

# Performance of ABS using LQR over PID Control on $\mu$ -split Roads

*ME41116 Vehicle Dynamics B*

Varun Kotian

(4980611)

Nikhil Nagendra

(5049628)

**Shantanu Shivankar**

**(5032814)**

Yen-Lin Wu

(4848489)



Source: Bosch

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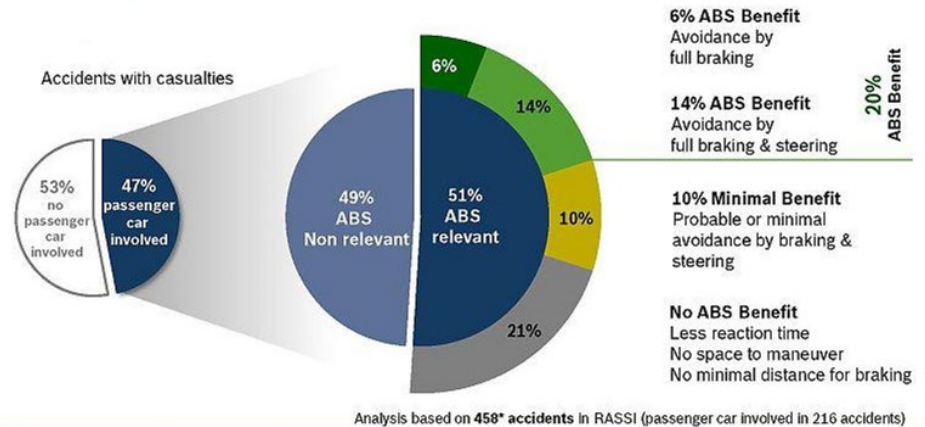
- Motivation
- LQR controller
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# Motivation

## NHSTA Passenger vehicle accidents - 2017

- Increases the Drivers ability to control the direction of car under hard braking.
- Its serves as a good point for selling Vehicles

### Accidents with casualties by avoidance maneuver

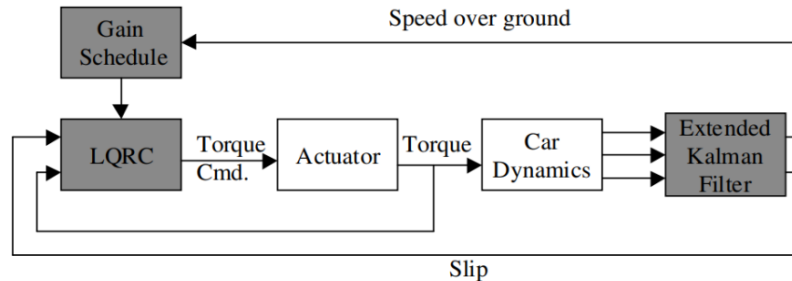


<http://www.cse.msu.edu/~cse470/F01/Projects/ABS/ABS4/web/do-requirements/do-requirements.html#:~:text=1.2%20Motivation,vehicle%20under%20hard%20braking%20conditions.>

Charles M. Farmer, Adrian K. Lund, Rebecca E. Trempe, Elisa R. Braver, Fatal crashes of passenger vehicles before and after adding antilock braking systems, Accident Analysis & Prevention, Volume 29, Issue 6, 1997,

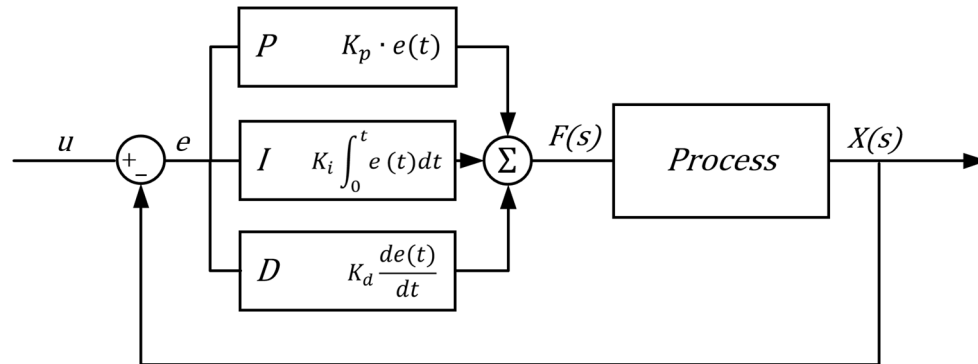
# What's LQR?

- LQR is used when
  - the system state : a set of linear differential equations
  - the system cost : quadratic equations.
- Cost function: 1) finds out the optimal gain
  - 2) uses gain as feedback to state space equation.
- Implementation **REQUIRES** precise set of state space equations
  - not feasible in real world: only limited to research purposes.



# PID Control

- A feedback control loop
- Applies accurate and responsive correction to a control function
- Challenges: **constant** parameters
  - Overshoot can only be corrected slowly
  - Latency when responding to (big) disturbances
- **Note:** In IPG, we use the default hydraulic controller which is a PID controller.



# Control Strategy LQR

$$\left\{ \begin{array}{l} m\dot{v} = -F_x \\ J\dot{\omega} = rF_x - T_b \text{sign}(\omega) \\ F_x = F_z \cdot \mu(\lambda, \mu_H, \alpha) \\ \lambda_x = \frac{v_n - \omega r}{v} \end{array} \right\} \Rightarrow \begin{array}{l} \dot{\lambda} = -\frac{1}{v} \left( \frac{1}{m}(1-\lambda) + \frac{r^2}{J} \right) F_z \mu(\lambda, \mu_H, \alpha) + \frac{1}{v} \frac{r}{J} T_b \\ \dot{v} = -\frac{1}{m} F_z \mu(\lambda, \mu_H, \alpha) \end{array}$$

The change in linear velocity if the vehicle velocity is very slow and cannot be used in the state space equation to control the input.

To make the model more robust and adjust to the every road friction it can encounter we include an integral function. State space equation is given as,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = A(v) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + B(v)(u - T_b^*) + W(v)\epsilon_\mu(x_2)$$

where,

$$\dot{x}_1 = \lambda - \lambda^* = x_2$$

The state, input and the error matrix,

$$A(v) = \begin{pmatrix} 0 & 1 \\ 0 & \frac{\alpha_1}{v} \end{pmatrix}, B(v) = \begin{pmatrix} 0 \\ \frac{\beta_1}{v} \end{pmatrix}, W(v) = \begin{pmatrix} 0 \\ \frac{1}{v} \end{pmatrix}$$

$$\begin{aligned} \alpha_1 &= -F_z^* \left( \frac{1}{m}(1-\lambda^*) + \frac{r^2}{J} \right) \frac{\partial \mu}{\partial \lambda}(\lambda^*, \mu_H^*, \alpha^*) \\ &\quad + F_z^* \frac{1}{m} \mu(\lambda^*, \mu_H^*, \alpha^*) \end{aligned} \quad \beta_1 = \frac{r}{J} > 0$$

The cost functions is given as,

$$J(x(t), u[t, \infty)) = \int_t^\infty \left( (x(\tau) - x^*)^T Q(v) (x(\tau) - x^*) + (u(\tau) - u^*)^T R(u(\tau) - u^*) \right) d\tau$$

Weights -

$$Q = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \quad R = 0.001$$

# Initial Conditions

- No estimation - direct measurement
- No noise in system
- Reference slip is constant
- Controller deactivation below 5 km/h

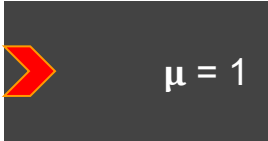
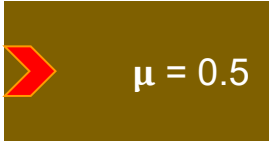
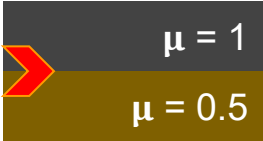
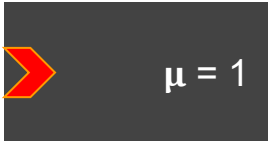
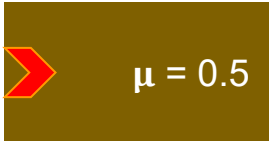
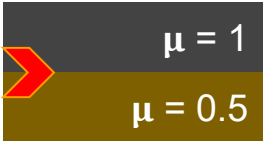

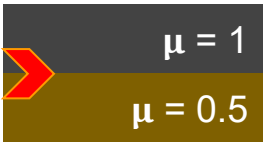
Vehicle Type	Audi TT (4WD, DCT)
Vehicle Mass	1.500 kg
Tyre	RT_225_50R17
Kinematic tyre radius	0.32 m

Parameters	Value
Mass	450 Kg
Inertia of the wheel	1.2 Kg-m <sup>2</sup>
Wheel effective radius	0.305 m
Initial velocity	120 Km/h
Final velocity	0 Km/h

Simulation System: i7 processor  
with 16GB RAM

# Experiments

- 8 tests:

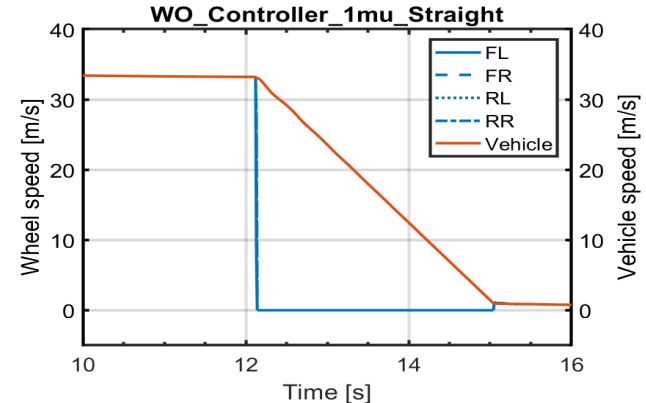
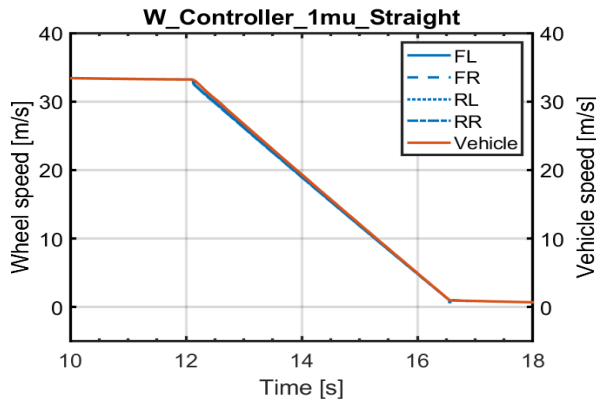
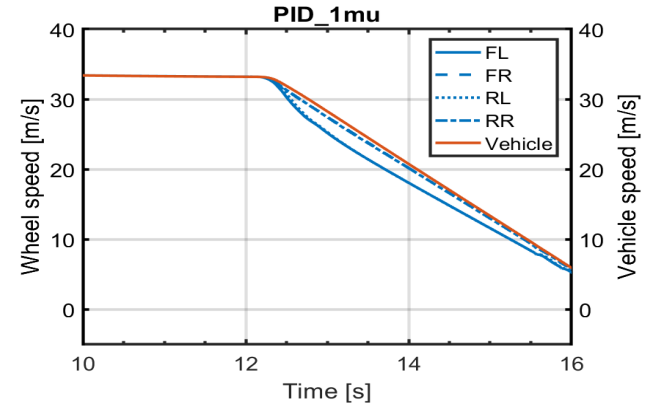
	Road type		
	High friction	Low friction	Mu Split
w/o controller:			
w/ LQR:			
w/ PID:			



# Simulation and Results

Braking on High friction surface ( $\mu = 1$ )

- The car travels straight for 300m at 120km/h and then full brake

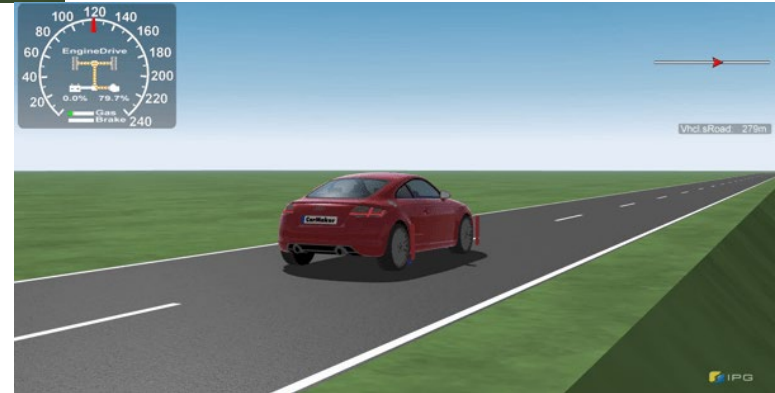


# Simulation and Results



← w/ Control ( $\mu = 1$ )

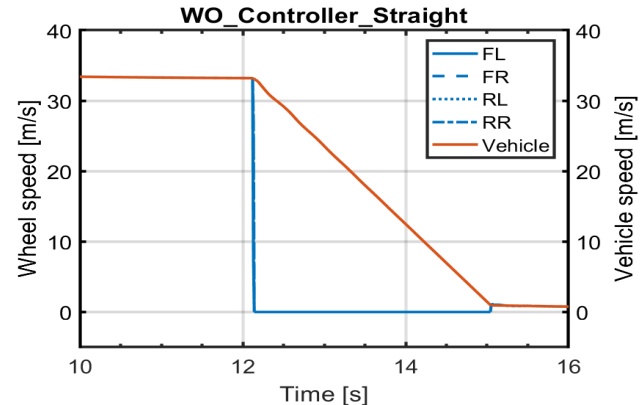
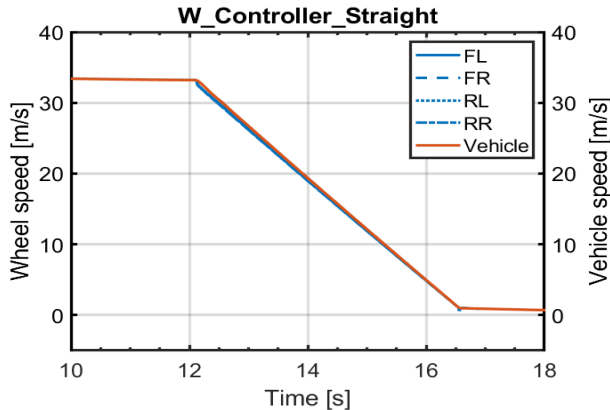
w/o Control ( $\mu = 1$ )



# Simulation and Results

Braking on Low friction surface ( $\mu = 0.5$ )

- The car travels straight for 300m at 120km/h and full brake



# Simulation and Results



← w/ Control ( $\mu = 0.5$ )

w/o Control ( $\mu = 0.5$ )



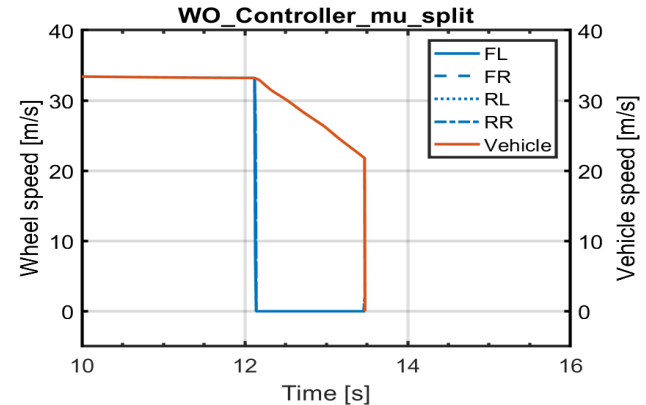
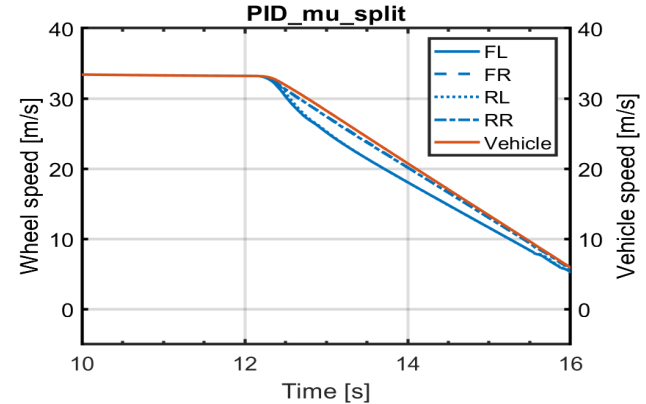
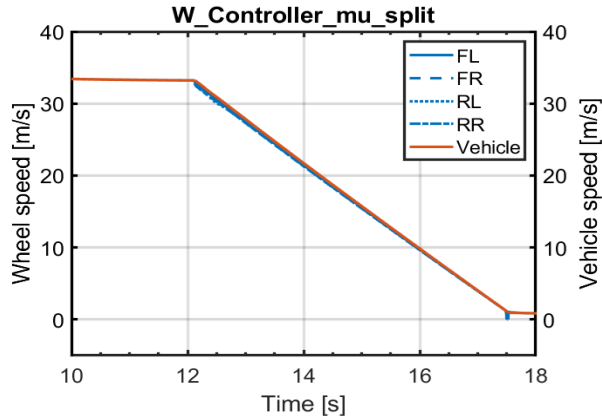
Note steering correction



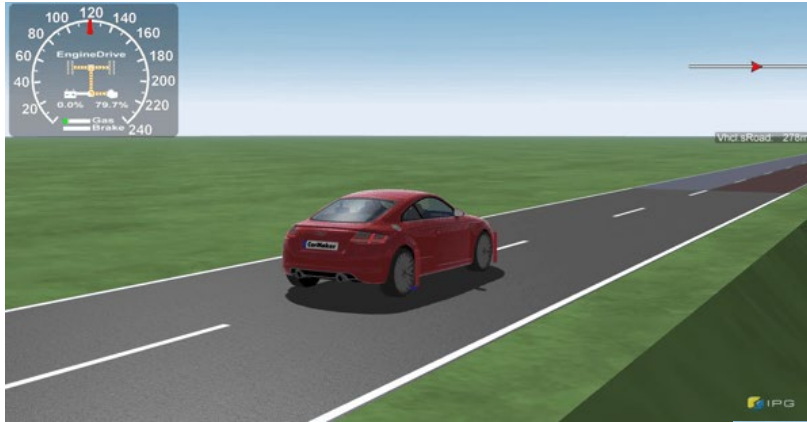
# Simulation and Results

## Braking on $\mu$ -split friction surface

- The car travels straight for 300m at 120 km/h and then full brake.



# Simulation and Results



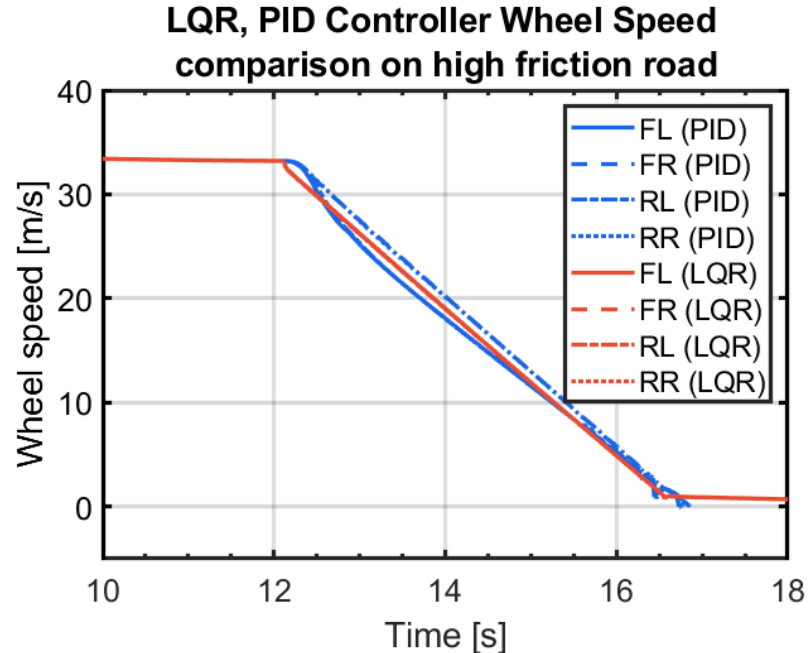
← w/ Control ( $\mu$  split)

w/o Control ( $\mu$  split)



# PID, LQR Comparison

- Braking from 120km/h at high friction ( $\mu = 1$ ) road.



# Conclusions

- LQR performance is better than PID for initial conditions
- Further tuning is necessary to obtain optimal performance of LQR
- Computation time fairly slower than PID control
- Inclusion of error dynamics can provide more accurate representation



# Future Work

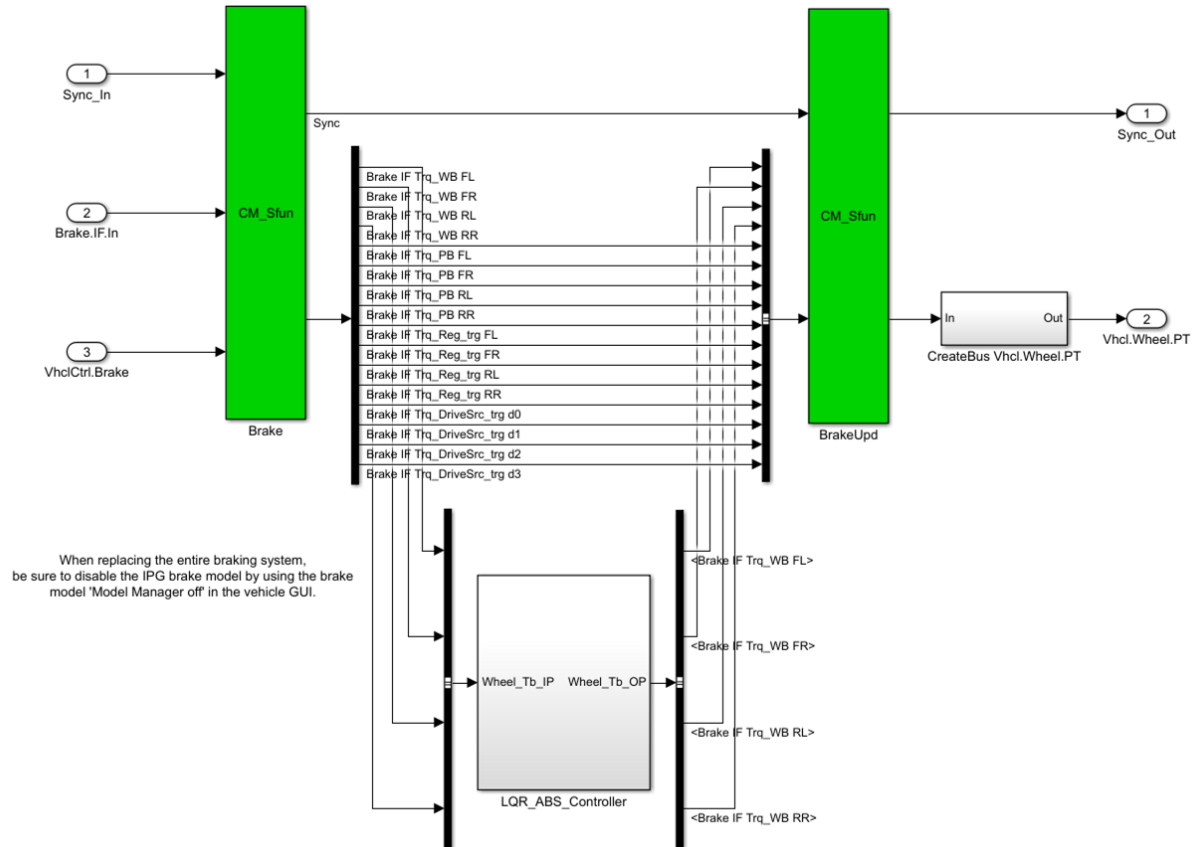
- Inclusion of sensor noise
- Saturation / Boundary conditions for input to be implemented
- Linking with Delft Tyre
- Optimise and tune weights for better performance of LQR control
- Implement non-linear MPC strategy and compare (possible future work)

# Thank you

Questions?

For Block Diagram - next  
slide

# Appendix



# Appendix

