Lesson 14

Percent Error

Goals

- Calculate the percent error, correct amount, or erroneous amount, given the other two of these three quantities, and explain (orally and using other representations) the solution method.
- Compare and contrast (orally) strategies used for solving problems about percent error with strategies used for solving problems about percent increase or decrease.

Learning Target

I can solve problems that involve percent error.

Lesson Narrative

In this lesson, students solve problems about **percent error**, a way to describe error, expressed as a percentage of the actual amount. They interpret situations and identify which value is the correct amount that the error should be compared to. They make sense of problems as they solve for the erroneous value, the correct value, or the percentage. They also consider error tolerance expressed as a maximum acceptable percent error to determine values that fall within or outside of the range. As students relate situations involving percent error to previous work they have done with percent increase or decrease, they are making use of structure.

Student Learning Goal

Let's use percentages to describe other situations that involve error.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 1)
- MLR6: Three Reads (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR3: Critique, Correct, Clarify
- MLR6: Three Reads
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Four-function calculators: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	5 min

Instructional Routines

Math Talk

ilclass.com/r/10694967

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Building on Student Thinking

If students try to figure out exact answers, encourage them to think about numbers that are *close to* the numbers in the problem in order to estimate the percentage for each question.

Warm-up

Math Talk: What Percentage?

10 min

Activity Narrative

This *Math Talk* focuses on finding the percentage that one number is of another number. It encourages students to think about multiplicative comparisons and to rely on relationships between the dividend and divisor to mentally solve problems.

Next, these expressions are used as examples to introduce the concept of percent error. Students reason that an error that is 50% of the correct value is more problematic than an error that is 0.5% of the correct value.

As students compare the previous expression to the next expression and determine how to scale the percentage, students need to look for and make use of structure.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find each percentage mentally.

A. 3 is what percent of 6?

50%

Sample reasoning: 3 is $\frac{1}{2}$ of 6, which is 50%.

B. 3 is what percent of 12?

25%

Sample reasonings:

- 3 is $\frac{1}{4}$ of 12, which is 25%.
- 12 is 2 times as big as 6. If the denominator increases by a factor of 2, then the denominator is reduced by $\frac{1}{2}$.

C. 3 is what percent of 120?

2.5%

Sample reasoning: 120 is 10 times as big as 12. If the denominator increases by a factor of 12, then the quotient is reduced by $\frac{1}{10}$.

D. 3 is what percent of 600?

0.5%.

Sample reasoning:

- 600 is 5 times as big as 10. If the denominator increases by a factor of 5, then the quotient is reduced by $\frac{1}{5}$.
- 600 is 100 times as big as 6. If the denominator increases by a factor of 100, then the quotient is reduced by $\frac{1}{100}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

After discussing students' strategies, use these calculations as examples to introduce percent error, where an error is expressed as a percentage of the correct amount.

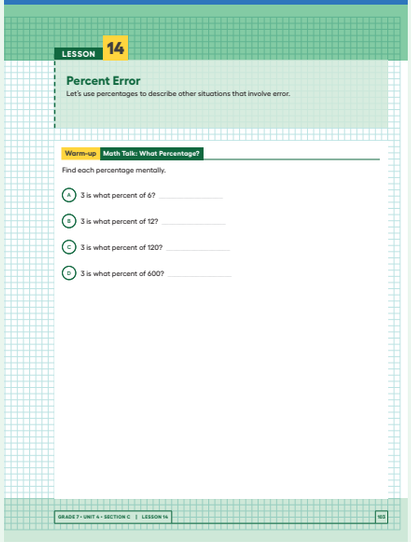
- *"If you bought a package that was supposed to contain 6 items, but it was missing 3 of them, how much would you care?"*

  3 is 50% of 6, so this is a significant error.

- *"If you bought a package that was supposed to contain 600 items, but it was missing 3 of them, how much would you care?"*

  3 is only 0.5% of 600, so this is a relatively minor error.

Student Workbook


LESSON 14

Percent Error

Let's use percentages to describe other situations that involve error.

Warm-up Math Talk: What Percentage?

Find each percentage mentally.

A. 3 is what percent of 6? _____

B. 3 is what percent of 12? _____

C. 3 is what percent of 120? _____

D. 3 is what percent of 600? _____

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Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 1)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.

Represent the same information through different modalities by using a tape diagram, double number line diagram, or a table to organize the information.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Building on Student Thinking

Students might struggle with figuring out how to calculate how much water the plant has been getting. Ask students,

"How much more water has the plant been getting? How do you calculate that total?"

Activity 1

Plants, Bicycles, and Crowds

15 min

Activity Narrative

In this activity, students solve three different problems involving percent error. The problems here are similar in structure to percent increase or decrease problems, but the language is different. Students interpret the language used for percent error, drawing parallels to the language used for percentage increase and decrease. This activity includes one of each type of problem:

- Finding the erroneous amount given the correct amount and the percent error—this is similar to finding the new value after a percent increase or decrease.
- Finding the correct amount given the erroneous amount and the percent error—this is similar to finding the original value before a percent increase or decrease.
- Finding the percent error given the erroneous amount and the correct amount—this is similar to finding the percentage that the increase or decrease is of the original amount.

Students connect the structure of percent increase and decrease to the situations involving percent error. In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Before students begin working, read the first question aloud. Ask several students to explain in their own words what information is given in the problem, and what it is asking them to find. The plant is supposed to get $\frac{3}{4}$ cup of water, but it is getting 25% more than that. We can think of this as $\frac{3}{4}$ cup increased by 25%.

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Student Task Statement

1. Instructions to care for a plant say to water it with $\frac{3}{4}$ cup of water every day. The plant has been getting 25% too much water. How much water has the plant been getting?

   $\frac{15}{16}$ of a cup (25% of $\frac{3}{4} = \frac{3}{16}$ so the plant has been getting $\frac{15}{16}$ cups of water each day because $\frac{3}{4} + \frac{3}{16} = \frac{15}{16}$.)

2. The pressure on a bicycle tire is 63 psi. This is 5% higher than what the manual says is the correct pressure. What is the correct pressure?

   60 psi (If the correct pressure is p, this means that $63 = p + 0.05p$, or $63 = 1.05p$, and $63 \div 1.05 = 60$.)

3. The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,486 people. What is the **percent error**?

   21% (The estimate is more than the exact amount by 514 people, and $\frac{514}{2,486} \approx 0.21$.)

Are You Ready for More?

A micrometer is an instrument that can measure lengths to the nearest micron. (A micron is a millionth of a meter.) Would this instrument be useful for measuring any of the following things? If so, what would the largest percent error be?

1. The thickness of an eyelash, which is typically about 0.1 millimeter.

   Yes, the instrument would be useful for this measurement. The biggest the error could be is half of a micron, or 0.0000005 meter. If we divide this by the thickness, which is 0.0001 meter, the percent error would be 0.5%.

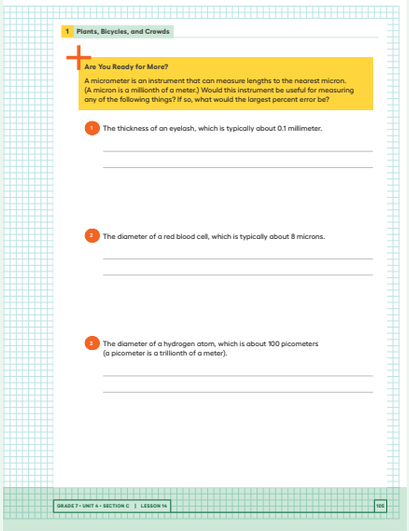
2. The diameter of a red blood cell, which is typically about 8 microns.

   Yes, the instrument would be useful for this measurement. The biggest the error could be is 0.5 micron. If we divide this by the length, which is 8 microns, the percent error would be 6.25%.

3. The diameter of a hydrogen atom, which is about 100 picometers (a picometer is a trillionth of a meter).

   No, the instrument would not be useful for this measurement. The diameter of a hydrogen atom is much smaller than a micron.

Student Workbook


1 Plants, Bicycles, and Crowds

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2. The diameter of a red blood cell, which is typically about 8 microns.

3. The diameter of a hydrogen atom, which is about 100 picometers (a picometer is a trillionth of a meter).

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Activity Synthesis

The purpose of this discussion is to highlight the connections between these problems involving percent error and the types of problems students solved previously involving percent increase and decrease. Invite students to share how they solved each problem. After each student shares, ask the class if they agree or disagree or if they had a different way to calculate the solution. Highlight efficient strategies, such as writing an equation. Ask students how the solution strategies here are similar to the ones used with percentage increase and decrease.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"To find the error, I subtracted 3,000 – 2,486 to get 514. To find the percent, I divided 514 by 3,000 to get 17%."

Ask,

💬 ***"What parts of this response are unclear, incorrect, or incomplete?"***

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

The key takeaway is that percent error is the amount of the error expressed as a percentage of the correct amount. In this case, the correct amount is the exact attendance, not the estimated crowd size, so it should be 514 ÷ 2,486.

Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 2, Student Task)

MLR6: Three Reads.

Keep student workbooks or devices closed. Display only the problem stem, without revealing the questions.

"We are going to read this situation 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions

"List some board lengths that should be accepted. List some board lengths that should be rejected."

Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Provide a paper which includes the meaning of percent error and how to convert a percentage to its decimal form for students to use as a reference.

Supports accessibility for: Memory, Organization

Building on Student Thinking

For the last question, some students may not realize that the board lengths were given in feet, while the amount of the error was given in inches. Consider asking

"How can we convert these lengths so that they have the same units?"

Activity 2
Sawmill

10 min

Activity Narrative

In this activity, students apply percent error to a quality control situation. It is very common that products in a factory are checked to make sure that they meet certain specifications. This activity is about a sawmill cutting boards to specific lengths. Boards that are too long or too short are rejected. Students decide whether boards should be accepted or rejected based on the intended length and an acceptable margin of error. As students determine values that are within, above, or below an acceptable range, they are making sense of how percent error characterizes the situation. They also persevere in solving when working backwards to determine the intended length given the percent error.

Launch

Arrange students in groups of 2.

Give students 4–5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Student Task Statement

1. A sawmill cuts boards that are 16 feet long. After they are cut, the boards are inspected and rejected if the length has a percent error of 1.5% or more.
   a. List some board lengths that should be accepted.

      Sample response: anything between 15.76 feet and 16.24 feet

   b. List some board lengths that should be rejected.

      Sample response: anything less than or equal to 15.76 feet or greater than or equal to 16.24 feet

2. The sawmill also cuts boards that are 10, 12, and 14 feet long. An inspector rejects a board that is 2.3 inches too long. What was the intended length of the board?

   The rejected board could have been intended to be 10 or 12 feet long since 1.5% of 10 feet is 0.15 foot (or 1.8 inches), and 1.5% of 12 feet is 0.18 foot (or 2.16 inches). It could not have been 14 feet since 1.5% of 14 feet is 0.21 foot (or 2.52 inches).

Activity Synthesis

The purpose of this discussion is to solidify students' understanding of how percent error can be used to establish an error tolerance. Invite students to share their responses for the first question about lengths that should be accepted or rejected. Record their responses on a number line, displayed for all to see. Ask students to describe any patterns they notice in the values that are accepted or rejected.

To help students understand these values in context, consider asking:

- *"Why should the mill accept boards that are a little longer or shorter than the intended length?"*

  It may be difficult to cut the boards to that exact length, especially as quickly as a sawmill probably needs to cut them, so some range of acceptable lengths is probably allowed.

- *"Why shouldn't the mill accept boards that are a lot longer or shorter than the intended length?"*

  People who are using the boards expect to get a consistent product. It would make them harder to transport, sell, or use if there was a lot of variability in the lengths.

- *"Why does it make sense for the range of acceptable lengths to be listed as a percent error rather than on a fixed length?"*

  While 2.88 inches may be ok for a board that should be about 192 inches long, if the board was supposed to be 5 inches long and it was allowed to be 2.88 inches longer, it would be more than 50% longer than intended.

Lesson Synthesis

Share with students,

- *"Today we solved problems involving percent error."*

To review these concepts, consider asking:

- *"How is percent error similar to percent increase or percent decrease? How are they different?"*

  If the incorrect value is higher than the correct value, it is similar to percent increase. If the incorrect value is lower, it is similar to percent decrease. Instead of comparing before and after a change, percent error is comparing an error to the correct amount.

- *"When working with percent error, what quantity in the problem corresponds to 100%?"*

  The correct value corresponds to 100%.

- *"Why is it helpful to express an error as a percentage of the correct value?"*

  It shows whether the error is relatively large or small in the context of the situation.

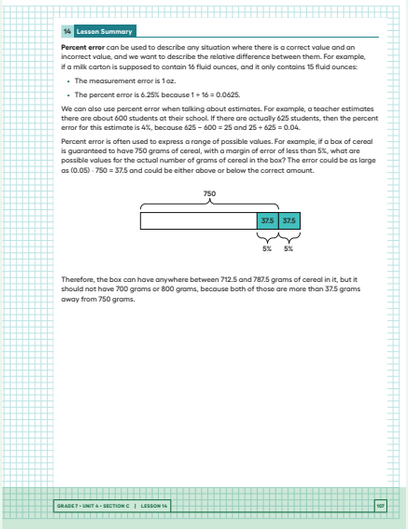
Responding To Student Thinking

Points to Emphasize

If students struggle with finding percent error from an expected value and an actual measurement, plan to focus on this as opportunities arise over the next several lessons. For example, this skill can be reviewed while discussing the following practice problem:

Grade 7, Unit 4, Lesson 15, Practice Problem 2

Student Workbook


14. Lesson Summary

Percent error can be used to describe any situation where there is a correct value and an incorrect value, and we want to describe the relative difference between them. For example, if a milk carton is supposed to contain 16 fluid ounces, and it only contains 15 fluid ounces:

- The measurement error is 1 oz.
- The percent error is 6.25% because 1 ÷ 16 = 0.0625.

We can also use percent error when talking about estimates. For example, a teacher estimates there are about 600 students at their school. If there are actually 625 students, then the percent error for this estimate is 4%, because 625 – 600 = 25 and 25 ÷ 625 = 0.04.

Percent error is often used to express a range of possible values. For example, if a box of cereal is guaranteed to have 750 grams of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as large as (0.05) · 750 = 37.5 and could be either above or below the correct amount.

Therefore, the box can have anywhere between 712.5 and 787.5 grams of cereal in it, but it should not have 700 grams or 800 grams, because both of those are more than 37.5 grams away from 750 grams.

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Lesson Summary

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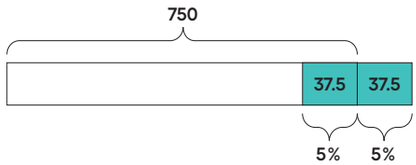


Therefore, the box can have anywhere between 712.5 and 787.5 grams of cereal in it, but it should not have 700 grams or 800 grams, because both of those are more than 37.5 grams away from 750 grams.

Cool-down

Yarn Weight

5 min

Students who answer "about 6.1%" may have divided by the erroneous weight rather than by the intended weight.

Student Task Statement

A ball of yarn is supposed to weigh 3.5 ounces. Priya measures it and finds that it weighs 3.3 ounces. What is the percent error?

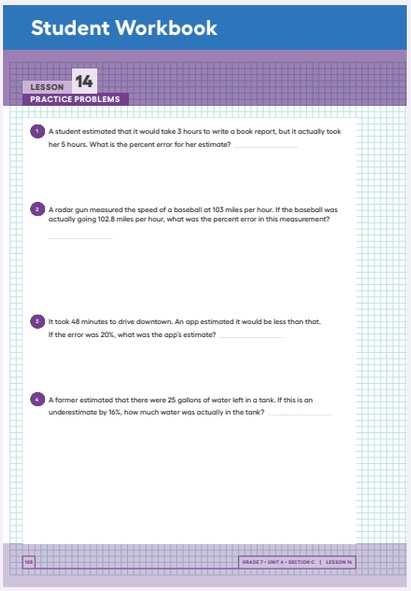
about 5.7%

Sample reasoning: $3.5 - 3.3 = 0.2$ and $0.2 \div 3.5 = 0.0\overline{571428}$

Lesson 14

Practice Problems

7 Problems


Student Workbook

LESSON 14
PRACTICE PROBLEMS

1. A student estimated that it would take 3 hours to write a book report, but it actually took her 5 hours. What is the percent error for her estimate? _____

2. A radar gun measured the speed of a baseball at 103 miles per hour. If the baseball was actually going 102.8 miles per hour, what was the percent error in this measurement? _____

3. It took 48 minutes to drive downtown. An app estimated it would be less than that. If the error was 20%, what was the app's estimate? _____

4. A farmer estimated that there were 25 gallons of water left in a tank. If this is an underestimate by 16%, how much water was actually in the tank? _____

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Problem 1

A student estimated that it would take 3 hours to write a book report, but it actually took her 5 hours. What is the percent error for her estimate?

40% (5 − 3 = 2 and 2 ÷ 5 = 0.4)

Problem 2

A radar gun measured the speed of a baseball at 103 miles per hour. If the baseball was actually going 102.8 miles per hour, what was the percent error in this measurement?

0.19% (103 − 102.8 = 0.2 and 0.2 ÷ 102.8 ≈ 0.0019)

Problem 3

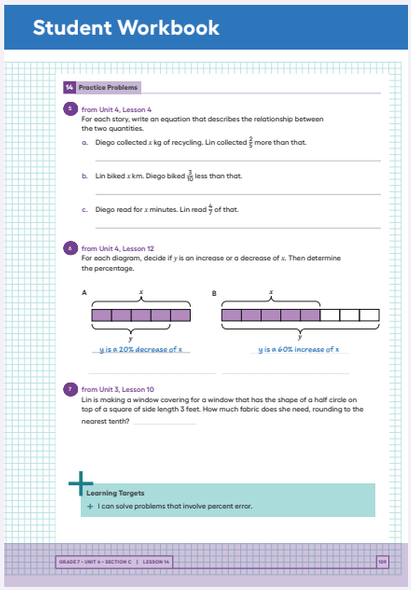
It took 48 minutes to drive downtown. An app estimated it would be less than that. If the error was 20%, what was the app's estimate?

38.4 minutes, because 48 − (0.2)48 = 38.4

Problem 4

A farmer estimated that there were 25 gallons of water left in a tank. If this is an underestimate by 16%, how much water was actually in the tank?

29.8 gallons (25 ÷ 0.84 ≈ 29.8)


Student Workbook

14 Practice Problems

5 from Unit 4, Lesson 4
For each story, write an equation that describes the relationship between the two quantities.
a. Diego collected x kg of recycling. Lin collected $\frac{2}{5}$ more than that.
b. Lin biked x km. Diego biked $\frac{3}{10}$ less than that.
c. Diego read for x minutes. Lin read $\frac{4}{7}$ of that.

6 from Unit 4, Lesson 12
For each diagram, decide if y is an increase or a decrease of x. Then determine the percentage.
A: y is a 20% decrease of x
B: y is a 60% increase of x

7 from Unit 3, Lesson 10
Lin is making a window covering for a window that has the shape of a half circle on top of a square of side length 3 feet. How much fabric does she need, rounding to the nearest tenth?

Learning Targets
+ I can solve problems that involve percent error.

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Problem 5

from Unit 4, Lesson 4

For each story, write an equation that describes the relationship between the two quantities.

a. Diego collected x kg of recycling. Lin collected $\frac{2}{5}$ more than that. $y = \frac{7}{5}x$

b. Lin biked x km. Diego biked $\frac{3}{10}$ less than that. $y = \frac{7}{10}x$

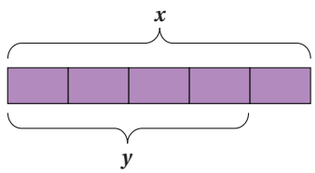
c. Diego read for x minutes. Lin read $\frac{4}{7}$ of that. $y = \frac{4}{7}x$

Problem 6

from Unit 4, Lesson 12

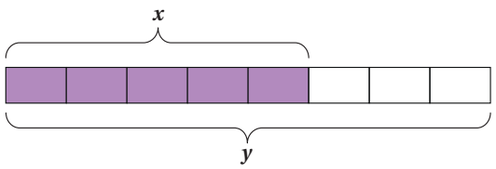
For each diagram, decide if y is an increase or a decrease of x. Then determine the percentage.

A



y is a 20% decrease of x

B



y is a 60% increase of x

Problem 7

from Unit 3, Lesson 10

Lin is making a window covering for a window that has the shape of a half circle on top of a square of side length 3 feet. How much fabric does she need, rounding to the nearest tenth?

at least 12.5 square feet (The area of the square part of the window is 9 square feet, and the area of the half circle is a little bit more than 3.5 square feet because $\frac{1}{2} \cdot \pi \cdot (1.5)^2 \approx 3.5$.)