Lesson 13

The Shadow Knows

Goals

- Calculate the unknown heights of objects by using proportional reasoning and explain (orally) the solution method.
- Justify (orally) why the relationship between the height of objects and the length of their shadows cast by the sun is approximately proportional.

Learning Target

I can model a real-world context with similar triangles to find the height of an unknown object.

Lesson Narrative

The purpose of this lesson is for students to interpret real-world data and use what they've learned about similar triangles to justify why a given relationship is proportional.

In this lesson, students examine the lengths of shadows of different objects. There appears to be a proportional relationship between the height of an object and the length of its shadow. Students must justify the proportionality of the relationship by interpreting real-world data and making simplifying assumptions.

Students then use this relationship to predict the height of a lamppost given the length of its shadow. Finally, in an optional activity students go outside and take their own measurements of different objects and the lengths of their shadows and use this technique to estimate the height of a tall object.

Student Learning Goal

Let's use shadows to find the height of an object.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 3)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Required Materials

Materials to Gather

- Measuring tapes: Activity 2, Activity 3
- Rulers: Activity 2, Activity 3
- Yardsticks: Activity 2, Activity 3

Required Preparation

Activity 2:
Before doing the last activity, conduct the experiment ahead of time to ensure that shadow lengths will be cooperative at the time your class takes place. Also, make preparations to take your class outside. They will need measuring devices (tape measures, yard sticks, rulers) as well as a way to record their measurements. Provide access to tape measures, yardsticks, or rulers.

Activity 3:
Before doing the last activity, conduct the experiment ahead of time to ensure that shadow lengths will be cooperative at the time your class takes place. Also, make preparations to take your class outside. They will need measuring devices (tape measures, yard sticks, rulers) as well as a way to record their measurements.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis
5 min	15 min	15 min	20 min	10 min

Instructional Routines

Notice and Wonder

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Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 13

The Shadow Knows

Let's use shadows to find the height of an object.

Warm-up Notice and Wonder: Long Shadows and Short Shadows

What do you notice? What do you wonder?

Warm-up

Notice and Wonder: Long Shadows and Short Shadows

5 min

Activity Narrative

The purpose of this *Warm-up* is to show what happens when shadows are cast from a lamp, which will be useful when students see shadows cast from the sun in a later activity. While students may notice and wonder many things about these images, the length of the shadows (which are different for the pens near the lamp and appear to be the same for the pens in the sunshine) is the most important discussion point.

This *Warm-up* prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved.

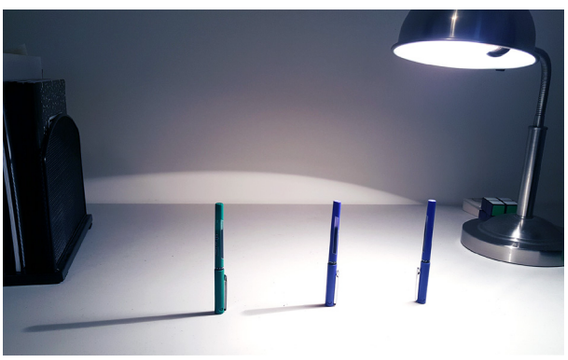
Launch

Arrange students in groups of 2. Display the image for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- The pens are the same in both photos.
- The pens have shadows in both photos.
- One photo was taken inside and one was taken outside.
- One light source is a lamp and the other is the sun.
- In the photo where the light source is a lamp, the light is coming from the right. In the other photo, the light is coming from the left.

- In the photo where the light source is a lamp, the shadows are all different lengths. In the photo where the light source is the sun, the shadows are all the same length.

Students may wonder:

- Are there times when the pens outdoors don't leave any shadow?
- Why do the pens near the lamp have different length shadows?

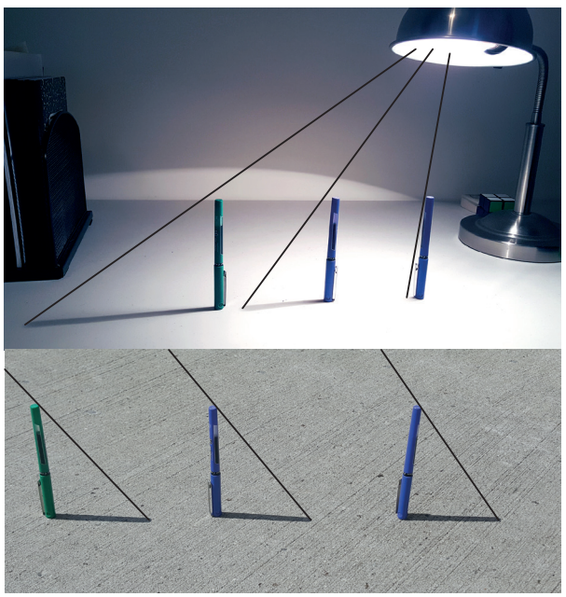
Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students:

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If time allows, display these images where the path of the light that reaches the top of each shadow is drawn in and ask students if there is anything new that they notice or wonder based on these new images.



Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.
Invite students to share an experience where they noticed a variety of shapes, lengths, and sizes of shadows.

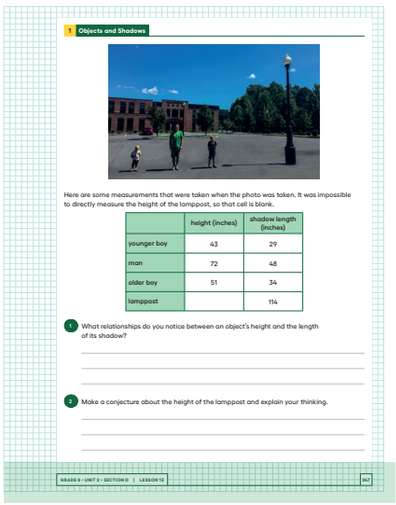
Supports accessibility for: Conceptual Processing, Memory

Building on Student Thinking

Some students may hesitate to identify the relationship as proportional. The task uses real measurements that were taken to the nearest inch and, because of rounding, the values in the table are not in a perfectly proportional relationship. Prompt students to start by coming up with a range of reasonable values for the height of the post. Also, share with them that the measurements were rounded to the nearest inch, so it's possible that the relationship is imperfect.

Some students may need help understanding the meaning of "conjecture." A simple definition to use is "a reasonable guess."

Student Workbook


1 Objects and Shadows

Here are some measurements that were taken when the photo was taken. It was impossible to directly measure the height of the lamppost, so that cell is blank.

	height (inches)	shadow length (inches)
younger boy	43	29
man	72	48
older boy	51	34
lamppost		114

1. What relationships do you notice between an object's height and the length of its shadow?

2. Make a conjecture about the height of the lamppost and explain your thinking.

GRADE 8 • UNIT 2 • SECTION D | LESSON 13

Activity 1

Objects and Shadows

15 min

Activity Narrative

In this activity, students explore the approximately proportional relationship between the height of an object and the length of its shadow by looking at a photo of three people, a lamppost, and their shadows taken on a sunny day. They use what they know about proportional relationships and the length of a shadow to find the height of an object that is difficult to measure directly.

The given measurements are real measurements rounded to the nearest inch, and therefore, the values in the table are not in a perfectly proportional relationship. The quotient of each shadow length and its corresponding object's height is around, but not exactly, $\frac{2}{3}$. Students engage in quantitative reasoning by exploring relationships with real-world measurements.

Launch

Display the photo in the task, and ask students to brainstorm how they might go about measuring the height of each person and the lamppost. It would be straightforward to measure the height of the people using a yardstick or tape measure, but difficult to measure the height of the lamppost. Tell students that even when something is too tall to measure directly, we can still figure out its height by using the length of its shadow (which is easy to measure since it's on the ground).

Draw students' attention to the measurements given in the table. Invite them to look for and use relationships in the table to conjecture about the height of the lamppost.

Student Task Statement

Here are some measurements that were taken when the photo was taken. It was impossible to directly measure the height of the lamppost, so that cell is blank.

	height (inches)	shadow length (inches)
younger boy	43	29
man	72	48
older boy	51	34
lamppost		114

1. What relationships do you notice between an object's height and the length of its shadow?

   Sample response: There appears to be a proportional relationship between the height of an object and the length of its shadow. All of the shadow lengths are approximately $\frac{2}{3}$ the height of the object.

2. Make a conjecture about the height of the lamppost and explain your thinking.

   Approximately 171 inches (or 14 feet 3 inches)

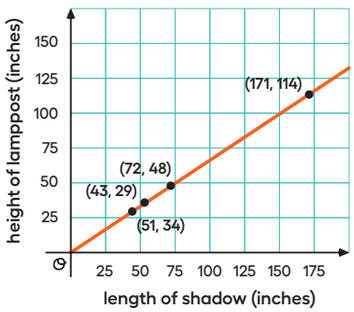
   Sample reasoning: $114 \cdot \frac{3}{2}$, $114 \div \frac{2}{3}$, 150% of 114.

Activity Synthesis

The goal of this discussion is for students to recognize a relationship that is approximately proportional and consider reasons why the relationship is not exactly proportional. Ask students to brainstorm possible reasons why the measurements in the table are not exactly proportional, highlighting these possible factors:

- The measurements were rounded, making them inexact.
- The ground might not be perfectly level.
- One of the people or the lamppost is at a slightly different angle to the ground.
- Anything that makes the real world differ slightly from a mathematical model.

If time allows, or if any students created a graph of the associated heights and shadow lengths, consider sharing this graph and asking how it could be used to determine the height of the lamppost. (This graph and the length of the shadow could be used to find the corresponding height.)



Building on Student Thinking

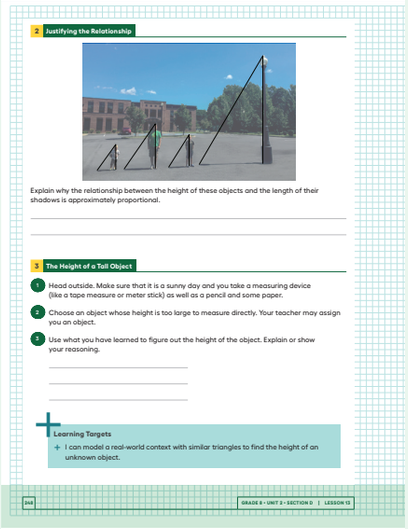
Some students may struggle with deciding what is important to include in their explanation. Prompt them to start by thinking about what they want to show. For example, in this activity, we want to show that there is a proportional relationship between the side lengths in some triangles. Then ask:

"What types of triangles have sides that are in proportion?"
"How do you know when triangles are similar triangles?"
"Which pairs of angles do you know are congruent?"

and

"Why are they congruent?"

Student Workbook



2 Justifying the Relationship

Explain why the relationship between the height of these objects and the length of their shadows is approximately proportional.

3 The Height of a Tall Object

1. Head outside. Make sure that it is a sunny day and you take a measuring device (like a tape measure or meter stick) as well as a pencil and some paper.
2. Choose an object whose height is too large to measure directly. Your teacher may assign you an object.
3. Use what you have learned to figure out the height of the object. Explain or show your reasoning.

Learning Targets

+ I can model a real-world context with similar triangles to find the height of an unknown object.

Activity 2

Justifying the Relationship

15 min

Activity Narrative

The purpose of this activity is for students to write a mathematical justification for the proportional relationship between heights and shadow lengths in the photo. A version of the photo is provided with some line segments drawn to strongly suggest an argument based on similar triangles.

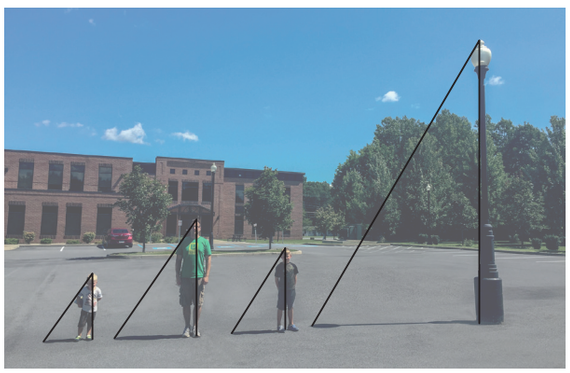
The sun is so far away relative to its size that the rays that reach Earth are extremely close to parallel. Share this information with the students or ask them to think about the shadows of the pens in the *Warm-up*.

Launch

Provide access to tape measures, yardsticks, or rulers. Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time, followed by a partner then whole-class discussion.

Student Task Statement



Explain why the relationship between the height of these objects and the length of their shadows is approximately proportional.

Sample reasoning: The angle between the sun's rays and the shadow on the ground are all congruent corresponding angles, since the sun's rays are approximately parallel. The lamppost and the people are all perpendicular to the ground, making right angles that are also congruent. The right triangles formed by the sun's rays, the lamppost or person, and their shadows are similar because they all have 2 pairs of corresponding angles that are congruent. Therefore, their corresponding sides are proportional.

Activity Synthesis

The goal of this discussion is for students to share and refine their explanations. Invite 2–3 students to share their thinking. Ask other students to restate, support, refine, or disagree with their statements by asking questions such as

- *"How do you know that the triangles are similar?"*

and

- *"Which pairs of angles are congruent?"*

Activity 3: Optional
The Height of a Tall Object

20 min

Activity Narrative

The goal of this optional activity is to solve a real-world problem by finding the height of some tall object. This activity should be done on a sunny day and tried out ahead of time to ensure that the shadows created in your part of the world at the time your class takes place are cooperative!

Find a tall object, or let students choose a tall object outside, that all students will find the height of (for example, a flagpole, a building, or a tree). The object should be on level ground, perpendicular to the ground, and tall enough that its height can't easily be measured directly. Students will work outside with tape measures (or other measuring devices) to use what they've learned in this lesson to determine the height of the tall object.

Launch

Arrange students in groups of 2–4. Explain they will apply what they have learned about shadow lengths of different objects to estimate the height of an object outside. Either tell them which object to use or explain the parameters for choosing an appropriate object.

Provide access to tape measures, yardsticks, or rulers.

Give students 10 minutes to take their measurements and perform any calculations followed by a whole-class discussion.

Student Task Statement

1. Head outside. Make sure that it is a sunny day and you take a measuring device (like a tape measure or meter stick) as well as a pencil and some paper.
2. Choose an object whose height is too large to measure directly. Your teacher may assign you an object.
3. Use what you have learned to figure out the height of the object. Explain or show your reasoning.

Answers vary

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to why the relationship between the height of the objects and the length of their shadows is approximately proportional. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Access for Students with Diverse Abilities (Activity 3, Launch)

Engagement: Develop Effort and Persistence.
Provide guides or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a guide that chunks the activity into a set of manageable tasks and contains an exemplar, as well as a checklist to help students track what they have completed.

Supports accessibility for: Attention, Social-Emotional Functioning

Activity Synthesis

The purpose of this discussion is for students to share their strategies for determining the height of a tall object. Invite 2–3 groups to share their thinking. Here are some questions for discussion:

- *"What object did you use that had a height that you could measure with the tools available?"*

  A person, a fence, a street sign, etc.

- *"How do you know the relationship between the object you measured is proportional to the height of the tall object you couldn't measure?"*

  Since the sun's rays are approximately parallel to the ground and the objects are perpendicular to the ground, we know that the triangles created will be similar and their side lengths will be proportional.

Lesson Synthesis

The goal of this discussion is for students to understand how mathematical models can be used to make accurate guesses or predictions about quantities that are difficult or impossible to measure directly. Here are some questions for discussion:

- *"Why did we need to make a mathematical model of the situation?"*

  The objects were too large for us to measure directly.

- *"What were some assumptions we had to make in order for our model to work?"*

  We had to assume that the light rays coming from the sun are parallel, that the people and the lamppost are perpendicular to the ground, and that the ground is level.

- *"Why isn't the data always perfect?"*

  Measurement error may occur, measurements might get rounded, and the angle between an object and the ground may not be exactly 90 degrees.

- *"How do we know our predictions are accurate?"*

  Earlier we observed a relationship between the height of an object and the length of its shadow. Using what we know about similar triangles, we can predict the same relationship to hold for a very tall object nearby at the same time of day.

An interesting historical connection: over 2,000 years ago, the ancient Greek mathematician Eratosthenes also studied shadows closely (in a slightly different way) and used this to estimate the circumference of Earth with an error of less than 2%!