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Application of the State-Dependent Nonlinear Model Predictive Control In Adaptive Cruise Control System

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Abstract— **In this paper the Nonlinear Model Predictive Control (NMPC) is applied in adaptive cruise control (ACC) system. State-dependent algorithm, as an approach to control the brake and throttle opening position is proposed. Two linear time invariant (LTI) discrete-time state space models, corresponding to modes of operation: accelerating - the throttle is active and braking - the brake is active, have been extracted from the full non-linear model of the vehicle and the power train. Those models are used to design the NMPC controller. From this prospective, a single but state-dependent MPC can be utilized in controlling the throttle and brake position which provides an easy approach to the control design process. The design is implemented in simulation environment to test its performance. Finally, a comparison between the application of two control methods including state-dependent NMPC and Linear Quadratic Control (LQC) are presented.**

I. INTRODUCTION

Adaptive Cruise Control (ACC) is the extension of the Cruise Control (CC) system , it is able to vary the velocity of the vehicle depending on the behaviour of the other vehicles moving in front of the vehicle equipped with ACC by applying the brake and modulating the throttle to produce the necessary power. This system uses the radar or other sensory devices to measure the distance between vehicles [3]. The extended version of the ACC is so-called ACC stop&go. In the urban area and traffic jam situation where driver needs frequently to apply the accelerator and brake pedal the stop&go system makes the driving less tire-some. Unlike the ACC normally operates for higher speeds rang stop&go works in the speed range of below 30km/h and slows down to stand-still.

This paper proposes the utilization of the Non-linear Model Predictive Control (NLMPC) based on state-dependent linear like representation of a nonlinear vehicle model for designing an ACC system which aims at controlling the velocity of the vehicle at high speed range. The velocity control is implemented by regulating both the throttle opening and the brake pedal position. For this, a NLMPC single-handedly undertakes controlling the brake and throttle so as to track the reference velocity. The reference velocity is calculated using another PI controller such that the deviation of the actual distance headway from desired distance is reduced.

The controller designed in this work is implemented and tested in a Matlab/Simulink environment to control the nonlinear model developed in [4]. Some simulation results are provided, to show the performance of the proposed controller. Also the comparison with another suggested structure in design of the ACC is carried out in which the throttle and brake are controlled by two separate controller i.e. throttle is regulated by Linear Quadratic Control (LQC) and a PI controller is opted to control the brake. Also this structure is required a sophisticated logical algorithm in order to proceed the switching between two controllers.

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II. VEHICLE MODEL

In this paper the following equations are used for representing the dynamic model of the vehicle [4] for the control design purpose:

$$I_{ei}\dot{N}_e = T_e(u_t, N_e) - (\frac{N_e}{K_{tc}})^2 \tag{1}$$

$$m\dot{v} = \frac{1}{r}\underbrace{[R_{tr}R_f C_{tr}(\frac{N_e}{K_{tc}})^2 + T_b]}_{T_{wheel}} - \underbrace{\frac{1}{2}\rho A C_d v^2}_{F_{aerodynamic}} - \underbrace{C_r mg\cos(\theta)}_{F_{rolling-resistance}} \pm \underbrace{mg\sin(\theta)}_{F_{gravitational}} \tag{2}$$

$$T_b = K_b P = K_b(1.5K_c u_b - \tau_{bs}\dot{P}_b) \leq 0 \tag{3}$$

Equation (1) explains the mathematical relationship of the engine and impeller, with N_e the engine speed measured in rpm, I_{ei} the summation of engine and impeller moment of inertia and T_e the engine torque. The parameters playing an important role in the performance of a torque converter are expressed as follows: the speed ratio $C_{sr} = \frac{N_t}{N_i}$, the torque ratio $C_{tr} = \frac{T_t}{T_i}$, the efficiency $\eta_e = C_{sr} \times C_{tr}$ and the capacity factor (K-factor) $K_{tc} = \frac{N_e}{\sqrt{T_i}}$. The speed ratio can be expressed as a function of the vehicle speed as in (4):

$$C_{sr} = \frac{vR_f R_{tr}}{2\pi N_e} \tag{4}$$

Knowing the value of the speed ratio helps us find the other parameters of the torque converter by interpolating the graph which illustrates the performance characteristic of a torque converter [2]. To present the model of the engine in the simulation, a look up table is applied which defines the amount of engine torque vs. engine rotation speed (rpm) and the percentage of throttle opening position u_t. In the model of power train, implementation of the gear shift is done through the shift logic based on the thresholds calculated by the respective blocks for up-shift and down-shift.

Equation (2) is to calculate the velocity of the vehicle by considering the torque produced on the wheel through the power train T_w, the braking torque T_b, aerodynamic

TABLE I
VALUES OF THE PARAMETERS IN (1)-(4)

Parameter	Numerical value
m	1500kg
θ	0
R_{tr}	[1:2.34 ; 2: 1.45; 3: 1.00; 4: 0.68]
R_f	3.28
I_{ei}	0.00093 kgm^2
$\frac{1}{2}\rho.A.c_d$	0.49 kg/m
τ_{bs}	0.2
K_c	1
C_r	0.015
K_b	20 Nm/bar
g	9.8 m/s^2

force $F_{aerodynamic}$ and the last two terms in (2) defining the rolling resistance $F_{rolling-resistance}$ and the gravitational forces $F_{gravitational}$ respectively. Consequently, the velocity of the vehicle, v can be obtained by integrating the acceleration. Here $\rho = 1.225 kg/m^3$ is the air density, C_d the drag coefficient depending on the body shape, v velocity of vehicle and A is the maximum vehicle cross area, R_{tr} the gear ratio, R_f the final drive ratio, m the total mass of the vehicle, g the gravitational acceleration, and θ presents the road slope.

Finally, (3) calculates the braking torque as a function of the brake position u_b varying from 0 to -1. where P_b is the amount of pressure produced behind the brake disk, τ_{bs} is the lumped lag obtained as the combination of the two lags relating to the dynamic of the servo valve and the hydraulic system, K_c is pressure gain. K_b is the lumped gain for entire brake system. The values of the parameters are given in table I.

III. ACC SYSTEM

ACC operates in two different modes depending on the situation of the traffic ahead; cruise control (CC) mode and ACC control mode (follow mode). It operates in the CC mode when the road in front of the ACC equipped vehicle is clear, i.e. there is no vehicle within clearance distance. In this situation vehicle travels at the desired cruising speed (reference speed) specified by the driver. Once it has approached other vehicles travelling at lower speed it switch to the time gap control (ACC) mode. In this mode ACC attempts to keep the vehicle within the desired distance headway by controlling the speed of the vehicle (Fig. 1). The distance headway can be customized by the driver in term of time headway. In the ACC mode the new reference speed is introduced into the CC system according to the speed of the leader vehicle in order to maintain the vehicle within the desired distance headway. The transition between the modes is performed automatically depending on the traffic condition ahead and the desired cruising speed. If during the ACC mode the speed of the ACC equipped vehicle reaches the desired cruising speed, it will enter the CC mode regardless

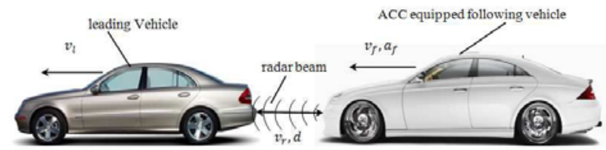

Fig. 1. An ACC equipped vehicle following another vehicle in front.

of the situation in front. In other words, if the desired cruising speed is less than the leader vehicle's speed, the system will switch to the CC mode, otherwise it keeps following the leader vehicle in the desired distance headway specified by driver [12], [11].

IV. CONTROLLER STRUCTURE

The ACC consists of two control loops (Fig. 2); First, the *inner-loop (low level) controller*, which is the typical cruise controller and trying to reduce the quantity of the error between actual velocity and reference velocity. Second, the *outer loop controller* adapting the functionality of CC according to the surrounding traffic ahead of the ACC equipped vehicle is to track the specified-driver distance headway (desired distance) from the vehicle ahead by introducing the new reference velocity into the inner-loop controller [12].

The desired distance headway, d_{des} can be computed using the following equation assuming that the follower and leader vehicles have the same speed $(v_f = v_l)$, which is known as Constant-Time Headway policy [1], [10]:

$$d_{des} = l + d_s + T_h v_f \tag{5}$$

where l is the vehicle length, d_s is the additional distance between two vehicle in order to avoid collision, v_f is vehicle velocity and T_h is constant-time headway which is specified by driver. Note that the time headway used here is approximated by the human reaction time i.e. 1.5-2 seconds [6]:

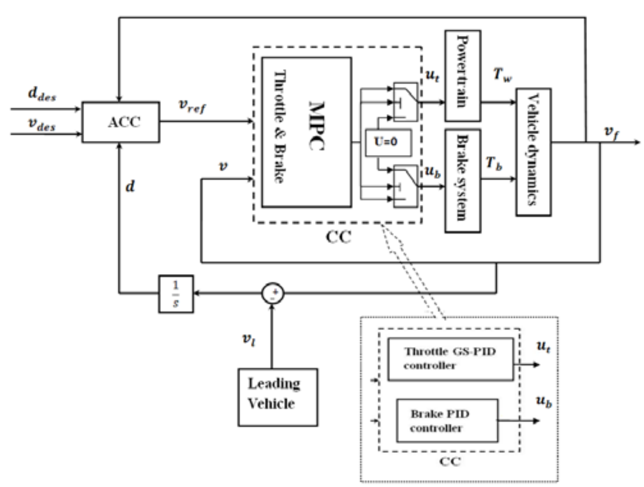

Fig. 2. Schematic block diagram of the ACC (inner loop & outer loop)- A single NMPC controls both brake and throttle in place of two separate control algorithms

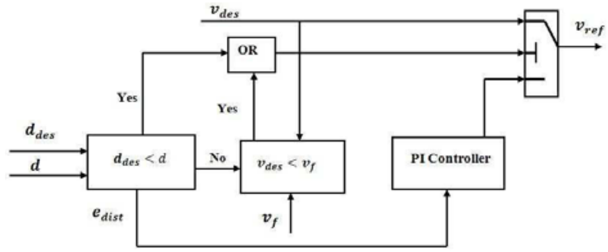

Fig. 3. An ACC structure with switching algorithm between cruising (desired) speed and the ACC mode.

In practice, the distance between vehicles is measured by radar. However, in this paper for carrying out the simulation test, the distance between the rear end of the follower and leader vehicle can be determined by taking the integral of their relative velocity:

$$d(t) = \int_0^t (v_f - v_l)dt \tag{6}$$

A. Inner loop controller

This level of the system undertakes to control the brake and throttle opening position. In order to conduct smooth and flawless distance and speed tracking, a single but state-dependent MPC is utilized in controlling the throttle and brake position. The switching between throttle and brake is processed depending on the controller calculated output. Therefore, in order to control each of these components - the throttle and the brake - a switching algorithm is devised in such a way that the controlling signal sent to either throttle or brake is set to zero while the other one becomes active. The constraint on the control signal is imposed inside the controller to restrict it between [-1,1]. This comes from the physical limitation on the throttle and brake system. Furthermore, the controller output is implemented in the switching logic which works as below:

$$\begin{cases} u_t = u, u_b = 0 & 0 \leq u \leq 1 \\ u_b = u, u_t = 0 & -1 \leq u < 0 \end{cases} \tag{7}$$

u_t and u_b are the throttle and brake controlling signal respectively. u is calculated by the MPC controller and assigns to either the throttle or the brake controlling signal depending on its value.

B. Outer loop controller

In order for the follower vehicle to track a leader vehicle within the desired distance, the speed of the vehicle needs to be controlled. Therefore, this level of controller, known as ACC, computes the new reference speed, $v_{ref}(t)$. This is implemented by use of a simple PI controller as follows:

$$v_{ref}(t) = K_p e_{dist}(t) + K_i \int_0^t e_{dist}(t)dt \tag{8}$$

where $v_{ref}(t)$ is the reference speed, K_p and K_i are proportional and integral gain of the PI controller, respectively. So, In order to avoid integrator windup which may be caused due to system saturation and result in hight overshoot in the system response, an anti windup with gain K_{AW} is required to be added in the feedback control loop. The values of K_p, K_i and K_{AW} are given in Table II. $e_{dist}(t)$ is the deviation of the actual distance headway from desired one:

$$e_{dist}(t) = d(t) - d_{des}(t) \tag{9}$$

As depicted in Fig. 3, the outer loop controller is able to switch between the ACC and CC modes as explained previously by validating various parameters including desired distance, actual distance, desired speed and velocity of the follower vehicle.

TABLE II
PI CONTROL INCLUDING ANTI WIND-UP IN DESIGN OF OUTER-LOOP CONTROLLER

Type of controller	K_i	K_p	K_{AW}
Outer loop	0.1	10	50

C. Non-Linear Model Predictive Control (NLMPC)

The State-Dependent State-Space (SDSS) representation is utilized to solve NMPC. In this way the Non-linear system is approximated with several state-dependent linear models at various operating point and used for solving the optimal control problem. To adopt this approach in design of the ACC system, in lieu of the models being functions of the states, they are correlated to the controlled input. Thus, Two linear models are found corresponding to operating of the brake and the throttle. So, the state-space model may vary depending on the value of the controlled inputs predicted over future horizon. The negative value of the controlled input at certain step of the future horizon ascertains the need to apply the brake and resulting in utilizing its associated state-space model in calculation of optimal control problem. The positive value identifies that the throttle needs to be taken in place.

The discrete time equation for non-linear system is assumed by following equation:

$$x_{k+1} = f_1(x_k) + f_2(x_k)u_k \tag{10}$$

To find the solution for above non-linear difference equation within optimal control context, it can be simply approximated by the following linear state-dependent state-space equation [7]:

$$x_{k+1} = A(x_k)x_k + B(x_k)u_k \tag{11}$$

*1) **NLMPC-Tracking problem:*** Having the equation of a linear system given as following state-space form:

$$\begin{aligned} x_{k+1} &= A_m x_k + B_m u_k \\ y_k &= C_m x_k \end{aligned} \tag{12}$$

with the state $x(k) \in^{n_x}$, the input $u(k) \in^{n_u}$ and the output $y(k) \in^{n_y}$. The linear state-space equation of a system can be extended by taking the integral action into consideration as following:

$$\underbrace{\begin{bmatrix} \Delta x_{k+1} \\ y_{k+1} \end{bmatrix}}_{x_{k+1}} = \underbrace{\begin{bmatrix} A_m & o_m^T \\ C_m A_m & I_{q\times q} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta x_k \\ y_k \end{bmatrix}}_{x_k} + \underbrace{\begin{bmatrix} B_m \\ B_m C_m \end{bmatrix}}_{B} \Delta u_k$$

$$y_k = \underbrace{\begin{bmatrix} o_m & I_{q\times q} \end{bmatrix}}_{C} \begin{bmatrix} \Delta x_k \\ y_k \end{bmatrix} \tag{13}$$

where $O_m = \overbrace{[0 \;\; 0...0]}^{n_x}$ and A, B, C are augmented matrices used in design of MPC. q counts the number of the output. Superscript T denotes matrix transpose. Δx_{k+1}, Δx_k and Δu_k are respectively the differences of the states and control variable and given as: $\Delta x_{k+1} = x_{k+1} - x_k$, $\Delta x_k = x_k - x_{k-1}$ and $\Delta u_k = u_k - u_{k-1}$.

To obtain an optimal control sequence, the following performance index must be minimized within a prediction horizon [9]:

$$J = (R_s - Fx_k)^T (R_s - Fx_k) - 2\Delta U^T \phi^T (R_s - Fx_k) + \Delta U^T (\phi^T \phi + \bar{R}) \Delta U \tag{14}$$

where $\Delta U \in^N$ is the control parameter vector and contains the optimal control elements over the future prediction horizon N. R_s contains the set-point information. $\bar{R}$ is a diagonal matrix and defined as below:

$$\bar{R} = r_w I_{N\times N} \tag{15}$$

where r_w is a positive value which is used for tunning so as to get the desirable performance of the system. The Solution to the above performance index is given as below:

$$\Delta U = (\phi^T \phi + \bar{R})^{-1} \phi^T (R_s - Fx_k) \tag{16}$$

In order to apply Model Predictive Control (MPC) control system to control the nonlinear system according to what developed for linear system [9], the non-linear equation need to be approximated with linear model in different operating point i.e. each operating point is dedicated with its own state, input and subsequently output. Thereby the augmented state-space matrices of each linear model (A,B) are the functions of the states calculated in their operating points. Therefore the non-linear equation of the system can be re-written in the state-dependent linear form as (11). So having the prediction of the future trajectory i.e. prediction of the future state and controlled input, and knowing the state-space linear model at various operating point the matrices F and ϕ can be calculated by (17) [8]. where

$$\left[\prod_{n=1}^{k} A_n\right] = \begin{cases} A_k A_{k-1} \cdots A_l & if\ l \le k \\ I & if\ l > k \end{cases} \tag{18}$$

*2) **Constraint on the controlled input:*** In the vehicle model, the throttle opening position u_t is varied from 0 to 1 and subsequently the brake pedal position u_b is restricted between 0 and -1. Therefore, the constraint needs to be imposed on the controlled input calculated by NMPC as -1 $\le u \le$ 1. In order to incorporate the constraint in the implementation of Model Predictive Control (MPC) the Hildreth's Quadratic Programming is utilized [9]. In our control design, the optimization problem is solved by taking into account the constraint on the amplitude of the control variable.

V. TESTING SCENARIO

The model developed in this paper will be tested using the following scenarios:

Velocity tracking mode: In this mode, depending on desired speed chosen by the driver, the controller endeavours to follow the reference speed. This scenario is defined to evaluate the performance of the controller under velocity tracking condition, as it is illustrated in Fig. 5. In this test, the reference speed is varying in an incremental order.

Distance tracking mode: This scenario is defined to examine the ACC performance in tracking the target vehicle within a safe distance, and to realize whether the controller is able to meet the requirement of the ACC system which is to control the velocity of the ACC vehicle to precisely track the desired distance headway (Fig. 6).

VI. SIMULATION

As described in the previous sections, in this work the ACC structure was built up to control the distance and speed of the vehicle. Two level of controller were introduced to fulfill this task; inner loop control loop implements the brake and throttle control. State-dependent NMPC was utilized to control these component by calculating the optimal control trajectory in future horizon. In order to design a NMPC based on state-dependent state-space model, two linear models corresponding to the operating of the brake and throttle were obtained as:

$$\begin{cases} x_{k+1} &= A_{mt} x_k + B_{mt} u_{tk} \qquad (I) \\ x_{k+1} &= A_{mb} x_k + B_{mb} u_{bk} \qquad (II) \end{cases} \tag{19}$$

the state-space linear model (I) is approximated when the throttle is only active, while model (II) is obtained for operating of the brake. u_t and u_b are the throttle and brake controlled input respectively. The subscript t and b denot that the state-space coefficients of each linear models $(A_{mt}, B_{mt}, A_{mb}, B_{mb})$ are correlated to the throttle and brake operating point respectively and their values are given as following matrices:

$$A_{mt} = \begin{bmatrix} 0.9512 & 0 & 0 \\ -0.0076 & 0.9986 & 0.0000 \\ -0.0144 & 3.6983 & 0.9511 \end{bmatrix} \quad B_{mt} = \begin{bmatrix} 0 \\ 0.0015 \\ 289.1885 \end{bmatrix}$$

$$A_{mb} = \begin{bmatrix} 0.9512 & 0 & 0 \\ -0.1530 & 1.000 & 0 \\ 0 & 0 & 0.9511 \end{bmatrix} \quad B_{mb} = \begin{bmatrix} -1.4631 \\ 0.1157 \\ 0 \end{bmatrix}$$

$$F_{k,N} = \begin{bmatrix} C\left[\prod_{n=1}^{1} A_{k+n}\right] \\ C\left[\prod_{n=1}^{2} A_{k+n}\right] \\ \vdots \\ C\left[\prod_{n=1}^{N} A_{k+n}\right] \end{bmatrix} \qquad \phi_{k,N} = \begin{bmatrix} C\left[\prod_{n=1}^{0} A_{k+n}\right] B_k & 0 & \cdots & 0 \\ C\left[\prod_{n=1}^{1} A_{k+n}\right] B_k & C\left[\prod_{n=2}^{1} A_{k+n}\right] B_{k+1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ C\left[\prod_{n=1}^{N-1} A_{k+n}\right] B_k & C\left[\prod_{n=2}^{N-1} A_{k+n}\right] B_{k+1} & \cdots & C\left[\prod_{n=N}^{N-1} A_{k+n}\right] B_{k+N-1} \end{bmatrix} \tag{17}$$

The states of the system are brake pressure, vehicle velocity and engine angular speed (rpm) as $x_k = [P \ \ v \ \ N]^T$. The system output y_k is obtained by having $C_m = [0 \ \ 1 \ \ 0]^T$. The corresponding augmented matrices for $(A_{mt}, B_{mt}, A_{mb}, B_{mb})$ are indicated with (A_t, B_t, A_b, B_b), i.e. the augmented matrices can be calculated using (13).

Firstly, assume that the vector of the future controls, i.e. $\Delta U_k(N) = [\Delta u(k+1) \quad \Delta u(k+2) \quad \ldots \quad \Delta u(k+N)]$, is known. Note that such vector is calculated as a part of MPC procedure. Having $\Delta U_k(N)$ calculated within the control horizon N, the vector containing the absolute values of the control signals can be calculated as following vector:

$$U_k(N) = [\underbrace{u(k) + \Delta u(k+1)}_{u(k+1)} \quad \underbrace{u(k+1) + \Delta u(k+2)}_{u(k+2)} \\ \ldots \underbrace{u(k+N-1) + \Delta u(k+N)}_{u(k+N)}] \tag{20}$$

In order to calculate matrices F_k and ϕ_k through (17), switching between linear models, i.e. the models have been approximated based on modes of operation-brake and throttle, should be implemented within prediction horizon ($n = 1, 2, \ldots, N$) as below:

$$(A(n), B(n)) = \begin{cases} A_t, B_t & 0 \leq u_n \leq 1 \\ A_b, B_b & -1 \leq u_n < 0 \end{cases} \tag{21}$$

The Schematic block diagram of NLMPC illustrating the steps for calculation of the controlled input is depicted in Fig. 4. In the simulation the values of r_w and N have been chosen as 10 and 30 respectively. Also, the set-point vector R_s is given as following matrix which introduces the constant reference speed v_{ref} within prediction horizon :

$$R_s = v_{ref} I_{N \times 1} \tag{22}$$

A different structure in design of the inner loop control is based on applying two separate controllers for brake and throttle [5]. In this method, the Linear Quadratic control is designed for regulating throttle while the brake is controlled by a conventional PI controller (Fig. 2). The state-space linear model utilized for designing of LQC is the same as the one used for NMPC i.e. the model is associated with throttle operating point.

The outer loop controller was developed to guarantee the well tracking the desired distance by introducing the new reference speed which is computed by use of the PI controller. Also this level customizes the ACC system by providing the automated switching between the CC and the ACC application.

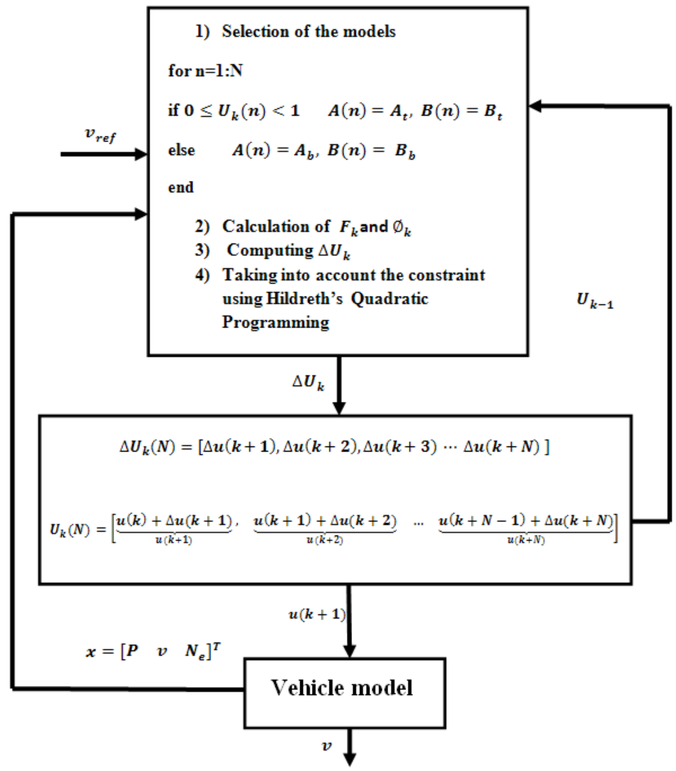

Fig. 4. Schematic diagram of NMPC structure illustrating the steps for calculation of the controlled input sequence $U_k(N)$ over the future horizon at each instance of time k. The first element of the sequence $u(k+1)$ is sent to the vehicle model.

Once the controller design task is accomplished, the system was tested against a various traffic scenarios. Fig. 5-a demonstrates the tracking of the varying cruise control speed while the system operates in CC mode. The aim of this scenario is to examine how accurately the output of the system i.e. vehicle velocity, is able to track the reference speed in CC mode. The response shows stable performance of the NMPC controller, i.e. no over shoot, no steady state error, suitable rising time and settling time. According to Fig. 5 the comparison carried out between two control structures in design of inner loop controller illustrates the better performance of NMPC. Furthermore, NMPC provides more desirable behavior in regulating the throttle position versus LQC (Fig. 5-b).

Fig. 6 shows the response of the system in distance tracking mode. The test was implemented assuming the initial condition at which the starting distance between the vehicles is $150m$ and also the initial travelling speed of the

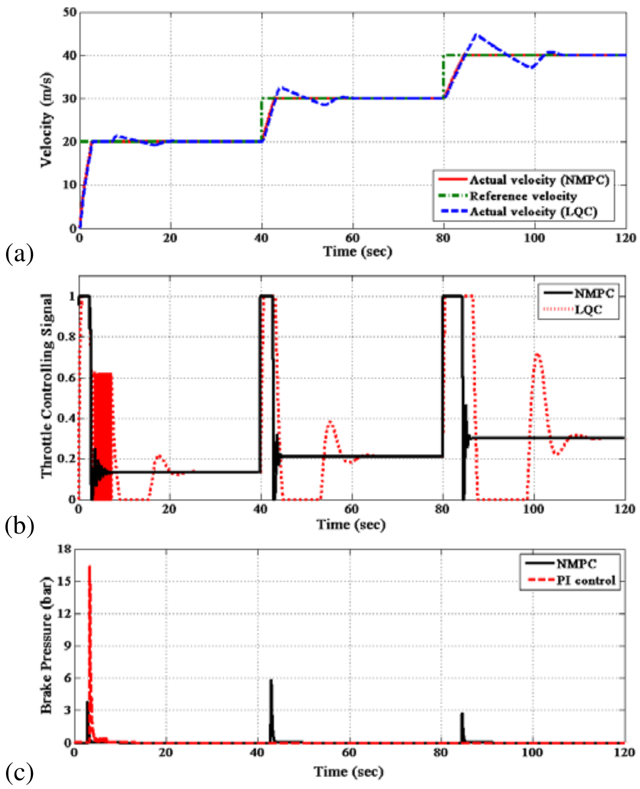

Fig. 5. Performance of the CC system during step change of the desired speed-The result of two control structures in design of inner loop controller are compared, (b) The controlled input (throttle opening position) into the system obtained by the NMPC and the LQC, (c) The brake pressure produced by use of the NMPC and the PI controller.

ACC equipped vehicle (following vehicle) set at the $60m/s$. As the result shows, after the distance headway between the vehicles being reduced, the following vehicle traces the leading vehicle by maintaining the desired distance which in turn results in both vehicles travelling at the same velocity (Fig. 6-b). The desired distance varies in a constant multiple of the following vehicle's velocity, which this constant term is so-called time headway. This value in our simulation was set at $1.5s$. Also the results of two control structures were compared for ACC mode as depicted in Fig. 6.

VII. Conclusion

In this paper state-dependent NMPC method was proposed in application of ACC system. In This approach the need of designing two separate controllers in order to regulate the brake and throttle opening position can be eliminated and both systems are controlled by a single state-dependent NMPC. An outer control loop is also presented which computes the velocity set-point by taking into account the distance to a vehicle in front and its speed. The LTI state-space models corresponding to brake and throttle operating points were extracted from nonlinear model. The test results show appropriate performance of the ACC simulation model against the described scenarios. Also, the comparison results prove better performance of proposed method over LQ controller. In this work the future reference trajectory in calculation of performance index has been assumed to be constant. However, the future reference velocity, calculated by outer control loop, is related to the predicted distance between the vehicles in future horizon and may be varied in each step of time. Therefore, In the future work the relevant algorithm to predict the future reference trajectory will be incorporated into analysis of NMPC which is expected to improve the performance of the ACC simulation.

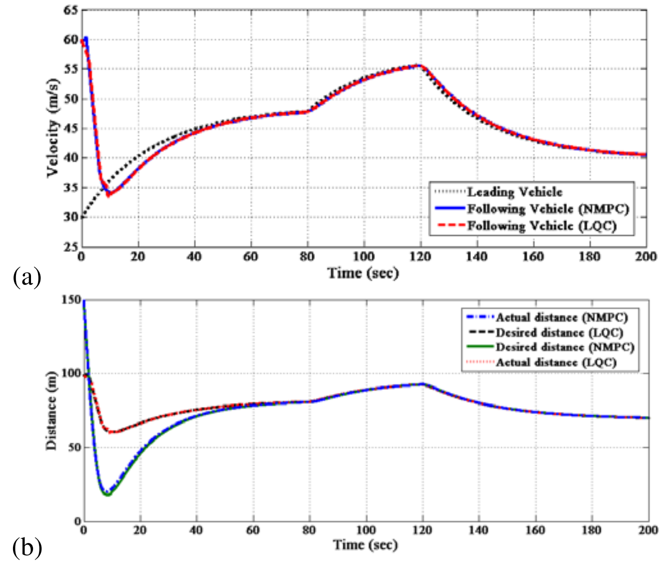

Fig. 6. (a) Velocity of the ACC equipped vehicle which is adapted in such a way to achieve desired headway distance, (b) Adapting the distance headway between follower and target vehicle.

References

[1] W. Zhou and S. Zhang, "Analysis of Distant Headways", Proc. of the Eastern Asia Society for Transportation Studies, 2003, vol. 4.

[2] J. Y. Wang, "Theory of Ground Vehicle", Third Ed., Wiley Inter Science, 2001.

[3] H. Winner, K. Winter and B. Lucas and et. al., "ACC Adaptive Cruise Control", The Bosch Yellow Jacket, 2003.

[4] P. Shakouri and A. Ordys and M. Askari and D. S. Laila, "Longitudinal vehicle dynamics using Simulink/Matlab", UKACC International conference on CONTROL, Coventry, 2010, Sep 6-10.

[5] P. Shakouri, A. Ordys and D. S. Laila, " Adaptive Cruise Control System: Comparing Gain-Scheduling PI and LQ Controllers", 18th IFAC World Congress, August 28- September 2, Milano, Italy, 2011

[6] J. J. Martinez and C. Canudas-de-Wit, "A Safe Longitudinal Control for Adaptive Cruise Control and Stop-and-Go Scenarios", IEEE Transactions on Control Systems Technology, vol. 15, 2007

[7] A. S. Dutka, A. W. Ordys and M. J. Grimble, "Optimized Disceret-Time State Dependent Riccati Equation Regulator", American Control Conference. June 8-10, 2005, Portland, OR, USA

[8] A. M. Youssef, A. W. Ordys, and M. J. Grimble, "Nonlinear Predictive Control for Fast Constrained Systems", IEEE Conference on Methods and Models in Automation and Robotics, August, Miedzyzdroje, Poland, 2004

[9] W. Liuping, "Model Predictive Control System Design and Implementation Using Matlab", Springer, 2nd Printing, 2009, pp. 1-100.

[10] J. Zhou and H. Peng, "Range Policy of Adaptive Cruise Control Vehicles for Improved Flow Stability and String Stability", IEEE Transaction on Intelligent Transportation System, T-ITS-04-03-0035.R2, 2003,

[11] J.Jonsson, "Fuel Optimized Predictive Following in Low Speed Condition, Master's thesis", Linkopings Univ., 2003.

[12] P. Riis, "Adaptive Cruise Controller Simulation as an Embedded Distributed System", MSc thesis, Linkoping Univ., 2007.