The Size of the Scale Factor

Goals

- Describe (orally and in writing) how scale factors of 1, less than 1, and greater than 1 affect the size of scaled copies.
- Explain and show (orally and in writing) how to recreate the original figure given a scaled copy and its scale factor.
- Recognize (orally and in writing) the relationship between a scale factor of a scaled copy to its original figure is the "reciprocal" of the scale factor of the original figure to its scaled copy.

Learning Targets

- I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1.
- I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure A.

Lesson Narrative

In this lesson, students deepen their understanding of scale factors in two ways:

- 1. They classify scale factors by size (less than 1, exactly 1, and greater than 1) and notice how each class of factors affects the scaled copies.
- 2. They see that the scale factor that takes an original figure to its copy and the one that takes the copy to the original are **reciprocals**. This means that the scaling process is reversible, and that if Figure B is a scaled copy of Figure A, then Figure A is also a scaled copy of Figure B.

Students also continue to apply scale factors and what they learned about corresponding distances and angles to draw scaled copies without a grid.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

 MLR8: Discussion Supports (Warm-up, Activity 1, Activity 2, Activity 3)

Instructional Routines

- Card Sort
- Math Talk
- MLR8: Discussion Supports

Required Materials

Materials to Gather

• Geometry toolkits: Activity 2, Activity 3

Materials to Copy

- Scaled Copies Cards (1 copy for every 3 students): Activity 1
- Scaling A Puzzle Cutouts (1 copy for every 3 students): Activity 2

Required Preparation

Activity 1:

For the blackline master, if possible, copy each complete set on a different color of paper so that a stray slip can quickly be put back.

Activity 2:

Print and cut puzzle pieces and blank squares from the Scaling a Puzzle blackline master. Make enough copies so that each group of 3 students has 1 original puzzle and 6 blank squares.

Lesson Timeline

10 min

Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Activity 3

10 min

Lesson Synthesis

Assessment

5_{min}

Cool-down

The Size of the Scale Factor

Lesson Narrative (continued)

Two of the activities are optional. In Scaling a Puzzle, students scale the 6 pieces of a puzzle individually and then assemble them to make a scaled copy of the puzzle. Students need to think strategically about which measurements to take in order to scale the pieces accurately. In Missing Figure, Factor, or Copy, students gain fluency in working with scale factor. Teachers can choose the activity that best fits the needs of their students.

Student Learning Goal

Let's look at the effects of different scale factors.

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Students might think that a product cannot be less than one of the factors, not realizing that one of the factors can be a fraction. Use examples involving smaller and familiar numbers to remind them that it is possible. Ask, for example,

"What number times 10 equals 5?"

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Warm-up

Math Talk: Missing Factor



Activity Narrative

This *Math Talk* focuses on solving equations of the form px = q. It encourages students to think about the relationship between multiplication and division and to rely on properties of operations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students examine the effects of reciprocal scale factors.

To solve these equations mentally, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A.8x = 4

 $\frac{1}{2}$ (or equivalent.) 8 ÷ 2 = 4 and dividing by 2 is the same as multiplying by $\frac{1}{2}$.

B. 8x = 1

 $\frac{1}{8}$ (or equivalent.) It takes 8 of $\frac{1}{8}$ to make I.

 $C_{\frac{1}{5}}x = 1$

5. There are 5 copies of $\frac{1}{5}$ in 1, so $5 \cdot \frac{1}{5} = 1$.

 $\mathbf{D}_{x} = x = 1$

 $\frac{5}{2}$ (or equivalent.) Five groups of $\frac{2}{5}$ make 2, so $\frac{5}{2}$ groups of $\frac{2}{5}$ make 1.

Activity Synthesis

To involve more students in the conversation, consider asking:

○ "Who can restate ______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaways are:

- Dividing 1 by a number gives the **reciprocal** of that number.
- Multiplying a number by its reciprocal equals 1.

Activity 1

Card Sort: Scaled Copies

15 min

Activity Narrative

In this activity students examine how the size of the scale factor is related to the original figure and the scaled copy. The activity serves several purposes:

- 1. To reinforce students' awareness of scale factors.
- **2.** To draw attention to how scaled copies behave when the scale factor is greater than 1, exactly 1, or less than 1.
- **3.** To help students notice that reciprocal scale factors reverse the scaling.

As students notice that groups of scale factors have similar effects on the scaled copies, they are looking for regularity in repeated reasoning.

This is the first *Card Sort* activity of the course. An important aspect of this routine is to allow students time at the start to sort the cards into categories of their choosing. This step gives students the opportunity to familiarize themselves with the content of the cards without the additional pressure of organizing them in a specific fashion. It also provides insight into the aspects of each card that students attend to and the language they have to describe their observations.

Monitor for different ways students choose to categorize the cards, especially for students who group the cards in terms of:

- Specific scale factors (such as 2, 3, $\frac{1}{2}$, and so on).
- Ranges of scale factors producing certain effects (for example, factors producing larger, unchanged, or smaller copies).
- Reciprocal scale factors (that is, one factor scales Figure A to B, and its reciprocal reverses the scaling).

Launch 228

Arrange students in groups of 2–3 and distribute the pre-cut cards. Tell them that in this activity, they will sort some cards into categories of their choosing. Allow students to familiarize themselves with the figures on the cards:

- Give students 3 minutes to sort the cards into categories of their choosing.
- Pause the class after students have sorted the cards.
- Select some groups that sorted their cards based on aspects other than the scale factor to share their categories and how they sorted their cards.

After a brief discussion, explain that on each card, Figure A is the original and Figure B is a scaled copy. Invite students to find another way to sort the cards, this time based on the scale factors. Give students 4–5 minutes of group work time, followed by whole-class discussion.

Instructional Routines

Card Sort

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Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.

Leverage choice around perceived challenge. Invite students to first sort the cards into groups based on difficulty of identifying the scale factor. Then direct them to sort one of those groups based on their scale factors.

Supports accessibility for: Organization, Social-Emotional Functioning

Building on Student Thinking

Students may sort by the types of figures rather than by how the second figure in each pair is scaled from the first. Remind students to sort based on how Figure A is scaled to create Figure B.

Students may think of the change in lengths between Figures A and B in terms of addition or subtraction, rather than multiplication or division. Remind students of an earlier lesson in which they explored the effect of subtracting the same length from each side of a polygon in order to scale it. What happened to the copy? (It did not end up being a polygon and was not a scaled copy of the original one.)

Students may be unclear as to how to describe how much larger or smaller a figure is, or may not recall the meaning of scale factor. Have them compare the lengths of each side of the figure. What is the common factor by which each side is multiplied?

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support small-group discussion: "I noticed ______, so I grouped ..." Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing

Student Task Statement

Your teacher will give you a set of cards. On each card, Figure A is the original and Figure B is a scaled copy.

1. Sort the cards based on their scale factors. Be prepared to explain your reasoning.

Grouping categories vary. Sample responses:

- · Scale factor of 2: Cards I and 9
- Scale factor of \(\frac{1}{2} \): Cards 3, 7, and II.
- Scale factor of 3: Cards 2, 5, and 10.
- Scale factor of \(\frac{1}{3}\): Cards 4, 6, and 13.
- · Scale factor of I: Cards 8 and I2.
- **2.** Examine cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? What do you notice about the scale factors?

The shapes on both cards are the same, but on Card IO, the scaled copy is larger and the scale factor is 3. On Card I3, the scaled copy is smaller and the scale factor is $\frac{1}{2}$.

3. Examine cards 8 and 12 more closely. What do you notice about the figures? What do you notice about the scale factors?

The original and the copy are the same size on Cards 8 and 12. The copy is identical to the original. The scale factor is I.

Are You Ready for More?

Triangle B is a scaled copy of Triangle A with scale factor $\frac{1}{2}$.

- 1. How many times bigger are the side lengths of Triangle B when compared with Triangle A? $\frac{l}{2}$
- **2.** Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to get Triangle C. How many times bigger will the side lengths of Triangle C be when compared with Triangle A? $\frac{1}{4}$
- **3.** Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale triangle A n times to get Triangle N, always using a scale factor of $\frac{1}{2}$. How many times bigger will the side lengths of Triangle N be when compared with Triangle A? $\left(\frac{1}{2}\right)^n$

Activity Synthesis

Select groups to share their categories and how they sorted the scaled copies. Discuss as many different types of categories as time allows, but ensure that at least one set of categories discussed distinguishes between ranges of scale factors (greater than 1, exactly 1, and less than 1).

If no groups sorted in terms of ranges of scale factors, consider asking:

"What can we say about the scale factors that produced larger copies? Smaller copies? Copies that are the same size as the original?"

If no groups sorted in terms of reciprocal scale factors, consider asking:

"Some cards had the same pair of figures on them, just in a reversed order (cards 1 and 7, cards 10 and 13). What do you notice about their scale factors?"

Highlight the two main ideas of the lesson:

- The effects of scale factors that are greater than 1, exactly 1, and less than 1.
- The reversibility of scaling—that is, if Figure B is a scaled copy of Figure A, then A is also a scaled copy of B. In other words, A and B are scaled copies of one another, and their scale factors are reciprocals.

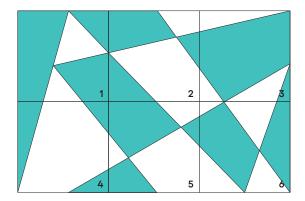
Activity 2: Optional

Scaling A Puzzle

15 min

Activity Narrative

This activity gives students a chance to apply what they know about scale factors, lengths, and angles and to create scaled copies without the support of a grid. Students work in groups of 6 to complete a jigsaw puzzle, with each group member scaling one piece of the puzzle with a scale factor of $\frac{1}{2}$. The group then assembles the scaled pieces and examines the accuracy of their scaled puzzle.



As students work, notice how they measure distances and whether they consider angles. Depending on how students determine scaled distances, they may not need to transfer angles. Look out for students who measure only the lengths of drawn segments rather than distances, e.g., between the corner of a square and where a segment begins. Suggest that they consider other measurements that might help them locate the beginning and end of a segment.

Launch 🙎 × 6

Arrange students in groups of 6. Give one pre-cut puzzle square to each student so that the students in each group have all the pieces 1–6. After students have answered the first question, give each student one blank square cut from the second section of the blackline master. (The sides of these squares are half as long as the sides of the puzzle squares.) Provide access to geometry toolkits.

Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Before students begin, ask them to predict which tools, from their geometry toolkits, they anticipate they will need, and to describe how they might use them. Demonstrate how to use their toolkits for students who are unfamiliar with using rulers or protractors correctly.

Supports accessibility for: Conceptual Processing, Language

Building on Student Thinking

Students may incorporate the scale factor when scaling line segments but neglect to do so when scaling distances between two points not connected by a segment. Remind them that all distances are scaled by the same factor.

Students may not remember to verify that the angles in their copies must remain the same as the original. Ask them to notice the angles and recall what happens to angles when a figure is a scaled copy.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support small-group discussion: "Why did you ... ?" and "First, I _____ because ...".

Advances: Speaking, Conversing, Representing

Student Task Statement

Your teacher will give you one of the six pieces of a puzzle.

1. If you drew scaled copies of your puzzle pieces using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original pieces? How do you know?

They would be smaller because $\frac{1}{2} < 1$

2. Create a scaled copy of each puzzle piece on a blank square with a scale factor of $\frac{1}{2}$.

Copies of puzzle pieces scaled by $\frac{1}{2}$

3. When everyone in your group is finished, put all 6 of the original puzzle pieces together like this:

1	2	3
4	5	6

Next, put all 6 of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem off? What might have caused those parts to be off?

Answers vary depending on how each student drew their pieces.

4. Revise any of the scaled copies that may have been drawn incorrectly. Revised copies of puzzle pieces.

5. If you were to lose one of the pieces of the original puzzle, but still had the scaled copy, how could you recreate the lost piece?

Use the copy and a scale factor of 2 to recreate the original puzzle piece. Sample explanation: 2 is the reciprocal of $\frac{1}{2}$, so it would scale the copy back to the original.

Activity Synthesis

Much of the conversations about creating accurate scaled copies will have taken place among partners, but consider coming together as a class to reflect on the different ways students worked. Ask questions such as:

"How is this task more challenging than creating scaled copies of polygons on a grid?"

"Besides distances or lengths, what helped you create an accurate copy?"

"How did you know or decide which distances to measure?"

"Before your drawings were assembled, how did you check if they were correct?"

Student responses to these questions may differ. For example, for Piece 6, the two lines can be drawn by measuring distances on the border of the puzzle piece; the angles work out correctly automatically. For Piece 1, however, to get the three lines that meet in a point in the middle of the piece just right, students can either measure angles, or extend those line segments until they meet the border of the piece and then measure distances.

Activity 3: Optional

Missing Figure, Factor, or Copy



Activity Narrative

In this activity, students practice identifying scale factors and drawing scaled copies. Each problem gives two of the three pieces of information (original figure, scaled copy, scale factor) and asks students to figure out the third.

Warm-up

Students work with these different scenarios on a grid, dealing with scale factors greater than 1, less than 1, and equal to 1. In addition, they create a scaled copy of a non-polygonal figure off of a grid. Especially in the problem that is off of a grid, students need to decide what tools to use to measure angles and side lengths in order to produce the scaled copy.

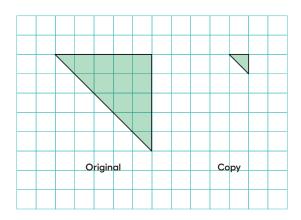
Launch 🙎

Arrange students in groups of 2. Provide access to geometry toolkits. Give students 3–4 minutes of quiet work time, followed by 2 minutes of group discussion and then whole-class discussion.

Student Task Statement

1. What is the scale factor from the original triangle to its copy?

<u>|</u>



Explain or show your reasoning.

The vertical and horizontal sides on the original are 5 grid units in length and the corresponding sides on the copy are I grid unit in length.

Instructional Routines

MLR8: Discussion Supports

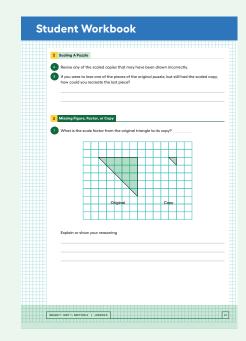
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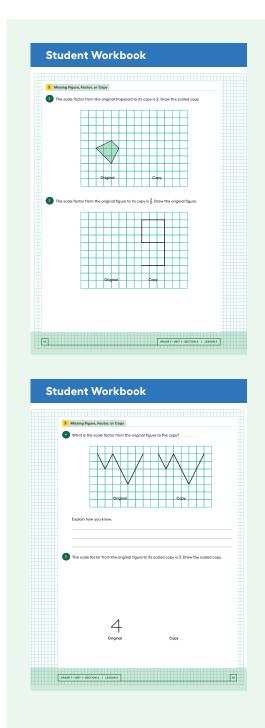
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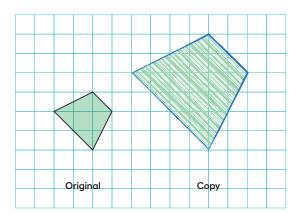
Building on Student Thinking

For the last problem, some students may have sketched the copy without checking angles. Ask them why measuring angles is important.

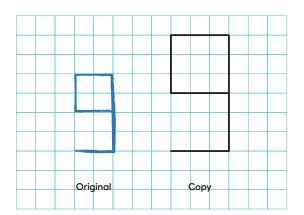




2. The scale factor from the original trapezoid to its copy is 2. Draw the scaled copy.

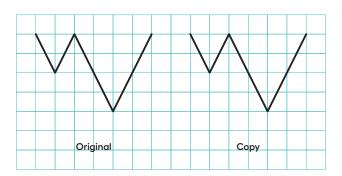


3. The scale factor from the original figure to its copy is $\frac{3}{2}$. Draw the original figure.



4. What is the scale factor from the original figure to the copy?

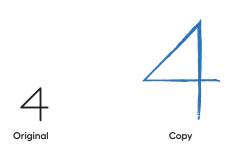
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Explain how you know.

The figure and the copy are the same size.

5. The scale factor from the original figure to its scaled copy is 3. Draw the scaled copy.



Activity Synthesis

The purpose of this discussion is to ensure that students understand the connections between the original figure, scaled copy, and scale factor. Given any two of the three, they should understand a method for finding the third.

Invite students to share how they solved each question. Consider asking:

"Did anyone solve the problem the same way, but would explain it differently?"

"Which problem was the most challenging? Why?"

The question with the missing original figure deserves special attention. This requires going backward because we have the scaled copy and the scale factor. This scenario also reinforces that the way to "undo" the scale factor of $\frac{3}{2}$ that has been applied to produce the copy is to apply a scale factor of $\frac{2}{3}$.

Lesson Synthesis

Share with students

O "Today we examined the effects of different scale factors more closely."

To help students generalize about the effects of different scale factors, consider asking students:

"What happens to the copy when it is created with a scale factor greater than 1? Less than 1? Exactly 1?"

When the scale factor is greater than I, the scaled copy is larger than the original. When it is less than I, the copy is smaller than the original. When it is exactly I, the copy is the same size as the original.

"How can we reverse the scaling to get back to the original figure when we have a scaled copy? Give an example."

Scaling can be reversed by using reciprocal factors. For example, if the scale factor from Figure A to Figure B is 4, then we can scale B back to A using a factor of $\frac{1}{4}$.

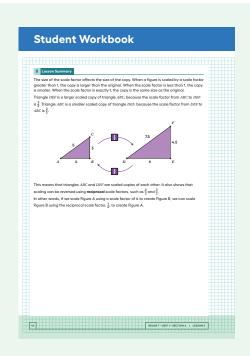
If Figure B is a scaled copy of Figure A, then A is also a scaled copy of B. They are scaled copies of each other.

Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports.

During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say ..." Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking



Responding To Student Thinking

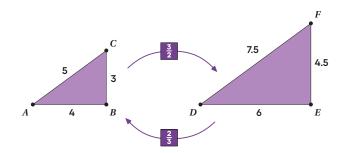
More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

The size of the scale factor affects the size of the copy. When a figure is scaled by a scale factor greater than 1, the copy is larger than the original. When the scale factor is less than 1, the copy is smaller. When the scale factor is exactly 1, the copy is the same size as the original.

Triangle DEF is a larger scaled copy of triangle ABC, because the scale factor from ABC to DEF is $\frac{3}{2}$. Triangle ABC is a smaller scaled copy of triangle DEF, because the scale factor from DEF to ABC is $\frac{2}{3}$.



This means that triangles ABC and DEF are scaled copies of each other. It also shows that scaling can be reversed using **reciprocal** scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

In other words, if we scale Figure A using a scale factor of 4 to create Figure B, we can scale Figure B using the reciprocal scale factor, $\frac{1}{4}$, to create Figure A.

Cool-down

Scaling a Rectangle

5 mir

Student Task Statement

A rectangle that is 2 inches by 3 inches will be scaled by a factor of 7.

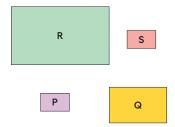
- **1.** What will the side lengths of the scaled copy be?
 - 14 inches by 21 inches, because $2 \cdot 7 = 14$ and $3 \cdot 7 = 21$
- **2.** Suppose you want to scale the copy back to its original size. What scale factor should you use?
 - $\frac{1}{7}$, because it is the reciprocal of 7

Practice Problems

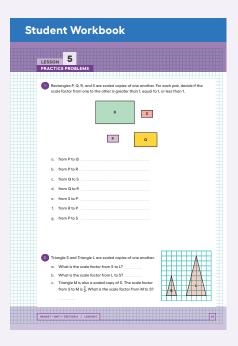
5 Problems

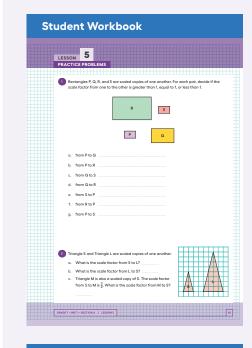
Problem 1

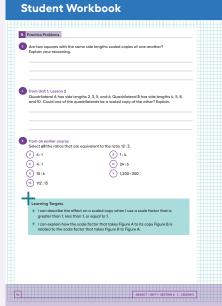
Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor from one to the other is greater than 1, equal to 1, or less than 1



- a. from P to Q
 - Greater than I
- **b.** from P to R
 - Greater than I
- c. from Q to S
 - Less than I
- d. from Q to R
 - Greater than I
- e. from S to P
 - Equal to 1
- f. from R to P
 - Less than I
- g. from P to S
 - Equal to 1

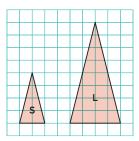






Problem 2

Triangle S and Triangle L are scaled copies of one another.



- a. What is the scale factor from S to L? 2
- **b.** What is the scale factor from L to S? $\frac{1}{2}$
- **c.** Triangle M is also a scaled copy of S. The scale factor from S to M is $\frac{3}{2}$. What is the scale factor from M to S?

2

The two scale factors are reciprocals of each other.

Problem 3

Are two squares with the same side lengths scaled copies of one another? Explain your reasoning.

Yes

There is a scale factor of I between them.

Problem 4

from Unit 1, Lesson 2

Quadrilateral A has side lengths 2, 3, 5, and 6. Quadrilateral B has side lengths 4, 5, 8, and 10. Could one of the quadrilaterals be a scaled copy of the other? Explain.

No

For the shortest sides to match up, the scale factor from A to B would have to be 2. But scaling the side of A with length 3 by a factor of 2 would give a side of length 6, which doesn't match any of the side lengths of B.

Problem 5

from an earlier course

Select all the ratios that are equivalent to the ratio 12:3.

- A. 6:1
- **B.** 1:4
- C. 4:1
- **D.** 24:6
- **E.** 15:6
- **F.** 1,200:300
- **G.** 112:13