Lesson 12

Using Equations for Lines

Goals

- Create an equation of a line with positive slope on the coordinate plane using knowledge of similar triangles.
- Generalize (orally) a process for dilating a slope triangle ABC on a coordinate plane with center of dilation A and scale factor s.
- Justify (orally) that a point (x, y) is on a line by verifying that the values of x and y satisfy the equation of the line.

Learning Target

I can find an equation for a line and use it to decide which points are on that line.

Lesson Narrative

The purpose of this lesson is for students to write equations for a line when no slope triangles are given. Students begin by revisiting the meaning of dilations and scale factors. Then students draw dilations of a triangle using the same center but different scale factors. Some students may find a pattern in the resulting coordinates, while others may use the definition of dilations to generalize what happens to a point dilated by any scale factor s.

Students are then given a line with a few labeled points and asked to use what they know about similar triangles to find an equation for the line. Students practice using this line to test whether given points are also on this line.

Student Learning Goal

Let's write equations for lines.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2
- Straightedges: Activity 2

Required Preparation

Warm-up:
Provide access to geometry toolkits.

Activity 1:
Provide access to geometry toolkits.

Activity 2:
Provide access to geometry toolkits (in particular, a straightedge is helpful).

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	15 min	10 min	5 min

Warm-up
Missing Center

5 min

Activity Narrative

The purpose of this activity is to revisit the meaning of dilations and the fact that the center of dilation, the point dilated, and the image all lie on the same line.

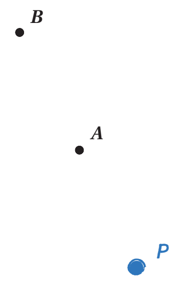
Launch

Provide access to geometry toolkits.

Give students 1–2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

A dilation with scale factor 2 sends A to B. Where is the center of the dilation?



Sample response: The center of dilation is on the same line as A and B, the same distance from B to A, but on the other side of A.

Activity Synthesis

The goal of this discussion is to review key ideas about dilations. Ask students:

- *"What do you know about centers of dilations that helped you solve this problem?"*

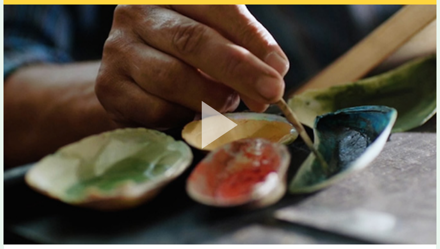
  The center of dilation always lies on the same line as a dilated point and its image.

- *"What do you know about scale factors that helped you solve this problem?"*

  The scale factor is 2, so the distance from the center to B had to be twice the distance from the center to A.

Inspire Math

New Perspective video

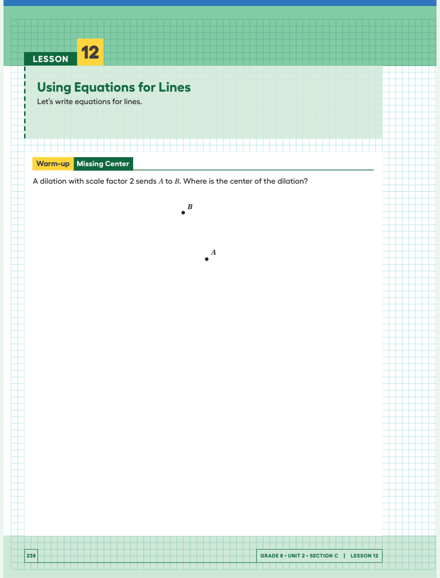


Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614228

Please log in to the site before using the QR code or URL.

Student Workbook



Activity 1

Dilations and Slope Triangles

15 min

Activity Narrative

The goal of this activity is to identify the coordinates of points on a line by looking at dilations of a single slope triangle. Since all dilations of the triangle are similar, their longest sides all lie on the same line, and the coordinates of the points on that line have a structure linked with the dilations used to produce them.

For the question where C is dilated by scale factor s, monitor for students who:

- Look for and express a pattern for the coordinates of the points from earlier questions. For example, a scale factor of 1 gives $C = (2, 2)$, a scale factor of 2 gives $C = (4, 3)$, and a scale factor of 2.5 gives $C = (5, 3.5)$. So the x-coordinate appears to be twice the scale factor while the y-coordinate appears to be 1 more than the scale factor.
- Use the structure of triangle ABC and the definition of dilations. For example, since the horizontal length of triangle ABC is 2 and the vertical length is 1, a dilation with scale factor s would make the horizontal length $2s$ and the vertical length s. Since the vertical side of triangle ABC is 1 unit above the x-axis, the dilation would result in a y-coordinate that is 1 greater than the dilated vertical length. No adjustment is needed for the dilated x-coordinate because the horizontal side of triangle ABC starts on the y-axis.

Launch

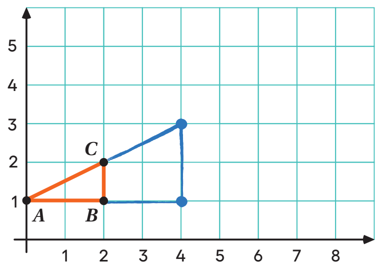
Arrange students in groups of 2. Provide access to geometry toolkits.

Give students 2–3 minutes of quiet work time followed by a partner then whole-class discussion.

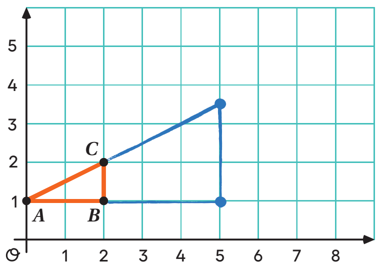
Student Task Statement

Here is triangle ABC.

1. Draw the dilation of triangle ABC with center (0, 1) and scale factor 2.



2. Draw the dilation of triangle ABC with center (0, 1) and scale factor 2.5.



3. For which scale factor does the dilation with center (0, 1) send point C to (9, 5.5)? Explain your reasoning.

4.5

Sample reasoning: The horizontal length of the original triangle is 2 units, and the corresponding length of the image triangle will be 9 units, and $9 \div 2 = 4.5$.

4. What are the coordinates of point C after a dilation with center (0, 1) and scale factor s?

$(2s, s + 1)$ (or equivalent)

Activity Synthesis

The goal of this discussion is for students to see that the structure of the coordinates of points on a line can be derived from properties of dilations. Begin by inviting previously selected students, as described in the *Activity Narrative*, to share their reasoning for the last question. Here are some questions for discussion:

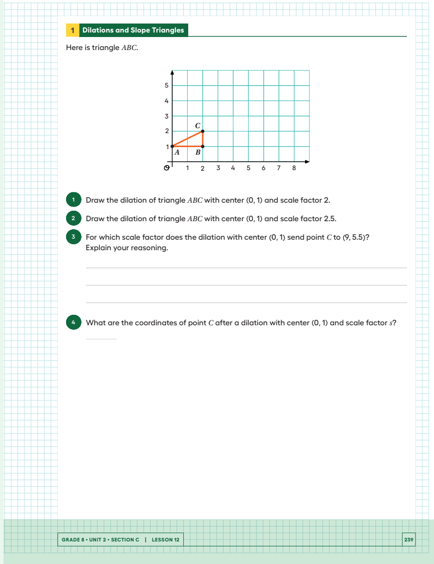
- *"What do the different strategies have in common? How are they different?"*

  Both use the structure of the grid. One strategy finds a pattern, and the other strategy uses the definition of dilations.

- *"Why do the different approaches lead to the same outcome?"*

  Finding a pattern helps to determine the coordinates for point C while thinking about dilations explains why the x coordinate doubles and the y coordinate is 1 more than the scale factor. The outcome applies to all scale factors in this situation, so a correct strategy will result in the same outcome.

Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect. Invite groups to prepare a visual display that shows the strategy they used to find the location of point C after a dilation with center (0, 1) and scale factor s. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each other's work. During the whole-class discussion, ask students:

"What kinds of additional details or language helped you understand the displays?"
"Were there any additional details or language that you have questions about?"
"Did anyone solve the problem the same way, but would explain it differently?"

Advances: Representing, Conversing

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify
This activity uses the Critique, Correct, Clarify math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Activity 2
Writing Relationships from Two Points

15 min

Activity Narrative

This activity has students write an equation satisfied by the points on a line and then use it to check whether or not specific points lie on that line.

Note that the y-intercept was intentionally left off of this activity's diagram so that students are encouraged to engage with thinking about similar triangles.

There are many slope triangles that students can draw, but the one joining (5, 3) and (7, 7) is the most natural for calculating the slope. Then (x, y) and either (5, 3) or (7, 7) can be used to find an equation. Monitor for students who use each of these points in their equation and invite them to present during the discussion.

This is the first time Math Language Routine 3: *Critique, Correct, Clarify* is suggested in this course. In this routine, students are given a "first draft" statement or response to a question that is intentionally unclear, incorrect, or incomplete. Students analyze and improve the written work by first identifying what parts of the writing need clarification, correction, or details, and then writing a second draft (individually or with a partner). Finally, the teacher scribes as a selected second draft is read aloud by its author(s), and the whole class is invited to help edit this "third draft" by clarifying meaning and adding details to make the writing as convincing as possible to everyone in the room. Typical prompts are "Is anything unclear?" and "Are there any reasoning errors?" The purpose of this routine is to engage students in analyzing mathematical writing and reasoning that is not their own, and to solidify their knowledge and use of language.

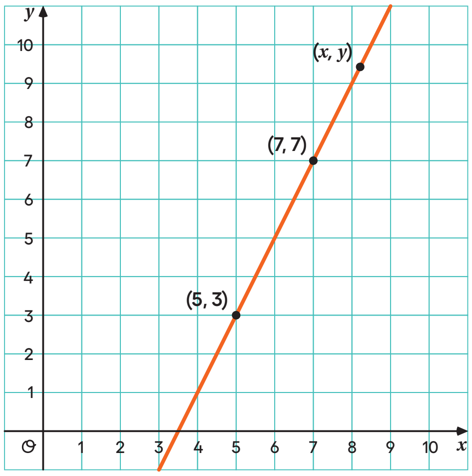
Launch

Provide access to geometry toolkits (in particular, a straightedge is helpful).

Give students 2–3 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

Here is a line.



1. Using what you know about similar triangles, find an equation for the line in the diagram.
   $\frac{y-7}{x-7} = 2$ or $\frac{y-3}{x-5} = 2$ (or equivalent)
2. What is the slope of this line? Does it appear in your equation?
   The slope is 2. Yes, the slope appears in my equation.
3. Is (9, 11) also on the line? Explain your reasoning.
   Yes, (9, 11) is also on the line.
   Sample reasoning: Looking at the graph, the point (9, 11) is on the line.
4. Is (100, 193) also on the line? Explain your reasoning.
   Yes, (100, 193) is on the line.
   Sample reasoning: When y is 193 and x is 100, my equation is true.

Are You Ready for More?

There are many different ways to write an equation for a line like the one in the *Student Task* you just completed. Does $\frac{y-3}{x-6} = 2$ represent that line? What about $\frac{y-6}{x-4} = 5$? What about $\frac{y+5}{x-1} = 2$? Explain your reasoning.

- No, the equation $\frac{y-3}{x-6} = 2$ does not represent the line.

Sample reasoning: This equation represents a line of slope 2 containing the point (6, 3). Since our line does not contain the point (6, 3), this is not our line.

- No, the equation $\frac{y-6}{x-4} = 5$ does not represent the line.

Sample reasoning: This equation represents a line of slope 5. Since our line has slope 2, this is not our line.

- Yes, the equation $\frac{y+5}{x-1} = 2$ represents the line.

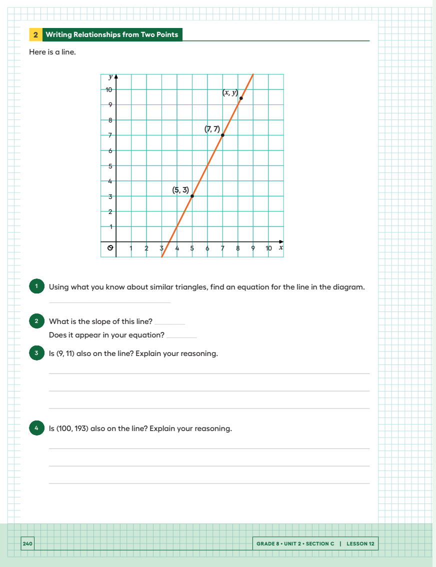
Sample reasoning: This equation represents a line of slope 2 containing the point (1, −5). Since our line contains the point (1, −5) and has slope 2, this is another possible equation for our line.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Develop Expression and Communication. Develop fluency with using slope triangles to find an equation for a given line. Provide access to a blank or partially completed copy of the graph with possible slope triangles already drawn in.

Supports accessibility for: Visual-Spatial Processing, Attention

Student Workbook


2 Writing Relationships from Two Points

Here is a line.

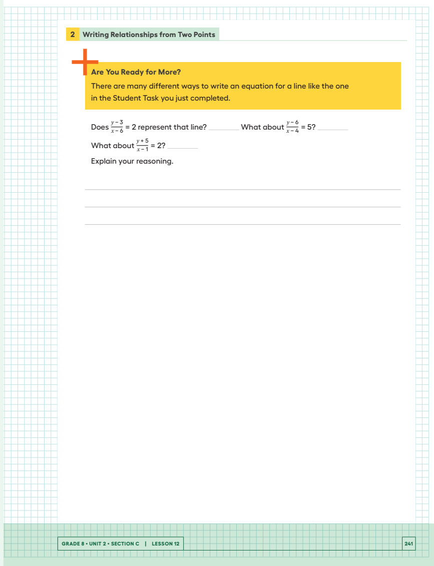
1 Using what you know about similar triangles, find an equation for the line in the diagram.

2 What is the slope of this line? Does it appear in your equation?

3 Is (9, 11) also on the line? Explain your reasoning.

4 Is (100, 193) also on the line? Explain your reasoning.

Student Workbook


2 Writing Relationships from Two Points

Are You Ready for More?

There are many different ways to write an equation for a line like the one in the Student Task you just completed.

Explain your reasoning.

Activity Synthesis

The purpose of this discussion is for students to examine multiple ways of finding an equation for a line and to understand how to use that equation to determine if a point lies on the line. Begin by inviting previously selected students to show how they arrived at their equation, making sure to have different equations shown as described in the *Activity Narrative*. Here are some questions for discussion:

"How are each of these equations similar?"

All of the equations have the slope somewhere in the equation. All of the equations have an x and a y.

"How are each of these equations different?"

Some equations used the point (5, 3) and some used the point (7, 7) for the slope triangle.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response for the last question about point (100, 193) by correcting errors, clarifying meaning, and adding details.

Display this first draft:

"When I put the numbers 100 and 193 into the equation $\frac{x-5}{y-3}=2$, it doesn't work."

Ask:

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement. If not mentioned by students, display and review these ideas:

- The meaning of slope and how it is seen in the equation
- How the numbers 100 and 193 are used in the equation
- How to use more precise language about what "doesn't work"

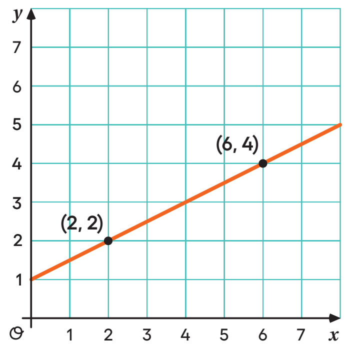
Give students 2–4 minutes to work with a partner to revise the first draft.

Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Highlight that using (x, y) and (5, 3) for a slope triangle gives an equation $\frac{y-3}{x-5}=2$ while using (x, y) and (7, 7) gives the equation $\frac{y-7}{x-7}=2$. These equations look different, but they will both work to check whether or not a point is on the line.

Using algebra to show that these two equations are equivalent is not necessary (or appropriate in grade 8), but students can see from the graphed line that either equation can be used to test whether or not a point is on the line. If students have not done so already when they share their solutions, draw and label the two slope triangles that correspond to these two equations.

Lesson Synthesis



The goal of this discussion is for students to understand how the coordinates of points on a line have a structure that is useful for checking whether or not a given point is on a line. Display this image of a line with two labeled points.

Discuss with students:

"What is the slope of this line?"

It's $\frac{1}{2}$ because a slope triangle for the two labeled points has horizontal side length 4 and vertical side length 2.

Draw the slope triangle to verify.

"What is an equation for the line?"

$\frac{y-2}{x-2} = \frac{1}{2}$ (or equivalent)

Label a new point on the line, and draw a slope triangle to show this relationship.

"What is another equation for this line?"

$\frac{y-4}{x-6} = \frac{1}{2}$ (or equivalent)

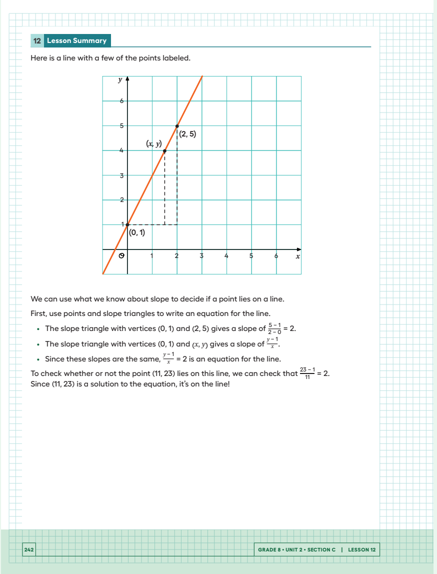
"How can you draw slope triangles to show that $\frac{4-y}{6-x} = \frac{1}{2}$ is also an equation for this line?"

Place a point (x, y) on the line to the left of $(6, 4)$ and draw in the horizontal and vertical sides.

"How can you find out whether or not the point (72,37) is on this line?

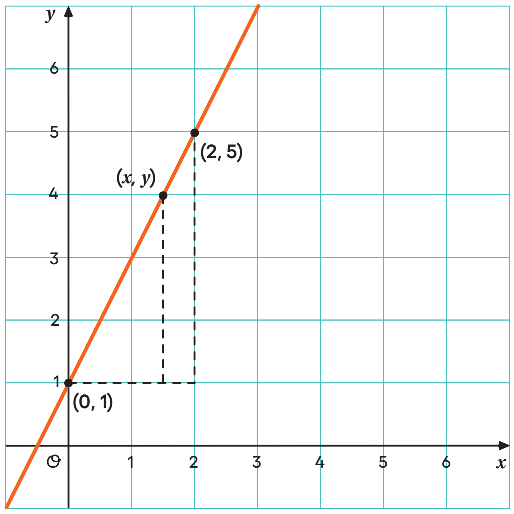
Check to see if the point $(72, 37)$ satisfies one of the equations of the line. Since $\frac{37-2}{72-2} = \frac{1}{2}$, the point $(72, 37)$ is on the line.

Student Workbook



Lesson Summary

Here is a line with a few of the points labeled.



We can use what we know about slope to decide if a point lies on a line.

First, use points and slope triangles to write an equation for the line.

- The slope triangle with vertices (0, 1) and (2, 5) gives a slope of $\frac{5-1}{2-0}=2$.
- The slope triangle with vertices (0, 1) and (x, y) gives a slope of $\frac{y-1}{x}$.
- Since these slopes are the same, $\frac{y-1}{x}=2$ is an equation for the line.

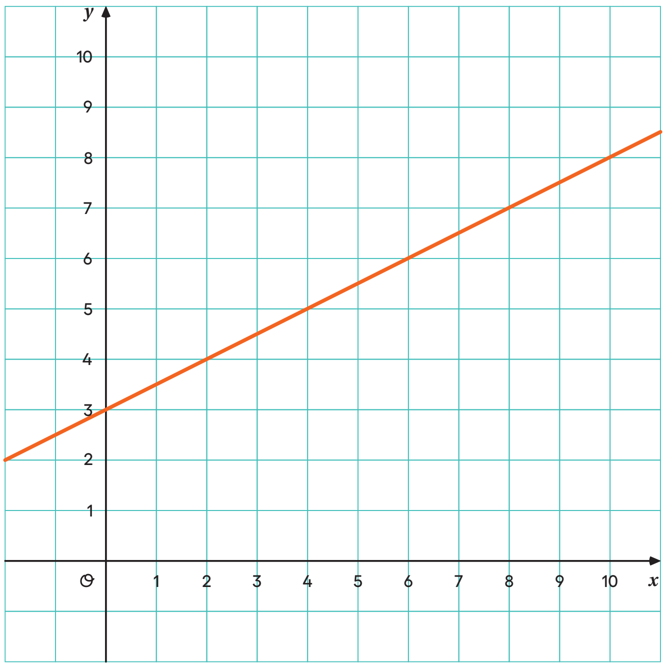
To check whether or not the point (11, 23) lies on this line, we can check that $\frac{23-1}{11}=2$. Since (11, 23) is a solution to the equation, it's on the line!

Cool-down

Is the Point on the Line?

5 min

Student Task Statement



Is the point (20, 13) on this line? Explain your reasoning.

Yes, point (20, 13) is on the line.

Sample reasoning: One possible equation for the line is $\frac{y-3}{x}=\frac{1}{2}$. Since $\frac{13-3}{20}=\frac{1}{2}$, the point (20, 13) is on this line.

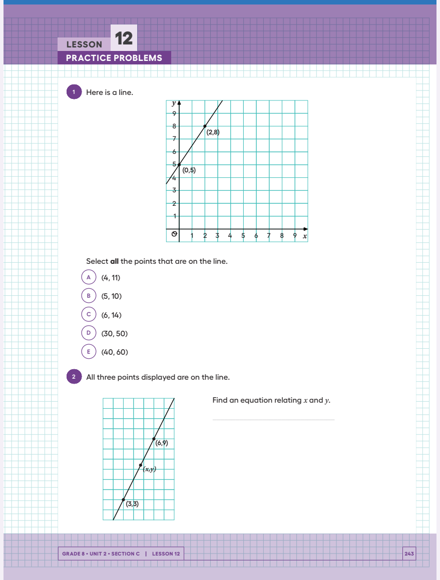
Responding To Student Thinking

Points to Emphasize
If students struggle with determining whether a point is on a given line, as opportunities arise over the next several lessons, revisit what must be true about the coordinates of different points in relation to an equation that describes a given line.

Practice Problems

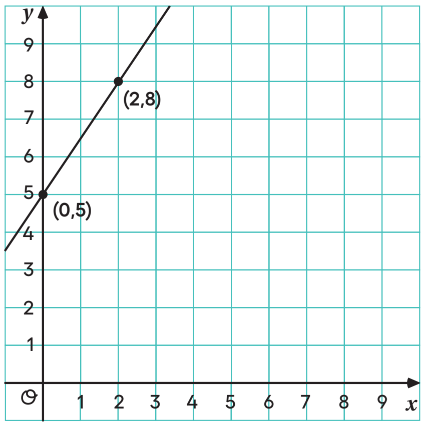
4 Problems

Student Workbook



Problem 1

Here is a line.



Select **all** the points that are on the line.

A. (4, 11)

B. (5, 10)

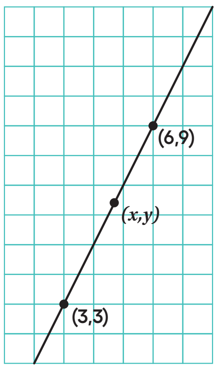
C. (6, 14)

D. (30, 50)

E. (40, 60)

Problem 2

All three points displayed are on the line. Find an equation relating x and y.

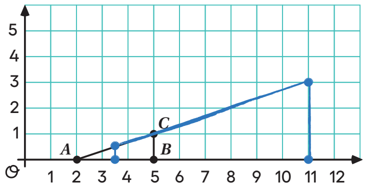


$\frac{y-3}{x-3} = 2$ (or equivalent) or $\frac{9-y}{6-x} = 2$ (or equivalent)

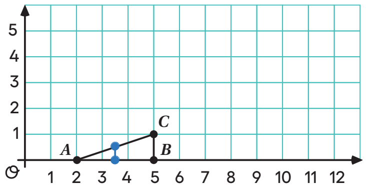
Problem 3

Here is triangle ABC.

a. Draw the dilation of triangle ABC with center (2, 0) and scale factor 3.



b. Draw the dilation of triangle ABC with center (2, 0) and scale factor $\frac{1}{2}$.



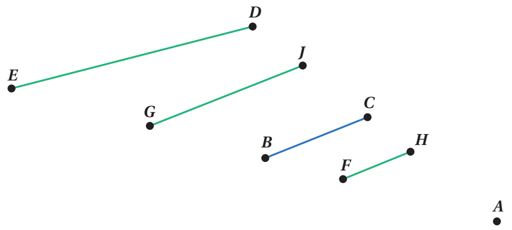
c. What are the coordinates of the image of point C when triangle ABC is dilated with center (2, 0) and scale factor s?

(2 + 3*s*, *s*) (or equivalent)

Problem 4

from Unit 2, Lesson 4

Here are some line segments.



a. Which segment is a dilation of segment BC using A as the center of dilation and a scale factor of $\frac{2}{3}$?

Segment *FH*

Sample reasoning: A scale factor of $\frac{2}{3}$ produces a parallel line segment with shorter length.

b. Which segment is a dilation of segment BC using A as the center of dilation and a scale factor of $\frac{3}{2}$?

Segment *GJ*

Sample reasoning: A scale factor of $\frac{3}{2}$ will produce a parallel line segment with longer length.

c. Which segment is not a dilation of segment BC? Explain your reasoning.

Segment *DE*

Sample reasoning: Dilations take lines to parallel lines, and *DE* is not parallel to *FH*.

Student Workbook