School of Science and Engineering
Division of Engineering Programs

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EGE416 Control Theory - Spring 2021

COURSE DESIGN PROJECT

Due Date: Thursday, May 6, 2021

Your Names: Niki Fokas 5/5/2021

Rules for submitting design project

- Groups of two students maximum
- 1 submission per group.
- Create a PDF file with **this cover sheet** and your work
- Upload your file to Blackboard → Course Content → Project → Design Project before:
- Deadline: 5 PM on Thursday, May 6, 2021.

Abstract: This project deals with the design of a controller to control the altitude of a quadrotor drone. Design specifications are in terms of a desired altitude step response.

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Nikiforos Fokas

Design of a Phase-Lead Controller for a Quadrotor Drone

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Introduction

The goal of this project is to develop a closed-loop phase-lead controller to regulate the altitude of a quadrotor drone (shown below in Figure 1.1). The drone uses four propellors each driven by a DC motor. The control input is the DC current of the propellor motors, the feedback information is the altitude of the drone. To simplify the design the following assumption was made: all four propellers are always synchronized, thus will they always generate the same amount of lift thrust. With this assumption we can ignore any changes in the pitch, roll, yaw, and horizontal movements of the drone.

Figure 1.1: A Quadrotor Drone [1]

Theory

The Laplace transform is an important mathematical tool for solving differential equations. For this project the Laplace transform was used to transform the various differential equations from the time domain to the s domain. By doing this, while in the s domain we can now use simple algebra to relate all the equations and turn a non-linear system into a linear one. From there a transfer function can be derived with relative ease.

The Root-Locus is used to graphically show how the complex poles of the transfer function change with gain in a closed-loop control system. Using the characteristic equation of a closed loop system 1+G(s)H(s) = 0, G(s)H(s) can be represented at K*N(s)/D(s) where K is the gain, N(s) is the numerator with factored s domain polynomials of n^{th} order, and D(s) is the denominator with factored s domain polynomials of m^{th} order. By subbing in G(s)H(s) in to 1 + K*N(s)/D(s) we get D(s) + KN(s) = 0.

The phase lead controller takes the form of $K_c\frac{aTs+1}{Ts+1}$ and is designed using the Root-Locus (RL) theory since this controller is already in RL form.

Model Linearization

Due to the weight of the drone, the model for the propulsion dynamics is non-linear. The first task (Task 1) is to linearize the model. This is done by calculating the offset values for the current i_0, the propeller torque τ_{p0}, the propeller angular velocity ω_{p0}, and the lift force F_0 that cancels the weight of the drone (W). Figure 1.2 below shows the force diagram of the drone.

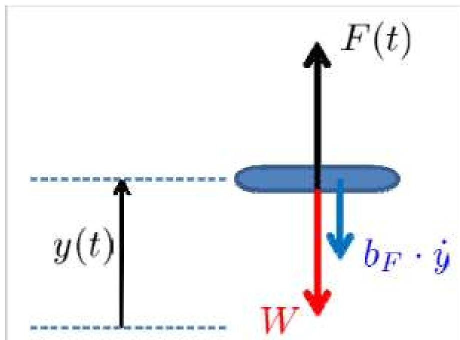


Figure 2.2: Force Diagram for the Drone [1]

Task 1

$$M\ddot{y} = F(t) - b_F\dot{y} - mg$$

$$F(t) = K_f \cdot \omega_p(t) = 1E\text{-}2 \cdot \omega_p(t)$$

$$\tau_p(t) = 2\,i(t)$$

$$J_p\,\dot{\omega}(t) = \tau_p(t) - b_p \cdot \omega_p(t)$$

$$2E\text{-}3\,\dot{\omega}(t) = 2i(t) - 4E\text{-}3\,\omega_p(t)$$

$$F(t) = 1E\text{-}2 \cdot \frac{-2E\text{-}3\,\dot{\omega}(t) + 2i(t)}{4E\text{-}3}$$

$$F(t) = \frac{-2E\text{-}3\,\dot{\omega}(t) + 2\,i(t)}{4E\text{-}1}$$

Figure 3: i_0 Offset Calculation [1]

$$F(0) = \frac{-2E{-3}\,\dot{\omega}(0) + 2\,i(0)}{4E{-1}}$$

$$F(0) = \frac{2 i_0}{.4}$$

$$F(0) = 5\, i_0$$

$$\parallel$$

$$9.8 \cdot .5 = 5\, i_0$$

$$i_0 = .98\text{ A}$$

$$\tau_{p_0} = 1.96\text{ N}\cdot\text{m}$$

$$\omega_{p_0} = \frac{1.96}{b_p} = \frac{1.96}{4E{-3}} = 490\ \frac{\text{rad}}{\text{s}}$$

$$F_0 = 490 \cdot 1E{-2} = 4.9\text{ N}$$

Figure 4: Calculated Offset Values [1]

Table 1: Calculated Offset Values

Offset	Value	Units
i_0	.98	A
τ_{p0}	1.96	N*m
ω_{p0}	490	Radians/s
F_0	4.9	N

Controller Design

The controller is designed based on Figure 2 below.

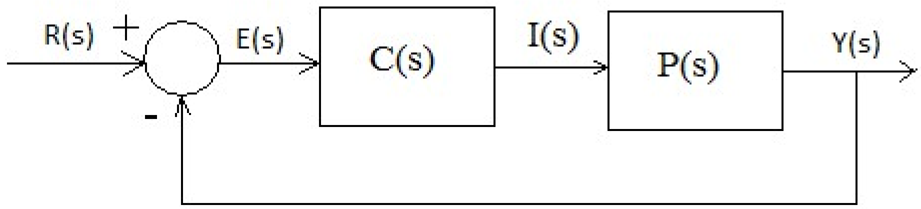


Figure 2: Closed Loop Control System [1]

Task 2

Figures 2.1 – 2.5 shown below are the calculations to compute the transfer function of block P(s) in Figure 2.

$$M\ddot{y} = f(t) - b_F\dot{y}$$

$$\mathcal{L}\left[M\ddot{y} = f(t) - b_F\dot{y}\right]$$

$$\Rightarrow M\cdot s^2 Y(s) = F(s) - b_F\cdot s\, Y(s)$$

$$F(t) = K_f\cdot\omega_p(t)$$

$$\mathcal{L}\left[F(t) = K_f\cdot\omega_p(t)\right]$$

$$\Rightarrow F(s) = K_f\cdot W_p(s)$$

Figure 2.1: Flight Dynamics Laplace Transforms [1]

$$J_p\dot{\omega}(t) = \tau_p(t) - b_p\cdot\omega_p(t)$$

$$\mathcal{L}\left[J_p\dot{\omega}_p(t) = \tau_p(t) - b_p\cdot\omega_p(t)\right]$$

$$\Rightarrow J_p\cdot s\, W_p(s) = \tau(s) - b_p W_p(s)$$

$$\Rightarrow (J_p s + b_p)W_p(s) = \tau(s)$$

$$\Rightarrow \omega_p(s) = \frac{\tau(s)}{J_p s + b_p}$$

Figure 2.2: Propulsion Dynamics Laplace Transforms [1]

$$\tau_p(t) = K_\tau \cdot i(t)$$
$$\mathcal{L}\left[\tau_p(t) = K_\tau \cdot i(t)\right]$$
$$\tau_p(s) = K_\tau \cdot I(s)$$
$$\omega_p(s) = \frac{K_\tau \cdot I(s)}{J_p \cdot s + b_p}$$

Figure 2.3: Propulsion Dynamics Laplace Transforms Cont. [1]

$$F(s) = \frac{K_F \cdot K_\tau \cdot I(s)}{J_p \cdot s + b_p}$$

$$M \cdot s^2 Y(s) = F(s) - b_F \cdot s\, Y(s)$$
$$\Rightarrow M \cdot s^2 Y(s) = \frac{K_F \cdot K_\tau \cdot I(s)}{J_p \cdot s + b_p} - b_F \cdot s\, Y(s)$$
$$(Ms^2 + b_F s)\, Y_s\ \frac{K_F \cdot K_\tau \cdot I(s)}{J_p \cdot s + b_p}$$
$$Y(s) = \frac{K_F \cdot K_\tau \cdot I(s)}{(Ms^2 + b_F \cdot s)(J_p s + b_p)}$$

Figure 2.4: Transfer Function Computation [1]

$$\frac{Y(s)}{I(s)} = \frac{K_F \cdot K_\tau}{(Ms^2 + b_F \cdot s)(J_p s + b_p)}$$
$$= \frac{1E{-}2 \cdot 2}{(.5s^2 + 5s)(2E{-}3 \cdot s + 4E{-}3)}$$
$$= \frac{2E{-}2}{.001 \cdot s(s+2)(s+10)}$$
$$= \frac{20}{s(s+2)(s+10)}$$

Figure 2.5: Plant Transfer Function [1]

Phase-Lead Controller Design

The design specification requires an Overshoot (OS) less than 5%, and a transient time of about 2.5 seconds.

Task 3

Task 3 is the design process of the phase lead controller (block C(s) in Figure 1) using the Root-Locus (RL) technique and zero-pole cancellation method with the following parameters OS = 4.3, and t_t = 2.5 seconds. The calculations are shown below in Figures 3.1 – 3.5, the results of the calculation are shown below in Table 2.

$$C(s) = K_c$$

$$P(s) = \frac{20}{s(s+2)(s+10)}$$

Design specs:

$$OS \approx 4.3\%,\ t_t = 2.5\,(s)$$

$$.043 = e^{-\pi \frac{a}{\beta}}$$

$$2.5 \approx \frac{5}{a}$$

$$a = 2$$

$$Ln(.043) = -\pi \cdot \frac{2}{\beta}$$

$$1 = \frac{2}{\beta}$$

$$\beta = 2$$

$$s = -a \pm j\beta$$

$$s = -2 \pm 2j$$

Figure 5.1: Finding the Desired S [2]

$$G(s) = C(s) \cdot P(s)$$

$$= K_c \frac{20}{s(s+2)(s+10)}$$

Evans

$$K_E = 20 \cdot K_c$$

$$\frac{K_E}{s(s+2)(s+10)} = -1$$

$n = 3$

$m = 0$

$$\rightarrow \frac{s + (s-2) + (s-10)}{n-m} =$$

$$\sigma_c = \frac{0 + -2 - 10}{3} = -4$$

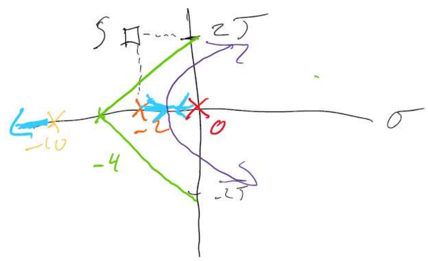


Figure 3.2: Calculation for the RL Plot [3]

Find Y

$$\measuredangle(s-0) + \measuredangle(s+2) + \measuredangle(s+10) = 180 - Y$$

$$\measuredangle(s-0) = 2.5,\ 2,\ 45^\circ = 180 - 45 = 135^\circ$$

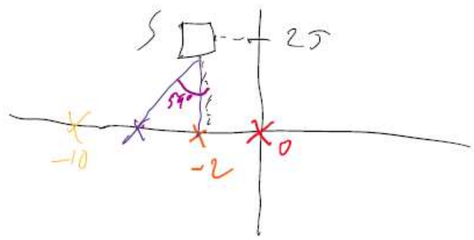
$$\measuredangle(s+2) = 90^\circ$$

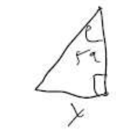
$$\measuredangle(s+10) = 2.5,\ 8 = 14.03^\circ \approx 14^\circ$$

$$135 + 90 + 14 = 180 - Y$$

$$Y = 59^\circ$$

Figure 3.3: Computing the Y for the Angle Condition [3]





$59 = \tan^{-1}\left(\frac{x}{2}\right)$

$\tan(59)\cdot 2 = x$

$x = 3.32$

$X = -2 - 3.32 = -5.32$

$P_c = -5.32 = -\frac{1}{T} \Rightarrow T = \frac{1}{5.32} = .188$

$Z_c = -2 = -\frac{1}{aT} \Rightarrow a = \frac{1}{.188\cdot 2} = 2.66$

$$C(s) = aK_c \frac{s + \frac{1}{aT}}{s + \frac{1}{T}}$$

$$C(s) = 2.66K_c \frac{s+2}{s+5.32}$$

Figure 3.6: Using Y to find a and T for C(s) [3]

$$G(s) = C(s) \cdot P(s)$$

$$\Rightarrow 2{,}66\, K_c \frac{s+2}{s+5{,}32} \cdot \frac{20}{s(s+2)(s+10)}$$

$$K_E = \frac{|s+10| \cdot |s+2| \cdot |s+0|\, |s+5{,}32|}{|s+2|}$$

$$\frac{|s+10| \cdot \cancel{|s+2|} \cdot |s+0|\, |s+5{,}32|}{\cancel{|s+2|}}$$

$$= \left(\sqrt{8^2+2^2}\right)\left(\sqrt{2^2+2^2}\right)\left(\sqrt{3.32^2+2^2}\right)$$

$$= 90.4$$

$$K_E = 20 \cdot 2.66 \cdot K_c$$

$$K_c = \frac{90.4}{20 \cdot 2.66}$$

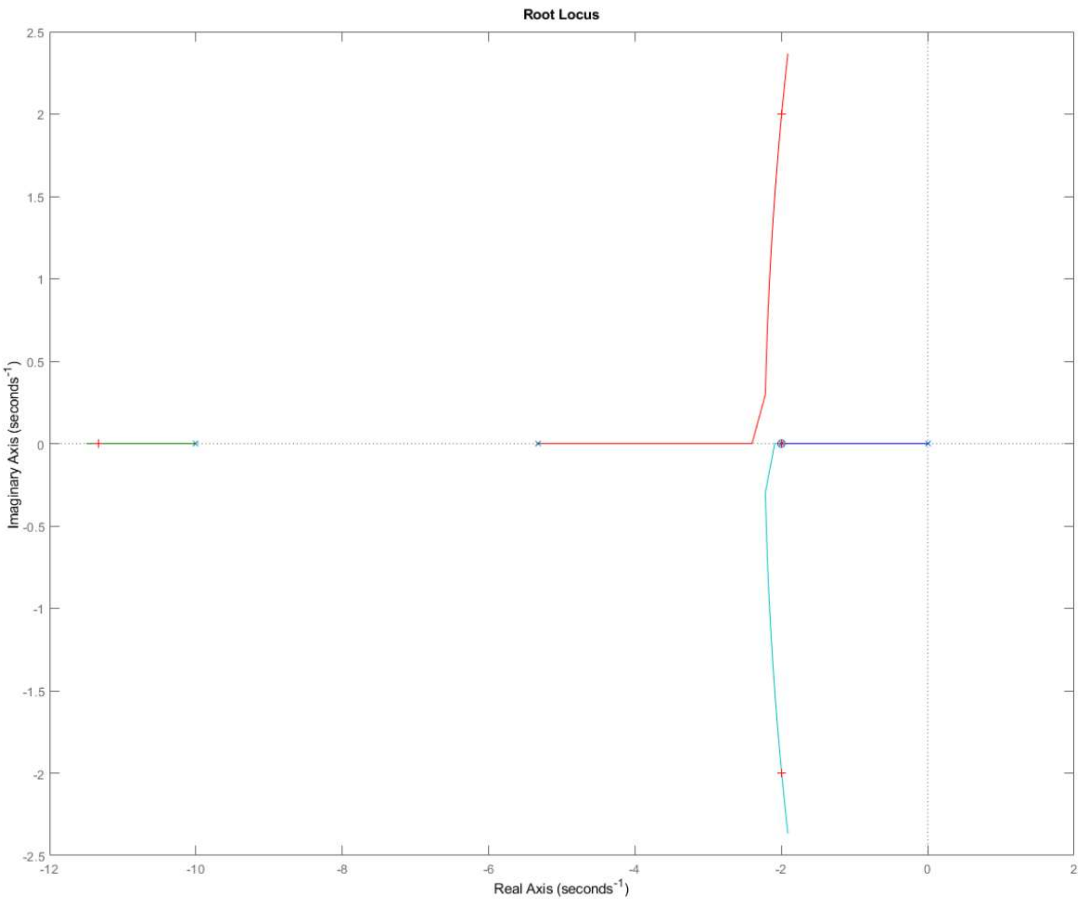
$$K_c = 1{,}7$$

Figure 3.7:Calculating the Controller Gain K_c [2]

Table 2: Calculated Parameters for C(s)

Variable	Value
a	2.66
T	.188
K_c	1.7

MATLAB Design Verification




```
selected_point = -1.9972 + 2.0000i
k = 1.6994
```

Figure 3.8:Root-Locus Plot with K_c

Figure 3.8 is the Root-Locus plot of the phase-lead controller with S = 2±2j and a controller gain K_c of 1.6994.

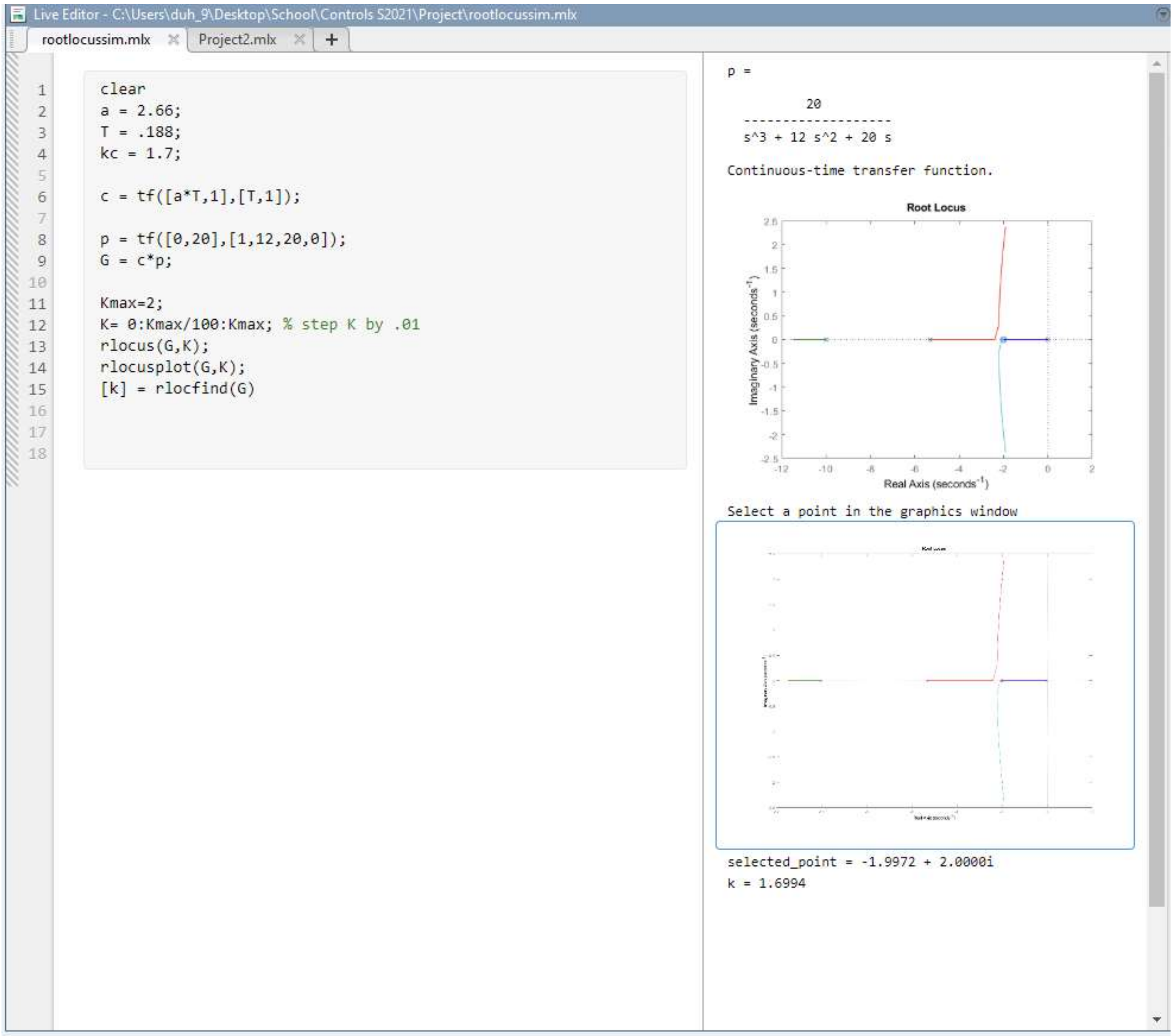


Figure 3.9: MATLAB Code with Output

Figure 3.9 shows the MATLAB code and the output for verification of the Root-Locus plot in Figure 3.8 [4]

Task 4

The final task is to simulate the step response of the completed controller design and compare it to the desired overshoot and transient time. This is shown below in Figure 4.1, Figure 4.2, and Table 3

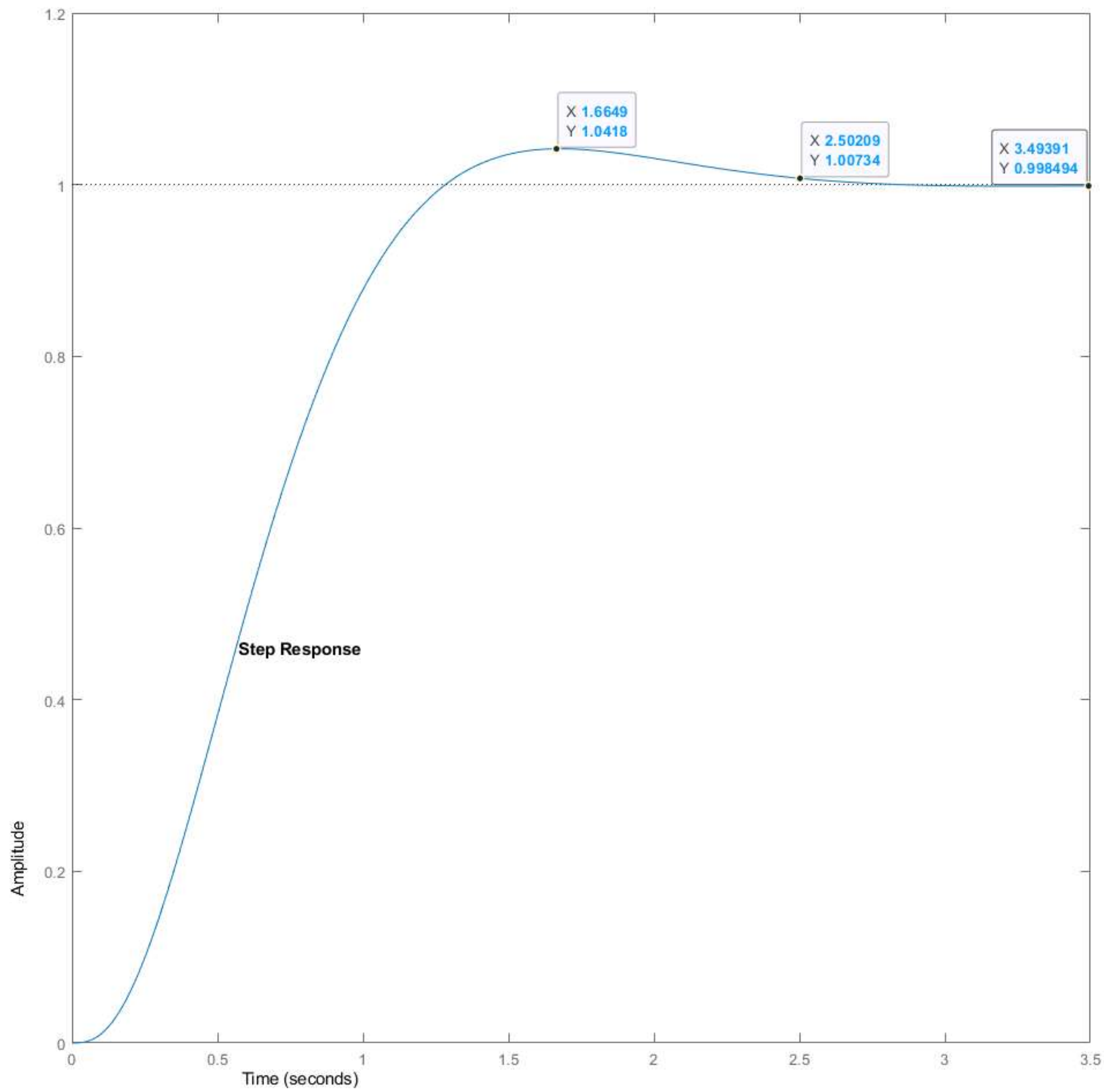


Figure 4.1: Controller Step Response

Figure 4.1 shows the peak value of the Overshoot at t = 1.6649 seconds with a value of 4.18% as well as a transient time (t_t) of 2.50209 seconds at Y = 1.00734.

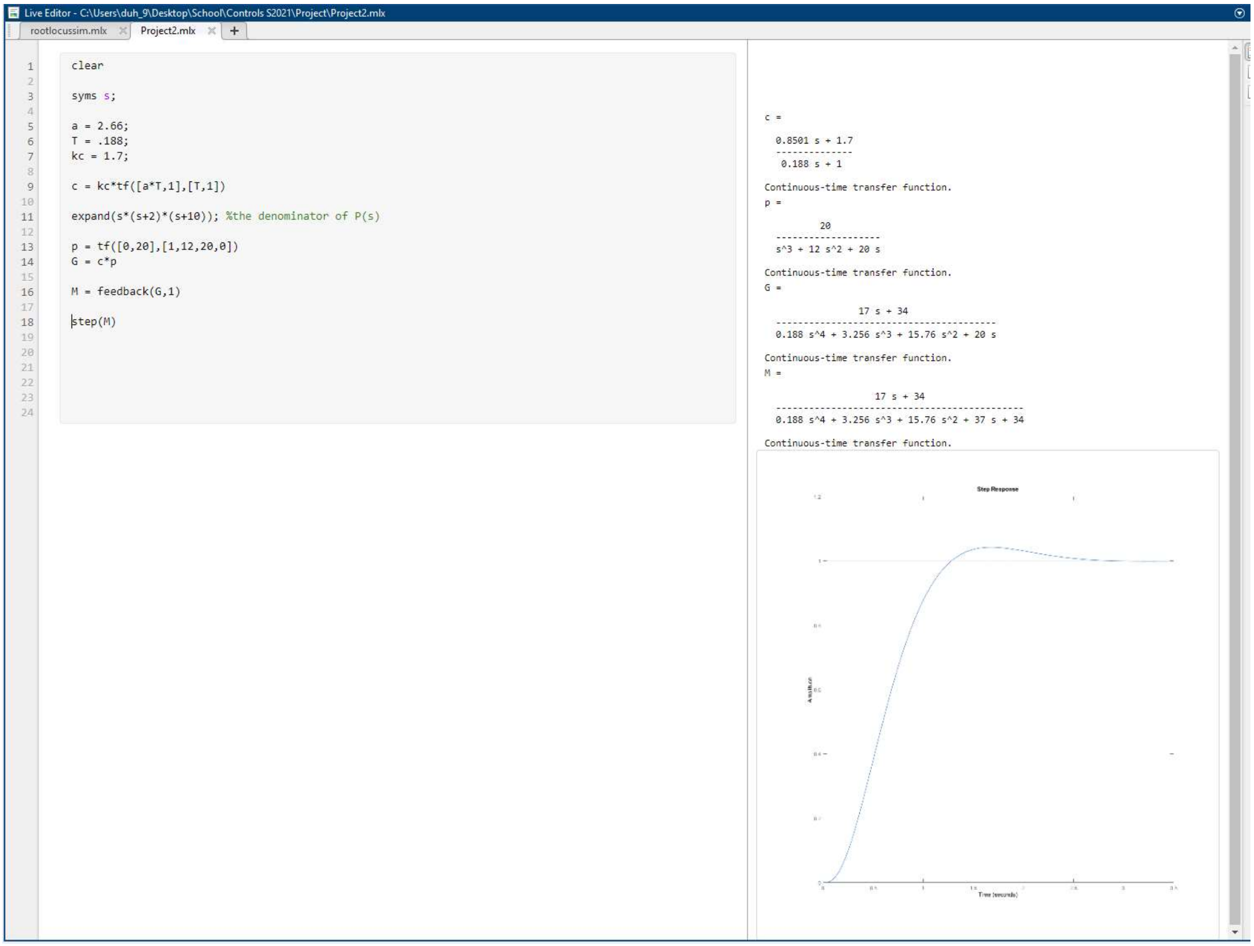


Figure 4.2: MATLAB Code for Task 4

Figure 4.2 show the MATLAB code and the output for verification of the plot in Figure 4.1

Table 3: Tabulated Results from Tasks 3&4

Parameter	Calculated	Simulated	Error
K_c	1.7	1.6994	.035 %
OS	4.3	4.18	2.79 %
t_t	2.5	~2.5	Insignificant

Conclusion

The calculated controller gain K_c was verified by the MATLAB Root-Locus simulation. The simulated step response agreed with the theoretical expectations because the simulated transient time matched the expected, and the simulated overshoot had an error of less than 3 %; this error is within expectation.

References

[1] J. Gonzalez, “S21_Control_Project”, [PDF]. Accessed on: April 10, 2021.

[2] J. Gonzalez, "Phase-Lead Controller", [*PowerPoint* slides]. Accessed on: April 10, 2021.

[3] J. Gonzalez, "Phase-Lead Controller- Bisector Approach", [*PowerPoint* slides]. Accessed on: April 10, 2021.

[4] J. Gonzalez, "Phase-Lead Controller- Bisector Approach with MATLAB Simulation", [*PowerPoint* slides]. Accessed on: April 10, 2021.