

A Model Predictive Cooperative Adaptive Cruise Control Approach

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Abstract—Reduction of fuel consumption is one of the primary goals of modern automotive engineering. While in the past the focus was on more efficient engine design and control there is an upcoming interest on economic context aware control of the complete vehicle. Technical progress will enable future vehicles to interact with other traffic participants and the surrounding infrastructure, collecting information which allow for reduction of fuel consumption by predictive vehicle control strategies. The principle of Model Predictive Control allows a straightforward integration of e.g. navigation systems, on-board radar sensors, V2V- and V2I-communication whilst regarding constraints and dynamic of the system.

This paper presents a Linear Model Predictive Control approach to Cooperative Adaptive Cruise Control, directly minimizing the fuel consumption rather than the acceleration of the vehicle. To this end the nonlinear static fuel consumption map of the internal combustion engine is included into the control design by a piecewise quadratic approximation. Inclusion of a linear spacing policy prevents rear end collisions. Simulation results demonstrate the fuel and road capacity benefits, for a single vehicle and for a string of vehicles, equipped with the proposed control, in comparison to vehicles operated by a non-cooperative adaptive cruise control. Full information on the speed prediction of the predecessor is assumed, hence the purpose of this paper is twofold. On the one hand, best achievable benefits, of the proposed control, due to perfect prediction are demonstrated. On the other hand, the paper studies the behavior of the considered control and the influence of the prediction horizon.

I. INTRODUCTION

Modern cars feature various Advanced Driving Assistance Systems (ADAS) aiming at safety and comfort, less attention was paid to fuel economic operation of the vehicle. Typical ADAS are Cruise Control (CC), maintaining a constant driving speed, and Adaptive Cruise Control (ACC) [1]–[5], managing a secure distance to preceding vehicles in addition. In recent times the focus has been extended to Cooperative Adaptive Cruise Control (CACC) [6]–[8] taking into account modern communication possibilities to achieve the goals of ACC. Communication and the increased information available will allow further degrees of freedom. A fuel efficient operation should be achieved by exploiting the additional information.

Approaching the ACC and CACC problem different control strategies have been suggested. While the studies related to CACC are more focused on string stability and basic frequency domain controllers, Model Predictive Control (MPC)

has been suggested for ACC and has been applied to CACC in [2]. In [1] a Pulse and Glide (PnG) algorithm utilizing a continuous variable transmission was adopted.

Minimizing the overall fuel consumption Q_f , driving along the road, from a starting point \mathcal{A} to a destination \mathcal{B} , would require an operating strategy minimizing

$$Q_f = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} q_f(v(t), a(t)) dt,$$

where q_f is the current fuel consumption depending on the vehicle's time varying speed $v(t)$ and acceleration $a(t)$. The integration limits $t_{\mathcal{A}}$ and $t_{\mathcal{B}}$ denote the time when the vehicle is located in \mathcal{A} or \mathcal{B} . Surrounding traffic as well as technical limitations have to be considered. Further the operating strategy has to ensure finiteness of $t_{\mathcal{B}}$.

Depending on whether the vehicle is situated on an urban street or freeway, driving at rush hour or off-peak time different driving situations, such as

- Start - Stop
- Driving on a road, without disturbance
- Driving on a road, disturbed by other vehicles

will become dominant [9], [10].

In this paper we will focus on driving, disturbed by a predecessor, in moderate non-congested traffic, subject to speed variations. The solution to the rather exhaustive problem stated above will be approached by minimizing $Q_{f,T}$, the fuel consumption within a finite time horizon T , while driving from \mathcal{A} to \mathcal{B} .

$$Q_{f,T} = \int_0^T q_f(v(t), a(t)) dt, \quad (1)$$

The considered scenario is shown in Fig. 1. All cars are assumed to be equipped with some distance measurement device, e.g. radar or likewise. Communication is established between succeeding vehicles. The connection might be direct or indirect, based on vehicle to vehicle (V2V) communication or via infrastructure along the road by means of vehicle to infrastructure (V2I) and infrastructure to vehicle (I2V) communication. The latter one would include each participating vehicle into a distributed network enabling centralized or distributed traffic prediction based on information received from the vehicles as well as from traffic regulation infrastructure. In any case the vehicles equipped with MPC based CACC (MPC-CACC) are assumed to be able to obtain a sufficiently accurate prediction of the preceding vehicle's speed v_p and position x_p within the finite prediction horizon T . The vehicle is assumed to drive with a fixed gear, hence

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gear changing is not considered by the optimization. Lane changes as well as overtaking maneuvers are not considered.

The main contribution of this paper is the development of a linear MPC based CACC. In contrast to previously proposed model predictive control approaches, aiming at higher fuel efficiency, a direct penalization of the approximate fuel consumption, rather than the acceleration, is incorporated. A full prediction of the predecessor's speed profile will be considered along physical limitations of the system. Exploiting a varying inter-vehicle distance will allow for significant fuel benefit. Since full information on the preceding vehicle, i.e. an accurate prediction of the speed profile, is assumed, the presented results reflect best possible achievable performance by the proposed control strategy.

In the next Section the nonlinear optimization problem will be developed. In Section III the before mentioned optimization problem is approximated and the linear MPC implementation is outlined. Section IV demonstrates the effectiveness of the proposed control by simulation results of two succeeding vehicles as well as a string of vehicles. Section V will conclude this paper.

II. PROBLEM STATEMENT

The basic dynamic equations of two succeeding vehicles are given by (2).

$$\begin{aligned}\Delta \dot{x} &= v_p - v \\ \dot{v} &= a\end{aligned}\quad (2)$$

Therein v and a are the velocity and acceleration of the vehicle under consideration and v_p is the velocity of the disturbing, preceding, car. The inter-vehicle distance is given by $\Delta x > 0$. Sometimes a so called generalized vehicle longitudinal dynamics is considered in addition, see e.g. [1] which accounts for the engine and power train dynamics.

Fuel consumption of a Diesel engine can be appropriately described by a static map depending on the engine torque and rotational engine speed. The transient fuel consumption, not covered by the static map, is usually about 4% [11]. The smooth, economic operation of the vehicle will support this approximation and further reduce the neglected amount of fuel. Assuming a fixed gear, i.e. a constant ratio between vehicle speed and engine speed, allows to recalculate the actual fuel consumption of the vehicle in terms of vehicle speed and acceleration (3).

$$q_f = q_f(v, a) \quad (3)$$

The corresponding static map is shown in Fig. 2. Therein 'BSFC' denotes the Brake-Specific-Fuel-Consumption, q_f .

Despite the major goal of minimized fuel consumption, an economic CACC has to achieve some further goals.

Prevent or reduce the risk of crashes; Different spacing policies can be adopted in order to prevent or at least reduce the risk of crashes. In [12] a historical review on car following models and their underlying spacing policies is given and in [13] collision avoidance capabilities of spacing policies are discussed. Basic dynamic considerations would

advise to use a spacing policy quadratic in vehicle speed. Nevertheless, the most common spacing policy used, for example in [6], is linearly depending on the vehicle speed v , managing constant time headway, i.e. the time it would take until the vehicle reaches the current position of the predecessor. The desired inter-vehicle distance Δx_{des} is given by (4).

$$\Delta x_{des} = \Delta x_0 + hv \quad (4)$$

Therein x_0 is the stand still inter-vehicle distance and h the time headway. However, we adopt (4) in terms of a minimum inter-vehicle distance, hence

$$\Delta x \geq \Delta x_{min,0} + h_{min}v. \quad (5)$$

Therein $\Delta x_{min,0}$ denotes the minimum inter-vehicle distance at stand still and h_{min} is the time-headway corresponding to the minimum inter-vehicle distance.

Traffic compatibility; Fuel benefit being the solely objective will lead to large inter-vehicle distances and eventually cause a coast down. Hence the traffic capacity would be decreased. In order to achieve traffic compatibility and prevent destructive driving maneuvers a maximum distance between succeeding vehicles has to be considered. While a smaller maximum distance will increase traffic capacity a larger one will enable a higher potential fuel savings. Contrary to a minimum distance a maximum distance can be relaxed arbitrarily without compromising safety. Thus, analog to the minimum inter-vehicle distance, the maximum inter-vehicle distance is enforced by the constraint

$$\Delta x \leq \Delta x_{max,0} + h_{max}v + \gamma r, \quad (6)$$

with $\Delta x_{max,0}$ being the maximum distance at stand still and h_{max} the time headway corresponding to the maximum inter-vehicle distance. Relaxation is incorporated by the slack variable γ and the relaxation parameter r . The higher this parameter is chosen, the more restrictive this constraints will be with respect to the optimization.

System-inherent limitations; Providing feasibility of control actions system inherent limitations like maximum accelerating or decelerating power as well as maximum and minimum speed of the vehicle have to be considered. Thus the constraints

$$a_{min}(v) \leq a \leq a_{max}(v) \quad (7)$$

$$v_{min} \leq v \leq v_{max} \quad (8)$$

have to be included. Therein a_{max} and a_{min} are the speed dependent maximum and minimum acceleration and v_{max} and v_{min} denote the velocity limits of the vehicle.

Comfort and approval of the driver; In addition to the essential goals, mentioned above, several constraints can be introduced to ensure ride comfort and the acceptance by a human driver. Those are for example limitations of acceleration or jerk not introduced by the capabilities of the vehicle.

Combining all these essential requirements, including the dynamics (2), minimization of the fuel consumption (1) turns into the nonlinear optimization problem (9) within prediction

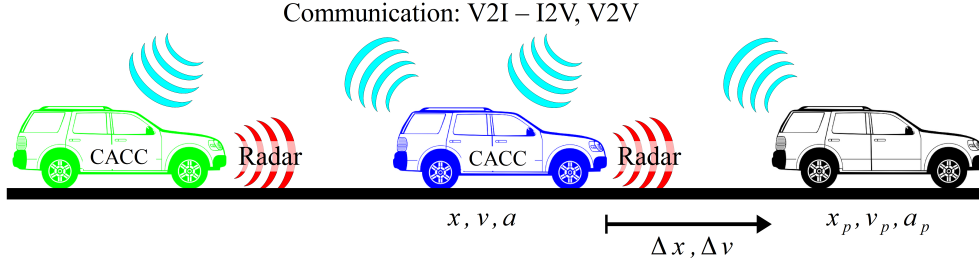


Fig. 1. Considered scenario.

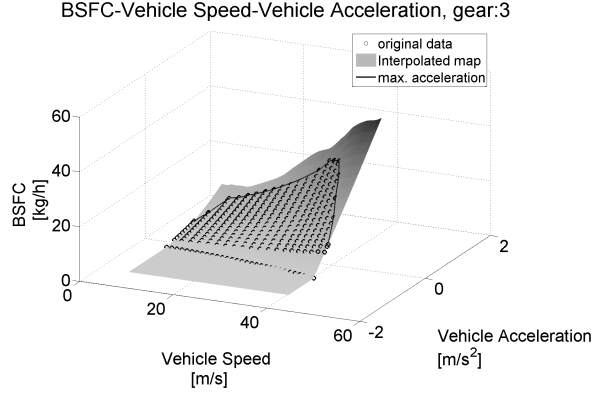


Fig. 2. Interpolated map of static fuel consumption, fixed gear.

horizon T subject to the dynamic (10) and (11) and the nonlinear constraints (12) to (17) with the initial conditions $\Delta x(0) = \Delta x_0$ and $v(0) = v_0$.

$$\min_{a(t)} \int_0^T (q_f(v(t), a(t)) + \gamma) dt \quad (9)$$

$$\text{s.t. } \Delta \dot{x}(t) = v_p(t) - v(t) \quad (10)$$

$$\dot{v}(t) = a(t) \quad (11)$$

$$\Delta x(t) \leq \Delta x_{\max,0} + h_{\max} v(t) + \gamma r \quad (12)$$

$$\Delta x(t) \geq \Delta x_{\min,0} + h_{\min} v(t) \quad (13)$$

$$a(t) \leq a_{\max}(v(t)) \quad (14)$$

$$a(t) \geq a_{\min}(v(t)) \quad (15)$$

$$v \leq v_{\max} \quad (16)$$

$$v \geq v_{\min} \quad (17)$$

III. CONTROL SOLUTION

The approximation of the nonlinear fuel consumption map as well as the nonlinear speed dependent maximum acceleration by piecewise quadratic or linear affine functions will allow to incorporate linear constraints as described in [14]. Consequently an implementation becomes possible within the linear model predictive control framework.

Approximation of nonlinear optimal control problem

A convex scalar function $f(x)$, with $x \in \mathbb{R}^n$, can be approximated by

$$f(x) \approx \max_{i=1 \dots j} \{c_i x + c_{i,c}\}, \quad (18)$$

with $c_i \in \mathbb{R}^n$, $c_{i,c} \in \mathbb{R}$ and $j \in \mathbb{N}$ the number of linear functions used. By the definition of the approximation it follows that each linear function corresponds to a not necessarily closed convex polytope region.

Since q_f is strictly positive an approximation of $\sqrt{q_f(v, a)}$ will be possible. Approximating $\sqrt{q_f(v, a)}$ rather than $q_f(v, a)$, by a convex piecewise linear function, according to (18), will allow a quadratic approximation and reduce the number of regions j_{q_f} . The approximation then reads

$$\sqrt{q_f(v, a)} \approx \max_{i=1 \dots j_{q_f}} \{c_{q_f,i,v} v + c_{q_f,i,a} a + c_{q_f,i,c}\}, \quad (19)$$

with $c_{q_f,i,v}$ and $c_{q_f,i,a}$ being the approximation parameters corresponding to vehicle speed and acceleration and $c_{q_f,i,c}$ the constant offset of the j_{q_f} linear functions used for the approximation. The approximation parameters are determined by minimization of the quadratic approximation error. Utilizing the approximation (19) minimization (9) can be augmented

$$\min_{a(t), \xi(t)} \int_0^T (\xi(t)^2 + \gamma) dt \quad (20)$$

$$\text{s.t. } \xi(t) \geq c_{q_f,i,v} v(t) + c_{q_f,i,a} a(t) + c_{q_f,i,c} \quad \forall i = 1 \dots j_{q_f}, \quad (21)$$

subject to the system dynamic and constraints (10) to (17).

Approximating $a_{\max}(v)$ and $a_{\min}(v)$ in a similar manner and replacing constraints (14) and (15) by a set of appropriate linear constraints a linear model predictive control problem with linear constraints is obtained.

Fig. 3 shows the quadratic approximation, resulting by (20) and (21). Different shades of gray indicate the three regions using a different quadratic approximation. Fig. 4 shows the approximations $a_{\max}(v)$ used for control design.

Implementation of Model Predictive Control

An overview on Model Predictive Control with constraints is given in [15] and [16]. Given the discrete time equivalent of (2) with sample time T_s

$$\begin{aligned} \Delta x(k+1) &= \Delta x(k) - T_s v(k) + T_s v_p(k) \\ v(k+1) &= v(k) + T_s a(k) \end{aligned} \quad (22)$$

with control input a and measured disturbance v_p , a model predictive control has been designed. Defining an auxiliary input ξ , approximating $\sqrt{q_f(v, a)}$, all the necessary

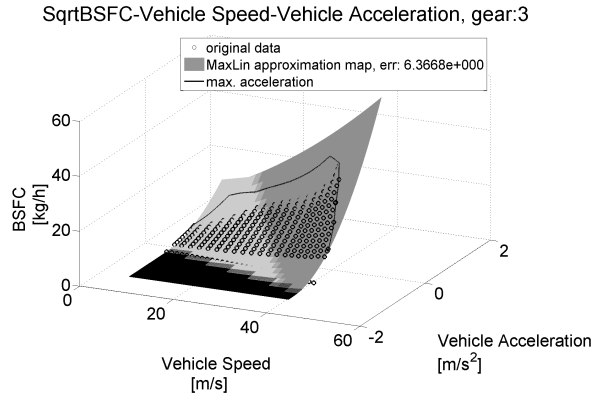


Fig. 3. Piecewise quadratic approximation of static fuel consumption, using 3 regions.

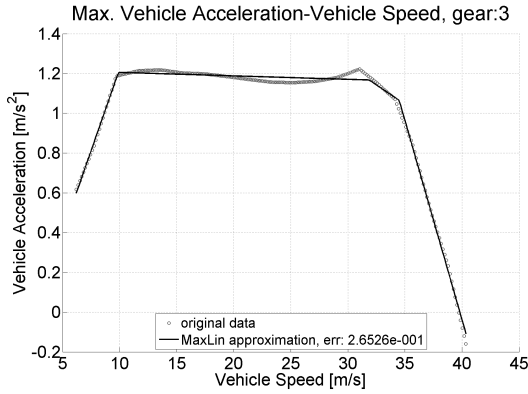


Fig. 4. Approximation of maximum acceleration by 4 linear constraints.

constraints (21) can be implemented as mixed state-input-measured-disturbance constraints. In order to reduce the computational burden input blocking was applied to the control input a (but not to the auxiliary input ξ). Further the sampling time for the internal prediction of the MPC was chosen $T_S = 1$ s, which is larger than the sampling time of 0.1s of the actual implementation. Thus the number of constraints is kept low while the sampling time, i.e. the response time of the control can be decreased. In addition to the hard constraint, representing the minimum inter-vehicle distance, a relaxed constraint was introduced, maintaining a larger distance to the preceding vehicle. In case of prediction errors this two-stage constraint will prevent infeasible optimization problems, thus the control will be more robust. Fig. 5 illustrates the implemented spacing policy.

Reference control

For comparison a simple ACC has been implemented. According to the constant time headway spacing policy given by (4) a desired inter-vehicle distance is maintained by a PI-control.

IV. RESULTS

In this section some simulation results will be presented. Fig. 6 shows the vehicle model implemented in MatLab Simulink utilizing the fuel consumption map shown in Fig. 2.

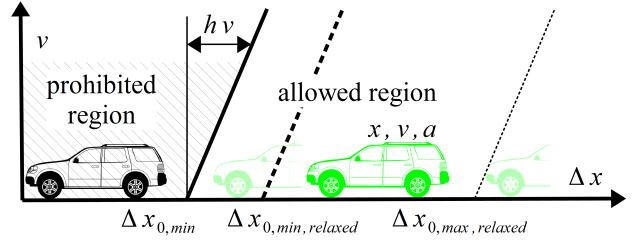


Fig. 5. Variable spacing policy implemented by MPC.

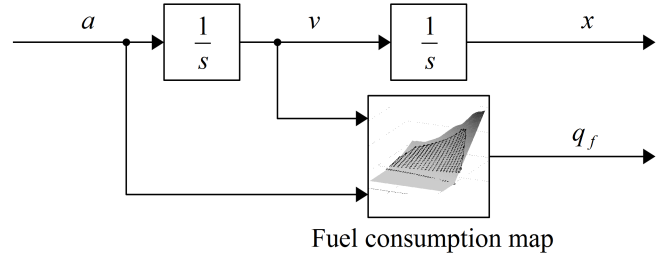


Fig. 6. Simulation model in MatLab Simulink.

Thus it is assumed that the vehicle is driving in a fixed gear. Two vehicles have been simulated. The first one was subjected to a given speed profile, controlled by an approximate inverse control. The speed profile was derived from the FTP75 test cycle, including some modifications in order to become feasible. Results have been obtained by simulating up to 1250 seconds, the Figures below are showing the time range from 250 to 750 seconds.

The second vehicle has been controlled either by a simple ACC or by the MPC based CACC. It was assumed that the speed of the preceding vehicle is known within the whole prediction horizon of the MPC. Table I lists the control parameters used. The prediction horizon has been chosen between 5 and 20 seconds. The optimized control action was calculated within a control horizon of 10 seconds (in the case of 5 second prediction horizon the control horizon was also adapted to 5 seconds) subject to input blocking.

Simulation results, applying the MPC-CACC with prediction horizon 15 seconds, are shown in the following. Fig. 7 depicts the inter-vehicle distance as well as speed and acceleration of the preceding and the controlled vehicle. Since a perfect prediction has been assumed the relaxed minimum inter-vehicle distance constrained is satisfied almost everywhere. Especially between 450 and 600 seconds one can observe the averaging behavior of the MPC-CACC in the speed profile. A comparison between the MPC-CACC and an PI-ACC equipped vehicle is shown in Fig. 8. The corresponding fuel benefit and the influence of the prediction horizon is presented in Table II. The In-Line benefit denotes the difference in fuel consumption between the preceding, disturbing vehicle and the host vehicle equipped with MPC-CACC or PI-ACC respectively. The average capacity given in vehicles per second and percent, is calculated assuming

TABLE I
CONTROL PARAMETERS

MPC-CACC	
Prediction Horizon (PH)	5/10/15/20s
Control Horizon (CH)	5/10/10/10s
Blocking, block size applied to the control input	2
Sample time of prediction model	1 s
Sampling time of implementation	0.1s
Minimum distance at $v = 0$	5m/10m
Maximum distance at $v = 0$	40m
Time headway	0.5s
PI-ACC reference control	
Proportional gain	10
Integral gain	1
Desired distance at $v = 0$	5m
Time headway	2s

an average vehicle length of 4 meters.

A comparison between a string of 5 vehicles equipped with MPC-CACC (with prediction horizon of 15 seconds) or PI-CACC respectively is given in Table III. The In-Line benefit denotes the difference in fuel consumption between two succeeding vehicles, while the benefit in the bottom row states the fuel savings between two corresponding vehicles equipped with MPC-CACC and PI-ACC. Averaging over all vehicles the capacity of the MPC-CACC equipped vehicles is 96.58% with respect to PI-ACC equipped ones.

V. CONCLUSION AND OUTLOOK

A Model Predictive Cooperative Adaptive Cruise Control has been developed. Simulation results show a significant cross benefit in fuel consumption, depending on the prediction horizon of the control, up to 16% in comparison to an ACC equipped vehicle and up to 20% with respect to an uncontrolled predecessor. The implemented variable spacing policy establishes a PnG-like behavior mitigating speed variations, thus decreasing fuel consumption. In a string of vehicles simulation results imply a reasonable benefit also for succeeding vehicles. Simulation results also indicate increased or equal traffic capacity with respect to PI-ACC equipped vehicles, depending on the penetration rate.

The linear Model Predictive Control approach enables a possible future online implementation and evaluation on a engine test bench, including a full vehicle simulation like AVL-InMotion, allowing for precise measurements of fuel consumption and emissions during dynamic operation of the engine. The restrictive assumption on a perfectly predicted predecessor, rendering the approach unsuitable for real applications should be approached by statistical methods like in [17] and [18], incorporating cooperative communication. A further increase of fuel benefit will be achieved by inclusion of shifting strategies, while adaptive spacing policies could allow for higher traffic capacity.

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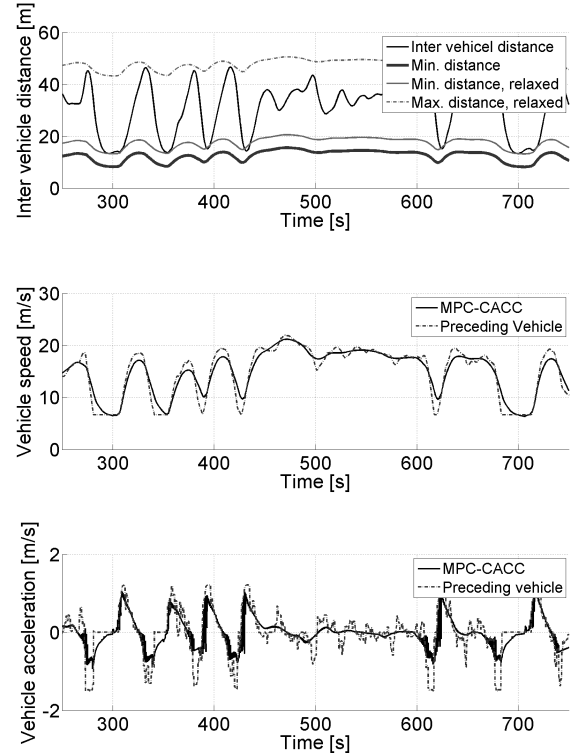


Fig. 7. Inter-vehicle distance including constraints, vehicle speed and acceleration.

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TABLE II
COMPARISON OF AVERAGE FUEL CONSUMPTION AND BENEFIT OF ACC AND MPC-CACC WITH DIFFERENT PREDICTION HORIZONS

	preceding vehicle	PI-ACC	MPC-CACC			
			PH=5	PH=10	PH=15	PH=20
I/100km	6.18	5.86	5.77	5.23	5.00	4.89
mpg(US)	38.30	40.10	40.73	44.93	47.00	48.06
In-Line Fuel Benefit in %		5.18	6.69	15.40	19.05	20.91
Fuel Benefit in %			1.52	10.71	14.57	16.52
Average Capacity in Vehicles/s		0.361	0.308	0.362	0.399	0.423
Average Capacity in %		100.0	85.2	100.1	110.4	117.0

TABLE III
COMPARISON OF AVERAGE FUEL CONSUMPTION AND BENEFIT OF ACC AND MPC-CACC WITHIN A STRING OF VEHICLES.

Vehicle #		1	2	3	4	5
PI-ACC	I/100km	5.86	5.67	5.54	5.43	5.34
	mpg(US)	40.10	41.45	42.42	43.28	44.01
	In-Line Benefit	3.24		1.99		
	in %	2.29		1.66		
MPC-CACC	I/100km	5.00	4.43	4.15	4.03	3.95
	mpg(US)	47.00	53.05	56.63	58.31	59.49
	In-Line Benefit	11.40		2.89		
	in %	6.32		1.99		
Benefit in %		14.57	21.95	25.07	25.80	25.94

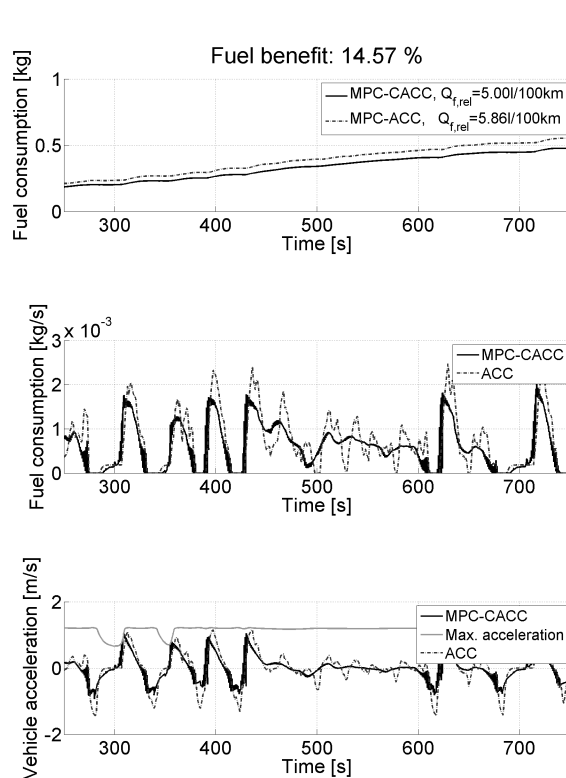


Fig. 8. Fuel consumption during driving cycle.

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