Lesson 5

Solving Any Linear Equation

Goals

- Calculate a value that is a solution to a linear equation in one variable, and explain (orally) the steps used to solve the equation.
- Create an expression to represent a number puzzle, and justify (orally) that it is equivalent to another expression.
- Justify (orally) that each step used in solving a linear equation maintains equality.

Learning Target

I can solve an equation where the variable appears on both sides.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR7: Compare and Connect

Lesson Narrative

The purpose of this lesson is to move toward a general method for solving linear equations. In the *Warm-up*, students solve equations mentally, including equations with negative coefficients, prompting a discussion of multiplying or dividing each side of an equation by a negative number. Next, students encounter several different structures of equations, and take turns suggesting moves for solving them. They apply their growing fluency with solving equations to explain the reasoning behind a numbers puzzle.

As students explain their reasoning for choosing a particular move when solving an equation and listen to the choices of their partner, they must explain and critique their arguments.

Student Learning Goal

Let's solve linear equations.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Math Talk: Equation Solving

5 min

Activity Narrative

This *Math Talk* focuses on solving equations with negative numbers. It encourages students to think about valid moves and to rely on what they know about creating equivalent equations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students solve equations in this lesson.

In describing their strategies, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.

Invite students to share their strategies, and record and display their responses for all to see.

Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A. $5 - x = 8$

$x = -3$

Sample reasoning: I subtracted 5 from each side, then multiplied by -1.

B. $-1 = x - 2$

$x = 1$

Sample reasoning: I added 2 to each side.

C. $-3x = 9$

$x = -3$

Sample reasoning: I know that -3 multiplied by -3 is 9, so x must be -3.

D. $-10 = -5x$

$x = 2$

Sample reasoning: I divided each side by -5.

Instructional Routines

Math Talk

ilclass.com/r10694967

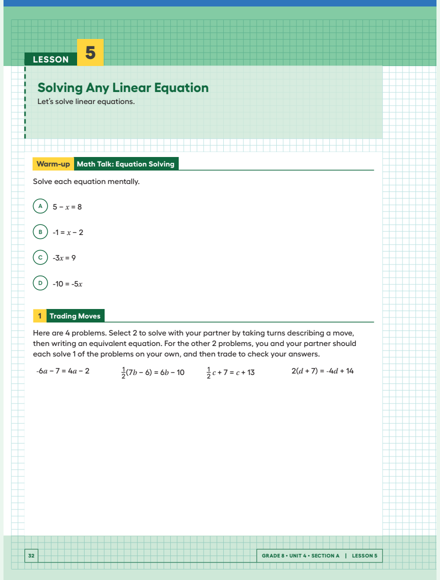
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 5

Solving Any Linear Equation

Let's solve linear equations.

Warm-up Math Talk: Equation Solving

Solve each equation mentally.

A $5 - x = 8$

B $-1 = x - 2$

C $-3x = 9$

D $-10 = -5x$

1 Trading Moves

Here are 4 problems. Select 2 to solve with your partner by taking turns describing a move, then writing an equivalent equation. For the other 2 problems, you and your partner should each solve 1 of the problems on your own, and then trade to check your answers.

$-6a - 7 = 4a - 2$ $\frac{1}{2}(7b - 6) = 6b - 10$ $\frac{1}{2}c + 7 = c + 13$ $2(d + 7) = -4d + 14$

Access for Multilingual Learners (Warm-up, Activity Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Activity Synthesis

To involve more students in the conversation, consider asking as the students share their ideas:

"Who can restate _____ 's reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____ 's strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Some students may reason about the value of x using logic. For example, in $-3x = 9$, the x must be -3 since $-3 \cdot -3 = 9$. Other students may reason about the value of x by changing the value of each side of the equation equally by, for example, dividing each side of $-3x = 9$ by -3 to get the result $x = -3$. Both of these strategies should be highlighted during the discussion where possible.

Activity 1

Trading Moves

15 min

Activity Narrative

The goal of this activity is for students to build fluency solving equations with variables on each side. Students describe each step in their solution process to a partner and justify how each of their changes maintains the equality of the two expressions.

Look for groups solving problems in different, but efficient, ways. For example, one group may distribute the $\frac{1}{2}$ on the left side in Problem 2, while another may multiply each side of the equation by 2 in order to re-write the equation with fewer factors on each side.

Launch

Arrange students in groups of 2. Instruct the class that they will receive 4 cards with problems on them and that their goal is to create a solution to the problems.

For the first two cards they draw, students will alternate solving the equation by stating to their partner the step that they plan to do to each side of the equation—and why—before writing down the step and passing the card. For the final two problem cards, each partner picks one and writes out its solution individually before trading to check each others' work.

To help students understand how they are expected to solve the first two problems, demonstrate the trading process with a student volunteer and a sample equation. Emphasize that the "why" justification should include how their step maintains the equality of the equation. Use MLR3 (*Clarify, Critique, Correct*) by reminding students to push each other to explain how their step guarantees that the equation is still balanced. For example, a student might say that they are combining two terms on one side of the equation, which maintains the equality because the value of that side does not change, only the appearance changes.

Distribute 4 slips from the blackline master to each group. Give time for groups to complete the problems, leaving at least 5 minutes for a whole-class discussion. If any groups finish early, make sure that they have checked their solutions. Then challenge them to find a new solution to one of the problems that uses fewer steps than their first solution used. Conclude with a whole-class discussion.

If time is a concern, give each group 2 cards rather than all 4, and have them do only the trading steps portion of the activity. In that case, make sure that all 4 cards are distributed throughout the class.

Give 6–7 minutes for groups to complete their problems. Make sure that each problem is discussed in a final whole-group discussion. Alternatively, extend the activity by selecting more problems for students to solve with their partners.

Student Task Statement

Here are 4 problems. Select 2 to solve with your partner by taking turns describing a move, then writing an equivalent equation. For the other 2 problems, you and your partner should each solve 1 of the problems on your own, and then trade to check your answers.

$-6a - 7 = 4a - 2$

$a = -\frac{1}{2}$ (or equivalent)

Sample reasoning: Add $6a$ and 2 to both sides, and then divide both sides by 10.

$\frac{1}{2}(7b - 6) = 6b - 10$

$b = 2.8$ (or equivalent)

Sample reasoning: Multiply both sides by 2, add $-7b$ to both sides, add 20 to both sides, and then divide both sides by 5.

$\frac{1}{2}c + 7 = c + 13$

$c = -12$

Sample reasoning: Multiply both sides by 2, and add $-c$ to both sides, and add -26 to both sides.

$2(d + 7) = -4d + 14$

$d = 0$

Sample reasoning: Use the distributive property of the left side, add $4d$ to both sides, add -14 to both sides, and then divide by 6.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Activity Synthesis

The goal of this discussion is for the class to see different, successful ways of solving the same equation. Record and display the student thinking that emerges during the discussion to help the class follow what is being said. To highlight some of the differences in solution paths, ask:

"Did your partner ever make a move different from the one you expected them to? Describe it."

"For the problem with $2(x + 7)$ on the left side, could you start by halving the value of each side? Why might you want to do this?"

Yes, it keeps the numbers smaller and easier to work with.

"How can you check that your solutions are correct?"

Substitute the final value for x into the original equation, and check that the value of each side of the equation is the same.

Activity 2

A Puzzling Puzzle

15 min

Activity Narrative

In this activity, students investigate a number puzzle. After the puzzle is demonstrated, students are asked to figure out how it works and encouraged to create an algebraic representation of the puzzle. The goal of this activity is to build student fluency working with equations with complex structure. This activity also looks ahead to the future work on functions where students will revisit some of these ideas and learn the language of inputs and outputs. More immediately, this activity points to the study of equations that do not have a single answer, which students will learn about in more depth later in this unit.

Monitor for student who use these strategies to write their expressions:

- Rewrite the expression in a simpler way after each instruction.
- Wait until the end to rewrite the expression.

Launch

Tell students to close their books or devices. Ask them to choose a positive number, but *not* to share the number with anyone else. Tell them that they will perform a sequence of operations on their number and then tell you their final answer.

Say each step of Tyler's number puzzle, giving students time to calculate their new number after each step. Select 5–6 students to share their final number, and after each, tell them their original number as quickly as you can.

Pause here, and ask students if they know how you are able to figure out their number so quickly. If no students notice that each number you say is always 6 more than the number given at the end of the steps, you may wish to record and display the pairs of numbers for all students to see, or call on more students so that everyone can hear more pairs of numbers.

When the class agrees that you are able to figure out their original numbers by adding 6 to their final numbers, tell them that the number puzzle is really Tyler's and that their task is to figure out how it works. Tell students to open their books or devices.

Give 3–4 minutes of quiet work time for students to write their explanations, and follow that with a whole-class discussion. Select work from students with different strategies, such as those described in the Activity Narrative, to share later.

Student Task Statement

Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to:

- Triple the number.
- Subtract 7.
- Double the result.
- Subtract 22.
- Divide by 6.

Clare says she now has -3. Tyler says her original number must have been a 3. How did Tyler know that? Follow the same instructions starting with x instead of a number. Explain or show your reasoning for why the last expression means that the person started with a number 6 greater than they ended with.

Sample responses:

x
$3x$
$3x - 7$
$6x - 14$
$6x - 36$
$x - 6$

x
$3x$
$3x - 7$
$2(3x - 7)$
$2(3x - 7) - 22$

$$\frac{2(3x-7)-22}{6} = \frac{6x-14-22}{6} = \frac{6x-36}{6} = x - 6$$

Activity Synthesis

The goal of this discussion is for students to discuss the different approaches to thinking through the number puzzle.

Display 2–3 approaches/representations from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the approaches have in common? How are they different?"

They are equivalent expressions, but one shows all of the instructions while the other is simpler to understand and some of the operations are done.

"Why do the different approaches lead to the same expression?"

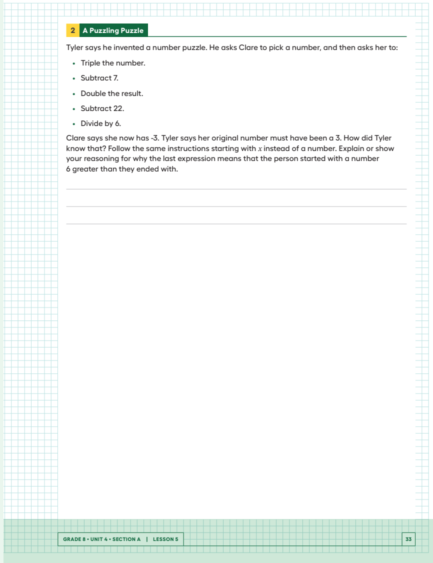
They are equivalent because you can use the distributive property and combine like terms as you go or all at once at the end.

Access for Multilingual Learners (Activity 2, Launch)

Representation: Internalize Comprehension.
Provide a blank two-column table for students to keep track of the moves and their numbers and variables.

Supports accessibility for: Organization, Attention

Student Workbook

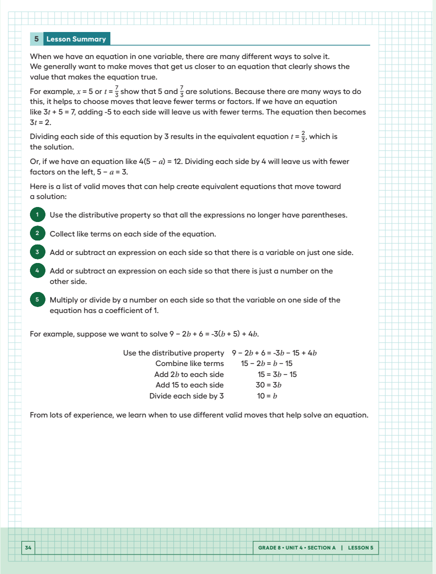

2 A Puzzling Puzzle

Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to:

- Triple the number.
- Subtract 7.
- Double the result.
- Subtract 22.
- Divide by 6.

Clare says she now has -3. Tyler says her original number must have been a 3. How did Tyler know that? Follow the same instructions starting with x instead of a number. Explain or show your reasoning for why the last expression means that the person started with a number 6 greater than they ended with.

Student Workbook



Lesson Synthesis

Give students 2–3 minutes to think about all the equations they solved in today's lesson and to describe to a partner things that they need to be careful about in the future.

Explain to students that some mathematicians describe their moves by writing them next to the new equivalent equation. This makes it easier to both identify wrong moves and to make moves visible to everyone. For example, this pair of equivalent expressions could be written either of these ways.

multiply by $\frac{1}{4}$ $\quad \begin{aligned} 4x &= \frac{1}{2} \\ x &= \frac{1}{8} \end{aligned} \quad$ multiply by $\frac{1}{4}$

$$\begin{array}{lr} & 4x = \frac{1}{2} \\ \text{Multiply each side by } \frac{1}{4} & x = \frac{1}{8} \end{array}$$

Lesson Summary

When we have an equation in one variable, there are many different ways to solve it. We generally want to make moves that get us closer to an equation that clearly shows the value that makes the equation true.

For example, $x = 5$ or $t = \frac{7}{3}$ show that 5 and $\frac{7}{3}$ are solutions. Because there are many ways to do this, it helps to choose moves that leave fewer terms or factors. If we have an equation like $3t + 5 = 7$, adding -5 to each side will leave us with fewer terms. The equation then becomes $3t = 2$.

Dividing each side of this equation by 3 results in the equivalent equation $t = \frac{2}{3}$, which is the solution.

Or, if we have an equation like $4(5 - a) = 12$. Dividing each side by 4 will leave us with fewer factors on the left, $5 - a = 3$.

Here is a list of valid moves that can help create equivalent equations that move toward a solution:

1. Use the distributive property so that all the expressions no longer have parentheses.
2. Collect like terms on each side of the equation.
3. Add or subtract an expression on each side so that there is a variable on just one side.
4. Add or subtract an expression on each side so that there is just a number on the other side.
5. Multiply or divide by a number on each side so that the variable on one side of the equation has a coefficient of 1.

$$\begin{array}{rrcl} \text{For example, suppose we want to solve} & 9 - 2b + 6 &=& -3(b + 5) + 4b. \\ \text{Use the distributive property} & 9 - 2b + 6 &=& -3b - 15 + 4b \\ \text{Combine like terms} & 15 - 2b &=& b - 15 \\ \text{Add } 2b \text{ to each side} & 15 &=& 3b - 15 \\ \text{Add 15 to each side} & 30 &=& 3b \\ \text{Divide each side by 3} & 10 &=& b \end{array}$$

From lots of experience, we learn when to use different valid moves that help solve an equation.

Cool-down

Check It

5 min

Student Task Statement

Noah tries to solve the equation $\frac{1}{2}(7x - 6) = 6x - 10$.

$$\frac{1}{2}(7x - 6) = 6x - 10$$
$$7x - 6 = 12x - 10$$
$$7x = 12x - 4$$
$$-5x = -4$$
$$x = \frac{4}{5}$$

Check Noah's work. If it is not correct, describe what is wrong and show the correct work.

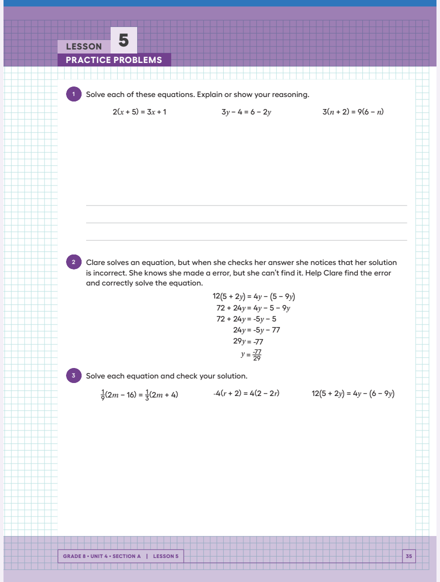
Sample response: Going from line 1 to line 2, Noah tried to multiply each side of the equation by 2, but did not multiply the 10. When you double each side of an equation, each term needs to be multiplied by 2.

$$\frac{1}{2}(7x - 6) = 6x - 10$$
$$7x - 6 = 12x - 20$$
$$7x = 12x - 14$$
$$-5x = -14$$
$$x = \frac{14}{5}$$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1 Solve each of these equations. Explain or show your reasoning.

$2(x+5)=3x+1$ $3y-4=6-2y$ $3(n+2)=9(6-n)$

2 Clare solves an equation, but when she checks her answer she notices that her solution is incorrect. She knows she made a error, but she can't find it. Help Clare find the error and correctly solve the equation.

3 Solve each equation and check your solution.

Lesson **5**

Practice Problems

5 Problems

Problem 1

Solve each of these equations. Explain or show your reasoning.

$2(x + 5) = 3x + 1$

$x = 9$

Sample reasoning: Distribute the coefficient 2 to each term in $(x + 5)$ on the left side, add -1 to each side, and then add $-2x$ to each side.

$3y - 4 = 6 - 2y$

$y = 2$

Sample reasoning: Add $2y$ to each side, then add 4 to each side, and then divide by 5.

$3(n + 2) = 9(6 - n)$

$n = 4$

Sample reasoning: Divide each side by 3, distribute the coefficient 3 on the right side, subtract 2 from each side, add $3n$ to each side, then divide each side by 4.

Problem 2

Clare solves an equation, but when she checks her answer she notices that her solution is incorrect. She knows she made a error, but she can't find it. Help Clare find the error and correctly solve the equation.

$$\begin{aligned} 12(5 + 2y) &= 4y - (5 - 9y) \\ 72 + 24y &= 4y - 5 - 9y \\ 72 + 24y &= -5y - 5 \\ 24y &= -5y - 77 \\ 29y &= -77 \\ y &= \frac{-77}{29} \end{aligned}$$

Sample response: Clare's error occurred in the transition from the 1st line to the 2nd line. She wrote $4y - 9y$ as $4y - 9y$ instead of $4y + 9y$ and $12(5) = 72$ instead of $12(5) = 60$. When corrected, the solution is $y = -\frac{65}{11}$.

Problem 3

Solve each equation and check your solution.

$\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$

$m = -7$

$-4(r + 2) = 4(2 - 2r)$

$r = 4$

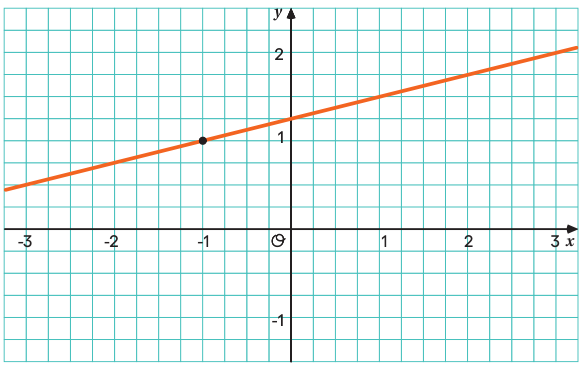
$12(5 + 2y) = 4y - (6 - 9y)$

$y = -6$

Problem 4

from Unit 3, Lesson 14

Here is the graph of a linear equation.



Select **all** true statements about the line and its equation.

- **A.** One solution of the equation is (3, 2).
- **B.** One solution of the equation is (-1, 1).
- **C.** One solution of the equation is $\left(1, \frac{3}{2}\right)$.
- **D.** There are exactly 2 solutions.
- **E.** There are infinitely many solutions.
- **F.** The equation of the line is y = $\frac{1}{4}x + \frac{5}{4}$.
- **G.** The equation of the line is y = $\frac{5}{4}x + \frac{1}{4}$.

Problem 5

from Unit 3, Lesson 9

A participant in a 21-mile walkathon walks at a steady rate of 3 miles per hour. He thinks, "The relationship between the number of miles left to walk and the number of hours I already walked can be represented by a line with slope -3." Do you agree with his claim? Explain your reasoning.

agree

Sample reasoning: The number of miles remaining decreases by 3 for every increase of 1 in the hours walked. Alternatively, points on a graph of remaining miles (y) and hours walked (x) could be (0, 21), (1, 18), (2, 15), (3, 12), etc., so the line slopes down.

Student Workbook