## **Introducing Proportional Relationships with Tables**

## Goals

- Comprehend that the phrase "proportional relationship" (in spoken and written language) refers to when two quantities are related by multiplying by a "constant of proportionality."
- Describe (orally and in writing) relationships between rows or between columns in a table that represents a proportional relationship.
- Explain (orally) how to calculate missing values in a table that represents a proportional relationship.

## **Learning Targets**

- I can use a table to reason about two quantities that are in a proportional relationship.
- I understand the terms proportional relationship and constant of proportionality.

#### **Access for Students with Diverse Abilities**

• Engagement (Activity 1, Activity 2)

#### **Access for Multilingual Learners**

- MLR6: Three Reads (Activity 2)
- MLR8: Discussion Supports (Activity 3)

#### **Instructional Routines**

- 5 Practices
- MLR6: Three Reads
- · Notice and Wonder

#### **Required Materials**

#### Materials to Gather

- · Measuring cup: Activity 2
- Measuring spoons: Activity 2

## **Lesson Narrative**

The purpose of this lesson is to introduce the concept of a **proportional relationship** by looking at tables of equivalent ratios. Students learn that all entries in one column of the table can be obtained by multiplying entries in the other column by the same number. This number is called the constant of proportionality. The activities use contexts that make using the constant of proportionality the more convenient approach, rather than reasoning about equivalent ratios.

The activities encourage students to look for and make use of structure in a proportional relationship. The last activity is optional because it provides an opportunity for additional practice with a new context.

#### **Lesson Timeline**

**Warm Up** 

15

**Activity 1** 

10

**Activity 2** 

10

**Activity 3** 

10

**Lesson Synthesis** 

**Assessment** 

**Cool Down** 

## **Introducing Proportional Relationships with Tables**

## **Lesson Narrative (continued)**

In these materials, when we say "b is proportional to a" we usually put a in the left hand column and b in the right hand column, so that multiplication by the constant of proportionality always goes from left to right. This is not a hard and fast rule, but it prepares students for later work on functions, where they will think of x as the independent variable and y as the dependent variable.

## Student Learning Goal

Let's solve problems involving proportional relationships using tables.

### Warm-up

## Notice and Wonder: Paper Towels by the Case



#### **Activity Narrative**

The purpose of this *Warm-up* is to elicit the idea of using a table to see patterns between related quantities, which will be useful when students discuss how to use tables to analyze proportional relationships in a later activity. While students may notice and wonder many things about these images, the relationships between the rows and the relationship between the columns are the important discussion points.

This prompt gives students opportunities to see and make use of structure. The specific structures they might notice are:

- Scale factors between any pair of rows (In other words, multiplying both values in one row by the same number gives the values in another row.)
- A unit rate between the two columns (For example, multiplying any value in the first column by 12 gives the corresponding number in the second column.)

## Launch 🙎

Arrange students in groups of 2. Display the table for all to see and read the problem stem aloud. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

## **Student Task Statement**

Here is a table that shows how many rolls of paper towels a store receives when they order different numbers of cases.

What do you notice? What do you wonder?

	number of cases they order	number of rolls of paper towel	
	1	12	
	3	36	
.2	5	60	
4	10	120	

## Students may notice:

- To go from one row to another, multiply both columns by the same number.
- To find the number of rolls, multiply the number of cases by 12.
- There are 12 rolls in a case.

#### Students may wonder:

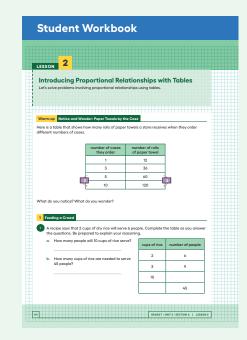
- How much does a case cost?
- · How many paper towels are on a roll?
- · Why would you need 120 rolls of paper towels?

#### **Instructional Routines**

# Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.





#### **Instructional Routines**

#### **5 Practices**

## ilclass.com/r/10690701

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## **Activity Synthesis**

Ask students to share the things they noticed and wondered. Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the relationship between the number of cases and the number of paper towels does not come up during the conversation, ask students to discuss this idea.

## **Activity 1**

#### Feeding a Crowd



## **Activity Narrative**

The purpose of this task is to introduce students to the idea of a proportional relationship. Students examine two different tables of equivalent ratios and use them to solve problems. In the *Activity Synthesis* they learn that there is a proportional relationship between two quantities when the quantities are characterized by a set of equivalent ratios.

The contexts and numbers used in this activity are intended to be accessible to all students. This way they can focus on its mathematical structure and the new terms introduced in the lesson without being distracted.

Monitor for groups who use these different strategies to complete the first table:

- Create a drawing or diagram that depicts 15 cups of rice and 45 people, organized into 3 people per cup
- Use a scale factor to scale from one row of the table to another (For example, since  $2 \cdot 5 = 10$ , then we can do  $6 \cdot 5$  to get 30.)
- Calculate and use a unit rate (In other words, one cup of rice serves 3 people, so 10 cups of rice must serve 30 people.)
- Apply the constant relationship between the columns of the table (In other words, multiply the number in the left column by 3 to get the number in the right column.)

Plan for students to present in this order, from concrete to more abstract.

### Launch

Explain that this task looks at two food items: rice and spring rolls. Say, "Rice is a big part of the traditions and cultures of many families. Does your family cook rice, and if so, how?" Invite a student to describe the process (measure rice, measure water, simmer for a while). You use more rice for more people and less rice for fewer people. If students have trouble understanding or representing the context, show them the measuring cup so that they have a sense of its size, or draw a literal diagram that looks something like this:



Similarly, ask students if they have ever eaten a spring roll and invite them to describe what they are. While some spring rolls can be very large, the ones referred to in this activity are smaller.

Keep students in the same groups.

Give students 5–6 minutes of partner work time. Select students with different strategies for completing the first table (such as those described in the *Activity Narrative*) to share later.

#### **Student Task Statement**

- 1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.
  - a. How many people will 10 cups of rice serve?

30

b. How many cups of rice are needed to serve 45 people?

15

cups of rice	number of people
2	6
3	9
10	30
15	45

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.

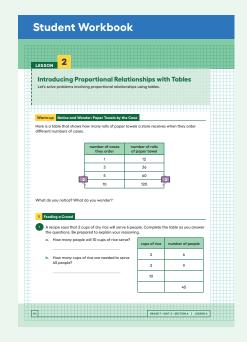
number of spring rolls	number of people
6	3
30	15
40	20
56	28

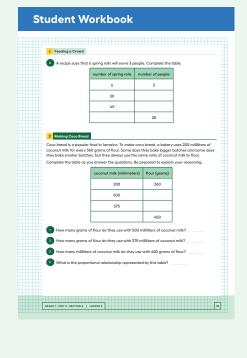
## Access for Students with Diverse Abilities (Activity 1, Launch)

## Engagement: Provide Access by Recruiting Interest.

Invite students to share their experience of making rice at home. How does their family cook rice? Based on their experience, about how many people does one cup of dry rice feed?

Supports accessibility for: Conceptual Processing, Memory





## Access for Multilingual Learners (Activity 2)

#### **MLR6: Three Reads**

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

#### **Instructional Routines**

# MLR6: Three Reads ilclass.com/r/10695568

Please log in to the site before using the QR code or URL.



## **Activity Synthesis**

The purpose of this discussion is to introduce the concept of a proportional relationship. Ask previously selected groups to share their reasoning for completing the first table. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

 $\bigcirc$  "Why do the different approaches lead to the same outcome?"

"What do you notice about the columns in the completed table?"

"How does the unit rate (people per cup of rice) show up in each method?"

After students have shared their reasoning, introduce the term **proportional relationship**. For example, say, "Whenever we have a situation like this where two quantities are always in the same ratio, we say there is a proportional relationship between the quantities." Display these statements for all to see:

"The relationship between the number of cups of rice and the number of people is a proportional relationship."

"The number of people is proportional to the number of cups of rice."

"There are 3 people for every 1 cup of rice."

Next, ask students to work with their partner to write a sentence that describes the proportional relationship in the second table. Then, invite students to share their sentences. Record and display their sentences for all to see.

#### Sample responses:

- The relationship between the number of spring rolls and the number of people is a proportional relationship.
- The number of people is proportional to the number of spring rolls.
- There are 2 spring rolls for each person.
- There are 0.5 people per spring roll. (The unit rate 0.5 people per spring roll may sound strange, but it means that 1 spring roll only halfway satisfies a person.)

#### **Activity 2**

### **Making Coco Bread**



#### **Activity Narrative**

The purpose of this activity is to introduce the term constant of proportionality. Students continue to find missing values in a table representing a proportional relationship. However, the numbers are purposefully chosen so that it is difficult to identify scale factors that scale one row to another. This encourages students to look for a unit rate relationship between the columns.

This is the first time Math Language Routine 6: *Three Reads* is suggested in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus.

During the first read, students focus on comprehending the situation. During the second read, students identify quantities. During the third read, the final prompt is revealed and students brainstorm possible starting points for answering the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing down a solution path. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

#### Launch

Keep students in the same groups. Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and without revealing the questions.

• In the first read, students read the problem with the goal of comprehending the situation.

For the first read, read the problem aloud while everyone else reads along, and then ask.

○ "What is this situation about? What is going on here?"

Allow 1 minute to discuss with a partner and then share with the whole class. A typical response may be,

"A bakery uses a recipe to make bread. The recipe includes coconut milk and flour."

Listen for and clarify any questions about the context.

• In the second read, students analyze the mathematical structure of the story by naming quantities.

Invite students to read the problem aloud with their partner, or select a student to read to the class, then prompt students by asking,

"What can be counted or measured in this situation?"

Give students 30 seconds of quiet think time, followed by another 30 seconds to share with their partner. A typical response may be:

"milliliters of coconut milk; cups of flour; size of the batches."

• In the third read, students brainstorm possible starting points for answering the questions.

Invite students to read the problem aloud with their partner, or select a different student to read to the class. After the third read, reveal the first question on finding the amount of flour to mix with 1,000 milliliters of coconut milk and ask,

"What are some ways we might get started on this?"

Instruct students to think of ways to approach the questions without actually solving.

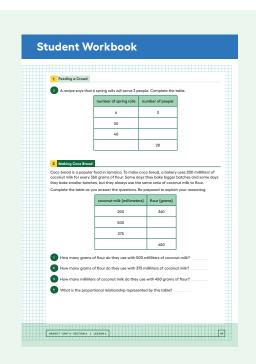
Give students 1 minute of quiet think time followed by another **minute** to discuss with their partner. Invite students to name some possible strategies referencing quantities from the second read.

Provide these sentence frames as partners discuss:

"To figure out how much flour is needed for a given amount of coconut milk ..."

"One way a diagram could help is ..."

"To calculate the unit rate we can ..."



As partners are discussing their solution strategies, select 1–2 students to share their ideas with the whole class. As students are presenting their strategies to the whole class, create a display that summarizes starting points for each question. (Stop students as needed before they share complete solutions or answers.)

Give students time to complete the rest of the activity followed by a wholeclass discussion.

## **Student Task Statement**

Coco bread is a popular food in Jamaica. To make coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

coconut milk (millimeters)	flour (grams)
200	360
500	
375	
	450

Complete the table as you answer the questions. Be prepared to explain your reasoning.

- How many grams of flour do they use with 500 milliliters of coconut milk?
   900 grams of flour
- 2. How many grams of flour do they use with 375 milliliters of coconut milk?
  675 grams of flour
- 3. How many milliliters of coconut milk do they use with 450 grams of flour?
  250 milliliters of coconut milk
- **4.** What is the proportional relationship represented by this table?

## Sample responses:

- The relationship between the milliliters of coconut milk and the grams of flour is proportional.
- The relationship between the amount of coconut milk and the amount of flour is proportional.
- The table represents a proportional relationship between the amount of coconut milk and the amount of flour.
- · The amount of flour is proportional to the amount of coconut milk.
- The bakery uses 1.8 grams of flour for every I milliliter of coconut milk.

### **Activity Synthesis**

The key takeaway from this discussion is that the constant of proportionality for this proportional relationship is 1.8.

Invite students to share their strategies for completing the table, especially the row with 375 milliliters of coconut milk. Highlight strategies that make use of the relationship between the columns of the table, that is, any value in the first column can be multiplied by 1.8 to get the corresponding value in the second column. Tell students that 1.8 is the constant of proportionality for this proportional relationship. Note that, like unit rate, the constant of proportionality can always be determined by finding how much of the second quantity there is per one of the first quantity.

Ask students to interpret the constant of proportionality in the context:

"What does the 1.8 tell us about the situation?"

There are 1.8 grams of flour per milliliter of coconut milk.

If time permits, consider inviting students to share their strategies for the question about the amount of coconut milk to use with 450 grams of flour. Possible strategies include:

- Multiplying the values in the second row of the table by the scale factor  $\frac{1}{2}$
- Dividing the given amount of flour by the constant of proportionality, 1.8

Highlight the structure that dividing any value in the second column by the constant of proportionality gives the corresponding value in the first column.

## **Activity 3: Optional**

## **Quarters and Dimes**

10 min

### **Activity Narrative**

The purpose of this activity is to practice finding and using the constant of proportionality in another context.

#### Launch

If needed, remind students of the value of a quarter and a dime before starting the activity.

#### **Student Task Statement**

4 quarters are equal in value to 10 dimes.

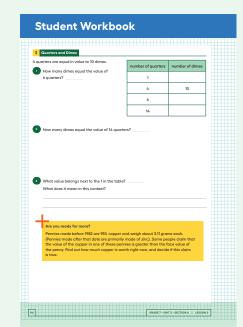
number of quarters	number of dimes
1	2.5
4	10
6	15
14	35

## Access for Students with Diverse Abilities (Activity 2, Synthesis)

## Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation such as: "First, I\_\_\_\_\_ because ..." "I noticed\_\_\_\_, so I ..." "Why did you ... ?" or "I agree/ disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning



## Access for Multilingual Learners (Activity 3, Synthesis)

#### MLR8: Discussion Supports.

Display sentence frames to support whole-class discussion: "\_\_\_\_\_ is equal in value to \_\_\_\_\_, because ..."

Advances: Speaking, Representing

1. How many dimes equal the value of 6 quarters?

15

2. How many dimes equal the value of 14 quarters?

35

**3.** What value belongs next to the 1 in the table? What does it mean in this context?

2.5, which means that 2.5 dimes are worth the same amount as I quarter

## **Are You Ready for More?**

Pennies made before 1982 are 95% copper and weigh about 3.11 grams each. (Pennies made after that date are primarily made of zinc). Some people claim that the value of the copper in one of these pennies is greater than the face value of the penny. Find out how much copper is worth right now, and decide if this claim is true.

The cost of copper fluctuates, so the answer depends on the current value of copper. Assuming that copper costs about \$5.00 per kg, the copper in a penny would be worth about  $0.95 \cdot 0.00311 \cdot 5 \approx 0.015$  or about 1.5 cents. As long as copper is worth at least \$3.39 per kilogram, then the copper in a pre-1982 penny will be worth more than I cent.

## **Activity Synthesis**

Ask students for the value that belongs next to the 1 in the table. Invite several students to explain the significance of this number.

#### **Lesson Synthesis**

Share with students,

"Today we looked at examples of proportional relationships, which are situations that are characterized by equivalent ratios. We found unknown values in the table."

Briefly revisit the two activities, demonstrating the use of the new terms. It would be helpful to display a completed table for each activity to facilitate the conversation. Consider asking students:

☐ "In the first activity, what were the proportional relationships?"

The number of people served was proportional to the amount of rice or the number of spring rolls.

"What was the constant of proportionality in each situation?"

3 for the rice, and  $\frac{1}{2}$  for the spring rolls

 $\bigcirc$  "In the second activity, what was the proportional relationship?"

The amount of flour was proportional to the amount of coconut milk.

"What was the constant of proportionality in that situation?"

1.8

## **Lesson Summary**

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**.

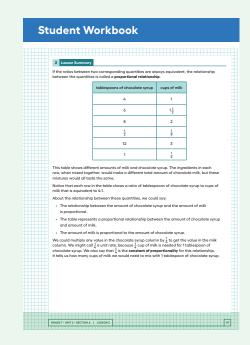
tablespoons of chocolate syrup	cups of milk
4	1
6	1 ½
8	2
1/2	<u>1</u> 8
12	3
1	1/4

This table shows different amounts of milk and chocolate syrup. The ingredients in each row, when mixed together, would make a different total amount of chocolate milk, but these mixtures would all taste the same.

Notice that each row in the table shows a ratio of tablespoons of chocolate syrup to cups of milk that is equivalent to 4:1.

About the relationship between these quantities, we could say:

- The relationship between the amount of chocolate syrup and the amount of milk is proportional.
- The table represents a proportional relationship between the amount of chocolate syrup and amount of milk.
- The amount of milk is proportional to the amount of chocolate syrup. We could multiply any value in the chocolate syrup column by  $\frac{1}{4}$  to get the value in the milk column. We might call  $\frac{1}{4}$  a *unit rate*, because  $\frac{1}{4}$  cup of milk is needed for 1 tablespoon of chocolate syrup. We also say that  $\frac{1}{4}$  is the **constant of proportionality** for this relationship. It tells us how many cups of milk we would need to mix with 1 tablespoon of chocolate syrup.



Cool-down

## **Responding To Student Thinking**

#### **More Chances**

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

#### Cool-down

#### **Green Paint**



#### **Student Task Statement**

When you mix two colors of paint in equivalent ratios, the resulting color is always the same. Complete the table as you answer the questions.

cups of blue paint	cups of yellow paint
2	10
1	5
3	15

**1.** How many cups of yellow paint should you mix with 1 cup of blue paint to make the same shade of green? Explain or show your reasoning.

You need 5 cups of yellow paint for I cup of blue paint.

You can see this by multiplying the first row by a factor of  $\frac{1}{2}$ . Alternatively, you have to multiply 2 by  $\frac{10}{2}$  = 5 to get 10. Multiplying I by 5 gives 5.

**2.** Make up a new pair of numbers that would make the same shade of green. Explain how you know they would make the same shade of green.

Any amounts equivalent to the ratio of 1 cup of blue paint to 5 cups of yellow paint.

Sample response: 3 cups of blue paint mixed with 15 cups of yellow paint will also make the same shade of green. This can be obtained by multiplying the second row by a factor of 3 or choosing 3 for blue and then multiplying that by 5.

- 3. What is the proportional relationship represented by this table?

  The relationship between the amount of blue paint and the amount of yellow paint is the proportional relationship represented by this table.
- 4. What is the constant of proportionality? What does it represent?
  The constant of proportionality is 5. It represents the cups of yellow

paint needed for I cup of blue paint.

#### **Practice Problems**

5 Problems

#### **Problem 1**

When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes. Use the information in the table to complete the statements. Some terms are used more than once.

cups of milk	tablespoons of chocolate syrup	
2	3	. /
8	12	7
1	<u>3</u> 2	
10	15	

- **a.** The table shows a proportional relationship between cups of milk and tablespoons of chocolate syrup.
- b. The scale factor shown is 4.
- **c.** The constant of proportionality for this relationship is  $\frac{3}{2}$ .
- d. The units for the constant of proportionality are tablespoons of chocolate syrup per cup of milk.

**Bank of Terms:** tablespoons of chocolate syrup, 4, cups of milk, cup of milk,  $\frac{3}{2}$ 

#### Problem 2

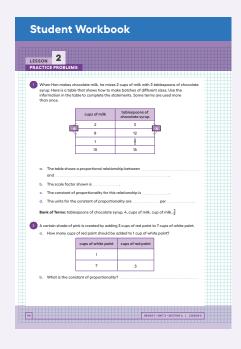
A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

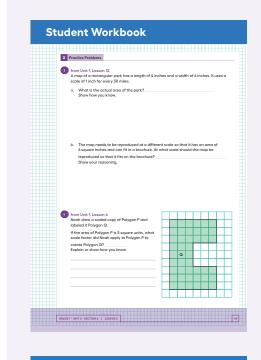
a. How many cups of red paint should be added to 1 cup of white paint?

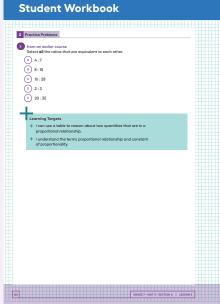
cups of white paint	cups of red paint
1	<u>3</u> 7
7	3

**b.** What is the constant of proportionality?

3







#### Problem 3

From Unit 1, Lesson 12

A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

a. What is the actual area of the park? 21,600 square miles Show how you know.

Sample reasoning: The area on the map is 24 square inches. I square incherences 100 square miles, since  $30 \cdot 30 = 90$ . The actual area is 24 · 900, which equals 21,600 square miles.

b. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? I inch to 60 miles
Show your reasoning.

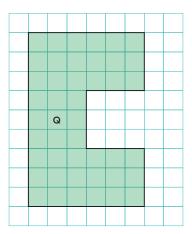
#### Sample reasonings:

- If 21,600 square miles need to be represented by 6 square inches, each square inches needs to represent 3,600 square miles: 21,600  $\div$  6 = 3,600. This means each 1-inch side of the square needs to be 60 miles.
- The area of this new map is  $\frac{1}{4}$  of the first map, since 6 is  $\frac{1}{4}$  of 24. This means each I inch square in this map has to represent 4 times as much area as in the first map.  $900 \cdot 4 = 3$ , 600. If each square inch represents 3,600 square miles, every I inch represents 60 miles.

## Problem 4

from Unit 1, Lesson 6

Noah drew a scaled copy of Polygon P and labeled it Polygon Q.



If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? The area of polygon Q is 45 square units, so the area has scaled by a factor of 1, since 5 · 1 = 45.

Explain or show how you know.

Since the area of a scaled copy varies from the original area by the square of the scale factor, the scale factor is 3.

## Problem 5

from an earlier course

Select all the ratios that are equivalent to each other.

**A.** 4:7

**B.** 8:15

**C.** 16:28

**D.** 2:3

**E.** 20:35