Lesson 7

Similar Polygons

Goals

- Comprehend the phrase "similar polygons" (in written and spoken language) to mean the polygons have congruent corresponding angles and proportional side lengths.
- Critique (orally) arguments that claim two polygons are similar.

Learning Targets

- I can use angle measures and side lengths to conclude that two polygons are not similar.
- I know the relationship between angle measures and side lengths in similar polygons.

Lesson Narrative

The focus of this lesson is on quadrilaterals and ways to decide whether or not they are similar. Since the transformations we have studied (translations, rotations, reflections, dilations) do not change angle measures, similar polygons will have congruent corresponding angles. Also, since dilations change all side lengths by the same scale factor, similar polygons will have proportional corresponding side lengths. While these two key ideas can determine whether two polygons *are not* similar, using them to determine that polygons *are* similar will not be formally proven until high school. In grade 8, the definition of similar figures is made using transformations.

Students begin the lesson by considering whether similarity and congruence are mutually exclusive. This prepares students to use what they know about side lengths and angle measures to determine if two figures are similar or not and explain their reasoning. In the last activity, students are given a card with a figure and must find another student in the room with a similar (but not congruent) figure, again explaining why their two figures are similar.

Student Learning Goal

Let's look at sides and angles of similar polygons.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer (Activity 1)
- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR2: Collect and Display

Required Materials

Materials to Copy

- Find Someone Similar Cards (1 copy for every 10 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	10 min	15 min	10 min	5 min

Warm-up
Math Talk: Congruence and Similarity

5 min

Activity Narrative

This *Math Talk* focuses on congruence and similarity. It encourages students to think about the similarities and differences between the two terms and to rely on what they know about dilations and rigid transformations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students determine if two figures are similar or not.

In explaining whether a statement is always true, sometimes true, or never true, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time and ask them to give a signal when they have an answer and a strategy.

Invite students to share their strategies and record and display their responses for all to see.

Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether each statement is always true, sometimes true, or never true.

A. If two figures are congruent, then they are similar.

Always true

Sample reasoning: Congruent figures can be taken from one to the other by using translations, rotations, and reflections. Similar figures use these same transformations, but also use dilations. If we don't do a dilation, the figures are still similar.

B. If two figures are similar, then they are congruent.

Sometimes true. Similar figures can be taken from one to the other using translations, rotations, reflections, and dilations. Congruent figures do not allow dilations.

Sample reasoning: Two figures can be similar but be different sizes, like a square with side length 1 and a square with side length 2. These figures are not congruent, but 2 squares with the same side length would be similar and congruent.

C. If a triangle is dilated with the center of dilation at one of its vertices, the side lengths of the new triangle will change.

Sometimes true

Sample reasoning: The side lengths of the new triangle will change based on the scale factor. The side lengths will not change when the scale factor is 1.

Instructional Routines

Math Talk

ilclass.com/r/10694967

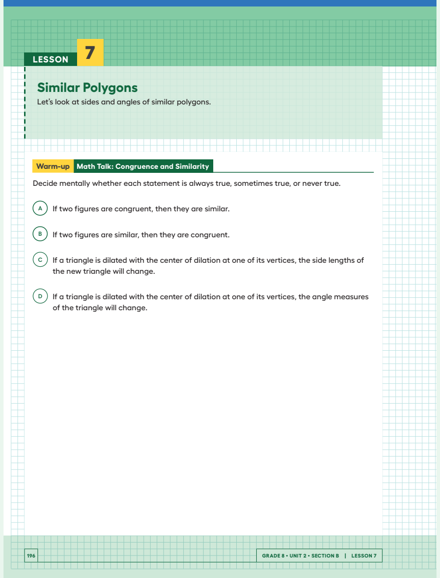
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 7

Similar Polygons

Let's look at sides and angles of similar polygons.

Warm-up Math Talk: Congruence and Similarity

Decide mentally whether each statement is always true, sometimes true, or never true.

A If two figures are congruent, then they are similar.

B If two figures are similar, then they are congruent.

C If a triangle is dilated with the center of dilation at one of its vertices, the side lengths of the new triangle will change.

D If a triangle is dilated with the center of dilation at one of its vertices, the angle measures of the triangle will change.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy, such as "First, I _____ because ..." or "I agree because ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

D. If a triangle is dilated with the center of dilation at one of its vertices, the angle measures of the triangle will change.

Never true

Sample reasoning: While a dilation will change the side lengths of the triangle (unless the scale factor is 1), the angle measures will always stay the same.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ___'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Are They Similar?

10 min

Activity Narrative

This activity focuses on common misconceptions about similar figures. Two polygons with proportional side lengths but different angles are not similar, and two polygons with the same angles but side lengths that are not proportional are also not similar. Reasoning through these problems will help further student thinking about when two polygons are not similar and give students an opportunity to critique the reasoning of others.

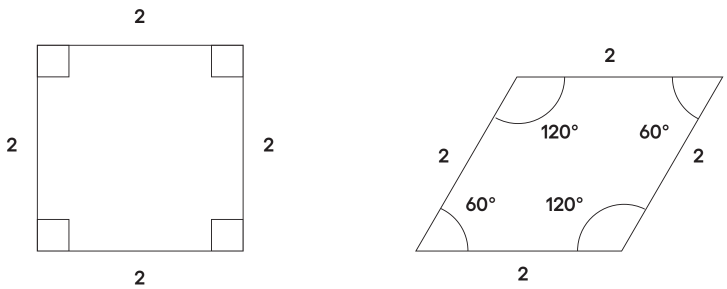
Launch

Ask students to share characteristics of similar polygons that are easy to recognize and record them for all to see. Students may say that similar polygons look like the same shape but could be different sizes (dilated by a given scale factor), or that similar polygons might be facing a different way (reflected or rotated). It is not necessary for students to discuss angle measures at this time. This discussion is meant to prime student thinking around similarity.

Arrange students in groups of 2.

Give students 3–4 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

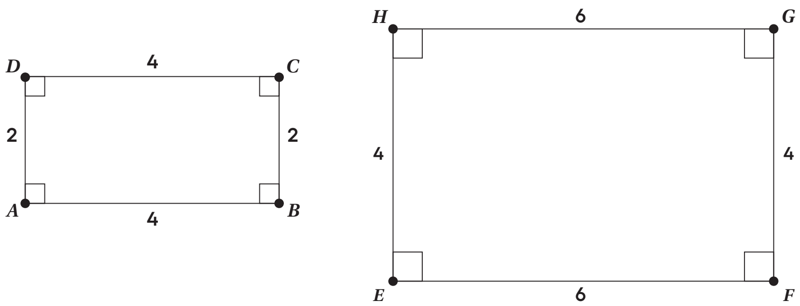


1. Let's look at a square and a rhombus.

Priya says, "These polygons are similar because their side lengths are all the same." Clare says, "These polygons are not similar because the angles are different." Do you agree with either Priya or Clare? Explain your reasoning.

Sample response: I agree with Clare. The angle measures in similar polygons are the same so these polygons can not be similar. Priya is incorrect because even though the side lengths of these polygons are the same, that is not enough to conclude that they are similar.

2. Now, let's look at rectangles $ABCD$ and $EFGH$.



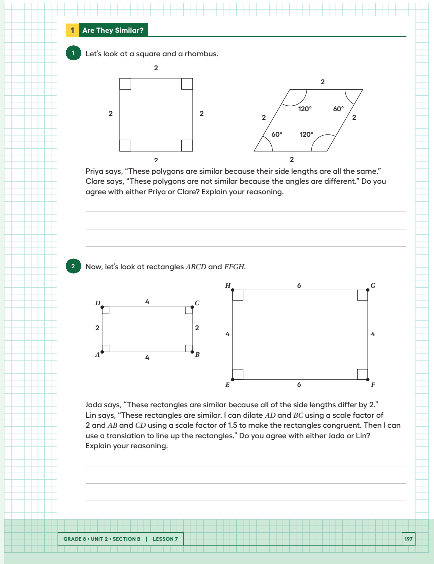
Jada says, "These rectangles are similar because all of the side lengths differ by 2." Lin says, "These rectangles are similar. I can dilate AD and BC using a scale factor of 2 and AB and CD using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles." Do you agree with either Jada or Lin? Explain your reasoning.

Sample response: I disagree with both Jada and Lin. Jada is right that the side lengths all differ by 2, but the scale factor needs to be the same for two rectangles to be similar. Lin is incorrect because the scale factor for dilating one set of sides can not be different from the scale factor for dilating the other set of sides.

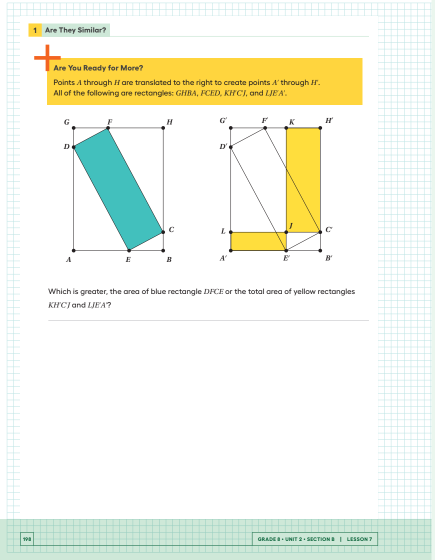
Building on Student Thinking

Some students may think the side lengths must be different in order for 2 figures to be similar, but a dilation does not need to be used in the sequence of transformations to show similarity. Prompt students to recall the *Warm-up* where students saw that congruent figures are always similar.

Student Workbook



Student Workbook



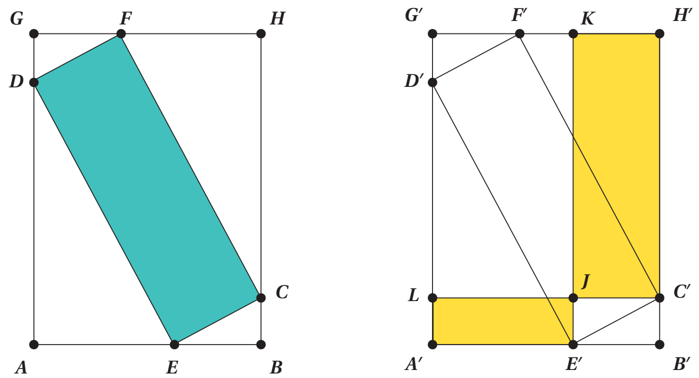
Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to whether they agree with Jada or Lin in the final question. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Are You Ready for More?

Points A through H are translated to the right to create points A' through H'. All of the following are rectangles: $GHBA$, $FCED$, $KH'C'J$, and $LJE'A'$.



Which is greater, the area of blue rectangle $DFCE$ or the total area of yellow rectangles $KH'C'J$ and $LJE'A'$?

They are equal. Rectangles $ABHG$ and $A'B'H'G'$ have the same area. The area of the blue rectangle $DECF$ can be found by subtracting the areas of the four right triangles in $ABHG$. The area of the two yellow rectangles can be found by subtracting the two smaller clear rectangles in $A'B'H'G'$. Since the area of rectangle $LJKG'$ is the same as the sum of the two larger right triangles from rectangle $ABHG$ and the area of the rectangle $E'B'C'J$ is the same as the sum of the two smaller right triangles from rectangle $ABHG$, the area of the blue rectangle is the same as the total area of the two yellow rectangles.

Activity Synthesis

The goal of this discussion is to make sure students understand that for similar figures, corresponding angles are congruent and corresponding side lengths are proportional.

Discuss with students:

- *"In the question about the square and the rhombus, both polygons had all the same side lengths but they still were not similar. What does this tell us about similar polygons?"*

  If corresponding angles are not congruent, then the figures cannot be similar.

- *"In the question about the rectangles, both polygons had all the same angles but they still were not similar. What does this tell us about similar polygons?"*

  If the side lengths are not all scaled by the same value, then the figures cannot be similar.

Activity 2
Find Someone Similar

15 min

Activity Narrative

In this activity, students practice identifying similar figures. Each student has a card with a figure on it and they identify someone with a similar (but not congruent) figure. Students work together to construct an argument for why their two figures are similar.

Monitor for students using these methods to identify a partner:

- Process of elimination (figures with different angle measures cannot be similar to each other).
- Looking for the same kind of figure (rectangle, square, rhombus).
- Looking for the same kind of figure with congruent corresponding angles.
- Looking for the same kind of figure with corresponding sides scaled by the same scale factor.

Launch

Distribute one card to each student. Explain that the task is for each student to find someone else in the class who has a similar (but not congruent) figure to their own, and be prepared to explain how they know the two figures are similar.

If the number of students in class is not a multiple of 10, ensure that any unused cards are matching pairs of similar figures. If there is an odd number of students, 1 or more students can be responsible for 2 cards.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to find a partner with a similar but not congruent polygon. Display words and phrases such as "Our figures have different angle measures so they can't be similar," "We don't have the same kind of figure so they can't be similar," or "We have the same kind of figure and corresponding sides have a scale factor of ..."

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display.
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 2, Launch)

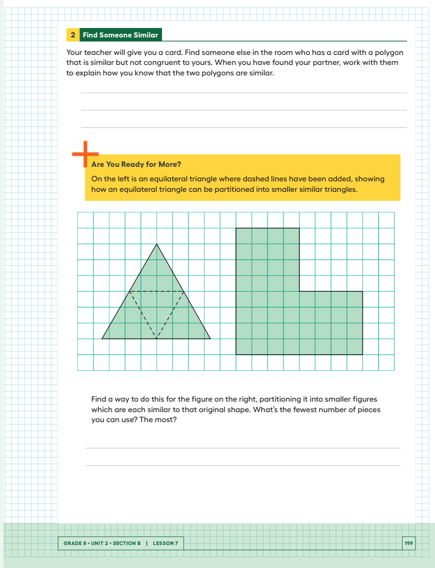
Representation: Internalize Comprehension.
Use multiple examples and non-examples to reinforce differences between similar and non-similar polygons.

Supports accessibility for: Conceptual Processing, Attention

Building on Student Thinking

Some students may have a hard time getting started. Prompt them to focus on properties of their figure that will be shared by a similar figure. For example, will a similar figure be a quadrilateral? Will a similar figure be square? A rectangle? A rhombus? Ask them to recall what is true about the angles in a similar figure.

Student Workbook


2 Find Someone Similar

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

Are You Ready for More?

On the left is an equilateral triangle where dashed lines have been added, showing how an equilateral triangle can be partitioned into smaller similar triangles.

Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?

GRADE 8 • UNIT 2 • SECTION B | LESSON 7

Student Task Statement

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

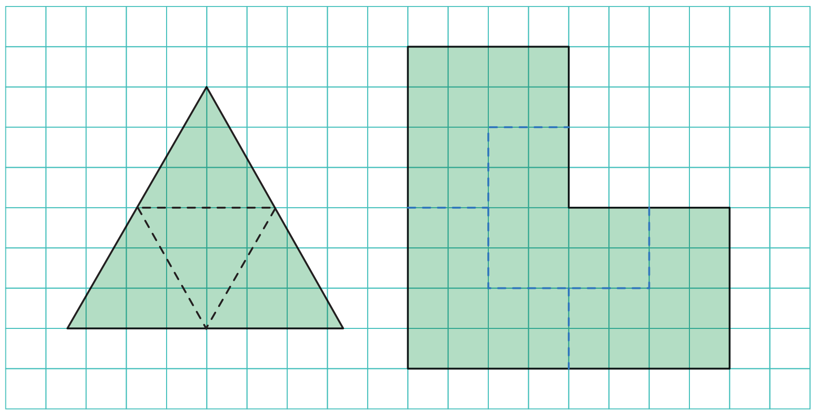
The similar quadrilaterals are:

- Rectangles with dimensions 6 by 8 and 9 by 12
  - Scale factor: $\frac{3}{2}$ or $\frac{2}{3}$ (or equivalent)
- Rectangles with dimensions 3 by 5 and 1.5 by 2.5
  - Scale factor: $\frac{1}{2}$ or 2 (or equivalent)
- Squares
  - Scale factor: $\frac{7}{8}$ or $\frac{8}{7}$ (or equivalent)
- Rhombuses with angles 60° and 120°
  - Scale factor: $\frac{9}{7}$ or $\frac{7}{9}$ (or equivalent)
- Rhombuses with angles 50° and 130°
  - Scale factor: $\frac{10}{9}$ or $\frac{9}{10}$ (or equivalent)

Are You Ready for More?

On the left is an equilateral triangle where dashed lines have been added, showing how an equilateral triangle can be partitioned into smaller similar triangles.

Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?



The fewest you can use is 4 pieces, as in the image, each a $\frac{1}{2}$-scale dilation of the original. There is no upper limit on the number of pieces you could use. For example, you could take the four $\frac{1}{2}$-scale pieces and divide them each using four $\frac{1}{4}$-scale pieces in exactly the same pattern, then cover each of those with four $\frac{1}{8}$-scale pieces, etc.

Activity Synthesis

The purpose of this discussion is to highlight different strategies for finding similar figures. For example, students can use the process of elimination (one shape is a rectangle and the other is not) or actively look for certain features (a rhombus whose angle measures are 60 degrees and 120 degrees).

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they determined that two figures were similar. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Discuss with students:

- *"Were the side lengths important?"*

  For some figures, such as the rectangles, the side lengths were important.

- *"Were the angles important?"*

  Yes. Several of the figures had different angles and that meant they were not similar.

- *"How did you know when two figures were not similar?"*

  The shapes were not the same. Corresponding angles didn't match. Corresponding sides were multiplied by different scale factors.

Some students may notice that *all* squares are similar since they have 4 right angles and proportional side lengths. If not mentioned by students, show how the first problem gives an example of rhombuses that are not similar to one another.

Lesson Synthesis

The main ideas to draw out of this lesson include:

Similar figures have congruent corresponding angles and proportional corresponding side lengths.

For some figures (like rectangles or squares), it is sufficient to focus on side lengths since corresponding angles are automatically congruent.

For some figures (like rhombuses), it is sufficient to focus on angles since corresponding side lengths are automatically proportional.

It is possible to determine that 2 polygons are not similar by identifying a single pair of corresponding angles whose measures are different or a pair of corresponding side lengths with a different ratio than another pair of corresponding side lengths.

Here are some questions for discussion:

- *"What is true about the corresponding angles of similar figures?"*

  They are congruent.

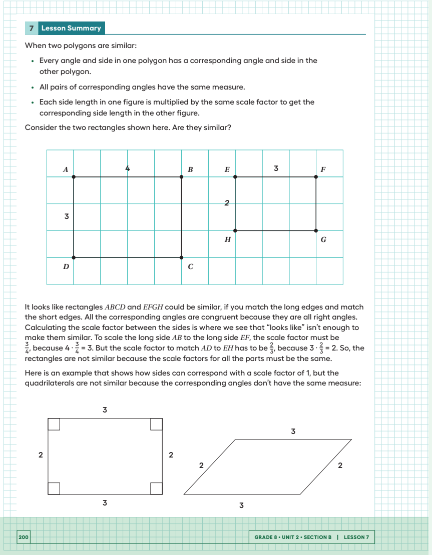
- *"Can you sketch an example of 2 figures with congruent corresponding angles that are similar? That aren't similar?"*

  "What is true about the corresponding sides of similar figures?"

  They are proportional.

- *"Can you sketch an example of 2 figures with proportional corresponding sides that are similar? That aren't similar?"*

Student Workbook


7 Lesson Summary

When two polygons are similar:

- Every angle and side in one polygon has a corresponding angle and side in the other polygon.
- All pairs of corresponding angles have the same measure.
- Each side length in one figure is multiplied by the same scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?

It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that "looks like" isn't enough to make them similar. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match AD to EH has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond with a scale factor of 1, but the quadrilaterals are not similar because the corresponding angles don't have the same measure:

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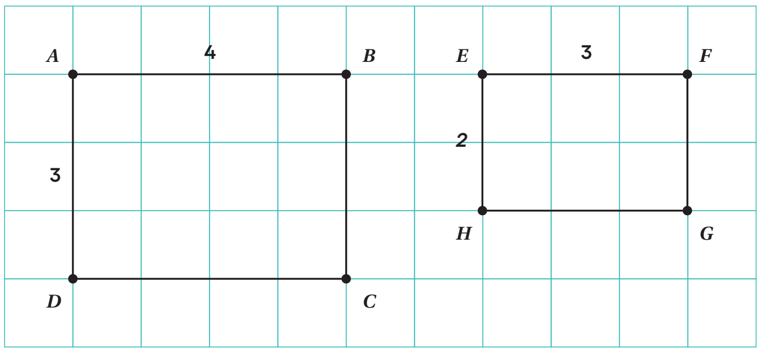
Lesson Summary

When two polygons are similar:

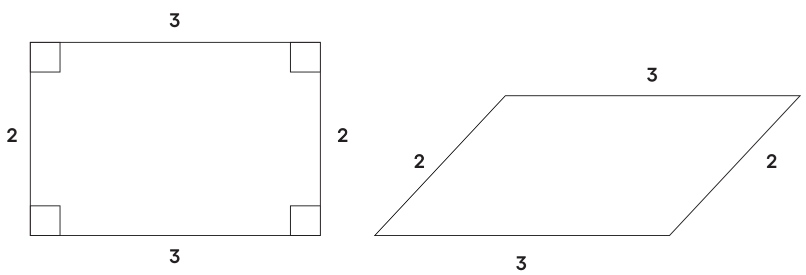
- Every angle and side in one polygon has a corresponding angle and side in the other polygon.
- All pairs of corresponding angles have the same measure.
- Each side length in one figure is multiplied by the same scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?



It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that "looks like" isn't enough to make them similar. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match AD to EH has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond with a scale factor of 1, but the quadrilaterals are not similar because the corresponding angles don't have the same measure:



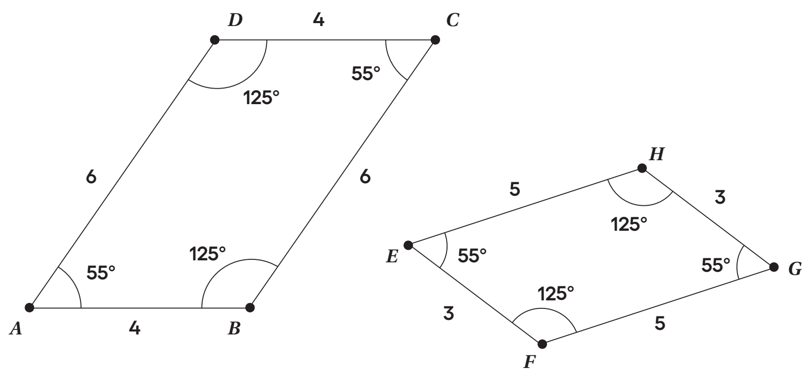
Cool-down

How Do You Know?

5 min

Student Task Statement

Are these 2 figures similar? Explain how you know.



The 2 figures are not similar.

Sample reasoning: Sides *CD* and *AB* are multiplied by a scale factor of $\frac{3}{4}$ to get sides *GH* and *EF*, but sides *AD* and *BC* are multiplied by a scale factor of $\frac{5}{6}$.

Responding To Student Thinking

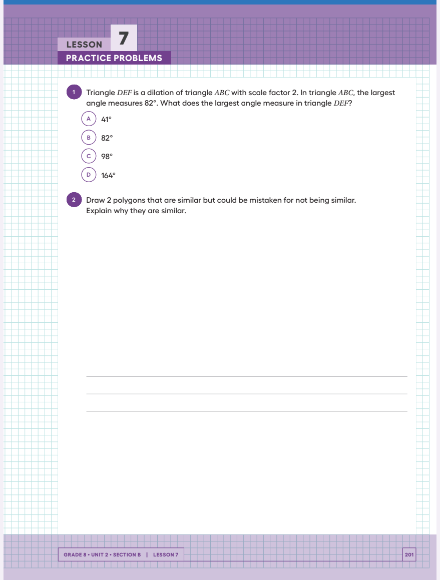
Points to Emphasize
If students struggle with determining whether or not two figures are similar, as opportunities arise over the next several lessons, revisit properties of similar figures. For example, in the activity referred to here, emphasize how corresponding side lengths must be proportional for two figures to be similar.

Unit 2, Lesson 9, Warm-up Two-three-four and Four-five-six

Practice Problems

6 Problems

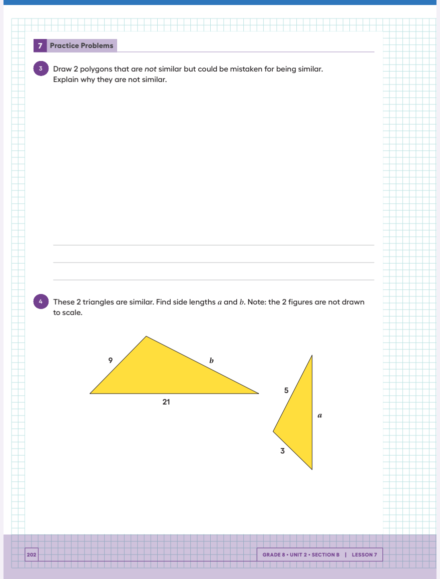
Student Workbook



LESSON 7
PRACTICE PROBLEMS

1. Triangle DEF is a dilation of triangle ABC with scale factor 2. In triangle ABC, the largest angle measures 82°. What does the largest angle measure in triangle DEF?
   - A 41°
   - B 82°
   - C 98°
   - D 164°
2. Draw 2 polygons that are similar but could be mistaken for not being similar. Explain why they are similar.

Student Workbook



7 Practice Problems

3. Draw 2 polygons that are *not* similar but could be mistaken for being similar. Explain why they are not similar.
4. These 2 triangles are similar. Find side lengths a and b. Note: the 2 figures are not drawn to scale.

Problem 1

Triangle DEF is a dilation of triangle ABC with scale factor 2. In triangle ABC, the largest angle measures 82°. What does the largest angle measure in triangle DEF?

A. 41°

B. 82°

C. 98°

D. 164°

Problem 2

Draw 2 polygons that are similar but could be mistaken for not being similar. Explain why they are similar.

Sample responses: 2 polygons with different orientations; 2 congruent polygons; two polygons where a reflection is part of the transformation from one to the other.

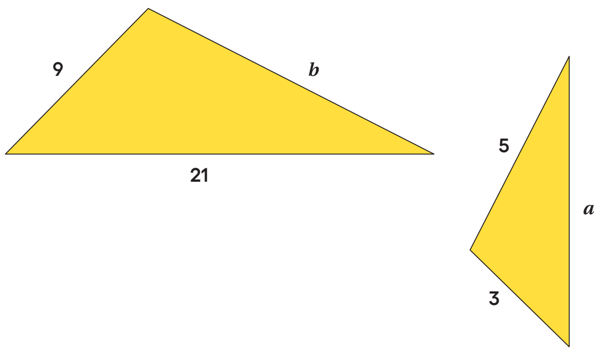
Problem 3

Draw 2 polygons that are *not* similar but could be mistaken for being similar. Explain why they are not similar.

Sample responses: Two polygons with the same angle measures but side lengths that are not proportional; 2 polygons with proportional side lengths but incorrect angle measures.

Problem 4

These 2 triangles are similar. Find side lengths a and b. Note: the 2 figures are not drawn to scale.

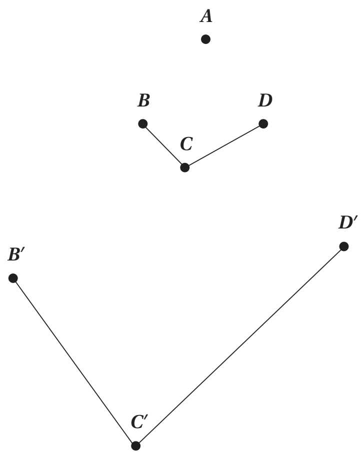


$a = 7$, $b = 15$

Problem 5

from Unit 2, Lesson 3

Jada claims that $B'C'D'$ is a dilation of BCD using A as the center of dilation.



What are some ways you can convince Jada that her claim is not true?

Sample responses: Below is a list of things which would have to be true if $B'C'D'$ is a dilation of BCD using A as the center of dilation. Any measurement which shows that one of the following is not true is a complete answer.

- Measure of angle BCD equals the measure of angle $B'C'D'$.
- $m\angle BCD = m\angle B'C'D'$
- A, B, and B' are collinear.
- A, C, and C' are collinear.
- A, D, and D' are collinear.
- $B'C'$ is parallel to BC.
- $C'D'$ is parallel to CD.

Problem 6

from Unit 1, Lesson 8

a. Draw a horizontal line segment AB.

Answers vary.

b. Rotate segment AB 90° counterclockwise around point A. Label any new points.

The segment is attached at point A and forms a right angle.

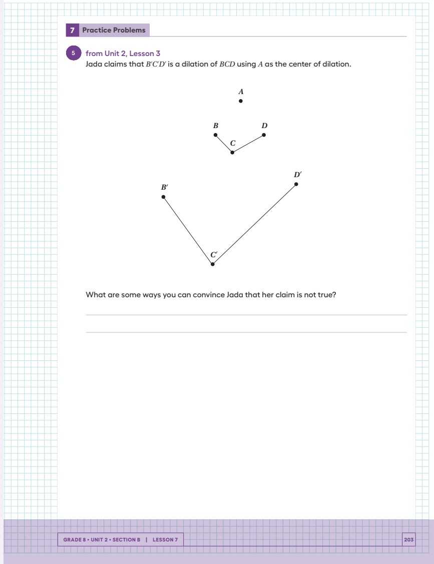
c. Rotate segment AB 90° clockwise around point B. Label any new points.

The segment is attached at point B and forms a right angle, parallel and in the same direction as the previous segment.

d. Describe a transformation on segment AB you could use to finish building a square.

Sample response: Translate A to C (the new point from part b).

Student Workbook



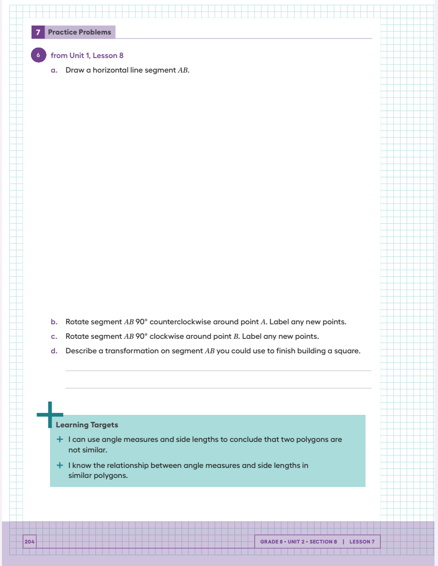
7 Practice Problems

5 from Unit 2, Lesson 3

Jada claims that $B'C'D'$ is a dilation of BCD using A as the center of dilation.

What are some ways you can convince Jada that her claim is not true?

Student Workbook



7 Practice Problems

6 from Unit 1, Lesson 8

a. Draw a horizontal line segment AB.

b. Rotate segment AB 90° counterclockwise around point A. Label any new points.

c. Rotate segment AB 90° clockwise around point B. Label any new points.

d. Describe a transformation on segment AB you could use to finish building a square.

Learning Targets

- I can use angle measures and side lengths to conclude that two polygons are not similar.
- I know the relationship between angle measures and side lengths in similar polygons.