Lesson 3

Grid Moves

Goals

- Describe (orally) the moves needed to perform a translation, rotation, or reflection.
- Draw and label the image of figures that result from translations, rotations, and reflections on a square or isometric grid.

Learning Targets

- I can decide which type of transformations will work to move one figure to another.
- I can use grids to carry out transformations of figures.

Student Learning Goal

Let's transform some figures on grids.

Lesson Narrative

Prior to this lesson, students have learned the names for the basic moves (translation, rotation, and reflection) and have learned how to identify them in pictures. In this lesson, they apply translations, rotations, and reflections to figures. They also label the image of a point P as P'. While not essential, this practice helps show the structural relationship between a figure and its image.

Students also encounter the isometric grid (one made of equilateral triangles with 6 meeting at each vertex). They perform translations, rotations, and reflections both on a square grid and on an isometric grid. Expect a variety of approaches, mainly making use of tracing paper but students may also begin to notice how the structure of the different grids helps draw images resulting from certain moves.

For classrooms using the digital version of the materials: This is the lesson where students learn to use the transformation tools in Geogebra.

Math Community

In this lesson, students review the themes that arose when they shared their initial thoughts in Exercise 1 about what they think it should look like and sound like to do math together as a community. Students then have a chance to both affirm and add to the ideas that were generated.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Notice and Wonder

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Sticky notes: Warm-up
- Geometry toolkits: Activity 1

Required Preparation

Lesson:

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles.

For classrooms using the print version of the materials: Access to tracing paper is particularly important. Each student will need about 10 small sheets of tracing paper (commercially available "patty paper" is ideal). If using large sheets of tracing paper, such as 8.5 inches by 11 inches, cut each sheet into fourths.

For classrooms using the digital version of the materials: If there is access to extra help from a tech-savvy person, this would be a good day to request their presence in class.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
10 min	25 min	10 min	5 min

Warm-up

Notice and Wonder: The Isometric Grid

10 min

Activity Narrative

The purpose of this *Warm-up* is to familiarize students with an isometric grid, which will be useful when students transform figures on an isometric grid in a later activity. While students may notice and wonder many things, characteristics such as the measures of the angles in the grid and the diagonal parallel lines are the important discussion points. Students are not expected to know that each angle in an equilateral triangle is 60 degrees, but after previous experience with supplementary angles, circles, and rotations, they may be able to explain why each smaller angle is 60 degrees. Many things they notice may be in comparison to the square grid paper which is likely more familiar.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

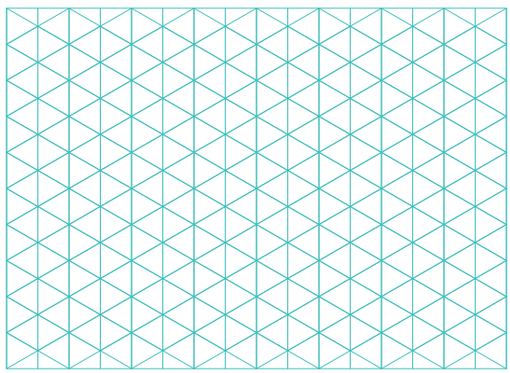
Launch

Arrange students in groups of 2. Display the isometric grid for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- There are three sets of parallel grid lines.
- The line segments form equilateral triangles.
- The individual angles in the equilateral triangles are 60 degrees.
- There are vertical lines but no horizontal lines.
- There are no 90-degree angles made by the grid lines.
- Each vertex has 6 line segments coming from it.
- The grid is made out of equilateral triangles instead of squares.

Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Student Workbook

Students may wonder:

- Why are there no 90-degree angles?
- Why are there no squares?
- Are we going to use this kind of grid?
- Why would we use this grid instead of the square grid?
- Why are there no horizontal lines?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image, and show where each of the features students notice is located on the actual grid itself, such as triangles, angles, and line segments. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement. If angle measures do not come up during the conversation, ask students to think about how they could figure out the measure of each angle. Some may measure with a protractor, and some may argue that since 6 angles share a vertex where each angle is identical, each angle measures 60° because $360 \div 6 = 60$. Establish that each angle measures 60°.

Math Community

After the *Warm-up,* display the class Math Community Chart for all to see and explain that the listed "doing math" actions come from the sticky notes students wrote in the first exercise.

Give students 1 minute to review the chart.

Then invite students to identify something on the chart they agree with and hope for the class or something they feel is missing from the chart and would like to add. Record any additions on the chart. Tell students that the chart will continue to grow and that they can suggest other additions that they think of throughout today's lesson during the *Cool-down*.

Activity 1
Image Information

25 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to interpret the information needed to perform a translation, rotation, or reflection, and draw the resulting image.

This activity works best when each student has access to tracing paper. In the digital version of the activity, students use an applet to perform transformations. The applet allows students to translate, rotate, and reflect using digital tools. The digital version may help students perform transformations accurately so they can focus on the mathematical analysis.

This activity is the first time students use prime notation, such as A' and B', to denote points in the image that correspond to points in the original figure.

Students make use of structure by using the mathematical properties of the grid lines to draw transformed figures.

Watch for student strategies including using tracing paper and using properties of the grids as students decide where to place the transformed figures. Tracing paper may be particularly useful for the isometric grid which may be unfamiliar to some students.

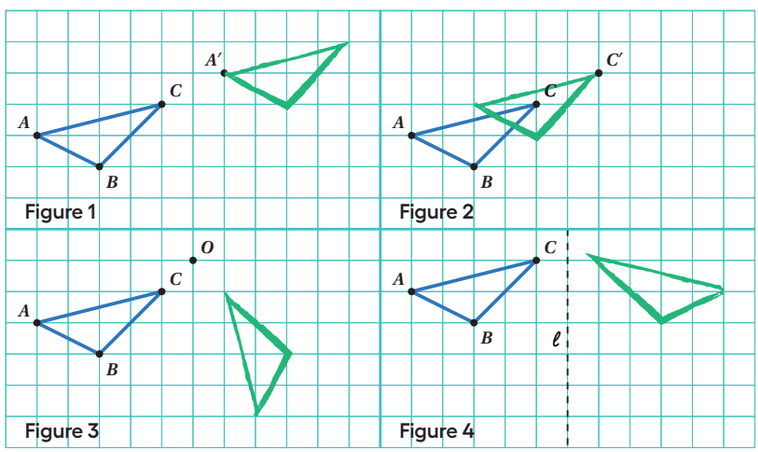
Launch

Point out A' in the first question. Tell students we call point A' "A prime" and that, after a transformation, it corresponds to point A in the original.

Give students 6–8 minutes of quiet work time for the first set of transformations. Invite students to share a strategy with the whole class for each transformation. Give students an additional 5–6 minutes of quiet work time for the last set of transformations.

Student Task Statement

Your teacher will give you tracing paper to carry out the moves specified. Use A', B', C', and D' to indicate vertices in the new figure that correspond to the points A, B, C, and D in the original figure.



Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.
Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "I turned the shape one space to the right" as "I rotated quadrilateral $ABCD$ 60° counterclockwise by using the 60° angle in each triangle in the isometric grid."

Advances: Speaking

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Begin with a physical demonstration of using tracing paper to perform each type of transformation to support connections between new situations and prior understandings. Consider using the prompts: "What does this demonstration have in common with previous activities where both images were given?" or "How does the point A correspond to the point A?"

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Workbook

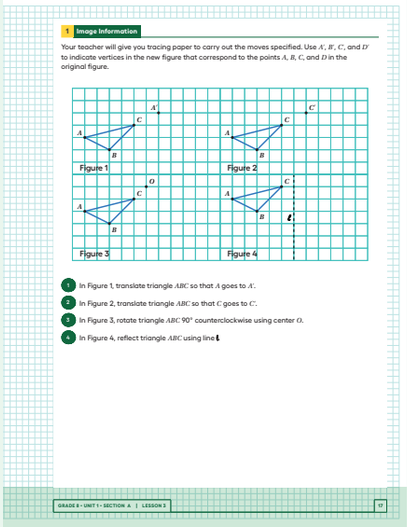

1 Image Information

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Figure 1 Figure 2 Figure 3 Figure 4

1. In Figure 1, translate triangle ABC so that A goes to A'.
2. In Figure 2, translate triangle ABC so that C goes to C'.
3. In Figure 3, rotate triangle ABC 90° counterclockwise using center O.
4. In Figure 4, reflect triangle ABC using line ℓ.

Student Workbook

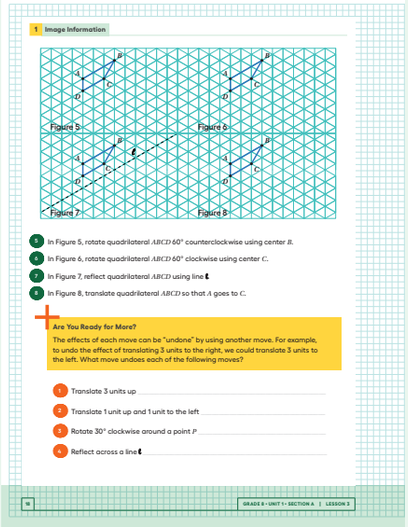

1 Image Information

Figure 5 Figure 6 Figure 7 Figure 8

5. In Figure 5, rotate quadrilateral $ABCD$ 60° counterclockwise using center B.
6. In Figure 6, rotate quadrilateral $ABCD$ 60° clockwise using center C.
7. In Figure 7, reflect quadrilateral $ABCD$ using line ℓ.
8. In Figure 8, translate quadrilateral $ABCD$ so that A goes to C.

Are You Ready for More?

The effects of each move can be "undone" by using another move. For example, to undo the effect of translating 3 units to the right, we could translate 3 units to the left. What move undoes each of the following moves?

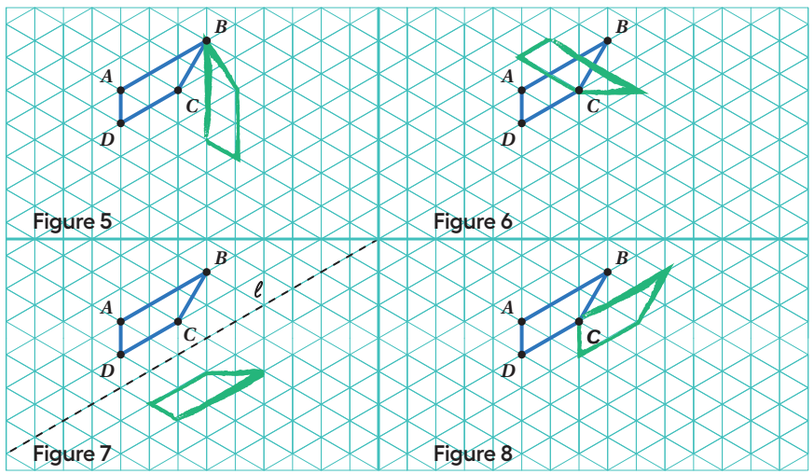
1. Translate 3 units up
2. Translate 1 unit up and 1 unit to the left
3. Rotate 30° clockwise around a point P
4. Reflect across a line ℓ

Building on Student Thinking

Students may struggle to understand the descriptions of the transformations to carry out. For these students, explain the transformations using the words they used in earlier activities, such as "slide," "turn," and "mirror image," to help them get started. Students may also struggle with reflections that are not over horizontal or vertical lines.

Some students may need to see an actual mirror to understand what reflections do and the role of the reflection line. If rectangular plastic mirrors are accessible, students can check their work by placing the mirror along the proposed mirror line.

Working with the isometric grid may be challenging, especially rotations and reflections across lines that are not horizontal or vertical. For the rotations, students can be asked what they know about the angle measures in an equilateral triangle. For reflections, the approach of using a mirror can work or students can look at individual triangles in the grid, especially those with a side on the line of reflection, and see what happens to them. After checking several triangles, they develop a sense of how these reflections behave.



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   translate 3 units down

2. Translate 1 unit up and 1 unit to the left

   translate 1 unit down and 1 unit to the right

3. Rotate 30° clockwise around a point P

   rotate 30° counterclockwise around P

4. Reflect across a line ℓ

   reflect again across ℓ

Activity Synthesis

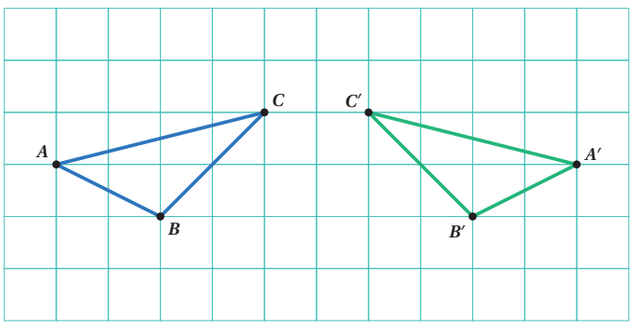
Ask students to share how they found the images, and highlight the information they needed in each to perform the transformation. Invite students who used tracing paper to share how they found the images and also ask students what mathematical patterns they found. For example, for the reflection in Figure 4, ask where some intersections of grid lines go (they stay on the same horizontal line and go to the other side of ℓ, the same distance away). How can this be used to identify the image of ABC?

Ask students how working on the isometric grid is similar to working on a regular grid and how it is different. Possible responses include:

- Translations work the same way, identifying how far and in which direction to move the shape.
- Rotations also work the same way but the isometric grid works well for multiples of 60 degrees (with center at a grid point), while the regular grid works well for multiples of 90 degrees (also with center at a grid point).
- Reflections on the isometric grid require looking carefully at the triangular pattern to place the reflection in the right place. Like for the regular grid, these reflections are difficult to visualize if the line of reflection is not a grid line.

Lesson Synthesis

Display this image for all to see:



Then read or display this statement:

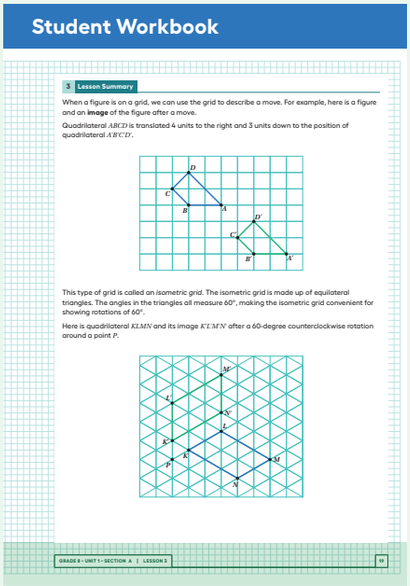
"Mai says that triangle $A'B'C'$ is a translation of triangle ABC 2 units to the right because C' is 2 units to the right of C.

Priya says that triangle $A'B'C'$ is a translation of triangle ABC 6 units to the right because B' is 6 units to the right of B.

Do you agree with either of them? Why or why not?"

Give students 1 minute of quiet think time, then 2 minutes to discuss with a partner, followed by a whole-class discussion.

Student Workbook



3 Lesson Summary

When a figure is on a grid, we can use the grid to describe a move. For example, here is a figure and an **image** of the figure after a move.

Quadrilateral $ABCD$ is translated 4 units to the right and 3 units down to the position of quadrilateral $A'B'C'D'$.

This type of grid is called an *isometric grid*. The isometric grid is made up of equilateral triangles. The angles in the triangles all measure 60°, making the isometric grid convenient for showing rotations of 60°.

Here is quadrilateral $KLMN$ and its image $K'L'M'N'$ after a 60-degree counterclockwise rotation around a point P.

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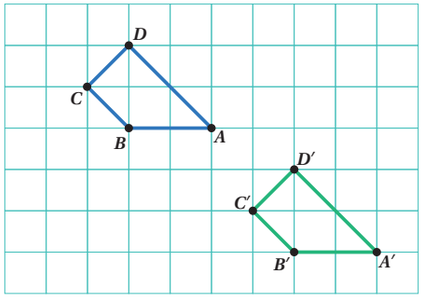

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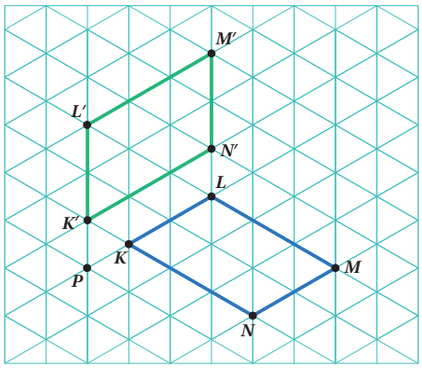
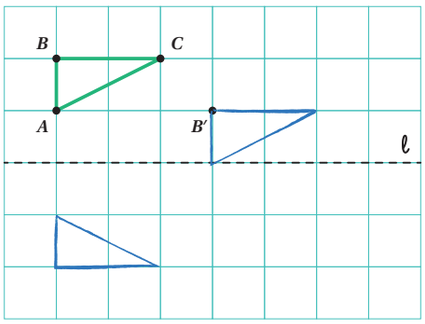


Cool-down

Triangle Images

5 min

Student Task Statement



1. Translate triangle ABC so that B goes to B'.
2. Reflect triangle ABC over line ℓ.

Math Community

Before distributing the *Cool-downs,* display the Math Community Chart and the community building question "Is there anything that you would like to add to the student 'Doing Math' section of the chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs,* identify themes from the community building question. Use the themes to add to or revise the student section of the Math Community Chart before Exercise 3.

Responding To Student Thinking

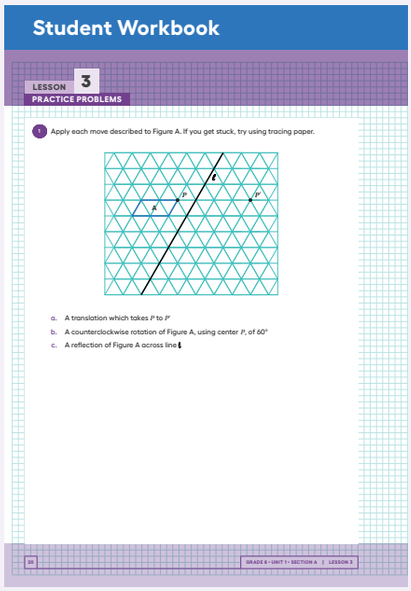
Points to Emphasize

If students struggle with drawing a translation or reflection, distinguish between the types of transformations as opportunities arise over the next several lessons. For example, during the activity referred to here, share visuals of the types of transformations that show both the vocabulary and a representation of the movement, such as those from the Student Lesson Summaries.

Unit 1, Lesson 4, Activity 2 Make That Move

Practice Problems

3 Problems


Student Workbook

LESSON 3
PRACTICE PROBLEMS

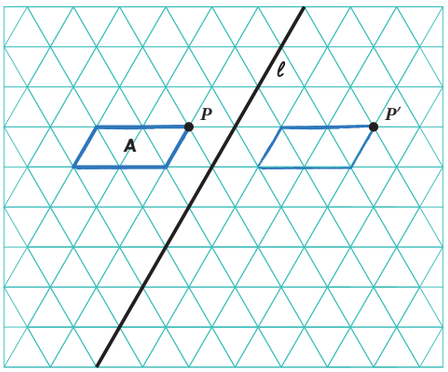
1 Apply each move described to Figure A. If you get stuck, try using tracing paper.

a. A translation which takes P to P'
b. A counterclockwise rotation of Figure A, using center P, of 60°
c. A reflection of Figure A across line ℓ

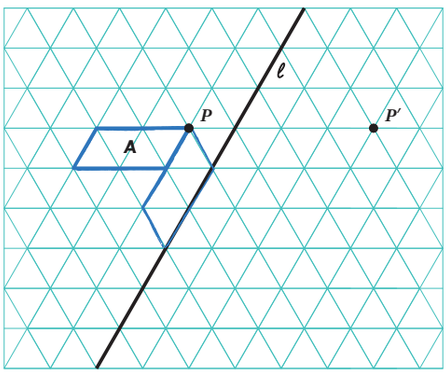
Problem 1

Apply each move described to Figure A. If you get stuck, try using tracing paper.

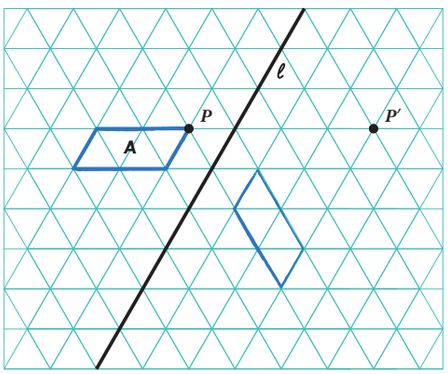
a. A translation which takes P to P'



b. A counterclockwise rotation of Figure A, using center P, of 60°

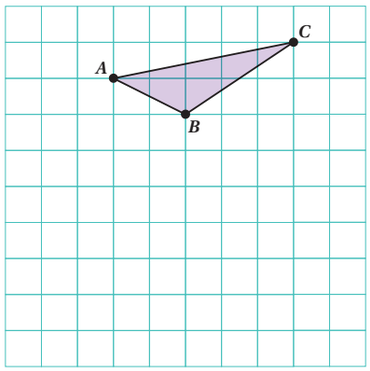


c. A reflection of Figure A across line ℓ.



Problem 2

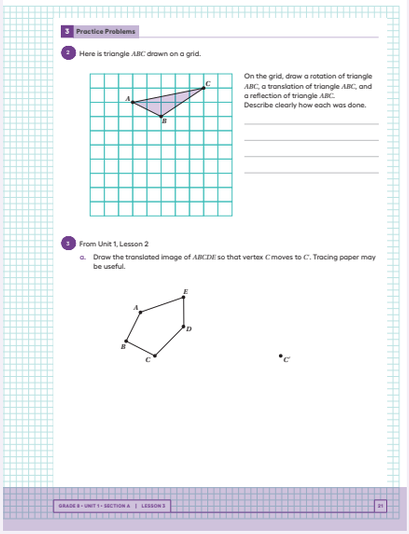
Here is triangle ABC drawn on a grid.



On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

Sample response: The rotation is a 90-degree counterclockwise rotation using center A. The translation is 4 units down and 3 to the left. The reflection is across a horizontal line through point B.

Student Workbook


3 Practice Problems

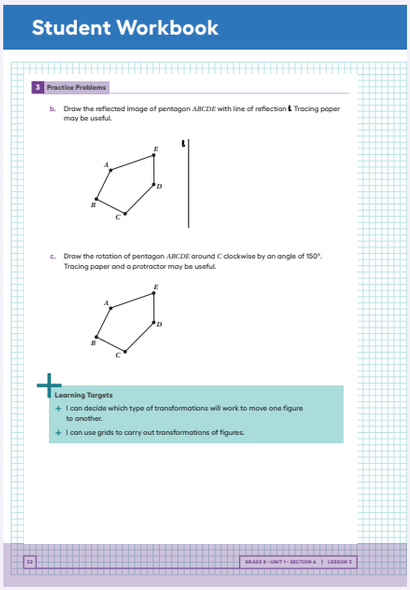
2 Here is triangle ABC drawn on a grid.

On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

3 From Unit 1, Lesson 2

a. Draw the translated image of $ABCDE$ so that vertex C moves to C'. Tracing paper may be useful.

Student Workbook


3 Practice Problems

b. Draw the reflected image of pentagon $ABCDE$ with line of reflection ℓ. Tracing paper may be useful.

c. Draw the rotation of pentagon $ABCDE$ around C clockwise by an angle of 150°. Tracing paper and a protractor may be useful.

Learning Targets
+ I can decide which type of transformations will work to move one figure to another.
+ I can use grids to carry out transformations of figures.

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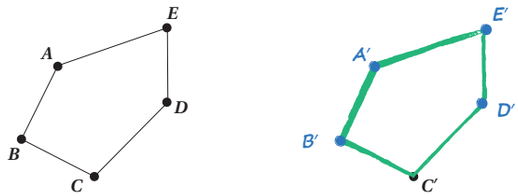
Problem 3

from Unit 1, Lesson 2

a. Draw the translated image of $ABCDE$ so that vertex C moves to C'. Tracing paper may be useful.



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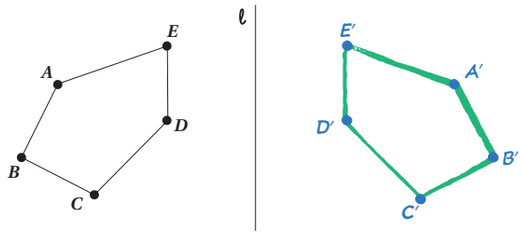

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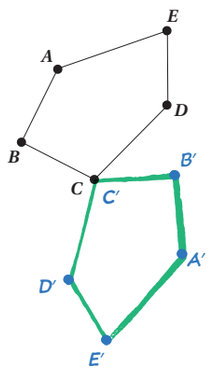

In the picture, angle DCD′ measures 150 degrees.