Lesson 2

Graphs of Proportional Relationships

Goals

- Compare graphs that represent the same proportional relationship using differently scaled axes.
- Create graphs representing the same proportional relationship using differently scaled axes, and identify which graph to use to answer specific questions.

Learning Targets

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR6: Three Reads (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Card Sort
- MLR6: Three Reads

Required Materials

Materials to Gather

- Straightedges: Lesson
- Straightedges: Activity 2

Materials to Copy

- Proportional Relationships Cards (1 copy for every 4 students): Activity 1

Lesson Narrative

The purpose of this lesson is for students to understand that while there are many ways to scale axes when graphing a proportional relationship, certain ranges for the axes are helpful for seeing specific information.

Students begin by considering a claim that the graph of one line is steeper than the graph of a second line. While one graph looks like a steeper line, by noticing the scale of the axes of each graph, it can be determined that the two lines actually have the same slope. Next, students sort graphs on cards based on what proportional relationship they represent. Each graph has a different scale, with some scales purposefully quite different, pressing the need to pay attention to scale and rely on mathematical definitions of steepness, not just visual ones.

Then students graph a proportional relationship on two differently scaled axes, comparing it to the graph of a nonproportional relationship. By looking at the same two relationships graphed at different scales, students see how the scale of the axes affects the information that can be determined.

Student Learning Goal

Let's think about scale.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	20 min	10 min	5 min

Warm-up
Two Perspectives

5 min

Activity Narrative

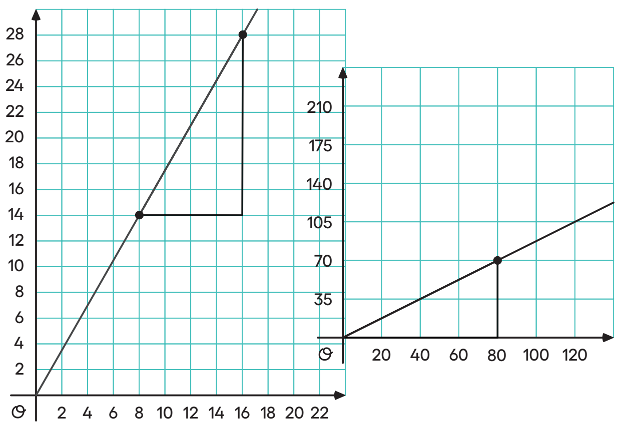
In this *Warm-up*, students work with the same proportional relationship shown on two sets of axes that are scaled differently. The purpose is to make explicit that the same proportional relationship can appear to have different steepness depending on the axes.

Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are two graphs that could represent a variety of different situations.

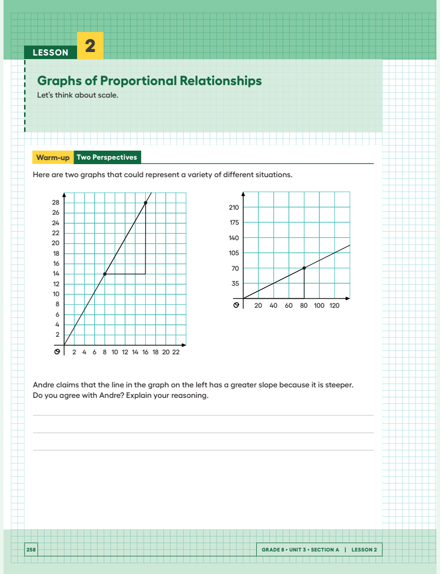


Andre claims that the line in the graph on the left has a greater slope because it is steeper. Do you agree with Andre? Explain your reasoning.

No, I do not agree with Andre.

Sample reasoning: Both lines have the same slope, even though the graph on the left looks steeper. Using the scales of the graphs to measure the vertical and horizontal change, the graph on the left has a slope of $\frac{14}{8} = \frac{7}{4}$ and the graph on the right has a slope of $\frac{70}{40} = \frac{7}{4}$.

Student Workbook



Instructional Routines

Card Sort

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Activity Synthesis

The goal of this discussion is to emphasize the importance of paying attention to scale when making sense of graphs. Display the two images from the activity for all to see. Identify 1–2 students to share their reasoning. Here are some questions for discussion:

"How can one graph look steeper yet still have the same slope as another graph?"

The two graphs are drawn using different scales, making them look different even though the value of their slopes is equivalent.

"Are the two slope triangles shown similar?"

Yes. The slope triangle on the left can be dilated and translated to match the slope triangle on the right, making the two triangles similar.

"What would happen if these 2 lines were graphed on the same set of axes?"

They would overlap and look like the same line.

Activity 1

Card Sort: Proportional Relationships

10 min

Activity Narrative

The purpose of this activity is for students to identify the same proportional relationship graphed using different scales. Students will first sort the graphs based on what proportional relationship they represent and then write an equation representing each relationship. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Monitor for and select groups using different strategies to match graphs to share later. For example, some groups may identify the unit rate for each graph in order to match while others may choose to write equations first and use those to match their graphs.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 4 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to place all the cards face up and start thinking about possible ways to sort the cards into categories.

- Pause the class and select 1–3 students to share the categories they identified.
- Discuss as many different categories as time allows.

Attend to the language that students use to describe their categories and graphs, giving them opportunities to describe their graphs more precisely. Highlight the use of terms like "slope," "scale," and "constant of proportionality." If necessary, remind students that the constant of proportionality is the number that values for one quantity are each multiplied by to get the values for the other quantity. After a brief discussion, invite students to open their books or devices and continue with the activity.

Student Task Statement

Your teacher will give you a set of cards. Each card contains a graph of a proportional relationship.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

Card sort and possible matching equation:

A: $y = 0.25x$ or $\frac{y}{x} = \frac{6}{24}$ (or equivalent)

B, E, H: $y = 3x$ or $\frac{y}{x} = \frac{6}{2}$ (or equivalent)

C, D, G, K: $y = 3.5x$ or $\frac{y}{x} = \frac{7}{2}$ (or equivalent)

I, L: $y = \frac{4}{3}x$ or $\frac{y}{x} = \frac{4}{3}$ (or equivalent)

F, J: $y = \frac{5}{2}x$ or $\frac{y}{x} = \frac{5}{2}$ (or equivalent)

Activity Synthesis

The goal of this discussion is for students to understand that looking only at the steepness of the line without paying attention to the numbers on the axes can hide the actual relationship between the two variables.

Once all groups have completed the *Card Sort*, ask previously selected groups to share their strategies for grouping the graphs. Discuss the following:

"Which graphs were tricky to group? Explain why."

"Did you need to make adjustments in your groups? What might have caused an error? What adjustments were made?"

Activity 2
Different Scales

20 min

Activity Narrative

In this activity, students graph a proportional relationship on two differently scaled axes and compare the proportional relationship to an already-graphed nonproportional relationship on the same axes. Students make sense of the intersections of the two graphs by reasoning about the situation and consider which scale is most helpful: zoomed in, or zoomed out. In this case, which graph is most helpful depends on the questions asked about the situation.

This is the first time *Three Reads* (Math Language Routine 6) is suggested in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation. During the second read, students identify important quantities. During the third read, the final prompt is revealed and students brainstorm possible strategies to answer the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing to a solution. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Access for Multilingual Learners (Activity 1, Student Task)

MLR8: Discussion Supports.
Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see:
"I noticed ____, so I matched …"
Encourage students to challenge each other when they disagree.

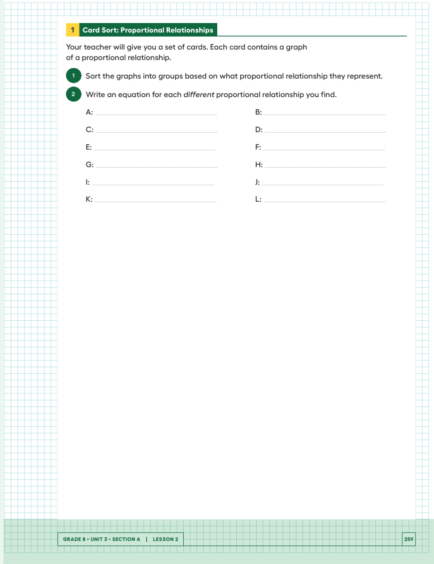
Advances: Speaking, Conversing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Student Workbook



1 Card Sort: Proportional Relationships

Your teacher will give you a set of cards. Each card contains a graph of a proportional relationship.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

A: ____ B: ____
C: ____ D: ____
E: ____ F: ____
G: ____ H: ____
I: ____ J: ____
K: ____ L: ____

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Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify. This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Launch

Arrange students in groups of 2. Provide access to straightedges.

Use MLR6 *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the graphs, without revealing the questions.

- In the first read, students read the problem with the goal of comprehending the situation.

Read the problem aloud while everyone else reads along, and then ask,

"What is this situation about?"

Allow 1 minute to discuss with a partner and then share with the whole class. A typical response may be,

"Two large water tanks are filling with water. One of them is filling at a constant rate, while the other is not. Both graphs represent Tank A. Tank B only has an equation."

Listen for and clarify any questions about the context.

- In the second read, students analyze the mathematical structure of the story by naming quantities.

Invite students to read the problem aloud with their partner, or select a student to read to the class, then prompt students by asking,

"What can be counted or measured in this situation?"

Give students 30 seconds of quiet think time, followed by another 30 seconds to share with their partner. A typical response may be

"Liters of water in Tank A; liters of water in Tank B; amount of time that has passed in minutes; constant rate of $\frac{1}{2}$ liters per minute."

- In the third read, students brainstorm possible solution strategies to answer the questions.

Invite students to read the problem aloud with their partner, or select a different student to read to the class. After the third read, reveal the first question on sketching and labeling a graph for Tank B on each of the axes and ask,

"What are some ways one might solve this?"

Instruct students to think of ways to approach the questions without actually solving. Give students 1 minute of quiet think time followed by another minute to discuss with their partner. Invite students to name some possible strategies referencing quantities from the second read. Provide these sentence frames as partners discuss:

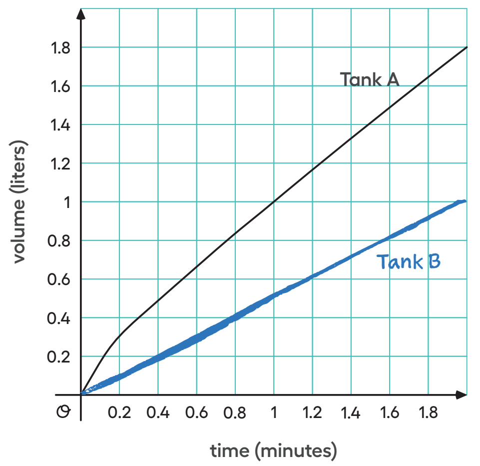
- "To draw a graph for Tank B, I would …"
- "One way to approach the question about finding the time when the tanks have the same amount of water would be to …" and "I would use the first/second graph to find …"

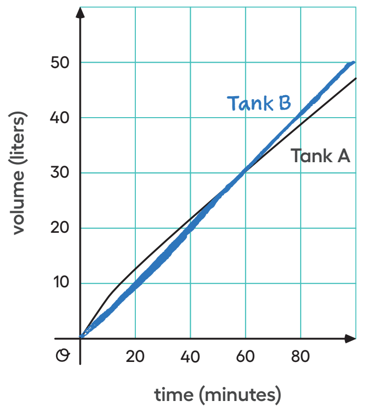
As partners are discussing their solution strategies, select 1–2 students to share their ideas with the whole class. As students are presenting their strategies to the whole class, create a display that summarizes the ideas for each question.

Give students time to complete the rest of the activity followed by a whole class discussion.

Student Task Statement

Two large water tanks are filling with water. Tank A is *not* filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of $\frac{1}{2}$ liters per minute. The relationship between its volume of water and time can be described by the equation $v = \frac{1}{2}t$, where t is the time in minutes, and v is the total volume in liters of water in the tank.





1. Sketch and label a graph of the relationship between the volume of water v and time t for Tank B on each of the coordinate planes.
2. Answer the following questions and say which graph you used to find your answer.
   a. After 30 seconds, which tank has the most water?

      Using the first graph, Tank A has more water after 30 seconds.

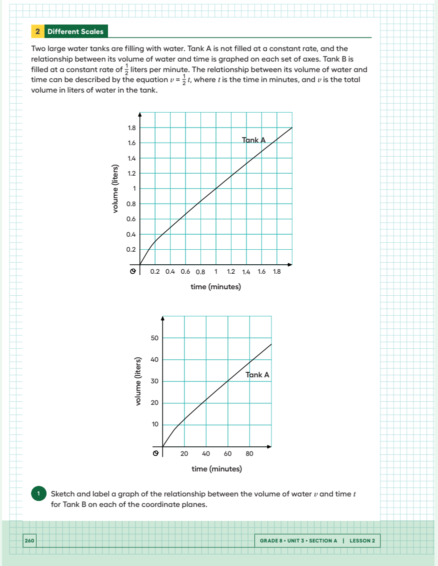
   b. At approximately what times do both tanks have the same amount of water?

      Using the second graph, at approximately 64 minutes both tanks have the same amount of water.

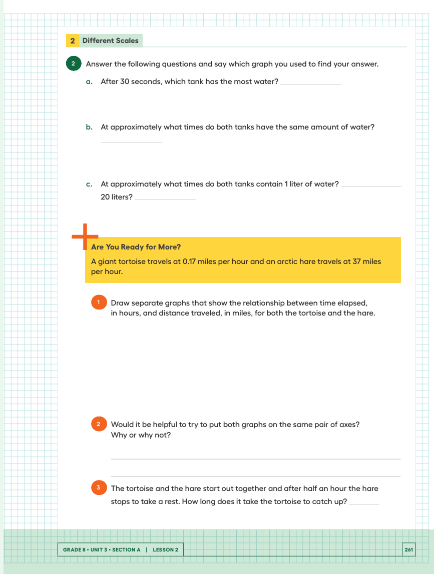
   c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

      Using the first graph, Tank A has 1 liter of water after 1 minute. Tank B has 1 liter of water after 2 minutes; using the second graph, Tank A has 20 liters of water at around 36 minutes while Tank B has 20 liters of water at 40 minutes.

Student Workbook



Student Workbook



Are You Ready for More?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

   Each axes should have "time elapsed (hours)" on the horizontal axis and "distance traveled (miles)" on the vertical axis. The scale for the giant tortoise graph is likely much smaller than the scale for the "arctic hare" graph.

2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?

   Sample response: No. Since the scales on the vertical axis are so different, it is very difficult to put both graphs on the same axes without one of the graphs being squashed up very close to an axis. This makes it difficult to read coordinate values from the graph.

3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

   After half an hour the hare has traveled $0.5 \cdot 37 = 18.5$ miles and the tortoise has traveled $0.5 \cdot 0.17 = 0.085$ miles, so the hare is $18.5 - 0.085 = 18.415$ miles ahead of the tortoise. Assuming the hare doesn't move, it will take the tortoise $\frac{18.415}{0.17} = 108.32$ hours to catch up, or about 4.5 days.

Activity Synthesis

The goal of this discussion is to emphasize how selecting an appropriate scale is important when creating a graph from scratch. Begin the discussion by asking students:

"What information can be seen using the second graph that can't be seen with the first?"

Tank A starts out with more water than Tank B at first, and then with less water than Tank B later on. The two tanks have the same amount of water after 60 minutes.

"If someone only looked at the first graph, what might they think (incorrectly)?"

The first graph makes it look like Tank A will always have more water than Tank B.

"In what situation might the first graph be more useful?"

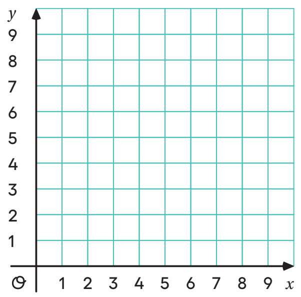
If someone wanted to answer a question about the tanks after only 1 or 2 minutes.

Explain that if making a graph from scratch, it is important to first check what questions are being asked. Some things to consider are:

- Should both axes have the same scale?
- How large are the numbers in the problem? Does each axis need to extend to 10 or 100?
- What will you count by? 1s? 5s? 10s?

Lesson Synthesis

Display this blank coordinate plane for all to see and provide pairs of students with graph paper.



Ask pairs to draw a copy of the coordinate plane and give a signal when they have finished. (Students may need to be warned to leave room on their graph paper for a second graph as sometimes students like to draw graphs that fill all the space they are given.) Invite a student to propose a proportional relationship that they consider to have a "steep" line for the class to graph on the axes.

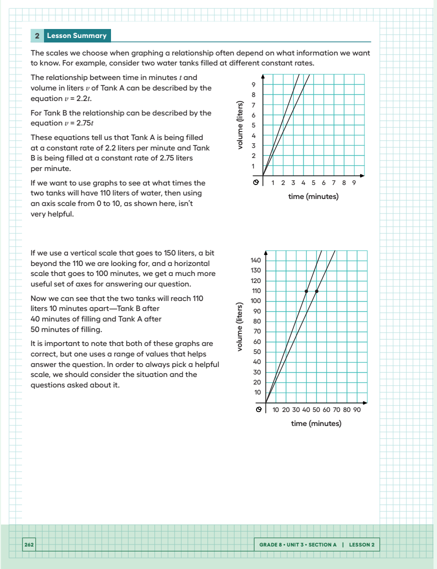
For example, say a student proposes $y = 6x$. Have students complete their graph in pairs, then add the line representing the equation to the graph on display. Next, ask students to make a second graph with the same horizontal scale, but with a vertical scale that makes $y = 6x$ not look as steep when graphed. After students have made the new graph, invite students to share and explain how they decided on their new vertical scale.

Conclude by reminding students that all these graphs of $y = 6x$ are correct since they all show a proportional relationship with a constant of proportionality equal to 6. Ask students,

"Can you think of a reason someone might want to graph this relationship with such a large vertical scale?"

If they needed to also graph something like $y = 60x$, they would need a pretty big vertical scale in order to see both lines.

Student Workbook



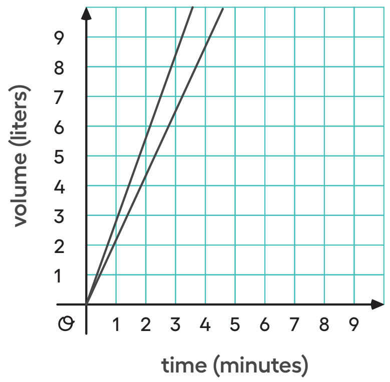
Lesson Summary

The scales we choose when graphing a relationship often depend on what information we want to know. For example, consider two water tanks filled at different constant rates.

The relationship between time in minutes t and volume in liters v of Tank A can be described by the equation $v = 2.2t$.

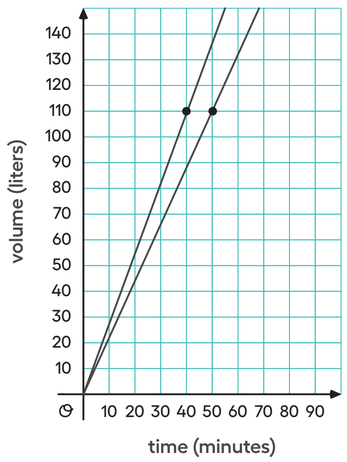
For Tank B the relationship can be described by the equation $v = 2.75t$.

These equations tell us that Tank A is being filled at a constant rate of 2.2 liters per minute and Tank B is being filled at a constant rate of 2.75 liters per minute.



If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.

If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.



Now we can see that the two tanks will reach 110 liters 10 minutes apart—Tank B after 40 minutes of filling and Tank A after 50 minutes of filling.

It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.

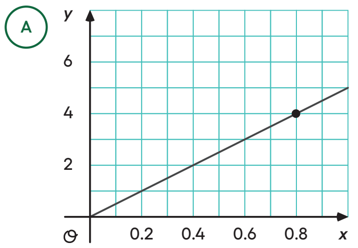
Cool-down

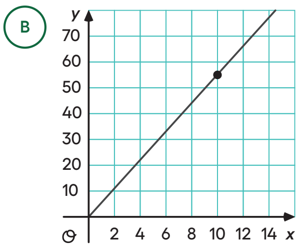
Different Axes

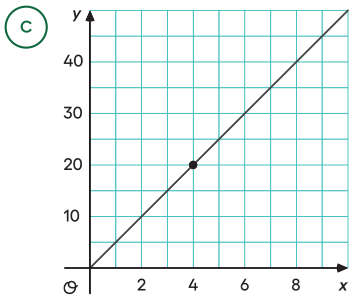
5 min

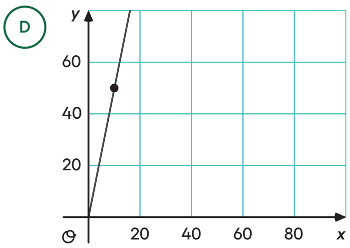
Student Task Statement

Which one of these relationships is different from the other three? Explain how you know.









Graph B is a representation of $y = 5.5x$ or $\frac{y}{x} = \frac{55}{10}$.

Graphs A, C, and D are all representations of $y = 5x$ or $\frac{y}{x} = 5$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 2

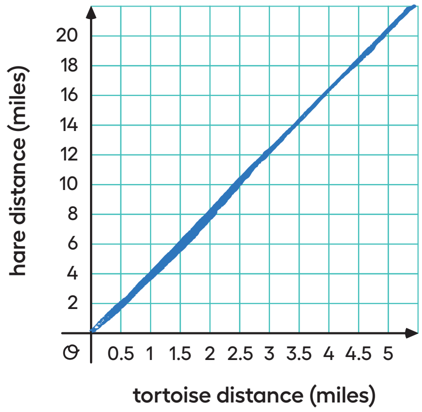
Practice Problems

4 Problems

Problem 1

The tortoise and the hare are having a race. After the hare runs 16 miles the tortoise has only run 4 miles.

This relationship can be described by the equation $y = 4x$, where x is the distance tortoise "runs" in miles, and y is the distance the hare runs in miles. Create a graph of this relationship.



A ray through (0, 0) and (2, 8)

Problem 2

The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched.

a. Complete the table.

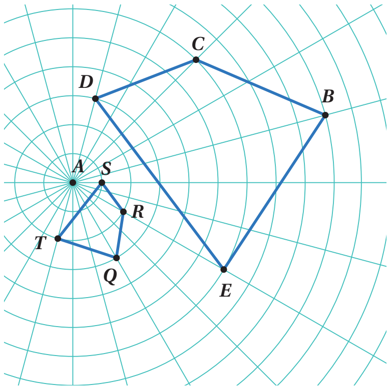
distance (cm)	weight (newtons)
20	28
55	77
100	140
1	$\frac{7}{5}$

b. Describe the scales you could use on the x- and y-axes of a coordinate plane that would show all the distances and weights in the table.

Sample response: from 0 to 100 on the horizontal axis (distance) and from 0 to 140 on the vertical axis (weight)

Problem 3

from Unit 2, Lesson 6



Describe a sequence of rotations, reflections, translations, and dilations that show one figure is similar to the other. Be sure to include the distance and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.

Sample response:

a. Begin with figure $BCDE$.

b. Dilate using A as the center of dilation with scale factor $\frac{1}{3}$.

c. Rotate using A as the center clockwise 75 degrees.

Problem 4

from Unit 2, Lesson 6

Andre said, "I found two figures that are congruent, so they can't be similar."

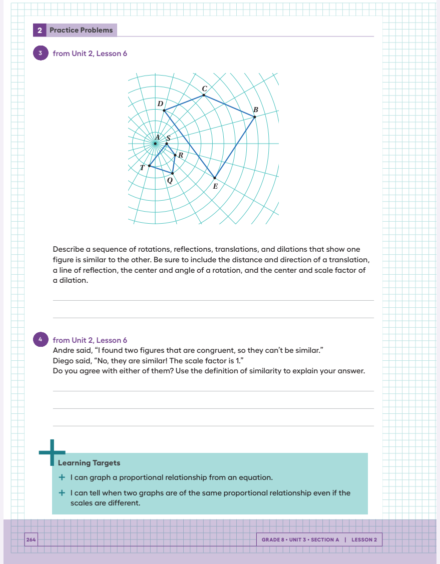
Diego said, "No, they are similar! The scale factor is 1."

Do you agree with either of them? Use the definition of similarity to explain your answer.

I agree with Diego.

Sample reasoning: Two figures are congruent if one can be moved to the other using a sequence of rigid transformations, and two congruent figures have a scale factor of 1. They are similar if one can be moved to the other using a sequence of rigid transformations and dilations. If two figures are congruent, then they are also similar.

Student Workbook


2 Practice Problems

3 from Unit 2, Lesson 6

Describe a sequence of rotations, reflections, translations, and dilations that show one figure is similar to the other. Be sure to include the distance and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.

4 from Unit 2, Lesson 6

Andre said, "I found two figures that are congruent, so they can't be similar."

Diego said, "No, they are similar! The scale factor is 1."

Do you agree with either of them? Use the definition of similarity to explain your answer.

Learning Targets

+ I can graph a proportional relationship from an equation.
+ I can tell when two graphs are of the same proportional relationship even if the scales are different.

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