Lesson 5

Coordinate Moves

Goals

- Determine (orally and in writing) coordinates that represent the image of a polygon in the coordinate plane after a transformation.
- Draw and label a diagram of a line segment rotated 90 degrees clockwise or counterclockwise about a given center.
- Generalize (orally and in writing) the process to reflect any point in the coordinate plane over the x-axis or y-axis.

Learning Target

I can apply transformations to points on a grid if I know their coordinates.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Lesson Narrative

Students continue to investigate the effects of transformations. The new feature of this lesson is the coordinate plane. In this lesson, students use coordinates to describe figures and their images under transformations in the coordinate plane. Reflections over the x-axis and y-axis have a very nice structure captured by coordinates. When we reflect a point like (2, 5) over the x-axis, the distance from the x-axis stays the same but instead of lying 5 units *above* the x-axis the image lies 5 units *below* the x-axis. That means the image of (2, 5) when reflected over the x-axis is (2, -5). Similarly, when reflected over the y-axis, (2, 5) goes to (-2, 5), the point 2 units to the left of the y-axis.

Using the coordinates to help understand transformations involves the mathematical practice standard of looking for and making use of structure (discovering the patterns coordinates obey when transformations are applied).

Student Learning Goal

Let's transform some figures and see what happens to the coordinates of points.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Translating Coordinates

5 min

Activity Narrative

The purpose of this *Warm-up* is to connect (x, y) coordinates with transformations.

There are many ways to express a translation because a translation is determined by two points P and Q once we know that P is translated to Q. There are many pairs of points that express the *same* translation. This is different from reflections which are determined by a unique line and rotations which have a unique center and a specific angle of rotation.

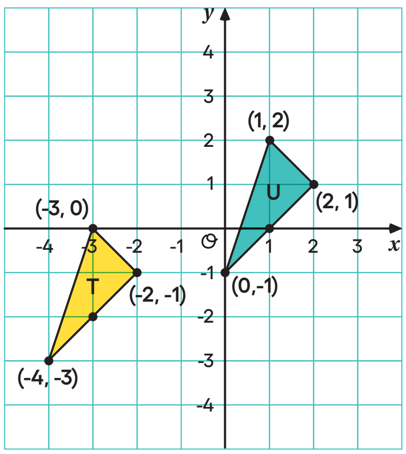
Launch

Ask students how they describe a translation. Is there more than one way to describe the same translation?

After they have thought about this for a minute, give them 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Select all of the translations that take Triangle T to Triangle U. There may be more than one correct answer.



A. Translate (-3, 0) to (1, 2).

B. Translate (2, 1) to (-2, -1).

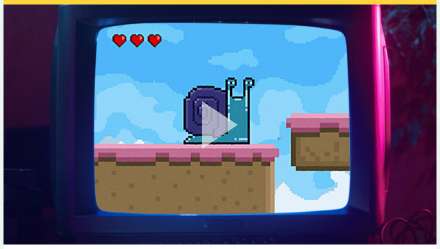
C. Translate (-4, -3) to (0, -1).

D. Translate (1, 2) to (2, 1).

A and C both take Triangle T to Triangle U.

Inspire Math

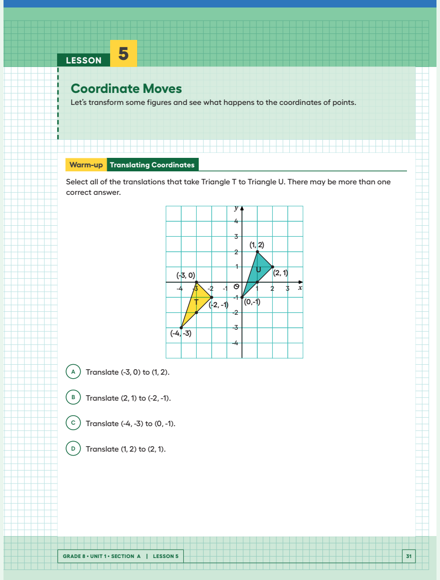
Video Games video



Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614199
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 5

Coordinate Moves

Let's transform some figures and see what happens to the coordinates of points.

Warm-up Translating Coordinates

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A Translate (-3, 0) to (1, 2).

B Translate (2, 1) to (-2, -1).

C Translate (-4, -3) to (0, -1).

D Translate (1, 2) to (2, 1).

Building on Student Thinking

Students may think that they need more information to determine the translation. Remind them that specifying one point can determine the distance and direction all of the other points move in a translation.

Instructional Routines

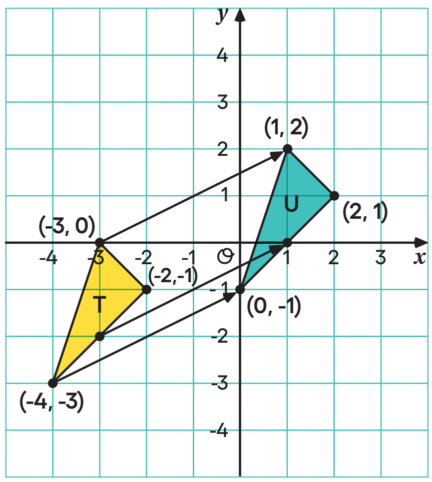
MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Activity Synthesis

Remind students that once you name a starting point and an ending point, that completely determines a translation because it specifies a distance and direction for *all* points in the plane. Appealing to their experiences with tracing paper may help. In this case, we might describe that distance and direction by saying "all points go up 2 units and to the right 4 units." Draw the arrow for the two correct descriptions and a third one not in the list, like this:



Point out that each arrow does, in fact, go up 2 and 4 to the right.

Activity 1

Reflecting Points on the Coordinate Plane

15 min

Activity Narrative

There is a digital version of this activity.

The goal of this activity is for students to work through multiple examples of specific points reflected over the x-axis and then generalize to describe where a reflection over the y-axis takes any point. They also consider reflections over the y-axis as they leverage the structure of the coordinate grid.

Monitor for students who use these different strategies:

- Count units or measure distances away from the axis of reflection to plot the new point
- Reason about the coordinate plane to determine the new coordinate

The key connection here is that when reflecting over the vertical or horizontal axis, one coordinate value must stay the same and one will be the opposite.

The goal of the activity is not to create a rule that students memorize. The goal is for students to notice the pattern of reflecting over an axis changing the sign of the coordinate without having to graph. The coordinate grid can sometimes be a powerful tool for understanding and expressing structure and this is true for reflections over both the x-axis and y-axis.

Launch

Tell students that they will have 5 minutes of quiet think time to work on the activity, and tell them to pause after the second question.

Select 2–3 students to share their strategies. Start with students who are measuring distances of points from the x-axis or counting the number of squares a point is from the x-axis and then counting out the same amount to find the reflected point. While these strategies work, they overlook the structure of the coordinate plane. Point out the role of the coordinate plane by selecting a student who noticed the pattern of changing the sign of the y-coordinate when reflecting over the x-axis.

Use *MLR7 Compare and Connect* when students present their strategies for reflecting points using the x-axis as the line of reflection, such as those described in the *Activity Narrative,* before continuing on. After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches and representations. Ask the following:

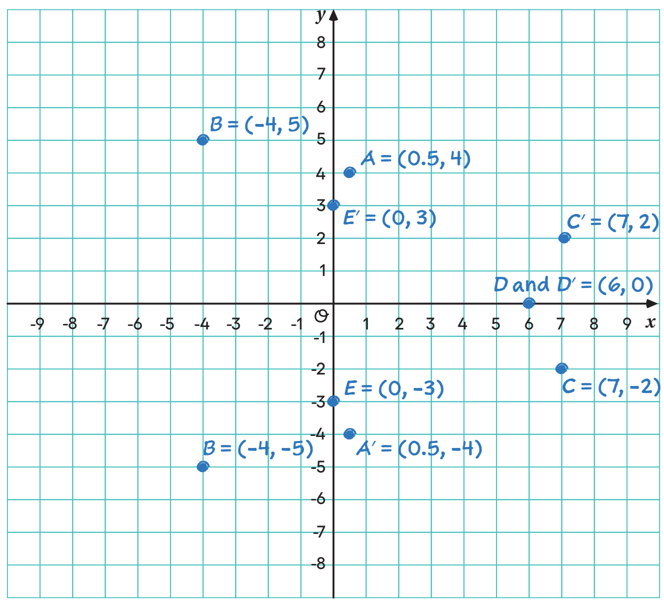
"What is the same and what is different about the strategies?"

"Did anyone solve the problem the same way, but would explain it differently?"

These exchanges strengthen students' mathematical language use and reasoning of reflections along the x-axis and y-axis.

After this initial discussion, give 2–3 minutes of quiet work time for the remaining questions, which ask them to generalize how to reflect a point over the y-axis.

Student Task Statement



1. Here is a list of points:
   A: (0.5, 4), B: (-4, 5), C: (7, -2), D: (6, 0)

   On the coordinate plane:

   a. Plot each point and label each with its coordinates.

   b. Using the x-axis as the line of reflection, plot the image of each point.

   c. Label the image of each point with its coordinates.

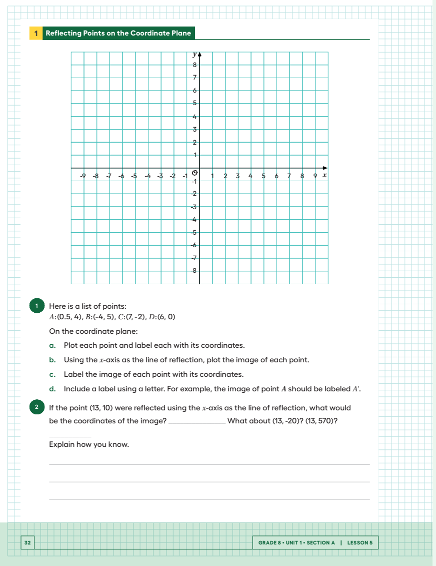
   d. Include a label using a letter. For example, the image of point A should be labeled A'.

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.
Use visible timers or audible alerts to help learners anticipate when to pause and prepare to transition to sharing strategies.

Supports accessibility for: Organization, Attention

Student Workbook


1 Reflecting Points on the Coordinate Plane

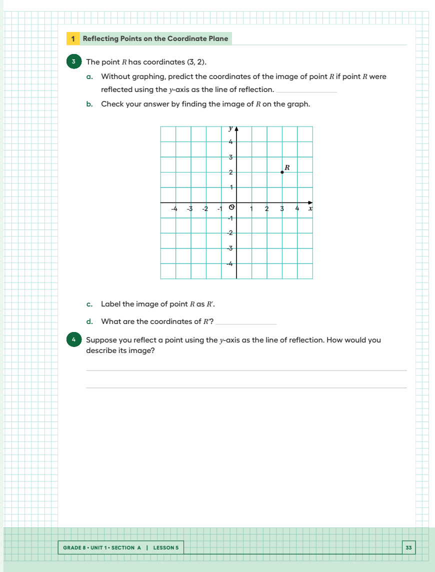
1 Here is a list of points:
A: (0.5, 4), B: (-4, 5), C: (7, -2), D: (6, 0)
On the coordinate plane:
a. Plot each point and label each with its coordinates.
b. Using the x-axis as the line of reflection, plot the image of each point.
c. Label the image of each point with its coordinates.
d. Include a label using a letter. For example, the image of point A should be labeled A'.

2 If the point (13, 10) were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? ________ What about (13, -20)? (13, 570)?
Explain how you know.

Building on Student Thinking

If any students struggle getting started because they are confused about where to plot the points, refer them back to the *Warm-up* and practice plotting a few example points with them.

Student Workbook


1 Reflecting Points on the Coordinate Plane

3 The point R has coordinates (3, 2).

a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

b. Check your answer by finding the image of R on the graph.

c. Label the image of point R as R'.

d. What are the coordinates of R'?

4 Suppose you reflect a point using the y-axis as the line of reflection. How would you describe its image?

2. If the point (13, 10) were reflected using the x-axis as the line of reflection, what would be the coordinates of the image? What about (13, -20)? (13, 570)? Explain how you know.

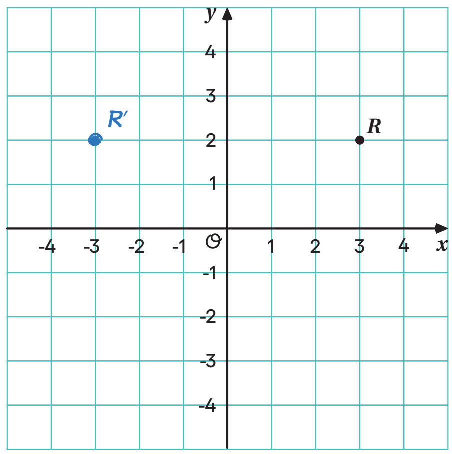
Using the x-axis as line of reflection, the reflection of (13, 10) is (13, -10), the reflection of (13, -20) is (13, 20), and the reflection of (13, 570) is (13, -570). Sample reasoning: Using the x-axis as the line of reflection does not move points horizontally but it does move points which are not on the x-axis vertically. In coordinates, the x-coordinate of the point stays the same while the y-coordinate changes sign.

3. The point R has coordinates (3, 2).

a. Without graphing, predict the coordinates of the image of point R if point R were reflected using the y-axis as the line of reflection.

Answers vary

b. Check your answer by finding the image of R on the graph.



c. Label the image of point R as R'.

d. What are the coordinates of R'?

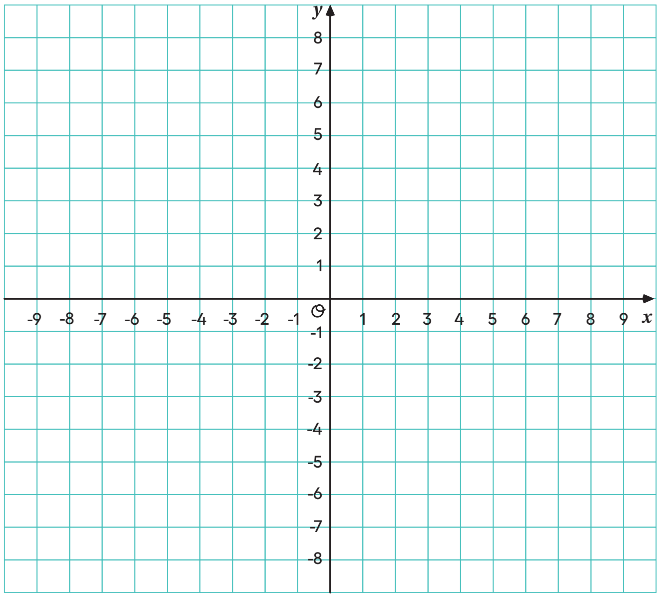
(-3, 2)

4. Suppose you reflect a point using the y-axis as the line of reflection. How would you describe its image?

Sample response: The point will have the same y-coordinate but the x-coordinate will change signs. The distance from the y-axis does not change and the y-coordinate does not change.

Activity Synthesis

Display a blank coordinate grid.



Questions for discussion:

"What was the same and what was different about reflecting over the y-axis instead of the x-axis?"

"When you have a point and an axis of reflection, how do you find the reflection of the point?"

"How can you use the coordinates of a point to help find the reflection?"

"Are some points easier to reflect than others? Why?"

"What patterns have you seen in these reflections of points on the coordinate grid?"

Activity 2

Transformations of a Segment

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students rotate a segment on a coordinate plane.

In general, it is difficult to use coordinates to describe rotations unless the center of the rotation is (0, 0) and the rotation is 90 degrees (clockwise or counterclockwise).

Unlike translations and reflections over the x- or y-axis, it is more difficult to visualize where a 90-degree rotation takes a point. Tracing paper is a helpful tool, as is an index card.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students in explaining the similarities and differences of the segment rotations for the last question: "_________ and _________ are the same/alike because …" or "_________ and _________ are different because …"

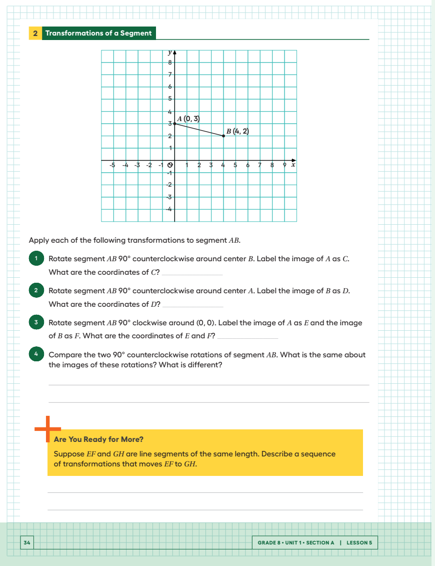
Advances: Speaking

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Develop Effort and Persistence.
Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.

Supports accessibility for: Language; Social-emotional skills; Attention

Student Workbook

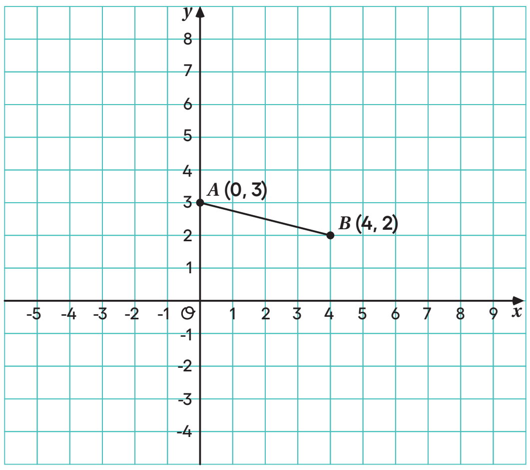


Launch

Demonstrate how to use tracing paper in order to perform a 90-degree rotation. It is helpful to put a small set of perpendicular axes (a + sign) on the piece of tracing paper and place their intersection point at the center of rotation. One of the small axes can be lined up with the segment being rotated and then the rotation is complete when the other small axis lines up with the segment.

An alternative method to perform rotations would be with the corner of an index card, which is part of the geometry toolkit.

Student Task Statement



Apply each of the following transformations to segment AB.

1. Rotate segment AB 90° counterclockwise around center B. Label the image of A as C. What are the coordinates of C?

   C: (3, -2)

2. Rotate segment AB 90° counterclockwise around center A. Label the image of B as D. What are the coordinates of D?

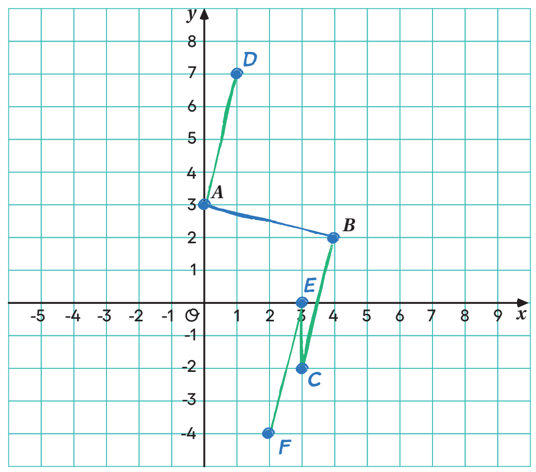
   D: (1, 7)

3. Rotate segment AB 90° clockwise around (0, 0). Label the image of A as E and the image of B as F. What are the coordinates of E and F?

   E: (3, 0), *F*: (2, -4)

4. Compare the two 90° counterclockwise rotations of segment AB. What is the same about the images of these rotations? What is different?

Sample response: The two counterclockwise rotations of AB are in different locations. The points A and B move different distances with the different rotations. One rotation can be mapped to the other by a translation.



Are You Ready for More?

Suppose EF and GH are line segments of the same length. Describe a sequence of transformations that moves EF to GH.

Sample response: Translate EF so that E lands on G ,and then rotate EF with center G until the image of F lands on H.

Activity Synthesis

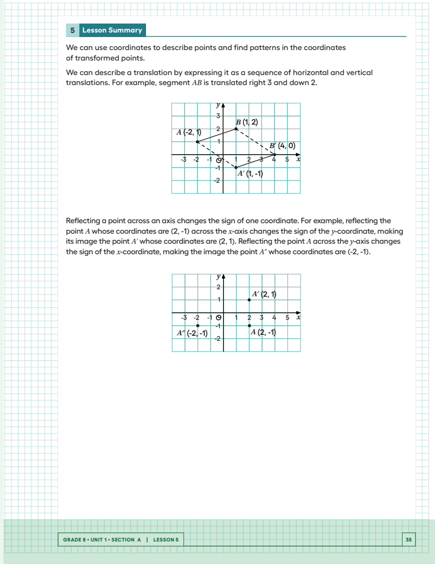
Ask students to describe or demonstrate how they found the rotations of segment AB. Make sure to highlight these strategies:

- Use tracing paper to enact a rotation through a 90-degree angle.
- Use an index card by placing the corner of the card at the center of rotation, aligning one side with the point to be rotated, and finding the location of the rotated point along an adjacent side of the card. (Each point's distance from the corner needs to be equal.)
- Use the structure of the coordinate grid so that a 90-degree rotation with center at the intersection of two grid lines will take horizontal grid lines to vertical grid lines and vertical grid lines to horizontal grid lines.

The third strategy should only be highlighted if students notice or use this in order to execute the rotation, with or without tracing paper. This last method is the most accurate because it relies on the structure of the coordinate grid.

If some students notice that the three rotations of segment AB are all parallel, this should also be highlighted.

Student Workbook



Lesson Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.

Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are (2, -1) across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are (2, 1). Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are (-2, -1).

GRADE 8 • UNIT 1 • SECTION A | LESSON 5

Lesson Synthesis

By this point, students should start to feel confident applying translations, reflections over either axis, and rotations of 90 degrees clockwise or counterclockwise to a point or shape in the coordinate plane.

To highlight working on the coordinate plane when doing transformations, ask:

"What are some advantages to knowing the coordinates of points when you are doing transformations?"

"What changes did we see when reflecting points over the x-axis? y-axis?"

"How do you perform a 90-degree clockwise rotation of a point with center (0, 0)?"

If time allows, ask students to apply a few transformations to a point. For example,

"What is the image of (1, 2) when it is ..."

"Reflected over the x-axis?"

(1, −2)

"Reflected over the y-axis?"

(−1, 2)

"Rotated 90 degrees clockwise with center (0, 0)?"

(2, −1)

Lesson Summary

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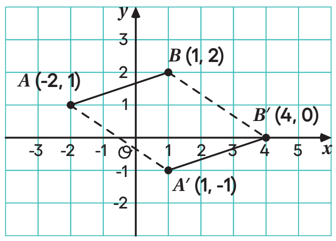
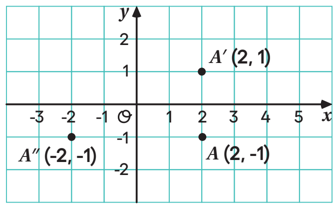
We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.



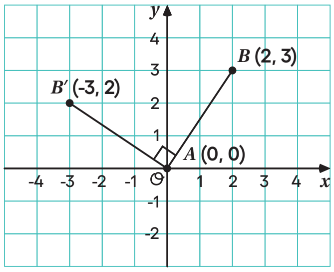
Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are (2, -1) across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are (2, 1). Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are (-2, -1).



Reflections across other lines are more complex to describe.

We don't have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a 90° rotation with center (0, 0) in a counterclockwise direction.

Point A has coordinates (0, 0). Segment AB is rotated 90° counterclockwise around A. Point B with coordinates (2, 3) rotates to point B' whose coordinates are (-3, 2).



Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

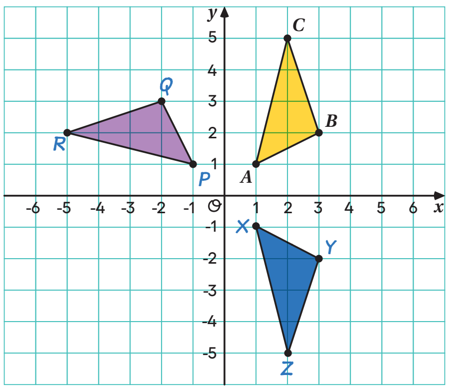
Cool-down

Rotation or Reflection

5 min

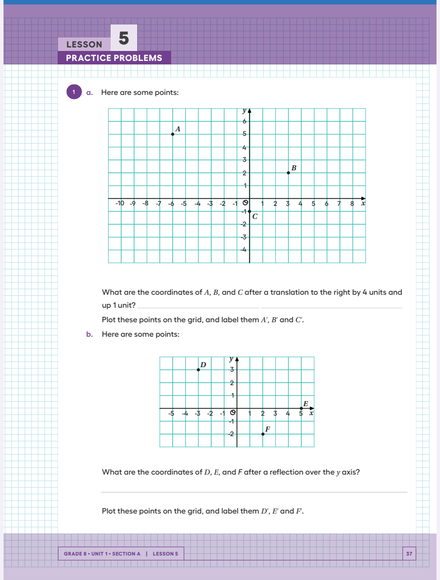
Student Task Statement

One of the triangles pictured is a rotation of triangle ABC and one of them is a reflection.

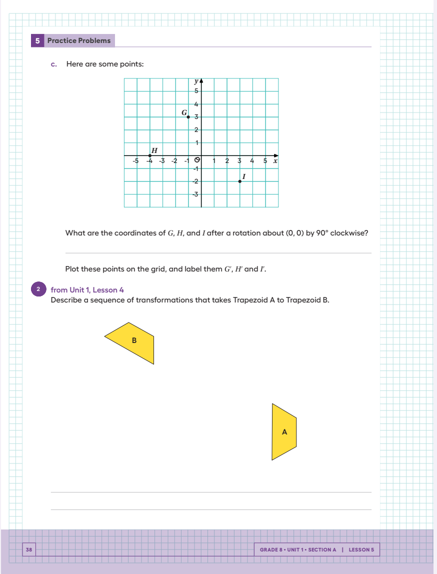


1. Label the rotated image PQR.
2. Label the reflected image XYZ.

Student Workbook

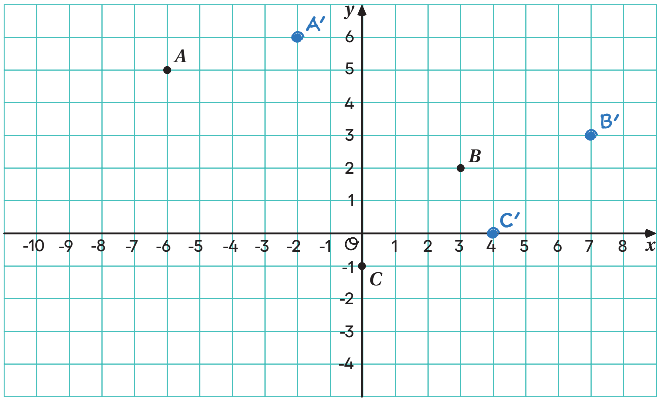


Student Workbook



Problem 1

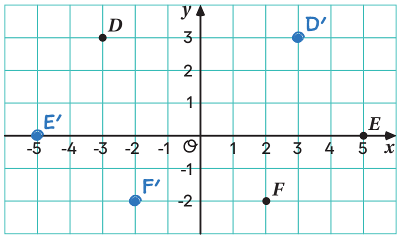
a. Here are some points:



What are the coordinates of A, B, and C after a translation to the right by 4 units and up 1 unit? Plot these points on the grid, and label them A', B' and C'.

A':(-2, 6), B':(7, 3), C':(4, 0)

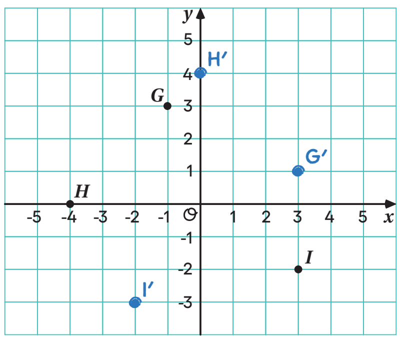
b. Here are some points:



What are the coordinates of D, E, and F after a reflection over the y-axis? Plot these points on the grid, and label them D', E' and F'.

D':(3, 3), E':(-5, 0), F':(-2, -2)

c. Here are some points:



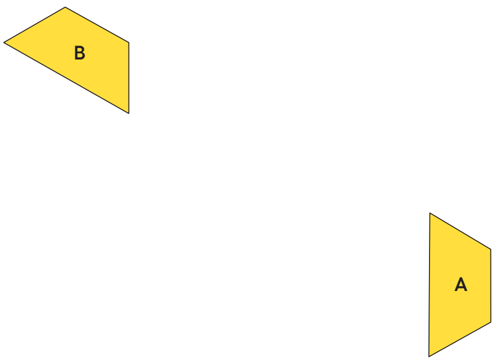
What are the coordinates of G, H, and I after a rotation about (0, 0) by 90° clockwise? Plot these points on the grid, and label them G', H' and I'.

G':(3, 1), H':(0, 4), I':(-2, -3)

Problem 2

from Unit 1, Lesson 4

Describe a sequence of transformations that takes Trapezoid A to Trapezoid B.

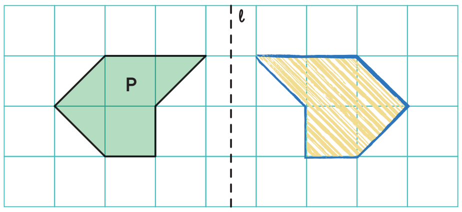


Sample response: Translate Trapezoid A up, then rotate it 60 degrees counterclockwise (with center of rotation the bottom vertex), and then translate it left.

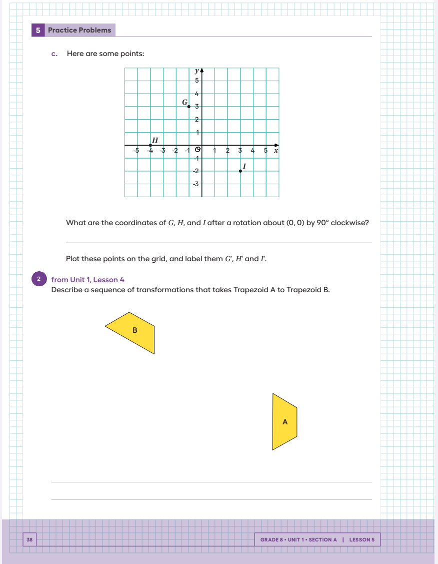
Problem 3

from Unit 1, Lesson 3

Reflect Polygon P using line ℓ.



Student Workbook



Student Workbook