Lesson 7

Representations of Linear Relationships

Goals

- Create an equation that represents a linear relationship.
- Generalize (orally and in writing) a method for calculating slope based on coordinates of two points.
- Interpret the slope and y-intercept of the graph of a line in context.

Learning Targets

- I can use patterns to write a linear equation to represent a situation.
- I can write an equation for the relationship between the total volume in a graduated cylinder and the number of objects added to the graduated cylinder.

Lesson Narrative

In this lesson, students develop an equation for a linear relationship in context by expressing regularity in repeated calculations. Students measure volume using a graduated cylinder. Identical objects are added to the cylinder, increasing the volume in the cylinder by the same amount with each object. Students graph the relationship, interpreting the initial water volume as the vertical intercept and the slope as the rate of change, or the amount by which the volume increases when one object is added.

In this lesson, students also continue to strengthen their understanding of how the slope and vertical intercept are connected in equations that represent a linear context by both interpreting an equation in context and creating a context to match an equation. This lesson continues to focus on positive slopes.

Student Learning Goal

Let's write equations from real situations.

Access for Students with Diverse Abilities

- Engagement (Activity 2)
- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Required Materials

Materials to Gather

- Graduated cylinders: Activity 1
- Straightedges: Activity 1
- Teacher's collection of objects: Activity 1
- Water: Activity 1

Required Preparation

Activity 2:
If doing the 20 minute version with a physical teacher demonstration or the 40 minute version with students collecting data, partially fill a graduated cylinder with water and immerse identical solid objects into the cylinder. Record the resulting volume in the cylinder after adding different numbers of objects and verify that the increase in volume is approximately linear and determine a good initial water volume so that there is enough water and room in the graduated cylinder to immerse 10 to 15 objects.

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	5 min

Warm-up
Which Holds More?

5 min

Activity Narrative

In this activity, students estimate the volume of different containers by reasoning about characteristics of their shape.

Launch

Display the image for all to see and ask students to estimate which container holds the most liquid and which holds the least. Tell students to give a signal when they have an estimate and can explain their reasoning.

Student Task Statement

Which container holds the most liquid? The least?

Answers vary.

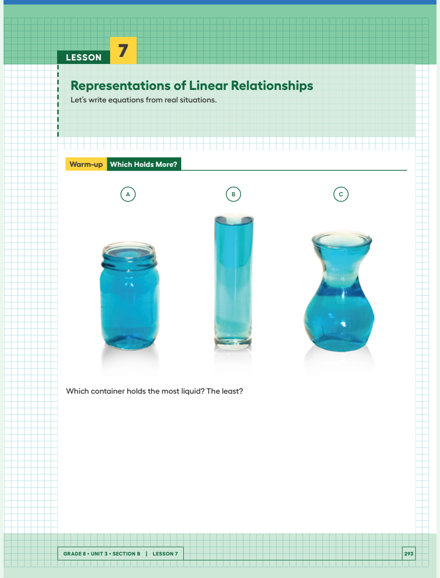
Activity Synthesis

Ask students to indicate which container they think holds the most liquid and record the responses for all to see. Invite a few students to share their reasoning and the characteristics of the container that were important in making their decision. If possible, record these characteristics on the images themselves during the discussion.

It turns out that B holds the least and A and C hold the same amount of liquid. If possible, consider showing the answer video.

Video ‘Glasses’ available here: **https://ilclass.com/r/16625310**.

Student Workbook


LESSON 7

Representations of Linear Relationships

Let's write equations from real situations.

Warm-up Which Holds More?

Which container holds the most liquid? The least?

GRADE 8 • UNIT 3 • SECTION B | LESSON 7

Activity 1

Rising Water Levels

20 min

Activity Narrative

There is a digital version of this activity.

The goal of this activity is to analyze a linear relationship with data gathered in context. Students examine data collected by submerging equal-size objects in a graduated cylinder that is partially filled with water, and then measuring the resulting volume by observing the level of the water.

This activity uses a 100 ml graduated cylinder filled with an initial amount of 60 ml of water and number cubes whose volume is approximately 3.7 ml each. Depending on the materials you gather, your measurements may be different. Be sure to leave enough space so many number cubes can be added before the water reaches the top.

There are three versions of this activity, two for a shorter 20-minute time frame and one for a longer 40-minute time frame. For the shorter versions, either the teacher performs a demonstration adding number cubes to the cylinder, and the whole class works with this data, or students work in groups with the digital applet. For the longer version, students gather their own data.

Note that for either of the non-digital versions, because of measurement error, the data may not lie exactly on a line, and different slope triangles may lead to slightly different values for the slope. Monitor for students who get slightly different values, and invite them to share during the discussion.

In the digital version of the activity, students use an applet to simulate dropping marbles into a graduated cylinder full of water. The applet allows students to set the starting water level, drop marbles into the water one by one, and reset the simulation as needed. This activity works best when each group has access to the applet. The digital version is beneficial because it requires less time, less setup and cleanup than physical manipulatives, and it allows students to focus on the linear relationship rather than taking precise measurements. If students don't have individual access, displaying the applet for all to see would be helpful during the launch.

Launch

Arrange students in groups of 2–3

Ask students,

"Have you ever noticed that when you put ice cubes in your drink, the level of the liquid goes up?"

Explain that today the class will investigate what happens when objects are dropped into a container with water.

For the 20 minute version, begin with either a physical demonstration or by displaying the applet with the digital version of the activity, adapted from an applet made in GeoGebra by John Golden. For a physical demonstration, fill a graduated cylinder with enough water to cover several identical objects, such as number cubes. Consider measuring the volume after putting in 1, 2, 5, 8, and 10 number cubes. Record the measurements for all to see or choose students to do so. After students have the information for the table, give them time to work in small groups to complete the activity.

For the 40 minute version, distribute materials to each group:

- 1 graduated cylinder
- 15 identical solid objects that fit into the cylinder and have a higher density than water and don't float (marbles, dice, cubes, hardware items such as nuts or bolts, etc.)

Tell students how much water (60 ml) to put initially into their cylinders. Give groups time to conduct the experiment, followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as a device that can run the digital applet.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Building on Student Thinking

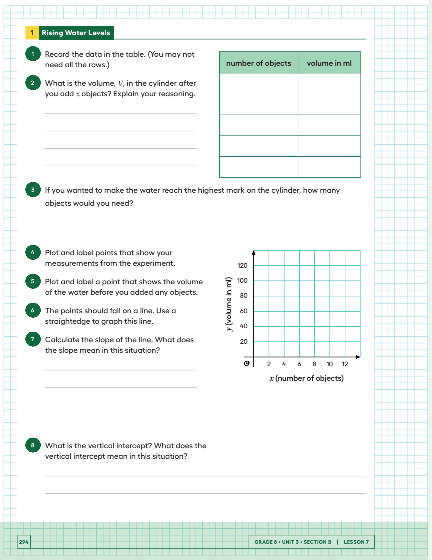
If students think the marks on the cylinder indicate the height of the water instead of the volume in the container, consider:

Asking

> *"If someone poured all the water from this cylinder into another cylinder with a larger diameter, would the height of the water be the same?"*

Explaining that milliliters are a measure of volume, but because the height of the water in a cylinder is proportional to the volume, an increase in the volume caused by each object will also result in an increase in the height of the water in the cylinder.

Student Workbook


1 Rising Water Levels

1. Record the data in the table. (You may not need all the rows.)

number of objects	volume in ml

2. What is the volume, V, in the cylinder after you add x objects? Explain your reasoning.
3. If you wanted to make the water reach the highest mark on the cylinder, how many objects would you need?
4. Plot and label points that show your measurements from the experiment.
5. Plot and label a point that shows the volume of the water before you added any objects.
6. The points should fall on a line. Use a straightedge to graph this line.
7. Calculate the slope of the line. What does the slope mean in this situation?
8. What is the vertical intercept? What does the vertical intercept mean in this situation?

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Student Task Statement

1. Record the data in the table. (You may not need all the rows.)

   This solution uses a 100 ml graduated cylinder, 60 ml as the initial amount of water, and number cubes whose volume is approximately 3.7 ml each. Measurements may be different if a different starting amount of water is used or different solid objects.

number of objects	volume in ml
3	71
7	86
11	100.7

2. What is the volume, V, in the cylinder after you add x objects? Explain your reasoning.

   The volume, V, is $60 + 3.7 \cdot x$.

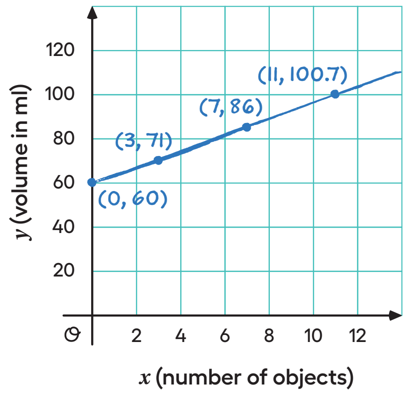
   Sample reasoning: Start with 60 ml of water, then add 3.7 times the number of objects, x, since each object adds 3.7 ml.

3. If you wanted to make the water reach the highest mark on the cylinder, how many objects would you need?

   11

4. Plot and label points that show your measurements from the experiment.

   Sample response:



5. Plot and label a point that shows the volume of the water before you added any objects.

   See graph

6. The points should fall on a line. Use a straightedge to graph this line.

   See graph

7. Calculate the slope of the line. What does the slope mean in this situation?

   The slope of the line is about 3.7 and means that each object adds about 3.7 ml of volume to the cylinder.

8. What is the vertical intercept? What does the vertical intercept mean in this situation?

   The vertical intercept is 60 and means that the cylinder had 60 ml of water in it before any objects were added.

Activity Synthesis

Display the equation $V = 3.7x + 60$ for all to see, or if a different amount of water or objects with a different volume was used, display the equation corresponding to the class data. Explain that this equation represents the situation they saw today. Discuss:

- *"What do the variables represent?"*

  V is the total volume in the cylinder in ml and x is the number of objects added to the cylinder.

- *"Where do you see a rate of change in this equation? What does it mean in this situation?"*

  The rate of change is 3.7 and is seen as the coefficient of the variable x. It is the volume of each object in ml or the increase of volume in the cylinder in ml each time an object is dropped in.

- *"What does the number 60 represent?"*

  60 is the initial amount of water in the cylinder in ml.

Emphasize that the equation $V = 3.7x + 60$ can be interpreted as saying:
total volume = (3.7 ml per object) · (number of objects) + initial volume

If necessary, discuss the precision of the computed slope. It makes sense that every object increases the volume by the same amount (because the number cubes, for example, are all the same size). For smaller objects or graduated cylinders with larger diameters, it may be difficult to accurately measure the change for 1 object. If students add objects in larger sets, the slope calculation will tell the amount of change for one object.

Activity 2
Telling Stories

10 min

Activity Narrative

The purpose of this activity is for students to consider how the parts of an equation are related to the linear context it describes. Without a graph, students must identify what the slope and vertical intercept of an equation mean.

Launch

Arrange students in groups of 2. Give students 3–4 of quiet work time followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Provide Access by Recruiting Interest.
Invite students to create stories for the second question that connect to their own lives.

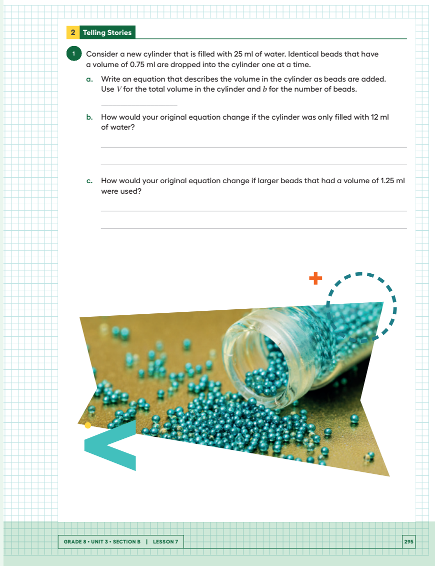
Supports accessibility for: Conceptual Processing, Memory

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.
Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Student Workbook


2 Telling Stories

1 Consider a new cylinder that is filled with 25 ml of water. Identical beads that have a volume of 0.75 ml are dropped into the cylinder one at a time.

a. Write an equation that describes the volume in the cylinder as beads are added. Use V for the total volume in the cylinder and b for the number of beads.

b. How would your original equation change if the cylinder was only filled with 12 ml of water?

c. How would your original equation change if larger beads that had a volume of 1.25 ml were used?

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Student Task Statement

1. Consider a new cylinder that is filled with 25 ml of water. Identical beads that have a volume of 0.75 ml are dropped into the cylinder one at a time.

   a. Write an equation that describes the volume in the cylinder as beads are added. Use V for the total volume in the cylinder and b for the number of beads.

   $V = 25 + 0.75b$ (or equivalent)

   b. How would your original equation change if the cylinder was only filled with 12 ml of water?

   $V = 12 + 0.75b$

   Sample response: The 25 would change to a 12, but everything else would stay the same.

   c. How would your original equation change if larger beads that had a volume of 1.25 ml were used?

   $V = 25 + 1.25b$

   Sample response: The 0.75 would change to 1.25, but everything else would stay the same.

2. A situation is represented by the equation $y = 5 + \frac{1}{2}x$.

   a. Create a story for this situation.

   Answers vary.

   b. What does the 5 represent in your situation?

   Answers vary, but responses should include a context where the 5 stays constant.

   c. What does the $\frac{1}{2}$ represent in your situation?

   Answers vary, but responses should include a context where something is increasing by $\frac{1}{2}$ each time.

Activity Synthesis

The goal of this discussion is for students to connect parts of an equation to the context it describes, without a graph. Invite as many students as time allows to share the stories they created for the second question. Then discuss:

- *"How does the 5 show up in each story? Do they have anything in common?"*

  Answers vary.

- *"How does the $\frac{1}{2}$ show up in each story? Do they have anything in common?"*

  Answers vary.

Lesson Synthesis

The goal of this discussion is for students to create and interpret an equation for a situation with linear growth. Tell students that there is a bucket of water that already contains 10 liters of water, and a water faucet is turned on, adding 2 liters of water to the bucket every minute. Ask students:

- *"Can a linear equation be used to represent this situation?"*

  Yes

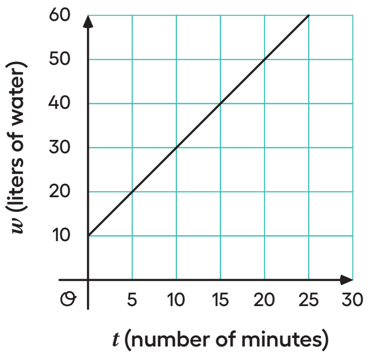
- *"Why or why not?"*

  The rate of change is constant since 2 more liters of water are added to the bucket each minute.

- *"What is an equation that could represent this situation?"*

  $w = 10 + 2t$ (or equivalent), where w is the liters of water in the bucket and t is the time in minutes since the faucet was turned on.

Then display this graph for all to see:



Ask students:

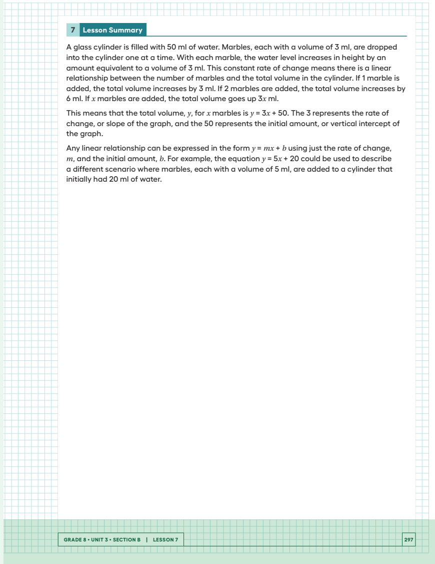
- *"What does the "10" from the equation mean and where can it be seen on the graph?"*

  The "10" represents the liters of water that were already in the bucket, and it can be seen on the graph as the vertical intercept.

- *"What does the "2" from the equation mean and where can it be seen on the graph?"*

  The "2" represents the liters of water that are added each minute, and can be seen on the graph as the slope of the line.

Student Workbook


7 Lesson Summary

A glass cylinder is filled with 50 ml of water. Marbles, each with a volume of 3 ml, are dropped into the cylinder one at a time. With each marble, the water level increases in height by an amount equivalent to a volume of 3 ml. This constant rate of change means there is a linear relationship between the number of marbles and the total volume in the cylinder. If 1 marble is added, the total volume increases by 3 ml. If 2 marbles are added, the total volume increases by 6 ml. If x marbles are added, the total volume goes up $3x$ ml.

This means that the total volume, y, for x marbles is $y = 3x + 50$. The 3 represents the rate of change, or slope of the graph, and the 50 represents the initial amount, or vertical intercept of the graph.

Any linear relationship can be expressed in the form $y = mx + b$ using just the rate of change, m, and the initial amount, b. For example, the equation $y = 5x + 20$ could be used to describe a different scenario where marbles, each with a volume of 5 ml, are added to a cylinder that initially had 20 ml of water.

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Responding To Student Thinking

Points to Emphasize

If most students struggle with calculating positive slopes, as opportunities arise over the next several lessons, revisit finding the slope of a line from a graph. For example, in the activity referred to here, have students calculate the slope of line a before sharing the equation of the line.

Unit 3, Lesson 8, Activity 2 Translating a Line

Lesson Summary

A glass cylinder is filled with 50 ml of water. Marbles, each with a volume of 3 ml, are dropped into the cylinder one at a time. With each marble, the water level increases in height by an amount equivalent to a volume of 3 ml. This constant rate of change means there is a linear relationship between the number of marbles and the total volume in the cylinder. If 1 marble is added, the total volume increases by 3 ml. If 2 marbles are added, the total volume increases by 6 ml. If x marbles are added, the total volume goes up $3x$ ml.

This means that the total volume, y, for x marbles is $y = 3x + 50$. The 3 represents the rate of change, or slope of the graph, and the 50 represents the initial amount, or vertical intercept of the graph.

Any linear relationship can be expressed in the form $y = mx + b$ using just the rate of change, m, and the initial amount, b. For example, the equation $y = 5x + 20$ could be used to describe a different scenario where marbles, each with a volume of 5 ml, are added to a cylinder that initially had 20 ml of water.

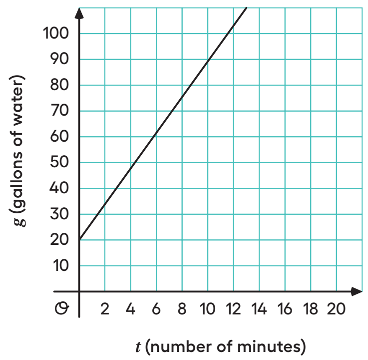
Cool-down

Filling a Tank

5 min

Student Task Statement

The graph shows the relationship between the gallons of water in a tank as it is filling.



1. What is the slope and what does it mean in this situation?

   The slope is 6 and means that 6 gallons of water are added to the tank each minute.

2. What is the vertical intercept and what does it mean in this situation?

   The vertical intercept is 20 and means that the tank already had 20 gallons in it before it started filling.

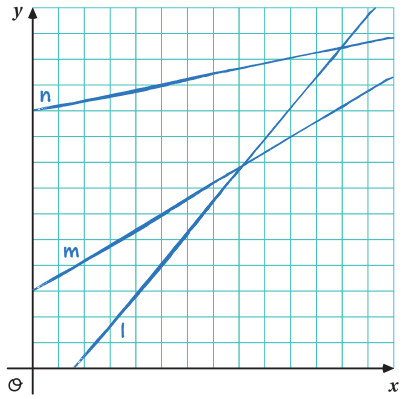
Practice Problems

4 Problems

Problem 1

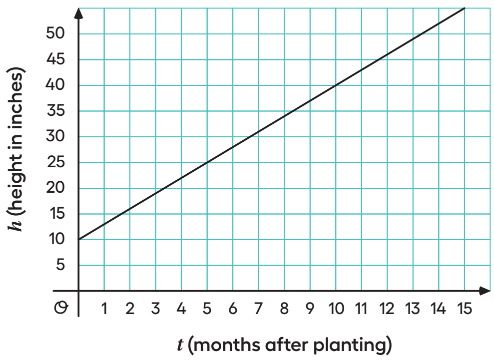
Create a graph that shows three lines with slopes $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{6}{5}$.

Sample response: line ℓ has slope $\frac{6}{5}$, line m has slope $\frac{3}{5}$, line n has slope $\frac{1}{5}$



Problem 2

The graph shows the height in inches, h, of a bamboo plant t months after it has been planted.



a. Write an equation that describes the relationship between h and t.

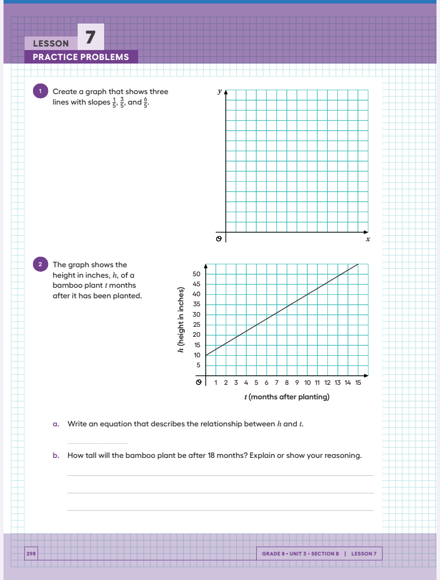
$h = 3t + 10$ (or equivalent)

b. How tall will the bamboo plant be after 18 months? Explain or show your reasoning.

The bamboo plant will be 64 inches tall after 18 months.

Sample reasoning: $64 = 3 \cdot 18 + 10$

Student Workbook



LESSON 7

PRACTICE PROBLEMS

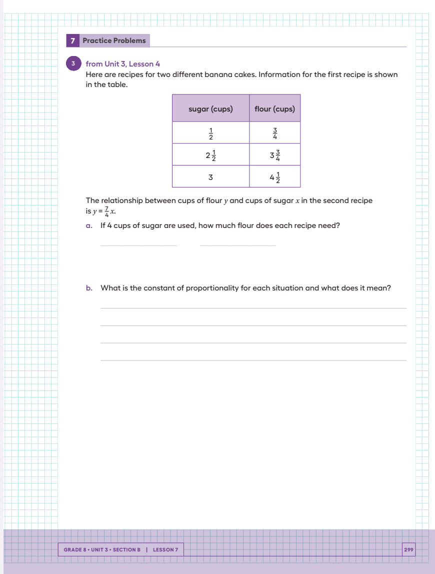
1 Create a graph that shows three lines with slopes $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{6}{5}$.

2 The graph shows the height in inches, h, of a bamboo plant t months after it has been planted.

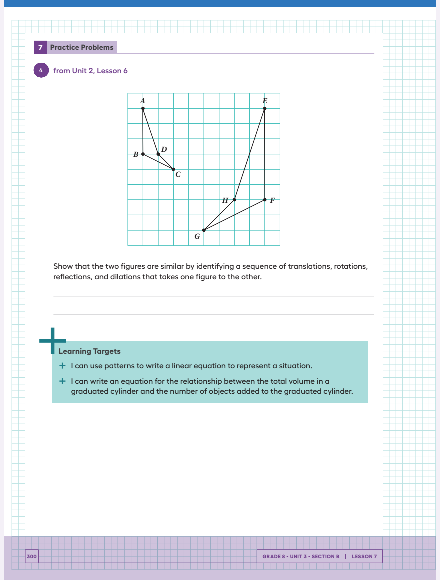
a. Write an equation that describes the relationship between h and t.

b. How tall will the bamboo plant be after 18 months? Explain or show your reasoning.

Student Workbook



Student Workbook



Problem 3

from Unit 3, Lesson 4

Here are recipes for two different banana cakes. Information for the first recipe is shown in the table.

sugar (cups)	flour (cups)
$\frac{1}{2}$	$\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{3}{4}$
3	$4\frac{1}{2}$

The relationship between cups of flour y and cups of sugar x in the second recipe is $y = \frac{7}{4}x$.

a. If 4 cups of sugar are used, how much flour does each recipe need?

First recipe: 6 cups

Second recipe: 7 cups

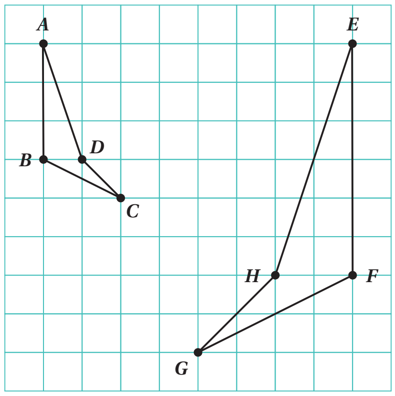
b. What is the constant of proportionality for each situation and what does it mean?

First recipe: $1\frac{1}{2}$ (or equivalent), which means the recipe needs $1\frac{1}{2}$ cups of flour per cup of sugar

Second recipe: $1\frac{3}{4}$ (or equivalent), which means the recipe needs $1\frac{3}{4}$ cups of flour per cup of sugar

Problem 4

from Unit 2, Lesson 6



Show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes one figure to the other.

Sample response: Translate H to D, reflect across a vertical line through D, and then dilate using a scale factor of $\frac{1}{2}$ centered at D.