NC STATE UNIVERSITY

ECE 726 – Advanced Feedback Control

Fall 2015

Model Predictive Control of Transitional Maneuvers for Adaptive Cruise Control Vehicles

Project Report

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1. Introduction

This project deals with the use of Model Predictive Control for computing the spacing-control laws for Transitional Maneuvers(TM) of vehicles with Adaptive Cruise Control (ACC) as proposed by Bageshwar *et al*. [1]. This is achieved by formulation of a TM as an Optimal Control Problem (OCP). The constraints used for the optimal control problem are the steady state following distance, collision avoidance and acceleration limits of the ACC vehicle. The spacing-control laws are then obtained by implementing a receding-horizon approach and the system performance is simulated for different scenarios.

2. Problem Definition

The control system of an ACC vehicle system is commonly modeled by a two-level control architecture which consists of an upper level and a lower level controller. The upper-level controller computes the acceleration commands for performing the TM, which are sent to the lower-level controller. The lower-level controller then uses the throttle and braking commands to track the spacing-control laws.

This project deals with use of Model Predictive Control (MPC) for designing of the upper-level controller. The following first-order ACC model is used to design the spacing-control laws -

$$\tau \frac{d}{dt}\ddot{x}(t) + \ddot{x}(t) = u(t)$$

$$\dot{x}(t) \geq 0$$

$$u_{min} \leq u \leq u_{max}$$

Where,

u (t) - acceleration commands computed by upper-level controller,

x,$\dot{x}$, $\ddot{x}$ - absolute position, velocity and acceleration of the ACC vehicle,

τ - Time-lag corresponding to bandwidth of lower-level controller.

Discrete time state-space representation of ACC vehicle model is given by:

$$\begin{pmatrix} x(t+T) \\ \dot{x}(t+T) \\ \ddot{x}(t+T) \end{pmatrix} = \begin{pmatrix} 1 & T & 0 \\ 0 & 1 & T \\ 0 & 0 & 1-\dfrac{T}{\tau} \end{pmatrix} \begin{pmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{T}{\tau} \end{pmatrix} u(t)$$

Where, T- sampling time of the system.

3. MPC Algorithm Synthesis

3.1 Coordinate frame for TMs

A coordinate frame having velocity of target is defined. In this frame the state variables of ACC vehicle are defined relative to Specified Inter-Vehicle Distance (SIVD), which is the origin of the frame. The objective of TM is then changed so as to maneuver ACC vehicle to the origin of this frame. The range rate should be zero when the vehicle reaches the origin.

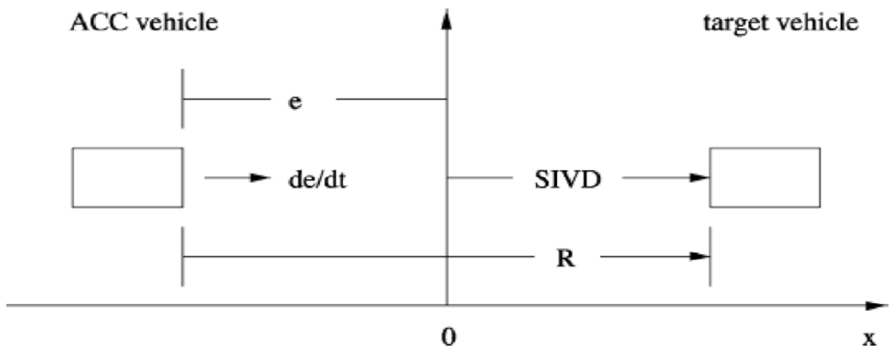

Fig. 1: Coordinate frame for TMs

ACC vehicle model using the coordinate frame is given by:

$$e_{k+1} = \phi e_k + \Gamma u_k$$

$$y_k = \Lambda e_k$$

Where, ϕ and Γ are as defined in the discrete time model.

$$\begin{pmatrix} e_k \\ \dot{e}_k \\ \ddot{e}_k \end{pmatrix} = \begin{pmatrix} -(R - SIVD) \\ \dot{R} \\ \ddot{x}_k \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

Where,

R - Range (relative distance between ACC and target vehicle),

$\dot{R}$ – Range Rate

3.2 Feasible Initial Conditions

Initial conditions refer to the values of R, $\dot{R}$ and ACC vehicle velocity and acceleration when target is first encountered. A scenario is considered feasible if the ACC vehicle can achieve the target vehicle velocity by decelerating from its initial velocity while avoiding collision with the target vehicle.

In order to find the feasible initial conditions, we need to solve the system equations to determine the minimum distance required by the vehicle to stop for any given initial condition.

The solution for the state equations of a system can be written as:

$$x(t) = \begin{pmatrix} x(t) \\ \dot{R}(t) \\ \ddot{x}(t) \end{pmatrix}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\eta)}Bu(\eta)d\eta$$

Where, 0 and t are set as the initial and final times of maximum deceleration maneuver respectively. Since both vehicles are moving with constant initial velocity, considering target vehicle to be moving with constant velocity v_0, the initial conditions can be written as –

$$x(t) = \begin{pmatrix} 0 \\ \dot{x}(0) - v_0 \\ 0 \end{pmatrix}$$

Given e^{At} and x(0) for the maximum deceleration maneuver, the solution of system of equations can be written as -

$$x(t) = (\dot{x}(t) - v_o)t + \left(\frac{1}{4} - \frac{t}{2} + \frac{t^2}{2} - \frac{1}{4}e^{-2t}\right)u_{min}$$

$$\dot{R}(t) = \dot{x}(0) - v_o + (-\frac{1}{2} + t + \frac{1}{2}e^{-2t})u_{min}$$

$$\ddot{x}(t) = (1 - e^{-2t})u_{min}$$

The SIVD is calculated by finding the value of x(t) and t at which the range rate becomes zero. If the initial range is greater than the SIVD obtained, the initial condition is said to be feasible.

3.3 Formulation of OCP

The control objectives and other requirements of the control problem are formulated as a Fixed Time Constrained OCP(FTCOCP) as follows. With an initial error of e(0), a control sequence is determined that minimizes the performance index

$$J = e_N^T S e_N + \sum_{k=0}^{N-1} \{ e_k^T Q e_k + u_k^T R\, u_k \}$$

subject to

$$e_{k+1} = \phi e_k + \Gamma u_k$$

$$e(0) \equiv \text{measured}$$

with the state and control constraints which formulate the collision avoidance, non-negative velocity constraint and acceleration limits of the ACC vehicle respectively

$$y_k = \begin{pmatrix} e_k \\ -\dot{e}_k \end{pmatrix} \leq \begin{pmatrix} SIVD \\ v_o \end{pmatrix} = S_c$$

$$u_{min} \leq u_k \leq u_{max}$$

and the final state constraint which formulates the establishment of SIVD with zero range-rate

$$e_N = \psi = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3.4 Formulation of MP

A three step procedure is used to reformulate the FTCOCP as a MP -

1. Predicted error e_k is expressed as a function of e(0) and the control sequence.
   $e_k = \Phi^k e(0) + \sum_{l=0}^{k-1} \phi^l \Gamma\, u_{k-l-1}$, where k=1,2,.....,N.
   The above equation can be expressed as $E = \bar{A}e(0) + \bar{B}U$. – **I**
2. The J (performance index), state constraints and control constraints are written in a matrix form similar to '**I**'.
3. Expression for e_k is directly substituted into J and the three constraint equations of FTCOCP.

The performance index in matrix form is expressed as:

$$\begin{aligned} J = {} & e_N^T S e_N + e_o^T Q e_N + \cdots + \cdots e_{N-1}^T Q e_0 + u_0^T R u_0 + \cdots + u_{N-1}^t R u_{N-1} \\ = {} & e_0^T Q e_0 + E^T \bar{Q} E + U^T \bar{R} U \quad \text{- } \mathbf{II} \end{aligned}$$

Where,

$$\bar{Q} = \begin{pmatrix} Q & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & Q & 0 \\ 0 & \dots & 0 & S \end{pmatrix}$$

$$\bar{R} = \begin{pmatrix} R & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & R & 0 \\ 0 & \dots & 0 & R \end{pmatrix}$$

By substituting **I** into **II**, we have,

$$\begin{aligned} J\big(U, e(0)\big) = {} & e_o^T Q e_0 + \{\bar{A}e(0) + \bar{B}U\}^T \bar{Q} \{\bar{A}e(0) + \bar{B}U\} \\ = {} & e_o^T \{Q + \bar{A}^T \bar{Q} \bar{A}\} e_0 + U^T H U + 2GU \end{aligned}$$

Where,

$$H = \bar{R} + \bar{B}^T \bar{Q} \bar{B}$$

$$G = e(0)^T \bar{A}^T \bar{Q} \bar{B}$$

Combining the above performance index with the state, control and terminal constraints, the fixed time constrained OCP can be written as:

$$\min_U \; U^T H U + 2G\big(e(0)\big) U$$

Subject to -

$L_{IN}U \leq M_{IN}(e(0))$ - obtained from state and control constraints

$L_{EQ}U = M_{EQ}\big(e(0)\big)$ - obtained from terminal state constraints.

Where,

$$L_{IN} = \begin{pmatrix} \bar{C}\,\bar{B} \\ I \\ -I \end{pmatrix}$$

$$M_{IN} = \begin{pmatrix} \bar{S}_c - \bar{C}\bar{A}e(0) \\ U_{max} \\ -U_{min} \end{pmatrix}$$

$$\bar{S}_c = \begin{pmatrix} S_c \\ \vdots \\ S_c \end{pmatrix}$$

And,

$$L_{EQ} = (\ \phi^{N-1}\Gamma\ \ \phi^{N-2}\Gamma \ldots \Gamma\ )$$

$$M_{EQ} = \ \psi - \phi^{N} e(0)$$

3.5 Receding-Horizon Control

The state feedback spacing-control law is computed using the receding-horizon approach. The range, range-rate, ACC vehicle velocity and accelerations are measured at each sampling time t and the SIVD is updated and then the error vector is computed. Using the error vector as initial condition the QP is solved for the entire time horizon of the FTCOCP:

$$\min_{U}\ U^{T}HU + 2G\big(e(t)\big)U$$

Subject to

$$L_{IN}U \leq M_{IN}(e(t))$$

$$L_{EQ}U = M_{EQ}\big(e(t)\big)$$

to obtain an open-loop control sequence U_{tol}.

4. Simulation of Spacing-Control Laws

The objective of the simulations is to evaluate the performance of spacing-control laws for different scenarios. The spacing-control laws are computed using MPC. A spacing-control law will be considered a success if it maneuvers the ACC vehicle to the SIVD with zero range-rate while avoiding collision with the target vehicle and satisfying other specified constraints. MATLAB is used for computing spacing-control laws and for simulation of ACC vehicle response to the spacing-control laws. The SIVD to be maintained by the steady-state spacing policy varies linearly with velocity of target vehicle.

4.1 Baseline Scenario

The ACC vehicle is assumed to be moving at constant speed before encountering the target vehicle with a speed of 30m/s. The first encounter range is set to be 110m. The vehicle has to stop completely behind the target vehicle which is assumed to be at rest and hence the SIVD to be achieved by the ACC vehicle is 0 m. The TM frame has zero velocity and target vehicle is at its origin. The initial error vector is defined as

$$e(0) = (-110\text{m}, 30\text{m/s}, 0)$$

This represents a fairly aggressive stopping scenario. The minimum feasibly SIVD for an initial range rate of 30 m/s is approximately 106 m. Thus the initial condition is barely feasible and hence a good candidate for evaluating the performance of the system.

4.2 Simulation of MPC spacing-control law

For all the simulation plots the 'X' mark is used to depict collision and dashed lines after the 'X' are used to represent the spacing-control laws after the collision. The sampling time T is chosen to be 0.1s and Q, R &S matrices are chosen to be identity matrices with dimension 3x3, 1x1 & 3x3 respectively, unless otherwise specified.

4.2.1 Unconstrained and Partially Constrained MPC

Fig.2 shows the response of ACC vehicle to spacing-control laws without including the collision avoidance and acceleration limits of the ACC vehicle in formulation of FTCOCP. Thus the formed QP includes the equality constraints only. The solid black line represents the spacing control law and response of ACC vehicle when no deceleration limits are applied. The blue line represents the spacing-control law and the response of the vehicle for vehicle with deceleration limits applied by checking for saturation. The red line represents the spacing-control law and the response of ACC vehicle for a vehicle with deceleration limits included in the formulation of FTCOCP. It is seen that in case of unlimited deceleration, the ACC vehicle successfully performs the TM but in case of saturated deceleration it is observed that the ACC vehicle fails in avoiding collision completely. And even for constrained deceleration, the range becomes marginally greater than 0, indicating a collision. However this marginal collision is not visible on the plot. Hence it can be concluded that the conditions of collision avoidance need to be explicitly included while formulating the FTCOCP.

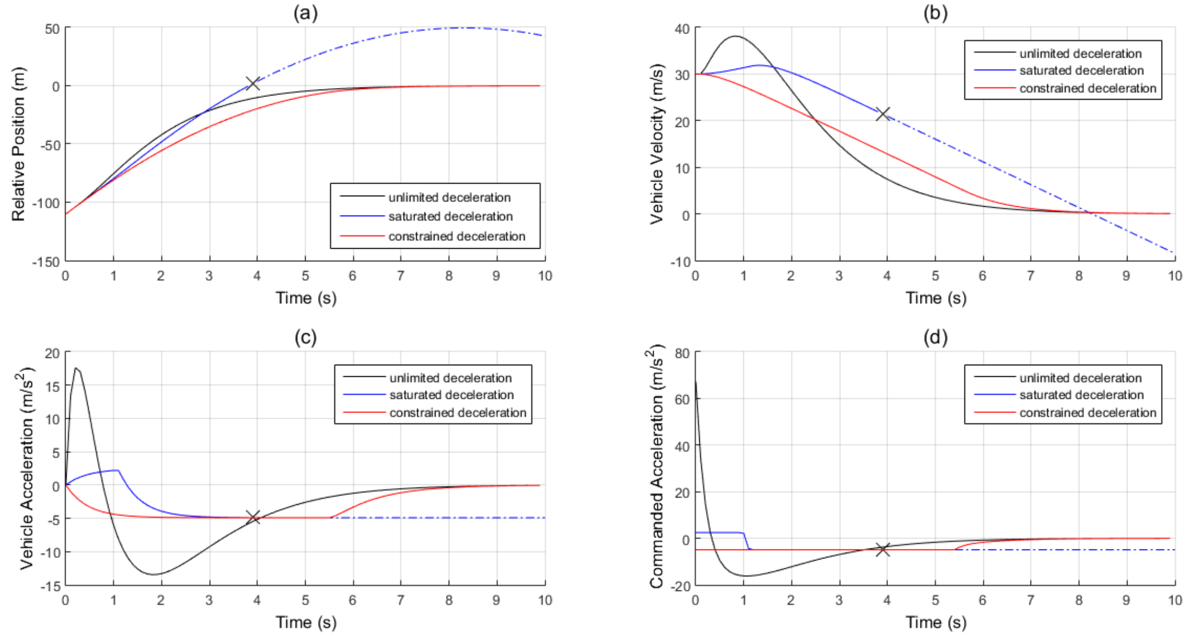

Fig. 2: ACC vehicle response without collision avoidance constraints in formulation of FTCOCP (a) Relative vehicle position, (b)Vehicle velocity, (c)Vehicle Acceleration and (d)Commanded Acceleration

4.2.2 Constant and Dynamic SIVD

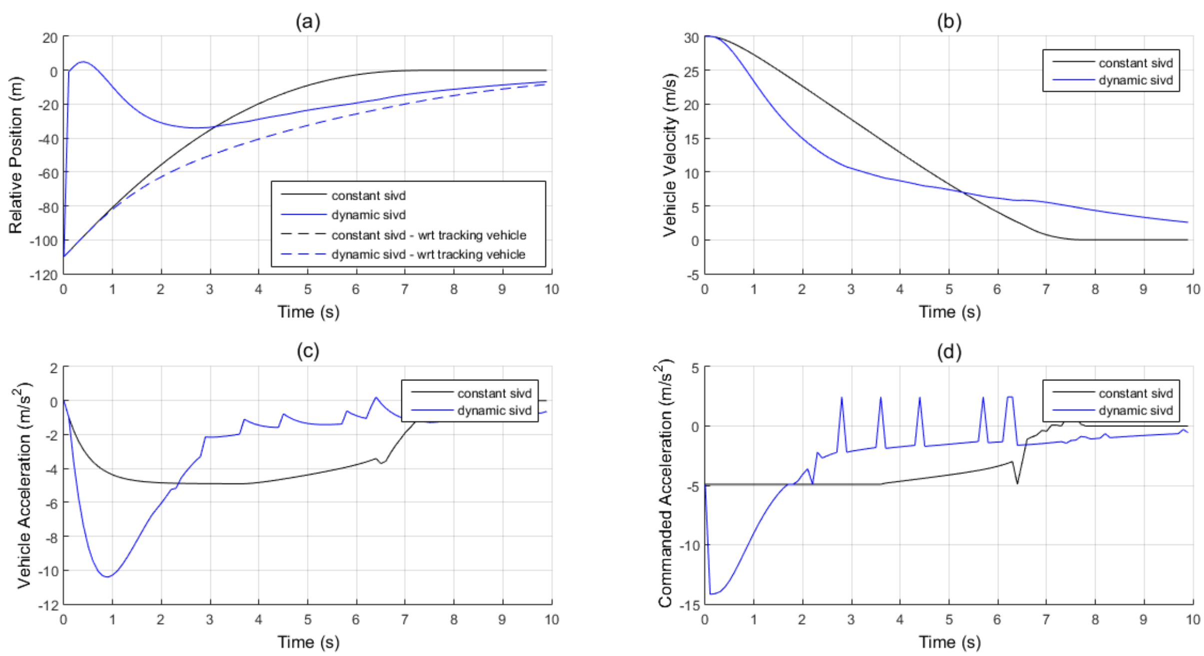

Fig.3: ACC vehicle response with all constraints in formulation of FTCOCP (a)Relative vehicle position, (b)Vehicle velocity, (c)Vehicle Acceleration and (d)Commanded Acceleration

Fig.3 shows the response of ACC vehicle to spacing control laws with all constraints applied in the formulation of FTCOCP. Thus the formed QP has both equality as well as inequality constraints. Two cases are considered here –

1. Static SIVD (=0 m, since target vehicle is at rest)
2. Dynamic SIVD as computed from equation. A.

For Case 1, the TM is a success for the ACC vehicle as represented by the black lines in Fig. 3. The solid lines represent the ACC vehicle's relative position with respect to the SIVD and the dashed lines represent the ACC vehicle's relative position with respect to the target vehicle in Fig. 3(a). Since the scenario considered for Case 2 is of extreme deceleration for ACC vehicle, sudden encounter of the target leads to rapid increase in SIVD as seen in Fig. 3(a) and thus the range reduces drastically in a short time span. Thus the vehicle can't decelerate rapidly enough and maintaining SIVD becomes infeasible, as can be observed from the violation of the input constraints in Fig. 3(c). The ACC vehicle doesn't collide with the target vehicle though, as can be seen from the plot of relative position of ACC with respect to target vehicle. Thus, we can conclude that this method of dynamically calculating SIVD can be overly conservative for extreme deceleration cases and can't be satisfied.

4.2.3 Variation of Time Lag of Lower-Level Controller

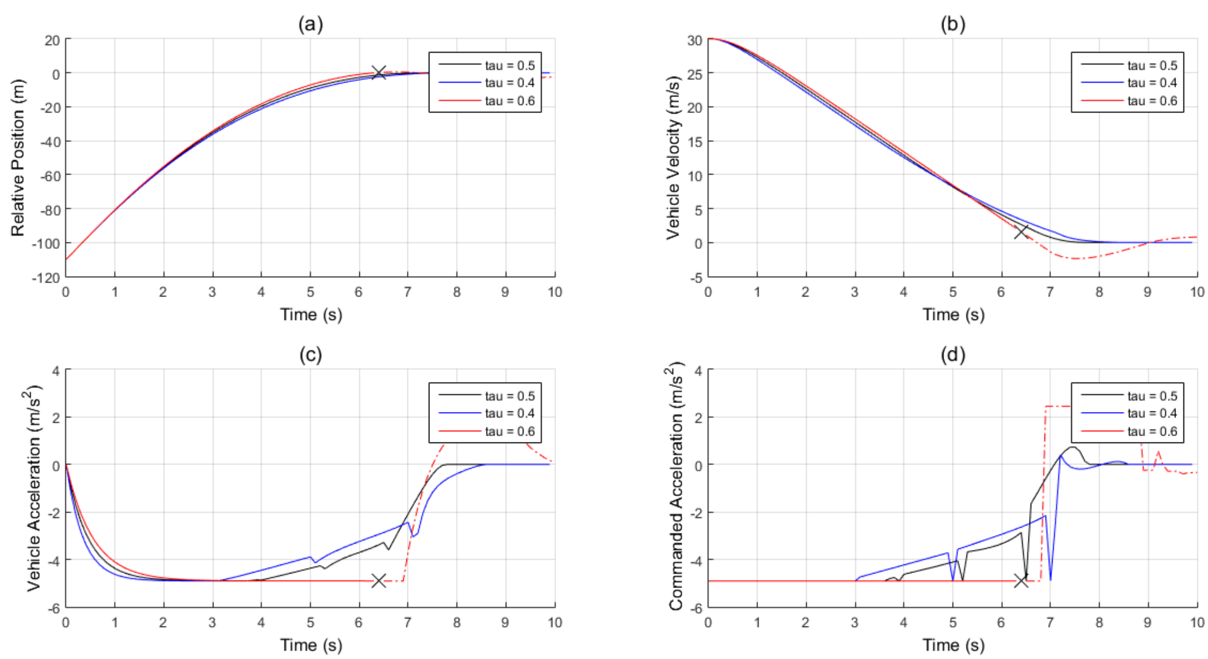

Fig.4: ACC vehicle response with variation in time-lag of lower-level controller of ACC. (a) Relative vehicle position, (b)Vehicle velocity, (c)Vehicle Acceleration and (d)Commanded Acceleration

Fig.4 shows response of ACC vehicle to spacing control laws with all constraints applied in formulation of FTCOCP. The case of static SIVD is considered and simulation is carried out by changing the bandwidth of lower-level controller of the ACC system. As observed from Fig. 4, the system performance deteriorates as the time-lag is increased. A greater time-lag leads to more spikes in commanded acceleration. It is observed that for time-lag of 0.6, the corresponding TM is a failure as collision occurs and a negative velocity, as seen in Fig. 4(b), is required to bring the vehicle back to the origin.

4.2.4 Variation of Control Horizon

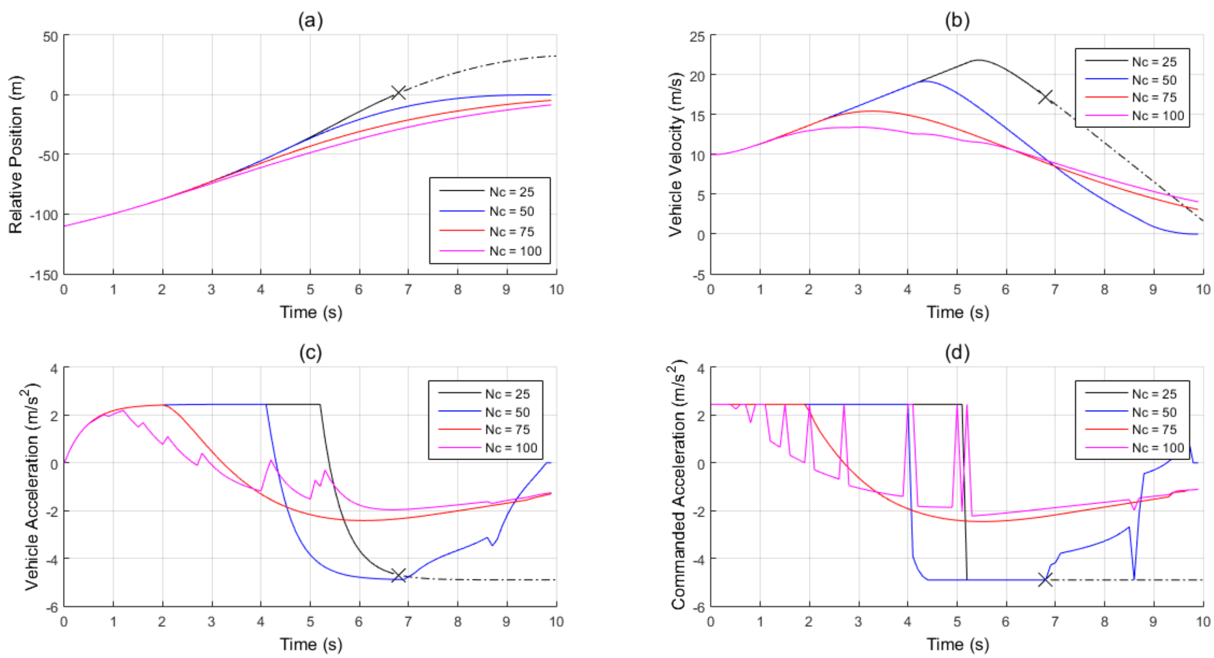

Fig. 5 ACC vehicle response with variation in control horizon (a) Relative vehicle position, (b)Vehicle velocity, (c)Vehicle Acceleration and (d)Commanded Acceleration

Fig. 5 shows response of ACC vehicle to spacing control laws while control horizon (N_c) is varied and prediction horizon (N_P) is set at 100. It is observed from Fig. 5 that the system is more conservative for longer control horizon. Also the computational cost reduces as the control horizon is reduced, but because of fewer degrees of freedom it becomes difficult to control the system and ensure feasibility throughout the horizon. Beyond a point, reduction in control horizon makes the system infeasible and results in a collision (N_c = 25) as it makes the system infeasible. Thus a balance is needed to be found between the two conflicting goals based on design requirements and knowledge of system operating conditions.

4.2.5 Variation of Relative Weights of States

Fig. 6 shows the response of ACC vehicle to spacing control laws when the relative weighting of range and range rate is varied while formulating the FTCOCP. An initial error vector of e0 = [-50; 5; 0] is chosen. This scenario is slightly less extreme as compared to the baseline scenario and prevents the controller from going into saturation and thus allowing us to observe the change in behavior with variation in parameters.

When Q_{pos} = 100*Q_{vel}, we observe that ACC vehicle reaches SIVD quickly, but the controller has to keep switching between extreme input values to maintain the position, and thus there are lot of oscillations in input. When Q_{vel} = 100*Q_{pos}, the controller is much more conservative and it takes longer to reach desired terminal state, but the input is much more stable. The best results are achieved when $Q_{pos} = Q_{vel}$, as ACC vehicle reaches SIVD and zero range rate at the earliest without a lot of oscillations in input.

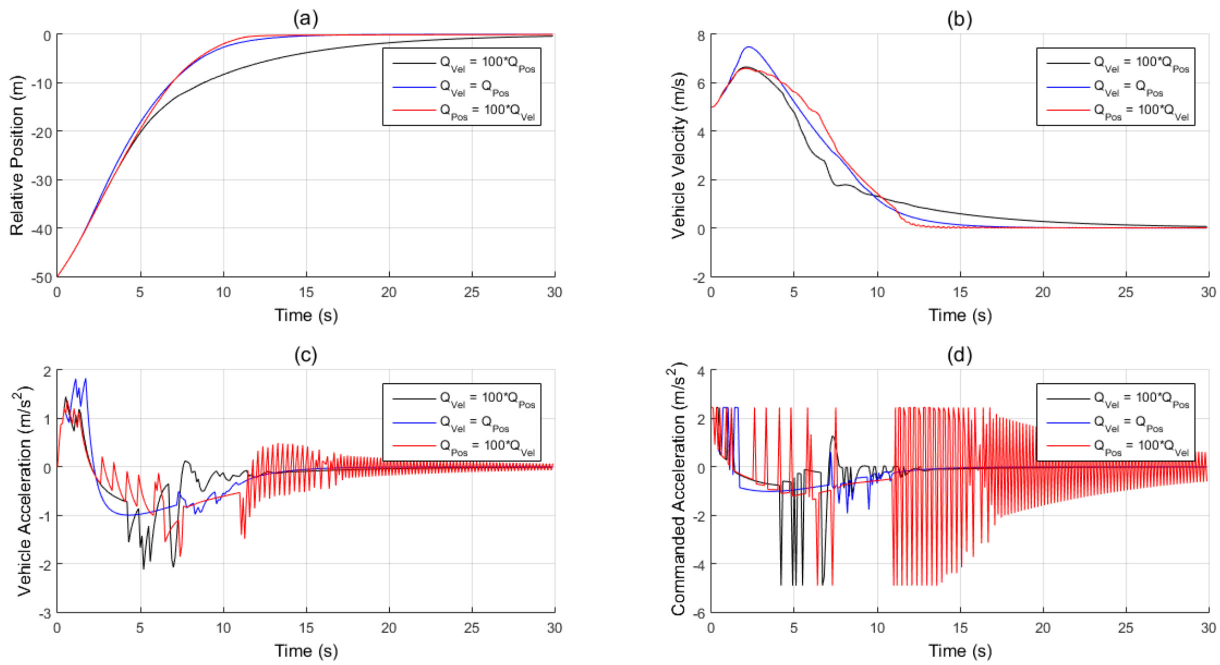

Fig. 6: ACC vehicle response with variation in relative weighting of desired position and velocity. (a) Relative vehicle position, (b) Vehicle velocity, (c) Vehicle Acceleration and (d) Commanded Acceleration

4.2.6 Encounter with accelerating target vehicle

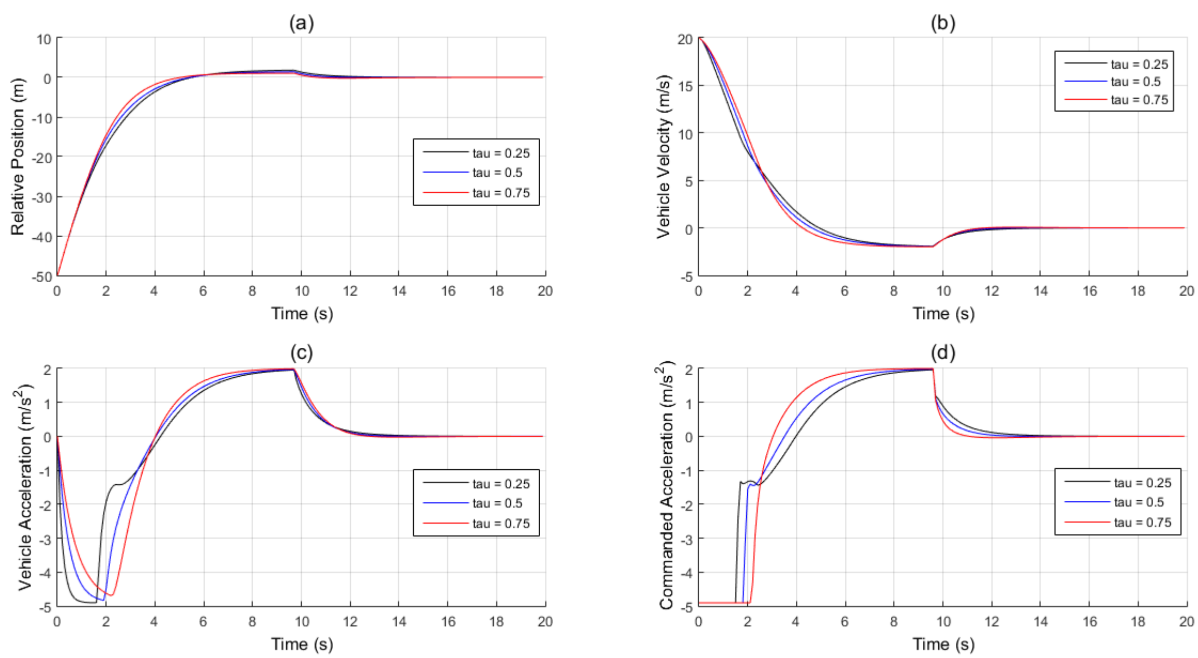

Fig.7 ACC vehicle response with all constraints included in formulation of FTCOCP while the target vehicle is accelerating

Fig. 7 shows response of ACC vehicle to spacing control laws when the target vehicle is accelerating and all constraints are included in formulation of the FTCOCP. We observe from Fig. 7(d) that the law initially commands a deceleration as the target is encountered but the SIVD goes on increasing due to the accelerating target vehicle and so the spacing-control law then commands the ACC vehicle to accelerate. We can observe from Fig. 7(b) that a negative range rate is maintained. This is done to ensure that the range decreases as the ACC vehicle tracks the increasing SIVD. Also from Fig. 7, it is observed that the change in time-lag due to lower-level controller does not affect the controller output considerably.

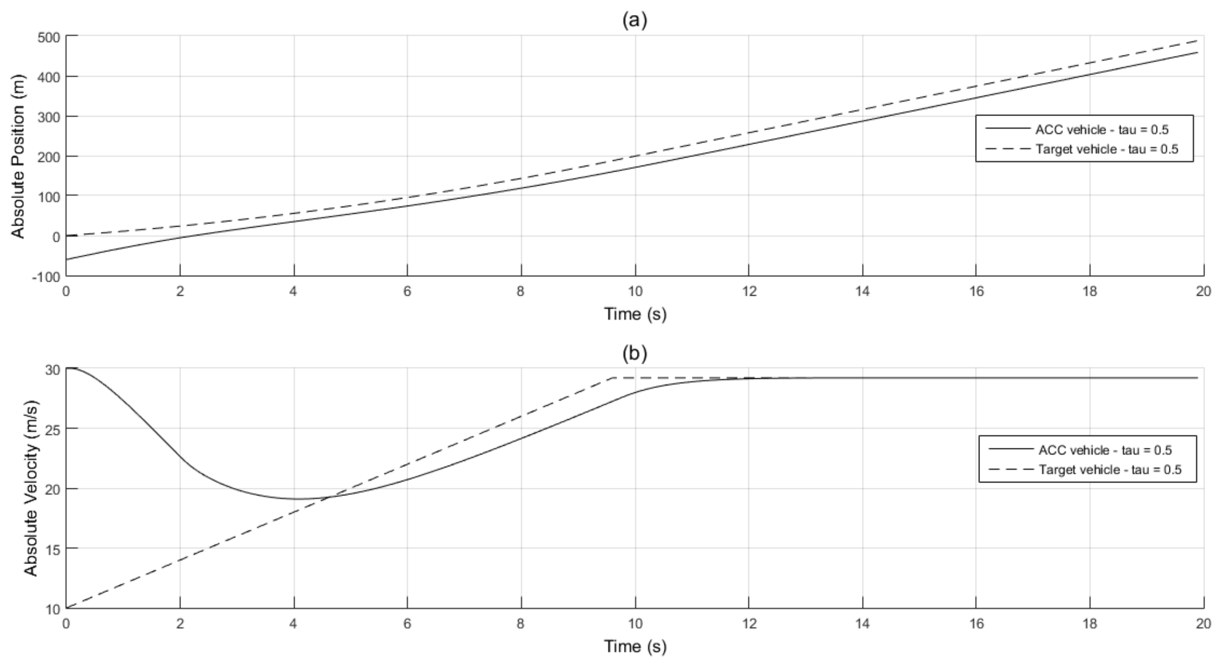

Fig.8 ACC vehicle response with all constraints included in formulation of FTCOCP while the target vehicle is accelerating (a)Absolute vehicle position, (b)Absolute Vehicle velocity.

In Fig. 8 the solid lines represent the absolute response of ACC vehicle to spacing control laws and the dashed lines represent the absolute response of target vehicle. From Fig. 8(a) we see that the ACC vehicle maintains the SIVD and from Fig. 8(b) we infer that the ACC vehicle first tracks the velocity of target vehicle then decreases making the range-rate negative and finally settles down to the steady-state value.

5. Conclusion

Upper-level controller for designing spacing control laws of ACC vehicle was designed and simulated under different scenarios using Model Predictive Control. The following conclusions can be drawn from the simulations –

- Successful execution of Transitional Maneuver cannot be ensured without explicitly incorporating collision avoidance state constraints.
- Computing SIVD dynamically at each time-step based on minimum stopping distance can result in an extremely conservative controller which is impractical.
- An increase in time-lag of lower-level controller decreases the set of feasible initial conditions. Thus a decrease in time-lag can greatly help in ensuring that TM can be feasible for extreme scenarios.
- The conflicting goals of reducing computational effort and increasing system stability need to be balanced depending on the design requirements and knowledge of operating scenarios.
- Best results are achieved in terms of settling time and smoothness of input when both range and range rate are weighted equally.

6. References

1. L. Bageshwar, L. Garrard and R. Rajamani "Model predictive control of transitional maneuvers for adaptive cruise control vehicles", *IEEE Trans. Veh. Technol.*, vol. 53, no. 5, pp.1573 -1585 2004
2. Abhishek Jain, 'Linear Model Predictive Control – Introduction and Parallels with LQR', Guest Lecture for Course : Advanced Feedback Control (ECE 726), NC State University

7. Division of Labour

The formulation of constraints and quadratic program for optimization was jointly handled by both the group members. Beyond that Aamod Velangi was in-charge of writing the project report and making the presentation. While Ajinkya Khade handled the coding of the entire problem and generation of plots for various simulation cases. It should be noted that the MPC controller design and simulation was done entirely using basic Matlab functions and without the use of Matlab MPC Toolbox, as against the previous discussion with the instructor.