Lesson 1

The Areas of Squares

Goal

Calculate the area of a tilted square on a grid by using decomposition, and explain (orally) the solution method.

Learning Targets

- I can find the area of a tilted square on a grid by using methods like "decompose and rearrange" and "surround and subtract."
- I can find the area of a triangle.

Lesson Narrative

Students learned previously how to find the area of a square given its side length. In this lesson, students investigate the areas of squares presented in different ways. The work of this lesson is intentionally focused on setting students up to see the geometric connection between a square's area and its side length, which will happen in a following lesson.

Students begin by comparing figures whose areas are easily determined by either composing and counting square units or by decomposing and rearranging into simpler, familiar shapes. Next, students manipulate two sets of shapes to see that if one set of shapes completely covers another set of shapes with no gaps and no overlaps, then both sets must have the same area. This idea will help students understand and explain informal proofs of the Pythagorean Theorem in later lessons.

Lastly, students find areas of "tilted" squares using a strategy where the tilted square is surrounded by a larger square whose area can easily be determined. The area of the 4 triangles that are not part of the tilted square are then subtracted from the area of the larger square.

One of the activities in this lesson works best when each student has access to devices that can run the applet. The applet allows students to manipulate small pieces digitally as they consider the area of the three squares without having to keep track of multiple small pieces of paper.

Student Learning Goal

Let's investigate the areas of squares.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Copy

- Making Squares Cutouts (1 copy for every 2 students): Activity 2

Activity 2:

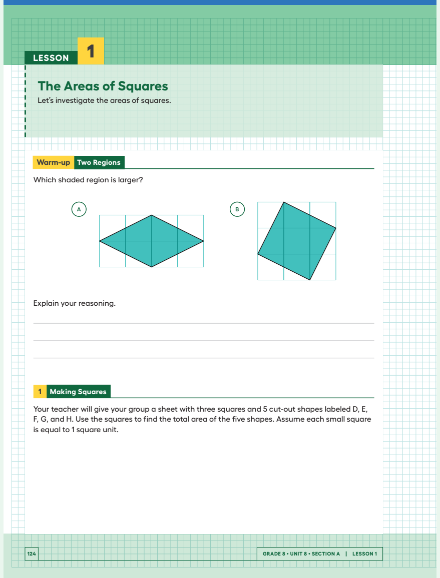
- For the print version, cut out the 5 shapes labeled D, E, F, G, and H.
- For the digital version of the activity, acquire devices that can run the applet.

Building on Student Thinking

If students think that the side length of either quadrilateral is 2 units because the sides are partitioned into two sub-pieces that look about 1 unit long, consider asking:

"How did you measure the side of each shape?"
"How could a compass or tracing paper be used to compare the side lengths to 2 units on the grid?"

Student Workbook



Warm-up

Two Regions

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to review how to find the area of a region on a grid by decomposing and rearranging pieces. They will use these techniques in a later lesson to understand and explain a proof of the Pythagorean Theorem.

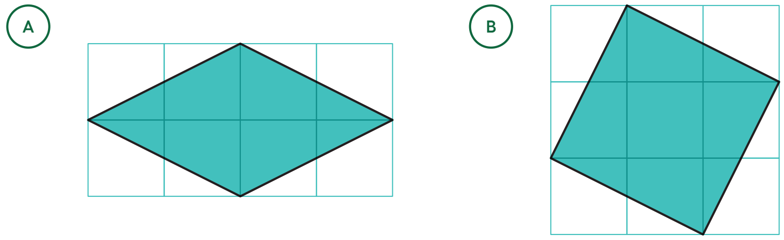
Monitor for students who use different strategies for finding area, including putting pieces together to make whole units in different ways and adding up the whole grid then subtracting the white space to indirectly figure out the area of the shaded region.

Launch

Display the image for all to see and ask students to vote on which of the shaded regions they predict has the larger area without doing any calculations. Record the results next to the displayed image. Then give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Which shaded region is larger?



Explain your reasoning.

Figure B has the larger shaded region because Figure A is 4 square units and Figure B is 5 square units.

Activity Synthesis

The goal of this discussion is to make sure students see different methods for determining the areas of different shapes.

Invite students who used the strategies described in the *Activity Narrative* to share their methods for finding area. Record and display their responses for all to see.

Activity 1
Making Squares

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students determine the total area of five shapes. How students determine the area of the shapes is left open-ended on purpose.

Monitor for groups who calculate the area of each shape individually, or fit the shapes into the squares and then calculate the area. It is possible to fit all five shapes into the two smaller squares or into the one larger square.

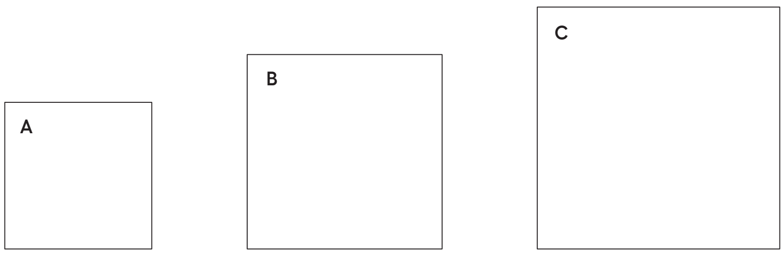
These shapes are part of a transformations-based proof of the Pythagorean Theorem that students will see in a future lesson. The five cut-out shapes in this activity can be reserved at the end of the activity for future use.

In the digital version of the activity, students use an applet to determine the total area of five shapes. The applet allows students to manipulate the shapes to fit them inside of squares with known areas. This activity works best when each student has access to the applet because preparation of paper manipulatives may be time-consuming. If students don't have individual access, displaying the applet for all to see would be helpful during the *Activity Synthesis*.

Launch

Display the image of three squares for all to see. Ask students which is larger: the combined area of the two smaller, Squares A and B, or the area of the one larger square, Square C?

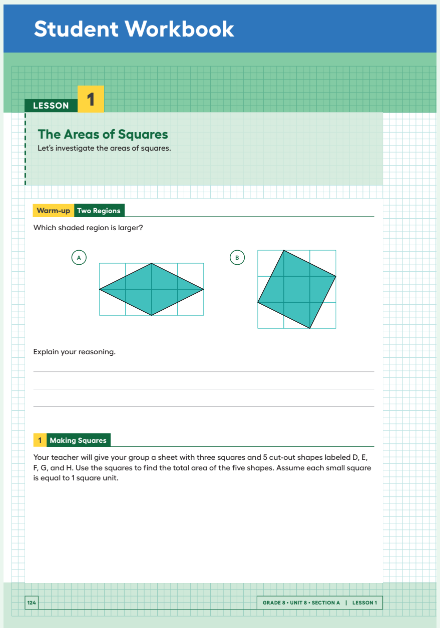
Give students 30–60 seconds of quiet think time, and then survey the class for their responses, displaying a tally of responses for all to see.



Next, display the image of three squares with grids for all to see. Tell students that these are the same three squares as before and repeat the previous question.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization


Student Workbook

LESSON 1

The Areas of Squares

Let's investigate the areas of squares.

Warm-up Two Regions

Which shaded region is larger?

Explain your reasoning.

1 Making Squares

Your teacher will give your group a sheet with three squares and 5 cut-out shapes labeled D, E, F, G, and H. Use the squares to find the total area of the five shapes. Assume each small square is equal to 1 square unit.

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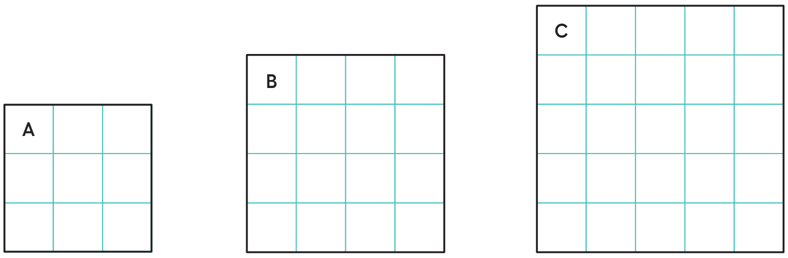
Give students 30–60 seconds of quiet think time, and then select 1–2 students to explain their reasoning.

9 square units plus 16 square units is the same as 25 square units, so the combined area of squares A and B is the same as the area of square C.



Arrange students in groups of 2 and distribute 1 copy of the three squares half-sheet and 5 pre-cut shapes from the blackline master to each group. The 5 pre-cut shapes are labeled on one side to facilitate conversation.

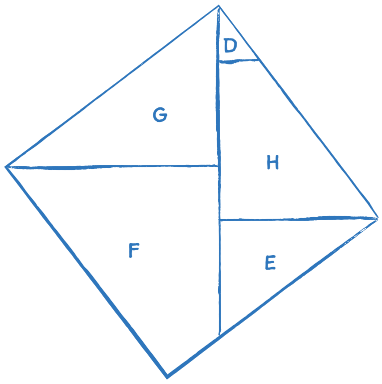
Student Task Statement

Your teacher will give your group a sheet with three squares and 5 cut-out shapes labeled D, E, F, G, and H. Use the squares to find the total area of the five shapes. Assume each small square is equal to 1 square unit.

The total area of the 5 shapes is 25 square units.

See the blackline master for how the 5 shapes fit into the two smaller squares.

Sample response:



Activity Synthesis

The purpose of this discussion is to prime student thinking about strategies for calculating area, particularly the use of rearranging and composition, in preparation for future activities. Ask groups previously identified to share their strategies.

If not brought up during the discussion, ask students how they know the total area is 25 square units. Make sure they understand that if the shapes fit in a square perfectly without overlaps or gaps, then the combined area of the shapes must be the same as the area of the square.

Activity 2
Decomposing to Find Area

15 min

Activity Narrative

The purpose of this activity is for students to find the areas of squares whose side lengths are not easily determined by inspection. The squares are represented in an increasingly abstract way: first showing all of the square units, next showing only the grid units along the edges, and finally providing labels only for the side lengths. This sequence provides students with practice decomposing and rearranging shapes, which will help them understand and explain a proof of the Pythagorean Theorem in a later lesson.

This sequence of tasks also provides practice for students in using the area and finding strategies they will need to understand and explain a proof of the Pythagorean Theorem in a later lesson.

After students find the area of the square in the first problem, pause for a class discussion about strategies used. In particular, students should see that the side length and area of the larger square (with sides that lie along grid lines surrounding the shaded "tilted" square) can easily be found by counting.

Monitor for students who find the area of the first shaded square by:

- Composing two triangles to make a rectangle and then subtracting the area of two rectangles from the larger surrounding square.
- Using the formula for the area of a triangle and then subtracting the area of four triangles from the larger surrounding square.

Make these strategies explicit to help students who are only able to count unit squares make a successful transition to the third square.

Launch

Give students 2–3 minutes of quiet work time for the first question and then pause for a brief whole-class discussion. Select students with different strategies, such as those described in the *Activity Narrative*, to share. While decomposing the shaded square and rearranging the pieces is a correct strategy, students need the subtraction strategy for more abstract work later, so it is better not to highlight the decomposing strategy because it does not generalize as well.

Display 2–3 approaches from selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"How are the two strategies the same? How are they different?"

"Are there any benefits or drawbacks to one representation compared to another?"

"Did anyone solve the problem the same way but would explain it differently?"

Follow the discussion with work time for students to finish the remaining questions.

Instructional Routines

MLR7: Compare and Connect

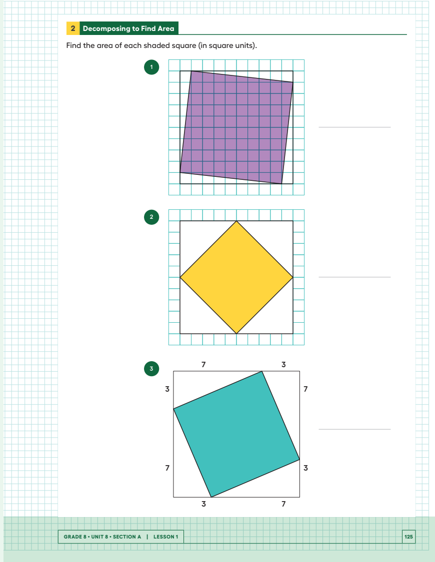
ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

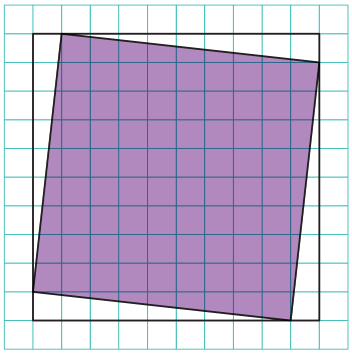
Student Workbook



Student Task Statement

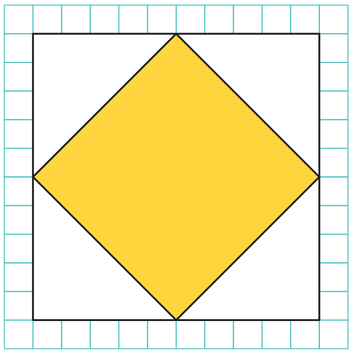
Find the area of each shaded square (in square units).

1.



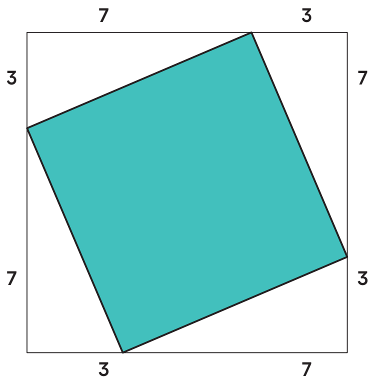
82 square units

2.



50 square units

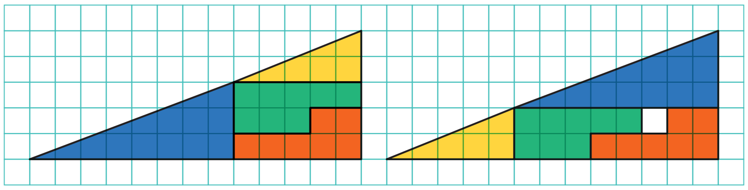
3.



58 square units

Are You Ready for More?

Any triangle with a base of 13 and a height of 5 has an area of $\frac{65}{2}$.



Both shapes in the figure have been partitioned into the same four pieces. Find the area of each of the pieces, and verify the corresponding parts are the same in each picture. There appears to be one extra square unit of area in the right figure. If all of the pieces have the same area, how is this possible?

There is at first an apparent paradox! Each of the two figures is made up of two triangles with areas 12 and 5, and two polygons with areas 8 and 7, for a total of 32. But in the right figure there seems to "magically" be room for an extra square unit of grid space!

The resolution is that, in fact, neither of the two figures are actually triangles! The slopes of the hypotenuses of the two triangles making up the figures are $\frac{2}{5}$ and $\frac{3}{8}$, which are relatively close but not equal, and so do not line up to make one straight hypotenuse for the larger apparent right triangle. So the only flaw in the argument is the expectation that the formula for the area of a triangle should apply.

Activity Synthesis

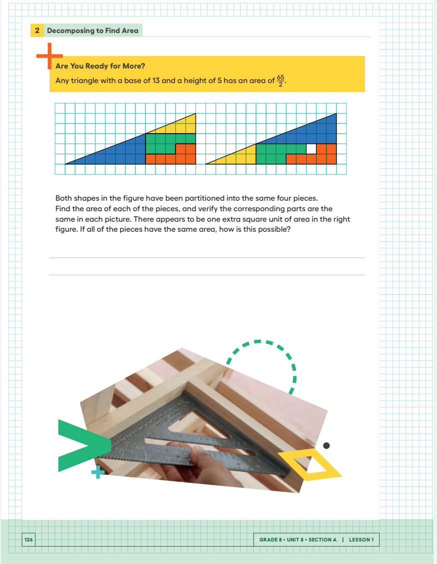
The goal of this discussion is to emphasize the strategy of decomposing the larger surrounding square and subtracting the area of the smaller triangles, either by rearranging the triangles into rectangles or by finding the areas of the triangles using the triangle area formula.

Select 2–3 students to explain how they found the area of the third square. Here are some questions for discussion:

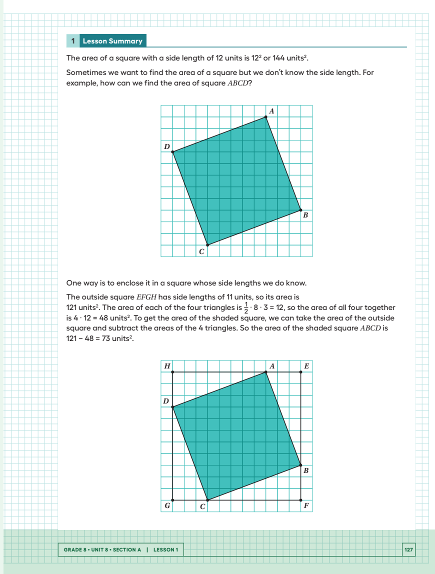
"Why did you choose that particular strategy?"

"Did anyone solve the problem the same way but would explain it differently?"

Student Workbook



Student Workbook


1 Lesson Summary

The area of a square with a side length of 12 units is 12^2 or 144 units^2.

Sometimes we want to find the area of a square but we don't know the side length. For example, how can we find the area of square $ABCD$?

One way is to enclose it in a square whose side lengths we do know.

The outside square $EFGH$ has side lengths of 11 units, so its area is 121 units^2. The area of each of the four triangles is $\frac{1}{2} \cdot 8 \cdot 3 = 12$, so the area of all four together is $4 \cdot 12 = 48$ units^2. To get the area of the shaded square, we can take the area of the outside square and subtract the areas of the 4 triangles. So the area of the shaded square $ABCD$ is $121 - 48 = 73$ units^2.

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Lesson Synthesis

The purpose of this discussion is to review strategies for finding the area of squares with side lengths not along gridlines. Here are some questions for discussion:

- *"If you have a square and know the side length, how can you find the area?"*

  multiply the side length by itself; square the side length

- *"What needs to be true about two congruent triangles in order to compose them into a rectangle?"*

  They need to be right triangles.

Lesson Summary

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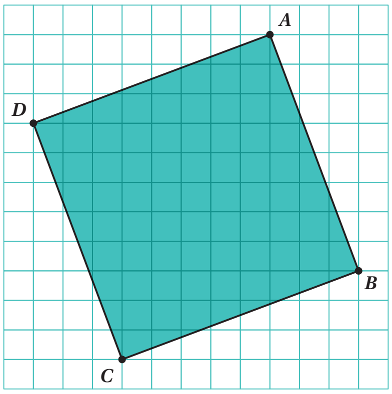

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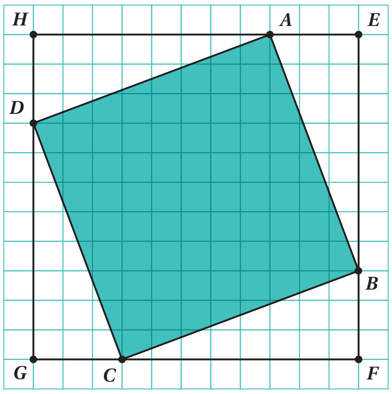


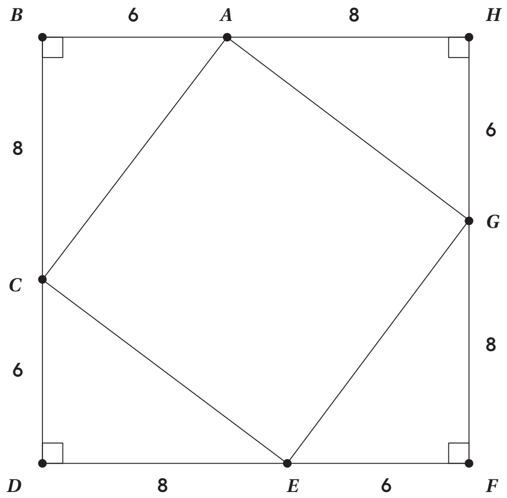
Cool-down

It's a Square

5 min

Student Task Statement

Find the area of square $ACEG$.

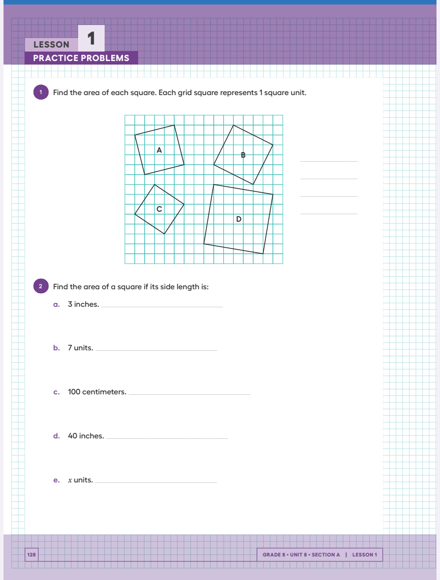


100 square units

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


LESSON 1
PRACTICE PROBLEMS

1 Find the area of each square. Each grid square represents 1 square unit.

2 Find the area of a square if its side length is:

a. 3 inches.

b. 7 units.

c. 100 centimeters.

d. 40 inches.

e. x units.

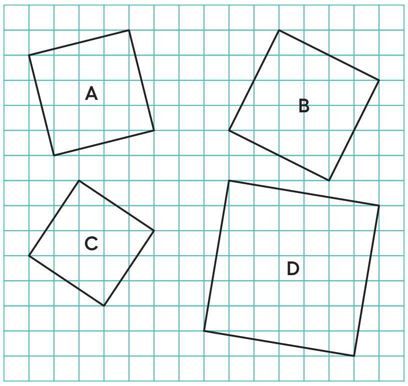
Lesson 1

Practice Problems

5 Problems

Problem 1

Find the area of each square. Each grid square represents 1 square unit.



A. 17 square units

B. 20 square units

C. 13 square units

D. 37 square units

Problem 2

Find the area of a square if its side length is:

a. 3 inches.

9 square inches

b. 7 units.

49 square units

c. 100 centimeters.

10,000 square centimeters

d. 40 inches.

1,600 square inches

e. x units.

x^2 square units

Problem 3

from Unit 7, Lesson 14

What is the value of $(3.1 \times 10^4) \cdot (2 \times 10^6)$?

A. 5.1×10^{10}

B. 5.1×10^{24}

C. 6.2×10^{10}

D. 6.2×10^{24}

Problem 4

from Unit 7, Lesson 15

Noah reads the problem, "Evaluate each expression and give the answer in scientific notation." The first problem part is: $5.4 \times 10^5 + 2.3 \times 10^4$.

Noah says, "I can rewrite 5.4×10^5 as 54×10^4. Now I can add the numbers: $54 \times 10^4 + 2.3 \times 10^4 = 56.3 \times 10^4$."

Do you agree with Noah's solution to the problem? Explain your reasoning.

I disagree with Noah's solution.

Sample reasoning: His calculations are correct, but his final answer is not in scientific notation. To finish the problem, he should convert his answer to the form 5.63×10^5.

Problem 5

from Unit 7, Lesson 6

Select **all** the expressions that are equivalent to 3^8.

A. $(3^2)^4$

B. 8^3

C. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

D. $(3^4)^2$

E. $\frac{3^6}{3^{-2}}$

F. $3^6 \cdot 10^2$

Student Workbook