Lesson 11

Representing Small Numbers on the Number Line

Goals

- Coordinate (orally and in writing) decimals and multiples of powers of 10 representing the same small number.
- Use number lines to represent (orally and in writing) small numbers as multiples of powers of 10 with negative exponents.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Lesson Narrative

In this lesson, students use the number line and negative exponents to explore very small numbers. They attend to precision when deciding how to label the powers of 10 on the number line and how to plot numbers correctly. Students write very small numbers as a multiple of a power of 10 and as a decimal value. While students explore different ways to express these numbers as a multiple of a power of 10, the use of a number line encourages the form $b \cdot 10^n$ where b is a value between 1 and 10.

Student Learning Goal

Let's visualize small numbers on the number line using powers of 10.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	20 min	10 min	5 min

Warm-up
Small Numbers on a Number Line

5 min

Activity Narrative

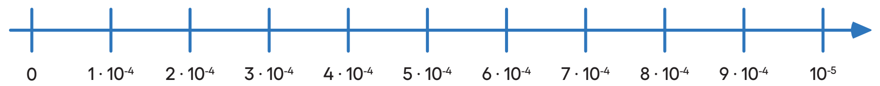
In this *Warm-up* students reason about expressions with negative exponents on a number line. They attend to the meaning of the numbers used to label the tick marks of the number line and how their values are related to the tick marks directly to the right or left.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Kiran drew this number line.



Andre said, "That doesn't look right to me."

Explain why Kiran is correct or explain how he can fix the number line.

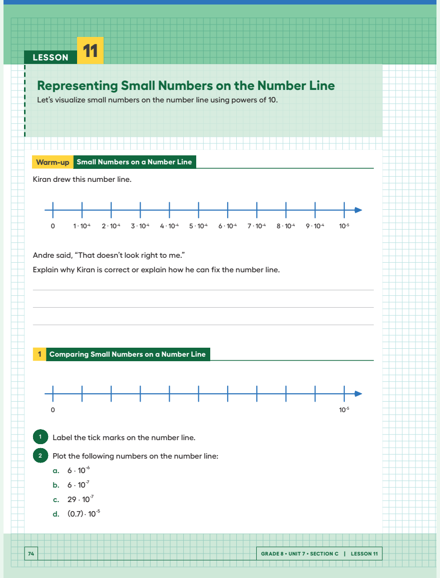
Sample response: Kiran can change each instance of 10^{-4} into 10^{-6} or change 10^{-5} to 10^{-3}.

Activity Synthesis

The goal of this discussion is to highlight the idea that the larger the size (or absolute value) of a negative exponent, the closer the value of the expression is to zero. This is because the negative exponent indicates the number of factors that are $\frac{1}{10}$. For example, 10^{-5} represents 5 factors that are $\frac{1}{10}$, and 10^{-6} represents 6 factors that are $\frac{1}{10}$, so 10^{-6} is 10 times smaller than 10^{-5}.

Invite students to share their reasoning about whether the number line is correct or not. If not brought up in students' explanations, show at least two correct ways in which the number line can be fixed — by changing the exponents for each tick mark to be one power of 10 smaller than 10^{-5} (10^{-6}), by changing 10^{-5} to be one power of 10 greater than the tick marks (10^{-3}), or by changing all of the exponents so that the tick marks are one power of 10 less than the power of 10 at the end of the number line.

Student Workbook


LESSON 11

Representing Small Numbers on the Number Line

Let's visualize small numbers on the number line using powers of 10.

Warm-up Small Numbers on a Number Line

Kiran drew this number line.

Andre said, "That doesn't look right to me."

Explain why Kiran is correct or explain how he can fix the number line.

1 Comparing Small Numbers on a Number Line

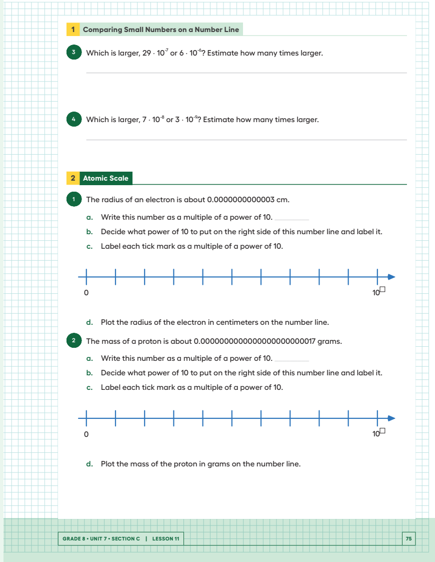
1 Label the tick marks on the number line.

2 Plot the following numbers on the number line:

Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.
Display sentence frames to to support students as they describe how they placed the values on the number line: "First, I ____ because …" "I noticed ____ so I …"
Advances: Speaking, Conversing

Student Workbook


1 Comparing Small Numbers on a Number Line

3 Which is larger, $29 \cdot 10^{-7}$ or $6 \cdot 10^{-6}$? Estimate how many times larger.

4 Which is larger, $7 \cdot 10^{-8}$ or $3 \cdot 10^{-9}$? Estimate how many times larger.

2 Atomic Scale

1 The radius of an electron is about 0.0000000000003 cm.

a. Write this number as a multiple of a power of 10. ______

b. Decide what power of 10 to put on the right side of this number line and label it.

c. Label each tick mark as a multiple of a power of 10.

0 $10^{\square}$

d. Plot the radius of the electron in centimeters on the number line.

2 The mass of a proton is about 0.0000000000000000000000017 grams.

a. Write this number as a multiple of a power of 10. ______

b. Decide what power of 10 to put on the right side of this number line and label it.

c. Label each tick mark as a multiple of a power of 10.

0 $10^{\square}$

d. Plot the mass of the proton in grams on the number line.

GRADE 8 • UNIT 7 • SECTION C | LESSON 11 75

Activity 1

Comparing Small Numbers on a Number Line

10 min

Activity Narrative

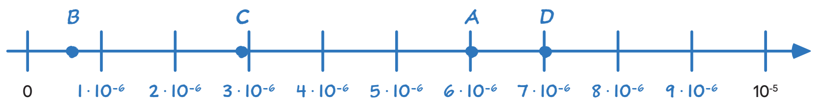
This activity encourages students to use the number line to make sense of powers of 10 and to think about how to rewrite expressions for very small numbers. Students use the structure of the number line to compare numbers and extend that understanding to estimate relative sizes of other numbers when no number lines are given.

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time, followed by a partner discussion and whole-class discussion. During the partner discussion, tell students to share their responses for the first two questions and to reach an agreement about where the numbers should be placed on the number line.

Student Task Statement

1. Label the tick marks on the number line.



2. Plot the following numbers on the number line:

A. $6 \cdot 10^{-6}$

B. $6 \cdot 10^{-7}$

C. $29 \cdot 10^{-7}$

D. $(0.7) \cdot 10^{-5}$

3. Which is larger, $29 \cdot 10^{-7}$ or $6 \cdot 10^{-6}$? Estimate how many times larger.

$6 \cdot 10^{-6}$ is about twice as large as $29 \cdot 10^{-7}$ because $29 \cdot 10^{-7} = (2.9) \cdot 10^{-6}$, which is roughly $3 \cdot 10^{-6}$.

4. Which is larger, $7 \cdot 10^{-8}$ or $3 \cdot 10^{-9}$? Estimate how many times larger.

$7 \cdot 10^{-8}$ is roughly 20 times as large as $3 \cdot 10^{-9}$ because 7 is roughly twice as much as 3, and 10^{-8} is 10 times as much as 10^{-9}.

Activity Synthesis

The goal of this discussion is for students to articulate how to rewrite very small numbers written using one power of 10 with a different power of 10.

Invite students to share how they rewrote $29 \cdot 10^{-7}$ using 10^{-6}. If not brought up in students' explanations, consider asking the following series of questions:

- *"What can 10^{-7} be multiplied by to get 10^{-6}?"*

  10 because the exponent is increasing by 1

- *"If 10^{-7} is multiplied by 10 to get 10^{-6}, the value of the expression will change. What can be done to keep the value of the expression the same?"*

  Divide the other factor by 10.

- *"What is the resulting equivalent expression?"*

  $2.9 \cdot 10^{-6}$

- *"How can $0.0123 \cdot 10^{-4}$ be written using 10^{-6}?*

  Since 10^{-4} is being divided by 10^2, the factor 0.0123 needs to be multiplied by 10^2, resulting in $1.23 \cdot 10^{-6}$.

Activity 2
Atomic Scale

20 min

Activity Narrative

In this activity, students convert small decimals into a multiple of a power of 10 with a negative exponent for the first time. The first problem leads to a product of an integer and a power of 10, and the second leads to a product of a decimal and a power of 10. Again, students build experience with scientific notation before the term is formally introduced.

Launch

Ask students to think of different ways the decimal 0.03 can be written (3 hundredths, $\frac{3}{100}$). If not brought up by students introduce the idea that 0.03 can also be written as $3 \cdot \frac{1}{100}$ or $3 \cdot \frac{1}{10} \cdot \frac{1}{10}$. So $0.03 = 3 \cdot \frac{1}{10} \cdot \frac{1}{10} = 3 \cdot 10^{-2}$.
Arrange students in groups of 2. Give students 10 minutes to work, followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
Chunk this task into more manageable parts to support use of structure. For example, check in with students within the first 2–3 minutes of work time. Ask students to share how they decide what power of 10 to put on the right side of this number line.
Supports accessibility for: Visual-Spatial Processing; Organization

Building on Student Thinking

If students write the mass of a proton as $17 \cdot 10^{-25}$ but find that there are not enough tick marks on the number line, consider asking:

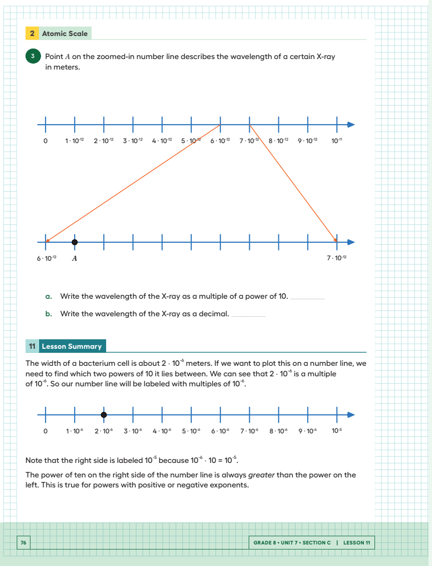
"Would 1.7 fit on the given number line?"

Yes.

"How could the expression be written with 1.7 as one of the factors while still keeping the value of the expression the same?"

Divide 17 by 10 and multiply 10^{-25} by 10, resulting in $(1.7) \cdot 10^{-24}$.

Student Workbook



2 Atomic Scale

3 Point A on the zoomed-in number line describes the wavelength of a certain X-ray in meters.

a. Write the wavelength of the X-ray as a multiple of a power of 10. ______

b. Write the wavelength of the X-ray as a decimal. ______

11 Lesson Summary

The width of a bacterium cell is about $2 \cdot 10^{-6}$ meters. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that $2 \cdot 10^{-6}$ is a multiple of 10^{-6}. So our number line will be labeled with multiples of 10^{-6}.

Note that the right side is labeled 10^{-5} because $10^{-6} \cdot 10 = 10^{-5}$.

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

76 GRADE 8 • UNIT 7 • SECTION C | LESSON 11

Student Task Statement

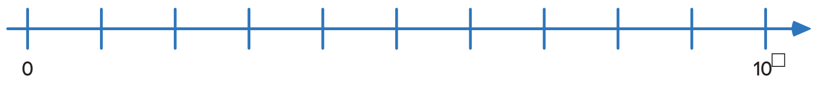
1. The radius of an electron is about 0.0000000000003 cm.

   a. Write this number as a multiple of a power of 10. $3 \cdot 10^{-13}$ cm

   b. Decide what power of 10 to put on the right side of this number line and label it.

   The power on the right side should be 10^{-12}.

   c. Label each tick mark as a multiple of a power of 10.



   The tick marks should be labeled in multiples of 10^{-13}.

   d. Plot the radius of the electron in centimeters on the number line.

   The radius of the electron in cm should be placed at the 3rd tick mark.

2. The mass of a proton is about 0.0000000000000000000000017 grams.

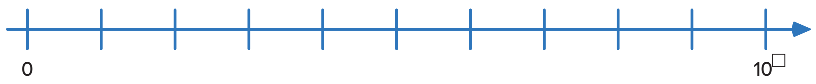
   a. Write this number as a multiple of a power of 10. $(1.7) \cdot 10^{-24}$ grams

   b. Decide what power of 10 to put on the right side of this number line and label it.

   The power on the right side should be 10^{-23}.

   c. Label each tick mark as a multiple of a power of 10.

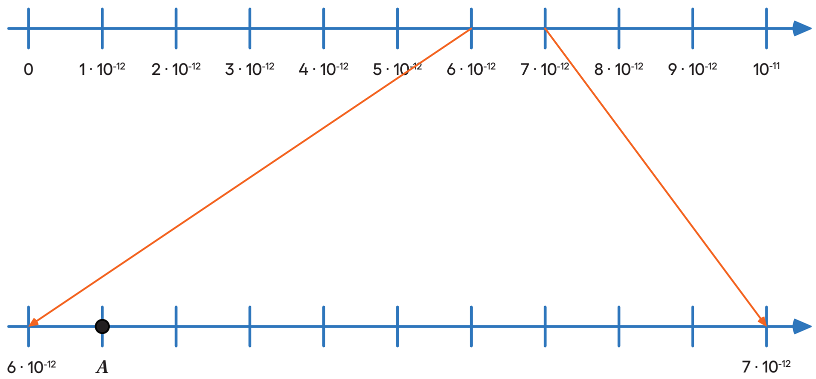
   The tick marks should be labeled in multiples of 10^{-24}.



   d. Plot the mass of the proton in grams on the number line.

   The mass of the proton in grams should be placed between the 1st and 2nd tick marks, closer to the 2nd than to the 1st.

3. Point A on the zoomed-in number line describes the wavelength of a certain X-ray in meters.



   a. Write the wavelength of the X-ray as a multiple of a power of 10.

   $(6.1) \cdot 10^{-12}$ meters

   b. Write the wavelength of the X-ray as a decimal.

   0.0000000000061 meters

Activity Synthesis

The goal of this discussion is for students to share the strategies they used when writing small decimals as a multiple of a power of 10. Some students may describe a strategy where they "move" the decimal point or count decimal places. Encourage students to develop a more precise explanation by emphasizing that each time a number is multiplied by $\frac{1}{10}$, its value decreases by one place value. So for example, the value 0.0000003 is equal to $3 \cdot 10^{-7}$ because 3 has been multiplied by $\frac{1}{10}$ seven times.

If time allows, have students practice writing decimals as a multiple of a given power of 10. Note that students are not expected to write these values using scientific notation at this time.

- *"What is 0.00014 written as a multiple of 10^{-5}?"*

  $14 \cdot 10^{-5}$

- *"What is 0.0000008 written as a multiple of 10^{-7}?"*

  $8 \cdot 10^{-7}$

- *"What is 0.00256 written as a multiple of 10^{-3}?"*

  $(2.56) \cdot 10^{-3}$

Lesson Synthesis

The purpose of this discussion is to compare representing very small numbers with representing very large numbers using multiples of powers of 10 or on a number line. One important concept to highlight is that it's always possible to change an expression that is a multiple of a power of 10 so that the leading factor is between 1 and 10.

Here are some questions for discussion:

- *"Given two points on the number line, how can we tell which point represents a larger value?"*

  The number on the right will always be larger.

- *"Would 10^5 appear to the left or to the right of 10^4 on a number line? Why?"*

  10^5 is larger than 10^4, so it would be to the right.

- *"Would 10^{-5} appear to the left or to the right of 10^{-4} on a number line? Why?"*

  10^{-5} is smaller than 10^{-4}, so it would be to the left.

- *"How does zooming in on the number line help express numbers between the tick marks?"*

  Zooming in allows us to subdivide the distance between two tick marks into 10 equal intervals, which allows us to to describe a number to an additional level of precision.

- *"Describe how to convert a number such as 0.000278 into a multiple of a power of 10."*

  The number is equivalent to $278 \cdot (0.000001)$ or $278 \cdot \frac{1}{100{,}000}$. The fraction $\frac{1}{100{,}000}$ is $\frac{1}{10^6}$ or 10^{-6}, so 0.000278 can be written as $278 \cdot 10^{-6}$.

If time allows, give students other small numbers that are written as decimals and ask them to write them as multiples of powers of 10, and vice versa.

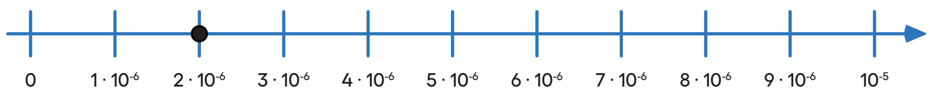
Responding To Student Thinking

Points to Emphasize
If most students struggle with writing small numbers as a multiple of a power of 10, focus on having students explain how they made their matches for cards with a negative exponent in the activity referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 7, Lesson 13, Activity 3 Card Sort: Scientific Notation Matching

Lesson Summary

The width of a bacterium cell is about $2 \cdot 10^{-6}$ meters. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that $2 \cdot 10^{-6}$ is a multiple of 10^{-6}. So our number line will be labeled with multiples of 10^{-6}.



Note that the right side is labeled 10^{-5} because $10^{-6} \cdot 10 = 10^{-5}$.

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

Cool-down

Describing Very Small Numbers

5 min

Student Task Statement

1. Write 0.00034 as a multiple of a power of 10.

   $(3.4) \cdot 10^{-4}$, $34 \cdot 10^{-5}$ (or equivalent)

2. Write $(5.64) \cdot 10^{-7}$ as a decimal.

   0.000000564

Lesson **11**

Practice Problems

5 Problems

Problem 1

Select **all** the expressions that are equal to $4 \cdot 10^{-3}$.

A. $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$

B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$

C. $4 \cdot 0.001$

D. $4 \cdot 0.0001$

E. 0.004

F. 0.0004

Problem 2

Write each expression as a multiple of a power of 10:

a. 0.04 $4 \cdot 10^{-2}$ (or equivalent)

b. 0.072 $72 \cdot 10^{-3}$ (or equivalent)

c. 0.0000325 $325 \cdot 10^{-7}$ (or equivalent)

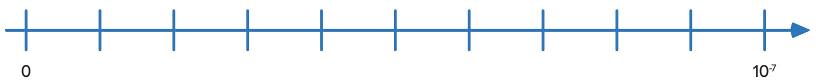
d. Three thousandths $3 \cdot 10^{-3}$ (or equivalent)

e. 23 hundredths $23 \cdot 10^{-2}$ (or equivalent)

f. 729 thousandths $729 \cdot 10^{-3}$ (or equivalent)

g. 41 millionths $41 \cdot 10^{-6}$ (or equivalent)

Problem 3



Plot the following points on the number line:

a. 10^{-8}

10^{-8} is the first tick mark after 0.

b. $3 \cdot 10^{-8}$

$3 \cdot 10^{-8}$ is the third tick mark after zero.

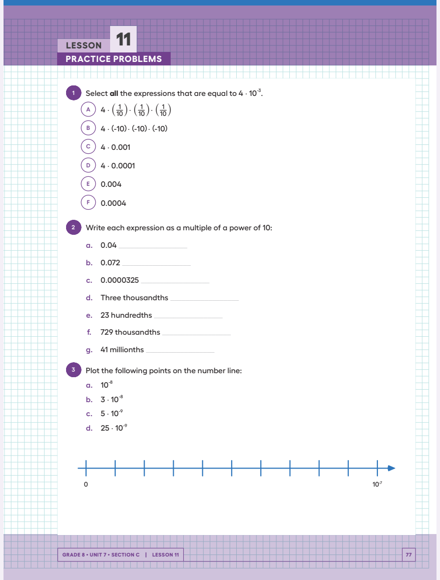
c. $5 \cdot 10^{-9}$

$5 \cdot 10^{-9}$ is halfway between zero and the next tick mark.

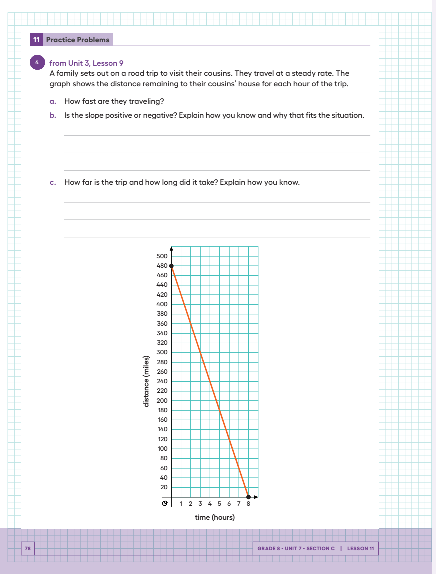
d. $25 \cdot 10^{-9}$

$25 \cdot 10^{-9}$ is halfway in between the second and third tick marks after zero.

Student Workbook



Student Workbook


11 Practice Problems

4 from Unit 3, Lesson 9

A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.

a. How fast are they traveling?

b. Is the slope positive or negative? Explain how you know and why that fits the situation.

c. How far is the trip and how long did it take? Explain how you know.

Problem 4

from Unit 3, Lesson 9

A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.

a. How fast are they traveling? 60 miles per hour

b. Is the slope positive or negative? Explain how you know and why that fits the situation.

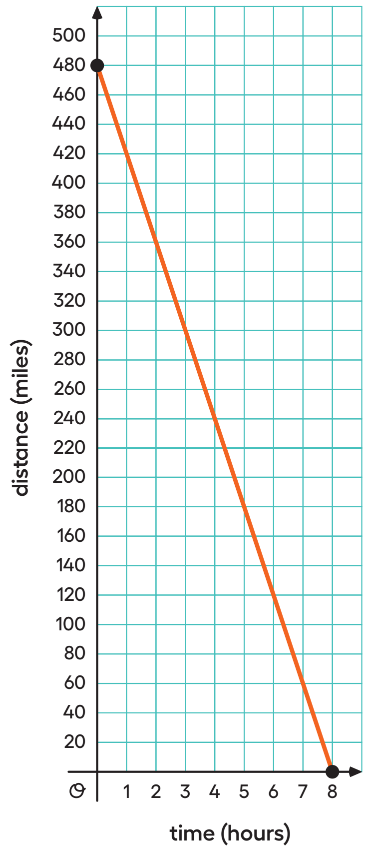
Negative

Sample reasoning: The slope is negative because the line moves down toward the right. It shows the change in remaining miles for each hour. There are 60 fewer miles remaining each hour, which means the car is traveling at a steady rate of 60 miles each hour.

c. How far is the trip and how long did it take? Explain how you know.

480 miles and 8 hours

Sample reasoning: The trip is 480 miles because the remaining distance was 480 miles when they started out (after 0 hours).
The trip took 8 hours because after 8 hours, there were 0 miles remaining.



Problem 5

from Unit 7, Lesson 8

Select **all** of the expressions that have the same value as $(3 \cdot 5)^2$.

A. 15^2

B. $6 \cdot 10$

C. $3 \cdot 5 \cdot 3 \cdot 5$

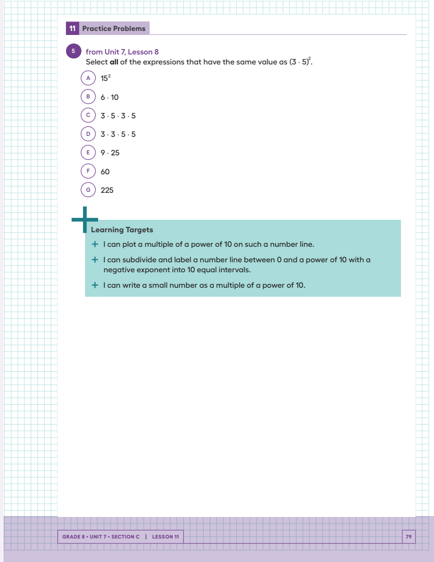
D. $3 \cdot 3 \cdot 5 \cdot 5$

E. $9 \cdot 25$

F. 60

G. 225

Student Workbook



11 Practice Problems

5 from Unit 7, Lesson 8

Select **all** of the expressions that have the same value as $(3 \cdot 5)^2$.

A. 15^2

B. $6 \cdot 10$

C. $3 \cdot 5 \cdot 3 \cdot 5$

D. $3 \cdot 3 \cdot 5 \cdot 5$

E. $9 \cdot 25$

F. 60

G. 225

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

GRADE 8 • UNIT 7 • SECTION C | LESSON 11

79