Lesson 15

Efficiently Solving Inequalities

Goals

- Compare and contrast (orally) solutions to equations and solutions to inequalities.
- Draw and label a graph on the number line that represents all the solutions to an inequality.
- Generalize (orally) that you can solve an inequality of the form $px + q > r$ or $px + q < r$ by solving the equation $px + q = r$ and then testing a value to determine the direction of the inequality in the solution.

Learning Targets

- I can graph the solutions to an inequality on a number line.
- I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

Lesson Narrative

In this lesson, students practice solving inequalities without any context to help support their number reasoning. First, students solve the corresponding equation to identify the boundary point between values that will make the inequality true and values that will make it false. Then students test various values above or below the boundary point to determine which values satisfy the original inequality. They use these test values to reason about the direction of the inequality symbol for the solution. Eventually, students see that they only need to test one value either above or below the boundary point to be able to determine the solution to the inequality. As students generalize a process for determining the solution to an inequality, they make use of repeated reasoning.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1 (Activity 1)
- Compare and Connect (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Lesson 15

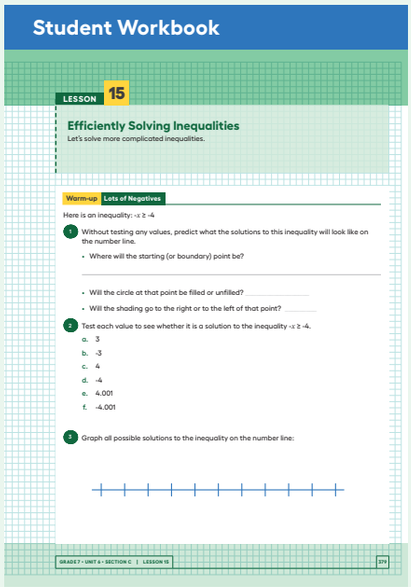
Efficiently Solving Inequalities

Lesson Narrative (continued)

It is important to understand that the goal is not to have students learn and practice an algorithm for solving inequalities like "whenever you multiply or divide by a negative, flip the inequality." Instead, students should understand that solving a related equation tells them the lower or upper bound of an inequality, and testing some values that are above or below the boundary number reveals which values make the inequality true. This way of reasoning about inequalities will serve students well long into their future studies, whereas students are very likely to forget a procedure memorized for a special case.

Student Learning Goal

Let's solve more complicated inequalities.


Student Workbook

LESSON 15

Efficiently Solving Inequalities

Let's solve more complicated inequalities.

Warm-up Lots of Negatives

Here is an inequality: $-x \geq -4$

1. Without testing any values, predict what the solutions to this inequality will look like on the number line.
   - Where will the starting (or boundary) point be?
   - Will the circle at that point be filled or unfilled?
   - Will the shading go to the right or to the left of that point?
2. Test each value to see whether it is a solution to the inequality $-x \geq -4$.
   a. 3
   b. -3
   c. 4
   d. -4
   e. 4.001
   f. -4.001
3. Graph all possible solutions to the inequality on the number line:

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Warm-up

Lots of Negatives

5 min

Activity Narrative

In this activity, students predict what the solutions to $-x \geq -4$ will look like, then test values, and then graph solutions.

Do not formalize a procedure for "flipping the inequality" when multiplying or dividing by a negative. Monitor for students who predict solution sets that are incorrect because of the sign.

Launch

Give students 3 minutes of quiet work time followed by a whole-class discussion.

Students are not expected to be able to make correct predictions in the first question. The purpose of this question is to prompt students to think about how negative values in an inequality affect the solution. It also emphasizes that we should not jump to conclusions about solutions without carefully studying what the inequality means.

Student Task Statement

Here is an inequality: $-x \geq -4$

1. Without testing any values, predict what the solutions to this inequality will look like on the number line.

- Where will the starting (or boundary) point be? 4
- Will the circle at that point be filled or unfilled? filled
- Will the shading go to the right or to the left of that point? left

2. Test each value to see whether it is a solution to the inequality $-x \geq -4$.

a. 3 yes

b. -3 yes

c. 4 yes

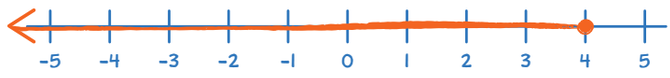
d. -4 yes

e. 4.001 no

f. -4.001 yes

3. Graph all possible solutions to the inequality on the number line:

Sample response:



Activity Synthesis

The purpose of the discussion is to highlight how negatives in the inequality sometimes make it hard to predict what the solutions will be.

Select students to share how their predictions differed from their final solutions.

To illustrate a simple case where solutions go in the opposite direction on the number line, ask how the solutions to $-x \geq -4$ are different from the solutions to $x \geq 4$.

Activity 1
Inequalities with Tables

15 min

Activity Narrative

In this activity, students complete tables to get a visual feel for the relationship between an inequality ($x - 3 > -2$) and its solution ($x > 1$). The second and third questions, taken together, demonstrate how a negative coefficient can make the solutions to an inequality go "the other way."

The work in this activity suggests a procedure for solving inequalities: solve the corresponding equation, then test a number on either side. But the purpose of this activity is not to teach students a procedure, but rather to provide underlying knowledge and experience.

In responding to the last question, students have an opportunity to refine their language or thinking to be more precise.

Launch

Arrange students in groups of 2. Display the table from the first question for all to see.

After students have had a chance to look at the table, ask them some familiarizing questions:

Instructional Routines

MLR1: Stronger and Clearer Each Time
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Access for Students with Diverse Abilities (Activity 1, Task Statement)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity. Consider pausing after each question for a brief class discussion before moving on.

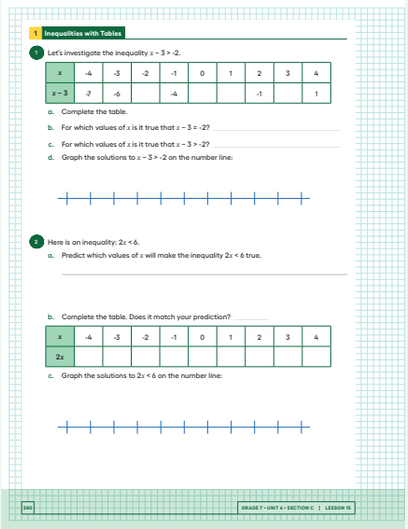
Supports accessibility for: Organization, Attention

Building on Student Thinking

Some students may say $x > 2$ shows the solutions for the inequality in the first question, since that is the place where the integer value of $x - 3$ first surpasses the number -2. Remind these students that there are values between 1 and 2. Ask them whether 1.1 is a solution, for example.

Some students may graph only whole-number (or only integer) solutions. Ask these students to think about whether values in between whole numbers (or integers) are also solutions.

Student Workbook


1 Inequalities with Tables

1. Let's investigate the inequality $x - 3 > -2$.

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7	-6		-4			-1		1

a. Complete the table.
b. For which values of x is it true that $x - 3 = -2$?
c. For which values of x is it true that $x - 3 > -2$?
d. Graph the solutions to $x - 3 > -2$ on the number line:

2. Here is an inequality: $2x < 6$.
a. Predict which values of x will make the inequality $2x < 6$ true.
b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$2x$									

c. Graph the solutions to $2x < 6$ on the number line:

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"How are the numbers in the top row and bottom row related?"

Each number in the bottom row is 3 less than the corresponding number in the top row.

"What numbers should go in the empty spots? How do you know?"

-5, -3, -2, and 0; These are 3 less than -2, 0, 1, and 3.

"Think about the equation $x - 3 = -2$. What value of x makes this equation true? Where do you see that fact in the table?"

$x = 1$; The column that has 1 in the top row has -2 in the bottom row.

Give students 5–6 minutes of partner work time, followed by whole-class discussion.

Student Task Statement

1. Let's investigate the inequality $x - 3 > -2$.

a. Complete the table.

b. For which values of x is it true that $x - 3 = -2$?

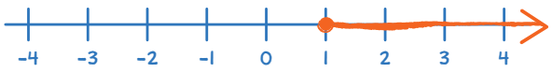
$x = 1$

c. For which values of x is it true that $x - 3 > -2$?

$x > 1$

d. Graph the solutions to $x - 3 > -2$ on the number line:

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7	-6	-5	-4	-3	-2	-1	0	1



2. Here is an inequality: $2x < 6$.

a. Predict which values of x will make the inequality $2x < 6$ true.

Sample response:

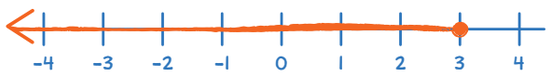
- I predict that any value less than 3 will work.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$2x$	-8	-6	-4	-2	0	2	4	6	8

The table may or many not match the prediction.

c. Graph the solutions to $2x < 6$ on the number line:



3. Here is an inequality: $-2x < 6$.

a. Predict which values of x will make the inequality $-2x < 6$ true.

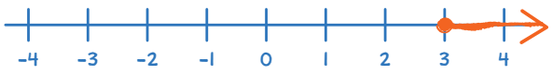
Sample response: Based on the solution to $2x < 6$, I predict that for $-2x < 6$, the solutions will be values less than -3.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$	8	6	4	2	0	-2	-4	-6	-8

The table may or may not match the prediction.

c. Graph the solutions to $-2x < 6$ on the number line:



4. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

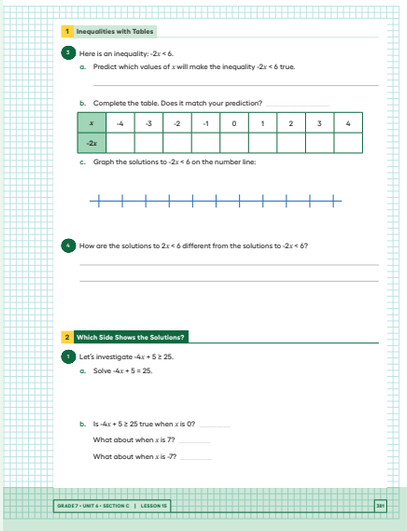
The solution to $2x < 6$ is all values less than 3, but the solution to $-2x < 6$ is all values greater than -3.

Activity Synthesis

The purpose of this discussion is to see that solving the associated equation to an inequality gives the value that is the boundary between solutions and non-solutions. Resist the temptation to summarize the last two problems into a procedure like "whenever you multiply or divide by a negative, flip the inequality."

In this activity, students have a table to check on which side of the boundary shows solutions and which side does not show solutions. In order to transition to the next activity, ask students whether they need to complete an entire table to test on which side of the boundary the solutions are. The goal is to help students understand that they only need to test one number. If that number is a solution, then all points on that same side of the boundary are solutions. If the point is not a solution, then the solutions are all the points on the other side of the boundary. The next activity will give students an opportunity to apply this insight and start to articulate such a procedure.

Student Workbook


1 Inequalities with Tables

3 Here is an inequality: $-2x < 6$.

a. Predict which values of x will make the inequality $-2x < 6$ true.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

c. Graph the solutions to $-2x < 6$ on the number line:

4 How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

2 Which Side Shows the Solutions?

1 Let's investigate $-4x + 5 \geq 25$.

a. Solve $-4x + 5 = 25$.

b. Is $-4x + 5 \geq 25$ true when x is 0?

What about when x is 7?

What about when x is -7?

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Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to

"How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?"

Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Activity 2

Which Side Shows the Solutions?

15 min

Activity Narrative

In this activity, students solidify a process for solving inequalities: first solve the associated equation to find the boundary point, then test a value to determine on which side of that boundary the solutions lie. When students apply the reasoning used for the more scaffolded problems to solve new problems, they express regularity in repeated reasoning.

Monitor for different choices students make in the process of determining solutions. Examples:

- When solving the associated equation, do they start by distributing or start by dividing both sides?
- When selecting points to test to determine the direction of the inequality, do they test more than one point or make an inference from just one?
- In particular, does anyone use 0 as their test value, since that makes the expression simpler to evaluate?

Launch

Keep students in groups of 2.

Give 5–10 minutes of quiet work time, time to share their responses and reasoning with a partner, and follow with a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

1. Let's investigate $-4x + 5 \geq 25$.

a. Solve $-4x + 5 = 25$.

$x = -5$

b. Is $-4x + 5 \geq 25$ true when x is 0?

no

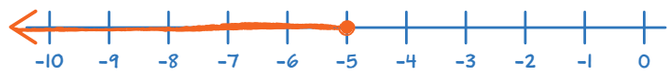
c. What about when x is 7?

no

d. What about when x is -7?

yes

e. Graph the solutions to $-4x + 5 \geq 25$ on the number line.



Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 2, Synthesis)

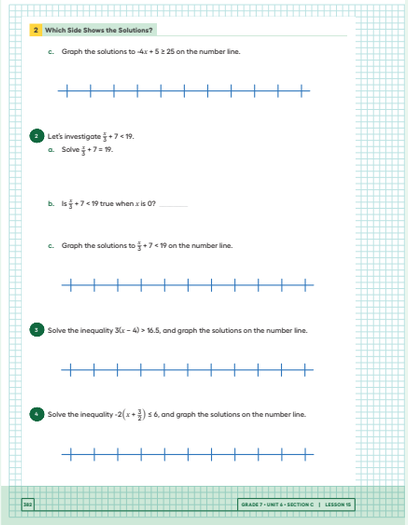
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


2 Which Side Shows the Solutions?

c. Graph the solutions to $-4x + 5 \geq 25$ on the number line.

2. Let's investigate $\frac{x}{3} + 7 < 19$.
a. Solve $\frac{x}{3} + 7 = 19$.

b. Is $\frac{x}{3} + 7 < 19$ true when x is 0? _____

c. Graph the solutions to $\frac{x}{3} + 7 < 19$ on the number line.

3. Solve the inequality $3(x - 4) > 16.5$, and graph the solutions on the number line.

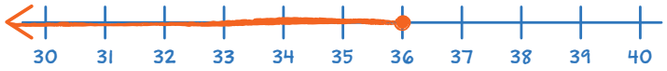
4. Solve the inequality $-2\left(x + \frac{3}{2}\right) \leq 6$, and graph the solutions on the number line.

2. Let's investigate $\frac{x}{3} + 7 < 19$.

a. Solve $\frac{x}{3} + 7 = 19$. $x = 36$

b. Is $\frac{x}{3} + 7 < 19$ true when x is 0? yes

c. Graph the solutions to $\frac{x}{3} + 7 < 19$ on the number line.



3. Solve the inequality $3(x - 4) > 16.5$, and graph the solutions on the number line.

$x > 9.5$

4. Solve the inequality $-2\left(x + \frac{3}{2}\right) \leq 6$, and graph the solutions on the number line.

$x \geq -4\frac{1}{2}$

Are You Ready for More?

Write at least three different inequalities whose solution is $x > -10$. Find one inequality with x on the left side that uses a $<$.

Sample responses: $2x > -20$, $x + 50 > 40$, $-5x < 50$, $\frac{x}{-6} < 60$.

Activity Synthesis

The goal of this discussion is to highlight that once we have found the boundary value for an inequality, there are a variety of valid options when testing one or more values on either side as solutions.

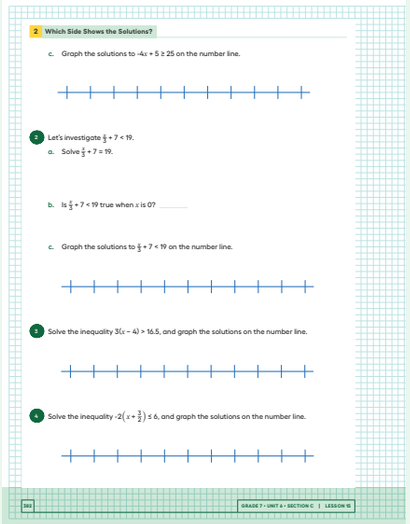
Display 2–3 approaches from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"What do the different ways of solving the associated equation have in common? How are they different?"*
- *"What do the different values being tested have in common? How are they different?"*
- *"Why do the different approaches lead to the same (or different) outcome(s)?"*

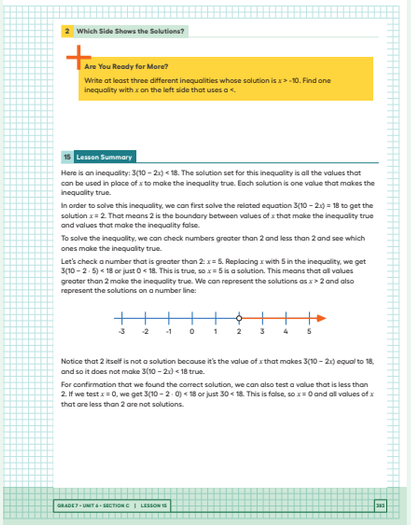
The key takeaway is that any inequality can be solved with the same general procedure, with choices to make along the way:

- Write and solve the related equation to find the boundary value.
- Choose one or more values on one side of the boundary value to test in the inequality.
- Optionally, choose one or more values on the other side of the boundary value to test in the inequality.
- Write and graph the solution.

Student Workbook



Student Workbook



Lesson Synthesis

Share with students, "Today we looked for efficient ways to solve inequalities."

To review these strategies, consider asking students:

- *"How can solving a related equation help us solve an inequality?"*

  The solution to the equation will be the boundary point between values that make the inequality true and values that make it false.

- *"How can we know whether the solution to an inequality is greater than or less than the boundary value?"*

  Test one number that is above or below the boundary value to see if it makes the inequality true. If it does, then the solution to the inequality is all the values on that same side of the boundary value.

- *"How many values do we need to test to determine the direction of the solution to the inequality? Explain your reasoning."*

  We only need to test one value. If the value we test does not make the inequality true, then the solution is all the values on the other side of the boundary point.

Lesson Summary

Here is an inequality: $3(10 - 2x) < 18$. The solution set for this inequality is all the values that can be used in place of x to make the inequality true. Each solution is one value that makes the inequality true.

In order to solve this inequality, we can first solve the related equation $3(10 - 2x) = 18$ to get the solution $x = 2$. That means 2 is the boundary between values of x that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let's check a number that is greater than 2: $x = 5$. Replacing x with 5 in the inequality, we get $3(10 - 2 \cdot 5) < 18$ or just $0 < 18$. This is true, so $x = 5$ is a solution. This means that all values greater than 2 make the inequality true. We can represent the solutions as $x > 2$ and also represent the solutions on a number line:

Notice that 2 itself is not a solution because it's the value of x that makes $3(10 - 2x)$ *equal* to 18, and so it does not make $3(10 - 2x) < 18$ true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test $x = 0$, we get $3(10 - 2 \cdot 0) < 18$ or just $30 < 18$. This is false, so $x = 0$ and all values of x that are less than 2 are not solutions.

Cool-down

Testing for Solutions

5 min

The purpose of this *Cool-down* is to check whether students can determine the direction of inequality for the solution to an inequality. The questions involve using algebra to find boundary points, then testing values of x. Since the boundary points are given, some students may skip directly to testing points.

Student Task Statement

For each inequality, decide whether the solution is represented by $x < 2.5$ or $x > 2.5$.

1. $-4x + 5 > -5$

$x < 2.5$

2. $-25 > -5(x + 2.5)$

$x > 2.5$

Responding To Student Thinking

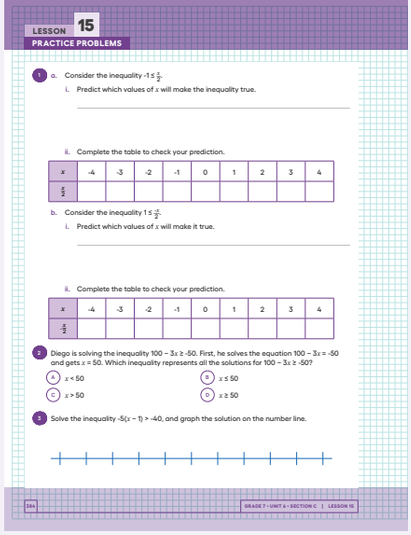
Points to Emphasize
If students struggle with testing points to determine the direction of the inequality in the solution, plan to revisit this concept when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about the direction of the solutions in this activity.

Lesson 15

Practice Problems

6 Problems

Student Workbook



LESSON 15
PRACTICE PROBLEMS

1 a. Consider the inequality $-1 \le \frac{x}{2}$.

i. Predict which values of x will make the inequality true.

ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$									

b. Consider the inequality $1 \le \frac{-x}{2}$.

i. Predict which values of x will make it true.

ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$-\frac{x}{2}$									

2 Diego is solving the inequality $100 - 3x \ge -50$. First, he solves the equation $100 - 3x = -50$ and gets $x = 50$. Which inequality represents all the solutions for $100 - 3x \ge -50$?

A. $x < 50$ B. $x \le 50$ C. $x > 50$ D. $x \ge 50$

3 Solve the inequality $-5(x - 1) > -40$, and graph the solution on the number line.

Problem 1

a. Consider the inequality $-1 \le \frac{x}{2}$.

i. Predict which values of x will make the inequality true.

Sample response: $x \ge -2$

ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$	-2	-1.5 (or $\frac{-3}{2}$)	-1	-0.5 (or $\frac{-1}{2}$)	0	0.5 (or $\frac{1}{2}$)	1	1.5 (or $\frac{3}{2}$)	2

b. Consider the inequality $1 \le \frac{-x}{2}$.

i. Predict which values of x will make it true.

Sample response: $x \le -2$

ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$-\frac{x}{2}$	2	1.5 (or $\frac{-3}{2}$)	1	0.5 (or $\frac{-1}{2}$)	0	-0.5 (or $\frac{1}{2}$)	-1	-1.5 (or $\frac{3}{2}$)	-2

Problem 2

Diego is solving the inequality $100 - 3x \ge -50$. First, he solves the equation $100 - 3x = -50$ and gets $x = 50$. Which inequality represents all the solutions for $100 - 3x \ge -50$?

A. $x < 50$

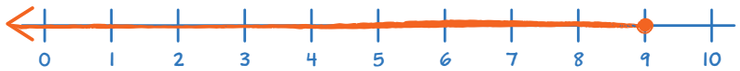
B. $x \le 50$

C. $x > 50$

D. $x \ge 50$

Problem 3

Solve the inequality $-5(x - 1) > -40$, and graph the solution on the number line.



$x < 9$

Problem 4

from Unit 6, Lesson 13

Select **all** values of x that make the inequality $-x + 6 \geq 10$ true.

A. -3.9 **B.** 4 **C.** -4.01 **D.** -4

E. 4.01 **F.** 3.9 **G.** 0 **H.** -7

Problem 5

from Unit 6, Lesson 13

Represent the solution for each inequality on the number line.

a. $x > 7$

b. $x \geq -4.2$

Problem 6

from Unit 4, Lesson 12

The price of a pair of earrings is $22 but Priya buys them on sale for $13.20.

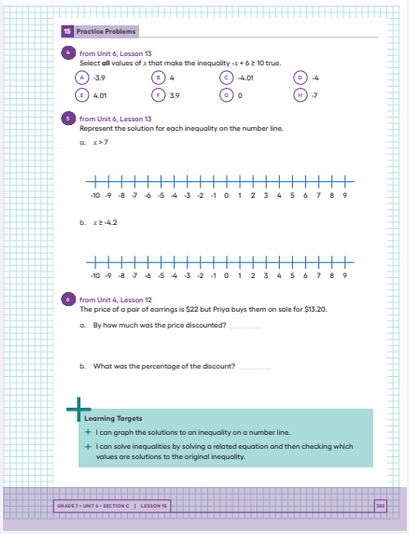
a. By how much was the price discounted?

$8.80

b. What was the percentage of the discount?

40%

Student Workbook


15 Practice Problems

A from Unit 6, Lesson 13
Select all values of x that make the inequality -x + 6 ≥ 10 true.
A -3.9 B 4 C -4.01 D -4
E 4.01 F 3.9 G 0 H -7

B from Unit 6, Lesson 13
Represent the solution for each inequality on the number line.
a. x > 7
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
b. x ≥ -4.2
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

A from Unit 4, Lesson 12
The price of a pair of earrings is $22 but Priya buys them on sale for $13.20.
a. By how much was the price discounted? _____
b. What was the percentage of the discount? _____

Learning Targets
+ I can graph the solutions to an inequality on a number line.
+ I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

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