Writing Systems of Equations

Goals

Categorize (in writing) systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety of strategies.

Create a system of equations that represents a situation and interpret (orally and in writing) the solution in context.

Learning Target

I can write a system of equations from a real-world situation.

Lesson Narrative

In this lesson, students use the Info Gap routine to work together to write systems of equations and solve them.

Two additional optional activities are included. The first optional activity focuses on writing systems of equations from a context and interpreting the meaning of the solution in terms of the context without actually solving the system. The second optional activity offers additional practice identifying the number of solutions for a system, then solving them.

Students must reason abstractly and quantitatively to write a system of equations from a context. Structure is important when recognizing the number of solutions for a system of equations and solving efficiently.

Student Learning Goal

Let's write systems of equations from real-world situations.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR4: Information Gap (Activity 1)
- MLR8: Discussion Supports

Instructional Routines

- MLR4: Information Gap Cards
- MLR8: Discussion Supports

Required Materials

Materials to Copy

• Racing and Play Tickets Cards (1 copy for every 4 students): Activity 1

Lesson Timeline







Activity 1



Activity 2



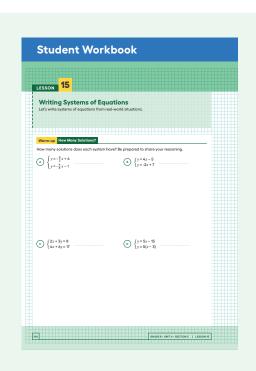
Activity 3



Lesson Synthesis



Cool-down



Warm-up

How Many Solutions?

5 min

Activity Narrative

This *Warm-up* asks students to connect the algebraic representations of systems of equations to the number of solutions. Efficient students will recognize that this can be done without solving the system, but rather by using slope, *y*-intercept, or other methods for recognizing the number of solutions.

Monitor for students who use these methods:

- 1. Solve the systems to find the number of solutions.
- **2.** Use the slope and *y*-intercept to determine the number of solutions.
- **3.** Manipulate the equations into another form, then compare the equations.
- **4.** Notice that the left side of the second equation in System C is double the left side of the first equation, but the right side is not.

Launch

Arrange students in groups of 2. Tell students that each number can be used more than once.

Allow students 2 minutes of work time followed by a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

How many solutions does each system have? Be prepared to share your reasoning.

A.
$$\begin{cases} y = -\frac{4}{3}x + 4 \\ y = -\frac{4}{7}x - 1 \end{cases} = 0$$

B.
$$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$$

c.
$$\begin{cases} 2x + 3y = 8 \\ 4x + 6y = 17 \end{cases} = 0$$

D.
$$\begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases}$$
 infinite

Lesson 15 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

Activity Synthesis

The goal of this discussion is to compare strategies that students use to find the number of solutions for a system of equations.

Invite a group of students to share their solution and reasoning. Then, ask:

"Did anyone solve the problem the same way, but would explain it differently?"

"How do the slope and intercept show up in each method?"

Activity 1

Info Gap: Racing and Play Tickets



Activity Narrative

In this activity, students apply what they know about systems of linear equations to solve a problem about a real-world situation, but they do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for the information that they need to solve the problem. This may take several rounds of discussion if their first requests do not yield the information that they need. It also allows students to refine the language that they use and to ask increasingly more precise questions until they get the information that they need.

Launch 🙎

Tell students that they will solve systems of equations from a situation. Display, for all to see, the *Info Gap* graphic that illustrates a framework for the routine.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.



Access for Multilingual Learners (Activity 1)

MLR4: Information Gap

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

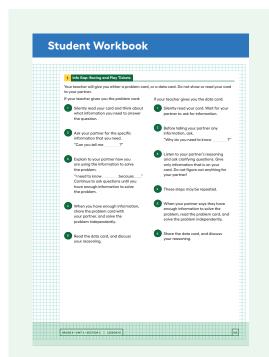
Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the Info Gap graphic visible throughout the activity, or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Lesson 15 Warm-up **Activity 1** Activity 2 Activity 3 Lesson Synthesis Cool-down



Student Task Statement

Your teacher will give you either a problem card, or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

- **1.** Silently read your card and think about what information you need to answer the question.
- **2.** Ask your partner for the specific information that you need. "Can you tell me ____?"
- 3. Explain to your partner how you are using the information to solve the problem. "I need to know_____ because ..."
 Continue to ask questions until you have enough information to solve the problem.
- 4. When you have enough information, share the problem card with your partner, and solve the problem independently.
- **5.** Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

- **1.** Silently read your card. Wait for your partner to ask for information.
- 2. Before telling your partner any information, ask, "Why do you need to know ____?"
- 3. Listen to your partner's reasoning and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
- 4. These steps may be repeated.
- 5. When your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
- **6.** Share the data card, and discuss your reasoning.

Problem/Data Card 1: 12 seconds

Problem/Data Card 2: 40 student tickets and 75 adult tickets

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- "Did you write 2 equations for each system and solve?"
 - I did, but didn't need to. For the race, I could've tried different numbers for x to find when the distances are equal.
- ☐ "Did you graph anything? If so, what was it useful for?"
 - I graphed the equations to check my answers.
- ☐ "Did you check your answers? How?"

I could check by graphing, but I could also substitute into the original equations to check that both equations are true for the values.

Activity 1

Activity 2: Optional

Situations and Systems



Activity Narrative

In this activity, students are presented with a number of scenarios that could be solved using a system of equations. Students are not asked to solve the systems of equations, because the focus at this time is for students to understand how to set up the equations for the system and to understand what the solution represents in context.

Warm-up

Launch

Arrange students in groups of 2. Suggest that groups split up the problems so that one person works on the first and third problem while their partner works on the second and fourth. Students may work with their partners to get help when they are stuck, but are encouraged to try to set up the equations on their own first. Partners should discuss their systems and interpretation of the solution after each has had a chance to work on their own.

Allow students 5–7 minutes of partner work time followed by a whole-class discussion.

Student Task Statement

For each situation:

- · Create a system of equations.
- Then, without solving, interpret what the solution to the system would tell you about the situation.
- 1. Lin's family is out for a bike ride when her dad stops to take a picture of the scenery. He tells the rest of the family to keep going and that he'll catch up. Lin's dad spends 5 minutes taking the photo and then rides at 0.24 miles per minute until he meets up with the rest of the family further along the bike path. Lin and the rest were riding at 0.18 miles per minute.

Sample response:
$$\begin{cases} d = 0.24t \\ d = 0.18t + 5 \cdot 0.18 \end{cases}$$

The solution would represent the time (t) that it would take for Lin's dad to catch up with the rest of the family.

2. Noah is planning a kayaking trip. Kayak Rental A charges a base fee of \$15 plus \$4.50 per hour. Kayak Rental B charges a base fee of \$12.50 plus \$5 per hour.

Sample response:
$$\begin{cases} y = 15 + 4.5x \\ y = 12.5 + 5x \end{cases}$$

The solution would represent the amount of time (x) spent with the kayak so that the cost (y) would be the same from each rental company.

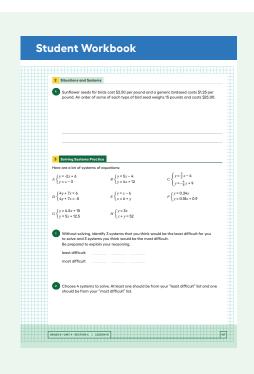
Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Provide Access by Recruiting Interest.

Leverage choice around perceived challenge. Invite students to select 2–3 of the situations to complete.

Supports accessibility for: Organization, Social-Emotional **Functioning**





Warm-up

3. Diego is crafting items in a video game. The crafting recipe calls for 3 pieces of wood for every piece of metal. Diego's character uses 52 item slots of wood and metal crafting materials while making the items.

Sample response:
$$\begin{cases} w = 3m \\ w + m = 52 \end{cases}$$

The solution would represent the number of pieces of wood (w) and the number of pieces of metal (m) that Diego used to make the items.

4. Sunflower seeds for birds cost \$2.50 per pound and a generic birdseed costs \$1.25 per pound. An order of some of each type of bird seed weighs 15 pounds and costs \$25.00.

Sample response:
$$\begin{cases} 2.5s + 1.25b = 25\\ s + b = 15 \end{cases}$$

The solution would represent the number of pounds of sunflower seeds (s) and the number of pounds of generic birdseed (b) purchased in this order.

Activity Synthesis

The focus of the discussion should be on making sense of the context and interpreting the solutions within the context of the problems.

Invite groups to share their systems of equations and interpretation of the solution for each problem. As groups share, record their systems of equations for all to see. When necessary, ask students to explain the meaning of the variables they used. For example, t represents the number of minutes the family rides after Lin's dad starts riding again after taking the picture.

To highlight the connections between the situations and the equations that represent them, ask:

 \bigcirc "How many solutions will each of these systems of equations have?"

Each system has exactly one solution. I can tell this because the slopes of each pair of equations are different.

"If Lin's dad biked 0.17 miles per minute instead of 0.24 miles per minute, how would that change the system of equations?"

The first equation would be d = 0.17t.

"How many solutions would there be for this new system where Lin's dad rides slower?"

Based on the equations there should still be one solution.

Activity 1

☐ "Would Lin's dad ever catch up with the family?"

He would not. He started farther back and rides slower than the family. The solution to the system would have a negative value for time, which does not make sense in the context of the problem.

Warm-up

If students disagree that there is a solution to the modified first problem in which Lin's dad rides slower than the family, you can display the graph of the modified system and point out the point where the lines intersect. So, although the system has a solution, it is disregarded in this context because it does not make sense.

Activity 3: Optional

Solving Systems Practice



Activity Narrative

In this activity, students solve a variety of systems of equations, some involving fractions, some involving substitution, and some involving inspection. This gives students a chance to solidify that learning by practicing the methods that they have learned for solving systems of equations. Some of the systems listed are ones that students could have used in an earlier activity in this lesson, to describe the situations there. In the discussion, students compare the systems here to the ones that they wrote in the earlier activity, and they interpret the answer in context.

Launch 🙎

Keep students in groups of 2.

Allow students 5–7 minutes of partner work time, and follow that with a whole-class discussion.

Student Task Statement

Here are a lot of systems of equations:

A.
$$\begin{cases} y = -2x + 6 \\ y = x - 3 \end{cases}$$

C.
$$\begin{cases} y = \frac{2}{3}x - 4 \\ y = -\frac{4}{7}x + 9 \end{cases}$$

E.
$$\begin{cases} y = x - 6 \\ x = 6 + y \end{cases}$$

G.
$$\begin{cases} y = 4.5x + 15 \\ y = 5x + 12.5 \end{cases}$$

B.
$$\begin{cases} y = 5x - 4 \\ y = 4x + 12 \end{cases}$$

D.
$$\begin{cases} 4y + 7x = 6 \\ 4y + 7x = -5 \end{cases}$$

F.
$$\begin{cases} y = 0.24x \\ y = 0.18x + 0.9 \end{cases}$$

H.
$$\begin{cases} y = 3x \\ y + y = 5 \end{cases}$$

1. Without solving, identify 3 systems that you think would be the least difficult for you to solve and 3 systems you think would be the most difficult. Be prepared to explain your reasoning.

Answers vary.

Instructional Routines

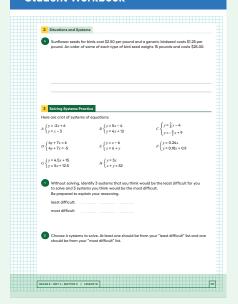
MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.



Student Workbook



Access for Multilingual Learners (Activity 3, Activity Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support whole-class discussion: "_____ is the least (or most) difficult to solve because _____." or "____ would make the problem easier to solve."

Advances: Conversing, Representing

2. Choose 4 systems to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Sample responses:

$$A. x = 3, y = 0$$

$$B. x = 16, y = 76$$

C.
$$x = 6\frac{1}{2}, y = \frac{1}{3}$$

D. No solution

E. Infinitely many solutions

$$F. x = 15, y = 3.6$$

$$G.x = 5, y = 37.5$$

$$H. x = 13, y = 39$$

Activity Synthesis

There are two key takeaways from this discussion. The first is to reinforce that for some systems, students can determine—by reasoning—whether it's possible for a solution to one equation to also be a solution to another equation. The second takeaway is that there are some systems that students will be able to solve only after learning techniques in future grades.

For each problem, ask students to indicate if they identified the system as least or most difficult. Record the responses for all to see.

Bring students' attention to this system:

$$\begin{cases} 4y + 7x = 6 \\ 4y + 7x = -5 \end{cases}$$

Ask students what the two equations in the system have in common with each other and to think about whether a solution to the first equation could also be a solution to the second. One can reason that there is no solution because 4y + 7x cannot be equal to both 6 and -5.

Ask students to return to the earlier activity and see if they can find any of those systems in these problems. (Lin's family ride is the sixth system. Noah's kayaking trip is the seventh system. Diego's crafting is the eighth system.) After students notice the connection, invite students who chose to solve those systems to interpret the numerical solution of each system in the context of the earlier activity.

Students may be tempted to develop the false impression that all systems where both equations are given as linear combinations can be solved by inspection. Conclude the discussion by displaying this system that students defined in the last activity about birdseed:

$$\begin{cases} 2.5x + 1.25y = 25\\ x + y = 15 \end{cases}$$

Tell students that this system has one solution and they will learn more sophisticated techniques for solving systems of equations like this in future grades.

Lesson Synthesis

To wrap up the lessons on solving systems of equations, consider displaying the three systems of equations and asking students how they might begin to solve the systems.

$$\begin{cases} y = 2x + 1 \\ y = \frac{1}{2}x + 10 \end{cases}$$
 (Both graphing and substitution methods work well)

$$\begin{cases} x = 5 - 2y \\ 2x + 6y = 16 \end{cases}$$
 (Substitution works best)

$$\begin{cases} 5x + 4y = 20 \\ 10x + 8y = 60 \end{cases}$$
 (Inspection may work best)

If there is time, consider assigning each system to small groups for them to solve and then to share their solutions with the class.

Lesson Summary

We have learned how to use algebra to solve many kinds of systems of equations that would be difficult to solve by graphing. For example, look at

$$\begin{cases} y = 2x - 3 \\ x + 2y = 7 \end{cases}$$

The first equation says that y = 2x - 3, so wherever we see y, we can substitute the expression 2x - 3 instead. So the second equation becomes x + 2(2x - 3) = 7.

We can solve for x:

$$x+4x-6=7$$
 distributive property
 $5x-6=7$ combine like terms
 $5x=13$ add 6 to each side
 $x=\frac{13}{5}$ multiply each side by $\frac{1}{5}$

We know that the *y*-value for the solution is the same for either equation, so we can use either equation to solve for it. Using the first equation, we get:

$$y=2(\frac{13}{5})-3$$
 substitute $x=\frac{13}{5}$ into the equation $y=\frac{26}{5}-3$ multiply $2(\frac{13}{5})$ to make $\frac{26}{5}$ $y=\frac{26}{5}-\frac{15}{5}$ rewrite 3 as $\frac{15}{5}$ $y=\frac{11}{5}$

If we substitute $x = \frac{13}{5}$ into the other equation, x + 2y = 7, we get the same y-value. So the solution to the system is $\left(\frac{13}{5}, \frac{11}{5}\right)$.

Student Workbook We have loaned how to use algebra to solve many kinds of systems of equations that would be difficult to solve by graphing. For example, look at $\begin{cases} y=2-3\\ (x-2)-3 \end{cases}$ The first equation can be fully 2x-3. As the second sequential becomes x=2(2x-3)-3. We can solve for x=x+4x-6-7 distribution property 5x-6-7 combine lise terms 5x-6-7 com

Responding To Student Thinking

Press Pause

If students continue to struggle to solve systems of equations algebraically, make time for students to practice solving equations with and without fractions involved. For example, use these practice problems:

Unit 4, Lesson 15, Practice Problem 3 Unit 4, Lesson 15, Practice Problem 4

Cool-down

Solve This

5 min

Student Task Statement

Solve

$$\begin{cases} y = \frac{3}{4}x \\ \frac{5}{2}x + 2y = 5 \end{cases}$$

$$x = \frac{5}{44}, y = \frac{15}{16}$$

Practice Problems

4 Problems

Problem 1

Kiran and his cousin work during the summer for a landscaping company. Kiran's cousin has been working for the company longer, so his pay is 30% more than Kiran's. Last week his cousin worked 27 hours, and Kiran worked 23 hours. Together, they earned \$493.85. What is Kiran's hourly pay? Explain or show your reasoning.

\$8.50

Sample reasoning: n = Kiran's hourly wage and c = Kiran's cousin's hourly wage. c = 1.3n and 27c + 23n = 493.85. Substituting 1.3n for c yields the equation 27(1.3n) + 23n = 493.85.

Problem 2

Clare and Noah play a game in which they earn the same number of points for each goal and lose the same number of points for each penalty. Clare makes 6 goals and 3 penalties, ending the game with 6 points. Noah earns 8 goals and 9 penalties and ends the game with -22 points.

- **a.** Write a system of equations that describes Clare's and Noah's outcomes. Use x to represent the number of points for a goal and y to represent the number of points for a penalty.
 - Clare: 6x 3y = 6
 - Noah: 8x 9y = -22
- **b.** Solve the system. What does your solution mean?

(4,6)

Sample response: A goal earns 4 points, and a penalty costs 6 points.

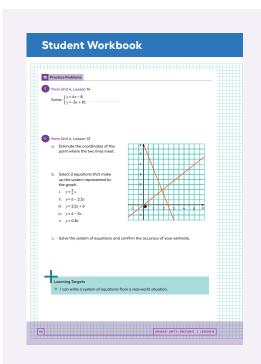
Problem 3

from Unit 4, Lesson 14

Solve:
$$\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$$

Sample reasoning: First solve 6x - 8 = -3x + 10 for x and substitute that value into either of the original equations to solve for y.

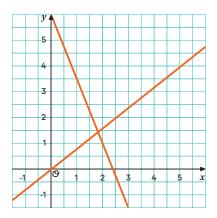




Problem 4

from Unit 4, Lesson 13

a. Estimate the coordinates of the point where the two lines meet.



Sample response: (1.8, 1.4)

b. Select 2 equations that make up the system represented by the graph.

i.
$$y = \frac{5}{4}x$$

ii.
$$y = 6 - 2.5x$$

iii.
$$y = 2.5x + 6$$

iv.
$$y = 6 - 3x$$

v.
$$y = 0.8x$$

c. Solve the system of equations and confirm the accuracy of your estimate.

$$x \approx 1.82$$
, $y \approx 1.46$ (the exact values are $x = \frac{20}{11}$ and $y = \frac{16}{11}$).

Sample reasoning: Find the x coordinate of the intersection point by solving 6-2.5x=0.8x. To find the y coordinate, substitute this value of x into either equation.