Performance Comparison of ABS using LQR and PID Control on Different Friction Surfaces

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***Abstract*—Linear quadratic regulator (LQR) controller is purposed for the control of Anti-lock Braking System (ABS), which is mandatory for road vehicles. It provides maximum braking force while maintains the vehicle controllability. This poses a primary challenge given the non-linearity between tire/road friction force and the wheel slip. This paper mainly addresses the behavior of ABS using LQR on $\mu - split$, high friction and low friction roads using IPG and Simulink for simulation. The source code can be found on:**
`https://github.com/wuyenlin/VDB_project/`.
***Index Terms*—Antilock braking system (ABS), LQR**

I. Introduction

An Anti-lock Braking System (ABS) is a system which controls the wheel slip of each wheel of a vehicle by maintaining the highest friction possible so that maneuverability of the vehicle is maintained even in poor road conditions. ABS controllers are highly dependent on tire characteristics and uncertain road properties. There are several methods to implement ABS control. Rule based PID controllers are the most widely used in the industry owing to their standardised implementation and easy tuning. However, more efficient and robust controllers that can be implemented. Some of them are, fuzzy logic, sliding mode, LQR and Model Predictive Control, to name a few. Each control logic has its own set of pros and cons. For more information on these controllers, interested reader is referred to [1]. In this report, we will look at primarily LQR-based control for ABS implementation. The first section describes the vehicle model and constraints wherein we have chosen a quarter car model for tuning the LQR controller. The next section describes the extension of the model into IPG Carmaker [2]. Section III discusses the results from the simulation. In the final section, conclusions are derived from the data.

A. Vehicle Model

To analyse the performances, a standard car model has to be defined. Two car models are selected - Quarter car model and IPG multi-body car model.

1) Quarter car Model: The quarter car model consist of a single wheel with no suspension or steering but with the braking present for a single wheel. The car consist of the following parameters:

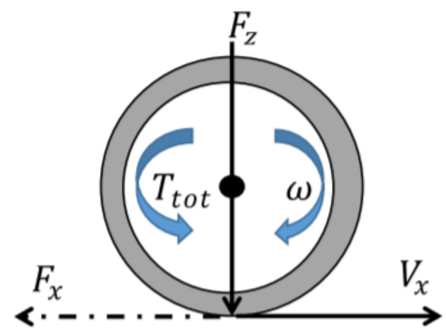

Figure 1. Quarter car dynamic Model

- Moment of inertia of wheel = 1.2 kgm^2
- Max braking torque = 1136 Nm
- Effective radius of wheel = 0.305 m
- Quarter mass of the vehicle = 450 kg
- Friction coefficient = 0.6

B. IPG Car Model

To simulate the controller in IPG with a full car model, an $Audi-TT(4WD-DCT)$, with vehicle mass of $1500kg$, was selected with the built in tire model RT/225/50R17 having an effective radius of $0.32m$. For other relevant parameters of the car for the ABS control, refer (A).

C. Tire Model

Tires are the main element of the braking system and along with its interaction with the road, play an important and key role in the braking performance of the vehicle. If the car is equipped with slick tires and driving on race track, they may attain a braking force of 5g without locking the wheel but the same car on wet roads might lock up the wheels at just 0.5g [3] of braking force. Therefore, race cars change their tire depending on the weather conditions but the passengers car are equipped with the same tire for the whole journey be it on dry asphalt (high grip) or wet mud (low grip) so the presence of ABS for controlling the wheel during the braking maneuver is essential.
A tire is a challenging element to model since the rubber element shows many varying properties depending on the environmental conditions. The empirical tire model introduced

by Pacjeka [4] fits the tire data curve using mathematical functions. However, this tire is tested under limited conditions which cannot always exist on regular roads. In this paper we use Delft TNO tire model for simulation in Simulink with a quarter car model and IPG built in tire model RT/225/50R17 for full car simulation. Figure 2 represents a typical tire data curve for longitudinal slip behaviour at different traction conditions.

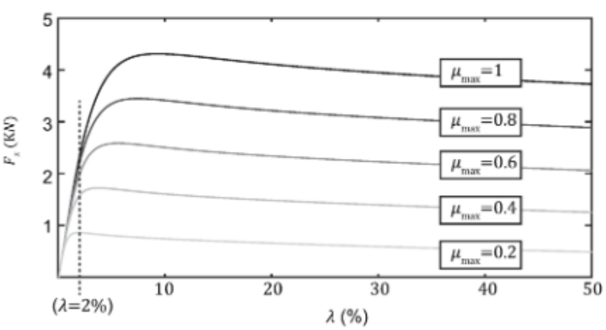

Figure 2. Fx vs Slip ratio for different μ [5]

D. Advanced ABS with continuous wheel slip control

Different control methods are used to control wheel slip necessary for ABS action. One method to control wheel slip is the usage of wheel deceleration method which involves the use of maximum allowed deceleration to control the wheel slip. The traditional ABS control strategy suffers from an inherent flaw. The continuous cycling of the brake pressure to keep the deceleration of the wheel under control prevents the vehicle from braking at the peak friction point, which results in reduced braking performance since the wheel is not always braking at the highest point of friction to ensure maximum longitudinal force. Also, this type of controller cannot be used in the case of combined turning and braking, which is common in many emergency braking situations. To counter this, another strategy called *continuous wheel slip control* is used. In this strategy the braking pressure is controlled by measuring the wheel slip of the tire (indirectly) using various sensors.

II. Control Method - LQR

Since we make use of the LQR control method for ABS controller, we first need to derive the state space equations which define the linear dynamics of the vehicle. As seen in [6], for LQR controller in the linear dynamics, the slip ratio λ_x is given by,

$$\lambda_x = \frac{v_x - \omega r_w}{v_x} \tag{1}$$

and the longitudinal dynamics of the wheel is given by the following equation,

$$\begin{aligned} m_w \dot{v_x} &= -F_x \\ J_w \dot{\omega} &= r_w F_x - T_b \operatorname{sign}(\omega) \end{aligned} \tag{2}$$

The tire friction force is given F_x

$$F_x = F_z \cdot \mu\left(\lambda_x, \mu_H, \alpha\right) \tag{3}$$

Considering only the longitudinal dynamics of the car, the slip angle $\alpha = 0$ can be assumed with $\mu_x = \mu$ and $v_x = x$. Combining equation (1) with (II) for $v > 0$ and $\omega > 0$, we have the following state space equation,

$$\dot{\lambda_x} = -\frac{1}{v_x}\left(\frac{1}{m_w}(1-\lambda_x) + \frac{r_w^2}{J_w}\right) F_z \mu\left(\lambda_x, \mu_H, \alpha\right) + \frac{1}{v_x}\frac{r_w}{J_w} T_b \tag{4}$$

$$\dot{v_x} = -\frac{1}{m_w} F_z \mu\left(\lambda_x, \mu_H, \alpha\right) \tag{5}$$

In this state space equation, $v_x \to 0$, the dynamics of the open loop system becomes infinite with infinite gain. Therefore, this controller should not be used for very low speed applications. Also, since the reaction of velocity of the car is quite slow as compared to the change in the slip ratio, the equation (4) for the modelling of the system is used.

The purpose of the controller is to control the wheel slip during the braking to a set point λ_x^* which is either fixed or varying depending on the tire road friction conditions.

The dynamics of the car during the braking given by equation (4) and (5) defines the control of the car based on the wheel slip and the deceleration of the car. The wheel slip changes more rapidly and is therefore a better candidate to determine the wheel locking as compared to the deceleration of the car which keeping the braking effort to the maximum. This leads to the use of the gain to control the rate of wheel slip $x_1 = \lambda_x - \lambda_x^*$.

To remove the effect of the errors accumulated overtime, we will use the the rate of change of slip error as a state space variable $x_2 = \dot{\lambda_x} - \lambda_x^* = \dot{x_1}$. The equation of the wheel slip control with the integral action is given as :

$$A(v) = \begin{pmatrix} 0 & 1 \\ 0 & \frac{\alpha_1}{v_x} \end{pmatrix}, B(v) = \begin{pmatrix} 0 \\ \frac{\beta_1}{v_x} \end{pmatrix}, W(v) = \begin{pmatrix} 0 \\ \frac{1}{v_x} \end{pmatrix} \tag{6}$$

where α_1 and β_1 are given as

$$\begin{aligned} \alpha_1 &= -F_z^*\left(\frac{1}{m_w}\left(1-\lambda_x^*\right) + \frac{r_w^2}{J_w}\right)\frac{\partial \mu}{\partial \lambda}\left(\lambda_x^*, \mu_H^*, \alpha^*\right) \\ &\quad + F_z^* \frac{1}{m_w}\mu\left(\lambda^*, \mu_H^*, \alpha^*\right) \\ \beta_1 &= \frac{r_w}{J_w} > 0 \end{aligned} \tag{7}$$

Therefore, the cost function of the LQR controller can be computed as,

$$\begin{aligned} J(x(t), u[t,\infty)) = \textstyle\int_t^\infty \Big(&\left(x(\tau) - x^*\right)^T Q(v)\left(x(\tau) - x^*\right) \\ &+ \left(u(\tau) - u^*\right)^T R\left(u(\tau) - u^*\right)\Big) d\tau \end{aligned} \tag{8}$$

For a constant v the feedback gain can be given by

$$\hat{u} = K(v)x \tag{9}$$

where the gain matrix $K(v)$ is given by $K(v) = -R^{-1}B^T(v)P(v)$.

The symmetric equation $P(v)$ is the solution of the algebraic Ricatti equation, in MATLAB which is solved using *idare* function.

$$\begin{aligned} &P(v)A(v) + A^T(v)P(v) \\ &-P(v)B(v)R^{-1}B^T(v)P(v) = -Q(v) \end{aligned} \tag{10}$$

III. Simulation and Results

A. Testing Strategy

As discussed in previous section, the LQR controller for quarter car model was designed and tested in MATLAB and Simulink to check for feasibility and to verify the performance of LQR controller. The controller was tuned to get an optimal performance for the defined conditions and was next implemented in the Simulink section of IPG Carmaker in order to link it to the full car model. This ensures that our controller is set up correctly and makes debugging easier for any errors due to incorrect control method implementation. Further tuning of the controller in IPG was required to obtain satisfactory performance. The controllers were simulated in a laptop having an Intel i7-processor and 16GB of RAM.

B. Quarter car model performance with controllers

Initially a quarter car model was prepared in order to test whether the LQR controller performs better than the PID controller. The details of the car model for both simulations are mentioned in I-A. For LQR controller in IPG, the weights were chosen as $Q = 1000 \times I$ and $R = 0.001$. Reference slip $K_{ref}/\lambda^* = 0.1$ as per paper [7]. A noise of 0.05 was added to λ in the subsequent simulations to check for the controller robustness. The LQR controller activates only when the brake is applied and when vehicle velocity is greater than 1 since it becomes unstable at zero velocity.

C. Full Car IPG Simulation

A track of width 6m was taken with 0.5m shoulder and a total length of 500 m. The vehicle from standstill first accelerates to a speed of $120 km/h$ and maintains it for a distance of 300 m after which it applies maximum brake on different conditions of road having a length of 200 m. The simulations were divided into 3 parts:

- low friction ($\mu = 0.5$)
- high friction ($\mu = 1$)
- μ-split condition (left: $\mu = 0.5$; right: $\mu = 1$)

Wheel speed, vehicle speed and wheel brake torques are reported in the following subsections.

1) High friction condition: For high friction ($\mu = 1$) conditions, the plots are shown below. The wheel velocities are able to track the vehicle velocity quite well and performs better than the PID controller in tracking. With noise, the LQR controller tracks the vehicle velocity as well, however, at low wheel velocities the controller becomes unstable due to poor tire dynamics and sub optimal weight tuning. In the subsequent results, the plots for no controller is not provided. Please refer to [2] for more detailed plots and videos.

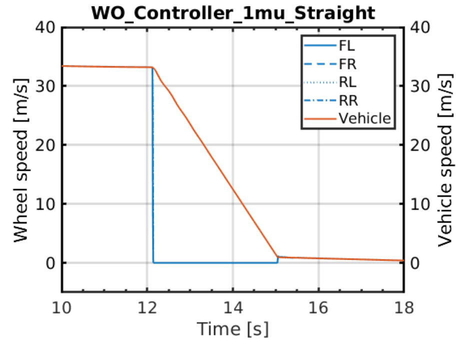

Figure 3. No controller performance at high friction($\mu = 1$) condition

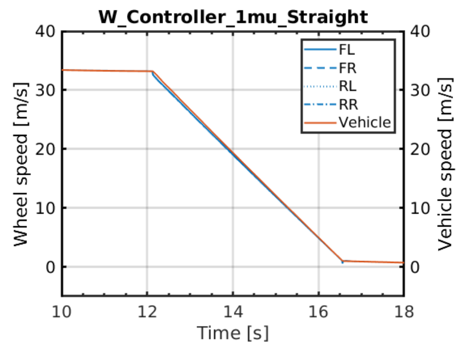

Figure 4. LQR controller performance at high friction($\mu = 1$) condition

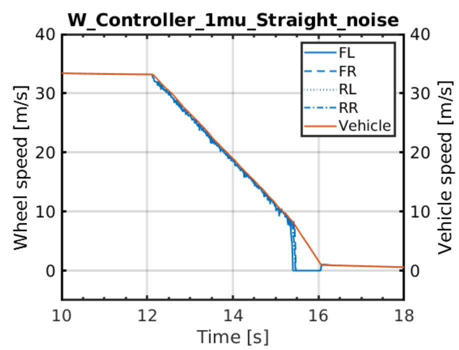

Figure 5. LQR controller - high friction($\mu = 1$) condition with noise

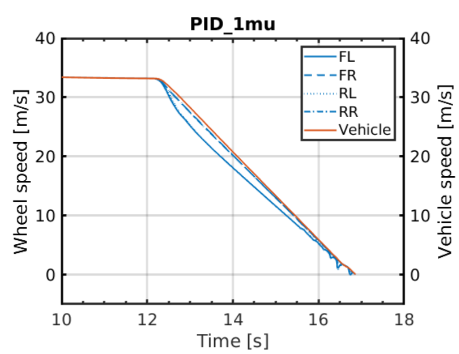

Figure 6. PID controller performance at high friction($\mu = 1$) condition

2) *Low friction condition:* For low friction ($\mu = 0.5$) conditions, the plots are shown below. The wheel velocities are able to track the vehicle velocity quite well and performs better than the PID controller in tracking similar to the high μ condition. With noise, the LQR controller tracks the vehicle velocity up to a certain velocity after which it fails as the controller becomes unstable due to sub optimal weight tuning.

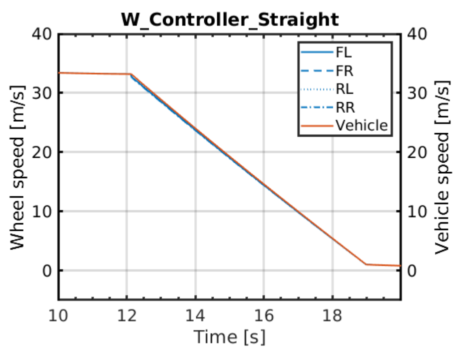

Figure 7. LQR controller performance at low friction($\mu = 0.5$) condition

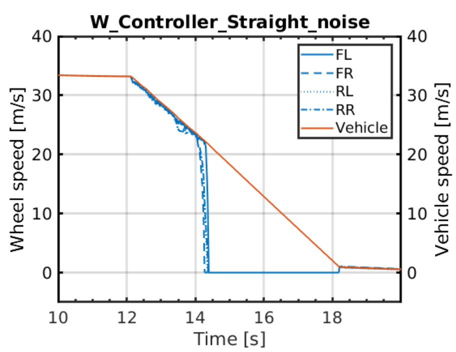

Figure 8. LQR controller - low friction($\mu = 0.5$) condition with noise

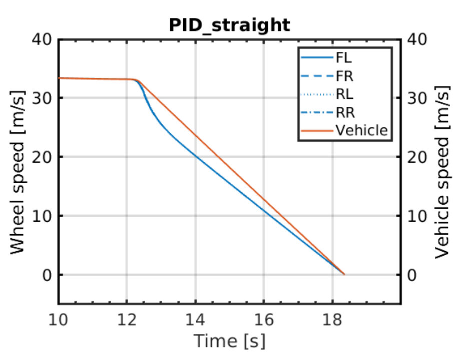

Figure 9. PID controller performance at low friction($\mu = 0.5$) condition

3) *$\mu - split$ condition:* For $\mu - split$ conditions, the plots are shown below. The wheel velocities are able to track the vehicle velocity quite well and performs better than the PID controller in tracking. With noise, the LQR controller tracks the vehicle velocity as well, however, at low wheel velocities the controller becomes unstable due to poor tire dynamics and weight tuning.

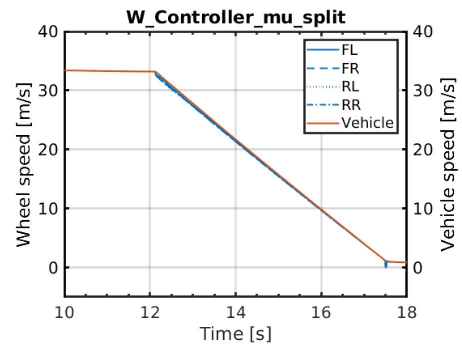

Figure 10. LQR controller performance at $\mu - split$ condition

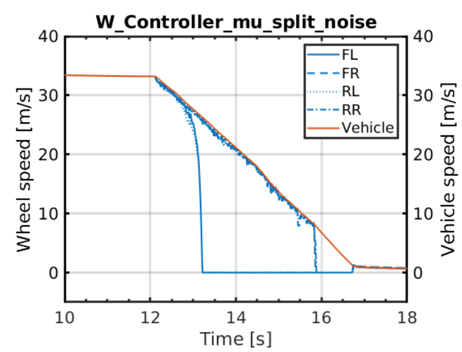

Figure 11. LQR controller - $\mu - split$ condition with noise

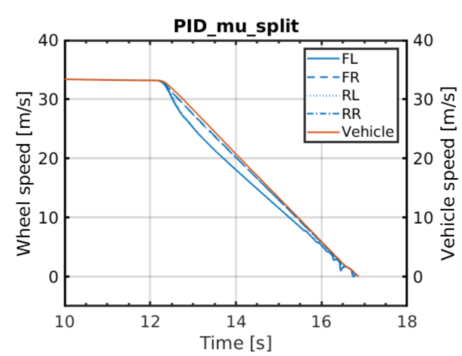

Figure 12. PID controller performance at $\mu - split$ condition

D. Braking torque with controller

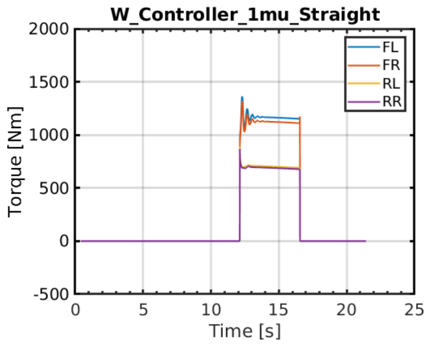

Figure 13. Brake Torque for high friction condition

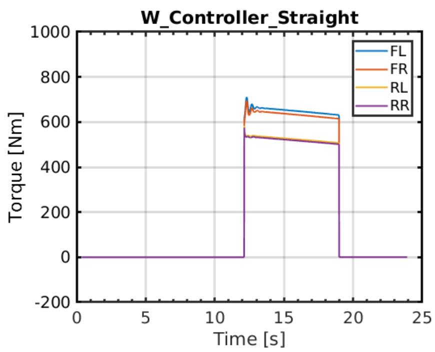

Figure 14. Brake Torque for low friction condition

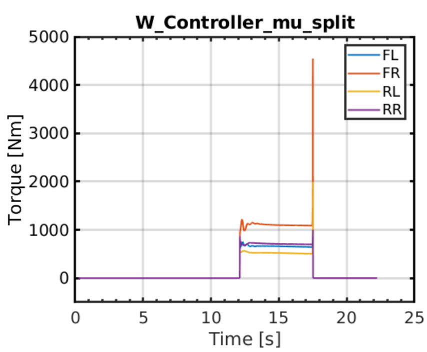

Figure 15. Brake Torque for $\mu - split$ condition

The plots above and below show the variation of brake torque (or brake pressure) with and without noise. There is a sudden increase in torque at the beginning of the maneuver since the car applies brake at maximum force and the controller stabilizes the braking torque.

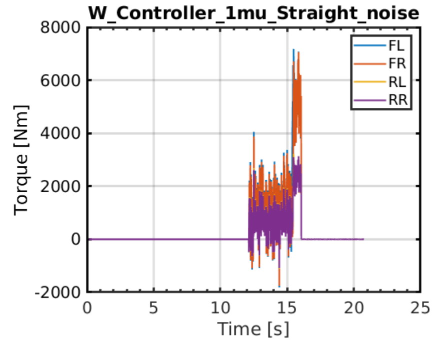

Figure 16. Brake Torque for high friction condition with noise

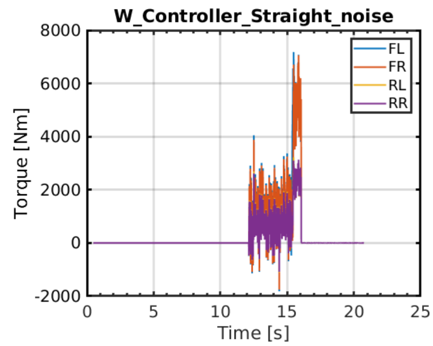

Figure 17. Brake Torque for low friction condition with noise

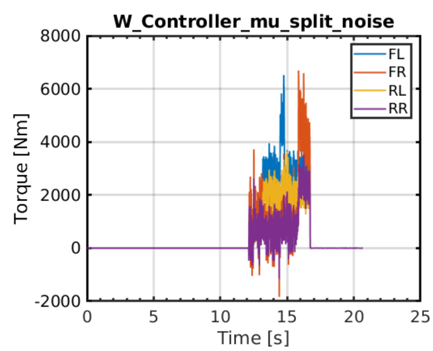

Figure 18. Brake Torque for $\mu - split$ condition with noise

In the plots with noise, there is a large deviation from the actual brake torque due to noise. As the velocity reduces below a certain value, i.e, at low velocities, the controller is unable to handle the unstable tire dynamics and fails.

E. Key performance indicators

Different parameters are used to establish the performance of the controller as stated in [8]. Steady state and transient state is compared between the 2 controllers in different road scenarios.

Table I
KEY PERFORMANCE INDICATORS ON $\mu = 0.5$

KPI criteria	LQR	PID
ABS Index of performance	1.487	1.278
Braking Distance	118.0411 (m)	105.722 (m)
Mean acceleration developed	4.667 (m/s^2)	4.874 (m/s^2)
ABS efficiency	0.952	0.994

Table II
KEY PERFORMANCE INDICATORS ON $\mu = 1$

KPI criteria	LQR	PID
ABS Index of performance	1.487	1.278
Braking Distance	77.926 (m)	66.961 (m)
Mean acceleration developed	7.251 (m/s^2)	7.417 (m/s^2)
ABS efficiency	0.736	0.755

Table III
KEY PERFORMANCE INDICATORS ON μ SPLIT

KPI criteria	LQR	PID
ABS Index of performance	2.510	2.129
Braking Distance	93.806 (m)	79.577 (m)
Mean acceleration developed	5.949 (m/s^2)	7.068 (m/s^2)
ABS efficiency	0.804	0.952

The above performance indicators are calculated only for the noiseless simulations.

IV. CONCLUSIONS

In this report we implement the LQR controller as the ABS controller based on the control logic of continuous wheel slip control in various road conditions. An Initial quarter car model is chosen to set up the basis for the comparison of the controller strategy and later they are implemented in the IPG carmaker. The modeling of the simple LQR controller is done on the a quarter car wheel model and the state space equation and the cost function are established. Later the LQR controller is modelled in Simulink along with the PID controller o test the theory of the controlling strategy.

The performance of LQR is worse than the PID controller for several reasons. The PID controller used in IPG carmaker takes into account the hydraulic modulator / actuator constraints. It also allows for the ABS to work at low speed conditions where the LQR controller would fail. Boundary conditions have also been implemented in the PID controller which enables it to provide realistic brake torque values. The LQR controller however, is much better in terms of velocity tracking as compared to PID controller and when tuned further to include low speeds and actuator constraints, the system will definitely perform better.

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APPENDIX

- v_x = Linear velocity of the car
- ω = Angular velocity of the wheel
- m_w = Mass of the wheel
- J_w = Moment of inertia of the wheel
- r_w = Effective radius of the wheel
- T_b = Braking torque
- λ_x = Longitudinal slip
- α = Slip angle
- μ_H = Road handling friction
- F_z = Normal force on the tire
- F_x = Longitudinal force on the tire
- μ_x = Longitudinal friction coefficient

A. Quarter Car Model

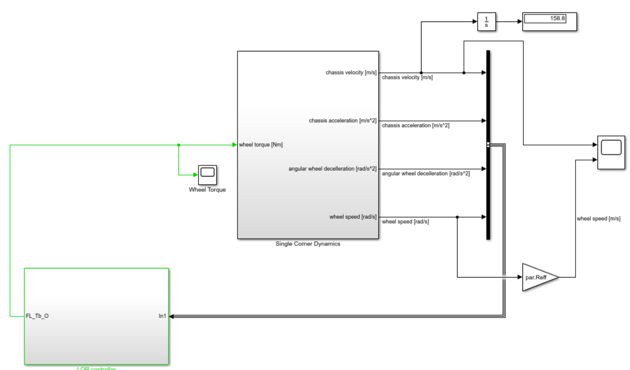

Figure 19. Quarter Car Model in Simulink

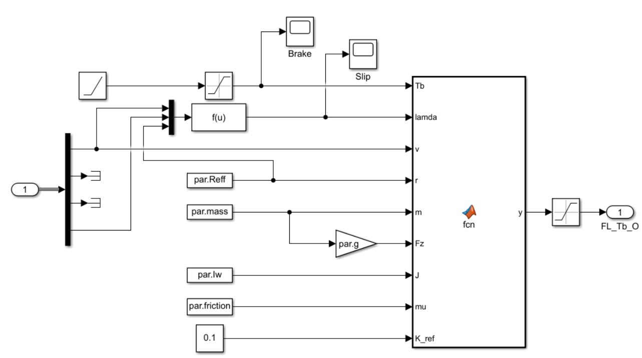

Figure 20. Implementation of LQR Controller

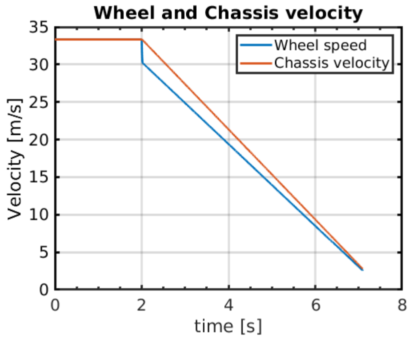

Figure 21. Result of ABS on Quarter Car Model using LQR

VEHICLE PARAMETERS IN IPG

Symbol	Description	Value	Unit
M	total vehicle mass	1499	[kg]
I_{xx}	Inertia along x-axis	430	$[\mathrm{kgm}^2]$
I_{yy}	Inertia along y-axis	1309	$[\mathrm{kgm}^2]$
I_{zz}	Inertia along z-axis	1461	$[\mathrm{kgm}^2]$
r_{eff}	Effective Wheel Radius	0.3035	[m]
m_{rim}	Rim mass	0.15	[kg]
m_{tire}	Tire mass	9.8	[kg]
l_f	Front Wheelbase to cg	1.48	[m]
l_r	Rear Wheelbase t o cg	1.41	[m]
h_{cg}	Height of CoG	0.54	[m]