Lesson 7

# Exploring the Area of a Circle

## Goals

- Create a table and a graph that represent the relationship between the diameter and area of circles of various sizes, and justify (using words and other representations) that this relationship is not proportional.
- Estimate the area of a circle on a grid by decomposing and approximating it with polygons.

## Learning Targets

- If I know a circle's radius or diameter, I can find an approximation for its area.
- I know whether or not the relationship between the diameter and area of a circle is proportional and can explain how I know.

## Lesson Narrative

In this lesson, students begin working with the area of circles. When we say **area of a circle** we technically mean "area of the region enclosed by a circle."

Students estimate the area inside different circles on a grid. This helps reinforce their understanding of the concept of area as the number of unit squares that cover a region. They use tables and graphs to analyze the measurements. Students see that, unlike circumference, the area of a circle is not proportional to the diameter.

The last activity is optional because it previews the relationship between area and radius, which will be explored in more depth in a later lesson. Students see that it takes a little more than 3 squares with side lengths equal to the circle's radius to completely cover a circle, leading to an approximate formula: the area of a circle is a little bigger than $3r^2$. At this point, it is a reasonable guess that the exact formula is $A = \pi r^2$, but the next lesson will focus on using informal dissection arguments to establish this formula.

## Student Learning Goal

Let's investigate the areas of circles.

## Lesson Timeline

| 5 min | 20 min | 20 min | 10 min | 5 min |
|---|---|---|---|---|
| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Cool-down |

Assessment: Cool-down

### Access for Students with Diverse Abilities

- Representation (Activity 2)

### Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR2: Collect and Display (Activity 1)

### Instructional Routines

- MLR2: Collect and Display
- Which Three Go Together?

### Required Materials

**Materials to Gather**

- Geometry toolkits: Activity 1, Activity 2

**Materials to Copy**

- Estimating Areas of Circles Handout (1 copy for every 12 students): Activity 1

### Required Preparation

**Activity 1:**
For the digital version of the activity, acquire devices that can run the applet.

**Inspire Math**

**Crop Circles video**

**Go Online**
Before the lesson, show this video to reinforce the real-world connection.

**ilclass.com/l/614205**

Please log in to the site before using the QR code or URL.

**Instructional Routines**

**Which Three Go Together?**

**ilclass.com/r/10690736**

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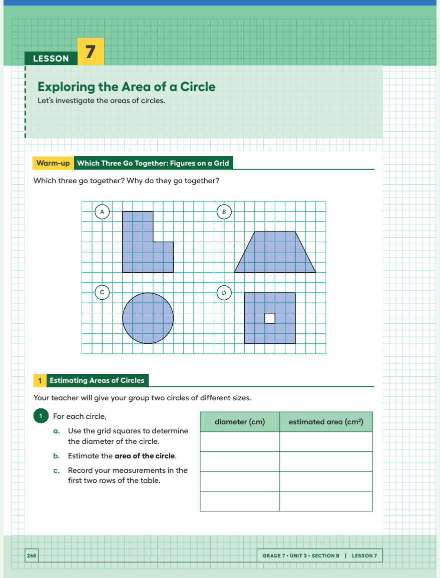
**Student Workbook**


LESSON 7

Exploring the Area of a Circle

Let's investigate the areas of circles.

Warm-up Which Three Go Together: Figures on a Grid

Which three go together? Why do they go together?

1 Estimating Areas of Circles

Your teacher will give your group two circles of different sizes.

1 For each circle,

a. Use the grid squares to determine the diameter of the circle.

b. Estimate the **area of the circle**.

c. Record your measurements in the first two rows of the table.

| diameter (cm) | estimated area (cm²) |
|---|---|
| | |
| | |
| | |
| | |

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## Warm-up

## Which Three Go Together: Figures on a Grid

**5 min**

### Activity Narrative

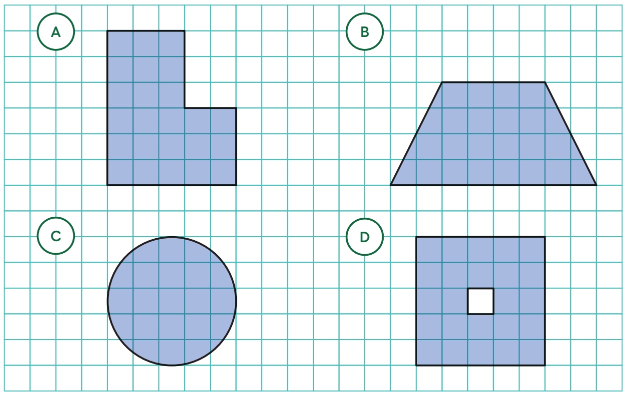
This *Warm-up* prompts students to compare four figures on a grid. In making comparisons, students have a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

### Launch

Arrange students in groups of 2–4. Display the figures for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three figures that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

### Student Task Statement

Which three go together? Why do they go together?




Sample responses:

A, B, and C go together because:

- They do not have a hole in the middle.

A, B, and D go together because:

- They have straight sides.
- We can figure out their areas.
- They have an area of 24 square units.

A, C, and D go together because:

- They are 5 units wide.
- They don't have any acute angles or obtuse angles.

B, C, and D go together because:

- They have line symmetry.

### Activity Synthesis

The goal of this discussion is to get students wondering about the area of Figure C.

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology that they use, such as "length," "width," "height," "area," "straight line," "angle," etc. and to clarify their reasoning as needed. Consider asking:

*"How do you know … ?"*

*"What do you mean by … ?"*

*"Can you say that in another way?"*

If determining the area of each figure does not come up during the conversation, ask students to discuss this idea. The key takeaway is that students have enough information to determine the area of Figures A, B, and D. They all have an area of 24 square units. The goal is to leave students wondering how they could approximate or determine the area of Figure C.

## Activity 1
## Estimating Areas of Circles

20 min

### Activity Narrative

**There is a digital version of this activity.**

In this activity, students estimate the area of various circles on a grid and see that the relationship between the diameter and area of a circle is not proportional. This echoes the earlier exploration comparing the length of a diagonal of a square to the area of the square, which was also not proportional.

Each group estimates the area of one smaller circle and one larger circle. After estimating the area of their circles, students graph the class's data on a coordinate plane to notice that the data points curve upward instead of making a straight line through the origin. As students measure multiple circles and notice patterns in their measurements, they express regularity in repeated reasoning.

In the digital version of the activity, students use an applet to graph the relationship between diameter and area of a circle. The applet allows students to enter values into a table and see the points plotted on a coordinate plane. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

**Instructional Routines**

**MLR2: Collect and Display**

**ilclass.com/r/10690754**

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**Access for Multilingual Learners (Activity 1)**

**MLR2: Collect and Display**
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

**Launch**

Arrange students in groups of 2. Distribute one page of the blackline master to each group.

Encourage students to look for strategies that will help them efficiently count the area of their assigned circles. Give students 5–6 minutes of partner work time, followed by whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

**Student Task Statement**

Your teacher will give your group two circles of different sizes.

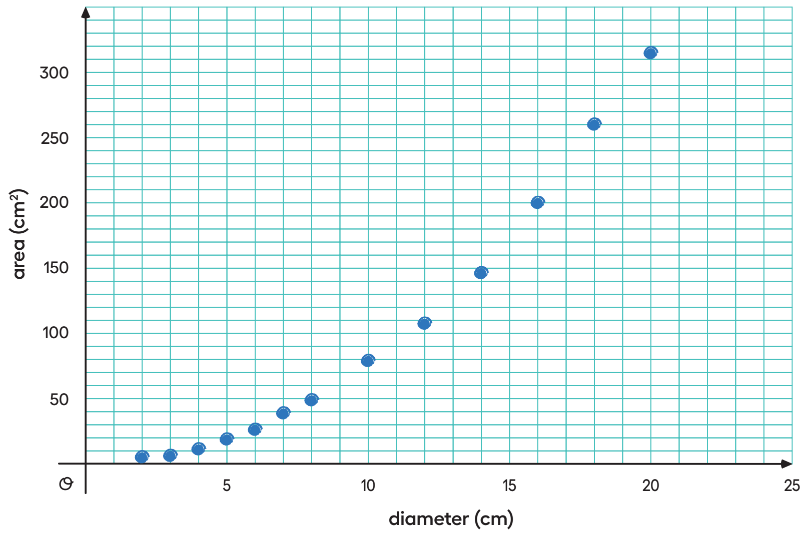
1. For each circle,
   a. Use the grid squares to determine the diameter of the circle.
   b. Estimate the **area of the circle**.
   c. Record your measurements in the first two rows of the table.

Each group has 2 of the rows from this table (and the values in the right column are approximate):

| diameter (cm) | estimated area (cm$^2$) |
|---|---|
| 2 | 3 |
| 16 | 200 |
| 3 | 7 |
| 12 | 108 |
| 4 | 12 |
| 20 | 312 |
| 5 | 19 |
| 10 | 78 |
| 6 | 27 |
| 18 | 250 |
| 7 | 38 |
| 14 | 147 |

2. Plot your diameter and area values on the coordinate plane. What do you notice?

Each group has 2 of the points from this graph.




Sample response: If you connected the two points with a straight line, it would not go through the origin. It would go below it.

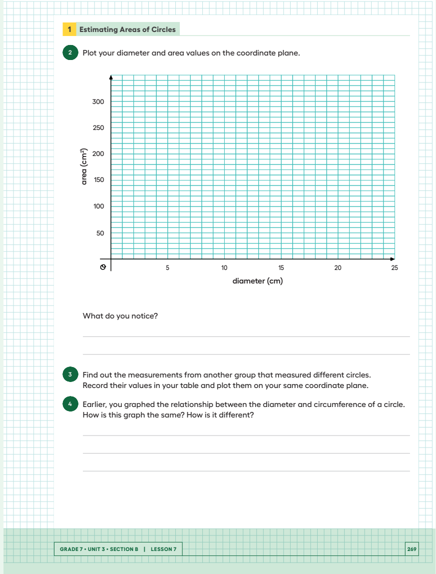
3. Find out the measurements from another group that measured different circles. Record their values in your table, and plot them on your same coordinate plane.

Two more rows from the above table and two more points from the above graph.

4. Earlier, you graphed the relationship between the diameter and circumference of a circle. How is this graph the same? How is it different?

Sample response: Both graphs go up from left to right. The circumference graph made a straight line through the origin, but this area graph curves upward. This graph does not represent a proportional relationship.

## Student Workbook




### Building on Student Thinking

Some students might be unsure about how to count the squares around the border of the circle that are only partially included. Let them come up with their own idea, but if they need additional support, suggest that they round up to a whole square when it looks like half or more of the square is within the circle and round down to no square when it looks like less than half the square is within the circle.

### Are You Ready for More?

How many circles of radius 1 unit can you fit inside each of the following so that they do not overlap?

1. a circle of radius 2 units? 2
2. a circle of radius 3 units? 7
3. a circle of radius 4 units? 11

If you get stuck, consider using coins or other circular objects.

## Activity Synthesis

The goal of this discussion is to help students see and express that the relationship between diameter and area of a circle is not proportional.

First, display the image of the coordinate plane for all to see. Ask students to share the diameter and estimated area of each circle. Plot these values on the grid.

Next, direct students' attention to the reference created using *Collect and Display*. Ask students to share what they notice about this graph of diameter and area and how it compares to the graph of diameter and circumference that they saw earlier. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

The key takeaways are:

- The graph shows that the area of the circle goes up faster the bigger the diameter; it curves upward.
- The relationship between the diameter and area of circles is not a proportional relationship.

If desired, emphasize that the relationship is not proportional by having students add a column to their table of measurements and calculate the quotient of the area divided by the diameter. Here is a table of sample values.

| diameter (cm) | estimated area ($cm^2$) | area ÷ diameter |
|---|---|---|
| 2 | 3 | 1.5 |
| 16 | 200 | 12.5 |
| 3 | 7 | 2.3 |
| 12 | 108 | 9.0 |
| 4 | 12 | 3.0 |
| 20 | 312 | 15.6 |
| 5 | 19 | 3.8 |
| 10 | 78 | 7.8 |
| 6 | 27 | 4.5 |
| 18 | 250 | 13.9 |
| 7 | 38 | 5.4 |
| 14 | 147 | 10.5 |

Ask students:

*"Are the values for area divided by diameter close to the same value for each row?"*

No

*"What does this tell us about the relationship between diameter and area of a circle?"*

There is no constant of proportionality.
It is not a proportional relationship.

## Activity 2: Optional
## Covering a Circle

20 min

### Activity Narrative

In this activity students compare the area of a circle to the area of a square that has side lengths equal to the circle's radius. The task is open-ended, so students can look for a very rough estimate or a more precise estimate. In either case, they find that the circle's area is greater than 2 times the square's, less than 4 times the square's, and close to 3 times the square's.

A video shows how to cut up 3 squares and place them inside the circle. Since there is a little white space still showing around the cut pieces, that means that the area of a circle with radius $r$ is a little bit more than $3r^2$.

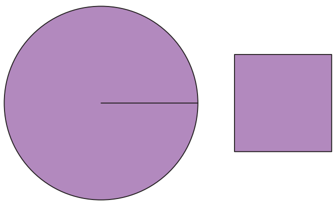
Students use appropriate tools strategically as they select items from their geometry toolkits and make choices about how to use the given figures to solve the problem.

### Launch

Keep students in the same groups. Provide access to geometry toolkits.

### Student Task Statement

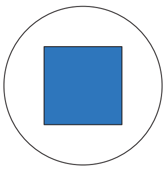
Here is a square whose side length is the same as the radius of the circle.



How many of these squares do you think it would take to cover the circle exactly?

Sample response:

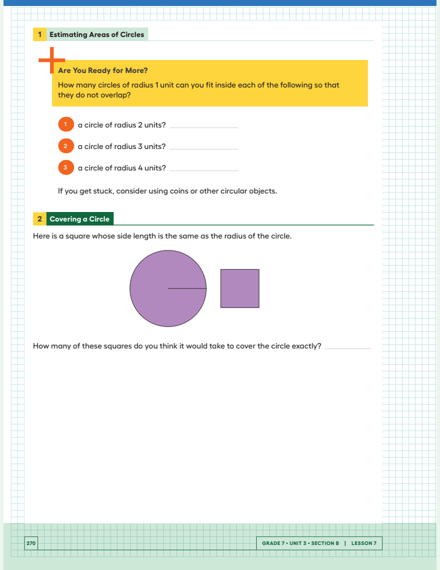
- It definitely takes more than 1 square. The square can be placed inside the circle and more white space remains.



### Access for Students with Diverse Abilities (Activity 2, Student Task)

**Representation: Access for Perception.**
Provide access to physical cutouts of the square or a digital version that students can manipulate. Ask students to identify correspondences between their specific case and circles in general.
*Supports accessibility for: Visual-Spatial Processing, Organization*

### Student Workbook



1 Estimating Areas of Circles

Are You Ready for More?

How many circles of radius 1 unit can you fit inside each of the following so that they do not overlap?

1 a circle of radius 2 units?

2 a circle of radius 3 units?

3 a circle of radius 4 units?

If you get stuck, consider using coins or other circular objects.

2 Covering a Circle

Here is a square whose side length is the same as the radius of the circle.

How many of these squares do you think it would take to cover the circle exactly?

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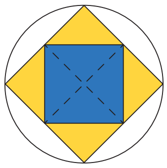
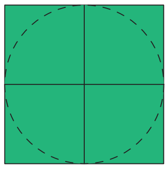
- It also takes more than 2 squares. The yellow square was cut and repositioned to fit within the circle. Some white space still remains.



- It takes less than 4 squares. The green squares completely cover the circle, and the corners go outside the circle.



- It takes somewhere close to 3 squares. It looks like the remaining white regions (inside the circle) are close in area to the green shaded regions (outside the circle but inside the square).

**Building on Student Thinking**

Students may focus solely on the radius of the circle and side length of the square, not relating their work to area. As these students work, ask them what they find as they try to cover the circle each time. Reinforce the idea that as they cover the circle, they are comparing the area of the circle and squares.

If students arrive at the idea that 4 squares suffice to completely cover the circle, ask them if there is any excess. Could they cover the square with $3\frac{1}{2}$ squares, for example?

## Activity Synthesis

The goal of this discussion is for students to recognize that the area of the circle is a little more than 3 times the area of the square radius.

First, ask the class:

*"Can 2 squares completely cover the circle?"*

no

*"Can 4 squares completely cover the circle?"*

yes

*"Can 3 squares completely cover the circle?"*

It's hard to tell for sure.

Expand the discussion of this third question by inviting students to share their strategies for comparing the area of the circle and square. The key takeaway is that it is reasonable to conclude that the area of the circle is approximately equal to 3 times the area of the square.

To illustrate that it would take a little bit more than 3 squares to cover the circle, consider showing this video.

Video 'Area of a Circle' available on Imagine Learning Classroom for this lesson.

There is a little white space remaining around the cut pieces, so the area of the circle is a little bit more than 3 times the area of one of those squares. At this point, some students may suggest that it takes exactly $\pi$ squares to cover the circle. This will be investigated in more detail in the next lesson. If not mentioned by students, it does not need to be brought up in this discussion.

**Access for Multilingual Learners (Activity 2, Synthesis)**

**MLR1: Stronger and Clearer Each Time.**
After the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "How many of these squares would it take to cover the circle exactly? How do you know?" Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

*Advances: Writing, Speaking, Listening*

## Lesson Synthesis

Share with students:

*"Today we measured the diameter and area of different circles. We saw that the relationship between these quantities is not proportional."*

To reinforce that the relationship between diameter and area of a circle is not proportional, consider asking students:

*"How did the graph help us decide that the relationship between diameter and area is not proportional?"*

When we plotted our measurements for diameter and area, the points curved upward instead of making a straight line.

*"How did the table help us decide that the relationship between diameter and area is not proportional?"*

When we divided the area by the diameter for each row, the quotients were not close to the same value.

**Student Workbook**


If time allows, consider using this quick activity to show that area scales by the (scale factor)$^2$. Tell students:

- *"First, find the scale factor between the diameters of the two circles that your group measured."*

  2, 3, 4, 5, or 8

- *"Next, find the factor that relates the areas of your two circles."*

  approximately 4, 9, 16, 25, or 64

- *"What do you notice about the values you found?"*

  The factor that relates the areas is approximately equal to the (scale factor)$^2$. Since the factors for the two quantities are not the same, the relationship is not proportional.

Consider recording each group's factors with arrows next to the corresponding rows of the table, for example:

| | diameter (cm) | area (sq. cm) | |
|---|---|---|---|
| $\cdot 2$ | 5 | 19 | $\cdot 4.1$ |
| | 10 | 78 | |
| $\cdot 3$ | 6 | 27 | $\cdot 9.3$ |
| | 18 | 250 | |
| $\cdot 5$ | 4 | 12 | $\cdot 26$ |
| | 20 | 312 | |

## Lesson Summary

The circumference $C$ of a circle is proportional to the diameter $d$, and we can write this relationship as $C = \pi d$. The circumference is also proportional to the radius of the circle, and the constant of proportionality is $2 \cdot \pi$ because the diameter is twice as long as the radius. However, the **area of a circle** is *not* proportional to the diameter (or the radius).

The area of a circle with radius $r$ is a little more than 3 times the area of a square with side $r$ so the area of a circle of radius $r$ is approximately $3r^2$. We saw earlier that the circumference of a circle of radius $r$ is $2\pi r$. If we write $C$ for the circumference of a circle, this proportional relationship can be written $C = 2\pi r$.

The area $A$ of a circle with radius $r$ is approximately $3r^2$. Unlike the circumference, the area is not proportional to the radius because $3r^2$ cannot be written in the form $kr$ for a number $k$. We will investigate and refine the relationship between the area and the radius of a circle in future lessons.

## Cool-down

## Areas of Two Circles

5 min

### Student Task Statement

- Circle A has a diameter of approximately 20 inches and an area of 300 $in^2$.
- Circle B has a diameter of approximately 60 inches.

Which of these could be the area of Circle B? Explain your reasoning.

**A.** About 100 $in^2$

**B.** About 300 $in^2$

**C.** About 900 $in^2$

**D.** About 2,700 $in^2$

Sample response: The diameter of Circle B is 3 times bigger than the diameter of Circle A, so the area of Circle B is larger than the area of Circle A. The pattern shows that the area grew quickly, so 900 is probably not large enough. The radius of Circle B is 30 inches, so the area is about $3 \cdot 30^2$ $in^2$ (and is definitely more than $30^2$ because a square of side 30 inches fits inside the circle with a lot of space left).

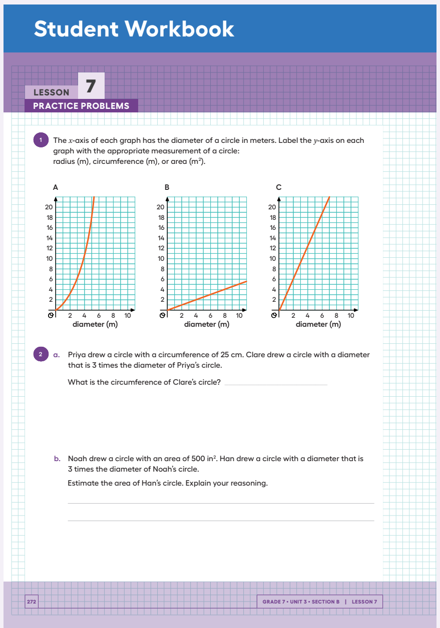
### Responding To Student Thinking

**Points to Emphasize**
If students struggle with recognizing that the relationship between diameter and area is not proportional, focus on this when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about the relationships in this activity:

Unit 3, Lesson 8, Activity 3 Objects for a Powwow

## Practice Problems

7 Problems


Student Workbook

LESSON 7
PRACTICE PROBLEMS

1. The $x$-axis of each graph has the diameter of a circle in meters. Label the $y$-axis on each graph with the appropriate measurement of a circle: radius (m), circumference (m), or area (m²).

2. a. Priya drew a circle with a circumference of 25 cm. Clare drew a circle with a diameter that is 3 times the diameter of Priya's circle.

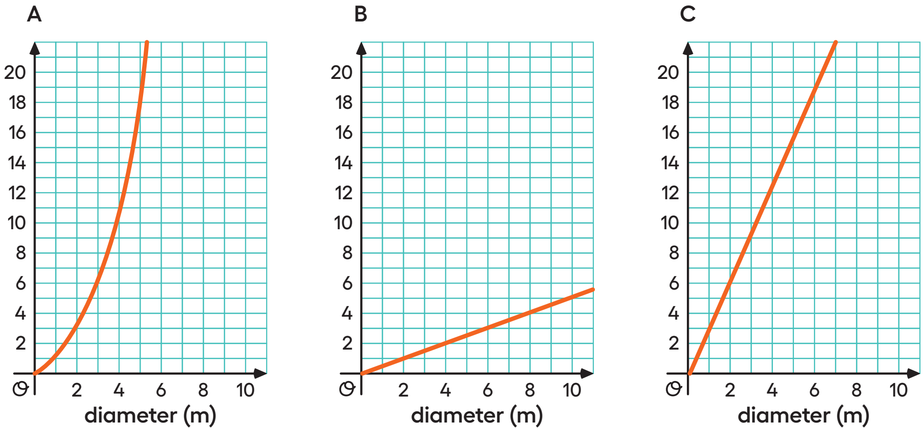
What is the circumference of Clare's circle?

b. Noah drew a circle with an area of 500 in². Han drew a circle with a diameter that is 3 times the diameter of Noah's circle.

Estimate the area of Han's circle. Explain your reasoning.

### Problem 1

The $x$-axis of each graph has the diameter of a circle in meters. Label the $y$-axis on each graph with the appropriate measurement of a circle: radius (m), circumference (m), or area (m$^2$).




The first graph shows the relationship between the diameter and area of a circle, because it is not a proportional relationship. The second graph shows the relationship between the diameter and the radius, because it is proportional and the constant of proportionality is $\frac{1}{2}$. The third graph shows the relationship between the diameter and the circumference, because it is proportional and the constant of proportionality is $\pi$.

### Problem 2

**a.** Priya drew a circle with a circumference of 25 cm. Clare drew a circle with a diameter that is 3 times the diameter of Priya's circle.

What is the circumference of Clare's circle? 75 cm

If the diameter is 3 times greater, then the circumference is also 3 times greater.

**b.** Noah drew a circle with an area of 500 in$^2$. Han drew a circle with a diameter that is 3 times the diameter of Noah's circle.

Estimate the area of Han's circle. Explain your reasoning.

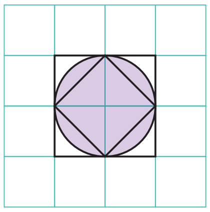
Sample responses:

- More than 2,000 in$^2$. Diameter and area are not proportional. The graph curves upward.
- About 4,500 in$^2$. If the diameter is 3 times greater, the area must be $3^2$, or 9 times greater.

## Problem 3

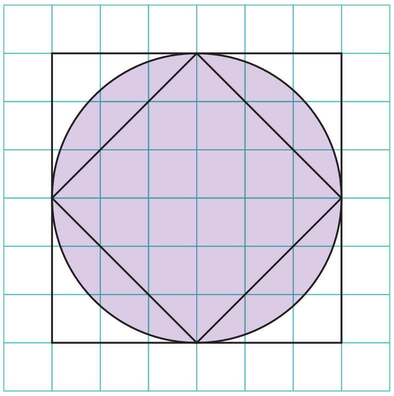
Each picture shows two squares and a circle.

**a.** Explain why the area of this circle is more than 2 square units but less than 4 square units.



The square inside the circle has an area of 2 square units because it is made of 4 triangles each with area $\frac{1}{2}$ square unit, and $\frac{4}{2} = 2$. The square outside the circle has an area of 4 square units, because $2^2 = 4$.

**b.** Explain why the area of this circle is more than 18 square units and less than 36 square units.

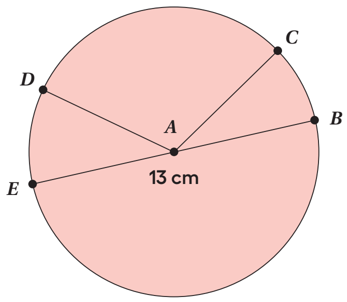


The square inside the circle has an area of 18 square units because $12 + \frac{12}{2} = 18$ (the square inside the circle contains 12 full grid squares and 12 half grid squares). The square outside the circle has an area of 36 square units because $6^2 = 36$.

## Problem 4

from Unit 3, Lesson 3

Point $A$ is the center of the circle, and the length of $EB$ is 13 centimeters.

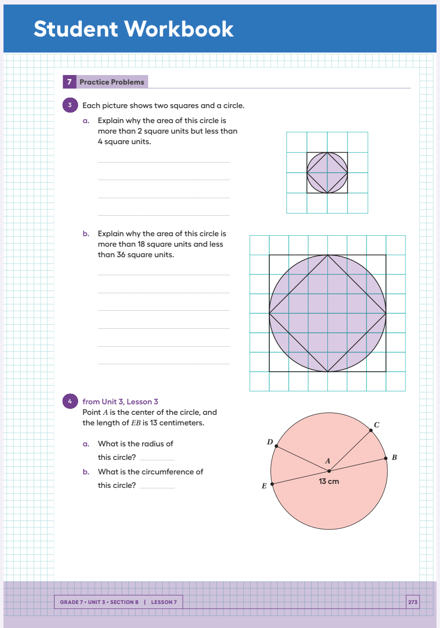


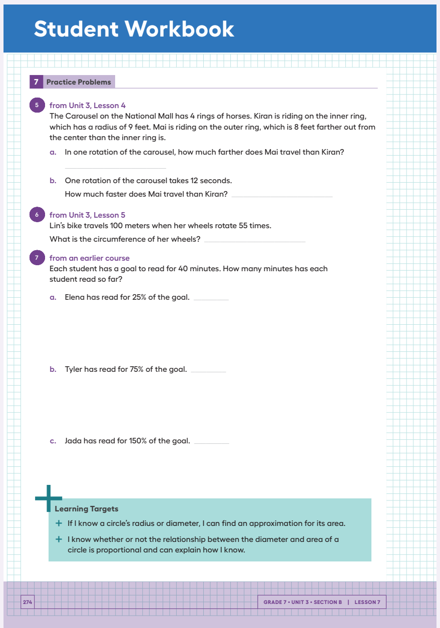

**a.** What is the radius of this circle?

7.5 cm, because $15 \div 2 = 7.5$

**b.** What is the circumference of this circle?

About 41 cm, because $13 \cdot \pi \approx 41$




Student Workbook

7 Practice Problems

5 from Unit 3, Lesson 4
The Carousel on the National Mall has 4 rings of horses. Kiran is riding on the inner ring, which has a radius of 9 feet. Mai is riding on the outer ring, which is 8 feet farther out from the center than the inner ring is.

a. In one rotation of the carousel, how much farther does Mai travel than Kiran?

b. One rotation of the carousel takes 12 seconds. How much faster does Mai travel than Kiran?

6 from Unit 3, Lesson 5
Lin's bike travels 100 meters when her wheels rotate 55 times. What is the circumference of her wheels?

7 from an earlier course
Each student has a goal to read for 40 minutes. How many minutes has each student read so far?

a. Elena has read for 25% of the goal.

b. Tyler has read for 75% of the goal.

c. Jada has read for 150% of the goal.

Learning Targets
+ If I know a circle's radius or diameter, I can find an approximation for its area.
+ I know whether or not the relationship between the diameter and area of a circle is proportional and can explain how I know.

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## Problem 5

from Unit 3, Lesson 4

The Carousel on the National Mall has 4 rings of horses. Kiran is riding on the inner ring, which has a radius of 9 feet. Mai is riding on the outer ring, which is 8 feet farther out from the center than the inner ring is.

**a.** In one rotation of the carousel, how much farther does Mai travel than Kiran?

about 106.8 – 56.5, or 50.3 feet farther

**b.** One rotation of the carousel takes 12 seconds. How much faster does Mai travel than Kiran?

about 50.3 ÷ 12, or 4.2 feet per second faster

## Problem 6

from Unit 3, Lesson 5

Lin's bike travels 100 meters when her wheels rotate 55 times. What is the circumference of her wheels?

about 1.82 meters because $100 \div 55 \approx 1.82$

## Problem 7

from an earlier course

Each student has a goal to read for 40 minutes. How many minutes has each student read so far?

**a.** Elena has read for 25% of the goal. 10 minutes

**b.** Tyler has read for 75% of the goal. 30 minutes

**c.** Jada has read for 150% of the goal. 60 minutes