Lesson 3

Reasoning about Equations with Tape Diagrams

Goals

- Coordinate tape diagrams and equations of the form $px + q = r$ or $p(x + q) = r$.
- Create a tape diagram to represent an equation of the form $px + q = r$ or $p(x + q) = r$, and use it to solve the equation.
- Identify equivalent equations, and justify (using words and other representations) that they are equivalent.

Learning Targets

- I can match equations and tape diagrams that represent the same situation.
- If I have an equation, I can draw a tape diagram that shows the same relationship.

Lesson Narrative

The purpose of this lesson is to make connections between a tape diagram and an equation of the form $px + q = r$ or $p(x + q) = r$. Students match tape diagrams to corresponding equations and sort them into categories. Then they draw tape diagrams to represent equations. Students use the tape diagram and the equation to reason about a solution, but this is not the time to teach particular methods for solving such equations. At this point in the unit, students should reason using any method that makes sense to them.

Student Learning Goal

Let's see how equations can describe tape diagrams.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2 Collect and Display (Activity 1)
- MLR1 (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Take Turns

Required Preparation

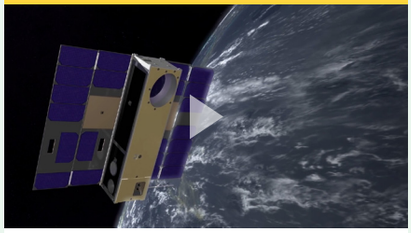
Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	10 min	10 min	10 min	5 min

Inspire Math

CubeSats video

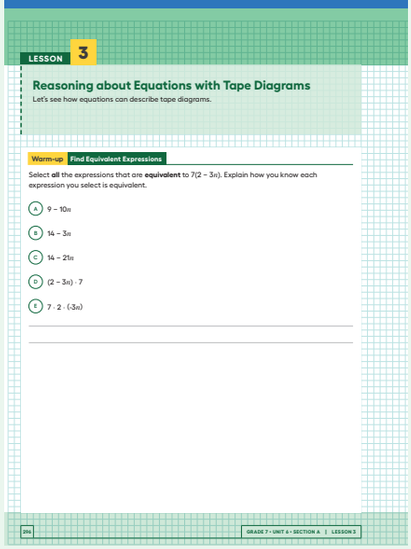


Go Online
Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614167

Please log in to the site before using the QR code or URL.

Student Workbook



LESSON 3

Reasoning about Equations with Tape Diagrams

Let's see how equations can describe tape diagrams.

Warm-up Find Equivalent Expressions

Select **all** the expressions that are **equivalent** to $7(2 - 3n)$. Explain how you know each expression you select is equivalent.

A. $9 - 10n$

B. $14 - 3n$

C. $14 - 21n$

D. $(2 - 3n) \cdot 7$

E. $7 \cdot 2 \cdot (-3n)$

Warm-up

Find Equivalent Expressions

10 min

Activity Narrative

In this activity, students remember what the distributive property is all about before they will be expected to use it in the process of solving equations of the form $p(x + q) = r$ later in this unit. If this activity indicates that students remember little of the distributive property from grade 6, heavier interventions may be needed.

Look for students who:

- Rule out expressions by testing values.
- Use the term *distributive property*.

In order to explain how they know each selected expression is equivalent, students need to attend to precision in the language used.

Launch

Ask students to think of anything they know about equivalent expressions. Ask if they can:

- Explain why $2x$ and $2 + x$ are not equivalent. (These expressions are equal when x is 2, but not equal for other values of x. Multiplying 2 by a number usually gives a different result than adding that number to 2.)
- Explain why $3 + x$ and $x + 3$ are equivalent. (These expressions are equal no matter the value of x. Also, addition is commutative.)
- Describe ways to decide whether expressions are equivalent. (Test some values, draw diagrams for different values, analyze them for properties of the operations involved.)

Arrange students in groups of 2.

Give 3 minutes of quiet work time and then invite students to share their responses with their partner, followed by a whole-class discussion.

Student Task Statement

Select **all** the expressions that are **equivalent** to $7(2 - 3n)$. Explain how you know each expression you select is equivalent.

A. $9 - 10n$

B. $14 - 3n$

C. $14 - 21n$

D. $(2 - 3n) \cdot 7$

E. $7 \cdot 2 \cdot (-3n)$

$14 - 21n$ is equivalent because of the distributive property.
$(2 - 3n) \cdot 7$ is equivalent because multiplication is commutative.

Activity Synthesis

The purpose of this discussion is to revisit ways of telling whether two expressions are equivalent, and to recall the distributive property.

Select a student who tested values to explain how they know two expressions are not equivalent. For example, $9 - 10n$ is not equivalent to $7(2 - 3n)$, because if we use 0 in place of n, $9 - 10 \cdot 0$ is 9 but $7(2 - 3 \cdot n)$ is 14. If no one brings this up, demonstrate an example.

Select a student who used the term "distributive property" to explain why $7(2 - 3n)$ is equivalent to $14 - 21n$ and ask them to explain what they mean by that term. In general, an expression of the form $a(b + c)$ is equivalent to $ab + ac$.

Activity 1

Matching Equations to Tape Diagrams

10 min

Activity Narrative

In this partner activity, students take turns matching a tape diagram to an equation. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Arrange students in groups of 2. Display the task for all to see. Tell students that for each tape diagram, there is an equation that it represents. If time allows, choose a student to be your partner and demonstrate how to set up and do the activity, otherwise share these steps:

- One partner picks a tape diagram.
- They identify an equation that the diagram represents, and explain why they think it is a match.
- The other partner listens and makes sure they agree with the match and the reasoning.
- If they don't agree, the partners discuss until they come to an agreement.
- For the next tape diagram, the students swap roles.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Building on Student Thinking

If students don't know where to begin, encourage them to describe the diagrams and equations in words. For example, Diagram D could be described as "two groups of $x + 5$ equal 19," and so could the equation $2(x + 5) = 19$.

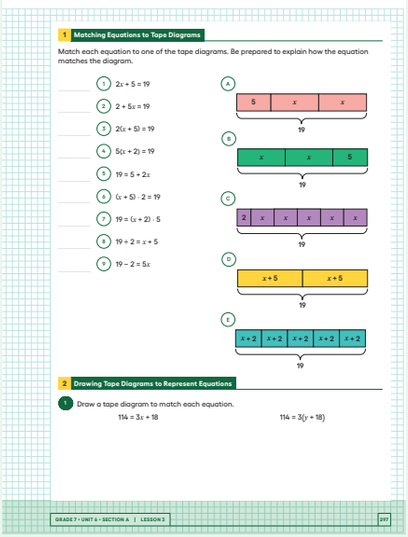
Access for Multilingual Learners (Activity 1, Task Statement)

MLR2: Collect and Display.

Circulate to listen for and collect the language that students use as they explain how each equation matches a diagram. On a visible display, record words and phrases, such as "two groups of x + 5 equal 19," along with the corresponding diagram or equation. Invite students to borrow language from the display as needed and update it throughout the lesson.

Advances: Conversing, Reading

Student Workbook


1 Matching Equations to Tape Diagrams

Match each equation to one of the tape diagrams. Be prepared to explain how the equation matches the diagram.

1. $2x + 5 = 19$
2. $2 + 5x = 19$
3. $2(x + 5) = 19$
4. $5(x + 2) = 19$
5. $19 = 5 + 2x$
6. $(x + 5) \cdot 2 = 19$
7. $19 = (x + 2) \cdot 5$
8. $19 \div 2 = x + 5$
9. $19 - 2 = 5x$

2 Drawing Tape Diagrams to Represent Equations

1. Draw a tape diagram to match each equation.

$114 = 3x + 18$ $114 = 3(y + 18)$

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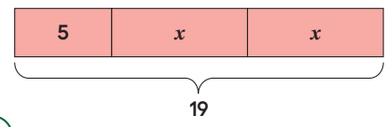
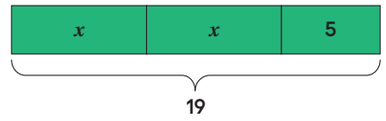
Student Task Statement

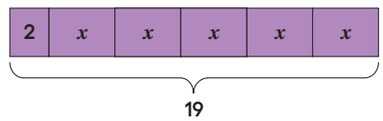
A



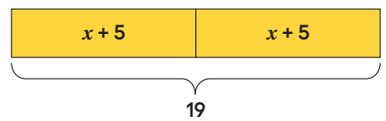
B



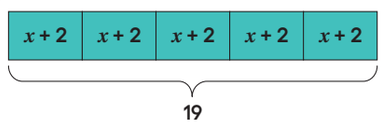
C



D



E



Match each equation to one of the tape diagrams. Be prepared to explain how the equation matches the diagram.

B or A **1.** $2x + 5 = 19$

C **2.** $2 + 5x = 19$

D **3.** $2(x + 5) = 19$

E **4.** $5(x + 2) = 19$

A or B **5.** $19 = 5 + 2x$

D **6.** $(x + 5) \cdot 2 = 19$

E **7.** $19 = (x + 2) \cdot 5$

D **8.** $19 \div 2 = x + 5$

C **9.** $19 - 2 = 5x$

Activity Synthesis

Once all groups have completed the matching, discuss the following:

"Which matches were tricky? Explain why."

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

If students express uncertainty about $2x + 5 = 19$, spend some time here. Some students are likely to match it to exactly one diagram and some students match it to both A and B. The point isn't that only one of these is right; it is to have the conversation about the idea of expressions or equations being identical vs. equivalent. Equivalent expressions or equivalent equations can have different literal interpretations, but when matching equations to tape diagrams, all that matters for the purposes of solving is that the equations are equivalent.

Activity 2

Drawing Tape Diagrams to Represent Equations

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students create a tape diagram after interpreting an equation. Students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language they use to describe their thinking.

In the digital version of the activity, students use an applet to create their tape diagrams. The applet allows students to divide the tape into parts and label the parts and total. The digital version may be helpful for students who struggle with drawing their own diagrams. However, even if the class is using the digital version, some students may prefer to draw the diagrams in their notebooks or on scratch paper.

Launch

Give students 5 minutes of quiet work time, followed by sharing their diagrams and methods for finding x and y with their partner. Follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, to highlight the meaning of coefficients in the equations, annotate the number of groups of repeated terms in the tape diagrams.

Supports accessibility for: Visual-Spatial Processing

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479

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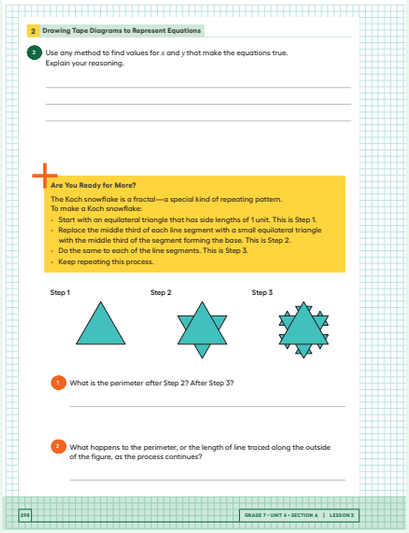
Access for Students with Diverse Abilities (Activity 2, Task Statement)

Representation: Develop Language and Symbols.

Provide students with access to a blank template of a tape diagram labeling the different parts with generalizations for what content will go inside. In addition, consider using a previous example situation or equation to make connections to the blank template.

Supports accessibility for: Language, Memory

Student Workbook


2 Drawing Tape Diagrams to Represent Equations

2 Use any method to find values for x and y that make the equations true. Explain your reasoning.

Are You Ready for More?

The Koch snowflake is a fractal—a special kind of repeating pattern. To make a Koch snowflake:

- Start with an equilateral triangle that has side lengths of 1 unit. This is Step 1.
- Replace the middle third of each line segment with a small equilateral triangle with the middle third of the segment forming the base. This is Step 2.
- Do the same to each of the line segments. This is Step 3.
- Keep repeating this process.

Step 1 Step 2 Step 3

1 What is the perimeter after Step 2? After Step 3?

2 What happens to the perimeter, or the length of line traced along the outside of the figure, as the process continues?

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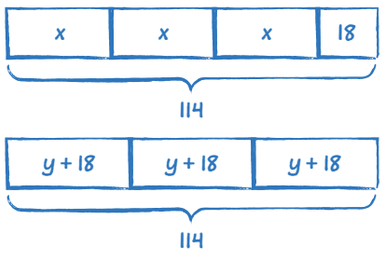
Student Task Statement

1. Draw a tape diagram to match each equation.

$114 = 3x + 18$ $114 = 3(y + 18)$

Sample responses:



2. Use any method to find values for x and y that make the equations true. Explain your reasoning.

$x = 32$

Sample reasoning: The three x's together equal 114 – 18, or 96, and 96 divided into 3 groups is 32.

$y = 20$

Sample reasoning: 114 is divided into 3 equal parts, so each part is 38. Each part is represented by $y + 18$, so y must be 20.

Are You Ready for More?

The Koch snowflake is a fractal—a special kind of repeating pattern. To make a Koch snowflake:

- Start with an equilateral triangle that has side lengths of 1 unit. This is Step 1.
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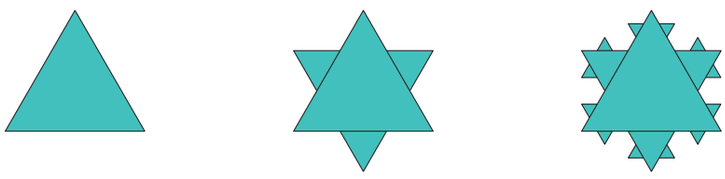
Step 1 Step 2 Step 3



1. What is the perimeter after Step 2? After Step 3?

4 units, $5\frac{1}{3}$ units

2. What happens to the perimeter, or the length of line traced along the outside of the figure, as the process continues?

Sample response: The perimeter increases as the process continues.

Activity Synthesis

The purpose of this discussion is for students to describe how their diagrams represent the two equations. Invite students to share their diagrams and explain their reasoning. If possible, display the students' diagrams for all to see. To involve more students in the conversation, consider asking:

- *"Who can restate_____'s reasoning in a different way?"*
- *"Do you agree or disagree? Why?"*
- *"What does this part of the diagram represent?"*
- *"What is the same about these diagrams? What is different?"*
- *"How else could we show this relationship?"*

Draw students' attention to the structural differences in the diagrams that correspond to the presence or absence of parentheses in the equation.

Lesson Synthesis

Display one or more tape diagrams students encountered or created during the lesson, along with their corresponding equations. Ask,

- *"What are some ways that tape diagrams represent equations?"*

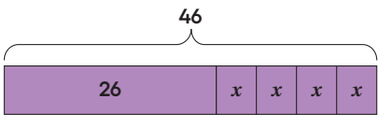
Responses to highlight:

- Multiplication in the equation is represented with multiple copies of the same piece in the diagram.
- The total amount is shown in both the equation and the diagram.
- An unknown amount is represented with a variable.
- Either the equation or the diagram can be used to reason about a solution to the equation.

Lesson Summary

We have seen how tape diagrams represent relationships between quantities. Because of the meaning and properties of addition and multiplication, more than one equation can often be used to represent a single tape diagram.

Let's take a look at two tape diagrams.



We can represent this diagram with several different equations. Here are some of them:

- $26 + 4x = 46$, because the parts add up to the whole.
- $4x + 26 = 46$, because addition is commutative.
- $46 = 4x + 26$, because if two quantities are equal, it doesn't matter how we arrange them around the equal sign.

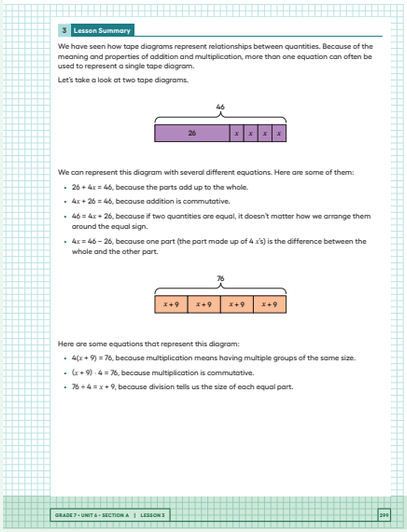
Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to how they found the values of x and y that make the equations true. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Workbook


3 Lesson Summary

We have seen how tape diagrams represent relationships between quantities. Because of the meaning and properties of addition and multiplication, more than one equation can often be used to represent a single tape diagram.

Let's take a look at two tape diagrams.

46
26 x x x x

We can represent this diagram with several different equations. Here are some of them:

- 26 + 4x = 46, because the parts add up to the whole.
- 4x + 26 = 46, because addition is commutative.
- 46 = 4x + 26, because if two quantities are equal, it doesn't matter how we arrange them around the equal sign.
- 4x = 46 − 26, because one part (the part made up of 4 x's) is the difference between the whole and the other part.

76
x + 9 x + 9 x + 9 x + 9

Here are some equations that represent this diagram:

- 4(x + 9) = 76, because multiplication means having multiple groups of the same size.
- (x + 9) · 4 = 76, because multiplication is commutative.
- 76 ÷ 4 = x + 9, because division tells us the size of each equal part.

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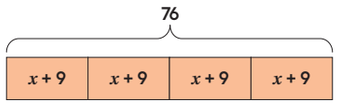
Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

- $4x = 46 - 26$, because one part (the part made up of 4 x's) is the difference between the whole and the other part.



Here are some equations that represent this diagram:

- $4(x + 9) = 76$, because multiplication means having multiple groups of the same size.
- $(x + 9) \cdot 4 = 76$, because multiplication is commutative.
- $76 \div 4 = x + 9$, because division tells us the size of each equal part.

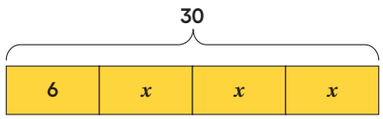
Cool-down

Three of These Equations Belong Together

5 min

Student Task Statement

Here is a diagram.



1. Which equation matches the diagram?

A. $6 + 3x = 30$

B. $6x + 3 = 30$

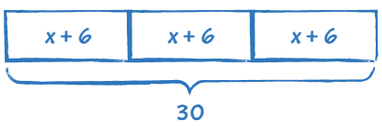
C. $3x = 30 + 6$

D. $30 = 3x - 6$

E. $6 + 3x = 30$

2. Draw a diagram that matches the equation $3(x + 6) = 30$.

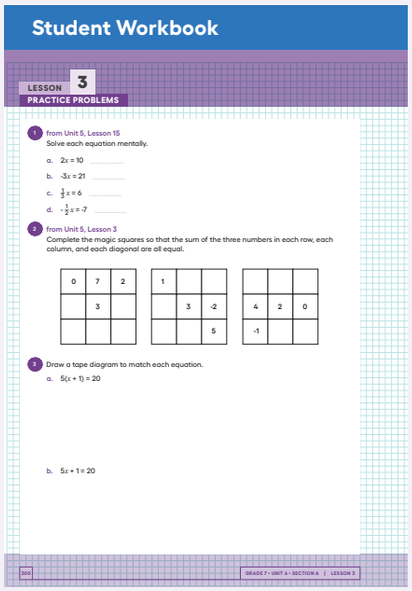
Sample response:



Lesson 3

Practice Problems

5 Problems



Problem 1

from Unit 5, Lesson 15

Solve each equation mentally.

a. $2x = 10$

5

b. $-3x = 21$

-7

c. $\frac{1}{3}x = 6$

18

d. $-\frac{1}{2}x = -7$

14

Problem 2

from Unit 5, Lesson 3

Complete the magic squares so that the sum of the three numbers in each row, each column, and each diagonal are all equal.

0	7	2
5	3	1
4	-1	6

1	2	6
8	3	-2
0	4	5

3	-2	5
4	2	0
-1	6	1

Problem 3

Draw a tape diagram to match each equation.

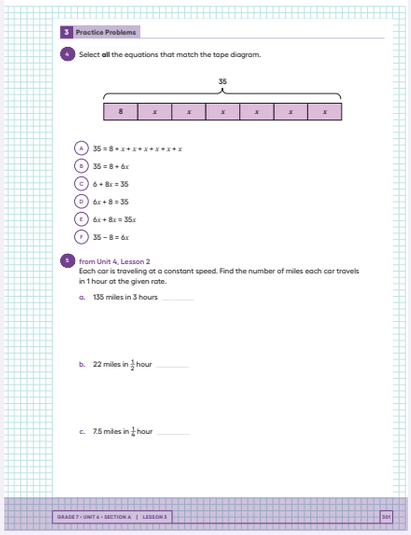
a. $5(x + 1) = 20$

A diagram showing 5 equal parts of $x + 1$ for a total of 20

b. $5x + 1 = 20$

A diagram showing 5 equal parts of x and one part of 1 for a total of 20

Student Workbook


3 Practice Problems

4. Select **all** the equations that match the tape diagram.

35

8 | x | x | x | x | x | x

A. 35 = 8 + x + x + x + x + x + x
B. 35 = 8 + 6x
C. 6 + 8x = 35
D. 6x + 8 = 35
E. 6x + 8x = 35x
F. 35 − 8 = 6x

5. from Unit 4, Lesson 2
Each car is traveling at a constant speed. Find the number of miles each car travels in 1 hour at the given rate.

a. 135 miles in 3 hours _____

b. 22 miles in $\frac{1}{2}$ hour _____

c. 7.5 miles in $\frac{1}{4}$ hour _____

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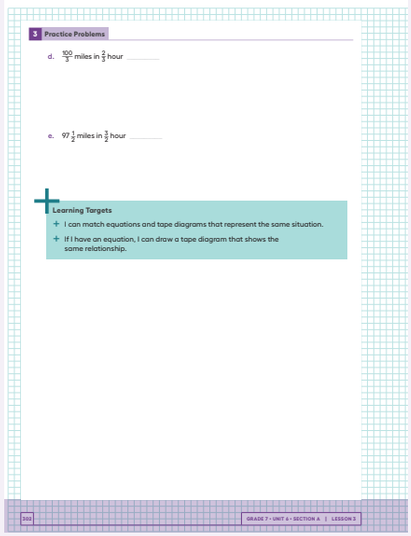
Student Workbook


3 Practice Problems

d. $\frac{100}{3}$ miles in $\frac{2}{3}$ hour _____

e. $97\frac{1}{2}$ miles in $\frac{3}{2}$ hour _____

Learning Targets
+ I can match equations and tape diagrams that represent the same situation.
+ If I have an equation, I can draw a tape diagram that shows the same relationship.

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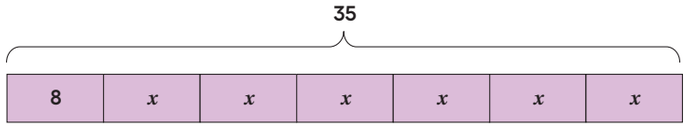
Problem 4

Select **all** the equations that match the tape diagram.



A. $35 = 8 + x + x + x + x + x + x$

B. $35 = 8 + 6x$

C. $6 + 8x = 35$

D. $6x + 8 = 35$

E. $6x + 8x = 35x$

F. $35 - 8 = 6x$

Problem 5

from Unit 4, Lesson 2

Each car is traveling at a constant speed. Find the number of miles each car travels in 1 hour at the given rate.

a. 135 miles in 3 hours

45 miles

b. 22 miles in $\frac{1}{2}$ hour

44 miles

c. 7.5 miles in $\frac{1}{4}$ hour

30 miles

d. $\frac{100}{3}$ miles in $\frac{2}{3}$ hour

50 miles

e. $97\frac{1}{2}$ miles in $\frac{3}{2}$ hour

65 miles