Lesson 14

Surface Area of Right Prisms

Goals

- Calculate the surface area of a prism, and explain (in writing) the solution method.
- Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- Interpret different methods for calculating the surface area of a prism, and evaluate (orally and in writing) their usefulness.

Learning Target

- I can find and use shortcuts when calculating the surface area of a prism.
- I can picture the net of a prism to help me calculate its surface area.

Lesson Narrative

In this lesson, students find **surface areas** of prisms, and they see that the structure of a prism allows for shortcuts in adding up the areas of the faces. They see that if the prism is sitting on its base, then the vertical sides can be unfolded into a single rectangle whose height is the height of the prism and whose length is the perimeter of the base. The purpose of the lesson is not to come up with a formula for the surface area of a prism, but to help students see and make use of the structure of the prism to find surface area efficiently.

Student Facing Learning Goal

Let's look at the surface area of prisms.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Required Materials

Materials to Gather

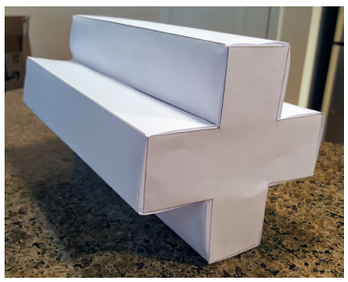
- Materials assembled from the blackline master: Lesson
- Materials assembled from the blackline master: Warm-up

Materials to Copy

- Multifaceted Cutouts (1 copy for every 1 student): Warm-up

Required Preparation

Lesson:
Assemble the net from the blackline master to make a prism with a base in the shape of a plus sign. Make sure to print the blackline master at 100% scale so the dimensions are accurate. This prism will be used for both the *Warm-up* and the following activity.



Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Multifaceted

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to recognize important parts of solids in anticipation of computing volume and surface area. The figure used in the next activity is introduced in this *Warm-up* as a way for students to start thinking about parts of solids and how we use them to compute surface area or volume.

Launch

Arrange students in groups of 2. Display the prism assembled from the blackline master for all to see. Give students 1 minute of quiet think time, followed by time to discuss their ideas with a partner. Follow this with a whole-class discussion.

Student Task Statement

Your teacher will show you a prism.

1. What are some things you could measure about the object?

   Sample response: You could measure the length of each of the edges of the object, the volume of the object, or the area of the faces.

2. What units would you use for these measurements?

   Sample response: Lengths could be measured in inches or centimeters. Volume could be measured in cubic inches, cubic centimeters, or milliliters. Area could be measured in square inches or square centimeters.

Activity Synthesis

Select students to share their responses. The goal of this discussion is to clarify what units would be reasonable and why. Here are some questions for discussion:

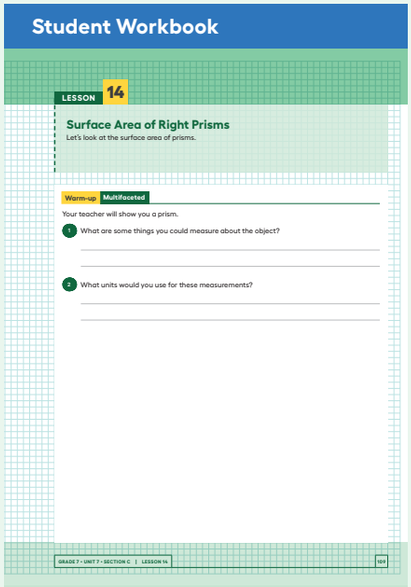
- *"What are some units that would not make sense for this situation?"*

  Miles, seconds

- *"Explain why those units would not make sense."*

  Miles would not be a good unit for length because miles are so large the measurement would be close to 0. Seconds would not be a good unit for volume because seconds measure time, and volume is a measurement of space.

Student Workbook


LESSON 14

Surface Area of Right Prisms

Let's look at the surface area of prisms.

Warm-up Multifaceted

Your teacher will show you a prism.

1. What are some things you could measure about the object?

2. What units would you use for these measurements?

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.
Keep books or devices closed. Display only the image and problem stem, without revealing the questions. Say,

"We are going to read this question 3 times."

After the 1st read, say,

"Tell your partner what this situation is about."

After the 2nd read, say,

"List the quantities. What can be counted or measured?"

For the 3rd read, reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Activity 1

So Many Faces

15 min

Activity Narrative

In this activity, students make sense of different methods for calculating the surface area of a figure. They then think about generalizing the methods to figure out if they would work for any prism. This activity connects to work they did with nets in a previous grade and builds upon strategies students might have to calculate surface area. Using the net helps students use the structure of a prism to make sense of the surface area. They do not need to generalize a formula for surface area at this time.

As students work on the task, monitor for students who understand the different methods and can explain if any of them will work for any other prisms.

Note: It is not important for students to learn the term "lateral area."

Launch

Arrange students in groups of 2. Display the prism assembled previously in the *Warm-up* for all to see. Ask students:

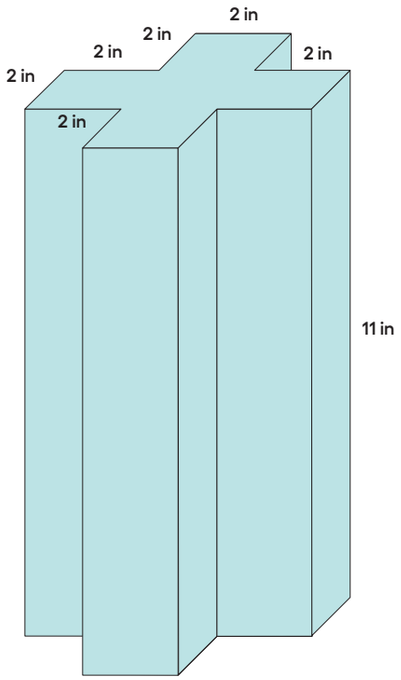
"How might we find surface area of this prism?"

If needed, remind students that the **surface area** of a polyhedron is the number of square units that covers all the faces of the polyhedron without any gaps or overlaps.

Invite students to share their ideas. Give students 1 minute of quiet think time to read Noah's method for calculating surface area, followed by time to discuss whether or not they agree with Noah. Repeat this process for the remaining two methods. After all three methods have been discussed, give students 1–2 minutes of quiet work time to answer the rest of the questions in the task statement.

Student Task Statement

Here is a picture of your teacher's prism:



Three students are trying to calculate the **surface area** of this prism.

- Noah says, "This is going to be a lot of work. We have to find the areas of 14 different faces and add them up."
- Elena says, "It's not so bad. All 12 rectangles are identical copies, so we can find the area for one of them, multiply that by 12, and then add on the areas of the 2 bases."
- Andre says, "Wait, I see another way! Imagine unfolding the prism into a net. We can use 1 large rectangle instead of 12 smaller ones."

1. Do you agree with any of them? Explain your reasoning.

Sample response: I agree with all three of them. Noah's method will work, but is not the most efficient. Elena's method is an improvement because we don't have to do the same calculation multiple times. Andre's method is more complicated to think about, but it should also work.

2. How big is the "1 large rectangle" that Andre is talking about? Explain or show your reasoning. If you get stuck, consider drawing a net for the prism.

The height of the rectangle will be the same as the height of the prism, 11 inches. The length of the rectangle must wrap around the entire base, so it will be the same as the perimeter of the base, 24 inches.

3. Will Noah's method always work for finding the surface area of any prism? Elena's method? Andre's method? Be prepared to explain your reasoning.

Noah's method will always work for any prism. Elena's method only works when each line segment in the base is the same length, so it will not work for all figures. Andre's method will work for all prisms because the long rectangle can fold around any base.

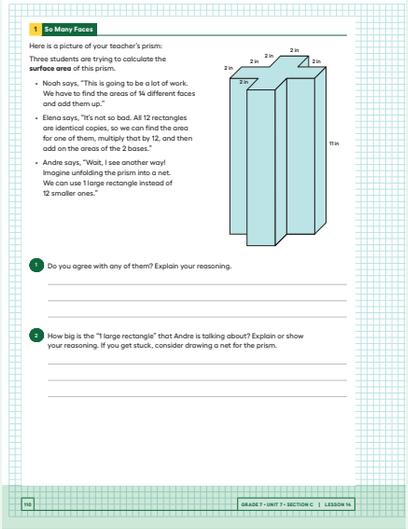
4. Which method do you prefer? Why?

Sample response: I prefer Andre's method because it is not too difficult once you understand it, and you need to find the areas of only two figures (the base and the long rectangle).

Building on Student Thinking

Students may think that Andre's method will not work for all prisms, because it will not work for solids that have a hole in their base and, therefore, more lateral area on the inside. Technically, those solids are not prisms, because their base is not a polygon. However, students could adapt Andre's method to find the surface area of a solid composed of a prism and a hole.

Student Workbook


1 So Many Faces

Here is a picture of your teacher's prism:

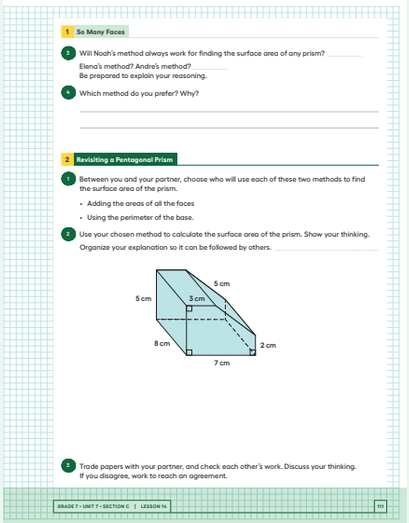
Three students are trying to calculate the **surface area** of this prism.

- Noah says, "This is going to be a lot of work. We have to find the areas of 14 different faces and add them up."
- Elena says, "It's not so bad. All 12 rectangles are identical copies, so we can find the area for one of them, multiply that by 12, and then add on the areas of the 2 bases."
- Andre says, "Wait, I see another way! Imagine unfolding the prism into a net. We can use 1 large rectangle instead of 12 smaller ones."

1 Do you agree with any of them? Explain your reasoning.

2 How big is the "1 large rectangle" that Andre is talking about? Explain or show your reasoning. If you get stuck, consider drawing a net for the prism.

Student Workbook


1 So Many Faces

3 Will Noah's method always work for finding the surface area of any prism? Elena's method? Andre's method? Be prepared to explain your reasoning.

4 Which method do you prefer? Why?

2 Revisiting a Pentagonal Prism

1 Between you and your partner, choose who will use each of these two methods to find the surface area of the prism.
- Adding the areas of all the faces
- Using the perimeter of the base.

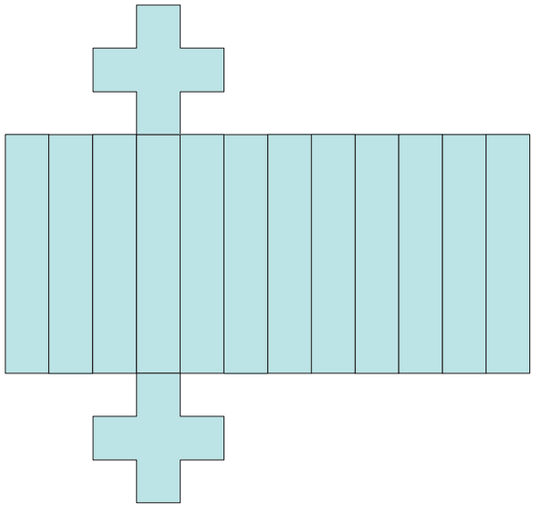
2 Use your chosen method to calculate the surface area of the prism. Show your thinking. Organize your explanation so it can be followed by others.

5 cm, 5 cm, 3 cm, 8 cm, 2 cm, 7 cm

3 Trade papers with your partner, and check each other's work. Discuss your thinking. If you disagree, work to reach an agreement.

Activity Synthesis

The purpose of this discussion is to show multiple methods of calculating surface area and when each method could work. Select previously identified students to share their reasoning. If not brought up in students' explanations, display the image for all to see and point out to students that the length of the "1 big rectangle" is equal to the perimeter of the base.



Students may have trouble generalizing which method would work for any prism. Here are some questions for discussion:

- *"Which of the students' methods will work for finding the surface area of this particular prism?"*

  all 3

- *"Which of the students' methods will work for finding the surface area of any prism?"*

  Noah's and Andre's

- *"Which of the students' methods will work for finding the surface area of other three-dimensional figures that are not prisms?"*

  only Noah's

If not mentioned by students, be sure that students understand:

- Noah's method will always work, but it can be inefficient if there are a lot of faces.
- Elena's method will not always work because the rectangles will not always be the same size, but we can notice that some shapes are the same and, therefore, not have to work out all of them individually.
- Andre's method does always work even if the rectangles have different widths. The length of the rectangle will be the same as the perimeter of the base, and the width of the rectangle will be the height of the prism.
- Prisms can always be cut into three pieces: two bases and one rectangle whose length is the perimeter of a base and whose width is the height of the prism. This can be more efficient than the other methods because students need to calculate only two areas (since the two bases will be identical copies).
- This method works only for prisms. For other shapes, such as pyramids, Noah's method of finding all the faces individually or Elena's method of combining those faces into identical copy groups will work. Solids with holes, such as the triangular prism with a square hole, can use a variation of Elena's method: two congruent triangles with holes for the bases, one rectangle for the outside side faces, and another rectangle for the faces that form the hole.

Explain to students that they will have the opportunity in the next activity to practice using any of these strategies.

Activity 2

Revisiting a Pentagonal Prism

15 min

Activity Narrative

In this activity, students are presented with a figure that was used in a previous lesson to explore volume. Here, they explore that figure's surface area and compare different methods of doing so based on their work in the previous task. Students work with a partner to share the task of investigating two methods to calculate the surface area.

As students work on the task, listen for students who find similarities and differences between the method they used and the one their partner used.

Launch

Arrange students in groups of 2. Tell students that they might recognize this figure from a previous lesson, but today they are going to compare two different methods for calculating its surface area. Give students 1–2 minutes of quiet work time, followed by time to trade their work with a partner to compare answers and methods. Follow this with a whole-class discussion.

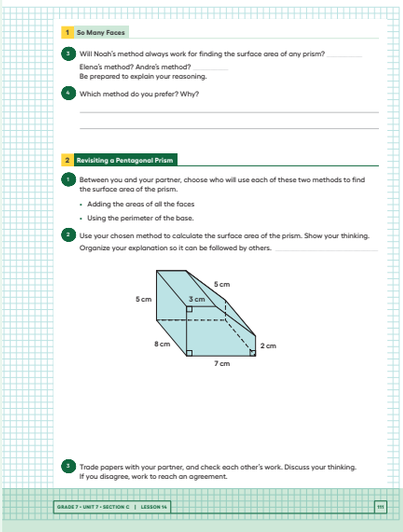
Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.
Display sentence frames to support partner discussion. Examples: "First, I ____ because ..." "I noticed ____ so I ... ____ reminds me of ____ because ..."
Advances: Writing, Conversing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code faces or edges of the figure.
Supports accessibility for: Visual-Spatial Processing

Student Workbook


1 So Many Faces

3 Will Noah's method always work for finding the surface area of any prism?
Elena's method? Andre's method?
Be prepared to explain your reasoning.

4 Which method do you prefer? Why?

2 Revisiting a Pentagonal Prism

1 Between you and your partner, choose who will use each of these two methods to find the surface area of the prism.
- Adding the areas of all the faces
- Using the perimeter of the base.

2 Use your chosen method to calculate the surface area of the prism. Show your thinking. Organize your explanation so it can be followed by others.

5 cm
5 cm
3 cm
8 cm
2 cm
7 cm

3 Trade papers with your partner, and check each other's work. Discuss your thinking. If you disagree, work to reach an agreement.

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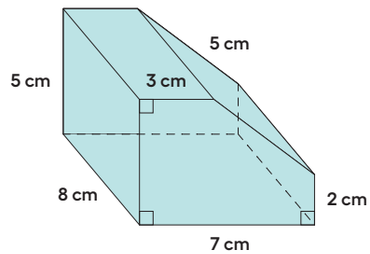
Student Task Statement

1. Between you and your partner, choose who will use each of these two methods to find the surface area of the prism.
   - Adding the areas of all the faces
   - Using the perimeter of the base.

   No written response required.

2. Use your chosen method to calculate the surface area of the prism. Show your thinking. Organize your explanation so it can be followed by others.



The surface area is 234 cm^2.

Sample responses:

- Adding the areas of all the faces: There are two pentagonal bases that can each be decomposed into two rectangles (3 cm by 5 cm, and 4 cm by 2 cm) and a right triangle (base 4 cm and height 3 cm). Each pentagon has an area of 29 cm^2, since $15 + 8 + 6 = 29$. There are five rectangular faces, each with a side that is 8 cm. Their combined area is 176 cm^2, since $(3 \cdot 8) + (5 \cdot 8) + (7 \cdot 8) + (2 \cdot 8) + (5 \cdot 8) = 24 + 40 + 56 + 16 + 40 = 176$. The sum of the areas of the bases and the rectangles is 234 cm^2, since $2(29) + 176 = 58 + 176 = 234$.
- Using the perimeter of the base: There are two pentagonal bases that have an area of 29 cm^2, since $15 + 8 + 6 = 29$. The perimeter of the pentagonal base is 22 cm, since $2 + 5 + 3 + 5 + 7 = 22$. All the rectangular faces, if unfolded, make a long rectangle that is 22 cm by 8 cm, so its area is 176 cm^2, $22 \cdot 8 = 176$. The sum of the areas of the two bases and the long rectangle is 234 cm^2, since $2(29) + 176 = 58 + 176 = 234$.

3. Trade papers with your partner, and check each other's work. Discuss your thinking. If you disagree, work to reach an agreement.

Are You Ready for More?

In a deck of cards, each card measures 6 cm by 9 cm.

1. When stacked, the deck is 2 cm tall, as shown in the first photo. Find the volume of this deck of cards.

   108 cm^3

2. Then the cards are fanned out, as shown in the second picture. The distance from the rightmost point on the bottom card to the rightmost point on the top card is now 7 cm instead of 2 cm. Find the volume of the new stack.

   108 cm^3

Activity Synthesis

The purpose of this discussion is to compare and contrast the two methods for finding the surface area of a prism. Select previously identified students to share the discussion they had with their partner. Here are some possible discussion questions:

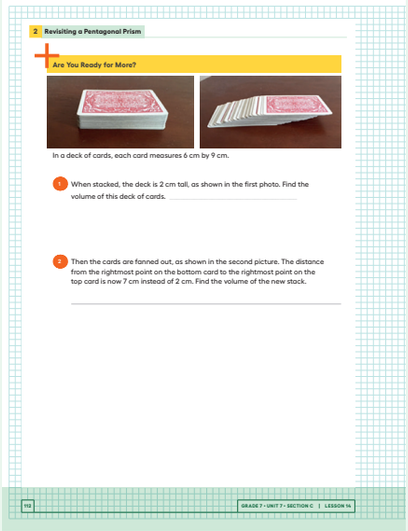
- *"How did you find the area of the base?"*
- *"How did you find any other areas that you needed, in order to solve the problem?"*
- *"When you used using the first method (calculating the area of all the faces), how many different shapes did you need to calculate the area of?"*
- *"When you used the second method (using the perimeter of the base), how many different shapes did you need to calculate the area of?"*
- *"Which method do you prefer for this problem? Why?"*
- *"Do you think you will prefer the same method for every problem? Why or why not?"*
- *"What would make you change methods?"*
- *"Do you need to know all of the measurements in the picture to solve for surface area?"*

  No, you need to know just the perimeter and area of the base and the height of the figure.

- *"Could you solve for volume with the measurements given in the picture? If so, are there any unnecessary measurements? If not, what else would you need to know?"*

  Yes, you can solve for the volume with the given measurements, and there are no unnecessary measurements.

Student Workbook


2 Revisiting a Pentagonal Prism

Are You Ready for More?

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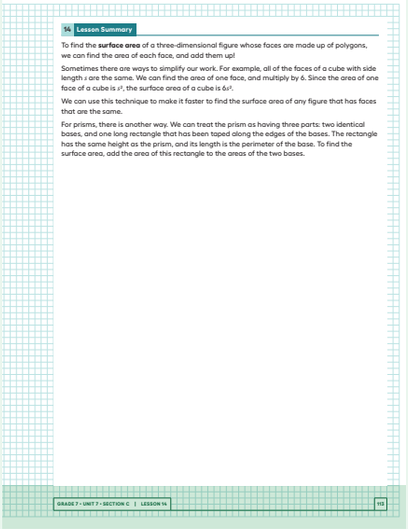
Student Workbook



14 Lesson Summary

To find the **surface area** of a three-dimensional figure whose faces are made up of polygons, we can find the area of each face, and add them up!

Sometimes there are ways to simplify our work. For example, all of the faces of a cube with side length s are the same. We can find the area of one face, and multiply by 6. Since the area of one face of a cube is s^2, the surface area of a cube is $6s^2$.

We can use this technique to make it faster to find the surface area of any figure that has faces that are the same.

For prisms, there is another way. We can treat the prism as having three parts: two identical bases, and one long rectangle that has been taped along the edges of the bases. The rectangle has the same height as the prism, and its length is the perimeter of the base. To find the surface area, add the area of this rectangle to the areas of the two bases.

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If not brought up in students' explanations, explain to students that the first method requires finding the area of 6 different shapes (there are 7 faces, but the two bases are the same). While the calculations using this method were simple, there were more pieces. The second method requires visualizing the solid in a different way, but we needed to find only the area of two different pieces (the long rectangle and the base).

Lesson Synthesis

The purpose of this discussion is for students to articulate their understanding of surface area of a prism. Here are some questions for discussion:

- *"What is surface area?"*

  The total area of all the exposed faces of an object.

- *"What are some methods for calculating surface area of prisms?"*

  Find the area of each face and add them for the total. Find groups of faces that have the same area and save some computation. Find the area of the bases and add that to the area of a "long rectangle."

Lesson Summary

To find the **surface area** of a three-dimensional figure whose faces are made up of polygons, we can find the area of each face, and add them up!

Sometimes there are ways to simplify our work. For example, all of the faces of a cube with side length s are the same. We can find the area of one face, and multiply by 6. Since the area of one face of a cube is s^2, the surface area of a cube is $6\,s^2$.

We can use this technique to make it faster to find the surface area of any figure that has faces that are the same.

For prisms, there is another way. We can treat the prism as having three parts: two identical bases, and one long rectangle that has been taped along the edges of the bases. The rectangle has the same height as the prism, and its length is the perimeter of the base. To find the surface area, add the area of this rectangle to the areas of the two bases.

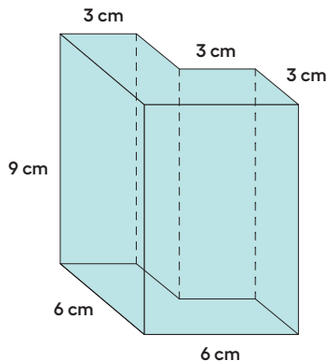
Cool-down

Surface Area of a Hexagonal Prism

5 min

Student Task Statement

Find the surface area of this prism. Show your reasoning. Organize your explanation so it can be followed by others.



The surface area is 270 cm^2.

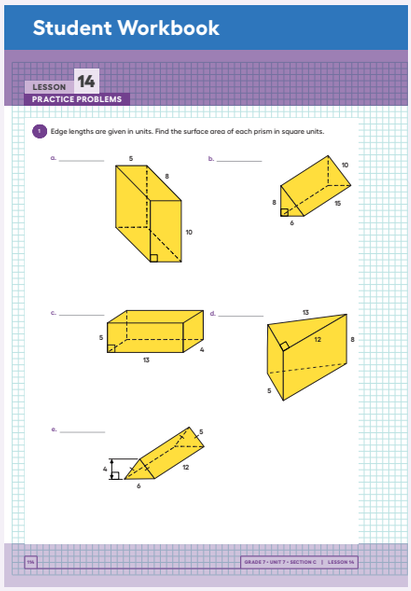
Possible strategy: The area of the base is 27 cm^2. The perimeter of the base is 24 cm, so the combined area of the sides is 216 cm^2, because $24 \cdot 9 = 216$. Therefore the total surface area is 270 cm^2, because $27 \cdot 2 + 216 = 270$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

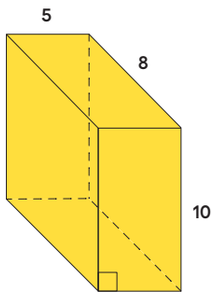
6 Problems



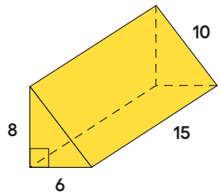
Problem 1

Edge lengths are given in units. Find the surface area of each prism in square units.

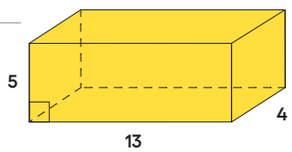
a. 340



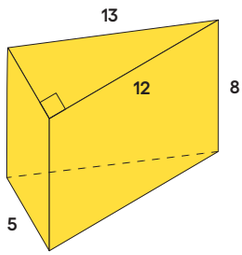
b. 408



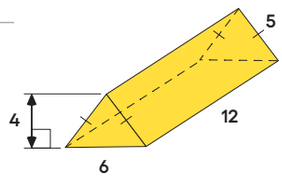
c. 274



d. 300



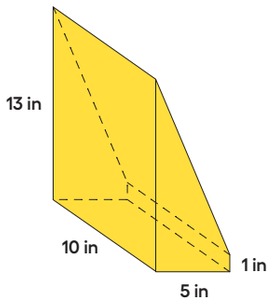
e. 216



Problem 2

from Unit 7, Lesson 13

A rectangular prism with dimensions 5 inches by 13 inches by 10 inches was cut to leave a piece as shown in the image.



a. What is the volume of this piece?

350 cubic inches

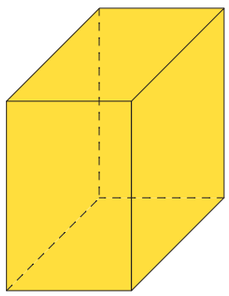
b. What is the volume of the other piece not pictured?

300 cubic inches

Problem 3

from Unit 7, Lesson 11

Priya says, "No matter which way you slice this rectangular prism, the cross section will be a rectangle." Mai says, "I'm not so sure." Describe a slice that Mai might be thinking of.

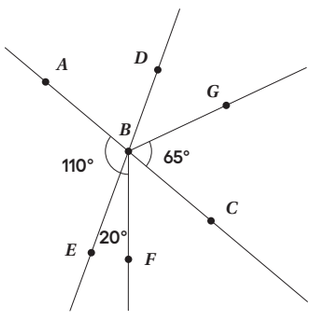


If you keep your slices parallel to a set of faces, then the cross section does have to be a rectangle. But if you can slice in any direction, you can get a triangle. Imagine slicing off one small corner of the prism.

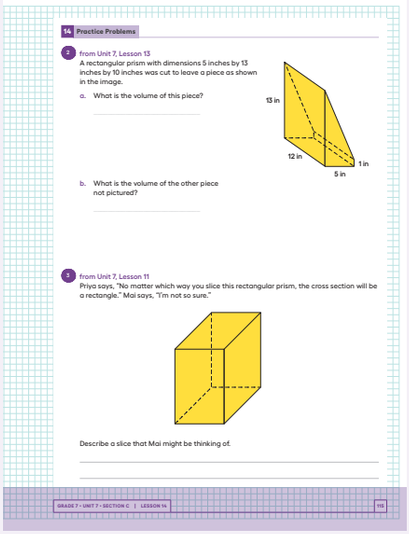
Problem 4

from Unit 7, Lesson 5

B is the intersection of line AC and line ED. Find the measurement of each of the angles.



Student Workbook


14 Practice Problems

2 from Unit 7, Lesson 13
A rectangular prism with dimensions 5 inches by 13 inches by 10 inches was cut to leave a piece as shown in the image.

a. What is the volume of this piece?

13 in
12 in
1 in
5 in

b. What is the volume of the other piece not pictured?

3 from Unit 7, Lesson 11
Priya says, "No matter which way you slice this rectangular prism, the cross section will be a rectangle." Mai says, "I'm not so sure."

Describe a slice that Mai might be thinking of.

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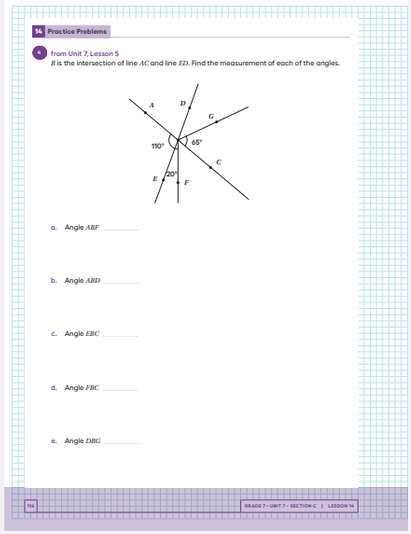
14 Practice Problems

5 from Unit 7, Lesson 5
B is the intersection of line AC and line ED. Find the measurement of each of the angles.

a. Angle ABF _____

b. Angle ABD _____

c. Angle EBC _____

d. Angle FBC _____

e. Angle DBG _____

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14 Practice Problems

5 from Unit 6, Lesson 20
Write each expression with fewer terms.

a. $12m - 4m$ _____

b. $12m - 5k + m$ _____

c. $9m + k - (3m - 2k)$ _____

6 from Unit 4, Lesson 9

a. Find 44% of 625 using the facts that 40% of 625 is 250 and 4% of 625 is 25. _____

b. What is 4.4% of 625? _____

c. What is 0.44% of 625? _____

Learning Targets
+ I can find and use shortcuts when calculating the surface area of a prism.
+ I can picture the net of a prism to help me calculate its surface area.

a. Angle ABF

130 degrees

Sample reasoning: Sum of angles ABE and EBF

b. Angle ABD

70 degrees

Sample reasoning: Supplementary with angle ABE

c. Angle EBC

70 degrees

Sample reasoning: Vertical with ABD

d. Angle FBC

50 degrees

Sample reasoning: Subtract the measurement of angle EBF from the measurement of angle EBC

e. Angle DBG

45 degrees

Sample reasoning: Subtract the measurements of angles ABD and CBG from 180°

Problem 5

from Unit 6, Lesson 20

Write each expression with fewer terms.

a. $12m - 4m$

$8m$

b. $12m - 5k + m$

$13m - 5k$

c. $9m + k - (3m - 2k)$

$6m + 3k$

Problem 6

from Unit 4, Lesson 9

a. Find 44% of 625 using the facts that 40% of 625 is 250 and 4% of 625 is 25.

275, because 44% of a number equals 40% of the number plus an additional 4% of the number.

b. What is 4.4% of 625?

27.5

c. What is 0.44% of 625?

2.75