Lesson 13

Decomposing Bases for Area

Goals

- Critique (orally) different methods for decomposing and calculating the area of a prism's base.
- Explain (orally and in writing) how to decompose and calculate the area of a prism's base, and then use it to calculate the prism's volume.

Learning Target

I can calculate the volume of a prism with a complicated base by decomposing the base into quadrilaterals or triangles.

Lesson Narrative

In this lesson, students continue working with the volume of right prisms. They encounter prisms in which the base is composed of triangles and rectangles, and they decompose the base to calculate the area. They also work with shapes such as heart-shaped boxes or house-shaped figures for which they have to identify the base in order to see the shape as a prism and calculate its volume using the structure of a prism.

Student Learning Goal

Let's look at how some people use volume.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	5 min

Inspire Math

3-D Printed Houses video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614217

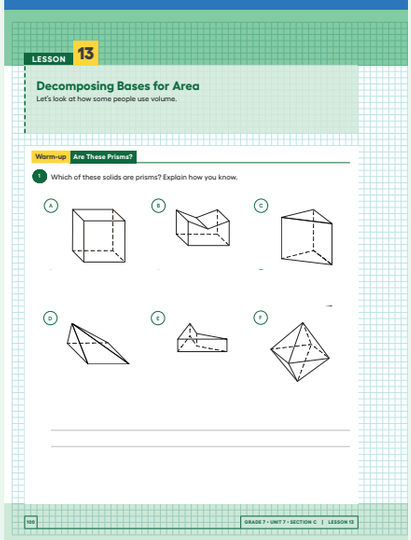
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students struggle to see why figure B is a prism, consider asking:

"How do you know if a solid is a prism?"
"Is the base of a prism always the bottom?"

Student Workbook


LESSON 13
Decomposing Bases for Area
Let's look at how some people use volume.
Warm-up Are These Prisms?
1 Which of these solids are prisms? Explain how you know.
A B C D E F
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Warm-up

Are These Prisms?

10 min

Activity Narrative

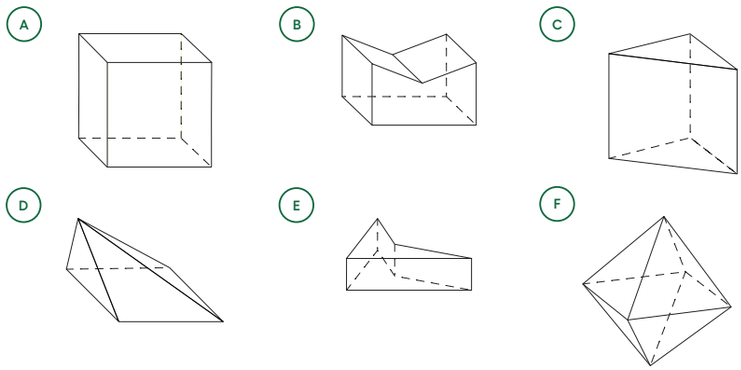
The purpose of this *Warm-up* is for students to recognize prisms and their bases. This concept reinforces what was discussed in the previous lesson where students found the volume of different prisms and non-prisms. Students first determine whether or not a given figure is a prism and then shade and describe the base of the prism. As students work on the task, monitor for students who are using precise language to describe the reason that a figure is a prism.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time, followed by time to discuss their answers with a partner. Follow this with a whole-class discussion.

Student Task Statement

1. Which of these solids are prisms? Explain how you know.



A, B, C, and E are prisms since there is a base shape that is the same on each end with vertices connected by line segments.

2. For each of the prisms, what does the base look like?

The base for A is a square (any of the faces). The base for B is a pentagon (in the front or back). The base for C is a triangle (on the top or bottom). The base for E is a quadrilateral (on the top or bottom).

a. Shade one base in the picture.

Answers vary.

b. Draw a cross section of the prism parallel to the base.

Students' drawings should be the same shape as the base of each prism: A: square, B: pentagon, C: triangle, E: quadrilateral

Activity Synthesis

The goal of this activity is to remind students that a figure is a prism if the cross section, when cut parallel to the base, has the same size and shape as the base of the figure. Select previously identified students to share their reasoning. Invite students to share the bases that they shaded and their drawings of the cross sections.

Activity 1

A Box of Chocolates

15 min

Activity Narrative

In this activity, students practice mentally decomposing, into simpler prisms, a more-complicated prism that has a non-rectangular base. The decomposition corresponds to a decomposition of a complicated two-dimensional figure into simpler two-dimensional figures. This expands on students' ability to calculate the area of a base of a figure and to rely on the structure of a prism to find its volume. This prepares students to calculate the volume of this figure and other figures in future lessons.

As students work in their groups, monitor for the different ways in which students are decomposing or constructing the base of the figure into more familiar shapes.

Launch

Arrange students in groups of 2. Introduce the context of a heart-shaped box of chocolates. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

- Display only the problem stem and related image, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation, before they compare questions with a partner.
- Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own question. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "volume," "composite shape," and "triangles."

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544

Please log in to the site before using the QR code or URL.

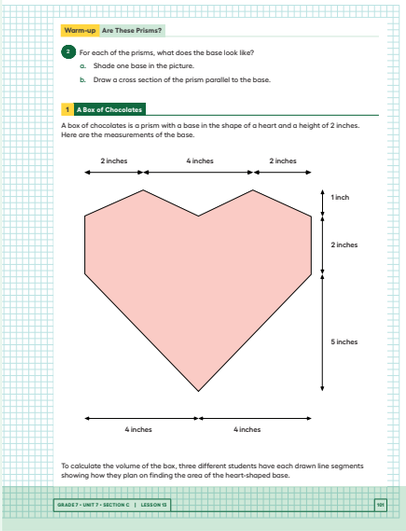
Access for Multilingual Learners (Activity 1)

MLR5: Co-Craft Questions. This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time.
Supports accessibility for: Organization, Attention

Student Workbook


Warm-up Are These Prisms?

2 For each of the prisms, what does the base look like?
a. Shade one base in the picture.
b. Draw a cross section of the prism parallel to the base.

1 A Box of Chocolates

A box of chocolates is a prism with a base in the shape of a heart and a height of 2 inches. Here are the measurements of the base.

2 inches 4 inches 2 inches
1 inch
2 inches
5 inches
4 inches 4 inches

To calculate the volume of the box, three different students have each drawn line segments showing how they plan on finding the area of the heart-shaped base.

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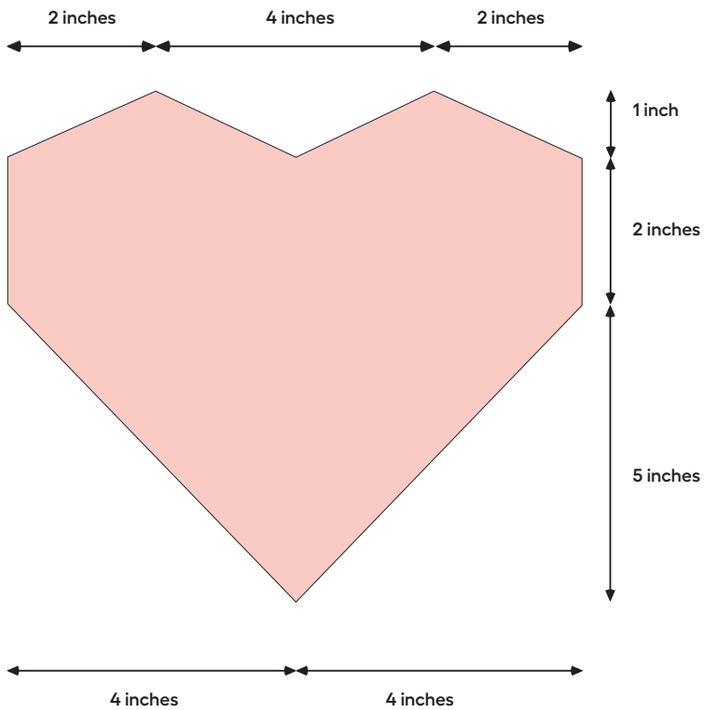
- Reveal the plans and questions. Give students 1–2 minutes to compare the provided question to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

💬 *"Is there a main mathematical concept that is present in both your question and those provided? If so, describe it."*

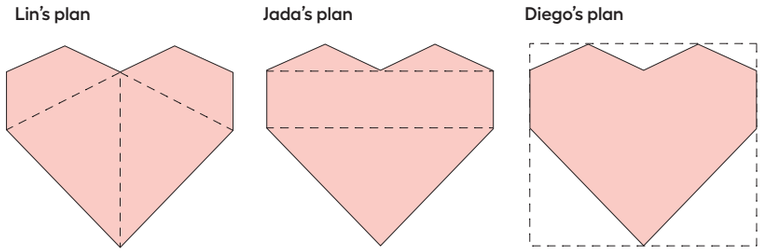
"How does your question relate to calculating the volume of prisms?"

Student Task Statement

A box of chocolates is a prism with a base in the shape of a heart and a height of 2 inches. Here are the measurements of the base.



To calculate the volume of the box, three different students have each drawn line segments showing how they plan on finding the area of the heart-shaped base.



1. For each student's plan, describe the shapes that the student must find the area of and the operations they must use to calculate the total area.

   Lin needs to add the areas of the 2 trapezoids and the 2 triangles. Jada needs to add the areas of the 3 triangles and of the rectangle. Diego needs to subtract the areas of the 5 triangles that are not part of the heart design from the area of the square that bounds it.

2. Although all three methods could work, one of them requires measurements that are not provided. Which one is it?

   Lin's plan requires measurements that are not given, specifically the bases of each figure and the heights of the trapezoids.

3. Between you and your partner, decide which of you will use which of the remaining two methods.

   Either Jada's plan or Diego's plan.

4. Using the quadrilaterals and triangles drawn in your selected plan, find the area of the base.

   The area of the base is 40 square inches.

   - Using Jada's plan, each of the top triangles has an area of 2 square inches because $\frac{1}{2} \cdot 1 \cdot 4 = 2$. The center rectangle has an area of 16 square inches because $2 \cdot (4 + 4) = 16$. The lower triangle has an area of 20 square inches, because $\frac{1}{2} \cdot (4 + 4) \cdot 5 = 20$. The total area is 40 square inches because $2 + 2 + 16 + 20 = 40$.
   - Using Diego's plan, the bounding square has an area of 64 square inches because $(1 + 2 + 5) \cdot (4 + 4) = 64$. The top 2 corner triangles each have an area of 1 square inch, because $\frac{1}{2} \cdot 2 \cdot 1 = 1$. The top center triangle has an area of 2 square inches, because $\frac{1}{2} \cdot 4 \cdot 1 = 2$. Each of the 2 lower corner triangles has an area of 10 square inches, because $\frac{1}{2} \cdot 4 \cdot 5 = 10$. The total area of all of the triangles is 24 square inches, because $1 + 1 + 2 + 10 + 10 = 24$. The total area of the heart-shaped base is 40 square inches, because $64 - 24 = 40$.

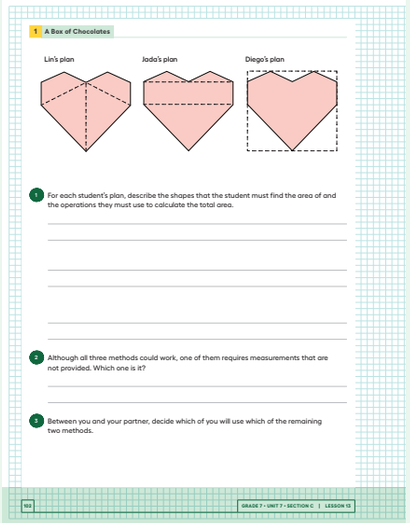
5. Trade with your partner, and check each other's work. If you disagree, work to reach an agreement.

   40 square inches

6. Return your partner's work. Calculate the volume of the box of chocolates.

   80 cubic inches, because $40 \cdot 2 = 80$

Student Workbook


1 A Box of Chocolates

Lin's plan Jada's plan Diego's plan

1 For each student's plan, describe the shapes that the student must find the area of and the operations they must use to calculate the total area.

2 Although all three methods could work, one of them requires measurements that are not provided. Which one is it?

3 Between you and your partner, decide which of you will use which of the remaining two methods.

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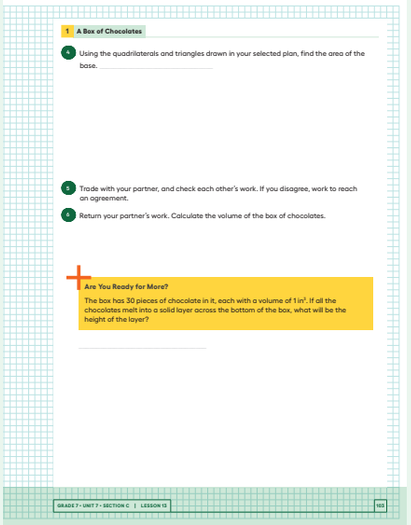
Student Workbook


1 A Box of Chocolates

4 Using the quadrilaterals and triangles drawn in your selected plan, find the area of the base.

5 Trade with your partner, and check each other's work. If you disagree, work to reach an agreement.

6 Return your partner's work. Calculate the volume of the box of chocolates.

Are You Ready for More?

The box has 30 pieces of chocolate in it, each with a volume of 1 in³. If all the chocolates melt into a solid layer across the bottom of the box, what will be the height of the layer?

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Are You Ready for More?

The box has 30 pieces of chocolate in it, each with a volume of 1 in^3. If all the chocolates melt into a solid layer across the bottom of the box, what will be the height of the layer?

$\frac{3}{4}$ in

Activity Synthesis

The purpose of this discussion is to clarify decomposing shapes to find area, which can then be used to calculate the volume. Select students to share whose method they decided to use and why. Ask students:

- *"Whose method could not be used? Explain how you know."*

  Lin's, because we don't know the base and height of the trapezoids.

- *"How did you find the area of the base?"*

  "What was different about the base of this figure in comparison to other bases we have worked with?"

  This base needed to be decomposed to calculate its area.

- *"What was your process to calculate the volume?"*

  I used the area of the base and multiplied it by the height of the figure.

- *"Why would a chocolatier want to know the volume of a heart shaped box like this?"*

  He may want to know how many candies can fit inside of a box.

Explain to students that they might encounter figures that have non-rectangular bases in future activities or lessons. It will be important for them to think about different strategies to calculate the area of the base.

Activity 2

Another Prism

10 min

Activity Narrative

In this activity, students practice finding the volume of another prism with a non-rectangular base by applying the formula Volume = (Area of the base) · (Height of the prism).

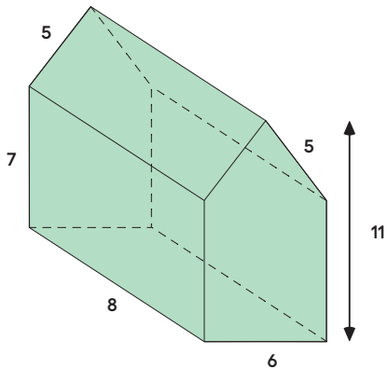
As students work on the task, monitor for students who decompose or compose the base of the figure into more familiar shapes.

Launch

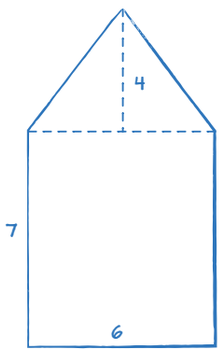
Arrange students in groups of 2. Give students 1–2 minutes of quiet work time followed by time, to discuss their work with a partner. Follow with a whole-class discussion.

Student Task Statement

A house-shaped prism is created by attaching a triangular prism on top of a rectangular prism.



1. Draw the base of this prism and label its dimensions.



2. What is the area of the base? Explain or show your reasoning.

54 square units

This shape can be divided into a rectangle and triangle. The area of the rectangle is $6 \cdot 7 = 42$. The area of the triangle is $\frac{1}{2} \cdot 4 \cdot 6 = 12$. So, the area of the base is $42 + 12 = 54$.

3. What is the volume of the prism?

432 cubic units

Since the volume of a prism is the area of the base times the height of the prism, the volume is $8 \cdot 54 = 432$.

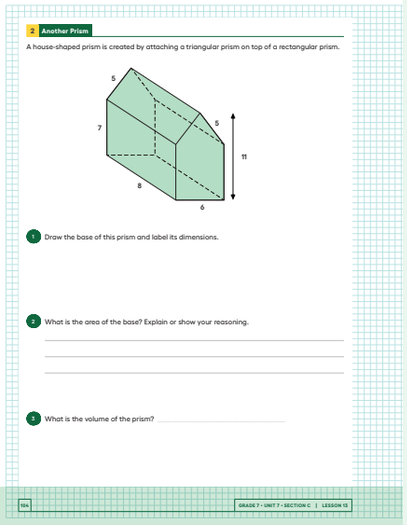
Activity Synthesis

The goal of this activity is to compare different methods of decomposing the base of the prism. Select previously identified students to share the different methods for calculating the area of base. If not brought up by students, explain to students that the base of this figure can either be decomposed into rectangles and triangles or be composed into a larger rectangle by adding two additional triangles. Ask students how they used the area of the base to calculate the volume of the figure (area of the base multiplied by the height).

Building on Student Thinking

If students mistake the rectangle for the base of the figure, ask students how we know that this figure is a prism and what the base of this figure needs to be in order to consider it a prism.

Student Workbook


2 Another Prism

A house-shaped prism is created by attaching a triangular prism on top of a rectangular prism.

1 Draw the base of this prism and label its dimensions.

2 What is the area of the base? Explain or show your reasoning.

3 What is the volume of the prism?

Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

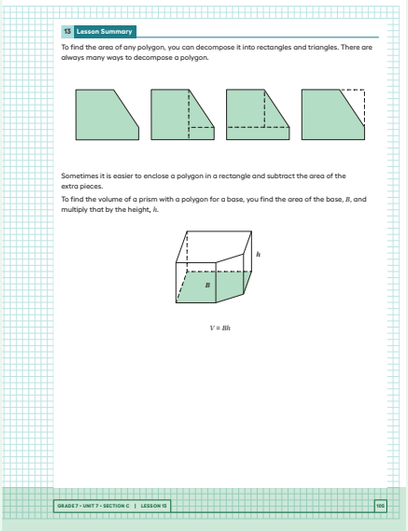
"How are the different methods of finding the area of the base the same? How are they different?"

and

"Are there any benefits or drawbacks to one method compared to another?".

Advances: Representing, Conversing

Student Workbook


13 Lesson Summary

To find the area of any polygon, you can decompose it into rectangles and triangles. There are always many ways to decompose a polygon.

Sometimes it is easier to enclose a polygon in a rectangle and subtract the area of the extra pieces.

To find the volume of a prism with a polygon for a base, you find the area of the base, B, and multiply that by the height, h.

$V = Bh$

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Lesson Synthesis

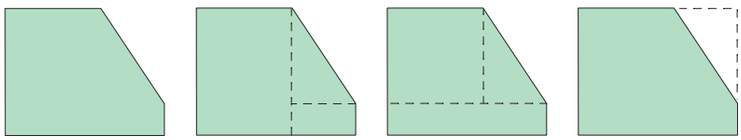
"When the base is not a rectangle or triangle, what are some methods for finding the area?"

We can cut the base apart into rectangles and triangles or imagine a larger shape that has been cut into the base.

Here, we mostly imagined cutting the base apart to find its area, but we could have imagined cutting the original object into smaller objects, and then finding the volume of each piece and adding them together.

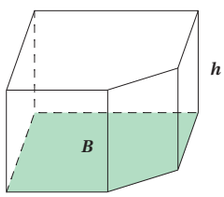
Lesson Summary

To find the area of any polygon, you can decompose it into rectangles and triangles. There are always many ways to decompose a polygon.



Sometimes it is easier to enclose a polygon in a rectangle and subtract the area of the extra pieces.

To find the volume of a prism with a polygon for a base, you find the area of the base, B, and multiply that by the height, h.



$V = Bh$

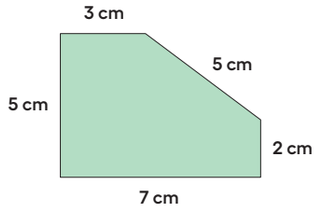
Cool-down

Volume of a Pentagonal Prism

5 min

Launch

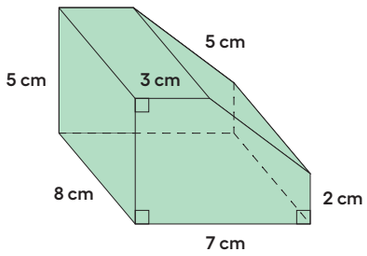
If desired, provide copies of the two-dimensional view of just the base of the prism.



Student Task Statement

Here is a prism with a pentagonal base. The height is 8 cm.

What is the volume of the prism? Show your thinking. Organize it so it can be followed by others.



The volume is 232 cm^3.

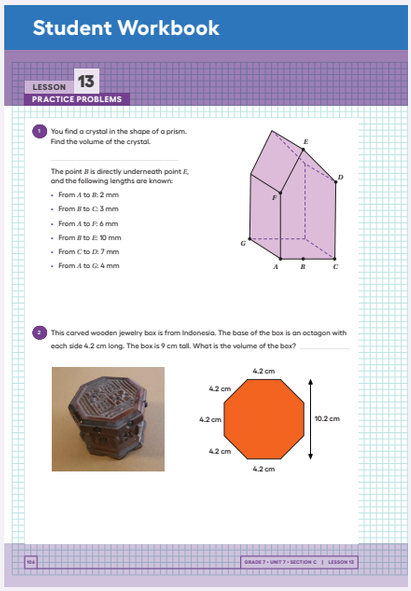
The area of the base is 29 cm^2 and can be found in multiple ways, but one way is to consider a 5 by 7 rectangle with a right triangle cut off, then $5 \cdot 7 - \frac{1}{2} \cdot 4 \cdot 3 = 29$. Since the height is 8 cm, the volume is calculated by $29 \cdot 8 = 232$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

5 Problems


Student Workbook

LESSON 13
PRACTICE PROBLEMS

1. You find a crystal in the shape of a prism. Find the volume of the crystal.

The point B is directly underneath point E, and the following lengths are known:

- From A to B: 2 mm
- From B to C: 3 mm
- From A to F: 6 mm
- From B to E: 10 mm
- From C to D: 7 mm
- From A to G: 4 mm

2. This carved wooden jewelry box is from Indonesia. The base of the box is an octagon with each side 4.2 cm long. The box is 9 cm tall. What is the volume of the box?

4.2 cm, 4.2 cm, 4.2 cm, 4.2 cm, 10.2 cm

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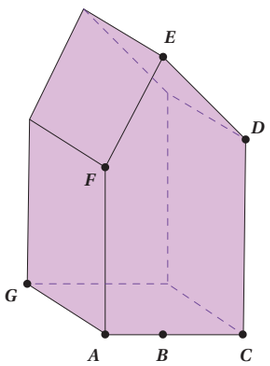
Problem 1

You find a crystal in the shape of a prism. Find the volume of the crystal.

166 cubic millimeters

The point B is directly underneath point E, and the following lengths are known:

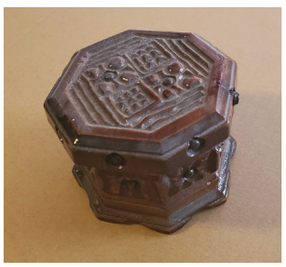
- From A to B: 2 mm
- From B to C: 3 mm
- From A to F: 6 mm
- From B to E: 10 mm
- From C to D: 7 mm
- From A to G: 4 mm

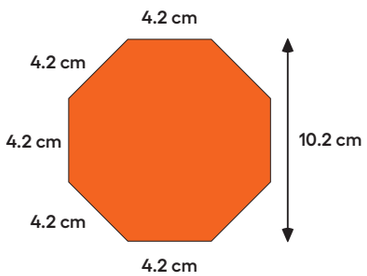


Problem 2

This carved wooden jewelry box is from Indonesia. The base of the box is an octagon with each side 4.2 cm long. The box is 9 cm tall. What is the volume of the box?

about 774 cm³, because the area of the base is about 86 cm², and 86 · 9 = 774.





Problem 3

from Unit 7, Lesson 9

A triangle has one side that is 7 cm long and another side that is 3 cm long.

a. Sketch this triangle and label your sketch with the given measurements. (If you are stuck, try using a compass or cutting some straws to these two lengths.)

Answers vary.

b. Draw one more triangle with these measurements that is not identical to your first triangle.

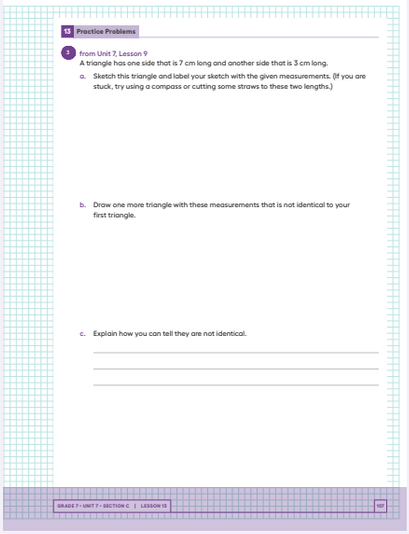
Answers vary.

c. Explain how you can tell they are not identical.

Responses vary.

Sample response: If I cut one of the triangles out and place it on top of the other triangle, the triangles do not match up.

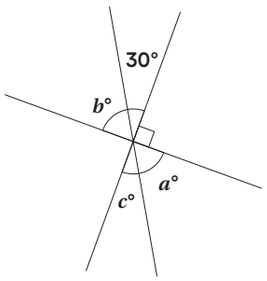
Student Workbook



Problem 4

from Unit 7, Lesson 4

Select **all** equations that represent a relationship between angles in the figure.



A. $90 - 30 = b$

B. $30 + b = a + c$

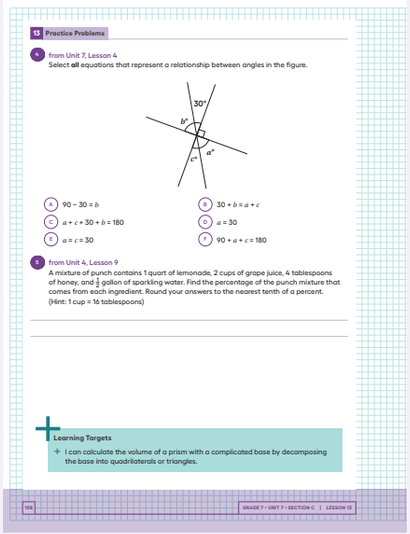
C. $a + c + 30 + b = 180$

D. $a = 30$

E. $a = c = 30$

F. $90 + a + c = 180$

Student Workbook



Problem 5

from Unit 4, Lesson 9

A mixture of punch contains 1 quart of lemonade, 2 cups of grape juice, 4 tablespoons of honey, and $\frac{1}{2}$ gallon of sparkling water. Find the percentage of the punch mixture that comes from each ingredient. Round your answers to the nearest tenth of a percent. (Hint: 1 cup = 16 tablespoons)

- Lemonade: 28.1%
- Grape juice: 14.0%
- Honey: 1.8%
- Sparkling water: 56.1%