Lesson 11

Applications of the Pythagorean Theorem

Goals

- Describe (orally) situations that use right triangles, and explain how the Pythagorean Theorem could help solve problems in those situations.
- Use the Pythagorean Theorem to solve problems within a context, and explain (orally) how to organize the reasoning.

Learning Target

I can use the Pythagorean Theorem to solve problems.

Lesson Narrative

This lesson gives students the opportunity to use the Pythagorean Theorem as a tool to solve different application problems. In the first activity, students make sense of a problem involving the distance and speed of two children walking and riding a bike along different sides of a triangular region. In the second activity they find internal diagonals of rectangular prisms.

By the end of this lesson, students should be ready to consider more complex problems involving the Pythagorean Theorem.

Student Learning Goal

Let's explore some applications of the Pythagorean Theorem.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR6: Three Reads (Activity 1)

Instructional Routines

- Math Talk

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Math Talk: Square Roots

5 min

Activity Narrative

This *Math Talk* focuses on estimating the decimal value of each square root expression. It encourages students to think about what the square root symbol means and to rely on what they know about perfect squares to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students apply the Pythagorean Theorem.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Mentally find the value of each expression to the nearest half.

A. $\sqrt{24}$

5

Sample reasoning: The square root of 25 is 5, and 24 is very close to 25.

B. $\sqrt{7}$

2.5

Sample reasoning: 7 is just a little over halfway between 2^2 and 3^2.

C. $\sqrt{42}$

6.5

Sample reasoning: 42 is almost halfway between 6^2 and 7^2.

D. $\sqrt{10} + \sqrt{97}$

13

Sample reasoning: The square root of 10 is a little more than 3, and the square root of 97 is a little less than 10.

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR8: Discussion Supports

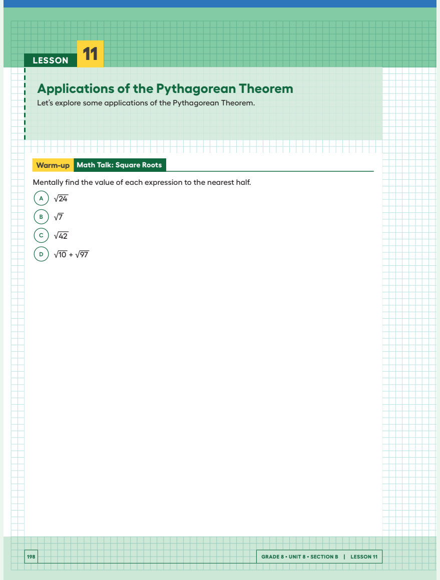
ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook


LESSON 11

Applications of the Pythagorean Theorem

Let's explore some applications of the Pythagorean Theorem.

Warm-up Math Talk: Square Roots

Mentally find the value of each expression to the nearest half.

A $\sqrt{24}$

B $\sqrt{7}$

C $\sqrt{42}$

D $\sqrt{10} + \sqrt{97}$

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.
Keep books or devices closed. Display only the problem stem and diagram, without revealing the questions.

"We are going to read this problem 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1
Cutting Corners

15 min

Activity Narrative

In this activity, students reason about distances and speeds and then use the Pythagorean Theorem to figure out who will win a race. Students must translate between the context and the geometric representation of the context and back. Monitor for students whose work is clearly labeled and organized to share during the whole-class discussion.

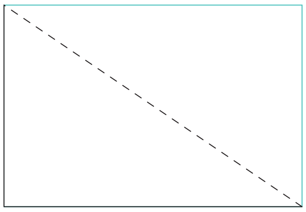
Launch

Arrange students in groups of 2. Provide students with access to calculators. Read the problem stem aloud, then ask the class who they think will win the race—Mai or Tyler. Display the results for all to see. Select 2–3 students to share their thinking. If not brought up in students' explanations, emphasize a few key points: Mai travels farther than Tyler, but she is also going faster, so it is not immediately clear who will win.

Give 2–3 minutes of quiet work time for the second problem followed by a partner discussion. Then follow with a whole-class discussion.

Student Task Statement

Mai and Tyler are standing at one corner of a large rectangular field and decide to race to the opposite corner. Since Mai has a bike and Tyler does not, they think it would be a fairer race if Mai rode along the sidewalk that surrounds the field (the bolded edges in the diagram) while Tyler ran the shorter distance directly across the field. The field is 100 meters long and 80 meters wide. Tyler can run at around 5 meters per second, and Mai can ride her bike at around 7.5 meters per second.



1. Before making any calculations, who do you think will win? By how much? Explain your thinking.

   Answers vary.

   Sample response: Mai is going faster, but she also has a longer distance to travel. I think that she might be fast enough to beat Tyler even if she is going a little farther, but she won't win by much.

2. Who wins? Show your reasoning.

   Mai wins. Mai has 180 meters to travel going 7.5 meters per second, which will take her 24 seconds since $\frac{180}{7.5} = 24$. Using the Pythagorean Theorem, Tyler travels $\sqrt{16{,}400}$ meters going 5 meters per second, which will take him approximately 25.6 seconds since $\frac{\sqrt{16{,}400}}{5} \approx 25.6$. Mai beats Tyler to the opposite corner by about 1.6 seconds.

Are You Ready for More?

A calculator may be necessary to answer the following questions. Round answers to the nearest hundredth.

1. If you could give the loser of the race a head start, how much time would they need in order for both people to arrive at almost the exact same time?

   Tyler needs roughly 1.6 seconds of head start to beat Mai. He travels approximately 128.06 meters going 5 meters per second, which will take him about 25.6 seconds. Mai finishes in 24 seconds.

2. If you could make the winner go slower, how slow would they need to go in order for both people to arrive at almost the exact same time?

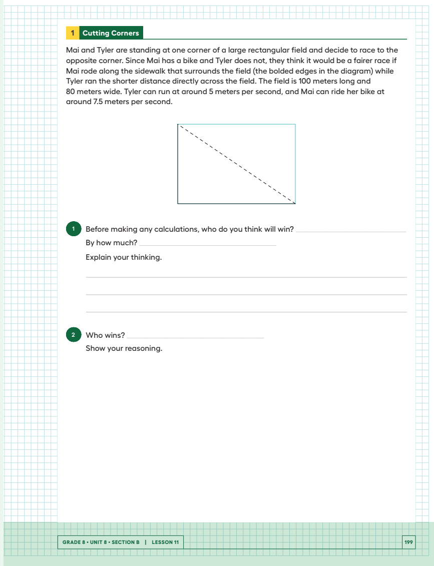
   About 7.03 meters per second. If Mai went 7.03 meters per second, then she would finish the race in $\frac{180}{7.03} \approx 25.6$ seconds.

Building on Student Thinking

If students mix up who is traveling which path at what speed, consider asking:

"Explain your reasoning to me for why Mai/Tyler wins."
"How could you label the diagram to help your thinking?"

Student Workbook



1 Cutting Corners

Mai and Tyler are standing at one corner of a large rectangular field and decide to race to the opposite corner. Since Mai has a bike and Tyler does not, they think it would be a fairer race if Mai rode along the sidewalk that surrounds the field (the bolded edges in the diagram) while Tyler ran the shorter distance directly across the field. The field is 100 meters long and 80 meters wide. Tyler can run at around 5 meters per second, and Mai can ride her bike at around 7.5 meters per second.

1 Before making any calculations, who do you think will win? By how much? Explain your thinking.

2 Who wins? Show your reasoning.

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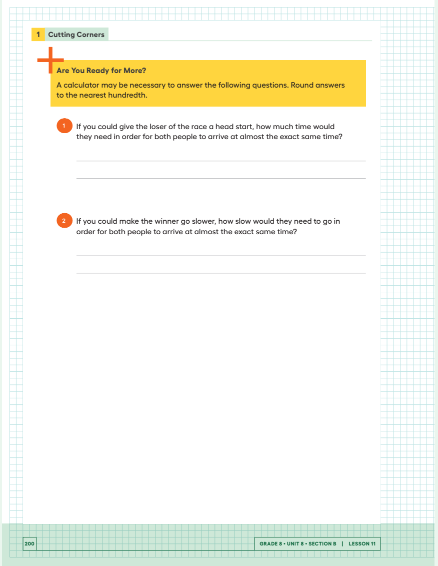
Student Workbook



1 Cutting Corners

Are You Ready for More?

A calculator may be necessary to answer the following questions. Round answers to the nearest hundredth.

1 If you could give the loser of the race a head start, how much time would they need in order for both people to arrive at almost the exact same time?

2 If you could make the winner go slower, how slow would they need to go in order for both people to arrive at almost the exact same time?

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Activity Synthesis

The purpose of this discussion is for students to see strategies for how they could organize their thinking, and how accurate their initial predictions were.

Select 1–2 previously identified students to share their work. Draw attention to how careful labeling of provided figures and organization of calculations when problems have multiple steps helps to solve problems and identify errors. For example, in problems where multiple calculations are completed using a calculator, it is easy to copy an incorrect number, such as writing 14,600 instead of 16,400 for the sum $100^2 + 80^2$.

Activity 2

Internal Dimensions

15 min

Activity Narrative

The purpose of this task is for students to use the Pythagorean Theorem with a three-dimensional figure. Students will need to picture or sketch the right triangles necessary to calculate and determine which rectangular prism has the longer diagonal length.

Monitor for groups who use a well-organized strategy to calculate the diagonals. For example, some groups may draw in the diagonal on the bottom face of the prism, label it with an unknown variable, and use the Pythagorean Theorem with the edge lengths of the bottom face to calculate the bottom diagonal length.

Launch

Arrange students in groups of 2. Provide access to calculators. If possible, use an actual rectangular prism, such as a small box, to help students understand what length the diagonal shows in the image.

Tell students to read and consider the first problem and then give a signal when they are ready. Ask the class which prism they think has the longer diagonal and display the results for all to see. Select 2–3 students to explain their reasoning. If not brought up in students' explanations, emphasize a few key points: Prism K has one edge with length 6 units, which is longer than any of the edges of Prism L, but it also has a side of length 4 units, which is shorter than any of the edges of Prism L. Prism K also has a smaller volume than Prism L.

Give students 1 minute of quiet think time to brainstorm how they will calculate the lengths of each diagonal followed by a partner discussion. Give students 3–4 minutes of quiet work time, and follow with a whole-class discussion.

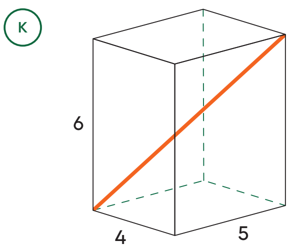
Access for Students with Diverse Abilities (Activity 2, Launch)

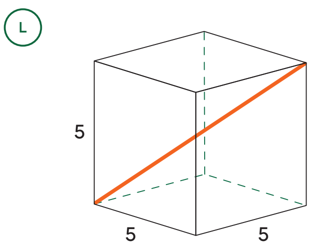
Representation: Access for Perception.
Provide access to 3-D models of rectangular prisms (for example, a tissue box) for students to view or manipulate. Ask students to identify correspondences between the concrete rectangular prism and the drawn rectangular prism.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Task Statement

Here are two rectangular prisms:





1. Which figure do you think has the longer diagonal? Why? Note that the figures are not drawn to scale.

   Sample response: I think Prism L has a longer diagonal since it has similar dimensions to Prism K but a larger volume.

2. Calculate the lengths of both diagonals. Which one is actually longer?

   Prism K has a diagonal of $\sqrt{77}$ units. Prism L has a diagonal of $\sqrt{75}$ units. Prism K is longer.

Activity Synthesis

Ask previously identified groups to share how they calculated the diagonal of one of the rectangular prisms. Display the figures for all to see and record students' reasoning next to each figure as each group shares.

If time allows and it is not brought up in students' explanations, point out how the diagonal length of the rectangular prisms is the square root of the sum of the squares of the three edge lengths. For example, to calculate the diagonal of Prism K, students would have to calculate $\sqrt{6^2 + (\sqrt{41})^2}$, but $\sqrt{41}$ is from $\sqrt{5^2 + 4^2}$, which means the diagonal length is really $\sqrt{6^2 + 5^2 + 4^2}$ since $\sqrt{6^2 + (\sqrt{41})^2} = \sqrt{6^2 + (\sqrt{5^2 + 4^2})^2} = \sqrt{6^2 + 5^2 + 4^2}$. So, for a rectangular prism with sides d, e, and f, the length of the diagonal of the prism is just $\sqrt{d^2 + e^2 + f^2}$.

Lesson Synthesis

The purpose of this discussion is to sum up some key points about using the Pythagorean Theorem. Here are some questions for discussion:

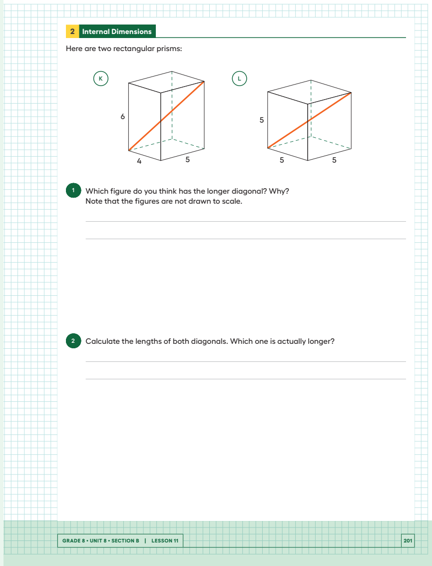
- *"How can you tell if you are trying to find a leg or a hypotenuse?"*

  I can draw a picture. I can figure out if I am trying to find the longest side or not.

- *"Can you use the Pythagorean Theorem to find a missing side of any triangle?"*

  No, it only works for right triangles.

Student Workbook

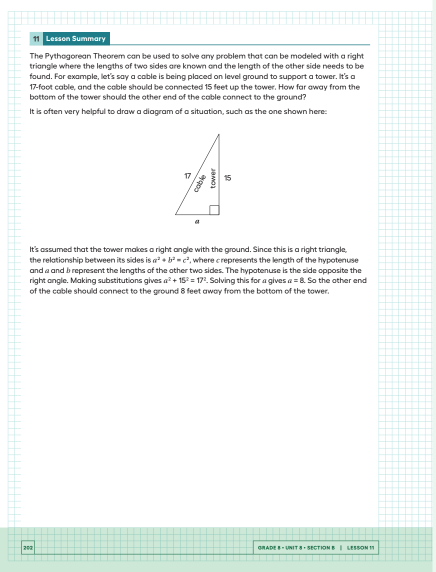

2 Internal Dimensions

Here are two rectangular prisms:

1 Which figure do you think has the longer diagonal? Why? Note that the figures are not drawn to scale.

2 Calculate the lengths of both diagonals. Which one is actually longer?

Student Workbook


11 Lesson Summary

The Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle where the lengths of two sides are known and the length of the other side needs to be found. For example, let's say a cable is being placed on level ground to support a tower. It's a 17-foot cable, and the cable should be connected 15 feet up the tower. How far away from the bottom of the tower should the other end of the cable connect to the ground?

It is often very helpful to draw a diagram of a situation, such as the one shown here:

It's assumed that the tower makes a right angle with the ground. Since this is a right triangle, the relationship between its sides is $a^2 + b^2 = c^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides. The hypotenuse is the side opposite the right angle. Making substitutions gives $a^2 + 15^2 = 17^2$. Solving this for a gives $a = 8$. So the other end of the cable should connect to the ground 8 feet away from the bottom of the tower.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery using the Pythagorean Theorem. If most students struggle, make time to revisit related work in this lesson. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 8, Lesson 11 Applications of the Pythagorean Theorem

Lesson Summary

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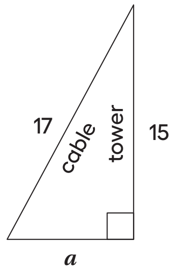

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Cool-down

How High Up?

5 min

Student Task Statement

An 11.5 m support pole is attached to a vertical utility pole to help keep it upright. The base of the support pole is 4.5 m from the base of the utility pole. How high up the utility pole does the support pole reach? Assume the vertical utility pole makes a right angle with the ground.

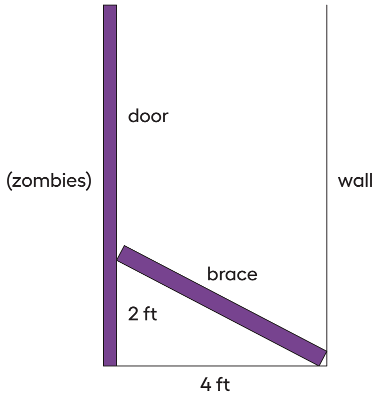
Approximately 10.58 m

Practice Problems

6 Problems

Problem 1

A man is trying to zombie-proof his house. He wants to cut a length of wood that will brace a door against a wall. The wall is 4 feet away from the door, and he wants the brace to rest 2 feet up the door. About how long should he cut the brace?



Around 4.5 feet

Sample reasoning: Solving $2^2 + 4^2 = c^2$, we get $c = \sqrt{20}$, which is approximately 4.5.

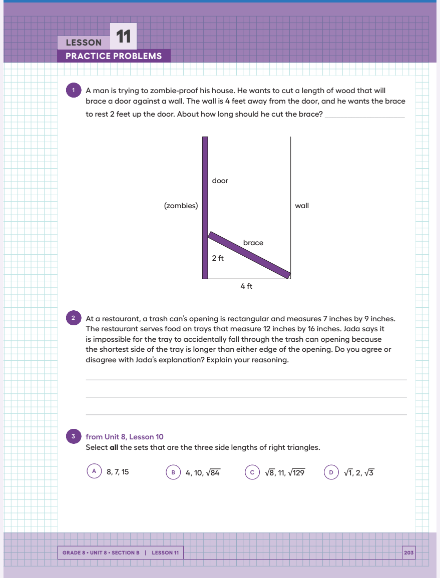
Problem 2

At a restaurant, a trash can's opening is rectangular and measures 7 inches by 9 inches. The restaurant serves food on trays that measure 12 inches by 16 inches. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening. Do you agree or disagree with Jada's explanation? Explain your reasoning.

I disagree

Sample reasoning: It is impossible for the tray to fall through the opening, but not for the reason Jada gives. The longest dimension of the trash can opening is the diagonal. The diagonal is $\sqrt{130}$ inches long, because $7^2 + 9^2 = 130$. The diagonal is between 11 and 12 inches long, because $11^2 < 130 < 12^2$. The tray cannot fall through the opening because the diagonal is a little shorter than the shortest dimension of the tray.

Student Workbook


LESSON 11
PRACTICE PROBLEMS

1 A man is trying to zombie-proof his house. He wants to cut a length of wood that will brace a door against a wall. The wall is 4 feet away from the door, and he wants the brace to rest 2 feet up the door. About how long should he cut the brace?

door
(zombies)
wall
brace
2 ft
4 ft

2 At a restaurant, a trash can's opening is rectangular and measures 7 inches by 9 inches. The restaurant serves food on trays that measure 12 inches by 16 inches. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening. Do you agree or disagree with Jada's explanation? Explain your reasoning.

3 from Unit 8, Lesson 10
Select **all** the sets that are the three side lengths of right triangles.

A 8, 7, 15
B 4, 10, $\sqrt{84}$
C $\sqrt{8}$, 11, $\sqrt{129}$
D $\sqrt{1}$, 2, $\sqrt{3}$

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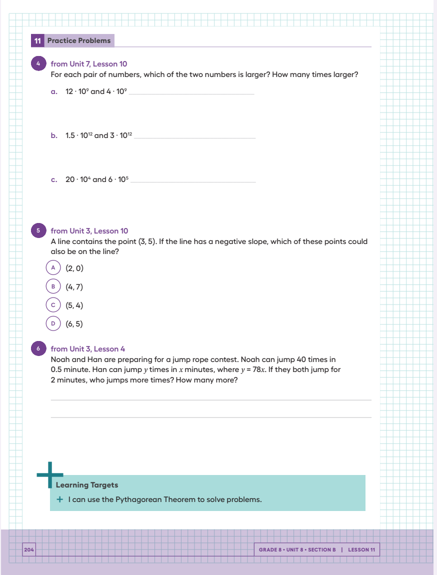
Student Workbook


11 Practice Problems

4 from Unit 7, Lesson 10
For each pair of numbers, which of the two numbers is larger? How many times larger?

a. $12 \cdot 10^9$ and $4 \cdot 10^9$

b. $1.5 \cdot 10^{12}$ and $3 \cdot 10^{12}$

c. $20 \cdot 10^4$ and $6 \cdot 10^5$

5 from Unit 3, Lesson 10
A line contains the point (3, 5). If the line has a negative slope, which of these points could also be on the line?

A (2, 0)

B (4, 7)

C (5, 4)

D (6, 5)

6 from Unit 3, Lesson 4
Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minute. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?

Learning Targets
+ I can use the Pythagorean Theorem to solve problems.

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Problem 3

from Unit 8, Lesson 10

Select **all** the sets that are the three side lengths of right triangles.

A. 8, 7, 15

B. 4, 10, $\sqrt{84}$

C. $\sqrt{8}$, 11, $\sqrt{129}$

D. $\sqrt{1}$, 2, $\sqrt{3}$

Problem 4

from Unit 7, Lesson 10

For each pair of numbers, which of the two numbers is larger? How many times larger?

a. $12 \cdot 10^9$ and $4 \cdot 10^9$

$12 \cdot 10^9$; 3 times larger

b. $1.5 \cdot 10^{12}$ and $3 \cdot 10^{12}$

$3 \cdot 10^{12}$; 2 times larger

c. $20 \cdot 10^4$ and $6 \cdot 10^5$

$6 \cdot 10^5$; 3 times larger

Problem 5

from Unit 3, Lesson 10

A line contains the point (3, 5). If the line has a negative slope, which of these points could also be on the line?

A. (2, 0)

B. (4, 7)

C. (5, 4)

D. (6 ,5)

Problem 6

from Unit 3, Lesson 4

Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minute. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?

Han jumps 4 more times, because Noah jumps 160 times, and Han jumps 156 times.