Lesson 4

Solving for Unknown Angles

Goals

- Coordinate (orally and in writing) diagrams and equations that represent the same relationship between angle measures.
- Solve multi-step problems involving complementary, supplementary, and vertical angles, and explain (orally) the reasoning.

Learning Targets

- I can reason through multiple steps to find unknown angle measures.
- I can recognize when an equation represents a relationship between angle measures.

Lesson Narrative

In previous lessons, students solved single-step problems about supplementary, complementary, and vertical angles. In this lesson, students apply these skills to find unknown angle measures in multi-step problems. In the *Information Gap* activity, students ask questions about the angle relationships and solve for missing measures in a diagram. Then they see that they can represent angle problems with equations. Students also work to construct arguments about angles and discuss them with their partners.

Student Learning Goal

Let's figure out some missing angles.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

- Math Community Chart: Activity 1
- Pre-printed slips, cut from copies of the blackline master: Activity 1

Materials to Copy

- Angle Finding Cards (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

Make 1 copy of the *Information Gap*: Angle Finding blackline master for every 2 students, and cut them up ahead of time.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down

Warm-up

Math Talk: Length Relationships

5 min

Activity Narrative

This *Math Talk* focuses on line segment measure relationships represented as equations. It encourages students to think about combinations of line segments. As students mentally solve problems, they rely on what they know about how segment lengths relate to each other. The strategies elicited here will be helpful later in the lesson when students explore the relationship of angle measures.

To compare line segment measure relationships, students need to look for and make use of structure.

In describing their strategies, students need to be precise in their word choice and use of language.

Launch

Remind students that we refer to a length of a segment by naming its endpoints. For example, AB means the length of the line segment from A to B.

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Here are some line segments.

Decide mentally whether each statement is true.

A. $CD + BC = BD$

True. Sample reasoning: *CD* and *BC* are next to each other and share the endpoint *C*, so when they are added together they make *BD*.

B. $AB + BD = CD + AD$

False. Sample reasoning: *AB* + *BD* makes the full segment *AD*, but *CD* + *AD* adds to the full segment *AD*, so they are not equal.

C. $AC - AB = AB$

False. Sample reasoning: *AC* is made of segments *AB* and *BC*, so when *AB* is removed *BC* remains, not *AB*.

D. $BD - CD = AC - AB$

True. Sample reasoning: *BD* − *CD* leaves *BC*, and *AC* − *AB* leaves *BC*, so they are equal.

Inspire Math

3-D Printed Houses video

Go Online
Before the lesson, show this video to introduce the real-world connection.

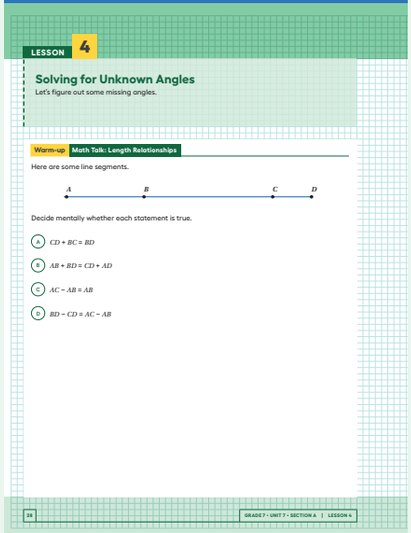
ilclass.com/l/614175

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook



Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class. *Advances: Speaking, Representing*

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards. This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate ____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to ____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

After each true equation, ask students if they could rely on the same reasoning to determine if other similar problems are equivalent. After each false equation, ask students how the problem could be changed to make the equation true.

Activity 1

Information Gap: Angle Finding

20 min

Activity Narrative

In this activity, students solve for an unknown angle measure in a multi-step problem but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Information Gap* structure requires students to make sense of a problems by determining what information is necessary, and then to ask for the information they need to solve the problem. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and to ask increasingly more precise questions until they get the information they need.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice. This norm should be one that they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

Tell students they will solve for an unknown angle measure given other angle measures. Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it. If desired, explain this variation from the typical *Information Gap*: Instead of the student with the problem card asking for each piece of information, the student with the data card chooses a piece of information to share. The student with the problem card still needs to explain how they can use each piece of information. If more information is needed to solve the problem, the student with the data card chooses another piece of information to share. Also, students need to listen to their partner carefully because they may be asked to explain their partner's reasoning to the class.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.
If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner! These steps may be repeated.

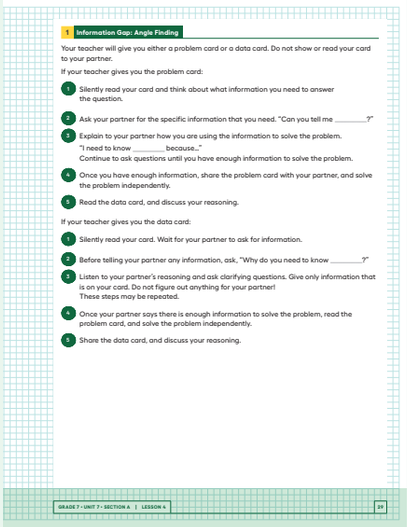
Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity or provide students with a physical copy.
Supports accessibility for: Memory, Organization

Building on Student Thinking

For the second set of cards, students may struggle to find the connection between the lower half of the figure and the upper half. Remind them that supplementary angles do not need to be next to one another, but they can be.

Student Workbook


1 Information Gap: Angle Finding

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because..." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

GRADE 7 • UNIT 7 • SECTION A | LESSON 4 29

4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: $b = 90°$

Sample responses:

- $c = 56$, $a = 90 - 56 = 34$, $b = 180 - 34 - 56 = 90$
- $d = 124$, $e = 180 - 124 = 56$, $c = 56$, $a = 90 - 56 = 34$, $b = 180 - 34 - 56 = 90$
- $a + c = 90$, $a + b + c = 180$, $b = 180 - 90 = 90$

Problem Card 2: $b = 27°$

Sample responses:

- $z = 63$, $x = 63$, $d = 180 - 63 = 117$, $c = 180 - 117 = 63$, $a = 90$, $b = 180 - 90 - 63 = 27$
- $z = 63$, $w = 180 - 63 = 117$, $x = 180 - 117 = 63$, $d = 180 - 63 = 117$, $e = 180 - 117 = 63$, $c = 63$, $a = 90$, $b = 360 - 117 - 63 - 90 - 63 = 2$

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"Did you need all of the information from the data card?"*

  No, it is possible to calculate the angle measure without using all of the information from the data card.

- *"How was your approach to the first card different from your approach to the second card?"*

Highlight for students that writing and solving equations is an efficient strategy to show their reasoning about multi-step angle problems.

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

- "I picked the norm '____' It really helped me/my group because ____."
- "I picked the norm '____' During the activity, I realized the norm '____' would be a better focus because ____."

Activity 2
What's the Match?

10 min

Activity Narrative

The purpose of this activity is for students to match relationships between angles in a figure with equations that can represent those relationships. This prepares students for writing and solving equations that represent relationships between angles in the next lesson. As students explain their reasoning, they construct logical arguments about the structure of the equation and the diagram.

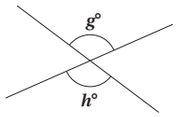
Launch

Keep students in the same groups. Give students 2–3 minutes of quiet work time, followed by partner and whole-class discussion.

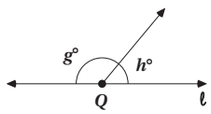
Student Task Statement

Match each figure to an equation that represents what is seen in the figure. For each match, explain how you know they are a match.

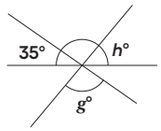
2 (A)



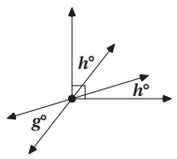
1 (B)



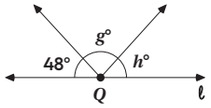
5 (C)



3 (D)



4 (E)



1. $g + h = 180$

   Sample reasoning: this is because g and h are supplementary angles, $g + h = 180$.

2. $g = h$

   Sample reasoning: g and h are vertical angles and vertical angles are congruent so $g = h$.

3. $2h + g = 90$

   Sample reasoning: the three angles in the figure form a right angle, and the angle that measures g degrees is vertical to one of those angles, which means $2h + g = 90$.

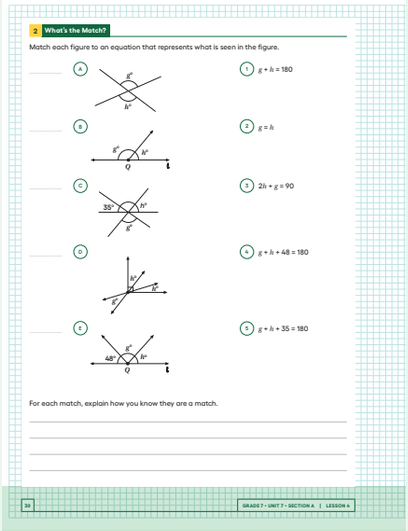
4. $g + h + 48 = 180$

   Sample reasoning: the angles all form a straight angle, which is why $g + h + 48 = 180$.

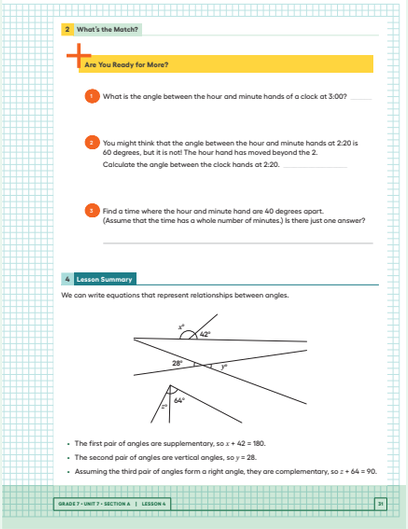
5. $g + h + 35 = 180$

   Sample reasoning: the three angles form a straight line. The angle in between the angle measuring 35° and the angle measuring h degrees is a vertical angle with the angle measuring g degrees, which means $g + h + 35 = 180$.

Student Workbook



Student Workbook


2 What's the Match?

Are You Ready for More?

1. What is the angle between the hour and minute hands of a clock at 3:00?

2. You might think that the angle between the hour and minute hands at 2:20 is 60 degrees, but it is not! The hour hand has moved beyond the 2. Calculate the angle between the clock hands at 2:20.

3. Find a time where the hour and minute hand are 40 degrees apart. (Assume that the time has a whole number of minutes.) Is there just one answer?

4 Lesson Summary

We can write equations that represent relationships between angles.

- The first pair of angles are supplementary, so $x + 42 = 180$.
- The second pair of angles are vertical angles, so $y = 28$.
- Assuming the third pair of angles form a right angle, they are complementary, so $z + 64 = 90$.

GRADE 7 • UNIT 7 • SECTION A | LESSON 4 31

Are You Ready for More?

1. What is the angle between the hour and minute hands of a clock at 3:00?

   90 degrees

2. You might think that the angle between the hour and minute hands at 2:20 is 60 degrees, but it is not! The hour hand has moved beyond the 2. Calculate the angle between the clock hands at 2:20.

   50 degrees

   Sample reasoning: At 2:20, the hour hand should be one-third of the way between the 2 and the 3. Since the angle between the 2 and the 3 is a 30-degree angle, the hour hand has moved 10 degrees toward the 3. Therefore, the angle is 50 degrees rather than 60.

3. Find a time where the hour and minute hand are 40 degrees apart. (Assume that the time has a whole number of minutes.) Is there just one answer?

   5:20 and 6:40

   Sample reasoning: These can be found by guess-and-check. It may help to realize that the hour hand of a clock moves at half a degree per minute, and the minute hand of a clock moves at 6 degrees per minute.

Activity Synthesis

The goal of this discussion is for students to articulate the angle relationships that they noticed in each figure and equation. Display the figures for all to see. Select previously identified students to share their explanations for each figure. If not mentioned in students' explanations, be sure that students see the vertical, supplementary, straight, and right angle relationships in the figures.

Lesson Synthesis

The purpose of this discussion is for students to connect angle relationships with equations that represent those relationships. Here are some questions to elicit student thinking:

- *"If you know that angles a and b are vertical, what equation could you use to represent this angle relationship?"*

  $a = b$

- *"If you know that angles c and d are complementary, what equation could you use to represent this angle relationship?"*

  $c + d = 90$

- *"If you know that angles e and f are supplementary, what equation could you use to represent this angle relationship?"*

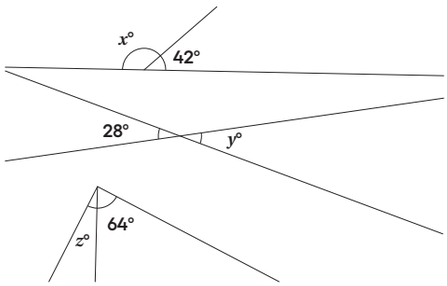
  $e + f = 180$

- *"How is it helpful to represent relationships between angles as an equation?"*

  If you know some of the information, you can put it in the equation and solve the equation to find what is missing. It can be easier to keep track of what you know as an equation.

Lesson Summary

We can write equations that represent relationships between angles.



- The first pair of angles are supplementary, so $x + 42 = 180$.
- The second pair of angles are vertical angles, so $y = 28$.
- Assuming the third pair of angles form a right angle, they are complementary, so $z + 64 = 90$.

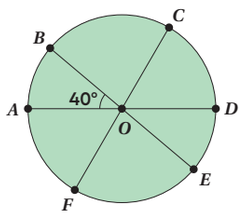
Cool-down

Missing Circle Angles

5 min

Student Task Statement

AD, BE, and CF are all diameters of the circle. The measure of angle AOB is 40 degrees. The measure of angle DOF is 120 degrees.



Find the measures of the angles:

1. BOC 80°

Sample reasoning: Given angle DOF = 120°, angle AOC = 120° because they are congruent vertical angles. Consequently, angles AOB + BOC = 120° because they are adjacent.

2. COD 60°

Sample reasoning: Angle COD and angle DOF are supplementary angles, so the sum of their measurements has to be 180°.

Responding To Student Thinking

Points to Emphasize

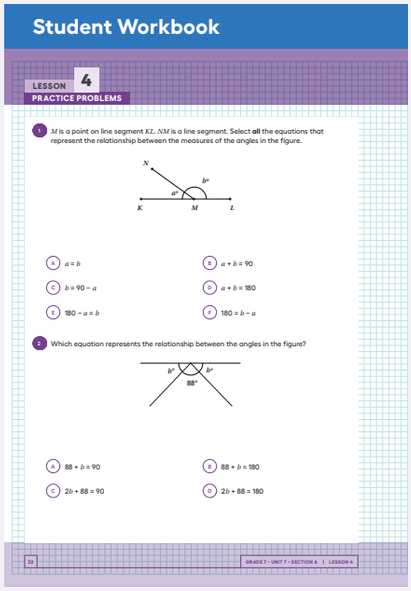
If most students struggle with using the given information to find the missing angle measures, revisit this idea when doing this activity:

Grade 7, Unit 7, Lesson 5, Activity 2
Calculate the Measure

Practice Problems

7 Problems

Student Workbook


LESSON 4
PRACTICE PROBLEMS

1. M is a point on line segment KL. NM is a line segment. Select **all** the equations that represent the relationship between the measures of the angles in the figure.

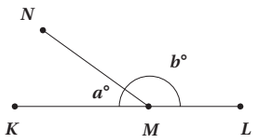
A. $a = b$
B. $a + b = 90$
C. $b = 90 - a$
D. $a + b = 180$
E. $180 - a = b$
F. $180 = b - a$

2. Which equation represents the relationship between the angles in the figure?

A. $88 + b = 90$
B. $88 + b = 180$
C. $2b + 88 = 90$
D. $2b + 88 = 180$

Problem 1

M is a point on line segment KL. NM is a line segment. Select **all** the equations that represent the relationship between the measures of the angles in the figure.



A. $a = b$

B. $a + b = 90$

C. $b = 90 - a$

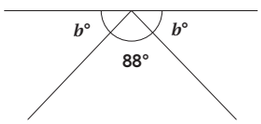
D. $a + b = 180$

E. $180 - a = b$

F. $180 = b - a$

Problem 2

Which equation represents the relationship between the angles in the figure?



A. $88 + b = 90$

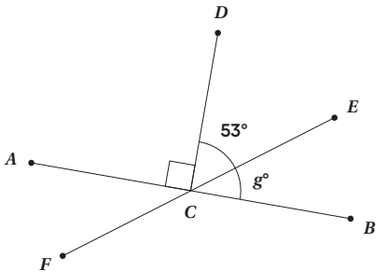
B. $88 + b = 180$

C. $2b + 88 = 90$

D. $2b + 88 = 180$

Problem 3

Segments AB, EF, and CD intersect at point C, and angle ACD is a right angle. Find the value of g.



37

Problem 4

from Unit 6, Lesson 12

Select **all** the expressions that are the result of decreasing x by 80%.

A. $\frac{20}{100}x$

B. $x - \frac{80}{100}x$

C. $\frac{100 - 20}{100}x$

D. $0.80x$

E. $(1 - 0.8)x$

Problem 5

from Unit 6, Lesson 8

Andre is solving the equation $4\left(x + \frac{3}{2}\right) = 7$. He says, "I can subtract $\frac{3}{2}$ from each side to get $4x = \frac{11}{2}$ and then divide by 4 to get $x = \frac{11}{8}$." Kiran says, "I think you made a mistake."

a. How can Kiran know for sure that Andre's solution is incorrect?

Answers vary.

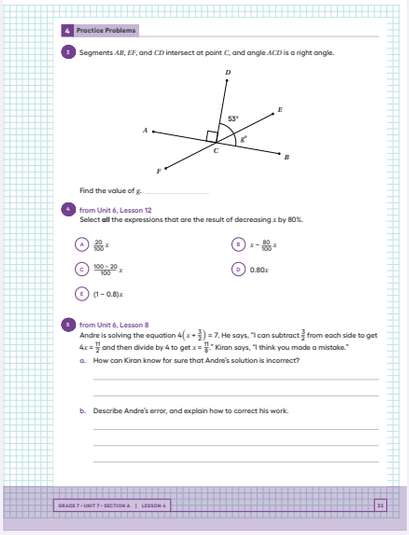
Sample response: He can substitute Andre's solution into the equation. If the solution is correct, the resulting equation will be true. $4\left(\frac{11}{8} + \frac{3}{2}\right)$ is $11\frac{1}{2}$, not 7, so the solution is incorrect.

b. Describe Andre's error, and explain how to correct his work.

Answers vary.

Sample response: Andre subtracted $\frac{3}{2}$ from each side, but that doesn't remove the $\frac{3}{2}$ from the equation because $\frac{3}{2}$ is part of an expression multiplied by 4. Andre could divide each side by 4 to get $x + \frac{3}{2} = \frac{7}{4}$ and then subtract $\frac{3}{2}$ on each side to get $x = \frac{1}{4}$. (Or, he could use the distributive property to write $4x + 6 = 7$, subtract 6 from each side to get $4x = 1$, and then divide by 4 on each side to get $x = \frac{1}{4}$.)

Student Workbook


4 Practice Problems

3 Segments AB, EF, and CD intersect at point C, and angle ACD is a right angle.

Find the value of g.

4 from Unit 6, Lesson 12
Select **all** the expressions that are the result of decreasing x by 80%.

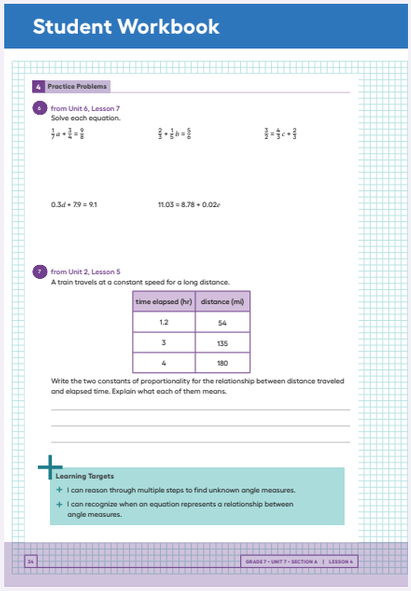
A $\frac{20}{100}x$ B $x - \frac{80}{100}x$ C $\frac{100-20}{100}x$ D $0.80x$ E $(1 - 0.8)x$

5 from Unit 6, Lesson 8
Andre is solving the equation $4\left(x + \frac{3}{2}\right) = 7$. He says, "I can subtract $\frac{3}{2}$ from each side to get $4x = \frac{11}{2}$ and then divide by 4 to get $x = \frac{11}{8}$." Kiran says, "I think you made a mistake."

a. How can Kiran know for sure that Andre's solution is incorrect?

b. Describe Andre's error, and explain how to correct his work.

GRADE 7 • UNIT 7 • SECTION A | LESSON 4 23


Student Workbook

4 Practice Problems

6 from Unit 6, Lesson 7
Solve each equation.

$\frac{1}{7}a + \frac{3}{4} = \frac{9}{8}$ $\frac{2}{3} + \frac{1}{5}b = \frac{5}{6}$ $\frac{3}{2} = \frac{4}{3}c + \frac{2}{3}$

$0.3d + 7.9 = 9.1$ $11.03 = 8.78 + 0.02e$

7 from Unit 2, Lesson 5
A train travels at a constant speed for a long distance.

time elapsed (hr)	distance (mi)
1.2	54
3	135
4	180

Write the two constants of proportionality for the relationship between distance traveled and elapsed time. Explain what each of them means.

Learning Targets
+ I can reason through multiple steps to find unknown angle measures.
+ I can recognize when an equation represents a relationship between angle measures.

24 GRADE 7 • UNIT 7 • SECTION A | LESSON 4

Problem 6

from Unit 6, Lesson 7

Solve each equation.

$\frac{1}{7}a + \frac{3}{4} = \frac{9}{8}$

$a = \frac{21}{8}$

$\frac{2}{3} + \frac{1}{5}b = \frac{5}{6}$

$b = \frac{5}{6}$

$\frac{3}{2} = \frac{4}{3}c + \frac{2}{3}$

$c = \frac{5}{8}$

$0.3d + 7.9 = 9.1$

$d = 4$

$11.03 = 8.78 + 0.02e$

$e = 112.5$

Problem 7

from Unit 2, Lesson 5

A train travels at a constant speed for a long distance.

time elapsed (hr)	distance (mi)
1.2	54
3	135
4	180

Write the two constants of proportionality for the relationship between distance traveled and elapsed time. Explain what each of them means.

45

The train travels 45 miles in 1 hour.

$\frac{1}{45}$

It takes $\frac{1}{45}$ hours for the train to travel 1 mile.