Lesson 3

Dilations with No Grid

Goals

- Create a dilation of a figure given a scale factor and center of dilation.
- Explain (orally) the effect of the scale factor on the size of the image of a polygon and its distance from the center of dilation.
- Identify the center, scale factor, and image of a dilation without a circular grid.

Learning Target

I can apply a dilation to a polygon using a ruler.

Lesson Narrative

In this lesson, students apply dilations to points with no grid. In order to perform a dilation, three pieces of information are needed: a center of dilation, a scale factor, and a point to be dilated.

Students begin by dilating points along a ray in preparation to perform dilations of points and polygons without a grid. They must select and use appropriate tools as they identify centers of dilation, scale factors, and images of dilation, and use dilations to make perspective drawings.

Student Learning Goal

Let's dilate figures not on grids.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2

Required Preparation

Warm-up:
Provide access to geometry toolkits.

Activity 1:
Provide access to geometry toolkits.

Activity 2:
Provide access to geometry toolkits.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	10 min	5 min

Warm-up

Points on a Ray

5 min

Activity Narrative

In this *Warm-up*, students apply a dilation to points on a ray without a circular grid. Without the grid, students need to come up with a way to measure distances. Strategies they might try include using a ruler, marking off distances on an index card, or folding paper in half. Providing access to geometry toolkits gives students an opportunity to use appropriate tools strategically.

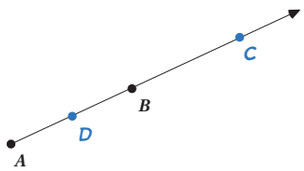
Launch

Provide access to geometry toolkits.

Give students 2 minutes of quiet work time followed by a whole class discussion.

Student Task Statement

1. Find a point on the ray whose distance from A is twice the distance from B to A and label it C.
2. Find a point on the ray whose distance from A is half the distance from B to A and label it D.



Activity Synthesis

The goal of this discussion is for students to understand that points C and D are dilations of point B, and to describe those dilations using precise language.

Invite students to present their methods for finding the points C and D, which may include the strategies mentioned in the *Activity Narrative*. Here are some questions for discussion:

- *"How is this work similar to previous work with dilations on a circular grid?"*

  The points lie on the same ray at different distances.

- *"How is this work different?"*

  There are no marked distances.

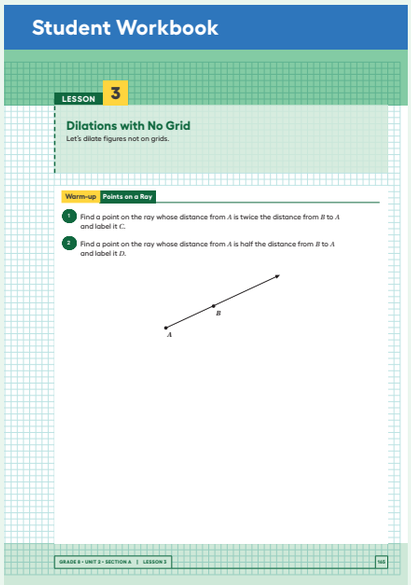
- *"How can we describe C as a dilation of B?"*

  C is a dilation of B with a center of dilation at A and a scale factor of 2.

- *"How can we describe D as a dilation of B?"*

  D is a dilation of B with a center of dilation at A and a scale factor of $\frac{1}{2}$.

Student Workbook


LESSON 3

Dilations with No Grid

Let's dilate figures not on grids.

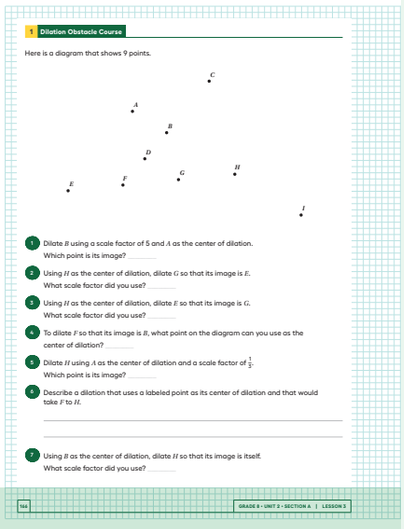
Warm-up: Points on a Ray

1. Find a point on the ray whose distance from A is twice the distance from B to A and label it C.
2. Find a point on the ray whose distance from A is half the distance from B to A and label it D.

Building on Student Thinking

Some students may think that for a point to be a dilation of itself, the scale factor is 0. Prompt them to consider multiplying the distance by 0. If they want the distance to be the same, they need to multiply by 1 instead.

Student Workbook


1 Dilation Obstacle Course

Here is a diagram that shows 9 points.

C
A
B
D
E
F
G
H
I

1. Dilate B using a scale factor of 5 and A as the center of dilation. Which point is its image? _____
2. Using H as the center of dilation, dilate G so that its image is E. What scale factor did you use? _____
3. Using H as the center of dilation, dilate E so that its image is G. What scale factor did you use? _____
4. To dilate F so that its image is B, what point on the diagram can you use as the center of dilation? _____
5. Dilate H using A as the center of dilation and a scale factor of $\frac{1}{3}$. Which point is its image? _____
6. Describe a dilation that uses a labeled point as its center of dilation and that would take F to H.

7. Using B as the center of dilation, dilate H so that its image is itself. What scale factor did you use? _____

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Activity 1

Dilation Obstacle Course

10 min

Activity Narrative

In this activity, students investigate dilations with no grid. Students make sense of a new problem as they perform dilations of points, determine scale factors, and find centers of dilation.

After doing a few of the problems, students may notice that the requested point is always one of the labeled points and then use this observation to expedite the work.

Launch

Arrange students in groups of 2.

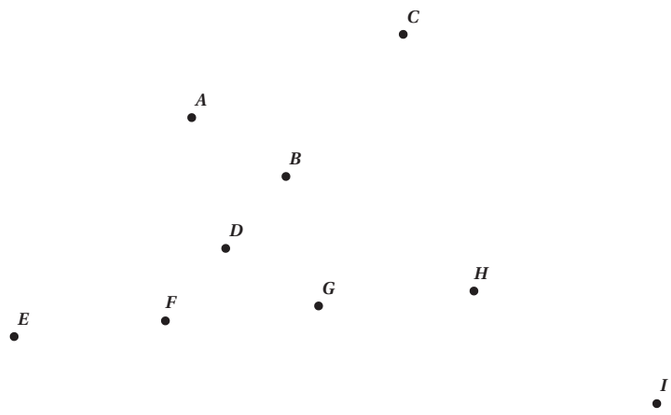
Give students 1–2 minutes to work on the first question and then pause for a whole-class discussion.

Remind students that the image of a point under dilation must lie on the same line as the point being dilated and the center of dilation. Demonstrate (or have a student demonstrate) how drawing a ray from point A through point B will also go through points H and I. Show that the length of AI is 5 times as long as the length of AB, either with a ruler or by marking intervals on the edge of a blank piece of paper from a geometry toolkit.

Give students 3–4 minutes to work on the remaining questions, followed by a whole-class discussion.

Student Task Statement

Here is a diagram that shows 9 points.



1. Dilate B using a scale factor of 5 and A as the center of dilation. Which point is its image? Point I
2. Using H as the center of dilation, dilate G so that its image is E. What scale factor did you use? 3
3. Using H as the center of dilation, dilate E so that its image is G. What scale factor did you use? $\frac{1}{3}$

4. To dilate F so that its image is B, what point on the diagram can you use as the center of dilation? point C (if restricted to positive scale factors)
5. Dilate H using A as the center of dilation and a scale factor of $\frac{1}{3}$. Which point is its image? point B
6. Describe a dilation that uses a labeled point as its center of dilation and that would take F to H.

   Dilate F using E as the center of dilation and a scale factor of 3.
7. Using B as the center of dilation, dilate H so that its image is itself. What scale factor did you use? 1

Activity Synthesis

The goal of this discussion is to make sure students understand how the location of the center of dilation and the scale factor work together, and to prepare students to draw their own dilations without a grid. Begin by discussing any strategies used to solve the problems. Ask students who noticed that all of the answers were labeled points to share how their observation helped them answer the questions. Then discuss with students:

- *"What do you notice about the scale factors for dilating G to E and dilating E back to G in the second and third questions?"*

  They both used H as the center of dilation and the scale factors are reciprocals of each other.
- *"What do you notice about the center of dilation, the point being dilated, and the image of the point after dilation?"*

  All 3 of those points lie on the same line.
- *"What scale factor results in an image that doesn't move?"*

  A scale factor of 1 does not move any points.

Show students 2 points A and B and explain that we want to dilate point B using A as the center and a scale factor of 3. Ask students to explain how to perform the dilation and if time allows, have a student demonstrate the process.

Be sure students understand that the image of point B will lie on the same line as points A and B and be located 3 times as far from A as B.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategies, such as "I know the scale factor is _____ because ..." or "I know this is the center of dilation because ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2, Narrative)

MLR7: Compare and Connect.
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Access for Perception.
Display or provide students with a physical copy of written directions for dilating a point on a plane with no grid and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.

Supports accessibility for: Language, Memory

Activity 2
Getting Perspective

20 min

Activity Narrative

In this activity, students find an appropriate way to take measurements in order to perform a dilation, most likely by using a ruler or the edge of an index card. Different students will work with different scale factors to produce perspective drawings of a box.

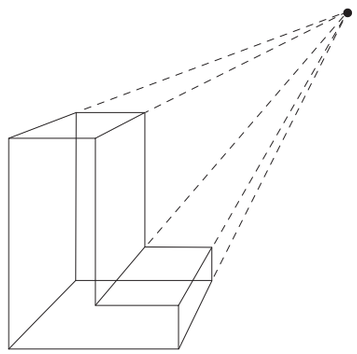
Monitor for students who draw these different representations:

- An accurate diagram using a scale factor greater than 1.
- An accurate diagram using a scale factor less than 1.

Launch

Provide access to geometry toolkits.

Give students 3–4 minutes to complete the first 2 questions. Pause to check student work and then display this perspective drawing for all to see.

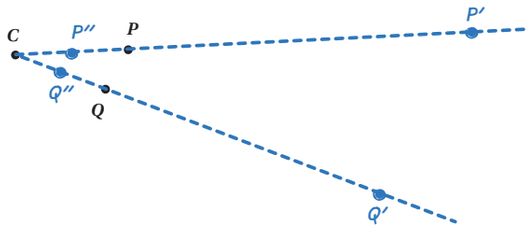


Tell students they will create their own perspective drawing and assign each student a scale factor to use. Appropriate scale factors include $\frac{1}{3}$, $\frac{1}{2}$, $1\frac{1}{2}$, and 2.

Select work from students with different scale factors, such as those described in the *Activity Narrative,* to share later.

Student Task Statement

1. Draw the images of points P and Q using C as the center of dilation and a scale factor of 4. Label the new points P' and Q'.



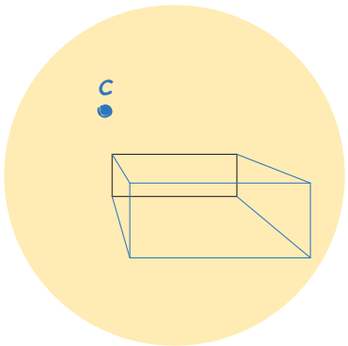
2. Draw the images of points P and Q using C as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q''.

   See image

   Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Let's make a perspective drawing. Here is a rectangle.

   Sample response (scale factor 1.5):



   a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it C.

   b. Use your center C and the scale factor you were given to draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.

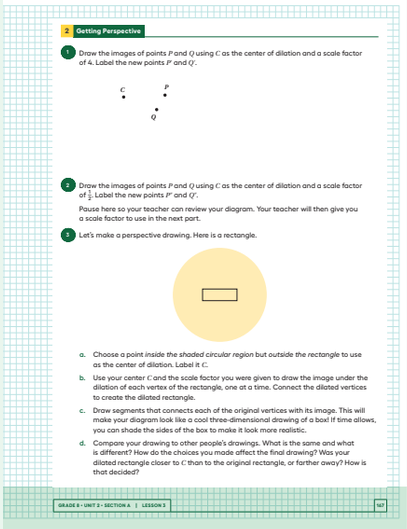
   c. Draw segments that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If time allows, you can shade the sides of the box to make it look more realistic.

   d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to C than to the original rectangle, or farther away? How is that decided?

Building on Student Thinking

Some students may try to make their drawing match the example drawing shown in the launch. For example, if the center of dilation shown is above and to the right, some students may place their center of dilation above and to the right of the rectangle. Emphasize that the center of dilation can be located anywhere, but its location will affect the resulting image.

Student Workbook


2 Getting Perspective

1. Draw the images of points P and Q using C as the center of dilation and a scale factor of 4. Label the new points P' and Q'.

C P Q

2. Draw the images of points P and Q using C as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q''.

   Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Let's make a perspective drawing. Here is a rectangle.

   a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it C.

   b. Use your center C and the scale factor you were given to draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.

   c. Draw segments that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If time allows, you can shade the sides of the box to make it look more realistic.

   d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to C than to the original rectangle, or farther away? How is that decided?

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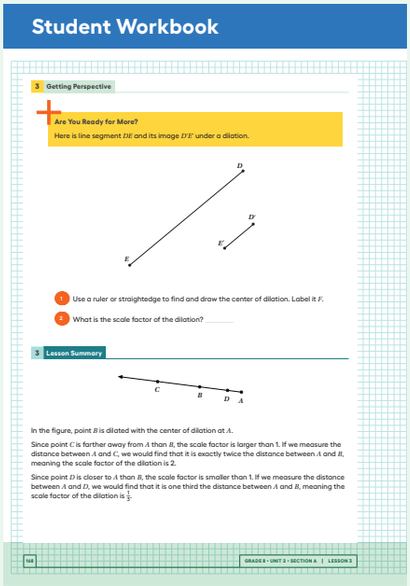
Student Workbook


3 Getting Perspective

Are You Ready for More?

Here is line segment DE and its image $D'E'$ under a dilation.

1 Use a ruler or straightedge to find and draw the center of dilation. Label it F.

2 What is the scale factor of the dilation? _____

3 Lesson Summary

In the figure, point B is dilated with the center of dilation at A.

Since point C is farther away from A than B, the scale factor is larger than 1. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B, meaning the scale factor of the dilation is 2.

Since point D is closer to A than B, the scale factor is smaller than 1. If we measure the distance between A and D, we would find that it is one third the distance between A and B, meaning the scale factor of the dilation is $\frac{1}{3}$.

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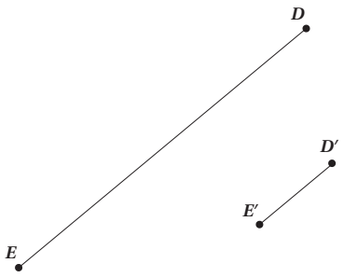
Are You Ready for More?

Here is line segment DE and its image $D'E'$ under a dilation.



1. Use a ruler or straightedge to find and draw the center of dilation. Label it F.

   Drawing of ray DD' and ray EE'. Their intersection is point F.

2. What is the scale factor of the dilation?

   $\frac{1}{4}$ (or equivalent)

Activity Synthesis

The goal of this discussion is for students to connect that scale factors less than 1 result in an image that is smaller and closer to the center of dilation while scale factors greater than 1 result in an image that is larger and farther away from the center of dilation.

Display 2–3 perspective drawings that all have a scale factor less than 1 from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the representations. Here are some questions for discussion:

- *"What do the dilations have in common?"*

  The images all got smaller. The images are all closer to the center of dilation.

- *"How are they different?"*

  The center of dilation is in a different location. They have different scale factors.

- *"How does the scale factor show up in each dilation?"*

  The scale factor determines how much smaller the dilation will be.

Then display 2–3 perspective drawings that all have a scale factor greater than 1 from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the representations. Here are some questions for discussion:

- *"What do these dilations have in common?"*

  The images all got bigger. The images are all farther away from the center of dilation.

- *"How are these dilations different from the first set?"*

  These dilations all got bigger while the other dilations all got smaller.

- *"How does the scale factor show up in each dilation?"*

  The scale factor determines how much larger or smaller the dilation will be.

Lesson Synthesis

The purpose of this discussion is to review the steps for dilating a point. Ask students to think about how they would explain the steps, and have them either write them down or share them with a partner. Ask a few students to share their steps. Ensure that all of the important aspects are mentioned:

You need to know which point you want to dilate, which point is the center of dilation, and what scale factor to use.

Use a straightedge to draw a ray from the center of dilation through the point you want to dilate.

Measure the distance from the center of dilation to the point being dilated. Multiply this distance by the scale factor. Place the new point at this distance from the center of dilation and also on the ray you drew. Note that there are many different ways to "measure" the distance.

If the scale factor is greater than 1, the new point will be farther from the center than the original point. If the scale factor is less than 1, the new point will be closer to the center than the original point.

Lesson Summary

In the figure, point B is dilated with the center of dilation at A.

Since point C is farther away from A than B, the scale factor is larger than 1. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B, meaning the scale factor of the dilation is 2.

Since point D is closer to A than B, the scale factor is smaller than 1. If we measure the distance between A and D, we would find that it is one third the distance between A and B, meaning the scale factor of the dilation is $\frac{1}{3}$.

Responding To Student Thinking

Points to Emphasize
If students struggle with identifying the center of a dilation, plan to revisit this when opportunities arise over the next several lessons. For example, in the activity referred to here, emphasize how the center of dilation, the point being dilated, and its image all lie on the same line.

Unit 2, Lesson 4, Activity 1 Dilations on a Grid

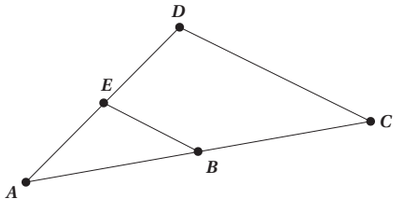
Cool-down

A Single Dilation of a Triangle

5 min

Student Task Statement

Lin drew a triangle and a dilation of the triangle with scale factor $\frac{1}{2}$:



1. What is the center of the dilation? Explain how you know.

   The center of dilation is *A*.

   Sample reasoning: The original and dilated points all lie on rays that start at *A*.

2. Which triangle is the original and which triangle is the dilation? Explain how you know.

   Triangle *ACD* is the original and triangle *ABE* is the dilation.

   Sample reasoning: Since the scale factor is less than 1, the dilated triangle is smaller than the original triangle.

Lesson 3

Practice Problems

6 Problems

Problem 1

Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with:

a. Scale factor 5? 15 cm

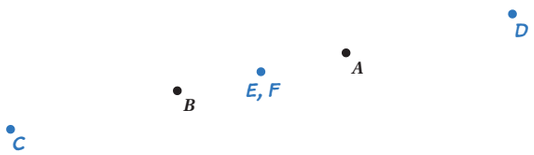
b. Scale factor 3.7? 11.1 cm

c. Scale factor $\frac{1}{5}$? $\frac{3}{5}$ cm

d. Scale factor s? $3s$ cm

Problem 2

Here are points A and B. Plot the points for each dilation described.



a. C is the image of B using A as the center of dilation and a scale factor of 2.

b. D is the image of A using B as the center of dilation and a scale factor of 2.

c. E is the image of B using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d. F is the image of A using B as the center of dilation and a scale factor of $\frac{1}{2}$.

Problem 3

Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

Answers vary

Problem 4

from Unit 2, Lesson 1

Triangle ABC and triangle DEF are scaled copies. Side AB measures 12 cm and is the longest side of ABC. Side DE measures 8 cm and is the longest side of DEF.

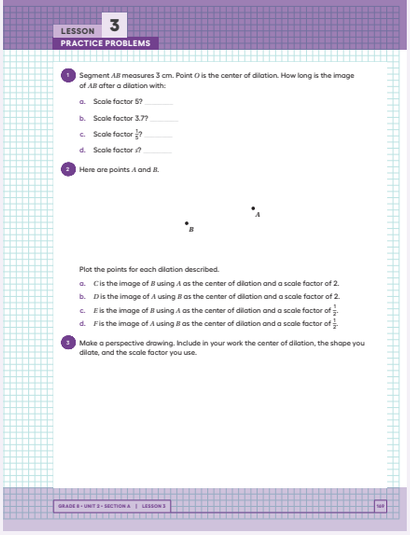
a. Triangle ABC is a scaled copy of triangle DEF with what scale factor?

$\frac{3}{2}$ (or equivalent)

b. Triangle DEF is a scaled copy of triangle ABC with what scale factor?

$\frac{2}{3}$ (or equivalent)

Student Workbook


LESSON 3
PRACTICE PROBLEMS

1 Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with:

a. Scale factor 5? _____

b. Scale factor 3.7? _____

c. Scale factor $\frac{1}{3}$? _____

d. Scale factor s? _____

2 Here are points A and B.

Plot the points for each dilation described.

a. C is the image of B using A as the center of dilation and a scale factor of 2.

b. D is the image of A using B as the center of dilation and a scale factor of 2.

c. E is the image of B using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d. F is the image of A using B as the center of dilation and a scale factor of $\frac{1}{2}$.

3 Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

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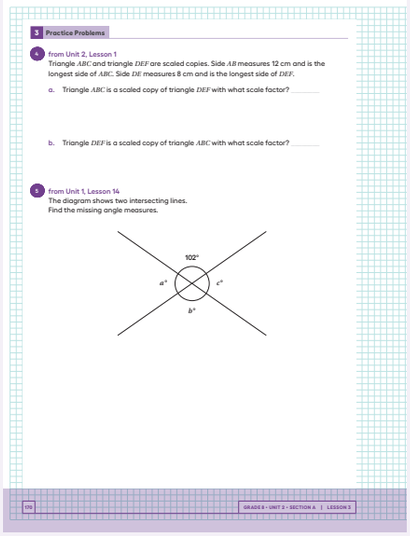
Student Workbook


3 Practice Problems

4 from Unit 2, Lesson 1
Triangle ABC and triangle DEF are scaled copies. Side AB measures 12 cm and is the longest side of ABC. Side DE measures 8 cm and is the longest side of DEF.

a. Triangle ABC is a scaled copy of triangle DEF with what scale factor? _____

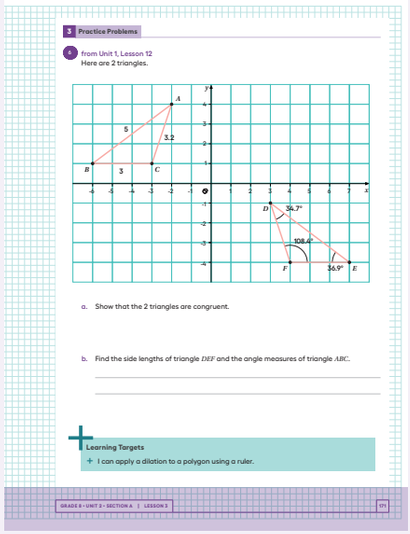
b. Triangle DEF is a scaled copy of triangle ABC with what scale factor? _____

5 from Unit 1, Lesson 14
The diagram shows two intersecting lines. Find the missing angle measures.

102°
$a°$
$c°$
$b°$

GRADE 8 • UNIT 2 • SECTION A | LESSON 3

Student Workbook


3 Practice Problems

6 from Unit 1, Lesson 12
Here are 2 triangles.

a. Show that the 2 triangles are congruent.

b. Find the side lengths of triangle DEF and the angle measures of triangle ABC.

Learning Targets
+ I can apply a dilation to a polygon using a ruler.

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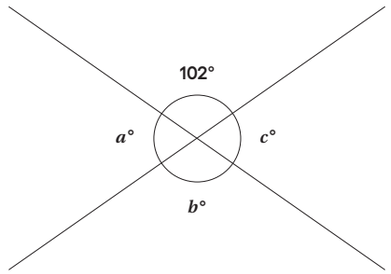
Problem 5

from Unit 1, Lesson 14

The diagram shows two intersecting lines.

Find the missing angle measures.

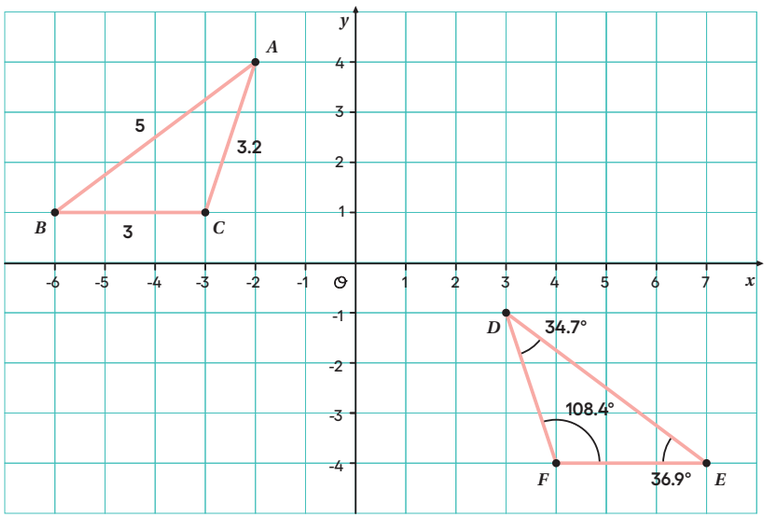


$a = 78$, $b = 102$, $c = 78$

Problem 6

from Unit 1, Lesson 12

Here are 2 triangles.



a. Show that the 2 triangles are congruent.

Sample response: Reflect triangle ABC across the y-axis and translate until A goes to D.

b. Find the side lengths of triangle DEF and the angle measures of triangle ABC.

$DE = 5$. $EF = 3$. $FD = 3.2$

- Angle ABC is 36.9°
- Angle BCA is 108.4°
- Angle CAB is 34.7°