Lesson 5

Representing Subtraction

Goals

- Generalize (orally and in writing) that subtracting a number results in the same value as adding the additive inverse.
- Interpret a number line diagram that represents subtracting signed numbers as adding with an unknown addend.
- Use a number line diagram to find the difference of signed numbers, and explain (orally) the reasoning.

Learning Targets

- I can explain the relationship between addition and subtraction of rational numbers.
- I can use a number line to subtract positive and negative numbers.

Student Learning Goal

Let's subtract signed numbers.

Lesson Narrative

In this lesson, students make sense of subtracting signed numbers by representing it on the number line. First, they rewrite subtraction equations as addition equations with a missing addend—for example, $-8 - 3 = ?$ can be thought of as $3 + ? = -8$. Students represent these equations on a number line and find the unknown value—to get from 3 to -8, the second arrow must be 11 units long and pointing to the left, so $? = -11$. From several such examples, students generalize that subtracting a number gets the same value as adding its opposite—in this case, $-8 - 3$ and $-8 + -3$ are both equal to -11. As students use the results from multiple subtraction expressions to notice a general pattern, they are making use of repeated reasoning.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Equivalent Equations

5 min

Activity Narrative

In this *Warm-up*, students make use of the relationship between addition and subtraction to write related equations.

Monitor for any students who create a number line diagram to help them generate equations that express the same relationship in a different way.

Launch

Give students 1 minute of quiet work time. Remind students that each new equation must include only the numbers in the original equation.

Student Task Statement

Consider the equation 2 + 3 = 5. Here are some more equations that express the same relationship in a different way:

$$3 + 2 = 5$$

$$5 - 3 = 2$$

$$5 - 2 = 3$$

For each equation, write two more equations that use the same numbers and express the same relationship in a different way.

1. 9 + (-1) = 8

Sample responses:

- $-1 + 9 = 8$
- $8 - 9 = -1$
- $8 - (-1) = 9$

2. -11 + x = 7

Sample responses:

- $x + (-11) = 7$
- $7 - x = -11$
- $7 - (-11) = x$

Activity Synthesis

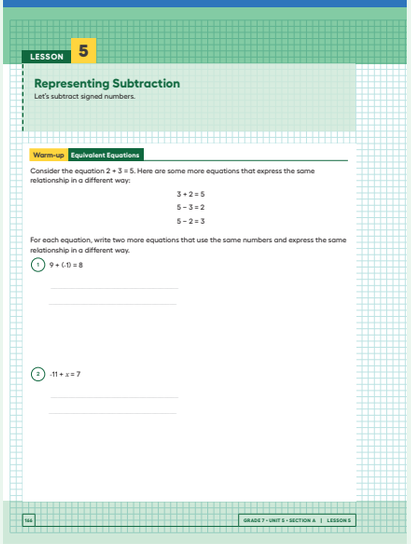
The purpose of this discussion is for students to share their responses and reasoning. Invite students to share their additional equations for each given equation. Display any number lines created by students for all to see. Use the number line to facilitate connections between addition equations and related subtraction equations.

The key ideas for students are that every addition equation has related subtraction equations and every subtraction equation has related addition equations.

Building on Student Thinking

If students struggle to come up with other equations, encourage them to represent the relationship using a number line diagram and then think about other operations they can use to show the same relationship with the same numbers.

Student Workbook


LESSON 5

Representing Subtraction

Let's subtract signed numbers.

Warm-up Equivalent Equations

Consider the equation 2 + 3 = 5. Here are some more equations that express the same relationship in a different way.

3 + 2 = 5
5 − 3 = 2
5 − 2 = 3

For each equation, write two more equations that use the same numbers and express the same relationship in a different way.

1. 9 + (-1) = 8

2. -11 + x = 7

Access for Students with Diverse Abilities (Activity 1, Launch)

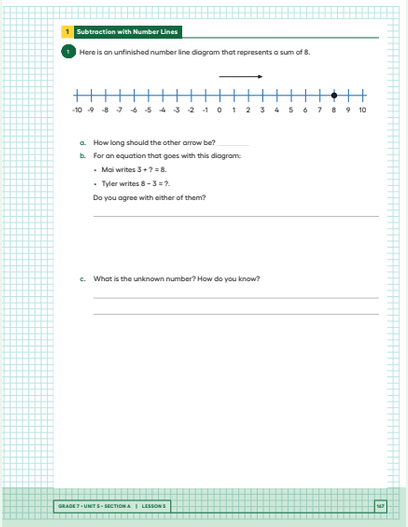
Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, reference previous activities where students used a number line to represent adding signed numbers to provide an entry point for this activity.

Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Building on Student Thinking

Some students may say they disagree with Tyler's equations for the number lines. Use fact families to help students see that subtraction equations are a valid way to represent problems that involve finding a missing addend given a sum. It may help to remind them of the work they did in the *Warm-up*.

Student Workbook


1 Subtraction with Number Lines

1 Here is an unfinished number line diagram that represents a sum of 8.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

a. How long should the other arrow be?

b. For an equation that goes with this diagram:

- Mai writes 3 + ? = 8.
- Tyler writes 8 − 3 = ?.

Do you agree with either of them?

c. What is the unknown number? How do you know?

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Activity 1

Subtraction with Number Lines

15 min

Activity Narrative

In this activity, students use their knowledge of the relationship between addition and subtraction and their understanding of how to represent the addition of signed numbers on a number line to begin subtracting signed numbers.

Students are given number line diagrams showing one addend and the sum. They examine how addition equations with missing addends can be written using subtraction by analyzing and critiquing the reasoning of others.

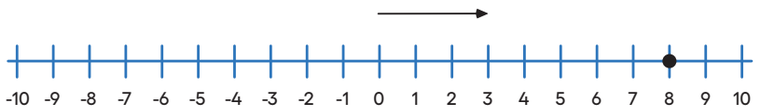
Launch

It may be useful to remind students how they represented addition on a number line in previous lessons. In particular, it is helpful to keep in mind that the two addends in an addition equation are drawn "tip-to-tail." Any number line diagrams created in the previous activity may be used as an illustration of this idea.

Ask students to complete the questions for the first diagram and pause for discussion. Then, give students quiet work time to complete the remaining problems, and follow with a whole-class discussion.

Student Task Statement

1. Here is an unfinished number line diagram that represents a sum of 8.



a. How long should the other arrow be? 5 units

b. For an equation that goes with this diagram:

- Mai writes 3 + ? = 8.
- Tyler writes 8 − 3 = ?.

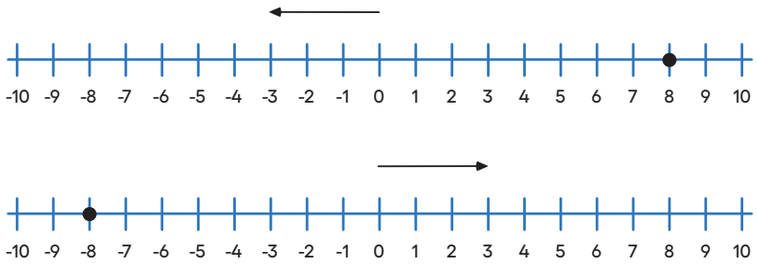
Do you agree with either of them?

Sample response: I agree with Mai because I want to know what to add to 3 to get 8, because 8 is the end point. I also agree with Tyler because the equations are equivalent.

c. What is the unknown number? How do you know?

+5, because to get from 3 to 8, we add on 5

2. Here are two more unfinished diagrams that represent sums.



For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

First Image: Mai would write (-3) + ? = 8

Second Image: Mai would write 3 + ? = (−8)

b. What equation would Tyler write if he used the same reasoning as before?

First Image: Tyler would write 8 − (-3) = ?

Second Image: Tyler would write (-8) − (3) = ?

c. How long should the other arrow be?

First Image: The other arrow should be 11 units long.

Second Image: The other arrow should be 11 units long.

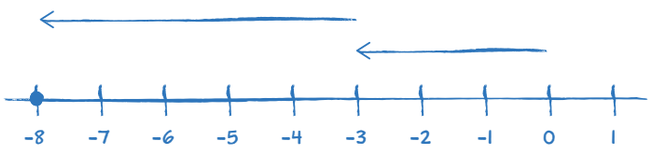
d. What number would complete each equation? Be prepared to explain your reasoning.

First Image: The number is 11, because the other arrow is 11 units long and pointing to the right.

Second Image: The number is -11, because the other arrow is 11 units long but pointing to the left.

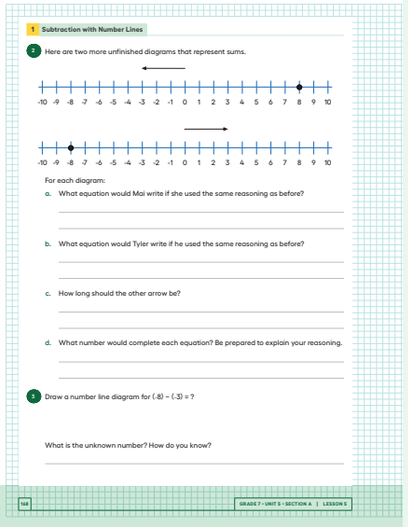
3. Draw a number line diagram for (-8) − (-3) = ? What is the unknown number? How do you know?

Sample response:



The unknown number is -5, because the arrow is 5 units long and pointing left.

Student Workbook


1 Subtraction with Number Lines

2 Here are two more unfinished diagrams that represent sums.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

d. What number would complete each equation? Be prepared to explain your reasoning.

3 Draw a number line diagram for (-8) − (-3) = ?

What is the unknown number? How do you know?

Activity Synthesis

The goal of this discussion is for students to be comfortable writing subtraction equations as an equivalent addition equation with a missing addend.

Invite at least one student to share the unknown number, or missing addend, for each problem. Ask students to share their reasoning until they come to an agreement. Display two related equations for all to see. For example, display 3 + ? = -8 and -8 − 3 = ?.

Then ask students what two related equations might look like if the two values were a and b. For example, using a and b, the equations might look like a + ? = b and $b - a$ = ?.

Display both sets of equations for all to see and use as a reference in the following activity.

Activity 2

We Can Add Instead

15 min

Activity Narrative

In this activity, students see more examples where subtracting a signed number is equivalent to adding its opposite. First, students match expressions and number line diagrams. Then they find an equivalent expression to a given addition or subtraction expression. Students use repeated reasoning when they notice that subtracting a number is equivalent to adding its opposite.

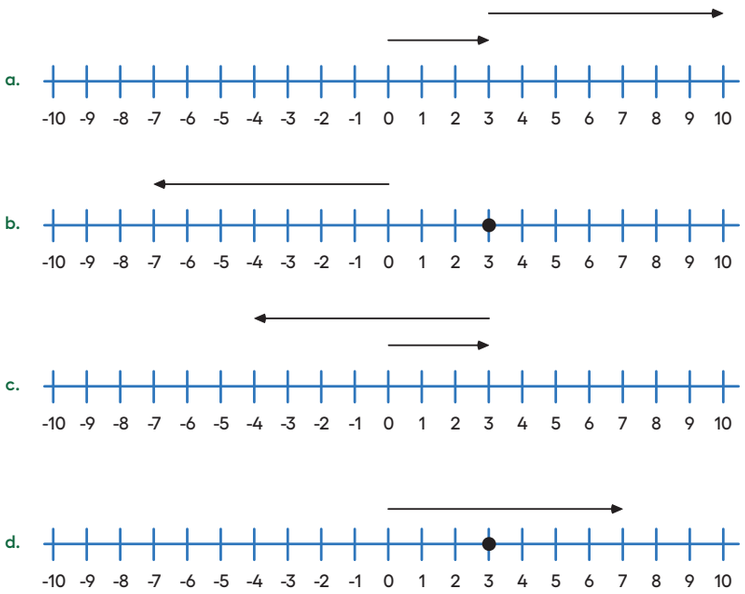
Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time, then have them check in with a partner. Have them continue to complete the activity, and follow with a whole-class discussion.

Student Task Statement

1. Match each diagram to one of these expressions:

3 + 7 a 3 − 7 d 3 + (-7) c 3 − (-7) b



2. Which expressions in the first question have the same value? What do you notice?

3 + 7 and 3 − (-7) have the same value. 3 + (-7) and 3 − 7 have the same value. Sample response: I notice that subtracting a number is the same as adding its opposite.

Pause here so your teacher can review your work.

3. Which expression has the same value as 8 − 12?

A. -8 + 12

C. 12 − 8

B. 8 + -12

D. 8 − -12

4. Which expression has the same value as 8 + -5?

A. 8 − 5

B. -5 + -8

C. 8 − -5

D. 8 + 5

5. Which expression has the same value as -5 − -9?

A. -9 − -5

B. -9 + 5

C. -5 + -9

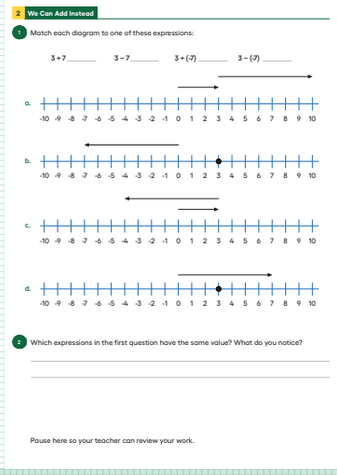
D. -5 + 9

Access for Students with Diverse Abilities (Activity 2, Student Task)

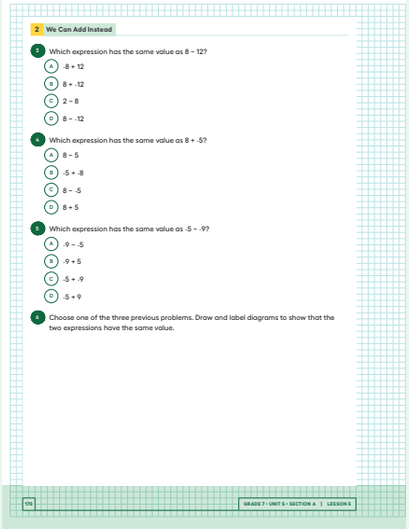
Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts: the first two questions, the next three questions, and the last question. Check in with students to provide feedback and encouragement after each chunk. Ensure that students have correctly identified the expressions that have the same value before they start drawing their diagrams for the last question.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook



Student Workbook



2 We Can Add Instead

3. Which expression has the same value as 8 − 12?
   - A. -8 + 12
   - B. 8 + -12
   - C. 2 − 8
   - D. 8 − -12

4. Which expression has the same value as 8 + -5?
   - A. 8 − 5
   - B. -5 + -8
   - C. 8 − -5
   - D. 8 + 5

5. Which expression has the same value as -5 − -9?
   - A. -9 − -5
   - B. -9 + 5
   - C. -5 + -9
   - D. -5 + 9

6. Choose one of the three previous problems. Draw and label diagrams to show that the two expressions have the same value.

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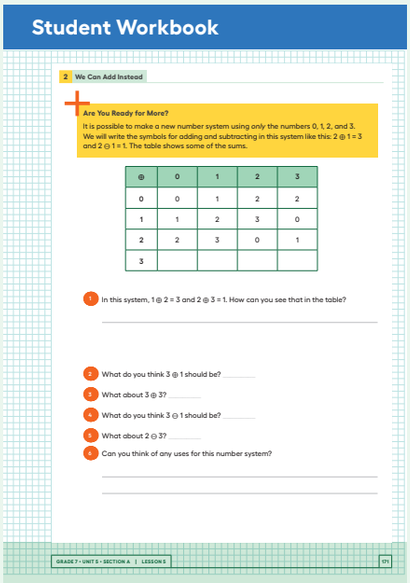
Student Workbook



2 We Can Add Instead

Are You Ready for More?

It is possible to make a new number system using only the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: 2 ⊕ 1 = 3 and 2 ⊖ 1 = 1. The table shows some of the sums.

⊕	0	1	2	3
0	0	1	2	2
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, 1 ⊕ 2 = 3 and 2 ⊕ 3 = 1. How can you see that in the table?
2. What do you think 3 ⊕ 1 should be?
3. What about 3 ⊕ 3?
4. What do you think 3 ⊖ 1 should be?
5. What about 2 ⊖ 3?
6. Can you think of any uses for this number system?

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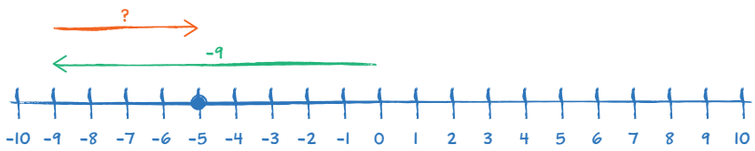
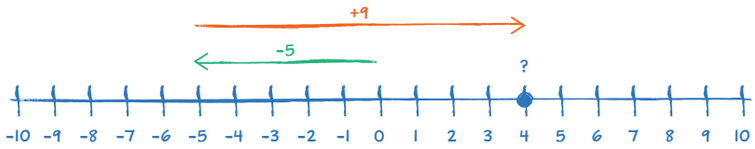
6. Choose one of the three previous problems. Draw and label diagrams to show that the two expressions have the same value.

Sample responses:

- −5 − −9 = ? is equivalent to −9 + ? = −5, which means ? = 4.



- −5 + 9 also equals 4.



Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: 2 ⊕ 1 = 3 and 2 ⊖ 1 = 1. The table shows some of the sums.

⊕	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, 1 ⊕ 2 = 3 and 2 ⊕ 3 = 1. How can you see that in the table?

1 ⊕ 2 = 3 is in the third row and fourth column, and 2 ⊕ 3 = 1 is in the fourth row and fifth column.

2. What do you think 3 ⊕ 1 should be?

Answers vary. However, if we want ⊕ to satisfy the commutative and associative properties, then 3 ⊕ 1 = 0.

3. What about 3 ⊕ 3?

Assuming commutativity and associativity, 3 ⊕ 3 = 2.

4. What do you think 3 ⊖ 1 should be?

Assuming ⊖ is the inverse of ⊕, 3 ⊖ 1 = 2.

5. What about 2 ⊖ 3?

Assuming ⊖ is the inverse of ⊕, 2 ⊖ 3 = 3.

6. Can you think of any uses for this number system?

Sample response: Think about a dial which can only make 90° clockwise turns. The fact that making three such turns followed by two more turns is the same as making only one turn (since making four of those five 90° turns leaves it in the same position) could be represented by the statement $3 \oplus 2 = 1$. If we thought of the $\ominus$ operation as turning the dial counterclockwise, then the statement $2 \ominus 3 = 3$ reflects that two clockwise turns followed by three counterclockwise turns has the same effect as doing 3 clockwise turns.

Activity Synthesis

The goal of this discussion is for students to see that subtracting a number results in the same value as adding its opposite. Begin by inviting students to share any patterns they noticed. If no student mentions it, point out that subtracting a number is the same as adding its opposite. Ask students to help you list all of the pairs that show this.

Then display the following expression for all to see: 3 – 7. Ask,

- *"How could it be written as a sum?"*

  3 + (-7)

Then ask students what the value of both expressions is.

3 – 7 = -4 and 3 + (-7) = -4

If time allows, ask students to rewrite each of the following subtraction expressions as an addition expression:

- 4 – 10. (4 + (-10))
- -2 – 6. (-2 + (-6))
- 9 – (-8). (9 + 8)

Emphasize that a number and its opposite always make a sum of 0. So subtracting a number is always the same as adding its opposite.

Lesson Synthesis

Share with students,

- *"Today we subtracted signed numbers by using number line diagrams and addition expressions with a missing addend."*

To reinforce the connection between subtraction and adding the opposite, consider asking:

- *"How are addition and subtraction related to each other?"*

  They are inverse operations. Subtracting one addend from the sum gives the other addend.

- *"How could we rewrite the expression -5 – 3 using addition?"*

  3 + ? = -5, or more simply -5 + (-3)

- *"How could we rewrite the expression 4 – -6 using addition?"*

  -6 + ? = 4, or more simply 4 + 6

- *"Does this work for all numbers?"*

  Yes.

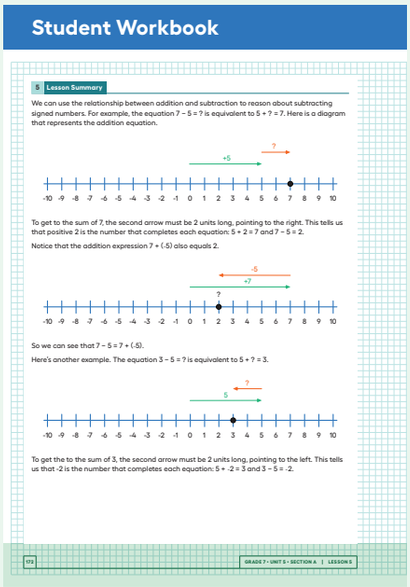
Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different number line diagrams students drew for the last problem. Ask,

"How are these representations the same?"
"How are they different?"
"How does addition or subtraction show up in each representation?"

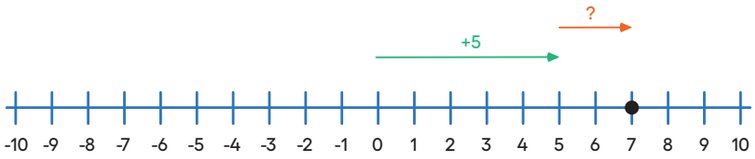
Advances: Representing, Conversing

Student Workbook



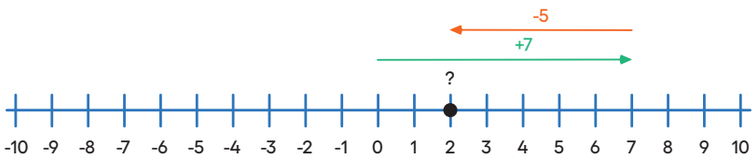
Lesson Summary

We can use the relationship between addition and subtraction to reason about subtracting signed numbers. For example, the equation 7 – 5 = ? is equivalent to 5 + ? = 7. Here is a diagram that represents the addition equation.



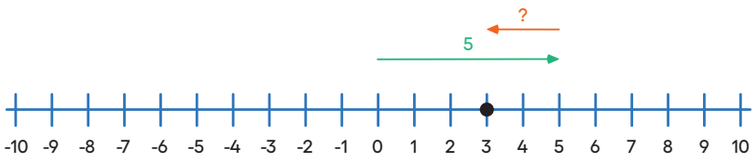
To get to the sum of 7, the second arrow must be 2 units long, pointing to the right. This tells us that positive 2 is the number that completes each equation: 5 + 2 = 7 and 7 – 5 = 2.

Notice that the addition expression 7 + (-5) also equals 2.



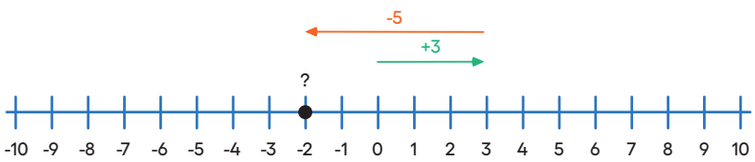
So we can see that 7 – 5 = 7 + (-5).

Here's another example. The equation 3 – 5 = ? is equivalent to 5 + ? = 3.



To get the to the sum of 3, the second arrow must be 2 units long, pointing to the left. This tells us that -2 is the number that completes each equation: 5 + -2 = 3 and 3 – 5 = -2.

Notice that the addition expression 3 + (-5) also equals -2.



So we can see that 3 – 5 = 3 + (-5).

This pattern always works. In general:

$$a - b = a + (-b)$$

Cool-down

Same Value

5 min

Student Task Statement

1. Which other expression has the same value as (-14) − 8? Explain your reasoning.

   a. (-14) + 8

   b. 14 − (-8)

   c. 14 + (-8)

   d. (-14) + (-8)

   Sample reasoning: Adding -8 results in the same value as subtracting 8.

2. Which other expression has the same value as (-14) − (-8)? Explain your reasoning.

   a. (-14) + 8

   b. 14 − (-8)

   c. 14 + (-8)

   d. (-14) + (-8)

   Sample reasoning: Subtracting -8 results in the same value as adding 8.

Responding To Student Thinking

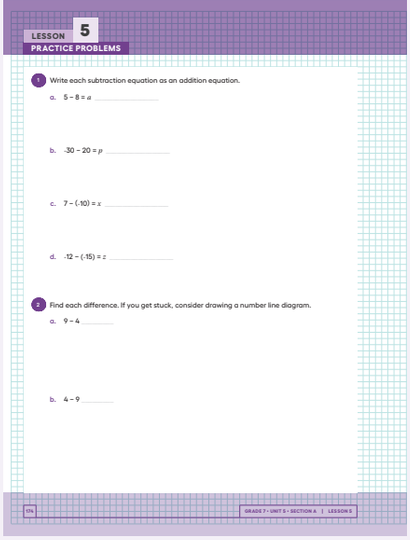
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 5

Practice Problems

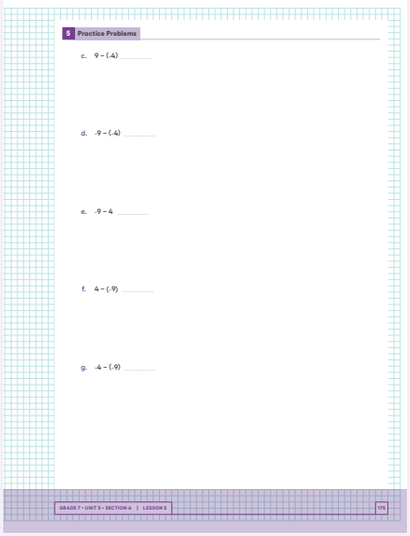
5 Problems

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1. Write each subtraction equation as an addition equation.
   a. $5 - 8 = a$ _____
   b. $-30 - 20 = p$ _____
   c. $7 - (-10) = x$ _____
   d. $-12 - (-15) = z$ _____
2. Find each difference. If you get stuck, consider drawing a number line diagram.
   a. $9 - 4$ _____
   b. $4 - 9$ _____

Student Workbook


5 Practice Problems

c. $9 - (-4)$ _____
d. $-9 - (-4)$ _____
e. $-9 - 4$ _____
f. $4 - (-9)$ _____
g. $-4 - (-9)$ _____

Problem 1

Write each subtraction equation as an addition equation.

a. $5 - 8 = a$

Sample responses:

- $5 + -8 = a$
- $8 + a = 5$

b. $-30 - 20 = p$

Sample responses:

- $-30 + -20 = p$
- $20 + p = -30$

c. $7 - (-10) = x$

Sample responses:

- $7 + 10 = x$
- $-10 + x = 7$

d. $-12 - (-15) = z$

Sample responses:

- $-12 + 15 = z$
- $-15 + z = -12$

Problem 2

Find each difference. If you get stuck, consider drawing a number line diagram.

a. $9 - 4$ 5

b. $4 - 9$ -5

c. $9 - (-4)$ 13

d. $-9 - (-4)$ -5

e. $-9 - 4$ -13

f. $4 - (-9)$ 13

g. $-4 - (-9)$ 5

Problem 3

from Unit 4, Lesson 10

A restaurant bill is $59, and you pay $72. What percentage gratuity did you pay?

22%, because 13 ÷ 59 ≈ 0.22

Problem 4

Find the solution to each equation mentally.

a. 30 + a = 40

$a = 10$

b. 500 + b = 200

$b = -300$

c. -1 + c = -2

$c = -1$

d. d + 3,567 = 0

$d = -3,567$

Problem 5

from Unit 2, Lesson 3

One kilogram is 2.2 pounds. Complete the tables. What is the interpretation of the constant of proportionality in each case?

a.

kilograms	pounds
1	2.2
7	15.4
30	66
0.5	1.1

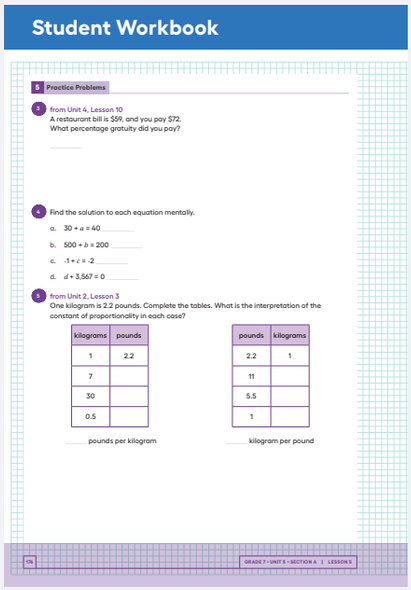
2.2 pounds per kilogram

b.

pounds	kilograms
2.2	1
11	5
5.5	2.5
1	0.45

0.45 kilogram per pound

Student Workbook


5 Practice Problems

3 from Unit 4, Lesson 10
A restaurant bill is $59, and you pay $72.
What percentage gratuity did you pay?

4 Find the solution to each equation mentally.
a. 30 + a = 40 _____
b. 500 + b = 200 _____
c. -1 + c = -2 _____
d. d + 3,567 = 0 _____

5 from Unit 2, Lesson 3
One kilogram is 2.2 pounds. Complete the tables. What is the interpretation of the constant of proportionality in each case?

kilograms	pounds
1	2.2
7	
30	
0.5	

_____ pounds per kilogram

pounds	kilograms
2.2	1
11	
5.5	
1	

_____ kilogram per pound