Negative Rates

Goals

- Apply operations with signed numbers to solve problems involving constant rates, and explain (orally) the solution method.
- Explain (orally and in writing) how signed numbers can be used to represent situations involving constant rates.
- Write an equation of the form y = -kx to represent a situation that involves descending at a constant rate.

Learning Targets

- I can solve problems that involve multiplying and dividing rational numbers.
- I can solve problems that involve negative rates.

Student Learning Goal

Let's apply what we know about signed numbers.

Lesson Narrative

In this lesson, students solve problems that involve proportional reasoning with negative rates of change. Earlier in the course students saw that proportional relationships can be represented with equations of the form y = kx, where k was a positive value. Now they apply their understanding of multiplying and dividing signed numbers to work with situations that involve negative values of k.

The first activity involves a fish tank that is being drained and filled. The second problem deals with historic voyages in a bathyscaphe (deep-sea submarine) and a high-altitude hot-air balloon. The activities in this lesson involve more reading than most lessons. Some math language routines are recommended to help all students have access to the activities.

As students interpret the rates to understand the situation, they are making sense of the problem and persevering in solving it.

Access for Students with Diverse Abilities

• Representation (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Warm-up)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

· MLR6: Three Reads

Lesson Timeline



Warm-up



Activity 1



Activity 2



Lesson Synthesis

Assessment



Cool-down

Instructional Routines

MLR6: Three Reads ilclass.com/r/10695568





Access for Multilingual Learners (Activity 1)

MLR6: Three Reads

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.



Warm-up

Shirts per Minute



Activity Narrative

In this *Warm-up*, students review "per" language by considering situations that deal with constant rates. This will be useful in following activities when students perform calculations with signed numbers involving rates.

Launch 🙎

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by 1 minute of partner discussion. Then follow with a whole-class discussion.

Student Task Statement

- 1. If you fold 5 shirts per minute for 8 minutes, how many shirts will you fold?
- **2.** If you hear 9 new songs per day for 3 days, how many new songs will you hear?

27 songs

- **3.** If you run 15 laps per practice, how many practices will it take you to run 30 laps?
 - 2 practices

Activity Synthesis

For each question, invite a student to share their response and reasoning. Resolve any disagreements that come up. Remind students that whenever we see the word "per," it means "for every 1."

Activity 1

Water Level in the Aquarium



Activity Narrative

In this activity, students use their knowledge of dividing and multiplying negative numbers to answer questions involving rates. In the first situation, students must make sense of the problem by determining whether a faulty aquarium system results in the aquarium filling or draining and persevere in solving the problem. In the second situation, students encounter another faulty aquarium and will need to convert between different rates to solve the problem.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem, without revealing the questions.

For the first read, read the problem aloud, then ask,

"What is this situation about?"

This problem is about fish in an aquarium. The fish need a certain amount of water to stay healthy, and one day the water level system stops working.

Listen for and clarify any questions about the context.

After the second read, ask students to list any quantities that can be counted or measured.

the liters of water that the aquarium can hold, the liters of water before the aquarium overflows, the liters of water the fish need to not get sick, the rate at which the faucet fills the aquarium, the rate at which the drain empties the aquarium, the length of time before the fish start getting sick, the length of time before the aquarium overflows

After the third read, reveal the question: How long will it take until the tank starts overflowing or the fish get sick? and ask,

"What are some ways we might get started on this?"

Invite students to name some possible starting points, referencing quantities from the second read.

We need to figure out if the aquarium is gaining or losing water by comparing the rates of the faucet and drain.

Student Task Statement

1. A large aquarium should contain 10,000 liters of water when it is filled correctly. It will overflow if it gets up to 12,000 liters. The fish will get sick if it gets down to 4,000 liters. The aquarium has an automatic system to help keep the correct water level. If the water level is too low, a faucet fills it. If the water level is too high, a drain opens.

One day, the system stops working correctly. The faucet starts to fill the aquarium at a rate of 30 liters per minute, and the drain opens at the same time, draining the water at a rate of 20 liters per minute.

a. Is the water level rising or falling? How do you know?

Represent the filling as 30 liters per minute and the drain as -20 liters per minute.

This means that the water level is rising at 30 + (-20) or 10 liters per minute.

b. How long will it take until the tank starts overflowing or the fish get sick?

The tank needs an extra 2,000 liters before it overflows. At IO liters per minute this takes $2000 \div 10$, or 200, minutes.

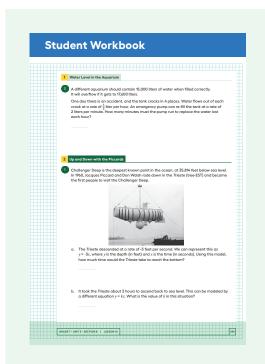
Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.

Represent the problem in multiple ways to support understanding of the situation. For example, use an illustration, storyboard, or animation.

Supports accessibility for: Conceptual Processing, Organization





2. A different aquarium should contain 15,000 liters of water when filled correctly. It will overflow if it gets to 17,600 liters.

One day there is an accident, and the tank cracks in 4 places. Water flows out of each crack at a rate of $\frac{1}{2}$ liter per hour. An emergency pump can re-fill the tank at a rate of 2 liters per minute. How many minutes must the pump run to replace the water lost each hour?

The tank has 4 cracks, which leak at 0.5 liters per hour. This is a total of -2 liters per hour. The emergency pump is 2 liters per minute, so it must run for I minute every hour to compensate.

Activity Synthesis

The purpose of this discussion is for students to share the strategies they used to solve each aquarium problem and to reflect on how they used integer arithmetic. Begin by arranging students in groups of 2. Ask partners to share their answers and reasoning for each problem. If time allows, instruct each group to create a visual display illustrating what is happening to one of the aquariums. Then consider discussing these questions:

"What were some assumptions you had to make to solve the first (or second) aquarium problem?"

I assumed that each aquarium was filled correctly when the automatic system stopped working or when the cracks appeared. I assumed the fish swimming in the first aquarium didn't affect the rate of water flowing from the faucet or the drain. I assumed that the cracks in the second pump were all below the level of the water in the aquarium and that no additional cracks formed.

"How did you use your knowledge of integer arithmetic to help you solve these problems?"

Activity 2

Up and Down with the Piccards



Activity Narrative

In this activity, students build on previous work with proportional relationships and their understanding of multiplying and dividing signed numbers to represent two historical scenarios involving ascent and descent. Students reason abstractly and quantitatively when they describe the situation using equations and interpret the meaning of their answer in context.

Monitor for students who convert between seconds and hours in their equations.

Launch 22

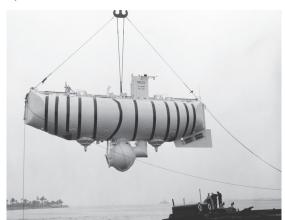
Arrange students in groups of 2. Introduce the activity by asking students where they think the deepest part of the ocean is. It may be helpful to display a map showing the location of the Challenger Deep, but this is not required. Explain that Jacques Piccard (ZHAHK pee-KAHR) had to design a specific type of submarine to make such a deep descent. Consider asking students,

"If sea level is represented by 0 feet, how can we represent the depth of a submarine descending from sea level to the bottom of Challenger Deep?"

We can use negative numbers to represent how many feet below sea level a submarine is.

Student Task Statement

 Challenger Deep is the deepest known point in the ocean, at 35,814 feet below sea level. In 1960, Jacques Piccard and Don Walsh rode down in the *Trieste* (tree-EST) and became the first people to visit the Challenger Deep.



a. The *Trieste* descended at a rate of -3 feet per second. We can represent this as y = -3x, where y is the depth (in feet) and x is the time (in seconds). Using this model, how much time would the *Trieste* take to reach the bottom?

It would take II,938 seconds, or about 3 hours and I9 minutes, to descend to the sea floor. Substituting the depth of -35,814 in for y gives the equation -35,814 = -3x. Solving the equation gives II,938 for x, because $-35,814 \div (-3) = 11,938$.

b. It took the *Trieste* about 3 hours to ascend back to sea level. This can be modeled by a different equation y = kx. What is the value of k in this situation?

 $k = \frac{35,814}{10,800}$. It took 10,800 seconds to go up 35,814 feet, because 3.60.60 = 10,800. (This is a vertical change of approximately 3.32 feet per second.)

- **2.** The design of the *Trieste* was based on the design of a hot air balloon built by Auguste Piccard, Jacques's father. In 1932, Auguste rode in his hotair balloon up to a record-breaking height.
 - **a.** Auguste's ascent took 7 hours and went up 51,683 feet. Write an equation y = kx to represent his ascent from his starting location.

The relationship is $y = \frac{51,683}{25,200}x$. It took 25,200 seconds to go up 51,683 feet, because $7 \cdot 60 \cdot 60 = 25,200$. (This is a vertical change of approximately 2.05 feet per second.)

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.

Use multimodal examples to help students create context for this activity. Use verbal descriptions along with gestures, drawings, or concrete objects to illustrate what is happening with the trench, ocean surface, submersible, and balloon.

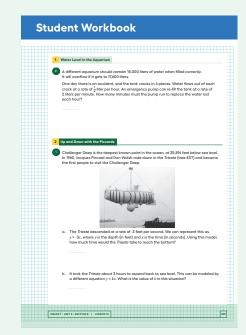
Advances: Listening, Representing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.

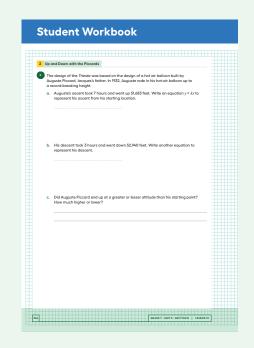
Ask students to read the problems aloud to their partner. Students who both listen to and read the information will benefit from extra processing time.

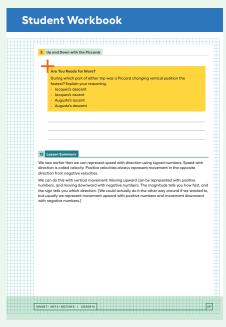
Supports accessibility for: Language, Attention



Building on Student Thinking

Some students may be confused by the correct answer to the last question, thinking that 51,683 – 52, 940 = –1,257 means that Auguste landed his balloon below sea level. Explain that Auguste launched his balloon from a mountain to help him reach as high of an altitude as possible. We have chosen to use zero to represent this starting point, instead of sea level, for this part of the activity. Therefore, a vertical position of -1,257 feet means that Auguste landed below his starting point, but not below sea level.





b. His descent took 3 hours and went down 52,940 feet. Write another equation to represent his descent.

The relationship is $y = \frac{-52,940}{10,800}x$, because it took 10,800 seconds to go down 52,940 feet. (This is a vertical change of approximately -4.9 feet per second.)

c. Did Auguste Piccard end up at a greater or lesser altitude than his starting point? How much higher or lower?

He ended up 1,257 feet lower, because 51,683 - 52,940 = -1,257.

Are You Ready for More?

During which part of either trip was a Piccard changing vertical position the fastest? Explain your reasoning.

- Jacques's descent
- · Jacques's ascent
- · Auguste's ascent
- · Auguste's descent

Auguste's descent was the fastest.

Sample reasoning: Auguste's descent was at a rate of approximately -4.9 feet per second. This rate had the largest magnitude of the 4 trips, meaning it changed vertical position the fastest. The sign of the rate signifies the direction of the change in vertical position.

Activity Synthesis

The purpose of this discussion is for students to share the equations they wrote to describe each situation. Begin by asking students to compare their solutions with a partner and notice what is the same and what is different.

Next, invite previously identified students to share their equations with the class. Highlight solutions that compute with negatives correctly, convert between seconds and hours, and state their assumptions clearly. To help students make sense of each equation, consider discussing the following questions:

- (*) "How can you tell from each equation whether they are going up or down?"
 - When the value of k is positive, they are going up, and when the value of k is negative, they are going down.
- "How can you tell from the equation the total time of their ascent or descent?"

The time of the ascent or descent is represented by x in each equation.

- "How can you tell from the equation the total distance traveled?"
 - The total distance traveled is represented by y in each equation.
- "What does 0 represent in each situation?"

For the submersible, O represents sea level. For the hot air balloon, O represents the starting elevation of the balloon.

Lesson Synthesis

Share with students,

"Today we used multiplication and division of signed numbers to solve problems."

If desired, use this short activity to review the different types of equations that students encountered in this lesson. Display the equations a = -6t and b = 5t, and ask students:

"Which equation could represent filling a tank?"

equation b

"Which equation could represent draining a tank?"

equation a

"Which equation could represent the temperature getting colder?"

equation a

"Which equation could represent the temperature getting warmer?"

equation b

"For which equation is the quantity changing faster?"

equation a

Lesson Summary

We saw earlier that we can represent speed with direction using signed numbers. Speed with direction is called *velocity*. Positive velocities always represent movement in the opposite direction from negative velocities.

We can do this with vertical movement: Moving upward can be represented with positive numbers, and moving downward with negative numbers. The magnitude tells you how fast, and the sign tells you which direction. (We could actually do it the other way around if we wanted to, but usually we represent movement upward with positive numbers and movement downward with negative numbers.)

Cool-down

Submarines

5 min

Student Task Statement

1. A submarine is descending to examine the seafloor 2,100 feet below the surface. It takes the submarine 2 hours to make this descent. Write an equation to represent the relationship between the submarine's elevation and time.

y = -1050x, where y is the elevation in feet and x is the time in hours

2. Another submarine's descent can be represented as y = -240x, where y is the elevation in feet and x is time in hours. How long will it take this submarine to make the descent?

8.75 hours



Responding To Student Thinking

Points to Emphasize

If most students struggle with writing and solving equations that involve negatives, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the multiplication and division equations in these activities:

Grade 7, Unit 5, Lesson 14, Activity 2 Solar Power

Grade 7, Unit 5, Lesson 16, Activity 1 Warmer or Colder Than Before?



Problem 1

Describe a situation where each of the following quantities might be useful.

a. -20 gallons per hour

Sample response: Water leaking out of a tank

b. -10 feet per minute

Sample response: An airplane descending

c. -0.1 kilograms per second

Sample response: Gravel being emptied out of a truck

Problem 2

A submarine is only allowed to change its depth by rising toward the surface in 60-meter stages. It starts off at -340 meters.

- **a.** At what depth is it after:
 - i. 1 stage

-280 m, because -340 + 60 = -280

ii. 2 stages

-220 m, because -340 + 120 = -220

iii. 4 stages

-100 m, because -340 + 240 = -100

b. How many stages will it take to return to the surface?

6, because $340 \div 60 = 5.7$, so the submarine would need 6 stages

Problem 3

Some boats were traveling up and down a river. A satellite recorded the movements of several boats.

- a. A motor boat traveled -3.4 miles per hour for 0.75 hours. How far did it go?
 - -2.55 miles, because $-3.4 \cdot 0.75 = -2.55$
- b. A tugboat traveled -1.5 miles in 0.3 hours. What was its velocity?
 - -5 miles per hour, because $-1.5 \div 0.3 = -5$
- **c.** What do you think that negative distances and velocities could mean in this situation?

Sample response: Someone had to choose one direction to be positive and the other to be negative. Positive distances could mean distances in the positive direction, and negative distances mean distances in the other direction. Positive velocities could mean it was moving in the positive direction, and negative velocities mean it was moving in the negative direction.

Problem 4

from Unit 5, Lesson 11

Evaluate each expression. When the answer is not a whole number, write your answer as a fraction.

a. -4 · -6

24

b. -24 $\cdot \frac{-7}{4}$

28

c. 4 ÷ -6

 $\frac{-2}{3}$ (or equivalent)

d. $\frac{4}{3} \div -24$

 $\frac{-1}{12}$ (or equivalent)

Problem 5

from Unit 5, Lesson 6

a. How much higher is 500 than 400 m?

100 m, because 400 + 100 = 500

b. How much higher is 500 than -400 m?

900 m, because -400 + 900 = 500

c. What is the change in elevation from 8,500 m to 3,400 m?

-5,100 m, because 8,500 + (-5,100) = 3,400

d. What is the change in elevation between 8,500 m and -300 m?

-8,800 m, because 8,500 + (-8,800) = -300

e. How much higher is -200 m than 450 m?

-650 m, because 450 + (-650) = -200

Problem 6

from Unit 4, Lesson 3

- **a.** A cookie recipe uses 3 cups of flour to make 15 cookies. If you had 4 cups of flour, how many cookies could you make with this recipe? (Assume you have enough of the other ingredients.)
 - 20, because I cup of flour makes 5 cookies, 4 cups of flour would make 20 cookies.
- **b.** A teacher uses 36 centimeters of tape to hang up 9 student projects. At that rate, how much tape would the teacher need to hang up 10 student projects?

40 centimeters, because 4 centimeters of tape is needed to hang up I student project, the teacher would need 40 centimeters of tape to hang IO student projects

