Lesson 5

More Dilations

Goals

- Describe (orally) a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points.
- Describe (orally) several dilations of one figure with the same center but different scale factors.
- Identify what information is needed to dilate a polygon on a coordinate grid. Ask questions to elicit that information.

Learning Target

I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)

Instructional Routines

- MLR4: Information Gap Cards
- Notice and Wonder

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Geometry toolkits: Activity 1

Materials to Copy

- Dilations Cards (1 copy for every 2 students): Activity 1

Required Preparation

Warm-up:
For the digital version of the activity, acquire devices that can run the applet.

Activity 1:
Provide access to geometry toolkits.

Lesson Narrative

The purpose of this lesson is to give students further practice in performing dilations on a coordinate grid using the *Information Gap* activity structure. Using this structure, students identify missing information needed to perform a dilation that may take several rounds of discussion. One important use of coordinates in geometry is to facilitate precise and concise communication about the location of points. This allows students to indicate where the center of the dilation is and also to communicate the vertices of the polygon that is dilated.

Math Community

The goal of today's exercise is to use the suggestions from the previous exercise to revise the "Norms" sections of the Math Community Chart and to invite students to reflect on one norm that will be a strength for them. Both activities begin to build shared accountability for and investment in the classroom norms.

Student Learning Goal

Let's dilate figures in the coordinate plane.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	5 min

Instructional Routines

Notice and Wonder

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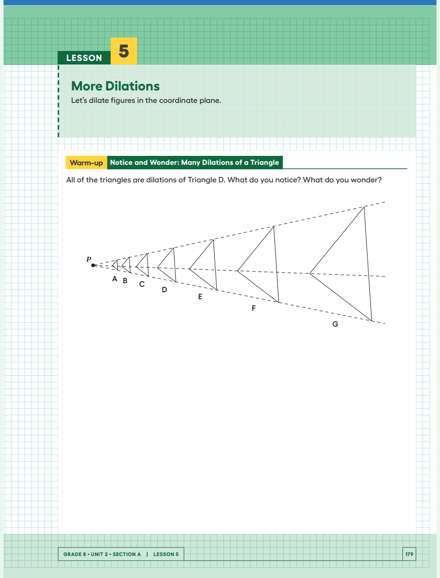
Please log in to the site before using the QR code or URL.

Student Workbook



Warm-up

Notice and Wonder: Many Dilations of a Triangle

5 min

Activity Narrative

There is a digital version of this activity.

In this *Warm-up,* students think about how the size of the scale factor impacts the dilation of a figure. While students may notice and wonder many things about the image, the fact that it shows several dilations of one triangle, all with the same center but different scale factors is the important discussion point.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

In the digital version of the activity, students use an applet to adjust the scale factor of a dilation. The applet allows students to instantly see how increasing or decreasing a scale factor impacts the dilation of a triangle.

Launch

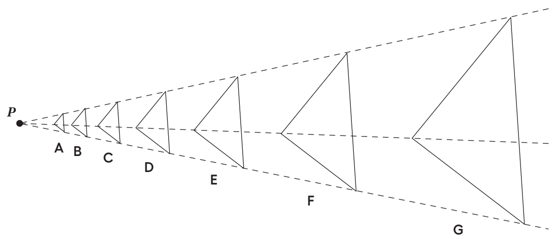
Arrange students in groups of 2. Display the image for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder.

Give students another minute to discuss their observations and questions.

Student Task Statement

All of the triangles are dilations of Triangle D. What do you notice? What do you wonder?



Students may notice:

- Triangles A, B, and C are smaller and closer to point *P* than Triangle D is.
- Triangles E, F, and G are larger and farther away from point *P* than Triangle D is.
- The center of dilation is point *P*.
- Corresponding sides of the triangles are parallel to each other.
- There is a dashed line through each set of corresponding vertices.
- The corresponding angles appear to be congruent.

Students may wonder:

- What scale factors were used?
- Are the triangles scaled copies of each other?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students:

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Math Community

After the *Warm-up*, display the Math Community Chart and a list of 2–5 revisions suggested by the class in the previous exercise for all to see. Remind students that norms are agreements that everyone in the class shares responsibility for, so everyone needs to understand and agree to work on upholding the norms. Briefly discuss any revisions and make changes to the "Norms" sections of the chart as the class agrees. Depending on the level of agreement or disagreement, it may not be possible to discuss all suggested revisions at this time. If that happens, plan to discuss the remaining suggestions over the next few lessons.

Tell students that the class now has an initial list of norms or "hopes" for how the classroom math community will work together throughout the school year. This list is just a start, and over the year it will be revised and improved as students in the class learn more about each other and about themselves and math learners.

Activity 1

Info Gap: Dilations

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to perform a dilation. Students will need to know the center of dilation, the coordinates of the polygon that they are dilating, and the scale factor.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for the information they need to solve the problem. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Monitor for how students request (and supply) information about the center of dilation and the location of the polygon that is being dilated. The coordinate grid helps name and communicate the location of points, which is essential in this activity.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards.
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Instructional Routines

MLR4: Information Gap Cards

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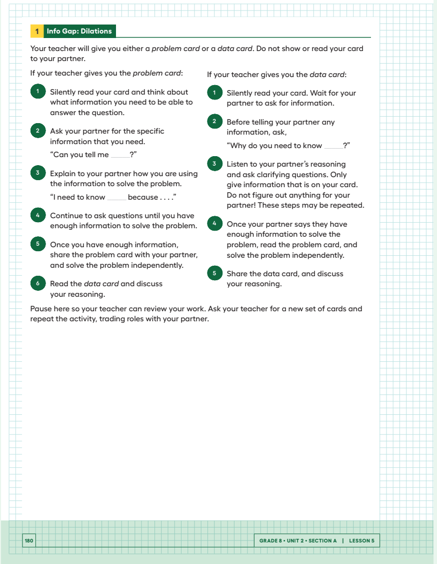
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the Info Gap graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Workbook



1 Info Gap: Dilations

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ___?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ___ because . . . ."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ___?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

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Launch

Provide access to geometry toolkits. Tell students they will perform dilations on polygons. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it. There is an extra set of cards available for demonstration purposes.

Arrange students in groups of 2 or 4. If students are new to the *Info Gap* routine, allowing them to work in groups of 2 for each role supports communication and understanding. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

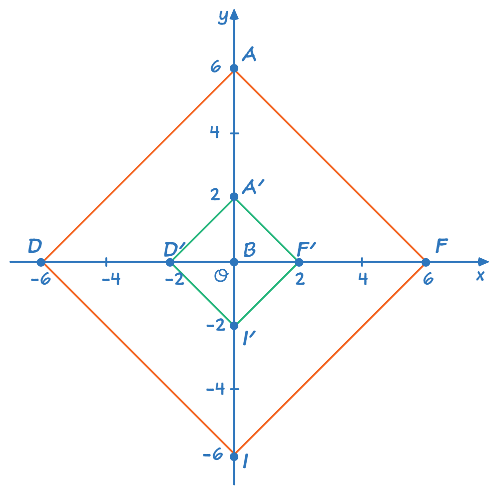
If your teacher gives you the *problem card:*

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the *data card* and discuss your reasoning.

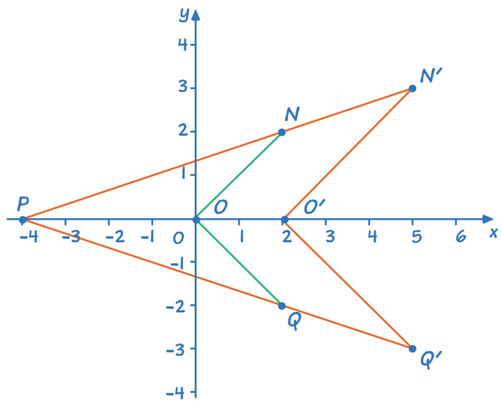
If your teacher gives you the *data card:*

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: A': (0, 2), F': (2, 0), I': (0, −2), D': (−2, 0). Image to help visualize:

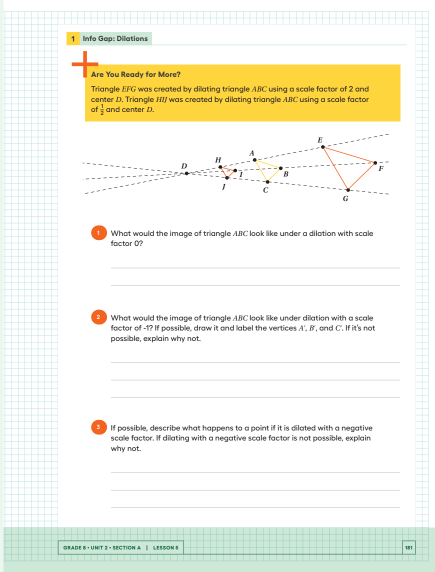


Problem Card 2: O': (2, 0), N': (5, 3), P: (−4, 0), Q': (5, −3). Image to help visualize:



Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Workbook


1 Info Gap: Dilations

Are You Ready for More?

Triangle EFG was created by dilating triangle ABC using a scale factor of 2 and center D. Triangle HIJ was created by dilating triangle ABC using a scale factor of $\frac{1}{2}$ and center D.

1 What would the image of triangle ABC look like under a dilation with scale factor 0?

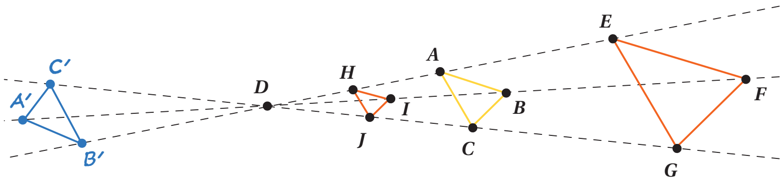
2 What would the image of triangle ABC look like under dilation with a scale factor of -1? If possible, draw it and label the vertices A', B', and C'. If it's not possible, explain why not.

3 If possible, describe what happens to a point if it is dilated with a negative scale factor. If dilating with a negative scale factor is not possible, explain why not.

GRADE 8 • UNIT 2 • SECTION A | LESSON 5

Are You Ready for More?

Triangle EFG was created by dilating triangle ABC using a scale factor of 2 and center D. Triangle HIJ was created by dilating triangle ABC using a scale factor of $\frac{1}{2}$ and center D.



1. What would the image of triangle ABC look like under a dilation with scale factor 0?

   If the distance from D to any vertex were multiplied by 0, the product would be 0. So, all 3 vertices of ABC would move to point D.

2. What would the image of triangle ABC look like under dilation with a scale factor of -1? If possible, draw it and label the vertices A', B', and C'. If it's not possible, explain why not.

   See image.

3. If possible, describe what happens to a point if it is dilated with a negative scale factor. If dilating with a negative scale factor is not possible, explain why not.

   The dilated point goes to the other side of the point of dilation.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"Other than the answer, what information did you ask for in order to solve the problem?"*

  the location of the vertices of the original image, the center of dilation, the scale factor

- *"How did using coordinates help in talking about the problem?"*

  Coordinates made it easy to name and locate points.

- *"If this same problem had a figure on a grid without coordinates, how would you talk about the points?"*

  We could describe points in relation to other points. For example a point might be 2 units to the right and 1 to the left of a known point.

💬 *"What if there had been no grid at all? Would you still have been able to request or provide the needed information to perform the transformation?"*

Maybe, but it would have been harder and not as precise.

Highlight for students that coordinates allow us to unambiguously provide the location of a polygon's vertices. In addition, reinforce the idea that in order to perform a dilation, we need to know the scale factor and the center of dilation. The coordinate grid again provides an efficient means to communicate the center of dilation.

Lesson Synthesis

The goal of this discussion is for students to see how coordinates are useful, especially for communicating location of points in the plane. Here are some questions for discussion:

💬 *"Why are coordinates useful? What are they good for?"*

Coordinates are an exceptionally powerful tool for communicating the location of points in the plane.

💬 *"How do coordinates communicate the specific location of a point in the plane?"*

There is only one point 3 units to the left of the origin and 2 units up from the origin, the point $(-3, 2)$.

💬 *"When performing a dilation, when can we use coordinates?"*

The location of a polygon can be communicated by giving the coordinates of the vertices. In a dilation, we also need to know the center of dilation [another point] and the scale factor.

Lesson Summary

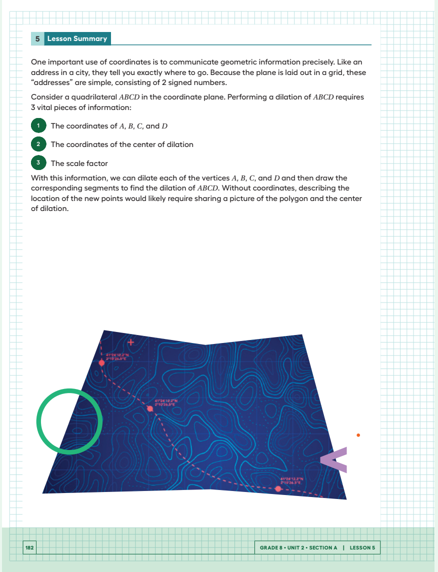
One important use of coordinates is to communicate geometric information precisely. Like an address in a city, they tell you exactly where to go. Because the plane is laid out in a grid, these "addresses" are simple, consisting of 2 signed numbers.

Consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires 3 vital pieces of information:

1. The coordinates of A, B, C, and D
2. The coordinates of the center of dilation
3. The scale factor

With this information, we can dilate each of the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.

Student Workbook



Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the question “What is one of our classroom norms that is a strength for you? Why?” Tell students that as a culmination to establishing the initial list of mathematical community norms, they are now asked to share one norm they think will be a strength for them. To help students understand what the question is asking, share a personal example. For example,

“I think that ‘Ask clarifying questions’ is a norm that is a strength for me because I am good at asking questions when I don’t think I understand how someone else is thinking about a problem. Instead of just telling you what I think you should do, I make sure to ask questions until I understand what YOU are doing.”

Display these prompts for all to see:

- “One of our classroom norms that will be a strength for me is ___.”
- “I think this will be a strength for me because ___.”

Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify which norms students feel more confident about and which norms were not listed as strengths by many students. In some cases, students may not think a norm is a strength because they are not sure what that norm looks like or sounds like. So, focus on identifying those norms in the class when they happen.

For example, during group work students ask a quiet group member which representation they prefer, and that student shares a third representation that the group had not even considered. Asking the quiet student illustrates a norm like “we invite others into the math.” Pointing out that action when it happens helps students understand the norm and see how it can benefit the math thinking of the entire group. This understanding and appreciation can promote the use of that norm in the math community.

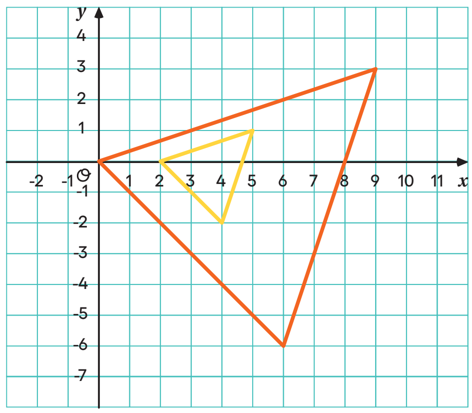
Cool-down

Identifying a Dilation

5 min

Student Task Statement

The smaller triangle is dilated to create the larger triangle. The center of dilation is plotted, but not labeled.



Describe this dilation. Be sure to include all of the information someone would need to perform the dilation.

Sample response: The triangle being dilated has vertices at $(2, 0)$, $(4, -2)$, and $(5, 1)$. The center of dilation is $(3, 0)$ and the scale factor is 3.

Responding To Student Thinking

Points to Emphasize

If students struggle with describing the dilation, as opportunities arise over the next several lessons, emphasize the three pieces needed to perform a dilation described in the *Student Lesson Summary* of the lesson referred to here.

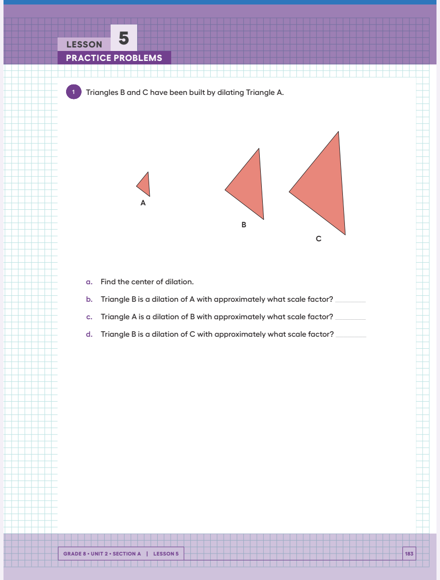
Unit 2, Lesson 5 More Dilations

Lesson 5

Practice Problems

4 Problems

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1 Triangles B and C have been built by dilating Triangle A.

a. Find the center of dilation.

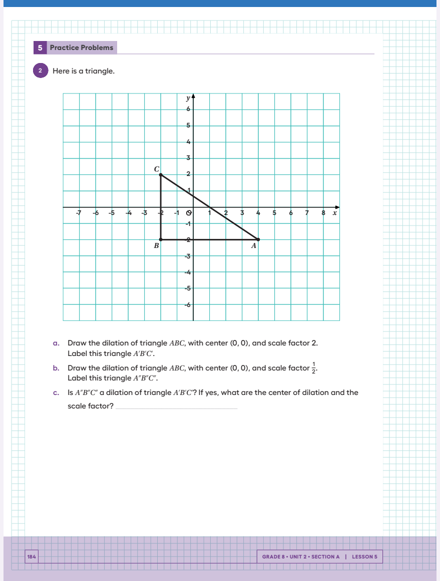
b. Triangle B is a dilation of A with approximately what scale factor?

c. Triangle A is a dilation of B with approximately what scale factor?

d. Triangle B is a dilation of C with approximately what scale factor?

GRADE 8 • UNIT 2 • SECTION A | LESSON 5

Student Workbook


5 Practice Problems

2 Here is a triangle.

a. Draw the dilation of triangle ABC, with center (0, 0), and scale factor 2. Label this triangle $A'B'C'$.

b. Draw the dilation of triangle ABC, with center (0, 0), and scale factor $\frac{1}{2}$. Label this triangle $A''B''C''$.

c. Is $A''B''C''$ a dilation of triangle $A'B'C'$? If yes, what are the center of dilation and the scale factor?

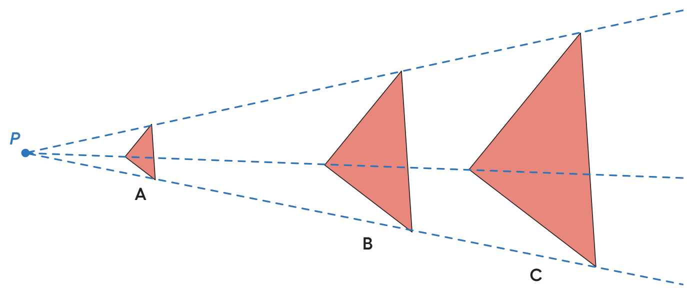
GRADE 8 • UNIT 2 • SECTION A | LESSON 5

Problem 1

Triangles B and C have been built by dilating Triangle A.

a. Find the center of dilation.

Point P denotes the center of dilation:



b. Triangle B is a dilation of A with approximately what scale factor?

3 (or equivalent)

c. Triangle A is a dilation of B with approximately what scale factor?

$\frac{1}{3}$ (or equivalent)

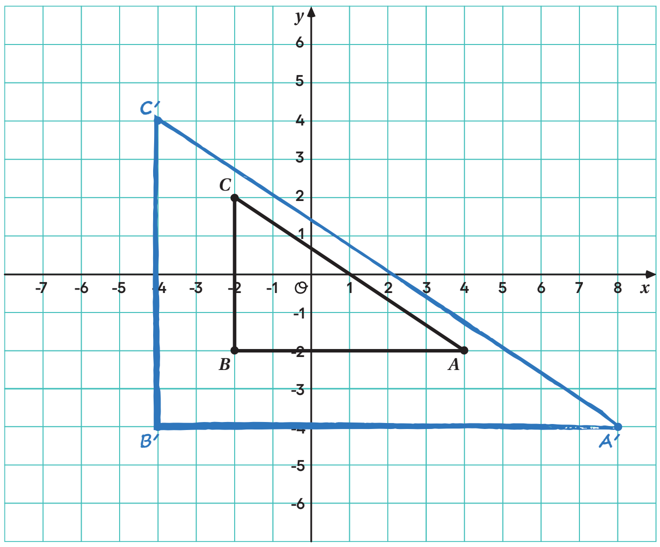
d. Triangle B is a dilation of C with approximately what scale factor?

$\frac{2}{3}$ (or equivalent)

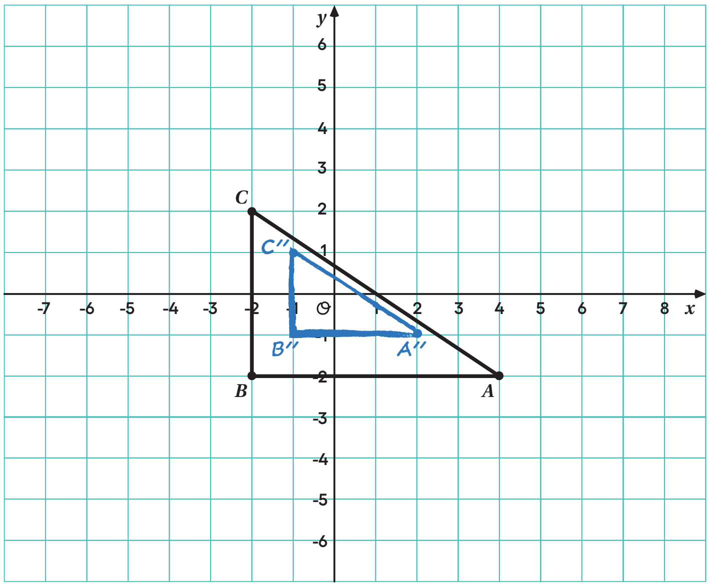
Problem 2

Here is a triangle.

a. Draw the dilation of triangle ABC, with center (0, 0), and scale factor 2. Label this triangle $A'B'C'$.



b. Draw the dilation of triangle ABC, with center (0, 0), and scale factor $\frac{1}{2}$. Label this triangle $A''B''C''$.

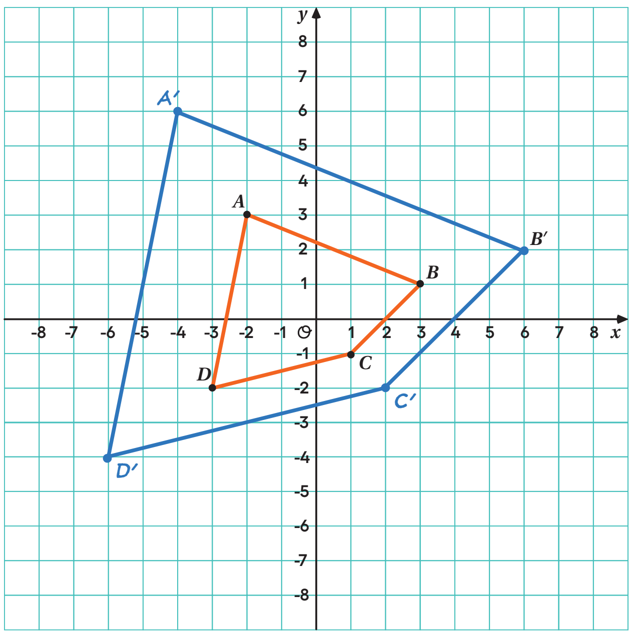


c. Is $A''B''C''$ a dilation of triangle $A'B'C'$? If yes, what are the center of dilation and the scale factor?

Yes, $A''B''C''$ is a dilation of $A'B'C'$ with center (0, 0) and scale factor $\frac{1}{4}$.

Problem 3

Quadrilateral $ABCD$ is dilated with center (0, 0), taking B to B'. Draw $A'B'C'D'$.



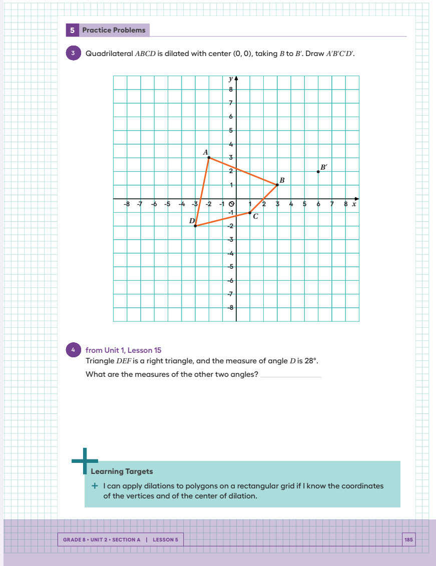
Problem 4

from Unit 1, Lesson 15

Triangle DEF is a right triangle, and the measure of angle D is 28°. What are the measures of the other two angles?

90° and 62°

Student Workbook


Practice Problems

Quadrilateral $ABCD$ is dilated with center (0, 0), taking B to B'. Draw $A'B'C'D'$.

from Unit 1, Lesson 15

Triangle DEF is a right triangle, and the measure of angle D is 28°. What are the measures of the other two angles?

Learning Targets

I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.