Lesson 12

Equations of All Kinds of Lines

Goals

- Create multiple representations of a linear relationship, including a graph, equation, and table.
- Generalize (in writing) that a set of points of the form (x, b) satisfy the equation $y = b$ and that a set of points of the form (a, y) satisfy the equation $x = a$.

Learning Targets

- I can write equations of lines that have a positive or a negative slope.
- I can write equations of vertical and horizontal lines.

Lesson Narrative

In this lesson, students extend previous work writing equations of lines to include equations for horizontal and vertical lines. Horizontal lines can still be written in the form $y = mx + b$ but because $m = 0$ in this case, the equation simplifies to $y = b$. Students interpret this to mean that, for a horizontal line, the y value does not change, but x can take any value. This structure is identical for vertical lines except that now the equation has the form $x = a$ and it is the x value that does not change while y can take any value.

Note that while the equation of a vertical line cannot be written in the form $y = mx + b$, it can be written in the form $Ax + By = C$, with $B = 0$. In this lesson, students encounter a context where this form arises naturally: A rectangle with length ℓ, width w, and perimeter of 50 can be described by the equation $2\ell + 2w = 50$.

Student Learning Goal

Let's write equations for vertical and horizontal lines.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- Which Three Go Together?

Required Materials

Materials to Gather

- Straightedges: Activity 1, Activity 2

Required Preparation

Activity 2:
Cut a piece of string 50 cm long.

Lesson:
Take a piece of string 50 centimeters long and tie the ends together to be used as demonstration in the third activity.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up

Which Three Go Together: Pairs of Lines

5 min

Activity Narrative

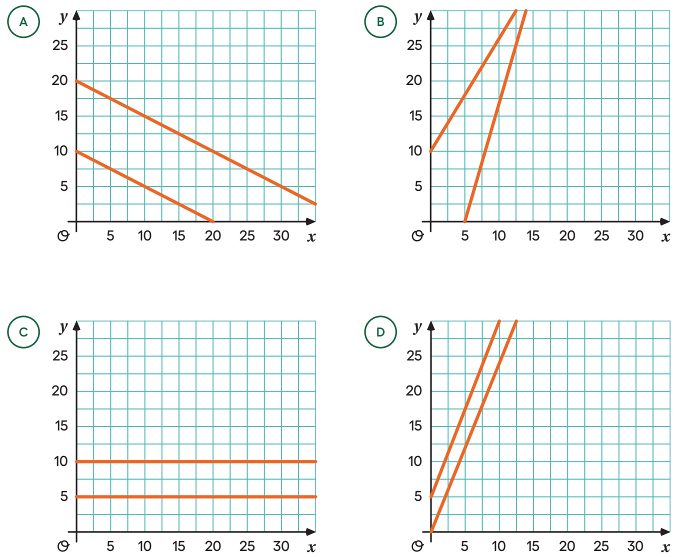
This *Warm-up* prompts students to compare four images. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the images for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three pairs of lines that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- they all have one line that goes through the point $(0, 10)$.
- they all have one line with a vertical intercept of 10.

A, B, and D go together because:

- they all have lines that are slanted upwards or downwards.
- they all have lines with a non-zero slope.

A, C, and D go together because:

- they all have a pair of parallel lines.
- they all have lines with non-negative y-intercepts.

B, C, and D go together because:

- they all have lines with a non-negative slope.

Instructional Routines

Which Three Go Together?

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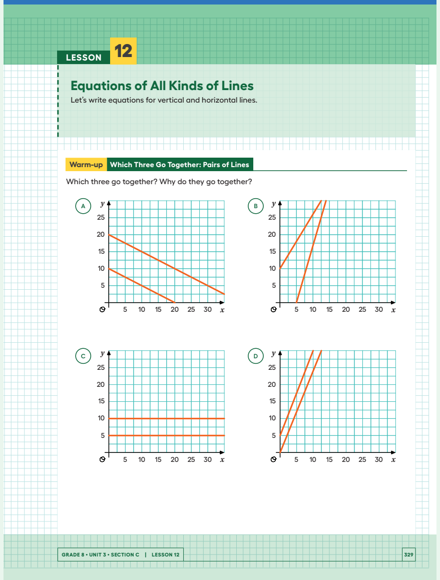
Please log in to the site before using the QR code or URL.

Student Workbook



Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display.

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "parallel," "intersect," "origin," "coordinate," "ordered pair," "quadrant," or "slope," and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Activity 1

All the Same

15 min

Activity Narrative

In this activity, students create equations that represent horizontal and vertical lines, and then graph them. Horizontal lines can be thought of as being described by equations of the form $y = mx + b$ where $m = 0$. Vertical lines, on the other hand, cannot be described by an equation of the form $y = mx + b$, and students must explain their reasoning when identifying an equation that can describe a vertical line.

Launch

Arrange students in groups of 2–4. Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to describe a set of points with the same x- or y-coordinate. Display words and phrases such as "horizontal," "vertical," and "in a line."

Begin by displaying the blank coordinate plane from the *Student Task Statement,* or similar, for all to see. Split the class in half and invite the first half to plot a point on the display whose y-coordinate is -4. Ask students what they notice about these points and record their observations on the display. (All of the points are in a row. All of the points make a line that is horizontal. The slope of the line is 0.) Give students 1 minute to discuss with their groups and answer the second question about which equation could represent this line.

Next, invite the second half of the class to plot a point on the display whose x-coordinate is 3. Ask students what they notice about these points and record their observations on the display. (All of the points are in a row again. All of the points make a line that is vertical.) Give students 1 minute to discuss with their groups and answer the fourth question about which equation could represent this line.

Give students 2 minutes to complete the rest of the questions, followed by a whole-class discussion.

Student Task Statement

1. Plot at least 10 points whose y-coordinate is -4. What do you notice about them?

   Answers vary.

   Sample responses: Points all lie on a horizontal line that crosses the y-axis at -4, points all lie on a line parallel to and 4 units down from the x-axis.

2. Which equation makes the most sense to represent all of the points with y-coordinate -4? Explain how you know.

   $x = -4$ $\quad$ $y = -4x$ $\quad$ $y = -4$ $\quad$ $x + y = -4$

   $y = -4$

   Sample reasoning: This is the only equation that is true for every point that was graphed and for all points for which the y-coordinate is -4.

3. Plot at least 10 points whose x-coordinate is 3. What do you notice about them?

   Sample responses: Points all lie on a vertical line that crosses the x-axis at 3, points all lie on a line parallel to and 3 units to the right of the y-axis.

4. Which equation makes the most sense to represent all of the points with x-coordinate 3? Explain how you know.

   $x = 3$ $\quad$ $y = 3x$ $\quad$ $y = 3$ $\quad$ $x + y = 3$

   $x = 3$

   Sample reasoning: This is the only equation that is true for every point that was graphed and for all points for which the x-coordinate is 3.

5. Graph the equation $x = -2$.

   A vertical line through (-2, 0) is drawn.

6. Graph the equation $y = 5$.

   A horizontal line through (0, 5) is drawn.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.

Provide a blank two-column table for students to list x- and y-values.

Supports accessibility for: Organization, Attention

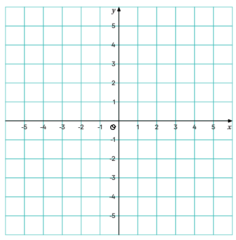
Building on Student Thinking

If students ask what the slope of a vertical line is, consider asking:

"How can a slope triangle be used to determine the slope?"

- Demonstrating how to use two points on a vertical line to attempt to calculate the slope. The vertical change can be calculated but the horizontal change will be 0. Since slope is the vertical change divided by the horizontal change, attempting to calculate the slope will require dividing by 0, making the slope of a vertical line undefined.

Student Workbook



1 All the Same

1. Plot at least 10 points whose y-coordinate is -4. What do you notice about them?

2. Which equation makes the most sense to represent all of the points with y-coordinate -4? Explain how you know.

   $x = -4$ $\quad$ $y = -4x$ $\quad$ $y = -4$ $\quad$ $x + y = -4$

3. Plot at least 10 points whose x-coordinate is 3. What do you notice about them?

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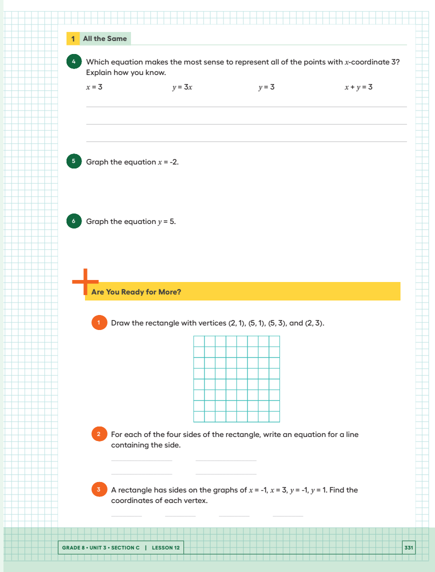
Student Workbook


1 All the Same

4 Which equation makes the most sense to represent all of the points with x-coordinate 3? Explain how you know.

$x = 3$ $y = 3x$ $y = 3$ $x + y = 3$

5 Graph the equation $x = -2$.

6 Graph the equation $y = 5$.

Are You Ready for More?

1 Draw the rectangle with vertices (2, 1), (5, 1), (5, 3), and (2, 3).

2 For each of the four sides of the rectangle, write an equation for a line containing the side.

3 A rectangle has sides on the graphs of $x = -1$, $x = 3$, $y = -1$, $y = 1$. Find the coordinates of each vertex.

Are You Ready for More?

1. Draw the rectangle with vertices (2, 1), (5, 1), (5, 3), and (2, 3).

   Rectangle with vertices at (2, 1), (5, 1), (5, 3), (2, 3) is drawn.

2. For each of the four sides of the rectangle, write an equation for a line containing the side.

   $x = 2$ $y = 1$

   $x = 5$ $y = 3$

3. A rectangle has sides on the graphs of $x = -1$, $x = 3$, $y = -1$, $y = 1$. Find the coordinates of each vertex.

   (-1, -1) (3, -1) (3, 1) (-1, 1)

Activity Synthesis

The goal of this discussion is for students to articulate their reasoning about the equations that represent vertical and horizontal lines.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their equation and reasoning for the horizontal line ($y = -4$) and then the vertical line ($x = 3$). Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. Discuss:

- *"Why does the equation for the points with y-coordinate -4 not contain the variable x?"*

  x can take any value while y is always -4. The only constraint is on y and there is no dependence of x on y.

- *"Why does the equation for the points with x-coordinate 3 not contain the variable y?"*

  y can take any value while x is always 3. The only constraint is on x and there is no dependence of y on x.

- *"What does this say about the relationship between the quantities represented by x and y in these situations?"*

  Changes in one do not affect the other. One is not dependent on the other.

- *"What would be some real-world examples of situations that could be represented by these types of equations?"*

  Everyone pays the same fee regardless of age. Bus tickets cost the same no matter how far the trip. A student remains the same distance from home as the hours pass during the school day.

Activity 2

Same Perimeter

15 min

Activity Narrative

In this activity, students analyze a line and an equation defining the line in a geometric context. By finding and graphing pairs of numbers for the width and length of rectangles that all have a perimeter of 50, students observe that this set of numbers lie on a line.

Writing an equation for this relationship introduces students to equations of the form $Ax + By = C$. This form of equation will be used in later lessons and it is not necessary for students to know this general form or to manipulate this form of equation at this time. The goal is for students to see multiple ways to describe a line and how each part of each equation relates to the situation.

Monitor for students who come up with different equations for the line. An equation of the form $2\ell + 2w = 50$, where ℓ is the length of the rectangle, and w is its width, makes sense in this context of the perimeter of a rectangle being 50 units. An equation of the form $w = 25 - \ell$ also makes sense because the y-intercept of the graph is 25 and its slope is -1.

Launch

Give students 1 minute to sketch a rectangle whose perimeter is 50 units and label the lengths of its sides. Invite 2–3 students to share the lengths and widths they found. For example, some lengths and widths could be 10 and 15, 5 and 20, or 1 and 24.

If possible, take a 50-cm long string with its ends tied together and demonstrate how a given perimeter can yield several different rectangles by varying the width and length. Ensure that everyone understands that rectangles have 4 sides, that rectangles have two pairs of congruent sides, and that there is more than one rectangle whose perimeter is 50 units.

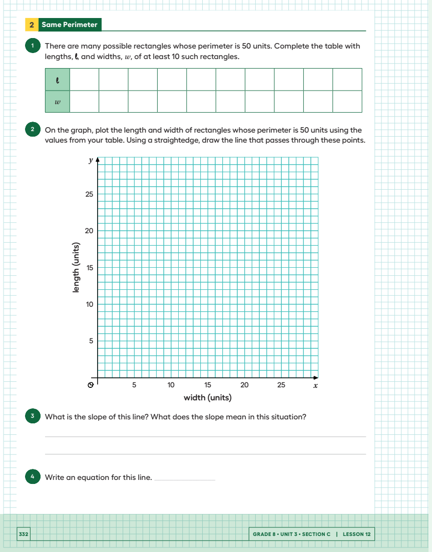
Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.

Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of lines in a geometric context. Terms may include “width,” “length,” “perimeter,” “rectangle,” “intercept,” and “slope.”

Supports accessibility for: Conceptual Processing, Language

Student Workbook


2 Same Perimeter

1 There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, ℓ, and widths, w, of at least 10 such rectangles.

2 On the graph, plot the length and width of rectangles whose perimeter is 50 units using the values from your table. Using a straightedge, draw the line that passes through these points.

3 What is the slope of this line? What does the slope mean in this situation?

4 Write an equation for this line.

Student Task Statement

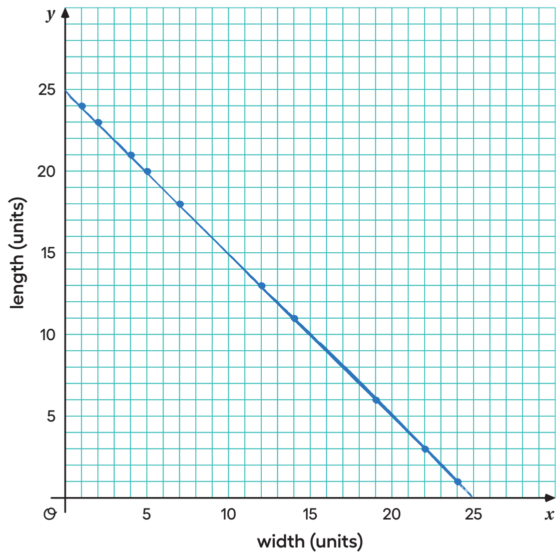
1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, ℓ, and widths, w, of at least 10 such rectangles.

   Sample response:

ℓ	1	4	7	12	24	19	2	5	14	22
w	24	21	18	13	1	6	23	20	11	3

2. On the graph, plot the length and width of rectangles whose perimeter is 50 units using the values from your table. Using a straightedge, draw the line that passes through these points.

   Sample response:



3. What is the slope of this line? What does the slope mean in this situation?

   The slope is -1.

   Sample response: For every 1 unit that the width increases, the length decreases by 1 unit.

4. Write an equation for this line.

   Sample responses:

   - $2\ell + 2w = 50$
   - $\ell + w = 25$
   - $\ell = 25 - w$
   - $\ell = -w + 25$

Activity Synthesis

The goal of this discussion is for students to observe how different forms of equations can reveal different information about a situation. Invite students who wrote $2\ell + 2w = 50$ (or equivalent) for an equation of the line to share and explain their reasoning. If no student wrote this equation, suggest it now. Invite students who wrote $\ell = 25 - w$ (or equivalent) to share and explain their reasoning. If not brought up in students' explanations, discuss:

- *"How does the equation $2\ell + 2w = 50$ describe this situation?"*

  A rectangle has 2 sides of length ℓ and 2 sides of length w and their sum must equal 50 units.

- *"How does the equation $\ell = 25 - w$ describe this line?"*

  The vertical intercept is 25 and the slope is -1.

- *"What does a vertical intercept of 25 mean in this situation?"*

  The rectangle has to have a length that is less than 25 since the width has to be positive.

- *"What are some advantages for using each form of equation?"*

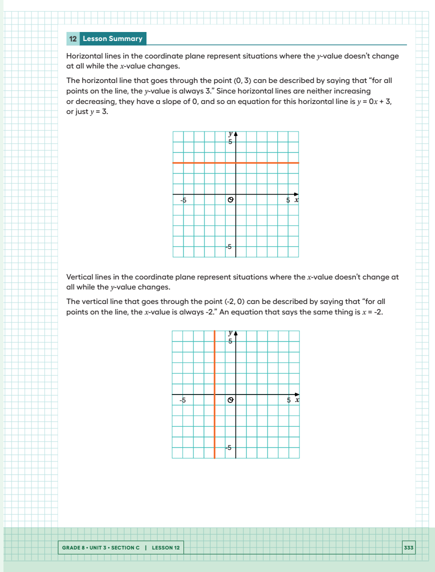
  The first equation makes it easier to see the relationship between the length and width while the second equation makes it easier to see what the line looks like.

Lesson Synthesis

The purpose of this discussion is for students to describe the similarities and differences between horizontal and vertical lines. Begin by creating a two-column table with the headings "horizontal lines" and "vertical lines" for all to see. Draw an example of each type of line in the appropriate column. Then ask students to identify other characteristics of each type of line. Here are some important points to emphasize:

- For horizontal lines the y-value is always the same and for vertical lines the x-value is always the same.
- For horizontal lines the equation will look like y = a number and for vertical lines the equation will look like x = a number.
- The slope of a horizontal line is always 0.

Student Workbook


12 Lesson Summary

Horizontal lines in the coordinate plane represent situations where the y-value doesn't change at all while the x-value changes.

The horizontal line that goes through the point (0, 3) can be described by saying that "for all points on the line, the y-value is always 3." Since horizontal lines are neither increasing or decreasing, they have a slope of 0, and so an equation for this horizontal line is $y = 0x + 3$, or just $y = 3$.

Vertical lines in the coordinate plane represent situations where the x-value doesn't change at all while the y-value changes.

The vertical line that goes through the point (-2, 0) can be described by saying that "for all points on the line, the x-value is always -2." An equation that says the same thing is $x = -2$.

Lesson Summary

Horizontal lines in the coordinate plane represent situations where the y-value doesn't change at all while the x-value changes.

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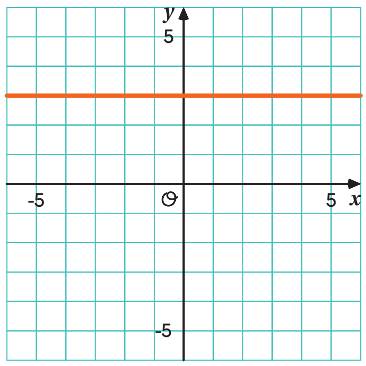
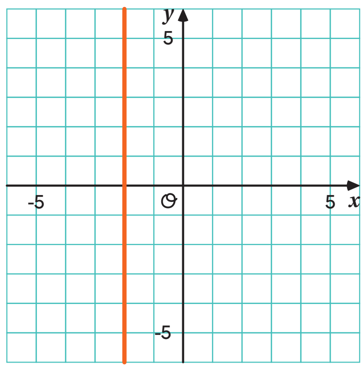


Vertical lines in the coordinate plane represent situations where the x-value doesn't change at all while the y-value changes.

The vertical line that goes through the point (-2, 0) can be described by saying that "for all points on the line, the x-value is always -2." An equation that says the same thing is $x = -2$.



Cool-down

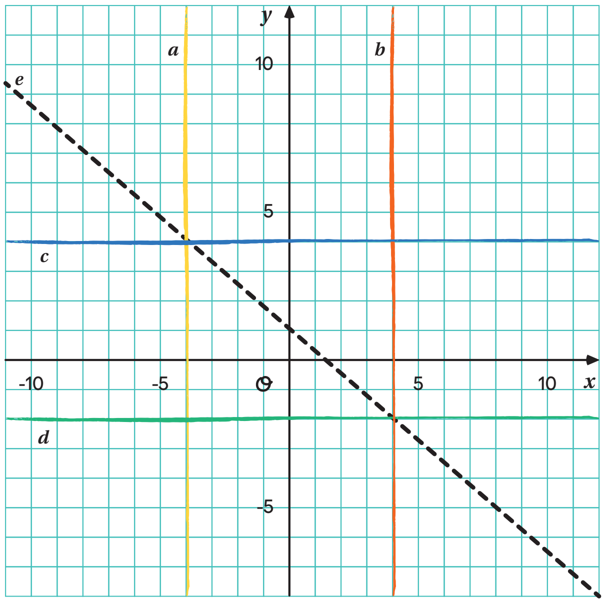
Five Lines

5 min

Students write equations for lines that are horizontal, vertical, or have negative slope.

Student Task Statement

Here are 5 lines in the coordinate plane:



Write equations for lines a, b, c, d, and e.

- line a: $x = -4$
- line b: $x = 4$
- line c: $y = 4$
- line d: $y = -2$
- line e: $y = \frac{-3}{4}x + 1$ (or equivalent)

Responding To Student Thinking

Points to Emphasize

If most students struggle with writing equations for horizontal and vertical lines, when discussing solutions to the activity referred to here, revisit what equations of horizontal and vertical lines look like.

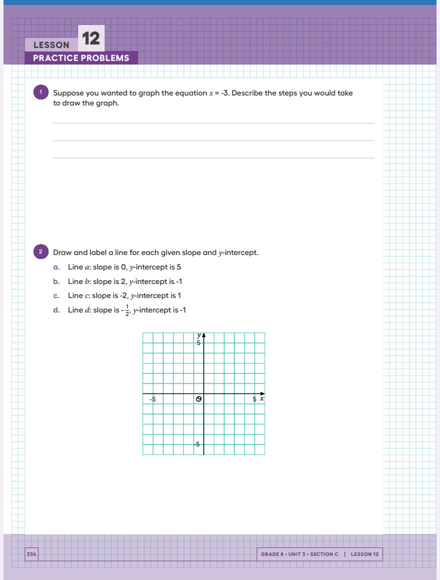
Unit 3, Lesson 14, Activity 1 Solutions in the Coordinate Plane

Lesson 12

Practice Problems

4 Problems

Student Workbook


LESSON 12
PRACTICE PROBLEMS

1. Suppose you wanted to graph the equation $x = -3$. Describe the steps you would take to draw the graph.

2. Draw and label a line for each given slope and y-intercept.
   a. Line a: slope is 0, y-intercept is 5
   b. Line b: slope is 2, y-intercept is -1
   c. Line c: slope is -2, y-intercept is 1
   d. Line d: slope is $-\frac{1}{2}$, y-intercept is -1

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Problem 1

Suppose you wanted to graph the equation $x = -3$. Describe the steps you would take to draw the graph.

Sample response: Draw a vertical line that passes through the point (-3, 0).

Problem 2

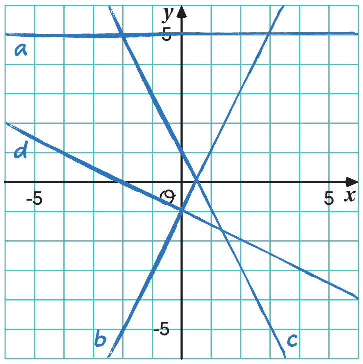
Draw and label a line for each given slope and y-intercept.

a. Line a: slope is 0, y-intercept is 5

b. Line b: slope is 2, y-intercept is -1

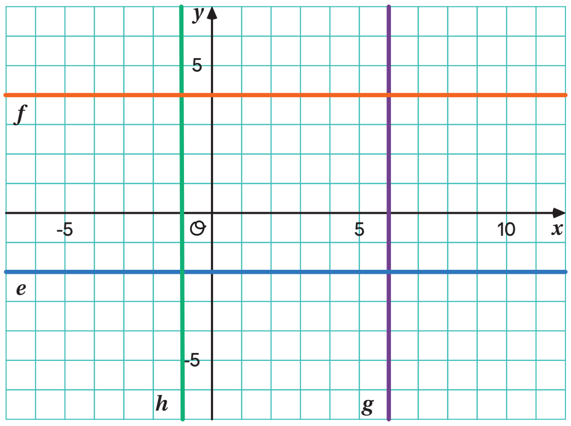
c. Line c: slope is -2, y-intercept is 1

d. Line d: slope is $-\frac{1}{2}$, y-intercept is -1



Problem 3

Write an equation for each line.



e: $y = -2$

f: $y = 4$

g: $x = 6$

h: $x = -1$

Problem 4

from Unit 3, Lesson 7

A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ of an inch. They have a choice of which type of paper to print the book on.

a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the width of the book in inches, y, if it has x hundred pages, printed on bond paper.

$y = \frac{1}{2} + \frac{1}{4}x$ (or equivalent)

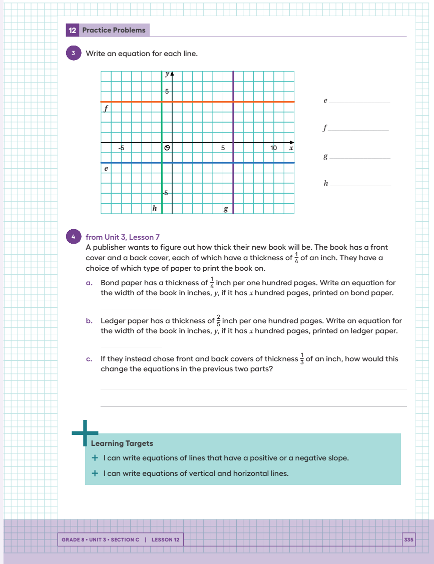
b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the width of the book in inches, y, if it has x hundred pages, printed on ledger paper.

$y = \frac{1}{2} + \frac{2}{5}x$ (or equivalent)

c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

Part a: $y = \frac{2}{3} + \frac{1}{4}x$, part b: $y = \frac{2}{3} + \frac{2}{5}x$

Student Workbook


12 Practice Problems

3 Write an equation for each line.

4 from Unit 3, Lesson 7

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a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the width of the book in inches, y, if it has x hundred pages, printed on bond paper.

b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the width of the book in inches, y, if it has x hundred pages, printed on ledger paper.

c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

Learning Targets

+ I can write equations of lines that have a positive or a negative slope.
+ I can write equations of vertical and horizontal lines.