Assignment 4

Comparison of Normal Distributions with Different Standard Deviations

1. Introduction

In probability theory and statistics, the Normal Distribution (also known as Gaussian Distribution) is one of the most fundamental continuous probability distributions. It plays a central role in statistical inference, hypothesis testing, quality control, economics, engineering, and data science.

A normal distribution is completely determined by two parameters:

- Mean (μ) — Measure of central tendency
- Standard Deviation (σ) — Measure of dispersion

In this assignment, we analyze and compare three normal distributions that share the same mean ($\mu = 55$) but differ in standard deviation ($\sigma = 4, 10, 15$). The objective is to determine which distribution is more stable, consistent, and statistically preferable under different conditions.

2. Given Data

The statistical parameters provided are:

- Mean (μ) = 55

Standard Deviations:

- Case A: $\sigma_1 = 4$
- Case B: $\sigma_2 = 10$
- Case C: $\sigma_3 = 15$

All three distributions are centered at 55. Therefore, the peak of all curves occurs at $x = 55$. However, the variability around the mean differs significantly.

3. Theoretical Background

3.1 Definition of Normal Distribution

A normal distribution is a continuous probability distribution defined by the Probability Density Function (PDF):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where:

- x = Random variable
- μ = Mean
- σ = Standard deviation
- π = 3.14159
- e = Euler’s constant

This function describes a symmetric bell-shaped curve.

3.2 Properties of Normal Distribution

1. The curve is symmetric about the mean.
2. Mean = Median = Mode.
3. Total area under the curve equals 1.
4. The curve is asymptotic to the x-axis.
5. Shape depends entirely on σ.

3.3 Mean (μ)

The mean represents the central value of the distribution.

In all three cases:

$$\mu = 55$$

Thus, the central tendency is identical across all distributions.

3.4 Standard Deviation (σ)

Standard deviation measures the average distance of observations from the mean.

$$\sigma = \sqrt{\frac{1}{N}\sum(x-\mu)^2}$$

It indicates how tightly or loosely the values cluster around the mean.

- Smaller σ → Less variability
- Larger σ → Greater variability

4. Analysis Using Empirical Rule (68–95–99.7 Rule)

The Empirical Rule states that in a normal distribution:

- 68% of values lie within ±1σ
- 95% lie within ±2σ
- 99.7% lie within ±3σ

4.1 Case A: σ = 4

±1σ Range

$$55 \pm 4 = 51 \text{ to } 59$$

±2σ Range

$$55 \pm 8 = 47 \text{ to } 63$$

±3σ Range

$$55 \pm 12 = 43 \text{ to } 67$$

Interpretation

- Very narrow interval.
- Data points are highly concentrated.
- Minimal deviation from mean.

- Highly predictable system.

This distribution represents maximum stability.

4.2 Case B: σ = 10

±1σ Range

$$55 \pm 10 = 45 \text{ to } 65$$

±2σ Range

$$55 \pm 20 = 35 \text{ to } 75$$

±3σ Range

$$55 \pm 30 = 25 \text{ to } 85$$

Interpretation

- Moderate spread.
- Balanced variability.
- Suitable for real-world situations where moderate variation exists.

4.3 Case C: σ = 15

±1σ Range

$$55 \pm 15 = 40 \text{ to } 70$$

±2σ Range

$$55 \pm 30 = 25 \text{ to } 85$$

±3σ Range

$$55 \pm 45 = 10 \text{ to } 100$$

Interpretation

- Very wide range.
- High dispersion.
- Less predictability.
- Higher uncertainty.

5. Graphical Interpretation

When plotted on a graph:

- $\sigma = 4$ produces a tall, narrow bell curve.
- $\sigma = 10$ produces a moderately wide curve.
- $\sigma = 15$ produces a flatter and broader curve.

As standard deviation increases:

- Peak height decreases.
- Spread increases.
- Probability density becomes more dispersed.

Although the mean remains constant, the shape of the curve changes significantly.

6. Statistical Comparison

Case	Mean (μ)	Standard Deviation (σ)	Dispersion	Predictability	Stability
A	55	4	Very Low	Very High	Very High
B	55	10	Moderate	Moderate	Moderate
C	55	15	High	Low	Low

7. Variance Comparison

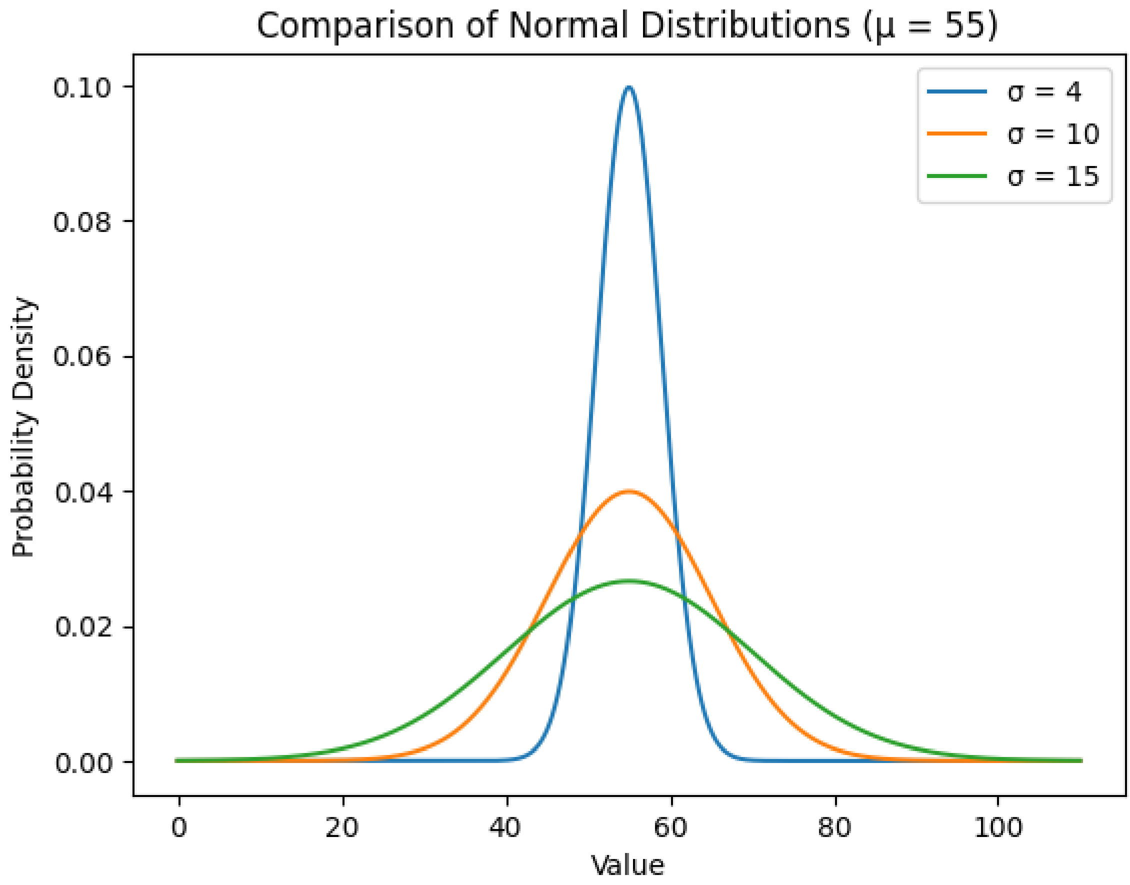
Variance is the square of standard deviation.

- Case A: $\sigma^2 = 16$
- Case B: $\sigma^2 = 100$
- Case C: $\sigma^2 = 225$

Higher variance indicates greater spread and instability.

8. Graphical Representation

Figure 1: Comparison of Normal Distributions with $\mu = 55$ and different σ values (4, 10, 15).



Interpretation of the Graph:

- The curve with $\sigma = 4$ is tall and narrow → highest stability.
- The curve with $\sigma = 10$ is moderately spread.
- The curve with $\sigma = 15$ is flat and wide → highest variability.
- As σ increases, spread increases and peak height decreases.

9. Practical Interpretation

If this distribution represents:

Example 1: Machine Output

Lower σ is preferred because consistency is important.

Example 2: Student Marks

Lower σ means students perform uniformly.

Example 3: Investment Returns

Higher σ means higher risk and uncertainty.

Thus, preference depends on context.

Which Distribution is Better?

If the objective is:

- High consistency
- Low risk
- Greater reliability
- Stable performance

Then:

$\sigma = 4$ is the best distribution

Because it has the smallest variability.

However, in real-world data, moderate variation ($\sigma = 10$) may be more realistic.

10. Conclusion

This study compares three normal distributions with identical mean ($\mu = 55$) but different standard deviations.

Key Findings:

- Standard deviation controls spread, not center.
- Smaller σ produces tighter clustering.

- Larger σ produces greater dispersion.
- Predictability decreases as σ increases.

Among the three distributions, $\sigma = 4$ represents the most stable and reliable distribution due to minimal variability.

Therefore, when stability and consistency are required, a smaller standard deviation is statistically preferable.