Lesson 12

How Much Will Fit?

Goals

- Draw a cylinder, and label its height and radius, then describe (in writing) the shape of the "base" of the figure.
- Estimate the volumes of various containers using different units of measure, and explain (orally) the reasoning.

Learning Targets

- I know that volume is the amount of space contained inside a three-dimensional figure.
- I recognize the following three-dimensional shapes: cylinder, cone, rectangular prism, and sphere.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Poll the Class

Required Preparation

Activity 1:
If possible, bring in containers, dried rice, and measuring tools.

Activity 2:
If possible, collect solid objects of various sizes for students to pass around at the start of the *Launch*.

Lesson Narrative

The purpose of this lesson is to remind students of the tangible meaning of volume: that it's the amount of space contained in a three-dimensional figure. Students estimate the amount of stuff different containers hold, recalling units of measurement commonly used for volume, like fluid ounces, cups, liters, gallons, cubic feet, and cubic centimeters (also known as milliliters). They revisit the names of figures learned prior to this unit: cylinders, cones, rectangular prisms, and spheres, and see some physical containers that can be modeled with these. It is important for students to make these connections between physical and mathematical objects so that, later on, real-world objects can be modeled with idealized figures.

Students also learn a method for quickly drawing a cylinder. Later in the unit, they also learn quick methods for sketching a cone and a sphere. This skill was included both because it is a handy thinking tool to have access to in problem solving and also because it helps students better understand the meaning of terms, like "radius" and "height," as they apply to these mathematical objects.

Student Learning Goal

Let's reason about the volume of different shapes.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	10 min	10 min	5 min

Warm-up
Two Containers

5 min

Activity Narrative

Previously, students studied the relationship between volume and height of liquid when poured into a cylindrical container. The purpose of this *Warm-up* is to shift students' attention toward other types of containers and to consider how the volume of two containers differs. This *Warm-up* is direct preparation for the following activity in which students reason about volumes of several container types and refamiliarize themselves with the language of three-dimensional objects.

Launch

Tell students to close their student workbooks or devices. Arrange students in groups of 2. Display the image of the two containers filled with beans for all to see.

Give partners 1 minute to estimate how many beans are in each container.

Poll the class for their estimates, and display them for all to see, in particular, the range of values expressed.

Tell students that the smaller container holds 200 beans. Ask students to open their student workbooks or devices and reconsider their estimate for the large container now that they have more information.

Give 1–2 minutes for students to write down a new estimate.

Follow with a whole-class discussion.

Student Task Statement

Your teacher will show you some containers. The small container holds 200 beans. Estimate how many beans the large jar holds.

Sample response: approximately 1,000 beans

Instructional Routines

Poll the Class

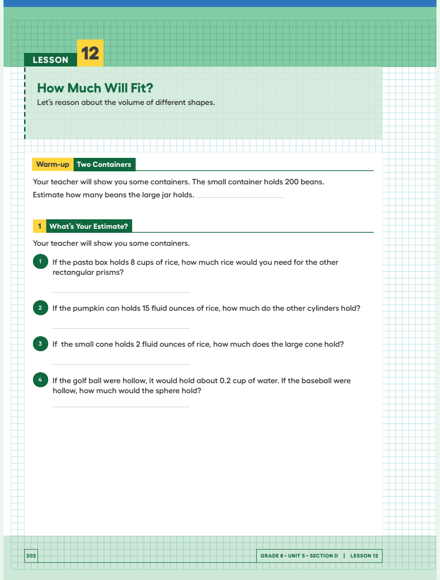
ilclass.com/r/10694985
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students are not sure how to start estimating the amount of beans in the larger jar once the number of beans in the smaller jar is known, consider asking:

"Tell me what you notice is similar or different between the two jars."
"How could estimating some dimensions (for example, height and width) help you?"

Student Workbook



LESSON 12

How Much Will Fit?

Let's reason about the volume of different shapes.

Warm-up Two Containers

Your teacher will show you some containers. The small container holds 200 beans.

Estimate how many beans the large jar holds. ____________

1 What's Your Estimate?

Your teacher will show you some containers.

1. If the pasta box holds 8 cups of rice, how much rice would you need for the other rectangular prisms?
2. If the pumpkin can holds 15 fluid ounces of rice, how much do the other cylinders hold?
3. If the small cone holds 2 fluid ounces of rice, how much does the large cone hold?
4. If the golf ball were hollow, it would hold about 0.2 cup of water. If the baseball were hollow, how much would the sphere hold?

202 GRADE 8 • UNIT 5 • SECTION D | LESSON 12

Activity Synthesis

Poll the class for their new estimates for the number of beans in the larger container, and display these next to the original estimates for all to see. Tell the class that the large container actually holds about 1,000 beans.

Discuss:

- *"How did you and your partner calculate your estimate for the large jar?"*

  We estimated the large jar holds 900 beans since the large jar is about 3 times taller than the smaller jar, and it's about 1.5 times wider, and $200 \cdot 3 \cdot 1.5 = 900$.

- *"Is there a more accurate way to measure the difference in volume between the two containers than 'number of beans?'"*

  Yes, we could use something smaller than beans, such as rice or water, so there is less air.

- *"What are some examples of units used to measure volume? Where have you seen them used in your life?"*

  Cups, tablespoons, gallons, liters, cubic centimeters, etc. Drinks often have fluid ounces, gallons, or liters written on them. Recipes may use cups or tablespoons.

Activity 1

What's Your Estimate?

15 min

Activity Narrative

The purpose of this activity is for students to practice using precise language to describe how they estimated volumes of objects. Starting from an object of known volume, students must consider the difference in dimensions between the two objects. The focus here is on strategies to estimate the volume and units of measure used, not on exact answers or calculating volume using a formula (which will be the focus of later lessons).

Launch

Arrange students in groups of 2. For each option, select groups who have clear strategies to estimate volume of contents inside the container or have an estimate that is very close to the actual volume to share later.

Option 1: Bring in real containers, and ask students to estimate how much rice each would hold, one at a time, preferably with one container whose volume is stated so students have a visual reference for their estimates. Also bring plenty of dried rice and measuring tools, such as tablespoons or cups. After collecting students' estimates, demonstrate how much rice each container holds using whichever units of measure the class deems reasonable. Note that 1 tablespoon is 0.5 ounce, or around 15 milliliters. 1 cup is 8 ounces, or around 240 milliliters. 1 milliliter is the same as 1 cubic centimeter.

Option 2: Display images one at a time for all to see.

Give students 1–2 minutes to work with their partner and write down an estimate for the objects of unknown volumes in the picture.

Follow with a whole-class discussion.

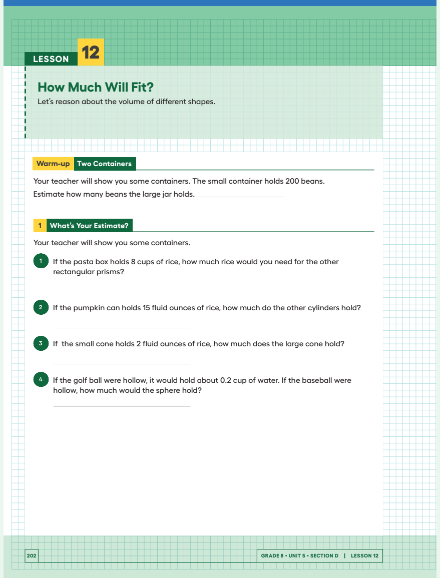
Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide access to three-dimensional models of the objects shown in the images for students to manipulate. Ask students to use the three-dimensional models to estimate the quantities in the other figures. Encourage students to place objects with known volumes inside or next to others to ensure an accurate estimate.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Building on Student Thinking

Students may think there is a single right answer. Measurements are always approximate. Some of the measurements given by the authors of this activity were calculated using estimates from the photos, and may not be very precise. Measurements listed on the sides of packages are more accurate, but actual contents may vary slightly.

Student Workbook


LESSON 12

How Much Will Fit?

Let's reason about the volume of different shapes.

Warm-up Two Containers

Your teacher will show you some containers. The small container holds 200 beans.
Estimate how many beans the large jar holds.

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4. If the golf ball were hollow, it would hold about 0.2 cup of water. If the baseball were hollow, how much would the sphere hold?

202 GRADE 8 • UNIT 5 • SECTION D | LESSON 12

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each strategy that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Student Task Statement

Your teacher will show you some containers.

1. If the pasta box holds 8 cups of rice, how much rice would you need for the other rectangular prisms?

   Pudding: 1.5 cups; pasta: 8 cups; chicken stock: 32 fluid ounces (or 4 cups). (Answers are from the information on the container.)

2. If the pumpkin can holds 15 fluid ounces of rice, how much do the other cylinders hold?

   Tuna: 5.5 fluid ounces; oatmeal: $5\frac{2}{3}$ cups (or about 45 fluid ounces); pumpkin: 15 fluid ounces. (Answers are from the information on the container.)

3. If the small cone holds 2 fluid ounces of rice, how much does the large cone hold?

   Cone with uneven top: approximately 3 fluid ounces (holds more if tilted); cone with smooth top: approximately 2 fluid ounces.

4. If the golf ball were hollow, it would hold about 0.2 cup of water. If the baseball were hollow, how much would the sphere hold?

   Baseball: approximately 0.9 cup; golf ball: approximately 0.2 cup.

   Some answers are more exact than others.

Activity Synthesis

For each set of containers, display the image, and ask previously identified groups to share their strategies for estimating the volume. Once strategies for each set of containers are shared, discuss:

- *"How do the estimates differ if we measure using water instead of rice?"*

  Measuring with rice leaves a bit of empty space between the grains, while water, being liquid, leaves no empty space, so it's more accurate.

- *"If the containers we used were much larger (like water tanks), would our units of measure change? Why?"*

  If we were measuring larger volumes, we might want to use larger units, like gallons. 4,000 ml sounds big, but it's only a bit more than 1 gallon, which isn't that much water.

Conclude the discussion by asking students to compare some other units of measure for volume that they know of. Ask students to recall what they know about unit conversion between some of the following units of measure:

- Fluid ounces, quarts, cups, liters, and milliliters
- Cubic feet, cubic meters, and cubic yards
- Note that cubic centimeters are special because 1 cc = 1 ml.

If it comes up, here is information about *ounces*: Units called "ounces" are used to measure both volume and weight. It is important to be clear about what quantity you are measuring! To differentiate between them, people refer to the units of measure for volume as "fluid ounces." For water, 1 fluid ounce is very close to 1 ounce by weight. This is not true for other substances! For example, mercury is much denser than water. 1 fluid ounce of mercury weighs about 13.6 ounces! Motor oil is less dense than water (that's why it floats), so 1 fluid ounce of oil weighs only about 0.8 ounces. The metric system is not so confusing for quantities that would be measured in ounces, since it's common to measure mass instead of weight and measure it in grams, whereas volume is measured in milliliters.

Activity 2

Do You Know These Figures?

10 min

Activity Narrative

The purpose of this activity is for students to learn or remember the names of the figures, and some features of those figures, that they worked with earlier and to practice a quick method for sketching a cylinder. Students start by determining the shapes that are the faces of the four figures. They also determine which shape would be considered the base of each figure shown. This allows students to connect what they have previously learned about two-dimensional figures to the three-dimensional figures in this activity and will help later when establishing volume formulas for cylinders, cones, and spheres.

The last question introduces students to a way to sketch a cylinder. This is a skill they will continue to use throughout the unit when working on problems that do not provide a visual example of a situation.

Launch

It is strongly recommended that you provide physical, solid objects for students to hold and look at. If using physical objects, pass around the objects for students to see and feel before starting the activity.

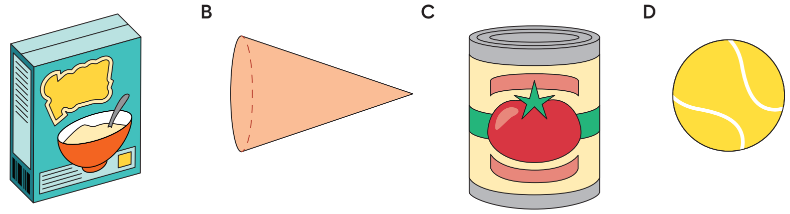
Give students 3–5 minutes of quiet work time.

Select students who sketch cylinders that are different sizes or drawn sideways to share later. Follow with a whole-class discussion.

Student Task Statement

1. What shapes are the faces of each type of object shown here? For example, all six faces of a cube are squares.

A B C D



Rectangular prism: rectangles; cone: circle and a curved surface; cylinder: 2 circles and a curved surface; sphere: one curved surface.

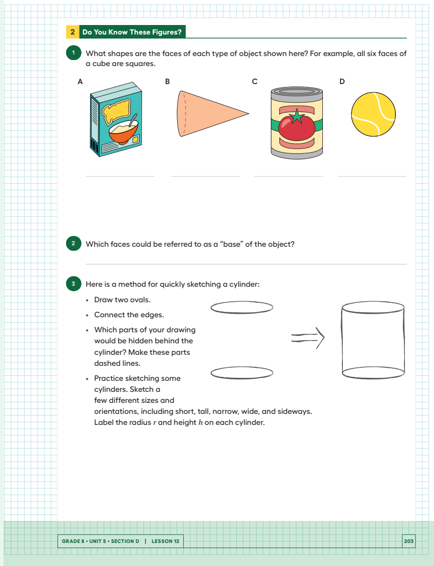
2. Which faces could be referred to as a "base" of the object?

Rectangular prism: any rectangular face can be called the base; cone: the circular face is the base; cylinder: either circular face can be called the base; sphere: has no base.

Building on Student Thinking

If students struggle to visualize the shapes of the faces, position the object so that students can only see two dimensions. Ask students what two-dimensional shape they see.

Student Workbook


2 Do You Know These Figures?

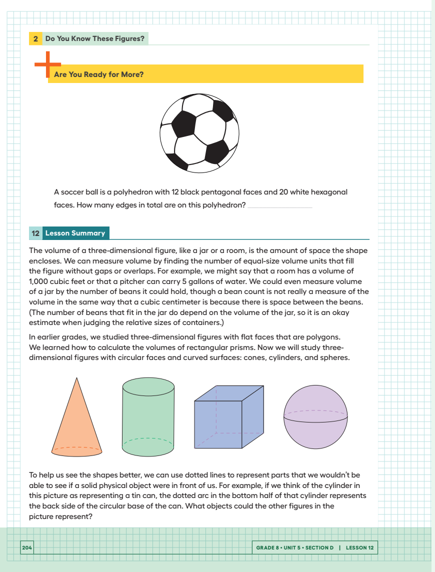
1 What shapes are the faces of each type of object shown here? For example, all six faces of a cube are squares.

A B C D

2 Which faces could be referred to as a "base" of the object?

3 Here is a method for quickly sketching a cylinder:
- Draw two ovals.
- Connect the edges.
- Which parts of your drawing would be hidden behind the cylinder? Make these parts dashed lines.
- Practice sketching some cylinders. Sketch a few different sizes and orientations, including short, tall, narrow, wide, and sideways. Label the radius r and height h on each cylinder.

Student Workbook


2 Do You Know These Figures?

Are You Ready for More?

A soccer ball is a polyhedron with 12 black pentagonal faces and 20 white hexagonal faces. How many edges in total are on this polyhedron? ______

12 Lesson Summary

The volume of a three-dimensional figure, like a jar or a room, is the amount of space the shape encloses. We can measure volume by finding the number of equal-size volume units that fill the figure without gaps or overlaps. For example, we might say that a room has a volume of 1,000 cubic feet or that a pitcher can carry 5 gallons of water. We could even measure volume of a jar by the number of beans it could hold, though a bean count is not really a measure of the volume in the same way that a cubic centimeter is because there is space between the beans. (The number of beans that fit in the jar do depend on the volume of the jar, so it is an okay estimate when judging the relative sizes of containers.)

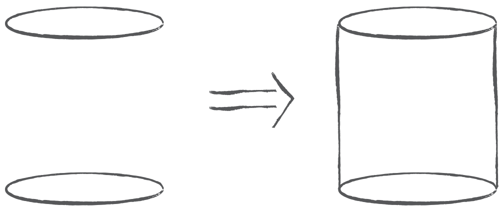
In earlier grades, we studied three-dimensional figures with flat faces that are polygons. We learned how to calculate the volumes of rectangular prisms. Now we will study three-dimensional figures with circular faces and curved surfaces: cones, cylinders, and spheres.

To help us see the shapes better, we can use dotted lines to represent parts that we wouldn't be able to see if a solid physical object were in front of us. For example, if we think of the cylinder in this picture as representing a tin can, the dotted arc in the bottom half of that cylinder represents the back side of the circular base of the can. What objects could the other figures in the picture represent?

204 | GRADE 8 • UNIT 5 • SECTION D | LESSON 12

3. Here is a method for quickly sketching a cylinder:

- Draw two ovals.
- Connect the edges.
- Which parts of your drawing would be hidden behind the cylinder? Make these parts dashed lines.



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  Answers vary.

Are You Ready for More?

A soccer ball is a polyhedron with 12 black pentagonal faces and 20 white hexagonal faces. How many edges in total are on this polyhedron?

Since each pentagonal face has 5 edges and there are 12 pentagonal faces, there are 60 edges on pentagonal faces. Similarly, there are $20 \cdot 6 = 120$ edges on the 20 hexagonal faces. Combined, this would make 180 edges, except that every edge is counted twice by this process. Dividing by 2, we conclude that there are 90 edges on a soccer ball.

Activity Synthesis

If using physical objects, display each object one at a time for all to see. If using images, display the images for all to see, and refer to each object one at a time. Ask students to identify:

- Which figure the object is an example of.
- The different shapes that make up the faces of the figure.
- The shape that is the base of the figure.

Invite previously selected students to share their sketches of cylinders. The goal is to ensure that students see a variety of cylinders: short, tall, sideways, narrow, etc. If no student drew a “sideways” cylinder, sketch one for all to see, and make sure students understand that even though it is sideways, the height is still the length perpendicular to the base.

Tell students that we will be working with these different three-dimensional figures for the rest of this unit. Consider posting a display in the classroom that shows a diagram of each object labeled with its name, and where appropriate, with one side labeled “base.” As volume formulas are developed, the formulas can be added to the display.

Lesson Synthesis

The goal of this discussion is to remind students that the volume of a three-dimensional figure is the amount of space it encloses. Ask students:

“What are some shapes you worked with in today’s lesson?”

cylinders, cones, spheres, rectangular prisms

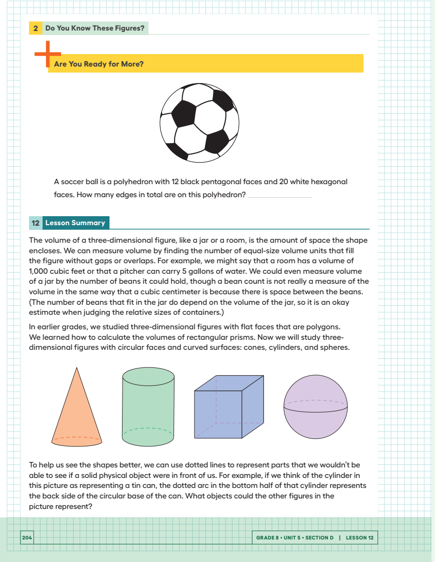
“What are some different units of measure we use to calculate volume of these figures?”

cubic feet, fluid ounces, gallons, cubic nanometers, rice

“What are some different examples of objects we see in our world that are very similar to the figures? What types of things go inside them?”

A cell phone is like a rectangular prism and is filled with wires, a battery, speakers, a microphone, and other electronics. A soccer ball is like a sphere and is filled with air. A chili pepper jar is like a cylinder and is filled with ground up dried chilis.

Student Workbook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In earlier grades, we studied three-dimensional figures with flat faces that are polygons. We learned how to calculate the volumes of rectangular prisms. Now we will study three-dimensional figures with circular faces and curved surfaces: cones, cylinders, and spheres.

To help us see the shapes better, we can use dotted lines to represent parts that we wouldn't be able to see if a solid physical object were in front of us. For example, if we think of the cylinder in this picture as representing a tin can, the dotted arc in the bottom half of that cylinder represents the back side of the circular base of the can. What objects could the other figures in the picture represent?

204 GRADE 8 • UNIT 5 • SECTION D | LESSON 12

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

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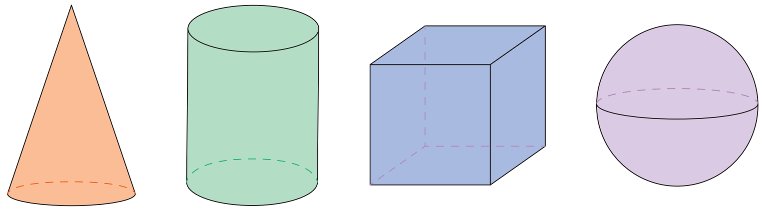

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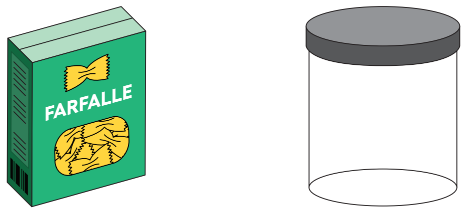
Cool-down

Rectangle to Round

5 min

Student Task Statement



Here is a box of pasta and a cylindrical container. The two objects are the same height, and the cylinder is just wide enough for the box to fit inside with all 4 vertical edges of the box touching the inside of the cylinder. If the box of pasta fits 8 cups of rice, estimate how many cups of rice will fit inside the cylinder. Explain or show your reasoning.

Sample response: About 11 cups of rice since it should be a little more than the box

Lesson 12

Practice Problems

6 Problems

Problem 1

a. Sketch a cube, and label its side length as 4 cm (this will be Cube A).

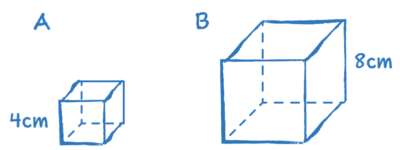
Cube A is sketched with a labeled side length of 4 cm.

b. Sketch a cube with sides that are twice as long as Cube A and label its side length (this will be Cube B).

Cube B is sketched with a labeled side length of 8 cm.

c. Find the volumes of Cube A and Cube B.

Cube A: 64 cm^3, Cube B: 512 cm^3



Problem 2

Two paper drink cups are shaped like cones. The small cone can hold 6 oz of water. The large cone is $\frac{4}{3}$ the height and $\frac{4}{3}$ the diameter of the small cone. Which of these could be the amount of water the large cone holds?

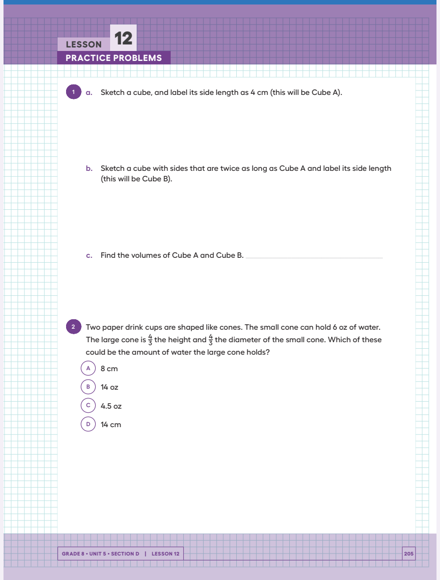
A. 8 cm

B. 14 oz

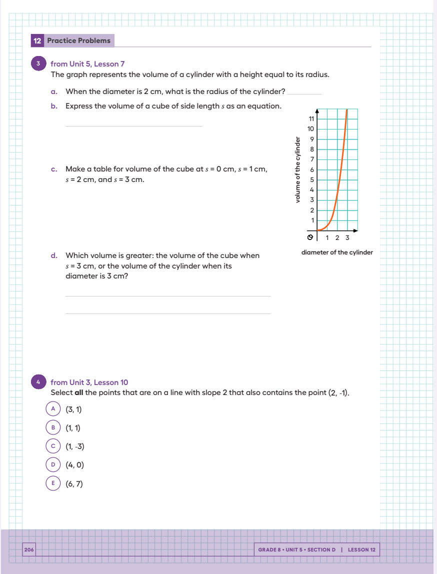
C. 4.5 oz

D. 14 cm

Student Workbook



Student Workbook


12 Practice Problems

3 from Unit 5, Lesson 7

The graph represents the volume of a cylinder with a height equal to its radius.

a. When the diameter is 2 cm, what is the radius of the cylinder?

b. Express the volume of a cube of side length s as an equation.

c. Make a table for volume of the cube at s = 0 cm, s = 1 cm, s = 2 cm, and s = 3 cm.

d. Which volume is greater: the volume of the cube when s = 3 cm, or the volume of the cylinder when its diameter is 3 cm?

4 from Unit 3, Lesson 10

Select **all** the points that are on a line with slope 2 that also contains the point (2, -1).

A (3, 1)

B (1, 1)

C (1, -3)

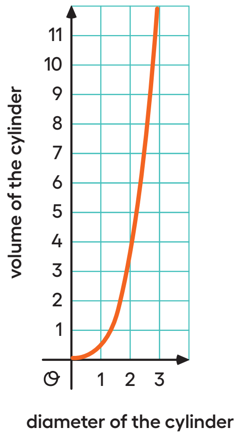
D (4, 0)

E (6, 7)

Problem 3

from Unit 5, Lesson 7

The graph represents the volume of a cylinder with a height equal to its radius.



a. When the diameter is 2 cm, what is the radius of the cylinder?

1 cm

b. Express the volume of a cube of side length s as an equation.

$V = s^3$

c. Make a table for volume of the cube at s = 0 cm, s = 1 cm, s = 2 cm, and s = 3 cm.

s (cm)	volume of cube (cm^3)
0	0
1	1
2	8
3	27

d. Which volume is greater: the volume of the cube when s = 3 cm, or the volume of the cylinder when its diameter is 3 cm?

The volume of the cube at s = 3 cm. Sample reasoning: Its volume is 27 cm^3, while the volume of the cylinder when its diameter is 3 cm is $\frac{27\pi}{8}$ cm^3, or about 10.6 cm^3.)

Problem 4

from Unit 3, Lesson 10

Select **all** the points that are on a line with slope 2 that also contains the point (2, -1).

A. (3, 1)

B. (1, 1)

C. (1, -3)

D. (4 ,0)

E. (6, 7)

Problem 5

Several glass aquariums of various sizes are for sale at a pet shop. They are all shaped like rectangular prisms. A 15-gallon tank is 24 inches long, 12 inches wide, and 12 inches tall. Match the dimensions of the other tanks with the volume of water they can each hold.

A. Tank 1: 36 inches long, 18 inches wide, and 12 inches tall

B. Tank 2: 16 inches long, 8 inches wide, and 10 inches tall

C. Tank 3: 36 inches long, 12 inches wide, and 12 inches tall

D. Tank 4: 20 inches long, 10 inches wide, and 12 inches tall

1. 5 gallons

2. 10 gallons

3. 20 gallons

4. 30 gallons

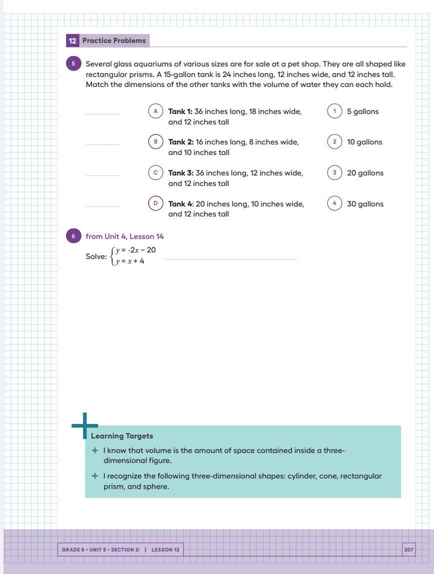
Problem 6

from Unit 4, Lesson 14

Solve: $\begin{cases} y = -2x - 20 \\ y = x + 4 \end{cases}$

(-8, -4)

Student Workbook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D Tank 4: 20 inches long, 10 inches wide, and 12 inches tall

1 5 gallons
2 10 gallons
3 20 gallons
4 30 gallons

6 from Unit 4, Lesson 14

Solve: $\begin{cases} y = -2x - 20 \\ y = x + 4 \end{cases}$

Learning Targets
+ I know that volume is the amount of space contained inside a three-dimensional figure.
+ I recognize the following three-dimensional shapes: cylinder, cone, rectangular prism, and sphere.