Assignment -5

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Download all python codes from

```
https://github.com/K.NIKHITHA/tree/main/
    Assignment−5/Codes
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and latex-tikz codes from

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https://github.com/K.NIKHITHA/tree/main/
    Assignment6
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1 Question No. 2.69(a)

Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum $y^2 = 12x$.

2 solution

Given parabola is

$$y^2 = 12x \tag{2.0.1}$$

$$\implies y^2 - 12x = 0 \tag{2.0.2}$$

Vector form of given parabola is

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -6 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.3}$$

$\therefore$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, f = 0 \tag{2.0.4}$$

$\because |\mathbf{V}| = 0$ and $\lambda_1 = 0$ i.e. it is in standard form
$\therefore$

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\eta = \mathbf{u}^T \mathbf{p_1} = -6 \tag{2.0.6}$$

The vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} -12 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

The focal length β is given by

$$\beta = \frac{1}{4} \left| \frac{2\eta}{\lambda_2} \right| = \frac{1}{4} \left| \frac{-12}{1} \right| = 3 \tag{2.0.9}$$

The focus $\mathbf{F}$ is given by

$$\mathbf{F} = \mathbf{c} + \frac{-2\eta \begin{pmatrix} 1 & 0 \end{pmatrix}^T}{4} \tag{2.0.10}$$

$$\implies \mathbf{F} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{F} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Axis of parabola is given by

$$k(\mathbf{V}\mathbf{c} + \mathbf{u})^T \mathbf{x} = 0 \quad (k \in \mathbb{R}) \tag{2.0.13}$$

$$\implies k \begin{pmatrix} -6 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.14}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.15}$$

Directrix of parabola is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T (\mathbf{x} + \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{2.0.16}$$

$$\implies \begin{pmatrix} -6 & 0 \end{pmatrix} (\mathbf{x} + \mathbf{3}) = 0 \tag{2.0.17}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -3 \tag{2.0.18}$$

Latus rectum of parabola is given by

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T (\mathbf{x} - \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{2.0.19}$$

$$\implies \begin{pmatrix} -6 & 0 \end{pmatrix} (\mathbf{x} - \mathbf{3}) = 0 \tag{2.0.20}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 3 \tag{2.0.21}$$

Length of latus rectum l is

$$l = \left\| \beta (\mathbf{V}\mathbf{c} + \mathbf{u})^T \right\| \tag{2.0.22}$$

$$\implies l = \left\| 3 \begin{pmatrix} -6 & 0 \end{pmatrix} \right\| \tag{2.0.23}$$

$$\implies l = 18 \tag{2.0.24}$$

Plot of given parabola

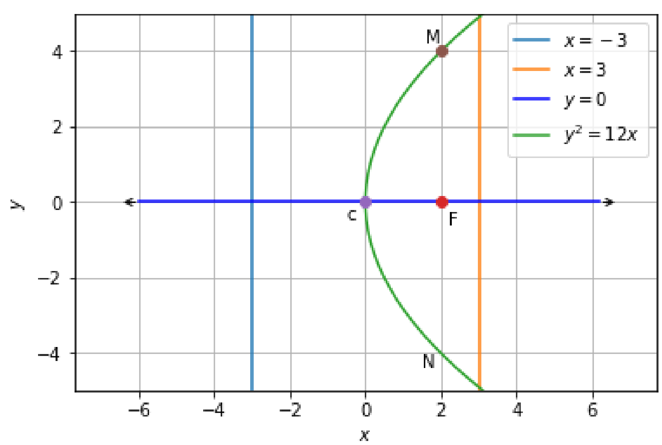

Fig. 2.1: Parabola $y^2 = 12x$