Lesson 2

Introduction to Functions

Goals

- Comprehend the structure of a function as having one and only one output for each allowable input.
- Describe (orally and in writing) a context using function language, e.g., "[The output] is a function of [the input]" or "[The output] depends on [the input]."
- Identify (orally) rules that produce exactly one output for each allowable input and rules that do not.

Learning Targets

- I know that a function is a rule with exactly one output for each allowable input.
- I know that if a rule has exactly one output for each allowable input, then the output depends on the input.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display

Lesson Narrative

The purpose of this lesson is to introduce students to the term "function." A **function** is a rule that produces a single output for a given input.

A key aspect of functions is that for each input, there is one and only one output. The *Warm-up* primes students for this idea by inviting them to consider a list of positive and negative numbers, including 3 and -3, and to consider what happens to the list after each number is squared.

In the next activities, students first consider if a given statement is enough to uniquely answer the question that follows, and if it is, they draw an input-output diagram. Then, they use the statements and diagrams to describe the inputs and outputs more precisely by using function language, such as "The output is a function of the input" and "The output depends on the input".

In the optional activity, students see how the same function can be described in different ways but still have the same input-output pairs. This helps solidify the notion of a function as something different from the method of calculating its values.

Student Learning Goal

Let's learn what a function is.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	5 min	10 min	5 min

Student Workbook


LESSON 2

Introduction to Functions

Let's learn what a function is.

Warm-up Square Me

Here are some numbers in a list:

$1, -3, -\frac{1}{2}, 3, 2, \frac{1}{4}, 0.5$

1. How many different numbers are in the list?
2. Make a new list containing the squares of all these numbers.
3. How many different numbers are in the new list?
4. Explain why the two lists do not have the same number of different numbers.

1 You Know This, Do You Know That?

Say yes or no for each question. If yes, draw an input-output diagram. If no, give examples of two different outputs that are possible for the same input.

1. A bookshelf is 5.5 feet tall. Do you know its height in inches?

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Warm-up
Square Me

5 min

Activity Narrative

The purpose of this *Warm-up* is to remind students that two different numbers can have the same square. This is an example of two inputs having the same output for a given rule—in this case, the rule is "Square the number." Later activities in the lesson explore rules that have multiple outputs for the same input.

Launch

Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Here are some numbers in a list:

$$1, -3, -\frac{1}{2}, 3, 2, \frac{1}{4}, 0.5$$

1. How many different numbers are in the list?

   7

2. Make a new list containing the squares of all these numbers.

   $1, 9, \frac{1}{4}, 9, 4, \frac{1}{16}, 0.25$

3. How many different numbers are in the new list?

   5

4. Explain why the two lists do not have the same number of different numbers.

   Sample response: Some numbers in the list are different but have the same square. This can happen because a negative times a negative is a positive. For example, -3 squared is 9, and 3 squared is also 9.

Activity Synthesis

The focus of this discussion should be on the final question, which, even though the language isn't used in the problem, helps prepare students for thinking about the collection of values that make up the inputs and outputs for given rules. Here, the input is a list of 7 unique values, while the output has only 5 unique values.

Invite students to share their responses to the second problem, and display this new list of numbers along with the original list for all to see. Next, invite different students to share their explanations from the forth problem. Emphasize the idea that when we square a negative number, we get a positive number. This means two different numbers can have the same square, or using the language of inputs and outputs, two different inputs can have the same output for a rule.

If time allows, ask,

"Can you think of other rules where different inputs can have the same output?"

After 30 seconds of quiet think time, select students to share their rules. They may recall the previous lesson where they encountered the rule "Write 7," which has only 1 output for all inputs, and the rule "Name the digit in the tenths place," which has only 10 unique outputs for all inputs.

Activity 1

You Know This, Do You Know That?

15 min

Activity Narrative

In this activity students are presented with a series of questions similar to "A table is 60 inches wide. Do you know its width in feet?" For some of the questions, the answer is yes (because we can convert from inches to feet by dividing by 12). In other cases, the answer is no (for example, "A person is 14 years old. Do you know their height?"). The purpose is to develop students' understanding of the structure of a function as something that has one and only one output for each allowable input. In cases where the answer is yes, students draw an input-output diagram with the rule in the box. In cases where the answer is no, they give examples of an input with two or more outputs. In the *Activity Synthesis*, the word "function" is introduced to students for the first time.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.
Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language, Attention

Student Workbook

LESSON 2

Introduction to Functions

Let's learn what a function is.

Warm-up Square Me

Here are some numbers in a list:

$1, -3, -\frac{1}{2}, 3, 2, \frac{1}{4}, 0.5$

1. How many different numbers are in the list? ______
2. Make a new list containing the squares of all these numbers. ______
3. How many different numbers are in the new list? ______
4. Explain why the two lists do not have the same number of different numbers.

1 You Know This, Do You Know That?

Say yes or no for each question. If yes, draw an input-output diagram. If no, give examples of two different outputs that are possible for the same input.

1. A bookshelf is 5.5 feet tall. Do you know its height in inches? ______

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Student Workbook

1 You Know This, Do You Know That?

2. A number is 5. Do you know its square? ______
3. The square of a number is 16. Do you know the number? ______
4. A square has a perimeter of 12 cm. Do you know its area? ______
5. A rectangle has an area of 16 cm^2. Do you know its length? ______

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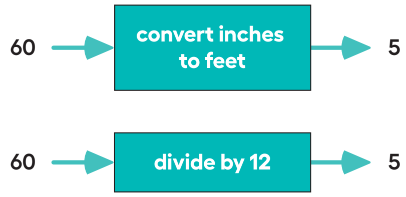
Launch

Display the example statement (but not the input-output diagram) for all to see.

Example: A table is 60 inches wide. Do you know its width in feet?

Give students 30 seconds of quiet think time, and ask them to be prepared to justify their response. Select students to share their answers, recording and displaying different justifications for all to see.

Display the following input-output diagrams for all to see. Ask students if the rules in the diagrams match the justifications they just heard:



Tell students that they will draw input-output diagrams like these as part of the task. Answer any questions students might have about the input-output diagrams. Be sure students understand that if they answer yes to the question, they will need to draw the input-output diagram, and if they answer no, they need to give an example of why the question does not have one answer.

Give students 8–10 minutes of quiet work time for the problems, and follow with a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to decide if the answer to the question is yes or no. Display words and phrases, such as "There is more than one possible output," "If the input is a number, the output is one-fifth times that number," and "This rule works for any input but zero since we cannot divide by zero."

Student Task Statement

Say yes or no for each question. If yes, draw an input-output diagram. If no, give examples of two different outputs that are possible for the same input.

1. A bookshelf is 5.5 feet tall. Do you know its height in inches?

   Yes, multiply a bookshelf's height in feet by 12 to get its height in inches. Since 5.5(12) = 66, the bookshelf is 66 inches tall.

2. A number is 5. Do you know its square?

   Yes, the square of 5 is 25.

3. The square of a number is 16. Do you know the number?

No, there are two different numbers whose square is 16, namely 4 and -4.

4. A square has a perimeter of 12 cm. Do you know its area?

Yes, a square with perimeter of 12 cm must have a side length of 3 cm, so it's area is 9 cm^2.

$$9 \rightarrow \boxed{\text{find the side length, and square it}} \rightarrow 12$$

5. A rectangle has an area of 16 cm^2. Do you know its length?

No, a rectangle with length 8 cm and width 2 cm has area 16 cm^2, as does a rectangle with length 16 cm and width 1 cm.

6. You are given a number. Do you know the number that is $\frac{1}{5}$ as big?

Yes, for any number, we can simply divide that number by 5, or multiply it by $\frac{1}{5}$.

$$\text{input} \rightarrow \boxed{\text{multiply by } \tfrac{1}{5}} \rightarrow \tfrac{1}{5} \text{ times the input}$$

7. You are given a number. Do you know its reciprocal?

Yes, we know the reciprocal as long as the number isn't 0 (since 0 doesn't have a reciprocal).

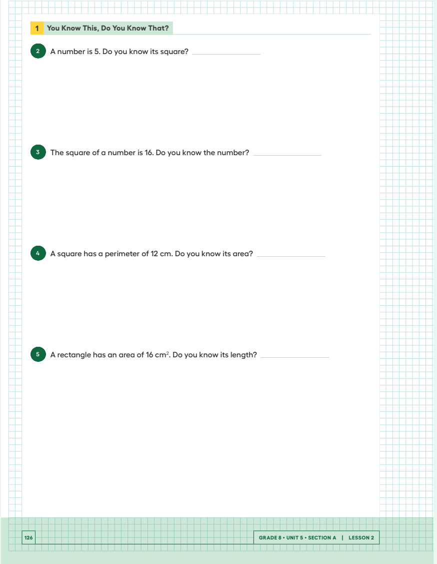
$$x \rightarrow \boxed{\text{divide 1 by the input}} \rightarrow \frac{1}{x}$$

Activity Synthesis

The goal of this discussion is for students to understand that functions are rules that have one distinct output for each input.

Direct students' attention to the reference created using *Collect and Display*. Ask previously selected students to share the rules they identified for questions they answered yes to. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. For example, the display may already have a rule for finding the area of the square from its perimeter as "Find the side length, then square it," and next to it can be added "Divide by four, then square it." For the final two problems where the input is not a specific value, if no student used a letter to stand in for the input, invite them to consider how to do so now, and record their thinking on the display. For example, next to "$\frac{1}{5}$ of a number" could go "$\frac{1}{5}n$."

Student Workbook


1 You Know This, Do You Know That?

2 A number is 5. Do you know its square?

3 The square of a number is 16. Do you know the number?

4 A square has a perimeter of 12 cm. Do you know its area?

5 A rectangle has an area of 16 cm^2. Do you know its length?

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Student Workbook


1 You Know This, Do You Know That?

6 You are given a number. Do you know the number that is $\frac{1}{5}$ as big?

7 You are given a number. Do you know its reciprocal?

2 Using Function Language

Here are the questions from the previous activity. For the ones you said yes to, write a statement like "The height a rubber ball bounces to depends on the height it was dropped from" or "Bounce height is a **function** of drop height." For all of the ones you said no to, write a statement like "The day of the week does not determine the temperature that day" or "The temperature that day is not a function of the day of the week."

1 A bookshelf is 5.5 feet tall. Do you know its height in inches?

2 A number is 5. Do you know its square?

3 The square of a number is 16. Do you know the number?

4 A square has a perimeter of 12 cm. Do you know its area?

5 A rectangle has an area of 16 cm^2. Do you know its length?

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Tell students that each time they answered one of the questions with a yes, the sentence defined a *function*, and that one way to represent a function is by writing a rule to define the relationship between the input and the output. **Functions** are special types of rules where each input has only one possible output. Functions are useful since once we know an input, we can find the single output that goes with it. Contrast this with something like a rolling a number cube where the input "roll" has many possible outputs. For the questions students responded to with no, these are not functions because there is no single output for each input.

To highlight how rules that are not functions do not determine outputs in a unique way, end the discussion by asking:

- *"Was the problem in the* Warm-up *where you had to square numbers an example of a function?"*

  Yes, each input has only one output, even though some inputs have the same output.

- *"Is the reverse—that is, knowing what number was squared to get a specific number—a function?"*

  No, if a number squared is 16, we don't know if the number was 4 or -4.

Activity 2
Using Function Language

15 min

Activity Narrative

In this activity students revisit the questions in the previous activity and start using the language of functions to describe the way one quantity depends on another. For the "yes" questions, students write a statement like "[The output] depends on [the input]" or "[The output] is a function of [the input]." For the "no" questions, they write a statement like "[The output] does not depend on [the input]." Students will use this language throughout the rest of the unit and course when describing functions.

Depending on the time available and students' needs, only a subset of the questions, such as just the odds, may be assigned.

Launch

Display the example statement from the previous activity ("A table is 60 inches wide. Do you know its width in feet?") for all to see. Tell students that since the answer to this question is yes, we can write a statement like "The width in feet depends on the width in inches" or "The width in feet is a function of the width in inches."

Arrange students in groups of 2.

Give students 5–8 minutes of quiet work time and then additional time to share their responses with their partner.

If they have a different response than their partner, encourage them to explain their reasoning and try to reach agreement. Follow with a whole-class discussion.

Student Task Statement

Here are the questions from the previous activity. For the ones you said yes to, write a statement like “The height a rubber ball bounces to depends on the height it was dropped from” or “Bounce height is a **function** of drop height.” For all of the ones you said no to, write a statement like “The day of the week does not determine the temperature that day” or “The temperature that day is not a function of the day of the week.”

1. A bookshelf is 5.5 feet tall. Do you know its height in inches?

   Yes, height in feet depends on the height in inches.

2. A number is 5. Do you know its square?

   Yes, the square of a number depends on the number.

3. The square of a number is 16. Do you know the number?

   No, knowing the square of a number does not determine the number.

4. A square has a perimeter of 12 cm. Do you know its area?

   Yes, the area of a square is determined by the perimeter of the square.

5. A rectangle has an area of 16 cm^2. Do you know its length?

   No, knowing the area of a rectangle does not determine its length.

6. You are given a number. Do you know the number that is $\frac{1}{5}$ as big?

   Yes, every number determines the number that is $\frac{1}{5}$ as large.

7. You are given a number. Do you know its reciprocal?

   Yes, every nonzero number determines its reciprocal.

Activity Synthesis

The goal of this discussion is for students to use the language like “[The output] depends on [the input]” and “[The output] is a function of [the input]” while recognizing that a “function” means each input gives exactly one output.

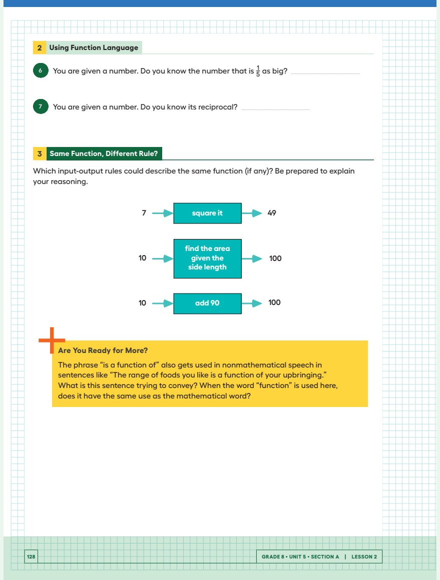
Begin the discussion by asking students if any of them had a different response from their partner that they were not able to reach agreement on. If any groups say yes, ask both partners to share their responses. Next, select groups to briefly share their responses for the other questions, and address any questions. For example, students may have a correct answer but be unsure since they used different wording than the person who shared their answer verbally with the class.

If time permits, give groups 1–2 minutes to invent a new question like the ones in the task that is not a function.

Select 2–3 groups to share their question, and ask a different group to explain why it is not a function using language like “[The input] does not determine [the output] because …”

Student Workbook

Student Workbook



Access for Multilingual Learners (Activity 3, Launch)

Writing, Listening, Speaking: MLR1: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their explanations for

> *"Which input-output rules could describe the same function (if any)?"*

Give students time to meet with 2–3 partners to share and get feedback on their responses. Provide prompts for feedback that will help students strengthen their ideas and clarify their language (for example,

> *"Can you say more about why?"*
> *"Can you give an example?"*

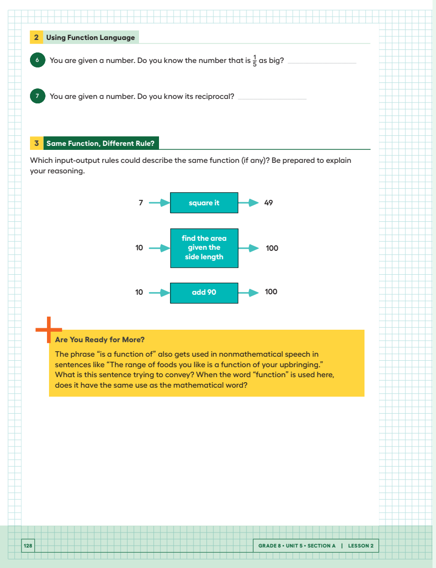
and

> *"How could you say that another way?").*

Give students 1–2 minutes to revise their writing based on the feedback they received.

Design Principle(s): Optimize output (for explanation)

Student Workbook


2 Using Function Language
You are given a number. Do you know the number that is $\frac{1}{3}$ as big?
You are given a number. Do you know its reciprocal?
3 Same Function, Different Rule?
Which input-output rules could describe the same function (if any)? Be prepared to explain your reasoning.
Are You Ready for More?
The phrase "is a function of" also gets used in nonmathematical speech in sentences like "The range of foods you like is a function of your upbringing." What is this sentence trying to convey? When the word "function" is used here, does it have the same use as the mathematical word?

Activity 3: Optional
Same Function, Different Rule?

5 min

Activity Narrative

The activity calls back to a previous lesson in which students filled out tables of values from input-output diagrams. Here, students determine if a rule is describing the same function but with different words, giving them an opportunity to look for and make use of the structure of a function.

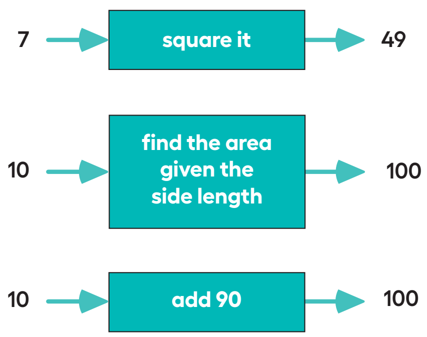
Students are given three different input-output diagrams and need to determine which rules could describe the same function. A key point in this activity is that context plays an important role. For example, if the first rule is limited to positive inputs and the second rule is about sides of squares (which also has only positive inputs), then the two input-output rules describe the same function.

Launch

Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Which input-output rules could describe the same function (if any)? Be prepared to explain your reasoning.



The first two rules define the same function if we are talking about the area of a square. Since the area of a square is the square of its side length, the second rule is the same as taking the input and squaring it to find the output. On the other hand, even though adding 90 to 10 gives the same result as squaring 10, this third rule does not define the same function as the first two. For example, adding 90 to 20 gives 110, which is not the same as squaring 20, or finding the area of the square with side length 20.

Are You Ready for More?

The phrase "is a function of" also gets used in nonmathematical speech in sentences like "The range of foods you like is a function of your upbringing." What is this sentence trying to convey? When the word "function" is used here, does it have the same use as the mathematical word?

Answers vary.

Activity Synthesis

The goal of this discussion is for students to explain how two different rules can describe the same function and that two functions are the same if and only if all of their input-output pairs are the same.

Consider asking some of the following questions:

- *"Do the latter two input-output rules describe the same function since they both take an input of 10 to an output of 100?"*

  No, every input-output pair needs to match in order for the two rules to describe the same function, not just a few input-output pairs.

- *"Do any of the input-output rules describe the same function?"*

  Yes, if the first is restricted to positive inputs and the second is about areas of squares, then they share the same input-output pairs and describe the same function.

Access for Multilingual Learners (Activity 3, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to

"Which input-output rules could describe the same function (if any)?"

Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Lesson Synthesis

The purpose of this lesson was to define functions as rules that assign exactly one output to each allowable input. We say things like "the output is a **function** of the input" and "the output depends on the input" when talking about the relationship between inputs and outputs of functions.

To highlight the language and definition of functions from the lesson, ask:

- *"How else could we describe the function with the rule 'Double the input'?"*

  Multiply the input by 2, or if the input is x, multiply by $2x$.

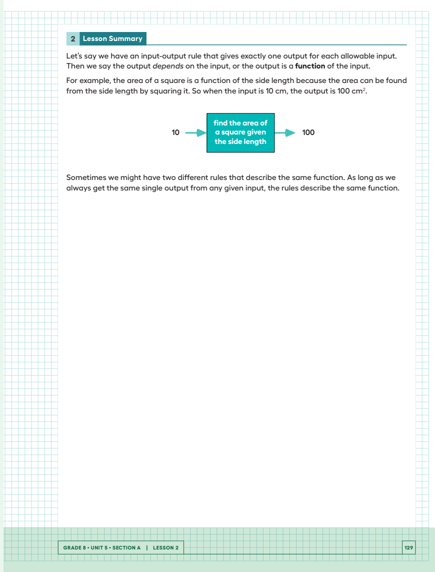
- *"Does the description 'the radius of a circle with circumference C' define a function? Why or why not?"*

  Yes, this description defines a function because the circumference depends on the radius of the circle and each radius gives only one circumference.

- *"Why does the description 'A person has \$5 in their pocket. How many dollar bills do they have in their pocket?' not define a function?"*

  The total amount of money in the pocket does not determine the number of dollar bills in the pocket. There are many ways to have a total of \$5, from a single \$5 bill to 500 pennies.

Student Workbook


2 Lesson Summary

Let's say we have an input-output rule that gives exactly one output for each allowable input. Then we say the output depends on the input, or the output is a **function** of the input.

For example, the area of a square is a function of the side length because the area can be found from the side length by squaring it. So when the input is 10 cm, the output is 100 cm².

10 → find the area of a square given the side length → 100

Sometimes we might have two different rules that describe the same function. As long as we always get the same single output from any given input, the rules describe the same function.

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

Let's say we have an input-output rule that gives exactly one output for each allowable input. Then we say the output *depends* on the input, or the output is a **function** of the input.

For example, the area of a square is a function of the side length because the area can be found from the side length by squaring it. So when the input is 10 cm, the output is 100 cm^2.

10 → find the area of a square given the side length → 100

Sometimes we might have two different rules that describe the same function. As long as we always get the same single output from any given input, the rules describe the same function.

Cool-down

Wait Time

5 min

Student Task Statement

You are in line to watch the volleyball championship. You are told that you will have to wait for 50 minutes in line before they open the doors to the gym and you can find a seat. Determine whether:

1. You know the number of seconds you have to wait.

   Yes

   Sample response: The number of seconds to wait depends on the number of minutes to wait.

   50 → multiply by 60 → 3,000

2. You know the number of people in line.

   No, if I know how many minutes I have to wait in line, I do not necessarily know how many people are in line.

   Sample response: The number of people who have to wait cannot be determined by the amount of time someone has to wait. For example, there could be 50 people waiting, or there could be 100 people waiting.

For each statement, if you answer yes, draw an input-output diagram, and write a statement that describes the way one quantity depends on another.

If you answer no, give an example of 2 outputs that are possible for the same input.

Lesson **2**

Practice Problems

4 Problems

Problem 1

Here are several function rules. Calculate the output for each rule when you use -6 as the input.

rule 1

→ **subtract 7** → 13

rule 2

→ **square the input** → 36

rule 3

→ **divide by 3** → -2

rule 4

→ **divide the input into 3** → $-\frac{1}{2}$

rule 5

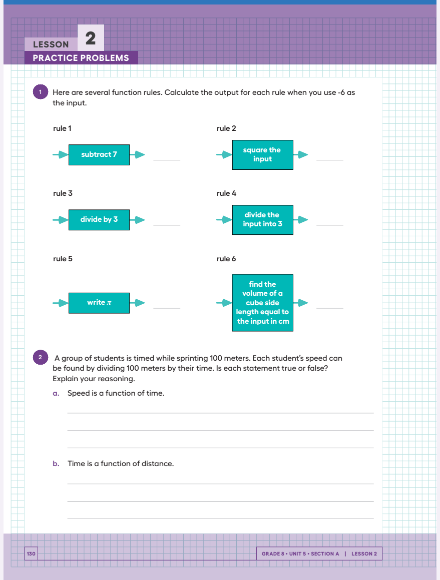
→ **write π** → π

rule 6

→ **find the volume of a cube side length equal to the input in cm** → ____

- Rule 6: -6 is not a valid input for this rule since it doesn't make sense to express a side length with a negative number.

Student Workbook


LESSON 2
PRACTICE PROBLEMS

1 Here are several function rules. Calculate the output for each rule when you use -6 as the input.

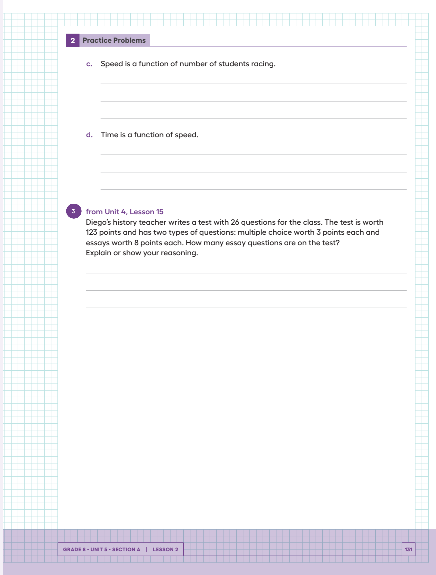
rule 1: subtract 7
rule 2: square the input
rule 3: divide by 3
rule 4: divide the input into 3
rule 5: write π
rule 6: find the volume of a cube side length equal to the input in cm

2 A group of students is timed while sprinting 100 meters. Each student's speed can be found by dividing 100 meters by their time. Is each statement true or false? Explain your reasoning.

a. Speed is a function of time.

b. Time is a function of distance.

Student Workbook


2 Practice Problems

c. Speed is a function of number of students racing.

d. Time is a function of speed.

3 from Unit 4, Lesson 15

Diego's history teacher writes a test with 26 questions for the class. The test is worth 123 points and has two types of questions: multiple choice worth 3 points each and essays worth 8 points each. How many essay questions are on the test? Explain or show your reasoning.

Problem 2

A group of students is timed while sprinting 100 meters. Each student's speed can be found by dividing 100 meters by their time. Is each statement true or false? Explain your reasoning.

a. Speed is a function of time.

True

Sample reasoning: For each time, one speed is generated.

b. Time is a function of distance.

False

Sample reasoning: For each distance (100 meters), many times are generated.

c. Speed is a function of number of students racing.

False

Sample reasoning: The number of students racing does not affect any student's speed, and the same speed may be reached for more than one student in a group of the same size.

d. Time is a function of speed.

True

Sample reasoning: For each speed calculated, there is only one possible time.

Problem 3

from Unit 4, Lesson 15

Diego's history teacher writes a test with 26 questions for the class. The test is worth 123 points and has two types of questions: multiple choice worth 3 points each and essays worth 8 points each. How many essay questions are on the test? Explain or show your reasoning.

9 essay questions

Sample reasoning: Use m to represent the number of multiple-choice questions and e for the number of essay questions. Write the system as $m + e = 26$ and $3m + 8e = 123$, and solve it by substituting $m = 26 - e$ into the second equation.

Problem 4

These tables correspond to inputs and outputs.

Table A:

input	output
-2	4
-1	1
0	0
1	1
2	4

Table B:

input	output
4	-2
1	-1
0	0
1	1
4	2

Table C:

input	output
1	0
2	0
3	0

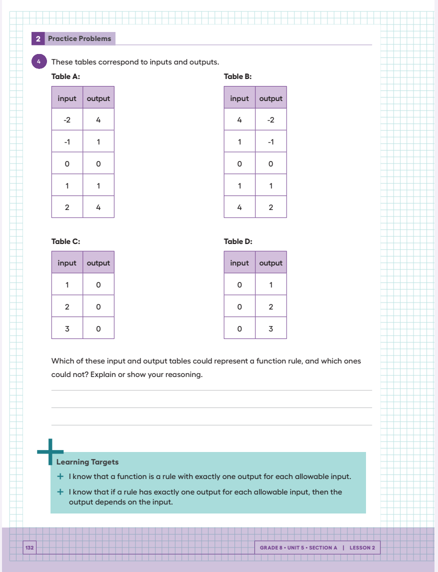
Table D:

input	output
0	1
0	2
0	3

Which of these input and output tables could represent a function rule, and which ones could not? Explain or show your reasoning.

Table A and Table C represent functions, but Table B and Table D do not. Sample reasoning: Tables B and D have multiple outputs for the same input, but functions take each input to only one output. On the other hand, a function rule can take different inputs to the same output.

Student Workbook


2 Practice Problems

4 These tables correspond to inputs and outputs.

Table A:

input	output
-2	4
-1	1
0	0
1	1
2	4

Table B:

input	output
4	-2
1	-1
0	0
1	1
4	2

Table C:

input	output
1	0
2	0
3	0

Table D:

input	output
0	1
0	2
0	3

Which of these input and output tables could represent a function rule, and which ones could not? Explain or show your reasoning.

Learning Targets

+ I know that a function is a rule with exactly one output for each allowable input.
+ I know that if a rule has exactly one output for each allowable input, then the output depends on the input.