Lesson 14

Edge Lengths and Volumes

Goals

- Comprehend the term "cube root of n" (in spoken language) and the notation $\sqrt[3]{n}$ (in written language) to mean the edge length of a cube whose volume is n cubic units.
- Coordinate representations of a cube root, including cube root notation, decimal representation, the edge length of a cube of given volume, and a point on the number line.

Learning Targets

- I can approximate cube roots.
- I know what a cube root is.
- I understand the meaning of expressions like $\sqrt[3]{5}$.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Math Community Chart: Activity 3

Materials to Copy

- Rooted in the Number Line Cards (1 copy for every 2 students): Activity 3

Lesson Narrative

In this lesson, students are formally introduced to **cube roots** and cube root notation where $\sqrt[3]{n}$ is the length of the edge of a cube whose volume is n cubic units. The cube root of a number n is the number whose cube is n.

Students begin by ordering solutions to equations of the form $a^2 = 9$ and $b^3 = 8$. Since they are already familiar with square roots and square root notation, this allows for a smooth transition to the parallel definition of cube roots. In the next activity, students complete a table connecting the edge length of a cube with its volume and a volume equation.

In the final activity, a *Card Sort* helps students make connections between square roots and cube roots as values, as solutions to equations, and as points on the number line. Students will have an opportunity to continue working with cube roots in the next lesson.

Student Learning Goal

Let's explore the relationship between volume and edge lengths of cubes.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	10 min	15 min	10 min	5 min

Warm-up
Ordering Squares and Cubes

10 min

Activity Narrative

The purpose of this *Warm-up* is to introduce students to cube roots by providing an opportunity to use cube root language and notation during the discussion. Encourage students to use estimated values for d and e to order the values before using a calculator. As students work, identify students who use different strategies for ordering, such as using a number line or using multiplication to guess and check.

Launch

Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time, and follow with a whole-class discussion.

For this activity, it is best if students do not have access to a calculator with a square root button. Encourage them to use estimation to order the values.

Student Task Statement

Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest.

A. $a^2 = 9$

B. $b^3 = 8$

C. $c^2 = 10$

D. $d^3 = 9$

E. $e^2 = 8$

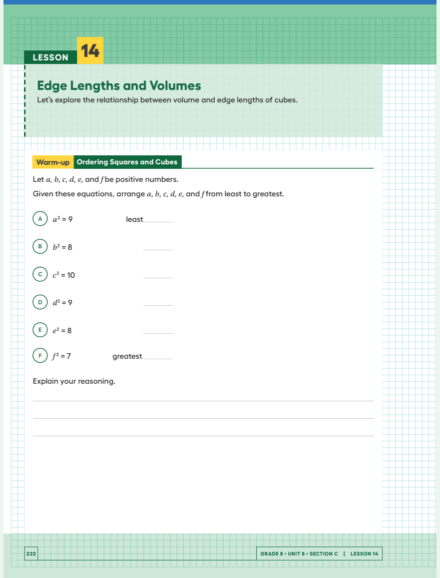
F. $f^3 = 7$

Explain your reasoning.

The order from least to greatest is f, b, d, e, a, c.

Sample reasoning: We know that $a = \sqrt{9}$, which is equal to 3. Since $e = \sqrt{8}$, it is slightly less than 3. Since $c = \sqrt{10}$, it is slightly greater than 3. We know that $b = \sqrt[3]{8}$, which is equal to 2 because $2^3 = 8$. Since $d = \sqrt[3]{9}$, it is slightly greater than 2. Since $f = \sqrt[3]{7}$, it is slightly less than 2.

Student Workbook


LESSON 14

Edge Lengths and Volumes

Let's explore the relationship between volume and edge lengths of cubes.

Warm-up Ordering Squares and Cubes

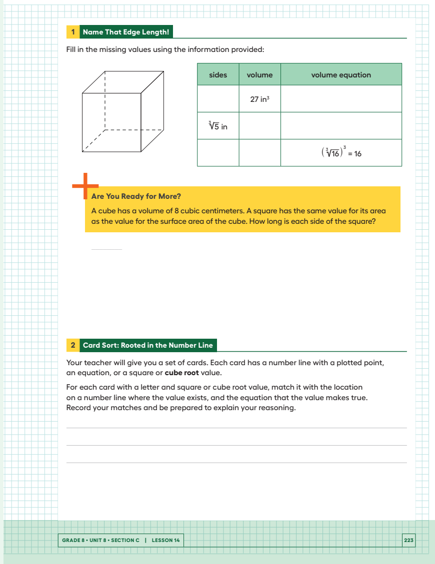
Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest.

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, review activities from previous lessons in which students explored square roots by completing a table based on the side length and the area of a square.
Supports accessibility for: Social-emotional skills, Conceptual processing

Student Workbook



1 Name That Edge Length!

Fill in the missing values using the information provided:

sides	volume	volume equation
	27 in^3	
$\sqrt[3]{5}$ in		
		$(\sqrt[3]{16})^3 = 16$

Are You Ready for More?

A cube has a volume of 8 cubic centimeters. A square has the same value for its area as the value for the surface area of the cube. How long is each side of the square?

2 Card Sort: Rooted in the Number Line

Your teacher will give you a set of cards. Each card has a number line with a plotted point, an equation, or a square or **cube root** value.

For each card with a letter and square or cube root value, match it with the location on a number line where the value exists, and the equation that the value makes true. Record your matches and be prepared to explain your reasoning.

GRADE 8 • UNIT 8 • SECTION C | LESSON 14 223

Activity Synthesis

The purpose of this discussion is to introduce cube root and cube root notation. Ask students to share their order of a, b, c, d, e, and f from least to greatest. Record and display their responses for all to see. If the class is in agreement, select previously identified students to share their strategies for ordering the values. If the class is in disagreement, ask students to share their reasoning until an agreement is reached.

Introduce students to **cube root** language and notation. Remind students that they previously learned that the equation $c^2 = 10$ has a solution $c = \sqrt{10}$. Similarly, we can say that the equation $d^3 = 9$ has a solution $d = \sqrt[3]{9}$.

Ask students to write a solution to $f^3 = 7$. ($f = \sqrt[3]{7}$)

Finally, tell students that while square roots are a way to write the exact value of the side length of a square with a known area, cube roots are a way to write the exact value of the edge length of a cube with a known volume.

Activity 1

Name That Edge Length!

10 min

Activity Narrative

The purpose of this task is for students to understand the relationship between the volume and edge lengths of cubes with that of cube roots.

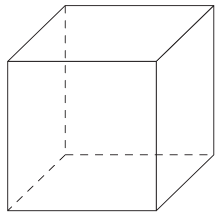
Launch

Students should not have access to a calculator since the goal of this activity is for students to write exact values using cube root notation.

Give students 2–3 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Fill in the missing values using the information provided:



sides	volume	volume equation
3 in	27 in^3	$3^3 = 27$
$\sqrt[3]{5}$	5 in^3	$(\sqrt[3]{5})^3 = 5$
$\sqrt[3]{16}$ in	16 in^3	$(\sqrt[3]{16})^3 = 16$

Are You Ready for More?

A cube has a volume of 8 cubic centimeters. A square has the same value for its area as the value for the surface area of the cube. How long is each side of the square?

The cube has sides of length 2 centimeters and surface area 24 square centimeters, so the square has side lengths $\sqrt{24}$ centimeters.

Activity Synthesis

The purpose of this discussion is for students to make the connection that knowing the volume of a cube is sufficient information for stating the exact side length of the cube using cube roots.

Here are some questions for discussion:

- *"Which two consecutive integers is $\sqrt[3]{5}$ between?"*

  $\sqrt[3]{5}$ is between 1 and 2 because $1^3 = 1$ and $2^3 = 8$.

- *"Which two consecutive integers is $\sqrt[3]{16}$ between?"*

  $\sqrt[3]{16}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$.

- *"Name another volume for a cube with edge lengths between 2 and 3."*

Sample response: A cube with volume 26 has edge lengths between 2 and 3 since $3^3 = 27$, meaning $\sqrt[3]{26}$ has a value slightly less than 3.

Activity 2

Card Sort: Rooted in the Number Line

15 min

Activity Narrative

Students sort square or cube root values, equations of the form $x^2 = n$ or $x^3 = n$, and number lines into matching sets of three during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to place all the cards face up and start thinking about possible ways to sort the cards into categories.

Pause the class and select 1–3 students to share the categories they identified.

Discuss as many different categories as time allows.

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports. Display sentence frames to support students as they justify their reasoning for each match: "The solution to ____ is ____ and it is between ____ and ____ on the number line because …"
Advances: Speaking, Conversing

Building on Student Thinking

If students mix up cube and square roots, consider asking:

"Can you explain how you made some of your matches?"

"What is the same and what is different about the way square and cube roots are written?"

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches. For example, give students the cards with the equations and root values and provide the number lines after the first set of cards has been matched.
Supports accessibility for: Conceptual Processing, Organization, Memory

Student Task Statement

Your teacher will give you a set of cards. Each card has a number line with a plotted point, an equation, or a square or **cube root** value.

For each card with a letter and square or cube root value, match it with the location on a number line where the value exists, and the equation that the value makes true. Record your matches and be prepared to explain your reasoning.

The blackline master shows the solution to the matching.

Activity Synthesis

Once all groups have completed the *Card Sort,* select groups to share one of their sets of three cards and how they matched the value, number line, and equation. Discuss the following:

"Which matches were tricky? Explain why."

Two of the number lines have values between 4 and 5, and it took some extra reasoning to figure out that the one with the value closer to 4 was $\sqrt{18}$ while the one with the point closer to 5 was $\sqrt[3]{100}$.

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

I initially matched $\sqrt[3]{50}$ with the number line close to 7, but then I noticed it was a cube root and not a square root, so I adjusted my match.

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

"I noticed our norm '_____' in action today and it really helped me/my group because _____."

Lesson Synthesis

The purpose of this discussion is to give students practice talking about cube roots in the context of shapes. For example, a cube with volume 64 cubic units has an edge length of 4 units, which is $\sqrt[3]{64}$ because $4^3 = 64$. Display each question one at a time, giving a brief quiet think time and then time to share with a partner before asking 1–2 students to share their thinking with the class:

"If a cube has a volume of 27 cubic inches, what is its side length?"

The cube with volume 27 cubic inches has an edge length of 3 inches since $\sqrt[3]{27} = 3$.

"What is the solution to $x^3 = 150$, and what two integers would it fall between on a number line?"

The solution to $x^3 = 150$ is $\sqrt[3]{150}$ and it is between 5 and 6 on a number line because $5^3 = 125$ and $6^3 = 216$.

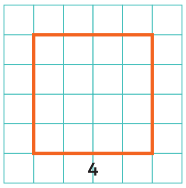
Lesson Summary

For a square, its side length is the square root of its area. For example, this square has an area of 16 square units and a side length of 4 units.

Both of these equations are true:

$4^2 = 16$

$\sqrt{16} = 4$

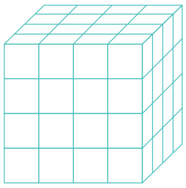


For a cube, the edge length is the **cube root** of its volume. For example, this cube has a volume of 64 cubic units and an edge length of 4 units.

Both of these equations are true:

$4^3 = 64$

$\sqrt[3]{64} = 4$



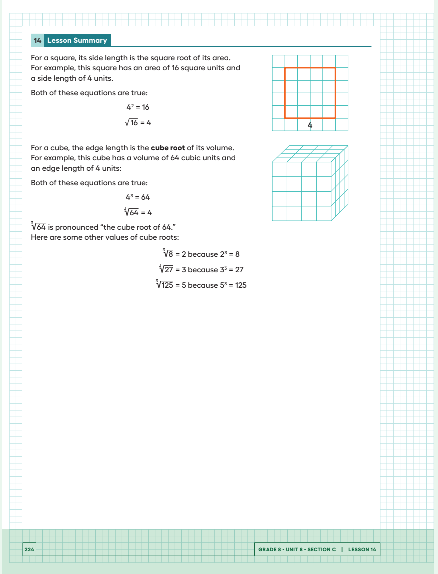
$\sqrt[3]{64}$ is pronounced "the cube root of 64." Here are some other values of cube roots:

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[3]{27} = 3$ because $3^3 = 27$

$\sqrt[3]{125} = 5$ because $5^3 = 125$

Student Workbook



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Roots, Sides, and Edges

5 min

Student Task Statement

Plot each value on the number line.

1. $\sqrt{36}$

   6

2. the edge length of a cube with volume 12 cubic units

   between 2 and 3

3. the side length of a square with area 70 square units

   between 8 and 9

4. $\sqrt[3]{36}$

   between 3 and 4

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 a. What is the volume of a cube with an edge length of

i. 4 centimeters?

ii. $\sqrt[3]{11}$ feet?

iii. s units?

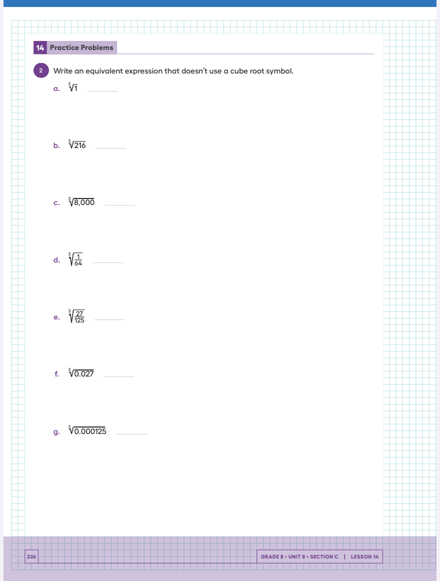
b. What is the edge length of a cube with a volume of

i. 1,000 cubic centimeters?

ii. 23 cubic inches?

iii. v cubic units?

Student Workbook


14 Practice Problems

2 Write an equivalent expression that doesn't use a cube root symbol.

a. $\sqrt[3]{1}$

b. $\sqrt[3]{216}$

c. $\sqrt[3]{8{,}000}$

d. $\sqrt[3]{\frac{1}{64}}$

e. $\sqrt[3]{\frac{27}{125}}$

f. $\sqrt[3]{0.027}$

g. $\sqrt[3]{0.000125}$

Lesson 14

Practice Problems

5 Problems

Problem 1

a. What is the volume of a cube with an edge length of

i. 4 centimeters?

64 cubic centimeters

ii. $\sqrt[3]{11}$ feet?

11 cubic feet

iii. s units?

s^3 cubic units

b. What is the edge length of a cube with a volume of

i. 1,000 cubic centimeters?

10 centimeters

ii. 23 cubic inches?

$\sqrt[3]{23}$ inches

iii. v cubic units?

$\sqrt[3]{v}$ units

Problem 2

Write an equivalent expression that doesn't use a cube root symbol.

a. $\sqrt[3]{1}$

1

b. $\sqrt[3]{216}$

6

c. $\sqrt[3]{8{,}000}$

20

d. $\sqrt[3]{\frac{1}{64}}$

$\frac{1}{4}$

e. $\sqrt[3]{\frac{27}{125}}$

$\frac{3}{5}$

f. $\sqrt[3]{0.027}$

0.3

g. $\sqrt[3]{0.000125}$

0.05

Problem 3

from Unit 8, Lesson 13

Find the distance (in units) between each pair of points. If you get stuck, try plotting the points on graph paper.

a. $X = (5, 0)$ and $Y = (-4, 0)$

9 units

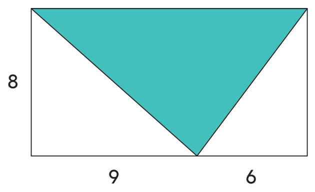
b. $K = (-21, -29)$ and $L = (0, 0)$

$\sqrt{1{,}282}$ units

Problem 4

from Unit 8, Lesson 10

Here is a 15-by-8 unit rectangle divided into triangles. Is the shaded triangle a right triangle? Explain or show your reasoning.



No

Sample reasoning: Use the Pythagorean Theorem to find the length of the interior sides of the triangle: The lengths are $\sqrt{145}$ units and 10 units. The longest side of the triangle is 15 units, the length of the rectangle. Now check whether this triangle's side lengths make $a^2 + b^2 = c^2$. Because $145 + 100 = 245$, not 225, the converse of the Pythagorean Theorem states this triangle is not a right triangle.

Problem 5

from Unit 7, Lesson 2

Express each of the following as a single power of 10:

a. 100

10^2

b. 100,000

10^5

c. 100,000,000

10^8

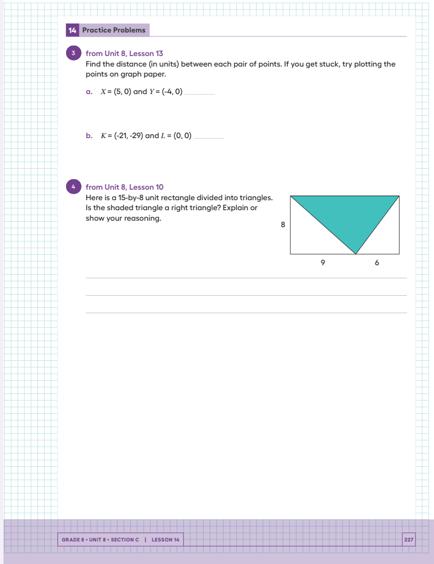
d. $100 \cdot 10$

10^3

e. $1{,}000 \cdot 1{,}000$

10^6

Student Workbook



14 Practice Problems

3 from Unit 8, Lesson 13

Find the distance (in units) between each pair of points. If you get stuck, try plotting the points on graph paper.

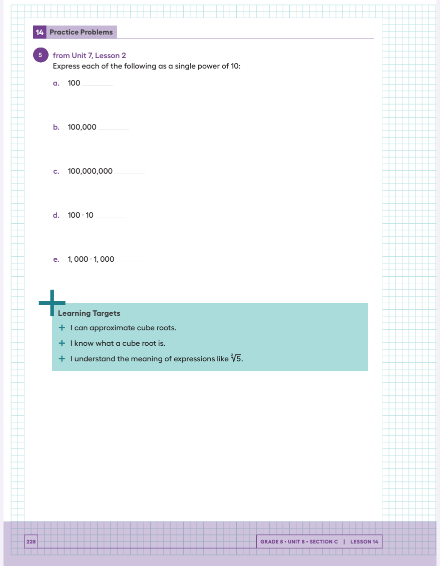
a. $X = (5, 0)$ and $Y = (-4, 0)$

b. $K = (-21, -29)$ and $L = (0, 0)$

4 from Unit 8, Lesson 10

Here is a 15-by-8 unit rectangle divided into triangles. Is the shaded triangle a right triangle? Explain or show your reasoning.

Student Workbook



14 Practice Problems

5 from Unit 7, Lesson 2

Express each of the following as a single power of 10:

a. 100

b. 100,000

c. 100,000,000

d. $100 \cdot 10$

e. $1{,}000 \cdot 1{,}000$

Learning Targets

+ I can approximate cube roots.

+ I know what a cube root is.

+ I understand the meaning of expressions like $\sqrt[3]{5}$.