**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: Since work beings 10 mins after the car is dropped, the time left to complete work is 50 mins. Probability that Service Manager cannot meet his commitment =P(X>50)=1. Pr(x<=50) (X is the time taken to complete work). Convert 50 to z-score Standard normal variable Z=(X-\mu)/\sigma=(x-45)/8

P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)=0.73237=73.237 (the number in z-table)

Probability that service manager will not meet his commitment is: 100-73.237=26.763%=0.2676

So, the answer is B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and44.

Ans: Probability of employees >44=Pr(x>44)=1-Pr(x\_{n}=44) Z=(X-\mu)/\sigma=

(x-38)/6 Pr(x<=44)=Pr(z<=(44-38)/6)=Pr(Z<=1)=0.84134=84.134%

Probability that employees will be greater than 44=100-84.134 = 15.866

Probability of employess between 38 & 44=Pr(x<=44)-Pr(x>=38)

Here, Pr(x<=44)=0.84134

Pr(x>=38)=Pr(z>=(38\cdot38)/6)=Pr(z>=0)=0.5

Therefore, Pr(x<=44)-Pr(x>=38)=0.84134-0.5-0.3413434.134%

So, the statement "More employees at the processing center are older than 44 than between 38 and 44" is TRUE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Probability of employess less than 30=Pr(X<30) Z=(X-\mu)/\sigma=(30-38)/6 r(x<30)=Pr(2<(30-38)/6)=

Pr(z<-1.3333)=.09176=9.178

So, the number of employees with probability 0.0917 of them being under 30=400^{\*}0.0917=36.68=36

The statement of "training program for employees under the age of 30 at the center would be expected to attract about 36 employees" is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
2. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
3. 90.5, 105.9
4. 80.2, 119.8
5. 22, 78
6. 48.5, 151.5
7. 90.1, 109.9

Ans: The probability of getting value between a & b is 0.99

So, the probability of getting value outside a & b is 1-0.99 = 0.01

The probability towards left of a = -0,01/2=-0.05

The probability towards right of b = 0.01/2=0.05

Since we have probabilities of a & b, we need calulcate the probability of X - the random

variable at a & b which has these probabilities

By finding Standard Normal Variable (z), need to calculate X:

Z=(X-Mue)/Sigma

For a probability of 0.005, z values is -2.57

Ζ\* σ+μ=χ

-(-2.57) 20+100=151.4

(-2.57)\*20+100=48.6

Option D is the correct answer

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: To find the Rupee range that contains 95% probability for the annual profit of the company, we need to find the 2.5th and 97.5th percentiles of the total profit distribution.

Mean of Total Profit = Mean(Profit1) + Mean(Profit2) = 5 + 7 = 12

Variance of Total Profit = Variance(Profit1) + Variance(Profit2) = 32 + 42 = 74

Standard Deviation of Total Profit = sqrt(Variance of Total Profit) ≈ sqrt(74) ≈ 8.60

95% Confidence Interval = Mean ± 1.96 \* Standard Deviation

= 12 ± 1.96 \* 8.60

= 12 ± 16.86

≈ ( -4.86, 28.86 ) Million Rupees

Converting to Rupees using the exchange rate of $1 = Rs. 45:

≈ ( -218.7, 1298.7 ) Million Rupees

Therefore, the Rupee range (centered on the mean) that contains 95% probability for the annual profit of the company is approximately Rs. (-218.7, 1298.7) Million.

B. To find the 5th percentile of profit (in Rupees) for the company, we need to find the value below which 5% of the observations fall.

Using the z-score formula: z = (X - Mean) / Standard Deviation

For the 5th percentile, z = -1.645 (from standard normal distribution table)

X = Mean + z \* Standard Deviation

= 12 - 1.645 \* 8.60

= 12 - 14.16

≈ -2.16 Million

Converting to Rupees using the exchange rate of $1 = Rs. 45:

≈ -2.16 \* 45 ≈ -97.2 Million Rupees

Therefore, the 5th percentile of profit for the company is approximately Rs. -97.2 Million.

C. To determine which of the two divisions has a larger probability of making a loss in a given year, we need to calculate the probability of each division making a loss.

For Profit1:

P(Profit1 < 0) = P(Z < (0 - 5) / sqrt(32))

= P(Z < -1.5625)

≈ 0.0594 (from standard normal distribution table)

For Profit2:

P(Profit2 < 0) = P(Z < (0 - 7) / sqrt(42))

= P(Z < -1.6677)

≈ 0.0475 (from standard normal distribution table)

Therefore, Profit1 has a larger probability of making a loss in a given year (approximately 5.94% compared to 4.75% for Profit2).