Lesson 14

Estimating with Scientific Notation

Goals

- Generalize (orally and in writing) a process of multiplying and dividing numbers in scientific notation.
- Use scientific notation and estimation to compare quantities and interpret (orally and in writing) results in context.

Learning Targets

- I can multiply and divide numbers given in scientific notation.
- I can use scientific notation and estimation to compare very large or very small numbers.

Lesson Narrative

In this lesson, students perform operations with numbers expressed in scientific notation, use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times as much one quantity is than the other.

Next, students use the *Info Gap* structure to compare planetary diameters and masses. Students will need to request information about the diameters and masses of certain planets as well as calculate how many of one will equal the other. The focus is on estimation strategies and interpreting their results in context.

An optional activity allows for additional practice multiplying and dividing large and small numbers in context.

Student Learning Goal

Let's multiply and divide with scientific notation to answer questions about animals, careers, and planets.

Instructional Routines

- MLR4: Information Gap Cards

Access for Multilingual Learners

- MLR4: Information Gap (Activity 2)

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Required Materials

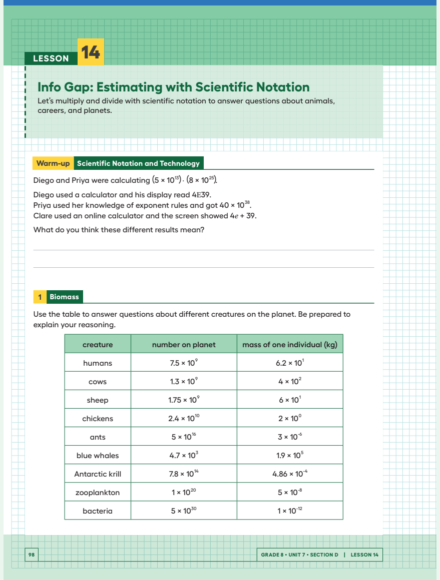
Materials to Copy

- Distances in the Solar System Cards (1 copy for every 4 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	25 min	10 min	5 min

Student Workbook



LESSON 14

Info Gap: Estimating with Scientific Notation

Let's multiply and divide with scientific notation to answer questions about animals, careers, and planets.

Warm-up Scientific Notation and Technology

Diego and Priya were calculating $(5 \times 10^{13}) \cdot (8 \times 10^{25})$.

Diego used a calculator and his display read 4E39.
Priya used her knowledge of exponent rules and got 40×10^{38}.
Clare used an online calculator and the screen showed $4e + 39$.

What do you think these different results mean?

1 Biomass

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number on planet	mass of one individual (kg)
humans	7.5×10^{9}	6.2×10^{1}
cows	1.3×10^{9}	4×10^{2}
sheep	1.75×10^{9}	6×10^{1}
chickens	2.4×10^{10}	2×10^{0}
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^{3}	1.9×10^{5}
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

98 GRADE 8 • UNIT 7 • SECTION D | LESSON 14

Warm-up

Scientific Notation and Technology

5 min

Activity Narrative

This activity introduces students to scientific notation that has been generated by technology. Technology may include physical and online calculators, apps, spreadsheet programs, and other digital tools that may display very large or very small numbers.

Launch

Give students 2 minutes of quiet think time followed by a whole-class discussion.

Student Task Statement

Diego and Priya were calculating $(5 \times 10^{13}) \cdot (8 \times 10^{25})$.

Diego used a calculator and his display read 4E39.
Priya used her knowledge of exponent rules and got 40×10^{38}.
Clare used an online calculator and the screen showed $4e + 39$.

What do you think these different results mean?

Sample response: I think the "E" in Diego's result and the "e" in Clare's result both mean "$\times 10$". If we wrote Priya's answer in scientific notation, it would be 4×10^{39}, so that means all 3 of them got the same result.

Activity Synthesis

The goal of this discussion is for students to understand the different ways scientific notation might be expressed by technology. Invite students to share their reasoning about Diego's result. Here are some questions for discussion:

- *"How are Diego's and Priya's answers the same?"*

  They both have the same value.

- *"How are Diego's and Priya's answers different?"*

  They have different factors. Diego's result has a letter in it.

If time allows, perform the calculation from the *Task Statement* on a calculator for all to see. Note that different calculators and programs may display scientific notation slightly differently. For example, some technology may use E, EE, or e.

Activity 1: Optional
Biomass

15 min

Activity Narrative

In this optional activity, students answer questions about small and large quantities in context. They use scientific notation as a tool to describe quantities, make estimates, and make comparisons. Use this activity if students would benefit from additional practice multiplying numbers expressed in scientific notation.

Launch

Arrange students in groups of 2 to allow for partner discussions as they work. Give groups 8–10 minutes to work, followed by a brief whole-class discussion.

Student Task Statement

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number on planet	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. Which creature is least numerous? Estimate how many times more ants there are than this creature.

   Blue whales are the least numerous. There are about 10^{13}, or 10 trillion, times as many ants as blue whales because $\frac{5 \times 10^{16}}{5 \times 10^3}$. This uses 5×10^3 to estimate the number of blue whales.

2. Which creature is the least massive? Estimate how many times more massive a human is than this creature.

   Bacteria is the least massive. A human is about 60 trillion times more massive because $\frac{6.2 \times 10^1}{1 \times 10^{-12}} = 6.2 \times 10^{13}$.

Student Workbook


1 Biomass

1 Which creature is least numerous? Estimate how many times more ants there are than this creature.

2 Which creature is the least massive? Estimate how many times more massive a human is than this creature.

3 Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?

4 Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

GRADE 8 • UNIT 7 • SECTION D | LESSON 14

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

3. Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?

   The total human mass is about 3 times as massive as the total ant mass. The total human mass is 7.5×10^9 times 6.2×10^1 kg per human, which is approximately 45×10^{10} kg. The total ant mass is 5×10^{16} times 3×10^{-6}, which is 15×10^{10} kg.

4. Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

   The total mass of krill is about 400 times the total mass of blue whales. The total krill mass is 7.8×10^{14} times 4.86×10^{-4}, which is approximately 8×10^{14} times 5×10^{-4} or 400 billion kg. The total mass of blue whales is 4.7×10^3 times 1.9×10^5, which is approximately 5×10^3 times 2×10^5 or 1 billion kg.

Activity Synthesis

The purpose of this discussion is for students to share their reasoning and strategies for comparing numbers using scientific notation. Begin by inviting students to share their responses to each question. To involve more students in the conversation, consider asking:

"Do you agree or disagree? Why?"

"Who can restate ____'s reasoning in a different way?"

"Does anyone want to add on to ____'s reasoning?"

Activity 2

Info Gap: Distances in the Solar System

25 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to describe quantities, make estimates, and make comparisons.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch

Ask students,

"How many humans do you think there are for each cat in the world?"

Ask for estimates that are too high, too low, and as reasonable as possible.

Explain that large numbers like populations are often estimated using scientific notation. There are an estimated 8.1×10^9 humans and an estimated 4.5×10^8 cats in the world. Guide students through the example: $\frac{8.1 \times 10^9}{4.5 \times 10^8} = \frac{8.1}{4.5} \times 10^{9-8} = 1.8 \times 10^1 = 18$. So there are roughly 18 humans for each cat.

Tell students that making reasonable estimates will help to answer the question in the *Task Statement*. For example, estimating the number of humans for each cat could have looked like $\frac{8.1 \times 10^9}{4.5 \times 10^8} \approx \frac{8 \times 10^9}{4 \times 10^8} = 2 \times 10^1 = 20$. In this case, the final estimate of 20 is not far from the original estimate of 18.

Tell students that they will be making estimates like this one about humans and cats based on very large numbers written in scientific notation. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1:

1. The Sun is about 100 times the width of Earth because $\frac{1.392 \times 10^6}{1.28 \times 10^4} \approx \frac{10^6}{10^4} = 10^2$.
2. The Sun is about 300,000 times as massive as Earth because $\frac{1.989 \times 10^{30}}{5.98 \times 10^{24}} \approx \frac{2 \times 10^{30}}{6 \times 10^{24}} = \frac{1}{3} \times 10^6$, which is $333{,}333.\overline{3}$.

Problem Card 2:

1. Neptune is about 100 times as far from Earth as Venus because $\frac{4.3 \times 10^9}{4 \times 10^7} \approx \frac{10^9}{10^7} = 10^2$.
2. It would take around 300 Mercuries to equal the mass of Neptune because $\frac{1.024 \times 10^{26}}{3.3 \times 10^{23}} \approx \frac{1 \times 10^{26}}{3 \times 10^{23}} = \frac{1}{3} \times 10^3$, which is $333.\overline{3}$.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy. *Supports accessibility for: Memory, Organization*

Student Workbook


2 Info Gap: Distances in the Solar System

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

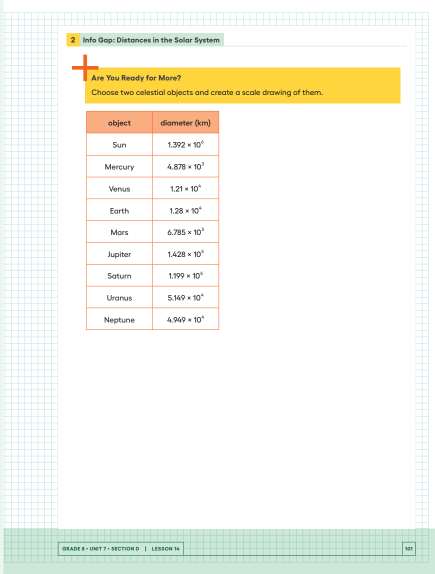
1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ____ because . . . ."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

100 GRADE 8 • UNIT 7 • SECTION D | LESSON 14

Student Workbook


2 Info Gap: Distances in the Solar System

Are You Ready for More?

Choose two celestial objects and create a scale drawing of them.

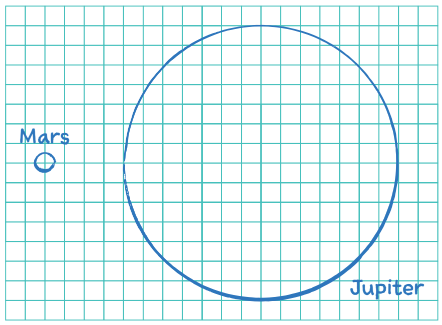
Are You Ready for More?

Choose two celestial objects and create a scale drawing of them.

object	diameter (km)
Sun	1.392×10^6
Mercury	4.878×10^3
Venus	1.21×10^4
Earth	1.28×10^4
Mars	6.785×10^3
Jupiter	1.428×10^5
Saturn	1.199×10^5
Uranus	5.149×10^4
Neptune	4.949×10^4

Answers vary.

Sample response:



The diameter of Mars is 6.785×10^3 kilometers and the diameter of Jupiter is 1.428×10^5 kilometers. Since $\frac{1.428 \times 10^5}{6.785 \times 10^3} \approx 0.21 \times 10^2$, the diameter of Jupiter is about $0.21 \times 10^2 = 21$ times the diameter of Mars.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"Why are some of the estimates in the class different?"

Different people used different numbers to estimate. For example some people may have estimated 1.392 as 1.4 while another person may have used 1.5.

"How did you come up with your estimates?"

I rounded the factor between 1 and 10 to the nearest whole number or half number.

"How did you compare powers of 10?"

Using exponent rules, I knew that to divide two powers of 10, I could find the difference between their exponents.

"Are the estimates still useful even if they are all different?"

Yes, the estimates may not be exactly the same, but they are close especially relative to the quantities that they are comparing.

Lesson Synthesis

The purpose of this discussion is for students to describe a process for multiplying and dividing numbers written in scientific notation. Begin by displaying this equation for all to see: $\frac{7 \times 10^6}{2 \times 10^4} = (7 \div 2) \times 10^{6-4}$.

Ask students if they think the statement is true or false and to explain their reasoning.

The statement is true.

Next display this equation for all to see: $(4 \times 10^5) \cdot (4 \times 10^4) = 4 \times 10^{20}$.

Ask students if they think this statement is true or false and to explain their reasoning.

The statement is false.

Ask students how to make the statement true and record responses for all to see.

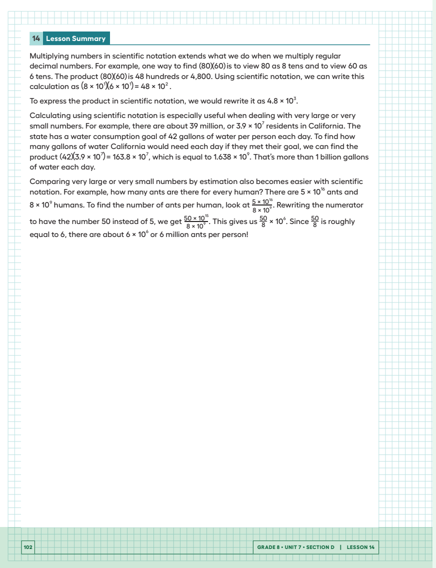
The result should be 16×10^9 because $4 \times 4 = 16$ and $10^5 \times 10^4 = 10^9$.

Ask students to rewrite the result in scientific notation ($16 \times 10^9 = 1.6 \times 10^{10}$.)

Ask students to describe a strategy for multiplying or dividing numbers written in scientific notation and record responses for all to see. Here are some strategies students may describe:

- To find the product of two numbers written in scientific notation, multiply the two factors that are a decimal number and multiply the two powers of 10 by adding their exponents. Write the result in scientific notation by making sure the first factor is greater than or equal to 1 but less than 10 and adjusting the power of 10 accordingly.
- To find the quotient of two numbers written in scientific notation, divide the two factors that are a decimal number and divide the two powers of 10 by subtracting their exponents. Write the result in scientific notation by making sure the first factor is greater than or equal to 1 but less than 10 and adjusting the power of 10 accordingly.

Student Workbook



Responding To Student Thinking

Points to Emphasize
If most students struggle with estimating how many times larger one value is compared to another value, revisit dividing and estimation with scientific notation in this activity:

Unit 7, Lesson 16, Activity 1 Old Hardware, New Hardware

Lesson Summary

Multiplying numbers in scientific notation extends what we do when we multiply regular decimal numbers. For example, one way to find (80)(60) is to view 80 as 8 tens and to view 60 as 6 tens. The product (80)(60) is 48 hundreds or 4,800. Using scientific notation, we can write this calculation as $(8 \times 10^1)(6 \times 10^1) = 48 \times 10^2$. To express the product in scientific notation, we would rewrite it as 4.8×10^3.

Calculating using scientific notation is especially useful when dealing with very large or very small numbers. For example, there are about 39 million, or 3.9×10^7 residents in California. The state has a water consumption goal of 42 gallons of water per person each day. To find how many gallons of water California would need each day if they met their goal, we can find the product $(42)(3.9 \times 10^7) = 163.8 \times 10^7$, which is equal to 1.638×10^9. That's more than 1 billion gallons of water each day.

Comparing very large or very small numbers by estimation also becomes easier with scientific notation. For example, how many ants are there for every human? There are 5×10^{16} ants and 8×10^9 humans. To find the number of ants per human, look at $\frac{5 \times 10^{16}}{8 \times 10^9}$. Rewriting the numerator to have the number 50 instead of 5, we get $\frac{50 \times 10^{15}}{8 \times 10^9}$. This gives us $\frac{50}{8} \times 10^6$. Since $\frac{50}{8}$ is roughly equal to 6, there are about 6×10^6 or 6 million ants per person!

Cool-down

Estimating with Scientific Notation

5 min

Student Task Statement

1. Estimate how many times larger 6.1×10^7 is than 2.1×10^{-4}. Explain or show your reasoning.

   6.1×10^7 is about 300 billion times larger than 2.1×10^{-4}

   Sample reasoning: $\frac{6.1 \times 10^7}{2.1 \times 10^{-4}} \approx \frac{6 \times 10^7}{2 \times 10^{-4}} = 3 \times 10^{7-(-4)} = 3 \times 10^{11}$.

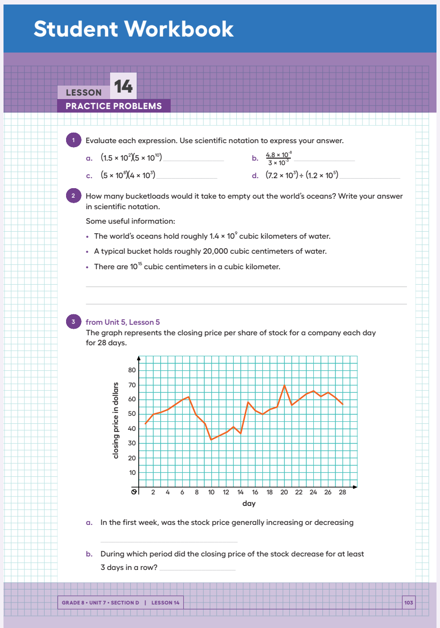
2. Estimate how many times larger 1.9×10^{-8} is than 4.2×10^{-13}. Explain or show your reasoning.

   1.9×10^{-8} is about 50,000 times larger than 4.2×10^{-13}

   Sample reasoning: $\frac{1.9 \times 10^{-8}}{4.2 \times 10^{-13}} \approx \frac{2 \times 10^{-8}}{4 \times 10^{-13}} = 0.5 \times 10^5 = 5 \times 10^4$.

Practice Problems

5 Problems


Student Workbook

LESSON 14
PRACTICE PROBLEMS

1 Evaluate each expression. Use scientific notation to express your answer.

a. $(1.5 \times 10^2)(5 \times 10^{10})$
b. $\frac{4.8 \times 10^{-8}}{3 \times 10^{-3}}$
c. $(5 \times 10^8)(4 \times 10^3)$
d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$

2 How many bucketloads would it take to empty out the world's oceans? Write your answer in scientific notation.

Some useful information:

- The world's oceans hold roughly 1.4×10^9 cubic kilometers of water.
- A typical bucket holds roughly 20,000 cubic centimeters of water.
- There are 10^{15} cubic centimeters in a cubic kilometer.

3 from Unit 5, Lesson 5

The graph represents the closing price per share of stock for a company each day for 28 days.

a. In the first week, was the stock price generally increasing or decreasing

b. During which period did the closing price of the stock decrease for at least 3 days in a row?

GRADE 8 • UNIT 7 • SECTION D | LESSON 14

Problem 1

Evaluate each expression. Use scientific notation to express your answer.

a. $(1.5 \times 10^2)(5 \times 10^{10})$ 7.5×10^{12}

b. $\frac{4.8 \times 10^{-8}}{3 \times 10^{-3}}$ 1.6×10^{-5}

c. $(5 \times 10^8)(4 \times 10^3)$ 2×10^{12}

d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$ 6×10^{-2}

Problem 2

How many bucketloads would it take to empty out the world's oceans? Write your answer in scientific notation.

Some useful information:

- The world's oceans hold roughly 1.4×10^9 cubic kilometers of water.
- A typical bucket holds roughly 20,000 cubic centimeters of water.
- There are 10^{15} cubic centimeters in a cubic kilometer.

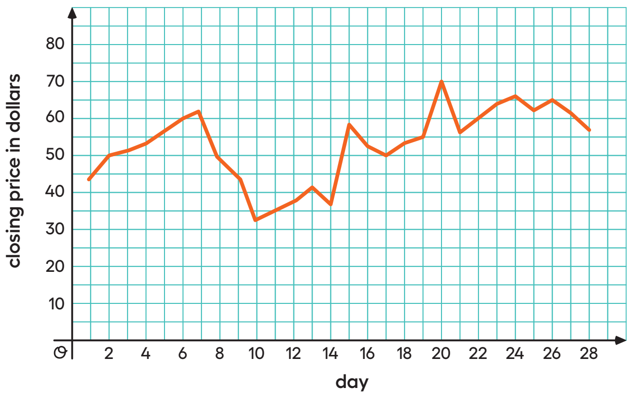
7×10^{19} buckets

Sample reasoning: The world's oceans hold 1.4×10^{24} cubic centimeters of water, found by multiplying 1.4×10^9 by 10^{15}. Then divide by 2×10^4 to get 0.7×10^{20}. In scientific notation, this quotient is 7×10^{19}.

Problem 3

from Unit 5, Lesson 5

The graph represents the closing price per share of stock for a company each day for 28 days.

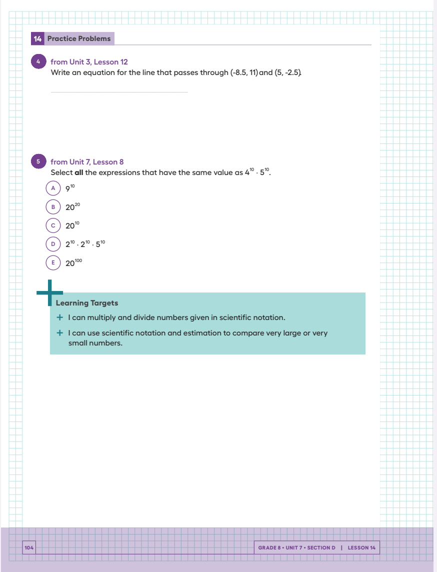


a. In the first week, was the stock price generally increasing or decreasing?

Increasing

b. During which period did the closing price of the stock decrease for at least 3 days in a row? Days 7 to 10

Student Workbook


14 Practice Problems

4 from Unit 3, Lesson 12

Write an equation for the line that passes through (-8.5, 11) and (5, -2.5).

5 from Unit 7, Lesson 8

Select **all** the expressions that have the same value as $4^{10} \cdot 5^{10}$.

Learning Targets

+ I can multiply and divide numbers given in scientific notation.
+ I can use scientific notation and estimation to compare very large or very small numbers.

GRADE 8 • UNIT 7 • SECTION B | LESSON 14

Problem 4 from Unit 3, Lesson 12

Write an equation for the line that passes through (-8.5, 11) and (5, -2.5).

$y = -x + 2.5$ (or equivalent)

Problem 5 from Unit 7, Lesson 8

Select **all** the expressions that have the same value as $4^{10} \cdot 5^{10}$

A. 9^{10}

B. 20^{20}

C. 20^{10}

D. $2^{10} \cdot 2^{10} \cdot 5^{10}$

E. 20^{100}