Lesson 6

More Linear Relationships

Goals

- Describe (orally and in writing) how the slope and vertical intercept influence the graph of a line.
- Identify and interpret the positive vertical intercept of the graph of a linear relationship.

Learning Targets

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.

Lesson Narrative

In this lesson, students are introduced to the term **vertical intercept**, or y-intercept, as the point where the graph of the linear relationship touches the vertical or y-axis.

Students begin by examining a graph that shows the relationship between the total number of pages read and the number of days, in preparation to identify a common error when graphing linear relationships. By noticing that the values for initial number of pages read and pages read per day have been swapped in the graph, students are shown how the y-intercept and slope influence the shape and location of a line: the y-intercept indicates where the line meets the y-axis, while the slope determines how steep the line is.

Next, students match situations to graphs and then interpret the slope and vertical intercept in terms of the situation.

Student Learning Goal

Let's explore some more relationships between two variables.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- Card Sort
- MLR6: Three Reads
- Notice and Wonder

Required Materials

Materials to Copy

- Slopes, Vertical Intercepts, and Graphs Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

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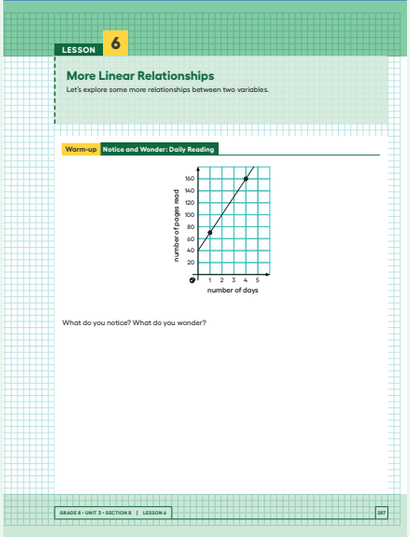
Student Workbook


LESSON 6

More Linear Relationships

Let's explore some more relationships between two variables.

Warm-up Notice and Wonder: Daily Reading

What do you notice? What do you wonder?

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Warm-up

Notice and Wonder: Daily Reading

5 min

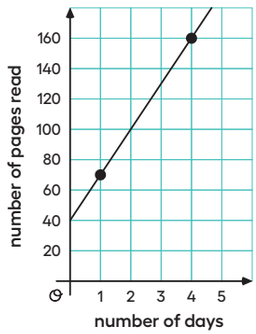
Activity Narrative

This *Warm-up* prompts students to make sense of a problem before solving it by observing an image and familiarizing themselves with a context and the mathematics that might be involved. The same image will be seen in a following activity.

Launch

Arrange students in groups of 2. Display the graph for all to see. Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement



What do you notice? What do you wonder?

Students may notice:

- The graph is about pages read each day.
- The same number of pages was read each day.
- 160 pages had been read on day 4.
- The number of pages read doesn't start at 0.

Students may wonder:

- Why doesn't the number of pages read start at 0?
- What are the 2 dots on the graph for?
- How many days will it take to have a total of 300 pages read?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the graph. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Activity 1
Summer Reading

15 min

Activity Narrative

This activity introduces students to the vertical intercept for a line and how to interpret it in context. They investigate the vertical intercept and slope together and observe what happens when their values are switched.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the diagram, without revealing the questions.

- For the first read, read the problem aloud then ask,

💬 *"What is this situation about?"*

Lin made a graph to track the number of pages she read for a summer reading assignment, but after 4 days, her graph doesn't match the page she is on.

Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured

Lin has already read 30 pages. She will read 40 pages each day following. She is on page 70 after day 1 and reaches page 190 after day 4.

- After the third read, reveal the question

"Why doesn't Lin's reading progress match her graph?"

and ask,

💬 *"What are some ways one might get started on this?"*

Invite students to name some possible starting points, referencing quantities from the second read.

Lin has already read 30 pages.

Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 1)

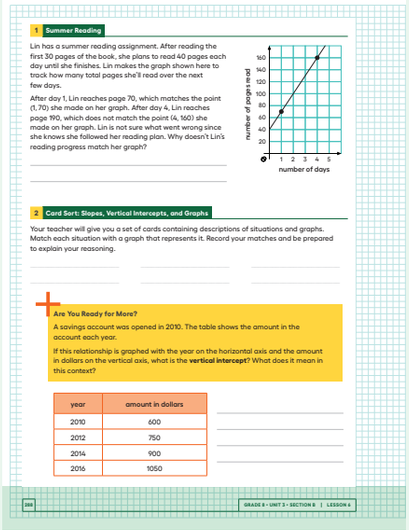
MLR6: Three Reads.
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.
Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of slope and intercepts. Terms may include: "vertical intercept" and "y-intercept."

Supports accessibility for: Conceptual Processing, Language

Student Workbook


1 Summer Reading

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.

After day 1, Lin reaches page 70, which matches the point (1, 70) she made on her graph. After day 4, Lin reaches page 190, which does not match the point (4, 160) she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan. Why doesn't Lin's reading progress match her graph?

number of pages read

number of days

2 Card Sort: Slopes, Vertical Intercepts, and Graphs

Your teacher will give you a set of cards containing descriptions of situations and graphs. Match each situation with a graph that represents it. Record your matches and be prepared to explain your reasoning.

Are You Ready for More?

A savings account was opened in 2010. The table shows the amount in the account each year.

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the **vertical intercept**? What does it mean in this context?

year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

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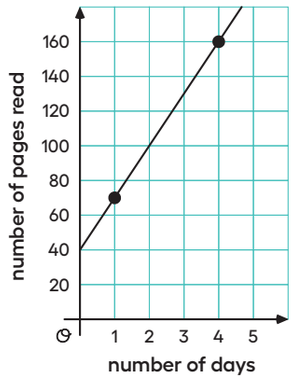
Student Task Statement

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.



After day 1, Lin reaches page 70, which matches the point (1, 70) she made on her graph. After day 4, Lin reaches page 190, which does not match the point (4, 160) she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan. Why doesn't Lin's reading progress match her graph?

Sample response: Lin had already read 30 pages and then read 40 pages each day after the first. The graph shows that she had started out reading 40 pages and then read 30 pages each day after. Lin's graph has the values for the initial number of pages and pages read each day switched.

Activity Synthesis

The purpose of this discussion is to introduce the term "vertical intercept." Begin by inviting 1–2 students to share why Lin's reading progress does not match the graph she made.

Then define the **vertical intercept** as the point where a line crosses the vertical axis. Note for students that sometimes "y-intercept" is used to refer to the numerical value of the y-coordinate in situations where the name of the variable graphed on the vertical axis is y. "Vertical intercept" can also be used to refer to this numerical value. Ask students:

- *"What is the vertical intercept for the graph Lin made? What does it represent in this context?"*

  The vertical intercept is 40. It represents that Lin had initially read 40 pages.

- *"What should the vertical intercept be?"*

  It should be 30 because Lin had already read only 30 pages of the book.

- *"What is the slope of the line in the graph Lin drew? What does it represent?"*

  The slope is 30. It means that Lin read 30 pages each day after the first.

- *"What should the slope be?"*

  The slope should be 40 because Lin's plan was to read 40 pages each day.

If time allows, have students draw a line that matches Lin's reading plan and progress on the same coordinate plane.

Activity 2

Card Sort: Slopes, Vertical Intercepts, and Graphs

15 min

Activity Narrative

This activity focuses on interpreting the slope of a graph and where it crosses the y-axis in context. Students sort different graphs and descriptions. They match each graph with a situation it could represent, and then use the context to interpret the meaning of the slope and the vertical intercept. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to place all the cards face up and start thinking about possible ways to sort the cards into categories.

- Pause the class and select 1–3 students to share the categories they identified.
- Discuss as many different categories as time allows.

Attend to the language that students use to describe their categories, giving them opportunities to describe their graphs and descriptions more precisely. Highlight the use of terms like "slope" and "vertical intercept." After a brief discussion, invite students to continue with the activity.

Student Task Statement

Your teacher will give you a set of cards containing descriptions of situations and graphs. Match each situation with a graph that represents it. Record your matches and be prepared to explain your reasoning.

Situation A: Graph 2
Situation B: Graph 6
Situation C: Graph 1
Situation D: Graph 3
Situation E: Graph 5
Situation F: Graph 4

Instructional Routines

Card Sort

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their reasoning. For example, "I noticed _____ so I matched ..." or "_____ and _____ are the same/alike because ..." Encourage students to challenge each other when they disagree.
Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code matching quantities in the problem stem and diagram.

Supports accessibility for: Visual-Spatial Processing

Are You Ready for More?

A savings account was opened in 2010. The table shows the amount in the account each year.

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the **vertical intercept**? What does it mean in this context?

year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

The vertical intercept corresponds to the amount in the year 0. The amount goes up by 150 dollars every two years, which is the same as 75 dollars per year. If the graph is extended backwards to the year 0, the amount goes down by 2010 × 75 = 150, 750 dollars from its value in 2010, so the vertical intercept would be -150, 150. It doesn't really make sense to extend the graph back that far because the account was not open then. The intercept has no useful meaning in this context

Activity Synthesis

The focus of this discussion is on the interpretation of the slope and the vertical intercept. For each situation, invite students to share how they determined the matching graph.

The slopes of the 6 lines given on the situation cards are all different, so the matching part of the task can be accomplished by examining just the slopes. Then for each situation, discuss:

- *"What does the slope mean in this situation?"*

  Situation A: the cost per month of the streaming service
  Situation B: an increase of 1 in side length adds 4 to the perimeter
  Situation C: the amount of money Diego adds to his piggy bank each week
  Situation D: the amount of money Noah adds to his piggy bank each month
  Situation E: the amount of money Elena adds to her piggy bank each day
  Situation F: the cost per month for internet service

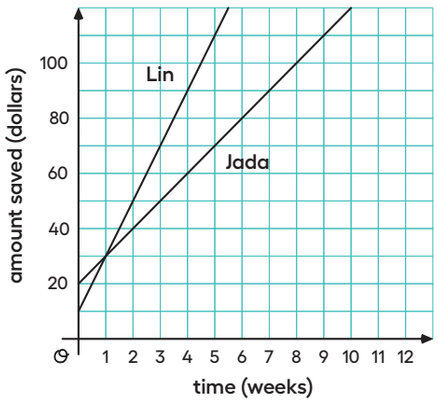
- *"What is the vertical intercept in this situation and what does it represent?"*

  Situation A; 40, the cost of the tablet was $40.
  Situation B; 0, the perimeter of a square with side length 0 is 0.
  Situation C; 10, Diego initially had $10 in his piggy bank.
  Situation D; 40, Noah started out with $40 in his piggy bank.
  Situation E; 9, Elena originally had $9 in her piggy bank.
  Situation F; 0, Lin's mom paid no money before paying for internet service.

Lesson Synthesis

The purpose of this discussion is to review how the slope and vertical intercept are related to the graph of a line drawn on a set of axes.

Display the graph for all to see and explain that it shows how Jada and Lin are saving some of the money they earn in the summer helping out their neighbors to use during the school year. Jada starts by putting $20 into a savings jar and plans to save $10 a week. Lin starts by putting $10 into a savings jar and plans to save $20 a week. Here are graphs of how much money each of them will save if they follow their plan:

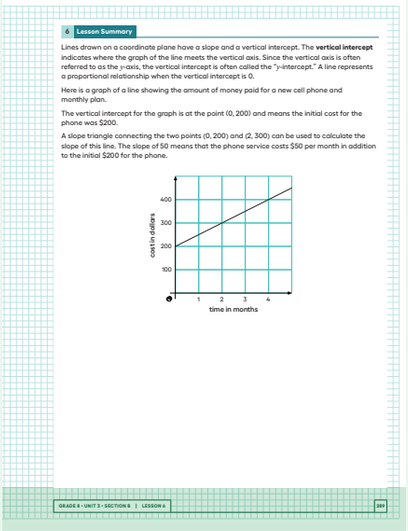


Invite students to share strategies for how to determine which graph represents which person. Emphasize the following ideas:

- The vertical intercept represents how much money each girl starts with and can be seen where the line meets the vertical axis.
- Since Lin starts out with less money, her vertical intercept will be lower than Jada's.
- The slope represents the rate of change for each girl, or in this situation, the amount of money saved each week.
- Since Lin plans on saving more money per week, her line will have a greater slope and her graph will look steeper.

When students are in agreement, label each line with the person it represents (After 1 week, Lin is the top line and Jada is the bottom line). As students share their ideas about vertical intercepts, label the vertical intercepts on the graph. If time allows and students would benefit from an additional demonstration, show how to calculate the slope for one of the lines by drawing a slope triangle connecting two points such as (4, 60) and (8, 100) on the graph of Jada's line. The slope of 10 matches that Jada is planning to save $10 each week.

Student Workbook


6 Lesson Summary

Lines drawn on a coordinate plane have a slope and a vertical intercept. The **vertical intercept** indicates where the graph of the line meets the vertical axis. Since the vertical axis is often referred to as the y-axis, the vertical intercept is often called the "y-intercept." A line represents a proportional relationship when the vertical intercept is 0.

Here is a graph of a line showing the amount of money paid for a new cell phone and monthly plan.

The vertical intercept for the graph is at the point (0, 200) and means the initial cost for the phone was $200.

A slope triangle connecting the two points (0, 200) and (2, 300) can be used to calculate the slope of this line. The slope of 50 means that the phone service costs $50 per month in addition to the initial $200 for the phone.

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

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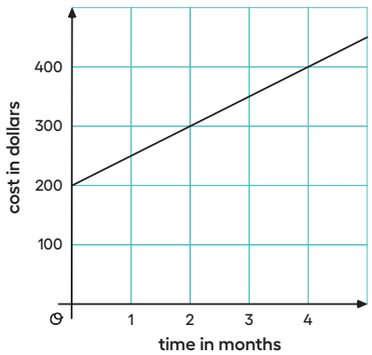


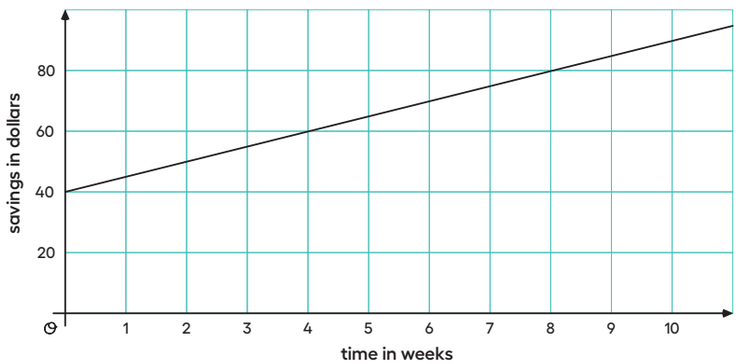
Cool-down

Savings

5 min

Student Task Statement

The graph shows the savings in Andre's bank account.



1. Calculate the slope and explain what it represents in this situation.

   The slope is 5 and means that Andre saves 5 dollars every week.

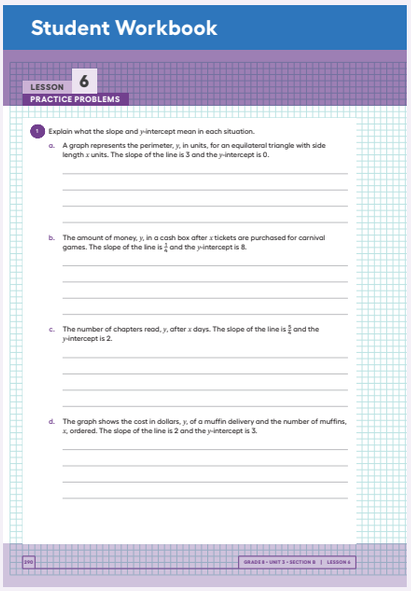
2. Determine the vertical intercept and explain what it represents in this situation.

   The vertical intercept is 40 and means that Andre initially had 40 dollars in his bank account.

Lesson 6

Practice Problems

3 Problems



Problem 1

Explain what the slope and y-intercept mean in each situation.

a. A graph represents the perimeter, y, in units, for an equilateral triangle with side length x units. The slope of the line is 3 and the y-intercept is 0.

Sample response: The slope of 3 shows that the triangle has 3 sides. For each increase of 1 unit of each side length, the perimeter increases by 3 units. The y-intercept of 0 shows that the relationship is proportional—a triangle with sides of length 0 has a perimeter of length 0.

b. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the y-intercept is 8.

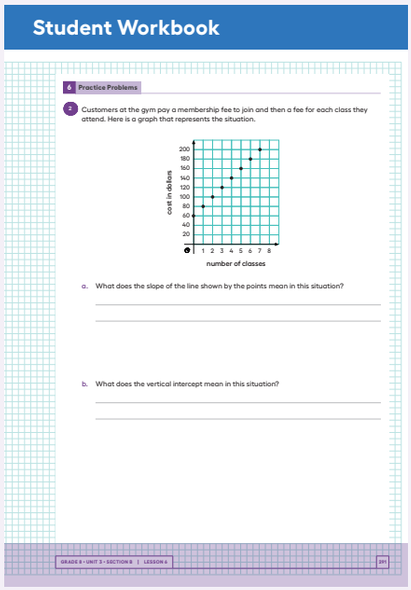
Sample response: The slope of $\frac{1}{4}$ means that each ticket costs \$0.25. The y-intercept of 8 represents the \$8 already in the cash box before any tickets are purchased.

c. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$ and the y-intercept is 2.

Sample response: The slope of $\frac{5}{4}$ shows that 5 chapters are read every 4 days. The y-intercept might show that 2 chapters were read before beginning to read 5 chapters every 4 days, or it might show that an additional 2 chapters were read on the first day.

d. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2 and the y-intercept is 3.

Sample response: The slope shows that \$2 are added for each muffin ordered. The y-intercept of 3 may represent a \$3 delivery fee or tip for the order.


Student Workbook

6 Practice Problems

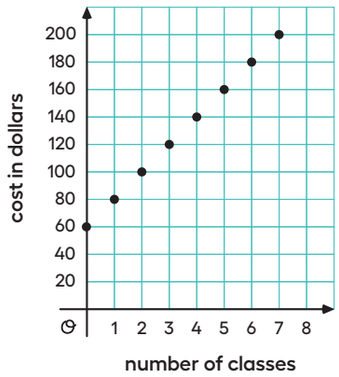
2 Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.

a. What does the slope of the line shown by the points mean in this situation?

b. What does the vertical intercept mean in this situation?

Problem 2

Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.



a. What does the slope of the line shown by the points mean in this situation?

Sample response: The cost for each class, which is $20.

b. What does the vertical intercept mean in this situation?

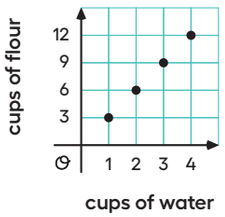
Sample response: The membership fee to join, which is $60.

Problem 3

from Unit 3, Lesson 4

The graph shows the relationship between the number of cups of flour and the number of cups of water in a recipe for making tortillas.

Recipe 1:



The table shows the amounts of flour and water needed for a different tortilla recipe.

Recipe 2:

water (cups)	flour (cups)
$2\frac{1}{2}$	1
5	2
$7\frac{1}{2}$	3

a. How much flour is needed for each recipe if 6 cups of water are used?

Recipe 1: 18 cups of flour

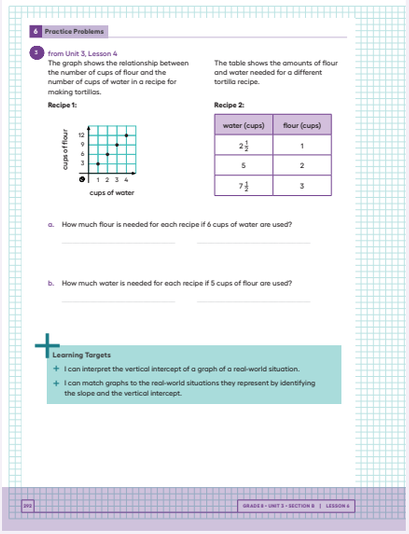
Recipe 2: $2\frac{2}{5}$ cups of flour (or equivalent)

b. How much water is needed for each recipe if 5 cups of flour are used?

Recipe 1: $\frac{5}{3}$ cups of water (or equivalent)

Recipe 2: $12\frac{1}{2}$ cups of water (or equivalent)

Student Workbook


6 Practice Problems

3 from Unit 3, Lesson 4

The graph shows the relationship between the number of cups of flour and the number of cups of water in a recipe for making tortillas.

Recipe 1:

cups of flour

cups of water

The table shows the amounts of flour and water needed for a different tortilla recipe.

Recipe 2:

water (cups)	flour (cups)
$2\frac{1}{2}$	1
5	2
$7\frac{1}{2}$	3

a. How much flour is needed for each recipe if 6 cups of water are used?

b. How much water is needed for each recipe if 5 cups of flour are used?

Learning Targets

+ I can interpret the vertical intercept of a graph of a real-world situation.
+ I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.