Lesson 5

Introduction to Linear Relationships

Goals

- Compare and contrast (orally and in writing) proportional and nonproportional linear relationships.
- Interpret (orally and in writing) features of the graph (i.e., slope and y-intercept) of a non-proportional linear relationship.

Learning Target

I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.

Lesson Narrative

In this lesson, students examine **linear relationships** with positive rates of change. They consider a situation where the height of a stack of styrofoam cups is not proportional to the number of cups in the stack. Using graphs, tables, and equations, they observe that each cup increases the height of the stack by the same amount. Since one quantity has a constant rate of change with respect to the other, this is an example of a **linear relationship.**

Using this context, students identify and describe similarities and differences between linear relationships and proportional relationships. They observe that the **rate of change** of the relationship and the *slope* of a line representing the relationship have the same value.

The meaning of the vertical intercept of the graph comes up briefly and will be revisited more fully in upcoming lessons, so it is not necessary for students to identify it at this time. In this lesson, the focus is on rate of change in linear relationships that are not proportional.

The activities in this lesson were written using a particular type of cup. Photos are included of all measurements needed, so this lesson can be used without any additional preparation. However, if desired, the lesson can be modified so that students measure stacks of actual cups. Teachers wanting to use real cups should measure the height of two stacks ahead of time, find the rate of change and make sure that it is approximately constant. Note that rounding error will likely play a role, so some flexibility in answers will be necessary if students measure actual stacks of cups.

Student Learning Goal

Let's explore some relationships between two variables.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Warm-up)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR5: Co-Craft Questions

Required Materials

Materials to Gather

- Graph paper: Activity 1
- Straightedges: Activity 1, Activity 2

Warm-up
Stacks of Cups

5 min

Activity Narrative

This *Warm-up* helps students to make sense of a context they will see in a following activity.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Introduce the context image. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Sample responses:

- How tall is each stack of cups?
- How much taller is the stack of cups on the right?
- How tall is one cup?
- How much taller does a stack get when one cup is added?
- How many cups will it take for the stack to be 100 cm tall?
- How many cups will it take for a stack to be as tall as me?

Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "rate of change," "proportional" or "nonproportional," and "linear."

Instructional Routines

MLR5: Co-Craft Questions

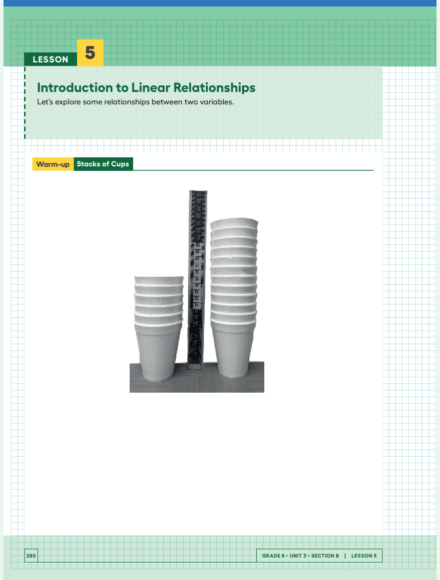
ilclass.com/r/10695544

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Warm-up)

MLR5: Co-Craft Questions.
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook



Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.
Use a display of foam cups next to a ruler, as depicted, to demonstrate the context.
Supports accessibility for: Conceptual Processing, Language, Memory

Building on Student Thinking

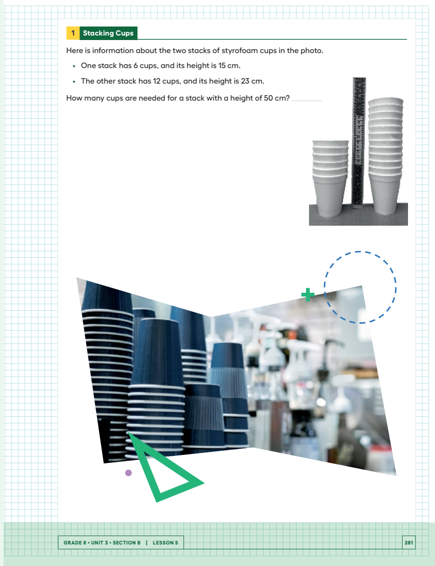
If students compute $\frac{15}{6} = 2.5$ or $\frac{23}{12} = 1.9$ and use this as the increase per cup, consider asking:

"How did you determine the increase per cup?"

"Does your strategy match if you find the increase per cup of the other stack?"

Students may also be looking for exact numbers, even though measurement is approximate. Reassure them that numbers that approximately agree are close enough.

Student Workbook


1 Stacking Cups

Here is information about the two stacks of styrofoam cups in the photo.

- One stack has 6 cups, and its height is 15 cm.
- The other stack has 12 cups, and its height is 23 cm.

How many cups are needed for a stack with a height of 50 cm?

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Activity 1
Stacking Cups

15 min

Activity Narrative

This activity presents students with a situation that is not proportional since there is a non-zero starting amount, and it leads to the definition of a linear relationship. Students explore rate of change, which in this situation is the increase in the height of the stack each time a cup is added to the existing first cup. Students can use any method or representation to solve this problem.

Monitor for students who use different strategies such as making tables or graphs or writing an equation.

Launch

Arrange students in groups of 2–4. Provide access to straightedges and graph paper. Allow 2 minutes of individual think time before students work together in groups.

Encourage students to approach this problem using any method. If needed, ask students to consider the following:

- The height each cup adds
- If the first cup is different from the others
- Creating a graph or table to help reason about the problem

Student Task Statement

Here is information about the two stacks of styrofoam cups in the photo.

- One stack has 6 cups, and its height is 15 cm.
- The other stack has 12 cups, and its height is 23 cm.

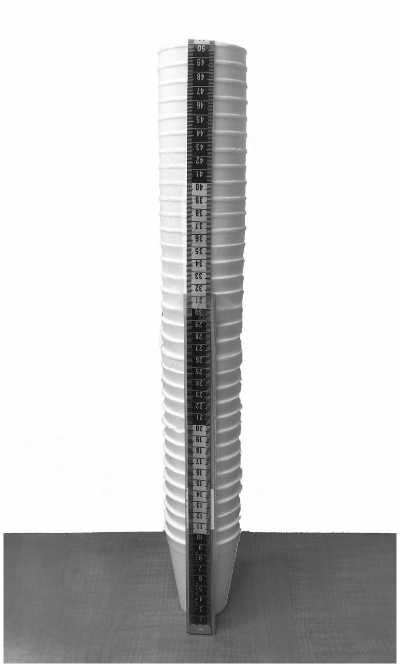
How many cups are needed for a stack with a height of 50 cm?

33 cups (32.25 cups reach 50 cm in height)

Activity Synthesis

Poll the class about the number of cups they came up with to reach a total height of 50 cm. If necessary, discuss the meaning of non-whole number answers and whether they are appropriate in this situation.

Display this photo. If the class used real cups, stack enough of them to get to a height of 50 cm.



Invite students who used the different strategies mentioned in the *Activity Narrative* to share their reasoning with the class.

Ask students if they think the relationship is proportional. If there is disagreement, encourage students to explain their reasoning, but come to the conclusion that it is not proportional. If necessary, show how doubling the number of cups from 6 to 12 did not double the height.

Then explain to students that this situation is an example of a **linear relationship**: when one quantity has a constant rate of change with respect to the other. In this situation, the total height of the stack has a constant rate of change with respect to the number of cups added. In other words, when the quantity of cups changes by a certain amount, the total height of the stack changes by a set amount.

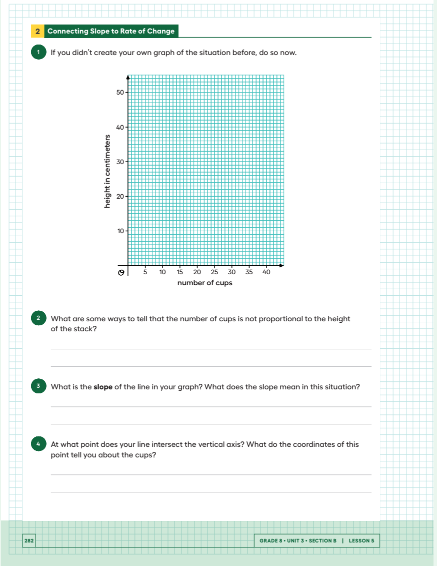
Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

"How are the strategies the same?"
"How are they different?"
"Are there any benefits or drawbacks to one representation compared to another?"
"How do these different representations show the same information?"

Advances: Representing, Conversing

Student Workbook


2 Connecting Slope to Rate of Change

1. If you didn't create your own graph of the situation before, do so now.

2. What are some ways to tell that the number of cups is not proportional to the height of the stack?

3. What is the **slope** of the line in your graph? What does the slope mean in this situation?

4. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the cups?

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Activity 2

Connecting Slope to Rate of Change

15 min

Activity Narrative

In this activity, students continue to examine the relationship between the number of cups and the height of the stack by creating a graph of the relationship.

Note that students' graphs may consist of discrete points corresponding to coordinate pairs (number of cups, height) or of the entire line as shown in the solution. It is understood that only the points that represent a whole number of cups have a valid interpretation in the context. This continuous graphical representation of a linear relationship, whether the context is continuous or discrete, is very common and will be seen throughout this unit.

Students connect the slope of the line in their graph with the rate of change, which is the additional height of the stack per cup added. They also find where their graph intersects the y-axis and interpret this value in terms of the situation: the height of the bottom part of the first cup, below the rim.

Monitor for students who use different slope triangles when determining the slope of the line in the graph.

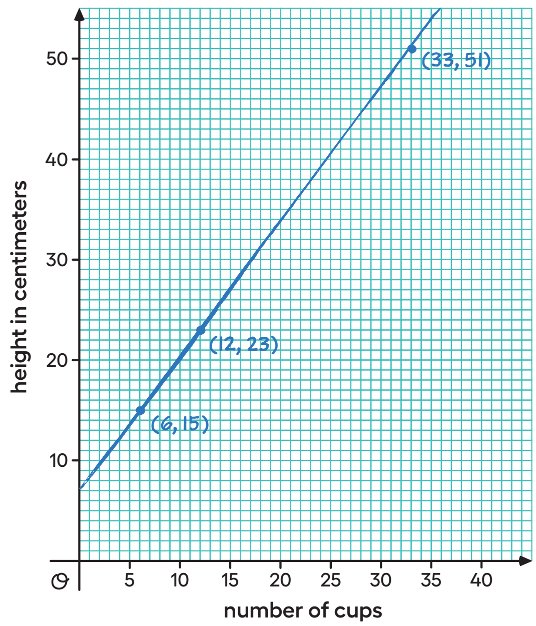
Launch

Give students 8–10 minutes of group work time and follow with a whole-class discussion.

If students need a reminder of how to find the slope of a line, encourage them to draw a slope triangle.

Student Task Statement

1. If you didn't create your own graph of the situation before, do so now.



2. What are some ways to tell that the number of cups is not proportional to the height of the stack?

   Sample responses: If the relationship were proportional, then the line would pass through the origin. Since the line does not pass through the origin, the relationship is not proportional. Proportional relationships have constant quotients of coordinate pairs: $\frac{15}{6}$ is not equal to $\frac{23}{12}$.

3. What is the **slope** of the line in your graph? What does the slope mean in this situation?

   The slope is $\frac{4}{3}$.

   Every cup after the first one adds $\frac{4}{3}$ centimeters to the height. Or every rim, including the first one, adds $\frac{4}{3}$ centimeters to the height.

4. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the cups?

   The line intersects the vertical axis at the point $(0,7)$.

   The coordinates of this point tell that if there are 0 cups, the height would be 7 centimeters. This does not make sense since the number of cups in the stack is 0. However, considering that each cup has a rim, 7 centimeters is the height from the bottom of the first cup to its rim.

Activity Synthesis

The goal of this discussion is for students to see that the rate of change in a linear relationship has the same value as the slope of the line representing the relationship.

Begin by inviting 1–2 previously selected students to share their graph, including slope triangles, for all to see. Discuss with students:

"What is the same and what is different about these graphs?"

The graphs all show the same line. The graphs used different slope triangles to calculate slope, but they all came up with the same value of $\frac{4}{3}$.

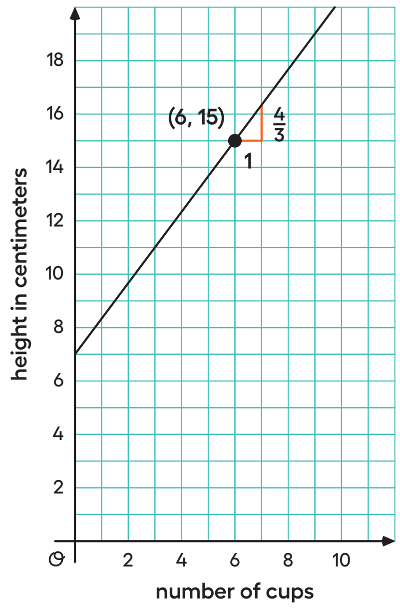
"What does the point on the graph (0,7) mean?"

It tells the height of the first cup not including the rim.

"How can you see on the graph the amount that one cup adds to the height?"

In a slope triangle with a horizontal distance of 1 on the graph, the vertical distance will show the amount each cup adds to the height.

Display this image:



It is important to emphasize here that $\frac{4}{3}$ is *not* the constant of proportionality, since this is *not* a proportional relationship. This value is how much each cup adds to the height of the stack, and is called the "rate of change." The **rate of change** in a linear relationship between x and y is the increase in y when x increases by 1. Note that the rate of change of the relationship has the same value as the *slope* of the line representing the relationship.

Lesson Synthesis

The focus of this discussion is the transition from proportional relationships to linear relationships that are not proportional. Emphasize the following ideas:

- There are linear relationships that are not proportional.
- The rate of change of a linear relationship is the same value as the slope of a line representing the relationship.
- The rate of change in the context of a situation is the ratio of how one quantity changes as the other quantity changes, or how much one quantity changes when the other increases by 1.

Create a classroom display that highlights how to tell if a linear relationship is proportional or not. Consider organizing the information in a table or using any format that students find useful. Discuss these questions and add student responses and examples to the display:

"How can we tell if a linear relationship is proportional or not? From the graph? From a table? From the context?"

On a graph, check whether the line goes through the origin; in a table, check whether the value of the dependent variable is 0 when the value of the independent variable is 0; in a context, check that it makes sense when both variables are 0.

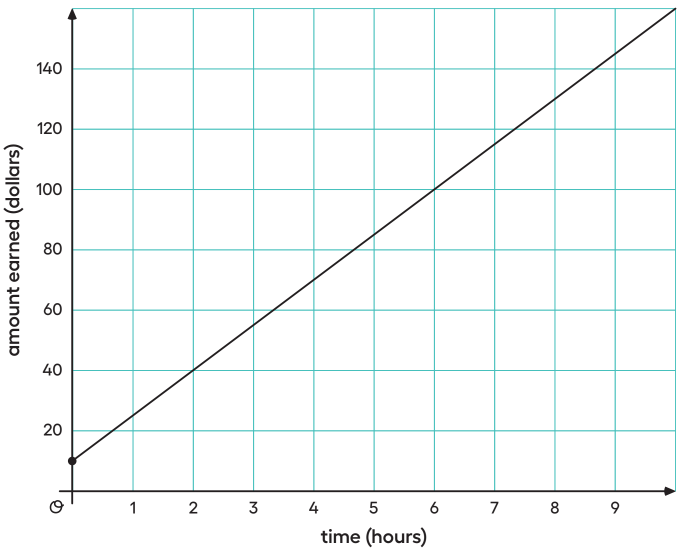
"What does the rate of change of a linear relationship tell us?"

the slope of the graph, the amount y changes when x increases by 1

"What are some examples of linear relationships that are proportional? That are not proportional?"

Lesson Summary

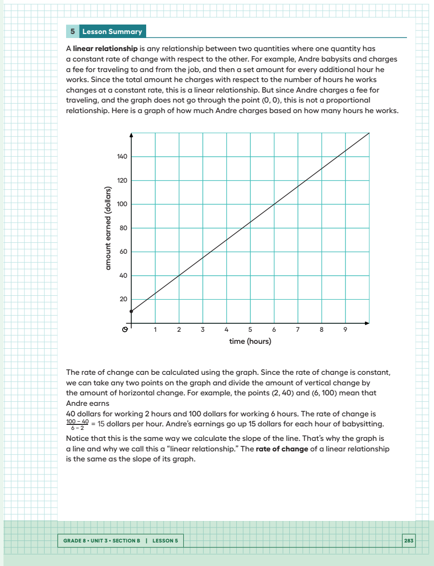
A **linear relationship** is any relationship between two quantities where one quantity has a constant rate of change with respect to the other. For example, Andre babysits and charges a fee for traveling to and from the job, and then a set amount for every additional hour he works. Since the total amount he charges with respect to the number of hours he works changes at a constant rate, this is a linear relationship. But since Andre charges a fee for traveling, and the graph does not go through the point (0, 0), this is not a proportional relationship. Here is a graph of how much Andre charges based on how many hours he works.



The rate of change can be calculated using the graph. Since the rate of change is constant, we can take any two points on the graph and divide the amount of vertical change by the amount of horizontal change. For example, the points (2, 40) and (6, 100) mean that Andre earns 40 dollars for working 2 hours and 100 dollars for working 6 hours. The rate of change is $\frac{100-40}{6-2} = 15$ dollars per hour. Andre's earnings go up 15 dollars for each hour of babysitting.

Notice that this is the same way we calculate the slope of the line. That's why the graph is a line and why we call this a "linear relationship." The **rate of change** of a linear relationship is the same as the slope of its graph.

Student Workbook


5 Lesson Summary

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

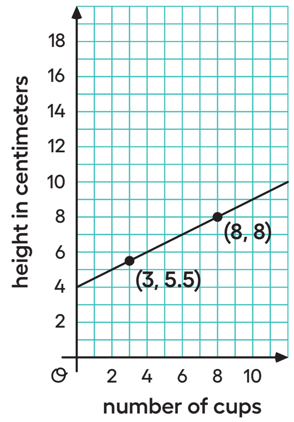
Cool-down

Stacking More Cups

5 min

Student Task Statement

A different style of cup is stacked. The graph shows the height of the stack in centimeters for different numbers of cups. How much does each cup after the first add to the height of the stack? Explain your reasoning.



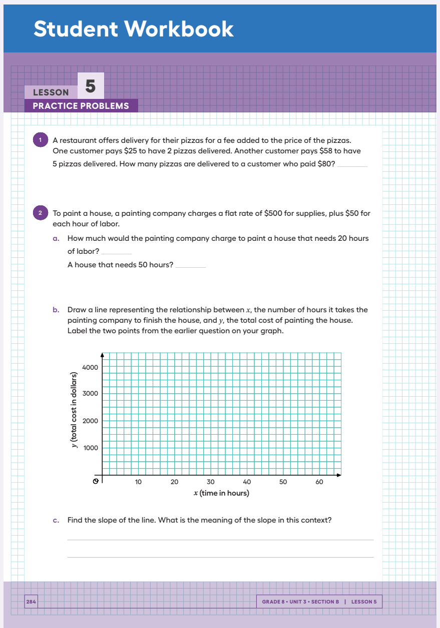
Each cup after the first adds 0.5 centimeters (or equivalent).

Since 5 cups add 2.5 centimeters to the height of the stack, each cup adds 0.5 centimeters.

Lesson 5

Practice Problems

4 Problems


Student Workbook

LESSON 5
PRACTICE PROBLEMS

1. A restaurant offers delivery for their pizzas for a fee added to the price of the pizzas. One customer pays $25 to have 2 pizzas delivered. Another customer pays $58 to have 5 pizzas delivered. How many pizzas are delivered to a customer who paid $80?

2. To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

a. How much would the painting company charge to paint a house that needs 20 hours of labor?

A house that needs 50 hours?

b. Draw a line representing the relationship between x, the number of hours it takes the painting company to finish the house, and y, the total cost of painting the house. Label the two points from the earlier question on your graph.

c. Find the slope of the line. What is the meaning of the slope in this context?

Problem 1

A restaurant offers delivery for their pizzas for a fee added to the price of the pizzas. One customer pays $25 to have 2 pizzas delivered. Another customer pays $58 to have 5 pizzas delivered. How many pizzas are delivered to a customer who paid $80?

7 pizzas

Problem 2

To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

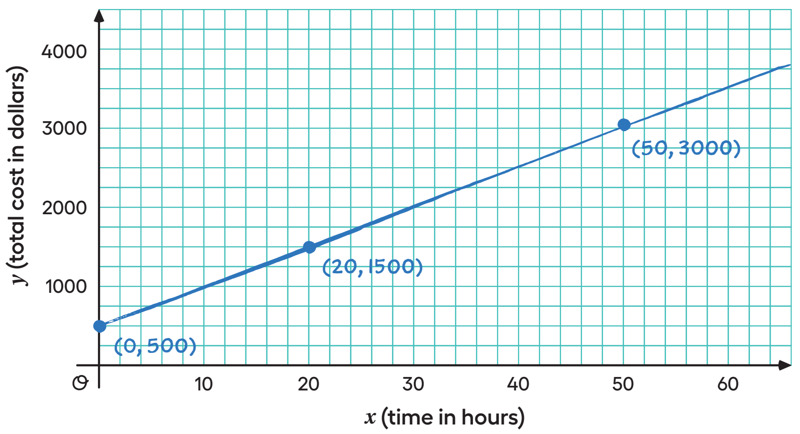
a. How much would the painting company charge to paint a house that needs 20 hours of labor?

$1,500

A house that needs 50 hours?

$3,000

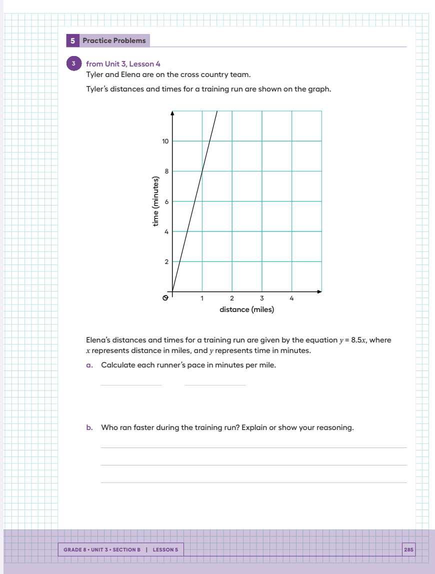
b. Draw a line representing the relationship between x, the number of hours it takes the painting company to finish the house, and y, the total cost of painting the house. Label the two points from the earlier question on your graph.



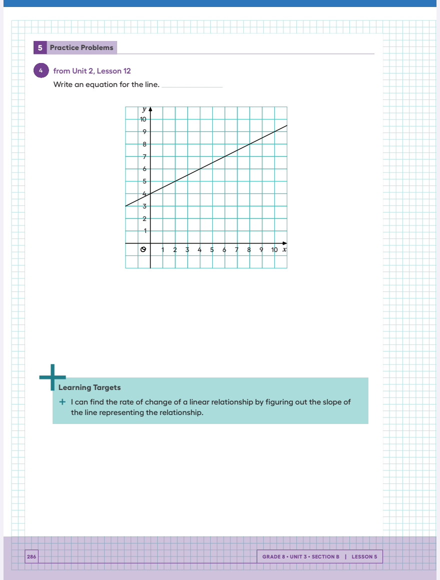
c. Find the slope of the line. What is the meaning of the slope in this context?

The slope of the line is 50, which is the same as the price per hour, in dollars, that the painting company charges for labor.

Student Workbook



Student Workbook

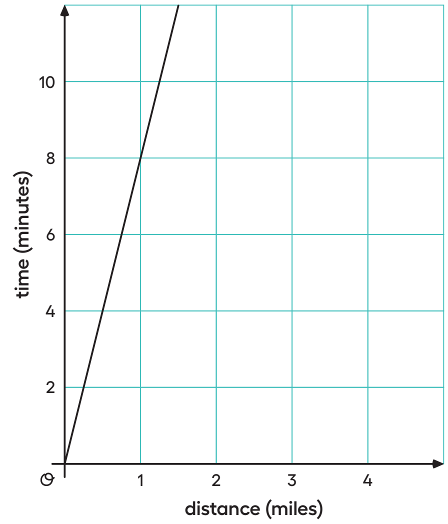


Problem 3

from Unit 3, Lesson 4

Tyler and Elena are on the cross country team.

Tyler's distances and times for a training run are shown on the graph.



Elena's distances and times for a training run are given by the equation $y = 8.5x$, where x represents distance in miles, and y represents time in minutes.

a. Calculate each runner's pace in minutes per mile.

Tyler had a pace of $8\frac{1}{3}$ minutes per mile, and Elena had a pace of $8\frac{1}{2}$ minutes per mile.

b. Who ran faster during the training run? Explain or show your reasoning.

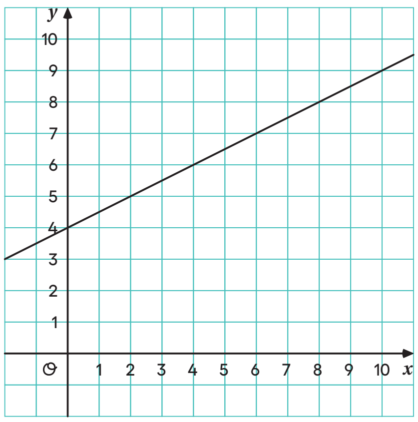
Tyler ran faster.

Sample reasoning: Tyler took fewer minutes to run 1 mile.

Problem 4

from Unit 2, Lesson 12

Write an equation for the line.



$\frac{y-5}{x-2} = \frac{1}{2}$ or $y = \frac{1}{2}x + 4$ (or equivalent)