Lesson 12

Systems of Equations

Goals

- Comprehend that solving a system of equations means finding values of the variables that make both equations true at the same time.
- Coordinate (orally and in writing) graphs of parallel lines and a system of equations that has no solutions.
- Create a graph of two lines that represents a system of equations in context.

Learning Targets

- I can explain the solution to a system of equations in a real-world context.
- I can explain what a system of equations is.
- I can make graphs to find an ordered pair that two real-world situations have in common.

Lesson Narrative

This lesson formally introduces the concept of a **system of equations** with different contexts. Students recognize that they have found solutions to systems of equations using graphs in the past few lessons by examining the intersection of graphed lines. Then students consider a system that has no solution and recognize that a system with no solution is represented by parallel lines. As students write equations for a situation, they must reason abstractly and quantitatively.

Student Learning Goal

Let's learn what a system of equations is.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Instructional Routines

- MLR6: Three Reads

Required Materials

Materials to Gather

- Straightedges: Activity 1, Activity 2

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson:
Provide access to straightedges for students to accurately draw graphs of lines.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	5 min

Warm-up
Staying Hydrated

5 min

Activity Narrative

In the *Warm-up,* students are given a situation and asked to describe the graph without actually graphing the lines. Identify students who correctly use mathematically correct terminology such as "y-intercept," "slope," "x-intercept," and "intersection" to describe the graph.

Launch

Arrange students in groups of 2.

Give 3 minutes of quiet work time followed by time for a brief partner discussion for the last question.

Student Task Statement

Diego and Lin are drinking water. Lin's glass has 12 ounces and she drinks $\frac{1}{4}$ ounce per second. Diego's glass has 20 ounces and he drinks $\frac{2}{3}$ ounce per second.

1. How long will it take Lin and Diego to finish their water?

   Lin will take 48 seconds and Diego will take 30 seconds.

2. Without graphing, explain what the graphs in this situation would look like. Think about slope, intercepts, axis labels, units, and intersection points to guide your thinking.

   Sample response: The horizontal axis is labeled "time (seconds)" and the vertical axis is labeled "amount of water left (ounces)." The line for Lin's graph starts at (0, 12) and decreases to the right to the point (48, 0). The line for Diego's graph starts at (0, 20) and decreases more steeply than Lin's line to the point (30, 0). The two lines intersect somewhere.

3. Discuss your description with your partner. If you disagree, work to reach an agreement.

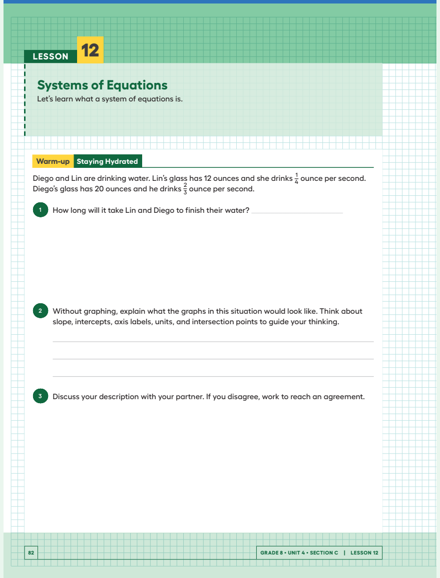
   No response needed.

Activity Synthesis

The purpose of the discussion is for students to practice describing graphs in words using correct mathematical terminology.

Select previously identified students to share their descriptions of the graphs. After each student shares, ask the class if the description is clear and to identify any vocabulary they heard that made the description precise or any vocabulary that is unclear. If any vocabulary needs to be reinforced for student understanding, this is a good time to discuss these words.

Student Workbook


LESSON 12

Systems of Equations

Let's learn what a system of equations is.

Warm-up Staying Hydrated

Diego and Lin are drinking water. Lin's glass has 12 ounces and she drinks $\frac{1}{4}$ ounce per second. Diego's glass has 20 ounces and he drinks $\frac{2}{3}$ ounce per second.

1. How long will it take Lin and Diego to finish their water?

2. Without graphing, explain what the graphs in this situation would look like. Think about slope, intercepts, axis labels, units, and intersection points to guide your thinking.

3. Discuss your description with your partner. If you disagree, work to reach an agreement.

Instructional Routines

MLR6: Three Reads

ilclass.com/r/10695568

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR6: Three Reads
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Activity 1
Passing on the Trail

20 min

Activity Narrative

There is a digital version of this activity.

In this activity, students start with an equation relating distance and time for Han's hike and enough information to write a second equation relating distance and time for Jada's hike. After writing Jada's equation and graphing both lines, students then use the lines to identify the point of intersection and make sense of the point's meaning in the context.

This activity is a culmination of students' work writing, solving, and graphing equations along with the thinking that they have done on what it means for an equation to be true. From this foundation, students are ready to understand solving systems of equations from an algebraic standpoint in the following lessons. Fluently solving systems algebraically is not expected at this time.

In the digital version of the activity, students use an applet to graph the distance of Han and Jada from the parking lot over time. The applet allows students to precisely graph the lines and find the point of intersection. Use the digital version if students have difficulty graphing precisely enough to find an accurate point of intersection.

Launch

Provide students with access to straightedges. Keep students in groups of 2.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem, without revealing the questions.

For the first read, read the problem aloud, and then ask,

"What is this situation about?"

> 2 students on a hike

Listen for and clarify any questions about the context.

After the second read, ask students to list any quantities that can be counted or measured.

> Jada's distance from the parking lot, Jada's hiking speed, Han's equation for distance from the parking lot

After the third read, reveal the question:

"What is an equation for Jada's distance from the parking lot as she heads toward the lake?"

Ask,

"What are some ways that we might get started on this?"

Invite students to name some possible starting points, referring to quantities from the second read.

> She starts 0.6 miles away and gets farther away, so I should use the relationship between distance, rate, and time to add to that distance.

To further understand the situation, consider asking:

💬 *"When $t = 0$, where is Han? Where is Jada?"*

Han is at the lake. Jada is 0.6 miles from the parking lot.

💬 *"For times shortly after 0, is d decreasing or increasing for Han? Is d decreasing or increasing for Jada?"*

Decreasing for Han and increasing for Jada.

Give 2–3 minutes of quiet work time, and ask students to pause after they have completed the first problem to discuss their equation with a partner before starting to graph the equations. Give 5–7 minutes for students to complete the remaining problems with their partners, and follow that with a whole-class discussion.

Student Task Statement

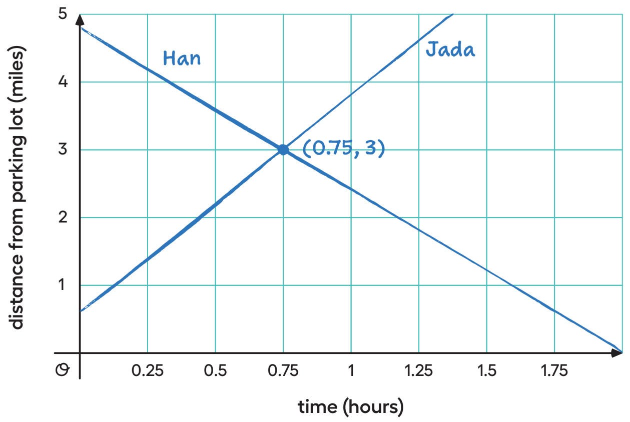
There is a hiking trail near the town where Han and Jada live that starts at a parking lot and ends at a lake. Han and Jada both decide to hike from the parking lot to the lake and back, but they start their hikes at different times.

At the time that Han reaches the lake and starts to turn back, Jada is 0.6 miles away from the parking lot and hiking at a constant speed of 3.2 miles per hour toward the lake. Han's distance, d, from the parking lot can be expressed as $d = -2.4t + 4.8$, where t represents the time in hours since he left the lake.

1. What is an equation for Jada's distance from the parking lot as she heads toward the lake?

   $d = 3.2t + 0.6$ (or equivalent)

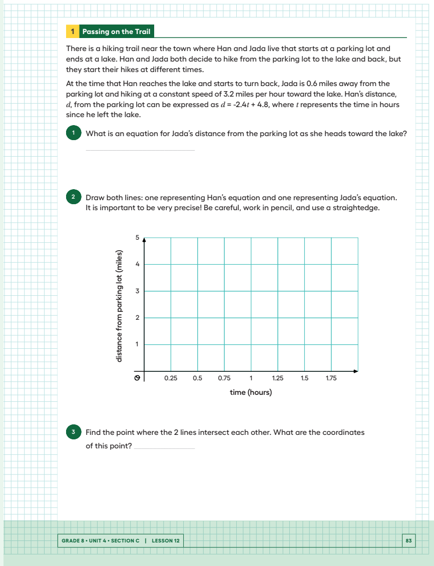
2. Draw both lines: one representing Han's equation and one representing Jada's equation. It is important to be very precise! Be careful, work in pencil, and use a straightedge.



3. Find the point where the 2 lines intersect each other. What are the coordinates of this point?

   $(0.75, 3)$

Student Workbook



1 Passing on the Trail

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1. What is an equation for Jada's distance from the parking lot as she heads toward the lake?

2. Draw both lines: one representing Han's equation and one representing Jada's equation. It is important to be very precise! Be careful, work in pencil, and use a straightedge.

distance from parking lot (miles) 5 4 3 2 1 0 0.25 0.5 0.75 1 1.25 1.5 1.75 time (hours)

3. Find the point where the 2 lines intersect each other. What are the coordinates of this point?

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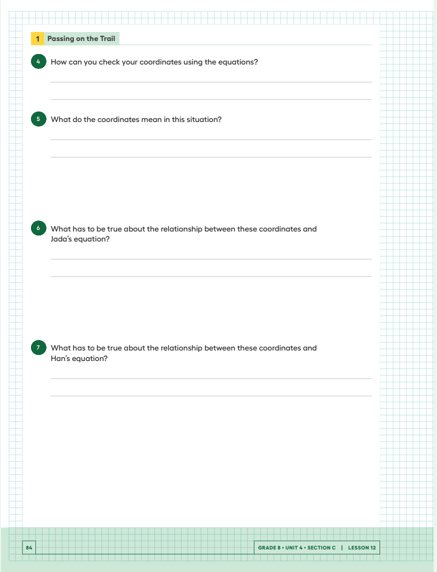
Student Workbook


1 Passing on the Trail

4 How can you check your coordinates using the equations?

5 What do the coordinates mean in this situation?

6 What has to be true about the relationship between these coordinates and Jada's equation?

7 What has to be true about the relationship between these coordinates and Han's equation?

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4. How can you check your coordinates using the equations?

   Sample response: I can substitute the coordinates into each equation and check if they are both true.

5. What do the coordinates mean in this situation?

   Sample response: 0.75 hours or 45 minutes after Han leaves the lake, he passes Jada on the trail. This happens at a distance of 3 miles from the parking lot.

6. What has to be true about the relationship between these coordinates and Jada's equation?

   These values of t and d make Jada's equation true.

7. What has to be true about the relationship between these coordinates and Han's equation?

   These values of t and d also make Han's equation true.

Activity Synthesis

The purpose of this discussion is to strengthen the connection between graphs and equations and to formally introduce the vocabulary for systems of equations.

Begin the discussion by asking groups to share the intersection point.

Display a graph of the two equations for all to see alongside the system of equations:

$$\begin{cases} d = -2.4t + 4.8 \\ d = 3.2t + 0.6 \end{cases}$$

Explain to students that this is called a **system of equations,** and that "solving a system of equations" means to find the values of the variables that make both equations true at the same time. Point out that in this problem, the solution to the system of equations is the point where Han and Jada are at the same distance from the parking lot at the same time (point to (0.75, 3)).

Tell students that it is also possible to solve a system of equations without graphing. Because they will be the same distance from the parking lot at the same time, the values for d will be equal. Because they both have an expression that is equal to d, we can set those expressions equal to one another to get the equation $-2.4t + 4.8 = 3.2t + 0.6$. Solving this equation for t will give the value for t when they are the same distance from the parking lot at the same time.

Ask students to solve this equation and confirm that $t = 0.75$, which is the same value they found earlier by carefully graphing the lines of each equation. Emphasize that the intersection point gave a value of both t and d. To get the same information from the equations, it is important to substitute t back into one of the equations to find the value for d. Because Han and Jada are at the same distance from the parking lot when $t = 0.75$, it doesn't matter which equation is used to find the value of d.

Activity 2
Stacks of Cups

10 min

Activity Narrative

There is a digital version of this activity.

Students explore a system of equations with no solutions, in the familiar context of cup stacking. The context reinforces a discussion about what it means for a system of equations to have no solutions, both in terms of a graph and in terms of the equations. Over the next few lessons, the concept of one solution, no solutions, and infinitely many solutions will be abstracted to problems without context. In those situations, it may be useful to refer back to the context in this activity and others as a way to guide students toward abstraction.

In the digital version of the activity, students use an applet to graph the heights of two stacks of cups. The applet allows students to precisely graph the lines to see that they never intersect. Use the digital version if students have difficulty graphing precisely enough to show that the lines do not intersect or if appropriate tools for graphing are not readily available.

Launch

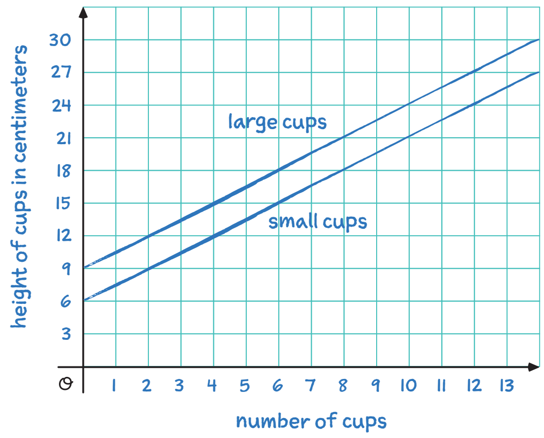
Give students 5–7 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

A stack of n small cups has a height, h, in centimeters, of $h = 1.5n + 6$.
A stack of n large cups has a height, h, in centimeters, of $h = 1.5n + 9$.

1. Graph the equation for each type of cup on the same set of axes. Make sure to label the axes and decide on an appropriate scale.

   Sample response:



2. For what number of cups will the two stacks have the same height?

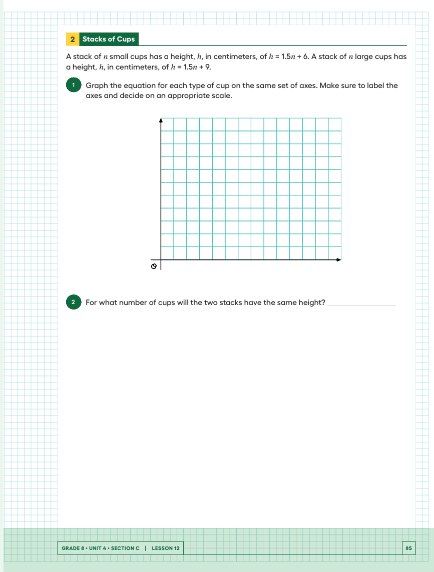
   There is no number of cups for which the two stacks have the same height.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Check in, and provide each group with feedback that encourages collaboration and community. For example, identify characteristics from each student's graph for the group to discuss—such as the scale on the axes, the intercept, and the slope.

Supports accessibility for: Social-Emotional Functioning, Organization

Student Workbook



Activity Synthesis

The key point for discussion is to connect what students observe about the graph to the idea (from earlier lessons) that equations can have no solution. Graphically, students see that the lines are parallel and are always separated by a distance that represents 3 cm. This means that a stack of n large cups will always be 3 cm taller than a stack of n small cups. Connecting this to the equations, this means that there is no value of n that is a solution to both $1.5n + 6$ and $1.5n + 9$ at the same time. By the end of the discussion, students should understand that these statements about this situation are equivalent:

- The lines don't intersect.
- The lines are parallel.
- There is no value of n for which the stacks have the same height.
- There is no value of n that makes $1.5n + 6 = 1.5n + 9$ true.
- Invite students to explain how they used the graph or equations to answer the second question. Ask other students if they answered the question with a different line of reasoning. If not brought up by students, demonstrate that setting the expression for the height of the large cup, $1.5n + 9$, equal to the expression for the height of the small cup, $1.5n + 6$, and subtracting $1.5n$ from both sides gives $6 = 9$, which is false no matter what value of n is used.

Lesson Synthesis

To highlight some of the main concepts from the lesson, ask:

"Suppose Jada and Han meet up with another person at the exact same time they meet each other along their hikes."

"What must be true about the graph of that person's distance from the parking lot over time?"

That person's line intersects Jada's and Han's lines at the point $(0.75, 3)$.

"What information is known, and what information might you need, to write an equation representing the third person's distance from the parking lot?"

I would need to know either something about the speed of the third person or the person's distance from the parking lot at another point in time.

"What is a system of equations?"

Two or more equations for which you want to find values for all of the variables so that all of the equations are true.

"What does the solution to a system of equations represent?"

The values for all of the variables that make all of the equations true.

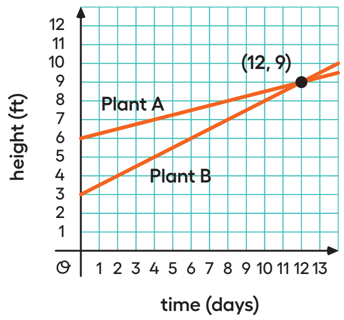
Lesson Summary

A **system of equations** is a set of 2 or more equations, where the variables represent the same unknown values. For example, suppose that two different kinds of bamboo are planted at the same time. Plant A starts at 6 ft tall and grows at a constant rate of $\frac{1}{4}$ foot each day. Plant B starts at 3 ft tall and grows at a constant rate of $\frac{1}{2}$ foot each day. Because Plant B grows faster than Plant A, it will eventually be taller, but when?

We can write equations $y = \frac{1}{4}x + 6$ for Plant A and $y = \frac{1}{2}x + 3$ for Plant B, where x represents the number of days after being planted, and y represents height. We can write this system of equations.

$$\begin{cases} y = \frac{1}{4}x + 6 \\ y = \frac{1}{2}x + 3 \end{cases}$$

Solving a system of equations means to find the values of x and y that make both equations true at the same time. One way we have seen to find the solution to a system of equations is to graph both lines and find the intersection point. The intersection point represents the pair of x and y values that makes both equations true. Here is a graph for the bamboo example:

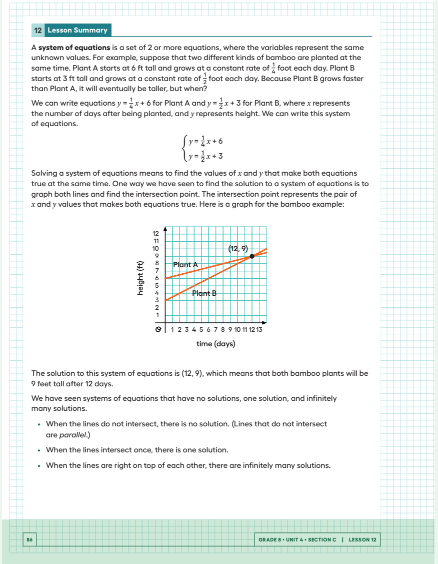


The solution to this system of equations is (12, 9), which means that both bamboo plants will be 9 feet tall after 12 days.

We have seen systems of equations that have no solutions, one solution, and infinitely many solutions.

- When the lines do not intersect, there is no solution. (Lines that do not intersect are *parallel*.)
- When the lines intersect once, there is one solution.
- When the lines are right on top of each other, there are infinitely many solutions.

Student Workbook



Responding To Student Thinking

Press Pause
If students struggle to graph the situations, pause after the following *Warm-up* to create a table of values that could help interpret the situation. If students struggle to interpret the point of intersection as a solution to the system, pause after the following *Warm-up* to interpret the intersection point for each of Lin's and Diego's situations so that students understand it has the same meaning for each student and is a solution to the system.

Unit 4, Lesson 13, Warm-up Ask About This Graph

Cool-down

5 min

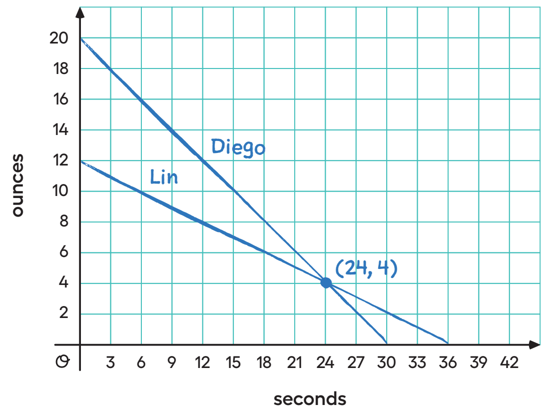
Finishing Their Water Again

Student Task Statement

Lin's glass has 12 ounces of water and she drinks it at a rate of $\frac{1}{3}$ ounce per second.

Diego's glass has 20 ounces and he drinks it at a rate of $\frac{2}{3}$ ounce per second.

1. Graph this situation on the axes provided.



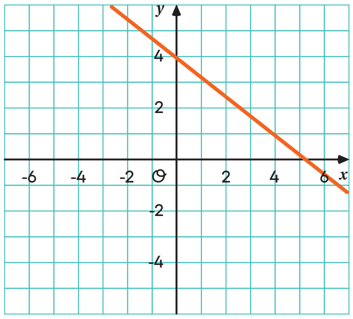
2. What does the graph tell you about the situation and how many solutions there are?

Sample response: There is one solution at (24, 4) meaning that after 24 seconds both of them have 4 ounces of water left.

Practice Problems

Problem 1

Here is the graph for one equation in a system of equations:



Write a second equation for the system so that:

a. The system has infinitely many solutions.

$y = \frac{-3}{4}x + 4$ (or equivalent)

b. The system has no solutions, and the line representing the new equation goes through the point (0, 1).

$y = \frac{-3}{4}x + 1$ (or equivalent)

c. The system has a solution at (4, 1), and the line representing the new equation goes through the point (0, 2).

$y = \frac{-1}{4}x + 2$ (or equivalent)

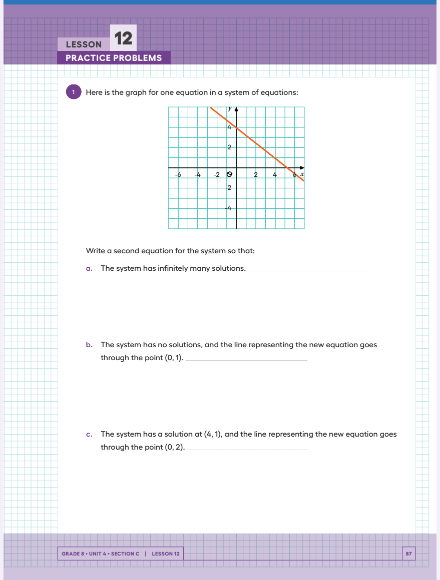
Problem 2

Create a second equation so the system has no solutions.

$$\begin{cases} y = \frac{3}{4}x - 4 \\ \end{cases}$$

any line with a slope of $\frac{3}{4}$, excluding the line $y = \frac{3}{4}x - 4$

Student Workbook



LESSON 12 PRACTICE PROBLEMS

1 Here is the graph for one equation in a system of equations:

Write a second equation for the system so that:

a. The system has infinitely many solutions.

b. The system has no solutions, and the line representing the new equation goes through the point (0, 1).

c. The system has a solution at (4, 1), and the line representing the new equation goes through the point (0, 2).

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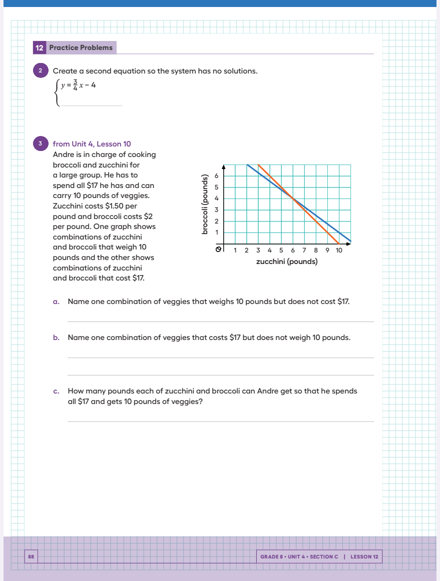
Student Workbook



12 Practice Problems

2 Create a second equation so the system has no solutions.

$\begin{cases} y = \frac{3}{4}x - 4 \\ \end{cases}$

3 from Unit 4, Lesson 10

Andre is in charge of cooking broccoli and zucchini for a large group. He has to spend all $17 he has and can carry 10 pounds of veggies. Zucchini costs $1.50 per pound and broccoli costs $2 per pound. One graph shows combinations of zucchini and broccoli that weigh 10 pounds and the other shows combinations of zucchini and broccoli that cost $17.

a. Name one combination of veggies that weighs 10 pounds but does not cost $17.

b. Name one combination of veggies that costs $17 but does not weigh 10 pounds.

c. How many pounds each of zucchini and broccoli can Andre get so that he spends all $17 and gets 10 pounds of veggies?

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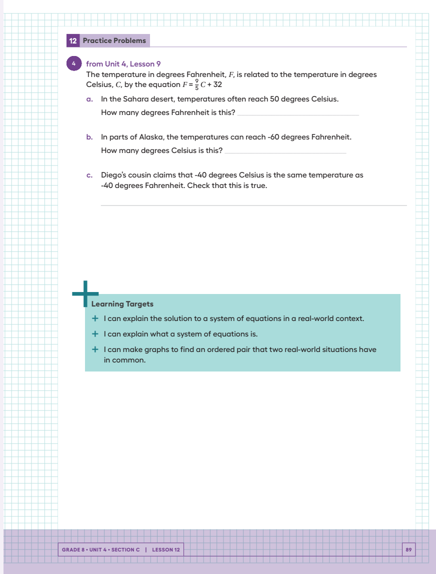
Student Workbook


12 Practice Problems

4 from Unit 4, Lesson 9

The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, by the equation $F = \frac{9}{5}C + 32$

a. In the Sahara desert, temperatures often reach 50 degrees Celsius. How many degrees Fahrenheit is this?

b. In parts of Alaska, the temperatures can reach -60 degrees Fahrenheit. How many degrees Celsius is this?

c. Diego's cousin claims that -40 degrees Celsius is the same temperature as -40 degrees Fahrenheit. Check that this is true.

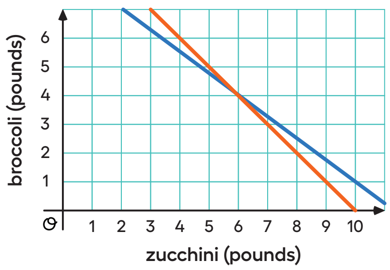
Learning Targets

+ I can explain the solution to a system of equations in a real-world context.
+ I can explain what a system of equations is.
+ I can make graphs to find an ordered pair that two real-world situations have in common.

Problem 3

from Unit 4, Lesson 10

Andre is in charge of cooking broccoli and zucchini for a large group. He has to spend all \$17 he has and can carry 10 pounds of veggies. Zucchini costs \$1.50 per pound and broccoli costs \$2 per pound. One graph shows combinations of zucchini and broccoli that weigh 10 pounds and the other shows combinations of zucchini and broccoli that cost \$17.



a. Name one combination of veggies that weighs 10 pounds but does not cost \$17.

Sample response: 4 pounds of zucchini and 6 pounds of broccoli weigh 10 pounds, but do not cost \$17 because (4, 6) is not on the line of combinations that cost \$17.

b. Name one combination of veggies that costs \$17 but does not weigh 10 pounds.

Sample response: 2 pounds of zucchini and 7 pounds of broccoli together cost \$17 because (2, 7) is on the \$17 line, but they only weigh 9 pounds.

c. How many pounds each of zucchini and broccoli can Andre get so that he spends all \$17 and gets 10 pounds of veggies?

6 pounds of zucchini, and 4 pounds of broccoli

Problem 4

from Unit 4, Lesson 9

The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, by the equation $F = \frac{9}{5}C + 32$

a. In the Sahara desert, temperatures often reach 50 degrees Celsius. How many degrees Fahrenheit is this?

122 degrees Fahrenheit

b. In parts of Alaska, the temperatures can reach -60 degrees Fahrenheit. How many degrees Celsius is this?

$-51\frac{1}{9}$ degrees Celsius

c. Diego's cousin claims that -40 degrees Celsius is the same temperature as -40 degrees Fahrenheit. Check that this is true.

$-40 = \frac{9}{5}(-40) + 32$ is true, so Diego's cousin is right.