Lesson 3

Tessellating Polygons

Goal

Generalize (orally) that any triangle or quadrilateral can be used to tessellate the plane.

Learning Target

I can create tessellations with other polygons.

Lesson Narrative

In this third in the sequence of three optional lessons, students examine tessellations using non-regular polygons. Students first study triangles and show that any triangle can tessellate the plane. This work calls back to an important idea that students studied in grade 6: two copies of a triangle can be put together to make a parallelogram. Next, students show that all quadrilaterals can also tessellate the plane by using rigid motions and the fact that the sum of the angles in a quadrilateral is always 360°. Lastly, students return to pentagons. While not all pentagons can tessellate the plane, students investigate a pentagon whose tessellation leads to some rotational thinking. Through experimenting with these shapes, students construct arguments about which polygons can tessellate the plane and why and apply their knowledge of interior angles of polygons to support their arguments.

Student Learning Goal

Let's make tessellations with different polygons.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Poll the Class

Required Materials

Materials to Gather

- Tracing paper: Activity 1, Activity 2, Activity 3

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Activity 3:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

10 min	15 min	20 min
Activity 1	Activity 2	Activity 3

Activity 1
Triangle Tessellations

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students experiment with copies of a triangle (no longer equilateral) and discover that it is always possible to build a tessellation of the plane. A key in finding a tessellation with copies of a triangle is to experiment with organizing copies of the triangle, and then reasoning that two copies of a triangle can always be arranged to form a parallelogram. Students may not remember this construction from the sixth grade, but with copies of the triangle to experiment with, they will find the parallelogram or a different method. These parallelograms can then be put together in an infinite row, and these rows can then be stacked upon one another to tessellate the plane. Applying this process to a variety of triangles gives the opportunity to apply repeated reasoning to develop the concept that all triangles can tessellate the plane.

In the digital version of the activity, students use an applet to tessellate the triangles. The applet allows students to work with many copies of each triangle without tracing. The digital version may be preferable if time is limited.

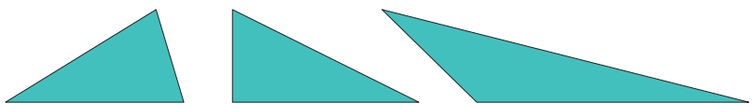
Launch

Assign different triangles to different students or groups of students. Provide access to tracing paper.

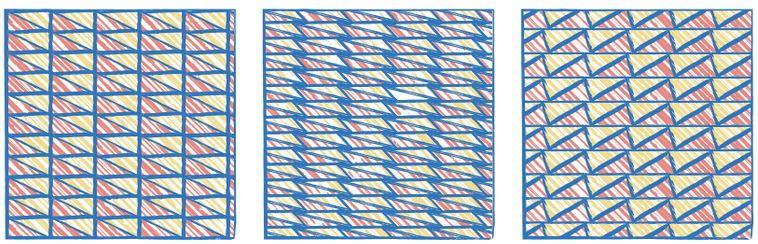
If students finish early, consider asking them to work on building a different tessellation or coloring their tessellation.

Student Task Statement

Your teacher will assign you one of the three triangles. You can use the picture to draw copies of the triangle on tracing paper. Your goal is to find a tessellation of the plane with copies of the triangle.

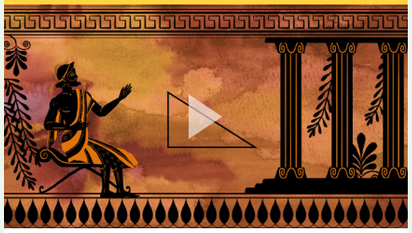


Sample response for each type of triangle:



Inspire Math

Pythagoras video



Go Online

Before the lesson, show this video to **introduce** the real-world connection.

ilclass.com/r/614195

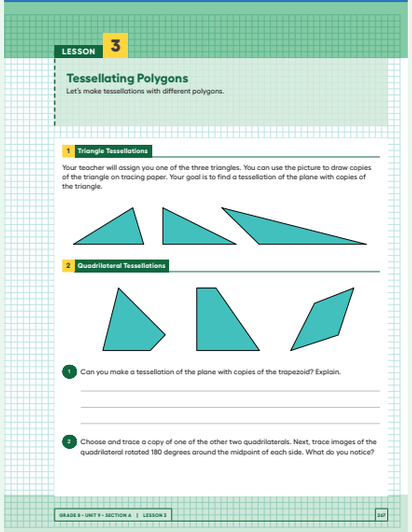
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students struggle to put together copies of their triangle in a way that can be continued to tessellate the plane, consider asking:

"Tell me more about how you have put triangles together."
"What are ways you can combine just two copies of the triangle?"

Student Workbook


LESSON 3

Tessellating Polygons

Let's make tessellations with different polygons.

1 Triangle Tessellations

Your teacher will assign you one of the three triangles. You can use the picture to draw copies of the triangle on tracing paper. Your goal is to find a tessellation of the plane with copies of the triangle.

2 Quadrilateral Tessellations

1. Can you make a tessellation of the plane with copies of the trapezoid? Explain.

2. Choose and trace a copy of one of the other two quadrilaterals. Next, trace images of the quadrilateral rotated 180 degrees around the midpoint of each side. What do you notice?

Instructional Routines

Poll the Class

ilclass.com/r/10694985
Please log in to the site before using the QR code or URL.

Activity Synthesis

Invite several students to share their tessellations for all to see.

Consider asking the following questions to help summarize the lesson:

- *"Were you able to make a tessellation with copies of your triangle?"*

Most students should respond "yes."

- *"How did you know that you could continue your pattern indefinitely to make a tessellation?"*

All parallelograms can be used to tessellate the plane as they can be placed side by side to make infinite "rows" or "columns," and then these rows or columns can be displaced to fill up the plane.

Share some of the tessellation ideas students come up with and relate them back to previous work, that is the tessellation of the plane with rectangles and parallelograms.

If time allows, ask:

- *"Will all triangles tessellate? How do you know?"*

Yes, any triangle will tessellate because any triangle can be rotated with itself to form a parallelogram that will tessellate.

- *"What do you notice about the angles of the triangles that are in a tessellation?"*

The angles that meet at each vertex must add to 360°, a full circle. The sum of the angles in any triangle is 180°, which is half of 360.

Activity 2

Quadrilateral Tessellations

15 min

Activity Narrative

There is a digital version of this activity.

The previous activity showed how to make a tessellation with copies of a triangle. A natural question is whether or not it is possible to tessellate the plane with copies of a single quadrilateral. Students have already investigated this question for some special quadrilaterals (squares, rhombuses, regular trapezoids), but what about for an arbitrary quadrilateral? This activity gives a positive answer to this question. As students discuss this question, they develop arguments and critique each other's reasoning. Pentagons are then investigated in the next activity, and there students will find that some pentagons can tessellate the plane while others can not.

In order to show that the plane can be tessellated with copies of a quadrilateral, students will experiment with rigid motions and copies of a quadrilateral.

This activity can be made more open ended by presenting students with a polygon and asking them if it is possible to tessellate the plane with copies of the polygon.

In the digital version of the activity, students use an applet to tessellate quadrilaterals. The applet allows students to work with many copies of each quadrilateral without tracing. The digital version may be preferable if time is limited.

Launch

Begin the activity with,

"*Any triangle can be used to tile the plane (some of them in many ways). Do some quadrilaterals tessellate the plane?*"

Yes: squares, rectangles, rhombuses, and parallelograms.

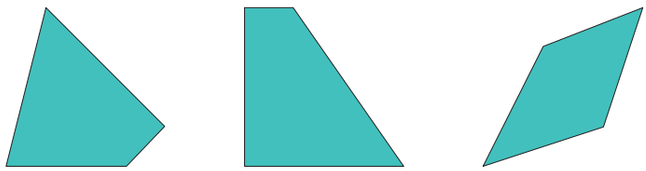
Next, ask,

"*Can any quadrilateral be used to tessellate the plane?*"

Give students a moment to ponder, and then poll the class for the number of "yes" and "no" responses. Record the responses for all to see. This question will be revisited in the *Activity Synthesis*.

Provide access to tracing paper.

Student Task Statement

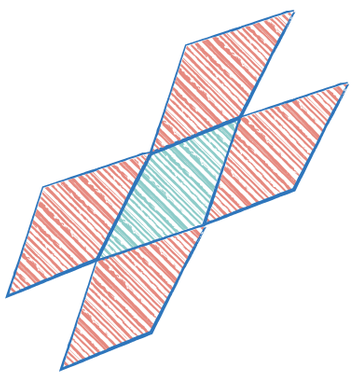


1. Can you make a tessellation of the plane with copies of the trapezoid? Explain.

   Yes, rotating the trapezoid 180 degrees about the slanted side to the base makes a rectangle. The plane can be tessellated with copies of this rectangle. Rotating the trapezoid 180 degrees about the other side to the base also makes a parallelogram, and then the plane can be tessellated with copies of the parallelogram.

2. Choose and trace a copy of one of the other two quadrilaterals. Next, trace images of the quadrilateral rotated 180 degrees around the midpoint of each side. What do you notice?

   It looks like there is room to fit copies of the quadrilateral in each of the wedges at the corners. When they are filled in, it will look like a checkerboard.



Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Provide Access by Recruiting Interest.
Provide choice and autonomy. Provide access to tracing paper, paper cut outs of the shapes, or other physical manipulatives for students to use while solving the problems.

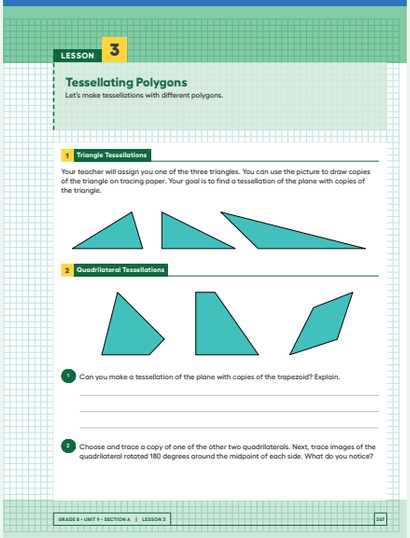
Supports accessibility for: Visual-Spatial Processing, Organization

Building on Student Thinking

If students have trouble determining the pattern because their figures are not traced accurately, consider asking:

"What do you know about the angles in a quadrilateral?"
"How does your knowledge of quadrilateral angles connect to trying to tessellate a quadrilateral?"

Student Workbook


LESSON 3

Tessellating Polygons

Let's make tessellations with different polygons.

1 Triangle Tessellations

Your teacher will assign you one of the three triangles. You can use the picture to draw copies of the triangle on tracing paper. Your goal is to find a tessellation of the plane with copies of the triangle.

2 Quadrilateral Tessellations

1 Can you make a tessellation of the plane with copies of the trapezoid? Explain.

2 Choose and trace a copy of one of the other two quadrilaterals. Next, trace images of the quadrilateral rotated 180 degrees around the midpoint of each side. What do you notice?

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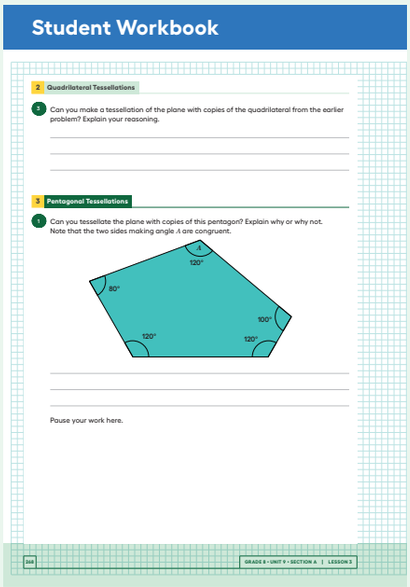
Student Workbook


2 Quadrilateral Tessellations

3. Can you make a tessellation of the plane with copies of the quadrilateral from the earlier problem? Explain your reasoning.

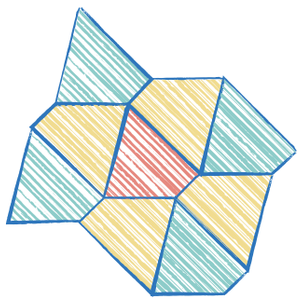
3 Pentagonal Tessellations

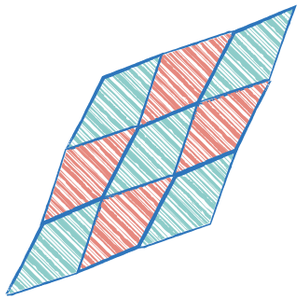
1. Can you tessellate the plane with copies of this pentagon? Explain why or why not. Note that the two sides making angle A are congruent.

Pause your work here.

3. Can you make a tessellation of the plane with copies of the quadrilateral from the earlier problem? Explain your reasoning.

Yes, continuing these 180-degree rotations about midpoints of sides of the quadrilateral unit it fills in the whole plane. At each vertex, the 4 angles of the quadrilateral meet to make a full 360-degree circle.





Activity Synthesis

Invite some students to share their tessellations.

Discussion questions include:

- *"Were you able to tessellate the plane with copies of the trapezoid?"*

  Yes, 2 of them can be put together to make a parallelogram, and the plane can be tessellated with copies of this parallelogram.

- *"What did you notice about the quadrilateral and the 180-degree rotations?"*

  They fit together with no gaps and no overlaps and leave space for 4 more quadrilaterals.

- *"How do you know that there are no overlaps?"*

  The sum of the angles in a quadrilateral is 360 degrees. At each vertex in the tessellation, copies of the 4 angles of the quadrilateral come together.

Revisit the question from the start of the activity,

- *"Can any quadrilateral be used to tessellate the plane?"*

Invite students to share if their answer has changed and explain their reasoning.

Activity 3
Pentagonal Tessellations

20 min

Activity Narrative

There is a digital version of this activity.

All triangles and all quadrilaterals give tessellations of the plane. For the quadrilaterals, this was complicated and depended on the fact that the sum of the angles in a quadrilateral is 360 degrees. Regular pentagons that do not tessellate the plane have been seen in earlier activities. The goal of this activity is to study some types of pentagons that *do* tessellate the plane. Students make use of structure when they relate the pentagons in this activity to the hexagonal tessellation of the plane, which they have seen earlier.

This activity can be made more open ended by presenting students with a polygon and asking them if it is possible to tessellate the plane with copies of the polygon.

In the digital version of the activity, students use an applet to tessellate a pentagon. The applet allows students to work with many copies of the pentagon without tracing. The digital version may be preferable if time is limited.

Launch

Ask students:

- *"Can you tessellate the plane with regular pentagons?"*

  No

- *"Can you think of a type of pentagon that could be used to tessellate the plane?"*

  A square base with a 45-45-90 triangle on top, for example.

Arrange students in groups of 2. Provide access to tracing paper.

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 3)

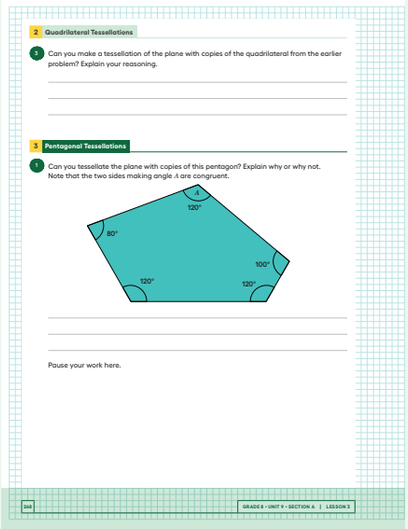
MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Building on Student Thinking

If students struggle tracing the rotated hexagon, consider asking:

"What do you already know about this hexagon?"
"What happens to the segments making angle A when the hexagon is rotated about A by 120 (or 240) degrees?"

Student Workbook


2 Quadrilateral Tessellations

3 Can you make a tessellation of the plane with copies of the quadrilateral from the earlier problem? Explain your reasoning.

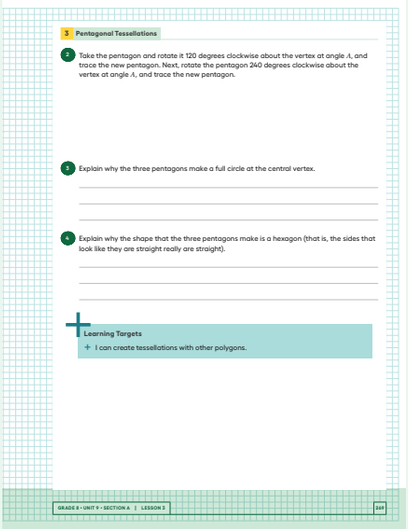
3 Pentagonal Tessellations

1 Can you tessellate the plane with copies of this pentagon? Explain why or why not. Note that the two sides making angle A are congruent.

A
120°
80°
100°
120°
120°

Pause your work here.

Student Workbook


3 Pentagonal Tessellations

2 Take the pentagon and rotate it 120 degrees clockwise about the vertex at angle A, and trace the new pentagon. Next, rotate the pentagon 240 degrees clockwise about the vertex at angle A, and trace the new pentagon.

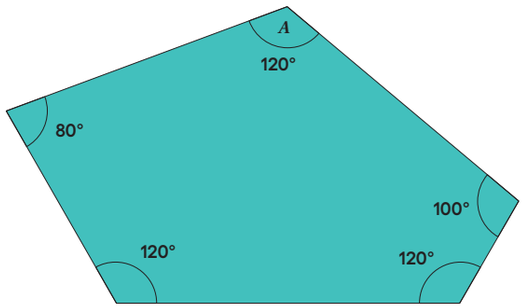
3 Explain why the three pentagons make a full circle at the central vertex.

4 Explain why the shape that the three pentagons make is a hexagon (that is, the sides that look like they are straight really are straight).

Learning Targets
+ I can create tessellations with other polygons.

Student Task Statement

1. Can you tessellate the plane with copies of this pentagon? Explain why or why not. Note that the two sides making angle A are congruent.



Yes, I can make a hexagon from 3 pentagons and can then tessellate the plane with those hexagons.

Pause your work here.

2. Take the pentagon and rotate it 120 degrees clockwise about the vertex at angle *A*, and trace the new pentagon. Next, rotate the pentagon 240 degrees clockwise about the vertex at angle *A*, and trace the new pentagon.

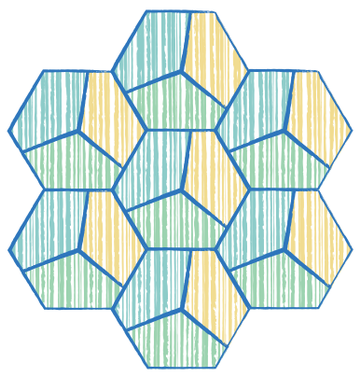
See picture for student response to question 4 showing several of the hexagons made by putting together 3 pentagons.

3. Explain why the three pentagons make a full circle at the central vertex.

The angles that are joined at the central vertex each measure 120 degrees, and there are 360 degrees in the full circle.

4. Explain why the shape that the three pentagons make is a hexagon (that is, the sides that look like they are straight really are straight).

Three of the 6 sides of the hexagon are corresponding sides of the pentagon after a rotation. For the other 3 sides, the angles measuring 100 degrees and 80 degrees are supplementary, and so the 2 segments making those sides are colinear. All angles of the hexagon measure 120 degrees, so it is a regular hexagon.



Activity Synthesis

Students may be successful in building a tessellation in the first question. The following questions guide them through a method while also asking for mathematical justification. Students who are successful in the first question can verify that their tessellation uses the strategy indicated in the following questions, and they will still need to answer the last two questions.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their explanation that the three pentagons make a hexagon. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- *"_____ makes sense, but what do you mean when you say …?"*
- *"Can you describe that another way?"*
- *"How do you know …? What else do you know is true?"*

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2 – 3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time*, invite some students to share their tessellations.

Some questions to discuss include:

- *"Does the hexagon made by 3 copies of the pentagon tessellate the plane?"*

  Yes.

- *"How do you know?"*

  I checked experimentally, or I noticed that all of the angles in the hexagon are 120 degrees.

- *"Why was it important that the 2 sides of the pentagons making the 120-degree angles are congruent?"*

  So that when I rotate my pentagon, those 2 sides match up with each other perfectly

- *"What is special about this pentagon?"*

  two sides are congruent, 3 angles measure 120 degrees