Lesson 8

Scale Drawings and Maps (Optional)

Goals

- Justify (orally and in writing) which of two objects was moving faster.
- Use a scale drawing to estimate the distance an object traveled, as well as its speed or elapsed time, and explain (orally and in writing) the solution method.

Learning Target

I can use a map and its scale to solve problems about traveling.

Lesson Narrative

This lesson is optional. In this lesson, students apply what they have learned about scale drawings to solve problems involving constant speed. Students are given a map with scale as well as a starting and ending point. In addition, either they are given the time the trip takes and are asked to estimate the speed, or they are given the speed and asked to estimate how long the trip takes. In both cases, they need to make strategic use of the map and scale, and they will need to estimate distances because the roads are not straight.

This lesson builds on grade 6 work involving travel at a constant speed. For example, if a car travels at 30 mph, there is a ratio between the time of travel and the distance traveled. This can be represented in a ratio table, or on a graph, or with an equation. If d is the distance traveled in miles, and t is the amount of time in hours, then traveling at 30 mph can be represented by the equation $d = 30t$. Students may or may not use this representation as they work on the activities in this lesson. The work in this lesson can help prepare students for studying proportional relationships in a later unit.

Student Learning Goal

Let's use scale drawings to solve problems.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- 5 Practices
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Required Preparation

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	10 min	5 min

Warm-up

A Train and a Car

5 min

Activity Narrative

This *Warm-up* serves two purposes. It refreshes the concept of distance, rate, and time of travel from grade 6, preparing students to use scale drawings to solve speed-related problems. It also allows students to estimate decimal calculations.

Students are likely to approach the question in a few different ways. Monitor for students who:

- Estimate the speed of the train in miles per hour and compare this to the speed of the car.
- Find the distance the car travels in 4 hr and compare this to the distance the train travels.

Launch

Give students 3 minutes of quiet think time. Ask students to give a signal when they have an answer and explanation. Follow with a whole-class discussion.

Student Task Statement

Two cities are 243 mi apart.

- It takes a train 4 hr to travel between the two cities at a constant speed.
- A car travels between the two cities at a constant speed of 65 miles per hour.

Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

The car is traveling faster. Sample reasoning: The speed of the train in miles per hour is $243 \div 4$. This is $(240 \div 4) + (3 \div 4) = 60\frac{3}{4}$, and that's slower than the car. Alternatively, in 4 hr, the car would travel $4 \cdot 65$ or 260 mi, and that's farther than the distance between the cities. So again, the conclusion is that the car is traveling faster.

Activity Synthesis

Invite students to share their strategies. Make sure to highlight different strategies, such as calculating the train's speed and calculating how far the car would travel in 4 hr.

Record and display student explanations for all to see. To involve more students in the conversation, consider asking:

- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*

Student Workbook


LESSON 8

Scale Drawings and Maps

Let's use scale drawings to solve problems.

Warm-up A Train and a Car

Two cities are 243 mi apart.

- It takes a train 4 hr to travel between the two cities at a constant speed.
- A car travels between the two cities at a constant speed of 65 miles per hour.

Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Activity 1

Biking through Kansas

10 min

Activity Narrative

In this activity, students use a map with a graphic scale to find the distance of travel and then calculate the amount of time a trip will take. As students think about how to use the graphic scale to get the information needed to solve the problem, they are reasoning quantitatively and abstractly and choosing tools strategically.

Monitor for students who use these approaches to determine the distance between the two cities:

- Copy the graphic scale onto another paper and use it to mark 4-mile increments along the route
- Measure the straight-line distance between the cities with a ruler
- Measure the route with a ruler, adjusting the angle of the ruler as they go
- Use a flexible tool (for example, a strip of paper or a piece of string) to approximate the route and then straighten out the tool to measure the route's distance with a ruler

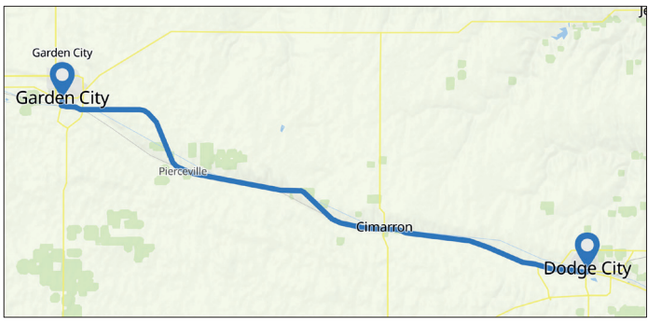
Launch

Tell students that they will now use a scale drawing (a map) to solve a problem about biking a long distance. Ask students what is the farthest they have ever biked. How long did it take? Do they know someone who has biked farther or for longer? If so, how far and how long?

Keep students in the same groups. Give students 4–5 minutes of quiet work time followed by partner and whole-class discussion.

Student Task Statement

A cyclist rides at a constant speed of 15 miles per hour. At this speed, about how long would it take the cyclist to ride from Garden City to Dodge City, Kansas?



Anywhere from 3 hr 10 min to 3 hr 30 min is reasonable. Sample responses:

- Using the scale, Dodge City appears to be about 50 mi from Garden City. In 3 hr, the cyclist would ride 45 mi, and the remaining 5 mi would take $\frac{1}{3}$ of an hour or 20 minutes. It would take the cyclist about 3 hr and 20 min.
- 15 mph is 15 mi in 60 min or 1 mi every 4 min. So 4 mi takes 16 min. The (4-mi) scale fits a little more than 12 times, so that means the trip will take a little more than 12 · 16 minutes. That's 192 minutes or 3 hours and 12 minutes.

Are You Ready for More?

Jada finds a map that says, "Note: This map is not to scale." What do you think this means? Why is this information important?

Sample response: It means that there is no one scale factor that relates distances on the map to distances in the place represented by the map. Some distances are distorted. If Jada were using her map to calculate how long it would take her to travel from one point to another on the map, her prediction may be inaccurate.

Activity Synthesis

The goal of this discussion is to highlight different strategies for estimating the distance between the two cities and dealing with the fact that the road is not straight. Invite students to share how they estimated the distance between the two cities (and how long it takes the cyclist to travel this distance). Ask students to consider the different distances that students estimated the trip to be. What are some reasons for the differences? Possible explanations include:

- There is error inherent in the process of measurement.
- The road is not straight and so distance needs to be approximated.
- For students who lay out the scale over and over again to cover the distance, it is difficult to estimate the fraction of the scale at the last step.

Because of these different sources of inaccuracy, reporting the distance as 50 mi is reasonable; reporting it as 52 mi would require a lot of time and measurements; reporting it as 51.6 mi is not reasonable with the given scale and map.

Building on Student Thinking

The road from Garden City to Dodge City has many twists and bends. Students may not be sure how to treat these. Tell them to make their best estimate. Measuring many small segments of the road will have the advantage that those short segments are straight but it is time consuming. A good estimate will be sufficient here.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect. After several strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask:

"Why do the different approaches lead to different outcomes?"
"Are there any benefits or drawbacks to one approach compared to another?"
"How was the scale used in each approach?"

Advances: Representing, Conversing

Student Workbook

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, draw students' attention to the warm-up and remind them how they calculated the speed of the train in miles per hour. Ask students how they can use this method to calculate the speed of the driver from Point A to Point B.

Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Instructional Routines

5 Practices

ilclass.com/r/10690701

Please log in to the site before using the QR code or URL.

Activity 2
Driving on I-90

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use a map with a graphic scale to find the distance of travel and then calculate the speed of travel. They compare two different trips and determine which vehicle was traveling faster. Minimal scaffolding is given here, allowing students to persevere in problem solving.

There are several different approaches that students could use to compare the speed of the car with the speed of the helicopter. Monitor for students who:

- Compare the *time* it would take each vehicle to travel the same distance.
- Compare the *distance* each vehicle could travel in the same amount of time.
- Compare the *speed* of each vehicle in *miles per minute*.
- Compare the *speed* of each vehicle in *miles per hour*.

Plan to have students present their strategies in order from most common to least common.

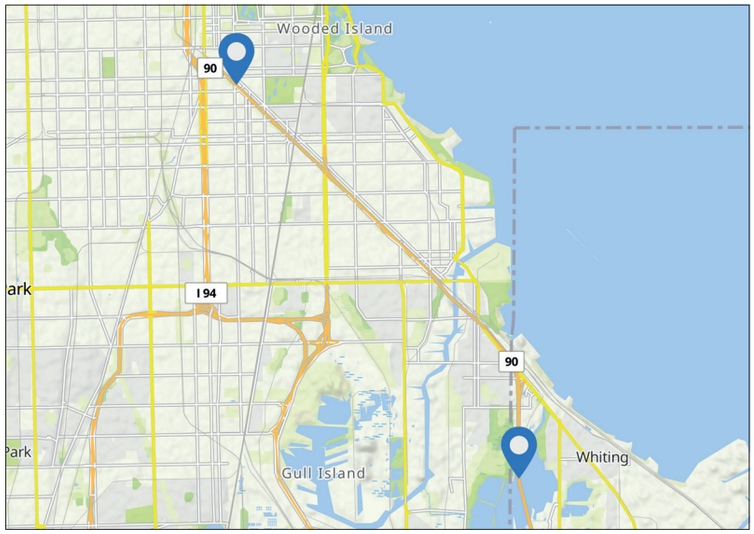
In the digital version of the activity, students use an applet to measure a map with a graphic scale. The applet allows students to add points and line segments and to measure distances. The digital version may help students measure quickly and accurately so they can focus more on the mathematical analysis.

Launch

Tell students that they will now use a scale drawing (a map) to solve a problem about speed of travel. Arrange students in groups of 2 and provide access to geometry toolkits. Give students 6–7 minutes to work on the problem either individually or with their partner.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement



1. A driver is traveling at a constant speed on Interstate 90 outside Chicago. If she traveled from Point A to Point B in 10 minutes, at what speed was she driving? Explain your reasoning.

   51 miles per hour (or equivalent)

   Sample reasoning: The distance from A to B along the road is about 8.5 times the length of the segment representing 1 mile, so the distance is about 8.5 miles. There are 6 sets of 10 minutes in 1 hour. $8.5 \cdot 6 = 51$, so she was traveling about 51 miles per hour.

2. A helicopter flew directly from Point A to Point B in 9 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.

   The helicopter traveled faster than the driver.

   Possible strategies:

   - The helicopter traveled about 8 mi in 9 minutes. At this rate, how long would it take the helicopter to travel 8.5 mi?

distance (miles)	time (minutes)
8	9
1	1.125
8.5	?

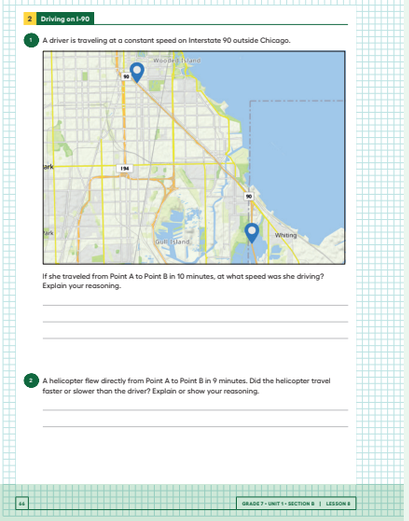
÷ 8, · 8.5 (between rows 1 and 2); ÷ 8, · 8.5 (between rows 2 and 3)

   It would take the helicopter 9.5625 minutes to travel 8.5 mi. This is less than the 10 minutes it took the driver, so the helicopter is traveling faster.

Building on Student Thinking

Students may think that the car drove in a straight line from A to B. Point out the path of the interstate on the map. Consider asking whether this path is longer or shorter than the straight line distance.

Student Workbook


2 Driving on I-90

1. A driver is traveling at a constant speed on Interstate 90 outside Chicago.

If she traveled from Point A to Point B in 10 minutes, at what speed was she driving? Explain your reasoning.

2. A helicopter flew directly from Point A to Point B in 9 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.

- At these rates, how far could each vehicle travel in 90 minutes?

distance (miles)	time (minutes)
8	9
?	90

distance (miles)	time (minutes)
8.5	10
?	90

The helicopter could travel 80 mi in 90 minutes. The car could travel 76.5 mi in 90 minutes. The helicopter travels a farther distance, so it is faster.

- The helicopter traveled about 8 mi in 9 minutes. How fast is this in miles per hour?

distance (miles)	time (minutes)
8	9
?	60

$\cdot 6\frac{2}{3}$ $\cdot 6\frac{2}{3}$

The helicopter was traveling $53\frac{1}{3}$ miles per hour, which is faster than the car's 51 miles per hour.

Activity Synthesis

The purpose of this discussion is to remind students of various strategies for reasoning about situations involving constant speed. First, ask students what distance the driver traveled from Point A to Point B. Make sure students are in agreement that this distance is about 8.5 mi before continuing the discussion.

Invite previously selected students to share their method for comparing the speeds of the car and helicopter. Sequence the discussion of the strategies in order from most common to least common. (It is not necessary to demonstrate every possible method. The goal is for students to see several different ways to apply ratio and rate reasoning to analyze this problem.)

After each student shares, consider organizing their reasoning into a table of equivalent ratios, displayed for all to see. This will make it easier for students to see connections between the various methods. Connect the different responses to the learning goals by asking questions such as:

- *"What do the approaches have in common? How are they different?"*
- *"Did this method compare the distance, speed, or elapsed time for the two vehicles?"*
- *"What role did scaling up (multiplication) play in each approach?"*

The main takeaway is that to determine which of two objects is moving faster, you can identify which one travels more distance in the same amount of time or which one travels the same distance in less time.

Lesson Synthesis

Share with students

- *"Today we saw that we can use a map with a scale to estimate the actual distance between two places."*

To review the different types of problems that we can solve based on a map, consider asking students:

- *"Once we know the distance between two places, how can we calculate how long the trip would take?"*

  We can divide the distance by the average speed to find the total time.

- *"If we know how long the trip took, how can we determine the average speed?"*

  We can divide the distance by the total time to find the average speed.

If desired, use this example to review these concepts:

- *"Two cities are 130 mi apart. At a constant speed the trip takes two hours. What was the speed?"*

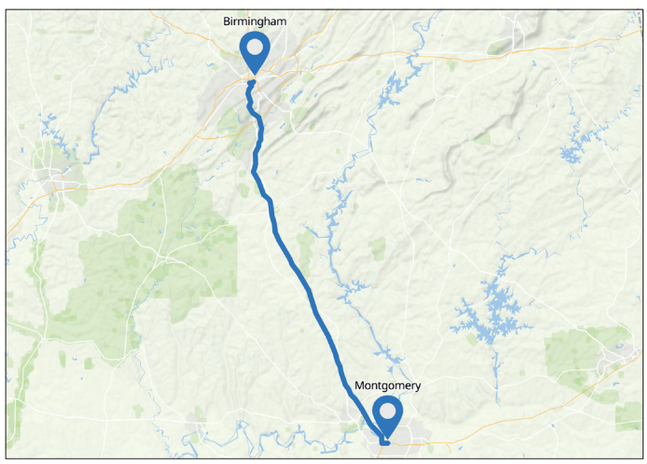
  65 miles per hour, because 130 ÷ 2 = 65

- *"Two locations are 35 mi apart. At 70 miles per hour, how long does the trip take?"*

  $\frac{1}{2}$ hour, because 35 ÷ 70 = $\frac{1}{2}$

Lesson Summary

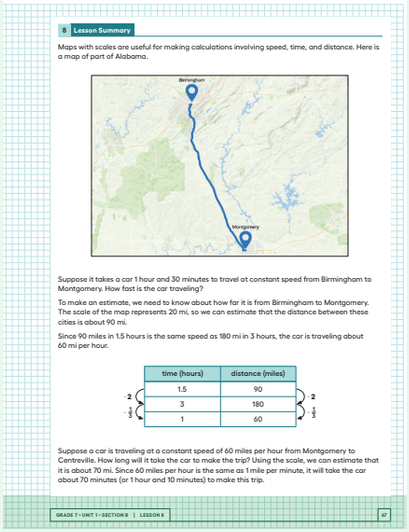
Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.



Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 mi, so we can estimate that the distance between these cities is about 90 mi.

Student Workbook


8 Lesson Summary

Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.

Birmingham
Montgomery

Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 mi, so we can estimate that the distance between these cities is about 90 mi.

Since 90 miles in 1.5 hours is the same speed as 180 mi in 3 hours, the car is traveling about 60 mi per hour.

time (hours)	distance (miles)
1.5	90
3	180
1	60

×2 ×2 ÷2 ÷2

Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 mi. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.

GRADE 7 • UNIT 1 • SECTION B | LESSON 8 67

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of using scale drawings to find actual distances. If students struggle, make time to revisit related work in the activities referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 1, Lesson 7, Activity 2 Tall Structures

Unit 1, Lesson 8 Scale Drawings and Maps

Since 90 miles in 1.5 hours is the same speed as 180 mi in 3 hours, the car is traveling about 60 mi per hour.

time (hours)	distance (miles)
1.5	90
3	180
1	60

(·2, ·$\frac{1}{3}$ between rows; ·2, ·$\frac{1}{3}$ between rows)

Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 mi. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.

Cool-down

Walking Around the Botanical Garden

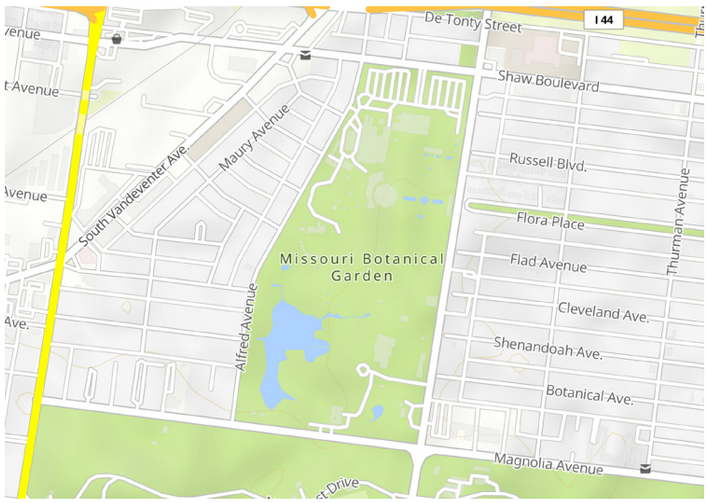
5 min

Launch

Provide access to geometry toolkits. Make sure students know where the boundaries of the Botanical Garden are on the map.

Student Task Statement

Here is a map of the Missouri Botanical Garden. Clare walked all the way around the garden.



1. What is the actual distance around the garden? Show your reasoning.

   It takes about 14 segments of the scale to measure the perimeter of the garden, and 14 · 600 = 8,400. So the distance around is about 8,400 feet.

2. It took Clare 30 minutes to walk around the garden at a constant speed. At what speed was she walking? Show your reasoning.

   If she walks for 30 minutes, that means she was traveling at about 280 feet per minute (8,400 ÷ 30 = 280), or about 16,800 feet per hour (280 · 60 ≈ 16,800).

Lesson 8

Practice Problems

4 Problems

Problem 1

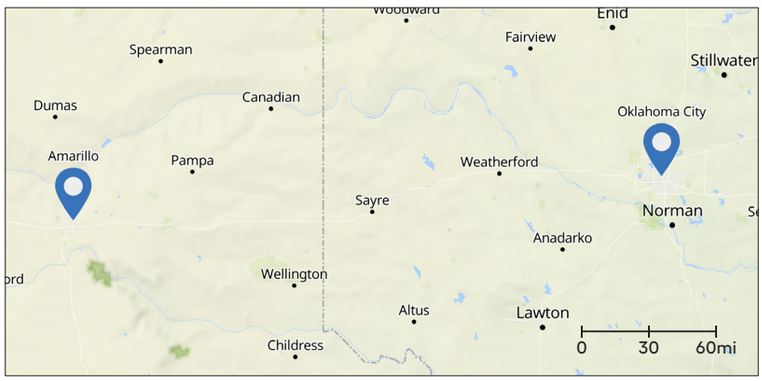
a. A whale swims at a constant speed of 4 meters per second. How far does it travel in 40 seconds? 160 meters

b. A horse runs at a constant speed of 5 meters per second. How much time does it take for the horse to travel 50 meters? 10 seconds

c. A goose flies at a constant speed, traveling 201 meters in 15 seconds. What is its speed in meters per second? 13.4 meters per second

Problem 2

Here is a map that shows parts of Texas and Oklahoma.



a. About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

About 260 mi (but the road is not straight, so it is hard to tell the exact distance from the map)

b. Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.

No, a traveler driving at a constant speed of 70 miles per hour can go only 210 miles in 3 hours, and the distance between the cities is definitely farther than that.

Student Workbook


LESSON 8
PRACTICE PROBLEMS

1 a. A whale swims at a constant speed of 4 meters per second. How far does it travel in 40 seconds? _____

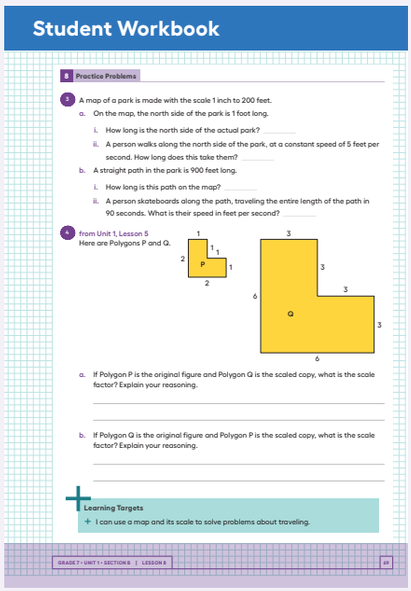
b. A horse runs at a constant speed of 5 meters per second. How much time does it take for the horse to travel 50 meters? _____

c. A goose flies at a constant speed, traveling 201 meters in 15 seconds. What is its speed in meters per second? _____

2 Here is a map that shows parts of Texas and Oklahoma.

a. About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

b. Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.


Student Workbook

8 Practice Problems

3. A map of a park is made with the scale 1 inch to 200 feet.
   a. On the map, the north side of the park is 1 foot long.
      i. How long is the north side of the actual park? _____
      ii. A person walks along the north side of the park, at a constant speed of 5 feet per second. How long does this take them? _____
   b. A straight path in the park is 900 feet long.
      i. How long is this path on the map? _____
      ii. A person skateboards along the path, traveling the entire length of the path in 90 seconds. What is their speed in feet per second? _____

4. from Unit 1, Lesson 5
   Here are Polygons P and Q.

   a. If Polygon P is the original figure and Polygon Q is the scaled copy, what is the scale factor? Explain your reasoning.

   b. If Polygon Q is the original figure and Polygon P is the scaled copy, what is the scale factor? Explain your reasoning.

Learning Targets
+ I can use a map and its scale to solve problems about traveling.

GRADE 7 • UNIT 1 • SECTION B | LESSON 8 49

Problem 3

A map of a park is made with the scale 1 inch to 200 feet.

a. On the map, the north side of the park is 1 foot long.

i. How long is the north side of the actual park?

2,400 feet (or equivalent)

ii. A person walks along the north side of the park, at a constant speed of 5 feet per second. How long does this take them?

480 seconds (or equivalent)

b. A straight path in the park is 900 feet long.

i. How long is this path on the map?

4.5 inches (or equivalent)

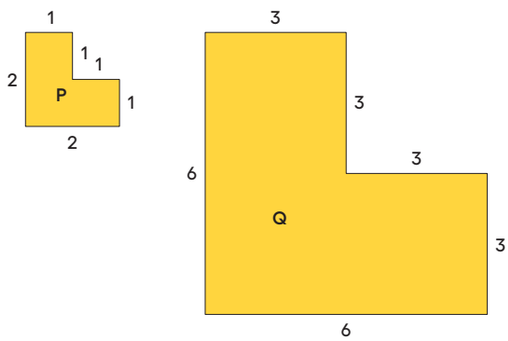
ii. A person skateboards along the path, traveling the entire length of the path in 90 seconds. What is their speed in feet per second?

10 feet per second

Problem 4

from Unit 1, Lesson 5

Here are Polygons P and Q.



a. If Polygon P is the original figure and Polygon Q is the scaled copy, what is the scale factor? Explain your reasoning.

3 because the sides of Polygon Q are 3 times as long as the sides of Polygon P

b. If Polygon Q is the original figure and Polygon P is the scaled copy, what is the scale factor? Explain your reasoning.

$\frac{1}{3}$ because the sides of Polygon P are $\frac{1}{3}$ the length of the sides of Polygon Q