq}}{\partial a_{ij}} = \sum_{n=1}^{3}\sum_{m=1}^{3} b_{m,n} \frac{\partial a_{m+p-1,n+q-1}}{\partial a_{i,j}}$$

$$\frac{\partial a_{m+p-1,n+q-1}}{\partial a_{i,j}} = \begin{cases} 1 & \text{if} \quad m = i-p+1, n = j-q+1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial c_{p,q}}{\partial a_{i,j}} = b_{i-p+1,j-q+1}$$

$$
\begin{align}
\frac{\partial E}{\partial a_{ij}} &= \sum_{p=1}^{3}\sum_{q=1}^{3} \frac{\partial L}{\partial c_{pq}} \frac{\partial c_{pq}}{\partial a_{ij}} \tag{1} \\
&= \sum_{p=1}^{3}\sum_{q=1}^{3} 1 \cdot b_{i+1-p,j+1-q} \tag{2} \\
&= \sum_{u=1}^{3}\sum_{v=1}^{3} b_{(i+u-2)-1,(j+v-2)-1} \text{(By change of variable } u = 4-p, v = 4-q) \tag{3} \\
& \tag{4}
\end{align}
$$

Comparing with the convolution formula, we see that the backprop matrix for convolution input can itself be visualized as a convolution of the filter with a 0-padding of size 2 with a matrix of 1s.

$$
\frac{\partial E}{\partial A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{11} & b_{12} & b_{13} & 0 & 0 \\ 0 & 0 & b_{21} & b_{22} & b_{23} & 0 & 0 \\ 0 & 0 & b_{31} & b_{32} & b_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}
$$

# 2 Pooling

a. 2D Pooling Modules

- MaxPool2d
- AvgPool2d
- LPPool2d

b. Pooling Expressions

- MaxPool2d: $Y_{i,j}^k = \max\left\{X_{a,b}^k | (a,b) \in S_{i,j}^k\right\}$
- AvgPool2d: $Y_{i,j}^k = \frac{1}{|S_{i,j}^k|} \sum_{(a,b)\in S_{i,j}^k} X_{a,b}^k$
- LPPool2d: $Y_{i,j}^k = \left(\sum_{(a,b)\in S_{i,j}^k} \left(X_{a,b}^k\right)^p\right)^{1/p}$

c. $\begin{pmatrix} 114 & 87 \\ 114 & 81 \end{pmatrix}$

d. Let $L(X^k, S^k_{i,j}, p)$ denote the output of LP Pooling module with parameter p.

- MaxPool2d: $\lim_{p\to+\infty} L\left(X^k, X^k_{i,j}, p\right)$
- AvgPool2d: $\frac{1}{|S^k_{i,j}|} L\left(X^k, S^k_{ij}, 1\right)$