Lesson 5

How Do We Choose?

Goals

- Comprehend the terms "majority" and "supermajority" (in spoken and written language).
- Critique (using words and other representations) a statement reporting the results of a vote.
- Interpret (orally and in writing) voting situations involving two choices through the use of ratios and percentages.

Lesson Narrative

This is the first in this series of optional lessons about voting systems. The activities in this lesson are about voting on issues where there are two choices. Students use equivalent ratio concepts and skills developed in grade 6 to compare voting results of two groups, to determine whether an issue wins an election with a supermajority rule, and discover that a few people can determine the results of an election when very few people vote.

Most of the activities use students' skills from earlier units to reason about ratios in the context of real-world problems. While some of the activities do not involve much computation, they all require serious thinking. In many activities, students have to make choices about how to understand election results and justify their methods.

Most importantly, this lesson addresses topics that are important for citizens in a democracy to understand. Students may want to discuss the unfairness of the results when only a few people vote. Consider collaborating with a civics or government teacher to learn how the fictional middle-school situations in this lesson relate to real-world elections, being mindful of the impact of elections on the lives of students and their families.

Student Learning Goal

Let's vote and choose a winner!

Lesson Timeline

Activity 1	Activity 2	Activity 3	Activity 4
15 min	10 min	10 min	20 min

Access for Students with Diverse Abilities

- Representation (Activity 4)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR5: Co-Craft Questions (Activity 1)
- MLR6: Three Reads (Activity 4)

Instructional Routines

- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR6: Three Reads

Required Materials

Materials to Gather

- Four-function calculators: Activity 3
- Colored pencils: Activity 4
- Graph paper: Activity 4
- Scissors: Activity 4

Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 1)

MLR5: Co-Craft Questions. This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Activity 1

Which Was "Yessier"?

15 min

Activity Narrative

This activity gives students a chance to recall and use various ways to reason about ratios in the context of a voting problem. Two classes voted on a yes-or-no question. Both classes voted yes. As students think about how to quantify the degree of preference for something, they practice making sense of a problem and persevering in solving it.

Monitor for students who use different ways to make sense of the problem.

Launch

Arrange students in groups of 2. Introduce the context of two classes voting on whether to answer math questions in the form of poetry. Use *Co-Craft Questions* to orient students to the context, and elicit possible mathematical questions.

- Display only the problem stem and related table, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation, and then to compare their questions with a partner.
- Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

💬 *"What do these questions have in common? How are they different?"*

Listen for and amplify language related to the learning goal, such as deciding which result is more popular.

- Reveal the question "Was one class more in favor of math poetry, or were they equally in favor? Find two or more ways to answer the question." and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

💬 *"Which of your questions is most similar to or different from the ones given? Why?"*

Give groups 2–3 minutes to answer the question, and follow that with a whole-class discussion.

Student Task Statement

Two sixth-grade classes, A and B, voted on whether to give the answers to their math problems in the form of poetry. The "yes" choice was more popular in both classes.

	yes	no
class A	24	16
class B	18	9

Was one class more in favor of math poetry, or were they equally in favor? Find 2 or more ways to answer the question.

Sample responses:

- Compare equivalent ratios where one quantity is the same for both classes. For example, Class A voted 24 yes to 16 no, which is equivalent to 3 yes to 2 no. Class B voted 18 yes to 9 no, which is equivalent to 3 yes to 1.5 no. For the same number of yeses there are fewer nos in Class B, so Class B is more in favor of answering math problems in poetry.
- Compare the unit rates. For each no in Class A, there are 1.5 yeses ($24 \div 16 = 1.5$). For each no in class B, there are 2 yeses ($18 \div 9 = 2$). This means class B is more in favor.
- Compare the percentages of yes votes. In Class A, $\frac{24}{40}$ or $\frac{6}{10}$ of the class voted yes, so 60% of the class voted yes. In Class B, $\frac{18}{27}$ or $\frac{2}{3}$ of the class voted yes, so about 66.7% voted yes. Class B has a higher percentage of yes votes.

Activity Synthesis

The goal of the discussion is to connect the idea of voting to rate comparisons. Students should recognize that the situation is mathematically the same as other rate comparison problems, such as comparing the tastes or colors of two mixtures.

Invite several students to present different methods. If no students compare the ratio of yes votes to all votes or use percentages, be sure to present such solutions.

Activity 2: Optional

Painting the School Purple

10 min

Activity Narrative

This activity offers another opportunity to compare and interpret ratios in the context of voting on a binary question. Unlike in the "Which Was Yessier?" activity, the numbers in the ratios here are harder to manipulate mentally, making it more difficult to find equivalent ratios for comparison. The impracticality may motivate students to calculate percentages or unit rates in order to compare the voting results.

Building on Student Thinking

If students compare only the yes votes rather than reasoning about ratios, or if they compare additively (by finding the differences between the yes votes and no votes) rather than comparing multiplicatively, consider using ratios that are more extreme to illustrate the issues. For instance:

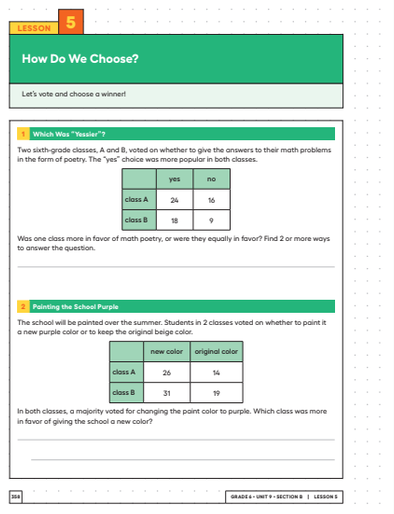
If students look only at the yes votes, ask:

> *"Suppose Class P voted 10 yes to 10 no, and Class Q voted 15 yes to 20 no, would you say that class Q likes the proposal better because it has more yes votes than class P does? Why or why not?"*

If students compare additively, ask:

> *"Suppose Class X voted 30 yes to 10 no, and Class Y voted 15 yes to 0 no. Is Class X more in favor of the proposal because it has 20 more yes votes than no votes, while Class Y has 15 more yes votes than no votes? Why or why not?"*

Student Workbook



LESSON 5

How Do We Choose?

Let's vote and choose a winner!

1 Which Was "Yessier"?

Two sixth-grade classes, A and B, voted on whether to give the answers to their math problems in the form of poetry. The "yes" choice was more popular in both classes.

	yes	no
class A	24	16
class B	18	9

Was one class more in favor of math poetry, or were they equally in favor? Find 2 or more ways to answer the question.

2 Painting the School Purple

The school will be painted over the summer. Students in 2 classes voted on whether to paint it a new purple color or to keep the original beige color.

	new color	original color
class A	26	14
class B	31	19

In both classes, a majority voted for changing the paint color to purple. Which class was more in favor of giving the school a new color?

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display. This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Building on Student Thinking

If students are computing percentages incorrectly, consider asking:

"Can you explain how you calculate percentages?"
"How could knowing the total number of votes in each class help you calculate percentages?"

As students consider equivalent ratios, find percentages or unit rates, and interpret the numbers in terms of favorability, students practice reasoning abstractly and quantitatively.

Launch

Arrange students in groups of 2–4. Provide access to four-function calculators.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to compare the voting results. Display words and phrases such as "ratio," "higher (or lower) percentage," "more (or less) likely," "greater (or smaller) fraction," "unit rate," and "for each vote (to change or keep the paint color)."

Student Task Statement

The school will be painted over the summer. Students in 2 classes voted on whether to paint it a new purple color or to keep the original beige color.

	new color	original color
class A	26	14
class B	31	19

In both classes, a majority voted for changing the paint color to purple. Which class was more in favor of giving the school a new color?

Class A was more in favor of a new color.
Sample reasoning:

- Comparing fractions of "new color" votes out of all votes: Class A has a greater fraction of students voting for a new color ($\frac{26}{40}$ or $\frac{65}{100}$) compared to what Class B has ($\frac{31}{50}$ or $\frac{62}{100}$).
- Comparing percentages: Class A has a higher percentage of "new color" votes (65%) compared to what Class B has (62%).
- Comparing unit rates:
- For each "new color" vote, there was 0.54 "original color" vote in Class A and 0.61 in Class B, so Class B has a stronger preference for the original color.
- Comparing unit rates per "original color" vote: For each "original color" vote, there were 1.9 "new color" votes in Class A and 1.6 in Class B, so Class A has a stronger preference for a new color.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to show different ways of solving the problem, including using unit rates and percentages. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

As students share, ask students to explain their thinking. Correct ideas with incorrect calculations are still worth sharing.

Activity 3
Supermajorities

10 min

Activity Narrative

This activity introduces the idea of requiring a supermajority. A supermajority is a voting rule that is used for issues where it is important to have more than just barely above half of the voters agreeing. To win, a choice must have at least the given fraction of the votes. In this activity, two supermajority rules are given: one as a fraction, one as a percentage. Students find a fraction of the total votes and a percentage of the total votes. They then compare the fraction and the percentage. Students reason abstractly and quantitatively while deciding if votes have reached the needed supermajority. This activity encourages students to think about votes in ratios and percentages.

Launch

Arrange students in groups of 2–4. Provide access to a four-function calculator.

Explain the difference between a majority and supermajority:

- *"In many voting situations, a choice that wins a majority of the votes wins. A majority is more than half the votes. So if 1,000 votes were cast, a majority is any number over 500, so 501 is the smallest number of votes that can win.*

  Many groups have special election rules for very important issues. Sometimes they need a supermajority: To win, you need more than a certain fraction that is more than half. For raising taxes, some governments need a $\frac{2}{3}$ supermajority. To change (amend) the U.S. Constitution, an amendment must get a $\frac{2}{3}$ supermajority of both the Senate and House of Representatives, and be ratified by $\frac{3}{4}$ of the states. Sometimes supermajorities are described as percentages, such as 60%."

Student Task Statement

1. Another school is also voting on whether to change their school's color to purple. Their rules need a $\frac{2}{3}$ supermajority to change the colors. A total of 240 people voted, and 153 voted to change to purple. Were there enough votes to make the change?

   153 votes are not enough to win.

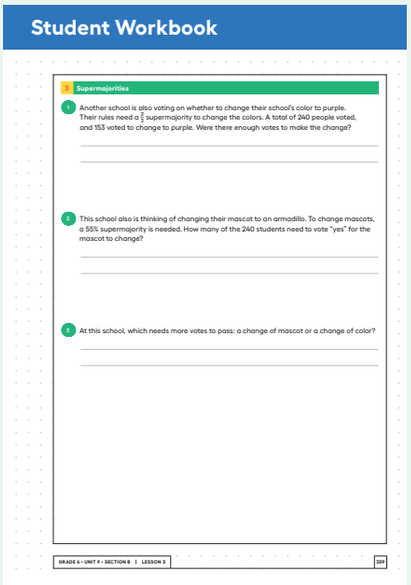
2. This school also is thinking of changing their mascot to an armadillo. To change mascots, a 55% supermajority is needed. How many of the 240 students need to vote "yes" for the mascot to change?

   132 votes are needed to change the mascot.

3. At this school, which needs more votes to pass: a change of mascot or a change of color?

   A $\frac{2}{3}$ supermajority needs more votes than a 55% supermajority.

Student Workbook


3 Supermajorities

1 Another school is also voting on whether to change their school's color to purple. Their rules need a $\frac{2}{3}$ supermajority to change the colors. A total of 240 people voted, and 153 voted to change to purple. Were there enough votes to make the change?

2 This school also is thinking of changing their mascot to an armadillo. To change mascots, a 55% supermajority is needed. How many of the 240 students need to vote "yes" for the mascot to change?

3 At this school, which needs more votes to pass: a change of mascot or a change of color?

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Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 4)

MLR6: Three Reads.
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Activity Synthesis

The goal of this discussion is for students to share their methods and to make sure that they understand how fractions and percentages are used in determining supermajorities.

For each question and as time allows, select 1–2 students to present their method. Here are some questions for discussion:

- *"Why are supermajorities used?"*

  Sometimes people want to make sure the people voting really support a choice. In that case, it is better to have more than half of the people vote for it.

- *"In the last question, we saw that a $\frac{2}{3}$ supermajority needs more support than a 55% supermajority. Will a supermajority represented by a fraction always need more support than one represented by a percentage?"*

  No, it depends on the value of each supermajority. This is why we change them to the same form of number to compare the values.

Activity 4

Best Restaurant

20 min

Activity Narrative

This activity shows how a few people can make a decision if many people don't vote. The mathematics involves repeated "percent of" operations, each percentage giving a smaller amount than the previous step. The main issue in this problem is to identify "percent of what?" for each percentage. The first percentage is 25% of the people in town who subscribe to the newspaper. The second percentage is 20% of the result of the previous number, and the third is 80% of the second result. Students reason abstractly and quantitatively when they work with these percentages of percentages.

Students construct viable arguments in their written explanation when they clearly show their calculations by writing expressions and equations and make a diagram that accurately shows the sizes of all the groups in the problem. The diagram might be on a 10 × 10 grid or on a tape diagram. Graph paper is a good way to make sure that the sizes are right. Tape diagrams can also be made with a folded strip of paper, if students are accustomed to folding fractions.

Launch

Arrange students in groups of 2–4. Tell students that sometimes local newspapers or magazines ask their readers to vote for their favorite businesses. In this activity, they think about whether this is a good way to decide which businesses are the best or are the most popular. (In other words, is this a scientific survey?) Make graph paper, colored pencils or markers, and scissors available.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem without revealing the question.

- For the first read, read the problem aloud then ask,

"*What is this situation about?*"

A town is voting on which restaurant is best.

Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured

25% of the people subscribe to the newspaper, 20% of the subscribers voted, 80% of the voters chose Darnell's

- After the third read, reveal the question:

"*Do you think Darnell's sign is making a correct and reliable statement?*"

and ask,

"*What are some ways we might get started on this?*"

Invite students to name some possible starting points, referring to quantities from the second read

We can find out what percentage of the town actually voted for Darnell's.

Select students who use different numbers of people to represent the whole town and students who create different types of diagrams. Ask them to share during the discussion.

Student Task Statement

A town's newspaper held a contest to decide the best restaurant in town. Only people who subscribe to the newspaper can vote. 25% of the people in town subscribe to the newspaper. 20% of the subscribers voted. 80% of the people who voted liked Darnell's BBQ Pit best.

Darnell put a big sign in his restaurant's window that said, "80% say Darnell's is the best!"

Do you think Darnell's sign is making a correct and reliable statement? Support your answer with:

- Some calculations
- An explanation in words
- A diagram that accurately represents the people in town, the newspaper subscribers, the voters, and the people who liked Darnell's best

Sample response: Darnell's sign is very misleading because only 4% of the people in town actually voted for Darnell's.

Access for Students with Diverse Abilities (Activity 4, Student Task)

Representation: Internalize Comprehension.
Provide a blank square grid and tape diagrams for students to annotate with details to show how each value is represented. For example, number of all people in town, number of newspaper subscribers in town, number of newspaper subscribers who voted.

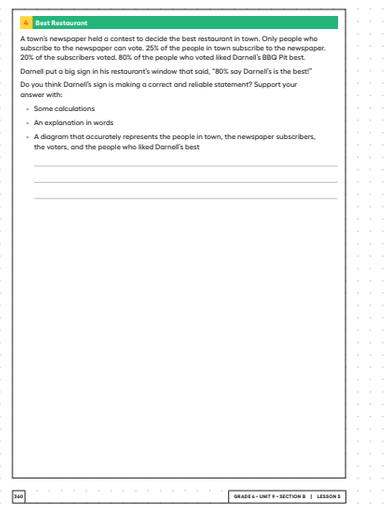
Supports accessibility for: Organization, Attention

Building on Student Thinking

If students struggle with approaching this problem without knowing the number of people in the town, consider asking:

"How could you approach the problem if you knew the number of people in the town?"
"How can you use the percentages that you know in order to follow your plan?"

Student Workbook


4 Best Restaurant

A town's newspaper held a contest to decide the best restaurant in town. Only people who subscribe to the newspaper can vote. 25% of the people in town subscribe to the newspaper. 20% of the subscribers voted. 80% of the people who voted liked Darnell's BBQ Pit best.

Darnell put a big sign in his restaurant's window that said, "80% say Darnell's is the best!"

Do you think Darnell's sign is making a correct and reliable statement? Support your answer with:

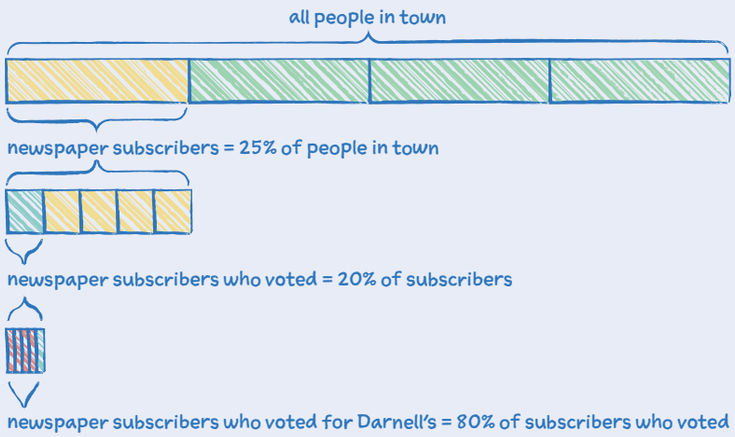
- Some calculations
- An explanation in words
- A diagram that accurately represents the people in town, the newspaper subscribers, the voters, and the people who liked Darnell's best

If there are 100 people in the town, then 25 of them subscribe to the paper. 20% of 25 people is 5, which is the number of subscribers who voted. 80% of 5 is 4, which is the number of subscribers who voted for Darnell's. So 4% of the people in town thought Darnell's is best. Darnell's sign is misleading. Some correct statements:

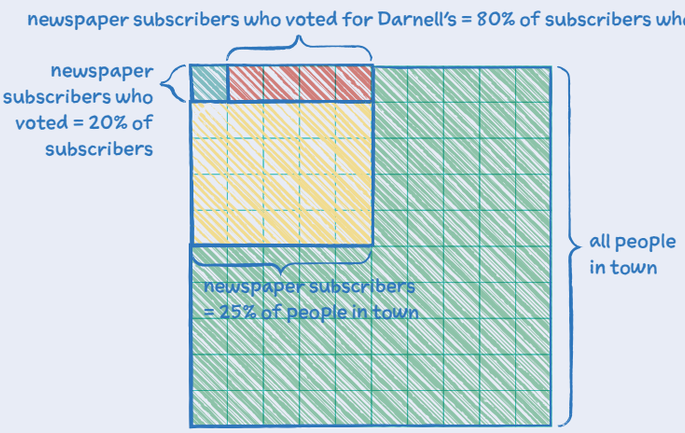
- 80% of people who voted for Best Restaurant liked Darnell's best.
- Among the voters, there were 4 times as many people who thought Darnell's was best, compared to all the other restaurants.

If there are some other number of people in the town, this reasoning still works. One person in the earlier calculation now represents a group of $\frac{1}{100}$ of the people in town.

A tape diagram that shows this reasoning should show the fraction or percentages of the whole town and also the percentage of the previous set. An accurate diagram can be drawn on graph paper, or folded from a strip of paper. A sequence of diagrams, such as the pictures shown, are more useful to show the steps of reasoning, as opposed to a single tape or area diagram.



An area diagram might be on a 10 × 10 grid, each square representing 1% of the town's people.



Activity Synthesis

The goal of this discussion is for students to share their diagrams and how they made sense of "the whole" at each step. In particular, this problem requires students to revise their idea of what is "the whole" three times: Initially, it's the number of people in the town. Then it's the number of subscribers, and then the number of voters. "Percent of what?" is a useful question to ask.

Ask previously selected students to share the number of people they used for the whole town and what percentage they got (4%). Next, choose several diagrams to display and discuss. Ask what part of the diagram represents each quantity:

- All the people in town
- 1% of the people in town
- The people who subscribe to the newspaper (25% of the people in town)
- The people who voted (20% of the subscribers, which is 5% of the people in town)
- The people who voted for Darnell's (80% of those who voted, 4% of the people in town)

Discuss the fact that Darnell's sign is misleading, asking students whom you noticed had interesting or well-stated answers.