Memory Test (Optional)

Goals

- Describe (orally) connections between sampling and probability.

- Generate a random sample, and use it to make inferences (in writing) about the population.

- Justify (orally and in writing) whether a given method produces a random sample.

Learning Target

I can compare two groups by taking a random sample, calculating important measures, and determining whether the populations are meaningfully different.

Lesson Narrative

This lesson is optional. It gives students a chance to use the material they have learned in the unit with the final goal of comparing two populations, but it may be shortened or skipped due to time constraints.

In this lesson, students apply what they have learned about probability, sampling, and comparing populations to analyze two data sets. Half of the class works with one data set while the other half of the class works with another. Students choose their own tools for selecting a sample at random and calculate the mean, MAD, and a proportion to summarize their sample. Then students compare their results with a partner that had the other data set to construct an argument for whether there is a meaningful difference between the sets.

Student Learning Goal

Let's put it all together.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 3)

Required Materials

Materials to Gather
- Paper bags: Activity 1, Activity 2
- Number cubes: Activity 2
- Paper clips: Activity 2

Materials to Copy
- Collecting a Sample Handout (1 copy for every 2 students): Activity 1

Required Preparation

Activity 1:
Provide one data set and one spinner for each student.

Activity 2:
If the spinners are used to select a random sample, provide a paper clip and sharpened pencil to use with the spinners. If possible, provide access to other tools for selecting a random sample from a 10-by-10 grid such as a 10-sided polyhedra or colored cubes and paper bags.
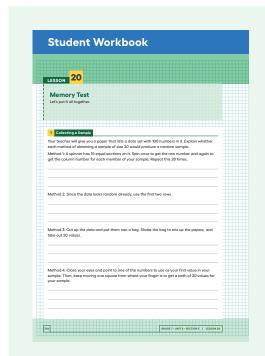
Lesson Timeline

5 min	**10** min	**10** min	**5** min	**10** min
Activity 1	Activity 2	Activity 3	Activity 4	Lesson Synthesis

Student Workbook

LESSON 20

Memory Test
Let's put it all together.

1 Collecting a Sample

Your teacher will give you a paper that lists a data set with 100 numbers in it. Explain whether each method of obtaining a sample of size 20 would produce a random sample.

Method 1: A spinner has 10 equal sections on it. Spin once to get the row number and again to get the column number for each member of your sample. Repeat this 20 times.

Method 2: Since the data looks random already, use the first two rows.

Method 3: Cut up the data and put them into a bag. Shake the bag to mix up the papers, and take out 20 values.

Method 4: Close your eyes and point to one of the numbers to use as your first value in your sample. Then, keep moving one square from where your finger is to get a path of 20 values for your sample.

GRADE 7 • UNIT 8 • SECTION E | LESSON 20

Activity 1: Optional
Collecting a Sample

5 min

Activity Narrative

In this activity, students review methods of obtaining samples that are fair and random. Students must make sense of the situation and will continue to work with the data to solve problems in later activities.

Launch 👥

Arrange students in groups of 2. Each group gets both sets of data from the blackline master, one data set for each partner. Students will not need the spinners from the blackline master for this activity, but the spinners are included for use later in the lesson. Partners may work together to answer the questions, but should not share their data set with one another until told to do so in a later activity.

Student Task Statement

Your teacher will give you a paper that lists a data set with 100 numbers in it. Explain whether each method of obtaining a sample of size 20 would produce a random sample.

Method 1: A spinner has 10 equal sections on it. Spin once to get the row number and again to get the column number for each member of your sample. Repeat this 20 times.

Sample response: This would produce a random sample since each row and column has an equal chance of being selected.

Method 2: Since the data looks random already, use the first two rows.

Sample response: This would not produce a random sample since all of the values do not have an equal chance of being selected.

Method 3: Cut up the data and put them into a bag. Shake the bag to mix up the papers, and take out 20 values.

Sample response: This would produce a random sample since all the papers are the same size and each value has an equal chance of being selected.

Method 4: Close your eyes and point to one of the numbers to use as your first value in your sample. Then, keep moving one square from where your finger is to get a path of 20 values for your sample.

Sample response: This would not produce a random sample since the path limits the values you can get in your sample. For example, the four corners could not all be in the same sample of 20.

Activity Synthesis

The purpose of the discussion is to help students solidify their understanding of methods for selecting random samples.

Consider these questions for discussion:

💬 *"Can you think of other methods for selecting a random sample that are not listed here?"*

Roll a polyhedron with 10 equal faces showing the numbers 1 through 10 to get the row, and roll again to get the column.

💬 *"What do you need to look for when determining if a sample is random?"*

Are all values equally likely to be included in the random sample?

Activity 2: Optional
Sample Probabilities

10 min

Activity Narrative

In this activity, students begin by practicing their understanding of proportions and probabilities by examining the data set they have available. In the fourth problem, students obtain a sample from the population using tools they choose and examine the sample they selected to compare it to the expected proportions and probabilities calculated in the first three problems.

The problems are intended for students to use their own data set to answer. Although they are kept in pairs for the entire lesson, this activity should be done individually.

Launch 👥

Keep students in the same groups of 2.

Give students 5–7 minutes of quiet work time, and follow with a whole-class discussion.

If possible, allow students to use their chosen method of random sampling to obtain a sample of 10 for this activity. Have items such as paper clips, scissors, 10-sided polyhedra, and other materials available for student use. The blackline master for the first activity in this lesson contains accurate spinners that could be used to select a random sample.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. After students have solved the first 2–3 problems, check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook

Student Task Statement

Continue working with the data set your teacher gave you in the previous activity. The data marked with a star all came from students at Springfield Middle School.

1. When you select the first value for your random sample, what is the probability that it will be a value that came from a student at Springfield Middle School?

   $\frac{20}{100} = \frac{1}{5} = 0.2$

2. What proportion of your entire sample would you expect to be from Springfield Middle School?

   The proportion I would expect is about 0.2.

3. If you take a random sample of size 10, how many scores would you expect to be from Springfield Middle School?

   From a random sample of 10, I expect there to be around 2 scores from Springfield Middle School.

4. Select a random sample of size 10.

   Sample response: 53, 46, 63, 42, 41, 33, 71, 39, 40, 18

5. Did your random sample have the expected number of scores from Springfield Middle School?

   Sample response: No, I only had 1 value come from Springfield Middle School.

Activity Synthesis

The purpose of this discussion is to connect the ideas of probability and random sampling from the unit.

Consider these questions for discussion:

💬 *"How is selecting a sample at random connected to probability?"*

A random sample should give each value an equal chance of being chosen. Therefore, each value has a $\frac{1}{100}$ probability of being chosen.

💬 *"How could we simulate the probability of getting at least 2 values in the sample of 10 from Springfield Middle School?"*

Since 20% of the values come from Springfield Middle School, we could put 10 blocks in a bag with 2 colored red to represent Springfield Middle School. Draw a block from the bag, and if it is red, it represents a score from Springfield Middle School. Replace the block and repeat. Get a sample of 10 and see if the sample has at least 2 red blocks. Repeat this process many times, and use the fraction of times there are at least 2 red blocks as an estimate for the probability that a random sample will have at least 2 scores from Springfield Middle School.

Activity 3: Optional
Estimating a Measure of Center for the Population

10 min

Activity Narrative

In this activity, students practice estimating a measure of center for the population using the data from a sample. The variability is also calculated to be used in the following activity to determine if there is a meaningful difference between the measure of center for the population they used to select their sample and the measure of center for another population.

Launch 👥

Keep students in groups of 2. Students should work with their partner for the first question, then individually for the last two problems. Follow up with a whole-class discussion.

Student Task Statement

1. Decide which measure of center makes the most sense to use based on the distribution of your sample. Discuss your thinking with your partner. If you disagree, work to reach an agreement.

   Sample response: We chose to use mean since there were no values far from the center of the data.

2. Estimate this measure of center for your population based on your sample.

   The mean is 44.6.

3. Calculate the measure of variability for your sample that goes with the measure of center that you found.

   The MAD is 10.92.

Activity Synthesis

The purpose of the discussion is for students to make clear their reasoning for choosing a particular measure of center and reiterate the importance of variability when comparing groups from samples.

Consider these questions for discussion:

💬 *"Which measure of center did your group choose? Explain your reasoning."*

A median should be used if there are a few values far from the center that overly influence the mean in that direction. If the data is not approximately symmetric, a median should be used as well. In other cases, the mean is probably a better choice for the measure of center.

💬 *"Why is it important to measure variability in the data when estimating a measure of center for the population using the data from a sample?"*

To use the general rule, the difference in means must be greater than 2 MADs to determine a meaningful difference. If there is small variation, then the samples may have come from a population that also has a small variation, so differences among groups may be more clearly defined.

Access for Multilingual Learners
(Activity 3, Student Task)

MLR8: Discussion Supports.
During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say …" Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Student Workbook

Activity 4: Optional

Comparing Populations

5 min

Activity Narrative

In this activity, students use the values computed in the previous activity to determine if there is a meaningful difference between two populations. Following the comparison of the groups, students are told that the populations from which they selected a sample are identical but shuffled. Students should use their understanding of sampling to construct an argument for why their means are not exactly the same, but reasonably close.

Launch 👥

Keep students in the same groups of 2 established at the beginning of this lesson. Allow students 3 minutes of partner work time, and follow with a whole-class discussion.

Student Task Statement

Using only the values you computed in the previous two activities, compare your sample to your partner's.

Is it reasonable to conclude that the measures of center for each of your populations are meaningfully different? Explain or show your reasoning.

Sample response: They are not meaningfully different. The difference in means is 4.9, but the larger of the two MADs is 10.9, so they are less than 2 MADs apart.

Activity Synthesis

Ask each group to share whether they found a meaningful difference.

Tell students,

💬 *"With your partner, compare the starred data for the two groups. What do you notice?"*

Tell students that the two populations are actually identical but rearranged. Ask,

💬 *"Did any groups get different means for your samples? Explain why that might have happened even though the populations are the same."*

Two random samples from the population will usually not contain the same values, so different means are probably expected.

One thing to note: The general rule is designed to say whether the two populations have a meaningful difference or if there is not enough evidence to determine if there is a meaningful difference. On its own, the general rule cannot determine if two populations are identical from only a sample. If the means are less than 2 MADs apart, there is still a chance that there is a difference in the populations, but there is not enough evidence in the samples to be convinced that there is a difference.

Lesson Synthesis

The main learning points of this lesson are:

- Probability and random samples are connected through the equal likelihood of individuals from the population being selected.

- It is important to select samples through a random process in order to compare two populations.

Consider asking these discussion questions:

💬 *"Why was it important to select a random sample from the population data you had?"*

A random sample gives us the best chance of being representative of the population.

💬 *"An astronomer has enough money to use a space telescope to collect data from a distant star 10 times this year and 10 times next year. Describe a process the astronomer could use to compare the data from the 2 years."*

Select the 10 days from each year at random. Determine the correct measure of center and variation. Use a general rule to compare the measure of center for each year based on the sample characteristics.