Lesson 13

Solving Systems of Equations

Goals

- Coordinate (orally) the solution of an equation with variables on each side to the solution of a system of two linear equations.
- Create a graph of a system of equations, and identify (orally and in writing) the number of solutions of the system of equations.

Learning Targets

- I can graph a system of equations.
- I can solve systems of equations using algebra.

Lesson Narrative

In this lesson, students begin to solve systems algebraically by using a simple form of substitution. Then, they match systems to their graphs to check that their algebraic solutions are reasonable.

Students revisit the conditions for which equations have 0, 1, or infinitely many solutions and adapt the conditions to apply to systems of equations. By examining the slope and y-intercept of the lines in the system, students can determine the number of solutions for the system based on the structure of the equations.

Student Learning Goal

Let's solve systems of equations.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR5: Co-Craft Questions (Warm-up)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Scissors: Activity 2
- Straightedges: Activity 2

Materials to Copy

- Different Types of Systems Handout (1 copy for every 2 students): Activity 2

Required Preparation

Activity 2:
Provide access to straightedges for drawing accurate graphs and scissors for groups who wish to cut apart the graphs on the blackline master. For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Instructional Routines

MLR5: Co-Craft Questions

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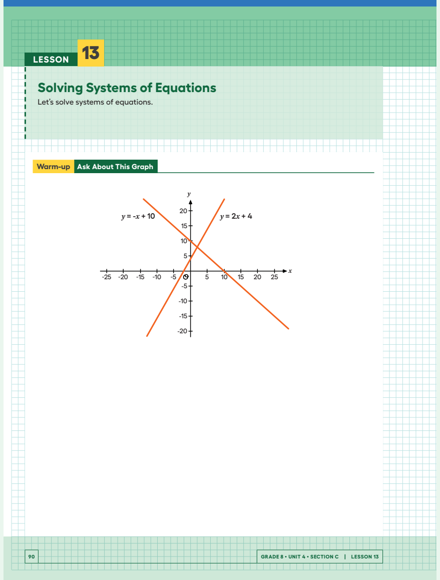
Access for Multilingual Learners (Warm-up)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook



Warm-up

Ask About This Graph

5 min

Activity Narrative

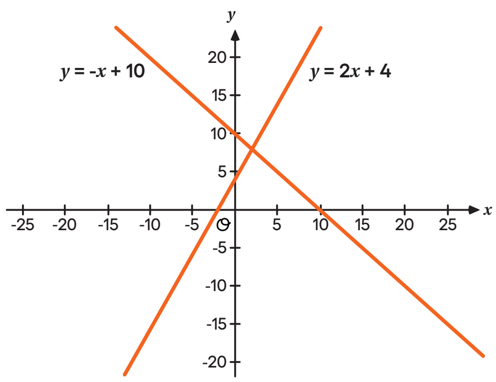
The purpose of this *Warm-up* is to get students thinking about the types of questions that they might ask when looking at two lines on a graph. Based on their recent work, students may be curious about the intersection points or other points on and off the lines.

Launch

Arrange students in groups of 2. Display the graph for all to see. Use Co-Craft Questions to orient students to the context and to elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement



Sample responses:

- What are the coordinates of the intersection point?
- Give an example of a point that is on only 1 line. What does that mean for the equation associated with each line?
- Give an example of a point that is not on either line. What does that mean for the equations?
- Write a situation that would result in these 2 lines.

Activity Synthesis

Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "make the equation(s) true," "solve the system," and "points on (or off) the line."

Activity 1

Matching Graphs to Systems

15 min

Activity Narrative

This activity represents the first time that students are explicitly asked to solve a system of equations using algebraic methods. Students also match systems of equations to their graphs so that they have a way to check that their algebraic solutions are correct. Students do not, however, do this in order to shortcut the algebraic process because the graphs themselves do not include enough detail to accurately guess the coordinates of the solution.

Launch

Keep students in groups of 2.

Give 2–3 minutes of quiet work time for the first problem, and then ask students to pause their work.

Select 1–2 students per figure to explain how they matched it to one of the systems of equations. For example, a student may identify the system matching Figure A as the only system with an equation that has a negative slope.

Give students 5–7 minutes of work time with their partner to complete the activity, and follow that with a whole-class discussion.

If students finish early and have not already done so on their own, ask them how they could check their solutions, and encourage them to do so.

Display the system of equations.

$$\begin{cases} y = x + 1 \\ y = -3x + 9 \end{cases}$$

Ask students,

"What must be true about the x and y values of the solution to this system of equations?"

The values must make both equations true.

"What does that mean about the left side of each equation?"

They are equal, $y = y$.

"What does that mean about the right side of each equation?"

They must also be equal.

Write $x + 1 = -3x + 9$ and ask students to solve this equation ($x = 2$).

Ask students how to find the y value for the solution when we know that $x = 2$ (substitute it for x in either equation). Show students that $y = 3$ when $x = 2$ for both equations so this must be the solution to the system.

Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the intercepts in the graph and the related value in the equation.

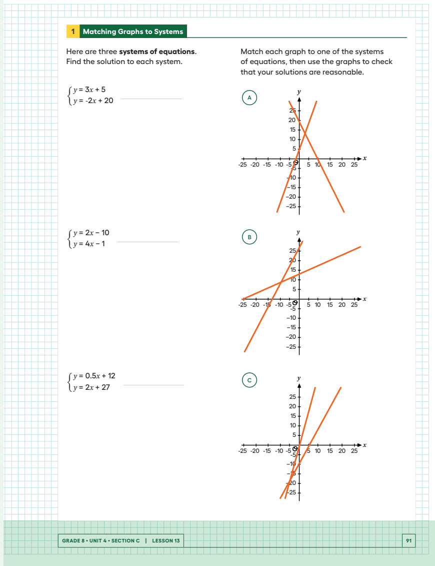
Supports accessibility for: Visual-Spatial Processing

Access for Multilingual Learners (Activity 1, Student Task)

MLR8: Discussion Supports.
Students should take turns finding a match and explaining their reasoning to their partner. Display the sentence frame for all to see: "I noticed ____ , so I matched …" Encourage students to challenge each other when they disagree.

Advances: Conversing, Representing

Student Workbook



Student Task Statement

Here are three **systems of equations.** Find the solution to each system.

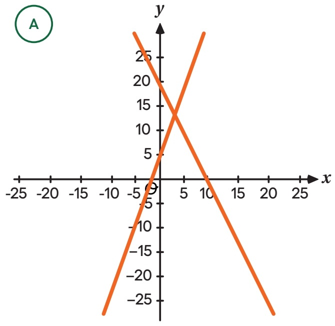
Match each graph to one of the systems of equations, then use the graphs to check that your solutions are reasonable.

$$\begin{cases} y = 3x + 5 \\ y = \text{-}2x + 20 \end{cases}$$ (3, 14)

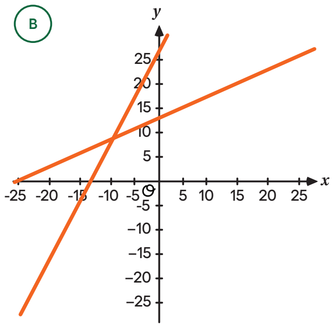
$$\begin{cases} y = 2x - 10 \\ y = 4x - 1 \end{cases}$$ (-4.5, -19)

$$\begin{cases} y = 0.5x + 12 \\ y = 2x + 27 \end{cases}$$ (-10, 7)

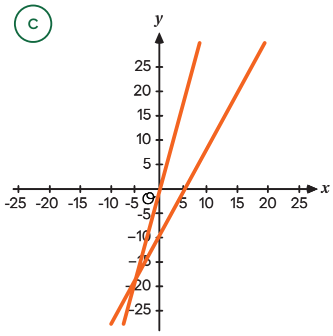
A



B



C



Activity Synthesis

The goal of this discussion is to deliberately connect the current topic of systems of equations to an earlier topic of solving equations with variables on each side.

For each of these questions, give students 30 seconds of quiet think time, and then invite students to explain their answer.

- *"Do you need to see the graphs of the equations in a system in order to solve the system?"*

  No, but seeing the graphs makes me feel more confident that my answer is correct.

- *"How do you know that your solution is correct?"*

  I know my solution is correct because I substituted my values for x and y into the equations and they made both equations true.

- *"How does solving systems of equations compare to solving equations with variables on both sides like we did in earlier lessons?"*

  They are very similar, only with a system of equations you are finding an x and a y to make both equations true and not just an x to make one equation true.

- *"When you solved equations with variables on both sides, some had one solution, some had no solutions, and some had infinite solutions. Do you think systems of equations can have no solutions or infinite solutions?"*

  Yes. We have seen some graphs of parallel lines where there were no solutions and some graphs of lines that are on top of one another where there are infinite solutions.

Activity 2
Different Types of Systems

15 min

Activity Narrative

There is a digital version of this activity.

Although students have encountered equations with different numbers of solutions in earlier activities, this is the first activity where students connect systems of equations with their previous thinking about equations that have no solution, one solution, or infinitely many solutions. The purpose of this activity is for students to connect the features of the graph of the equations of a system to the number of solutions of a system. Although students are not asked to solve the systems of equations, they may choose to rewrite the equations in equivalent forms as they work to graph the lines.

Depending on instructional time available, you may wish to alter the activity and ask students to solve one or more of the systems of equations algebraically.

In the digital version of the activity, students use an applet to graph systems of equations and draw conclusions about the possible types of solutions. The applet allows students to graph systems of equations quickly and accurately. This activity works best when each group has access to the applet because it allows groups to focus on understanding the three different types of solutions. If groups don't have access to the applet, displaying it for all to see would be helpful during the synthesis.

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Launch

Arrange students in groups of 2–3. Provide each group with access to straightedges and scissors as well as one copy of the blackline master. Encourage partners to split the work by cutting apart the problems, each taking one to three graphs, and then trading pages within their group to check the work.

Give 4–6 minutes for groups to complete the graphs, and remind students to use straightedges for precision while graphing.

Remind students of the activity that they did sorting equations with a single variable, where each equation had either one solution, no solution, or infinitely many solutions. Display the equation form $ax + b = cx + d$. Ask students how they could determine how many solutions the equation will have without solving it.

- There is 1 solution if the coefficients of x are not equal.
- There are no solutions if the coefficients of x are equal and the constant terms are different.
- There are infinitely many solutions if the coefficients of x are equal and the constant terms are equal.

Display this list for all to see for the rest of the activity.

Tell them that, just like one variable equations, systems of equations can also have either one solution, no solution, or infinitely many solutions.

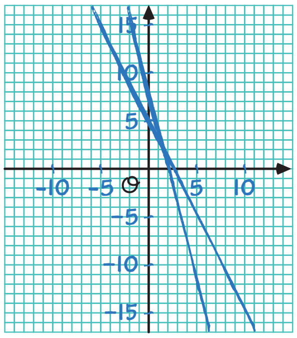
Before beginning the final problem, have each group trade their work with another group and place a question mark next to the graphs that they are not sure are correct. Give groups 3–4 minutes to revise as needed and to write their descriptions for the second problem. Follow that with a whole-class discussion.

Student Task Statement

Your teacher will give you a page with some systems of equations.

1. Graph each system of equations carefully on the provided coordinate plane.

A



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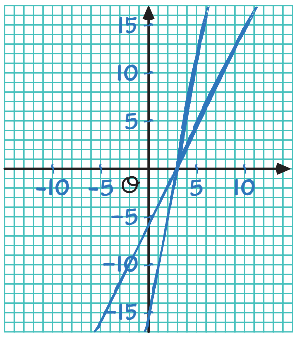

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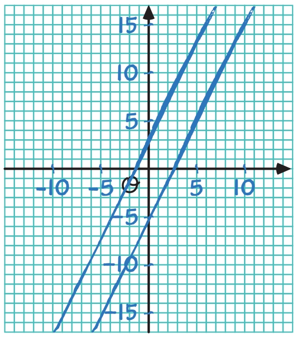

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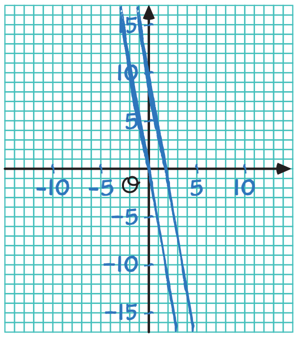
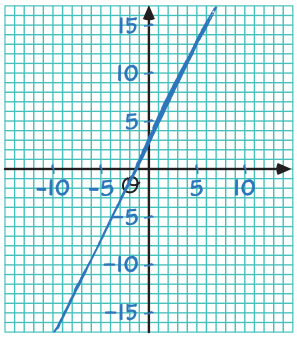


E



2. Describe what the graph of a system of equations looks like when it has

a. 1 solution.

Two lines that cross at a single point, which is the only solution that they have in common. Graphs A and B have one solution.

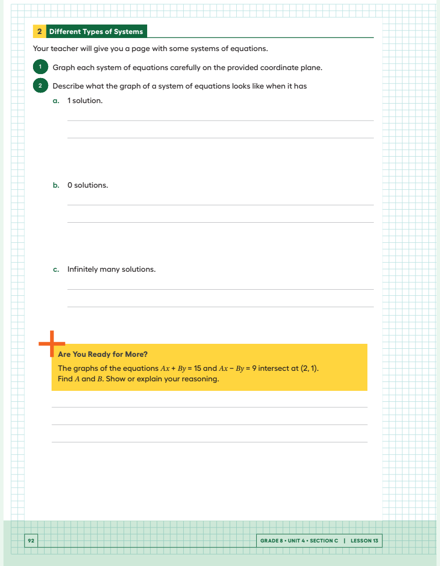
b. 0 solutions.

Two distinct lines that are parallel. They have no solutions in common. Graphs C and D have no solution.

c. Infinitely many solutions.

A single line because both equations have the same set of solutions. Graph E has infinitely many solutions.

Student Workbook


2 Different Types of Systems

Your teacher will give you a page with some systems of equations.

1. Graph each system of equations carefully on the provided coordinate plane.

2. Describe what the graph of a system of equations looks like when it has

a. 1 solution.

b. 0 solutions.

c. Infinitely many solutions.

Are You Ready for More?

The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$. Find A and B. Show or explain your reasoning.

GRADE 8 • UNIT 4 • SECTION C | LESSON 13

Are You Ready for More?

The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at (2, 1). Find A and B. Show or explain your reasoning.

$A = 6, B = 3$

Sample reasoning: If the lines intersect at (2, 1) then that point is on both lines. So we can substitute $x = 2$, $y = 1$ into both equations and write $2A + B = 15$, $2A - B = 9$. Then there are different ways we can solve it.

- Because $2A - B$ has the same value as 9, we can add $2A - B$ to one side of the first equation and 9 to the other.
- Subtracting 6 from each side of the first equation we get $2A + B - 6 = 9$. Then, set the 2 equations equal $2A + B - 6 = 2A - B$. This gives us that $B = 3$ and we can find A using that value.

Activity Synthesis

The goal of this discussion is for students to draw conclusions about the relationship between the number of solutions a system of equations has and the appearance of the graphs of the equations in the system, as well as the form of the equations.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the description of a graph based on the number of solutions by correcting errors, clarifying meaning, and adding details.

Display this first draft:

"There's 1 solution when it makes an X. There's no solution when the lines are next to each other. There are infinitely many solutions when there's only 1 line."

Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, and then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Direct students' attention to the list created at the beginning of the activity. Ask students,

"How do the coefficients of x appear in the graph?"

It is the slope of the lines.

"How do the constant terms appear in the graph?"

They are the y-intercept.

Create a new column next to the 3 statements in the display. Ask students how they can use the terms "slope" and "intercept" to update the language from the display. For example,

There is 1 solution if the coefficients of x are not equal.

could become,

There is 1 solution if the slopes of the lines are not equal.

Invite groups of students to rewrite the other 2 statements, and add the new versions to the second column.

Tell students to check that the new list of conditions about the number of solutions for a system of equations matches (makes sense with) their graphs.

Lesson Synthesis

To highlight the connection between the number of solutions to a system of equations and features of its graph and equations, ask,

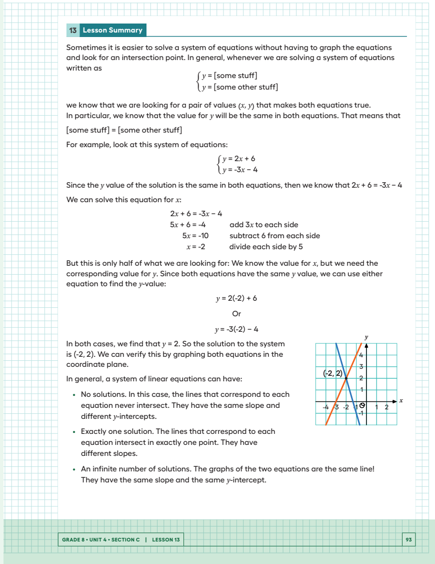
- ***"How can you know the number of solutions for a system of equations from its graph?"***

  If the two lines intersect at a point, there is one solution. If the two lines are parallel and do not intersect, there are no solutions. If the two lines are drawn through the same points, there are infinitely many solutions.

If students do not make the connection themselves, remind them of their earlier conclusions about the number of solutions that an equation in one variable has.

If time allows, assign a number of solutions (one, none, or infinite) to each group, and ask them to write a system of equations that would have that number of solutions. Have a few groups share their systems and describe how the graphs of the systems would look. In particular, ask each group to describe how the slope and y-intercept of their written lines would be seen in the graph and how the number of solutions would appear on the graph. Following the description, display the graph of the system using a digital resource, if possible, or a general sketch on a set of displayed axes.

Student Workbook



Lesson Summary

Sometimes it is easier to solve a system of equations without having to graph the equations and look for an intersection point. In general, whenever we are solving a system of equations written as

$$\begin{cases} y = \text{[some stuff]} \\ y = \text{[some other stuff]} \end{cases}$$

we know that we are looking for a pair of values (x, y) that makes both equations true. In particular, we know that the value for y will be the same in both equations. That means that

$$\text{[some stuff]} = \text{[some other stuff]}$$

For example, look at this system of equations:

$$\begin{cases} y = 2x + 6 \\ y = -3x - 4 \end{cases}$$

Since the y value of the solution is the same in both equations, then we know that

$$2x + 6 = -3x - 4$$

We can solve this equation for x:

$$\begin{aligned} 2x + 6 &= -3x - 4 & \\ 5x + 6 &= -4 & \text{add } 3x \text{ to each side} \\ 5x &= -10 & \text{subtract 6 from each side} \\ x &= -2 & \text{divide each side by 5} \end{aligned}$$

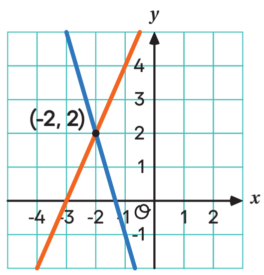
But this is only half of what we are looking for: We know the value for x, but we need the corresponding value for y. Since both equations have the same y value, we can use either equation to find the y-value:

$$y = 2(-2) + 6$$

or

$$y = -3(-2) - 4$$

In both cases, we find that $y = 2$. So the solution to the system is (-2, 2). We can verify this by graphing both equations in the coordinate plane.



In general, a system of linear equations can have:

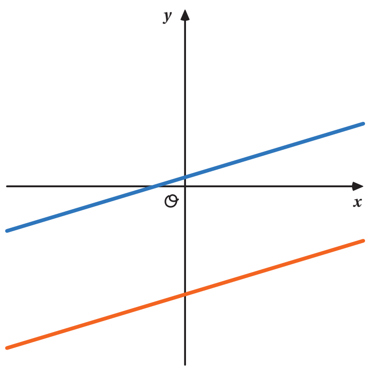
- No solutions. In this case, the lines that correspond to each equation never intersect. They have the same slope and different y-intercepts.
- Exactly one solution. The lines that correspond to each equation intersect in exactly one point. They have different slopes.
- An infinite number of solutions. The graphs of the two equations are the same line! They have the same slope and the same y-intercept.

Cool-down

Two Lines

5 min

Student Task Statement



1. Given the lines shown here, what are two possible equations for this system of equations?

   Any two equations with the same positive slope for each linear equation yet one with a negative y-intercept and the other a positive y-intercept

2. How many solutions does this system of equations have? Explain your reasoning.

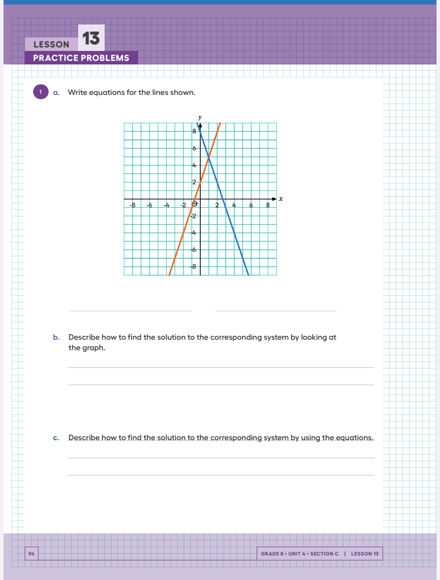
   0

   Sample reasoning: Since the lines are parallel and do not intersect, there are no solutions to the system of equations.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook

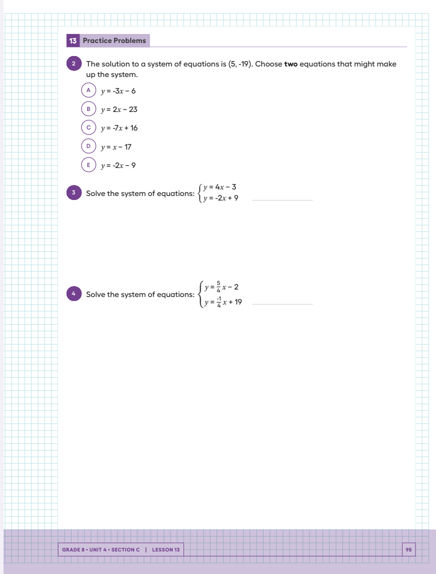

LESSON 13
PRACTICE PROBLEMS

1 a. Write equations for the lines shown.

b. Describe how to find the solution to the corresponding system by looking at the graph.

c. Describe how to find the solution to the corresponding system by using the equations.

Student Workbook


13 Practice Problems

2 The solution to a system of equations is (5, -19). Choose **two** equations that might make up the system.

A. $y = -3x - 6$
B. $y = 2x - 23$
C. $y = -7x + 16$
D. $y = x - 17$
E. $y = -2x - 9$

3 Solve the system of equations: $\begin{cases} y = 4x - 3 \\ y = -2x + 9 \end{cases}$

4 Solve the system of equations: $\begin{cases} y = \frac{5}{4}x - 2 \\ y = \frac{-1}{4}x + 19 \end{cases}$

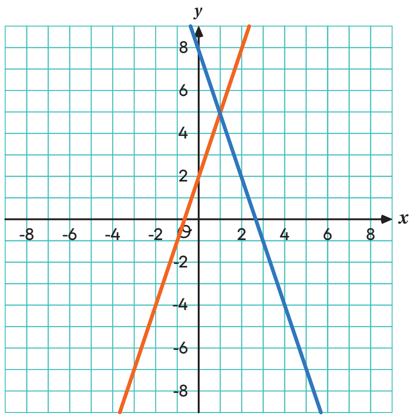
Lesson **13**

Practice Problems

5 Problems

Problem 1

a. Write equations for the lines shown.



$y = 3x + 2$ and $y = 8 - 3x$ (or equivalent)

b. Describe how to find the solution to the corresponding system by looking at the graph.

Sample response: Identify the coordinate point where the two lines intersect, (1, 5).

c. Describe how to find the solution to the corresponding system by using the equations.

Sample response: Set the two expressions for y equal to each other and solve: $3x + 2 = 8 - 3x$, $6x = 6$, $x = 1$, $y = 3(1) + 2 = 5$.

Problem 2

The solution to a system of equations is (5, -19). Choose **two** equations that might make up the system.

A. $y = -3x - 6$

B. $y = 2x - 23$

C. $y = -7x + 16$

D. $y = x - 17$

E. $y = -2x - 9$

Problem 3

Solve the system of equations: $\begin{cases} y = 4x - 3 \\ y = -2x + 9 \end{cases}$

(2, 5)

Problem 4

Solve the system of equations: $\begin{cases} y = \frac{5}{4}x - 2 \\ y = \frac{-1}{4}x + 19 \end{cases}$

$\left(14, 15\frac{1}{2}\right)$

Problem 5

from Unit 4, Lesson 6

Here is an equation: $\frac{15(x-3)}{5} = 3(2x-3)$

a. Solve the equation by using the distributive property first.

$x = 0$

Sample response:

$\frac{15(x-3)}{5} = 3(2x-3)$	
$\frac{15x-45}{5} = 6x - 9$	distributive property on each side
$3x - 9 = 6x - 9$	divide each term in the numerator of the left side by 5
$3x = 6x$	add 9 to each side
$0 = 3x$	subtract $3x$ on each side
$0 = x$	divide each side by 3

b. Solve the equation without using the distributive property.

$x = 0$

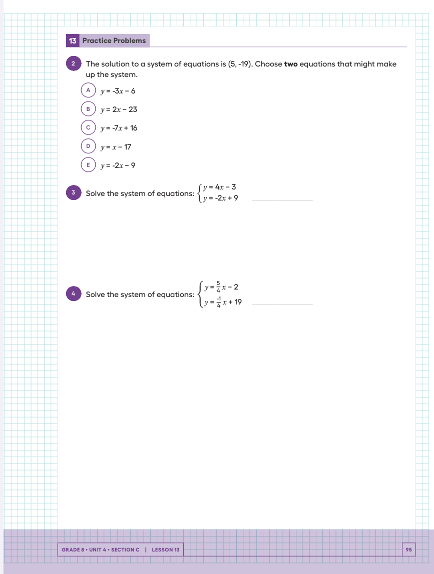
Sample response:

$\frac{15(x-3)}{5} = 3(2x-3)$	
$15(x-3) = 15(2x-3)$	multiply each side by 5
$x - 3 = 2x - 3$	divide each side by 15
$x = 2x$	add 3 to each side
$0 = x$	subtract x on each side

c. Check your solution.

This equation is true, so $x = 0$ is the solution.

Student Workbook



Student Workbook