Lesson 12

What is Surface Area?

Goals

- Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method.
- Comprehend that the term "surface area" (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.

Learning Target

I know what the surface area of a three-dimensional object means.

Lesson Narrative

This lesson introduces students to the concept of **surface area**. They use what they learned about the area of rectangles to find the surface areas of prisms with rectangular **faces**.

Students begin by exploring surface area in concrete terms, by thinking about the number of square sticky notes it would take to cover a filing cabinet. First, they make an estimate, and then they think about what information is needed to calculate the actual number of sticky notes. Because no techniques are given, students need to make sense of the problem and persevere in solving it. As they analyze the situation, think about the problem in context, and consider the mathematics strategies that they know, students practice modeling with mathematics.

Students learn that "surface area" (in square units) is the number of unit squares it takes to cover all the faces of a three-dimensional figure without gaps or overlaps.

Later in the lesson, students determine the surface area of rectangular prisms that are built from snap cubes.

Student Learning Goal

Let's cover the surfaces of some three-dimensional objects.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder
- Poll the Class

Required Materials

Materials to Gather

- Snap cubes: Activity 2

Required Preparation

Activity 2:
Prepare 12 cubes per student and extra copies of isometric dot paper.

Lesson:
Build several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes for the *Cool-down*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down

Instructional Routines

Notice and Wonder

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Instructional Routines

Poll the Class

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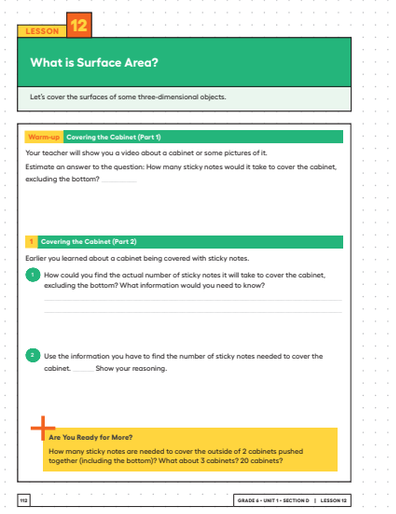
Student Workbook


LESSON 12

What is Surface Area?

Let's cover the surfaces of some three-dimensional objects.

Warm-up Covering the Cabinet (Part 1)

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom? _____

1 Covering the Cabinet (Part 2)

Earlier you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes needed to cover the cabinet. _____ Show your reasoning.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

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Warm-up

Covering the Cabinet (Part 1)

5 min

Activity Narrative

This activity prepares students to think about surface area, which they explore in this lesson and in upcoming lessons. Students watch a video of a cabinet being gradually tiled with non-overlapping sticky notes. The cabinet was left only partially tiled, which raises the question of the number of sticky notes it takes to cover the entire rectangular prism. Students estimate the answer to this question.

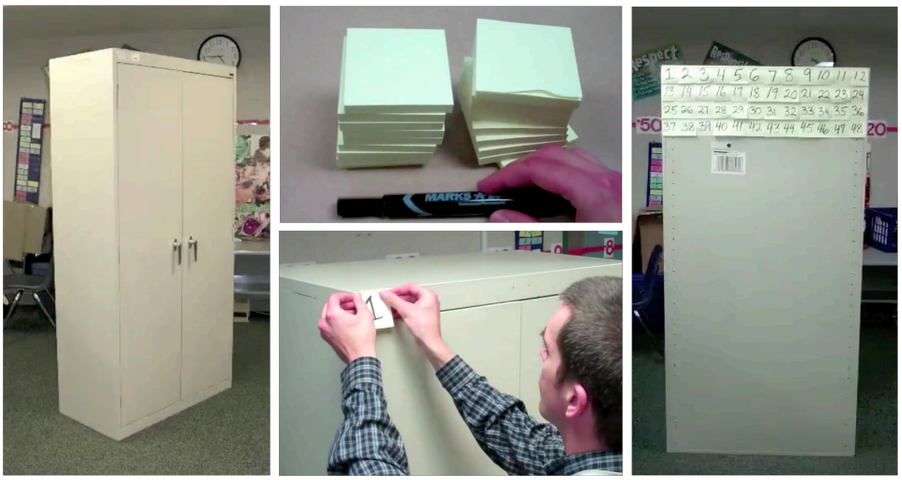
This activity was inspired by Andrew Stadel. Media used with permission. **http://www.estimation180.com/filecabinet**.

Launch

Arrange students in groups of 2. Show the video of a teacher beginning to cover a large cabinet with sticky notes or display the following still images for all to see. Before starting the video or displaying the image, ask students to be prepared to share one thing they notice and one thing they wonder.

Video 'File Cabinet – Act 1' available here:

https://ilclass.com/r/16059297.



Give students a minute to share their observation and question with a partner. Invite a few students to share their questions with the class. If the question, "How many sticky notes would it take to cover the entire cabinet?", is not mentioned, ask if anyone wondered how many sticky notes it would take to cover the entire cabinet.

Give students a minute to make an estimate.

Student Task Statement

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

Estimates vary. The actual number of sticky notes is 935. Good estimates are in the 800–1,200 range.

Activity Synthesis

Poll the class for students' estimates, and record them for all to see. Invite a couple of students to share how they made their estimate. Explain to students that they will now think about how to answer this question.

Activity 1
Covering the Cabinet (Part 2)

20 min

Activity Narrative

After making an estimate of the number of sticky notes on the cabinet in the *Warm-up*, students now brainstorm ways to find that number more accurately. They then go about calculating an answer. The activity prompts students to transfer their understanding of the area of polygons to find the surface area of a three-dimensional object.

Students learn that the surface area of a three-dimensional figure is the total area of all its faces. Because the area of a region is the number of square units it takes to cover the region without gaps and overlaps, surface area can be thought of as the number of square units that are needed to cover all sides of an object without gaps and overlaps. The square sticky notes illustrate this idea in a concrete way.

As students work, notice the various approaches that they take to determine the number of sticky notes needed to tile the faces of the cabinet (excluding the bottom). Identify students with different strategies to share later.

Launch

Arrange students in groups of 2–4. Give students 1 minute of quiet time to think about the first question and another minute to share their responses with their group. Ask students to pause afterward.

Select some students to share how they might figure out the number of sticky notes and what information they would need. Students may ask for some measurements:

- The measurements of the cabinet in units of sticky notes: 24 by 12 by 6 sticky notes.
- The measurements of the cabinet in inches or centimeters: Tell students that you don't have that information and prompt them to think of another piece of information that they could use.
- The measurements of each sticky note: 3 inches by 3 inches.

If no students mention needing the edge measurements of the cabinet in terms of sticky notes, let them begin working on the second question and provide the information when they realize that it is needed. Give students 8–10 minutes for the second question.

Instructional Routines

MLR7: Compare and Connect

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Building on Student Thinking

Students may treat all sides as if they were congruent rectangles. That is, they find the area of the front of the cabinet and then just multiply by 5, or act as if the top is the only side that is not congruent to the others. If there is a real cabinet (or any other large object in the shape of a rectangular prism) in the classroom, consider showing students that only the sides opposite each other can be presumed to be identical.

Students may neglect the fact that the bottom of the cabinet will not be covered. Point out that the bottom is inaccessible because of the floor.

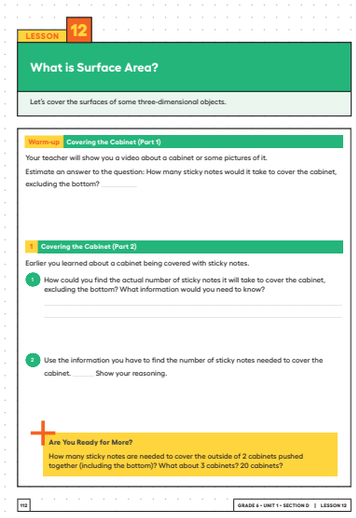
Access for Multilingual Learners (Activity 1, Student Task)

MLR7: Compare and Connect. After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask:

- "What did the approaches have in common? How were they different?"
- "In each approach, where do we see the number of sticky notes for opposite faces of the cabinet?" (Some students may find the number of sticky notes on each of the five faces of the cabinet and add them. Others may see that opposite faces are congruent and only find the number of sticky notes on three faces and double the number for two of them.)
- "Why did the different approaches lead to the same outcome?"

Advances: Representing, Conversing

Student Workbook


LESSON 12

What is Surface Area?

Let's cover the surfaces of some three-dimensional objects.

Warm-up Covering the Cabinet (Part 1)

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1 Covering the Cabinet (Part 2)

Earlier you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes needed to cover the cabinet. _____ Show your reasoning.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

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Student Task Statement

Earlier you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

   Find the area of each side of the cabinet, excluding the bottom, and add them together. Needed information: Measurements of the cabinet edge lengths in sticky notes.

2. Use the information you have to find the number of sticky notes needed to cover the cabinet. Show your reasoning.

   Reasoning may be a combination of the following two strategies:

   - Multiply the number of sticky notes along each edge of each side. Add all of the products.
   - Multiply the edge lengths of each side of the cabinet to find the area of each side. Add all of the areas.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

Two cabinets: 1,582 sticky notes. Three cabinets: 2,229 sticky notes. Twenty cabinets: 13,228 sticky notes.

Activity Synthesis

Invite previously identified students or groups to share their answer and strategy. On a visual display, record each answer and each distinct process for determining the surface area (that is, multiplying the side lengths of each rectangular face and adding up the products). After each presentation, poll the class on whether others had the same answer or process.

Play the video that reveals the actual number of sticky notes needed to cover the cabinet. If students' answers vary from that shown on the video, discuss possible reasons for the differences. (For example, students may not have accounted for the cabinet's door handles. Some may have made a calculation error.)

Tell students that the question they have been trying to answer is one about the surface area of the cabinet. Explain that the **surface area** of a three-dimensional figure is the total area of all its surfaces. We call the flat surfaces on a three-dimensional figure its **faces**.

The surface area of a rectangular prism would then be the combined area of all six of its faces. In the context of this problem, we excluded the bottom face because it is sitting on the ground and will not be tiled with sticky notes. Discuss:

- *"What unit of measurement are we using to represent the surface area of the cabinet?"*

  Square sticky notes

- *"Would the surface area, in terms of the number of square units, change if we used larger or smaller sticky notes? How?"*

  Yes, if we use larger sticky notes, we would need fewer. If we use smaller ones, we would need more.

Activity 2
Prisms Built from Cubes

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students reason about the surface area of rectangular prisms built from cubes. To verify and find surface area, students may count the number of square units on the visible faces and double the number of squares on each face (or double the total number of squares on the three visible faces). They may also use the squares to determine the edge lengths of the prisms, multiply them to find the area of each face, and then combine the areas.

In *Are You Ready for More?*, students are prompted to build a different prism from 12 cubes, draw it on isometric dot paper, and find its surface area. If physical cubes aren't available, consider using the digital version, in which students can use an applet to build a prism.

Launch

Display the image of the first prism in the activity and read the first question aloud. Remind students that we refer to the flat surfaces of a three-dimensional figure as "faces." Tell students that in this activity, we call the area of each face of a single cube, "1 square unit." Point to a single square on the displayed image to clarify 1 square unit on the prism.

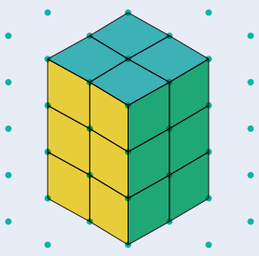
Give students 4–5 minutes of quiet work time to complete the activity.

Give 12 cubes to each student who opts to do the extension. If students are using snap cubes, tell them that we will pretend that all of the faces are completely smooth and not to worry about the "innies and outies" of the snap cubes. Consider doing a quick demonstration on how to draw a simple prism on isometric dot paper. (Start with one cube and then add a cube in each dimension.)

Student Task Statement

1. Here is a sketch of a rectangular prism built from 12 cubes. It has six **faces**, but you can see only three of them in the sketch.

   Show that it has a **surface area** of 32 square units.



Sample response: There are 2 faces with 4 square units each ($2 \cdot 4 = 8$) and 4 faces with 6 square units each ($4 \cdot 6 = 24$). $8 + 24 = 32$

Access for Students with Diverse Abilities (Activity 2, Student Task)

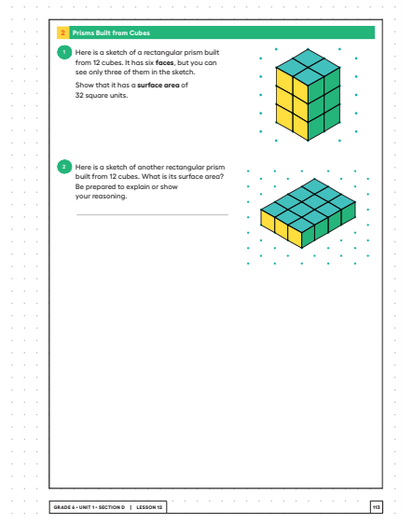
Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: Students can share their learning orally using virtual or concrete manipulatives such as snap cubes.

Supports accessibility for: Language, Fine Motor Skills

Building on Student Thinking

Students may count the faces of the individual snap cubes rather than faces of the completed prism. Help them understand that the faces are the visible ones on the outside of the figure.

Student Workbook


2 Prisms Built from Cubes

1 Here is a sketch of a rectangular prism built from 12 cubes. It has six **faces**, but you can see only three of them in the sketch.

Show that it has a **surface area** of 32 square units.

2 Here is a sketch of another rectangular prism built from 12 cubes. What is its surface area? Be prepared to explain or show your reasoning.

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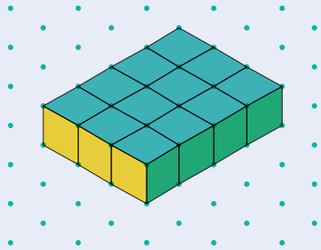
2. Here is a sketch of another rectangular prism built from 12 cubes. What is its surface area? Be prepared to explain or show your reasoning.



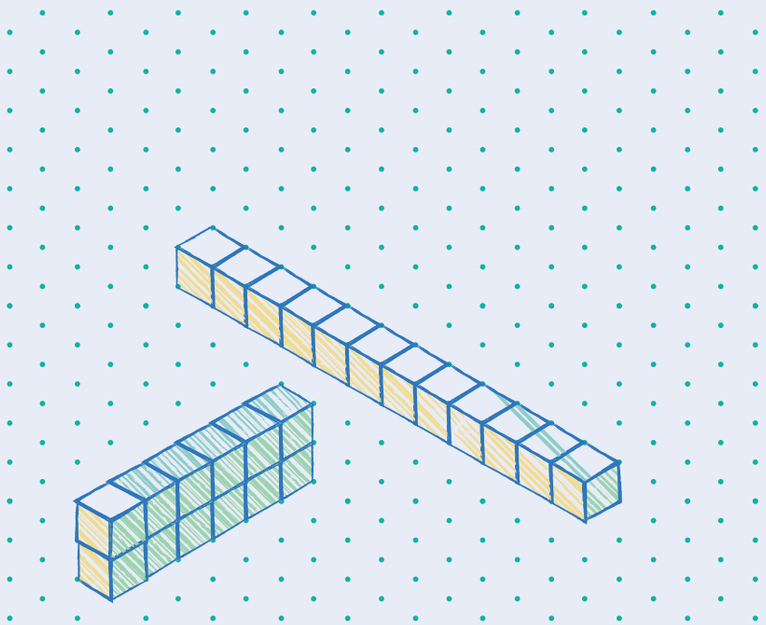
38 square units

Are You Ready for More?

Is it possible to use 12 cubes to build a rectangular prism that has a greater surface area than either prism shown earlier? Explain or show your reasoning. You can draw prisms on the dot paper if it helps.

Yes

Drawings vary, but all prisms should have one edge length that is 1 unit.



Sample reasoning:

- A prism that is 12 units by 1 unit by 1 unit has a surface area of 50 square units. $(4 \cdot 12) + (2 \cdot 1) = 48 + 2 = 50$
- A prism that is 6 units by 2 units by 1 unit has a surface area of 40 square units. $(2 \cdot 12) + (2 \cdot 6) + (2 \cdot 2) = 24 + 12 + 4 = 40$

Activity Synthesis

Select 1 or 2 students to share how they know the surface area of the first prism is 32 square units. Use students' explanations to highlight the meaning of surface area. Emphasize that the areas of all the faces need to be accounted for, including those we cannot see when looking at a two-dimensional drawing.

Select 1 or 2 students to briefly share their reasoning about the area of the second prism.

Point out that, in this activity, each face of their prism is a rectangle. We can find the area of each rectangle (by multiplying its base by its corresponding height) and then add the areas of all the faces to figure out the surface area. Explain that later, when we encounter non-rectangular prisms, we can likewise reason about the area of each face. We can find the areas of faces that are not rectangles the way we reasoned about the area of polygons earlier in the unit.

Lesson Synthesis

In this lesson, students found the surface areas of a cabinet and of rectangular prisms built out of cubes. Discuss with students:

- *"What does it mean to find the surface area of a three-dimensional figure?"*

  It means finding the number of unit squares that cover the entire surface of the object without gaps or overlaps.

- *"How can we find the number of unit squares that cover the entire surface of an object?"*

  We can count them, or we can find the area of each face of the object and add the areas of all faces.

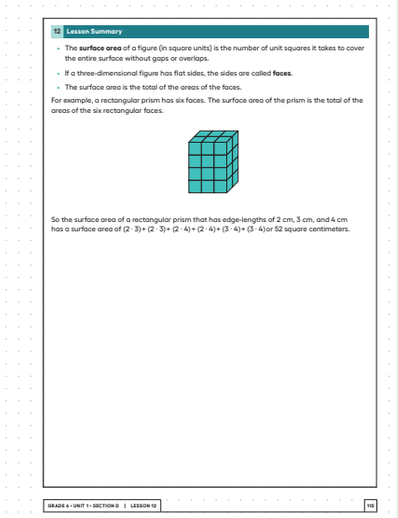
- *"How are finding surface area and finding area alike? How are they different?"*

  They both involve finding the number of unit squares that cover a region entirely without gaps and overlaps. Both have to do with two-dimensional regions. Finding area involves a single polygon. Finding surface area means finding the sum of the areas of multiple polygons (faces) of which a three-dimensional figure is composed.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


12 Lesson Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.

So the surface area of a rectangular prism that has edge-lengths of 2 cm, 3 cm, and 4 cm has a surface area of (2 · 3) + (2 · 3) + (2 · 4) + (2 · 4) + (3 · 4) + (3 · 4) or 52 square centimeters.

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Lesson Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
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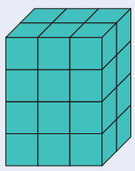


So the surface area of a rectangular prism that has edge-lengths of 2 cm, 3 cm, and 4 cm has a surface area of
(2 · 3) + (2 · 3) + (2 · 4) + (2 · 4) + (3 · 4) + (3 · 4) or 52 square centimeters.

Cool-down

A Snap Cube Prism

5 min

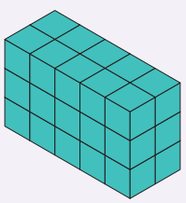
Launch

Prepare several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes. Display one for all to see and pass the rest around for students to examine, if needed.

Student Task Statement

A rectangular prism is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units?

62 square units



Explain or show your reasoning.

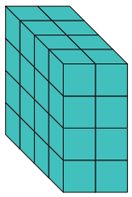
Sample reasoning: $2 \cdot [(3 \cdot 5) + (2 \cdot 5) + (2 \cdot 3)] = 62$

Lesson 12

Practice Problems

Problem 1

What is the surface area of this rectangular prism?



A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units

Problem 2

Which description can represent the surface area of this trunk?

A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

Problem 3

Which figure has a greater surface area?

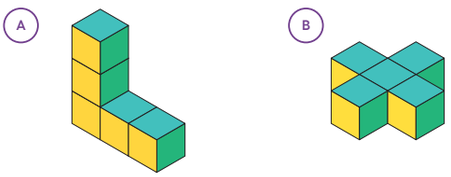
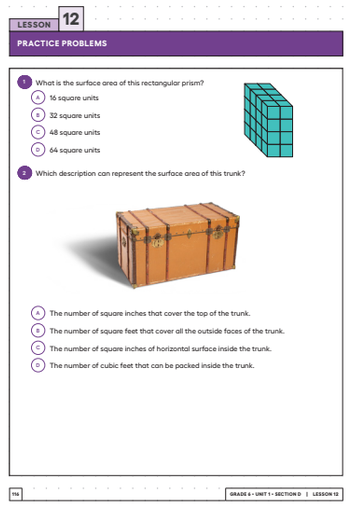


Figure A and Figure B have the same surface area of 22 square units.

Student Workbook


LESSON 12

PRACTICE PROBLEMS

1. What is the surface area of this rectangular prism?
   - A. 16 square units
   - B. 32 square units
   - C. 48 square units
   - D. 64 square units
2. Which description can represent the surface area of this trunk?
   - A. The number of square inches that cover the top of the trunk.
   - B. The number of square feet that cover all the outside faces of the trunk.
   - C. The number of square inches of horizontal surface inside the trunk.
   - D. The number of cubic feet that can be packed inside the trunk.

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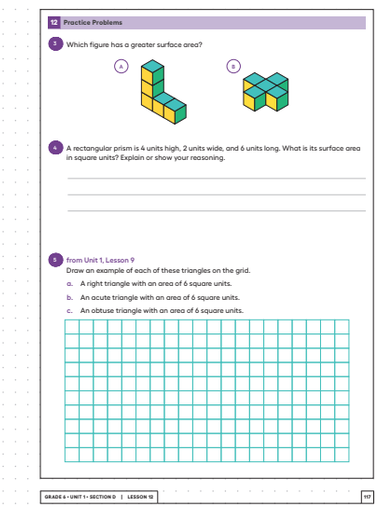
Student Workbook


12 Practice Problems

3. Which figure has a greater surface area?
   - A.
   - B.
4. A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.
5. from Unit 1, Lesson 9
   Draw an example of each of these triangles on the grid.
   - a. A right triangle with an area of 6 square units.
   - b. An acute triangle with an area of 6 square units.
   - c. An obtuse triangle with an area of 6 square units.

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Problem 4

A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

88 square units

Sample reasoning: Two faces are 4 units by 2 units, amounting to 16 square units. Two faces are 4 units by 6 units, amounting to 48 square units. Two faces are 2 units by 6 units, amounting to 24 square units. 16 + 48 + 24 = 88

Problem 5

from Unit 1, Lesson 9

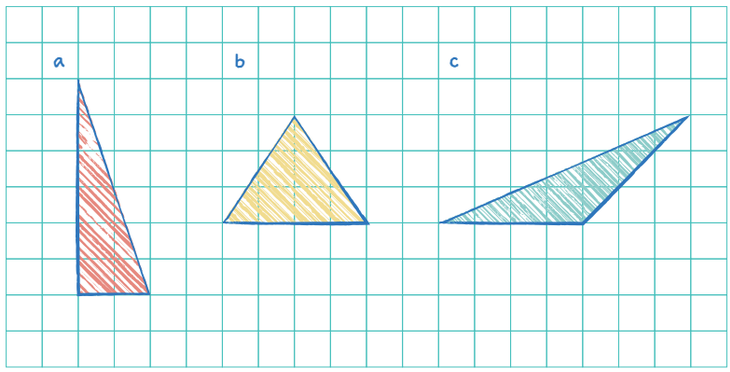
Draw an example of each of these triangles on the grid.

a. A right triangle with an area of 6 square units.

b. An acute triangle with an area of 6 square units.

c. An obtuse triangle with an area of 6 square units.

Sample response:



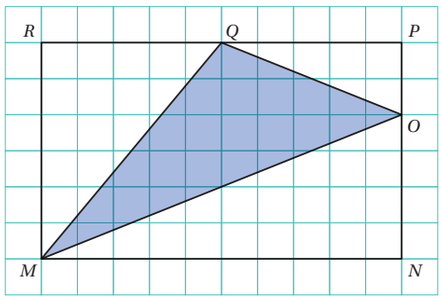
Problem 6

from Unit 1, Lesson 10

Find the area of triangle MOQ in square units.

20 square units

Show your reasoning.



Sample reasoning: The area of triangle MOQ can be found by subtracting the areas of the three right triangles from the area of rectangle $MNPR$.

- The area of rectangle $MNPR$ is $10 \cdot 6$ or 60 square units.
- The area of triangle QRM is $\frac{1}{2} \cdot 6 \cdot 5$ or 15 square units.
- The area of triangle MNO is $\frac{1}{2} \cdot 10 \cdot 4$ or 20 square units.
- The area of triangle OPQ is $\frac{1}{2} \cdot 2 \cdot 5$ or 5 square units. $60 - (15 + 20 + 5) = 20$

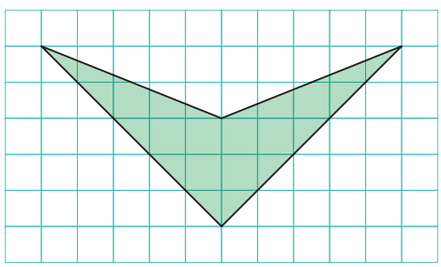
Problem 7

from Unit 1, Lesson 3

Find the area of this shape.

15 square units

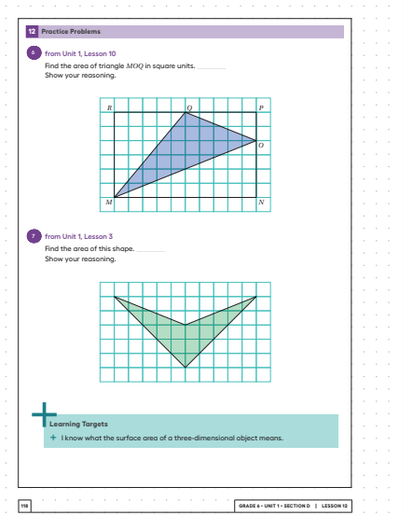
Show your reasoning.



Sample reasoning:

- The shape can be decomposed into two identical triangles with a vertical cut down the middle. Each triangle has base 3 units and height 5 units, so its area is $\frac{1}{2} \cdot 3 \cdot 5$, or 7.5 square units. $2 \cdot (7.5) = 15$
- The shape can be decomposed into two identical triangles and rearranged into a parallelogram with a base of 3 units and a height of 5 units. $3 \cdot 5 = 15$

Student Workbook


12 Practice Problems

6 from Unit 1, Lesson 10
Find the area of triangle MOQ in square units.
Show your reasoning.

7 from Unit 1, Lesson 3
Find the area of this shape.
Show your reasoning.

Learning Targets
+ I know what the surface area of a three-dimensional object means.