Lesson 9

Slopes Don't Have to Be Positive

Goals

- Create a graph of a line representing a linear relationship with a non-positive rate of change.
- Interpret the slope of a non-increasing line in context.

Learning Targets

- I can create a graph of a situation that has a negative slope.
- I can determine if a situation or a graph has a slope that is positive, negative, or zero and explain how I know.

Lesson Narrative

In this lesson, students are introduced to lines with non-positive slopes. Students begin by reflecting on similarities and differences between lines that slope in different directions. Then they are introduced to the context of a fare card with decreasing funds. They graph this relationship and see that the line appears to be going "downhill" from left to right and reason that it makes sense for the slope to be negative in terms of the context.

Students are then introduced to a related context of a fare card but with a constant balance. They interpret a graph of this situation and reason that it makes sense that a flat graph has a slope of zero.

Student Learning Goal

Let's find out what a negative slope means.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

Instructional Routines

- Notice and Wonder
- Which Three Go Together?

Required Materials

Materials to Gather

- Straightedges: Activity 1

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	15 min	10 min	10 min	5 min

Assessment

Warm-up

Which Three Go Together: Intersecting Lines

5 min

Activity Narrative

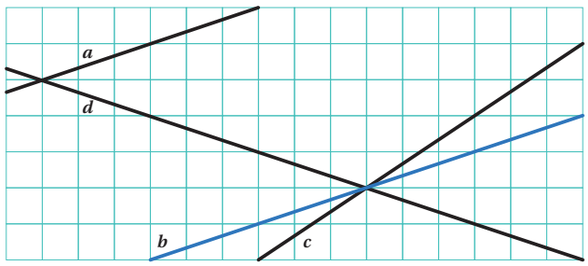
This *Warm-up* prompts students to compare four lines. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the lines in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the graphs for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three lines that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

Lines *a*, *b*, and *c* go together because:

- they are all going up from left to right.

Lines *a*, *b*, and *d* go together because:

- slope triangles for these lines are all similar.

Lines *a*, *c*, and *d* go together because:

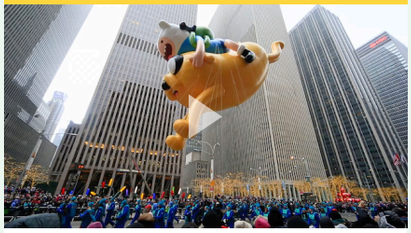
- none of these lines are parallel.
- they are all the same color.

Lines *b*, *c*, and *d* go together because:

- they all go through the same point.
- none of these lines are parallel.

Inspire Math

Helium video



Go Online

Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/r/614206

Please log in to the site before using the QR code or URL.

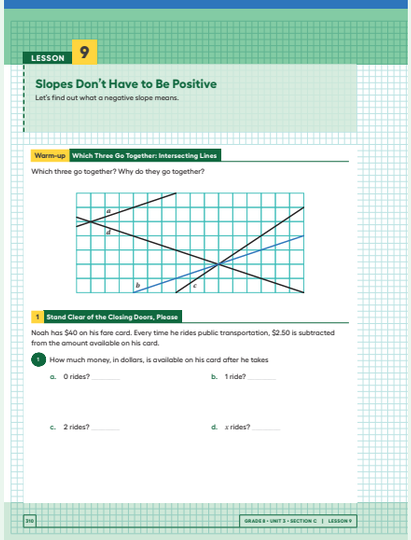
Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 9

Slopes Don't Have to Be Positive

Let's find out what a negative slope means.

Warm-up Which Three Go Together: Intersecting Lines

Which three go together? Why do they go together?

1 Stand Clear of the Closing Doors, Please

Noah has $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes
   a. 0 rides?
   b. 1 ride?
   c. 2 rides?
   d. x rides?

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "parallel," "intersect," and "slope triangle," and to clarify their reasoning as needed. Consider asking:

- *"How do you know ...?"*
- *"What do you mean by ...?"*
- *"Can you say that in another way?"*

Activity 1

Stand Clear of the Closing Doors, Please

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students see negative slopes for the first time as they answer questions about a public transportation fare card. After computing the amount left on the card after 0, 1, and 2 rides, they express regularity in repeated reasoning to represent the amount remaining on the card after x rides. Students reason about why a negative value for the slope of this line makes sense.

While the language is not introduced in the *Task Statement*, the value of x for which the money on the card is 0 is called the x-intercept or horizontal intercept. Unlike the y-intercept, which can be seen in the equation $y = 40 - 2.5x$, the x-intercept has to be calculated. It is the value of x for which $0 = 40 - 2.5x$.

In the digital version of the activity, students use an applet to graph the relationship between the number of rides Noah has taken on public transportation and the amount of money left on his fare card. The applet allows students to graph this relationship quickly and precisely. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Launch

If your students are unlikely to be familiar with public transportation, provide some information about how a fare card works. If possible, prepare some photos related to purchasing and using a fare card. (Some example images are provided.)

Explain to students that someone who wants to ride a bus or train often uses a card like this. The rider adds money to the card and the cost of each ride they take is subtracted from the balance on the card. Eventually, the money on the card runs out, and more must be added before taking additional rides.

Arrange students in groups of 2. Provide access to straightedges.
Give students 4–5 minutes of quiet work time, followed by a partner then whole-class discussion.

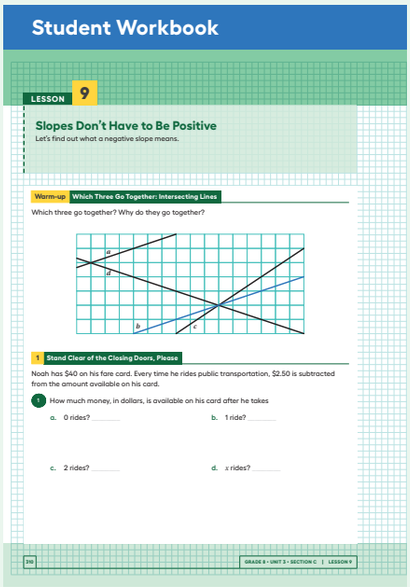
Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.

Provide access to tools and assistive technologies such as a device that can run the digital applet.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Student Workbook


LESSON 9

Slopes Don't Have to Be Positive

Let's find out what a negative slope means.

Warm-up Which Three Go Together: Intersecting Lines

Which three go together? Why do they go together?

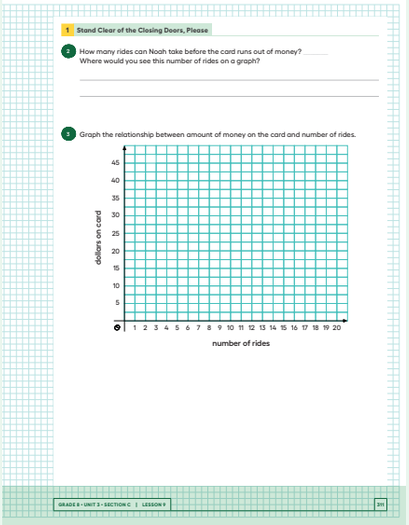
1 Stand Clear of the Closing Doors, Please

Noah has $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes
   a. 0 rides? _____ b. 1 ride? _____
   c. 2 rides? _____ d. x rides? _____

GRADE 8 • UNIT 3 • SECTION C | LESSON 9

Student Workbook


1 Stand Clear of the Closing Doors, Please

2. How many rides can Noah take before the card runs out of money? _____
   Where would you see this number of rides on a graph?

3. Graph the relationship between amount of money on the card and number of rides.

dollars on card

number of rides

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Student Task Statement

Noah has $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes

a. 0 rides?

40 dollars

b. 1 ride?

37.50 dollars

c. 2 rides?

35 dollars

d. x rides?

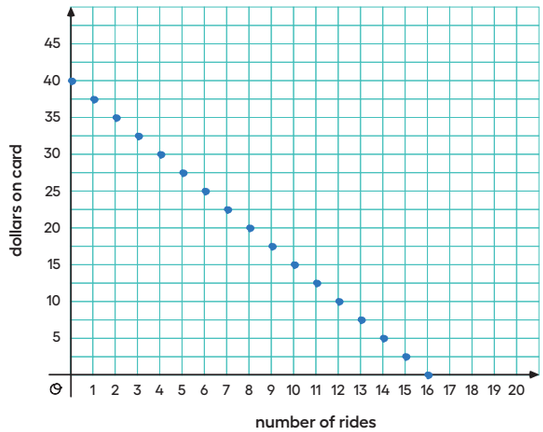
40 – 2.5x (or equivalent) dollars

2. How many rides can Noah take before the card runs out of money? Where would you see this number of rides on a graph?

16 rides.

This is the point (16,0) on the graph, or when the line touches the x-axis.

3. Graph the relationship between amount of money on the card and number of rides.



Sample response: A graph that shows a line going through these points is also acceptable.

Activity Synthesis

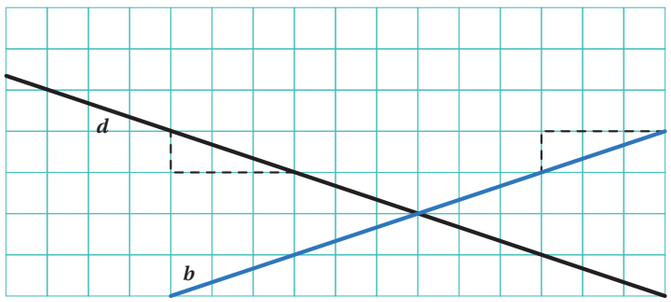
The purpose of this discussion is for students to understand what a negative slope means in context. Begin by asking students,

- *"Why does it make sense to say the slope of this graph is -2.5 rather than 2.5?"*

Ensure that students can articulate some version of "For every ride, the amount on the card decreases by $2.50, and this is why it makes sense that the slope is -2.5."

Invite students to share their answers to the question about when Noah will run out of money on his card. (After 16 rides, the card has no money left, which can be seen at the point (16, 0).) Explain that just like the point (0, 40) on the graph is called the vertical intercept, the point (16, 0) is called the horizontal intercept. In this situation, the vertical intercept tells how much money Noah started with on the card, and the horizontal intercept tells how many rides Noah can take before the card has no more money on it.

Display this image of lines b and d from the *Warm-up* and draw in two slope triangles as shown.



Ask students what they notice about the vertical and horizontal lengths of the two slope triangles. Explain that even though they both have a vertical length of 1 and horizontal length of 3, they don't have the same slope. Ask students which line has a slope of $\frac{1}{3}$ and which line has a slope of $\frac{-1}{3}$. Validate students' use of informal language to describe the differences between the two lines. For example, students may say that if you put a pencil on a line and move it along the line from left to right, line b goes "uphill" but line d goes "downhill."

Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "Why does it make sense to say the slope of this graph is -2.5 rather than 2.5?" Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.

Provide students with a graphic, such as a two-column table, to record what they notice and wonder prior to being expected to share these ideas with others.

Supports accessibility for: Visual-Spatial Processing, Organization

Building on Student Thinking

If students have trouble writing an equation to represent the situation, consider asking:

"What do you notice about the 3 points on the line that you plotted and labeled?"

"How can you figure out the y-value if you know the x-value?"

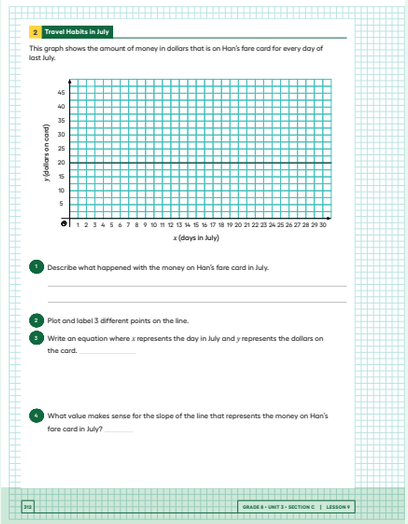
Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Student Workbook


2 Travel Habits in July

This graph shows the amount of money in dollars that is on Han's fare card for every day of last July.

y (dollars on card)

x (days in July)

1. Describe what happened with the money on Han's fare card in July.

2. Plot and label 3 different points on the line.

3. Write an equation where x represents the day in July and y represents the dollars on the card.

4. What value makes sense for the slope of the line that represents the money on Han's fare card in July?

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Activity 2

Travel Habits in July

10 min

Activity Narrative

This activity introduces the idea of a slope of zero by considering a fare card where the amount on the card does not go up or down at all.

Launch

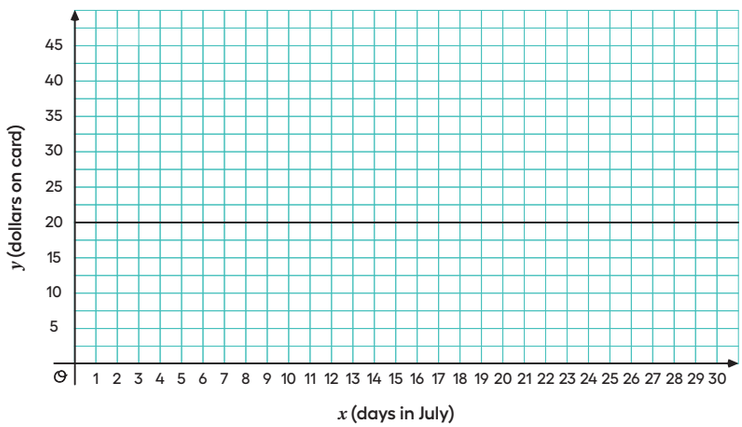
Tell students to close their books or devices (or to keep them closed). Display the graph for all to see. Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the graph.

If the fact that the x-axis represents different days in July rather than the number of rides does not come up during the conversation, ask students to discuss this idea.

Tell students to open their books or devices and give them 3–4 minutes to complete the task, followed by a partner then whole-class discussion.

Student Task Statement

This graph shows the amount of money in dollars that is on Han's fare card for every day of last July.



1. Describe what happened with the money on Han's fare card in July.

   Sample response: The amount of money on Han's fare card did not change at all in July.

2. Plot and label 3 different points on the line.

   Answers vary. See graph.

3. Write an equation where x represents the day in July and y represents the dollars on the card.

   $y = 20$ (or equivalent)

4. What value makes sense for the slope of the line that represents the money on Han's fare card in July?

   Sample response: A slope of 0 makes sense for this line because the money on the card is not increasing or decreasing.

Are You Ready for More?

A loan was taken out and is being paid back in multiple payments. Which of the following situations would have a graph with a positive slope and which would have a negative slope? Explain your reasoning.

1. Amount paid on the vertical axis and time since payments started on the horizontal axis.

   Positive slope

   Sample reasoning: Initially the amount paid is 0 and as time passes, the amount paid increases.

2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

   Positive slope

   Sample reasoning: If the time remaining until the loan is paid off is 0, then the amount owed is also zero. This is represented with a point on the origin of the graph. If time remaining is greater than 0, then something is owed. This is represented with a point where both coordinates are positive. The graph has to slope upward.

3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

   Negative slope

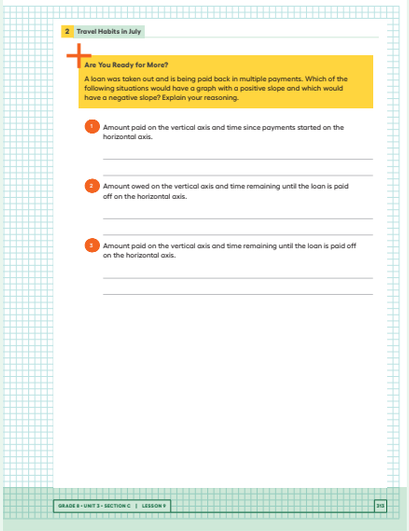
   Sample reasoning: If the time remaining until the loan is paid off is 0, then the amount paid is the whole loan. This is represented with a positive point on the vertical axis. If the time remaining is the entire period of the loan, then the amount paid is 0. This is represented with a positive point on the horizontal axis. The graph has to slope downward.

Activity Synthesis

The goal of this discussion is for students to understand a context where a slope of 0 makes sense. Display the graph from the task for all to see. Ask students what they think the slope of the graph is and to explain their reasoning. Ensure that students understand that a slope of 0 makes sense because no money is added or subtracted each day. If students have been thinking in terms of "uphill" and "downhill" lines, they might describe this line as "flat," indicating that the slope can't be positive or negative, so 0 makes sense.

If not mentioned by students, ask what would happen if they tried to create a slope triangle for this line. They may claim that it would be impossible, but suggest thinking of a slope "triangle" where the vertical segment has length 0. Thinking of slope as the quotient of horizontal displacement by vertical displacement for two points on a line is very effective here. The vertical displacement is 0 for any two points on this line, and so the quotient or slope is also 0. If possible, display and demonstrate this with the following: **ilclass.com/l/717494**

Student Workbook


2 Travel Habits in July

Are You Ready for More?

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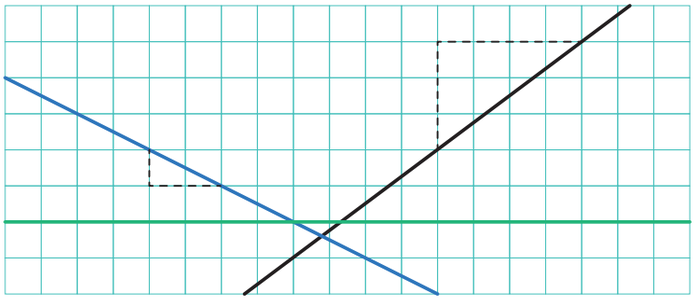
1. Amount paid on the vertical axis and time since payments started on the horizontal axis.

2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

Lesson Synthesis

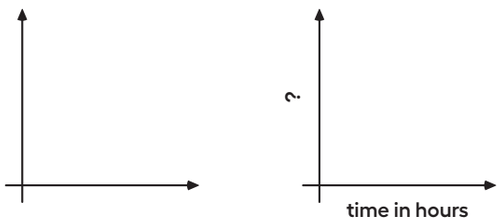
The goal of this discussion is for students to determine cues that identify whether a graphed line has a positive slope, a negative slope, or zero slope. Display the graph for all to see.



Create a classroom display that is divided into 3 sections and invite students to explain how they know if a line has a positive, negative, or zero slope. Record their responses on chart paper, and label the image when appropriate. Important ideas to record include:

- As the x value increases, the y value increases (positive slope), decreases (negative slope), or stays the same (zero slope).
- From left to right, the graph of the line looks like it is going uphill (positive slope), downhill (negative slope), or flat (zero slope).

If time allows, give students an opportunity to practice thinking about contexts with positive, negative, and zero slopes. Display a blank set of coordinate axes and give students a label for the horizontal axis, such as time in hours. Ask students to think of labels for the vertical axis that might result in lines with positive, negative, or zero slopes. For example, graphing the total miles driven during a road trip might have a positive slope, while graphing miles left until reaching the destination might have a negative slope.

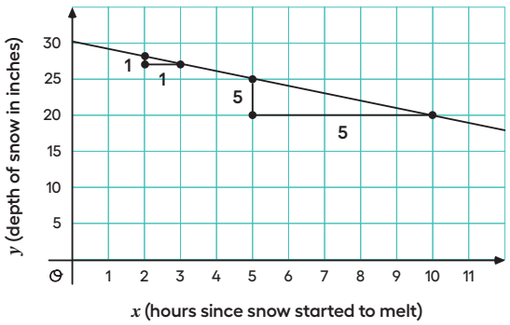


Lesson Summary

At the end of winter in Maine, the snow on the ground was 30 inches deep. Then there was a particularly warm day and the snow melted at the rate of 1 inch per hour. The graph shows the relationship between the time since the snow started to melt and the depth of the remaining snow.

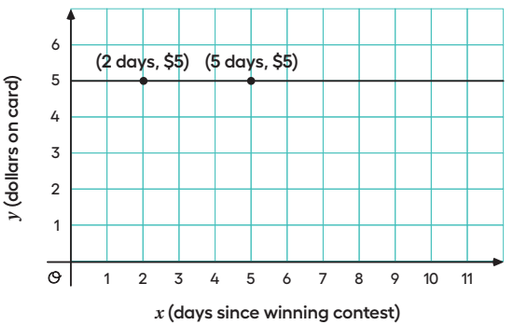
Graphs with a negative slope often describe situations where some quantity is decreasing over time.

Since the depth of the snow decreases by 1 inch per hour, the rate of change is -1 inch per hour and the slope of this graph is -1. The vertical intercept is 30 since the snow was 30 inches high before it started to melt.

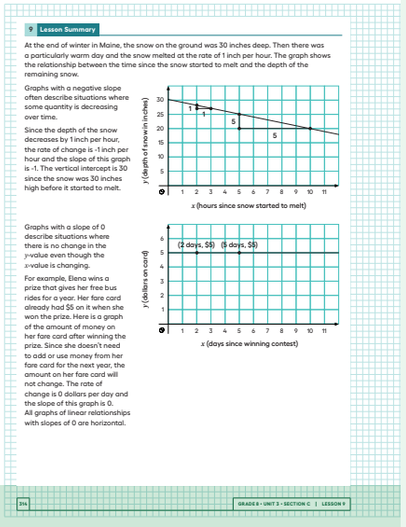


Graphs with a slope of 0 describe situations where there is no change in the y-value even though the x-value is changing.

For example, Elena wins a prize that gives her free bus rides for a year. Her fare card already had $5 on it when she won the prize. Here is a graph of the amount of money on her fare card after winning the prize. Since she doesn't need to add or use money from her fare card for the next year, the amount on her fare card will not change. The rate of change is 0 dollars per day and the slope of this graph is 0. All graphs of linear relationships with slopes of 0 are horizontal.



Student Workbook



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

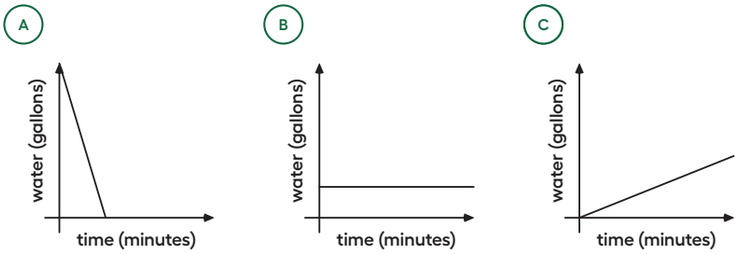
Cool-down

The Slopes of Graphs

5 min

Student Task Statement

Match each graph with the situation that could describe the line.



Situation 3 Situation 2 Situation I

1. A tank is set up to collect rainwater. During a storm, 3 gallons of rainwater is collected each minute.
2. After the storm, no water is used and no additional water is collected.
3. Several days later, rainwater from the tank is used to irrigate a garden at a rate of 8 gallons of water per minute.

Lesson 9

Practice Problems

4 Problems

Problem 1

Suppose that during its flight, the elevation e (in feet) of a certain airplane and its time since takeoff t are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

a. The plane is cruising at an altitude of 37,000 feet above sea level.

zero

b. The plane is descending at a rate of 1,000 feet per minute.

negative

c. The plane is ascending at a rate of 2,000 feet per minute.

positive

Problem 2

from Unit 3, Lesson 7

A group of hikers park their car at a trail head and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles, d, from the car after h hours of hiking.

a. How far is the campsite from their car? Explain how you know.

4 miles

Sample reasoning: The y-intercept represents this initial distance from the car before the start of the hike.

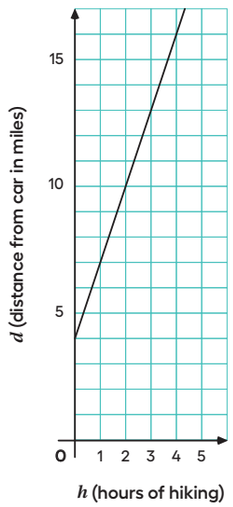
b. Write an equation that describes the relationship between d and h.

$d = 4 + 3h$ (or equivalent)

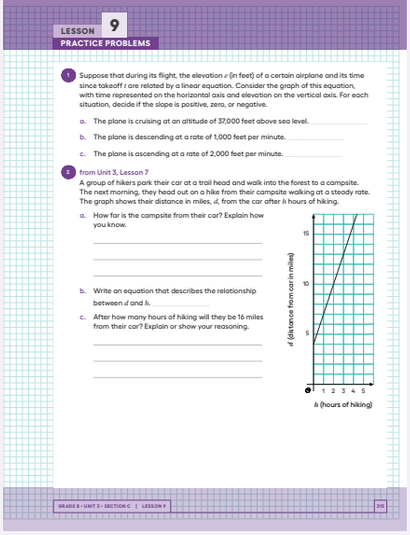
c. After how many hours of hiking will they be 16 miles from their car? Explain or show your reasoning.

4 hours

Sample reasoning: On the graph, $d = 16$ when $h = 4$. The equation can be used to solve for h when $d = 16$. If $16 = 4 + 3h$, then $12 = 3h$, and $h = 4$.



Student Workbook


LESSON 9
PRACTICE PROBLEMS

1 Suppose that during its flight, the elevation e (in feet) of a certain airplane and its time since takeoff t are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

a. The plane is cruising at an altitude of 37,000 feet above sea level.

b. The plane is descending at a rate of 1,000 feet per minute.

c. The plane is ascending at a rate of 2,000 feet per minute.

2 from Unit 3, Lesson 7
A group of hikers park their car at a trail head and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles, d, from the car after h hours of hiking.

a. How far is the campsite from their car? Explain how you know.

b. Write an equation that describes the relationship between d and h.

c. After how many hours of hiking will they be 16 miles from their car? Explain or show your reasoning.

15
10
5
0
1 2 3 4 5
d (distance from car in miles)
h (hours of hiking)

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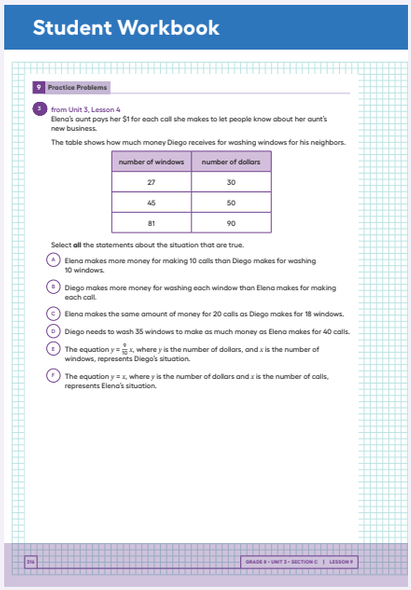

Student Workbook

9 Practice Problems

3 from Unit 3, Lesson 4
Elena's aunt pays her $1 for each call she makes to let people know about her aunt's new business.

The table shows how much money Diego receives for washing windows for his neighbors.

number of windows	number of dollars
27	30
45	50
81	90

Select **all** the statements about the situation that are true.

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.

B. Diego makes more money for washing each window than Elena makes for making each call.

C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.

D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.

E. The equation $y = \frac{9}{10}x$, where y is the number of dollars, and x is the number of windows, represents Diego's situation.

F. The equation $y = x$, where y is the number of dollars and x is the number of calls, represents Elena's situation.

Problem 3

from Unit 3, Lesson 4

Elena's aunt pays her $1 for each call she makes to let people know about her aunt's new business.

The table shows how much money Diego receives for washing windows for his neighbors.

number of windows	number of dollars
27	30
45	50
81	90

Select **all** the statements about the situation that are true.

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.

B. Diego makes more money for washing each window than Elena makes for making each call.

C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.

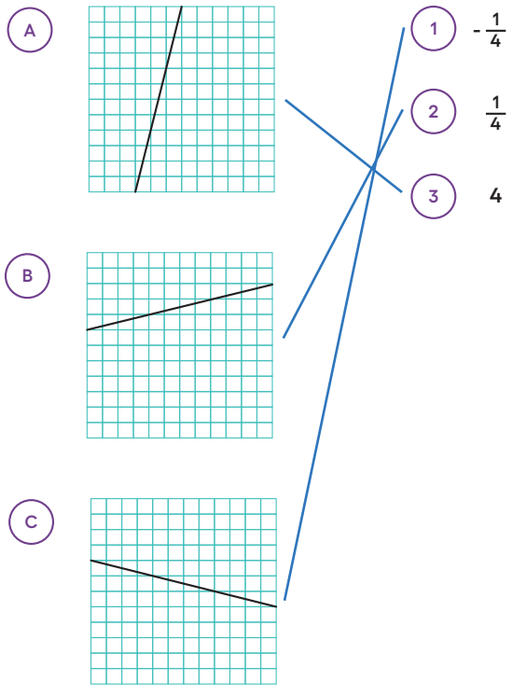
D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.

E. The equation $y = \frac{9}{10}x$, where y is the number of dollars, and x is the number of windows, represents Diego's situation.

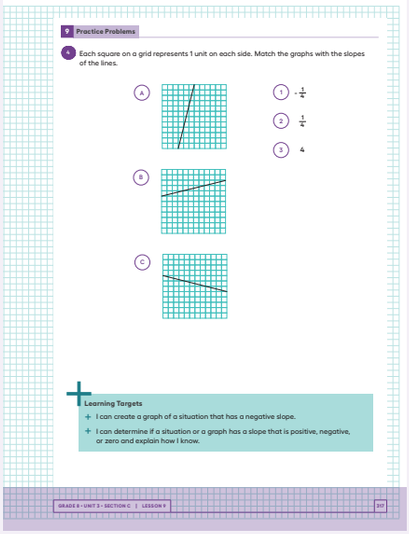
F. The equation $y = x$, where y is the number of dollars and x is the number of calls, represents Elena's situation.

Problem 4

Each square on a grid represents 1 unit on each side. Match the graphs with the slopes of the lines.



Student Workbook


9 Practice Problems

4 Each square on a grid represents 1 unit on each side. Match the graphs with the slopes of the lines.

A
B
C

1 $-\frac{1}{4}$
2 $\frac{1}{4}$
3 4

Learning Targets

+ I can create a graph of a situation that has a negative slope.
+ I can determine if a situation or a graph has a slope that is positive, negative, or zero and explain how I know.

GRADE 8 • UNIT 3 • SECTION C | LESSON 9