Comparing Proportional Relationships

Goals

- Compare the rates of change for two proportional relationships, given multiple representations.

- Interpret multiple representations of a proportional relationship in order to answer questions (in writing), and explain the solution method.

- Present a comparison of two proportional relationships (using words and multiple other representations).

Learning Target

I can compare proportional relationships represented in different ways.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Required Materials

Materials to Gather

- Math Community Chart: Activity 1

- Tools for creating a visual display: Activity 1

Lesson Narrative

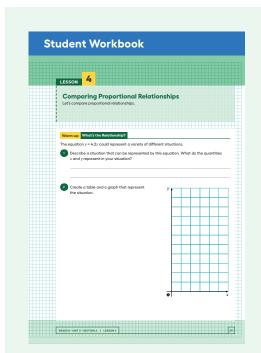In this lesson students compare two situations that are represented in different ways. For example, one situation might specify a rate of change, while the other is represented by a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they've garnered from each representation.

Student Learning Goal

Let's compare proportional relationships.

Lesson Timeline

10 min	**25** min	**10** min	**5** min
Warm-up	Activity 1	Lesson Synthesis	Cool-down

Assessment

Warm-up

What's the Relationship?

10 min

Activity Narrative

The purpose of this *Warm-up* is for students to describe a situation presented by an equation using other representations. Students decide on a context and then create a table and a graph, scaling the axes appropriately to the situation. Moving between representations of a proportional relationship here will be useful in a following activity where students compare proportional relationships represented in different ways.

Launch 👥

Arrange students in groups of 2. Give them 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

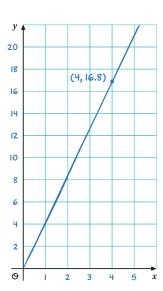The equation $y = 4.2x$ could represent a variety of different situations.

1. Describe a situation that can be represented by this equation. What do the quantities x and y represent in your situation?

   Sample response: A frog jumps 16.8 feet in 4 seconds, where x represents the time in seconds and y represents how far the frog jumps in feet.

2. Create a table and a graph that represent the situation.

   Sample response:

x	y
3	12.60
4	16.80

Activity Synthesis

Invite several students to share their situations and display their graphs for all to see. Ask:

💬 *"What does the rate of change represent in this situation?"*

   "How did you decide on the scale for your axes?"

Activity 1
Comparing Two Different Representations

25 min

Activity Narrative

In this activity, students consider representations of two different proportional relationships and make comparisons between them. They work in groups to compare the relationships and reason quantitatively to answer context specific questions. Groups make a visual display for their problem set to explain each of their responses and convince others of their accuracy.

Launch 👥

Math community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Tell students that the activity today is about summer jobs. Invite students to share their experiences with a summer job or any other experience they may have earning money, such as doing chores for a neighbor or selling handmade items. If desired, update the contexts of the question sets so they are more familiar to students or to better reflect the experiences of the class.

Then arrange students in groups of 2–3. Provide each group with tools for creating a visual display. Assign each group one of the three question sets (or have groups to choose). Tell groups they will make a visual display for their responses to the questions. The display should clearly demonstrate their reasoning and use multiple representations in order to be convincing. Let them know that there will be a gallery walk when they finish for the rest of the class to inspect the accuracy of their solutions.

If time allows, consider asking groups to complete all three problems and make a visual display for just one.

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.
In small groups, tell students to keep books or devices closed. Display only the *Task Statement* and two descriptions, without revealing the questions. Say,

   "We are going to read this information 3 times."

After the 1st read:

   "Tell your partner what this situation is about."

After the 2nd read:

   "List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask,

   "What are some ways one might get started on this?"

Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Develop Expression and Communication.
Provide students with alternatives to writing on paper: students can share their learning using digital technology, such as creating a digital presentation or slideshow.
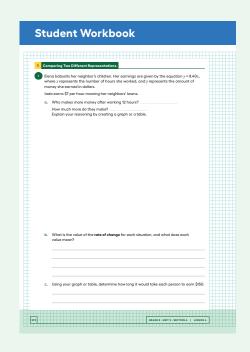
Supports accessibility for: Language, Fine Motor Skills

Building on Student Thinking

Some students may confuse the values for the rate of change of a situation. For example, Lemonade Recipe 1's equation, $y = 4x$, shows that the rate of change is 4 cups of water per cup of lemonade mix. Students may switch these values and think that the rate of change is 4 cups lemonade mix per cup of water. Consider asking:

"How did you find the rate of change and what does it mean?"

Prompting students to list a few additional values or sketch a graph to see if it matches their interpretation of the rate of change.

Student Workbook

Student Task Statement

1. Elena babysits her neighbor's children. Her earnings are given by the equation $y = 8.40x$, where x represents the number of hours she worked, and y represents the amount of money she earned in dollars.

   Jada earns $7 per hour mowing her neighbors' lawns.

   a. Who makes more money after working 12 hours?

      Elena.

      How much more do they make?

      Elena will make $16.80 more than Jada after working 12 hours.

      Explain your reasoning by creating a graph or a table.

      Sample reasoning: A graph shows hours worked along the x-axis and money earned in dollars along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 12 hours. The difference between the y-coordinates of these two points is $16.80.

   b. What is the value of the **rate of change** for each situation, and what does each value mean?

      Sample response: For Elena, the value of the rate of change is 8.4, which means that she earns $8.40 per hour babysitting. For Jada, the value of the rate of change is 7, which means that she earns $7.00 per hour mowing lawns.

   c. Using your graph or table, determine how long it would take each person to earn $150.

      Sample response: It will take Elena almost 18 hours and Jada almost 21.5 hours.

2. Clare and Han have summer jobs stuffing envelopes for two different companies.

   Han earns $15 for every 300 envelopes he finishes.

   Clare's earnings can be seen in the table.

number of envelopes	money earned in dollars
400	40
900	90

   a. By creating a graph, show how much money each person makes after stuffing 1,500 envelopes.

      Clare makes $150.00 and Han earns $75.00.

      Sample response: A graph shows envelopes stuffed along the x-axis and money earned in dollars along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 1,500 envelopes.

   b. What is the value of the rate of change for each situation, and what does each value mean?

      Sample response: For Clare, the value of the rate of change is $\frac{1}{10}$, which means that she makes $0.10 per stuffed envelope. For Han, the value of the rate of change is $\frac{15}{300}$, which means that he makes $0.05 per stuffed envelope.

c. Using your graph, determine how much more money one person makes relative to the other after stuffing 1,500 envelopes. Explain or show your reasoning.

After stuffing 1,500 envelopes, Clare makes $75 more than Han. Sample reasoning: At 1,500 envelopes, the difference between the y-coordinates of these two points is $75.

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn't use too much lemonade mix (lemon juice and sugar) but still tastes good.

Recipe 1 is given by the equation $y = 4x$, where x represents the amount of lemonade mix in cups, and y represents the amount of water in cups.

Recipe 2 is given in the table.

lemonade mix (cups)	water (cups)
10	50
13	65
21	105

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe?

Recipe 1: 64 cups of water, Recipe 2: 80 cups of water.

Explain your reasoning by creating a graph or a table.

Sample reasoning: A graph shows cups of lemonade mix along the x-axis and cups of water along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 16 cups of lemonade mix.
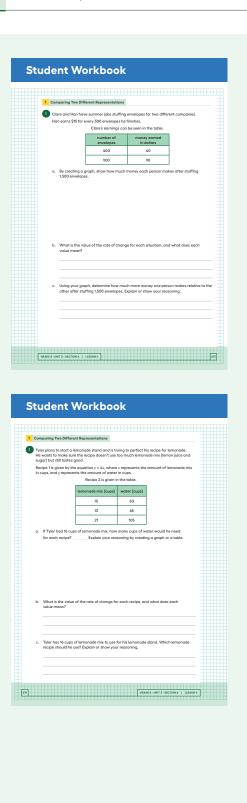
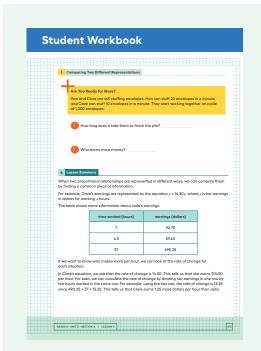b. What is the value of the rate of change for each recipe, and what does each value mean?

Sample response: For Recipe 1, the value of the rate of change is 4, which means that there are 4 cups of water per cup of lemonade mix. For Recipe 2, the value of the rate of change is 5, which means there are 5 cups of water per cup of lemonade mix.

c. Tyler has 16 cups of lemonade mix to use for his lemonade stand. Which lemonade recipe should he use? Explain or show your reasoning.

Sample responses:

○ Recipe 1 uses 4 cups of water for every cup of mix, while Recipe 2 uses 5 cups of water for every cup of mix. That means Recipe 2 will taste more watered down, so Tyler should use Recipe 1.

○ If Tyler uses all 16 cups of mix, he will use 64 cups of water with Recipe 1 and 80 cups of water with Recipe 2. Tyler should use Recipe 2 since Recipe 2 will make more lemonade to sell.

Student Workbook

Are You Ready for More?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?

$33\frac{1}{3}$ minutes

Sample reasoning: Working together they can stuff 30 envelopes per minute, so it takes them $\frac{1,000}{30} = 33\frac{1}{3}$ minutes to finish the pile.

2. Who earns more money?

Han and Clare earn the same amount of money.

Sample reasoning: Han stuffs twice as many envelopes as Clare, but he only earns half as much.

Activity Synthesis

Begin with a gallery walk for students to see how other groups answered the same set of questions they did. In small groups, invite students who created a display for the same set of problems to discuss what is the same and what is different about their work and representations on the posters. Here are some questions for discussion:

💬 *"What representations did you choose to answer the questions? Why did you pick them?"*

"What representation did you not use? Why?"

"How did you decide what scale to use when you made your graph?"

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

💬 *"I noticed our norm '_____' in action today and it really helped me/my group because_____."*

Lesson Synthesis

The goal is for students to discuss strategies they can use when comparing two proportional relationships. Have students conduct a second gallery walk to see how other groups answered questions about the two contexts they did not make a display for. Here are some questions for discussion:

💬 *"What was the same and different about the other problem sets?"*

Answers vary.

💬 *"What do you need in order to compare two proportional relationships?"*

the rate of change

💬 *"What type of wording in a problem statement or description of a situation tells you that you have a rate of change?"*

phrases like "per," "every," and "for each"

💬 *"How did you decide which representation to use to solve the different types of problems?"*

Answers vary.

Lesson Summary

When two proportional relationships are represented in different ways, we can compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation $y = 14.50x$, where y is her earnings in dollars for working x hours.

The table shows some information about Jada's earnings.

time worked (hours)	earnings (dollars)
7	92.75
4.5	59.63
37	490.25

If we want to know who makes more per hour, we can look at the rate of change for each situation.

In Clare's equation, we see that the rate of change is 14.50. This tells us that she earns $14.50 per hour. For Jada, we can calculate the rate of change by dividing her earnings in one row by the hours worked in the same row. For example, using the last row, the rate of change is 13.25 since 490.25 ÷ 37 = 13.25. This tells us that Clare earns 1.25 more dollars per hour than Jada.

Cool-down   **5** min

Different Salt Mixtures

Student Task Statement

Here are recipes for two mixtures of salt and water that taste different.

Information about Mixture A is shown in the table.

salt (teaspoons)	water (cups)
4	5
7	$8\frac{3}{4}$
9	$11\frac{1}{4}$

Mixture B can be described by the equation $y = 2.5x$, where x is the number of teaspoons of salt, and y is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.

   Mixture A uses 4 more teaspoons of salt than Mixture B.

   Sample reasoning: Mixture A would use 8 teaspoons of salt because I can double the row with 4 and 5 to get 8 and 10. Mixture B would use 4 teaspoons of salt because 10 = 2.5(4).

2. Which mixture tastes saltier? Explain your reasoning.

   Mixture A tastes saltier because it uses more salt for the same amount of water.

   Sample reasoning: Mixture A uses 8 teaspoons of salt for 10 cups of water and Mixture B only uses 4 teaspoons of salt for the same amount of water.
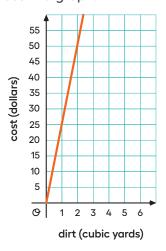
Problem 1

A contractor must haul a large amount of dirt to a worksite. She collected information from two hauling companies.

EZ Excavation gives its prices in a table.

Happy Hauling Service gives its prices in a graph.

dirt (cubic yards)	cost (dollars)
8	196
20	490
26	637

dirt (cubic yards)

a. How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.

EZ Excavation: $980, Happy Hauling Service: $1,000.

Sample reasoning: According to the table, EZ Excavation charges $490 for 20 cubic yards of dirt, so it would cost double that, or $980, for 40 cubic yards of dirt. According to the graph, Happy Hauling charges $25 for 1 cubic yard of dirt, so 40 cubic yards of dirt would cost $1,000 since $40 · 25 = $1,000.

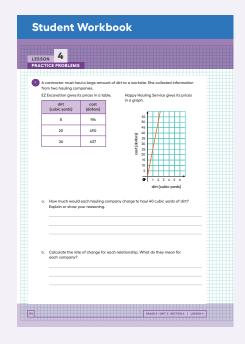**b.** Calculate the rate of change for each relationship. What do they mean for each company?

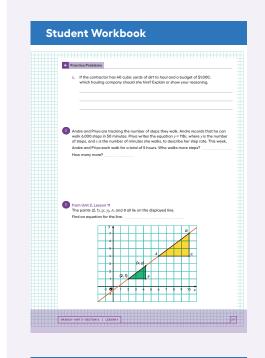EZ Excavation: 24.50 dollars per cubic yard, Happy Hauling Service: 25 dollars per cubic yard.

Sample response: For each cubic yard of dirt hauled to a worksite, the company charges a set amount of dollars.
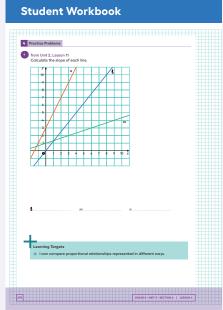
c. If the contractor has 40 cubic yards of dirt to haul and a budget of $1,000, which hauling company should she hire? Explain or show your reasoning.

Either company

Sample reasoning: Both companies would meet the budget requirement, yet EZ Excavation will cost $980 for 40 cubic yards and be under budget, while Happy Hauling would cost $1,000 and use all of the budget.

Problem 2

Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6,000 steps in 50 minutes. Priya writes the equation $y = 118x$, where y is the number of steps, and x is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?
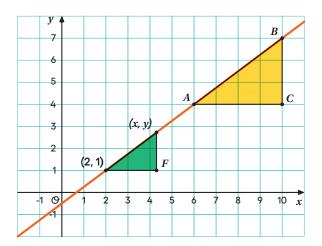
Andre walks 600 more steps than Priya.

Problem 3   from Unit 2, Lesson 11

The points $(2, 1)$, (x, y), A, and B all lie on the displayed line. Find an equation for the line.
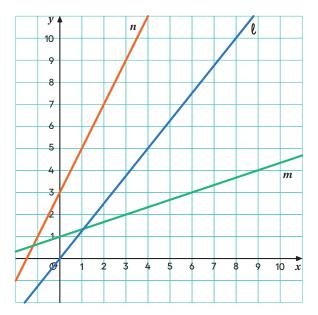
$\frac{y-1}{x-2} = \frac{3}{4}$ (or equivalent)

Problem 4   from Unit 2, Lesson 11

Calculate the slope of each line.

Line ℓ: $\frac{5}{4}$ (or equivalent)
Line m: $\frac{1}{3}$ (or equivalent)
Line n: 2 (or equivalent)