

## Dividing by Unit and Non-Unit Fractions

## Goals

- Interpret and critique explanations (in spoken and written language, as well as in other representations) of how to divide by a fraction.
- Use a tape diagram to represent dividing by a non-unit fraction  $\frac{a}{b}$  and explain (orally) why this produces the same result as multiplying the number by  $b$  and dividing by  $a$ .
- Use a tape diagram to represent dividing by a unit fraction  $\frac{1}{b}$  and explain (orally and in writing) why this is the same as multiplying by  $b$ .

## Learning Targets

- I can divide a number by a non-unit fraction by reasoning with the numerator and denominator, which are whole numbers.
- I can divide a number by a unit fraction  $\frac{1}{b}$  by reasoning with the denominator, which is a whole number.

## Lesson Narrative

In this lesson, students begin by using tape diagrams to recall and visualize two ideas from grade 5:

- Dividing by a whole number  $b$  has the same outcome as multiplying by the unit fraction  $\frac{1}{b}$ .
- Dividing by a unit fraction  $\frac{1}{b}$  has the same outcome as multiplying by the whole number  $b$ .

They apply the second idea to find the value of other expressions of the form  $a \div \frac{1}{b}$  without drawing tape diagrams.

Next, students use the same diagrams to analyze the effects of dividing by non-unit fractions. Through repetition, they notice a pattern in the steps of their reasoning and structure in the visual representation of these steps.

## Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

## Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

## Required Materials

## Materials to Gather

- Colored pencils: Activity 2, Activity 3

## Lesson Timeline

10  
min

Warm-up

15  
min

Activity 1

10  
min

Activity 2

10  
min

Lesson Synthesis

## Assessment

5  
min

Cool-down

## Dividing by Unit and Non-Unit Fractions

### Lesson Narrative (continued)

Students see that division of a number by a non-unit fraction can be thought of as having two steps:

1. Finding how many unit fractions there are (dividing the number by the unit fraction).
2. Putting the unit fractions into groups the size of the non-unit fraction to see how many groups there are (dividing the result by the numerator of the non-unit fraction).

For instance, to divide by  $\frac{2}{5}$  is equivalent to dividing by  $\frac{1}{5}$ , and then again by

2. Because dividing by a unit fraction  $\frac{1}{5}$  is equivalent to multiplying by 5, to divide a number by  $\frac{2}{5}$ , we can multiply it by 5 and divide the result by 2.

### Student Learning Goal

Let's look for patterns when we divide by a fraction.

**Student Workbook**

**LESSON 10**

**Dividing by Unit and Non-Unit Fractions**

Let's look for patterns when we divide by a fraction.

**Work with a partner:** One person solves the problems labeled "Partner A" and the other person solves those labeled "Partner B." Write an equation for each question. If you get stuck, consider drawing a diagram.

**1. Partner A:**

How many 3s are in 12?  
Division equation: \_\_\_\_\_



How many 4s are in 12?  
Division equation: \_\_\_\_\_



How many 6s are in 12?  
Division equation: \_\_\_\_\_



**Warm-up****Dividing by a Whole Number**

10 min

**Activity Narrative**

In this warm-up, students use tape diagrams to revisit the idea that dividing by a whole number is equivalent to multiplying by a unit fraction. Though this is a review of a grade 5 expectation, connecting the division problems to diagrams allows students to see the equivalence in the related division and multiplication problems. It also prepares students to apply the same reasoning and representations to division by non-unit fractions later.

**Launch**

Arrange students in groups of 2. Ask one person in each group to draw diagrams and answer the questions for Partner A, and the other to take on the questions for Partner B. Give students a few minutes of quiet time to complete the first two questions, and then ask them to compare their responses and collaborate on the last two questions.

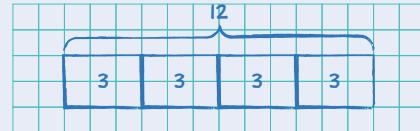
**Student Task Statement**

Work with a partner. One person solves the problems labeled "Partner A" and the other person solves those labeled "Partner B." Write an equation for each question. If you get stuck, consider drawing a diagram.

**1. Partner A:**

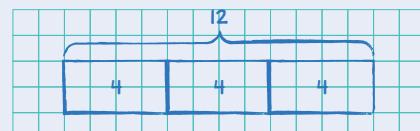
How many 3s are in 12?

Division equation:  $12 \div 3 = 4$



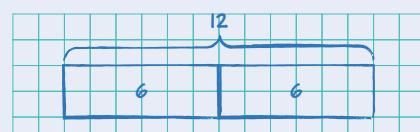
How many 4s are in 12?

Division equation:  $12 \div 4 = 3$



How many 6s are in 12?

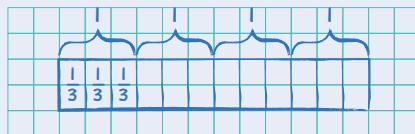
Division equation:  $12 \div 6 = 2$



**Partner B:**

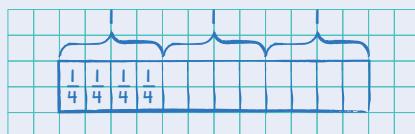
What is 12 groups of  $\frac{1}{3}$ ?

$$\text{Multiplication equation: } 12 \cdot \frac{1}{3} = 4$$



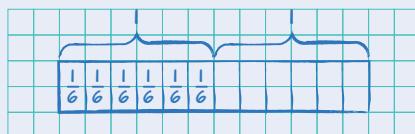
What is 12 groups of  $\frac{1}{4}$ ?

$$\text{Multiplication equation: } 12 \cdot \frac{1}{4} = 3$$



What is 12 groups of  $\frac{1}{6}$ ?

$$\text{Multiplication equation: } 12 \cdot \frac{1}{6} = 2$$



- 2.** What do you notice about the diagrams and equations? Discuss with your partner.

Sample responses:

- The quotients in the division equations have the same value as the products in the multiplication equations.
- Both sets of problems use the number 12. The division problems have 3, 4, and 6 as divisors, and the multiplication problems have a factor that is a fraction with those numbers in the denominator ( $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ ).
- For each pair, the diagrams are divided into the same number of major parts but they show different information.
- Dividing by a whole number gives the same result as multiplying by a fraction with that number as the denominator.

- 3.** Complete this sentence based on what you noticed:

Dividing by a whole number  $a$  produces the same result as multiplying by  $\frac{1}{a}$ .

**Student Workbook**

## Warm-up

Dividing by a Whole Number

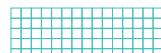
Partner B:

What is 12 groups of  $\frac{1}{3}$ ?

Multiplication equation:

What is 12 groups of  $\frac{1}{4}$ ?

Multiplication equation:

What is 12 groups of  $\frac{1}{6}$ ?

Multiplication equation:



2. What do you notice about the diagrams and equations? Discuss with your partner.

3. Complete this sentence based on what you noticed:

Dividing by a whole number  $a$  produces the same result as multiplying by \_\_\_\_\_.

GRADE 6 • UNIT 4 • SECTION C | LESSON 10

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## Activity Synthesis

Invite a couple of students to share their observations about their group's diagrams and answers. Students should notice that the answers for the three division problems match those for the multiplication ones, even though the questions were not the same, and their diagrams show groups of different sizes. Ask a few students to share their response to the last question.

Consider displaying the following image to reinforce the idea that dividing by a whole number  $a$  has the same effect as multiplying by  $\frac{1}{a}$ .

$$12 \div 3 = 4$$

A horizontal tape diagram divided into 3 equal segments. Above the tape, a bracket labeled "12" spans the entire length. Below the segments, the number "3" is written above each segment. To the left of the first segment, the equation  $12 \div 3 = 4$  is written.

$$12 \cdot \frac{1}{3} = 4$$

A horizontal tape diagram divided into 3 equal segments. Above the tape, three brackets labeled "1" are placed above each segment. Below the segments, the number "1" is written above each segment. To the left of the first segment, the equation  $12 \cdot \frac{1}{3} = 4$  is written.

$$12 \div 4 = 3$$

A horizontal tape diagram divided into 4 equal segments. Above the tape, a bracket labeled "12" spans the entire length. Below the segments, the number "4" is written above each segment. To the left of the first segment, the equation  $12 \div 4 = 3$  is written.

$$12 \cdot \frac{1}{4} = 3$$

A horizontal tape diagram divided into 4 equal segments. Above the tape, four brackets labeled "1" are placed above each segment. Below the segments, the number "1" is written above each segment. To the left of the first segment, the equation  $12 \cdot \frac{1}{4} = 3$  is written.

$$12 \div 6 = 2$$

A horizontal tape diagram divided into 6 equal segments. Above the tape, a bracket labeled "12" spans the entire length. Below the segments, the number "6" is written above each segment. To the left of the first segment, the equation  $12 \div 6 = 2$  is written.

$$12 \cdot \frac{1}{6} = 2$$

A horizontal tape diagram divided into 6 equal segments. Above the tape, six brackets labeled "1" are placed above each segment. Below the segments, the number "1" is written above each segment. To the left of the first segment, the equation  $12 \cdot \frac{1}{6} = 2$  is written.

## Activity 1

## Dividing by Unit Fractions

15  
min

## Activity Narrative

In this activity, students use tape diagrams and one interpretation of division to divide a number by unit fractions. They do this as a first step toward generalizing the reasoning for dividing any two fractions. By reasoning repeatedly and noticing a pattern, students arrive at the conclusion that  $a \div \frac{1}{b}$  is equivalent to  $a \cdot b$ .

As students work, monitor for those who are able to apply the reasoning in the first two problems to subsequent problems without the help of diagrams. Select several students to share later.

## Launch

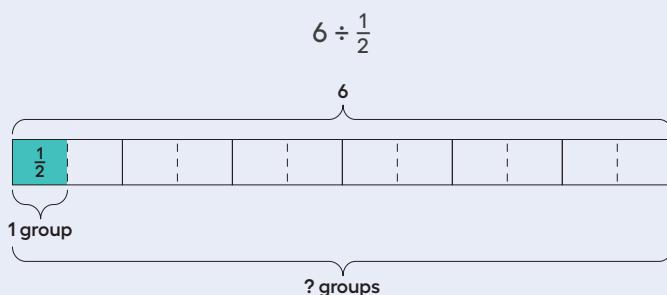


Arrange students in groups of 2. Give students 4–5 minutes of quiet work time for the first set of questions. Then give them time to look for a pattern and to discuss their observations with a partner. Advise partners to move on to the remainder of the activity only after they have identified and articulated a pattern for finding the value of each division expression.

Provide access to colored pencils. Some students may find it helpful to identify whole groups and partial groups on a tape diagram by coloring.

**Student Task Statement**

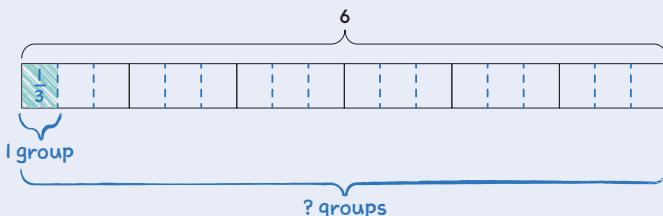
To find the value of  $6 \div \frac{1}{2}$ , Elena thought, "How many  $\frac{1}{2}$ s are in 6?" and then she drew this tape diagram. It shows 6 ones, with each one partitioned into 2 equal pieces.



- 1.** For each division expression, complete the diagram using the same method as Elena. Then, find the value of the expression.

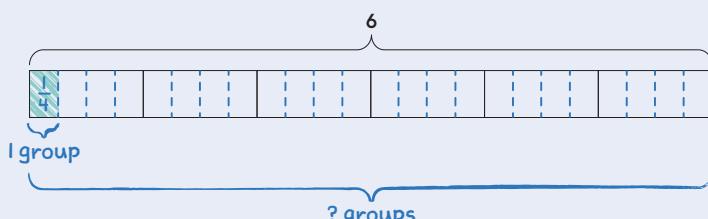
a.  $6 \div \frac{1}{3}$

Value of the expression: 18



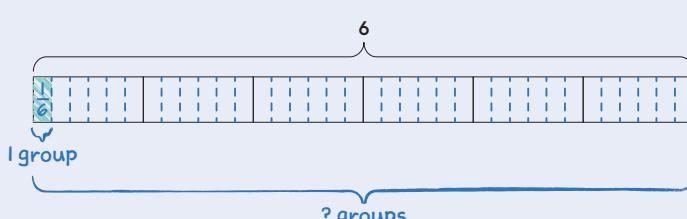
b.  $6 \div \frac{1}{4}$

Value of the expression: 24



c.  $6 \div \frac{1}{6}$

Value of the expression: 36

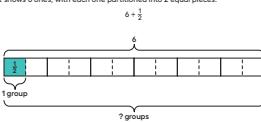


- 2.** Look for a pattern in the expressions and their values. Talk to your partner about how to find how many halves, thirds, fourths, or sixths were in 6 wholes, without counting all the parts.

**Sample response:** We can divide each 1 whole in the tape diagram into the same number of pieces as in the number in the denominator. If the fraction was  $\frac{1}{4}$ , each 1 whole would be broken into 4 pieces. To find how many pieces are in 6, we can multiply 4 by 6.

**Student Workbook**

**Dividing by Unit Fractions**  
To find the value of  $6 \div \frac{1}{2}$ , Elena thought, "How many  $\frac{1}{2}$ s are in 6?" and then she drew this tape diagram. It shows 6 ones, with each one partitioned into 2 equal pieces.



- For each division expression, complete the diagram using the same method as Elena. Then, find the value of the expression.

a.  $6 \div \frac{1}{3}$

Value of the expression: \_\_\_\_\_

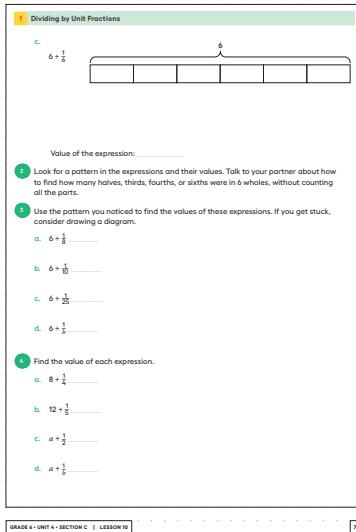
b.  $6 \div \frac{1}{4}$

Value of the expression: \_\_\_\_\_

**Access for Multilingual Learners  
(Activity 1, Synthesis)**
**MLR8: Discussion Supports.**

Invite students to repeat their response and reasoning using mathematical language: “Can you say that again, using words such as ‘denominator,’ ‘numerator,’ ‘unit fraction,’ and ‘partitioned?’”

*Advances: Speaking, Representing*

**Student Workbook**

3. Use the pattern you noticed to find the values of these expressions. If you get stuck, consider drawing a diagram.

a.  $6 \div \frac{1}{8}$

48

b.  $6 \div \frac{1}{10}$

60

c.  $6 \div \frac{1}{25}$

150

d.  $6 \div \frac{1}{b}$

$6 \cdot b$

4. Find the value of each expression.

a.  $8 \div \frac{1}{4}$

32

b.  $12 \div \frac{1}{5}$

60

c.  $a \div \frac{1}{2}$

$a \cdot 2$

d.  $a \div \frac{1}{b}$

$a \cdot b$

**Activity Synthesis**

The goal of the discussion is to make explicit the connection between division by a unit fraction and multiplication by the whole number that is its denominator, using tape diagrams to highlight this connection.

Invite students to share their completed diagrams for  $6 \div \frac{1}{3}$ ,  $6 \div \frac{1}{4}$ , and  $6 \div \frac{1}{6}$ . (If needed, remind students that fractions such as  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$  are called “unit fractions.”) Ask questions such as:

“Where in each diagram do we see division by the unit fraction  $\frac{1}{3}$ ,  $\frac{1}{4}$ , or  $\frac{1}{6}$ ?”

The tape is broken into equal parts. The unit fraction is the size of each part.

“Where in each diagram do we see the multiplication of 6 by 3, 4, or 6?”

In each diagram, we see the 6 ones divided into multiple equal parts: 6 groups of 3 thirds in the first diagram, 6 groups of fourths in the second, and 6 groups of sixths in the third.

“How many parts would be on a tape diagram that represents 6 divided by  $\frac{1}{10}$ ?”

60 tenths

“What about 6 divided by  $\frac{1}{50}$ ?”

300 fiftieths

Next, ask previously selected students to share their responses to the last set of questions. Emphasize that when we divide a number by a unit fraction  $\frac{1}{b}$ , we end up with  $b$  times as many parts, so dividing by  $\frac{1}{b}$  is the same as multiplying by  $b$ .

**Activity 2****Dividing by Non-unit Fractions**10  
min**Activity Narrative**

In this activity, students extend their reasoning about division by unit fractions to division by non-unit fractions. Specifically, they explore how to represent the numerator of the fraction in the tape diagram and study its effect on the quotient. Students generalize their observations as operational steps and then as an expression. Along the way, they practice looking for and making use of structure.

As students work, monitor for those who effectively show the fractional divisor on their diagrams, as well as those who could explain why the steps make sense. Select them to share their diagrams or reasoning later.

**Launch**

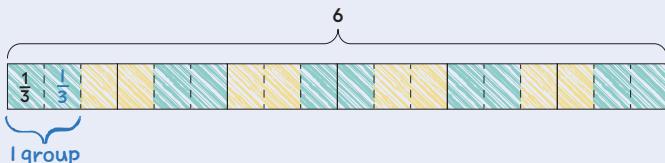
Keep students in groups of 2.

- Give students **1 minute** of quiet time to complete the diagram for  $6 \div \frac{2}{3}$  and another minute to discuss with their partner Elena's reasoning about that division. Then, give students **4–5 minutes** of partner work time for the remainder of the activity. Allow a few minutes for a whole-class discussion.

Provide continued access to colored pencils.

**Student Task Statement**

- 1.** To find the value of  $6 \div \frac{2}{3}$ , Elena started by drawing this diagram.



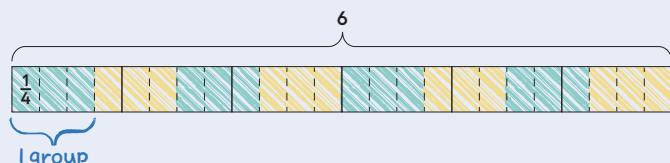
- a.** Complete the diagram to show how many  $\frac{2}{3}$ s are in 6.  
**b.** Elena says, "To find  $6 \div \frac{2}{3}$ , I can take the value of  $6 \div \frac{1}{3}$  and then either multiply it by  $\frac{1}{2}$  or divide it by 2."

Discuss with your partner why Elena's method works.

**Sample reasoning:**  $\frac{2}{3}$  is two  $\frac{1}{3}$ s. If we put two  $\frac{1}{3}$ s in a group, we would have half as many pieces as we did for  $6 \div \frac{1}{3}$ .

- 2.** Use the diagram and Elena's method to find the value of each expression. Think about how to find that value without counting all the pieces in the diagram.

**a.**  $6 \div \frac{3}{4}$



Value of the expression: 8

**Access for Students with Diverse Abilities (Activity 2, Student Task)****Action and Expression: Provide Access for Physical Action.**

Give students who need support with fine-motor skills the option of representing division of a whole number by a fraction kinesthetically on a larger scale. For example, use a row of classroom floor tiles as a tape diagram where each tile represents a fractional part, and allow students to mark equal-size groups using masking tape.

*Supports accessibility for: Fine Motor Skills, Visual-Spatial Processing*

**Student Workbook**

To find the value of  $6 \div \frac{2}{3}$ , Elena started by drawing this diagram.

a. Complete the diagram to show how many  $\frac{2}{3}$ s are in 6.  
b. Elena says, "To find  $6 \div \frac{2}{3}$ , I can take the value of  $6 \div \frac{1}{3}$  and then either multiply it by  $\frac{1}{2}$  or divide it by 2."  
Discuss with your partner why Elena's method works.

1. Use the diagram and Elena's method to find the value of each expression. Think about how to find that value without counting all the pieces in the diagram.

a.  $6 \div \frac{2}{3}$

Value of the expression: \_\_\_\_\_

b.  $6 \div \frac{3}{4}$

Value of the expression: \_\_\_\_\_

## Student Workbook

**3** Dividing by Non-unit Fractions

$6 \div \frac{4}{3}$

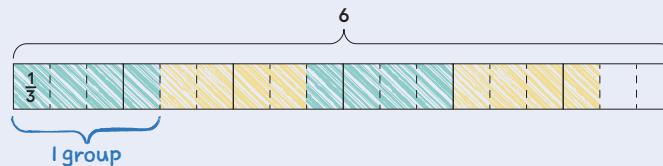
Value of the expression:

- 1 Elena noticed that she always took the same two steps to show division by a fraction on a tape diagram. She said:
- 2 First, I would partition each 1 whole into as many parts as the number in the denominator. For  $6 \div \frac{3}{4}$ , that number is 4, so the diagram would have 4 times as many parts.
- 3 Next, I would put a certain number of those parts into one group. For  $6 \div \frac{3}{4}$ , I would put 3 of the  $\frac{1}{4}$ s into each group and see how many groups there are.
- 4 Which expression represents the result of taking these two steps to find  $6 \div \frac{3}{4}$ ? Be prepared to explain your reasoning.

A  $6 \div 4 \cdot 3$   
 B  $6 \div 4 + 3$   
 C  $6 + 4 \div 3$   
 D  $6 - 4 \cdot 3$

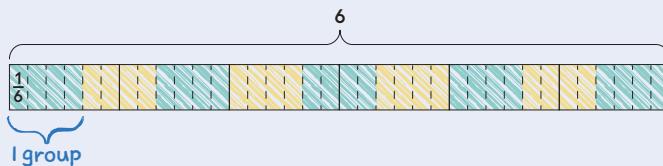
**Are You Ready for More?**  
Find the missing value.

b.  $6 \div \frac{4}{3}$



Value of the expression:  $\frac{9}{2}$  (or  $4\frac{1}{2}$ )

c.  $6 \div \frac{6}{6}$



Value of the expression:  $\frac{9}{1}$

3. Elena noticed that she always took the same two steps to show division by a fraction on a tape diagram. She said:

"First, I would partition each 1 whole into as many parts as the number in the denominator. For  $6 \div \frac{3}{4}$ , that number is 4, so the diagram would have 4 times as many parts.

Next, I would put a certain number of those parts into one group. For  $6 \div \frac{3}{4}$ , I would put 3 of the  $\frac{1}{4}$ s into each group and see how many groups there are."

Which expression represents the result of taking these two steps to find  $6 \div \frac{3}{4}$ ? Be prepared to explain your reasoning.

A.  $6 \div 4 \cdot 3$

B.  $6 \div 4 \div 3$

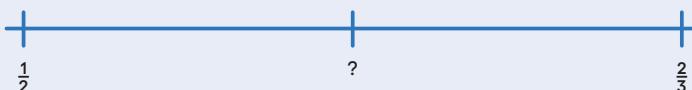
**C.  $6 \cdot 4 \div 3$**

D.  $6 \cdot 4 \cdot 3$

**Sample reasoning:** Dividing each 1 whole into 4 parts makes 4 times as many pieces in the diagram. Making every 3 of those pieces into a group makes  $\frac{1}{3}$  as many groups as there were pieces in the diagram. To get the number of groups, multiply the number of wholes—which is 6—by  $\frac{1}{3}$  or (divide it by 3).

### Are You Ready for More?

Find the missing value.

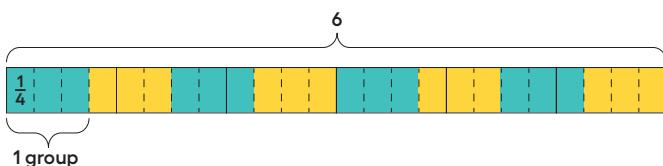
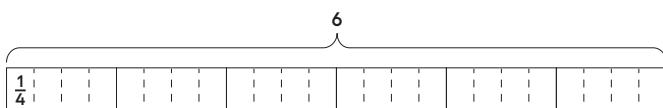
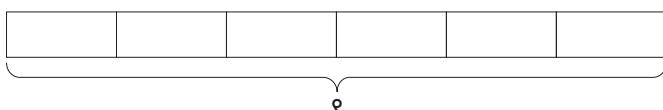


$\frac{7}{12}$

### Activity Synthesis

Focus the discussion on clarifying why Elena's two steps make sense and the expression that represents her process of finding  $6 \div \frac{3}{4}$ .

Invite previously selected students to share their tape diagram, explanation, and expression for finding  $6 \div \frac{3}{4}$ , or display the following diagrams for all to see.



Discuss questions such as:

- “Where in the diagrams can we see Elena’s first step—dividing each 1 whole into as many parts as the number in the denominator?”

Each of the 6 wholes is partitioned into 4 parts.
- “Where can we see ‘4 times as many parts?’”

Instead of having 6 wholes, there are now 24 fourths.
- “Where can we see Elena’s second step—putting a certain number of the parts into groups and seeing how many groups there are?”

Every 3 one-fourths make a group, so there are 8 groups in 24 fourths.
- “Why does  $6 \cdot 4 \div 3$  make sense?”

We multiplied 6 by 4 to get the number of fourths, and then divided it by 3 to get the number of groups containing 3 one-fourths.

### Lesson Synthesis

The key takeaway from this lesson is that we can divide a number by unit and non-unit fractions—without always relying on diagrams—by looking for and making use of structure.

Ask students questions such as:

- “What did we notice about the result of dividing a number by a unit fraction? Can you explain with an example?”

It has the same outcome as multiplying by the denominator of the fraction. Dividing by  $\frac{1}{5}$  is the same as multiplying by 5.
- “What observations did we make when dividing a number by a non-unit fraction? Can you explain with an example?”

It has the same outcome as multiplying by the denominator of the fraction and dividing by the numerator. Dividing by  $\frac{4}{6}$  is the same as multiplying by 6 and dividing by 4.

## Student Workbook

**10 Lesson Summary**

To answer the question "How many  $\frac{1}{3}$ s are in 4?" or "What is  $4 \div \frac{1}{3}$ ?", we can reason that there are 3 thirds in 1, so there are  $(4 \cdot 3)$  thirds in 4. In other words, dividing 4 by  $\frac{1}{3}$  has the same result as multiplying 4 by 3.

$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction  $\frac{1}{b}$  is the same as multiplying the number by  $b$ . How can we reason about  $4 \div \frac{2}{3}$ ? We already know that there are  $(4 \cdot 3)$  or 12 groups of  $\frac{1}{3}$ s in 4. To find how many  $\frac{2}{3}$ s are in 4, we need to put together every 2 of the  $\frac{1}{3}$ s into a group. Doing this results in half as many groups, which is 6 groups. In other words,

$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by  $\frac{a}{b}$  is the same as multiplying the number by  $b$  and then dividing by  $a$ , or multiplying the number by  $b$  and then by  $\frac{1}{a}$ .

If time permits, discuss the limits of diagrams and the benefits of looking for a pattern. Discuss questions such as:

Q **"How can we use a diagram to find the value of  $10 \div \frac{1}{25}$ ?"**

Partition each 1 whole into 25 parts and count the parts.

Q **"What about  $10 \div \frac{8}{25}$ ?"**

Put the  $\frac{1}{25}$ s into groups of 8 and count the groups.

Q **"Would you use a diagram to find such quotients? Why or why not?"**

No, it is not practical. There are too many parts to show, wholes to partition, or groups to count.

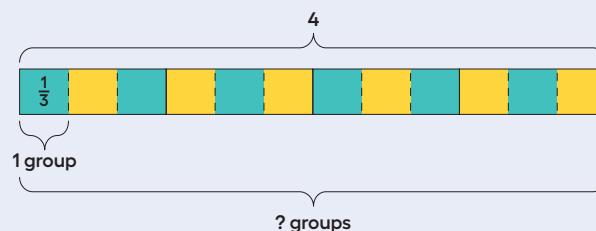
Q **"Why might it make sense to look for a pattern instead of relying on diagrams?"**

The larger the dividend or the smaller the unit fractions, the more cumbersome the diagram would be. Noticing a pattern and using it is more efficient.

## Lesson Summary

To answer the question "How many  $\frac{1}{3}$ s are in 4?" or "What is  $4 \div \frac{1}{3}$ ?", we can reason that there are 3 thirds in 1, so there are  $(4 \cdot 3)$  thirds in 4.

In other words, dividing 4 by  $\frac{1}{3}$  has the same result as multiplying 4 by 3.

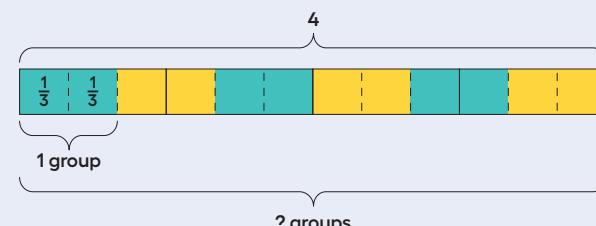


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction  $\frac{1}{b}$  is the same as multiplying the number by  $b$ .

How can we reason about  $4 \div \frac{2}{3}$ ?

We already know that there are  $(4 \cdot 3)$  or 12 groups of  $\frac{1}{3}$ s in 4. To find how many  $\frac{2}{3}$ s are in 4, we need to put together every 2 of the  $\frac{1}{3}$ s into a group. Doing this results in half as many groups, which is 6 groups. In other words,



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

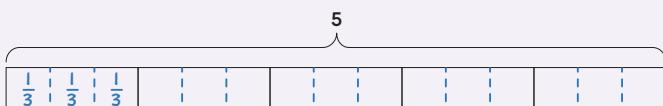
In general, dividing a number by  $\frac{a}{b}$  is the same as multiplying the number by  $b$  and then dividing by  $a$ , or multiplying the number by  $b$  and then by  $\frac{1}{a}$ .

**Cool-down****Dividing by  $\frac{1}{3}$  and  $\frac{3}{5}$** 5  
min**Student Task Statement**

1. Explain or show how you could find  $5 \div \frac{1}{3}$ . You can use this diagram if it is helpful.

**Sample reasoning:**

$5 \div \frac{1}{3}$  can mean “How many  $\frac{1}{3}$ s (thirds) are in 5?” There are 3 thirds in 1, so in 5, there are 5 times as many thirds. Five times as many is  $5 \cdot 3$ , so there are 15 thirds in 5.



2. Find  $12 \div \frac{3}{5}$ . Try not to use a diagram, if possible. Show your reasoning.

20.

**Sample reasoning:**  $12 \div \frac{3}{5} = 12 \cdot 5 \cdot \frac{1}{3} = 20$

**Responding To Student Thinking****Points to Emphasize**

If students struggle with dividing a whole number by a non-unit fraction, continue using tape diagrams as needed to support sense making. For example, ask students to create diagrams to represent and reason about the division expressions in these practice problems:

Grade 6, Unit 4, Lesson 10, Practice Problem 3

## Student Workbook

LESSON 10  
PRACTICE PROBLEMS

1 Priya is sharing 24 apples equally with some friends. She uses division to find out how many people can have a share if each person gets a particular number of apples. For example,  $24 \div 4 = 6$  means that if each person gets 4 apples, then 6 people can have apples. Here are some other calculations:

$$24 \div 4 = 6 \quad 24 \div 2 = 12 \quad 24 \div \frac{1}{2} = ?$$

a. Priya thinks the "?" represents a number less than 24. Do you agree? Explain or show your reasoning.

b. The value of  $24 \div \frac{1}{2}$  tells us how many people can enjoy the apples that Jada has if each person gets half an apple. What is that value?

2 Here is a centimeter ruler.

a. Use the ruler to find  $1 \div \frac{1}{10}$  and  $4 \div \frac{1}{10}$ .



b. What calculation did you do each time?

c. Use this pattern to find  $18 \div \frac{1}{10}$ .

GRADE 4 • UNIT 4 • SECTION C | LESSON 10

## Problem 1

Priya is sharing 24 apples equally with some friends. She uses division to find out how many people can have a share if each person gets a particular number of apples. For example,  $24 \div 4 = 6$  means that if each person gets 4 apples, then 6 people can have apples. Here are some other calculations:

$$24 \div 4 = 6$$

$$24 \div 2 = 12$$

$$24 \div 1 = 24$$

$$24 \div \frac{1}{2} = ?$$

- a. Priya thinks the "?" represents a number less than 24. Do you agree? Explain or show your reasoning.

Disagree

Sample reasoning:

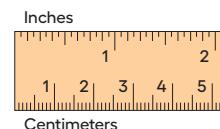
- o As the amount for each person gets smaller, more people can have apples.
- o There is a pattern in the numbers: When the number of apples per person is halved, the number of people doubles. Since 1 apple per person means 24 people can enjoy an apple, then  $\frac{1}{2}$  apple per person means 48 (twice as many) people can enjoy some apple.

- b. The value of  $24 \div \frac{1}{2}$  tells us how many people can enjoy the apples that Jada has if each person gets half an apple. What is that value?

48 people

## Problem 2

Here is a centimeter ruler.



- a. Use the ruler to find  $1 \div \frac{1}{10}$  and  $4 \div \frac{1}{10}$ .

10 and 40

- b. What calculation did you do each time?

Each time the dividend was multiplied by 10.

- c. Use this pattern to find  $18 \div \frac{1}{10}$ .

180

- d. Explain how you could find  $4 \div \frac{2}{10}$  and  $4 \div \frac{8}{10}$ .

Take the answer from  $4 \div \frac{1}{10}$  and divide it by 2 or 8, getting 20 and 5, respectively.

## Lesson 10 Practice Problems

### Problem 3

Find each quotient.

a.  $5 \div \frac{1}{10}$

50

b.  $5 \div \frac{3}{10}$

$\frac{50}{3}$  or  $16\frac{2}{3}$

c.  $5 \div \frac{9}{10}$

$\frac{50}{9}$  or  $5\frac{5}{9}$

### Problem 4

Use the fact that  $2\frac{1}{2} \div \frac{1}{8} = 20$  to find  $2\frac{1}{2} \div \frac{5}{8}$ . Explain or show your reasoning.

4

Sample reasoning: There are 20 groups of  $\frac{1}{8}$  in  $2\frac{1}{2}$ . If each group is 5 times as large, then the number of groups will need to be divided by 5.  $20 \div 5 = 4$ .

### Problem 5

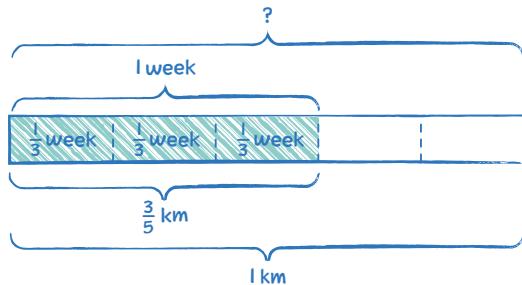
from Unit 4, Lesson 9

Consider the problem: It takes one week for a crew of workers to pave  $\frac{3}{5}$  kilometer of a road. At that rate, how long will it take to pave 1 kilometer?

Write a multiplication equation and a division equation to represent the question. Then find the answer and show your reasoning.

- $\frac{3}{5} \cdot ? = 1$  (or equivalent),  $1 \div \frac{3}{5} = ?$
- $1\frac{2}{3}$  weeks

Sample reasoning:



### Student Workbook

#### Practice Problems

- d. Explain how you could find  $4 + \frac{2}{10}$  and  $4 \div \frac{8}{10}$ .

1. Find each quotient.

a.  $5 \div \frac{1}{10}$

b.  $5 \div \frac{3}{10}$

c.  $5 \div \frac{9}{10}$

2. Use the fact that  $2\frac{1}{2} \div \frac{1}{8} = 20$  to find  $2\frac{1}{2} \div \frac{5}{8}$ . Explain or show your reasoning.

3. from Unit 4, Lesson 9

Consider the problem: It takes one week for a crew of workers to pave  $\frac{3}{5}$  kilometer of a road. At that rate, how long will it take to pave 1 kilometer? Write a multiplication equation and a division equation to represent the question. Then find the answer and show your reasoning.

## Lesson 10 Practice Problems

**Student Workbook**

**10 Practice Problems**

from Unit 4, Lesson 7

A box contains  $1\frac{3}{4}$  pounds of baking soda. Jada used  $\frac{7}{8}$  pound for a science project. What fraction of the baking soda in the box did she use? Explain or show your reasoning. Draw a diagram if you find it helpful.

$\frac{1}{2}$

**Learning Targets**

- + I can divide a number by a non-unit fraction by reasoning with the numerator and denominator, which are whole numbers.
- \* I can divide a number by a unit fraction  $\frac{1}{n}$  by reasoning with the denominator, which is a whole number.

GRADE 6 • UNIT 4 • SECTION C | LESSON 10

### Problem 6

from Unit 4, Lesson 7

A box contains  $1\frac{3}{4}$  pounds of baking soda. Jada used  $\frac{7}{8}$  pound for a science project. What fraction of the baking soda in the box did she use? Explain or show your reasoning. Draw a diagram if you find it helpful.

$$\frac{1}{2}$$

**Sample reasoning:**

- $1\frac{3}{4}$  is  $\frac{7}{4}$ , and  $\frac{7}{8}$  is half of  $\frac{7}{4}$ .
- The question can be represented with:  $? \cdot \frac{7}{4} = \frac{7}{8}$ . The “?” has to be  $\frac{1}{2}$  so that the product is  $\frac{7}{8}$ .

### Problem 7

from Unit 3, Lesson 14

Calculate each percentage mentally.

a. 25% of 400

100

b. 50% of 90

45

c. 75% of 200

150

d. 10% of 8,000

800

e. 5% of 20

1