Lesson 10

Drawing Triangles (Part 2)

Goals

- Draw triangles with two given side lengths and one angle measure or with three given angle measures, and describe (orally) how many different triangles could be drawn with the given conditions.
- Use drawings to justify (in writing) whether two given side lengths and one angle measure determine one unique triangle.

Learning Target

Given two side lengths and one angle measure, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.

Lesson Narrative

In this lesson, students continue their work on drawing triangles with specified angle and side measures. Students focus on drawing a triangle given two side lengths and an angle, and given three angles. They continue to gain experience with a compass, ruler, and protractor. They continue to notice from their drawings when the conditions determine one triangle, more than one, or none, and they increase their precision of language as they describe the conditions to each other.

Students are not expected to know rules about which conditions determine each possibility.

Student Learning Goal

Let's draw some more triangles.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

- Compasses: Warm-up, Activity 1, Activity 2
- Geometry toolkits: Activity 1, Activity 2

Required Preparation

Lesson:

To help students see how they can use a compass to draw different triangles with two of the same side lengths, you might ask students to use the Revisiting How Many Can You Draw? resource in their student workbooks. This is optional.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Using a Compass to Estimate Length

5 min

Activity Narrative

The purpose of this *Warm-up* is to remind students that a compass is useful for transferring a length in general, and not just for drawing circles. As students discuss answers with their partners, monitor for students who can clearly explain how they can use a compass to compare the length of the third side.

Launch

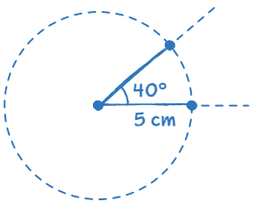
Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by time to discuss their answers with their partner. Follow with a whole-class discussion. Provide access to geometry toolkits and compasses.

Student Task Statement

1. Draw a 40° angle.

   See image.

2. Use a compass to make sure both sides of your angle have a length of 5 centimeters.

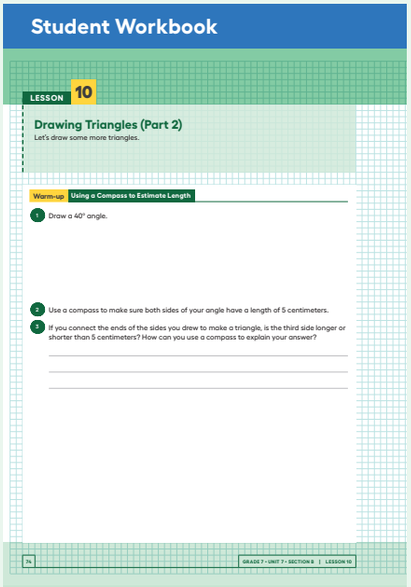


3. If you connect the ends of the sides you drew to make a triangle, is the third side longer or shorter than 5 centimeters? How can you use a compass to explain your answer?

   Shorter

   Sample reasoning: The third side is shorter than 5 cm. I know this because I can use my compass that is set to a radius of 5 cm and place it at the end of one of the two sides. When I draw another circle, the end of the other side of the angle is inside that new circle, so the distance between the two ends must be less than 5 cm.

Student Workbook


LESSON 10

Drawing Triangles (Part 2)

Let's draw some more triangles.

Warm-up Using a Compass to Estimate Length

1. Draw a 40° angle.
2. Use a compass to make sure both sides of your angle have a length of 5 centimeters.
3. If you connect the ends of the sides you drew to make a triangle, is the third side longer or shorter than 5 centimeters? How can you use a compass to explain your answer?

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Instructional Routines

MLR1: Stronger and Clearer Each Time

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time.
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to associate symbols with concrete objects or values. Provide access to fasteners and to cardboard strips of the required length and fasteners to create the triangles.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Activity Synthesis

The purpose of this discussion is for students to share their observations and reasoning about using a compass to draw a triangle. Ask previously identified students to share their responses to the final question. Display their drawing of the angle for all to see. If not mentioned in students' explanations, demonstrate for all to see how to use the compass to estimate the length of the third side of the triangle.

Activity 1

Revisiting How Many Can You Draw?

15 min

Activity Narrative

There is a digital version of this activity.

Students continue to practice drawing triangles from given conditions and categorizing their results. This activity focuses on the inclusion of a single angle and two sides. Again, they do not need to memorize which conditions result in unique triangles, but they should begin to notice how some conditions (such as the equal side lengths) result in certain requirements for the completed triangle.

When students articulate their reasoning, they have an opportunity to attend to precision in the language they use to describe their thinking. They might first propose less formal or imprecise language, and after sharing with a partner, revise their explanation to be clearer and stronger.

There is an optional resource in the student workbook that can help students organize their work.

Launch

Arrange students in groups of 2. Remind students of the activity in a previous lesson where they used the strips and fasteners to draw triangles on their paper. Ask what other tool also helps them find all the points that are a certain distance from a center point (a compass). Ask students to use the Revisiting How Many Can You Draw? resource in their student workbooks, if desired. Provide access to geometry toolkits and compasses.

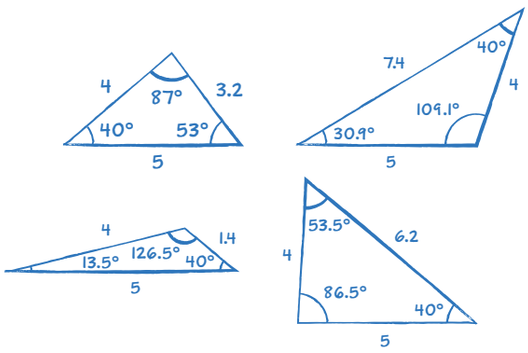
Give students 7–8 minutes of partner work time, followed by a whole-class discussion.

Student Task Statement

1. Draw as many different triangles as you can with each of these sets of conditions:

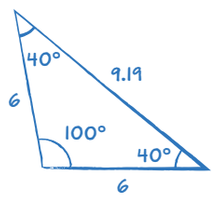
a. One angle measures 40°, one side measures 4 cm, and one side measures 5 cm.

Sample response: There are 4 different triangles that can be drawn from the given conditions.



b. Two sides measure 6 cm, and one angle measures 100°.

Sample response: There is only one triangle that can be drawn from the given conditions.



2. Did either of these sets of measurements determine one unique triangle? How do you know?

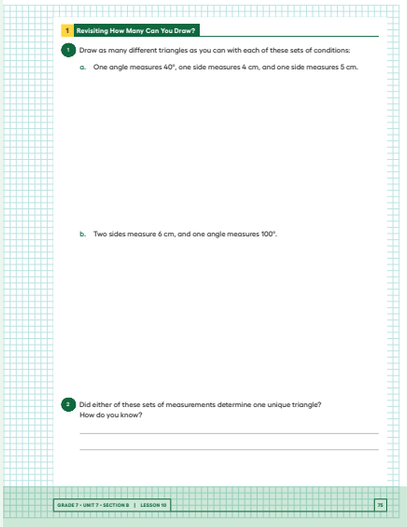
The second set of measurements determined one unique triangle. There was no other way I could draw it.

Building on Student Thinking

Some students may draw two different orientations of the same triangle for the first set of conditions, with the 40° angle in between the 4-cm and 5-cm sides. Prompt them to use tracing paper to check whether their two triangles are really different (not identical copies).

If students struggle to create more than one triangle from the first set of conditions, prompt them to write down the order they already used for their measurements and then to brainstorm other possible orders they could use.

Student Workbook


1 Revisiting How Many Can You Draw?

1. Draw as many different triangles as you can with each of these sets of conditions:

a. One angle measures 40°, one side measures 4 cm, and one side measures 5 cm.

b. Two sides measure 6 cm, and one angle measures 100°.

2. Did either of these sets of measurements determine one unique triangle? How do you know?

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Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.
Break the class into small discussion groups, and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Attention; Social-Emotional Skills

Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to "Did either of these sets of measurements determine one unique triangle? How do you know?" In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.
If time allows, display these prompts for feedback:

- " ____ makes sense, but what do you mean when you say … ?"
- "Can you describe that another way?"
- "How do you know … ? What else do you know is true?"

Close the partner conversations, and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words that they got from their partners in order to make their next draft stronger and clearer.

If the Revisiting How Many Can You Draw? resource was used, ask students:

💬 *"Which configurations made identical triangles?"*

the top left and bottom left

💬 *"Which configurations made more than one triangle?"*

the bottom right

If not mentioned by students, explain to students that the top left and bottom left configurations result in the same triangle, because in both cases the 40° angle is in between the 4-cm and 5-cm sides, and that the bottom right configuration results in two different triangles, because the arc intersects the ray in two different places.

Activity 2
Three Angles

15 min

Activity Narrative

There is a digital version of this activity.

This activity focuses on including three angle conditions. The goal is for students to notice that some angle conditions result in a large number of possible triangles (all scaled copies of one another) or are impossible to create. Students are not expected to learn that the angle measures in a triangle must have a sum of 180 degrees, but they are not barred from noticing this fact.

Launch

Arrange students in groups of 2. Tell students that they should attempt to create a triangle with the given specifications. If they can create one, they should attempt to either create at least one more or justify to themselves why there is only one. If they cannot create any, they should show some valid attempts to include as many pieces as they can and be ready to explain why they cannot include the remaining conditions.

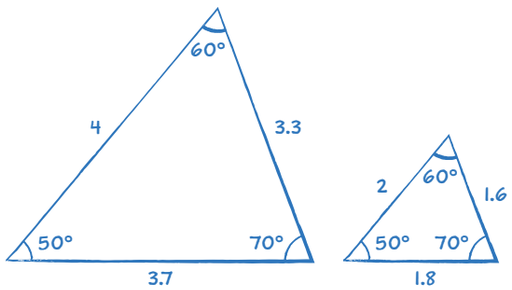
Give students 5 minutes of quiet work time, followed by time to discuss with a partner the triangles that they individually made . Follow with a whole-class discussion. Provide access to geometry toolkits and compasses.

Student Task Statement

1. Draw as many different triangles as you can with each of these sets of measurements:

a. One angle measures 50°, one measures 60°, and one measures 70°.

Sample response:



There are many possible triangles that can be made with these angle measurements, because we don't know any of the side lengths. All the possible triangles, however, are scaled copies of each other.

b. One angle measures 50°, one measures 60°, and one measures 100°.

There is no way to draw a triangle with these angle measurements, because once the first two angles are drawn, the third angle is already set.

- If I start with 50° and 60°, then the third angle is always 70°. I can't make it 100°.
- If I start with 50° and 100°, then the third angle is always 30°. I can't make it 60°.
- If I start with 60° and 100°, then the third angle is always 20°. I can't make it 50°.

2. Did either of these sets of measurements determine one unique triangle? How do you know?

Neither of these sets of measurements determine I unique triangle.

Building on Student Thinking

If students struggle to get started, remind them of Lin's technique of using the protractor and a ruler to make an angle that can move along a line.

Student Workbook


2 Three Angles

1. Draw as many different triangles as you can with each of these sets of measurements:
   a. One angle measures 50°, one measures 60°, and one measures 70°.
   b. One angle measures 50°, one measures 60°, and one measures 100°.
2. Did either of these sets of measurements determine one unique triangle? How do you know?

Are You Ready for More?

Using only a compass and the edge of a blank index card, draw a perfectly equilateral triangle. (Note! The tools are part of the challenge! You may not use a protractor! You may not use a ruler!)

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Are You Ready for More?

Using *only* a compass and the edge of a blank index card, draw a perfectly equilateral triangle. (Note! The tools are part of the challenge! You may not use a protractor! You may not use a ruler!)

Draw a line segment, and label the endpoints A and B. Open the compass so that the radius is the same length as the line segment you drew. Place the pointy end of the compass on A, and draw an arc. Next, put the pointy end on B, and draw an arc. The intersection of the arcs is the other vertex of the equilateral triangle.

Activity Synthesis

Select students to share their drawings and display them for all to see. Ask students:

- *"Were there any sets of measurements that produced a unique triangle?"*

  no

- *"Why is there more than one triangle that can be made with the measurements in the first problem?"*

  because there are no side lengths mentioned, so we can create scaled copies of the triangles with the same angles but with shorter or longer side lengths

- *"Which combinations of angles could not be drawn?"*

  the angles in the second problem, 50°, 60°, 100°

Consider displaying the applet for all to see while discussing the last questions:

The Geogebra applet 'Three Angles' is available here: **ilclass.com/l/395064**

Lesson Synthesis

Here are some questions for discussion:

- How was a compass useful in drawing triangles today?

  It helps find all the points a certain distance away.

- What strategies did you use to include two given side lengths and a given angle?

  Draw one of the side lengths, and use a protractor to draw the angle at one end. Then use a compass to finish the picture.

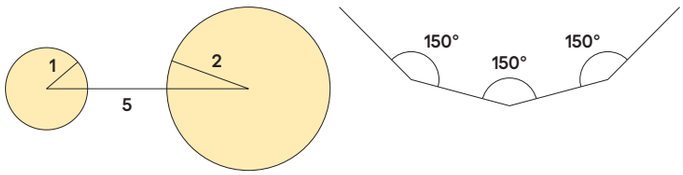
- What strategies did you use to include three given angles?

  Draw one angle, and then use a protractor and ruler to slide along one side of the first angle.

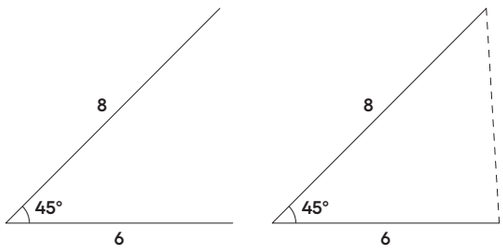
Lesson Summary

A triangle has six measures: three side lengths and three angle measures.

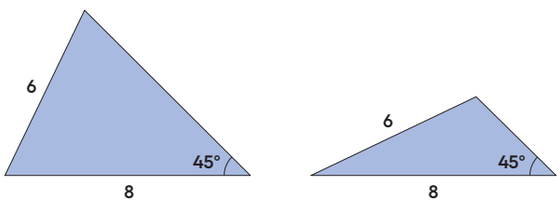
If we are given three measures, then sometimes there is no triangle that can be made. For example, there is no triangle with side lengths 1, 2, 5, and there is no triangle with all three angles measuring 150°.



Sometimes, only one triangle can be made. By this we mean that any triangle we make will be the same, having the same six measures. For example, if a triangle can be made with three given side lengths, then the corresponding angles will have the same measures. Another example is shown here: an angle measuring 45° between two side lengths of 6 and 8 units. With this information, one unique triangle can be made.

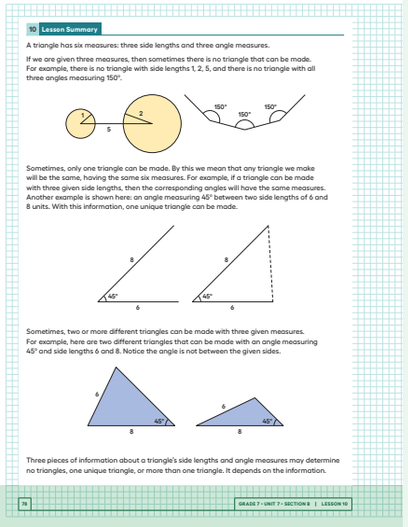


Sometimes, two or more different triangles can be made with three given measures. For example, here are two different triangles that can be made with an angle measuring 45° and side lengths 6 and 8. Notice the angle is not between the given sides.



Three pieces of information about a triangle's side lengths and angle measures may determine no triangles, one unique triangle, or more than one triangle. It depends on the information.

Student Workbook



Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of drawing triangles to fit measurement requirements. If most students struggle, make time to revisit related work in the Lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 7, Lesson 9 Drawing Triangles (Part 1)

Cool-down

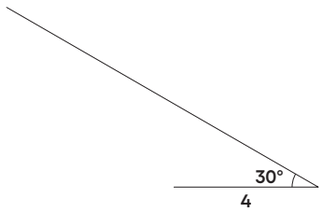
Finishing Noah's Triangle

5 min

Student Task Statement

Noah is trying to draw a triangle with a 30° angle and side lengths of 4 cm and 6 cm.

- He uses his ruler to draw a 4 cm line segment.
- He uses his protractor to draw a 30° angle on one end of the line segment.



1. What should Noah do next? Explain and show how he can finish drawing the triangle.

   Noah should use a compass to draw a circle with radius 6 cm and center at one end of the 4-cm side. He should then draw segments connecting both ends of the 4-cm side to the point where the circle and ray cross, and that will complete the triangle.

2. Is there a different triangle Noah could draw that would answer the question? Explain or show your reasoning.

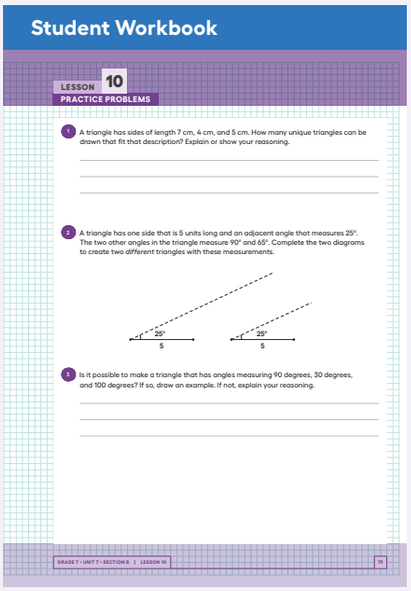
   Yes

   Noah could try beginning with the same setup he has already drawn again, but this time center the circle on the other end of the 4-cm side. He could also start with the 6-cm side drawn instead of the 4-cm side and follow the same process.

Lesson 10

Practice Problems

6 Problems



Problem 1

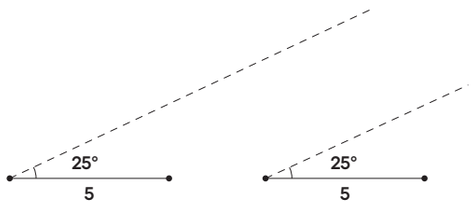
A triangle has sides of length 7 cm, 4 cm, and 5 cm. How many unique triangles can be drawn that fit that description? Explain or show your reasoning.

You can draw only one unique triangle with those same 3 measures.

If you start by drawing the 7-cm side and then draw circles of radii 4 cm and 5 cm at each endpoint, the circles will cross at two places. Connecting the endpoints of the 7-cm side to those crossing points will produce two identical triangles, each having side lengths 7 cm, 4 cm, and 5 cm. There are no other points that could be the third vertex of the triangle.

Problem 2

A triangle has one side that is 5 units long and an adjacent angle that measures 25°. The two other angles in the triangle measure 90° and 65°. Complete the two diagrams to create two *different* triangles with these measurements.



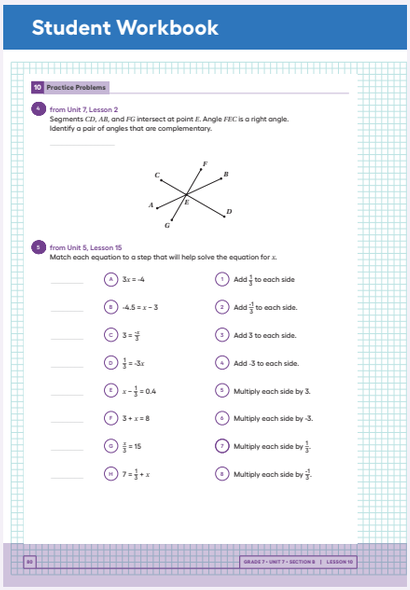
Answers vary.

Problem 3

Is it possible to make a triangle that has angles measuring 90 degrees, 30 degrees, and 100 degrees? If so, draw an example. If not, explain your reasoning.

No, if you try to draw a triangle that has a 90-degree angle on the end of a side and a 100-degree angle on the other end of the same side, there is no way to make the other two sides meet to form a triangle.

Student Workbook


10 Practice Problems

3 from Unit 7, Lesson 2
Segments CD, AB, and FG intersect at point E. Angle FEC is a right angle. Identify a pair of angles that are complementary.

4 from Unit 5, Lesson 15
Match each equation to a step that will help solve the equation for x.

A	$3x = -4$	1	Add $\frac{1}{3}$ to each side
B	$-4.5 = x - 3$	2	Add $\frac{-1}{3}$ to each side.
C	$3 = \frac{-x}{3}$	3	Add 3 to each side.
D	$\frac{1}{3} = -3x$	4	Add -3 to each side.
E	$x - \frac{1}{3} = 0.4$	5	Multiply each side by 3.
F	$3 + x = 8$	6	Multiply each side by -3.
G	$\frac{x}{3} = 15$	7	Multiply each side by $\frac{1}{3}$.
H	$7 = \frac{1}{3} + x$	8	Multiply each side by $\frac{-1}{3}$.

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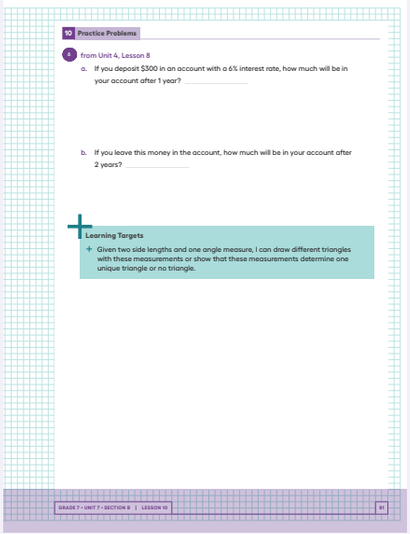
Student Workbook


10 Practice Problems

4 from Unit 4, Lesson 8
a. If you deposit $300 in an account with a 6% interest rate, how much will be in your account after 1 year?

b. If you leave this money in the account, how much will be in your account after 2 years?

Learning Targets
+ Given two side lengths and one angle measure, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.

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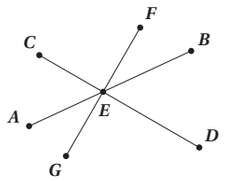
Problem 4

from Unit 7, Lesson 2

Segments CD, AB, and FG intersect at point E. Angle FEC is a right angle. Identify a pair of angles that are complementary.



Any of these pairs are complementary:

- FEB and DEB
- CEA and AEG
- FEB and CEA
- DEB and AEG

Problem 5

from Unit 5, Lesson 15

Match each equation to a step that will help solve the equation for x.

A.	$3x = -4$	**1.**	Add $\frac{1}{3}$ to each side.
B.	$-4.5 = x - 3$	**2.**	Add $\frac{-1}{3}$ to each side.
C.	$3 = \frac{-x}{3}$	**3.**	Add 3 to each side.
D.	$\frac{1}{3} = -3x$	**4.**	Add -3 to each side.
E.	$x - \frac{1}{3} = 0.4$	**5.**	Multiply each side by 3.
F.	$3 + x = 8$	**6.**	Multiply each side by -3.
G.	$\frac{x}{3} = 15$	**7.**	Multiply each side by $\frac{1}{3}$.
H.	$7 = \frac{1}{3} + x$	**8.**	Multiply each side by $\frac{-1}{3}$.

Problem 6

from Unit 4, Lesson 8

a. If you deposit $300 in an account with a 6% interest rate, how much will be in your account after 1 year?

$318

b. If you leave this money in the account, how much will be in your account after 2 years?

$337.08