Lesson 10

Meet Slope

Goals

- Comprehend the term "slope" to mean a number that tells how steep a line is. To find the slope, divide the vertical change by the horizontal change for any two points on the line.
- Draw a line on a grid given its slope and describe (orally) observations about lines with the same slope.
- Justify (orally) that all "slope triangles" on one line are similar.

Learning Targets

- I can draw a line on a grid with a given slope.
- I can find the slope of a line on a grid.

Lesson Narrative

In this lesson, students use dilations and triangle similarity to understand why the **slope** of a line can be calculated by dividing the vertical change by the horizontal change for any two points on the line. Students also see that this ratio determines how steep the line is.

Students begin by finding dilations of a triangle with one vertical side and one horizontal side. All the dilations use the same center but different scale factors. Students observe that regardless of the scale factor used, all the resulting triangles are similar, and their longest sides all lie on the same line. They are slope triangles. By analyzing examples, students see that all slope triangles for a given line are similar, and so any slope triangle can be used to find the slope of a line. The quotient of the vertical side length and the horizontal side length will always be the same.

Students then use slope triangles to draw lines with a given slope. They determine that lines with the same slope are parallel and that as the slope increases from 0, the lines look steeper (from left to right).

Student Learning Goal

Let's learn about the slope of a line.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up
- Tracing paper: Warm-up
- Straightedges: Activity 2

Required Preparation

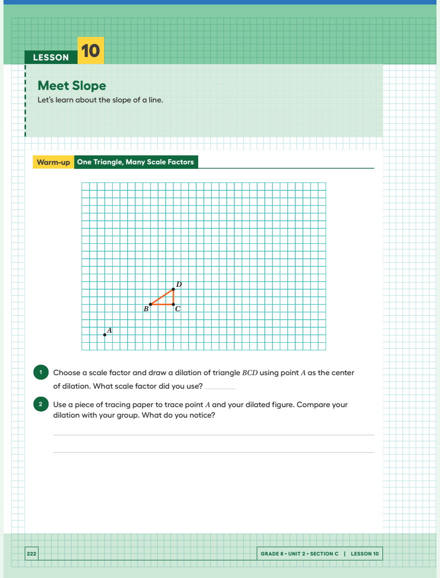
Warm-up:
Provide access to geometry toolkits, making sure tracing paper is available for each student.

Activity 2:
Provide access to straightedges.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	10 min	10 min	5 min

Student Workbook


LESSON 10

Meet Slope

Let's learn about the slope of a line.

Warm-up One Triangle, Many Scale Factors

1 Choose a scale factor and draw a dilation of triangle BCD using point A as the center of dilation. What scale factor did you use?

2 Use a piece of tracing paper to trace point A and your dilated figure. Compare your dilation with your group. What do you notice?

Warm-up
One Triangle, Many Scale Factors

10 min

Activity Narrative

The goal of this *Warm-up* is to revisit dilations and similar triangles in preparation for understanding slope and slope triangles, which will be introduced in a following activity.

Launch

Arrange students in groups of 3–4. Provide access to geometry toolkits, making sure tracing paper is available for each student. Display the image from the task for all to see.

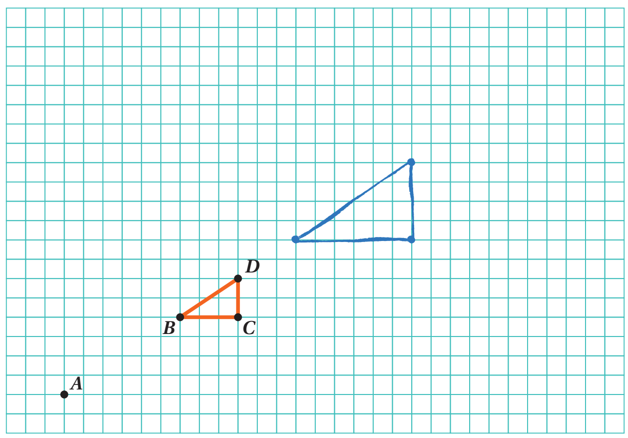
Give students 2–3 minutes to choose a scale factor and draw the dilation using that scale factor and point A as the center.

Monitor for students who use a variety of scale factors, such as $\frac{1}{3}$, $\frac{1}{2}$, 2, 2.5, and 3. Encourage students who choose a scale factor of 1 to select an additional scale factor to draw a dilation for. Pause for a partner then whole-class discussion.

Student Task Statement

1. Choose a scale factor and draw a dilation of triangle BCD using point A as the center of dilation. What scale factor did you use?

   Sample response: I used a scale factor of 2.



2. Use a piece of tracing paper to trace point A and your dilated figure. Compare your dilation with your group. What do you notice?

   Sample responses:

   - The triangles are all similar.
   - The triangles are all scaled copies of each other.
   - Scale factors less than 1 resulted in smaller triangles closer to point A.
   - Scale factors greater than 1 resulted in larger triangles farther away from point A.
   - The quotient of the horizontal and vertical sides is the same for all of the triangles.

Activity Synthesis

The goal of this discussion is to show how dilations of a triangle with the same center but different scale factors will result in a series of similar triangles, all having their longest side along the same line.

Display 3–4 dilated triangles from previously selected students who used different scale factors. Ask students to share what they noticed in their groups and record the observations for all to see.

If not mentioned by students that the triangles are similar, suggest it now. Ask students how they would be able to tell that the triangles are similar. (Since the triangles are all dilations of triangle BCD, they are all similar to each other.)

Have students stack their tracing papers containing point A and their dilated triangle so that all of the point As and triangle BCDs are on top of each other. Ask students what they notice. (The longest side of the triangles all line up.)

Activity 1
Similar Triangles on the Same Line

15 min

Activity Narrative

The purpose of this activity is to show that right triangles with their longest side along the same line are similar. This fact about the triangles is used to define slope.

To show that two triangles are similar, students need to use the structure of the grid. They can use it to describe a sequence of transformations or to show that the triangles share two pairs of congruent angles because the vertical or horizontal grid lines are parallel.

Launch

Arrange students in groups of 2. Assign one partner triangles ABC and CDE and the other partner triangles ABC and FGH.

Give students 3–4 minutes of quiet work time to construct an argument for why their two triangles are similar.

Pause for a partner discussion before having pairs complete the activity.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time.
This activity uses the Stronger and *Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 1, Student Task)

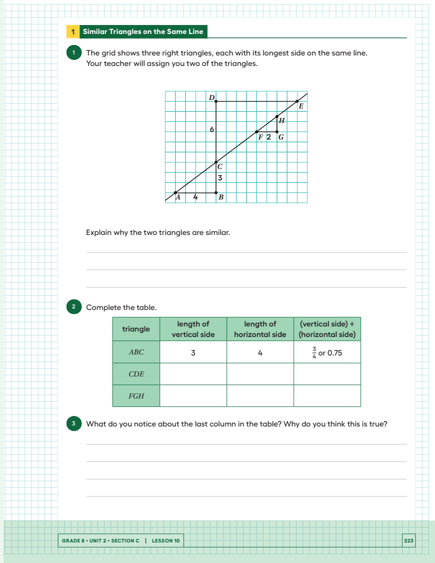
Engagement: Provide Access by Recruiting Interest.
Provide choice. Invite students to decide which pair of triangles they will show are similar.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Some students may struggle to get started. Prompt them by asking how to show that two triangles are similar (There is a sequence of translations, rotations, reflections, and dilations taking one to the other, or the two figures share 2 pairs of congruent angles.)

Student Workbook



1 Similar Triangles on the Same Line

1. The grid shows three right triangles, each with its longest side on the same line. Your teacher will assign you two of the triangles.

Explain why the two triangles are similar.

2. Complete the table.

triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC	3	4	$\frac{3}{4}$ or 0.75
CDE			
FGH			

3. What do you notice about the last column in the table? Why do you think this is true?

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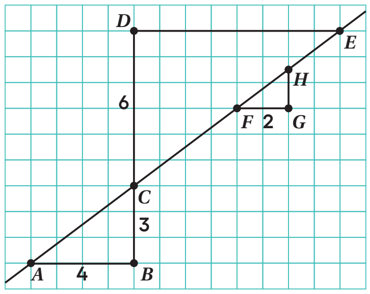
Student Task Statement

1. The grid shows three right triangles, each with its longest side on the same line. Your teacher will assign you two of the triangles. Explain why the two triangles are similar.



Sample responses:

- Triangle ABC is similar to triangle CDE because you can rotate triangle ABC 180° around point C and then perform a dilation using scale factor 2 and C as the center of dilation. This sequence maps ABC to CDE.
- Triangle ABC is similar to triangle FGH because they share two pairs of congruent angles. Angle ABC and angle FGH are both right angles, so they are congruent. The vertical sides of the triangles are along the grid, so they are all parallel. Since angle ACB and angle FHG are corresponding angles for transversal EA, they are congruent.

2. Complete the table.

triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC	3	4	$\frac{3}{4}$ or 0.75
DEF	6	8	$\frac{6}{8}$ or 0.75
FGH	1.5	2	$\frac{3}{4}$ or 0.75

3. What do you notice about the last column in the table? Why do you think this is true?

Sample response: All of the values are the same. When two triangles are similar, the ratio between corresponding side lengths is the same. Since these triangles are all similar, dividing the vertical side by the horizontal side will always result in equivalent ratios.

Activity Synthesis

The goal of this discussion is for students to see that slope is a natural consequence of triangle similarity.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to why dividing the vertical side by the horizontal side results in quotients that are all equivalent. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

Display these prompts for feedback:

- "_____ makes sense, but what do you mean when you say ...?"
- "Can you describe that another way?"
- "How do you know ...? What else do you know is true?"

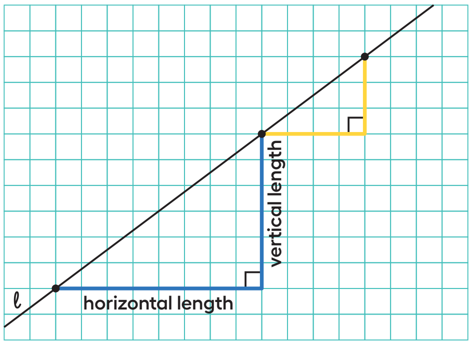
Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time*, explain that slope triangles like these can be constructed for every non-vertical, non-horizontal line. Choose any two points on the line. Then draw horizontal and vertical lines from the points to form a right triangle. The quotient of the lengths of each slope triangle's vertical side and horizontal side will always be the same. This ratio is called the **slope** of the line, and it tells how steep the line is. In this activity, the slope can be written as 0.75 or $\frac{3}{4}$, or any equivalent value.

Make clear to students that the mathematical convention is to define slope using the vertical change divided by the horizontal change and not the other way around. Display the following diagram, or create a similar display. Post this diagram for reference along with accompanying text:

Slope is a number that tells how steep a line is. To find the slope, divide the vertical change by the horizontal change for any two points on the line. The slope of line can be written as $\frac{6}{8}$, $\frac{3}{4}$, 0.75, or any equivalent value.



Activity 2

Multiple Lines with the Same Slope

10 min

Activity Narrative

In this activity, students use slope triangles to draw a line with a given slope and observe two important properties:

- Lines with the same slope are parallel.
- As the slope of a line increases from 0, so does its steepness (from left to right).

Monitor for students who draw slope triangles of different sizes when drawing lines with slope $\frac{1}{2}$.

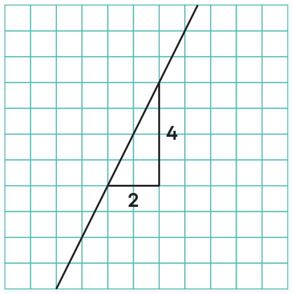
Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Access for Perception.
Students may benefit from watching or hearing the demonstration from the launch of how to use a slope triangle to draw a line with a given slope more than once.

Supports accessibility for: Language, Attention

Launch

Display this image for all to see, or draw a line with a slope of 2 and include a slope triangle.



Discuss with students:

"How can you tell that this line has a slope of 2?"

Dividing the vertical length of the slope triangle by the horizontal length gives the quotient is 2.

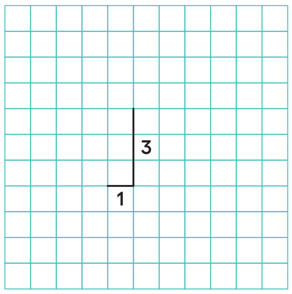
"Could a different slope triangle be used? Explain your reasoning."

Yes, all slope triangles of a line are similar. So, dividing the vertical length by the horizontal length for any slope triangle of this line will give the same quotient, 2.

"What do you know about a line that has a slope of 3?"

For any slope triangle of the line, the vertical length will be three times the horizontal length.

Now display this image for all to see, or draw and label two legs of a triangle as shown.



Discuss with students how this information could be used to draw a line with a slope of 3. (Use a ruler or straightedge to draw a line through what would be the third side of the triangle.) Remind students that any slope triangle whose vertical length divided by its horizontal length equals 3 could also be used, such as a triangle with a vertical side 6 units long and a horizontal side 2 units long.

Provide access to straightedges.

Give students 3–4 minutes of quiet work time to complete the task followed by a whole-class discussion.

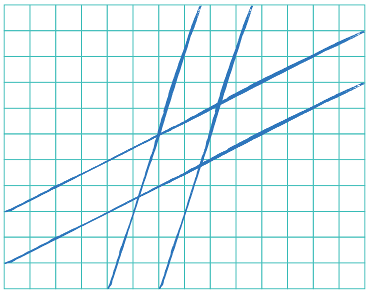
Student Task Statement

1. Draw two lines with a **slope** of 3. What do you notice about the two lines?

   See image. Sample response: The pairs of lines with the same slope are parallel.

2. Draw two lines with a slope of $\frac{1}{2}$. What do you notice about the two lines?

   See image. Sample response: Slope triangles for lines with the same slope are similar.



Are You Ready for More?

As you learn more about lines, you will occasionally have to consider perfectly vertical lines as a special case and treat them differently. Think about applying what you have learned in the last couple of activities to the case of vertical lines. What is the same? What is different?

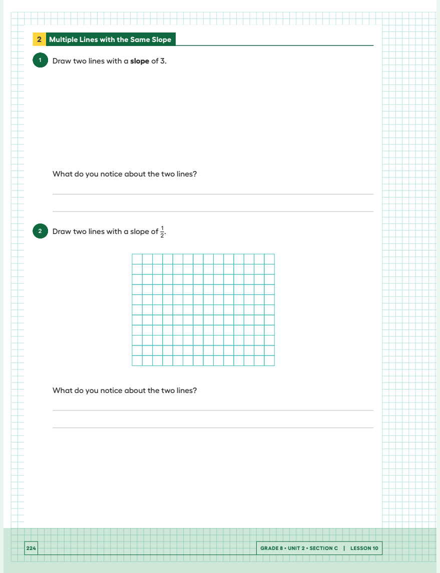
Sample response: Geometrically, vertical lines are no different than any other line. A vertical line can be rotated to a non-vertical line. Just as lines with the same slope are parallel, all vertical lines are also parallel. But some things are quite different. For example, the notion of a slope triangle doesn't make much sense for vertical lines because there is no "horizontal change" to use as the base of the triangle, and trying to define a slope using slope $= \frac{\text{vertical change}}{\text{horizontal change}}$ would involve dividing by zero. For this reason, the slope of a vertical line is not defined.

Building on Student Thinking

Some students may draw lines with slope -3 and $-\frac{1}{2}$. Display these alongside lines of slope 3 or slope $\frac{1}{2}$, and ask students to describe how they are alike and different.

For now, leave the question open about whether such lines have the same or different slopes. Future lessons will focus on distinguishing positive from negative slopes.

Student Workbook


2 Multiple Lines with the Same Slope

1 Draw two lines with a **slope** of 3.

What do you notice about the two lines?

2 Draw two lines with a slope of $\frac{1}{2}$.

What do you notice about the two lines?

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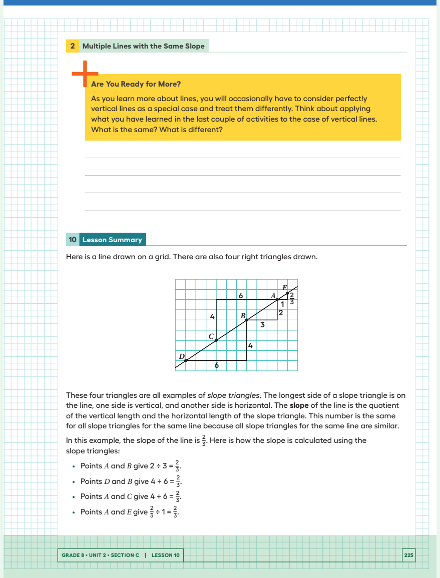
Student Workbook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10 Lesson Summary

Here is a line drawn on a grid. There are also four right triangles drawn.

These four triangles are all examples of *slope triangles*. The longest side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The **slope** of the line is the quotient of the vertical length and the horizontal length of the slope triangle. This number is the same for all slope triangles for the same line because all slope triangles for the same line are similar.

In this example, the slope of the line is $\frac{2}{3}$. Here is how the slope is calculated using the slope triangles:

- Points A and B give $2 \div 3 = \frac{2}{3}$.
- Points D and B give $4 \div 6 = \frac{2}{3}$.
- Points A and C give $4 \div 6 = \frac{2}{3}$.
- Points A and E give $\frac{2}{3} \div 1 = \frac{2}{3}$.

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to help students to formulate statements about what they notice about two lines with the same slope and lines with different slopes:

"Two lines with the same slope are____ because ...,"

"The lines with slopes equal to 3 are____than lines with slope 1/2, because ..."

Advances: Speaking, Conversing

Activity Synthesis

The goals of this discussion are for students to see that since all slope triangles for a line are similar, any slope triangle can be used to draw the line. Students should also notice that lines with the same slope are parallel, and that as the slope increases from 0, the line appears steeper from left to right.

Ask previously selected students, as described in the *Activity Narrative*, to share how they drew their lines with a slope of $\frac{1}{2}$. Demonstrate how it does not matter if you draw a slope triangle with a vertical length of 1 and a horizontal length of 2, or a triangle with vertical and horizontal lengths of 3 and 6, or 5 and 10. Explain that the quotient of side lengths is the important feature, since any triangle drawn to match a given slope will be similar to any other triangle drawn to match the same slope. Here are some questions for discussion:

- *"What did you notice about the two lines you drew with a slope of 3? With a slope of $\frac{1}{2}$?"*

  Lines with the same slope are parallel. Slope triangles for the lines with the same slope are similar.

- *"What did you notice about the lines with a slope of 3 compared to the lines with a slope of $\frac{1}{2}$?*

  The lines with a slope of 3 look "steeper" than the lines with a slope of $\frac{1}{2}$.

Lesson Synthesis

The goal of this discussion is to review the definition of slope. Discuss:

- *"What is a slope triangle for a line?"*

  A triangle whose longest side is on the line and whose other sides are horizontal and vertical.

- *"How can you use a slope triangle to find the slope of a line?"*

  Divide the length of the vertical side by the length of the horizontal side.

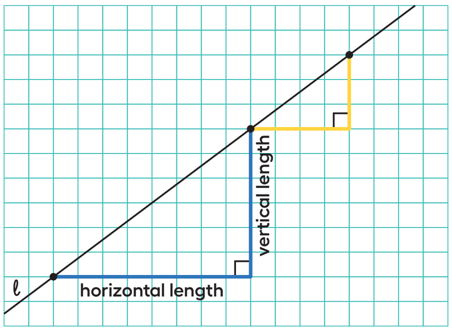
- *"Does it matter which two points on a line you use to create a slope triangle? Why?"*

  No. Any two slope triangles for a line are similar. So the quotient of the two corresponding sides will always give the same value, regardless of which two points you use to draw the slope triangle.

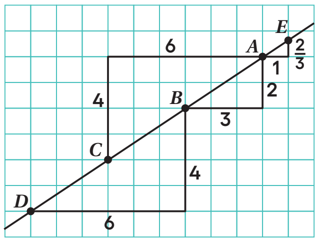
- *"Why are any two slope triangles for the same line similar?"*

  They can be created by performing a dilation, so they will always be similar.

Refer to the classroom display created in an earlier activity as needed.



Lesson Summary



Here is a line drawn on a grid. There are also four right triangles drawn.

These four triangles are all examples of slope triangles. The longest side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The slope of the line is the quotient of the vertical length and the horizontal length of the slope triangle. This number is the same for all slope triangles for the same line because all slope triangles for the same line are similar.

In this example, the slope of the line is $\frac{2}{3}$. Here is how the slope is calculated using the slope triangles:

- Points A and B give $2 \div 3 = \frac{2}{3}$.
- Points D and B give $4 \div 6 = \frac{2}{3}$.
- Points A and C give $4 \div 6 = \frac{2}{3}$.
- Points A and E give $\frac{2}{3} \div 1 = \frac{2}{3}$.

Cool-down

Finding Slope and Graphing Lines

5 min

Student Task Statement

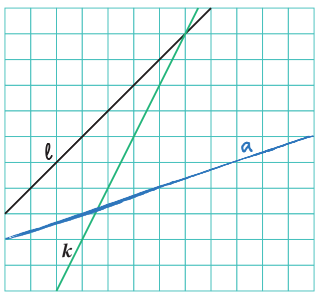
Lines ℓ and k are graphed.

1. Which line has a slope of 1, and which has a slope of 2?

   Line ℓ has a slope of 1, and line k has a slope of 2.

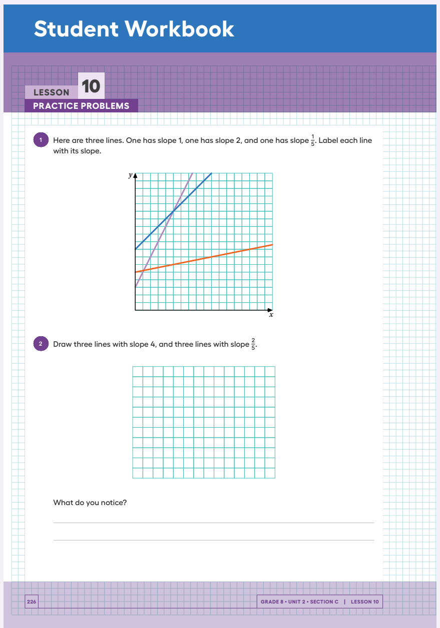
2. Use a ruler or straightedge to help you graph a line whose slope is $\frac{3}{5}$. Label this line a.

   Sample response:



Responding To Student Thinking

Points to Emphasize
If students struggle with drawing a line with a given slope, over the next several lessons, revisit how slope triangles can be used to help draw lines. For example, in the activity referred to here, invite multiple students to share their strategies for drawing a line only given the slope.
Unit 2, Lesson 11, Warm-up Different Slopes of Different Lines



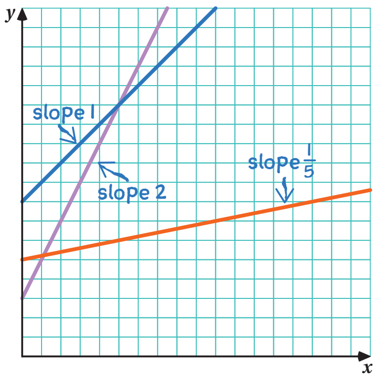
Lesson 10

Practice Problems

4 Problems

Problem 1

Here are three lines. One has slope 1, one has slope 2, and one has slope $\frac{1}{5}$. Label each line with its slope.



Problem 2

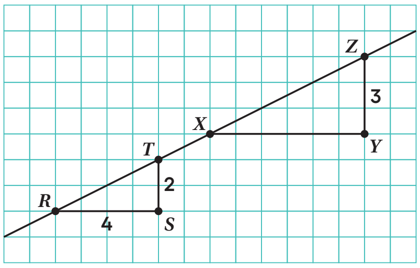
Draw three lines with slope 4, and three lines with slope $\frac{2}{5}$. What do you notice?

The three lines in each set should be parallel.

Sample response: The lines with slope 4 look steeper than those with slope $\frac{2}{5}$.

Problem 3

The grid shows two right triangles, each with its longest side on the same line.



a. Explain how you know the two triangles are similar.

Sample response: Translating R to X and dilating with center X and scale factor $\frac{3}{2}$ shows there is a sequence of translations, rotations, reflections, and dilations taking one triangle to the other.

b. How long is XY?

6 units

c. For each triangle, calculate (vertical side) ÷ (horizontal side).

For both triangles, the quotient is $\frac{1}{2}$ (or equivalent).

d. What is the slope of the line? Explain how you know.

The slope of the line is $\frac{1}{2}$.

Sample reasoning: The quotient of the vertical and horizontal side length for either slope of the triangle is $\frac{1}{2}$.

Problem 4

from Unit 2, Lesson 9

Triangle A has side lengths 3 units, 4 units, and 5 units. Triangle B has side lengths 6 units, 7 units, and 8 units.

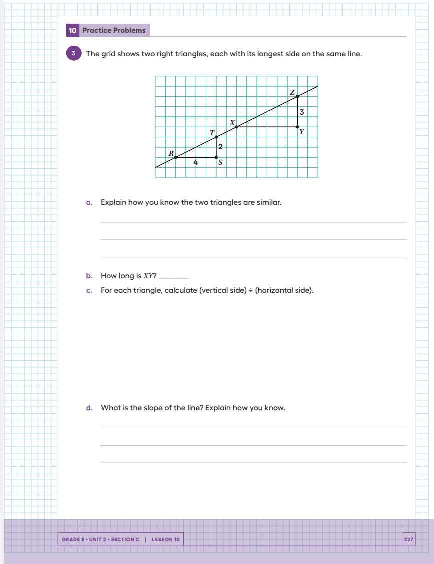
a. Explain how you know that Triangle B is not similar to Triangle A.

Sample response: The shortest side in Triangle B is twice as long as the shortest side in Triangle A, but the longest side in Triangle B is only 1.6 times as long as the corresponding side in Triangle A. These different ratios mean the triangles cannot be similar.

b. Give possible side lengths for Triangle B so that it is similar to Triangle A.

Sample response: 6 units, 8 units, and 10 units

Student Workbook



Student Workbook