Lesson 9

Part of a Percent

Goals

- Comprehend that percentages do not have to be a whole number.
- Recognize that 0.1% of a number is $\frac{1}{10}$ of 1% of the number.
- Use reasoning about place value to calculate percentages that are not whole numbers, and explain (orally) the strategy.

Learning Targets

- I can find fractional percentages (like 12.5% or 0.4%) of quantities.
- I understand that to find 0.1% of a quantity, I have to multiply by 0.001.

Lesson Narrative

In this lesson, students consider how to calculate a fractional percentage using a whole number percentage as a reference. For example, if they know that 1% of 200 is 2, they can use the structure of the base-ten system to reason that 0.1% of 200 is 0.2, and 0.01% of 200 is 0.02.

First, students use place value and properties of operations to calculate percentages without a context. Next, they apply this reasoning about fractional percentages to situations involving percent increase. This prepares students for working with more complex contexts, such as interest rates, taxes, tips, and measurement error.

Student Learning Goal

Let's explore percentages smaller than 1%.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

- 5 Practices
- Math Talk
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Math Talk: What Percentage?

5 min

Activity Narrative

This *Math Talk* focuses on dividing by 100. It encourages students to think about place value and to rely on properties of operations to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students use decimals to represent percentages that are not whole numbers.

As students use results from the previous expression to help evaluate the next expression, they are making use of repeated reasoning.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A. 14 ÷ 100

0.14

Sample reasoning: 0.14 is $\frac{1}{100}$ of 14 because the decimal point is two places to the left.

B. 7 ÷ 100

0.07

Sample reasoning: 7 is $\frac{1}{2}$ of 14, and 0.07 is $\frac{1}{2}$ of 0.14.

C. 3.5 ÷ 100

0.035

Sample reasoning: 3.5 is $\frac{1}{2}$ of 7, and 0.035 is $\frac{1}{2}$ of 0.07.

D. 103.5 ÷ 100

1.035

Sample reasoning: 1.035 is $\frac{1}{100}$ of 103.5 because the decimal point is two places to the left.

Inspire Math

Sport Success video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

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Instructional Routines

Math Talk

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Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time, and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization, Attention

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

The key takeaway is to remind students that numbers can have more than two decimal places. Up to this point, students have mostly worked mostly with percentages that are whole numbers, such as 14% and 7%. Representing these percentages as decimals always gives values with two decimals places, like 0.14 and 0.07. This could lead students to overgeneralize and think that 3.5% would be represented as 0.35. Focusing on the relative size of 3.5% compared to 7% can help students see why the decimal representation is actually 0.035.

Activity 1

Fractions of a Percent

15 min

Activity Narrative

In this activity, students reason about percentages that are not whole numbers. Students use what they know about place value and properties of operations to calculate fractional percentages of numbers, making use of structure. Similar to a *Math Talk*, the problems are sequenced so that students can use the result of the previous problem to help answer the next problem.

Monitor for students who:

- Use 1% to find 0.1%.
- Make substitutions with known quantities to help compute unknown quantities.
- Use the result of the previous problem to help answer the next problem.

Launch

Give students 1–2 minutes of quiet work time for the first problem.

Ask students to pause their work and share what they noticed about these expressions and their answers. A key takeaway is that students can use the result of the previous problem to help them reason about the next expression. For example, 3% is $\frac{1}{10}$ of 30%, and $\frac{1}{10}$ of 18 is 1.8. Avoid using the terminology "moving the decimal."

Give students 4–5 minutes of quiet work time to finish the rest of the problems.

Student Task Statement

1. Find each percentage of 60. What do you notice about your answers?

30% of 60	3% of 60	0.3% of 60	0.03% of 60
18	1.8	0.18	0.018

I notice that each percentage is $\frac{1}{10}$ of the previous percentage.

2. 20% of 5,000 is 1,000, and 21% of 5,000 is 1,050. Find each percentage of 5,000, and be prepared to explain your reasoning. If you get stuck, consider using the double number line diagram.

a. 1% of 5,000 50

Sample reasoning: 21% − 20% = 1%, and 1,050 − 1,000 = 50

b. 0.1% of 5,000 5

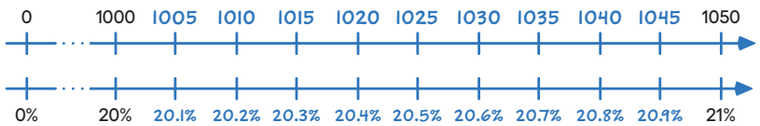
Sample reasoning: 5 is $\frac{1}{10}$ of 50

c. 20.1% of 5,000 1,005

Sample reasoning: 1,000 + 5 = 1,005

d. 20.4% of 5,000 1,020

Sample reasoning: 1,000 + 4 • 5 = 1,020



3. 15% of 80 is 12, and 16% of 80 is 12.8. Find each percentage of 80, and explain your reasoning.

a. 15.1% of 80

12.08

Sample reasoning: One percent of 80 is 0.8 because 16% − 15% = 1% and 12.8 − 12 = 0.8. So 0.1% of 80 is 0.08 because $\frac{1}{10} \cdot 0.8 = 0.08$. Then we add 15% of 80 to 0.1% of 80, which is 12 + 0.08.

b. 15.7% of 80

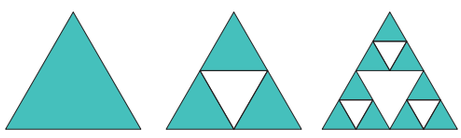
12.56

Sample reasoning: 12 + 7 • 0.08 = 12.56

Are You Ready for More?

To make Sierpinski's triangle,

- Start with an equilateral triangle. This is step 1.
- Connect the midpoints of every side, and remove the middle triangle, leaving three smaller triangles. This is step 2.
- Do the same to each of the remaining triangles. This is step 3.
- Keep repeating this process.



Building on Student Thinking

When students calculate the various percentages of 60, they may make mistakes in the place value of the answers. Refer students to the previous activity's discussion. You may also want to ask students to calculate 10% of 60 and use that answer to calculate 30%.

If students get stuck calculating various percents of 5,000, recommend that they use the double number line provided. Ask them:

"What percentages are visible in the bottom number line?"
"How much is that 1% in reference to the top number line?"
"How can we use that 1% to figure out the other percentages?"

Use these same questions if students get stuck calculating 15.1% and 15.7%.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question, about 15.7% of 80. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Instructional Routines

5 Practices

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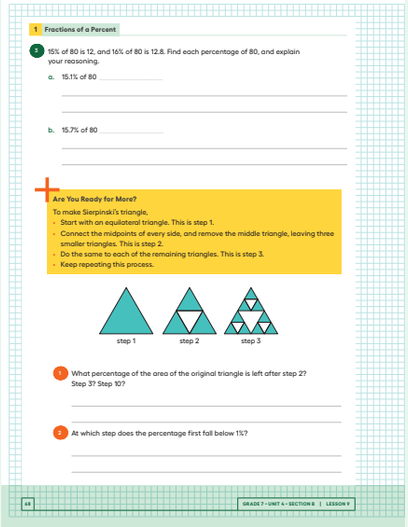
Student Workbook


1 Fractions of a Percent

3 15% of 80 is 12, and 16% of 80 is 12.8. Find each percentage of 80, and explain your reasoning.

a. 15.1% of 80

b. 15.7% of 80

Are You Ready for More?

To make Sierpinski's triangle,
- Start with an equilateral triangle. This is step 1.
- Connect the midpoints of every side, and remove the middle triangle, leaving three smaller triangles. This is step 2.
- Do the same to each of the remaining triangles. This is step 3.
- Keep repeating this process.

step 1 step 2 step 3

1 What percentage of the area of the original triangle is left after step 2? Step 3? Step 10?

2 At which step does the percentage first fall below 1%?

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Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.

Encourage students to begin partner discussions by reading their written responses for the last question aloud. If time allows, invite students to revise or add to their responses based on the conversation that follows.

Advances: Conversing, Speaking

1. What percentage of the area of the original triangle is left after step 2? Step 3? Step 10?

   Step 2: 75%; Step 3: 56.25%; Step 10: about 7.51%

2. At which step does the percentage first fall below 1%?

   Step 18

Activity Synthesis

Invite students to share how they reasoned about the problems. Highlight strategies by students who recognized that they can use 1% of a number to calculate 0.1% of a number and make multiples of that to get, for example, 0.7% of a number.

Activity 2

Population Growth

15 min

Activity Narrative

In this activity, students find the new value after a percent increase that involves a fractional percentage. The first problem asks about an 8% increase, and the second problem asks about a 0.8% increase, so students can use the relationship between these two percentages to help solve the second problem.

Monitor for students who:

- Calculate the percentage first and then add that to the original amount.
- Multiply the original amounts by 1.08 and 1.008, respectively.

Plan to have students present in this order to support moving them from less efficient to more efficient methods.

As students explain why either city could be considered to have grown by more, they are attending to precision.

Launch

Arrange students in groups of 2.

Give students 4 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

1. The population of City A was approximately 243,000 people. It increased by 8% in one year. What was the new population?

   approximately 262,000 (1.08 · 243,000 ≈ 262,000)

2. The population of City B was approximately 7,150,000. It increased by 0.8% in one year. What was the new population?

   approximately 7,210,000 (1.008 · 7,150,000 ≈ 7,210,000)

3. Lin says that City A grew by more. Andre says that City B grew by more. Do you agree with either of them? Explain your reasoning.

   Sample responses:

   Lin and Andre are both correct.

   - Thinking about the growth in terms of addition, City B grew by more because 57,200 is more than 19,440.
   - Thinking about the growth in terms of multiplication, City A grew by more because 8% is greater than 0.8%.

Activity Synthesis

The purpose of this discussion is to compare and contrast the 8% increase with the 0.8% increase. First, invite previously selected students to share how they calculated the new populations for each city. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions, such as:

- *"How are these strategies the same? How are they different?"*
- *"How does the 8% increase (or 0.8% increase) show up in each method?"*
- *"Why do the different approaches lead to the same outcome?"*

The key takeaway is that the same strategies work for finding 0.8% as for 8%. We can multiply first and then add, or we can add first and then multiply. The multiplier is just different by a factor of 10 (0.008 instead of 0.08).

Next, invite students to share their reasoning about which city grew by more. Either city could be the correct answer depending on whether students are viewing the growth additively or multiplicatively. Make sure that some students support each perspective. If needed, assign some students to support Lin's statement and others to support Andre's statement. The goal is for students to articulate more precisely what is meant by the phrase "grew by more" in each interpretation. To involve more students in the conversation, consider asking:

- *"Do you agree or disagree? Why?"*
- *"Who can restate _____'s reasoning in a different way?"*
- *"Does anyone want to add on to _____'s reasoning"*

Building on Student Thinking

Students who want to multiply by $1 + \frac{p}{100}$ may have trouble determining where to put the decimal. Have them think about the problem in steps.

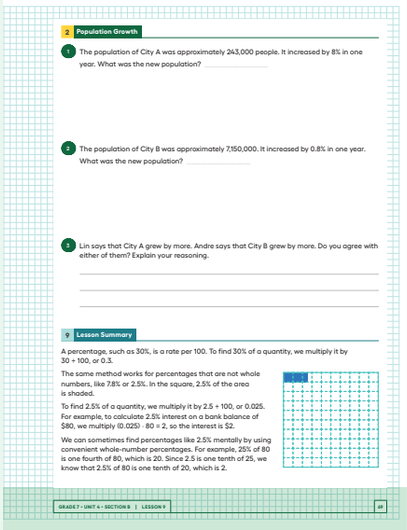
How can you find 8%?

Multiply by 0.08.

How can you find 0.8%?

Multiply by 0.008.

Student Workbook



2 Population Growth

1. The population of City A was approximately 243,000 people. It increased by 8% in one year. What was the new population?

2. The population of City B was approximately 7,150,000. It increased by 0.8% in one year. What was the new population?

3. Lin says that City A grew by more. Andre says that City B grew by more. Do you agree with either of them? Explain your reasoning.

9 Lesson Summary

A percentage, such as 30%, is a rate per 100. To find 30% of a quantity, we multiply it by 30 ÷ 100, or 0.3.

The same method works for percentages that are not whole numbers, like 7.8% or 2.5%. In the square, 2.5% of the area is shaded.

To find 2.5% of a quantity, we multiply it by 2.5 ÷ 100, or 0.025. For example, to calculate 2.5% interest on a bank balance of $80, we multiply (0.025) · 80 = 2, so the interest is $2.

We can sometimes find percentages like 2.5% mentally by using convenient whole-number percentages. For example, 25% of 80 is one fourth of 80, which is 20. Since 2.5 is one tenth of 25, we know that 2.5% of 80 is one tenth of 20, which is 2.

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Responding To Student Thinking

Points to Emphasize

If students struggle with percentages that are not whole numbers, plan to review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the percentages in this activity:

Grade 7, Unit 4, Lesson 10, Activity 3 Dining at a Restaurant

Lesson Synthesis

Share with students,

- *"Today we worked with percentages that were not whole numbers."*

To review the reasoning developed in this lesson, consider asking students:

- *"How are these percentages related to each other: 40%, 4%, 0.4%, 0.04%?"*

  Each percentage is $\frac{1}{10}$ of the previous one.

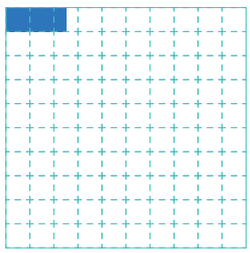
- *"How can we use 40% of a number to help calculate the other percentages?"*

  4% of that number will be $\frac{1}{10}$ of what 40% of that number was. 0.4% of that number will be $\frac{1}{100}$, and so on.

- *"If we know 1% of a number, how can we use that to help us calculate 0.5% of a number?"*

  0.5% of a number is half of 1% of that number.

Lesson Summary



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We can sometimes find percentages like 2.5% mentally by using convenient whole-number percentages. For example, 25% of 80 is one fourth of 80, which is 20. Since 2.5 is one tenth of 25, we know that 2.5% of 80 is one tenth of 20, which is 2.

Cool-down

Percentages of 750

5 min

Student Task Statement

A school has 750 students.

1. If the number increases by 4%, how many students will there be? Explain or show your reasoning.

   780 students

   Sample reasoning: 750 · 1.04 = 780

2. If the number increases by 0.4%, how many students will there be? Explain or show your reasoning.

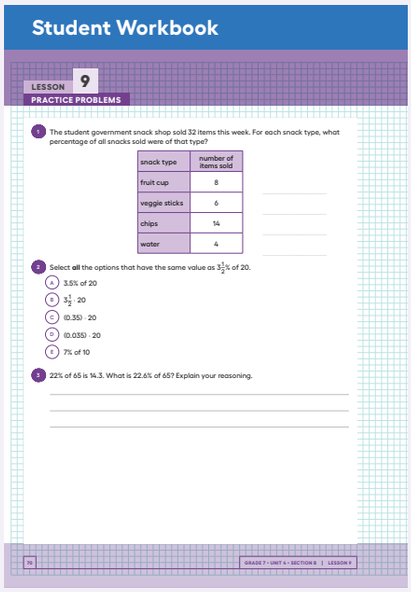
   753 students

   Sample reasoning: 750 · 1.004 = 753

Lesson 9

Practice Problems

6 Problems



Problem 1

The student government snack shop sold 32 items this week. For each snack type, what percentage of all snacks sold were of that type?

snack type	number of items sold
fruit cup	8
veggie sticks	6
chips	14
water	4

Fruit cup: 25%, veggie sticks: 18.75%, chips: 43.75%, water: 12.5%

Problem 2

Select **all** the options that have the same value as $3\frac{1}{2}$% of 20.

A. 3.5% of 20

B. $3\frac{1}{2} \cdot 20$

C. $(0.35) \cdot 20$

D. $(0.035) \cdot 20$

E. 7% of 10

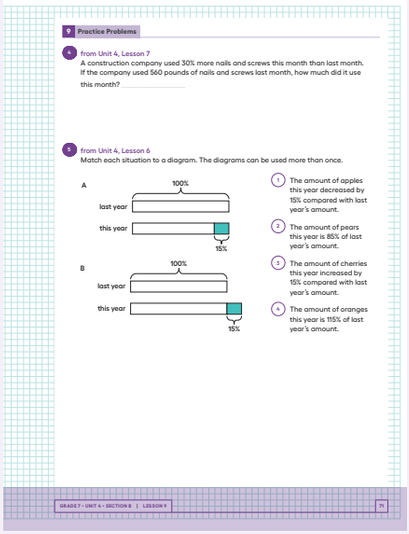
Problem 3

22% of 65 is 14.3. What is 22.6% of 65? Explain your reasoning.

14.69

Sample reasoning: 22.6% of 65 is 22% of 65 (or 14.3) and an additional 0.6% of 65. Since we know 1% of 65 is 0.65, and 0.1% of 65 is 0.065, then 0.6% of 65 is $6 \cdot (0.065) = 0.39$. So 22.6% of 65 is 14.69, because $14.3 + 0.39 = 14.69$.

Student Workbook


9 Practice Problems

4 from Unit 4, Lesson 7
A construction company used 30% more nails and screws this month than last month. If the company used 560 pounds of nails and screws last month, how much did it use this month?

5 from Unit 4, Lesson 6
Match each situation to a diagram. The diagrams can be used more than once.

A
100%
last year
this year
15%

B
100%
last year
this year
15%

1. The amount of apples this year decreased by 15% compared with last year's amount.
2. The amount of pears this year is 85% of last year's amount.
3. The amount of cherries this year increased by 15% compared with last year's amount.
4. The amount of oranges this year is 115% of last year's amount.

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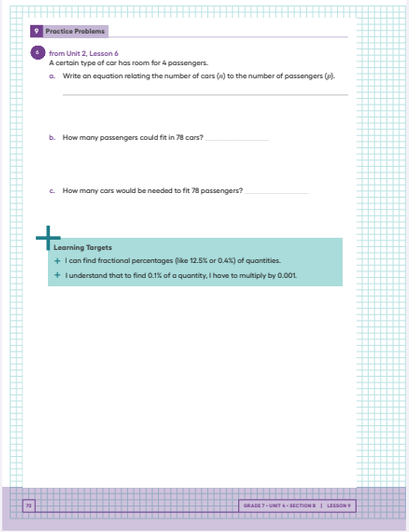
Student Workbook


9 Practice Problems

6 from Unit 2, Lesson 6
A certain type of car has room for 4 passengers.
a. Write an equation relating the number of cars (n) to the number of passengers (p).

b. How many passengers could fit in 78 cars?

c. How many cars would be needed to fit 78 passengers?

Learning Targets
+ I can find fractional percentages (like 12.5% or 0.4%) of quantities.
+ I understand that to find 0.1% of a quantity, I have to multiply by 0.001.

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Problem 4

from Unit 4, Lesson 7

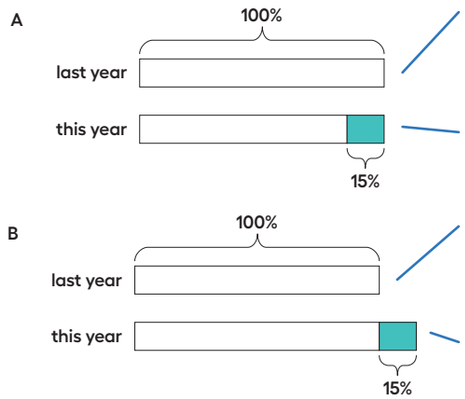
A construction company used 30% more nails and screws this month than last month. If the company used 560 pounds of nails and screws last month, how much did it use this month?

728 pounds

Problem 5

from Unit 4, Lesson 6

Match each situation to a diagram. The diagrams can be used more than once.



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2. The amount of pears this year is 85% of last year's amount.
3. The amount of cherries this year increased by 15% compared with last year's amount.
4. The amount of oranges this year is 115% of last year's amount.

Problem 6

from Unit 2, Lesson 6

A certain type of car has room for 4 passengers.

a. Write an equation relating the number of cars (n) to the number of passengers (p).

$p = 4n$

b. How many passengers could fit in 78 cars?

312 passengers, because $4 \cdot 78 = 312$

c. How many cars would be needed to fit 78 passengers?

20 cars, because $78 \div 4 = 19.5$ and we can't use half of a car