Comparing Relationships with Equations

Goals **Learning Target**

- Compare and contrast (orally) equations that do and do not represent proportional relationships.
- Generalize that an equation equivalent to the form y = kx can represent a proportional relationship.
- Use a table to determine whether a given equation represents a proportional relationship, and justify (in writing) the decision.

I can decide if a relationship represented by an equation is proportional or not.

Lesson Narrative

In this lesson, students examine equations of proportional and nonproportional relationships. They use the equation to complete a table of values and then divide the pairs of values on each row to determine whether the relationship is proportional. The focus is on looking for structure, seeing the connection between the form of the equation and the kind of relationship it represents. Students should see by the end of this lesson that equations of the form y = kx characterize proportional relationships (for k > 0).

The last activity is optional because it provides an opportunity for additional practice determining whether equations represent proportional relationships, this time without a context to help students focus on the structure of the equations.

Student Learning Goal

Let's develop methods for deciding if a relationship is proportional.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 3)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR3: Critique, Correct, Clarify (Activity 3)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- · MLR3: Critique, Correct, Clarify
- MLR7: Compare and Connect
- · Notice and Wonder

Required Materials

Materials to Gather

• Snap cubes: Activity 2

Required Preparation

Lesson:

Calculators can optionally be made available to take the focus off computation.

Lesson Timeline



Warm-up

15

Activity 1



Activity 2

10

Activity 3

10

Lesson Synthesis



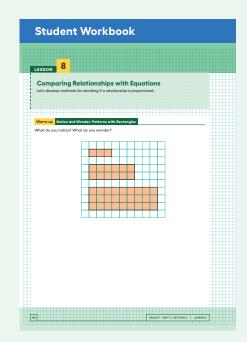
Cool-down

Instructional Routines

Notice and Wonder ilclass.com/r/10694948







Warm-up

Notice and Wonder: Patterns with Rectangles



Activity Narrative

The purpose of this *Warm-up* is to elicit comparisons of lengths and areas, which will be useful when students decide whether such relationships are proportional in a later activity. While students may notice and wonder many things about these images, the important discussion points are the way that the side lengths, perimeter, and area are increasing.

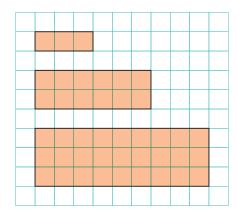
When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch 🙎

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- · There are three rectangles on the grid.
- The rectangles are scaled copies.
- The rectangles increase in size from top to bottom.
- The width increases by 3 each time.
- · The height increases by I each time.
- The small rectangle could fit into the medium rectangle 4 times, and it could fit into the large rectangle 9 times.

Students may wonder:

- By how much does the area increase each time?
- By how much does the perimeter increase each time?
- · Why are the rectangles getting larger?
- · Does this pattern keep going?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the idea of continuing the pattern does not come up during the conversation, ask students to discuss this idea:

"What predictions can you make about future rectangles in the set if the pattern continues?"

Activity 1

More Conversions

15 min

Activity Narrative

In this activity students use equations to create tables and decide whether the relationships are proportional. Students have previously looked at measurement conversions that can be represented by proportional relationships. This task introduces a measurement conversion that is not associated with a proportional relationship.

Monitor for students who use these different strategies to justify whether each relationship is proportional:

- Look for patterns in the table, such as scale factors between rows or constant of proportionality between columns
- Use the structure of the equation, whether it represents multiplying one quantity by a constant of proportionality to get the other quantity

Launch 🙎

Arrange students in groups of 2. Provide access to calculators, if desired. Give students 5 minutes of quiet work time followed by partner and whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Instructional Routines

MLR7: Compare and Connect

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Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.

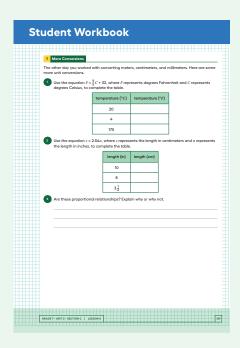
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Building on Student Thinking

Some students may struggle with the fraction $\frac{9}{5}$ in the temperature conversion. Teachers can prompt them to convert the fraction to its decimal form, 1.8, before trying to evaluate the equation for the values in the table.

Some students may think of the two scales on a thermometer like a double number line diagram, leading them to believe that the relationship between degrees Celsius and degrees Fahrenheit is proportional. Point out that when a double number line is used to represent a set of equivalent ratios, the tick marks for 0 on each line need to be aligned.



Student Task Statement

The other day you worked with converting meters, centimeters, and millimeters. Here are some more unit conversions.

1. Use the equation $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, to complete the table.

temperature (°C)	temperature (°F)
20	68
4	39.2
175	347

2. Use the equation c = 2.54n, where c represents the length in centimeters and n represents the length in inches, to complete the table.

length (in)	length (cm)
10	25.4
8	20.32
3 ½	8.89

3. Are these proportional relationships? Explain why or why not.

The temperature conversion does not determine a proportional relationship because the number of degrees Fahrenheit per degree Celsius is not the same. The length conversion does determine a proportional relationship because the number of centimeters per inch is the same.

Activity Synthesis

The goal of this discussion is to start to move students from determining whether a relationship is proportional by examining a table, to making the determination from the equation. Display 2–3 approaches from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the strategies have in common? How are they different?"
"How does the constant of proportionality show up in each method?"

"Are there any benefits or drawbacks to one approach compared to another?"

The key takeaway is that the equation for the proportional relationship is of the form y = kx, while the equation for the nonproportional relationship is not.

- When we use an equation of the form y = kx to create a table, the values in the columns will be related by the constant of proportionality k.
- When we use an equation that is not of the form y = kx to create a table, the values in the columns will likely not be related by a constant of proportionality. (There's a small possibility that a given equation could be equivalent to an equation of the form y = kx, but showing this would require algebraic skills that students have not yet developed. Examples: y = 5(x + 2) 10 or $y = (x + 3)^2 x^2 9$)

Activity 2

Total Edge Length, Surface Area, and Volume



Activity Narrative

This activity uses a geometric context to give students more practice with deciding whether relationships are proportional. The context of surface area and volume of a cube should be familiar from grade 6.

The units for the quantities are purposely not given in the task statement to avoid giving away which relationships are not proportional. However, discussion should raise the possible units of measurement for edge length, surface area, and volume.

Launch

Arrange students in groups of 2. Display a cube (such as a cardboard box) for all to see and ask:

"How many edges are there?"

"How long is one edge?"

"How many faces are there?"

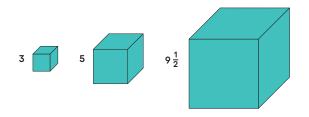
"How large is one face?"

Give students 5 minutes of quiet work time, followed by partner and wholeclass discussion.

Watch carefully as students work and be ready to provide guidance or equations as needed, so students can get to the central purpose of the task, which is noticing the correspondences between the nature of relationships and the form of their equations.

Student Task Statement

Here are some cubes with different side lengths. Complete each table. Be prepared to explain your reasoning.



Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display

This activity uses the Collect and Display math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.

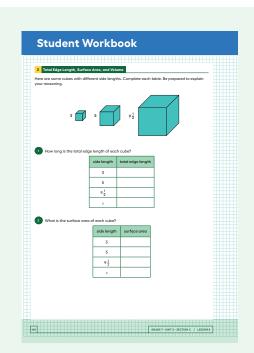
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Give students a physical cube to help develop understanding of "side length," "surface area," and "volume."

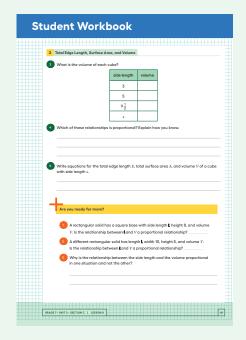
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Building on Student Thinking

Some students may struggle to complete the tables. Teachers can use nets of cubes (flat or assembled) partitioned into square units to reinforce the process for finding total edge length, surface area, and volume of the cubes in the task. Snap cubes would also be appropriate supports.

If difficulties with the fractional side length $9\frac{1}{2}$ keep students from being able to find the surface area and volume or write the equations, the teacher can tell those students to replaceh 9 $\frac{1}{2}$ with 10 and retry their calculations. Their answers for surface area and volume will be different for that row in the table, but their equations and proportionality decisions will be the same. That way they can still learn the connection between the form of the equations and the nature of the relationships.





Students may construct the third column to reach this conclusion.

1. How long is the total edge length of each cube?

side length	total edge length	total edge length side length
3	36	12
5	60	12
9 1/2	114	12
S	12s	12

A cube has 12 edges.

2. What is the surface area of each cube?

side length	surface area	surface area side length
3	54	18
5	150	30
9 1/2	541 <u>1</u>	57
s	6 <i>s</i> ²	6s

A cube has 6 faces each with an area of s^2 square units.

3. What is the volume of each cube?

side length	volume	volume side length
3	27	9
5	125	25
9 1/2	857 3	90 4
s	5 ³	5 ²

The bottom layer of a cube fits s^2 cubic units and s of these layers make up the cube.

4. Which of these relationships is proportional? Explain how you know.

The relationship between side length and total edge length is proportional because the ratio in the third column is 12 for every side length. The relationships between side length and surface area and between side length and volume are not proportional because the ratios in the third column of tables 2 and 3 are not the same for each side length.

- **5.** Write equations for the total edge length E, total surface area A, and volume V of a cube with side length s.
 - \circ E = 12s
 - $o A = 6s^2$
 - o $V = S^3$

Are You Ready for More?

1. A rectangular solid has a square base with side length ℓ , height 8, and volume V. Is the relationship between ℓ and V a proportional relationship?

no

2. A different rectangular solid has length ℓ , width 10, height 5, and volume V. Is the relationship between ℓ and V a proportional relationship?

yes

3. Why is the relationship between the side length and the volume proportional in one situation and not the other?

In one situation, there are two unknown dimensions and in the other, there is only one. So even though these situations look very similar, the relationship is different.

Activity Synthesis

The goal of the discussion is for students to recognize that the proportional relationship has an equation of the form y = kx while the nonproportional relationships do not.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share what they notice about the equations for the relationships. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. Display words and phrases such as "factor," "number next to the variable," "coefficient," "exponent," "operation," "not proportional." Make sure students see that the equation for the proportional relationship is of the form y = kx, and the others are not.

If time permits, consider asking:

"What could be possible units for the side lengths?"

linear measurements: centimeters, inches

"Then what would be the units for the surface area?"

square units: square centimeters, square inches

"What would be the units for the volume?"

cubic units: cubic centimeters, cubic inches

Connect the units of measurements with the structure of the equation for each quantity: the variable that represents the side length and the units are raised to the same power.

Activity 3: Optional

All Kinds of Equations

10 min

Activity Narrative

This activity involves checking for a constant of proportionality in tables generated from simple equations. Students should be able to evaluate these equations from their work with expressions and equations in grade 6. The purpose of this activity is to generalize about the forms of equations that do and do not represent proportional relationships.

Access for Multilingual Learners (Activity 3)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique*, *Correct*, *Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Internalize Self Regulation.

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to any four of the six equations. In particular, recommend students start with y = 4x and $y = \frac{x}{4}$.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students might struggle to see that the two proportional relationships have equations of the form y = kx and to characterize the others as not having equations of that form. Students do not need to completely articulate this insight for themselves. This synthesis should emerge in the whole-class discussion.

Student W	orkbook		
3 All Kinds of Equations			
Here are six different equations		4	
y = 4 + x	y = 4x	$y = \frac{4}{x}$	
$y = \frac{x}{4}$	y = 4+	y = x4	
Predict which of these equ	uations represent a proportion	I relationship.	
Complete each table usin	g the equation that represents	de a calada a cala	
v = 4 + x	y = 4x	y = 4	
x y y	x y y	x y 7	
2	2	2	
3	3	3	
	 		
5	5	5	
$y = \frac{x}{4}$	y = 4°	$y = x^{\Delta}$	
x y y/x	x y y/x	x y <u>x</u>	
2	2	2	
3	3	3	
4	4	4	
5	5	5	
Do these results change y	our answer to the first question	? Explain your reasoning.	
Mhat do the equations of			
What do the equations of the proportional relationships have in common?			
162	GRADET - UNIT 2 - SECTION C LESSON B		

The relationships in this activity are presented without a context so that students can focus on the structure of the equations without being distracted by what the variables represent.

Launch 228

Activity 1

Arrange students in groups of 2–3. Provide access to calculators.

If time is limited, consider instructing group members to each complete 2–3 of the tables and then share their results with each other.

Give students 5 minutes of quiet work time followed by partner and wholeclass discussion.

Student Task Statement

Here are six different equations.

$$y = 4 + x$$

$$y = 4x$$

$$y = \frac{4}{x}$$

$$y = \frac{x}{4}$$

$$y = 4^x$$

$$y = x^2$$

1. Predict which of these equations represent a proportional relationship.

Sample response: u = 4x and $u = \frac{x}{4}$ represent proportional relationships.

Sample response: y = 4x and $y = \frac{x}{4}$ represent proportional relationships, but the others do not.

2. Complete each table using the equation that represents the relationship.

$$y = 4 + x$$

x	у	$\frac{y}{x}$
2	6	3
3	7	2 3
4	8	2
5	9	I 4 5

$$y = 4x$$

x	у	$\frac{y}{x}$
2	8	4
3	12	4
4	16	4
5	20	4

$$y = \frac{4}{x}$$

x	у	$\frac{y}{x}$
2	2	1
3	4/3	49
4	ı	14
5	<u>4</u> 5	4 25

$$y = \frac{x}{4}$$

x	у	$\frac{y}{x}$
2	1/2	1/4
3	3 4	1/4
4	1	14
5	<u>5</u>	14
		-

$y = 4^x$		
X	у	$\frac{y}{x}$
2	16	8
3	64	21 1 3
4	256	64
5	1,024	204 4 5

$y = x^4$		
У	$\frac{y}{x}$	
16	8	
81	27	
256	64	
625	125	
	y 16 81 256	

3. Do these results change your answer to the first question? Explain your reasoning.

Warm-up

- Sample response: No, they just confirm that y = 4x and $y = \frac{x}{4}$ represent proportional relationships, but the others do not.
- 4. What do the equations of the proportional relationships have in common?

The equations representing proportional relationships:

- Can be written in the form y = kx.
- · Do not contain any exponents or addition operations.
- Do not involve dividing by x.

Activity Synthesis

Invite students to share what the equations for proportional relationships have in common, and by contrast, what is different about the other equations.

Use *Critique*, *Correct*, *Clarify* to give students an opportunity to improve a sample written response about which of these equations represent a proportional relationship, by correcting errors, clarifying meaning, and adding details.

- Display this first draft:
- "Just the equation that uses multiplication is proportional, and all the other equations are not proportional."

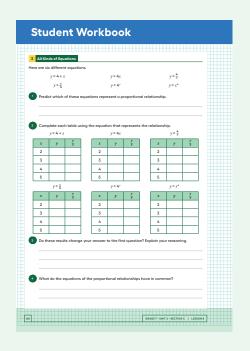
Ask, "What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

Select 1–2 individuals or groups to read their revised draft aloud slowly
enough to record for all to see. Scribe as each student shares, then invite
the whole class to contribute additional language and edits to make the
final draft even more clear and more convincing.

The key takeaway is that any equation that can be written in the form y = kx represents a proportional relationship. At first glance, the equation $y = \frac{x}{4}$ does not look like our standard equation for a proportional relationship, y = kx. Suggest to students that they rewrite the equation using the constant of proportionality they found after completing the table: y = 0.25x which can also be expressed $y = \frac{1}{4}x$. If students do not express this idea themselves, remind them that they can think of dividing by 4 as multiplying by $\frac{1}{4}$.



Lesson Synthesis

Share with students,

"Today we learned some ways to tell whether an equation could represent a proportional relationship."

To help students generalize about equations of proportional relationships, consider asking students:

"What are some equations we have seen for proportional relationships?"

$$c = 2.54n$$
 $E = 12s$ $y = 4x$
 $y = \frac{1}{4}x$ $y = \frac{1}{2}n$ $t = 2.5d$
 $c = 62.5w$ $r = 0.014c$ $r = 0.875w$

"What do you notice about the form of these equations?"

They are of the form y = kx, where k is the constant of proportionality. They have one variable being multiplied by a number to equal the other variable.

"Why do these equations all have this form?"

In a proportional relationship, any value for one quantity can be multiplied by the constant of proportionality to find the corresponding value for the other quantity.

"What are some equations that we have seen for relationships that are not proportional?"

$$F = \frac{9}{5}C + 32$$
 $A = 6s^2$ $V = s^3$ $y = 4 + x$

"How are these equations different from the ones for the proportional relationships?"

They include other operations, such as addition or exponents.

Students might say that equations of the form $y = \frac{x}{n}$, where n is a constant, are also proportional. This is true, however we do not need this additional form to represent proportional relationships of this type. If this comes up, point out that these equations could also be written as $y = \frac{1}{n}x$, which is in the form of y = kx. In this form, we see that $k = \frac{1}{n}$.

Lesson Summary

If two quantities are in a proportional relationship, then their quotient is always the same. This table represents different values of a and b, two quantities that are in a proportional relationship.

а	b	$\frac{b}{a}$
20	100	5
3	15	5
11	55	5
1	5	5

Notice that the quotient of b and a is always 5. To write this as an equation, we could say $\frac{b}{a}$ = 5. If this is true, then b = 5a. (This doesn't work if a = 0, but it works otherwise.)

If quantity y is proportional to quantity x, we will always see that $\frac{y}{x}$ has a constant value. This value is the constant of proportionality, which we often refer to as k. We can represent this relationship with the equation $\frac{y}{x} = k$ (as long as x is not 0) or y = kx.

Note that if an equation cannot be written in this form, then it does not represent a proportional relationship.

Cool-down

Tables and Chairs

5 min

Student Task Statement

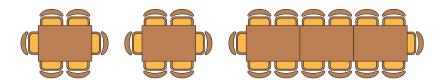
Andre is setting up rectangular tables for a party. He can fit 6 chairs around a single table. Andre lines up 10 tables end-to-end and tries to fit 60 chairs around them, but he is surprised when he cannot fit them all.

- **1.** Write an equation for the relationship between the number of chairs c and the number of tables t when:
- the tables are apart from each other:

When the tables are apart: c = 6t or $t = \frac{1}{6}c$

• the tables are placed end-to-end:

When the tables are together: c = 4t + 2 or $t = \frac{1}{4}c - \frac{1}{2}$



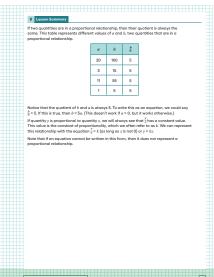
Responding To Student Thinking

Points to Emphasize

If students struggle with explaining whether or not a relationship is proportional, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about how they knew that each situation was proportional.

Unit 2, Lesson 9, Activity 2 Info Gap: Biking and Rain

Student Workbook



2. Is the first relationship proportional? Explain how you know.

This relationship is proportional.

Sample reasoning:

- It can be represented with an equation of the form c = kt (or t = kc).
- · There are 6 chairs per table no matter how many tables.
- **3.** Is the second relationship proportional? Explain how you know.

This relationship is not proportional.

Sample reasoning:

- The number of chairs per table changes depending on how many tables there are.
- · The quotient of chairs and tables is not constant.
- The relationship cannot be expressed with an equation of the form c = kt.

As shown in this table, the number of chairs per table is the same when the tables are apart, but it is not the same if the tables are pushed together.

With tables apart:

tables	chairs	chairs tables
ı	6	6
2	12	6
3	18	6
4	24	6
10	60	6
t	6t	6

With tables end-to-end:

tables	chairs	chairs tables
1	6	6
2	10	5
3	14	4.667
4	18	4.5
10	42	4.2
t	4t+2	

Practice Problems

4 Problems

Problem 1

The relationship between a distance in yards (y) and the same distance in miles (m) is described by the equation y = 1,760m.

a. Find measurements in yards and miles for distances by completing the table.

distance measured in miles	distance measured in yards
1	1,760
5	8,800
2	3,520
10	17,600

b. Is there a proportional relationship between a measurement in yards and a measurement in miles for the same distance? Explain why or why not.

There is a proportional relationship.

The constant of proportionality is 1,760 yards per mile.

Problem 2

Decide whether or not each equation represents a proportional relationship.

a. The remaining length (L) of 120-inch rope after x inches have been cut off: 120 - x = L

no

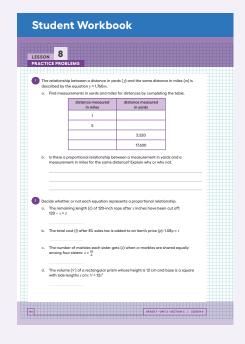
b. The total cost (t) after 8% sales tax is added to an item's price (p): 1.08p = t yes

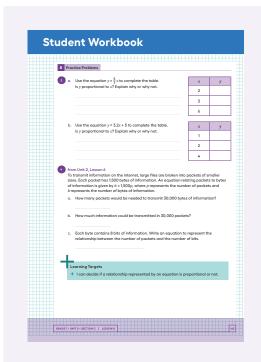
c. The number of marbles each sister gets (x) when m marbles are shared equally among four sisters: $x = \frac{m}{4}$

yes

d. The volume (V) of a rectangular prism whose height is 12 cm and base is a square with side lengths s cm: $V = 12 s^2$

no





Problem 3

a. Use the equation $y = \frac{5}{2}x$ to complete the table. Is y proportional to x? Explain why or why not.

Yes, there is a proportional relationship between x and y since $\frac{y}{x} = \frac{5}{2}$ in each row.

x	у
2	5
3	<u>15</u> 2
6	15

b. Use the equation y = 3.2x + 5 to complete the table. Is y proportional to x? Explain why or why not.

No, there is no proportional relationship between x and y. In the first row $\frac{y}{x} = 8.2$ but in the second row $\frac{y}{x} = 5.7$.

x	у
1	8.2
2	11.4
4	17.8

Problem 4

from Unit 2, Lesson 6

To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. An equation relating packets to bytes of information is given by b = 1,500p, where p represents the number of packets and p represents the number of bytes of information.

a. How many packets would be needed to transmit 30,000 bytes of information?

20 packets

- **b.** How much information could be transmitted in 30,000 packets? 45,000,000 bytes
- **c.** Each byte contains 8 bits of information. Write an equation to represent the relationship between the number of packets and the number of bits.

$$x = 12,000p$$

LESSON 8 • PRACTICE PROBLEMS