Lesson 8

Translating to $y = mx + b$

Goals

- Coordinate (orally) features of the equation $y = b + mx$ to the graph, including lines with a negative y-intercept.
- Create and compare (orally and in writing) graphs that represent linear relationships with the same rate of change but different initial values.

Learning Targets

- I can explain where to find the slope and vertical intercept in both an equation and its graph.
- I can write equations of lines using $y = mx + b$.

Lesson Narrative

This lesson introduces the idea that any line in the plane can be considered a vertical translation of a line through the origin. The notion of a negative y-intercept is also introduced.

Students begin by identifying lines that could be images of a given line under a translation, observing that they will be parallel to the original line. Next students graph two situations on the same coordinate plane that involve the same rate of change but with different initial values, observing how the graph of one line can be described as a translation of the other.

Then students identify equations that all represent the same line. In previous lessons, the terms in the expression were more likely to be arranged as $b + mx$ because the situation involved a starting amount and then added on a multiple. In this lesson, $mx + b$ is more likely because the situation involves starting with a proportional relationship and shifting it up or down.

Finally, students match cards that have lines presented as equations, graphs, descriptions, and tables.

Student Learning Goal

Let's see what happens to the equations of translated lines.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- Card Sort

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up

Materials to Copy

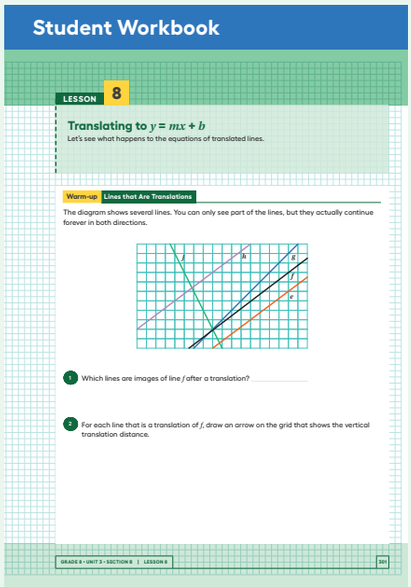
- Translating a Line Cards (1 copy for every 2 students): Activity 2

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	15 min	15 min	10 min	5 min
				Cool-down


Student Workbook

LESSON 8

Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.

Warm-up Lines that Are Translations

The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.

1. Which lines are images of line f after a translation?

2. For each line that is a translation of f, draw an arrow on the grid that shows the vertical translation distance.

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Warm-up

Lines that Are Translations

5 min

Activity Narrative

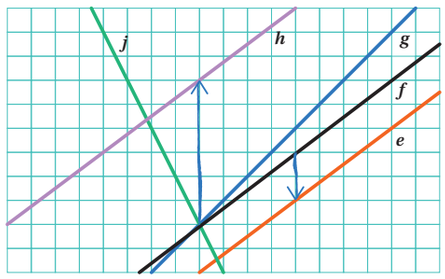
The purpose of this *Warm-up* is to remind students that the translation of a line is parallel to the original line. They begin by inspecting several lines to decide which are translations of a given line. Students then describe the translations by specifying the number of units and the direction, in preparation to see the equation $y = mx + b$ as a translation of $y = mx$.

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time and access to geometry toolkits. Ask them to share their responses with a partner afterwards.

Student Task Statement

The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.



1. Which lines are images of line f after a translation?

   Lines h and e

   Sample reasoning: They are parallel to line f, and translated lines are parallel to the original.

2. For each line that is a translation of f, draw an arrow on the grid that shows the vertical translation distance.

   Line h is line f translated up 6 units. Line e is line f translated down 2 units.

Activity Synthesis

Invite students to share how they determined that lines h and e are translations of f. Emphasize that lines h and e are parallel to f, and line f matching up with the other lines would require a rotation or reflection. If possible, demonstrate the transformations using a clear transparency or tracing paper. If using a transparency or tracing paper to demonstrate the translations, it is helpful to draw a dot for a specific point on both the underlying graph and on the transparency as a reference point.

Activity 1
Increased Savings

15 min

Activity Narrative

There is a digital version of this activity.

The goal of this activity is for students to think about translations of lines in a context by examining two scenarios. Graphically, the two lines representing the relationships are parallel. The lines have the same slope but different vertical intercepts. Students will observe this structure in the equations that they write for the two lines.

Even though babysitters are often paid for increments of 1 hour or ½ hour and in increments of $1, a continuous line is used to represent this relationship. If a student brings up the idea that it would be better to represent the relationship using discrete points rather than a line, acknowledge the observation but suggest that a continuous line is an acceptable representation. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation.

Monitor for students who use these approaches as they graph the two earnings scenarios, starting with an approach that looks at the arithmetic relationship, followed by an approach that looks at the geometric relationship, and concluding with a more abstract approach that combines the first two:

- Making a table showing earnings for different numbers of hours worked, and then graphing
- Plotting points directly
- Writing an equation that represents the situation and then graphing the solutions to the equation

In the digital version of the activity, students use an applet to graph two linear relationships. The applet allows students to quickly and accurately create a graph of two scenarios. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Instructional Routines

5 Practices

ilclass.com/r/10690701

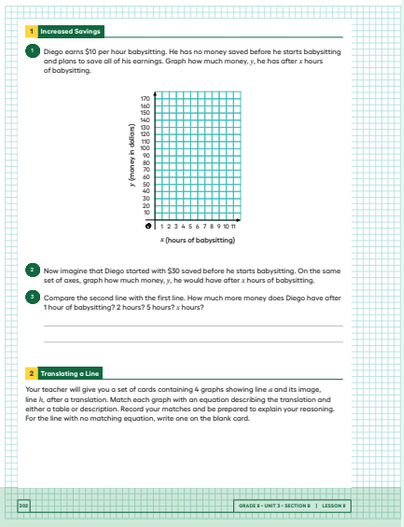
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a device that can run the digital applet.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Student Workbook


1 Increased Savings

1. Diego earns $10 per hour babysitting. He has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.

y (money in dollars)

x (hours of babysitting)

2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.

3. Compare the second line with the first line. How much more money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

2 Translating a Line

Your teacher will give you a set of cards containing 4 graphs showing line a and its image, line b, after a translation. Match each graph with an equation describing the translation and either a table or description. Record your matches and be prepared to explain your reasoning. For the line with no matching equation, write one on the blank card.

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Launch

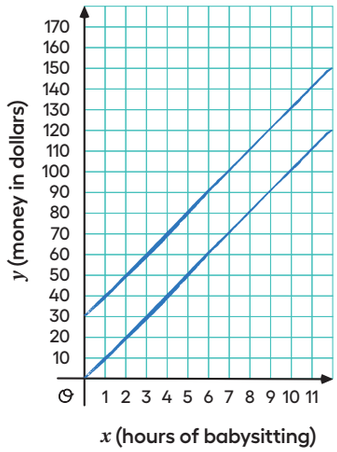
Give students 5 minutes of quiet work time, followed by partner then whole-class discussion.

Select students with different approaches, such as those described in the *Activity Narrative,* to share later.

Student Task Statement

1. Diego earns $10 per hour babysitting. He has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.

See line through (0, 0) on graph



2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.

See line through (0, 30) on graph

3. Compare the second line with the first line. How much *more* money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

Diego will always have $30 more dollars regardless of how many hours he babysits.

Activity Synthesis

The purpose of this discussion is for students to connect the graphical translation of a line to a context. Invite previously selected students to share their approaches to graphing each line. Sequence the discussion in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"How much more money does Diego have after 1 hour of babysitting in the second situation compared to the first? After 2 hours? After 5 hours? After x hours?"*

  Diego always has $30 more.

- *"Where can this extra $30 be seen in each approach?"*

  Table: The number of dollars saved after a given number of hours is always 30 more in the second situation

  Graph: The first line is translated up by 30 units, and the vertical intercept is at (0, 30)

  Equation: The second equation is the same as the first but with a "+30."

- *"Where can the $10 that Diego earns each hour be seen in each approach?"*

  Table: The number of dollars saved goes up by 10 each time the number of hours goes up by 1

  Graph: The slope of both lines is 10

  Equation: Both equations have the term "10x."

Instructional Routines

Card Sort

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Activity 2

Translating a Line

15 min

Activity Narrative

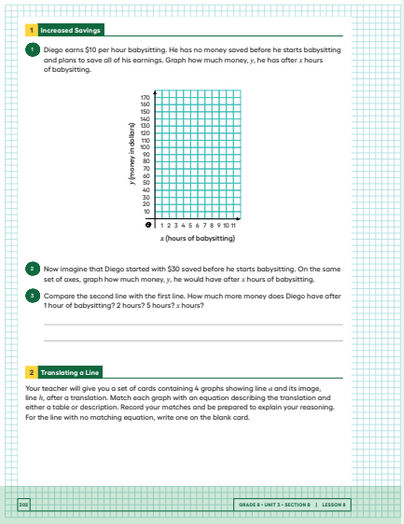
This activity continues to examine parallel lines, including situations where the vertical intercept is negative. Students sort cards with lines represented graphically, algebraically, with a table of values, or with a verbal description. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code connections between lines and their matching equation.

Supports accessibility for: Visual-Spatial Processing

Student Workbook


1 Increased Savings

1. Diego earns $10 per hour babysitting. He has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.

y (money in dollars)
x (hours of babysitting)

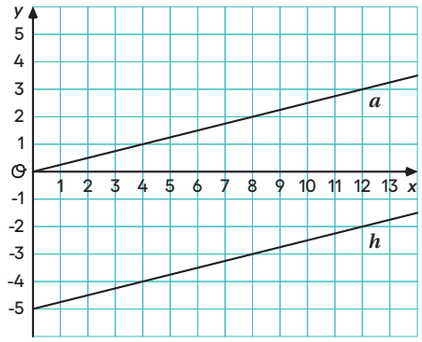
2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.
3. Compare the second line with the first line. How much more money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

2 Translating a Line

Your teacher will give you a set of cards containing 4 graphs showing line a and its image, line h, after a translation. Match each graph with an equation describing the translation and either a table or description. Record your matches and be prepared to explain your reasoning. For the line with no matching equation, write one on the blank card.

Launch

Display the image for all to see.



Ask students how this pair of lines is the same and different from the lines in the previous activity. (Instead of being translated up, the line that goes through the origin (line a in this case) was translated down.) Display the equation $y = \frac{1}{4}x$ for all to see and tell students that this is one equation that represents line a.

Then display this set of equations and ask students which equations represent line h:

A. $y = \frac{1}{4}x - 5$

B. $y = \frac{1}{4}x + 5$

C. $\frac{1}{4}x - 5 = y$

D. $y = -5 + \frac{1}{4}x$

E. $-5 + \frac{1}{4}x = y$

F. $y = 5 - \frac{1}{4}x$

Give students 2 minutes of quiet think time to decide which of the equations also represents line h.

equations A, C, D, and E

Discuss what is the same an what is different about the equivalent equations.

They all have y by itself on one side, but it could be on the left or right side of the equation; they all have the term "$\frac{1}{4}x$," and it is always positive; they all have a "-5," and this term could come before or after the $\frac{1}{4}x$.

Arrange students in groups of 2 and distribute pre-cut cards. Give students time to complete the activity followed by a whole-class discussion.

Student Task Statement

Your teacher will give you a set of cards containing 4 graphs showing line a and its image, line h, after a translation. Match each graph with an equation describing the translation and either a table or description. Record your matches and be prepared to explain your reasoning. For the line with no matching equation, write one on the blank card.

The blackline master shows the correct matches by column. The equation for the line with no matching equation is $y = \frac{1}{2}x$ (or equivalent).

Are You Ready for More?

A student says that the graph of the equation $y = 3(x + 8)$ is the same as the graph of $y = 3x$, only translated upwards by 8 units. Do you agree? Explain your reasoning.

I disagree with the student.

Sample reasoning: When I use the distributive property for $3(x + 8)$ I get $3x + 24$, so the equation is $y = 3x + 24$. This is the graph of $y = 3x$ translated up by 24 units.

Activity Synthesis

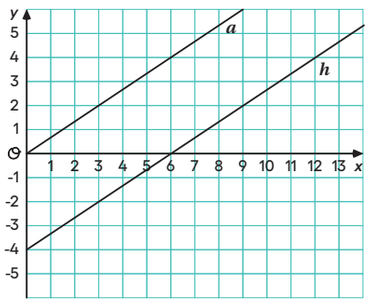
Once all groups have completed the *Card Sort*, discuss the following:

- *"Which matches were tricky? Explain why."*
- *"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"*
- *"What clues did you look for to see which equation went with each graph?"*

Invite students to share their version of the missing equation. Record responses for all to see. Ensure students understand that $y = \frac{1}{2}x - 4$ and $y = -4 + \frac{1}{2}x$ are equivalent.

Lesson Synthesis

Display this graph of two lines on the same set of axes for all to see:



Discuss with students:

- *"How can one line be thought of as a transformation of the other?"*

  One line is translated up or down to match the other line.

- *"What is the equation of line a?"*

  $y = \frac{2}{3}x$

- *"How is the equation of line h different?"*

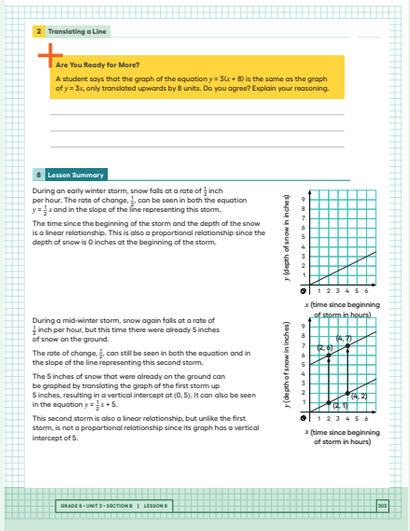
  It has a "-4" in it.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display the following sentence frames for all to see: "I noticed _____, so I matched ..." "The equation _____ matches line h on this card because ..." or "This description matches line h on this card because ..." Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing

Student Workbook



2 Translating a Line

Are You Ready for More?

A student says that the graph of the equation $y = 3(x + 8)$ is the same as the graph of $y = 3x$, only translated upwards by 8 units. Do you agree? Explain your reasoning.

3 Lesson Summary

During an early winter storm, snow falls at a rate of $\frac{1}{2}$ inch per hour. The rate of change, $\frac{1}{2}$, can be seen in both the equation $y = \frac{1}{2}x$ and in the slope of the line representing this storm.

The time since the beginning of the storm and the depth of the snow is a linear relationship. This is also a proportional relationship since the depth of snow is 0 inches at the beginning of the storm.

y (depth of snow in inches)

x (time since beginning of storm in hours)

During a mid-winter storm, snow again falls at a rate of $\frac{1}{2}$ inch per hour, but this time there were already 5 inches of snow on the ground.

The rate of change, $\frac{1}{2}$, can still be seen in both the equation and in the slope of the line representing this second storm.

The 5 inches of snow that were already on the ground can be graphed by translating the graph of the first storm up 5 inches, resulting in a vertical intercept at (0, 5). It can also be seen in the equation $y = \frac{1}{2}x + 5$.

This second storm is also a linear relationship, but unlike the first storm, is not a proportional relationship since its graph has a vertical intercept of 5.

y (depth of snow in inches)

(4, 7) (2, 6) (4, 2) (2, 1)

x (time since beginning of storm in hours)

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"What would the equation be for a line that was a translation of line a 2 units up?"

$y = \frac{2}{3}x + 2$

"What would the graph of the line $y = \frac{2}{3}x - 9$ look like?"

It would be parallel to line a and be translated down 9 units so that the vertical intercept was at (0, -9).

Lesson Summary

During an early winter storm, snow falls at a rate of $\frac{1}{2}$ inch per hour. The rate of change, $\frac{1}{2}$, can be seen in both the equation $y = \frac{1}{2}x$ and in the slope of the line representing this storm.

The time since the beginning of the storm and the depth of the snow is a linear relationship. This is also a proportional relationship since the depth of snow is 0 inches at the beginning of the storm.

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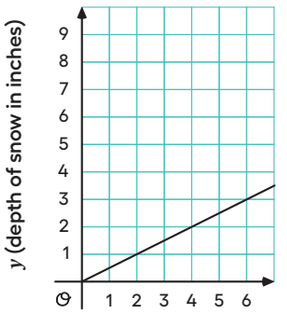
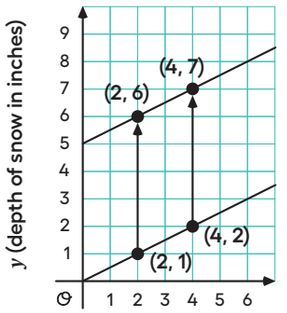




Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- What norm(s) should stay the way they are?
- What norm(s) do you think should be made more clear? How?
- What norms are missing that you would add?
- What norm(s) should be removed?

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Similarities and Differences in Two Lines

5 min

Student Task Statement

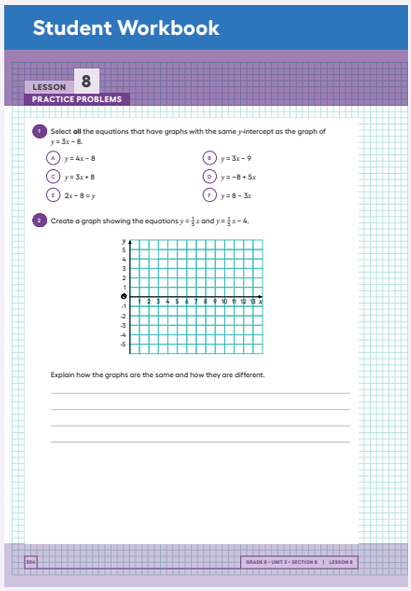
Describe how the graph of $y = 2x$ is the same and different from the graph of $y = 2x - 7$.

Sample responses:

- Both lines have a slope of 2, but one line has a y-intercept of 0 while the other has a y-intercept at -7.
- Both lines have the same slope but different vertical intercepts.
- The lines are parallel to each other, with one line being a translation of the other line.
- Both lines have the same rate of change, but cross the y-axis (or x-axis) at different points.

Practice Problems

5 Problems


Student Workbook

LESSON 8
PRACTICE PROBLEMS

1. Select **all** the equations that have graphs with the same *y*-intercept as the graph of $y = 3x - 8$.

A. $y = 4x - 8$
B. $y = 3x - 9$
C. $y = 3x + 8$
D. $y = -8 + 5x$
E. $2x - 8 = y$
F. $y = 8 - 3x$

2. Create a graph showing the equations $y = \frac{1}{3}x$ and $y = \frac{1}{3}x - 4$.

Explain how the graphs are the same and how they are different.

Problem 1

Select **all** the equations that have graphs with the same *y*-intercept as the graph of $y = 3x - 8$.

A. $y = 4x - 8$

B. $y = 3x - 9$

C. $y = 3x + 8$

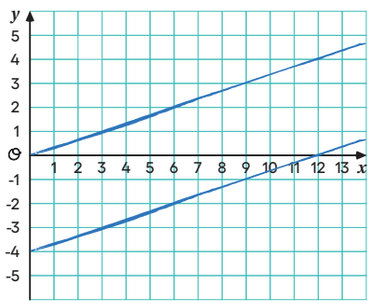
D. $y = -8 + 5x$

E. $2x - 8 = y$

F. $y = 8 - 3x$

Problem 2

Create a graph showing the equations $y = \frac{1}{3}x$ and $y = \frac{1}{3}x - 4$. Explain how the graphs are the same and how they are different.



Sample responses: The graphs have the same slope of $\frac{1}{3}$ but different *y*-intercepts. One graph has a *y*-intercept of 0 and the other has a *y*-intercept of -4. Each point (x, y) on the graph of $y = \frac{1}{3}x$ is translated down by 4 to get a corresponding point on the second graph.

Problem 3

An internet company charges $70 per month for internet service to existing customers.

a. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by an existing customer.

$y = 70x$

b. For new customers, there is a one-time $100 installation fee. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a new customer.

$y = 70x + 100$ (or equivalent)

c. Explain how graphs of the lines representing each situation would be the same and how they would be different.

Sample response: The lines representing each situation would be parallel. They would have the same slope but different y-intercepts. The line representing new customers would be parallel to the line representing existing customers but would be translated up by 100.

Problem 4

from Unit 3, Lesson 6

A mountain road is 5 miles long and gains elevation at a constant rate. At the start of the road, the elevation is 4,800 feet above sea level. After 2 miles, the elevation is 5,500 feet above sea level.

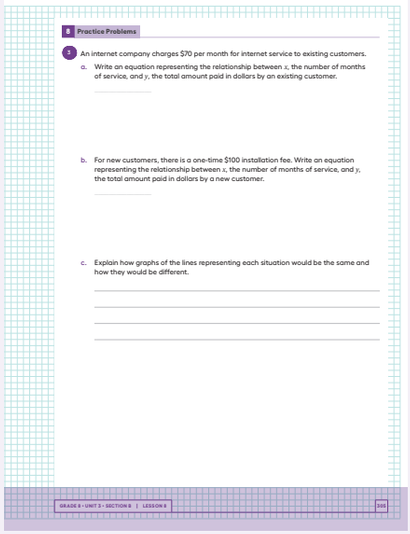
a. Find the elevation of the road after 5 miles.

6,550 feet above sea level

b. What is the vertical intercept and what does it represent in this situation?

The vertical intercept is 4,800 and represents the elevation at the start of the mountain road.

Student Workbook


8 Practice Problems

3 An internet company charges $70 per month for internet service to existing customers.

a. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by an existing customer.

b. For new customers, there is a one-time $100 installation fee. Write an equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by a new customer.

c. Explain how graphs of the lines representing each situation would be the same and how they would be different.

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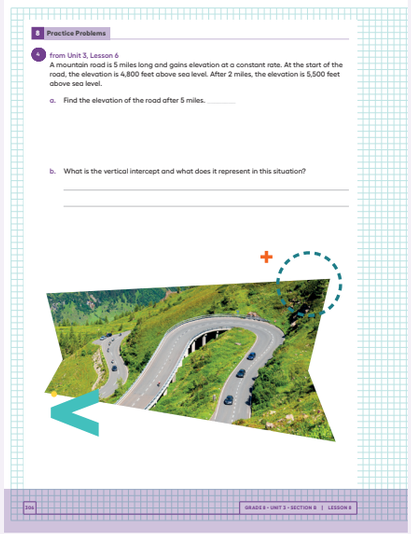
Student Workbook


8 Practice Problems

4 from Unit 3, Lesson 6
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a. Find the elevation of the road after 5 miles.

b. What is the vertical intercept and what does it represent in this situation?

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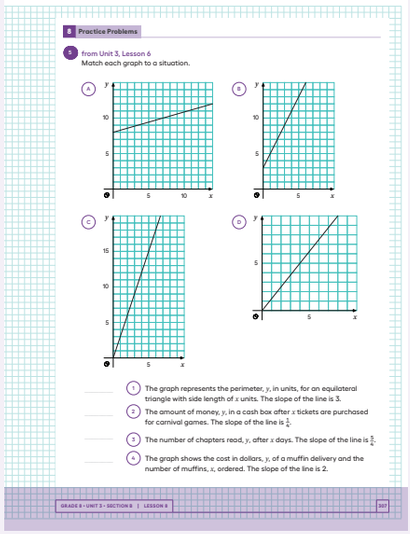
Student Workbook


8 Practice Problems

5 from Unit 3, Lesson 6
Match each graph to a situation.

A B C D

1. The graph represents the perimeter, y, in units, for an equilateral triangle with side length of x units. The slope of the line is 3.
2. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$.
3. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$.
4. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2.

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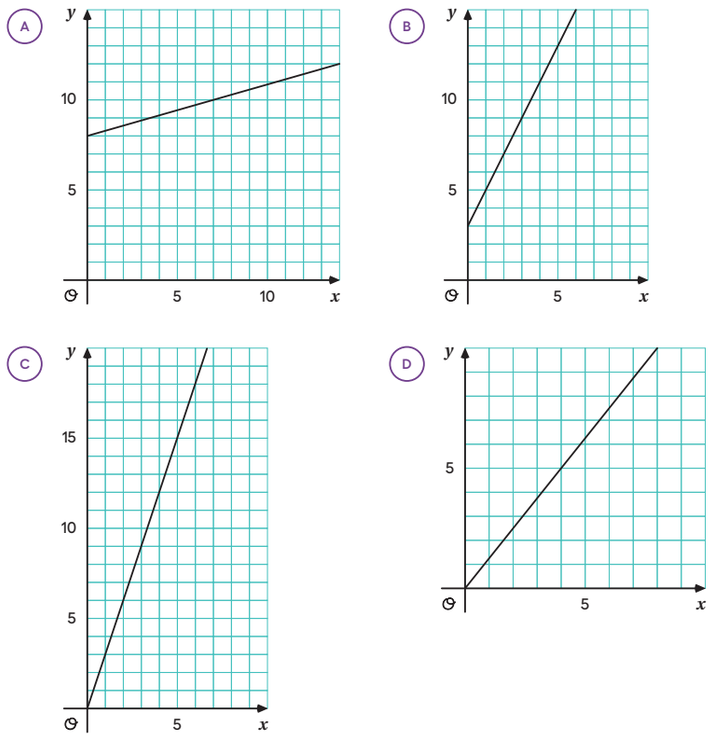
Problem 5

from Unit 3, Lesson 6

Match each graph to a situation.



C **1.** The graph represents the perimeter, y, in units, for an equilateral triangle with side length of x units. The slope of the line is 3.

A **2.** The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$.

D **3.** The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$.

B **4.** The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2.