Lesson 17

More about Sampling Variability (Optional)

Goals

- Compare and contrast (orally) a distribution of sample means and the distribution of the population.
- Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a larger random sample.
- Interpret (orally and in writing) a dot plot that displays the means of multiple samples from the same population.

Learning Targets

- I can use the means from many samples to judge how accurate an estimate for the population mean is.
- I know that as the sample size gets bigger, the sample mean is more likely to be close to the population mean.

Lesson Narrative

This lesson is optional. It goes beyond necessary grade-level standards to examine the accuracy of estimates for population characteristics based on many samples. The lesson builds a solid foundation for future grades to build upon, but may be shortened or skipped due to time constraints.

In this lesson, students continue to look at multiple samples from the same population. Examining the structure of dot plots composed of the means from several samples, they see that different samples from the same population can have different means, but that most of these means cluster around the mean of the population. They consider how far off their estimate might be if they didn't know the mean of the population but they did know the sample mean. Additionally, students see that larger samples usually produce sample means that are less variable from one another and can more accurately estimate population means.

Student Learning Goal

Let's compare samples from the same population.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Required Materials

Materials to Gather

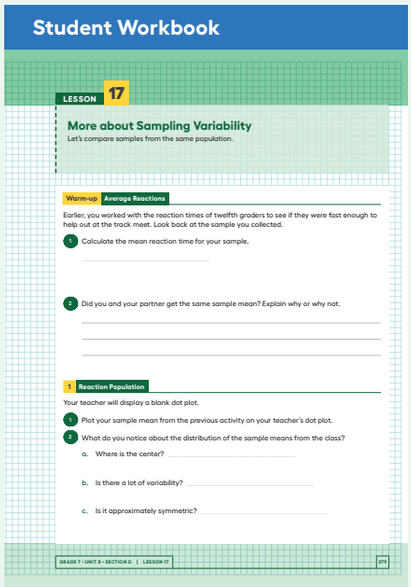
- Sticky notes: Activity 1

Required Preparation

Activity 1:
Prepare a large number line for the class to use as a dot plot of their sample means from the *Warm-up* of this lesson. Provide students with sticky notes to include as dots for this dot plot.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	5 min


Student Workbook

LESSON 17

More about Sampling Variability

Let's compare samples from the same population.

Warm-up Average Reactions

Earlier, you worked with the reaction times of twelfth graders to see if they were fast enough to help out at the track meet. Look back at the sample you collected.

1. Calculate the mean reaction time for your sample.

2. Did you and your partner get the same sample mean? Explain why or why not.

1 Reaction Population

Your teacher will display a blank dot plot.

1. Plot your sample mean from the previous activity on your teacher's dot plot.
2. What do you notice about the distribution of the sample means from the class?
   a. Where is the center?
   b. Is there a lot of variability?
   c. Is it approximately symmetric?

GRADE 7 • UNIT 8 • SECTION D | LESSON 17 239

Warm-up

Average Reactions

10 min

Activity Narrative

Students calculate the mean of a sample collected in an earlier lesson to compare with their partners. Students experience firsthand that different samples from the same population can produce different results. In later activities students will use the data they have collected here to develop a deeper understanding of sampling variability.

Launch

Arrange students in groups of 2 so that different partners are used from the ones used in the earlier activity analyzing reaction times of 12th graders at a track meet.

Remind students that the numbers came from a survey of all 120 seniors from a certain school. The numbers represent their reaction time in seconds during an activity in which they clicked a button as soon as they noticed that a square changed color. Those 120 values are the population for this activity.

Give students 2 minutes of quiet work time followed by partner work time. Follow with a whole-class discussion.

Student Task Statement

Earlier, you worked with the reaction times of twelfth graders to see if they were fast enough to help out at the track meet. Look back at the sample you collected.

1. Calculate the mean reaction time for your sample.

   Sample response: 0.43 seconds

2. Did you and your partner get the same sample mean? Explain why or why not.

   Sample response: The two means were slightly different, but both close to 0.43 seconds. Because there are different samples, the means are slightly different.

Activity Synthesis

The purpose of the discussion is for students to think about how the data they collected relates to the population data.

Some questions for discussion:

- *"Based on the information you currently know, estimate the mean of the population. Explain your reasoning."*

  Since both my partner and I got means close to 0.4, I think the population mean will be a little greater than 0.4.

"If each person selected 40 reaction times for their sample instead of 20, do you think this would provide a better estimate for the population mean?"

Since the sampling method is random, and thus fair, it should produce a better estimate.

Activity 1: Optional
Reaction Population

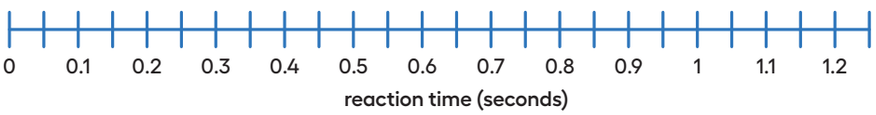
15 min

Activity Narrative

In this activity, a dot plot is created by the class that includes all of the calculated sample means. Students then compare that display to the data from the entire population to better understand the information that can be gained from a sample mean. In the discussion that follows the activity, students look at similar displays of sample means for samples of different sizes. Students should see that larger samples tend to more accurately estimate the population mean than smaller samples.

Launch

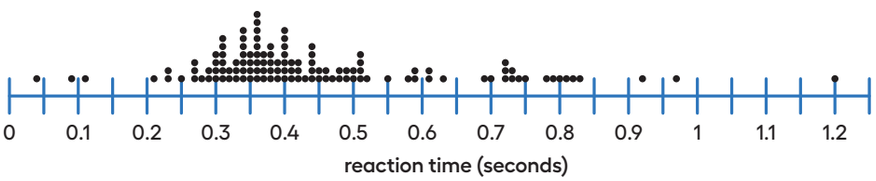
Display the basis for a dot plot of sample means for all to see.



Allow students 5 minutes to create the class dot plot and complete the first set of questions.

Ask students to include their sample means from the previous task to the display for all to see (since the samples originated in pairs, there will be many repeats. It is OK to include the repeats or ensure only 1 of the original partners includes their point on the display). They may do this by adding a dot to the display on the board, placing a sticky note in the correct place, or passing a large sheet of paper around the class.

After students have had a chance to answer the first set of questions, pause the class to display the dot plot of the reaction times for the entire population of 120 seniors for all to see.



Provide students 5 minutes to complete the problems.

Follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
To support organization, provide students with a graphic organizer to describe the connections between sample size and mean distribution shape.

Supports accessibility for: Language, Organization

Building on Student Thinking

The center of the dot plot created by the class can be thought of as a mean of sample means. The phrase can be difficult for students to think through, so remind them what the dot they added to the dot plot represents and how they calculated that value. Consider displaying the class dot plot for the rest of the unit to help students remember this example to understand similar dot plots.

Student Workbook


1 Reaction Population

3 The population mean is 0.442 seconds. How does this value compare to the sample means from the class?

Pause here so your teacher can display a dot plot of the reaction times of the population.

4 What do you notice about the distribution of the population?

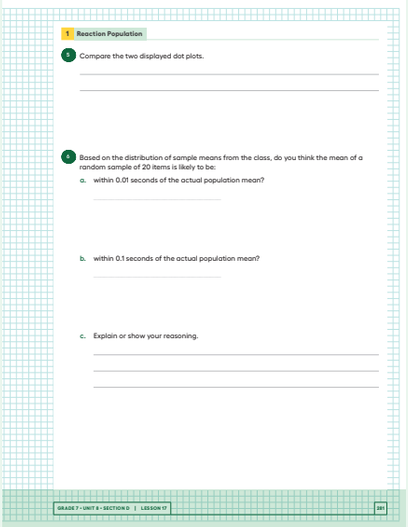
a. Where is the center?

b. Is there a lot of variability?

c. Is it approximately symmetric?

260 GRADE 7 • UNIT 8 • SECTION D | LESSON 17

Student Workbook


1 Reaction Population

5 Compare the two displayed dot plots.

6 Based on the distribution of sample means from the class, do you think the mean of a random sample of 20 items is likely to be:

a. within 0.01 seconds of the actual population mean?

b. within 0.1 seconds of the actual population mean?

c. Explain or show your reasoning.

GRADE 7 • UNIT 8 • SECTION D | LESSON 17 261

Student Task Statement

Your teacher will display a blank dot plot.

1. Plot your sample mean from the previous activity on your teacher's dot plot.

   No response required.

2. What do you notice about the distribution of the sample means from the class?

   a. Where is the center?

      Sample response: The center of the distribution is about 0.44 seconds.

   b. Is there a lot of variability?

      Sample response: There is not a lot of variability. Most of the values are very close to 0.44 seconds.

   c. Is it approximately symmetric?

      Sample response: The distribution is approximately symmetric.

3. The population mean is 0.442 seconds. How does this value compare to the sample means from the class?

   Pause here so your teacher can display a dot plot of the reaction times of the population.

   Sample response: The center of the class distribution of sample means is about the same as the population mean.

4. What do you notice about the distribution of the population?

   a. Where is the center?

      Sample response: The center of the population distribution is around 0.44 seconds.

   b. Is there a lot of variability?

      Sample response: There is relatively large variability.

   c. Is it approximately symmetric?

      Sample response: The distribution is approximately symmetric, but there are some times that are much longer than the bulk of the data.

5. Compare the two displayed dot plots.

   Sample response: The center of the two dot plots are in about the same place, but the dot plot of the sample means shows much less variability.

6. Based on the distribution of sample means from the class, do you think the mean of a random sample of 20 items is likely to be:

   a. within 0.01 seconds of the actual population mean?

      Sample response: Most of the dots in our sample mean plot are not within 0.01 seconds of the center, so I do not think the mean for a random sample of 20 would be that close.

   b. within 0.1 seconds of the actual population mean?

   Explain or show your reasoning.

   Sample response: Most of the dots in our sample mean plot are within 0.1 seconds of the center, so I think a random sample mean would probably be within 0.1 seconds of the population mean.

Activity Synthesis

Display the dot plot of sample means for all to see throughout the unit. It may be helpful to refer to this display when viewing dot plots of sample means in later lessons.

The purpose of the discussion is for students to understand that the centers of the two dot plots in the activity are close, if not the same, but the sample means have less variability, and the shapes of the population distribution and the sample mean distribution are probably different. Additionally, students should use the dot plots shown here to see that larger samples have sample means that are still centered around the population mean, but have less variability than the means from smaller samples. Therefore, larger samples should provide better estimates of the population mean than smaller samples do.

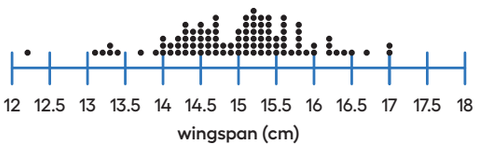
Items for further discussion:

- *"If the sample size were increased to 30 instead of 20, what do you think the distribution might look like?"*

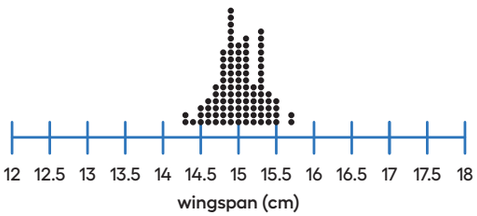
  There would be less variability in the same means, but they should be centered around the same value.

For a different set of data representing the wingspan in centimeters of a certain species of bird, compare these dot plots.

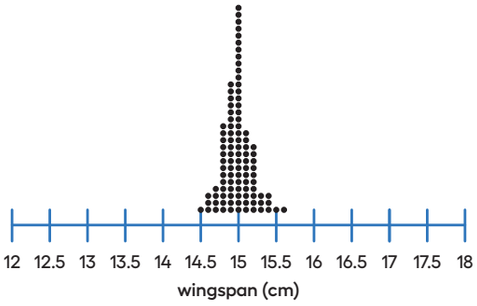
Here is a dot plot of the population data.



Here is a dot plot of the sample means for 100 different random samples each of size 10.



Here is a dot plot of the sample means for 100 different random samples each of size 30.



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support whole-class discussion: "When the sample size is 10, I noticed that the distribution ..." "When the sample size is 30, I noticed that the distribution ..." and "It's better to have a larger/smaller sample size to estimate the population mean because ..."

Advances: Conversing, Representing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Develop Language and Symbols.
Make connections between representations visible. Ask students to explain the meaning of a particular dot on one of the dot plots. Ensure students understand that a single dot represents a mean, not the age of one person.

Supports accessibility for: Language, Conceptual Processing

💬 *"What do you notice about the distribution of the sample means as the sample size increases?"*

The variability decreases, but the center stays in about the same place.

💬 *"Use the dot plots to explain why the sample mean from a random sample of size 30 would be a better estimate of the population mean than the sample mean from a random sample of size 10."*

Since the variability is less, the sample mean is probably closer to the true mean with size 30 than with size 10.

Activity 2: Optional

How Much Do You Trust the Answer?

10 min

Activity Narrative

In this activity, students continue to look at how variability in the sample means can be used to think about the accuracy of an estimate of a population mean. If the means from samples tend to be very spread out, then there is reason to believe that the mean from a single sample may not be especially close to the mean for the population. If the means from samples tend to be tightly grouped, then there is reason to believe that the mean from a single sample is a good estimate of the mean for the population.

Launch

Arrange students in groups of 2.

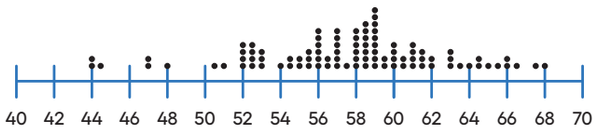
Remind students about the context of earlier lessons in which they examined ages of people watching various shows.

Tell students that, although the dot plots seem to have a similar shape, attention should be given to the scale of the horizontal axis.

Allow students 3–5 minutes of quiet work time followed by partner discussion. Then follow with a whole-class discussion.

Student Task Statement

Here are the mean ages for 100 different samples of viewers from each show.



sample means for *Trivia the Game Show*

sample means for *Science Experiments YOU Can Do*

sample means for *Learning to Read*

1. For each show, use the dot plot to estimate the population mean.

   a. *Trivia the Game Show* 58 years old

   b. *Science Experiments YOU Can Do* 12 years old

   c. *Learning to Read* 6 years old

2. For each show, are most of the sample means within 1 year of your estimated population mean?

   Science Experiments YOU Can Do and *Learning to Read* have most of the sample means within 1 year of the estimated population mean.

3. Suppose you take a new random sample of 10 viewers for each of the 3 shows. Which show do you expect to have the new sample mean closest to the population mean? Explain or show your reasoning.

   Learning to Read should produce the best estimates of population mean since it has the least variability in the sample means.

Are You Ready for More?

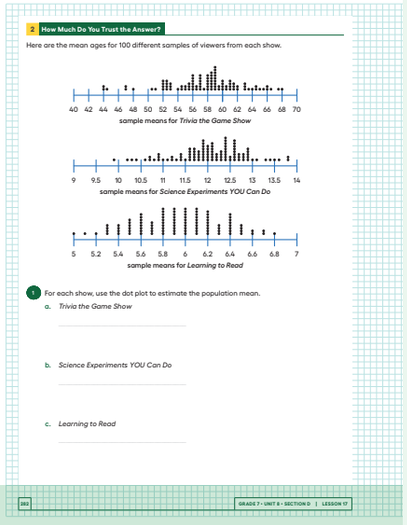
Market research shows that advertisements for retirement plans appeal to people between the ages of 40 and 55. Younger people are usually not interested, and older people often already have a plan. Is it a good idea to advertise retirement plans during any of these three shows? Explain your reasoning.

Sample response: It might be worth it to advertise during *Trivia the Game Show*. Although the mean appears to be about 59 years, about a fourth of the viewers are still in the desired age range of 40 to 55 years. It might depend on the total number of viewers to make the decision.

Building on Student Thinking

Students may get confused about what the dot plots represent. Refer to the class dot plot of sample means from the reaction time activity to help them understand that a single dot on the dot plot represents a mean from a single sample. Each dot plot shows means from several different samples.

Student Workbook



Student Workbook

Activity Synthesis

The purpose of the discussion is to think about what the information in the given dot plots tells us about the accuracy of an estimate of a population mean based on the sample mean from a single sample.

Consider asking these questions for discussion:

- *"In the first dot plot for 'Trivia the Game Show,' what does the dot at 68 represent?"*

  There is a sample of 10 viewers that has a mean age of 68 years.

- *"Based on the dot plot shown for 'Learning to Read,' do you think any 7 year olds are included in the data?"*

  Yes. Because one of the sample means is 6.8, there are probably a few 7 year olds and some 6 year olds in that sample.

- *"Why is this kind of dot plot useful?"*

  It shows how variable the means from samples can be, so it gives a good idea of how accurate a sample mean might be as an estimate of a population mean.

- *"Why might it be difficult to obtain a dot plot like this?"*

  Each dot represents a sample of 10. It might be hard to get 100 different samples of size 10.

Lesson Synthesis

Consider asking these discussion questions to emphasize the main ideas of the lesson:

- *"In the first dot plot for Trivia the Game Show, what does the dot at 68 represent?"*

  There is a sample of 10 viewers that has a mean age of 68 years.

- *"Based on the dot plot shown for Learning to Read, do you think any 7-year-olds are included in the data?"*

  Yes. Because one of the sample means is 6.8, there are probably a few 7-year-olds and 6-year-olds in that sample.

- *"Why is this kind of dot plot useful?"*

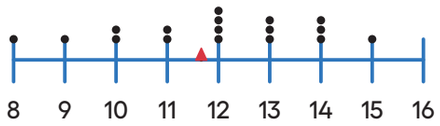
  It shows how variable the means from samples can be, so it gives a good idea of how accurate a sample mean might be as an estimate of a population mean.

- *"Why might it be difficult to obtain a dot plot like this?"*

  Each dot represents a sample of 10. It might be hard to get 100 different samples of size 10.

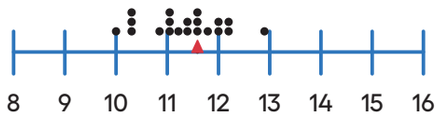
Lesson Summary

A population of hummingbirds has a mean weight of 11.6 grams. This dot plot shows the weights of 18 hummingbirds selected from the population that have the same mean weight as the population. The mean weight is indicated with a triangle.



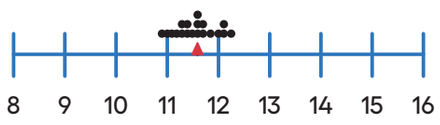
hummingbird weights (grams)

20 more samples, each with 5 hummingbirds, are selected from the original population. For each sample, the mean of the 5 birds is calculated and plotted on this dot plot. The triangle represents the population mean of 11.6 grams. Notice that the sample means are fairly close to the population mean, but that they are not exactly the same.



means of samples of size 5

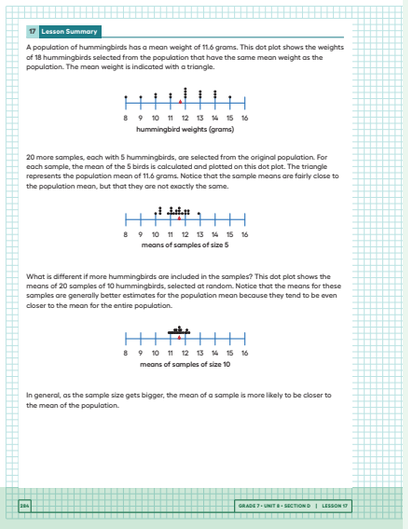
What is different if more hummingbirds are included in the samples? This dot plot shows the means of 20 samples of 10 hummingbirds, selected at random. Notice that the means for these samples are generally better estimates for the population mean because they tend to be even closer to the mean for the entire population.



means of samples of size 10

In general, as the sample size gets bigger, the mean of a sample is more likely to be closer to the mean of the population.

Student Workbook


17 Lesson Summary

A population of hummingbirds has a mean weight of 11.6 grams. This dot plot shows the weights of 18 hummingbirds selected from the population that have the same mean weight as the population. The mean weight is indicated with a triangle.

8 9 10 11 12 13 14 15 16

hummingbird weights (grams)

20 more samples, each with 5 hummingbirds, are selected from the original population. For each sample, the mean of the 5 birds is calculated and plotted on this dot plot. The triangle represents the population mean of 11.6 grams. Notice that the sample means are fairly close to the population mean, but that they are not exactly the same.

8 9 10 11 12 13 14 15 16

means of samples of size 5

What is different if more hummingbirds are included in the samples? This dot plot shows the means of 20 samples of 10 hummingbirds, selected at random. Notice that the means for these samples are generally better estimates for the population mean because they tend to be even closer to the mean for the entire population.

8 9 10 11 12 13 14 15 16

means of samples of size 10

In general, as the sample size gets bigger, the mean of a sample is more likely to be closer to the mean of the population.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

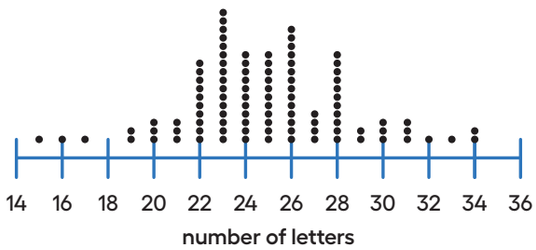
Cool-down

How Much Mail?

5 min

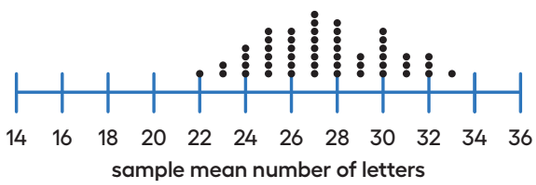
Student Task Statement

Jada collects data about the number of letters people get in the mail each week. The population distribution is shown in the dot plot.

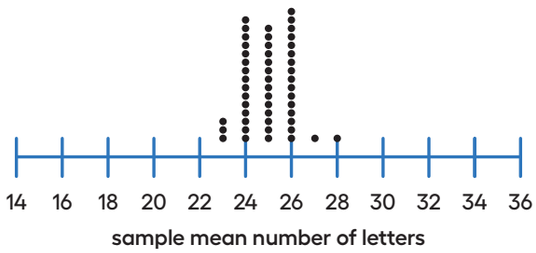


Which of these dot plots are likely to represent the means from samples of size 10 from this population? Explain your reasoning.

Dot Plot 1



Dot Plot 2



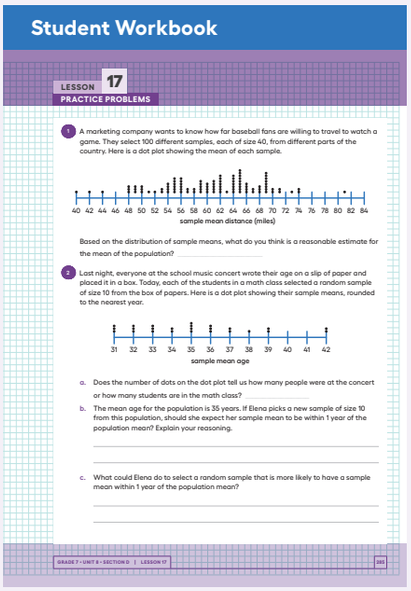
Dot Plot 2

Sample reasoning: It has the same center, but less variability than the population data. Dot Plot I also has less variability, but has a different center than the population data, so it is probably not generated by sample means from the original population.

Lesson 17

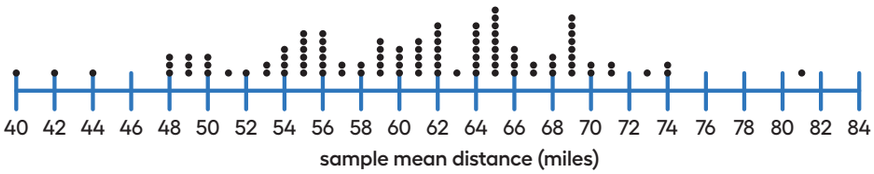
Practice Problems

5 Problems



Problem 1

A marketing company wants to know how far baseball fans are willing to travel to watch a game. They select 100 different samples, each of size 40, from different parts of the country. Here is a dot plot showing the mean of each sample.

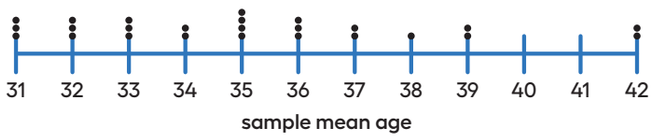


Based on the distribution of sample means, what do you think is a reasonable estimate for the mean of the population?

Reasonable answers are between 59 and 65 miles.

Problem 2

Last night, everyone at the school music concert wrote their age on a slip of paper and placed it in a box. Today, each of the students in a math class selected a random sample of size 10 from the box of papers. Here is a dot plot showing their sample means, rounded to the nearest year.



a. Does the number of dots on the dot plot tell us how many people were at the concert or how many students are in the math class?

It tells us how many students are in the math class.

b. The mean age for the population is 35 years. If Elena picks a new sample of size 10 from this population, should she expect her sample mean to be within 1 year of the population mean? Explain your reasoning.

No. Only 9 of the 25 sample means in the dot plot are within 1 year of the population mean. While it could happen, it is more likely Elena's sample mean will be more than 1 year away from the population mean.

c. What could Elena do to select a random sample that is more likely to have a sample mean within 1 year of the population mean?

She could select more than 10 papers for her sample.


Student Workbook

17 Practice Problems

3 from Unit 8, Lesson 16
A random sample of people were asked which hand they prefer to write with. "L" means they prefer to use their left hand, and "R" means they prefer to use their right hand. Here are the results:

L R R R R R R R

R R L R R R R

Based on this sample, estimate the proportion of the population that prefers to write with their left hand.

4 from Unit 8, Lesson 15
Andre would like to estimate the mean number of books the students at his school read over the summer break. He has a list of the names of all the students at the school, but he doesn't have time to ask every student how many books they read.

What should Andre do to estimate the mean number of books?

5 from Unit 8, Lesson 10
A hockey team has a 75% chance of winning against the opposing team in each game of a playoff series. To win the series, the team must be the first to win 4 games.

a. Design a simulation for this event.

b. What counts as a successful outcome in your simulation?

c. Estimate the probability using your simulation.

Problem 3

from Unit 8, Lesson 16

A random sample of people were asked which hand they prefer to write with. "L" means they prefer to use their left hand, and "R" means they prefer to use their right hand. Here are the results:

L	R	R	R	R	R	R	R
R	R	L	R	R	R	R	

Based on this sample, estimate the proportion of the population that prefers to write with their left hand.

$\frac{2}{15}$ (or about 13%)

Problem 4

from Unit 8, Lesson 15

Andre would like to estimate the mean number of books the students at his school read over the summer break. He has a list of the names of all the students at the school, but he doesn't have time to ask every student how many books they read.

What should Andre do to estimate the mean number of books?

Sample response: Andre can estimate the mean by first using the list he has to select a random sample of the students, then asking the students in the sample how many books they read over the summer. He can choose a sample size, and it would be better to take a larger sample if possible because this will probably give him a more accurate estimate.

Problem 5

from Unit 8, Lesson 10

A hockey team has a 75% chance of winning against the opposing team in each game of a playoff series. To win the series, the team must be the first to win 4 games.

a. Design a simulation for this event.

Sample response: Make a spinner with 4 equal sections labeled 1, 2, 3, and 4. Spin the spinner, and record the outcomes as wins if the spinner lands on 1, 2, or 3 or losses if the spinner lands on 4, until one team wins the series. Repeat this process several times.

b. What counts as a successful outcome in your simulation?

Sample response: Count the times that 1, 2, or 3 appear as winning a game and 4 as losing. A trial is a success if 1, 2, or 3 appear four times before 4 appears four times.

c. Estimate the probability using your simulation.

The actual probability is over 99%.