Lesson 15

Using Linear Relations to Solve Problems

Goals

- Describe (orally) limitations of a graphical representation of a situation based on real-world constraints on the quantities.
- Interpret the graph of a linear equation in context, including slope, intercept, and solution, in contexts using multiple representations of non-proportional linear relationships.

Learning Target

I can write linear equations to reason about real-world situations.

Lesson Narrative

In this culminating lesson for the unit, students interpret representations of linear relationships in real-world contexts. They study graphs that represent linear situations, encountering lines with positive, negative, and zero slopes graphed on the same set of axes for the first time. Students work with equations written in both $y = mx + b$ and $Ax + By = C$ formats, identifying situations when one form of equation may provide more contextual information. In this activity, students solidify what it means to be a solution to an equation by recalling how to substitute values in an equation or by looking at whether points lie on or off the line.

Student Learning Goal

Let's write equations for real-world situations and think about their solutions.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Lesson Timeline

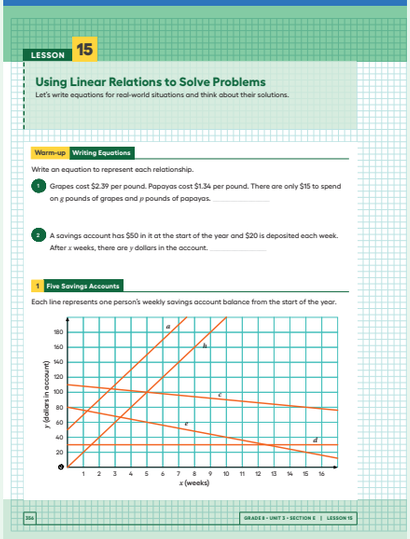
Warm-up	Activity 1	Activity 2	Lesson Synthesis
5 min	20 min	10 min	10 min

Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports. Display sentence frames to support small-group discussion of the question, "What can we say about the points where two lines cross?": "Lines ____ and ____ cross at the point ____ and this tells me that ... I know this because ..."

Advances: Speaking, Conversing

Student Workbook


LESSON 15

Using Linear Relations to Solve Problems

Let's write equations for real-world situations and think about their solutions.

Warm-up Writing Equations

Write an equation to represent each relationship.

1. Grapes cost $2.39 per pound. Papayas cost $1.34 per pound. There are only $15 to spend on g pounds of grapes and p pounds of papayas.

2. A savings account has $50 in it at the start of the year and $20 is deposited each week. After x weeks, there are y dollars in the account.

1 Five Savings Accounts

Each line represents one person's weekly savings account balance from the start of the year.

Warm-up

Writing Equations

5 min

Activity Narrative

In this *Warm-up,* students write equations representing linear relationships. While the relationships can be represented by any form of linear equation, these two situations lend themselves to using equations of the form $y = mx + b$ and $Ax + By = C$. This activity encourages students to think about strategies for writing linear equations.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one situation at a time and ask students to write an equation that represents the relationship. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy for how they came up with their equation.
- Invite students to share their equations and strategies and record and display their responses for all to see.

Student Task Statement

Write an equation to represent each relationship.

1. Grapes cost $2.39 per pound. Papayas cost $1.34 per pound. There are only $15 to spend on g pounds of grapes and p pounds of papayas.

   $2.39g + 1.34p = 15$ (or equivalent)

2. A savings account has $50 in it at the start of the year and $20 is deposited each week. After x weeks, there are y dollars in the account.

   $y = 20x + 50$ (or equivalent)

Activity Synthesis

The purpose of this discussion is for students to hear and explain strategies for writing equations to represent situations. Consider discussing:

- *"How are the equations for the two situations similar? How are they different?"*

  Both equations have two variables. Both equations include numbers used in the descriptions. Both equations describe a linear relationship. One equation has both variables on one side while the other equation has variables on both sides. The slope and vertical intercept are indicated in one of the equations but not the other.

- *"What are some strategies that helped you to write your equations?"*

  Make a table of possible values; find an initial value and a rate of change; compare the situation to similar situations from previous lessons and activities.

- *"Is the slope for each of these equations positive or negative? Why does that make sense with the situation?"*

  For the fruit, the slope is negative, which makes sense because if more of one fruit is bought, less can be bought of the other. For the savings account, the slope is positive, which makes sense because the more weeks go by, the more money will be in the account.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Internalize Self-Regulation.
Provide students an opportunity to self-assess and reflect on their own progress. For example, ask students how confident they are that their description matches the line they chose in the first question.

Supports accessibility for: Organization, Conceptual Processing

Activity 1
Five Savings Accounts

20 min

Activity Narrative

In this activity, students are given an image with five graphs representing changes in five different savings account balances over time. By writing equations to represent each graph, and interpreting what specific points represent for each account, students contextualize their understanding of linear relationships.

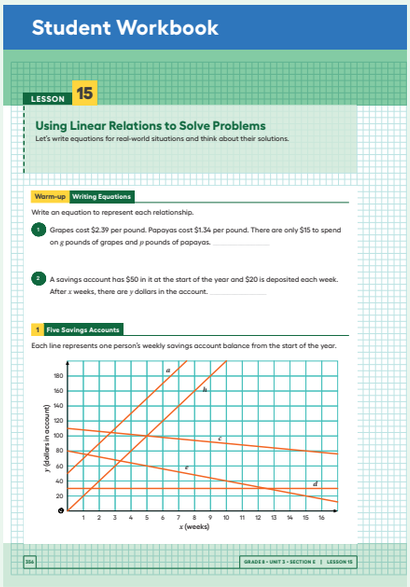
By considering the meaning of non-intersecting lines or the point of intersection between two lines, this activity lays the foundation for thinking about systems of equations in following units.

Launch

Display the image from the task for all to see and explain that each line represents one person's weekly savings account balance from the start of the year. Ask students to consider line a and invite 2–3 students to describe what line a shows in the context of a savings account. If not mentioned by students, explain that this line represents the situation from the *Warm-up* that they wrote an equation for. Instruct students not to choose line a for the first question.

Arrange students in groups of 3–4. Give students 10 minutes of work time followed by a whole-class discussion.

Student Workbook



LESSON 15

Using Linear Relations to Solve Problems

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Warm-up Writing Equations

Write an equation to represent each relationship.

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2. A savings account has $50 in it at the start of the year and $20 is deposited each week. After x weeks, there are y dollars in the account.

1 Five Savings Accounts

Each line represents one person's weekly savings account balance from the start of the year.

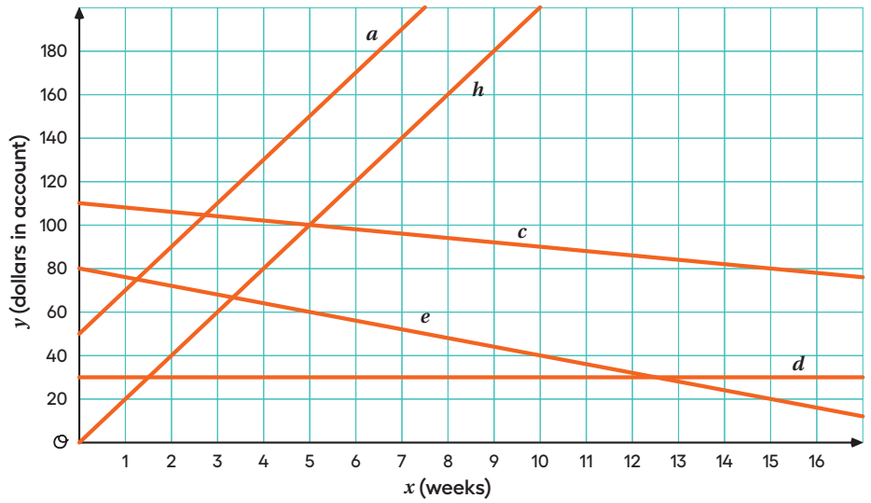
Student Workbook

1 Five Savings Accounts

1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.
2. Share your story with your group and see if anyone can guess your line.
3. Write an equation for each line on the graph.
4. Predict the balance in each account after 20 weeks.
5. For which equation is (5, 100) a solution? What does this solution represent in this situation?

Student Task Statement

Each line represents one person's weekly savings account balance from the start of the year.



1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.

   Sample responses:

   Line c: The account starts with $110 and money is withdrawn at a rate of $2 per week.
   Line d: The account stays at $30 the entire time.
   Line e: The account starts with $80 and is decreasing in value by $4 per week.
   Line h: The account starts with no money and $20 is added each week.

2. Share your story with your group and see if anyone can guess your line.

3. Write an equation for each line on the graph.

   Line a: $y = 20x + 50$ (or equivalent)
   Line c: $y = 110 - 2x$ (or equivalent)
   Line d: $y = 30$ (or equivalent)
   Line e: $y = 80 - 4x$ (or equivalent)
   Line h: $y = 20x$ (or equivalent)

4. Predict the balance in each account after 20 weeks.

   Account a: $450
   Account c: $70
   Account d: $30
   Account e: $0
   Account h: $400

5. For which equation is (5, 100) a solution? What does this solution represent in this situation?

   The point (5, 100) is a solution to the equations represented by line h and line c.

   Sample response: This point means that after 5 weeks, both accounts have $100.

Activity Synthesis

The purpose of this discussion is for students to share the stories they created for each graph. While students will touch on the meaning of the point of intersection between two lines in preparation for future work, the focus of this activity is on the interpretation of slope, vertical intercept, and specific points in context.

Invite students to share their stories for each line. After each story, discuss the following questions:

- *"What part of this story is represented by the slope?"*
- *"Is the slope positive, negative, or zero? Why does this make sense in this situation?"*
- *"What part of this story is represented by the vertical intercept?"*

After a story for each line has been shared, discuss the answers to the last question. Emphasize that (5, 100) represents a point in time when both accounts have the same value, and this point is also a solution to both of the equations represented by each line.

If time allows, ask students to add a line to the graphs in the task for each of the following and then write an equation for the line:

- A savings account that starts with $10 and has $5 added each week
- A situation with a y-intercept of 100 and a slope of -10
- A line parallel to line d
- A line that intersects line c at (10, 90)

Activity 2

Fabulous Fish

10 min

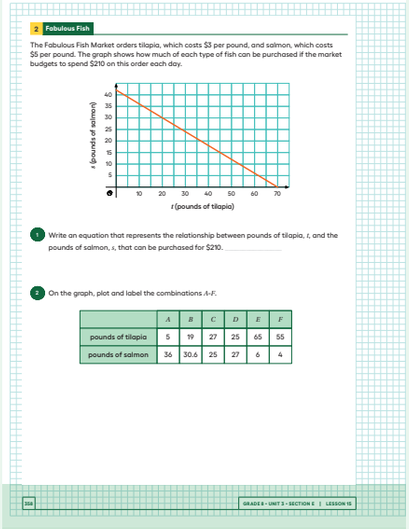
Activity Narrative

In this activity, students write an equation to represent the combinations of fish that a market can order each day. Students plot points and interpret the meaning of points that are or are not on the graph of the line representing the equation.

Launch

Arrange students in groups of 2. Give them 3–4 minutes of quiet work time followed by a whole-class discussion.

Student Workbook


2 Fabulous Fish

The Fabulous Fish Market orders tilapia, which costs $3 per pound, and salmon, which costs $5 per pound. The graph shows how much of each type of fish can be purchased if the market budgets to spend $210 on this order each day.

s (pounds of salmon)
t (pounds of tilapia)

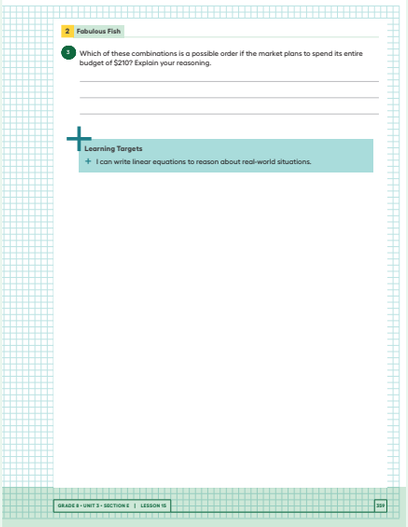
1. Write an equation that represents the relationship between pounds of tilapia, t, and the pounds of salmon, s, that can be purchased for $210.

2. On the graph, plot and label the combinations A–F.

	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

GRADE 8 • UNIT 3 • SECTION E | LESSON 15

Student Workbook


2 Fabulous Fish

3. Which of these combinations is a possible order if the market plans to spend its entire budget of $210? Explain your reasoning.

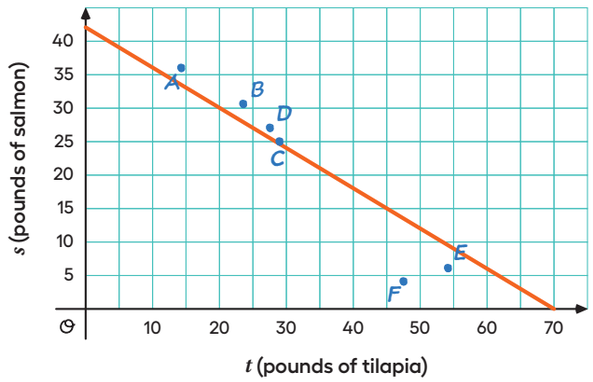
Learning Targets

+ I can write linear equations to reason about real-world situations.

GRADE 8 • UNIT 3 • SECTION E | LESSON 15

Student Task Statement

The Fabulous Fish Market orders tilapia, which costs $3 per pound, and salmon, which costs $5 per pound. The graph shows how much of each type of fish can be purchased if the market budgets to spend $210 on this order each day.



1. Write an equation that represents the relationship between pounds of tilapia, t, and the pounds of salmon, s, that can be purchased for $210.

$3t + 5s = 210$ (or equivalent) or $s = 42 - 0.6t$ (or equivalent)

2. On the graph, plot and label the combinations A–F.

	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

3. Which of these combinations is a possible order if the market plans to spend its entire budget of $210? Explain your reasoning.

Combinations B and D are possible.

Sample reasoning: The points B and D lie on the line so they must be solutions to the equation that represents the line. All the other points are not on the line, so they cannot be solutions.

Activity Synthesis

The purpose of this discussion is to consider what different features of the given graph mean in the context of this situation. Begin by asking students:

- *"Does it make sense for this graph to be only in the first quadrant?"*

  Yes, because purchasing a negative amount of fish is not possible, so having all positive values makes sense.

- *"Does it make sense for this situation to be represented with a line?"*

  Yes, since the fish is priced by weight, a fraction of a pound is reasonable.

- *"What do the intercepts represent in this situation?"*

  The vertical intercept represents the pounds of salmon that can be purchased if no tilapia is ordered. The horizontal intercept represents the pounds of tilapia that can be purchased if no salmon is ordered.

Lesson Synthesis

The goal of this discussion is to review how to determine if a pair of numbers is a solution to an equation. Begin by asking students to brainstorm ideas with a partner. Invite students to share their thinking and record student responses for all to see. Consider collecting student responses in a way that can be displayed in the classroom for upcoming units.

Strategies to emphasize include:

- Substituting the values for the variable they each represent in the equation to calculate if the pair of values makes the equation true
- Graphing the line of the equation and plotting the pair of values as a point to see if the point is on the line