Lesson 11

Comparing Groups

Goals

- Calculate the mean and mean absolute deviation for a data set, and interpret (orally) these measures.
- Compare and contrast (orally and in writing) populations represented on dot plots in terms of their shape, center, spread, and visual overlap.
- Justify (in writing) whether two populations are "very different" based on the difference in their means expressed as a multiple of the mean absolute deviation.

Learning Targets

- I can calculate the difference between two means as a multiple of the mean absolute deviation.
- When looking at a pair of dot plots, I can determine whether the distributions are very different or have a lot of overlap.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- 5 Practices
- Notice and Wonder

Lesson Narrative

In this lesson, students review measures of center and variability from grade 6. They also work to informally decide whether or not two distributions are very different from each other. This lesson introduces the idea of expressing the difference between the centers of two distributions as a multiple of a measure of variability as a way to help students make this determination.

An optional activity provides additional practice comparing two groups based on their means and mean absolute deviations.

Student Learning Goal

Let's compare two groups.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	10 min	5 min

Warm-up

Notice and Wonder: Comparing Heights

5 min

Activity Narrative

The purpose of this *Warm-up* is to collect ideas about comparing two data sets, which will be useful when students quantify how to compare groups in a later activity. While students may notice and wonder many things about these dot plots, the ways students describe how the volleyball team is taller is the important discussion point.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

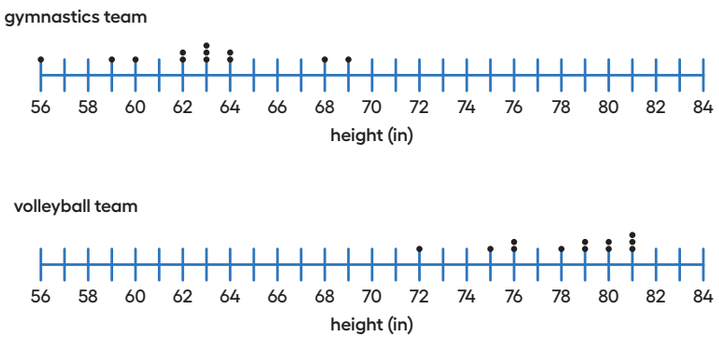
Launch

Arrange students in groups of 2. Display the dot plots for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Things students may notice:

- The volleyball team is much taller than the gymnastics team.
- The gymnastics team's heights are all under 70 inches tall.
- The volleyball team's heights are all over 70 inches tall.

Things students may wonder:

- How much taller are the volleyball players?
- Are players on volleyball teams usually that much taller than gymnasts, or is this just for these two teams?
- Why are gymnasts typically short and volleyball players typically tall?

Instructional Routines

Notice and Wonder

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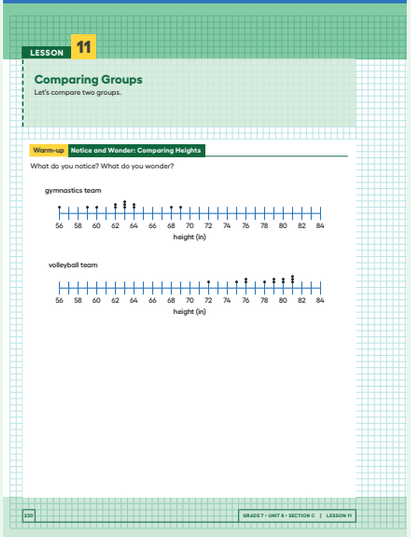
Student Workbook



Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the dot plots. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If an attempt to use numerical values to compare the height of the teams does not come up during the conversation, ask students to discuss this idea.

Activity 1

More Team Heights

15 min

Activity Narrative

In this activity, students are asked to compare the heights of two groups of people. The wording of the questions allows for multiple interpretations and any reasonable answer should be accepted as long as the argument is supported. This activity also provides an opportunity to remind students of how to analyze dot plots as well as how to calculate the measures of center and variability of the data.

Launch

Keep students in groups of 2.

Display the dot plots from the *Warm-up* activity and help students see that the data sets given in their books or devices match the numbers shown in the dot plots.

For the problem addressing the tennis and badminton teams, you may suggest that each student creates a dot plot of one of the groups and then compares it with their partner.

Allow students 10 minutes of partner work time followed by a whole-class discussion.

Student Task Statement

1. How much taller is the volleyball team than the gymnastics team?

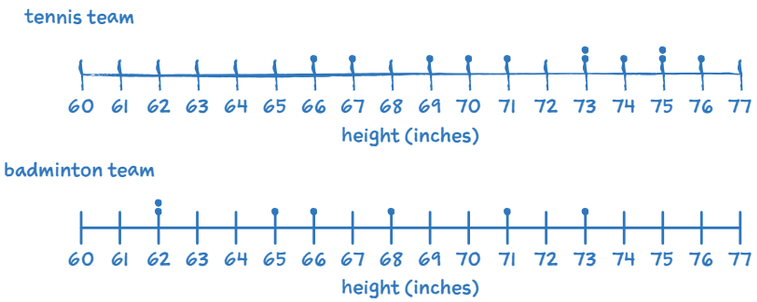
- Gymnastics team's heights (in inches): 56, 59, 60, 62, 62, 63, 63, 63, 64, 64, 68, 69
- Volleyball team's heights (in inches): 72, 75, 76, 76, 78, 79, 79, 80, 80, 81, 81, 81

Sample responses:

- The total height of the volleyball team is 185 inches, or about 15 feet 5 inches, greater than the total height of the gymnastics team. The height of everyone on each team is added, then the values for the teams are subtracted: 938 − 753 = 185.
- The mean height of the volleyball team is about 15.4 inches greater than the mean height of the gymnastics team. Mean height of volleyball team, because $753 \div 12 = 62\frac{3}{4}$. Mean height of gymnastics team: $938 \div 12 = 78\frac{1}{6}$. Difference in the means: $78\frac{1}{6} - 62\frac{3}{4} = 15\frac{5}{12}$, or about 15.4 inches.
- The median height of the volleyball team is 16 inches greater than the median height of the gymnastics team, and this may be more important than the difference in the means because the shape of the distribution for the volleyball team is not symmetric.
- The tallest person on the volleyball team is 12 inches taller than the tallest person on the gymnastics team.
- The shortest person on the volleyball team is 16 inches taller than the shortest person on the gymnastics team.
- The tallest person on the volleyball team is 25 inches taller than the shortest person on the gymnastics team.
- The shortest person on the volleyball team is 3 inches taller than the tallest person on the gymnastics team.

2. Make dot plots to compare the heights of the tennis and badminton teams.

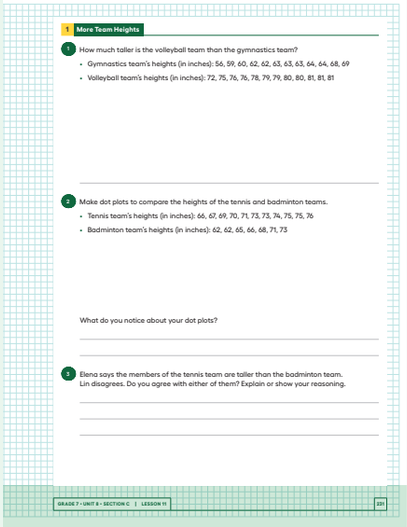
- Tennis team's heights (in inches): 66, 67, 69, 70, 71, 73, 73, 74, 75, 75, 76
- Badminton team's heights (in inches): 62, 62, 65, 66, 68, 71, 73



What do you notice about your dot plots?

The center of the distribution as well as the minimum and maximum heights for the tennis team are all greater than the center, minimum, and maximum for the badminton team. However, there is a lot of overlap between the two distributions. Unlike with the gymnastics and volleyball teams, the tallest person on the badminton team is taller than the shortest person on the tennis team.

Student Workbook


1 More Team Heights

1. How much taller is the volleyball team than the gymnastics team?
   - Gymnastics team's heights (in inches): 56, 59, 60, 62, 62, 63, 63, 63, 64, 64, 68, 69
   - Volleyball team's heights (in inches): 72, 75, 76, 76, 78, 79, 79, 80, 80, 81, 81, 81

2. Make dot plots to compare the heights of the tennis and badminton teams.
   - Tennis team's heights (in inches): 66, 67, 69, 70, 71, 73, 73, 74, 75, 75, 76
   - Badminton team's heights (in inches): 62, 62, 65, 66, 68, 71, 73

   What do you notice about your dot plots?

3. Elena says the members of the tennis team are taller than the badminton team. Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

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Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

- The total height of the tennis team is 322 inches, or 26 feet 10 inches, greater than the total height of the badminton team. However, it does not make sense to compare the totals, because the tennis team has more people.
- The mean height of the tennis team is about 5 inches greater than the mean height of the badminton team.
- The median height of the tennis team is 7 inches greater than the median height of the badminton team.
- The tallest person on the tennis team is only 3 inches taller than the tallest person on the badminton team.
- The shortest person on the tennis team is 4 inches taller than the shortest person on the badminton team.
- The tallest person on the tennis team is 14 inches taller than the shortest person on the badminton team.
- The shortest person on the tennis team is only 7 inches shorter than the tallest person on the badminton team because there is so much overlap between the two distributions.

3. Elena says the members of the tennis team are taller than the badminton team. Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

Sample responses:

- I agree with Elena because the center of the distribution for the tennis team is greater than the center for the badminton team.
- I agree with Lin because the two distributions overlap so much. Some of the people on the badminton team are taller than some of the people on the tennis team. The difference in centers is not big enough to matter. If the people from both teams were mixed together into one group, one would not be able to determine who was a member of each team.

Activity Synthesis

The purpose of this discussion is for students to think about ways to approach comparing two groups as well as have an opportunity to review dot plots, measures of center, and measures of variation from prior grades.

Ask,

💬 *"What are some ways we can compare groups of things?"*

At this stage, students are only expected to informally compare the groups. Although a consistent general rule for comparing groups will be introduced in later lessons, this activity is about getting a general idea that some groups (like the gymnastics and volleyball teams) have a rather clear difference, while others (like the tennis and badminton teams) may be more alike.

Ask students about the distribution of the data shown in the dot plots. Make sure to highlight the shape, center, and spread. Review how to find the **mean** as a measure of the center of a data set. Review how to calculate the **mean absolute deviation** (MAD) as a measure of the variability of a data set. Students may mention median and **interquartile range** (IQR) as other ways to measure center and variability. Although median and IQR are not needed in this activity, it may be useful to review how to calculate those values as well. Both measures of center and variability will be used later in the unit.

Introduce the idea of judging how much two data sets overlap.

Activity 2
Family Heights

15 min

Activity Narrative

In this activity, students use a measure of center and a measure of variability to compare two groups more formally. In the discussion following the activity involving describing the difference of the measures of center as a multiple of the variability, students are shown one quantifiable method of determining whether the two groups are relatively close or relatively very different. The important idea for students to grasp from this activity is that the measures of center and measures of variability of the groups work together to give an idea of how similar or different the groups are.

Monitor for groups who use different strategies to compare the heights. Here are some strategies from less to more formal:

1. Create dot plots of the data and look at the overlap visually.
2. Compute measures of center to compare the groups numerically.
3. Also compute measures of variability to compare the measures of center for each group accounting for the spread of values.

Allow students access to calculators to compute measures of center and measures of variability so that students can use those values without using too much time calculating in other ways.

Launch

Keep students in groups of 2.

Allow students 1 minute of quiet think time to examine ways of approaching the problem followed by partner work time and a whole-class discussion.

Select students with different strategies, such as those described in the Activity Narrative, to share later.

Student Task Statement

Compare the heights of these two families. Explain or show your reasoning.

- The heights (in inches) of Noah's family members: 28, 39, 41, 52, 63, 66, 71
- The heights (in inches) of Jada's family members: 49, 60, 68, 70, 71, 73, 77

Sample response:

The mean height of Noah's family members is about 51.43 inches. The mean height of Jada's family members is about 66.86 inches, which is greater than for Noah's family. The difference between these means is 66.86 – 51.43, or 15.43 inches.

There is also some overlap in the heights of the two families. The tallest person in Noah's family is 22 inches taller than the shortest person in Jada's family.

Instructional Routines

5 Practices

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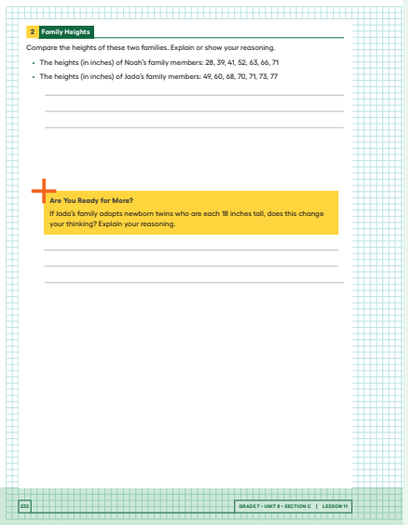
Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.
Activate or supply background knowledge. To help students recall the terms measure of center, ask what terms they have used to represent typical values such as mean or median.

Supports accessibility for: Memory, Language

Student Workbook


2 Family Heights

Compare the heights of these two families. Explain or show your reasoning.

- The heights (in inches) of Noah's family members: 28, 39, 41, 52, 63, 66, 71
- The heights (in inches) of Jada's family members: 49, 60, 68, 70, 71, 73, 77

Are You Ready for More?

If Jada's family adopts newborn twins who are each 18 inches tall, does this change your thinking? Explain your reasoning.

Are You Ready for More?

If Jada's family adopts newborn twins who are each 18 inches tall, does this change your thinking? Explain your reasoning.

Sample responses:

- Yes. After the addition of the new babies, the mean height for Jada's family drops to 56 inches, which is much closer to the mean height of Noah's family of about 51 inches.
- No. With the addition of the new babies, the mean does not make sense to use as a measure of center for Jada's family anymore. We should use the median. The median of Noah's family is 52 inches, and the new median for Jada's family is 68 inches. There is still a large difference between the two groups.

Activity Synthesis

The purpose of this discussion is for students to begin to be more formal in their comparison of data from different groups. In particular, a general rule is established that will be used in this unit.

Invite previously selected groups to share how they compared the groups. Sequence the discussion of the strategies in the order listed in the Activity Narrative. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"How does the inclusion of a numerical value for a measure of center help the argument that Jada's family tends to be taller?"*

  By including calculated numbers, it can reduce bias about how different the groups are.

- *"In addition to the measure of center, how does a measure of variability help show how different the groups are?"*

  Along with the measure of center, it gives a number to how much overlap we expect from the groups.

Explain that the difference between the means is not enough information to know whether or not the data sets are very different. One way to express the amount of overlap is to divide the difference in means by the (larger) mean absolute deviation.

Demonstrate for students how to do this calculation for the volleyball and gymnastics teams:

- The difference in means is more than 6 times the measure of variability: $15.42 \div 2.46 \approx 6.3$.

Leave the calculation for the two teams displayed. Ask students to do the calculation for Jada's and Noah's families.

- The difference in means is a little more than 1 time the measure of variability: $15.43 \div 13.22 \approx 1.2$.

As a general rule, we will consider it a large difference between the data sets if the difference in means is more than twice the mean absolute deviation. If the mean absolute deviation is different for each group, use the larger one for this calculation.

For students who ask why twice the MAD is used rather than some other value, defer the question for later in the unit.

Activity 3: Optional
Track Length

10 min

Activity Narrative

Students begin by matching information about a set of data to its dot plot and calculating the mean and mean absolute deviation for the remaining dot plots. Then they compare the data sets pairwise using the mean and MAD values. In the discussion, students are introduced to a way to add extra information to dot plots to visualize the general rule given in the previous activity.

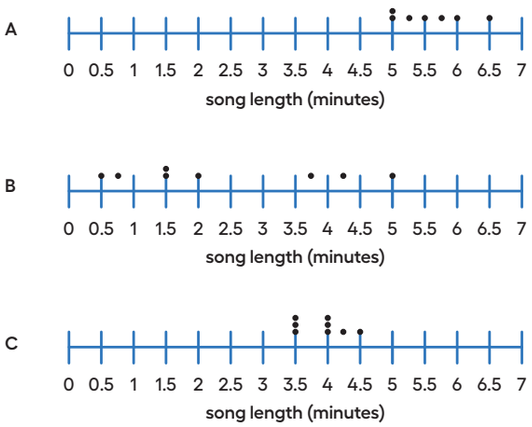
Launch

Keep students in groups of 2.

Give students 5 minutes of partner work time followed by a whole-class discussion.

Student Task Statement

Here are three dot plots that represent the lengths, in minutes, of songs on different albums.



1. One of these data sets has a mean of 5.57 minutes and another has a mean of 3.91 minutes.

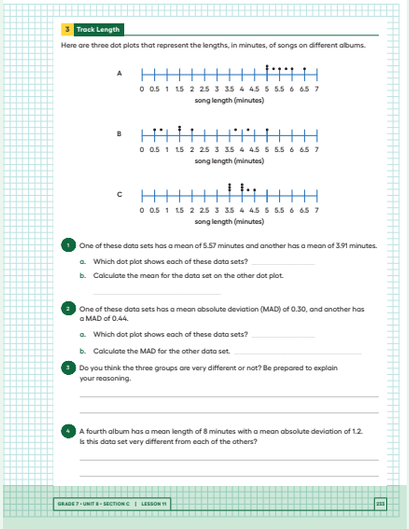
a. Which dot plot shows each of these data sets?

Dot Plot A has a mean of 5.57 minutes, and Dot Plot C has a mean of 3.91 minutes.

b. Calculate the mean for the data set on the other dot plot.

Dot Plot B has a mean of 2.41 minutes.

Student Workbook


3 Track Length

Here are three dot plots that represent the lengths, in minutes, of songs on different albums.

A
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7
song length (minutes)

B
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7
song length (minutes)

C
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7
song length (minutes)

1. One of these data sets has a mean of 5.57 minutes and another has a mean of 3.91 minutes.
   a. Which dot plot shows each of these data sets?
   b. Calculate the mean for the data set on the other dot plot.

2. One of these data sets has a mean absolute deviation (MAD) of 0.30, and another has a MAD of 0.44.
   a. Which dot plot shows each of these data sets?
   b. Calculate the MAD for the other data set.

3. Do you think the three groups are very different or not? Be prepared to explain your reasoning.

4. A fourth album has a mean length of 8 minutes with a mean absolute deviation of 1.2. Is this data set very different from each of the others?

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2. One of these data sets has a mean absolute deviation (MAD) of 0.30, and another has a MAD of 0.44.

a. Which dot plot shows each of these data sets?

Dot Plot C has a MAD of 0.30 minutes, and Dot Plot A has a MAD of 0.44 minutes.

b. Calculate the MAD for the other data set.

Dot Plot B has a MAD of 1.44 minutes.

3. Do you think the three groups are very different or not? Be prepared to explain your reasoning.

Sample response: Because Dot Plot B has such a large mean absolute deviation, it is hard to say that it is very different from Dot Plot C, but it is very different from Dot Plot A. Because the MADs are much smaller for the other two albums,
it is easier to say that the length of the tracks from Dot Plot C and Dot Plot A are very different.

4. A fourth album has a mean length of 8 minutes with a mean absolute deviation of 1.2. Is this data set very different from each of the others?

Sample response: The large mean for the fourth album makes it very different from the others, even with a large MAD.

Activity Synthesis

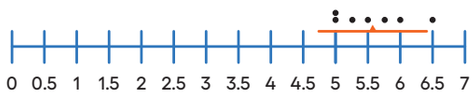
The purpose of the discussion is to help students visualize the calculations they performed.

Ask,

💬 *"Before calculating any of the values, would you have guessed that dot plots B and C would not be very different, but A would be very different from the other two? Explain your reasoning."*

Yes, since there is some overlap between the data of dot plots B and C, but the data in A does not overlap very much with B and not at all with C.

Demonstrate a technique using Dot Plot A for students to see the general rule on the dot plots. Mark the mean of the data set on the dot plot with a triangle. Then draw a line segment from the triangle so that its length is 2 MADs in each direction. On Dot Plot A, draw a triangle at 5.57 and a line segment from 4.69 (5.57 − 2 · 0.44) to 6.45 (5.57 + 2 · 0.44) as in the image here.



Tell students to draw a triangle for the mean and a segment representing 2 MADs in each direction on all three dot plots. Ask students,

💬 *"How can you use these markings to see the general rule from these pictures?"*

If any of the means are within 2 MADs of the mean of another group, then there is not a large difference between the two groups.

Lesson Synthesis

Consider asking these questions for discussion:

- *"What does a dot plot tell you?"*

  It shows the data over a number line to give a visual understanding of the frequency for each value.

- *"What are some measures of center, and how are they calculated?"*

  A mean is the sum of all values divided by the number of values. A median is the middle number in the list when the list is ordered.

- *"Why is a measure of center useful for comparing two groups?"*

  It gives a single value that is typical for a group, so it is easier to compare than using all of the data for each group.

- *"Why is a measure of variability also needed when comparing two groups?"*

  While a measure of center is helpful in getting started, if the data are very spread out, the center may not be very typical, and two groups may overlap a lot in their values.

- *"What is the general rule we will use to determine whether two groups have a large difference or not?"*

  If the difference between the two measures of center are more than double the greater measure of variability, then the two groups are meaningfully different.

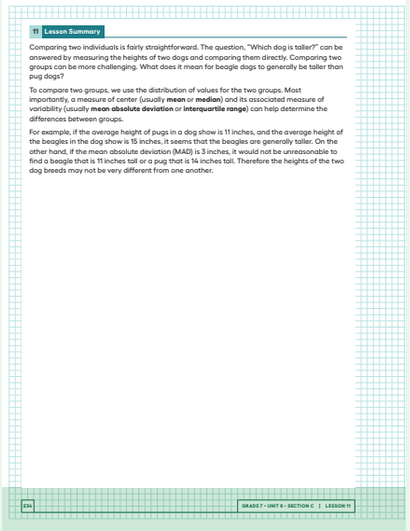
Lesson Summary

Comparing two individuals is fairly straightforward. The question, "Which dog is taller?" can be answered by measuring the heights of two dogs and comparing them directly. Comparing two groups can be more challenging. What does it mean for beagle dogs to generally be taller than pug dogs?

To compare two groups, we use the distribution of values for the two groups. Most importantly, a measure of center (usually **mean** or **median**) and its associated measure of variability (usually **mean absolute deviation** or **interquartile range**) can help determine the differences between groups.

For example, if the average height of pugs in a dog show is 11 inches, and the average height of the beagles in the dog show is 15 inches, it seems that the beagles are generally taller. On the other hand, if the mean absolute deviation (MAD) is 3 inches, it would not be unreasonable to find a beagle that is 11 inches tall or a pug that is 14 inches tall. Therefore the heights of the two dog breeds may not be very different from one another.

Student Workbook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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Prices of Homes

5 min

Student Task Statement

Noah's parents are interested in moving to another part of town. They look up all the prices of the homes for sale and record them in thousands of dollars.

Neighborhood 1 (mean: 75, MAD: 15)

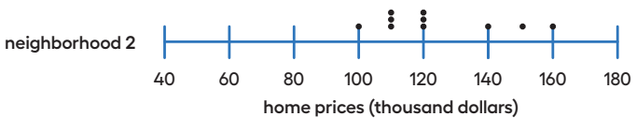
Neighborhood 2 (mean: 124, MAD: 15.6)

neighborhood 1

80 55 80 120 60 90 60 80 55 70

neighborhood 2

110 120 160 110 100 110 140 150 120 120



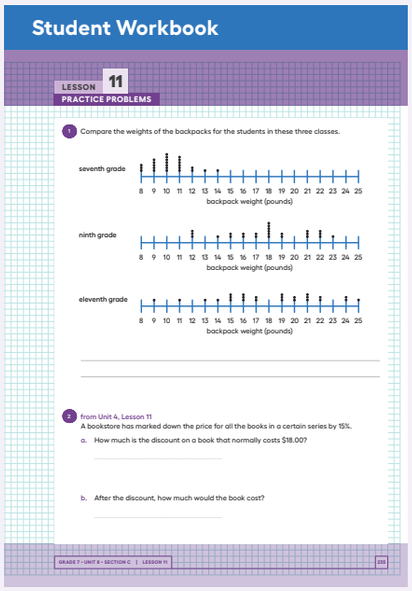
Decide whether the two groups are very different or not.

They are very different since the difference in means is 49, which is more than 3 times the larger MAD.

Lesson 11

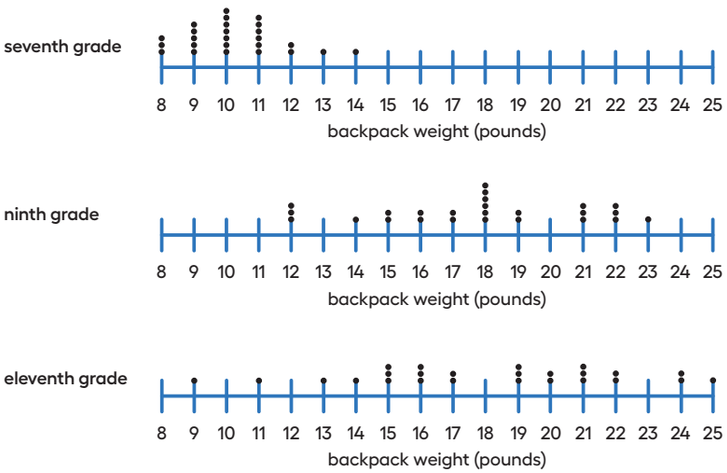
Practice Problems

4 Problems



Problem 1

Compare the weights of the backpacks for the students in these three classes.



Sample response:

The backpacks of the seventh graders tend to weigh less than the backpacks of the ninth graders. A typical weight for the seventh graders' packs is about 10 pounds, compared to a typical weight of about 18 pounds for the ninth graders' packs. The weights were also less variable for the seventh graders than the ninth graders. Similar things can be said when comparing seventh and eleventh graders' backpacks.

The distribution of weights for the ninth graders and the eleventh graders are similar, but the eleventh graders had a slightly larger spread. Both distributions are centered at around 18 pounds, and backpack weights varied quite a bit from student to student in both of these grades.

Problem 2

from Unit 4, Lesson 11

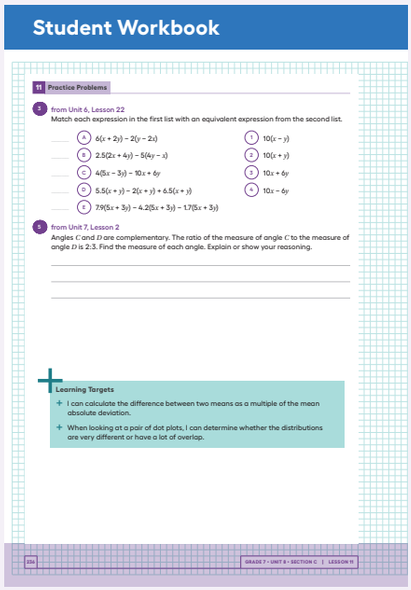
A bookstore has marked down the price for all the books in a certain series by 15%.

a. How much is the discount on a book that normally costs $18.00?

$2.70, because $18 \cdot 0.15 = 2.7$

b. After the discount, how much would the book cost?

$15.30, because $18 - 2.7 = 15.3$


Student Workbook

11 Practice Problems

3 from Unit 6, Lesson 22
Match each expression in the first list with an equivalent expression from the second list.

A $6(x + 2y) - 2(y - 2x)$ — 1 $10(x - y)$
B $2.5(2x + 4y) - 5(4y - x)$ — 2 $10(x + y)$
C $4(5x - 3y) - 10x + 6y$ — 3 $10x + 6y$
D $5.5(x + y) - 2(x + y) + 6.5(x + y)$ — 4 $10x - 6y$
E $7.9(5x + 3y) - 4.2(5x + 3y) - 1.7(5x + 3y)$

4 from Unit 7, Lesson 2
Angles C and D are complementary. The ratio of the measure of angle C to the measure of angle D is 2:3. Find the measure of each angle. Explain or show your reasoning.

Learning Targets
+ I can calculate the difference between two means as a multiple of the mean absolute deviation.
+ When looking at a pair of dot plots, I can determine whether the distributions are very different or have a lot of overlap.

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Problem 3

from Unit 6, Lesson 21

Match each expression in the first list with an equivalent expression from the second list.

2	**A.** $6(x + 2y) - 2(y - 2x)$	**1.** $10(x - y)$
1	**B.** $2.5(2x + 4y) - 5(4y - x)$	**2.** $10(x + y)$
4	**C.** $4(5x - 3y) - 10x + 6y$	**3.** $10x + 6y$
2	**D.** $5.5(x + y) - 2(x + y) + 6.5(x + y)$	**4.** $10x - 6y$
3	**E.** $7.9(5x + 3y) - 4.2(5x + 3y) - 1.7(5x + 3y)$	

Problem 4

from Unit 7, Lesson 2

Angles C and D are complementary. The ratio of the measure of angle C to the measure of angle D is 2:3. Find the measure of each angle. Explain or show your reasoning.

Angle C: 36°, Angle D: 54°

Sample reasoning: The two angle measures must add to 90° and since $\frac{2}{5}$ of 90° comes from angle C, it must have measure 36°. Then angle D comes from 90 − 36 = 54.