Lesson 8

Linear Functions

Goals

- Comprehend that any linear function can be represented by an equation in the form $y = mx + b$, where m and b are the rate of change and initial value of the function, respectively.
- Coordinate (orally and in writing) the graph of a linear function and its rate of change and initial value.

Learning Targets

- I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative.
- I can explain in my own words how the graph of a linear function relates to its rate of change and initial value.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Lesson Narrative

The purpose of this lesson is for students to connect linear relationships to functions and to understand that any linear function can be represented by $y = mx + b$.

In the first activity, students see that a proportional relationship between two quantities can be viewed as a function. They see that either quantity can be chosen as the independent variable and that the only difference in the equation and the graph is the constant of proportionality, which is visible on the graph as the slope of the line through the origin.

In the next activities, students investigate and make connections between linear functions as represented by graphs, descriptions, and by the equation $y = mx + b$. They interpret the slope of the line as the rate of change m of the dependent variable with respect to the independent variable and the vertical intercept of the line as the initial value b of the function. Students also compare properties of linear functions represented in different ways to determine, for example, which function has the greater rate of change.

The last activity is optional. Consider using this activity if students need more practice comparing linear functions represented in different ways.

Student Facing Learning Goal

Let's investigate linear functions.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	10 min	5 min

Warm-up
Bigger and Smaller

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to reason about the values that can be assigned to graphs depending on which feature of the graph, such as slope or y-intercept, the viewer focuses on. Since there are no numbers or other labels on the graph, it is important for students to explain how they know the sign of the slope and y-intercept based on the position of the graph.

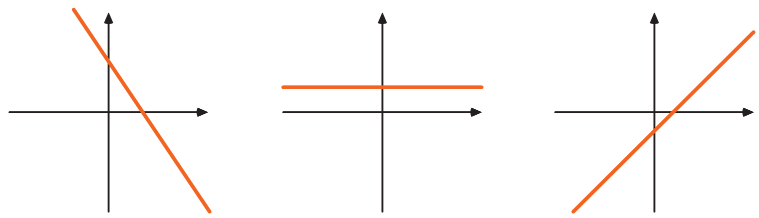
Launch

Display the three graphs for all to see. Tell students that all three graphs have the same scale.

Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Diego said that these graphs are ordered from smallest to largest. Mai said they are ordered from largest to smallest. But these are graphs, not numbers! What do you think Diego and Mai are thinking?



Sample reasoning:

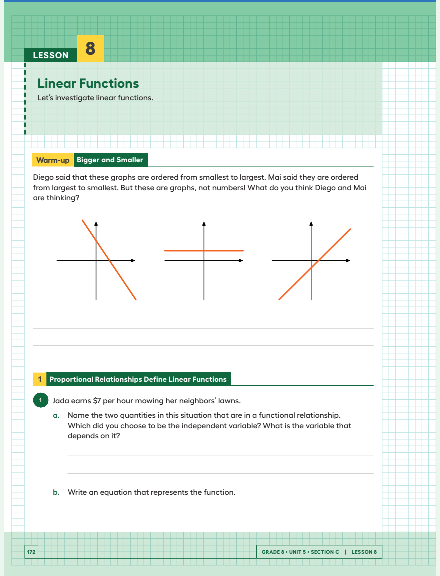
Diego:

- In the first graph, the line decreases when we read left to right. That means it has a negative slope. The second line stays horizontal the entire time, so it must have a slope of zero. The third graph increases as we read left to right, so it has a positive slope. That means the slopes are ordered from least to greatest.
- Diego is looking at the right side of each graph where the line "ends up."

Mai:

- The y-intercept of the first graph is positive and higher than the second graph. The y-intercept of the last graph is negative, so the y-intercepts are ordered from greatest to least.
- Mai is looking at the left side of each graph where the line "starts."

Student Workbook



Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.
Display sentence frames to support students as they justify the independent and dependent variables they selected. Examples: "______ is the independent variable because______" or "______ depends on ______ because ______."
Advances: Writing, Representing

Activity Synthesis

Display the three graphs for all to see. Invite students to share what they think Diego and Mai are thinking, and record student responses next to the graphs. Encourage students to reference the graphs in their explanation and to use precise language, like "y-intercept" and "slope." Emphasize that even though there are no numbers shown, we can tell the sign of the slope and the sign of the y-intercept by looking at the position of the line.

Activity 1

Proportional Relationships Define Linear Functions

15 min

Activity Narrative

The purpose of this activity is for students to recognize proportional relationships as linear functions. They also practice writing equations and using the language of functions to describe two proportional situations. A key part of this activity is students making connections between the situation and the possible equations and graphs that can be created to represent the situation.

Monitor for students who assign the independent and dependent variables in opposite ways. For example:

- For the first problem, some may have written $M = 7t$, and some may have written $t = \frac{1}{7}M$.
- For the second problem, some may have written $f = 3y$, and some may have written $y = \frac{1}{3}f$.

Launch

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time and then time to share responses with their partner. Follow with a whole-class discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially of students who haven't shared recently.

Student Task Statement

1. Jada earns \$7 per hour mowing her neighbors' lawns.

   a. Name the two quantities in this situation that are in a functional relationship. Which did you choose to be the independent variable? What is the variable that depends on it?

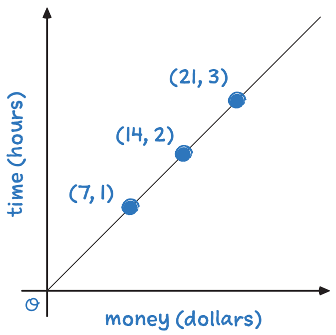
   The time, t, that Jada spends mowing lawns is in a functional relationship with the amount of money, M, that Jada earns. We can choose to think of t as a function of M, or vice versa.

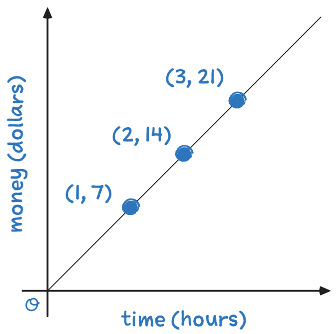
   b. Write an equation that represents the function.

   $M = 7t$ if we chose M as the dependent variable and t as the independent variable, or $t = \frac{M}{7}$ if we chose t as the dependent variable and M as the independent variable.

   c. Here is a graph of the function. Label the axes. Label at least two points with input-output pairs.

   Sample responses:





2. To convert feet to yards, we multiply the number of feet by $\frac{1}{3}$.

   a. Name the two quantities in this situation that are in a functional relationship. Which did you choose to be the independent variable? What is the variable that depends on it?

   The value of a measurement in yards, y, is in a functional relationship with the value of that same measurement in feet, f. We can choose to think of y as a function of f, or vice versa.

   b. Write an equation that represents the function.

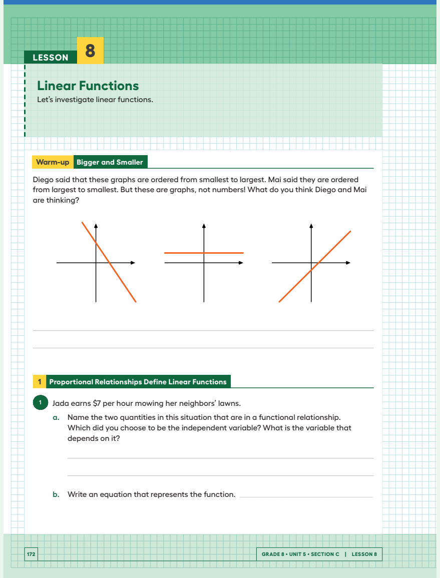
   $f = 3y$ if we choose f as the dependent variable and y as the independent variable, or $y = \frac{f}{3}$ if we choose y as the dependent variable and f as the independent variable.

Building on Student Thinking

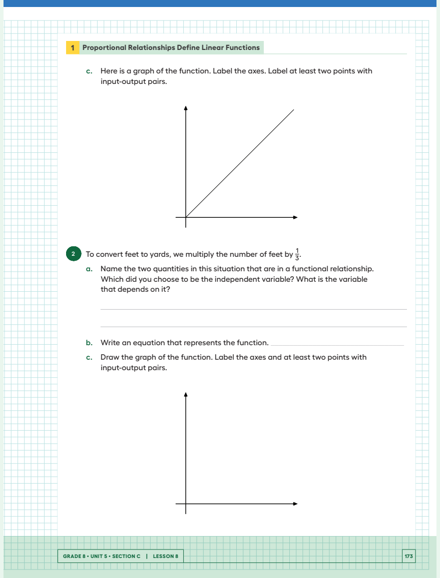
If two partners identify the same equation but use different letters for the variables or different scales on their axes and then try to compare their work, they may think one of them is incorrect. Consider asking:

"Tell me more about how you picked your variables."
"What is the same and what is different about the scales on your graphs?"

Student Workbook

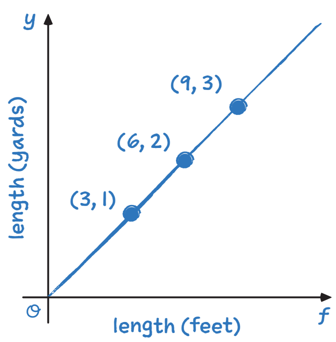


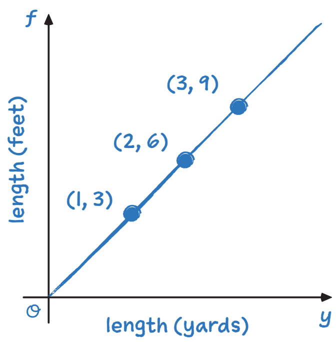
Student Workbook



c. Draw the graph of the function. Label the axes and at least two points with input-output pairs.

Sample responses:





Activity Synthesis

The goal of this discussion is for students to understand that proportional relationships are functions and to connect the parts of functions to what they know about proportional relationships.

Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"What do the graphs have in common? How are they different?"*

  Each graph uses the same variables, but the axes are switched.

- *"What do the equations have in common? Which is more useful?"*

  Both equations for Jada describe the same thing: time spent mowing lawns and money earned. The equation $t = \frac{1}{7}M$ is more helpful if Jada knows how much money she wants to earn and is trying to figure out how much time it will take while, $M = 7t$ is more helpful if Jada knows how long she spends mowing laws and is trying to calculate what she is owed.

Conclude the discussion by asking,

- *"How do we know that each of these situations are represented by functions?"*

  For each valid input, there is only one output. For example, no matter which equation I use for the relationship between feet and yards, a specific number of feet will always equal the same number of yards.

Activity 2

Is It Filling Up or Draining Out?

15 min

Activity Narrative

The purpose of this activity is to connect features of an equation representing a function to what that means in a context.

Students start with two functions about water tanks represented in different ways. They identify key facts about the situations, write an equation for one of the tanks, and practice interpreting information about contexts from equations and graphs. This thinking gives students the opportunity to connect initial value and slope, which they learned about in a previous unit, to the general form of the linear equation and to the fact that linear relationships are functions.

Launch

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time and then time to share their responses with their partner and reach an agreement on their answers.

Encourage partners to talk about specific parts of the graph and equation that indicate whether the tank is filling up or draining out. Follow with a whole-class discussion.

Student Task Statement

There are four tanks of water.

- The amount of water in gallons, A, in Tank A is a function of time in minutes, t, and can be represented by the equation $A = 200 + 8t$.
- The amount of water in gallons, B, in Tank B is a function of time in minutes, t. The amount of water starts at 400 gallons and is decreasing at 5 gallons per minute.

1. Which tank started out with more water?

   Tank B

   The two equations tell us that when $t = 0$, the volume of water in Tank A is 200 gallons, and the volume of water in Tank B is 400 gallons.

2. Write an equation representing the relationship between B and t.

   $B = 400 - 5t$

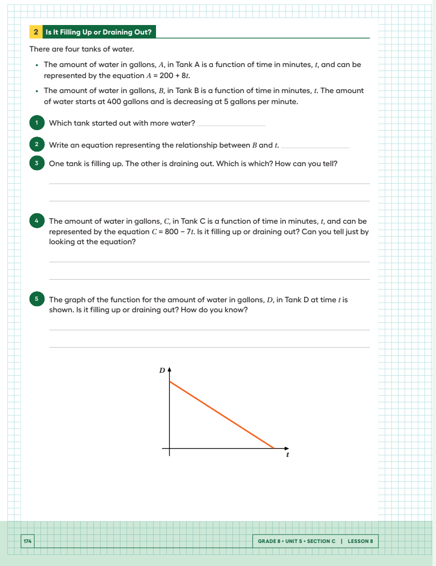
3. One tank is filling up. The other is draining out. Which is which? How can you tell?

   Tank A is filling up, and Tank B is draining out. As time goes on, corresponding to larger values of t, the value of A gets bigger, but the value of B gets smaller.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. For example, present one question at a time. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook



2 Is It Filling Up or Draining Out?

There are four tanks of water.

- The amount of water in gallons, A, in Tank A is a function of time in minutes, t, and can be represented by the equation $A = 200 + 8t$.
- The amount of water in gallons, B, in Tank B is a function of time in minutes, t. The amount of water starts at 400 gallons and is decreasing at 5 gallons per minute.

1. Which tank started out with more water? ______
2. Write an equation representing the relationship between B and t. ______
3. One tank is filling up. The other is draining out. Which is which? How can you tell?
4. The amount of water in gallons, C, in Tank C is a function of time in minutes, t, and can be represented by the equation $C = 800 - 7t$. Is it filling up or draining out? Can you tell just by looking at the equation?
5. The graph of the function for the amount of water in gallons, D, in Tank D at time t is shown. Is it filling up or draining out? How do you know?

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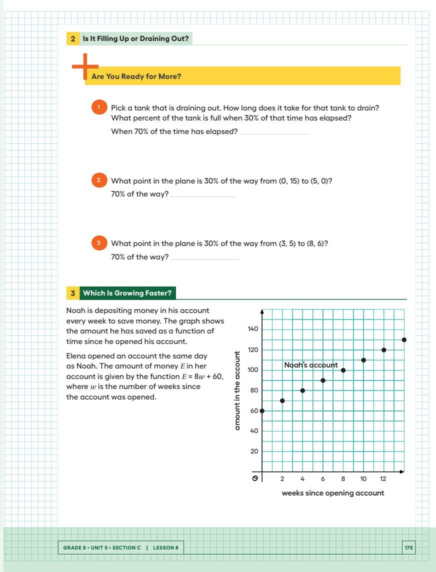
Student Workbook


2 Is It Filling Up or Draining Out?

Are You Ready for More?

1. Pick a tank that is draining out. How long does it take for that tank to drain? What percent of the tank is full when 30% of that time has elapsed?
When 70% of the time has elapsed? ____

2. What point in the plane is 30% of the way from (0, 15) to (5, 0)?
70% of the way? ____

3. What point in the plane is 30% of the way from (3, 5) to (8, 6)?
70% of the way? ____

3 Which Is Growing Faster?

Noah is depositing money in his account every week to save money. The graph shows the amount he has saved as a function of time since he opened his account.

Elena opened an account the same day as Noah. The amount of money E in her account is given by the function $E = 8w + 60$, where w is the number of weeks since the account was opened.

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4. The amount of water in gallons, C, in Tank C is a function of time in minutes, t, and can be represented by the equation $C = 800 - 7t$. Is it filling up or draining out? Can you tell just by looking at the equation?

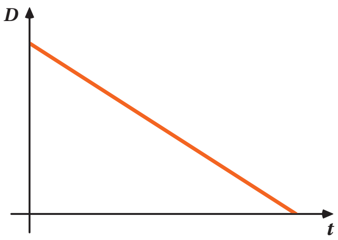
   Draining out

   As t increases, the value of C decreases since we are subtracting larger values from 800. We can see that Tank C is draining because we are subtracting multiples of t instead of adding them.

5. The graph of the function for the amount of water in gallons, D, in Tank D at time t is shown. Is it filling up or draining out? How do you know?

   Draining out

   As time increases, the value of D goes down.



Are You Ready for More?

1. Pick a tank that is draining out. How long does it take for that tank to drain? What percent of the tank is full when 30% of that time has elapsed? When 70% of the time has elapsed?

   Sample response:

   It takes 80 minutes for Tank B to drain, because $400 \div 5 = 80$. The tank is 70% full after 30% of that time has elapsed. The tank is 30% full after 70% of that time has elapsed.

2. What point in the plane is 30% of the way from (0, 15) to (5, 0)? 70% of the way?

   The point (1.5, 10.5) is 30% of the way from (0, 15) to (5, 0) because $0.3 \cdot 5 = 1.5$ and $0.7 \cdot 15 = 10.5$.
   The point (3.5, 4.5) is 70% of the way from (0, 15) to (5, 0) because $0.7 \cdot 5 = 3.5$ and $0.3 \cdot 15 = 4.5$.

3. What point in the plane is 30% of the way from (3, 5) to (8, 6)? 70% of the way?

   The point (4.5, 5.3) is 30% of the way from (3, 5) to (8, 6). The x-coordinates are 5 units apart because $8 - 3 = 5$. 30% of 5 is 1.5, and $3 + 1.5 = 4.5$.
   The y-coordinates are 1 unit apart, because $6 - 5 = 1$. 30% of 1 is 0.3, and $5 + 0.3 = 5.3$.
   The point (6.5, 5.7) is 70% of the way from (3, 5) to (8, 6). 70% of 5 is 3.5, and $3 + 3.5 = 6.5$. 70% of 1 is 0.7, and $5 + 0.7 = 5.7$.

Activity Synthesis

The goal of this discussion is for students to make connections between $y = mx + b$ and linear functions.

Consider asking some of the following questions to begin the discussion:

- ***"When you were writing an equation for Tank B, how did you decide if the value for m in $y = mx + b$ was positive or negative?"***

  Since the water in Tank B is decreasing, it has a negative rate of change.

- ***"What is similar and what is different between the equation for Tank B and the equation for Tank C? How would their graphs be similar and different?"***

  Both equations have negative rates of change and positive vertical intercepts. The initial value and rate of change for Tank C are greater than those for Tank B. Graphed, the vertical intercept of Tank C would be higher than Tank B, and the slope for Tank C would decrease steeper than the slope of Tank B.

- ***"For the last question, what in the graph tells you that Tank D is draining out? What would a graph that has a tank filling up look like? What would be different?"***

  I know it is draining out because the graph is going down from left to right. If the tank were filling up, the graph would be going up from left to right.

Tell students that a linear function can always be represented with an equation of the form $y = mx + b$. The slope of the line, m, is the rate of the change of the function, and the initial value of the function, b, is the vertical intercept.

If time allows, give students the following scenario to come up with a possible equation for Tank D:

- ***"Tank D started out with more water than Tank B but less water than Tank C. The water is draining from Tank D faster than from Tank B but slower than Tank C. What is a possible equation for the graph of the function for the amount of water D in Tank D over time t?"***

  Students should choose an initial value between 400 and 800 and a constant rate of change between -7 and -5. One possible such equation might be $D = 600 - 6t$.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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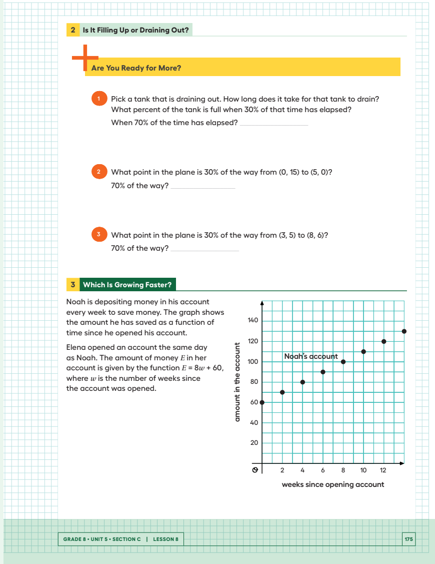
Student Workbook



Activity 3: Optional

Which Is Growing Faster?

10 min

Activity Narrative

The purpose of this activity is for students to connect their work with linear equations to functions. The two linear functions in this activity are represented differently, and students are asked to compare various features of each representation.

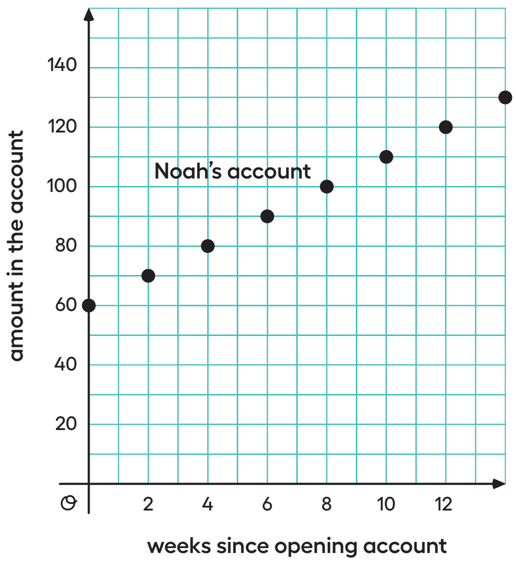
Identify students who use different methods to answer the questions. For example, students may write an equation to represent Noah's account, and others may make a table to show the value in each account at different numbers of weeks by reasoning about the rate of change and the amount in each account when they were opened.

Launch

Give students 3–5 minutes of quiet work time. Follow with a whole-class discussion.

Student Task Statement

Noah is depositing money in his account every week to save money. The graph shows the amount he has saved as a function of time since he opened his account.



Elena opened an account the same day as Noah. The amount of money E in her account is given by the function $E = 8w + 60$, where w is the number of weeks since the account was opened.

1. Who started out with more money in their account? Explain how you know.

   They are the same.

   Sample reasoning: At the left edge of the graph, representing the time when they opened the accounts, Noah had $60. When t is 0, the money in Elena's account when it was opened, $E = 8 \cdot 0 + 60$, so she also had $60.

2. Who is saving money at a faster rate? Explain how you know.

   Elaine is saving money at a faster rate.

   Sample reasoning: Every 2 weeks, Noah's account increases by 10, while Elena's account goes up by 8 each week, so she makes 16 in 2 weeks.

3. How much will Noah save over the course of a year if he does not make any withdrawals? How long will it take Elena to save that much?

   In addition to the 60 he opened the account with, Noah will save 260 over a year since he saves 10 every 2 weeks and there are 52 weeks in the year. It will take Elena about 33 weeks to save the same amount of 260. She also started with $60, and 260 ÷ 8 = 32.5, so rounding up, it will take 33 weeks.

Activity Synthesis

Display the graph of Noah's savings over time and the equation for the amount of money in Elena's account for all to see. Select students previously identified to share their responses.

Consider asking the following questions to help student make connections between the different representations:

"How did you determine the amount Noah saved in a year?"

I used the graph to figure out that Noah saves $5 each week and multiplied that by 52 weeks.

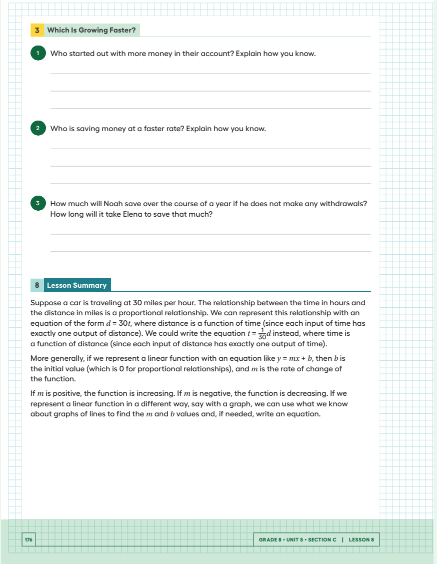
"What equations could you use to solve the last question?"

I could use the equation $N = 60 + 5w$ for the amount of money in Noah's account after w weeks. When $w = 52$, Noah has $320. If I solve the equation $320 = 8w + 60$ for w, I would know how many weeks it would take Elena to have $320 in her account.

"How could you solve the last question without using an equation?"

I could extend the graph out to 52 weeks and plot the value of each account over the year.

Student Workbook


3 Which Is Growing Faster?

1. Who started out with more money in their account? Explain how you know.

2. Who is saving money at a faster rate? Explain how you know.

3. How much will Noah save over the course of a year if he does not make any withdrawals? How long will it take Elena to save that much?

8 Lesson Summary

Suppose a car is traveling at 30 miles per hour. The relationship between the time in hours and the distance in miles is a proportional relationship. We can represent this relationship with an equation of the form $d = 30t$, where distance is a function of time (since each input of time has exactly one output of distance). We could write the equation $t = \frac{1}{30}d$ instead, where time is a function of distance (since each input of distance has exactly one output of time).

More generally, if we represent a linear function with an equation like $y = mx + b$, then b is the initial value (which is 0 for proportional relationships), and m is the rate of change of the function.

If m is positive, the function is increasing. If m is negative, the function is decreasing. If we represent a linear function in a different way, say with a graph, we can use what we know about graphs of lines to find the m and b values and, if needed, write an equation.

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Access for Multilingual Learners (Activity 3, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "Who started out with more money in their account? Explain how you know." Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Lesson Synthesis

To help students make connections between work they have done previously with linear equations and functions, consider asking some of the following questions:

- *"How can we tell if a linear function is increasing from an equation? From a graph?"*

  In a linear equation $y = mx + b$, if m is positive, the linear function is increasing. In a graph, if the line is going up from left to right, then the function is increasing.

- *"How can we tell from the graph if the initial value of the function is positive? How can we tell from the equation?"*

  If the graph of the function crosses the vertical axis above 0, then the initial value is positive. In the equation, $y = mx + b$ of a linear function, the b is positive when the initial value is positive.

Lesson Summary

Suppose a car is traveling at 30 miles per hour. The relationship between the time in hours and the distance in miles is a proportional relationship. We can represent this relationship with an equation of the form $d = 30t$, where distance is a function of time (since each input of time has exactly one output of distance). Or we could write the equation $t = \frac{1}{30}d$ instead, where time is a function of distance (since each input of distance has exactly one output of time).

More generally, if we represent a linear function with an equation like $y = mx + b$, then b is the initial value (which is 0 for proportional relationships), and m is the rate of change of the function. If m is positive, the function is increasing. If m is negative, the function is decreasing. If we represent a linear function in a different way, say with a graph, we can use what we know about graphs of lines to find the m and b values and, if needed, write an equation.

Cool-down

Beginning to See Daylight

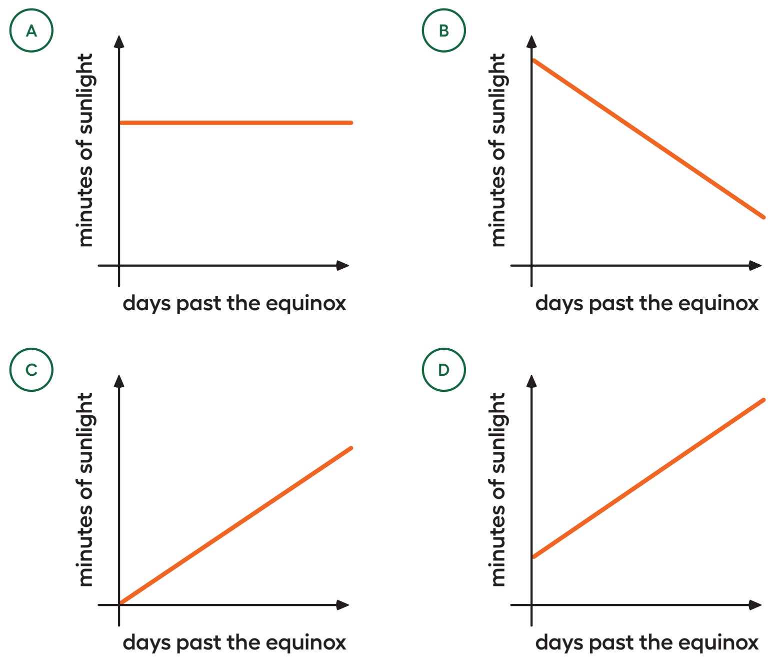
5 min

Launch

Explain that an equinox is the day when there is an approximately equal amount of daylight and darkness.

Student Task Statement

In a certain city in France, they gain 2 minutes of daylight each day after the spring equinox (usually in March), but after the autumnal equinox (usually in September), they lose 2 minutes of daylight each day.



1. Which of the graphs is most likely to represent the graph of daylight for the month after the spring equinox?

   D

2. Which of the graphs is most likely to represent the graph of daylight for the month after the autumnal equinox?

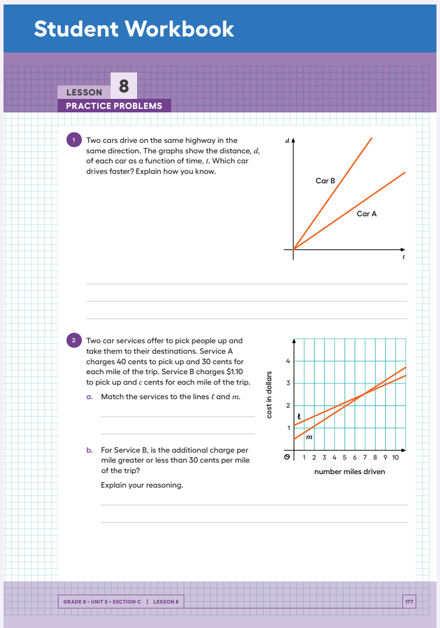
   B

3. Why are the other graphs not likely to represent either month?

   Graph A does not make sense because there is a constant amount of daylight. Graph C does not make sense because it goes through the origin, meaning it started with 0 minutes of daylight.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.



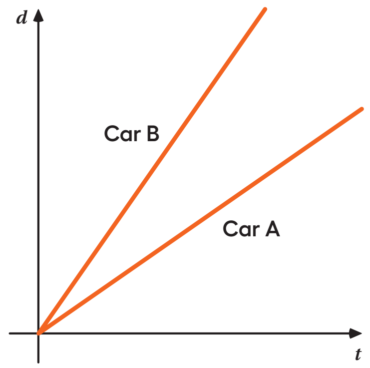
Lesson 8

Practice Problems

4 Problems

Problem 1

Two cars drive on the same highway in the same direction. The graphs show the distance, d, of each car as a function of time, t. Which car drives faster? Explain how you know.

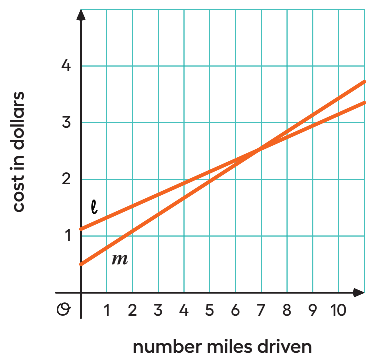


Car B

Sample reasoning: The two cars began at the same place, but after any amount of time, Car B has traveled farther than Car A. Graphically, the slope of the line corresponding to Car B is greater than the slope of the line corresponding to Car A, so the rate of change of distance per time (speed) is higher for Car B.

Problem 2

Two car services offer to pick people up and take them to their destinations. Service A charges 40 cents to pick up and 30 cents for each mile of the trip. Service B charges $1.10 to pick up and c cents for each mile of the trip.



a. Match the services to the lines ℓ and m.

Service A is represented by line m. Service B is represented by line ℓ.

b. For Service B, is the additional charge per mile greater or less than 30 cents per mile of the trip? Explain your reasoning.

Less than 30 cents per mile

Sample reasoning: Line ℓ is not increasing as quickly as line m.

Problem 3

Kiran and Clare like to race each other home from school. They run at the same speed, but Kiran's house is slightly closer to school than Clare's house is. On a graph, their distances (in meters) from their homes is a function of the time (in seconds) from when they begin the race.

a. As you read the graphs left to right, would the lines go up or down?

down

b. What is different about the lines representing Kiran's run and Clare's run?

Sample response: Clare's line would be higher up since she started farther away from her house.

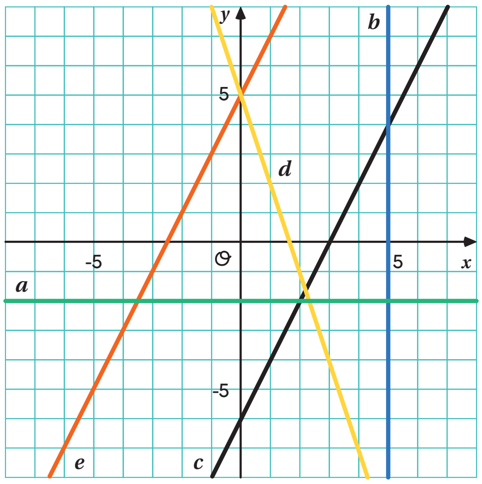
c. What is the same about the lines representing Kiran's run and Clare's run?

Sample response: The lines would have the same slope since they run at the same speed.

Problem 4

from Unit 3, Lesson 12

Write an equation for each line.



a. Line a: $y = -2$

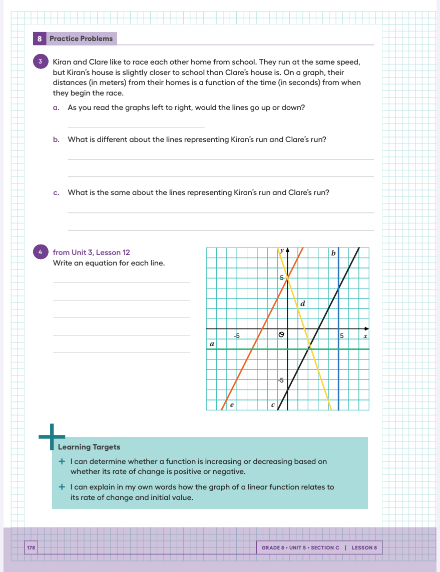
b. Line b: $x = 5$

c. Line c: $y = 2x - 6$

d. Line d: $y = -3x + 5$

e. Line e: $y = 2x + 5$

Student Workbook



8 Practice Problems

3 Kiran and Clare like to race each other home from school. They run at the same speed, but Kiran's house is slightly closer to school than Clare's house is. On a graph, their distances (in meters) from their homes is a function of the time (in seconds) from when they begin the race.

a. As you read the graphs left to right, would the lines go up or down?

b. What is different about the lines representing Kiran's run and Clare's run?

c. What is the same about the lines representing Kiran's run and Clare's run?

4 from Unit 3, Lesson 12

Write an equation for each line.

Learning Targets

+ I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative.

+ I can explain in my own words how the graph of a linear function relates to its rate of change and initial value.