Lesson 1

Projecting and Scaling

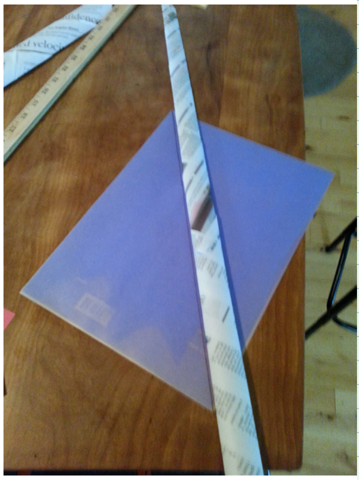
Goals

- Describe (orally) features of scaled copies of a rectangle.
- Identify rectangles that are scaled copies of one another.

Learning Target

I can decide if one rectangle is a scaled copy of another rectangle.

Lesson Narrative



In this lesson, students build on the ideas from grade 7 that side lengths of scaled copies are proportional, and that the constant of proportionality relating the original lengths to the corresponding lengths in the scaled copy is the scale factor.

Students arrange a set of scaled copies of rectangles to share one angle and observe that the opposite vertices all lie on the same line. In future lessons, students will see that this is an example of a dilation, a geometric process that produces scaled copies, though it is not necessary that they make this connection in this lesson. A dilation scales the distance of all points from the center of dilation, which in this context is the shared vertex of the shared angle. An optional activity recalls work from grade 7 about scaled copies of rectangles.

Math Community

Today's activity is for students to individually reflect on the norms generated so far. During *the Cool-down*, students provide feedback on the norms, sharing those they agree with and those they feel need revision or removal. These suggestions will inform the next version of the classroom norms.

Student Learning Goal

Let's explore scaling.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR2: Collect and Display

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Blank paper: Activity 1
- Long straightedge: Activity 1
- Scissors: Activity 1
- Rulers: Activity 2

Required Preparation

Warm-up:
Make a space for students to place their sticky notes at the end of the *Warm-up*. For example, hang a sheet of chart paper on a wall near the door.

Activity 1:
If creating sets of rectangles for students ahead of time, prepare and label one set A–E for each pair of students:

- A: One full sheet, 8.5" x 11"
- B: One half sheet, 8.5" x 5.5"
- C: One quarter sheet, 4.25" x 5.5"
- D: One eighth sheet, 4.25" x 2.75"
- E: One sixteenth sheet, 2.125" x 2.75"

Each pair of students will also need a long straightedge (at least 14 inches long), such as a meter or yardstick. Alternatively, long straightedge can be created from newspaper by folding it as shown.

Lesson Timeline

10 min	Warm-up
20 min	Activity 1
10 min	Activity 2
10 min	Lesson Synthesis
5 min	Cool-down (Assessment)

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Warm-up

Math Talk: Remembering Fraction Division

10 min

Activity Narrative

This is the first *Math Talk* activity in the course. See the launch for extended instructions for facilitating this activity successfully.

This *Math Talk* focuses on reviewing fraction division. It encourages students to think about the meaning of division and to rely on what they know about the structure of mixed numbers to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students perform similar calculations dividing a mixed number by a whole number. While many strategies may emerge, the focus of these problems is for students to recall and rehearse a reliable way to divide a mixed number by a whole number.

In explaining their strategy, students need to be precise in their word choice and use of language.

Launch

This is the first time students do the *Math Talk* instructional routine in this course, so it is important to explain how it works before starting.

Explain that a *Math Talk* has four problems, revealed one at a time. For each problem, students have a minute to quietly think and are to give a signal when they have an answer and a strategy. The teacher then selects students to share different strategies (likely 2–3, given limited time), and might ask questions such as “Who thought about it in a different way?” The teacher then records the responses for all to see and might ask clarifying questions about the strategies before revealing the next problem.

Consider establishing a small, discreet hand signal that students can display when they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A. $6\frac{1}{4} \div 2$

$3\frac{1}{8}$ or $\frac{25}{8}$. Sample reasoning: $6 \div 2 = 3$ and $\frac{1}{4} \div 2 = \frac{1}{8}$

B. $10\frac{1}{7} \div 5$

$2\frac{1}{35}$ or $\frac{71}{35}$. Sample reasoning: $10 \div 5 = 2$ and $\frac{1}{7} \div 5 = \frac{1}{35}$

C. $4\frac{1}{3} \div 8$

$\frac{13}{24}$. Sample reasoning: $4\frac{1}{3} = \frac{13}{3}$, and $\frac{13}{3} \div 8 = \frac{13}{3} \cdot \frac{1}{8} = \frac{13}{24}$

D. $8\frac{1}{2} \div 11$

$\frac{17}{22}$. Sample reasoning: $8\frac{1}{2} = \frac{17}{2}$, and $\frac{17}{2} \div 11 = \frac{17}{2} \cdot \frac{1}{11} = \frac{17}{22}$

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ___'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Math Community

After the *Warm-up,* display the Math Community Chart. Remind students that norms are agreements that everyone in the class shares responsibility for, so it is important that everyone understands the intent of each norm and can agree with it. Tell students that today's *Cool-down* includes a question asking for feedback on the drafted norms. This feedback will help identify which norms the class currently agrees with and which norms need revising or removing.

Activity 1

Sorting Rectangles

20 min

Activity Narrative

The purpose of this activity is to investigate scaled copies of rectangles. Students see that if one rectangle is a scaled copy of another, then they can be arranged so that the diagonal of the larger rectangle contains the diagonal of the smaller rectangle.

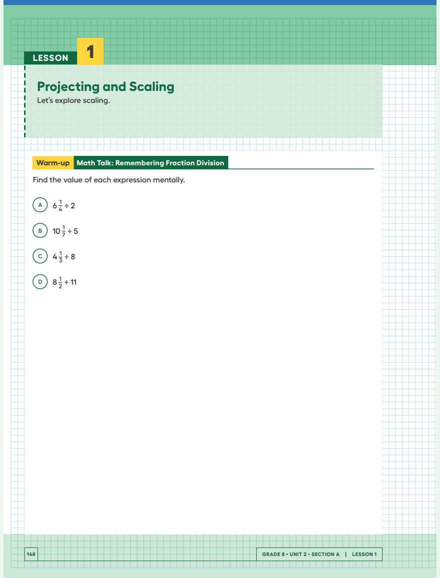
Encourage students to use what they know about how the rectangles were created rather than measuring each new rectangle (which is likely to introduce errors). Making use of structure helps students build their understanding of what makes something a scaled copy.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy, such as "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Student Workbook



Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display. This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Launch)

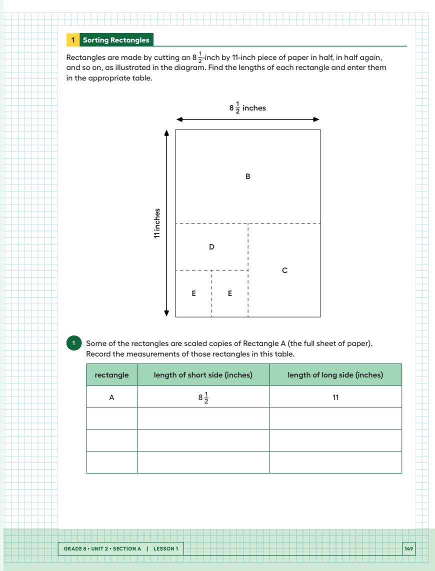
Action and Expression: Provide Access for Physical Action.
Provide access to pre-cut materials to reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Supports accessibility for: Fine Motor Skills, Organization, Visual-Spatial Processing

Building on Student Thinking

Some students may forget how to check if two rectangles are scaled copies of one another. Prompt them to compare measurements of corresponding sides to see if they have the same scale factor. Students may recall that scaled copies have corresponding angles of the same measure, but they may not recall that equal angle measurements don't necessarily mean you have scaled copies.

Student Workbook



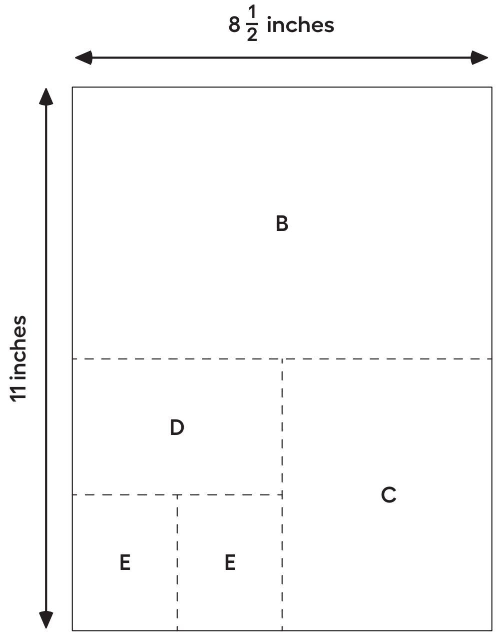
Launch

Arrange students in groups of 2. Provide each group with 2 whole sheets of paper, scissors, and a long straightedge. Direct students to create rectangles by folding and cutting an $8\frac{1}{2}$-inch by 11-inch piece of paper in half, in half again, and so on, as illustrated in the *Task Statement*. (Alternatively, these rectangles could be pre-cut and a set of 5 pre-cut rectangles provided to each group.)

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to determine which rectangles are scaled copies of the full sheet of paper (Rectangle A). Display words and phrases such as "Rectangle E is the same as Rectangle C but smaller" and "The side lengths of Rectangle C are half of the side lengths of Rectangle A."

Student Task Statement

Rectangles are made by cutting an $8\frac{1}{2}$-inch by 11-inch piece of paper in half, in half again, and so on, as illustrated in the diagram. Find the lengths of each rectangle and enter them in the appropriate table.



1. Some of the rectangles are scaled copies of Rectangle A (the full sheet of paper). Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)
A	$8\frac{1}{2}$	11
C	$4\frac{1}{4}$	$5\frac{1}{2}$
E	$2\frac{1}{8}$	$2\frac{3}{4}$

2. Some of the rectangles are *not* scaled copies of Rectangle A (the full sheet of paper). Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)
B	$5\frac{1}{2}$	$8\frac{1}{2}$
D	$2\frac{3}{4}$	$4\frac{1}{4}$

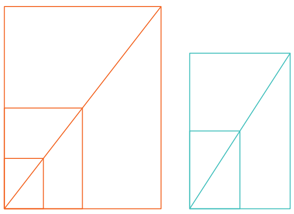
3. Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice?

Sample response: Each measurement in Triangle C is $\frac{1}{2}$ of the corresponding measurement in Triangle A. Each measurement in Triangle E is $\frac{1}{2}$ of the corresponding measurement in Triangle C, or $\frac{1}{4}$ of the corresponding measurement in Triangle A. The quotient of the lengths of a short side and a long side is $\frac{17}{22}$.

4. Look at the measurements for the rectangles that are not scaled copies of the full sheet. What do you notice?

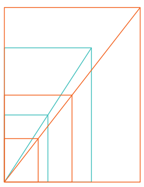
Sample response: Each measurement in Triangle D is $\frac{1}{2}$ of the corresponding measurement in Triangle B. The quotient of the lengths of a short side and a long side is $\frac{11}{17}$.

5. Stack the rectangles that are scaled copies of the full sheet so that they all line up at a corner, as shown in the diagram. Do the same with the other set of rectangles. On each stack, draw a line from the bottom left corner to the top right corner of the biggest rectangle. What do you notice?



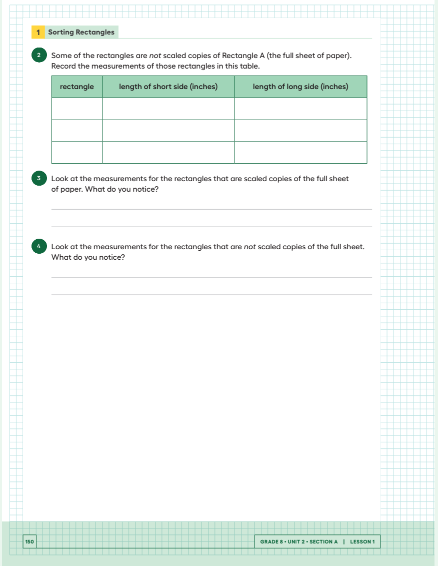
The diagonal goes through two vertices of each rectangle in the pile.

6. Stack *all* of the rectangles from largest to smallest so that they all line up at a corner. Compare the lines that you drew. Can you tell, from the drawn lines, which set each rectangle came from? What do you notice?



The diagonals of the second set lie above the diagonals of the first set.

Student Workbook



1 Sorting Rectangles

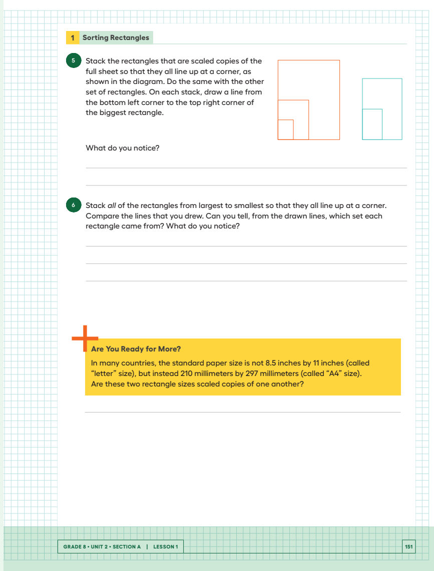
2 Some of the rectangles are *not* scaled copies of Rectangle A (the full sheet of paper). Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)

3 Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice?

4 Look at the measurements for the rectangles that are *not* scaled copies of the full sheet. What do you notice?

Student Workbook



1 Sorting Rectangles

5 Stack the rectangles that are scaled copies of the full sheet so that they all line up at a corner, as shown in the diagram. Do the same with the other set of rectangles. On each stack, draw a line from the bottom left corner to the top right corner of the biggest rectangle.

What do you notice?

6 Stack *all* of the rectangles from largest to smallest so that they all line up at a corner. Compare the lines that you drew. Can you tell, from the drawn lines, which set each rectangle came from? What do you notice?

Are You Ready for More?

In many countries, the standard paper size is not 8.5 inches by 11 inches (called "letter" size), but instead 210 millimeters by 297 millimeters (called "A4" size). Are these two rectangle sizes scaled copies of one another?

Are You Ready for More?

In many countries, the standard paper size is not 8.5 inches by 11 inches (called "letter" size), but instead 210 millimeters by 297 millimeters (called "A4" size). Are these two rectangle sizes scaled copies of one another?

No. Converting from millimeters to inches, A4 paper is about 8.27 inches by 11.69 inches. Since it is both taller and less wide than letter paper, it can not be a scaled copy.

Activity Synthesis

The purpose of this discussion is for students to describe features of scaled copies.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they decided that the $4\frac{1}{4}$-inch by $5\frac{1}{2}$-inch rectangle (C) was a scaled copy of the $8\frac{1}{2}$-inch by 11-inch rectangle (A). Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. For example, the display may say "Rectangle E is the same as rectangle C but smaller," and be revised to a more precise phrase, such as "Both dimensions of Rectangle E are multiplied by 2 to get the corresponding dimensions of Rectangle C."

Also discuss:

- *"How did you decide that the $5\frac{1}{2}$-inch by $8\frac{1}{2}$-inch rectangle (B) is not a scaled copy of the $8\frac{1}{2}$-inch by 11-inch rectangle (A)?"*

  There is no single number that you can multiply by $5\frac{1}{2}$ to get $8\frac{1}{2}$ and also multiply by $8\frac{1}{2}$ to get 11.

- *"What happened when we stacked all of the rectangles from largest to smallest so that they all lined up at a corner?"*

  The diagonals of the rectangles that are scaled copies of one another match up, creating 2 sets of scaled copies.

Activity 2: Optional
Scaled Rectangles

10 min

Activity Narrative

This optional activity revisits previous grade level standards by examining scaled copies that are created by evenly dividing a rectangle into smaller rectangles. Use this activity if students need additional work with scale factor. There are no given dimensions for any of the rectangles, so students must use their understanding of scale factors and the fact that the rectangles are divided evenly. This offers students additional opportunities to see and make use of structure.

Launch

Arrange students in groups of 2. Tell them they will examine a different set of rectangles and determine scale factors for pairs that are scaled copies of one another.

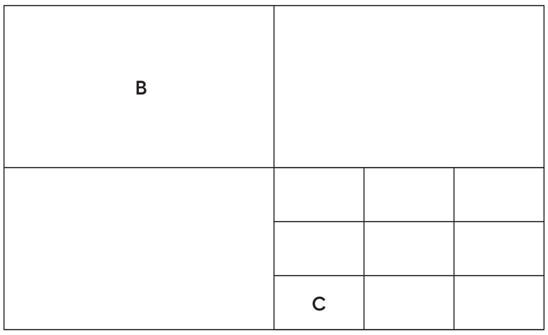
First, ask students to interpret what is meant by "evenly divided." (Rectangle R is cut exactly in half vertically and horizontally, and one of its quadrants is cut exactly in thirds vertically and horizontally.) Students may want to use a ruler to validate their understanding of what "evenly divided" means.

Give students 3–4 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

Here is a picture of Rectangle R, which has been evenly divided into smaller rectangles. Two of the smaller rectangles are labeled B and C.

Rectangle R



1. Is B a scaled copy of R? If so, what is the scale factor?

   Yes, the scale factor is $\frac{1}{2}$ (or equivalent).

2. Is C a scaled copy of B? If so, what is the scale factor?

   Yes, the scale factor is $\frac{1}{3}$ (or equivalent).

3. Is C a scaled copy of R? If so, what is the scale factor?

   Yes, the scale factor is $\frac{1}{6}$ (or equivalent).

Access for Multilingual Learners (Activity 2, Launch)

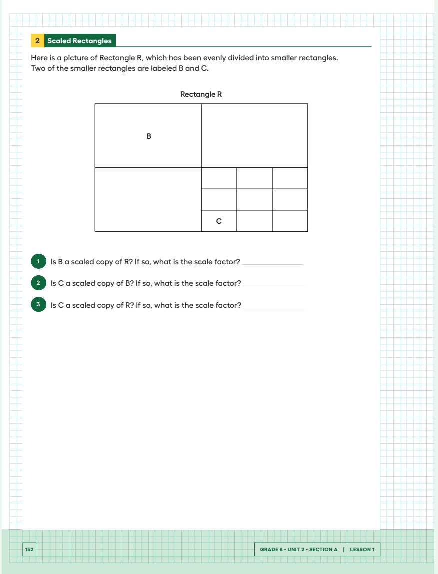
MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the image of Rectangle R without revealing the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask:

> *"What do these questions have in common? How are they different?"*

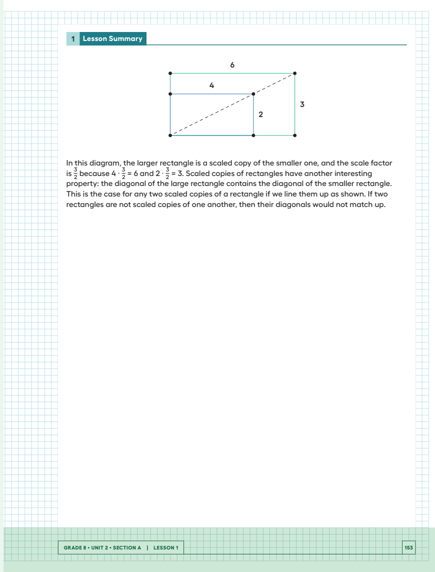
Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Student Workbook



Lesson Summary



Activity Synthesis

The goal of this discussion is to recall the connection between scale factor and a scaled copy. Discuss with students:

- *"Why is Rectangle B a scaled copy of Rectangle R?"*

  The length and width of Rectangle R have been multiplied by the same number because the rectangles were evenly divided.

- *"How are the scale factors from R to B and B to C related to the scale factor from R to C?"*

  The scale factor from R to C is the product of the scale factors from R to B and B to C.

- *"If we were to stack Rectangle R, Rectangle B, and Rectangle C so that they all shared one vertex, would the diagonal of Rectangle R contain the diagonals of Rectangles B and C?"*

  Yes.

Lesson Synthesis

The goal of this discussion is for students to describe some features of scaled copies. Here are some questions for discussion:

- *"How can we tell if one rectangle is a scaled copy of another?"*

  Every length in one rectangle is multiplied by the same scale factor to get the corresponding length in the other rectangle.

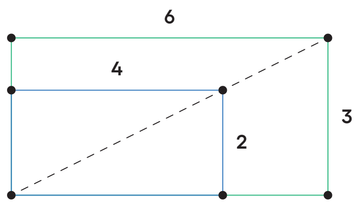
- *"Is it possible to make a scaled copy of a figure without straight sides, such as a circle?"*

  Yes, multiply the radius by the scale factor.

- *"When making a scaled copy of a figure, what properties of the figure change? What properties stay the same?"*

  Unless the scale factor is 1, the side lengths change and the angle measures stay the same.

Lesson Summary



In this diagram, the larger rectangle is a scaled copy of the smaller one, and the scale factor is $\frac{3}{2}$ because $4 \cdot \frac{3}{2} = 6$ and $2 \cdot \frac{3}{2} = 3$. Scaled copies of rectangles have another interesting property: the diagonal of the large rectangle contains the diagonal of the smaller rectangle. This is the case for any two scaled copies of a rectangle if we line them up as shown. If two rectangles are not scaled copies of one another, then their diagonals would not match up.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet. Make sure students know they can make suggestions for both student and teacher norms.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about.

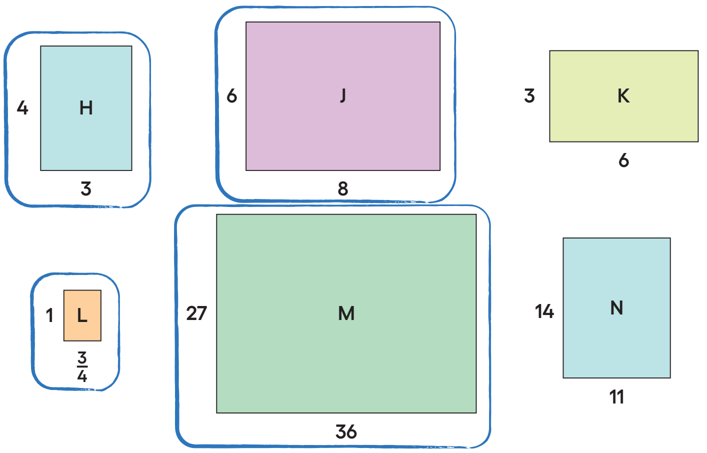
Cool-down

Scaled Copies

5 min

Student Task Statement

Rectangle G measures 9 inches by 12 inches. Which of these rectangles are scaled copies of Rectangle G?



Responding To Student Thinking

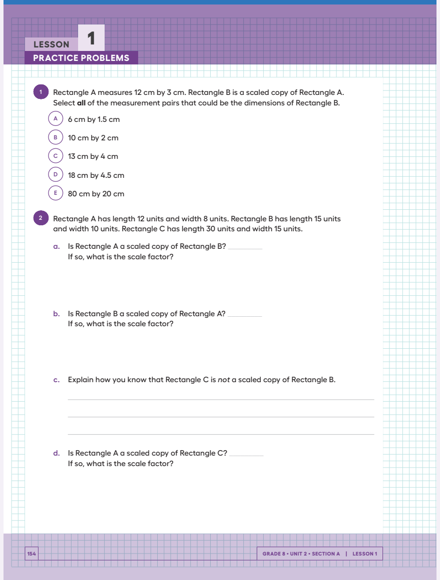
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 1

Practice Problems

5 Problems

Student Workbook


LESSON 1
PRACTICE PROBLEMS

1 Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A 6 cm by 1.5 cm
B 10 cm by 2 cm
C 13 cm by 4 cm
D 18 cm by 4.5 cm
E 80 cm by 20 cm

2 Rectangle A has length 12 units and width 8 units. Rectangle B has length 15 units and width 10 units. Rectangle C has length 30 units and width 15 units.

a. Is Rectangle A a scaled copy of Rectangle B?
If so, what is the scale factor?

b. Is Rectangle B a scaled copy of Rectangle A?
If so, what is the scale factor?

c. Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

d. Is Rectangle A a scaled copy of Rectangle C?
If so, what is the scale factor?

Problem 1

Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A. 6 cm by 1.5 cm

B. 10 cm by 2 cm

C. 13 cm by 4 cm

D. 18 cm by 4.5 cm

E. 80 cm by 20 cm

Problem 2

Rectangle A has length 12 units and width 8 units. Rectangle B has length 15 units and width 10 units. Rectangle C has length 30 units and width 15 units.

a. Is Rectangle A a scaled copy of Rectangle B? Yes

If so, what is the scale factor?

Sample response: The scale factor is $\frac{4}{5}$ (or equivalent).

b. Is Rectangle B a scaled copy of Rectangle A? Yes

If so, what is the scale factor?

Sample response: The scale factor is $\frac{5}{4}$ (or equivalent).

c. Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

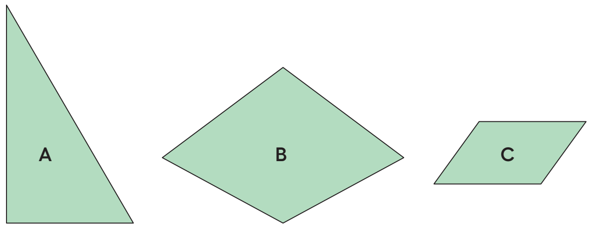
Rectangle C's length is double that of Rectangle B, but its width is not double.

d. Is Rectangle A a scaled copy of Rectangle C? If so, what is the scale factor?

No

Problem 3

Here are 3 polygons.



a. Draw a scaled copy of Polygon A with scale factor $\frac{1}{2}$.

The scaled copy of Polygon A should be a right triangle with each side half as long as the original.

b. Draw a scaled copy of Polygon B with scale factor 2.

The scaled copy of Polygon B should be a quadrilateral with each side twice as long as the original.

c. Draw a scaled copy of Polygon C with scale factor $\frac{1}{4}$.

The scaled copy of Polygon C should be a parallelogram with each side one-fourth the length of the original.

Problem 4

from Unit 1, Lesson 15

Which of these sets of angle measures could be the 3 angles in a triangle?

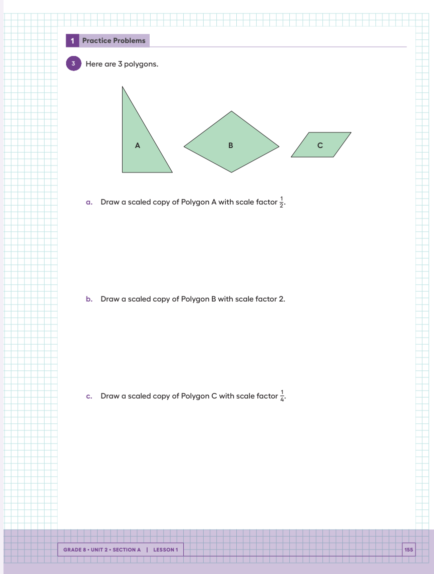
A. 40°, 50°, 60°

B. 50°, 60°, 70°

C. 60°, 70°, 80°

D. 70°, 80°, 90°

Student Workbook


1 Practice Problems

3 Here are 3 polygons.

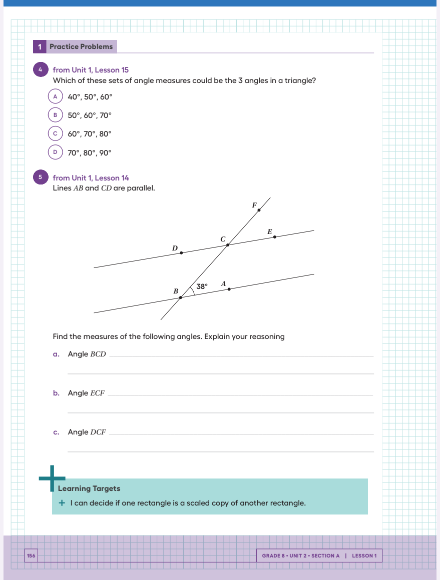
a. Draw a scaled copy of Polygon A with scale factor $\frac{1}{2}$.

b. Draw a scaled copy of Polygon B with scale factor 2.

c. Draw a scaled copy of Polygon C with scale factor $\frac{1}{4}$.

GRADE 8 • UNIT 2 • SECTION A | LESSON 1 155

Student Workbook


1 Practice Problems

4 from Unit 1, Lesson 15
Which of these sets of angle measures could be the 3 angles in a triangle?

A 40°, 50°, 60°
B 50°, 60°, 70°
C 60°, 70°, 80°
D 70°, 80°, 90°

5 from Unit 1, Lesson 14
Lines AB and CD are parallel.

Find the measures of the following angles. Explain your reasoning

a. Angle BCD

b. Angle ECF

c. Angle DCF

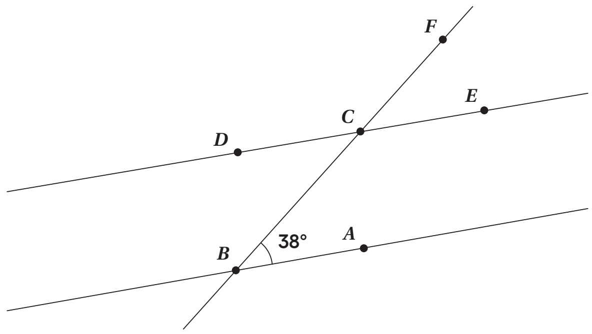
Learning Targets
+ I can decide if one rectangle is a scaled copy of another rectangle.

156 GRADE 8 • UNIT 2 • SECTION A | LESSON 1

Problem 5

from Unit 1, Lesson 14

Lines AB and CD are parallel. Find the measures of the following angles. Explain your reasoning.



a. Angle BCD

38 degrees

Sample reasoning: angle BCD and angle ABC are alternate interior angles for the parallel lines AB and CD cut by the transversal BC.

b. Angle ECF

38 degrees

Sample reasoning: angle ECF and angle BCD are a pair of vertical angles.

c. Angle DCF

142 degrees

Sample reasoning: angle DCF and angle ECF are supplementary angles.