Lesson 6

Even More Graphs of Functions

Goals

- Compare and contrast (orally) peers' graphs that represent the same context.
- Comprehend that graphs representing the same context can appear different, depending on the variables chosen.
- Draw the graph of a function that represents a context, and explain (orally) which quantity is a function of which.

Learning Target

I can draw the graph of a function that represents a real-world situation.

Student Learning Goal

Let's draw a graph from a story.

Lesson Narrative

The goal of this lesson is for students to focus on qualitative aspects of graphs. The axes in this lesson have no units or scales, allowing students to reason more generally about graphs that represent situations.

In the *Warm-up*, students analyze two different graphs that represent the same situation given as a series of photos. Depending on which quantities are chosen as the dependent and independent variables, both graphs describe different aspects of the same story. The two functions represented by the graphs have the same independent variable (time) but different dependent variables (distance from edge of lawn versus distance from the camera).

In the following activity, students identify independent and dependent variables from contexts and select an appropriate graph to match their choices. Different choices are possible, so students must be precise about which choice they are making and explain how the choice relates to the graph based on the situation.

In the final activity, students create a graph from a story. In doing so, students have to make many choices about the aspects of a situation they want to represent with a mathematical object—this is an important part of modeling with mathematics. Depending on the variables chosen, graphs of the same situation can appear to be different but still tell the same story.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 2

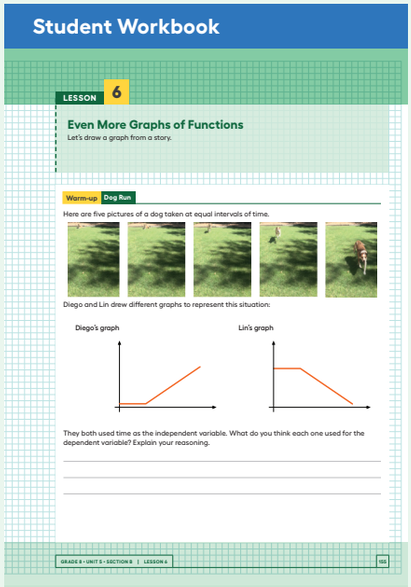
Required Preparation

Activity 2:
Students are asked to make displays of their work in groups of 2–3. Prepare materials, such as markers, chart paper, and board space, for creating this visual display.

For the digital version of the activity, acquire devices that can run the applet instead of preparing materials to create a visual display.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	20 min	10 min	5 min


Student Workbook

LESSON 6

Even More Graphs of Functions

Let's draw a graph from a story.

Warm-up Dog Run

Here are five pictures of a dog taken at equal intervals of time.

Diego and Lin drew different graphs to represent this situation:

Diego's graph

Lin's graph

They both used time as the independent variable. What do you think each one used for the dependent variable? Explain your reasoning.

GRADE 8 • UNIT 5 • SECTION B | LESSON 6

Warm-up

Dog Run

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to realize that different dependent variables can be used when making a model of a context. The choice of which variables we use affects how a graph representing the context looks.

Students begin by comparing two graphs and determining what they think the creator of each graph chose as their dependent variable. During the partner discussion, students should listen to their partner's argument and decide if they agree or disagree with what their partner is saying.

Launch

Arrange students in groups of 2.

Give 1–2 minutes of quiet work time, then ask students to share their responses with their partner to see if they agree or disagree about which variables Diego and Lin graphed.

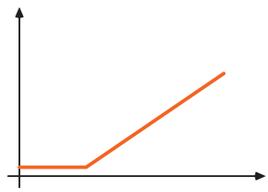
If partners do not agree, encourage students to make sense of one another's thinking and reach a consensus. Follow with a whole-class discussion.

Student Task Statement

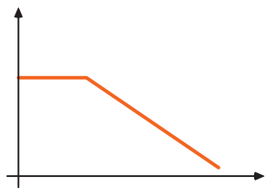
Here are five pictures of a dog taken at equal intervals of time.

Diego and Lin drew different graphs to represent this situation:

Diego's graph



Lin's graph



They both used time as the independent variable. What do you think each one used for the dependent variable? Explain your reasoning.

Sample response: Diego used the distance from the edge of the grass since it shows the value of something increasing, and Lin used the distance from the camera since it shows the value of something decreasing.

Activity Synthesis

The goal of this discussion is for students to understand that the same situation can be represented in different ways depending on what variables are chosen.

Select students to share the different variables that they think Lin and Diego used in their graphs. If any partners disagreed at first, ask those groups to share how they decided on their final response for what variables Diego and Lin were using.

Activity 1

Which Graph Is It?

10 min

Activity Narrative

The purpose of this activity is for students to sketch a graph showing the qualitative features of the function described in the problem.

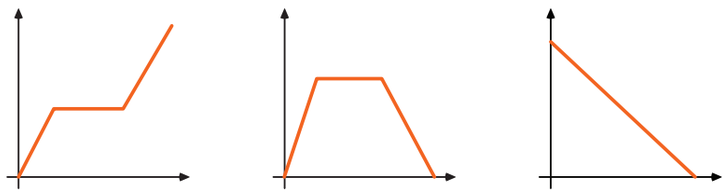
In the first problem, students identify possible independent and dependent variables for a situation and then identify which of three graphs matches their choice of variables. In the second problem, students choose the variables and make their own sketch of the context.

The problems are designed to have multiple correct solutions based on which quantities students identify in the descriptions. For example, Jada's information could be viewed from the perspective that the time it takes her to swim a lap depends on how much she practices. Alternatively, we could also say that the number of seconds she takes off her time depends on how much she practices. Each of these choices would lead to students selecting a different graph.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Arrange students in groups of 2. Display the statement and graphs for all to see:

Elena filled up the tub and gave her small dog a bath. Then she let the water out of the tub.



Ask groups to decide which graph best fits the provided context and what independent and dependent variables were used to create it.

Give 1 minute for partners to discuss, then select 2–3 groups to each share which graph and variables they chose.

The second graph shows the amount of water increasing, then staying steady, then decreasing over time. The amount of water in the bathtub is a function of time.

Select groups choosing different graphs for the first problem and groups sketching different graphs for the second problem to share during the *Activity Synthesis*.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Represent the same information through different modalities by using tables. Create a table to represent the independent and dependent variables.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Access for Multilingual Learners (Activity 1, Launch)

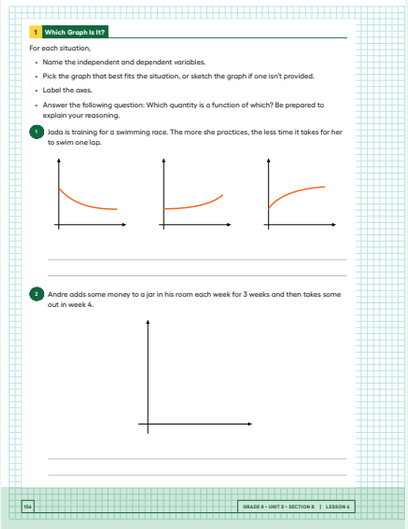
MLR8: Discussion Supports.
Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "The line goes up, is flat, then goes down" as "The line represents the amount of water in the tub. First it increases, then stays steady as the dog is bathed, then it decreases as the tub is drained."

Advances: Speaking, Listening

Building on Student Thinking

If students are not sure how to represent the "jump" when money is added to the jar, remind them of some of the function graphs they have seen in the past, such as with only discrete points plotted, and that graphs do not have to be a single connected line.

Student Workbook


1 Which Graph Is It?

For each situation,

- Name the independent and dependent variables.
- Pick the graph that best fits the situation, or sketch the graph if one isn't provided.
- Label the axes.
- Answer the following question: Which quantity is a function of which? Be prepared to explain your reasoning.

1. Jada is training for a swimming race. The more she practices, the less time it takes for her to swim one lap.

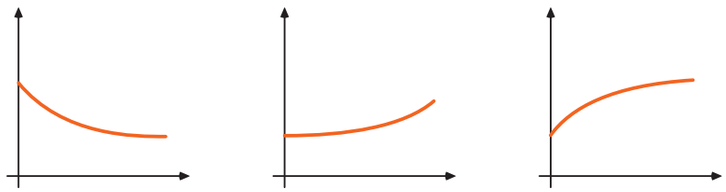
2. Andre adds some money to a jar in his room each week for 3 weeks and then takes some out in week 4.

Student Task Statement

For each situation,

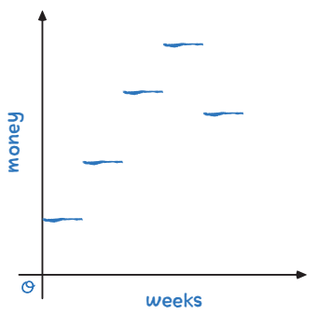
- Name the independent and dependent variables.
- Pick the graph that best fits the situation, or sketch the graph if one isn't provided.
- Label the axes.
- Answer the following question: Which quantity is a function of which? Be prepared to explain your reasoning.

1. Jada is training for a swimming race. The more she practices, the less time it takes for her to swim one lap.



Sample responses: If practice time is the independent variable and time to swim one lap is the dependent variable, then the first graph is the best choice because an increase in practice time corresponds to a decrease in lap completion time. (If the quantities are assigned in the other order, the first graph will also be the best choice.) If practice time is the independent variable and pace is the dependent variable, then the second graph could be a good choice. If practice time is the independent variable and time she takes off her total lap time is the dependent variable, then the third graph is the best choice because an increase in practice time leads to an increase in how much time she has dropped from her lap time.

2. Andre adds some money to a jar in his room each week for 3 weeks and then takes some out in week 4.

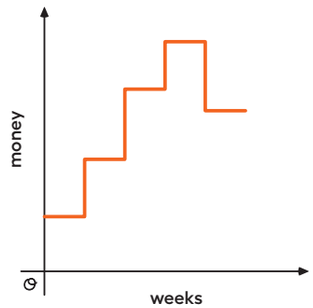


The graph shows money in the jar increasing once per week three times followed by a decrease in week 4. The amount of money in the jar is a function of the number of weeks.

Activity Synthesis

Invite previously identified groups to share their responses. If possible, display their labeled graphs for all to see. Invite 2–3 groups per graph to share their reasoning about the variables they used and why they picked or sketched the graph they did.

If time allows, begin the discussion of the last problem by displaying this graph and asking students what they think of it as a possible representation of the amount of money in Andre's savings jar:



Give students a brief quiet think time to consider the graph, then time to compare their answer with their partner. Invite 1–3 students to explain their thoughts about why it is or is not a good representation. (For example, the vertical lines would mean that at the same time the jar has two different amounts of money in it, which isn't possible, so this is not a good representation.) Remind students that functions have only one input for each output, so relationships whose graphs have vertical lines cannot be functions. A version of this graph with only horizontal lines does work.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies, such as the applet or graphing software. Some students may benefit from a checklist or list of steps to be able to use the applet or software.
Supports accessibility for: Organization; Conceptual processing; Attention

Activity 2
Sketching a Story about a Boy and a Bike

20 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to sketch a graph from a story. In order to make the sketch, students must select two quantities from the story to graph, decide which is the independent variable and which is the dependent variable, and create and label their axes based on their decisions.

Monitor for displays that are correct but different from each other in important ways for students to focus on during the discussion. For example, they may differ by what variables were graphed, such as:

- Distance from home as a function of time
- Distance from park as a function of time
- Total distance traveled as a function of time

In the digital version of the activity, groups use an applet to create their graphs and add labels as needed. The applet allows groups to efficiently revise and refine their graphs. Consider using the digital version if displaying the digital graphs is possible.

Launch

Arrange students in groups of 2–3. Distribute tools for creating a visual display. Before students begin, it may be necessary to demonstrate how to "create a set of axes" so that a first-quadrant graph can be sketched and is large enough to be seen from a distance.

Select work from students with different graphs, such as those described in the *Activity Narrative*, to share later.

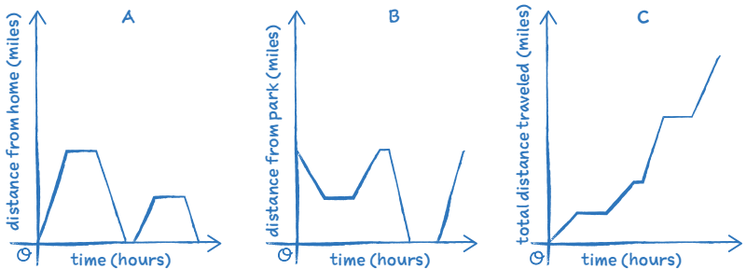
Student Task Statement

Your teacher will give you tools for creating a visual display. With your group, create a display that shows your response to each question.

Here is a story: "Noah was at home. He got on his bike and rode to his friend's house and stayed there for a while. Then he rode home. Then he rode to the park. Then he rode home again."

1. Create a set of axes, and sketch a graph of this story.

There are several ways of choosing which variables to use to tell the story, and once the variables are chosen, there are several ways to draw a graph to represent the story. The solutions here will reference the graphs A, B, and C, which are all possible graphical interpretations of the story.



2. What are the two quantities? Label the axes with their names and units of measure. (For example, if this were a story about pouring water into a pitcher, one of your labels might say "volume (liters).")

In A, the quantities are the time elapsed (in hours) and the distance Noah is from home (in miles).
In B, the quantities are the time elapsed (in hours) and the distance Noah is from the park (in miles).
In C, the quantities are the time elapsed (in hours) and the total distance Noah has traveled (in miles).

3. Based on your graph, is his friend's house or the park closer to Noah's home? Explain how you know.

In A, his friend's house is farther than the park. Sample reasoning: The graph indicates that while Noah is at his friend's house, he is further from home than when he is at the park.
In B, it is impossible to tell from the graph which is farther from Noah's house. Sample reasoning: The graph indicates that the friend's house is closer to the park than it is to Noah's house, but there is not enough information about the distance between Noah's house and his friend's house to answer the question.
In C, the park is closer than his friend's house. Sample reasoning: The graph of Noah's total distance traveled changes more when he travels to the park than it does when he travels to his friend's house.

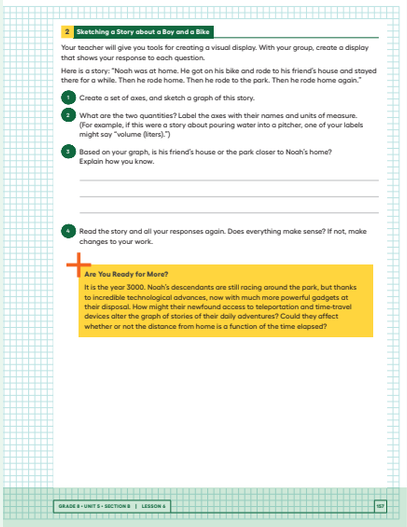
4. Read the story and all your responses again. Does everything make sense? If not, make changes to your work.

Building on Student Thinking

If groups try to start graphing before they have clearly articulated and labeled their axes with their chosen variables, ask,

"What do you plan to label your axes with?"
"Does everyone in your group agree on the labels?"

Student Workbook


2 Sketching a Story about a Boy and a Bike

Your teacher will give you tools for creating a visual display. With your group, create a display that shows your response to each question.

Here is a story: "Noah was at home. He got on his bike and rode to his friend's house and stayed there for a while. Then he rode home. Then he rode to the park. Then he rode home again."

1. Create a set of axes, and sketch a graph of this story.
2. What are the two quantities? Label the axes with their names and units of measure. (For example, if this were a story about pouring water into a pitcher, one of your labels might say "volume (liters).")
3. Based on your graph, is his friend's house or the park closer to Noah's home? Explain how you know.
4. Read the story and all your responses again. Does everything make sense? If not, make changes to your work.

Are You Ready for More?

It is the year 3000. Noah's descendants are still racing around the park, but thanks to incredible technological advances, now with much more powerful gadgets at their disposal. How might their newfound access to teleportation and time-travel devices alter the graph of stories of their daily adventures? Could they affect whether or not the distance from home is a function of the time elapsed?

GRADE 8 • UNIT 5 • SECTION B | LESSON 6 187

Are You Ready for More?

It is the year 3000. Noah's descendants are still racing around the park, but thanks to incredible technological advances, now with much more powerful gadgets at their disposal. How might their newfound access to teleportation and time-travel devices alter the graph of stories of their daily adventures? Could they affect whether or not the distance from home is a function of the time elapsed?

Answers vary.

Activity Synthesis

The goal of this discussion is for students to understand that the same situation can be graphed in different ways depending on the variables chosen and assumptions made.

Display 2–3 graphs from previously selected groups for all to see. If time allows, invite students to briefly describe their graphs, then use *Compare and Connect* to help students compare, contrast, and connect the different graphs. Here are some questions for discussion:

- *"What do the graphs have in common? How are they different?"*

  They are all about the same situation, but different groups thought about the situation from different points of view, so the graphs are not identical.

- *"How does time show up in each graph?"*

  Time is the independent variable for each graph.

- *"Why do the different approaches lead to different graphs?"*

  Our group assumed the park was farther from Noah's house than his friend's house, so the vertical height of our graph was shorter while he visited his friend's house. Other groups assumed the opposite, so the vertical height of their graphs was shorter while Noah was at the park.

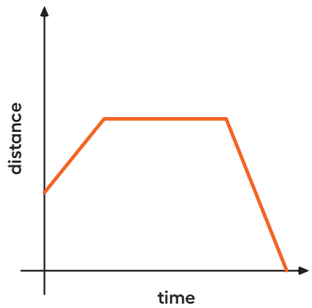
Conclude the discussion by refocusing students on the input-output pairs described by the different graphs. For example, on a graph where distance from home is a function of time, there should be three inputs where the output is zero since he starts at home, returns home from his friend's house, and ends at home.

Lesson Synthesis

Keep students in the same groups. Remind them of the multiple representations of the situation in the last activity. Tell students to imagine a situation that could be modeled in at least two different ways, depending upon which variables are chosen for the axes. Give time for students to write a clear explanation of the situation, the variables chosen, and how the choices would affect the appearance of the graph. If time allows, ask students to sketch an example of each of their graphs and share.

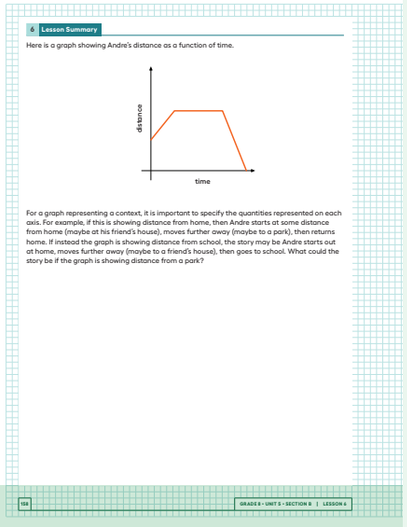
Lesson Summary

Here is a graph showing Andre's distance as a function of time.



For a graph representing a context, it is important to specify the quantities represented on each axis. For example, if this is showing distance from home, then Andre starts at some distance from home (maybe at his friend's house), moves further away (maybe to a park), then returns home. If instead
the graph is showing distance from school, the story may be Andre starts out at home, moves further away (maybe to a friend's house), then goes to school. What could the story be if the graph is showing distance from a park?

Student Workbook


6 Lesson Summary

Here is a graph showing Andre's distance as a function of time.

distance

time

For a graph representing a context, it is important to specify the quantities represented on each axis. For example, if this is showing distance from home, then Andre starts at some distance from home (maybe at his friend's house), moves further away (maybe to a park), then returns home. If instead the graph is showing distance from school, the story may be Andre starts out at home, moves further away (maybe to a friend's house), then goes to school. What could the story be if the graph is showing distance from a park?

Cool-down

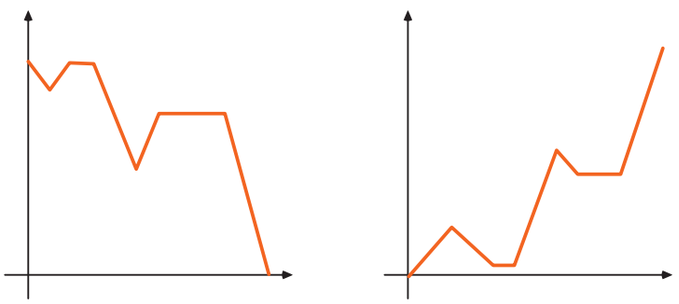
Walking Home from School

5 min

Student Task Statement

Elena starts to walk home from school but has to turn around and go back because she left something in her locker. On her way back home (the second time), she runs into her friend who invites her to the library to do homework with her. She stays at the library and then heads home to do her chores. Determine:

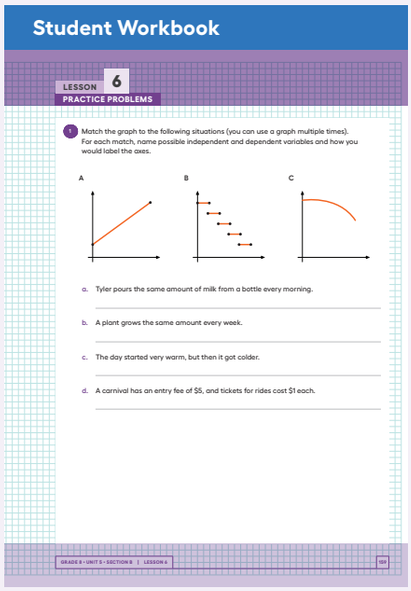
- Which graph fits Elena's story.
- What the two quantities are.
- Which quantity is a function of which.



The first graph most directly reflects Elena's story if the vertical axis represents Elena's distance from home and the horizontal axis represents the time since she started to walk home from school the first time. The graph then demonstrates that the distance from home is a function of the time elapsed.

Practice Problems

4 Problems


Student Workbook

LESSON 6
PRACTICE PROBLEMS

1 Match the graph to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.

A B C

a. Tyler pours the same amount of milk from a bottle every morning.

b. A plant grows the same amount every week.

c. The day started very warm, but then it got colder.

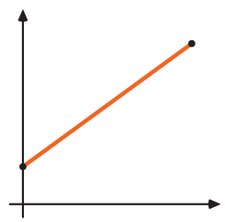
d. A carnival has an entry fee of $5, and tickets for rides cost $1 each.

GRADE 8 • UNIT 5 • SECTION B | LESSON 6

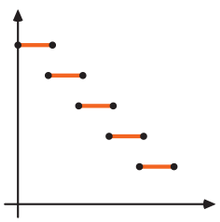
Problem 1

Match the graph to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.

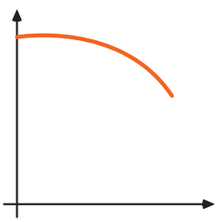
A



B



C



a. Tyler pours the same amount of milk from a bottle every morning.

B

Independent variable and horizontal-axis label: time (hours)

Dependent variable and vertical-axis label: amount of milk in bottle (ounces)

b. A plant grows the same amount every week.

A

Independent variable and horizontal-axis label: time (weeks)

Dependent variable and vertical-axis label: height of plant (inches)

c. The day started very warm, but then it got colder.

C

Independent variable and horizontal-axis label: time (hours)

Dependent variable and vertical-axis label: temperature (degrees Celsius)

d. A carnival has an entry fee of $5, and tickets for rides cost $1 each.

A

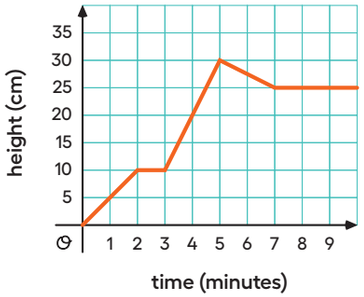
Independent variable and horizontal-axis label: number of rides

Dependent variable and vertical-axis label: cost to attend carnival in dollars

Problem 2

Jada fills her aquarium with water.

The graph shows the height of the water, in cm, in the aquarium as a function of time in minutes. Invent a story of how Jada fills the aquarium that fits the graph.



Sample response: Jada turns on the water faucet, and the water in the aquarium is increasing at a constant rate for the first two minutes to a height of 10 cm. Then Jada's mom calls her to take out the trash, so she turns off the faucet for the minute it takes her to take out the trash. After she comes back, she turns on the water higher than before, and the water increases to a height of 30 cm in the next two minutes. This is high enough, and Jada turns off the water. Unfortunately, there is a slow leak, and the water height decreases to 25 cm. After two minutes, Jada notices the leak. She stops it, and the water stays constant after that.

Problem 3

from Unit 5, Lesson 4

Recall the formula for area of a circle.

a. Write an equation relating a circle's radius, r, and area, A.

$A = \pi r^2$

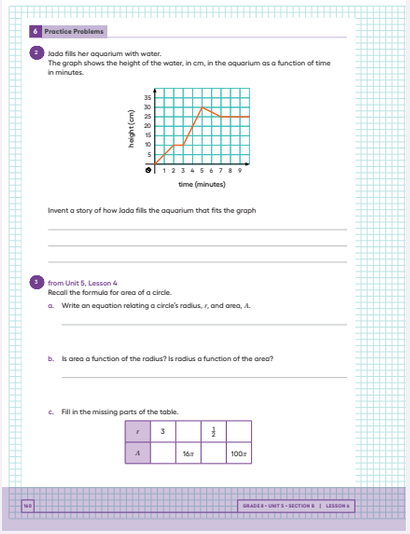
b. Is area a function of the radius? Is radius a function of the area?

Sample response: Yes for both. Any radius results in one and only one area. Any area results in one and only one radius, assuming that radii have to be positive.

c. Fill in the missing parts of the table.

r	3	4	$\frac{1}{2}$	10
A	9π	16π	$\frac{1}{4}\pi$	100π

Student Workbook


6 Practice Problems

2 Jada fills her aquarium with water. The graph shows the height of the water, in cm, in the aquarium as a function of time in minutes.

height (cm) 35 30 25 20 15 10 5 0 1 2 3 4 5 6 7 8 9 time (minutes)

Invent a story of how Jada fills the aquarium that fits the graph

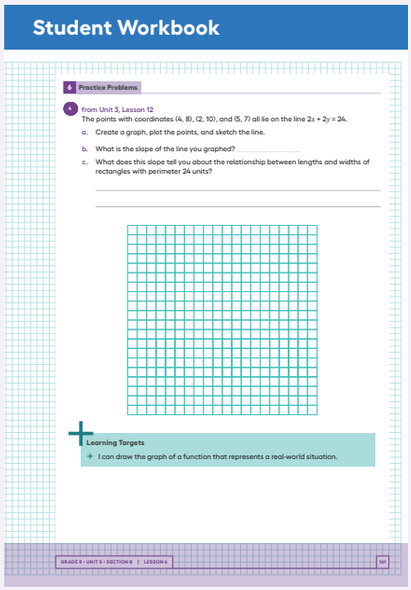
3 from Unit 5, Lesson 4
Recall the formula for area of a circle.
a. Write an equation relating a circle's radius, r, and area, A.

b. Is area a function of the radius? Is radius a function of the area?

c. Fill in the missing parts of the table.

r	3		$\frac{1}{2}$	
A		16π		100π

162 GRADE 8 • UNIT 5 • SECTION B | LESSON 6


Student Workbook

6 Practice Problems

A from Unit 3, Lesson 12
The points with coordinates (4, 8), (2, 10), and (5, 7) all lie on the line $2x + 2y = 24$.
a. Create a graph, plot the points, and sketch the line.
b. What is the slope of the line you graphed?
c. What does this slope tell you about the relationship between lengths and widths of rectangles with perimeter 24 units?

Learning Targets
+ I can draw the graph of a function that represents a real-world situation.

GRADE 8 • UNIT 3 • SECTION B | LESSON 6 181

Problem 4

from Unit 3, Lesson 12

The points with coordinates (4, 8), (2, 10), and (5, 7) all lie on the line $2x + 2y = 24$.

a. Create a graph, plot the points, and sketch the line.

b. What is the slope of the line you graphed?

-1

c. What does this slope tell you about the relationship between lengths and widths of rectangles with perimeter 24 units?

Sample response: A slope of -1 means that for rectangles of perimeter 24 units, every extra unit of length put into the width is one less unit of length that can be put into the length.