Finding Differences

Goals

- Compare and contrast (orally) subtraction expressions that have the same numbers in the opposite order.
- Recognize that the "difference" of two numbers can be positive or negative, depending on the order they are listed, while the "distance" between two numbers is always positive.
- Subtract signed numbers, and explain (orally) the reasoning.

Learning Target

I can subtract positive and negative numbers.

Student Learning Goal

Let's bring addition and subtraction together.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- 5 Practices
- · Math Talk
- MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

• Math Community Chart: Activity 2

Lesson Narrative

In this lesson, students build fluency with subtracting signed numbers. They learn that "the difference of a and b" means a – b. They see that the difference between two numbers can be positive or negative depending on the relative position of the numbers on the number line. For example, 9-7=2, while 7-9=-2. If students had previously understood subtraction to involve always subtracting the lesser number from the greater number, then this lesson is an opportunity for them to develop a more nuanced understanding of subtraction. As students write subtraction expressions to represent differences in elevation, they are reasoning abstractly and quantitatively.

Lesson Timeline

5 min

Warm-up

10 min

Activity 1

10 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Warm-up

Math Talk: Missing Addend



Activity Narrative

This *Math Talk* focuses on finding a missing addend. It encourages students to think about using subtraction to solve an addition equation and to rely on strategies they know for finding unknown values to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students find the difference between two values.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A.
$$247 + c = 458$$

$$c = 21$$

Sample reasoning: I can subtract 458 - 247 to find the value of c.

$$B.c + 389 = 721$$

$$c = 332$$

Sample reasoning: 389 is II less than 400 and 400 is 321 less than 721, which means 389 is II + 321 = 332 less than 721.

C.
$$c + 43.87 = 58.92$$

c = 15.05

Sample reasoning: 43.87 + 5 = 48.87; 48.87 + 10 = 58.87;

58.87 + 0.05 = 58.92; and 5 + 10 + 0.05 = 15.05.

D. $\frac{15}{8}$ + $c = \frac{51}{8}$

 $c = \frac{36}{8}$ (or equivalent)

Sample reasoning: Since both terms have the same denominator, we can subtract 5I-I5=36 to find the number of eighths.

Instructional Routines

Math Talk

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Please log in to the site before using the QR code or URL.



Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Instructional Routines

5 Practices

ilclass.com/r/10690701

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Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because ..." or "I noticed _______, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ______'s reasoning in a different way?"
"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Expressions with Altitude

10 min

Activity Narrative

In this activity, students find the difference between two values by subtracting one value from the other. Students return to the familiar context of climbing up and down a cliff to apply what they have learned about subtracting signed numbers. They reason abstractly and quantitatively as they represent the change in elevation with an expression and determine what the value of the expression means in this context. The familiar context of climbing up and down a cliff helps students write the numbers in a subtraction expression in the correct order.

Monitor for groups who use these different strategies for determining the change in the mountaineer's elevation, focusing on the situation with a beginning elevation of -200 feet and a final elevation of -50 feet. Here are some strategies students may use, ordered from less efficient to more efficient:

- Rewriting the subtraction problem as addition with an unknown addend and drawing a number line diagram to find the solution
- Drawing a number line diagram that shows the beginning and final elevations and reasoning about the signed distance between the numbers
- Rewriting the subtraction expression, a b, as adding the opposite, a + -b

Launch 🙎

Arrange students in groups of 2. Give students 3 minutes of quiet work time followed by a partner discussion. After students have come to agreement about the first few rows, tell them to complete the rest of the activity.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

A mountaineer is changing elevations. The table shows some beginning and final elevations.

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change (feet)
+400	+900	900 – 400	+500
+400	+50	50 - 400	-350
+400	-120	-120 - 400	-520
-200	+610	610 - (-200)	+810
-200	-50	-50 - (-200)	+150
-200	-500	-500 - (-200)	-300
-200	0	0 - (-200)	200

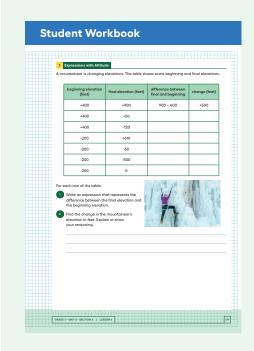
For each row of the table:

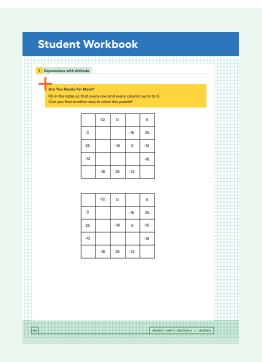
- 1. Write an expression that represents the difference between the final elevation and the beginning elevation.
- 2. Find the change in the mountaineer's elevation in feet. Explain or show your reasoning.



Sample responses:

- Row 1: I used a number line and saw that the mountaineer went up by 500 feet.
- Row 2: I subtracted the numbers and knew that since the mountaineer was at a lower elevation than when she started, the change would be negative.
- Row 3: The mountaineer would have to go down 400 feet to get to 0 and then another I20 feet down to get to -I20. That is a total of 520 down, or -520.
- Row 4: Subtracting -200 is like adding the opposite, or adding 200 to 610.
- Row 5: Since both elevations are negative, the mountaineer never crossed 0. She ended closer to 0, so she went up by I50 feet.
- Row 6: Subtracting -200 is like adding the opposite, or adding 200 to -500.
- Row 7: The distance between -200 and 0 is 200. Since the mountaineer started at a negative elevation, she had to travel in a positive direction to get to 0.





Are You Ready for More?

Fill in the table so that every row and every column sums to 0. Can you find another way to solve this puzzle?

Sample response:

-18	-12	0	25	5
0	5	-12	-18	25
25	0	-18	5	-12
-12	25	5	0	-18
5	-18	25	-12	0

Sample response:

-18	-12	0	25	5
0	-7	0	-18	25
25	0	-18	5	-12
-12	37	-7	0	-18
5	-18	25	-12	0

Activity Synthesis

The purpose of this discussion is to help students visualize how subtracting a number is equivalent to adding its opposite. Students should also understand how the beginning and final elevations relate to the order in which the numbers are subtracted.

Invite previously selected groups to share their reasoning for one of the rows where the mountaineer has a beginning elevation of -200 feet. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

For example, students who rewrite subtraction as addition with a missing addend may write an expression like -200 + ? = -50. They may use reasoning or draw an incomplete addition number line.

Students who draw a number line showing the beginning and final elevations may create something like this diagram to represent a beginning elevation of -200 feet and a final elevation of -50 feet.



Connect the number line diagram to the subtraction expression -50 – (-200). Note how the beginning and final elevations are shown and how the direction of the arrow between them represents a positive change in elevation of 150 feet. Connect the missing addend expression by showing that -200 + 150 = -50.

Connect the different responses to the learning goals by asking questions such as:

"Where can the mountaineer's beginning elevation be seen in each strategy?"

"Where can the mountaineer's ending elevation be seen in each strategy?"

"Where can the magnitude of the change in the mountaineer's elevation be seen in each strategy?"

"Where can the direction of the change in the mountaineer's elevation be seen in each strategy?"

"Which strategy seems more or less efficient for finding the difference between two numbers?"

Activity 2

Does the Order Matter?



Activity Narrative

In this activity, students work with a partner to evaluate related subtraction expressions. Through repeated reasoning, they notice that if the order of the two numbers in a subtraction expression is reversed, the values of the two expressions have the same magnitude but opposite signs.

No strategies are suggested for students as they evaluate each expression. Some strategies they may use include drawing a number line or using the additive inverse.

Launch 🞎

Math Community

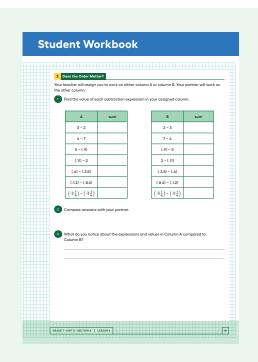
Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms, or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice. This norm should be one that they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing



Before working with the subtraction expressions in the task statement, consider telling students to close their books or devices and display these addition expressions for all to see. Discuss whether the order of the addends matters when adding signed numbers.

Α	В
3 + 2	2+3
4 + 7	7 + 4
5 + (-9)	(-9) + 5
(-11) + 2	2 + (-11)
(-6) + (-3.5)	(-3.5) + (-6)
(-1.2) + (-8.6)	(-8.6) + (-1.2)
$\left(-2\frac{1}{4}\right) + \left(-3\frac{1}{4}\right)$	$\left(-3\frac{1}{4}\right) + \left(-2\frac{1}{4}\right)$

Arrange students in groups of 2. Tell one student in each group to work on column A and the other student to work on column B. Give students quiet work time followed by time for partner discussion. Then follow with a whole-class discussion.

Student Task Statement

Your teacher will assign you to work on either column A or column B. Your partner will work on the other column.

1. Find the value of each subtraction expression in your assigned column.

A	sum
3 – 2	I
4 – 7	-3
5 - (-9)	14
(-11) – 2	-13
(-6) - (-3.5)	-2.5
(-1.2) - (-8.6)	7.4
$\left(-2\frac{1}{4}\right)-\left(-3\frac{1}{4}\right)$	I

В	sum
2 - 3	-1
7 – 4	3
(-9) – 5	-14
2 - (-11)	13
(-3.5) – (-6)	2.5
(-8.6) - (-1.2)	-7.4
$\left(-3\frac{1}{4}\right)-\left(-2\frac{1}{4}\right)$	-1

2. Compare answers with your partner

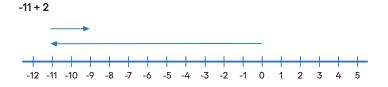
3. What do you notice about the expressions and values in Column A compared to Column B?

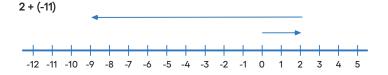
Sample response: Corresponding expressions from Column A and Column B have the same numbers, but the numbers are subtracted in the opposite order. The values of corresponding expressions are opposites (or additive inverses) of each other.

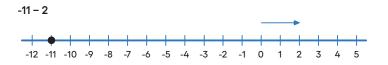
Activity Synthesis

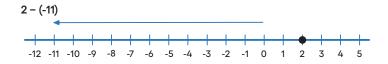
The purpose of this discussion is for students to discuss how changing the order of two numbers being subtracted will give the additive inverse of the original difference: a - b = -(b - a). The two differences have the same magnitude but opposite signs. On a number line diagram, the arrows are the same length but pointing in opposite directions.

Consider displaying these unfinished number line diagrams as specific examples that students can refer to during the whole-class discussion:









Discuss:

"Does changing the order of the numbers in an addition expression change the value? Why?"

No. Addition is commutative. It doesn't matter which arrow you draw first. When you put them tip-to-tail, they will end up in the same place.

"Does changing the order of the numbers in a subtraction expression change the value? Why?"

Yes. Subtraction is not commutative. Switching the arrow and the point changes the diagram.

Building on Student Thinking

Some students may try to interpret each subtraction expression as an addition equation with a missing addend and struggle to calculate the correct answer. Remind them that we saw another way to evaluate subtraction is by adding the additive inverse. Consider demonstrating how one of the subtraction expressions can be rewritten (for example, -11 - 2 = -11 + (-2)).

Some students may struggle with deciding whether to add or subtract the magnitudes of the numbers in the problem. Prompt them to sketch a number line diagram and notice how the arrows compare.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.

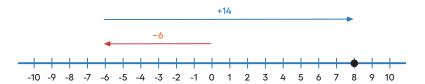
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question that asks students what they notice about the expressions and values in Column A compared to Column B. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

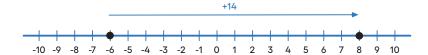
\wp	"The difference between x and y is 16. What is the difference between y and x ? How do you know?"				
	-16, because that is the opposite of 16				
\bigcirc	"The difference between $\it q$ and $\it r$ is -3.8. What is the difference between $\it r$ and $\it q$? How do you know?"				
	3.8, because that is the opposite of -3.8				
N	Math Community				
th ho	vite 2–3 students to share the norm they chose and how it supported e work of the group or a realization they had about a norm that would eve worked better in this situation. Provide these sentence frames to help udents organize their thoughts in a clear, precise way:				
Q	"I picked the norm '" It really helped me/my group because"				
	"I picked the norm '" During the activity, I realized the norm '" would be a better focus because"				
	esson Synthesis				
	are with students,				
Q	"Today we represented differences in elevation with subtraction expressions. We saw some patterns that happen when subtracting signed numbers."				
То	To review the distinction between difference and distance, consider asking:				
\bigcirc	"What is the difference between 12 and 10?"				
	12 - 10 = 2				
\bigcirc	"What is the difference between 10 and 12?"				
	10 - 12 = -2				
\bigcirc	"What is the distance between 12 and 10?"				
	12 - 10 = 2				
\bigcirc	"What is the distance between 10 and 12?"				
	10 - 12 = 2				
\bigcirc	"If the difference between x and y is -5, then:"				
	"What is the distance between x and y ?"				
	5				
\bigcirc	"What is the difference between y and x ?"				
	5				
\bigcirc	"What is the distance between y and x ?"				
	5				

Lesson Summary

To find the difference between two numbers, we subtract them. Usually, we subtract them in the order they are named. For example, "the difference of +8 and -6" means 8 - (-6). We can find the value of 8 - (-6) by thinking -6 + ? = 8. The second arrow must be 14 units long, pointing to the right.



The difference of two numbers tells us how far apart they are on the number line and in which direction. The difference of +8 and -6 is 14 because these numbers are 14 units apart, and 8 is to the right of -6.



If we subtract the same numbers in the opposite order, we get the opposite number. For example, "the difference of -6 and +8" means -6 – 8. This difference is -14 because these numbers are 14 units apart, and -6 is to the left of +8.



In general, the distance between two numbers a and b on the number line is |a-b|. Note that the *distance* between two numbers is always positive, no matter the order. But the *difference* can be positive or negative, depending on the order.

Cool-down

A Subtraction Expression

5 min

Student Task Statement

Select **all** of the choices that are equal to (-5) – (-12).

A.-7

B.7

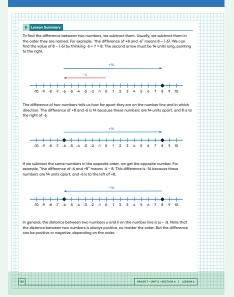
C. The difference between -5 and -12

D. The difference between -12 and -5

E. (-5) + 12)

F. (-5) + (-12)

Student Workbook



Responding To Student Thinking

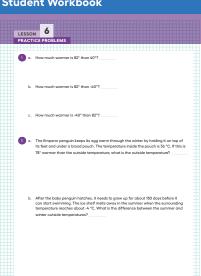
Points to Emphasize

If students struggle with subtracting signed numbers, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the differences they are asked to calculate in these activities:

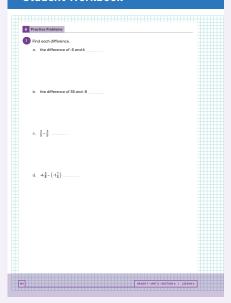
Grade 7, Unit 5, Lesson 7, Activity 1 Phone Inventory

Grade 7, Unit 5, Lesson 7, Activity 2 Climbing Mount Kilimanjaro 6





Student Workbook



Problem 1

a. How much warmer is 82° than 40°?

42° warmer

b. How much warmer is 82° than - 40°?

122° warmer

c. How much warmer is - 40° than 82°?

-122° warmer

Problem 2

a. The Emperor penguin keeps its egg warm through the winter by holding it on top of its feet and under a brood pouch. The temperature inside the pouch is 36 °C. If this is 78° warmer than the outside temperature, what is the outside temperature?

-42 °C, because 36 - 78 = -42

b. After the baby penguin hatches, it needs to grow up for about 150 days before it can start swimming. The ice shelf melts away in the summer when the surrounding temperature reaches about - 4 °C. What is the difference between the summer and winter outside temperatures?

Problem 3

Find each difference.

a. the difference of -5 and 6

-11

b. the difference of 35 and -8

43

c. $\frac{2}{5} - \frac{3}{5}$

 $-\frac{1}{5}$ (or equivalent)

d. $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$

 $-3\frac{1}{8}$ (or equivalent)

Problem 4

from Unit 4, Lesson 10

A family goes to a restaurant. When the bill comes, this is printed at the bottom of it:

Gratuity Guide for Your Convenience:

15% would be \$4.89

18% would be \$5.87

20% would be \$6.52

How much was the price of the meal? Explain your reasoning.

The bill was close to \$32.60.

Sample reasoning: We can't tell the exact amount because the suggested dollar amounts have been rounded to the hundredths place. $4.89 \div 0.15 = 32.60$, $5.87 \div 0.18 = 32.61$, and $6.52 \div 0.2 = 32.60$. The first and third quotients are exact while the middle quotient is rounded to the nearest hundredth.

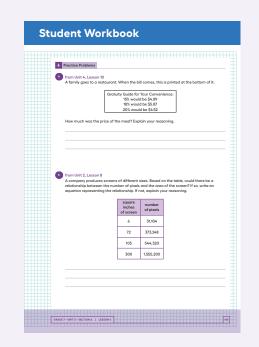
Problem 5

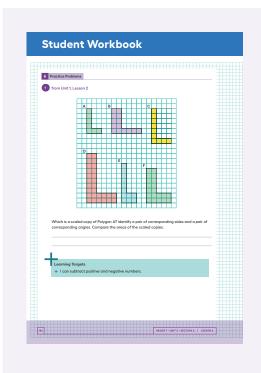
from Unit 2, Lesson 8

A company produces screens of different sizes. Based on the table, could there be a relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your reasoning.

square inches of screen	number of pixels	
6	31,104	
72	373,248	
105	544,320	
300	1,555,200	

It is a proportional relationship, which can be represented with the equation $p = 5,184 \cdot a$, where p represents the number of pixels and a is the area of the screen in square inches.





Problem 6

from Unit 1, Lesson 2

Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.

Sample response: Polygon D is a scaled copy of Polygon A. This is true because all of the side lengths are doubled. The area of Polygon D is 4 times the area of Polygon A.

