Designing Simulations

Goals

- Describe a multi-step experiment that could be used to simulate a compound event in a realworld situation, and justify (orally and in writing) that it represents the situation.
- Perform a simulation to estimate the probability of a compound event, and explain (orally and in writing) how the simulation could be improved.

Learning Target

I can design a simulation to estimate the probability of a multi-step real-world situation.

Lesson Narrative

In this lesson, students see that the probability of compound events can also be estimated using simulations. The last activity in this lesson is a culmination of all the work students have done with probability in this unit: each group works to design a simulation for a different situation. Students strategically choose tools like number cubes, spinners, or blocks to model chance experiments in the situations they are given. This is also an opportunity for students to practice communicating precisely about how their simulation is conducted and what their outcomes represent.

Student Learning Goal

Let's simulate some real-life scenarios.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

 MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

Math Talk

Required Materials

Materials to Gather

- Coins: Activity 1
- Compasses: Activity 2
- Math Community Chart: Activity 2
- Number cubes: Activity 2
- Paper bags: Activity 2
- Paper clips: Activity 2
- Protractors: Activity 2
- · Scissors: Activity 2
- Snap cubes: Activity 2
- Straightedges: Activity 2

Required Preparation

Activity 1:

Every 3 students need 2 coins

Activity 2:

Groups will need access to number cubes, protractors, rulers, compasses, paper clips, bags, snap cubes, and scissors to simulate their scenarios.

Lesson Timeline



Warm-up

10 min

Activity 1

20 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Warm-up

Math Talk: Division



Activity Narrative

This *Math Talk* focuses on dividing sums. It encourages students to think about division problems with the same solution and to rely on the structure of previous solutions to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students calculate means.

To divide sums with more terms, students need to look for and make use of structure.

In describing their strategies, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

$$A.(4.2 + 3) \div 2$$

3.6

Sample reasoning: $4.2 \div 2 + 3 \div 2$ or $7.2 \div 2$.

B.
$$(4.2 + 3.1 + 3.5) \div 3$$

3.6

Sample reasoning: Compared to the first problem, there is an additional 3.6 in the sum (0.1 added to 3 and another 3.5), and the total is divided by 3, so it should have the same solution (or $10.8 \div 3$).

$$C.(4.2 + 3.1 + 3.5 + 3.6) \div 4$$

3.6

Sample reasoning: Because there is an additional 3.6 in the addends from the previous problem, and there are now four addends with a divisor of 4, the result is the same as in the previous problem (or $14.4 \div 4$).

D.
$$(4.2 + 3.1 + 3.5 + 3.6 + 3.6) \div 5$$

3.6

Sample reasoning: Again, 3.6 is added to the numerator, and the total is divided by I more than the previous problem, so it has the same solution (or $18 \div 5$).

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.



Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization



Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Breeding Mice



Activity Narrative

In this activity, students revisit the idea of simulating real-life situations with chance experiments. In the next activity, they will design and run their own simulations for situations that involve multiple steps. Here, students are asked to use their understanding of experiments that have multiple steps to simulate a single part of a larger simulation: They flip two coins to represent a single offspring from a pair of mice. Since the outcome probabilities of the simulation and the real-life situation are the same, this is another option for creating simulations that represent real-life scenarios.

Launch

Arrange students in groups of 3. Provide 2 coins for each group.

Ask students:

"When flipping two coins, what is the probability of both landing heads up?"



For context, it might be helpful to explain that mice are often used in science experiments since they have similar genetics to humans, but are easier to maintain. Setting up a mating to work with a new generation of mice with specific combinations of genes can be costly and time-consuming, so it can help to simulate some outcomes before actually beginning the experiment. The word "offspring" refers to children.

Give students 5–7 minutes for group work, and follow with a whole-class discussion.

Student Task Statement

A scientist is studying the genes that determine the color of a mouse's fur. When two mice with brown fur breed, there is a 25% chance that each baby will have white fur. For the experiment to continue, the scientist needs at least 2 out of 5 baby mice to have white fur.

To simulate this situation, a coin can be flipped twice for each baby mouse.

- If the coin lands heads up both times, it represents a mouse with white fur.
- Any other result represents a mouse with brown fur.
- **1.** Each group member will simulate the mice having 5 baby mice three times. Write your own results for the fur color of the mice in the table.

Sample response:

	mouse 1	mouse 2	mouse 3	mouse 4	mouse 5	Do at least 2 have white fur?
simulation 1	ь	w	ь	Ь	Ь	no
simulation 2	ь	Ь	ь	Ь	w	no
simulation 3	ь	w	Ь	w	Ь	yes

2. Based on the results from everyone in your group, estimate the probability that the scientist's experiment will be able to continue.

Sample response: I estimate the probability to be $\frac{3}{9}$ since 3 of our group's 9 simulations had at least 2 white offspring.

3. How could you improve your estimate?

Sample response: If we did more trials, I would expect the estimate to improve.

Are You Ready for More?

For a certain pair of mice, the genetics show that each offspring has a probability of $\frac{1}{16}$ that it will be albino. Describe a simulation that could be used that would estimate the probability that at least 2 of the 5 offspring are albino.

Sample response: Flip 4 coins. If they are all heads, that offspring is albino. Do this 5 times to represent the 5 offspring. Repeat this process many times to simulate many groups of 5 offspring, and estimate the probability based on the cumulative fraction of groups that have at least 2 albino offspring.

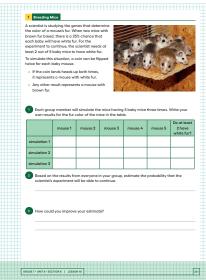
Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.

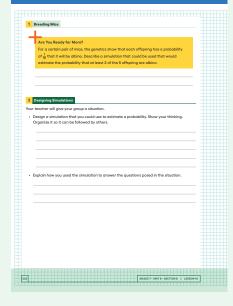
Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after 3–5 minutes of work time.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook



Student Workbook



Activity Synthesis

The purpose of the discussion is for students to articulate why the simulation is appropriate and think about other methods of simulating the same situation.

Consider asking these questions for discussion:

"How could we get a better estimate than what you got in your group?"
Repeat the experiment many more times or combine the data from the class.

Collect data from the class to find a better estimate.

- \bigcirc "What is the probability using the class data?"

 Answers vary. For reference, the actual probability is $\frac{47}{128} \approx 0.37$.
- "Notice that we used a two-part experiment (flipping two coins) to represent a single thing (one offspring). Why was this OK to do?"

The probability of getting HH on two coins is the same as the probability of getting a single offspring with white fur.

"Can you think of another method that would work to simulate a single offspring?"

We could use a spinner with 25% of the circle labeled "white" and 75% labeled "brown," or one white block and three brown blocks in a bag.

Activity 2

Designing Simulations

20 min

Activity Narrative

In this activity, each group is assigned a situation for which they will design and perform a simulation to estimate the probability. Students will give a short presentation on the methods and results of their simulation for the class after they have designed and run the simulation. Students will need to attend to precision for the simulation method they chose. At this stage, students have experienced a large number of simulation methods and should be able to design their own to represent the situations using the appropriate tools.



Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Arrange students in groups of 3. A copy of the Designing Simulations Cards can be found in the student workbook. Provide access to materials for simulation such as number cubes, compasses, protractors, rulers, paper bags, colored snap cubes, scissors, and coins.

Give students 15 minutes for group work, and follow with a whole-class discussion.

Student Task Statement

Your teacher will give your group a situation.

- Design a simulation that you could use to estimate a probability. Show your thinking. Organize it so it can be followed by others.
- Explain how you used the simulation to answer the questions posed in the situation.

Sample responses:

Simulation 1

- Flip 5 coins. Heads represents a unicorn and tails represents a dragon.

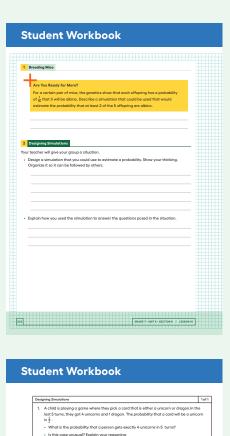
 If the coins land with 4 heads and I tail, it will match the situation.
- After doing this IO times, I of them had this result, so the probability should be about O.I. It is fairly unusual.

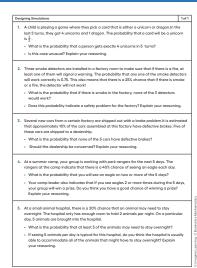
Simulation 2

- Spin 3 spinners that each have 75% of the circle marked as "working" and 25% as "not working." If all 3 of the spinners land on "not working," the fire is undetected.
- After doing this IO times, this never happened. Since it is possible, the
 estimate should be between O and O.I. It may not be a safety problem for
 the factory since fires are so rare and when there is a fire, at least one
 of the detectors will usually work.

Simulation 3

- In a bag, there are 9 slips of paper that say "brakes work" and I that
 says "defective brakes." Draw one paper out, replace it, and repeat for
 each of the five cars. If all 5 cars have working brakes, the dealership
 doesn't have to be concerned.
- After doing this IO times, this happened 6 times. The dealership should be concerned since having defective brakes is a big problem, and the probability that none of the cars have this problem is only about 0.6.





Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Internalize Comprehension.

Use color and annotations to illustrate student thinking. As students show their representations of simulations and explain their reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visualspatial processing; Conceptual processing

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students justifying their simulation from their interpretation of the situation: "This simulation was designed so that ..." and "The simulation and the actual event have the same probabilities because ..."

Advances: Speaking, Writing, Representing

Simulation 4

- In a bag there are 6 slips of paper that say "eagle" and 4 that say "no eagle." Draw one paper out, replace it, and repeat for each of the five days. If at least 2 days have an eagle, we will get the prize.
- After doing this IO times, this happened 7 times. There is a probability of about 0.7 that we will get the prize. I think the probability of 0.7 means we have a pretty good chance of seeing eagles 2 or more times and winning the prize.

Simulation 5

- In a bag there are 2 slips of paper that say "stay overnight" and 8 that say "release." Draw one paper out, replace it, and repeat for each of the five animals. If at least 3 of the animals have to stay overnight, the hospital will have a problem.
- After doing this IO times, this happened once. There is a probability of about O.I that the hospital will not have enough space for the animals.
 A probability of O.I that the hospital runs out of room means that the hospital will usually have enough room for the animals that come in each day.

Activity Synthesis

Ask each group to share their situation, their method of simulating the situation, and their results. Students should explain why their chosen method works to simulate the situation they were given. In particular, all important outcomes should be represented with the same probability as stated in the situation.

If all groups that have the same situation use the same simulation method, ask for ideas from the class about alternate methods that could be used for the situation.

For reference, the computed probabilities for each situation are:

1.
$$\frac{5}{32} \approx 0.16$$

2.
$$\frac{1}{64} \approx 0.02$$

3.
$$\frac{59049}{100000} \approx 0.59$$

4.
$$\frac{2072}{3125} \approx 0.66$$

5.
$$\frac{181}{3125} \approx 0.06$$

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

"I noticed our norm '____' in action today and it really helped me/my group because ..."

Lesson Synthesis

Consider these questions for discussion:

"What are some things you had to consider when designing your simulation?"

Among other things, the probability of the actual portion of the event should match the probability of the associated simulated event.

"What did you learn from the simulations the other groups did?"

"Were the results of any of the simulations surprising?"

"Why would it make sense to design and run a simulation rather than repeat the actual experiment multiple times?"

A simulation is useful when the actual experiment is costly in time or resources or cannot be controlled or repeated.

Lesson Summary

Many real-world situations are difficult to repeat enough times to get an estimate for a probability. If we can find probabilities for parts of the situation, we may be able to simulate the situation using a process that is easier to repeat.



For example, if we know that each egg of a fish in a science experiment has a 13% chance of having a mutation, how many eggs do we need to collect to make sure we have 10 mutated eggs? If getting these eggs is difficult or expensive, it might be helpful to have an idea about how many eggs we need before trying to collect them.

We could simulate this situation by having a computer select random numbers between 1 and 100. If the number is between 1 and 13, it counts as a mutated egg. Any other number would represent a normal egg. This matches the 13% chance of each fish egg having a mutation.

We could continue asking the computer for random numbers until we get 10 numbers that are between 1 and 13. How many times we asked the computer for a random number would give us an estimate of the number of fish eggs we would need to collect.

To improve the estimate, this entire process should be repeated many times. Because computers can perform simulations quickly, we could simulate the situation 1.000 times or more.



Responding To Student Thinking

Press Pause

By this point in the unit, there should be some student mastery of finding the probability of compound events. If students struggle, make time to revisit the work in the activity referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 8, Lesson 8, Activity 2 How Many Necklaces?

Cool-down

The Best Power-Up



Student Task Statement

Elena is programming a video game. She needs to simulate the power-up that the player gets when they reach a certain level. The computer can run a program to return a random integer between 1 and 100. Elena wants the best power-up to be given 15% of the time.

Explain how Elena could use the computer to simulate the player getting the best power-up at least 2 out of 3 times.

Sample response: Elena could have the computer generate 3 random integers between I and IOO. If at least 2 of the numbers are between I and I5, then the player gets the best power-up at least twice. She could repeat this process many times and estimate the probability as the proportion of trials for which at least 2 of the numbers are between I and I5.

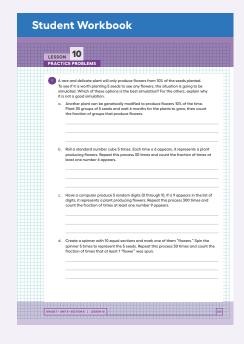
Practice Problems

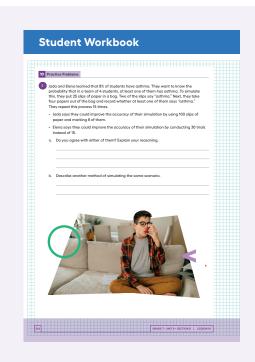
4 Problems

Problem 1

A rare and delicate plant will only produce flowers from 10% of the seeds planted. To see if it is worth planting 5 seeds to see any flowers, the situation is going to be simulated. Which of these options is the best simulation? For the others, explain why it is not a good simulation.

- **a.** Another plant can be genetically modified to produce flowers 10% of the time. Plant 30 groups of 5 seeds and wait 6 months for the plants to grow, then count the fraction of groups that produce flowers.
 - Sample response: Using another plant will probably be costly and take a long time, so it is not a good simulation.
- **b.** Roll a standard number cube 5 times. Each time a 6 appears, it represents a plant producing flowers. Repeat this process 30 times and count the fraction of times at least one number 6 appears.
 - Sample response: Rolling the standard number cube does not match the probability for each seed, so it will not produce a good simulation.
- **c.** Have a computer produce 5 random digits (0 through 9). If a 9 appears in the list of digits, it represents a plant producing flowers. Repeat this process 300 times and count the fraction of times at least one number 9 appears.
 - Sample response: Using the computer is the best simulation.
- **d.** Create a spinner with 10 equal sections and mark one of them "flowers." Spin the spinner 5 times to represent the 5 seeds. Repeat this process 30 times and count the fraction of times that at least 1 "flower" was spun.
 - Sample response: The spinner idea would work as a simulation, but it has only 30 trials and the computer has 300.





Problem 2

Jada and Elena learned that 8% of students have asthma. They want to know the probability that in a team of 4 students, at least one of them has asthma. To simulate this, they put 25 slips of paper in a bag. Two of the slips say "asthma." Next, they take four papers out of the bag and record whether at least one of them says "asthma." They repeat this process 15 times.

- Jada says they could improve the accuracy of their simulation by using 100 slips of paper and marking 8 of them.
- Elena says they could improve the accuracy of their simulation by conducting 30 trials instead of 15.
- **a.** Do you agree with either of them? Explain your reasoning.

I agree with Elena, but not with Jada.

Sample reasoning: Jada's suggestion would have the same probability of a success as the original simulation, so it would work, but would not produce more accuracy. More trials help keep uncommon outcomes from having a big impact on the estimated probability.

b. Describe another method of simulating the same scenario.

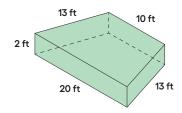
Sample response: Use a random digit list. For each trial, take 4 pairs of digits (00 through 99). Repeat the simulation many times. Use the proportion of times the pairs OI through O8 appeared in the outcomes to estimate the probability that at least one student on the team has asthma.

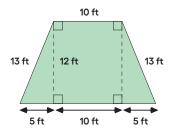
LESSON 10 • PRACTICE PROBLEMS

Problem 3

from Unit 7, Lesson 13

The three-dimensional figure is a trapezoidal prism. The trapezoid represents its base. Find the volume of this prism.





360 ft³

The area of the trapezoidal base is 180 ft², the height is 2 ft, and $180 \cdot 2 = 360$.

Problem 4

from Unit 6, Lesson 21

Match each expression in the first list with an equivalent expression from the second list.

A.
$$(8x + 6y) + (2x + 4y)$$

1.
$$10(x - y)$$

B.
$$(8x + 6y) - (2x + 4y)$$

2.
$$10(x - y)$$

C.
$$(8x + 6y) - (2x - 4y)$$

3.
$$6(x-\frac{1}{3}y)$$

D.
$$8x - 6y - 2x + 4y$$

E.
$$8x - 6y + 2x - 4y$$

5.
$$8x + 6y - 2x + 4y$$

F.
$$8x - (-6y - 2x + 4y)$$

