Lesson 11

On Both of the Lines

Goals

- Create a graph that represents two linear relationships in context, and interpret (orally and in writing) the point of intersection.
- Interpret a graph of two equivalent lines in context.

Learning Target

I can use graphs to find an ordered pair that two real-world situations have in common.

Lesson Narrative

The purpose of this lesson is to introduce students to the graphical interpretation of multiple equations written in the form $y = mx + b$. The equations arise in the context of distance and time in order to anchor student understanding of the shared solution as the point in time when distances are equal. Students are also introduced to a pair of situations represented by the same equation to illustrate how situations can arise with infinitely many solutions.

Student Learning Goal

Let's use lines to think about situations.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 2)

Instructional Routines

- MLR6: Three Reads
- Notice and Wonder

Required Materials

Materials to Gather

- Straightedges: Activity 1, Activity 2

Required Preparation

Lesson:
Provide students with access to straightedges for drawing accurate lines.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	10 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

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Warm-up

Notice and Wonder: Bugs Passing in the Night

10 min

Activity Narrative

The purpose of this *Warm-up* is to get students to think about a context that will be explored in the following activity and to reason about the speed, distance, and time that each animal is traveling in relation to one another. In the next activity, students will write equations for the bugs and graph these relationships.

By familiarizing students with a context and the mathematics that might be involved, this *Warm-up* prompts them to make sense of a problem before solving it.

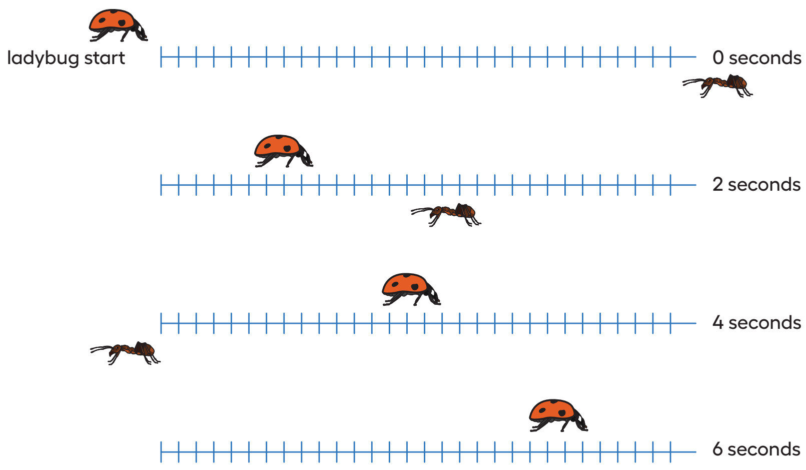
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing that they notice and at least one thing that they wonder about.

Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things that they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students might notice:

- The ladybug is moving from left to right and the ant is moving from right to left.
- The ladybug and the ant are each moving at a constant speed.
- The ladybug is moving 8 units every 2 seconds and the ant is moving 16 units every 2 seconds.
- In the picture at 6 seconds, the ant is no longer visible in the picture.
- At some time in between the 2 second picture and the 4 second picture, they pass each other.

Students might wonder:

- Where did the ant go in the last picture?
- At what time did they pass each other?
- At what tick mark did they pass each other?
- Did they wave as they passed each other?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on the display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

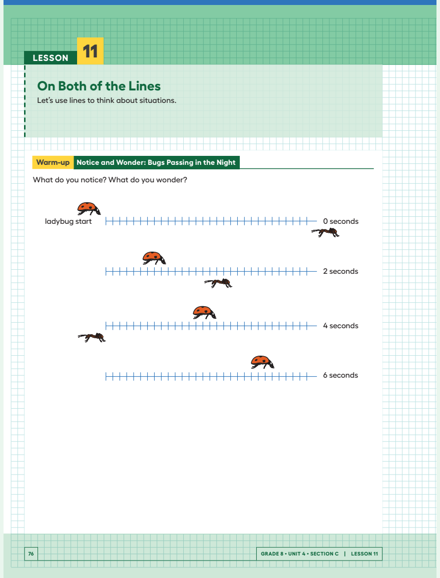
If the idea of when the bugs pass one another and their speeds does not come up during the conversation, ask students to discuss this idea.

Access for Students with Diverse Abilities (Activity 1, Launch)

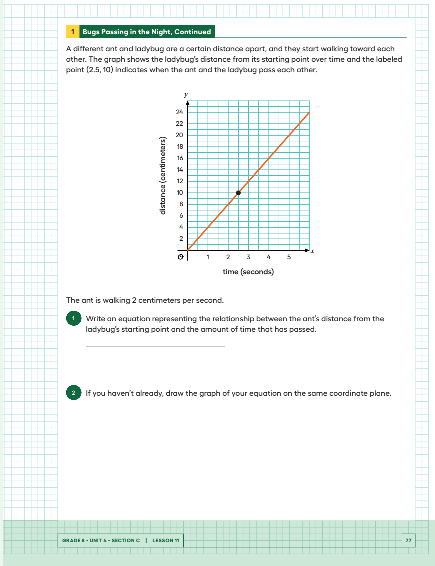
Representation: Internalize Comprehension.
Represent the same information through different modalities by using a number line and pictures similar to those in the *Warm-up*.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Workbook



Student Workbook


1 Bugs Passing in the Night, Continued

A different ant and ladybug are a certain distance apart, and they start walking toward each other. The graph shows the ladybug's distance from its starting point over time and the labeled point (2.5, 10) indicates when the ant and the ladybug pass each other.

The ant is walking 2 centimeters per second.

1. Write an equation representing the relationship between the ant's distance from the ladybug's starting point and the amount of time that has passed.

2. If you haven't already, draw the graph of your equation on the same coordinate plane.

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Activity 1

Bugs Passing in the Night, Continued

10 min

Activity Narrative

In this task, students find and graph a linear equation given only the graph of another equation, information about the slope, and the coordinates where the lines intersect. The purpose of this task is to check student understanding about the point of intersection in relationship to the context, while applying previously learned skills of equation writing and graphing.

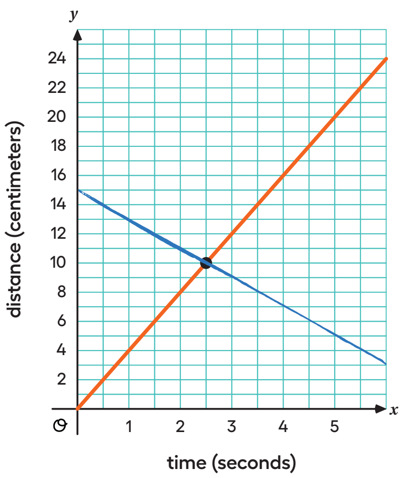
Launch

Display the graph from the task statement. Tell students that this activity is about a *different* ant and ladybug than those in the *Warm-up,* and that we are going to think about their distances using a coordinate plane.

Give 4–6 minutes for students to complete the problems, and follow that with a whole-class discussion.

Student Task Statement

A different ant and ladybug are a certain distance apart, and they start walking toward each other. The graph shows the ladybug's distance from its starting point over time and the labeled point (2.5, 10) indicates when the ant and the ladybug pass each other.



The ant is walking 2 centimeters per second.

1. Write an equation representing the relationship between the ant's distance from the ladybug's starting point and the amount of time that has passed.

   $d = -2t + 15$ (or equivalent)

2. If you haven't already, draw the graph of your equation on the same coordinate plane.

Activity Synthesis

The purpose of this discussion is to ensure that all students understand both how the labeled point in the task statement relates to the context and how to write and graph an equation from the given information.

Invite 2–3 groups with different approaches to share their solutions. Here are some questions for discussion:

- *"How does the ant's speed show up in each method?"*

  It is the slope, which is the coefficient of x in the equation and is related to the angle of the line in the graph.

- *"How can each method be used to find the starting distance between the 2 bugs?"*

  In one method, I counted back 2 cm per second to find the starting position. In the other method, I counted forward 2 cm per second to find the line, then drew the line and looked at where it crosses the vertical axis.

If students struggled to graph the ant's path, you may wish to conclude the discussion by asking students for different ways to add the graph of the ant's distance onto the coordinate plane. For example, some students may say to use the equation figured out in the first problem to plot points and then draw a line through them. Other students may suggest starting from the known point, (2.5, 10), and "working backward" to figure out that 1 second earlier at 1.5 seconds, the ant would have to be 12 centimeters away because $10 + 2 = 12$.

Activity 2

A Close Race

15 min

Activity Narrative

In previous lessons, students encountered equations with a single variable that had infinitely many solutions. In this activity, students interpret a system with infinitely many solutions. A race is described using different representations (a table and a description in words). Students graph the relationships given by the descriptions and notice that the lines overlap so that both relationships are true for any pair of values along the graphed line.

Instructional Routines

MLR6: Three Reads

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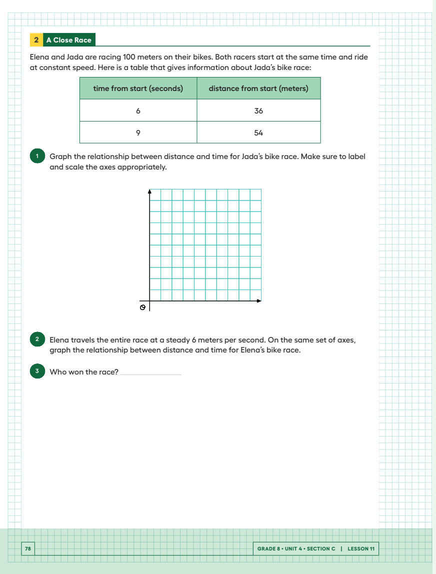
Access for Multilingual Learners (Activity 2)

MLR6: Three Reads

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Student Workbook


2 A Close Race

Elena and Jada are racing 100 meters on their bikes. Both racers start at the same time and ride at constant speed. Here is a table that gives information about Jada's bike race:

time from start (seconds)	distance from start (meters)
6	36
9	54

1 Graph the relationship between distance and time for Jada's bike race. Make sure to label and scale the axes appropriately.

2 Elena travels the entire race at a steady 6 meters per second. On the same set of axes, graph the relationship between distance and time for Elena's bike race.

3 Who won the race?

Launch

Allow students 7–10 minutes of silent work time followed by a whole-class discussion.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the table, without revealing the questions.

For the first read, read the problem aloud, and then ask,

"What is this situation about?"

friends racing bikes

Listen for and clarify any questions about the context.

After the second read, ask students to list any quantities that can be counted or measured.

the distance and times for 2 points in Jada's race

After the third read, reveal the question:

"Graph the relationship between distance and time for Jada's bike race."

Ask,

"What are some ways that we might get started on this?"

Invite students to name some possible starting points, referring to quantities from the second read.

plot the points

Student Task Statement

Elena and Jada are racing 100 meters on their bikes. Both racers start at the same time and ride at constant speed. Here is a table that gives information about Jada's bike race:

time from start (seconds)	distance from start (meters)
6	36
9	54

1. Graph the relationship between distance and time for Jada's bike race. Make sure to label and scale the axes appropriately.

   A graph of $d = 6t$ with the horizontal axis titled "time (seconds)" and the vertical axis titled "distance (meters)"

2. Elena travels the entire race at a steady 6 meters per second. On the same set of axes, graph the relationship between distance and time for Elena's bike race.

   A graph of $d = 6t$ (or equivalent)

   Elena's graph is the same line as Jada's.

3. Who won the race?

   Elena and Jada tied. Both racers traveled the same distance at the same speed.

Activity Synthesis

The key point for discussion is to connect what students observed about the graph that they made to the concept of "infinitely many solutions" encountered in earlier lessons. Graphically, students see that there are situations where two lines align "on top of each other." We can interpret each point on the line as representing a solution to both Elena's and Jada's equations. If t is the time from start and d is the distance from the start, the equation for Elena is $d = 6t$. The equation for Jada is also $d = 6t$. Every solution to Elena's equation is also a solution to Jada's equation, and every solution to Jada's equation is also a solution to Elena's equation. In this way, there are infinitely many points that are solutions to both equations at the same time.

Ensure that students clearly understand that just because there are infinitely many points that are solutions, it does not mean that any pair of values will solve both Elena's and Jada's equations. In this example, the pair of values must still be related by the equation $d = 6t$. So, pairs of values like (1, 6), (10, 60), and $\left(\frac{1}{2}, 3\right)$ are all solutions, but (1, 8) is not.

Lesson Synthesis

Display a set of axes for all to see. Ask each question one at a time, allowing students time to work through each problem. As students share their responses, add graphs of the lines described to the axes.

- *"A line goes through the point (2, 5) and has a slope of 1.5. What is an equation for this line?"*

  $y = 1.5x + 2$

- *"A second line goes through the point (2, 5) and has a y-intercept of (0, 10). What is an equation for this line?"*

  $y = -2.5x + 10$

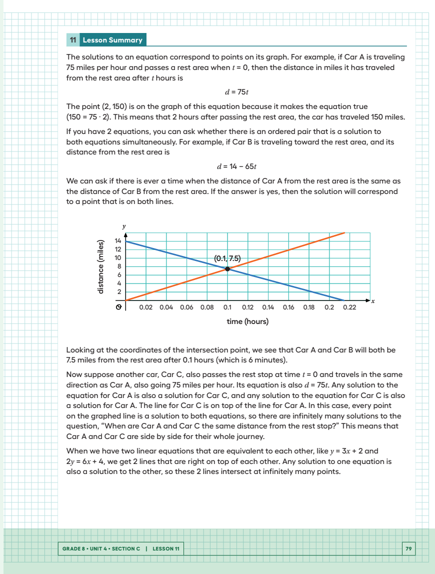
- *"What does the point (2, 5) represent for these lines?"*

  The pair of values that is true in both situations.

- *"A third line goes through this same point. How would that show up in a table representing the relationship for the third line?"*

  The number 2 would be in the x column right next to the number 5 in the y column.

Student Workbook



Lesson Summary

The solutions to an equation correspond to points on its graph. For example, if Car A is traveling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has traveled from the rest area after t hours is

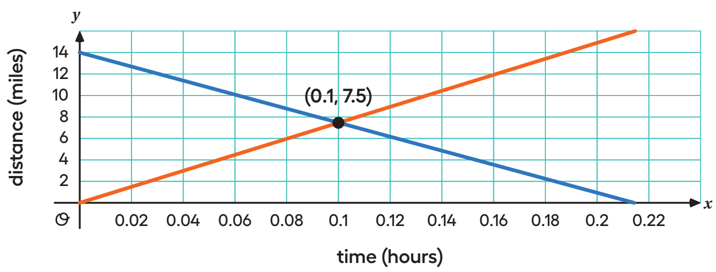
$$d = 75t$$

The point (2, 150) is on the graph of this equation because it makes the equation true ($150 = 75 \cdot 2$). This means that 2 hours after passing the rest area, the car has traveled 150 miles.

If you have 2 equations, you can ask whether there is an ordered pair that is a solution to both equations simultaneously. For example, if Car B is traveling toward the rest area, and its distance from the rest area is

$$d = 14 - 65t$$

We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is yes, then the solution will correspond to a point that is on both lines.



Looking at the coordinates of the intersection point, we see that Car A and Car B will both be 7.5 miles from the rest area after 0.1 hours (which is 6 minutes).

Now suppose another car, Car C, also passes the rest stop at time $t = 0$ and travels in the same direction as Car A, also going 75 miles per hour. Its equation is also $d = 75t$. Any solution to the equation for Car A is also a solution for Car C, and any solution to the equation for Car C is also a solution for Car A. The line for Car C is on top of the line for Car A. In this case, every point on the graphed line is a solution to both equations, so there are infinitely many solutions to the question, "When are Car A and Car C the same distance from the rest stop?" This means that Car A and Car C are side by side for their whole journey.

When we have two linear equations that are equivalent to each other, like $y = 3x + 2$ and $2y = 6x + 4$, we get 2 lines that are right on top of each other. Any solution to one equation is also a solution to the other, so these 2 lines intersect at infinitely many points.

Cool-down

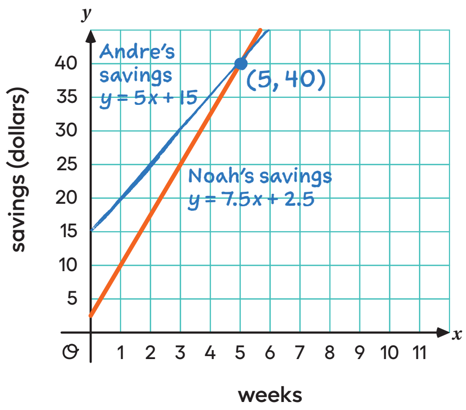
Saving Cash

5 min

Student Task Statement

Andre and Noah start tracking their savings at the same time. Andre starts with \$15 and deposits \$5 per week. Noah starts with \$2.50 and deposits \$7.50 per week. The graph of Noah's savings is given, and his equation is $y = 7.5x + 2.5$, where x represents the number of weeks and y represents his savings.

Write the equation for Andre's savings, and graph it alongside Noah's. What does the intersection point mean in this situation?



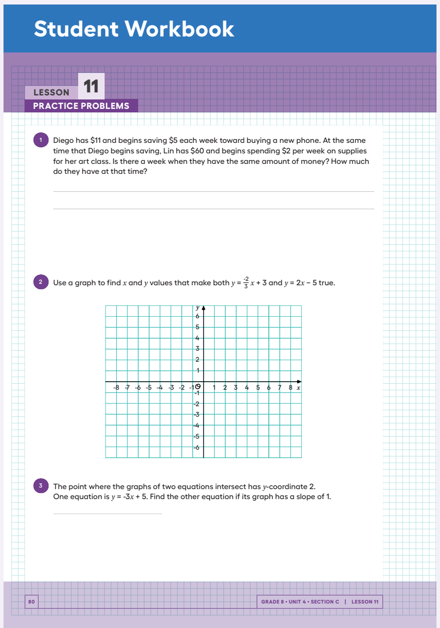
Sample response: The intersection at (5, 40) means that after 5 weeks, Noah and Andre each have \$40.

Responding To Student Thinking

Points to Emphasize

If students struggle to graph and interpret the relationship correctly, use this practice problem to discuss and compare two different savings plans:

Unit 4, Lesson 11, Practice Problem 1


Student Workbook

LESSON 11

PRACTICE PROBLEMS

1 Diego has $11 and begins saving $5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they have the same amount of money? How much do they have at that time?

2 Use a graph to find x and y values that make both $y = \frac{-2}{3}x + 3$ and $y = 2x - 5$ true.

3 The point where the graphs of two equations intersect has y-coordinate 2. One equation is $y = -3x + 5$. Find the other equation if its graph has a slope of 1.

Lesson **11**

Practice Problems

4 Problems

Problem 1

Diego has $11 and begins saving $5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they have the same amount of money? How much do they have at that time?

After 7 weeks they both have $46.

Problem 2

Use a graph to find x and y values that make both $y = \frac{-2}{3}x + 3$ and $y = 2x - 5$ true.

(3, 1)

Problem 3

The point where the graphs of two equations intersect has y-coordinate 2. One equation is $y = -3x + 5$. Find the other equation if its graph has a slope of 1.

$y = x + 1$

Sample reasoning: $2 = -3x + 5$ is true when $x = 1$, so the line needed has a slope of 1 and contains the point (1, 2).

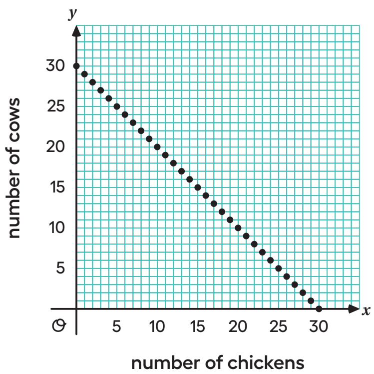
Problem 4

from Unit 4, Lesson 10

A farm has chickens and cows. All the cows have 4 legs and all the chickens have 2 legs. All together, there are 82 cow and chicken legs on the farm. Complete the table to show some possible combinations of chickens and cows to get 82 total legs.

number of chickens (x)	number of cows (y)
35	3
7	17
21	10
19	11
31	5

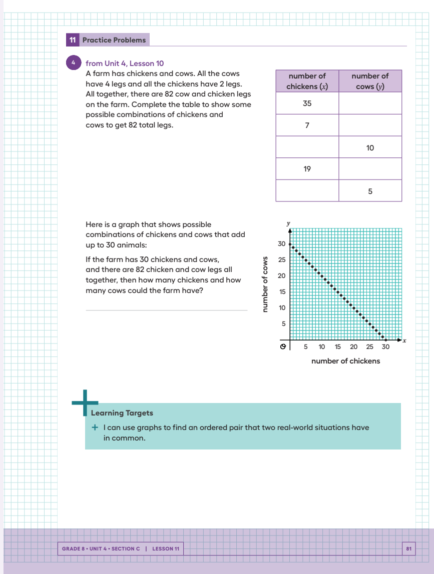
Here is a graph that shows possible combinations of chickens and cows that add up to 30 animals:



If the farm has 30 chickens and cows, and there are 82 chicken and cow legs all together, then how many chickens and how many cows could the farm have?

The farm could have 19 chickens and 11 cows.

Student Workbook



11 Practice Problems

4 from Unit 4, Lesson 10

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7	
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	5

Here is a graph that shows possible combinations of chickens and cows that add up to 30 animals:

If the farm has 30 chickens and cows, and there are 82 chicken and cow legs all together, then how many chickens and how many cows could the farm have?

Learning Targets

+ I can use graphs to find an ordered pair that two real-world situations have in common.

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