Lesson 11

What is the Same?

Goals

- Compare and contrast (orally and in writing) side lengths, angle measures, and other features of shapes using rigid transformations to explain why a shape is or is not congruent to another.
- Describe (orally and in writing) two figures that can be moved to one another using a rigid transformation as "congruent."
- Explain that congruent figures have equal corresponding side lengths, angle measures, and areas.

Learning Target

I can decide whether or not two figures are congruent using rigid transformations.

Student Learning Goal

Let's decide whether shapes are the same.

Lesson Narrative

In this lesson, students explore what it means for shapes to be "the same" and learn that the term **congruent** is a mathematical way to talk about figures being the same that has a precise meaning. Specifically, they learn that two figures are **congruent** if there is a sequence of translations, rotations, and reflections that moves one to the other. They learn that figures that are congruent can have different orientations, but corresponding lengths and angle measures are equal. Agreeing upon and formulating the definition of congruence requires careful use of precise language and builds upon all of the student experiences thus far in this unit, moving shapes and trying to make them match up.

As they work to decide whether or not pairs of shapes are congruent, students will look for and make use of structure. For shapes that are not congruent, what property can be identified in one that is not shared by the other? This could be an angle measure, a side length, or the size of the shape. For shapes that are congruent, is there any way to tell other than experimenting with tracing paper? In some cases, like the rectangles, students discover that looking at the length and width is enough to decide if they are congruent.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)

Instructional Routines

- 5 Practices

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2
- Math Community Chart: Activity 1, Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	10 min	5 min

What is the Same?

Lesson Narrative (continued)

In elementary grades, deciding if two shapes are the "same" usually involves making sure that they are the same general shape (for example, triangles or circles) and that the size is the same. As shapes become more complex and as we develop new ways to measure them (angles for example), something more precise is needed. The definition of congruence here states that two shapes are congruent if there is a sequence of translations, rotations, and reflections that matches one shape up exactly with the other. This definition has many advantages:

- It does not require measuring all side lengths or angles.
- It applies equally well to all shapes, not just polygons.
- It is precise and unambiguous: certain moves are allowed and two shapes are congruent when one can be moved to align exactly with the other.

The material treated here will be taken up again in high school (G-CO.B) from a more abstract point of view. In grade 8, it is essential for students to gain experience executing rigid motions with a variety of tools (tracing paper, coordinates, technology) to develop the intuition that they will need when they study these moves (or transformations) in greater depth later.

Math Community

Today's math community building time has two goals. The first is for students to make a personal connection to the math actions chart and to share on their *Cool-down* the math action that is most important to them. The second is to introduce the idea that the math actions that students have identified will be used to create norms for their mathematical community in upcoming lessons.

Warm-up

Find the Right Hands

5 min

Activity Narrative

In this activity, students consider shapes which are mirror images of each other. They identify which have the same orientation by selecting the figures which represent a right hand. Then, students discuss what is the same and what is different about the figures, identifying specific features that are the same among all of the figures.

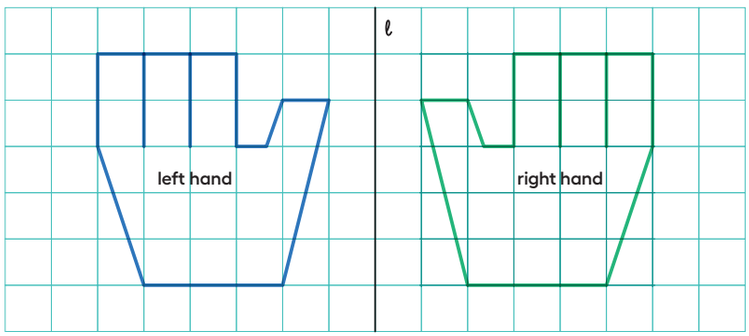
Providing access to tracing paper, rulers, and protractors in the geometry toolkit allows students the opportunity to choose appropriate tools strategically.

Launch

Arrange students in groups of 2, and provide access to geometry toolkits.

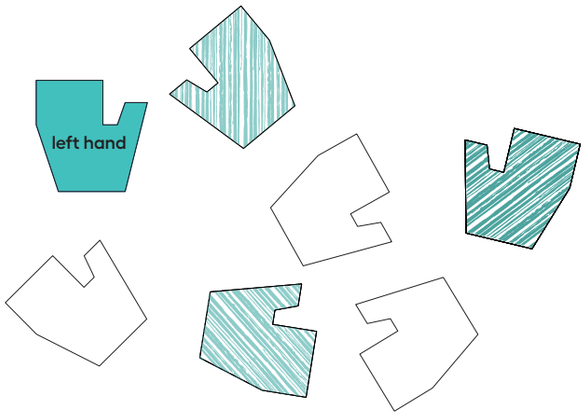
Give 2 minutes of quiet work time, followed by time for sharing with a partner and a whole-class discussion.

Show students this image or hold up both hands and point out that our hands are mirror images of each other. These are hands shown from the back. If needed, clarify for students that all of the hands in the task are shown from the back.

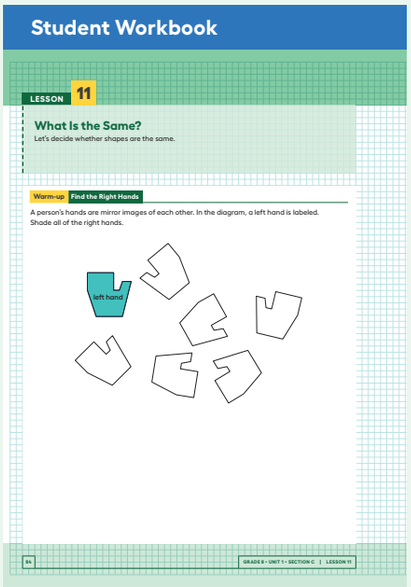


Student Task Statement

A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Shade all of the right hands.



Student Workbook


LESSON 11

What Is the Same?

Let's decide whether shapes are the same.

Warm-up Find the Right Hands

A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Shade all of the right hands.

left hand

Activity Synthesis

Ask students to think about the ways in which the left and right hands are the same, and the ways in which they are different.

Some ways that they are the same include:

- The side lengths and angles on the left and right hands match up with one another.
- If a left hand is flipped, it can match it up perfectly with a right hand (and vice versa).

Some ways that they are different include:

- They can not be lined up with one another without flipping one of the hands over.
- It is not possible to make a physical left and right hand line up with one another, except as "mirror images."

Math Community

After the *Warm-up,* display the revisions to the class Math Community Chart that were made from student suggestions in an earlier exercise. Tell students that over the next few exercises, this chart will help the class decide on community norms—how they as a class hope to work and interact together over the year. To get ready for making those decisions, students are invited at the end of today's lesson to share which "Doing Math" action on the chart is most important to them personally.

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "This pair of shapes is/is not the same because ..." or "If I translate/rotate/reflect, then ..."

Supports accessibility for: Language, Organization

Activity 1

Are They the Same?

15 min

Activity Narrative

In this activity, students express what it means for two shapes to be the same by considering carefully chosen examples. During the whole-class discussion, students come to a consensus about what it means for two shapes to be the same and are introduced to the word "congruent" to describe this relationship.

There may be discussion where a reflection is required to match one shape with the other. Students may disagree about whether or not these should be considered the same and discussion should be encouraged. As students discuss if they believe a pair of figures is the same or is not the same, they construct arguments and critique the reasoning of others.

Monitor for students who use these strategies to decide whether or not the shapes are the same and invite them to share during the discussion, ordered from less precise to more precise:

- Observing: This is often sufficient to decide that they are not the same. Encourage students to articulate what feature(s) of the shapes help them to decide that they are not the same.
- Measuring side lengths using a ruler or angles using a protractor: These students then use differences among these measurements to argue that two shapes are not the same.
- Using tracing paper to trace one figure and slide it to match with another figure: This is a version of applying a rigid transformation.

Providing access to tracing paper, rulers, and protractors in the geometry toolkit allows students the opportunity to choose appropriate tools strategically and is necessary for students to generate multiple strategies in this activity.

The routine of *Anticipate, Monitor, Select, Sequence, Connect (5 Practices)* requires a balance of planning and flexibility. The anticipated approaches might not surface in every class, and there may be reason to change the order in which strategies are presented. While monitoring, keep in mind the learning goal and adjust the order to ensure all students have access to the first idea presented (whether that be a common misconception or a different approach).

Launch

Give 5 minutes of quiet work time followed by a whole-class discussion.

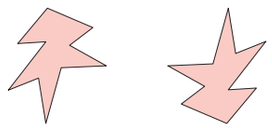
Provide access to geometry toolkits.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

For each pair of shapes, decide whether or not they are the same.

A



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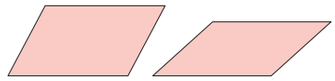
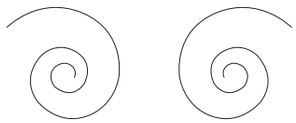
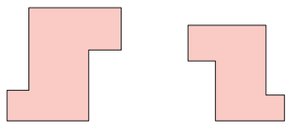


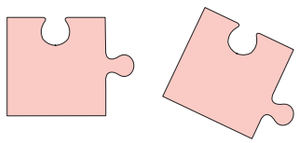
C



D



E



A. The two shapes are the same.

Sample reasoning: Rotating the shape on the left (by 180 degrees) around the top point and moving it down and to the right it matches up perfectly with the shape on the right.

B. They are not the same.

Sample reasoning: The side lengths of the shapes are the same but the angles are not. The shape on the right is more squished down (and has less area), so they are not the same.

C.
- They are not the same.

  Sample reasoning: The shapes are both spirals and look like they are the same size. Reflecting over a vertical line halfway between the two shapes, they appear to match up perfectly with one another.
- They are not the same.

  Sample reasoning: The spiral on the left moves in a clockwise direction while the spiral on the right moves in a counterclockwise direction.

D. They are not the same.

Sample reasoning: The general shapes are the same and the angles match up but the side lengths are different. The shape on the left is bigger than the shape on the right.

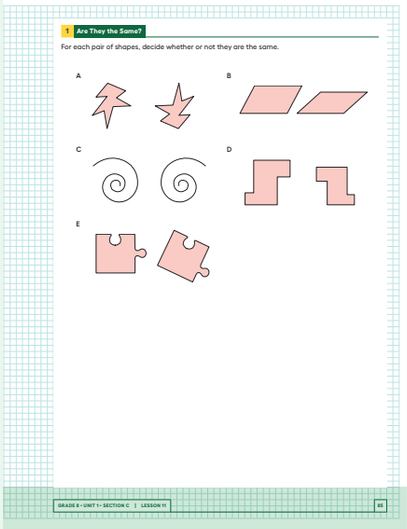
E. They are not the same.

Sample reasoning: The part that sticks out of the right side is higher on the first piece and lower on the second piece. Building a puzzle, both shapes would not fit in the same spot.

Building on Student Thinking

Students may think all of the shapes are the same because they are the same general shape at first glance. Ask these students to look for any differences they can find among the pairs of shapes.

Student Workbook


1 Are They the Same?

For each pair of shapes, decide whether or not they are the same.

A

B

C

D

E

Activity Synthesis

The purpose of this discussion is to build an understanding of congruence.

Invite previously selected students to share their reasoning. Sequence the discussion of the methods in the order listed in the *Activity Narrative:* making general observations, taking measurements, and applying rigid transformations with the aid of tracing paper. If possible, record and display their work for all to see.

The most general and precise of these criteria is the third which is the foundation for the mathematical definition of congruence: The other two are consequences. The moves allowed by rigid transformations do not change the shape, size, side lengths, or angle measures.

Connect the different responses to the learning goals by asking questions, such as the following:

- ***"When looking at the figures to determine if they are the same, what are you looking for?"***

  "How does having the same side lengths and angle measures show 2 figures are the same?"

  "Do reflections keep figures the same?"

There may be disagreement about whether or not to include reflections when deciding if two shapes are the same. Here are some reasons to include reflections:

- A shape and its reflected image can be matched up perfectly using a reflection.
- Corresponding angles and side lengths of a shape and its reflected image are the same.

Explain to students that people in the world can mean many things when they say two things are "the same." In mathematics there is often a need to be more precise, and this need can be met with the term "congruent".

We say that Figure A is *congruent* to Figure B if there is a rigid transformation that takes Figure A exactly to Figure B. This can be any sequence of translations, rotations, and reflections.

We can connect this definition of congruent with properties of rigid motions to conclude that:

- Corresponding sides of congruent figures are congruent.
- Corresponding angles of congruent figures are congruent.

Activity 2
Area, Perimeter, and Congruence

10 min

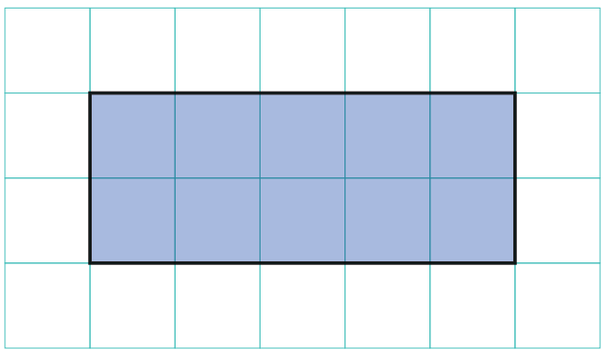
Activity Narrative

The purpose of this activity is for students to identify features that must be the same for congruent figures.

All of the figures in this activity have the same shape because they are all rectangles, but they are not all congruent. Students examine a set of rectangles and classify them according to their area and perimeter. Then they identify which ones are congruent. Because congruent shapes have the same side lengths, congruent rectangles have the same perimeter. But rectangles with the same perimeter are not always congruent. Congruent shapes, including rectangles, also have the same area. But rectangles with the same area are not always congruent. This allows students to highlight important features, like perimeter and area, which can be used to construct the argument that two shapes are not congruent.

Launch

Tell students that they will investigate what area and perimeter can tell us about two figures. If needed, display a rectangle like this one for all to see. Ask students to explain what perimeter means and how they can find the perimeter and area of this rectangle.



Arrange students in groups of 2. Provide access to geometry toolkits, including colored pencils. For the last question, encourage students to show that two figures are congruent using tools from the geometry toolkit.

Give 2 minutes for quiet work time followed by sharing with a partner and a whole-class discussion.

Access for Multilingual Learners (Activity 2, Student Task)

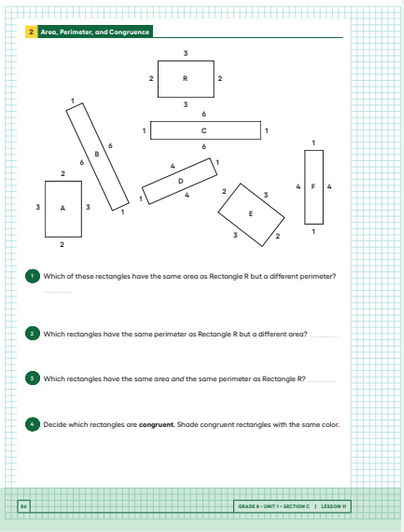
MLR2: Collect and Display.
Collect the language students use to compare the perimeter and area of rectangles. Display words and phrases such as “perimeter,” “area,” “congruent,” “the same,” “equal.” During the synthesis, invite students to suggest ways to update the display: “What are some other words or phrases we should include?” Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

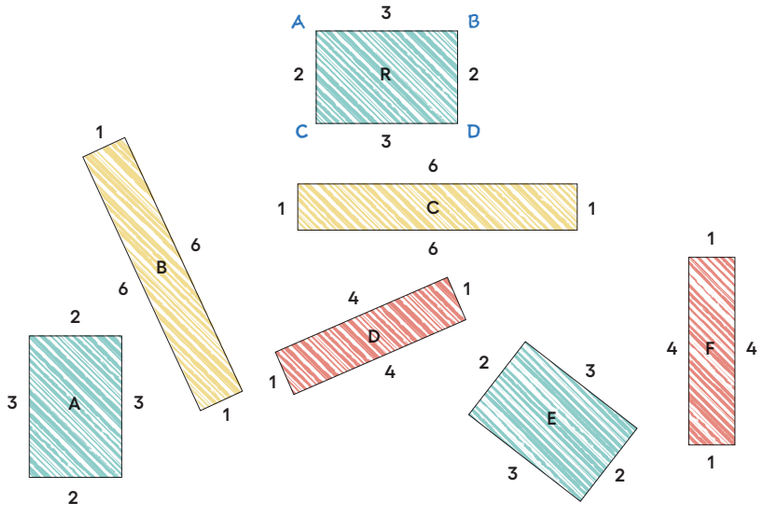
Building on Student Thinking

Watch for students who think about the final question in terms of "same shape and size." Remind them of the definition of congruence introduced in the last activity.

Student Workbook



Student Task Statement



The perimeter of Rectangle R is 10 units since 3 + 2 + 3 + 2 = 10 while its area is 6 square units since 2 • 3 = 6. All of the rectangles in the picture share at least one of these properties (either the perimeter or the area), but only the 2 unit by 3 unit rectangles share both.

1. Which of these rectangles have the same area as Rectangle R but a different perimeter?

   Rectangles B and C have the same area (6 square units) but a different perimeter (14 units).

2. Which rectangles have the same perimeter as Rectangle R but a different area?

   Rectangles D and F have the same perimeter (10 units) but a different area (4 square units).

3. Which rectangles have the same area *and* the same perimeter as Rectangle R?

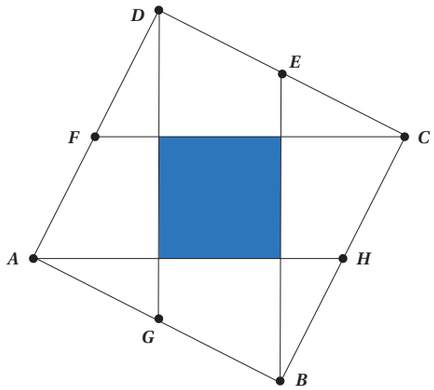
   Rectangles A and E have the same area and perimeter: only their position and orientation on the page is different.

4. Decide which rectangles are **congruent**. Shade congruent rectangles with the same color.

   The 2 by 3 rectangles are congruent to Rectangle R. In each case, Rectangle R can be translated and rotated so that it matches up perfectly with the 2-by-3 rectangle. The same argument shows that Rectangles B and C are congruent as are Rectangles D and F.

Are You Ready for More?

In square $ABCD$, points E, F, G, and H are midpoints of their respective sides. What fraction of square $ABCD$ is shaded? Explain your reasoning.



$\frac{1}{5}$ of square $ABCD$ is shaded. Reasoning varies.

Sample reasoning: Transform the unshaded pieces into four congruent squares that are each congruent to the shaded square.

It is interesting to generalize this problem such that points E, F, G, and H partition the sides of $ABCD$ in a ratio other than 1:1.

Activity Synthesis

Invite students who used the language of transformations to answer the final question to describe how they determined that a pair of rectangles are congruent.

Perimeter and area are two different ways to measure the size of a shape. Ask the students:

- *"Do congruent rectangles have the same perimeter? Explain your reasoning."*

  Yes, rigid motions do not change distances, and so congruent rectangles have the same perimeter.

- *"Do congruent rectangles have the same area? Explain your reasoning."*

  Yes, rigid motions do not change area or distances, therefore they do not change the length times the width in a rectangle.

- *"Are rectangles with the same perimeter always congruent?"*

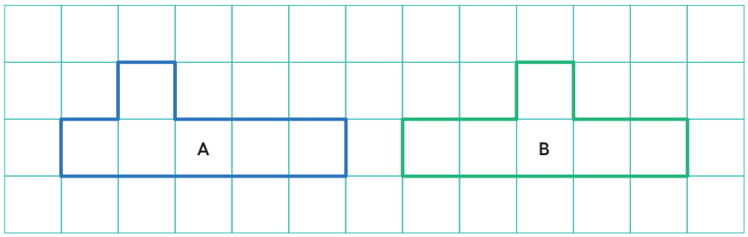
  No, rectangles D and F have the same perimeter but they are not congruent.

- *"Are rectangles with the same area always congruent?"*

  No, rectangles B and C have the same area but are not congruent.

One important takeaway from this lesson is that measuring perimeter and area is a good method to show that two shapes are *not* congruent if these measurements differ. When the measurements are the same, more work is needed to decide whether or not two shapes are congruent.

A risk of using rectangles is that students may reach the erroneous conclusion that if two figures have both the same area and the same perimeter, then they are congruent. If this comes up, challenge students to think of two shapes that have the same area and the same perimeter, but are not congruent. Here is an example:



Lesson Synthesis

Ask students to state their best definition of **congruent**.

Two shapes are congruent when there is a sequence of translations, rotations, and reflections that take one shape to the other.

Here are some questions for discussion:

- *"How can you check if two shapes are congruent?"*

  For rectangles, the side lengths are enough to tell. For more complex shapes, experimenting with transformations is needed.

- *"Are a shape and its mirror image congruent?"*

  Yes, because a reflection takes a shape to its mirror image.

- *"What are some ways to know that two shapes are not congruent?"*

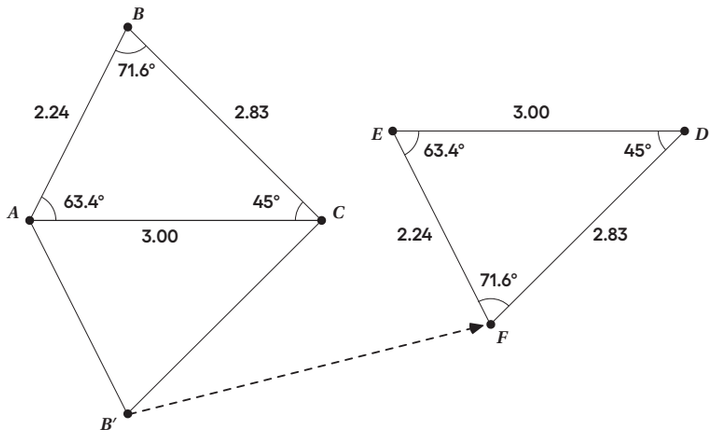
  Two shapes are not congruent if they have different areas, side lengths, or angles.

- *"What are some properties that are shared by congruent shapes?"*

  They have the same number of sides, same length sides, same angles, same area.

Lesson Summary

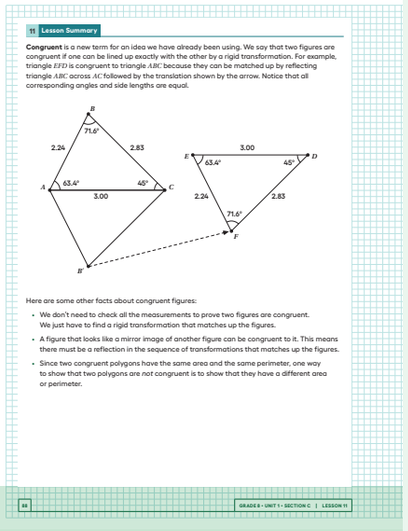
Congruent is a new term for an idea we have already been using. We say that two figures are congruent if one can be lined up exactly with the other by a rigid transformation. For example, triangle EFD is congruent to triangle ABC because they can be matched up by reflecting triangle ABC across AC followed by the translation shown by the arrow. Notice that all corresponding angles and side lengths are equal.



Here are some other facts about congruent figures:

- We don't need to check all the measurements to prove two figures are congruent. We just have to find a rigid transformation that matches up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different area or perimeter.

Student Workbook


11 Lesson Summary

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Here are some other facts about congruent figures:

- We don't need to check all the measurements to prove two figures are congruent. We just have to find a rigid transformation that matches up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different area or perimeter.

Responding To Student Thinking

Points to Emphasize
If students struggle with identifying congruence and rigid transformations, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, provide multiple opportunities for students to share their drawings on the tracing paper and to demonstrate their thinking or strategy for identifying the congruent pairs.

Unit 1, Lesson 12, Activity 2 Congruent Pairs (Part 1)

Math Community

Before distributing the *Cool-downs,* display the Math Community Chart and the community building question "Which 'Doing Math' action is most important to you, and why?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs,* review student responses to the community building question. Use the responses to draft a student norm and a teacher norm to use as an example in Exercise 6. For example, if "sharing ideas" is a common choice for students, a possible norm is "We listen as others share their ideas."

For the teacher norms section, if "questioning vs. telling" from the "Doing Math" section is key for your teaching practice, then one way to express that as a norm is "Ask questions first to make sure I understand how someone is thinking."

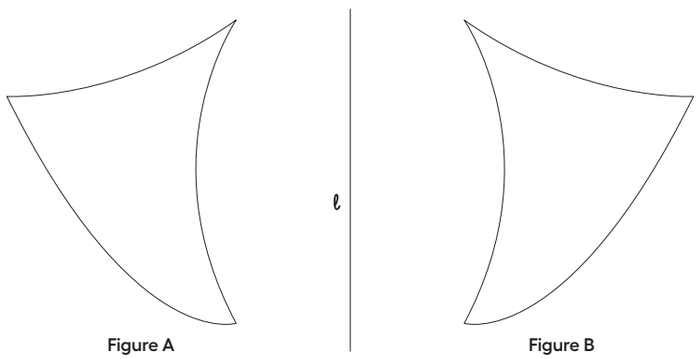
Cool-down

Mirror Images

5 min

Student Task Statement

Figure B is the image of Figure A when reflected across line ℓ. Are Figure A and Figure B congruent? Explain your reasoning.



Yes, they are congruent. There is a rigid transformation that takes one figure to the other, so they are congruent.

Lesson 11

Practice Problems

6 Problems

Problem 1

If two rectangles have the same perimeter, do they have to be congruent? Explain how you know.

no

Sample reasoning: For example, a rectangle with side lengths 3 inches and 4 inches is not congruent to a rectangle with side lengths 2 inches and 5 inches. Even though the angles of all rectangles have the same measure, when two figures are congruent all side lengths and angle measures are the same.

Problem 2

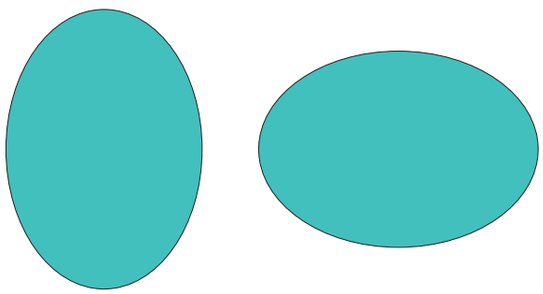
Draw two rectangles that have the same area, but are *not* congruent.

Sample response: A 2-by-6 rectangle and a 3-by-4 rectangle.

Problem 3

For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

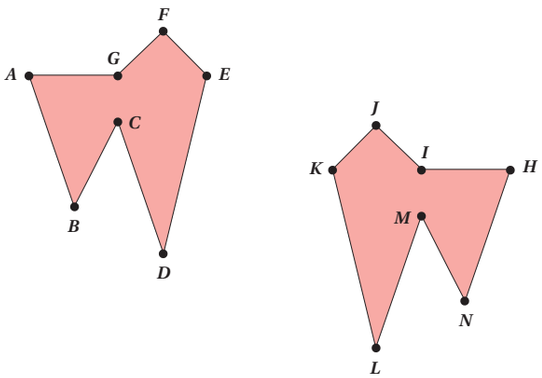
a.



yes

Sample reasoning: If the shape on the right is traced, it can be moved over and it appears to match up perfectly with the shape on the left. This can be done with a rotation (90 degrees clockwise) and then a translation.

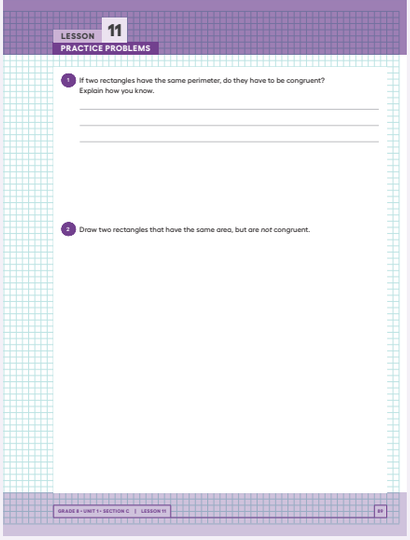
b.



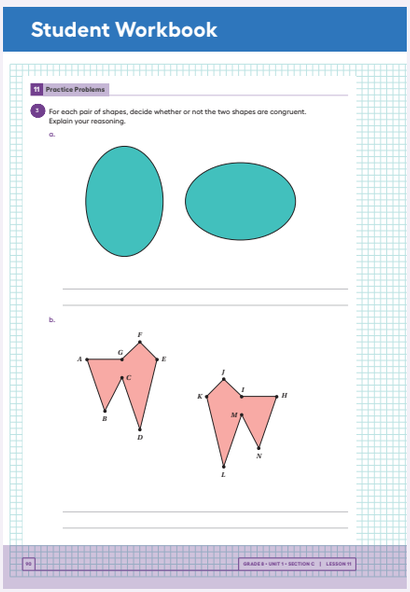
yes

Sample reasoning: If *ABCDEFG* is reflected about a vertical line and then translated, it appears to land on top of *HNMLKJI*.

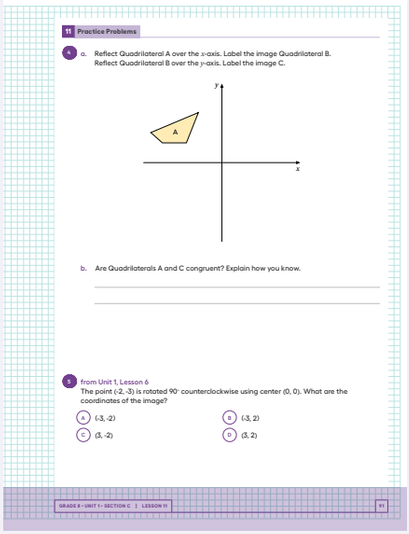
Student Workbook



Student Workbook



Student Workbook


11 Practice Problems

4 a. Reflect Quadrilateral A over the x-axis. Label the image Quadrilateral B. Reflect Quadrilateral B over the y-axis. Label the image C.

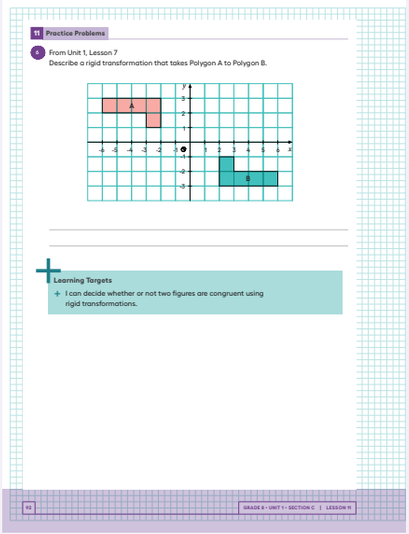
b. Are Quadrilaterals A and C congruent? Explain how you know.

5 from Unit 1, Lesson 6
The point (-2, -3) is rotated 90° counterclockwise using center (0, 0). What are the coordinates of the image?

(A) (-3, -2) (B) (-3, 2)
(C) (3, -2) (D) (3, 2)

GRADE 8 • UNIT 1 • SECTION C | LESSON 11 X1

Student Workbook


11 Practice Problems

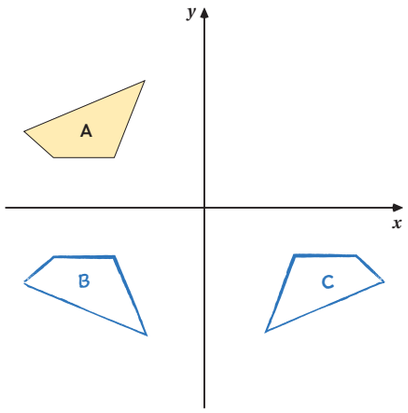
5 From Unit 1, Lesson 7
Describe a rigid transformation that takes Polygon A to Polygon B.

Learning Targets
+ I can decide whether or not two figures are congruent using rigid transformations.

X2 GRADE 8 • UNIT 1 • SECTION C | LESSON 11

Problem 4

a. Reflect Quadrilateral A over the x-axis. Label the image Quadrilateral B. Reflect Quadrilateral B over the y-axis. Label the image C.



b. Are Quadrilaterals A and C congruent? Explain how you know.

yes

Sample reasoning: There is a rigid transformation taking A to C so the two shapes are congruent.

Problem 5

from Unit 1, Lesson 6

The point (-2, -3) is rotated 90° counterclockwise using center (0, 0). What are the coordinates of the image?

A. (-3, -2)

B. (-3, 2)

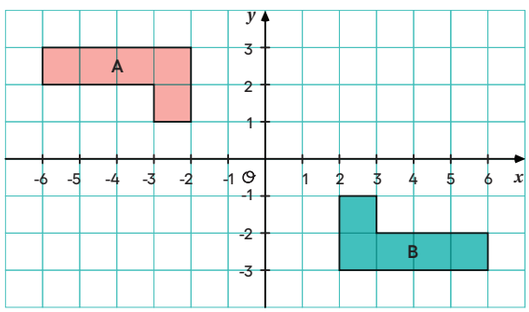
C. (3, -2)

D. (3, 2)

Problem 6

from Unit 1, Lesson 7

Describe a rigid transformation that takes Polygon A to Polygon B.



Sample response: Rotate Polygon A 180° around (0, 0).