Lesson 5

Square Roots on the Number Line

Goals

- Calculate an approximate value of a square root to the nearest tenth, and represent the square root as a point on the number line.
- Determine the exact length of a line segment on a coordinate grid and express the length (in writing) using square root notation.
- Explain (orally) how to verify that a value is a close approximation of a square root.

Learning Targets

- I can find a decimal approximation for square roots.
- I can plot square roots on the number line.

Lesson Narrative

In this lesson, students continue to transition from understanding square roots simply as side lengths to recognizing that all square roots are specific points on the number line.

Students begin with an image of a diagonal line segment that is also a radius of a circle centered around the origin. They can calculate the exact length of the line segment by drawing a square and finding its area, and they can approximate its length by looking at where the circle intersects the x-axis.

The next image of a diagonal line segment is similar to the first but without the circle. Students should notice that they can use the same strategies as earlier to find the exact and an approximate length. Finally, given an image that starts with a square that has its sides already sitting along the x- and y-axes, students are asked to explain whether or not the square's side length of 2.5 is a good approximation for $\sqrt{3}$. This type of reasoning allows students to think flexibly about the relationship between squares and square roots.

Student Learning Goal

Let's explore square roots.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)

Instructional Routines

- 5 Practices
- Notice and Wonder

Required Materials

Materials to Gather

- Compasses: Activity 2
- Geometry toolkits: Activity 2
- Four-function calculators: Activity 3

Warm-up
Notice and Wonder: Diagonals

5 min

Activity Narrative

The purpose of this *Warm-up* is to use the structure of the circle and a rotation to relate the length of the segment to a point on the number line, which will be useful when students locate square roots on a number line in a later activity. While students may notice and wonder many things about the image, seeing how a decimal approximation can be found by looking at where the circle intersects an axis is an important discussion point.

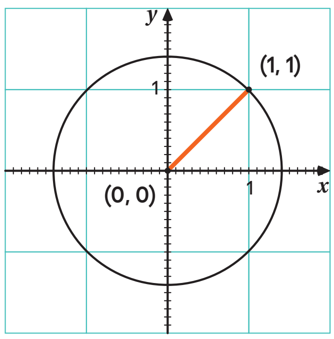
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Things students may notice:

- The center of the circle is at $(0, 0)$.
- There is a point labeled at $(1, 1)$ on the circle.
- There are many tick marks between 0 and 1.

Things students may wonder:

- How to find the distance across the circle.
- Where exactly does the circle land on the x- and y-axes?

Instructional Routines

Notice and Wonder

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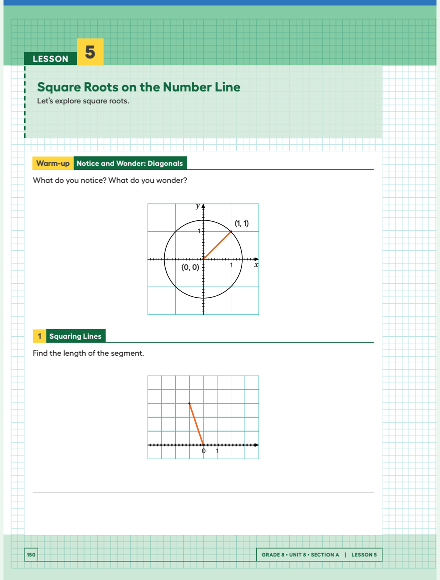
Please log in to the site before using the QR code or URL.

Student Workbook



Instructional Routines

5 Practices

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Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the length of the radius does not come up during the conversation, ask students to discuss how they could use the image to determine it. While some students may recognize the length from earlier activities, keep the discussion focused on strategies they could use to find the length.

Activity 1

Squaring Lines

15 min

Activity Narrative

The purpose of this activity is for students to connect values expressed using square roots with values expressed in decimal form by determining the length of a diagonal line segment on a grid.

Monitor for students who use the following strategies to find the length of the segment, ordered to show students a reasoning strategy that produces an exact square root length followed by concrete measurement strategies that help students understand the value in a decimal form that they may be more familiar with:

- Draw a tilted square and find the area
- Use tracing paper
- Use a compass to make a circle

Launch

Provide access to geometry toolkits and compasses, but do not provide access to a calculator with a square root button since part of this activity asks students to estimate the value of a square root. Students will be able to use a calculator in later lessons.

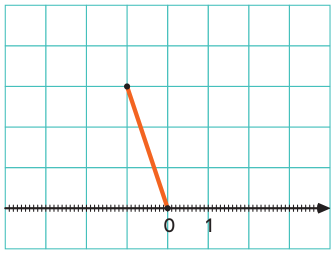
Begin by displaying the diagram for all to see. Ask students how this diagram is similar and how it is different from the diagram in the *Warm-up*. (Both diagrams show a line segment on the coordinate plane. This diagram does not have a circle drawn around it.)

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative,* to share later.

Student Task Statement

Find the length of the segment.



$\sqrt{10}$ units or about 3.1 units

Activity Synthesis

The purpose of this discussion is for students to see multiple ways the length of a line segment can be represented. This helps students transition from thinking of square roots as side lengths to thinking about them as values that can be plotted on the number line.

Invite previously selected students to share how they determined the length of the line segment. Sequence the discussion of the methods in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Since the task did not specify whether students should find an exact or approximate side length, some students will draw a square and use the area to find the exact side length, which is a familiar strategy. Other students may use tracing paper to use the number line as a ruler. Students who use a compass are finding another way to use the number line as a ruler.

Connect the different responses to the learning goals by asking questions such as:

- *"Which of these methods gives the most accurate length of the line segment?"*

  Finding the area of the square and taking the square root will give an exact answer.

- *"What do the different methods tell us about the exact value of $\sqrt{10}$?"*

  It is about 3.1.

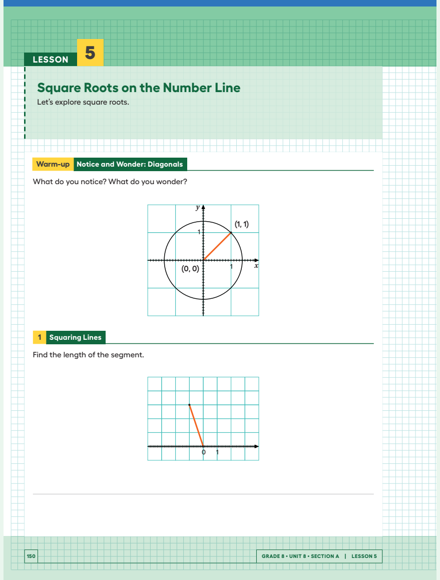
- *"How could you use the compass method to approximate the side lengths of other squares?"*

  Use the compass to measure a side length, then set the point at $(0, 0)$ and draw a circle, marking where it crosses the number line.

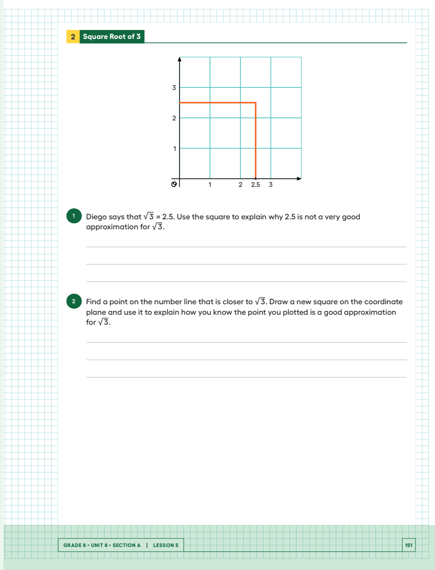
Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "First, I ____ because ..." and "I tried____, and what happened was ..."
Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook



Student Workbook



Activity 2

Square Root of 3

15 min

Activity Narrative

In previous activities and lessons, students used the areas of squares with whole number side lengths to find an approximation for the square root of an integer. In this activity, students start with the square root of an integer and then use a drawn square to explain why a given approximation of the square root is reasonable or not.

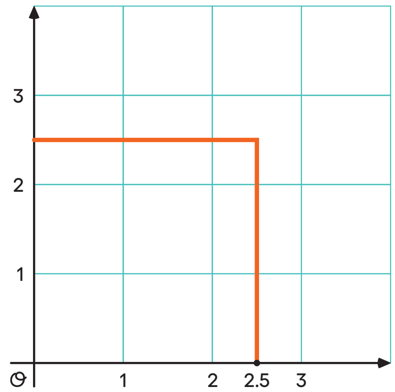
Launch

Arrange students in groups of 2. Provide access to four-function calculators without a square root button.

Display the diagram for all to see. Ask students what is the same and what is different about this diagram and diagrams they have seen in earlier activities. (This diagram also shows line segments on a coordinate plane. This diagram has 2 segments instead of 1. None of the line segments go through the origin.) If not mentioned by students, make sure to highlight how the vertices of the drawn square are not at the intersection of grid lines.

Give students 2–3 minutes of quiet work time followed by a partner then whole-class discussions.

Student Task Statement



1. Diego says that $\sqrt{3} \approx 2.5$. Use the square to explain why 2.5 is not a very good approximation for $\sqrt{3}$.

   Sample reasoning: The area of the square with sides $\sqrt{3}$ should be 3 square units. The area of the square drawn is larger than 4 square units because there are greater than 4 unit squares inside of it.

2. Find a point on the number line that is closer to $\sqrt{3}$. Draw a new square on the coordinate plane and use it to explain how you know the point you plotted is a good approximation for $\sqrt{3}$.

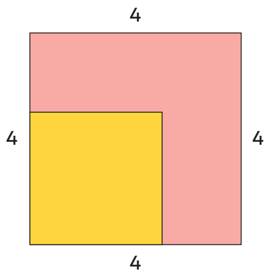
   Any value between 1.5 and 2 is reasonable.

   Sample response: A square drawn in with side length 1.7 has an area that is more than 2.25 but still less than 4 square units, making it a good approximation for $\sqrt{3}$.

Are You Ready for More?

A farmer has a grassy patch of land enclosed by a fence in the shape of a square with a side length of 4 meters. To make it a suitable home for some animals, the farmer would like to carve out a smaller square to be filled with water, as shown in the figure.

What should the side length of the smaller square be so that half of the area is grass and half is water?



The area enclosed by the fence is 16 square meters, so we want the area of both the grassy region and the water region to be 8 square meters. For the blue square in the figure to have an area of 8 square meters, the side length needs to be $\sqrt{8}$ meters, or about 2.8 meters.

Activity Synthesis

The goal of this discussion is to make sure students understand both a visual and an algebraic strategy for checking the value of square root approximations and to connect this thinking to the number line.

Display the diagram from the *Student Task Statement* for all to see. Invite 1–2 students to share their reasoning for why 2.5 is not a very good approximation for $\sqrt{3}$. Emphasize that squaring a point on the number line can be visualized as the area of a literal square sitting on the number line. This can help us estimate the value of a square root.

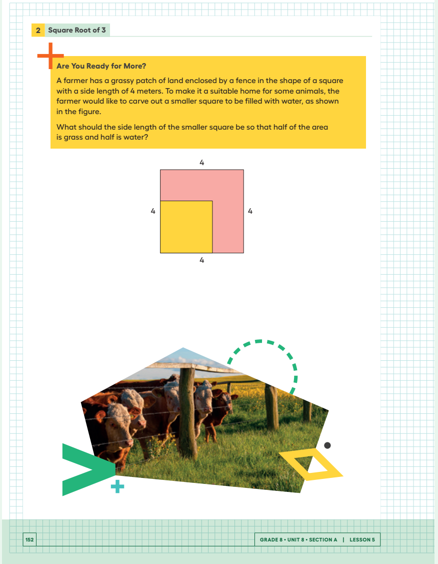
Ask students,

"2.5 might be a good approximation for the square root of what number?".

After a brief quiet think time, invite students to share their values. If not mentioned by students, make sure these two strategies are brought up:

- Estimate the area of the square by decomposing and rearranging squares and rectangles to get an area of 6.25 square units.
- Find the exact value by squaring. Since $2.5^2 = 6.25$, the square root of 6.25 is 2.5.

Student Workbook



Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to how they identified a point on the line that is closer to $\sqrt{3}$. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing, such as

> *"How do you know that $\sqrt{3}$ is between 1.5 and 2?"*

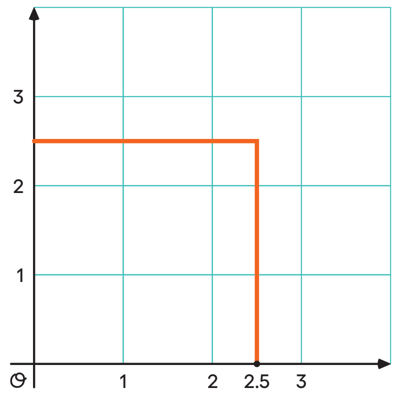
and

> *"How did you determine the area of the square that you drew?"*

Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Lesson Synthesis

The goal of this discussion is for students to see a strategy for determining if a value is a close approximation for a square root. Display this image for all to see:



Remind students how this square shows that 2.5 is not a good approximation for $\sqrt{3}$. Then ask students to check the squares of some numbers that are potential approximations of $\sqrt{3}$. For example, show how taking the square of 1.5 is 2.25, which is too low. Students should use each guess to refine their next guess—since 1.5 was too small, they may suggest trying 1.8 next.

Ask students to suggest decimal approximations, and check together as a class by finding their squares. Students might suggest an order like this:

$1.5^2 = 2.25$

$1.8^2 = 3.24$

$1.7^2 = 2.89$

$1.72^2 = 2.9584$

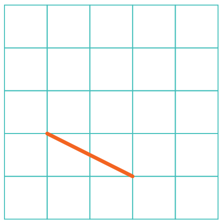
$1.73^2 = 2.9929$

So 1.73 is a pretty good approximation of $\sqrt{3}$. Reinforce this approximation by finding 1.73 on the x-axis of the diagram and drawing in the square. Briefly show students how to decompose and rearrange the square to see that the area of the new square is close to 3.

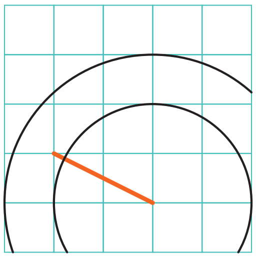
If time allows, ask students to use this strategy to find an approximation for $\sqrt{23}$ (approximately 4.7985).

Lesson Summary

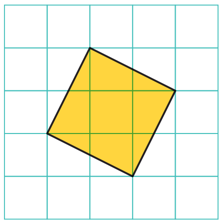
Here is a line segment on a grid. How can we determine the length of this line segment?



By drawing some circles, we can tell that it's longer than 2 units, but shorter than 3 units.



To find an exact value for the length of the segment, we can build a square on it, using the segment as one of the sides of the square.

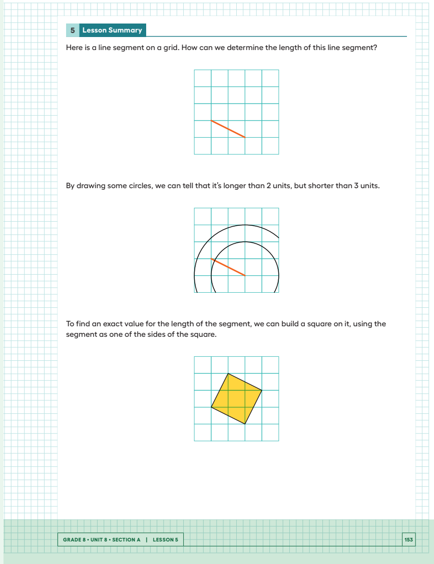


The area of this square is 5 square units. That means the exact value of the length of its side is $\sqrt{5}$ units.

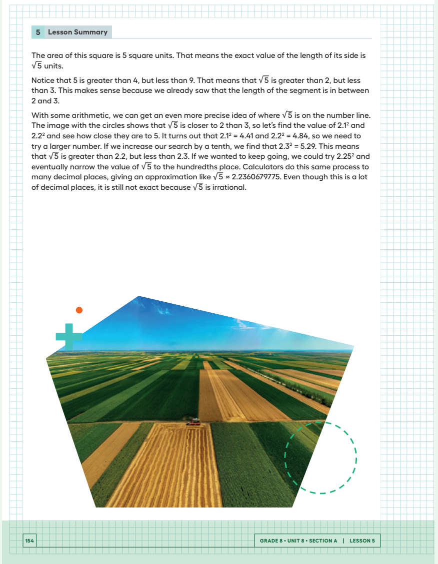
Notice that 5 is greater than 4, but less than 9. That means that $\sqrt{5}$ is greater than 2, but less than 3. This makes sense because we already saw that the length of the segment is in between 2 and 3.

With some arithmetic, we can get an even more precise idea of where $\sqrt{5}$ is on the number line. The image with the circles shows that $\sqrt{5}$ is closer to 2 than 3, so let's find the value of 2.1^2 and 2.2^2 and see how close they are to 5. It turns out that $2.1^2 = 4.41$ and $2.2^2 = 4.84$, so we need to try a larger number. If we increase our search by a tenth, we find that $2.3^2 = 5.29$. This means that $\sqrt{5}$ is greater than 2.2, but less than 2.3. If we wanted to keep going, we could try 2.25^2 and eventually narrow the value of $\sqrt{5}$ to the hundredths place. Calculators do this same process to many decimal places, giving an approximation like $\sqrt{5} \approx 2.2360679775$. Even though this is a lot of decimal places, it is still not exact because $\sqrt{5}$ is irrational.

Student Workbook



Student Workbook



Responding To Student Thinking

Points to Emphasize
If most students struggle with approximating square roots of irrational numbers, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, discuss the two whole numbers the square root of a number lies between and invite multiple students to share their thinking about its placement on a number line.

Unit 8, Lesson 6, Activity 2 Square Root Values

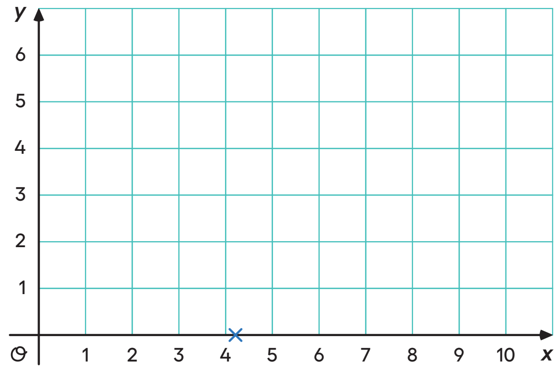
Cool-down

Approximating $\sqrt{18}$

5 min

Student Task Statement

Plot $\sqrt{18}$ on the x-axis.



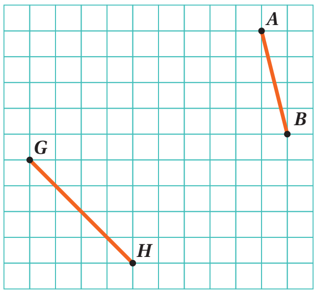
Lesson 5

Practice Problems

6 Problems

Problem 1

a. Find the exact length of each line segment.



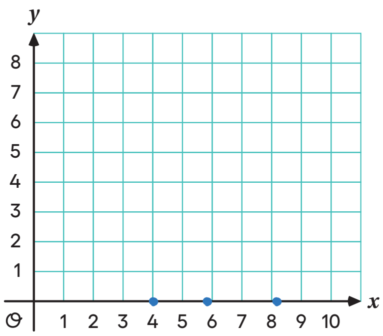
- $AB = \sqrt{17}$
- $GH = \sqrt{32}$

b. Estimate the length of each line segment to the nearest tenth of a unit. Explain your reasoning.

$AB \approx 4.1$, because $4.1^2 = 16.81$ and $4.2^2 = 17.64$. $GH \approx 5.7$, because $5.6^2 = 31.36$ and $5.7^2 = 32.49$

Problem 2

Plot each number on the x-axis: $\sqrt{16}$, $\sqrt{35}$, $\sqrt{66}$. Consider using the grid to help.



Sample response: $\sqrt{16}$ at 4, $\sqrt{35}$ at a little less than 6, $\sqrt{66}$ at a little more than 8

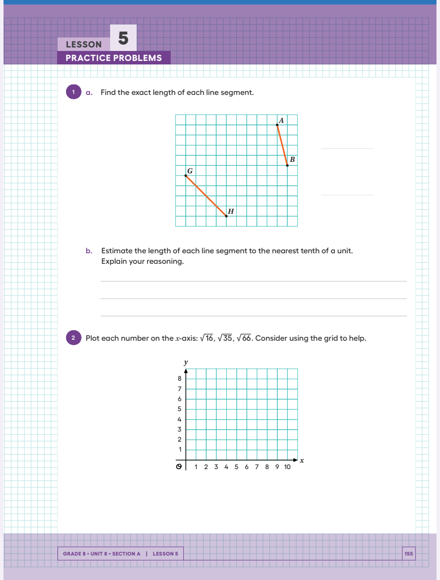
Problem 3

Use the fact that $\sqrt{7}$ is a solution to the equation $x^2 = 7$ to find a decimal approximation of $\sqrt{7}$, whose square is between 6.9 and 7.1.

Sample responses:

2.63, 2.64, 2.65, 2.66

Student Workbook



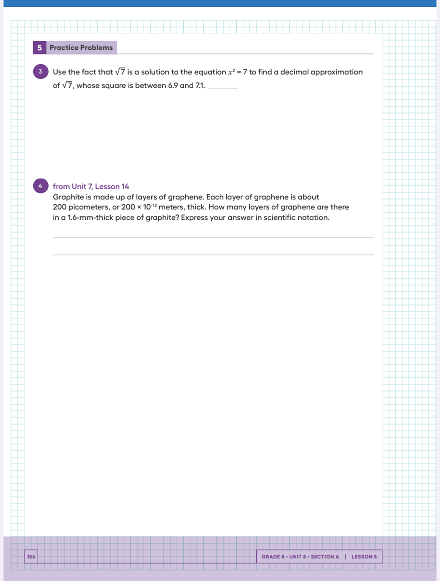
LESSON 5 PRACTICE PROBLEMS

1 a. Find the exact length of each line segment.

b. Estimate the length of each line segment to the nearest tenth of a unit. Explain your reasoning.

2 Plot each number on the x-axis: $\sqrt{16}$, $\sqrt{35}$, $\sqrt{66}$. Consider using the grid to help.

Student Workbook

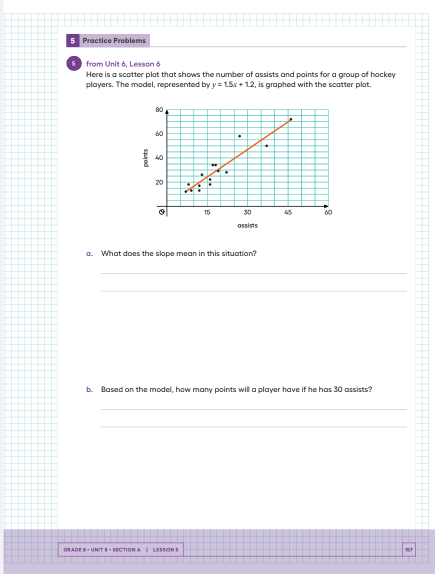


5 Practice Problems

3 Use the fact that $\sqrt{7}$ is a solution to the equation $x^2 = 7$ to find a decimal approximation of $\sqrt{7}$, whose square is between 6.9 and 7.1.

4 from Unit 7, Lesson 14
Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} meters, thick. How many layers of graphene are there in a 1.6-mm-thick piece of graphite? Express your answer in scientific notation.

Student Workbook


5 Practice Problems

5 from Unit 6, Lesson 6

Here is a scatter plot that shows the number of assists and points for a group of hockey players. The model, represented by $y = 1.5x + 1.2$, is graphed with the scatter plot.

a. What does the slope mean in this situation?

b. Based on the model, how many points will a player have if he has 30 assists?

Student Workbook


5 Practice Problems

6 from Unit 3, Lesson 5

The points (12, 23) and (14, 45) lie on a line. What is the slope of the line?

Learning Targets

+ I can find a decimal approximation for square roots.

+ I can plot square roots on the number line.

Problem 4

from Unit 7, Lesson 14

Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} meters, thick. How many layers of graphene are there in a 1.6-mm-thick piece of graphite? Express your answer in scientific notation.

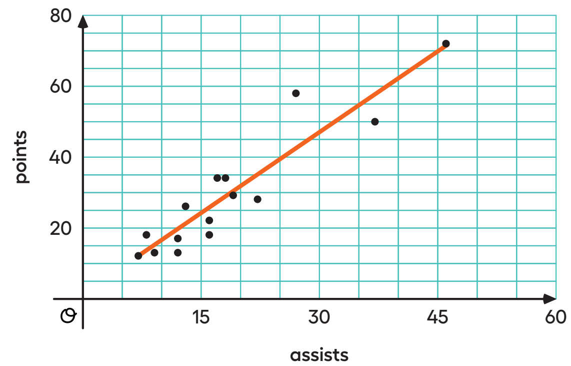
About 8×10^{6}

Sample reasoning: The thickness of the graphite is 1.6×10^{-3} meters. The number of layers of graphene is given by $\frac{1.6 \times 10^{-3}}{200 \cdot 10^{-12}} = 0.008 \times 10^{9}$. This number, in scientific notation, is 8×10^{6}, or about 8 million.

Problem 5

from Unit 6, Lesson 6

Here is a scatter plot that shows the number of assists and points for a group of hockey players. The model, represented by $y = 1.5x + 1.2$, is graphed with the scatter plot.



a. What does the slope mean in this situation?

Sample response: For every assist, a player's points goes up by 1.5.

b. Based on the model, how many points will a player have if he has 30 assists?

46.2 points

Problem 6

from Unit 3, Lesson 5

The points (12, 23) and (14, 45) lie on a line. What is the slope of the line?

$\frac{22}{2}$ (or equivalent)