Lesson 5

Defining Equivalent Ratios

Goals

- Generate equivalent ratios and justify that they are equivalent.
- Present (in words and through other representations) a definition of equivalent ratios, including examples and non-examples.

Learning Targets

- If I have a ratio, I can create a new ratio that is equivalent to it.
- If I have two ratios, I can decide whether they are equivalent to each other.

Lesson Narrative

In this lesson, students work with **equivalent ratios** more abstractly, both in the context of recipes and by using numbers without a context. They learn and articulate that all ratios that are equivalent to $a:b$ can be generated by multiplying both a and b by the same number.

As they connect concrete quantitative experiences to abstract representations, students develop their skills in reasoning abstractly and quantitatively. They continue to use diagrams, words, or a combination of both for their explanations.

Student Learning Goal

Let's investigate equivalent ratios some more.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1, Activity 2)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

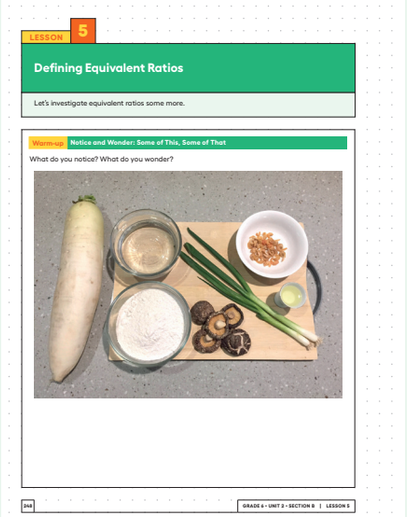
Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	15 min	15 min	10 min	5 min
				Cool-down

Student Workbook


LESSON 5

Defining Equivalent Ratios

Let's investigate equivalent ratios some more.

Warm-up Notice and Wonder: Some of This, Some of That

What do you notice? What do you wonder?

248 GRADE 6 • UNIT 2 • SECTION B | LESSON 5

Warm-up

Notice & Wonder: Some of This, Some of That

5 min

Activity Narrative

This *Warm-up* familiarizes students with a new context that they will encounter later in the lesson. It allows them to make sense of a situation and the mathematics that might be involved before solving problems. While students may notice and wonder many things about the image, they should recognize that the ingredients are likely parts of a recipe and the amounts are specified by the recipe.

Launch

Arrange students in groups of 2. Display the image for all to see.

Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing that they notice and one thing that they wonder.

Give students another minute to discuss their observations and questions.

Student Task Statement

What do you notice? What do you wonder?

Students may notice:

- These look like cooking ingredients.
- There are 5 mushrooms and 2 types of vegetables.
- There are some dry ingredients and some wet ones.
- One of the items looks like a root vegetable.
- There is a bowl of water and a small amount of oil.

Students may wonder:

- What dish or dishes are the ingredients for?
- What is the white vegetable on the left? How will it be prepared?
- What are the ingredients in the bowls?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

- *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Activity 1

Radish Cake

15 min

Activity Narrative

In this activity, students use a realistic food recipe to find equivalent ratios that represent different numbers of batches. Students use the original recipe to form ratios of ingredients that represent double, half, five times, and one-fifth of the recipe. Then, they examine given ratios of ingredients and determine how many batches the amounts represent.

Launch

Ask students to tell a partner about their favorite snack and its taste. (Is it savory, sweet, sour, spicy, or a mix?)

Introduce a traditional Chinese dish called *luo bo gao* (luoh-boe-gow, in Mandarin) or *lo bak go* (law-buhk-gaw, in Cantonese), which translates to radish cake or Chinese turnip cake. Explain that it is a popular savory dish enjoyed as an everyday snack and as an auspicious treat during Lunar New Year. The term "gao" or "go" is translated to "cake" in English. Though "cake" typically refers to a sweet baked good, the original term can refer to something sweet or savory.

The image in the *Warm-up* shows some of the ingredients of radish cake, the main two being daikon radish and rice flour. The dish originated in southern China where both radishes and rice are plentiful, but it is now enjoyed around the world.

Invite students to briefly share some treats that their family or community enjoy, either regularly or during special occasions. If anyone wishes to share what they know about radish cake, allow them to do so. Otherwise, consider reading the given cooking instructions with students or showing a video clip of the process.

Explain to students that they will think about the ratios of the ingredients and adjust them for making different numbers of batches of radish cake.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the corresponding quantities in the recipe and in the questions.

Supports accessibility for: Visual-Spatial Processing

Access for Multilingual Learners (Activity 1, Launch)

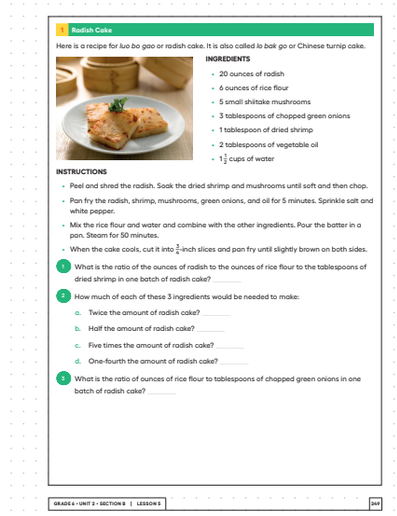
MLR8: Discussion Supports.
To help students understand ratios in the context of recipes and ingredients, act out or use images that demonstrate the meaning of the terms "ratio," "recipe," and "ingredients" in the context of cooking. Demonstrate combining specific ingredients in their stated ratios. This will help students connect the language found in a recipe with the ratio reasoning needed for different batches of that recipe.

Advances: Listening

Building on Student Thinking

Students who are not yet fluent in fraction multiplication from grade 5 may have difficulty understanding how to find half or one-fourth of the recipe ingredient amounts. Likewise, they may have difficulty identifying one-third of a batch. Suggest that they draw a picture of $\frac{1}{2}$ of 10, remind them that finding $\frac{1}{2}$ of a number is the same as dividing it by 2, or remind them that $\frac{1}{2}$ of a number means $\frac{1}{2}$ times that number.

Student Workbook



1 Radish Cake

Here is a recipe for *luo bo gao* or radish cake. It is also called *lo bak go* or Chinese turnip cake.

INGREDIENTS
- 20 ounces of radish
- 6 ounces of rice flour
- 5 small shiitake mushrooms
- 3 tablespoons of chopped green onions
- 1 tablespoon of dried shrimp
- 2 tablespoons of vegetable oil
- $1\frac{1}{2}$ cups of water

INSTRUCTIONS
- Peel and shred the radish. Soak the dried shrimp and mushrooms until soft and then chop.
- Pan fry the radish, shrimp, mushrooms, green onions, and oil for 5 minutes. Sprinkle salt and white pepper.
- Mix the rice flour and water and combine with the other ingredients. Pour the batter in a pan. Steam for 50 minutes.
- When the cake cools, cut it into $\frac{3}{4}$-inch slices and pan fry until slightly brown on both sides.

1. What is the ratio of the ounces of radish to the ounces of rice flour to the tablespoons of dried shrimp in one batch of radish cake? _____
2. How much of each of these 3 ingredients would be needed to make:
   a. Twice the amount of radish cake? _____
   b. Half the amount of radish cake? _____
   c. Five times the amount of radish cake? _____
   d. One-fourth the amount of radish cake? _____
3. What is the ratio of ounces of rice flour to tablespoons of chopped green onions in one batch of radish cake? _____

GRADE 6 • UNIT 2 • SECTION B | LESSON 5 2nd

Student Task Statement

Here is a recipe for *luo bo gao* or radish cake. It is also called *lo bak go* or Chinese turnip cake.

INGREDIENTS
- 20 ounces of radish
- 6 ounces of rice flour
- 5 small shiitake mushrooms
- 3 tablespoons of chopped green onions
- 1 tablespoon of dried shrimp
- 2 tablespoons of vegetable oil
- $1\frac{1}{2}$ cups of water

INSTRUCTIONS
- Peel and shred the radish. Soak the dried shrimp and mushrooms until soft and then chop.
- Pan fry the radish, shrimp, mushrooms, green onions, and oil for 5 minutes. Sprinkle salt and white pepper.
- Mix the rice flour and water and combine with the other ingredients. Pour the batter in a pan. Steam for 50 minutes.
- When the cake cools, cut it into $\frac{3}{4}$-inch slices and pan fry until slightly brown on both sides.

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1. What is the ratio of the ounces of radish to the ounces of rice flour to the tablespoons of dried shrimp in one batch of radish cake?

   20:6:1

2. How much of each of these 3 ingredients would be needed to make:

   a. Twice the amount of radish cake?

      40 ounces, 12 ounces, 2 tablespoons

   b. Half the amount of radish cake?

      10 ounces, 3 ounces, $\frac{1}{2}$ tablespoon

   c. Five times the amount of radish cake?

      100 ounces, 30 ounces, 5 tablespoons

   d. One-fourth the amount of radish cake?

      5 ounces, $1\frac{1}{2}$ ounces, $\frac{1}{4}$ tablespoon

3. What is the ratio of ounces of rice flour to tablespoons of chopped green onions in one batch of radish cake?

   6:3

4. How many batches of radish cake would you make if you used the following amounts of ingredients?

a. 18 ounces of rice flour and 9 tablespoons of green onions

3 batches

b. 36 ounces of rice flour and 18 tablespoons of green onions

6 batches

c. 2 ounces of rice flour and 1 tablespoon of green onions

$\frac{1}{3}$ batch

Activity Synthesis

Display the recipe for all to see. Ask students to share and explain their responses. List their responses—along with the specified number of batches—for all to see.

Ask students to analyze the list and describe how the ratio of quantities relates to the number of batches in each case. Elicit the idea that each quantity within the recipe was *multiplied* by a number to obtain each batch size, and that each ingredient amount is multiplied by the *same* value.

In finding one-half and one-fourth of a batch, students may speak in terms of dividing by 2 and dividing by 4. Point out that "dividing by 2" has the same outcome as "multiplying by one-half," and "dividing by 4" has the same outcome as "multiplying by one-fourth." Later, we will want to state our general definition of equivalent ratios as simply as possible: as *multiplying* both a and b in the ratio $a:b$ by the same number (not "multiplying or dividing").

Activity 2

What Are Equivalent Ratios?

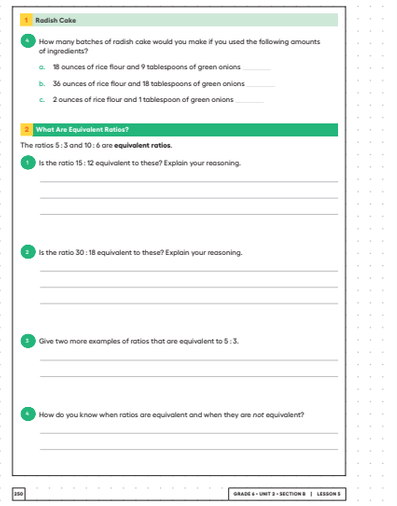
15 min

Activity Narrative

In this activity, students identify what equivalent ratios have in common, generate equivalent ratios, and write a definition for "equivalent ratios." Through repeated reasoning, students recognize that a ratio equivalent to $a:b$ can be generated by multiplying both a and b by the same number. As they prepare and present their best definition of equivalent ratios, examples of equivalent ratios, and an explanation of how they know that the examples are equivalent, students practice attending to precision.

This activity uses the *Compare and Connect* math language routine to support mathematically precise language in discussion.

Student Workbook



1 Radish Cake

4. How many batches of radish cake would you make if you used the following amounts of ingredients?
   a. 18 ounces of rice flour and 9 tablespoons of green onions _____
   b. 36 ounces of rice flour and 18 tablespoons of green onions _____
   c. 2 ounces of rice flour and 1 tablespoon of green onions _____

2 What Are Equivalent Ratios?

The ratios 5 : 3 and 10 : 6 are **equivalent ratios.**

1. Is the ratio 15 : 12 equivalent to these? Explain your reasoning.
2. Is the ratio 30 : 18 equivalent to these? Explain your reasoning.
3. Give two more examples of ratios that are equivalent to 5 : 3.
4. How do you know when ratios are equivalent and when they are *not* equivalent?

250 GRADE 6 • UNIT 2 • SECTION B | LESSON 5

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports. Display sentence frames to support students in explaining how they know when two ratios are or are not equivalent:

"I know that the ratio _____ : _____ is equivalent to $_____ : _____$ because ..."

"To find out if two ratios are equivalent, I would look at ... and check ..."

"I know that two ratios are (or are not) equivalent when ..."

Advances: Speaking, Writing

Building on Student Thinking

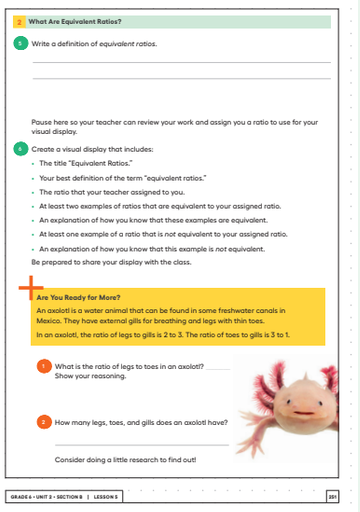
Students may incorporate recipes, specific examples, or batch thinking into their definitions. These are important ways of thinking about equivalent ratios, but challenge them to come up with a definition that talks only about the numbers involved and not about what the numbers represent.

If groups struggle to get started thinking generally about a definition, give them a head start with:

"A ratio is equivalent to a:b when ..."

If students include "or divide" in their definition, remind them that, for example, dividing by 5 gives the same result as multiplying by one-fifth. Therefore, we can use only the term "multiply" in our definition.

Student Workbook



2 What Are Equivalent Ratios?

5 Write a definition of *equivalent ratios*.

Pause here so your teacher can review your work and assign you a ratio to use for your visual display.

6 Create a visual display that includes:

- The title "Equivalent Ratios."
- Your best definition of the term "equivalent ratios."
- The ratio that your teacher assigned to you.
- At least two examples of ratios that are equivalent to your assigned ratio.
- An explanation of how you know that these examples are equivalent.
- At least one example of a ratio that is *not* equivalent to your assigned ratio.
- An explanation of how you know that this example is *not* equivalent.

Be prepared to share your display with the class.

Are You Ready for More?

An axolotl is a water animal that can be found in some freshwater canals in Mexico. They have external gills for breathing and legs with thin toes.

In an axolotl, the ratio of legs to gills is 2 to 3. The ratio of toes to gills is 3 to 1.

1 What is the ratio of legs to toes in an axolotl? Show your reasoning.

2 How many legs, toes, and gills does an axolotl have?

Consider doing a little research to find out!

GRADE 6 • UNIT 2 • SECTION B | LESSON 5 251

Launch

Summarize what we know so far about equivalent ratios:

- When we double or triple a color recipe, the ratios of the amount of ingredients in the mixtures are equivalent to those in the original recipe. For example, 24:9 and 8:3 are equivalent ratios, because we can think of 24:9 as a mixture that contains three batches of purple water where a single batch is 8:3.
- When we make multiple batches of a food recipe, we say the ratios of the amounts of the ingredients are equivalent to the ratios in a single batch. For example, the ratios 6:3, 2:1, and 18:9 are equivalent because they correspond to the amount of the ingredients in different numbers of batches of radish cake, and they all taste the same.

Tell students that in this activity they will write a definition for **equivalent ratios**.

Arrange students in groups of 3–4. Provide each group with tools for creating a visual display. Give students 3–4 minutes to answer the first five questions and ask them to pause for a brief whole-class discussion afterward. Next, assign each group a different ratio to use as an example in their explanation of equivalent ratios. Some possibilities:

4:5 3:2 5:6 3:4 2:5

Student Task Statement

The ratios 5:3 and 10:6 are **equivalent ratios**.

1. Is the ratio 15:12 equivalent to these? Explain your reasoning.

   15:12 is not equivalent to 5:3 because 15 is $5 \cdot 3$ but 12 is $3 \cdot 4$.

2. Is the ratio 30:18 equivalent to these? Explain your reasoning.

   30:18 is equivalent to 5:3 because 30 is $5 \cdot 6$ and 18 is $3 \cdot 6$.

3. Give two more examples of ratios that are equivalent to 5:3.

   Sample responses: 15:9, 20:12, and 50:30.

4. How do you know when ratios are equivalent and when they are *not* equivalent?

   Sample response: I know two ratios are equivalent if I multiply both parts of one ratio by the same number and get the other ratio.

5. Write a definition of *equivalent ratios*.

   Sample response: A ratio is equivalent to $a:b$ when both a and b are multiplied by the same number.

   Pause here so your teacher can review your work and assign you a ratio to use for your visual display.

6. Create a visual display that includes:

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- An explanation of how you know that these examples are equivalent.
- At least one example of a ratio that is *not* equivalent to your assigned ratio.
- An explanation of how you know that this example is *not* equivalent.

Be prepared to share your display with the class.

Answers vary.

Are You Ready for More?

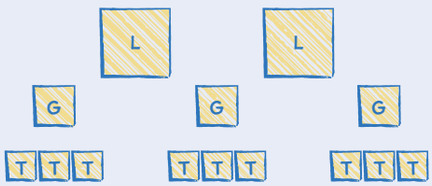
An axolotl is a water animal that can be found in some freshwater canals in Mexico. They have external gills for breathing and legs with thin toes.

In an axolotl, the ratio of legs to gills is 2 to 3. The ratio of toes to gills is 3 to 1.

1. What is the ratio of legs to toes in an axolotl? Show your reasoning.

2 to 9

Sample reasoning:



2. How many legs, toes, and gills does an axolotl have? Consider doing a little research to find out!

4 legs, 6 gills, and 18 toes

Activity Synthesis

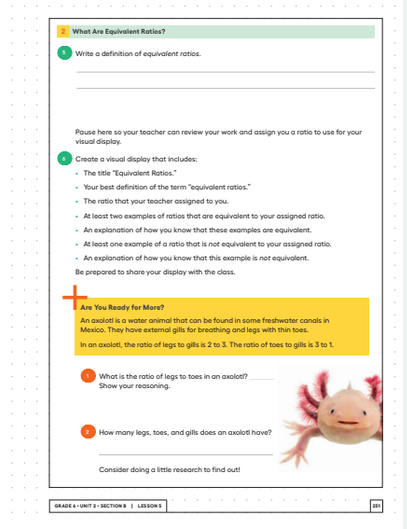
Invite each group to share their visual display and explain their definition. After all groups have presented their work, use *Compare and Connect* to help students compare, contrast, and connect the definitions of equivalent ratios and explanations about whether pairs of ratios are equivalent. Highlight phrases or explanations that are similar in each display. Ask questions such as:

- *"What do these definitions have in common?"*
- *"What details or language helped to make certain explanations clearer to you?"*
- *"What additions or adjustments might clarify these definitions (or explanations) even more?"*

Then, make one class display that incorporates all valid definitions. This display should be kept posted in the classroom for the remaining lessons within this unit. It should look something like:

Equivalent Ratio: A ratio is equivalent to $a:b$ when both a and b are multiplied by the same number.

Student Workbook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- An explanation of how you know that this example is *not* equivalent.

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Are You Ready for More?

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1 What is the ratio of legs to toes in an axolotl? Show your reasoning.

2 How many legs, toes, and gills does an axolotl have?

Consider doing a little research to find out!

GRADE 6 • UNIT 2 • SECTION B | LESSON 5 201

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of equivalent ratios. If students struggle to identify or create equivalent ratios, make time to revisit the work in the past few lessons. For example, do the optional activity and the practice problems in the lesson referred to here. Unit 2, Lesson 4, Activity 3 Perfect Purple Water Unit 2, Lesson 5 Defining Equivalent Ratios

Lesson Synthesis

In this lesson, students came to an understanding of what equivalent ratios are. Consider asking:

- *"If you want to make a different amount of a food recipe, how can you ensure that the resulting food will taste the same?"*

  Each ingredient in the recipe must be multiplied by the same value.

- *"How do we know if two ratios are equivalent?"*

  Multiplying the numbers in one ratio by the same value gives us the numbers in the other ratio.

- *"How can we generate an equivalent ratio?"*

  Multiply each number in the ratio by the same value.

Lesson Summary

All ratios that are **equivalent** to $a:b$ can be made by multiplying both a and b by the same number.

For example, the ratio 18:12 is equivalent to 9:6 because both 9 and 6 are multiplied by the same number: 2.

3:2 is also equivalent to 9:6, because both 9 and 6 are multiplied by the same number: $\frac{1}{3}$.

Is 18:15 equivalent to 9:6?

No, because 18 is 9 · 2, but 15 is *not* 6 · 2.

Cool-down

Why Are They Equivalent?

5 min

Student Task Statement

1. Write another ratio that is equivalent to the ratio 4:6.

   Sample responses:

   - 2:3
   - 16:24
   - 400:600

2. How do you know that your new ratio is equivalent to 4:6? Explain or show your reasoning.

   Sample responses:

   - 2:3 is equivalent to 4:6 because both 4 and 6 are multiplied by $\frac{1}{2}$.
   - 16:24 is equivalent to 4:6 because both 4 and 6 are multiplied by 4.
   - 400:600 is equivalent to 4:6 because both 4 and 6 are multiplied by 100.

Lesson 5

Practice Problems

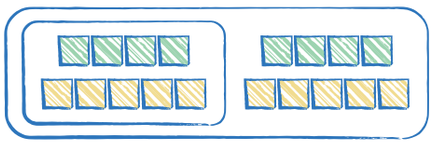
6 Problems

Problem 1

Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a diagram that shows why they are equivalent ratios.

a. 4 : 5 and 8 : 10

Sample response: The diagram shows that 8 to 10 is the same as 2 groups of 4 to 5 so these are equivalent ratios.



b. 18 : 3 and 6 : 1

$18 \cdot \frac{1}{3} = 6$ and $3 \cdot \frac{1}{3} = 1$

c. 2 : 7 and 10,000 : 35,000

$2 \cdot (5{,}000) = 10{,}000$ and $7 \cdot (5{,}000) = 35{,}000$

Problem 2

Explain why 6 : 4 and 18 : 8 are not equivalent ratios.

Sample response: 6:4 is not equivalent to 18 : 8 because 18 is $6 \cdot 3$, but 8 is not $4 \cdot 3$.

Problem 3

Are the ratios 3 : 6 and 6 : 3 equivalent? Why or why not?

No, they are not equivalent.

Sample reasoning: The ratio 3:6 represents 3 of one type of object for every 6 of another type of object, while the ratio 6:3 represents 6 of the first type of object for every 3 of the second type of object.

Problem 4

from Unit 2, Lesson 4

This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.

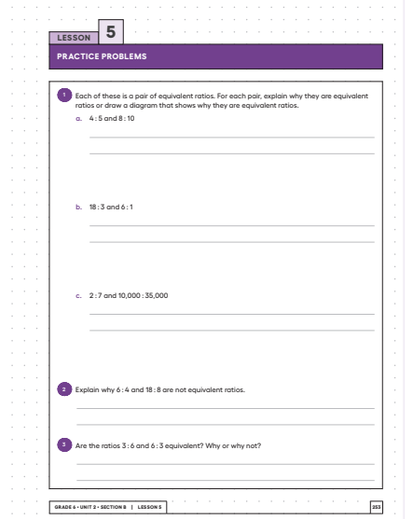
white paint (cups) □□□□□□□□□

yellow paint (cups) ■■■■■■■■■■■■■■■

white paint (cups) □□□

yellow paint (cups) ■■■■■

Student Workbook



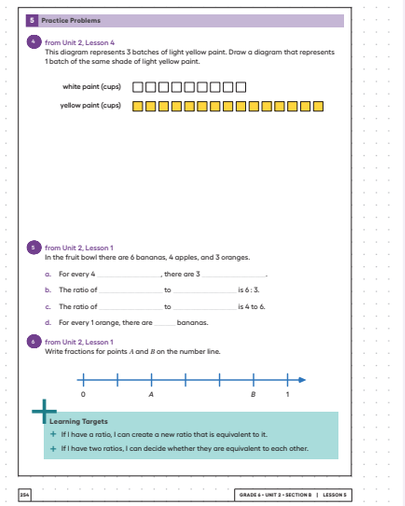
LESSON 5

PRACTICE PROBLEMS

1. Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a diagram that shows why they are equivalent ratios.
   a. 4 : 5 and 8 : 10
   b. 18 : 3 and 6 : 1
   c. 2 : 7 and 10,000 : 35,000
2. Explain why 6 : 4 and 18 : 8 are not equivalent ratios.
3. Are the ratios 3 : 6 and 6 : 3 equivalent? Why or why not?

GRADE 6 • UNIT 2 • SECTION B | LESSON 5 203

Student Workbook



5 Practice Problems

4 from Unit 2, Lesson 4
This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.

white paint (cups) □□□□□□□□□

yellow paint (cups) ■■■■■■■■■■■■■■■

5 from Unit 2, Lesson 1
In the fruit bowl there are 6 bananas, 4 apples, and 3 oranges.
a. For every 4 _____, there are 3 _____.
b. The ratio of _____ to _____ is 6 : 3.
c. The ratio of _____ to _____ is 4 to 6.
d. For every 1 orange, there are _____ bananas.

6 from Unit 2, Lesson 1
Write fractions for points A and B on the number line.

0 A B 1

Learning Targets
+ If I have a ratio, I can create a new ratio that is equivalent to it.
+ If I have two ratios, I can decide whether they are equivalent to each other.

204 GRADE 6 • UNIT 2 • SECTION B | LESSON 5

Problem 5

from Unit 2, Lesson 1

In the fruit bowl there are 6 bananas, 4 apples, and 3 oranges.

a. For every 4 apples, there are 3 oranges.

b. The ratio of bananas to oranges is 6:3.

c. The ratio of apples to bananas is 4 to 6.

d. For every 1 orange, there are 2 bananas.

Problem 6

from Unit 2, Lesson 1

Write fractions for points A and B on the number line.

$A = \frac{2}{6}$ or $\frac{1}{3}$

$B = \frac{5}{6}$