Lesson 12

More Applications of the Pythagorean Theorem

Goals

- Generate sets of three numbers that satisfy the Pythagorean Theorem.
- Identify what information is needed to solve a problem using the Pythagorean Theorem. Ask questions to elicit that information.

Learning Target

I can recognize situations where the Pythagorean Theorem can be used to solve a problem.

Lesson Narrative

This lesson offers students additional practice solving problems that use the Pythagorean Theorem. The first activity has students investigating Pythagorean triples. The main activity of the lesson uses the *Information Gap* routine, which requires students to make sense of problems and use precise language when asking for or giving information.

Student Learning Goal

Let's solve problems using the Pythagorean Theorem.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

- Math Community Chart: Activity 2

Materials to Copy

- Pythagorean Theorem Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	5 min

Instructional Routines

MLR4: Information Gap Cards

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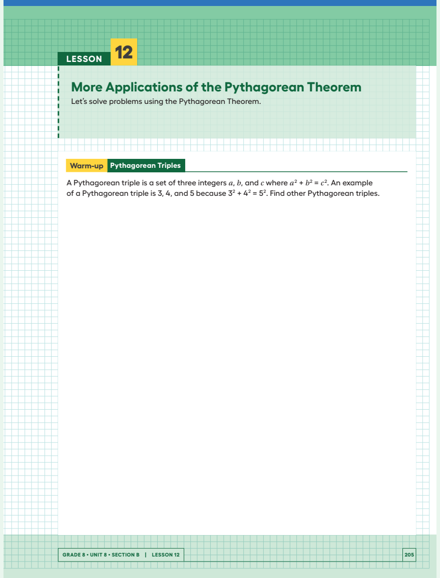
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Student Workbook



Warm-up
Pythagorean Triples

5 min

Activity Narrative

The purpose of this *Warm-up* is to get students thinking about the relationship between the values of a, b, and c in the Pythagorean Theorem by looking at Pythagorean triples.

Launch

Arrange students in groups of 2.

Give partners 2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

A Pythagorean triple is a set of three integers a, b, and c where $a^2 + b^2 = c^2$. An example of a Pythagorean triple is 3, 4, and 5 because $3^2 + 4^2 = 5^2$. Find other Pythagorean triples.

Common Pythagorean triples: 6, 8, 10; 5, 12, 13; 8, 15,17; 7, 24, 25; 20, 21, 29; 9, 40, 41

Activity Synthesis

The purpose of this discussion is to help students see patterns and similarity in right triangles. For example, one common Pythagorean triple is 3, 4, and 5, and by multiplying each value by a common factor, more triples like 6, 8, and 10 or 15, 20, and 25 can be made.

Ask students to share any Pythagorean triples they found and record and display these for all to see. Choose a few triples for the class to verify.
Ask students if they notice any patterns between the triples. If not mentioned by students, draw their attention to any triples that are multiples of each other.

Activity 1
Info Gap: Pythagorean Theorem

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to solve a problem that uses the Pythagorean Theorem.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice that they think will help themselves and their group during the activity. At the end of the activity, students can share what norms they chose and how the norm did or did not support their group.

Next, tell students they will solve problems that deal with right triangles. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ____?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ____ because ...”
   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

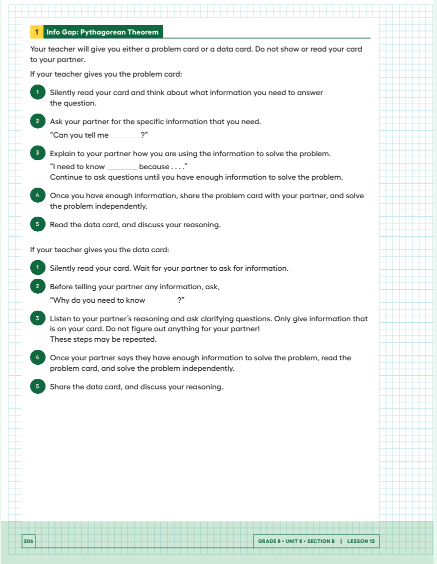
If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know____?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
   These steps may be repeated.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.
Supports accessibility for: Memory, Organization

Student Workbook



4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: The length across the diagonal of the gym is $\sqrt{9,556}$ or approximately 97.8 feet. Tyler would need 8.1 strings, so the school should buy 9 strings of light.

Problem Card 2: The ladder will reach $\sqrt{60}$ feet up the wall or approximately 7.7 feet.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"What strategies did you use to help you understand the problem?"

I drew a picture.

"Would the Pythagorean Theorem still be useful in a room or space that was not rectangular?"

The Pythagorean Theorem is only useful if the length to be determined is one of the legs of a right triangle.

Highlight for students the difference between finding the length of a leg compared to finding the length of a hypotenuse. If time allows and it is not mentioned by students, highlight the fact that Tyler and the school can not buy 8.1 strings of light and discuss whether it makes sense to buy 8 or 9 strings of light.

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

"I picked the norm '____.' It really helped me/my group because ____."

"I picked the norm '____.' During the activity, I realized the norm '____' would be a better focus because ____."

Lesson Synthesis

The purpose of this discussion is to have students see that there are many situations where the Pythagorean Theorem can be used to solve problems. Ask students,

"What situations can you think of that involve right triangles?"

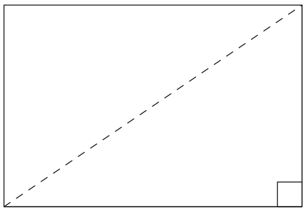
Give students 1 minute of quiet think time, and follow with a whole-class discussion. Record student responses for all to see.

If not mentioned by students, consider sharing some of these examples with students:

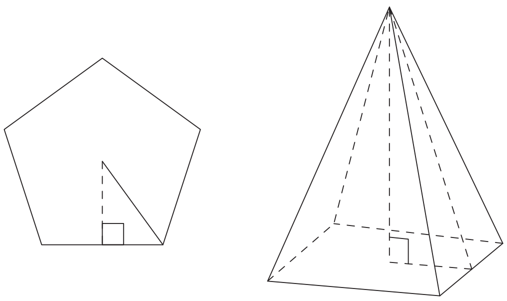
- Thick wires (called guy-wires) are used to keep utility poles upright, and their length depends on how high up the pole they attach and how far away from the pole they hook into the ground.
- Many rectangular device screens (such as for phones, tablets, and televisions) are sold based on diagonal length.

Lesson Summary

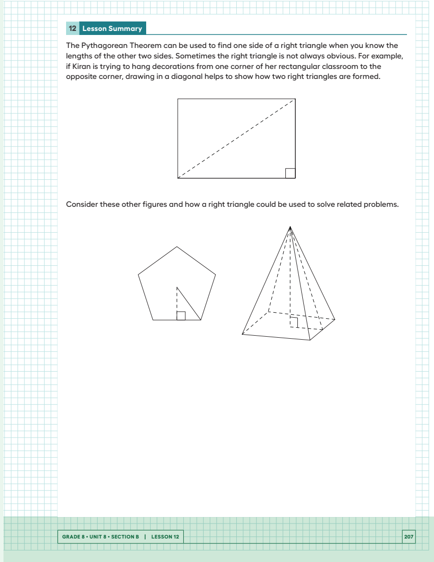
The Pythagorean Theorem can be used to find one side of a right triangle when you know the lengths of the other two sides. Sometimes the right triangle is not always obvious. For example, if Kiran is trying to hang decorations from one corner of her rectangular classroom to the opposite corner, drawing in a diagonal helps to show how two right triangles are formed.



Consider these other figures and how a right triangle could be used to solve related problems.



Student Workbook


12 Lesson Summary

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Consider these other figures and how a right triangle could be used to solve related problems.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery using the Pythagorean Theorem. If most students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 8, Lesson 11 Applications of the Pythagorean Theorem

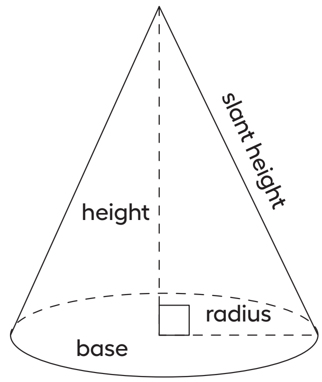
Cool-down

Diameter of a Cone

5 min

Student Task Statement

The height of a cone is 12 cm and its slant height is 13 cm. What is the diameter of the base of the cone?



10 cm

Lesson 12

Practice Problems

6 Problems

Problem 1

The dimensions of a rectangle are given. For each rectangle, find the exact length of its diagonal (a segment whose endpoints lie on opposite vertices of the rectangle).

a. 10 by 24

26

b. 16 by 30

34

c. 7 by 7

$\sqrt{98}$

d. $\sqrt{13}$ by 6

7

e. $\sqrt{3}$ by 1

2

f. $\frac{1}{2}$ by $\frac{\sqrt{3}}{2}$

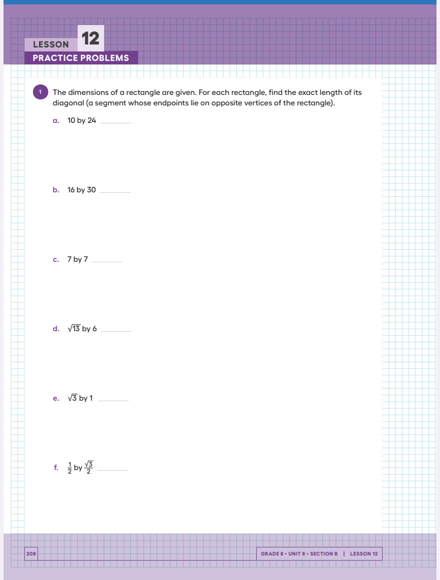
1

Problem 2

A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference in length between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round your answer to the nearest meter.

The walk through the park is about 68 m shorter than the walk around the park. Along the streets: $80 + 270 = 350$ meters. Along the diagonal: Solve $80^2 + 270^2 = x^2$ and get approximately 282 meters. $350 - 282 = 68$ meters.

Student Workbook


LESSON 12
PRACTICE PROBLEMS

1 The dimensions of a rectangle are given. For each rectangle, find the exact length of its diagonal (a segment whose endpoints lie on opposite vertices of the rectangle).

a. 10 by 24

b. 16 by 30

c. 7 by 7

d. $\sqrt{13}$ by 6

e. $\sqrt{3}$ by 1

f. $\frac{1}{2}$ by $\frac{\sqrt{3}}{2}$

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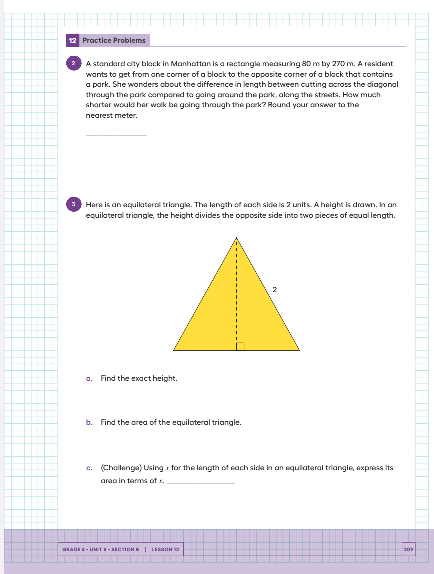
Student Workbook


12 Practice Problems

2 A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference in length between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round your answer to the nearest meter.

3 Here is an equilateral triangle. The length of each side is 2 units. A height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.

a. Find the exact height.

b. Find the area of the equilateral triangle.

c. (Challenge) Using x for the length of each side in an equilateral triangle, express its area in terms of x.

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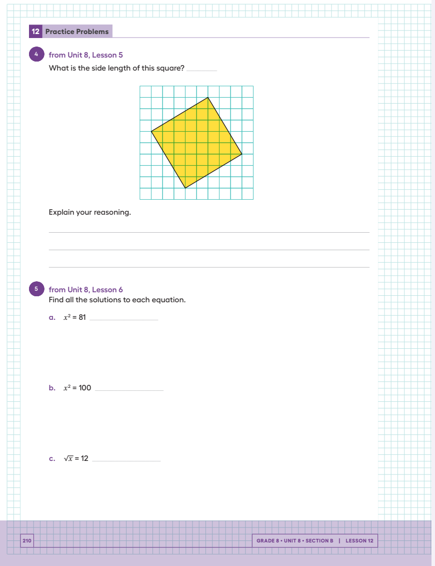
Student Workbook


12 Practice Problems

4 from Unit 8, Lesson 5
What is the side length of this square?

Explain your reasoning.

5 from Unit 8, Lesson 6
Find all the solutions to each equation.

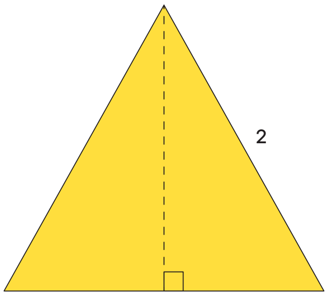
a. $x^2 = 81$

b. $x^2 = 100$

c. $\sqrt{x} = 12$

Problem 3

Here is an equilateral triangle. The length of each side is 2 units. A height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.



a. Find the exact height.

$\sqrt{3}$ units

b. Find the area of the equilateral triangle.

$\sqrt{3}$ units^2

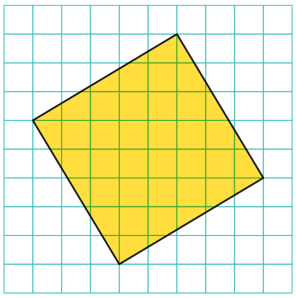
c. (Challenge) Using x for the length of each side in an equilateral triangle, express its area in terms of x.

The area is $\frac{x^2\sqrt{3}}{4}$ units^2

Problem 4

from Unit 8, Lesson 5

What is the side length of this square? Explain your reasoning.



The area of the square is 34 square units, so its side length is $\sqrt{34}$ units.

Problem 5

from Unit 8, Lesson 6

Find all the solutions to each equation.

a. $x^2 = 81$

9 and -9

b. $x^2 = 100$

10 and -10

c. $\sqrt{x} = 12$

144

Problem 6

from Unit 7, Lesson 2

What is the value of the "?" in the following equation?

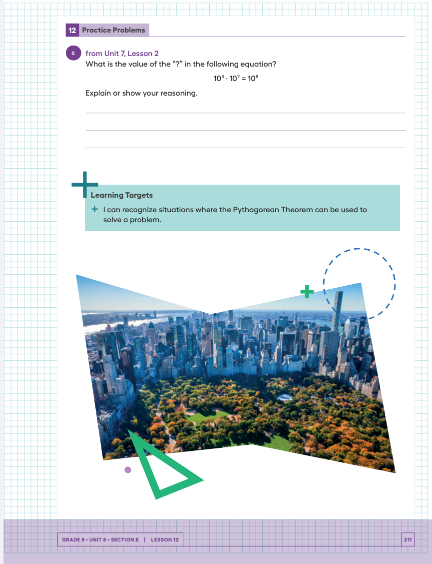
$10^3 \cdot 10^? = 10^8$

$? = 5$

Explain or show your reasoning.

Sample reasoning: Because $10^3 \cdot 10^5 = 10^8$.

Student Workbook


12 Practice Problems

6 from Unit 7, Lesson 2

What is the value of the "?" in the following equation?

$10^3 \cdot 10^? = 10^8$

Explain or show your reasoning.

Learning Targets

+ I can recognize situations where the Pythagorean Theorem can be used to solve a problem.