Lesson 4

# Making the Moves

## Goals

- Draw a transformation of a figure using information given orally.
- Explain (orally) the sequence of transformations that takes one figure to its image.
- Identify (orally and in writing) the features that determine a translation, rotation, or reflection.

## Learning Target

I can use the terms "translation," "rotation," and "reflection" to precisely describe transformations.

## Student Learning Goal

Let's draw and describe translations, rotations, and reflections.

## Lesson Narrative

In the previous lesson, students were introduced to the terms "translation," "rotation," and "reflection." In this lesson, students understand that:

- A translation is determined by two points that specify the distance and direction of the translation.
- A rotation is determined by a point, angle of rotation, and a direction of rotation.
- A reflection is determined by a line.

These moves are called **transformations** for the first time and students draw images of figures under these transformations. They also study where shapes go under sequences of these transformations and identify the steps in a **sequence of transformations** that takes one figure to another. Note the subtle shift in language. In the previous lesson, one shape "moves" to the other shape—it is as if the original shape has agency and does the moving. In this lesson, the transformation "takes" one shape to the other shape—this language choice centers the transformation itself as an object of study.

### Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

### Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 1)

### Required Materials

**Materials to Gather**

- Geometry toolkits: Activity 2

**Materials to Copy**

- Make that Move Cards (1 copy for every 4 students): Activity 1

**Activity 2:**
For the digital version of the activity, acquire devices that can run the applet.

## Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Assessment: Cool-down |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 10 min | 5 min |

# Making the Moves

## Lesson Narrative (continued)

Students using the print version may make use of tracing paper to experiment moving shapes. Students using the digital version have access to Geogebra applets with which to perform transformations. Whenever students choose to make use of an appropriate tool, they are engaging in the mathematical practice standard of using tools strategically. Students are also likely starting to begin thinking strategically about which transformations will take one figure to another, identifying properties of the shapes that indicate whether a translation, rotation, reflection or sequence of these will achieve this goal.

## Warm-up

# Reflection Quick Image

5 min

### Activity Narrative

**There is a digital version of this activity.**

In this *Warm-up,* students are asked to sketch a reflection of a given triangle and explain the strategies they used. The goal is to prompt students to notice and articulate that they can use the location of a single point and the fact that the image is a reflection of the triangle to sketch the image. To encourage students to use what they know about reflections and not count every grid line, flash the image for a few seconds and then hide it. Flash it once more for students to check their thinking.

### Launch

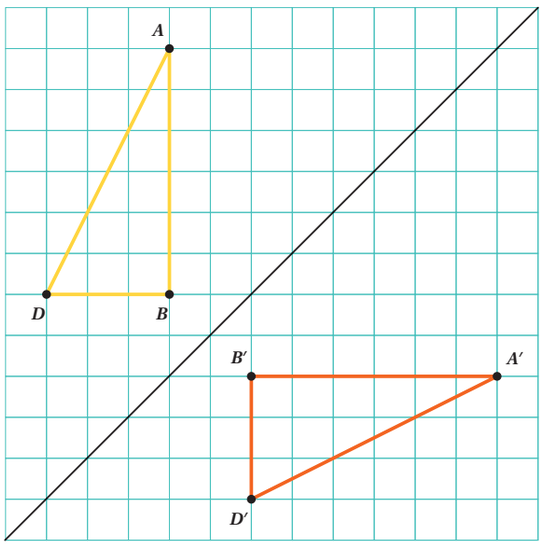
Before beginning, make sure students have their books or devices open to the correct page. Tell students that an image of a reflection of triangle $ABD$ will be shown for 3 seconds. Their job is to draw the image and explain any strategies they used.

Display the completed image for 3 seconds and then hide it.

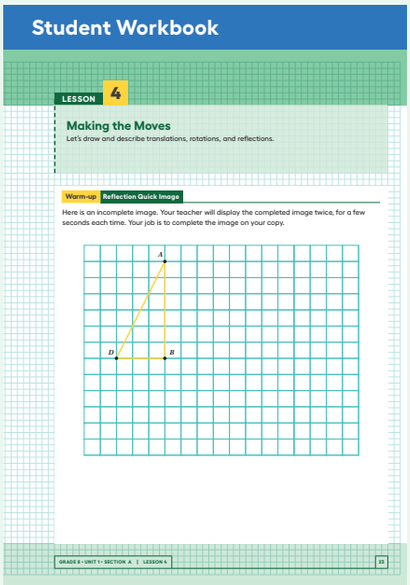
Do this twice.

Give students 1 minute of quiet work time after each flash of the image.

Encourage students to think about any shortcuts they used to draw the reflected image.



## Student Workbook


LESSON 4

Making the Moves

Let's draw and describe translations, rotations, and reflections.

Warm-up Reflection Quick Image

Here is an incomplete image. Your teacher will display the completed image twice, for a few seconds each time. Your job is to complete the image on your copy.

A

D

B

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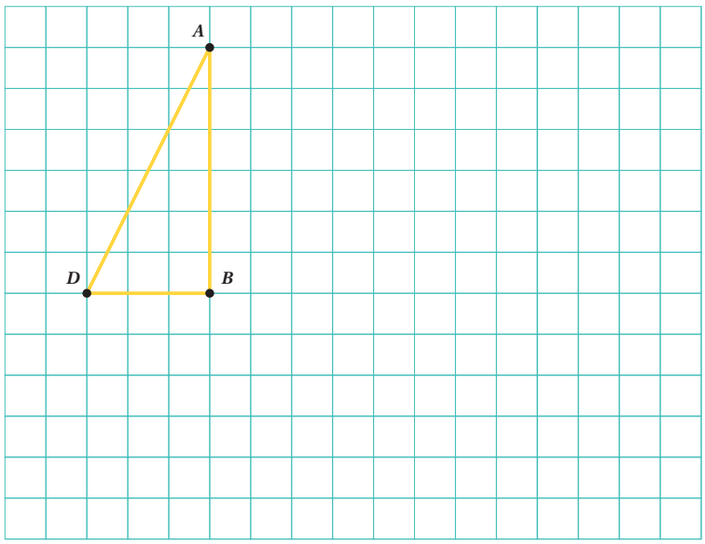
## Building on Student Thinking

Students may struggle drawing the image under transformation from the quick flashes of the image because they are trying to count the number of spaces each vertex moves. Encourage these students to use the line in the image to help them reflect the image.

## Student Task Statement

Here is an incomplete image. Your teacher will display the completed image twice, for a few seconds each time. Your job is to complete the image on your copy.




Sample response: The first flash showed where to put $B'$ and the second flash where to put $A'$. Once these were in place, there was only one place $D'$ could go, underneath the segment $A'B'$, so that triangle $A'B'D'$ is a reflected image of triangle $ABD$.

## Activity Synthesis

Select a few students to share strategies they used in sketching their figure. Consider asking some of the following questions:

- *"What was important in creating your sketch (what did you need)?"*
- *"What did you look for in the first flash? The second?"*
- *"What stayed the same and what is different in the shape and its image?"*
- *"How did you decide where to place the vertices of the image?"*
- *"How did you decide how long to make the sides?"*

## Activity 1

## Make That Move

15 min

## Activity Narrative

**There is a digital version of this activity.**

The purpose of this activity is for students to give precise descriptions of translations, rotations, and reflections. In previous lessons, students have identified and sketched transformations from written directions, however they have not used this more precise language to give descriptions of the three transformations. The images in this activity are given on grids to encourage students to describe the transformation in terms of specific points, lines or angles.

As students describe their transformations and sketch their images, monitor for students using precise descriptions with their partner in terms of specific points, lines, or angles.

## Launch

Introduce the word **transformation:** This is a word for a move (such as the reflection in the *Warm-up)* or for a sequence of moves. It's a mathematical word which will be used in place of "move," which has been used up to this point.

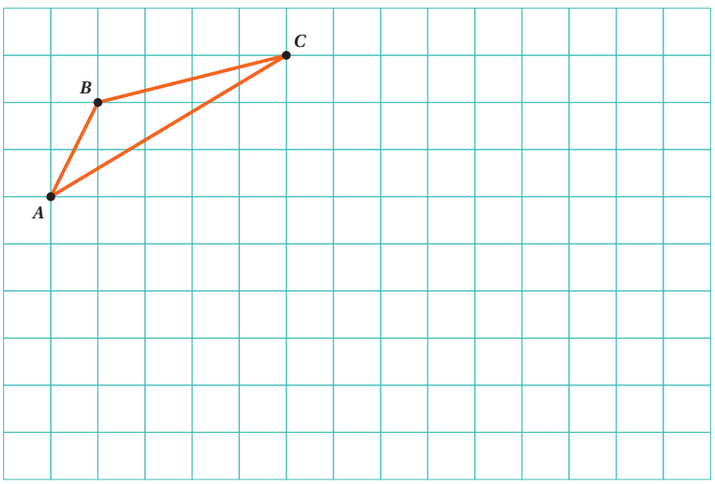
Arrange students in groups of 2. Display the original image that each student also has in front of them. Give each student one of the four transformation cards and tracing paper—make sure students know *not* to show their card to their partner! Tell students they will sketch a transformation based on directions given to them by a partner.

Each partner has Cards 1A and 1B or Cards 2A and 2B. Each A card is a translation, while the B cards show either a rotation or reflection. Tell students in the first round, those with the A cards give a precise description of the transformation displayed on their card to their partner. Their partners may use tracing paper to produce the image under transformation on the grid with the original image. When the sketch is complete, the student describing the transformation reveals their card, and together, students decide if the sketch is correct. In the second round, the roles are reversed. The students with the B cards describe their transformation while their partner sketches.

The student describing the transformation is allowed to repeat, revise, or add any important information as their partner sketches, however, they are not allowed to tell them to fix anything until they are finished. The student sketching should not speak, just sketch. This is to encourage the describer to use mathematical language. Refer students to the displays made in previous lessons with the terms "translation," "rotation," and "reflection" and accompanying visual example, and remind them that they can refer to the display when giving directions to their partner.

### Student Task Statement

Your partner will describe the image of this triangle after a certain **transformation**. Sketch it here.



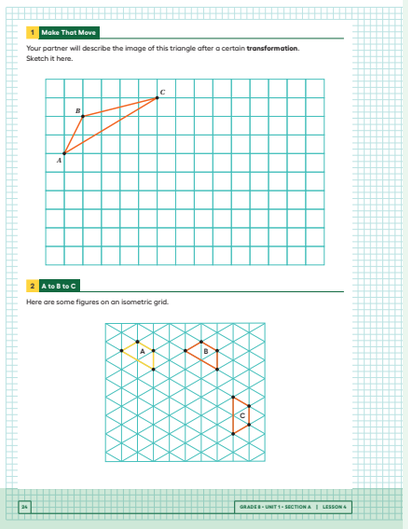

The correct transformations are shown on the cards.

### Access for Multilingual Learners (Activity 1, Launch)

**MLR2: Collect and Display.**
Collect the language students use to describe the image of the triangle after a transformation. Display words and phrases such as "slide right and then down," "the line of reflection is vertical," or "rotate 90 degrees clockwise." During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

*Advances: Conversing, Reading*

### Student Workbook


1 Make That Move

Your partner will describe the image of this triangle after a certain **transformation**. Sketch it here.

2 A to B to C

Here are some figures on an isometric grid.

### Building on Student Thinking

Students may get stuck thinking they need to use the precise terms for the transformation in their description. Encourage these students to describe it in a way that makes sense to them and to look for things they know about the specific points, lines, or angles on their card to help them.

**Access for Multilingual Learners (Activity 1)**

**MLR7: Compare and Connect**
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

### Activity Synthesis

Display the following questions for all to see and give groups 2 minutes to discuss:

- What pieces of your partner's description were helpful when you were sketching?
- What pieces did you find difficult to explain to your partner? Point to specific examples on your cards.
- When you were sketching, what questions would have been helpful to be able to ask the describer?

Ask selected students who were observed using precise descriptions and sketching based on those descriptions to explain why they used the information they did and how it was helpful in sketching. Focus on:

- The direction and distance of a translation
- The center and the measure of a rotation
- The line of a reflection

If time allows, ask students who were both using and not using tracing paper to explain their process.

Reinforce the term "transformation" as a term that encompasses translations, rotations, and reflections. Tell them that there are other types of transformations, but for now, we will focus on these three types.

## Activity 2

## A to B to C

15 min

### Activity Narrative

**There is a digital version of this activity.**

This activity introduces the term **sequence of transformations** and gives students an opportunity to describe a sequence of more than one transformation using precise language.

Each time a reflection is mentioned, ask students where the line of reflection is located and when a rotation is mentioned, ask for the center of the rotation and the number of degrees. Monitor for students who apply these different transformations:

- Reflect Figure B over line $\ell$ to Figure C
- Rotate Figure B about point $R$ to Figure C
- Choose a sequence of transformations from Figure A to B to C
- Choose a sequence of transformations using a different order or not going through Figure B.

The key connection is that different transformations or sequences of transformations may result in the same image. There may be more than one way to describe or perform a transformation.

This is the first time Math Language Routine 7: *Compare and Connect* is suggested in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and they record their method for all to see. They then compare and identify correspondences across strategies by means of a teacher-led gallery walk with commentary or teacher think-aloud (such as "I notice … I wonder …"). A typical discussion prompt is "What is the same and what is different?", comparing their own strategy to the others. The purpose of this routine is to allow students to make sense of mathematical strategies and, through constructive conversations, develop awareness of the language used as they compare, contrast, and connect other ways of thinking to their own.

## Launch

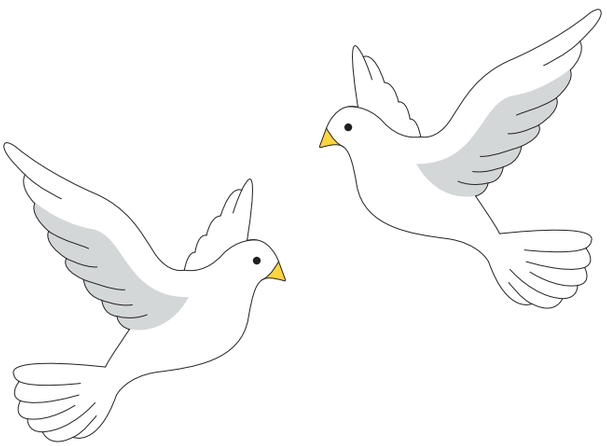
Arrange students in groups of 2, and provide access to their geometry toolkits. Display the image for all to see. Ask students if they can imagine a single translation, rotation, or reflection that would take one bird to another?

After a minute, verify that this is not possible.

Ask students to describe how we could use translations, rotations, and reflections to take one bird to the other. Collect a few different responses.

> One way would be to take the bird on the left, translate it up, and then reflect it over a vertical line.

Tell students when we do one or more transformations in a row to take one figure to another, it is called a **sequence of transformations**.



In the digital version of the activity, students use an applet to perform transformations digitally. The applet allows students to create transformations precisely. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Give students 2 minutes of quiet work time to engage in the task followed by 3 minutes to discuss their responses with a partner and complete any unfinished questions.

Follow with a whole-class discussion.

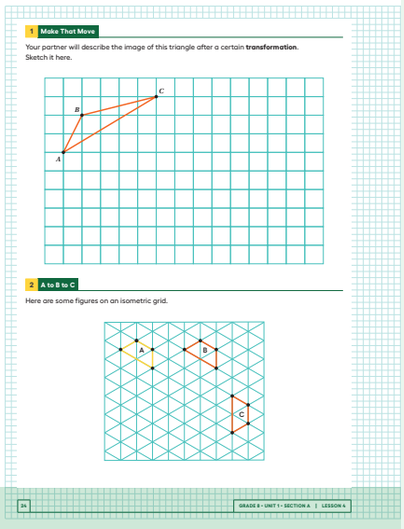
Select work from students with different strategies, such as those described in the *Activity Narrative,* to share later.

**Access for Students with Diverse Abilities (Activity 2, Launch)**

**Action and Expression: Provide Access for Physical Action.** Provide access to tools and assistive technologies, such as devices that can run the digital applet.

*Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization*

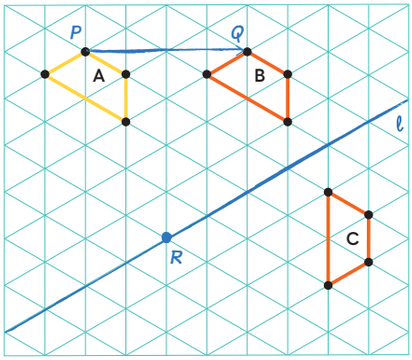
**Student Workbook**




**Student Task Statement**

Here are some figures on an isometric grid.

**1.** Name a transformation that takes Figure A to Figure B.




Sample response: Translate Figure A so that point $P$ goes to point $Q$.

**2.** Name a transformation that takes Figure B to Figure C.

Sample responses:

- Reflect Figure B across line $\ell$.
- Rotate Figure B 60° clockwise around point $R$ on line $\ell$.

**3.** What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Sample responses:

- Translate Figure A so that point $P$ goes to point $Q$. Then reflect Figure B across line $\ell$.
- Apply a translation so that Figure A goes to Figure B and then a reflection taking Figure B to Figure C.

## Are You Ready for More?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with...

- No rotations?

  Sample response: Translate $S$ to $V$ and then reflect over a gridline to make the figures align.

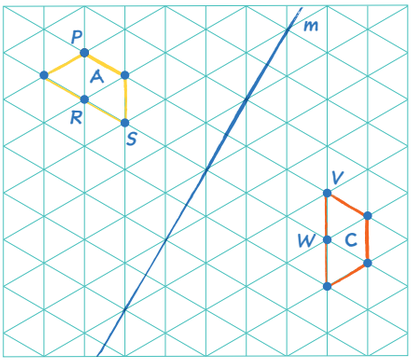
- No reflections?

  Sample response: Translate $R$ to $W$ and then rotate around $W$ to make the figures align.

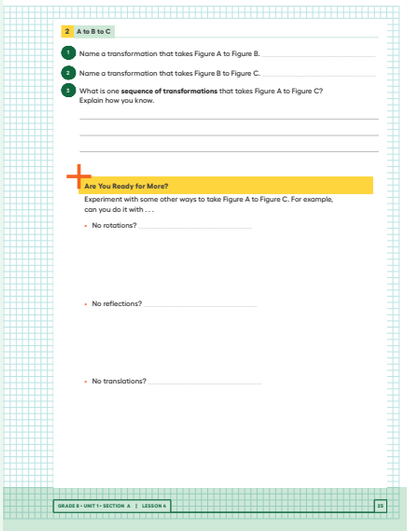
- No translations?

  Sample response: Reflect across the line $m$, taking $R$ to $W$, and then rotate around $W$ to make the figures align.

  Sample response: First translate $P$ to $S$ and then rotate 60° clockwise.




**Student Workbook**


2 A to B to C

1. Name a transformation that takes Figure A to Figure B.
2. Name a transformation that takes Figure B to Figure C.
3. What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Are You Ready for More?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with . . .

- No rotations?
- No reflections?
- No translations?

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## Activity Synthesis

The goal of this discussion is to show different transformations and sequences of transformations that result in the same image.

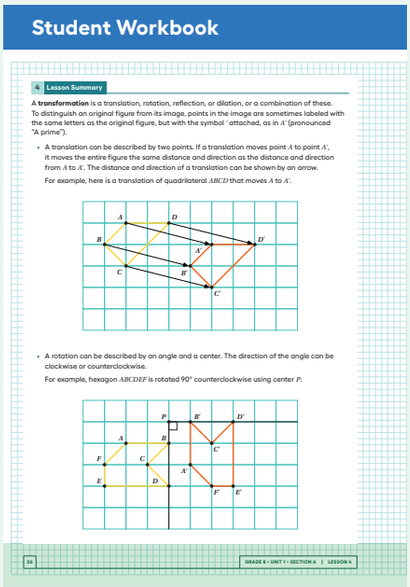
Select students with different correct responses to show their solutions. Be sure to highlight at least one rotation. If no students mention that, demonstrate a way to take Figure A to Figure C that involves a rotation.

It may be helpful to use the applet to facilitate discussion:

The Geogebra applet 'A to B to C' is available here: **ilclass.com/l/395288**

- Emphasize that there are many ways to describe the translation that takes Figure A to Figure B. All one needs is to identify a pair of corresponding points and name them in the correct order (and to use the word "translate").
- For students who use a reflection to take Figure B to Figure C, emphasize that reflections are determined by lines and we should name the line when we want to communicate about it.
- After a student or the teacher uses a rotation, emphasize that a rotation is defined by a center point and an angle (with a direction). The center point needs to be named and an angle with the correct measure needs to be named as well (as does the direction). Reinforce to students that when we do more than one transformation in a row, we call this a "sequence of transformations."

**Student Workbook**




Here are some questions for discussion:

- *"What do the sequences have in common? How are they different?"*
- *"When does the translation show up in each method?"*
- *"Why do the different approaches lead to the same outcome?"*

## Lesson Synthesis

The goal for this lesson is for students to begin to identify the features that determine a translation, rotation, or reflection. Refer to the class display produced in a previous lesson during the discussion. To highlight the features specific to each type of transformation, consider asking the following questions:

- *"If you want to describe a translation, what important information do you need to include?"*

  A translation is determined by two points that specify the distance and direction of the translation.

- *"If you want to describe a rotation, what important information do you need to include?"*

  A rotation is determined by a center point and an angle with a direction.

- *"If you want to describe a reflection, what important information do you need to include?"*

  A reflection is determined by a line.

- *"What does the word 'transformation' mean?"*

  Translations, rotations, and reflections, or any combination of these.

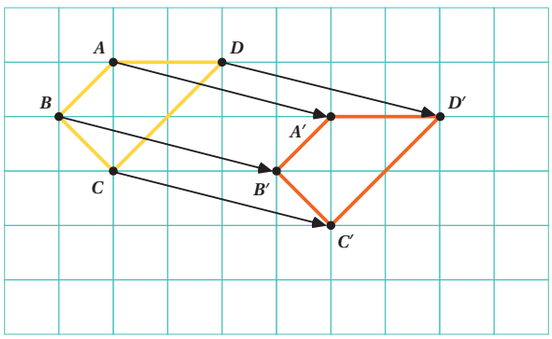
- *"What does 'sequence of transformations' mean?"*

  More than one applied one after the other.

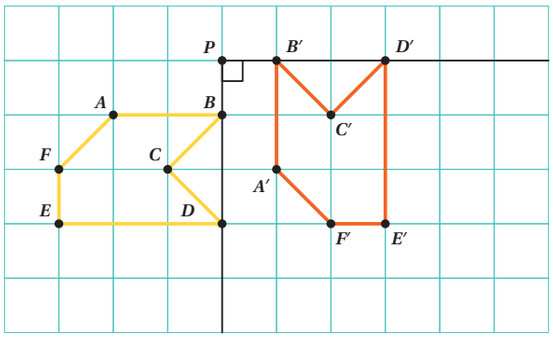
## Lesson Summary

A **transformation** is a translation, rotation, reflection, or dilation, or a combination of these. To distinguish an original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol ′ attached, as in $A'$ (pronounced "A prime").

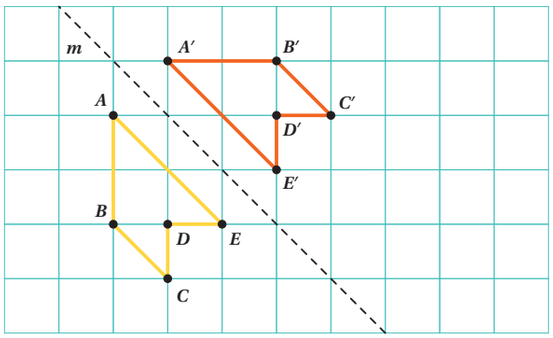
- A translation can be described by two points. If a translation moves point $A$ to point $A'$, it moves the entire figure the same distance and direction as the distance and direction from $A$ to $A'$. The distance and direction of a translation can be shown by an arrow.
  For example, here is a translation of quadrilateral $ABCD$ that moves $A$ to $A'$.



- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.
  For example, hexagon $ABCDEF$ is rotated 90° counterclockwise using center $P$.

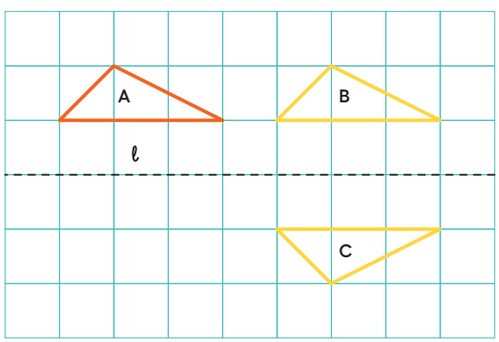



- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.
  For example, pentagon $ABCDE$ is reflected across line $m$.

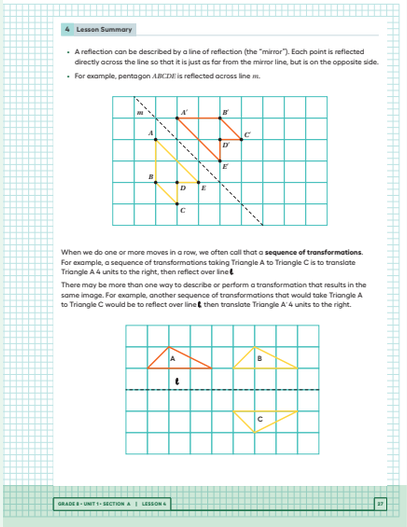



When we do one or more moves in a row, we often call that a **sequence of transformations**. For example, a sequence of transformations taking Triangle A to Triangle C is to translate Triangle A 4 units to the right, then reflect over line $\ell$.

There may be more than one way to describe or perform a transformation that results in the same image. For example, another sequence of transformations that would take Triangle A to Triangle C would be to reflect over line $\ell$, then translate Triangle A′ 4 units to the right.




**Student Workbook**


4 Lesson Summary

- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.
- For example, pentagon $ABCDE$ is reflected across line $m$.

When we do one or more moves in a row, we often call that a **sequence of transformations**. For example, a sequence of transformations taking Triangle A to Triangle C is to translate Triangle A 4 units to the right, then reflect over line $\ell$.

There may be more than one way to describe or perform a transformation that results in the same image. For example, another sequence of transformations that would take Triangle A to Triangle C would be to reflect over line $\ell$, then translate Triangle A′ 4 units to the right.



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**Responding To Student Thinking**

**Points to Emphasize**
If students struggle with identifying the type of transformation, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, allow multiple students to share their drawings of rotations and translations on the tracing paper.

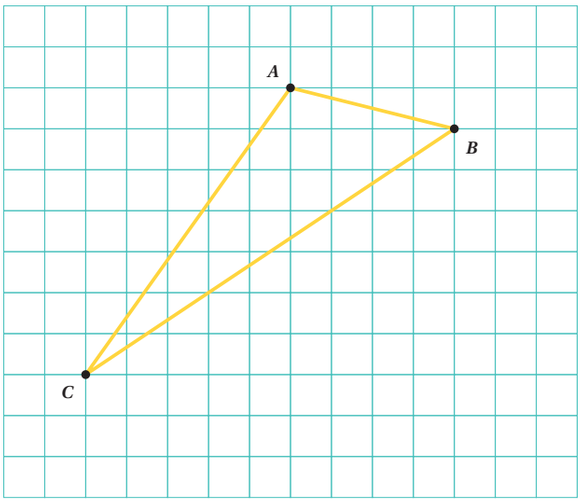
Unit 1, Lesson 5, Activity 3
Transformations of a Segment

Cool-down

## What Does It Take?

5 min

### Student Task Statement




For each description of a transformation, identify what information is missing.

1. Translate triangle $ABC$ to the right.

   Sample response: Distance—how many units to the right

2. Rotate triangle $ABC$ 90° around point $C$.

   Sample response: Direction—clockwise or counterclockwise

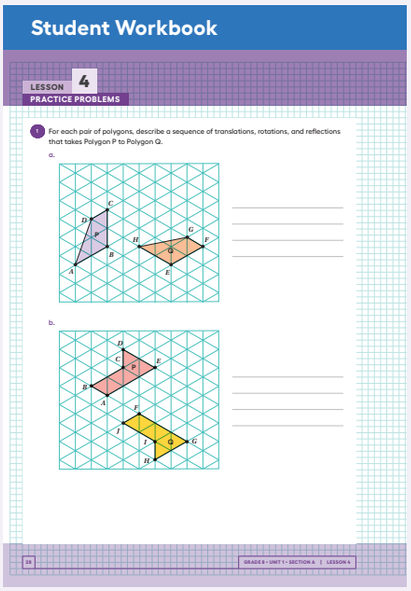
3. Reflect triangle $ABC$ over a line.

   Sample response: A drawing or description of where the line is

Lesson 4

## Practice Problems

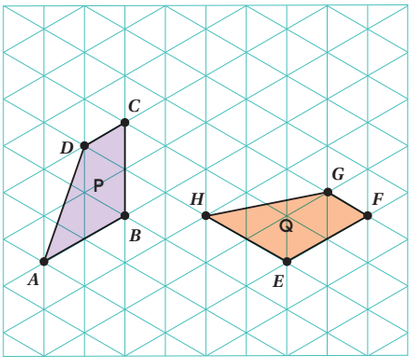
3 Problems




### Problem 1

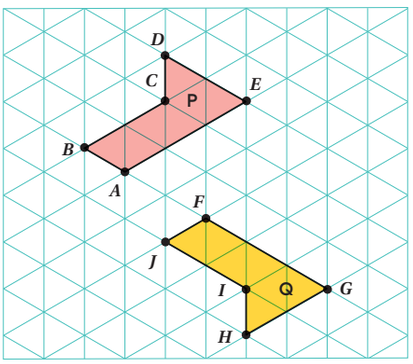
For each pair of polygons, describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.

**a.**



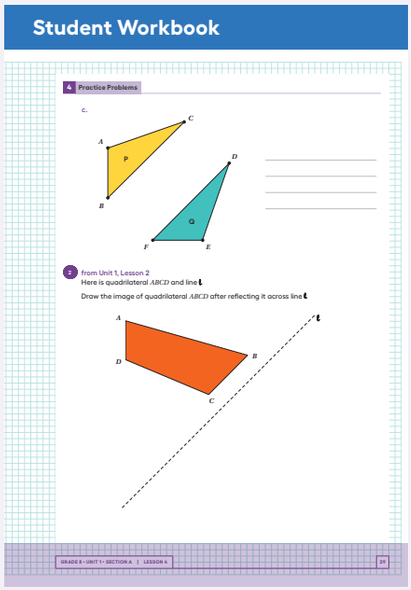

Sample response: *A* is translated to *H*, followed by a rotation 60 degrees clockwise with center *H*.

**b.**

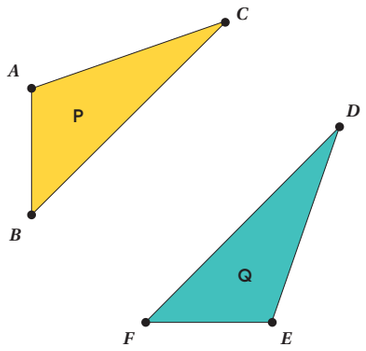



Sample response: Polygon P is reflected over line *AE*. *A* is then translated to *F* and a rotation of 60° clockwise with center *F* is applied.

Student Workbook




**c.**



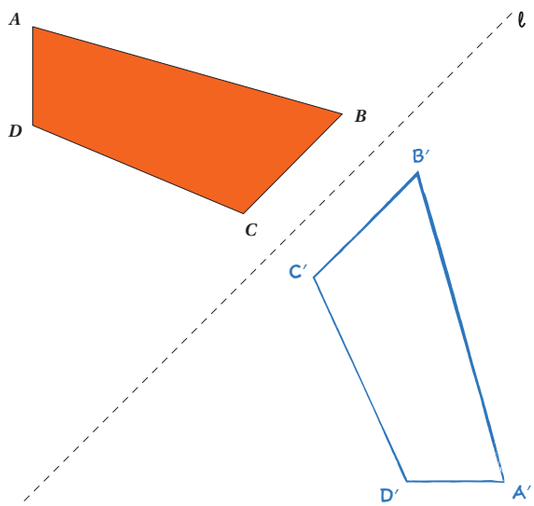

Sample response: *A* is translated to *E*, then apply a rotation with center *E* so that *B* lands on top of *F*. Finally the polygon is reflected over line *EF*.

## Problem 2

from Unit 1, Lesson 2

Here is quadrilateral $ABCD$ and line $\ell$.

Draw the image of quadrilateral $ABCD$ after reflecting it across line $\ell$.



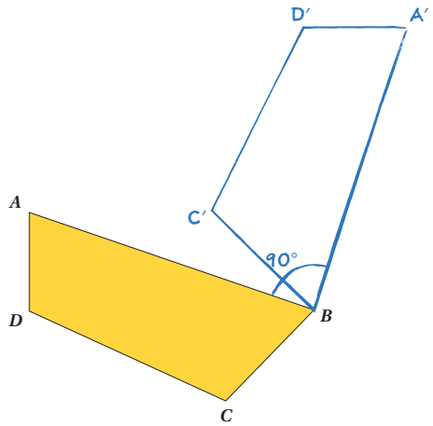
## Problem 3

from Unit 1, Lesson 2

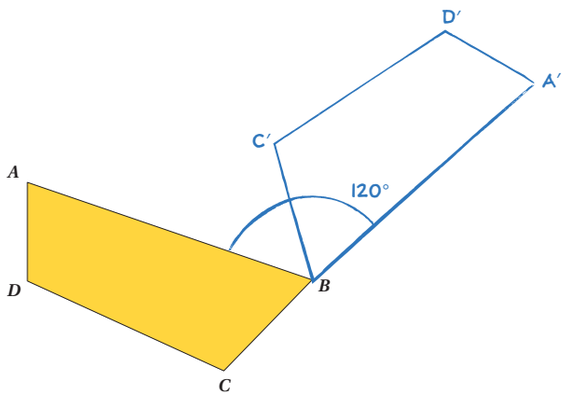
Here is quadrilateral $ABCD$.

Draw the image of quadrilateral $ABCD$ after each rotation using $B$ as center.

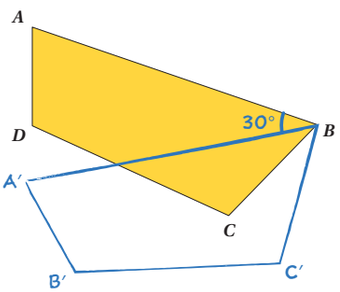
**a.** 90° clockwise



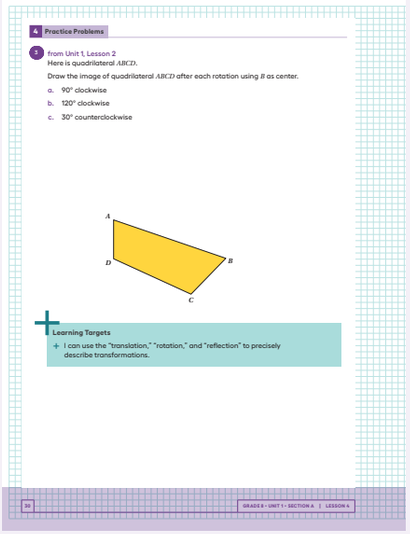

**b.** 120° clockwise




**c.** 30° counterclockwise




### Student Workbook


4. Practice Problems

3. from Unit 1, Lesson 2
Here is quadrilateral $ABCD$.
Draw the image of quadrilateral $ABCD$ after each rotation using $B$ as center.
a. 90° clockwise
b. 120° clockwise
c. 30° counterclockwise

Learning Targets
+ I can use the "translation," "rotation," and "reflection" to precisely describe transformations.