Lesson 7

Reasoning about Solving Equations (Part 1)

Goals

- Compare and contrast (orally) different strategies for solving an equation of the form $px + q = r$.
- Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form $px + q = r$.
- Interpret a balanced hanger diagram, and write an equation of the form $px + q = r$ to represent the relationship shown.

Learning Targets

- I can explain how a balanced hanger and an equation represent the same situation.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced hanger diagram.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Action and Expression (Activity 2)

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Lesson Narrative

The goal of this lesson is for students to understand that we can generally approach equations of the form $px + q = r$ by subtracting q from each side and dividing each side by p (or multiplying by $\frac{1}{p}$). Students only work with examples where p, q, and r are specific numbers, not represented by letters. This goal is accomplished by considering what can be done to a hanger to keep it balanced.

Students solve equations in this lesson in a different way than they did in the previous lessons of this unit. Here, they reason about things one could "do" to hangers while keeping them balanced alongside an equation that represents the hanger, so they are thinking about "doing" things to each side of an equation, rather than only reasoning with situations or diagrams or simply thinking "What value would make this equation true?"

Student Learning Goal

Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	15 min	10 min	5 min

Warm-up

Hanger Diagrams

10 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that different weights on each side of a hanger cause the hanger to be balanced or unbalanced, which will be useful when students use hanger diagrams to develop general strategies for solving equations in a later activity. While students may notice and wonder many things about this image, the possible weights on each hanger and the fact that one hanger is balanced and the other is not are the important discussion points.

Launch

Tell students to close their books or devices (or to keep them closed). Display this image for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder.

Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image.

If needed, explain that the photo shows two clothes hangers with a sock hung from each end of each hanger. The socks have different objects inside them that have different weights. If the contrast of "balanced" and "unbalanced" hangers does not come up during the conversation, ask students to discuss this idea. If possible, use a real clothes hanger to demonstrate.

Give students 3 minutes of quiet work time followed by a whole-class discussion.

Instructional Routines

Notice and Wonder

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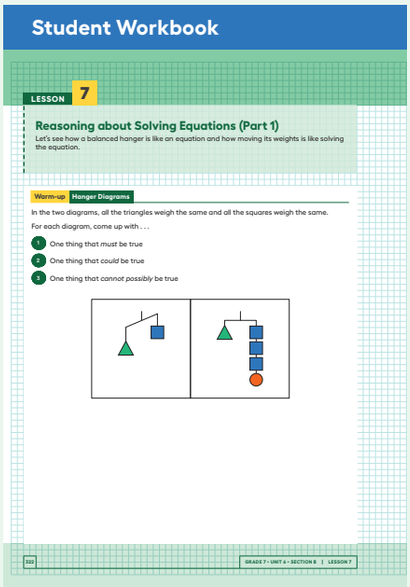

Student Workbook

LESSON 7

Reasoning about Solving Equations (Part 1)

Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.

Warm-up Hanger Diagrams

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

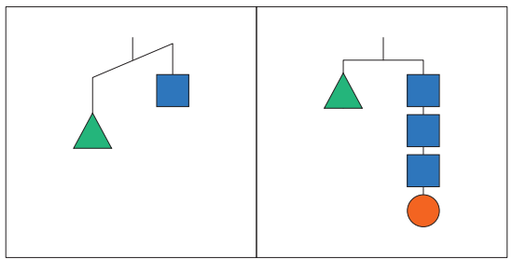
1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true

Student Task Statement

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with ...

1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true



Sample responses:

1. The triangle is heavier than the square; 1 triangle weighs the same as 3 squares and a circle.
2. The triangle weighs 32 ounces, the square weighs 10 ounces, and the circle weighs 2 ounces.
3. The triangle and the square weigh the same.

Activity Synthesis

The purpose of this discussion is to understand how the hanger diagrams work. Some possible questions for discussion:

- *"What are some things that must be true, could be true, and cannot possibly be true about the diagrams?"*

  "What does it mean when the diagram is balanced?"

  The weight on either side is equal.

- *"What does it mean when the diagram is unbalanced?"*

  The weight on the lower side is heavier than the weight on the higher side.

Activity 1
Hanger and Equation Matching

15 min

Activity Narrative

In this activity, students match hanger diagrams to equations. Then students use the diagrams and equations to find the unknown value in each diagram. This value is a solution of the equation.

Launch

Display the diagrams and explain that each square labeled with a 1 weighs 1 unit, and each shape labeled with a variable has an unknown weight. Shapes labeled with the same variable have the same weight.

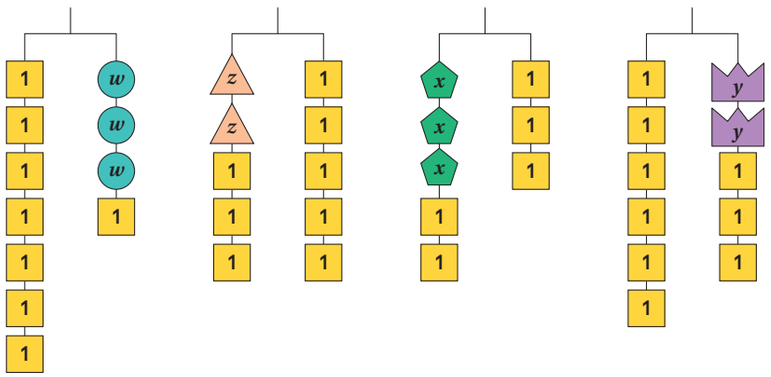
Arrange students in groups of 2.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Student Task Statement

On each balanced hanger, shapes with the same variable have the same weight.

A B C D



1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   a. $2 \boxed{z} + 3 = 5$

   b. $3 \boxed{x} + 3 = 3$

   c. $6 = 2 \boxed{y} + 3$

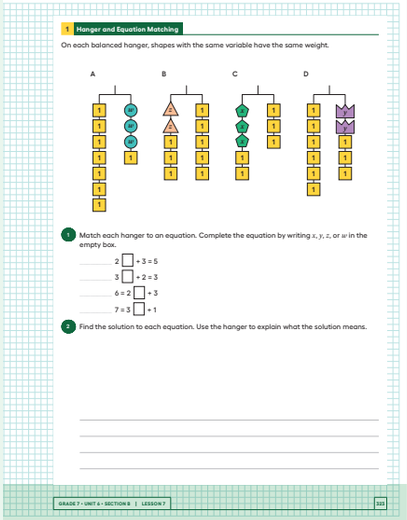
   d. $7 = 3 \boxed{w} + 1$

Access for Students with Diverse Abilities

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the variables in the hanger with the same variables in its corresponding equation.

Supports accessibility for: Visual-Spatial Processing

Student Workbook


1 Hanger and Equation Matching

On each balanced hanger, shapes with the same variable have the same weight.

A B C D

1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   ____ $2 \square + 3 = 5$

   ____ $3 \square + 2 = 3$

   ____ $6 = 2 \square + 3$

   ____ $7 = 3 \square + 1$

2. Find the solution to each equation. Use the hanger to explain what the solution means.

Instructional Routines

MLR8: Discussion Supports

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2. Find the solution to each equation. Use the hanger to explain what the solution means.

a. $w = 2$

Sample reasoning: I circle weighs the same as 2 squares.

b. $z = 1$

Sample reasoning: I triangle weighs the same as I square.

c. $x = \frac{1}{3}$

Sample reasoning: 3 pentagons weigh the same as I square.

d. $y = \frac{3}{2}$

Sample reasoning: 2 crowns weigh the same as 3 squares.

Activity Synthesis

Much discussion takes place between partners. Invite students to share how they found a solution to each equation.

Display one of the hanger diagrams alongside its matching equation.

Demonstrate removing the same number from each side (cross off shapes), and then dividing each side by the same thing (circle equal groups of shapes). Show how these moves correspond to doing the same thing to each side of the equation. (See the student Lesson Summary for an example of this.)

Help students make connections between the representations by asking questions, such as

- *"Where do you see division in both the hanger diagram and the equation?"*

Activity 2

Use Hangers to Understand Equation Solving

15 min

Activity Narrative

Students are presented with balanced hangers and are asked to write equations that represent them. They are then asked to explain how to use the diagrams, and then the equations, to reason about a solution. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

Launch

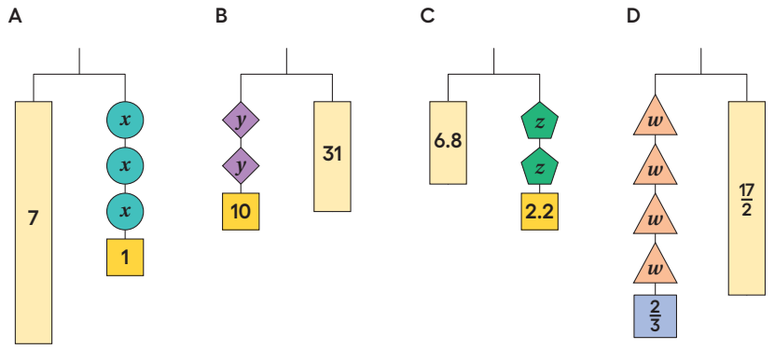
Draw students' attention to the diagrams in the *Task Statement*. Ensure they notice that the hangers are balanced and that each hanging piece is labeled with its weight. Some weights are labeled with numbers, but some are unknown, so they are labeled with a variable.

Keep students in the same groups.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Student Task Statement

Here are some balanced hangers diagrams where each piece is labeled with its weight in the same units.



For each diagram:

1. Write an equation.

   A: $7 = 3x + 1$ (or equivalent)

   B: $2y + 10 = 31$ (or equivalent)

   C: $6.8 = 2z + 2.2$ (or equivalent)

   D: $4w + \frac{3}{2} = \frac{17}{2}$ (or equivalent)

2. Explain how to figure out the weight of a piece labeled with a variable by reasoning about the diagram.

   Sample reasoning for Diagram A: Remove 1 unit of weight from each side of the hanger, leaving 6 units on the left and 3 x's on the right. Split each side into three equal groups, showing that each x piece weighs 2 units. So, $x = 2$.

3. Explain how to figure out the weight of a piece labeled with a variable by reasoning about the equation.

   Sample reasoning for $7 = 3x + 1$: Subtract 1 from each side, leaving $6 = 3x$. Divide each side by 3, leaving $2 = x$.

Activity Synthesis

The purpose of this discussion is to ensure that students:

1. Recognize the structure of equations of the form $px + q = r$ where p, q, and r are specific, given numbers.
2. Have a visual representation in their minds that can be used to support understanding of why for equations of this type, we can subtract q from each side and then divide each side by p to find the solution.

Invite students to demonstrate, side by side, how they reasoned with each diagram and its matching equation. For example, Diagram A can be shown next to the equation $7 = 3x + 1$. Cross out a piece representing 1 from each side, and write $7 - 1 = 3x + 1 - 1$, followed by $6 = 3x$. Encircle 3 equal groups on each side, and write $6 \div 3 = 3x \div 3$, followed by $2 = x$. Repeat for as many diagrams as time allows. If Diagrams A and B did not present much of a challenge for students, spend most of the time on Diagrams C and D.

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Action and Expression: Internalize Executive Functions.
To support organization, provide students with a graphic organizer with dedicated space to respond to each of the three prompts. Repeat this process for each hanger diagram.

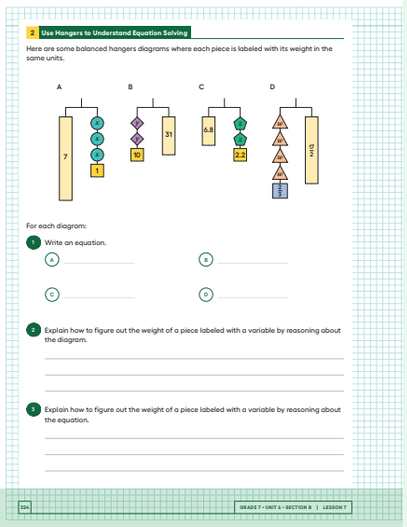
Supports accessibility for: Language, Organization

Access for Multilingual Learners (Activity 2, Synthesis)

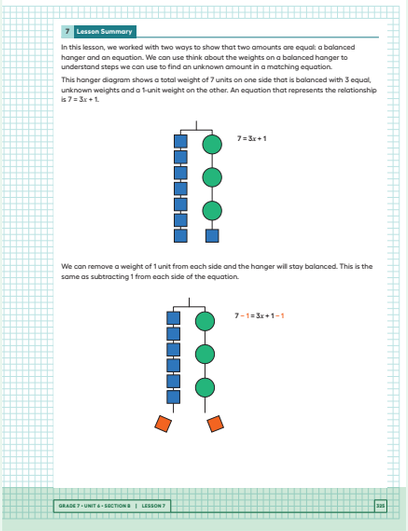
MLR8: Discussion Supports.
Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "dividing by the number in front" as "dividing by the coefficient."

Advances: Speaking, Representing

Student Workbook



Student Workbook



7 Lesson Summary

In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use think about the weights on a balanced hanger to understand steps we can use to find an unknown amount in a matching equation.

This hanger diagram shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.

$7 = 3x + 1$

We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.

$7 - 1 = 3x + 1 - 1$

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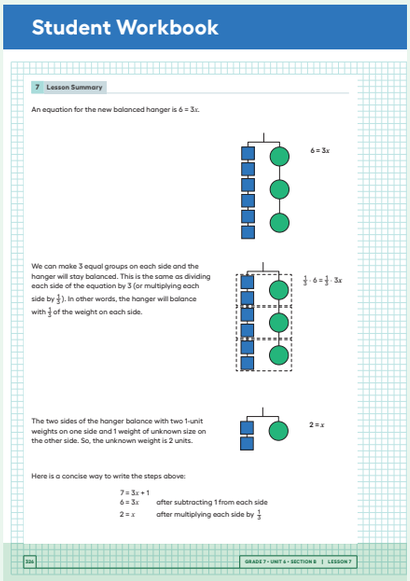
Student Workbook



7 Lesson Summary

An equation for the new balanced hanger is $6 = 3x$.

$6 = 3x$

We can make 3 equal groups on each side and the hanger will stay balanced. This is the same as dividing each side of the equation by 3 (or multiplying each side by $\frac{1}{3}$). In other words, the hanger will balance with $\frac{1}{3}$ of the weight on each side.

$\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$

The two sides of the hanger balance with two 1-unit weights on one side and 1 weight of unknown size on the other side. So, the unknown weight is 2 units.

$2 = x$

Here is a concise way to write the steps above:

$7 = 3x + 1$
$6 = 3x$ after subtracting 1 from each side
$2 = x$ after multiplying each side by $\frac{1}{3}$

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Lesson Synthesis

Display the equation $4x + 6 = 9 . 2$ for all to see. Ask students to work with their partner to draw a corresponding hanger diagram. Then one partner solves by reasoning about the equation, the other solves by reasoning about the diagram. Ask students to compare the two strategies and discuss how they are alike and how they are different.

Lesson Summary

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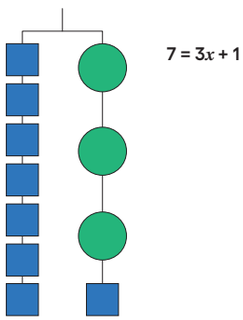

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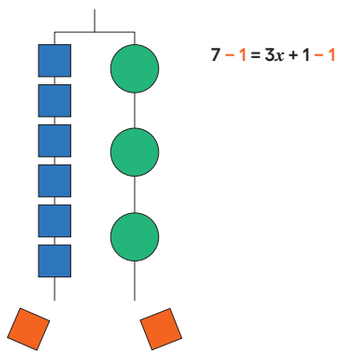

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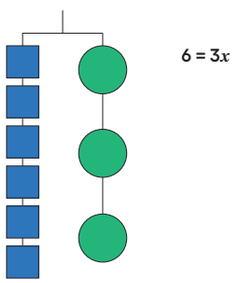

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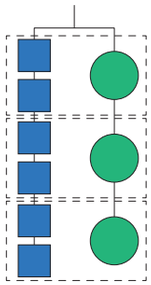
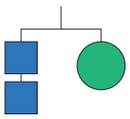


$6 = 3x$

The two sides of the hanger balance with two 1-unit weights on one side and 1 weight of unknown size on the other side. So, the unknown weight is 2 units.



$2 = x$

Here is a concise way to write the steps above:

$7 = 3x + 1$
$6 = 3x$ after subtracting 1 from each side
$2 = x$ after multiplying each side by $\frac{1}{3}$

Cool-down

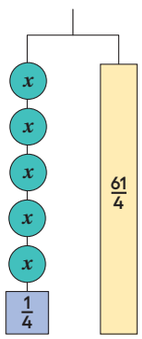
Solve the Equation

5 min

Student Task Statement

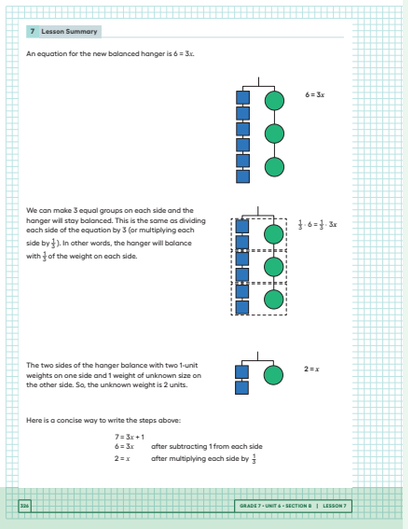
Solve the equation. If you get stuck, use the diagram.

$5x + \frac{1}{4} = \frac{61}{4}$



$x = 3$

Student Workbook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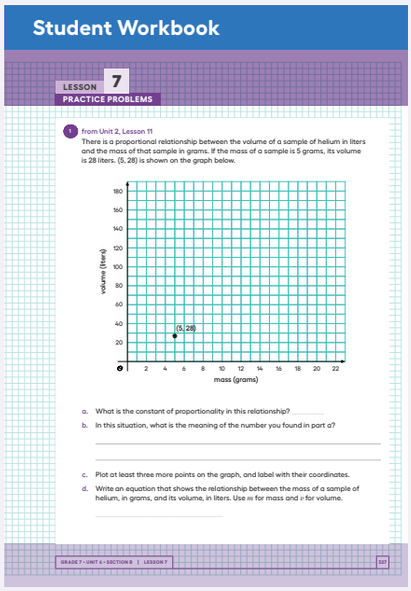
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

3 Problems


Student Workbook

LESSON 7
PRACTICE PROBLEMS

1 from Unit 2, Lesson 11
There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters. (5, 28) is shown on the graph below.

a. What is the constant of proportionality in this relationship?
b. In this situation, what is the meaning of the number you found in part a?
c. Plot at least three more points on the graph, and label with their coordinates.
d. Write an equation that shows the relationship between the mass of a sample of helium, in grams, and its volume, in liters. Use m for mass and v for volume.

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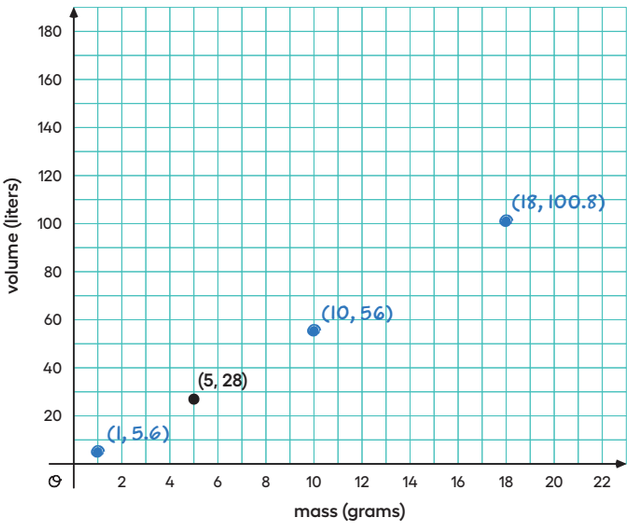
Problem 1

from Unit 2, Lesson 11

There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters. (5, 28) is shown on the graph below.



a. What is the constant of proportionality in this relationship?

5.6 liters per gram

b. In this situation, what is the meaning of the number you found in part a?

The volume of 1 gram of helium is 5.6 liters.

c. Plot at least three more points on the graph, and label with their coordinates.

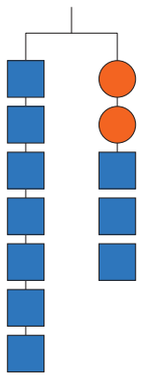
d. Write an equation that shows the relationship between the mass of a sample of helium, in grams, and its volume, in liters. Use m for mass and v for volume.

$v = 5.6m$ (or equivalent)

Problem 2

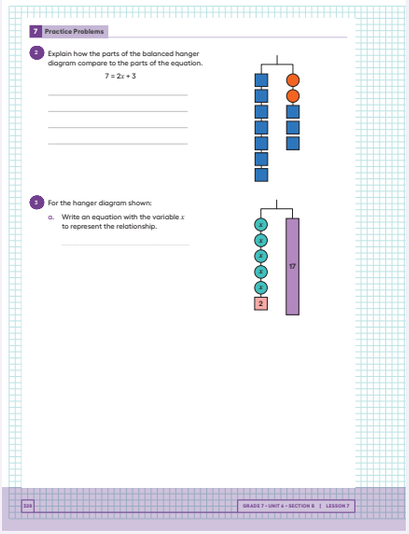
Explain how the parts of the balanced hanger diagram compare to the parts of the equation.

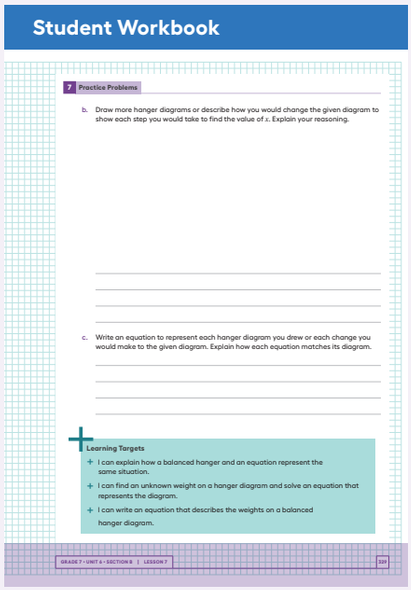
$7 = 2x + 3$



Sample response: The fact that the hanger is balanced (equal weights on each side) matches the equal sign in the equation (equal expressions on each side). On the left of the hanger there are 7 equal weights. The equation shows 7 on the left side, so we can assume that each square represents 1 unit. The right side of the hanger has 2 circles of unknown weight, which matches the 2x in the equation, for twice an unknown amount. The right side of the hanger also has 3 squares of unit weight, which matches the 3 on the right side of the equation. The weight of the 7 squares is the same as (equal sign) the weight of the 2 circles and 3 squares added together (the plus sign in the equation).

Student Workbook




Student Workbook

7 Practice Problems

b. Draw more hanger diagrams or describe how you would change the given diagram to show each step you would take to find the value of x. Explain your reasoning.

c. Write an equation to represent each hanger diagram you drew or each change you would make to the given diagram. Explain how each equation matches its diagram.

Learning Targets

+ I can explain how a balanced hanger and an equation represent the same situation.
+ I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
+ I can write an equation that describes the weights on a balanced hanger diagram.

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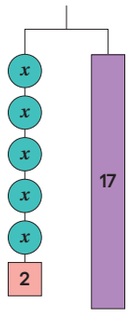
Problem 3

For the hanger diagram shown:



a. Write an equation with the variable x to represent the relationship.

$5x + 2 = 17$ (or equivalent)

b. Draw more hanger diagrams or describe how you would change the given diagram to show each step you would take to find the value of x. Explain your reasoning.

Sample response: Cross off 2 from each side to get a hanger diagram with 5 circles on the left and a rectangle labeled 15 on the right. Then divide both sides by 5 to get a hanger diagram with one circle on the left and a rectangle labeled 3 on the right.

c. Write an equation to represent each hanger diagram you drew or each change you would make to the given diagram. Explain how each equation matches its diagram.

Sample response: $5x = 15$. Subtracting 2 from each side of the equation is the same as removing 2 unit weights from each side. $x = 3$. Dividing both sides of the equation by 3 is the same as circling 3 equal groups on each side of the diagram.