Lesson 8

# Keeping Track of All Possible Outcomes

## Goals

- Compare and contrast (in writing) different methods for representing the sample space of a compound event, and evaluate (orally) their usefulness.
- Determine the total number of possible outcomes for a compound event, and justify the reasoning (orally, in writing, and using other representations).
- Interpret or create a list, table, or tree diagram that represents the sample space of a compound event.

## Learning Target

I can write out the sample space for a multi-step experiment, using a list, table, or tree diagram.

## Lesson Narrative

In this lesson, students practice listing the sample space for a compound event. They make use of the structure of tree diagrams, tables, and organized lists as methods of organizing this information. Students notice that the total number of outcomes in the sample space for an experiment, that can be thought of as being performed as a sequence of steps, can be found by multiplying the number of possible outcomes for each step in the experiment.

## Student Learning Goal

Let's explore sample spaces for experiments with multiple parts.

### Access for Students with Diverse Abilities

- Representation (Activity 1)

### Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

### Instructional Routines

- MLR7: Compare and Connect

## Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Assessment: Cool-down |
|---|---|---|---|---|
| 5 min | 15 min | 10 min | 10 min | 5 min |

## Warm-up

## How Many Different Bracelets?

5 min

### Activity Narrative

The purpose of this *Warm-up* is to elicit methods students are already using to organize their understanding of different outcomes. In this lesson, students are asked to use different structures to think about the different combinations that are possible from selecting one item from each of three categories.

Monitor for groups who use different strategies to organize their thinking about the different combinations, such as:

- List several options and count them.
- Group several options together and multiply.
- Draw links between some of the options.

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

### Launch

Arrange students in groups of 2. Tell students they should organize their work so it can be understood by others.

Give students 1 minute of quiet think time, 3 minutes for partner discussion, and follow up with a whole-class discussion.

Ask students if they have looked at the jewelry part of a store. Then ask what options are available for different bracelets.

Tell students that many Native American tribes, such as Navajo and Hopi, craft jewelry such as bracelets for trade, status, and fashion. The bracelets are often made with different types of featured stone, have designs, and come in different sizes.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

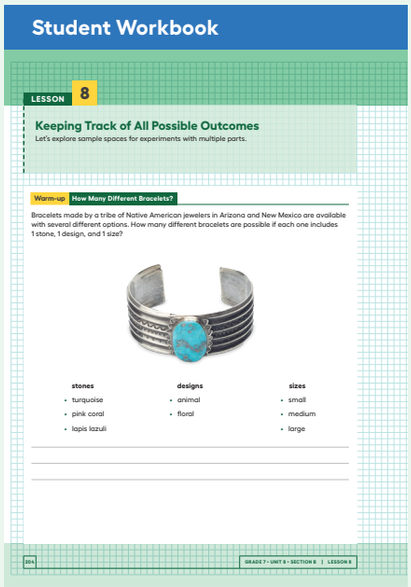
### Student Task Statement

Bracelets made by a tribe of Native American jewelers in Arizona and New Mexico are available with several different options. How many different bracelets are possible if each one includes 1 stone, 1 design, and 1 size?

| stones | designs | sizes |
|---|---|---|
| • turquoise | • animal | • small |
| • pink coral | • floral | • medium |
| • lapis lazuli | | • large |

There are 18 different bracelets ($3 \cdot 2 \cdot 3 = 18$).

### Student Workbook


LESSON 8

Keeping Track of All Possible Outcomes

Let's explore sample spaces for experiments with multiple parts.

Warm-up How Many Different Bracelets?

Bracelets made by a tribe of Native American jewelers in Arizona and New Mexico are available with several different options. How many different bracelets are possible if each one includes 1 stone, 1 design, and 1 size?

stones: turquoise, pink coral, lapis lazuli

designs: animal, floral

sizes: small, medium, large

**Instructional Routines**

**MLR7: Compare and Connect**

**lclass.com/r/10695592**
Please log in to the site before using the QR code or URL.

### Activity Synthesis

The goal of this discussion is for students to share their methods for organizing their thoughts about the different bracelets that are possible.

Display 2–3 approaches/representations from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"How did you know that you counted all of the different possible bracelets?"*

  We began with the turquoise option and wrote out all of the bracelets that have turquoise and animal designs, then all the bracelets that have turquoise and floral designs. We knew that there would be the same number of bracelets that had pink coral or lapis lazuli instead of turquoise, so we multiplied that by 3.

- *"How did you know you didn't repeat any bracelets?"*

  By carefully keeping the different options separate and going through them systematically, we made sure we didn't repeat.

## Activity 1
## Lists, Tables, and Trees

15 min

### Activity Narrative

In this activity, students learn three different methods for writing the sample spaces of multi-step experiments and explore their use in a few different situations. Since the calculated probability of an event depends on the number of outcomes in the sample space, it is important to be able to find this value in an efficient way. In the discussion, students will explore how different methods may be useful in different situations.

Monitor for students who use these different approaches for finding the sample space on the last question. Here are some ways students may arrange their thinking from less to more flexible:

- always use a list format to write out the sample space
- always use a tree diagram to write out the sample space
- change which representations they use for different questions

### Launch

Allow 10 minutes for students to answer the questions, and then follow with a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

## Student Task Statement

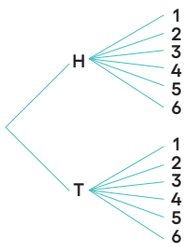
Consider the experiment: Flip a coin, and then roll a number cube.

Elena, Kiran, and Priya each use a different method for finding the sample space of this experiment.

- Elena carefully writes a list of all the options: Heads 1, Heads 2, Heads 3, Heads 4, Heads 5, Heads 6, Tails 1, Tails 2, Tails 3, Tails 4, Tails 5, Tails 6.
- Kiran makes a table:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| H | H1 | H2 | H3 | H4 | H5 | H6 |
| T | T1 | T2 | T3 | T4 | T5 | T6 |

- Priya draws a tree with branches in which each pathway represents a different outcome:




**1.** Compare the three methods. What is the same about each method? What is different? Be prepared to explain why each method produces all the different outcomes without repeating any.

Sample response: They all show the 12 possible outcomes. It can be difficult to keep track of all the outcomes when listing them like in Elena's method. Using Kiran's method to make a table only makes sense when there are 2 parts to the chance experiment. Drawing a tree like Priya uses a lot of paper.

**2.** Which method do you prefer for this situation?

Pause here so your teacher can review your work.

Sample response: Since there are not very many outcomes in this experiment, I might use Elena's method since it is quick and I am careful.

**3.** Find the sample space for each of these experiments using any method. Make sure you list every possible outcome without repeating any.

**a.** Flip a dime, then flip a nickel, and then flip a penny. Record whether each lands heads or tails up.

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

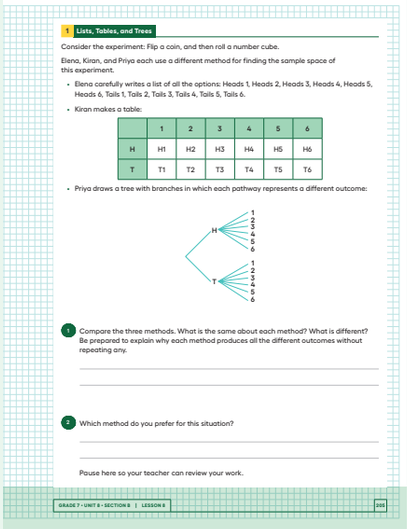
**b.** Han's closet has: a blue shirt, a green shirt, a white shirt, blue pants, khaki pants, and black pants. He must select one shirt and one pair of pants to wear for the day.

blue shirt and blue pants, blue shirt and khaki pants, blue shirt and black pants, green shirt and blue pants, green shirt and khaki pants, green shirt and black pants, white shirt and blue pants, white shirt and khaki pants, white shirt and black pants

### Building on Student Thinking

Some students may have trouble interpreting the tree diagram. Help students see that a single outcome is represented by following a branch from the point furthest to the left until they reach the end of a branch on the right side. It may help for students to write the full outcome on the diagram as well. In Priya's picture, next to the uppermost 1, a student could write H1.

### Student Workbook


1 Lists, Tables, and Trees

Consider the experiment: Flip a coin, and then roll a number cube.

Elena, Kiran, and Priya each use a different method for finding the sample space of this experiment.

- Elena carefully writes a list of all the options: Heads 1, Heads 2, Heads 3, Heads 4, Heads 5, Heads 6, Tails 1, Tails 2, Tails 3, Tails 4, Tails 5, Tails 6.
- Kiran makes a table:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| H | H1 | H2 | H3 | H4 | H5 | H6 |
| T | T1 | T2 | T3 | T4 | T5 | T6 |

- Priya draws a tree with branches in which each pathway represents a different outcome:

1. Compare the three methods. What is the same about each method? What is different? Be prepared to explain why each method produces all the different outcomes without repeating any.

2. Which method do you prefer for this situation?

Pause here so your teacher can review your work.

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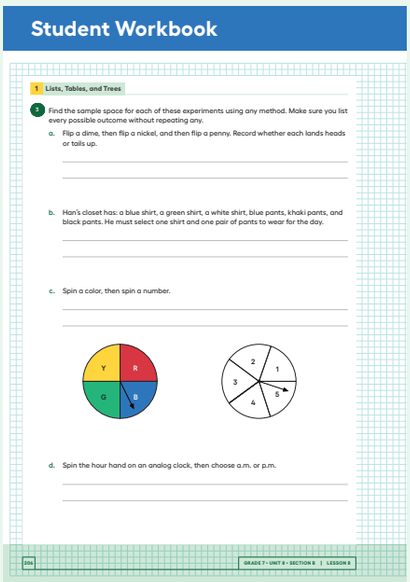
### Student Workbook


1 Lists, Tables, and Trees

3. Find the sample space for each of these experiments using any method. Make sure you list every possible outcome without repeating any.

a. Flip a dime, then flip a nickel, and then flip a penny. Record whether each lands heads or tails up.

b. Han's closet has: a blue shirt, a green shirt, a white shirt, blue pants, khaki pants, and black pants. He must select one shirt and one pair of pants to wear for the day.

c. Spin a color, then spin a number.

d. Spin the hour hand on an analog clock, then choose a.m. or p.m.

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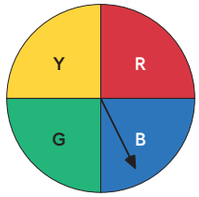
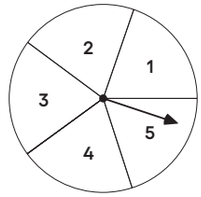
**Access for Students with Diverse Abilities (Activity 1, Synthesis)**

**Representation: Access for Perception.**
Use gestures to explain how to use the tree to find the sample space. For example, trace a path of branches in the tree to highlight a possible outcome.

*Supports accessibility for: Conceptual Processing, Language, Memory*

**c.** Spin a color, then spin a number.




R1, R2, R3, R4, R5, B1, B2, B3, B4, B5, G1, G2, G3, G4, G5, Y1, Y2, Y3, Y4, Y5

**d.** Spin the hour hand on an analog clock, then choose a.m. or p.m.

1 a.m., 2 a.m., 3 a.m., 4 a.m., 5 a.m., 6 a.m., 7 a.m., 8 a.m., 9 a.m., 10 a.m., 11 a.m., 12 a.m., 1 p.m., 2 p.m., 3 p.m., 4 p.m., 5 p.m., 6 p.m., 7 p.m., 8 p.m., 9 p.m., 10 p.m., 11 p.m., 12 p.m.

## Activity Synthesis

The purpose of this discussion is to think about the different methods of writing the sample space and when each might be useful. The discussion is also meant to make the connection between the methods and the number of outcomes in the sample space so that students can quickly find the number of outcomes without writing out all the possibilities.

Invite previously selected students to share their strategies for finding the sample spaces. Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

For students who used the same representation throughout, ask,

- *"Why did you choose to use this same strategy for all the questions? What are the benefits of this strategy? Did you encounter any problems using the strategy?" For students who changed strategies for different questions, ask, "How did you decide which strategy to use for each question?"*

Connect the different responses to the learning goals by asking questions such as:

- *"How would you represent the outcome R2 for the spinner using each method?"*

  In a list it would be one of the options written. In a table it would be on the R row and the 2 column. In a tree it is represented by following the branch to R and then to 2.

- *"How do you find the number of outcomes in the sample space using each method?"*

  Using the list, count each item on the list. From the table, multiply the number of rows by the number of columns to get the number of outcomes in the middle of the table. From the tree, count the number of branch endings.

- *"Are all three of the methods useful for each of the situations?"*

  The table does not work when there are 3 parts of the experiment, like flipping 3 coins.

## Activity 2
## How Many Necklaces?

10 min

### Activity Narrative

In this activity, students practice using their understanding of ways to calculate the number of outcomes in the sample space without writing out the entire sample space. Many situations with multiple steps have very large sample spaces for which it is not helpful to write out the entire sample space, but it is still useful to know the number of outcomes in the sample space. In this activity, students find the number of different necklaces that can be made from available options.

### Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion.

### Student Task Statement

1. A store that sells Native American jewelry offers beaded necklaces with 1 bead color, 1 pendant option, 1 length, and 1 style. How many different necklaces are possible? Explain your reasoning. You do not need to write out the sample space.
   - color: red, blue, black, pink, turquoise
   - pendant: large stone, silver design, none
   - length: 18 inches, 21 inches, 25 inches, 27 inches
   - style: one strand, two strand, three strands

   180 options

   Sample reasoning: $5 \cdot 3 \cdot 4 \cdot 3 = 180$.
2. Andre knows he wants a necklace that is red and 21 inches in length. He isn't sure about the other choices. How many of the different necklaces could Andre consider to fit his two requirements?

   9 necklaces, because there are 3 options for pendant and 3 options for style ($3 \cdot 3 = 9$)
3. If a necklace is made by randomly choosing each of the options, what is the probability it will be a necklace that fits Andre's requirements?

   $\frac{9}{180}$ (or equivalent), because there are 9 necklaces that fit Andre's requirements out of 180 possible options

### Are You Ready for More?

Describe a situation that involves three parts and has a total of 24 outcomes in the sample space.

Sample response: Flip a coin, then select between rock, paper, scissors, then select a letter from the word MATH. This has 24 outcomes because $2 \cdot 3 \cdot 4 = 24$.

### Building on Student Thinking

Some students may attempt to write out the entire sample space. Encourage them to write a few outcomes to get the idea of what is possible, but let them know that the answer for question 1 is over 100, so finding a pattern or way to calculate the answer might be more efficient.

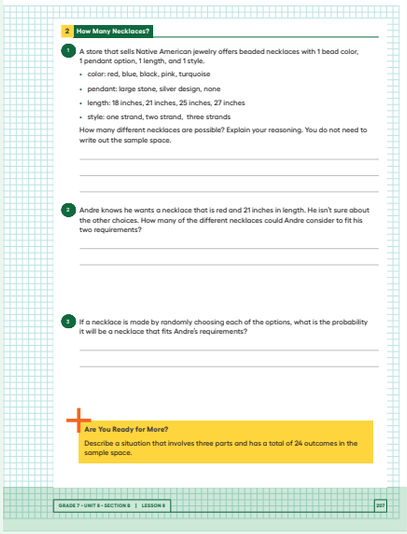
Some students may not count the none option for pendant as a distinct choice. Show these students that a necklace like "red, large stone, small, single strand" is different than "red, no pendant, small, single strand" and should be counted separately.

### Access for Multilingual Learners (Activity 2, Student Task)

**MLR8: Discussion Supports.** During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say ..." Original speakers can agree or clarify for their partner.

*Advances: Listening, Speaking*

### Student Workbook



2 How Many Necklaces?

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- color: red, blue, black, pink, turquoise
- pendant: large stone, silver design, none
- length: 18 inches, 21 inches, 25 inches, 27 inches
- style: one strand, two strand, three strands

How many different necklaces are possible? Explain your reasoning. You do not need to write out the sample space.

2 Andre knows he wants a necklace that is red and 21 inches in length. He isn't sure about the other choices. How many of the different necklaces could Andre consider to fit his two requirements?

3 If a necklace is made by randomly choosing each of the options, what is the probability it will be a necklace that fits Andre's requirements?

Are You Ready for More?

Describe a situation that involves three parts and has a total of 24 outcomes in the sample space.

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## Activity Synthesis

The purpose of the discussion is to help students understand the calculations behind the solutions of these problems.

Some questions for discussion:

- *"Describe how the tree of necklace options would look without drawing it out."*

  The first column could have the 5 options for color. Coming out from each of those options could be 3 branches for each of the pendant options. From each of these there would be 4 more branches for each of the length options. Each of those would have 3 branches for the style.

- *"How is the tree connected to the calculation of the size of the sample space?"*

  For the choice of color and pendant, there are 5 groups of 3 branches, making 15 options for those two choices. When including the length, there are 15 groups of 4 branches, making 60 options. For the style, there are 60 groups of 3 branches, making 180 options.

- *"If you are allowed a second choice of color for a design on the beads, are there more or fewer options?"*

  There are more options because each of the 180 options would allow for another choice of 4 options.

## Lesson Synthesis

Consider asking these discussion questions:

- *"What are some methods for writing out the sample space of a chance experiment that consists of multiple steps?"*

  Methods include trees, tables, and lists.

- *"How does the tree method relate to finding the number of outcomes in a sample space?"*

  Each path from the start to the end of each "branch" represents one outcome in the sample space, so counting all the paths will give the number of items in the sample space.

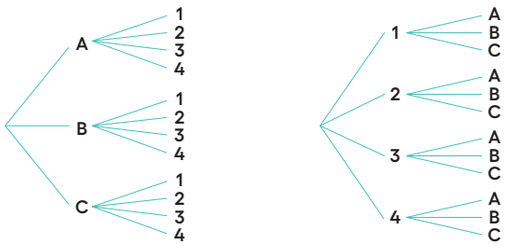
- *"Why is it important to know the number of outcomes in a sample space when finding probability?"*

  Probability can be found by $\frac{k}{n}$, where $k$ represents the number of outcomes in the event and $n$ represents the number of outcomes in the sample space.

## Lesson Summary

Sometimes we need a systematic way to count the number of outcomes that are possible in a given situation. For example, suppose there are 3 people (A, B, and C) who want to run for the president of a club and 4 different people (1, 2, 3, and 4) who want to run for vice president of the club. We can use a tree, a table, or an ordered list to count how many different combinations are possible for a president to be paired with a vice president.

With a tree, we can start with a branch for each of the people who want to be president. Then for each possible president, we add a branch for each possible vice president, for a total of $3 \cdot 4 = 12$ possible pairs. We can also start by counting vice presidents first and then adding a branch for each possible president, for a total of $4 \cdot 3 = 12$ possible pairs.



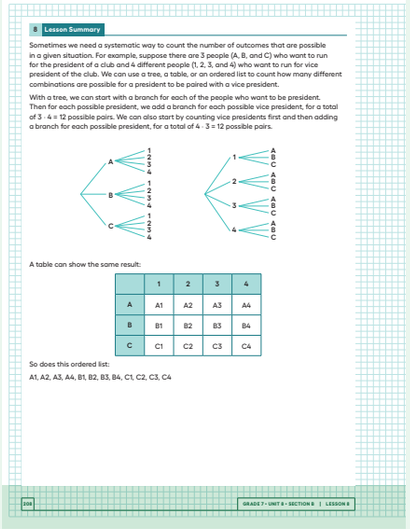

A table can show the same result:

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | A1 | A2 | A3 | A4 |
| B | B1 | B2 | B3 | B4 |
| C | C1 | C2 | C3 | C4 |

So does this ordered list:

A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

### Student Workbook


8 Lesson Summary

Sometimes we need a systematic way to count the number of outcomes that are possible in a given situation. For example, suppose there are 3 people (A, B, and C) who want to run for the president of a club and 4 different people (1, 2, 3, and 4) who want to run for vice president of the club. We can use a tree, a table, or an ordered list to count how many different combinations are possible for a president to be paired with a vice president.

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A table can show the same result:

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | A1 | A2 | A3 | A4 |
| B | B1 | B2 | B3 | B4 |
| C | C1 | C2 | C3 | C4 |

So does this ordered list:

A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

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**Responding To Student Thinking**

**More Chances**
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

## Shirt Options

5 min

### Student Task Statement

A school club is selling shirts for a fundraiser. Shirts come with one of each of these options:

- size: small, medium, large, extra large
- color: black, white
- design: logo on front, no logo on front

1. How many different shirts are available with these options? Explain or show your reasoning.

   16

   Sample reasoning: $4 \cdot 2 \cdot 2 = 16$

2. Diego wants a medium-sized shirt. How many shirts can he choose from with that requirement?

   4

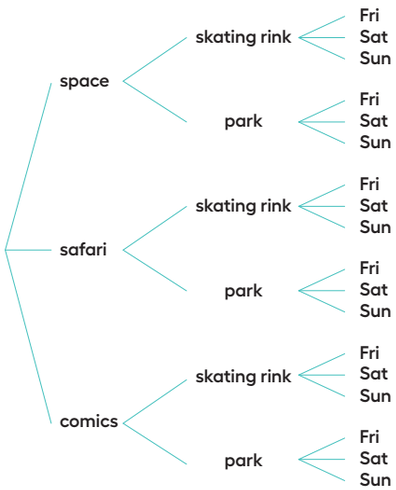
Lesson 8

## Practice Problems

4 Problems

### Problem 1

Noah is planning his birthday party. Here is a tree showing all of the possible themes, locations, and days of the week that Noah is considering.




**a.** How many themes is Noah considering?

3 themes

**b.** How many locations is Noah considering?

2 locations

**c.** How many days of the week is Noah considering?

3 days

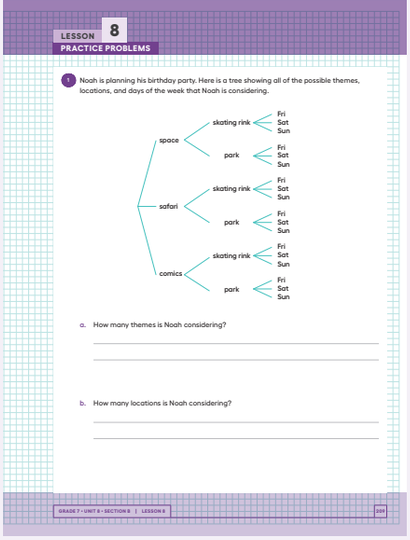
**d.** One possibility that Noah is considering is a party with a space theme at the skating rink on Sunday. Write two other possible parties Noah is considering.

Sample response: Noah is considering a comics themed party at the skating rink on Saturday or a safari themed party at the park on Friday.

**e.** How many different possible outcomes are in the sample space?

18 outcomes. The number of outcomes is given by $3 \cdot 2 \cdot 3$ or by counting the branches in the tree diagram.

**Student Workbook**


LESSON 8
PRACTICE PROBLEMS

1 Noah is planning his birthday party. Here is a tree showing all of the possible themes, locations, and days of the week that Noah is considering.

space — skating rink — Fri Sat Sun; park — Fri Sat Sun
safari — skating rink — Fri Sat Sun; park — Fri Sat Sun
comics — skating rink — Fri Sat Sun; park — Fri Sat Sun

a. How many themes is Noah considering?

b. How many locations is Noah considering?

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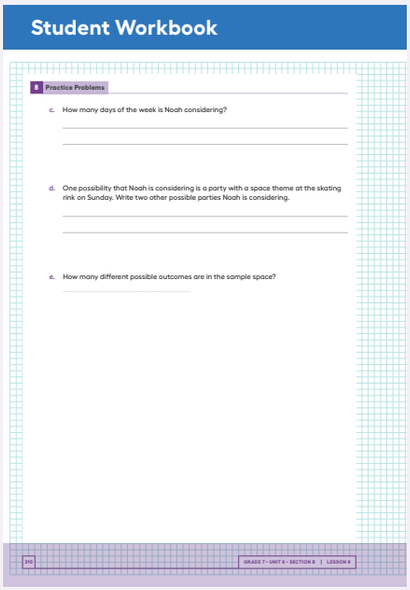
**Student Workbook**


8 Practice Problems

c. How many days of the week is Noah considering?

d. One possibility that Noah is considering is a party with a space theme at the skating rink on Sunday. Write two other possible parties Noah is considering.

e. How many different possible outcomes are in the sample space?

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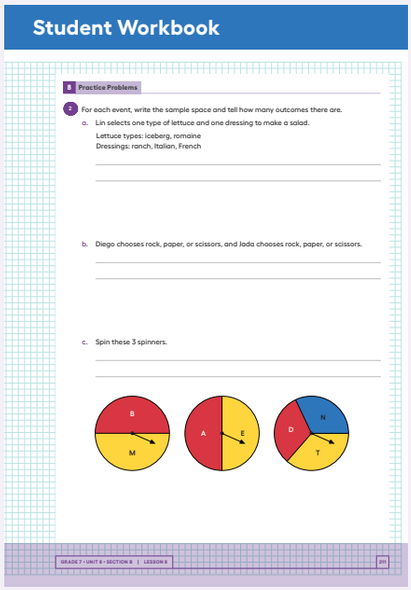
Student Workbook

8 Practice Problems

2 For each event, write the sample space and tell how many outcomes there are.

a. Lin selects one type of lettuce and one dressing to make a salad.

Lettuce types: iceberg, romaine
Dressings: ranch, Italian, French

b. Diego chooses rock, paper, or scissors, and Jada chooses rock, paper, or scissors.

c. Spin these 3 spinners.



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## Problem 2

For each event, write the sample space and tell how many outcomes there are.

**a.** Lin selects one type of lettuce and one dressing to make a salad.

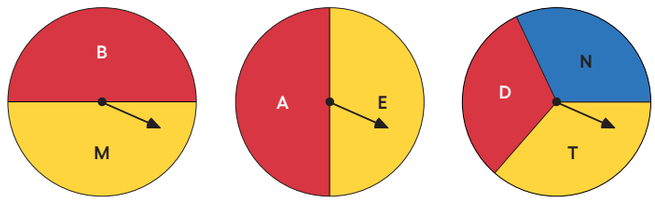
Lettuce types: iceberg, romaine
Dressings: ranch, Italian, French

6 outcomes: iceberg and Italian, iceberg and ranch, iceberg and French, romaine and Italian, romaine and ranch, romaine and French

**b.** Diego chooses rock, paper, or scissors, and Jada chooses rock, paper, or scissors.

9 outcomes: rr, rp, rs, pr, pp, ps, sr, sp, ss

**c.** Spin these 3 spinners.




12 outcomes: BAT, BET, MAT, MET, BAN, BEN, MAN, MEN, BAD, BED, MAD, MED

## Problem 3

from Unit 8, Lesson 7

A simulation is done to represent kicking 5 field goals in a single game with a 72% probability of making each one. A 1 represents making the kick, and a 0 represents missing the kick.

| trial | result |
|---|---|
| 1 | 10101 |
| 2 | 11010 |
| 3 | 00011 |
| 4 | 11111 |
| 5 | 10011 |

Based on these results, estimate the probability that 3 or more kicks are made.

$\frac{4}{5}$

## Problem 4

from Unit 8, Lesson 4

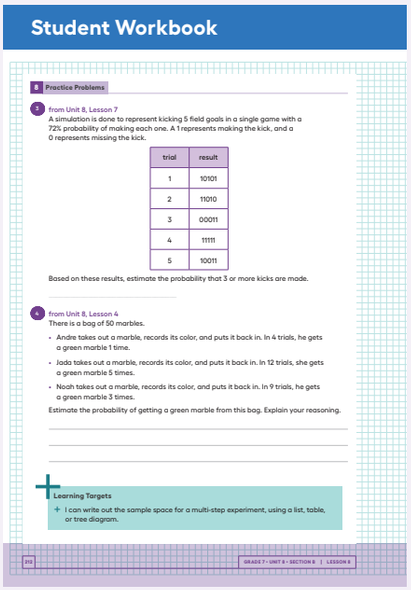
There is a bag of 50 marbles.

- Andre takes out a marble, records its color, and puts it back in. In 4 trials, he gets a green marble 1 time.
- Jada takes out a marble, records its color, and puts it back in. In 12 trials, she gets a green marble 5 times.
- Noah takes out a marble, records its color, and puts it back in. In 9 trials, he gets a green marble 3 times.

Estimate the probability of getting a green marble from this bag. Explain your reasoning.

Sample response: A good estimate of the probability of getting a green marble comes from combining Andre, Jada, and Noah's trials. They take a marble out of the bag a total of 25 times and get a green marble 9 of those times. So the probability of getting a green marble appears to be close to $\frac{9}{25} = 0.36$. Since there are 50 marbles in the bag, it is a reasonable estimate that 18 of the 50 marbles are green, though this is not guaranteed.

### Student Workbook


8 Practice Problems

3 from Unit 8, Lesson 7
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| trial | result |
|---|---|
| 1 | 10101 |
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| 4 | 11111 |
| 5 | 10011 |

Based on these results, estimate the probability that 3 or more kicks are made.

4 from Unit 8, Lesson 4
There is a bag of 50 marbles.
- Andre takes out a marble, records its color, and puts it back in. In 4 trials, he gets a green marble 1 time.
- Jada takes out a marble, records its color, and puts it back in. In 12 trials, she gets a green marble 5 times.
- Noah takes out a marble, records its color, and puts it back in. In 9 trials, he gets a green marble 3 times.

Estimate the probability of getting a green marble from this bag. Explain your reasoning.

Learning Targets
+ I can write out the sample space for a multi-step experiment, using a list, table, or tree diagram.