



## UNITS AND MEASUREMENT

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### 1.1 INTRODUCTION

Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called **unit**. The result of a measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit. Although the number of physical quantities appears to be very large, we need only a limited number of units for expressing all the physical quantities, since they are inter-related with one another. The units for the fundamental or base quantities are called **fundamental** or **base units**. The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called **derived units**. A complete set of these units, both the base units and derived units, is known as the **system of units**.

### 1.2 THE INTERNATIONAL SYSTEM OF UNITS

In earlier time scientists of different countries were using different systems of units for measurement. Three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.

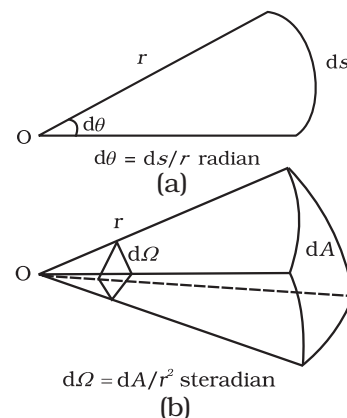
The base units for length, mass and time in these systems were as follows :

- In CGS system they were centimetre, gram and second respectively.
- In FPS system they were foot, pound and second respectively.
- In MKS system they were metre, kilogram and second respectively.

The system of units which is at present internationally accepted for measurement is the *Système Internationale d' Unites* (French for International System of Units), abbreviated as SI. The SI, with standard scheme of symbols, units and abbreviations, developed by the Bureau International des Poids et mesures (The International Bureau of Weights and Measures, BIPM) in 1971 were recently revised by the General Conference on Weights and Measures in November 2018. The scheme is now for

international usage in scientific, technical, industrial and commercial work. Because SI units used decimal system, conversions within the system are quite simple and convenient. We shall follow the SI units in this book.

In SI, there are seven base units as given in Table 1.1. Besides the seven base units, there are two more units that are defined for (a) plane angle  $d\theta$  as the ratio of length of arc  $ds$  to the radius  $r$  and (b) solid angle  $d\Omega$  as the ratio of the intercepted area  $dA$  of the spherical surface, described about the apex  $O$  as the centre, to the square of its radius  $r$ , as shown in Fig. 1.1(a) and (b) respectively. The unit for plane angle is radian with the symbol rad and the unit for the solid angle is steradian with the symbol sr. Both these are dimensionless quantities.



**Fig. 1.1** Description of (a) plane angle  $d\theta$  and (b) solid angle  $d\Omega$ .

**Table 1.1 SI Base Quantities and Units\***

Base quantity	SI Units		
	Name	Symbol	Definition
Length	metre	m	The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum $c$ to be 299792458 when expressed in the unit $\text{m s}^{-1}$ , where the second is defined in terms of the caesium frequency $\Delta\nu_{\text{Cs}}$ .
Mass	kilogram	kg	The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant $h$ to be $6.62607015 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\text{kg m}^2 \text{s}^{-1}$ , where the metre and the second are defined in terms of $c$ and $\Delta\nu_{\text{Cs}}$ .
Time	second	s	The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\text{Cs}}$ , the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to $\text{s}^{-1}$ .
Electric	ampere	A	The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge $e$ to be $1.602176634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\text{Cs}}$ .
Thermodynamic Temperature	kelvin	K	The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant $k$ to be $1.380649 \times 10^{-23}$ when expressed in the unit $\text{J K}^{-1}$ , which is equal to $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$ , where the kilogram, metre and second are defined in terms of $h$ , $c$ and $\Delta\nu_{\text{Cs}}$ .
Amount of substance	mole	mol	The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, $N_A$ , when expressed in the unit $\text{mol}^{-1}$ and is called the Avogadro number. The amount of substance, symbol $n$ , of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
Luminous intensity	candela	cd	The candela, symbol cd, is the SI unit of luminous intensity in given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}$ Hz, $K_{\text{cd}}$ , to be 683 when expressed in the unit $\text{lm W}^{-1}$ , which is equal to $\text{cd sr W}^{-1}$ , or $\text{cd sr kg}^{-1} \text{m}^{-2} \text{s}^3$ , where the kilogram, metre and second are defined in terms of $h$ , $c$ and $\Delta\nu_{\text{Cs}}$ .

\* The values mentioned here need not be remembered or asked in a test. They are given here only to indicate the extent of accuracy to which they are measured. With progress in technology, the measuring techniques get improved leading to measurements with greater precision. The definitions of base units are revised to keep up with this progress.

**Table 1.2** Some units retained for general use (Though outside SI)

Name	Symbol	Value in SI Unit
minute	min	60 s
hour	h	60 min = 3600 s
day	d	24 h = 86400 s
year	y	365.25 d = $3.156 \times 10^7$ s
degree	°	$1^\circ = (\pi/180)$ rad
litre	L	$1 \text{ dm}^3 = 10^{-3} \text{ m}^3$
tonne	t	$10^3 \text{ kg}$
carat	c	200 mg
bar	bar	$0.1 \text{ MPa} = 10^5 \text{ Pa}$
curie	Ci	$3.7 \times 10^{10} \text{ s}^{-1}$
roentgen	R	$2.58 \times 10^{-4} \text{ C/kg}$
quintal	q	100 kg
barn	b	$100 \text{ fm}^2 = 10^{-28} \text{ m}^2$
are	a	$1 \text{ dam}^2 = 10^2 \text{ m}^2$
hectare	ha	$1 \text{ hm}^2 = 10^4 \text{ m}^2$
standard atmospheric pressure	atm	$101325 \text{ Pa} = 1.013 \times 10^5 \text{ Pa}$

Note that when mole is used, the elementary entities must be specified. These entities may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.

We employ units for some physical quantities that can be derived from the seven base units (Appendix A 6). Some derived units in terms of the SI base units are given in (Appendix A 6.1). Some SI derived units are given special names (Appendix A 6.2) and some derived SI units make use of these units with special names and the seven base units (Appendix A 6.3). These are given in Appendix A 6.2 and A 6.3 for your ready reference. Other units retained for general use are given in Table 1.2.

Common SI prefixes and symbols for multiples and sub-multiples are given in Appendix A2. General guidelines for using symbols for physical quantities, chemical elements and nuclides are given in Appendix A7 and those for SI units and some other units are given in Appendix A8 for your guidance and ready reference.

### 1.3 SIGNIFICANT FIGURES

As discussed above, every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus

the first uncertain digit are known as **significant digits** or **significant figures**. If we say the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. Thus, the measured value has three significant figures. The length of an object reported after measurement to be 287.5 cm has four significant figures, the digits 2, 8, 7 are certain while the digit 5 is uncertain. Clearly, reporting the result of measurement that includes more digits than the significant digits is superfluous and also misleading since it would give a wrong idea about the precision of measurement.

The rules for determining the number of significant figures can be understood from the following examples. Significant figures indicate, as already mentioned, the precision of measurement which depends on the least count of the measuring instrument. **A choice of change of different units does not change the number of significant digits or figures in a measurement.** This important remark makes most of the following observations clear:

(1) For example, the length 2.308 cm has four significant figures. But in different units, the same value can be written as 0.02308 m or 23.08 mm or 23080  $\mu\text{m}$ .

All these numbers have the same number of significant figures (digits 2, 3, 0, 8), namely four.