Scales without Units

Goals

- Explain (orally and in writing) how to use scales without units to determine scaled or actual distances.
- Interpret scales expressed without units, e.g., "1 to 50," (in spoken and written language).

Learning Targets

- I can explain the meaning of scales expressed without units.
- I can use scales without units to find scaled distances or actual distances.

Lesson Narrative

In this lesson, students see that a scale can be expressed without units. For example, the scale "1 to 60" means that every length on the scale drawing represents an actual length that is 60 times its size, whatever the unit may be (inches, centimeters, etc.).

Expressing the scale without units, such as "1 to 60," highlights the scale factor that relates the scale drawing to the actual object. Each measurement on the scale drawing is multiplied by 60 to find the corresponding measurement on the actual object. This relates closely to the scaled copies that were examined earlier in the unit in which each copy was related to the original by a scale factor. Students gain a better understanding of both scaled copies and scale drawings as they recognize the common underlying structure.

Student Learning Goal

Let's explore a different way to express scales.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Materials to Gather

• Geometry toolkits: Activity 1, Activity 2

Lesson Timeline



<u>Warm</u>-up



Activity 1



Activity 2



Lesson Synthesis

Assessment



Cool-down

Building on Student Thinking

Students might think that when no units are given, they can choose their own units, using different units for the 1 and the 100. This is a natural interpretation given students' work so far. Make note of this misconception, but address it only if it persists beyond the lesson.



Warm-up

One to One Hundred



Activity Narrative

This Warm-up introduces students to a scale without units and invites them to interpret it using what they have learned about scales so far.

As students work and discuss, notice those who interpret the unitless scale as numbers having the same units, as well as those who see "1 to 100" as comparable to using a scale factor of 100. Invite them to share their thinking later.

Launch 🙎

Remind students that, until now, we have worked with scales that each specify two units—one for the drawing and one for the object it represents. Tell students that sometimes scales are given without units.

Arrange students in groups of 2. Give students 2 minutes of quiet think time and another minute to discuss their thinking with a partner.

Student Task Statement

A map of a park says its scale is 1 to 100.

1. What do you think that means?

Sample responses:

- Distances in the park are 100 times bigger than corresponding distances on the map.
- One unit on the map represents IOO units of distance in the park.
- **2.** Give an example of how this scale could tell us about measurements in the park.

Sample responses:

- If a path is 6 inches long on the map, then we could tell that the actual path is 600 inches long.
- We could use the scale to tell the size of the park. For example, if the park is 20 inches wide on the map, we can tell the actual park is 2,000 inches wide.

Activity Synthesis

Solicit students' ideas about what the scale means and ask for a few examples of how it could tell us about measurements in the park. If not already mentioned by students, point out that a scale written without units simply tells us how many times larger or smaller an actual measurement is compared to what is on the drawing. In this example, a distance in the park would be 100 times the corresponding distance on the map, so a distance of 12 cm on the map would mean 1,200 cm or 12 m in the park.

Explain that the distances could be in any unit, but because one is expressed as a number times the other, the unit is the same for both.

Tell students that we will explore this kind of scale in this lesson.

Activity 1

Apollo Lunar Module



Activity Narrative

In this activity, students use a scale drawing in their student workbooks and a scale expressed without units to calculate actual lengths. They also use actual heights to calculate corresponding scaled heights. Students make choices about which units to use. Their choice of units could influence the number of conversions needed and the efficiency of their paths (as shown in the sample student responses). As students choose units and calculate scaled heights, they are reasoning quantitatively and abstractly.

For the problem about the height of the spacecraft, monitor for students who:

- Measure the drawing in inches, calculate the actual height in inches, and then convert to meters.
- Measure the drawing in centimeters, calculate the actual height in centimeters, and then convert to meters.
- Measure the drawing in centimeters, convert to meters, and then calculate the actual height.

Launch

156

Tell students that Neil Armstrong and Buzz Aldrin were the first people to walk on the surface of the Moon. The Apollo Lunar Module was the spacecraft used by the astronauts when they landed on the Moon in 1969. (The landing module was one part of a larger spacecraft that was launched from Earth.) Consider displaying a picture of the landing module such as this one.

Solicit some guesses about the size of the spacecraft and about how the height of a person might compare to it. Explain to students that they will use a scale drawing of the Apollo Lunar Module to find out.

Arrange students in groups of 2. Provide access to centimeter and inch rulers.

Make sure that students understand what is meant by a "leg" of the spacecraft in the first question. The "legs" of the spacecraft are its landing gear. Students should measure one of the legs on the side of the spacecraft in the drawing because the one in the middle appears shorter due to foreshortening.

Give students 6–7 minutes of partner work time. Select work from students with different strategies, such as those described in the activity narrative, to share later.

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 1, Narrative)

MLR7: Compare and Connect

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, directing students to first find the length on the drawing. Then provide students with the scale of the drawing and prompt them to use that to find the actual length.

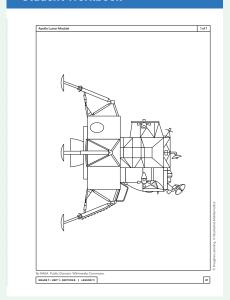
Supports accessibility for: Organization, Attention

Building on Student Thinking

If students are unsure how to begin finding the actual length of the landing gear or actual height of the spacecraft, suggest that they first find out the length on the drawing.

Students may measure the height of the spacecraft in centimeters and then simply convert it to meters without using the scale. Ask students to consider the reasonableness of their answer (which is likely around 0.14 m) and remind them to take the scale into account.

Student Workbook



Student Task Statement

Look at the drawing of the Apollo Lunar Module on the previous page. It is drawn at a scale of 1 to 50.

1. Estimate the actual length of one leg of the spacecraft to the nearest 10 centimeters. Explain or show your reasoning.

The legs of the spacecraft are about 350 cm long.

Sample reasonings:

- The leg is about 7 centimeters on the drawing, so the actual length is 7.50 or 350 centimeters
- The leg is about 2.75 inches on the drawing, so the actual length is 137.5 inches. $(2.75) \cdot 50 = 137.5$. Multiplying 137.5 by 2.54 gives the length in centimeters. $(137.5) \cdot (2.54) = 349.5$; this is 350 cm when rounded to the nearest 10 cm.
- **2.** Estimate the actual height of the spacecraft to the nearest meter. Explain or show your reasoning.

About 7 meters tall

Sample reasonings:

- The spacecraft is about 14 cm tall in the drawing. The actual height is 50 times 14 cm, which is 700 cm. 700 cm is 7 m.
- 14 cm is 0.14 m, because $14 \div 100 = 0.14$, and $(0.14) \cdot 50 = 7$, so the spacecraft is about 7 m tall.
- The spacecraft is about 5.5 inches on the drawing. $(5.5) \cdot 50 = 275$. The actual height is about 275 inches, which is 698.5 cm. $275 \cdot (2.54) = 698$. 5. 698.5 cm is 6.985 m, or about 7 m.

(Do not highlight this solution in class discussion.)

3. Neil Armstrong was 71 inches tall when he went to the Moon. How tall would he be in this scale drawing? Show your reasoning.

About 1.4 inches tall

71÷50≈1.4

4. Sketch a stick figure to represent yourself standing next to the Apollo Lunar Module. Make sure the height of your stick figure is to scale. Show how you determined your height on the drawing.

Answers vary.

Sample responses:

- My height is 5 feet and 2 inches, which equals 62 inches. $(5 \cdot 12) + 2 = 62$. My height on the drawing is about $1\frac{1}{4}$ inches, because $62 \div 50 \approx 1.24$.
- \circ 1 am 155 cm tall. 155 \div 50 = 3 . I. My height is 3.1 cm on the drawing.

Are You Ready for More?

The table shows the distance between the Sun and 8 planets in our solar system.

| planet | average distance (millions of miles) |
|---------|---|
| Mercury | 35 |
| Venus | 67 |
| Earth | 93 |
| Mars | 142 |
| Jupiter | 484 |
| Saturn | 887 |
| Uranus | 1,784 |
| Neptune | 2,795 |

1. If you wanted to create a scale model of the solar system that could fit somewhere in your school, what scale would you use?

Answers vary.

Sample response: The gymnasium has a space of about 100 feet by 100 feet. The largest distance we need to represent is between the Sun and Neptune and this is about 2,800 million (or 3 billion) miles. So if I foot represents about 30 million miles, the solar system will fit.

2. The diameter of Earth is approximately 8,000 mi. What would the diameter of Earth be in your scale model?

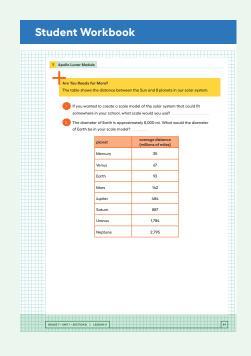
Answers vary.

Sample response: 8,000 miles is one thousandth of 8 million miles and one millionth of 8,000 million miles. So the diameter of Earth will be about 3 millionths of the distance from Neptune to the Sun. This distance is represented by about 100 feet on the scale model, so the diameter of Earth will be about 3 millionths of 100 feet. This is about $\frac{1}{2,500}$ of an inch. This is smaller than a fine grain of sand!

Activity Synthesis

The goal of this discussion is to highlight how units matter in problems involving a scale without units. First, poll the class on their answers to the first question. Highlight the multiplication of scaled measurements by 50 to find actual measurements.

Next, display 2–3 approaches to the second question from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches.



Instructional Routines

MLR8: Discussion Supports

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Here are some questions for discussion:

○ "How is the scale '1 to 50' used in each method?"

"Why do the different approaches lead to the same outcome?"

"Are there any benefits or drawbacks to one approach compared to another?"

The key takeaways are that when we have a scale without units:

- We can use any unit we want to measure the scale drawing.
- · We will get an actual measurement in that same unit.
- We may choose what unit we use to measure the drawing based on how we want to express the final answer.

(Since the question asks for a height in meters, using centimeters as the unit would be more efficient than using inches because fewer conversions would be required. If the question asked for actual height in feet, then inches would be a more strategic unit to use.)

Next, poll the class on their answers to the third question. Highlight that finding the length on the scale drawing involves dividing the actual measurement by 50 (or multiplying by $\frac{1}{50}$).

If time permits, display the image of the Apollo Lunar Module from the student workbook. Sketch a stick figure that is 1.4 inches tall to represent Neil Armstrong standing next to the Apollo Lunar Module. Select students who gave their heights in different units to share their solutions to the last problem. Sketch a stick figure with its height to scale for each student who shares an answer. Consider displaying a photograph of one of the astronauts next to the Lunar Module, such as shown here, as a way to visually check the reasonableness of students' solutions.

Activity 2

Same Drawing, Different Scales



Activity Narrative

In this activity, students explore the connection between a scale with units and one without units. Students are given two equivalent scales (one with units and the other without) and are asked to make sense of how the two could yield the same scaled measurements of an actual object. They also learn to rewrite a scale with units as a scale without units. As students recognize that different scales are equivalent, they are making use of structure.

Students will need to attend to precision as they work simultaneously with scales with units and scales without units. A scale of 1 inch to 16 feet is very different from a scale of 1 to 16, and students have multiple opportunities during the activity to address this subtlety.

As students work, identify groups that are able to reason clearly about why the two scales produce the same scale drawing. Two different types of reasoning to expect are:

- Using the two scales and the given dimensions of the parking lot to calculate and verify the student calculations.
- Thinking about the meaning of the scales that is, in each case, the actual measurements are 180 times the measurements on the scale drawing.

Launch

Ask students: "Is it possible to express the 1 to 50 scale of the Lunar Module as a scale with units? If so, what units would we use?" Solicit some ideas. Students are likely to say "1 inch to 50 inches," and "1 cm to 50 cm." Other units might also come up. Without resolving the questions, explain to students that their next task is to explore how a scale without units and one with units could express the same relationship between scaled lengths and actual lengths.

Keep students in the same groups. Provide access to rulers. Give partners 3–4 minutes to complete the first question and another 3–4 minutes of quiet work time for the last two questions.

Student Task Statement

A rectangular parking lot is 120 feet long and 75 feet wide.

- Lin made a scale drawing of the parking lot at a scale of 1 inch to 15 feet. The drawing she produced is 8 inches by 5 inches.
- Diego made another scale drawing of the parking lot at a scale of 1 to 180. The drawing he produced is also 8 inches by 5 inches.
- Explain or show how each scale would produce an 8 inch by 5 inch drawing.
 Sample responses:
 - In Lin's case, I in represents 15 ft, so 120 ft is 8 in (120 \div 15 = 8) and 75 ft is 5 in (75 \div 15 = 5). In Diego's case, I unit on the drawing represents 180 of the same unit in the actual distance, so I in represents 180 in. 180 in is equal to 15 ft (180 \div 12 = 15). Since the scale here is also I in to 15 ft, the drawing will also be 8 in by 5 in.
 - 120 ft is 1,440 in (120 \cdot 12 = 1,440) and 75 ft is 900 in (75 \cdot 12 = 900). If the scale is 1 to 180, the sides of the parking lot will be 1,440 \div 180 and 900 \div 180, or 8 in and 5 in, respectively.
- 2. Make another scale drawing of the same parking lot at a scale of 1 inch to 20 feet. Be prepared to explain your reasoning.

Drawing should show a 6 inch by $3\frac{3}{4}$ inch rectangle.

Sample reasoning: $120 \div 20 = 6.75 \div 20 = 3\frac{3}{4}$

3. Express the scale of 1 inch to 20 feet as a scale without units. Explain your reasoning.

I to 240

Sample reasoning: 20 ft is equivalent to 240 in, so I in on the drawing represents 240 in of actual distance.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. First provide students with the original measurements and the scale that Lin used and ask for the size of her scale drawing. Then do the same for Diego's scale before moving onto the rest of the activity. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

Some students may have trouble getting started. Suggest that they begin by treating each scale separately and find out, for instance, how a scale of 1 inch to 15 feet produces a drawing that is 8 inches by 5 inches.



Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

At the appropriate time, give groups 2–3 minutes to plan what they will say when they present to the class.

"Practice what you will say when you share your responses to the first question with the class. Talk about what is important to say, and decide who will speak."

Advances: Speaking, Conversing, Representing

Activity Synthesis

The purpose of this discussion is to clarify how two different scales (one with units and one without units) can be equivalent. Invite a few students to share their responses to the first question. Highlight how scaled lengths and actual lengths are related by a factor of 180 in both scales. This factor is shown explicitly in one scale but not in the other:

- In the case of "1 to 180" we know that actual lengths are 180 times as long as scaled lengths (or scaled lengths are $\frac{1}{180}$ of actual lengths), regardless of the unit used.
- In the case of "1 in to 15 ft" the actual lengths are *not* 15 times as long as their corresponding lengths on the drawing, because 15 feet is not 15 times the length of 1 inch. Converting the units helps us see the scale factor. Since 1 foot equals 12 inches and $15 \cdot 12 = 180$, the scale "1 in to 15 feet" is equivalent to the scale "1 in to 180 in" or "1 to 180."

Next, direct students' attention to the new scale drawing with the scale "1 in to 20 ft." Consider asking questions such as:

"Is the new scale drawing larger or smaller than the original scale drawings? How do you know?"

The new drawing is smaller. Each inch represents a larger actual distance, so we don't need as many inches to represent the entire parking lot. Also, the dimensions are smaller. The value 6 is less than 8, and $3\frac{3}{4}$ is less than 5.

○ "How can we express the scale '1 inch to 20 feet' without units?"

I to 240, because 20 feet is equal to 240 inches.

 $20 \cdot 12 = 240$

Lesson Synthesis

Share with students

"Today we worked with scale drawings in which the scale did not indicate any units."

To help students generalize about scales without units, consider asking:

"What does it mean when the scale on a scale drawing does not include units?"

When a scale does not show units, the same unit is used for both the scaled distance and the actual distance.

"How is a scale without units the same as a scale with units? How are they different?"

Both types of scale show the relationship between scaled and actual distances. When the scale doesn't specify a unit, you can use any unit to measure the scale drawing and then calculate the actual distance in that same unit.

If desired, use this example to review these concepts:

☐ "A scale drawing has the scale '1 to 500.' What does this mean?"

A scale of I to 500 means that I inch on the drawing represents 500 inches in actual distance, I centimeter on the drawing represents 500 centimeters in actual distance, etc.

"A segment on the drawing is 10 millimeters long. What actual distance does this represent?"

5,000 mm, because $10 \cdot 500 = 5,000$

"An actual distance is 1,000 feet long. How long is this distance on the drawing?"

2 feet, because 1,000 $\cdot \frac{1}{500}$ = 2

Lesson Summary

In some scale drawings, the scale specifies one unit for the distances on the drawing and a different unit for the actual distances represented. For example, a drawing could have a scale of 1 cm to 10 km.

In other scale drawings, the scale does not specify any units at all. For example, a map may simply say that the scale is 1 to 1,000. In this case, the units for the scaled measurements and actual measurements can be any unit, as long as the same unit is being used for both. If a map of a park has a scale 1 to 1,000, then 1 inch on the map represents 1,000 inches in the park, and 12 centimeters on the map represent 12,000 centimeters in the park. In other words, 1,000 is the scale factor that relates distances on the drawing to actual distances, and $\frac{1}{1,000}$ is the scale factor that relates an actual distance to its corresponding distance on the drawing.

A scale with units can be expressed as a scale without units by converting one measurement in the scale into the same unit as the other (usually the unit used in the drawing). For example, these scales are equivalent:

- 1 inch to 200 feet
- 1 inch to 2,400 inches (because there are 12 inches in 1 foot, and $200 \cdot 12 = 2,400$)
- 1 to 2,400

This scale tells us that all actual distances are 2,400 times their corresponding distances on the drawing, and distances on the drawing are $\frac{1}{2,400}$ times the actual distances that they represent.

Cool-down

Scaled Courtyard Drawings

5 min

Student Task Statement

Andre drew a plan of a courtyard at a scale of 1 to 60. On his drawing, one side of the courtyard is 2.75 inches.

1. What is the actual measurement of that side of the courtyard? Express your answer in inches and then in feet.

165 in, which is 13.75 ft. Sample reasoning: $2.75 \cdot 60 = 165.165 \div 12 = 13.75$.

2. If Andre made another courtyard scale drawing at a scale of 1 to 12, would this drawing be smaller or larger than the first drawing? Explain your reasoning.

It would be larger. Sample reasoning: A scale of I to I2 means the length on paper is $\frac{1}{12}$ of the original length (or I0 inches by I3.75 inches), so the drawing would be larger than one drawn at $\frac{1}{60}$ the original length.

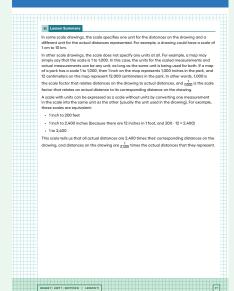
Responding To Student Thinking

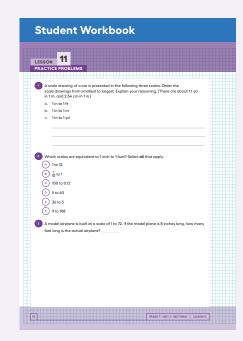
Points to Emphasize

If students struggle with using scales without units, focus on this when opportunities arise over the next several lessons. For example, in this activity, make sure to invite multiple students to share their thinking about how they found the measurements of the Tunisian flag:

Unit 1, Lesson 12, Activity 2 The World's Largest Flag

Student Workbook





Practice Problems

6 Problems

Problem 1

A scale drawing of a car is presented in the following three scales. Order the scale drawings from smallest to largest. Explain your reasoning. (There are about 1.1 yd in 1 m, and 2.54 cm in 1 in.)

- **a.** 1 in to 1 ft
- **b.** 1 in to 1 m
- **c.** 1 in to 1 yd

b, c, a

Sample reasonings:

- Of the three units, I ft is the smallest unit, and I m is the largest. Therefore,
 a drawing with scale I in to I ft will require the most number units (the
 largest), and a drawing with scale I in to I m will require the least (the
 smallest).
- Each scale was converted into a scale without units. I in to I ft is equivalent to I to I2. I in to I m is equivalent to 2.54 cm to IOO cm, which is roughly I to 39. And I in to I yd is equivalent to I to 36.

Problem 2

Which scales are equivalent to 1 inch to 1 foot? Select all that apply.

- **A.** 1 to 12
- **B.** $\frac{1}{12}$ to 1
- C. 100 to 0.12
- **D.** 5 to 60
- **E.** 36 to 3
- **F.** 9 to 108

Problem 3

A model airplane is built at a scale of 1 to 72. If the model plane is 8 inches long, how many feet long is the actual airplane?

48 feet

The actual airplane is 72 times the length of the model. $8 \cdot 72 = 576$. 576 inches is 48 feet, because 576 ÷ 12 = 48.

Problem 4

from Unit 1, Lesson 3

Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of A with a shortest side length equal to 2. Jada says, "Since the side lengths go down by 1 in this scaling, the perimeter goes down by 4 in total." Do you agree with Jada? Explain your reasoning.

No

The side lengths of B are not each I less than those of A. The side lengths of B are $\frac{2}{3}$ of those of A, so they must be 2, 4, 4, and 6. The perimeter of A is 24 and the perimeter of B is I6, which is 8 less in total.

Problem 5

from Unit 1, Lesson 6

Polygon B is a scaled copy of Polygon A using a scale factor of 5. Polygon A's area is what fraction of Polygon B's area? $\frac{1}{25}$

Problem 6

from Unit 1, Lesson 5

Figures R, S, and T are all scaled copies of one another. Figure S is a scaled copy of R using a scale factor of 3. Figure T is a scaled copy of S using a scale factor of 2. Find the scale factors for each of the following:

- a. From T to S
 - 1/2

- **b.** From S to R
 - 3

- c. From R to T
- 6

- d. From T to R
 - 16

