Lesson 15

Adding the Angles in a Triangle

Goals

- Describe (orally) that a straight angle can be decomposed into 3 angles to construct a triangle.
- Justify (orally and in writing) that the sum of angles in a triangle is 180 degrees using properties of rigid motions.

Learning Target

I can determine whether three angles could make a triangle using their sum.

Lesson Narrative

The purpose of this lesson is for students to observe that the sum of the interior angle measures of a triangle is 180 degrees.

In the optional activity, students work to identify a set of three congruent triangles, where each triangle in the set has a different interior angle measure marked. Once the set has been identified, students make observations about the angles in their congruent triangles, including the sum of the three angles.

Students decompose a straight angle into three angles and create a triangle using those angles. Using repeated reasoning as many different combinations of angles are used to create triangles, students observe a close connection between three positive numbers adding up to 180 and having a triangle with those three numbers as angle measures.

In this lesson, students informally describe the relationship they observe between angle measures in a triangle. They will justify this relationship formally in an upcoming lesson, so they do not need to show that the sum of the angles in a triangle is always 180 degrees at this time.

Student Learning Goal

Let's explore angles in triangles.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Required Materials

Materials to Gather

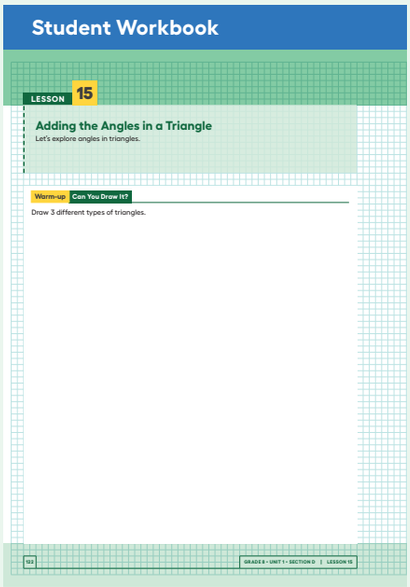
- Sticky notes: Warm-up
- Geometry toolkits: Activity 1, Activity 2
- Scissors: Activity 2

Materials to Copy

- Find All Three Cards (1 copy for every 15 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	25 min	10 min	5 min


Student Workbook

LESSON 15

Adding the Angles in a Triangle

Let's explore angles in triangles.

Warm-up Can You Draw It?

Draw 3 different types of triangles.

123 GRADE 8 • UNIT 1 • SECTION D | LESSON 15

Warm-up

Can You Draw It?

10 min

Activity Narrative

The purpose of this activity is for students to recall properties and types of triangles and specific language for describing them. In this activity, students draw a variety of triangles and sort them into categories of their choosing. Monitor for students choosing categories that compare angle measures and side lengths. Students may use informal language to describe their categories, such as "all of the sides are different lengths" rather than "scalene."

Select students who use these categories to share during the whole-class discussion:

- Equilateral triangles
- Right triangles
- Isosceles triangles
- Obtuse triangles

Launch

Arrange students in groups of 4. Distribute 3 sticky notes per student.

Give 1 minute of quiet work time for students to draw a triangle on each sticky note.

Tell each group to look at all of their triangles and sort them into categories.

Give 2–3 minutes of group time followed by a whole-class discussion.

Listen for language describing the side lengths and angles of triangles, such as "equilateral," "right," "obtuse," and "acute."

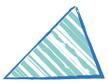
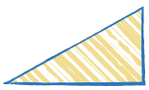
Student Task Statement

Draw 3 different types of triangles.

Answers vary

Activity Synthesis

Invite previously selected students to share the categories they chose for the triangles. As students describe categories, organize their descriptions for all to see in a table or chart like this one:

	acute (all angles acute)	right (has a right angle)	obtuse (has an obtuse angle)
scalene (side lengths all different)			
isosceles (at least two side lengths are equal)			
equilateral (three equal side lengths)		impossible	impossible

As students share, invite them to place their sticky note on the chart for all to see.

If any students drew triangles that fall under multiple categories, such as a right isosceles triangle, display it for all to see. If not, draw one for all to see. Ask students which categories the triangle belongs to. Students may describe an isosceles triangle as having two angles the same or two sides the same.

Ask students if they can draw any other triangles that fit into two categories. Examples include equilateral and acute, scalene and obtuse, scalene and right. Students may describe scalene as having all different side lengths or all different angles.

Ask students if it is possible to draw an equilateral triangle that has a right angle.

> No, 3 right angles would not form a triangle, and all the angles are the same in an equilateral triangle.

If time allows, continue filling out the table with an example of each type of triangle or an explanation of why that triangle is not possible.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.
Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.

Supports accessibility for: Language, Memory

Activity 1: Optional

Find All Three

15 min

Activity Narrative

This activity is optional because it provides additional opportunity for students to practice identifying congruent figures. In this matching activity, each student receives a card with a triangle that has one angle measure marked. Students work to find partners with triangles congruent to theirs, although there are three different angles marked for each set of congruent triangles.

Students may use strategies such as applying rigid transformations to find a matching triangle, or using the structure of triangles to identify them as acute, right, or isosceles to find matches with the same features.

Launch

Provide access to geometry toolkits. Distribute one card to each student, making sure that all three cards have been distributed for each triangle. If the number of students in the class is not a multiple of three, one or two students may need to take ownership over two cards showing congruent triangles.

Explain that there are two other students who have a triangle congruent to theirs that has been re-oriented in the coordinate plane through combinations of translations, rotations, and reflections. Instruct students to look at the triangle on their card and estimate the measures of the other two angles. With these estimates and their triangle in mind, students look for the two triangles congruent to theirs with one of the missing angles labeled.

Prepare and display a table for all to see with columns angle 1, angle 2, angle 3 and one row for each group of three students. It should look something like this:

student groups	angle 1	angle 2	angle 3

Once the three partners are together, they complete one row in the posted table for their triangle's angle measures. Whole-class discussion to follow.

Give students 1 minute quiet think time, then 3–5 minutes to find their matches and record.

Follow with a whole-class discussion.

Student Task Statement

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Mentally estimate the measures of the other two angles.
2. Find two other students with triangles congruent to yours but with a different angle labeled. Confirm that the triangles are congruent, that each card has a different angle labeled, and that the angle measures make sense.
3. Enter the three angle measures for your triangle on the table your teacher has posted.

The angle combinations are: 40°, 50°, 90°; 40°, 60°, 80°; 50°, 50°, 80°; 20°, 20°, 140°; 20°, 40°, 120°

Activity Synthesis

The goal of this discussion is for students to describe what they notice about angles in triangles. Here are some questions for discussion:

- *"How did you decide that you had a match?"*

  I used tracing paper to see that it matched, I estimated the angle measures and found them, or I looked for a triangle with the same features.

- *"What do you notice about the three angle measures for your triangle?"*

  They add up to 180 degrees.

- *"What do all of the triangles have in common?"*

  They all have angles that add up to 180 degrees.

If no students notice that the angles for each triangle have a sum of 180 degrees, add a column labeled "total" and ask students to find the sum of their group's triangle, then display it on the table for all to see.

Activity 2
Tear It Up

25 min

Activity Narrative

The purpose of this activity is for students to observe that three angles that total to 180 degrees can be used to create a triangle.

Students cut out three angles that form a line, and then try to use these three angles to make a triangle. Students also create their own three angles from a line and check whether they can construct a triangle with their angles. As students observe the triangles of others in their group as well as the triangles created from their chosen angles, they use repeated reasoning to conclude that 3 angles decomposed from a straight line can form a triangle.

Monitor for students who successfully make triangles out of each set of angles and select them to share (both the finished product and how they worked to arrange the angles) during the discussion. Also monitor how students divide the blank line into angles. It is helpful if the rays all have about the same length as in the pre-made examples.

Student Workbook

1 Find All Three

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labeled. Mentally estimate the measures of the other two angles.
2. Find two other students with triangles congruent to yours but with a different angle labeled. Confirm that the triangles are congruent, that each card has a different angle labeled, and that the angle measures make sense.
3. Enter the three angle measures for your triangle on the table your teacher has posted.

2 Tear It Up

Your teacher will give you a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

Are You Ready for More?

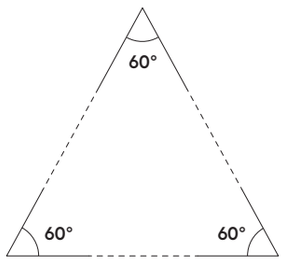
1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?
2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

Launch

Arrange students in groups of 4. Provide access to geometry toolkits, especially scissors.

Instruct students to cut the four individual pictures out of the black line master found in the student workbook. Each student will work with one of these. Instruct the student with the blank copy to use a straightedge to divide the line into three angles (different from the three angles that the other students in the group have). Demonstrate how to do this if needed.

If necessary, demonstrate "making a triangle" before beginning the activity so students understand the intent. With three cut-out 60-degree angles, for example, an equilateral triangle can be built. Here is a picture showing three 60-degree angles arranged so that they can be joined to form the three angles of an equilateral triangle. The students will need to arrange the angles carefully, and they may need to use a straightedge in order to add the dotted lines to join the angles and create a triangle.



Student Task Statement

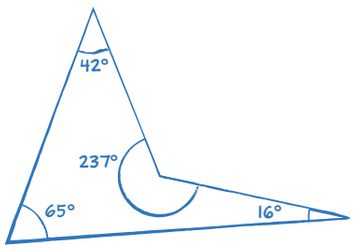
In your student workbook, you will see a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

Sample response: We were all able to build triangles with the given sets of angles. One is a right triangle, one acute, and one obtuse. The three angles we chose also made a triangle.

Are You Ready for More?

1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?
2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

The sum of interior angles in any quadrilateral is 360°. This is pretty clear with rectangles. Parallelograms have 2 pairs of equal supplementary angles, so they work too. In fact, it works for anything, even non-convex quadrilaterals.



Activity Synthesis

If time allows, have students do a "gallery walk" at the start of the discussion. Ask students to compare the triangle they made to the other triangles made from the same angles and be prepared to share what they noticed. (For example, students might notice that all the other triangles made with their angles looked pretty much the same, but were different sizes.) If students do not bring it up, direct them to notice that all of the "create three of your own angles" students were able to make a triangle, not just students with the ready-made angles.

Ask previously selected students to share their triangles and explain how they made the triangles. To make the triangles, some trial and error is needed. A basic method is to line up the line segments from two angles (to get one side of the triangle) and then try to place the third angle so that it lines up with the rays coming from the two angles already in place. Depending on the length of the rays, they may overlap, or the angles may need to be moved further apart.

Here are some questions for discussion:

- *"How do you know the three angles you were given sum to 180 degrees?"*

  They were adjacent to each other along a line.

- *"How do you know these can be the three angles of a triangle?"*

  We were able to make a triangle using these three angles.

- *"What do you know about the three angles of the triangle you made and why?"*

  Their measures sum to 180 because they were the same three angles that made a line.

Ask students if they think they can make a triangle with *any* set of three angles that form a line and poll the class for a positive or negative response. Tell them that they will continue to investigate this idea and emphasize that while experiments may lead us to believe this statement is true, the methods used are not very accurate and were only applied to a few sets of angles.

If time permits, perform a demonstration of the converse: Start with a triangle, tear off its three corners, and show that these three angles when placed adjacent to each other sum to a line.

Lesson Synthesis

The goal of this discussion is for students to articulate their observations that the sum of the angle measures in a triangle is 180 degrees.
Some guiding questions for the discussion include:

- *"What did we observe about the sum of the angles inside a triangle?"*

  The sum of the angles inside a triangle seem to always add up to 180 degrees.

- *"Are there pairs of angles that cannot be used to make a triangle?"*

  Yes, if the two angles are both bigger than or equal to 90 degrees, a triangle cannot be made.

Emphasize that we were able to see for multiple triangles that the sum of their angles is 180° *and* that using several sets of three angles adding to 180°, we were able to build triangles. In the next lesson we will investigate and explain this interesting relationship.

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "To arrange the angles, first, I ___ because ..." or "I noticed ___ so I ..."

Supports accessibility for: Language, Social-Emotional Functioning

Access for Multilingual Learners (Activity 2, Synthesis)

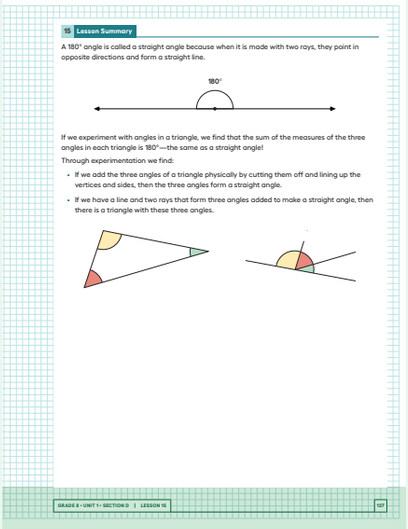
MLR7: Compare and Connect.
After the gallery walk, lead a discussion comparing, contrasting, and connecting the different strategies for creating triangles. To amplify student language, and illustrate connections, follow along and point to the relevant parts of the displays as students speak.

Advances: Representing, Conversing

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


15 Lesson Summary

A 180° angle is called a straight angle because when it is made with two rays, they point in opposite directions and form a straight line.

180°

If we experiment with angles in a triangle, we find that the sum of the measures of the three angles in each triangle is 180°—the same as a straight angle!

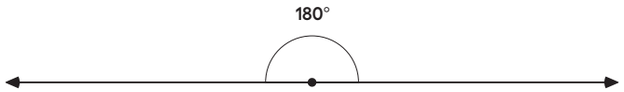
Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.

GRADE 8 • UNIT 1 • SECTION D | LESSON 15 127

Lesson Summary

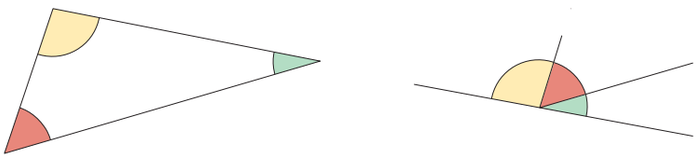
A 180° angle is called a straight angle because when it is made with two rays, they point in opposite directions and form a straight line.



If we experiment with angles in a triangle, we find that the sum of the measures of the three angles in each triangle is 180°—the same as a straight angle!

Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.



Cool-down

Three Angles

5 min

Student Task Statement

Tyler has 3 right angles. Can he use them to make a triangle? Explain your reasoning.

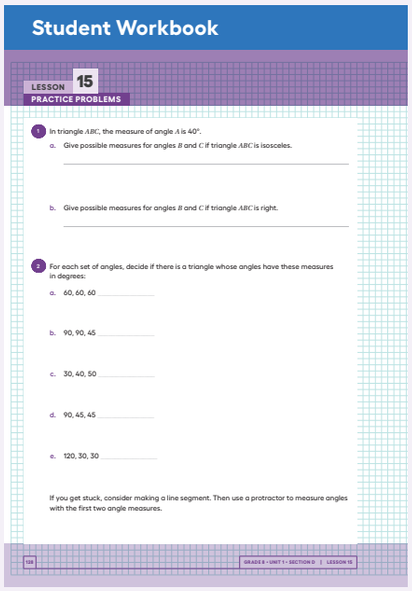
no

Sample reasoning: 3 right angles sums to more than 180 degrees, since $3 \cdot 90 = 270$.

Lesson 15

Practice Problems

5 Problems



Problem 1

In triangle ABC, the measure of angle A is 40°.

a. Give possible measures for angles B and C if triangle ABC is isosceles.

Both angles B and C would measure either 40° or 70°.

b. Give possible measures for angles B and C if triangle ABC is right.

Either angle B or angle C measures 50°, and the other angle measures 90°.

Problem 2

For each set of angles, decide if there is a triangle whose angles have these measures in degrees:

a. 60, 60, 60

b. 90, 90, 45

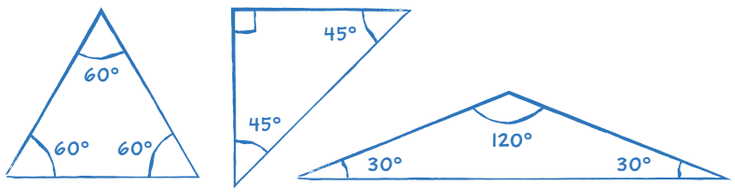
c. 30, 40, 50

d. 90, 45, 45

e. 120, 30, 30

If you get stuck, consider making a line segment. Then use a protractor to measure angles with the first two angle measures.

Triangles can be made with the sets of angles in a, d, and e but not with b, and c.



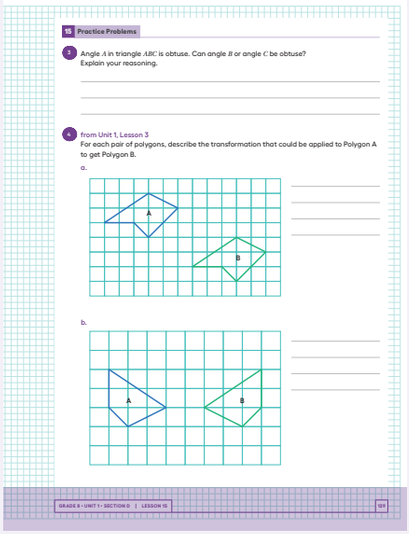
Problem 3

Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your reasoning.

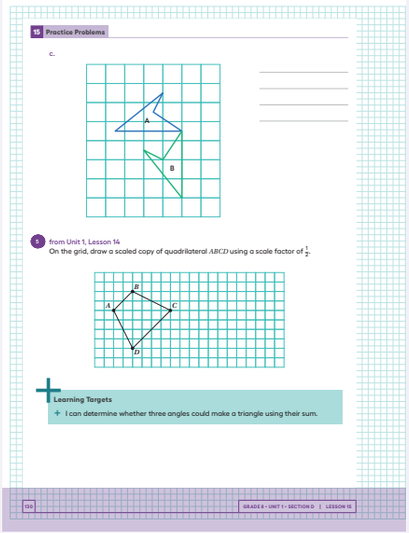
no

Sample reasoning: A triangle can not have two obtuse angles. If the obtuse angles were at vertices A and B, for example, then those angles do not meet at any point C.

Student Workbook



Student Workbook

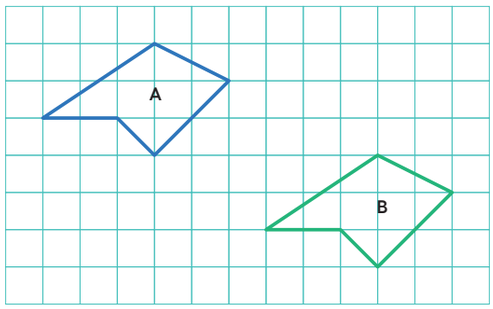


Problem 4

from Unit 1, Lesson 3

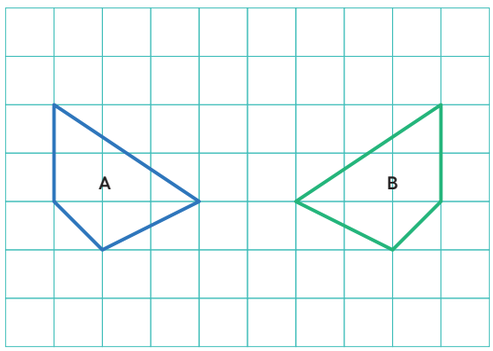
For each pair of polygons, describe the transformation that could be applied to Polygon A to get Polygon B.

a.



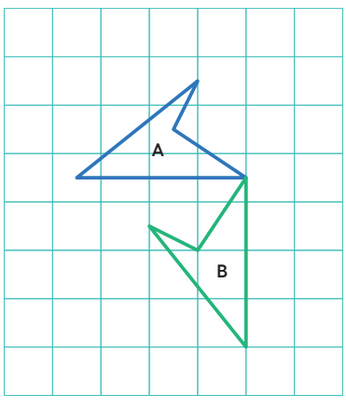
Sample response: translation down 3 units and right 6 units

b.



Sample response: reflection with a vertical line of reflection halfway between the two polygons

c.

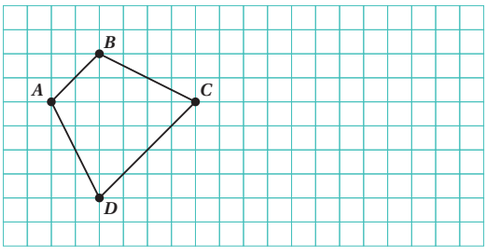


Sample response: rotation by 90 degrees counterclockwise with the vertex shared by the two polygons as the center of rotation

Problem 5

from Unit 1, Lesson 14

On the grid, draw a scaled copy of quadrilateral $ABCD$ using a scale factor of $\frac{1}{2}$.



Sample response: Each side is $\frac{1}{2}$ the length of the corresponding side on quadrilateral $ABCD$.

Student Workbook