Lesson 8

Multiplying Rational Numbers (Part 1)

Goals

- Explain (orally and in writing) how signed numbers can be used to represent positions and speeds in opposite directions.
- Generalize (orally) that the product of a negative number and a positive number is negative.
- Write a multiplication equation to represent a situation involving constant speed with direction.

Learning Targets

- I can multiply a positive number with a negative number.
- I can use rational numbers to represent speed and direction.

Student Learning Goal

- Let's use signed numbers to represent movement

Lesson Narrative

In this lesson, students explore multiplying a negative number by a positive number. To make sense of why such a product is negative, they use the context of constant velocity.

Scientists use the term velocity to describe the speed of an object in a specified direction. If one object is moving with a positive velocity, then any object moving in the opposite direction will have a negative velocity. Building on their previous work with constant speed, students calculate the final position for several different combinations of velocities and times. They see that the product of a negative velocity and a positive travel time results in a negative position relative to the starting point. As students reason through multiple examples to develop these generalizations about multiplication, they are making use of repeated reasoning.

Next, students apply their understanding of multiplying a negative number times a positive number to calculate the total amount of carbon dioxide that is absorbed or released by different objects in a year. A positive value represents carbon dioxide being released by the object, while a negative value represents carbon dioxide being absorbed.

Access for Students with Diverse Abilities

- Engagement (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)
- MLR6: Three Reads (Activity 1)

Instructional Routines

- MLR6: Three Reads

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	15 min	15 min	10 min	5 min

Assessment

Warm-up

Distance, Rate, Time

5 min

Activity Narrative

This activity reminds students of previous work they have done with constant speed situations, using $d = rt$ for the relationship between distance, rate, and time. This prepares students for representing movement in opposite directions using signed numbers in following activities.

Students may choose to use strategies such as creating a double number line or table of equivalent ratios to make sense of these problems and come up with a solution. While students are free to use these strategies, ensure that they also understand how to use $d = rt$ to represent the relationship between distance traveled, elapsed time, and rate of travel for constant speed situations.

Launch

Ask students what they remember about problems involving distance, rate, and time. They might offer that distances traveled and elapsed time create a set of equivalent ratios or that the elapsed time can be multiplied by the speed to give the distance traveled. Give students 1 minute of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. A car is traveling at a constant speed of 60 miles per hour. How far does the car travel in:
   a. 2 hours?

      120 miles

   b. 5 hours?

      300 miles

   c. x hours?

      60x miles

2. Create a representation that shows the relationship between the elapsed time and the distance traveled for this car.

   Sample responses:

   - $d = 60t$
   -

elapsed time (hours)	distance traveled (miles)
2	120
5	300
x	60x

Inspire Math

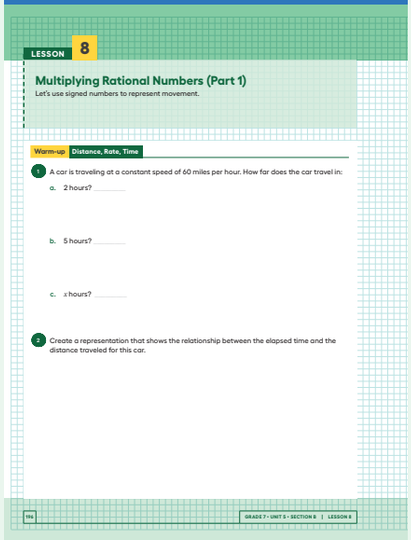
Icebergs video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614211

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 8

Multiplying Rational Numbers (Part 1)

Let's use signed numbers to represent movement.

Warm-up Distance, Rate, Time

1. A car is traveling at a constant speed of 60 miles per hour. How far does the car travel in:
   a. 2 hours?
   b. 5 hours?
   c. x hours?
2. Create a representation that shows the relationship between the elapsed time and the distance traveled for this car.

Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners

MLR6: Three Reads
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Begin with a physical demonstration of people moving with a constant speed in two different directions to support connections between new situations and prior understandings. Consider using the prompts:

"Does this situation remind anyone of something we have done before?"

"What factors determine the person's final position?"

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Activity Synthesis

The purpose of this discussion is to remind students of how the equation $d = rt$ can be used to solve problems involving movement at a constant speed. To find the distance traveled, we can multiply the rate of travel (or speed) by the elapsed time.

Consider drawing a diagram or table to facilitate the discussion of each problem and to remind students of the strategies they used while working with proportional relationships, such as using a scale factor or calculating the constant of proportionality. When relating distance and time in a constant speed situation, the speed is the constant of proportionality.

Activity 1
Velocity

15 min

Activity Narrative

The purpose of this activity is for students to encounter a concrete situation where multiplying two positive numbers results in a positive number and multiplying a negative and a positive number results in a negative number.

Students use their earlier understanding of a chosen zero point, a location relative to this as a positive or negative quantity, and a description of movement left or right along the number line as negative or positive. They extend their understanding to movement with positive and negative velocities and different times. This situation will produce negative or positive end points depending on whether the velocity is negative or positive. Looking at a number of different examples will help students generalize about the sign of the product of a negative number and a positive number.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the diagram, without revealing the questions.

For the first read, read the problem aloud then ask,

"What is this situation about?"

An engineer is recording the speed and direction (east or west) that cars are traveling.

Listen for and clarify any questions about the context.

After the second read, ask students to list any quantities that can be counted or measured.

the number of cars that pass by, the average speed of each car, the number of cars traveling east or west, the number or cars traveling faster or slower than the speed limit

After the third read, reveal the first question:

"A car is traveling east at 12 meters per second. Where will it be 10 seconds after it passes the camera?"

and ask,

"What are some ways we might get started on this?"

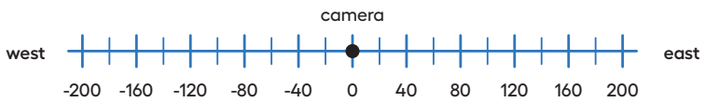
Invite students to name some possible starting points, referencing quantities from the second read.

We can draw a diagram on the number line. We can calculate how far the car will have traveled in 10 seconds.

Give students 6–7 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.



1. A car is traveling east at 12 meters per second. Where will it be 10 seconds after it passes the camera?

   the position +120 meters, which is east of the camera

2. A car is traveling west at -14 meters per second. Where will it be 10 seconds after it passes the camera?

   the position -140 meters, which is west of the camera

3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	+10	+120	12 · 10 = 120
car B	-14	+10	-140	-14 · 10 = -140
car C	+9	+5	+45	9 · 5 = 45
car D	-11	+8	-88	-11 · 8 = -88
car E	-15	+20	-300	-15 · 20 = -300
car F	+8	0	0	8 · 0 = 0

4. Complete the sentences. Be prepared to explain your reasoning.

- A positive number times a positive number equals a positive number.
- A negative number times a positive number equals a negative number.

Building on Student Thinking

Encourage students who get stuck to use the provided number line to represent each situation.

Student Workbook


1 Velocity

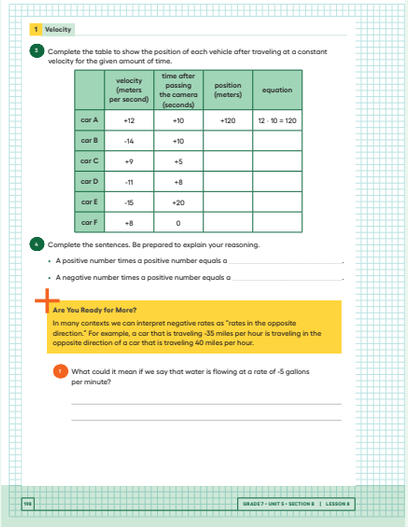
A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.

camera
west east
-200 -160 -120 -80 -40 0 40 80 120 160 200

1 A car is traveling east at 12 meters per second. Where will it be 10 seconds after it passes the camera?

2 A car is traveling west at -14 meters per second. Where will it be 10 seconds after it passes the camera?

Student Workbook



1 Velocity

3 Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	+10	+120	12 · 10 = 120
car B	-14	+10		
car C	+9	+5		
car D	-11	+8		
car E	-15	+20		
car F	+8	0		

4 Complete the sentences. Be prepared to explain your reasoning.

- A positive number times a positive number equals a _____.
- A negative number times a positive number equals a _____.

Are You Ready for More?

In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1 What could it mean if we say that water is flowing at a rate of -5 gallons per minute?

Are You Ready for More?

In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1. What could it mean if we say that water is flowing at a rate of -5 gallons per minute?

   Sample responses:

   - A tank is draining out 5 gallons of water per minute.
   - Due to tides, a creek is flowing upstream at a rate of 5 gallons per minute.

2. Make up another situation with a negative rate, and explain what it could mean.

   Answers vary.

Activity Synthesis

The purpose of this discussion is to emphasize that the product of two positive numbers is a positive number and that the product of a positive number and a negative number is a negative number. Begin by displaying the table and the number line from the *Task Statement* for all to see. Invite students to share their responses and reasoning for each car.

Demonstrate how cars with a positive velocity are moving towards the east and cars with a negative velocity are moving towards the west. Then place a point on the number line to represent each car's position (except Car E's position) at the given time. Discuss the following questions:

- *"Where would Car E be located?"*

  At the position -300 to the east of the camera.

- *"What do you notice about the product of two positive numbers?"*

  The product is also positive.

- *"What do you notice about the product of a positive number and a negative number?"*

  The product is negative.

- *"What does it mean for Car F to be at the position 0 in this situation?"*

  Car F is traveling east at a velocity of 8 meters per second. Its position is measured 0 seconds after passing the camera, meaning that its position is measured exactly as it is passing the camera.

Activity 2
Carbon Dioxide

15 min

Activity Narrative

In this activity, students interpret signed numbers in the context of the amount of carbon dioxide released in a year. Students reason that a positive number represents that an object releases carbon dioxide into the environment, while a negative number represents that an object absorbs carbon dioxide. This gives them opportunities to reason quantitatively about the situation, then perform operations with signed numbers as they reason abstractly.

Launch

To familiarize students with the table and get them started thinking about how to approach the questions in the task statement, consider asking:

- *"Some of the numbers in the table are negative. What do you think that means?"*

  Those objects absorb carbon dioxide instead of releasing it.

- *"If a family has 4 campfires in a year, about how much carbon dioxide does this release?"*

  36 kilograms, because $9 \cdot 4 = 36$

- *"If a trucking company owns 6 semi-trucks, about how much carbon dioxide is released by these trucks in one year?"*

  1.2 million kilograms, because $200{,}000 \cdot 6 = 1{,}200{,}000$

Access for Multilingual Learners (Activity 2, Launch)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the problem stem and table from the *Task Statement*, without revealing the questions. Give students 2–3 minutes to write a list of mathematical questions that could be asked about this situation, before comparing their questions with a partner. Invite each group to contribute one written question to a whole-class display. Ask the class to make comparisons among the shared questions and their own. Reveal the intended questions for this task, and invite additional connections.

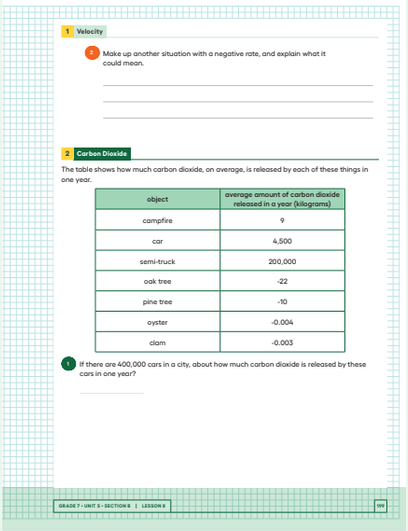
Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook



1 Velocity

2. Make up another situation with a negative rate, and explain what it could mean.

2 Carbon Dioxide

The table shows how much carbon dioxide, on average, is released by each of these things in one year.

object	average amount of carbon dioxide released in a year (kilograms)
campfire	9
car	4,500
semi-truck	200,000
oak tree	-22
pine tree	-10
oyster	-0.004
clam	-0.003

1. If there are 400,000 cars in a city, about how much carbon dioxide is released by these cars in one year?

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Student Workbook

2 Carbon Dioxide

2. If there are 10,000 clams in a clam bed, about how much carbon is released by this clam bed in one year?

3. If a park has 25 pine trees and 10 oak trees, about how much carbon dioxide is released by these trees in one year?

4. If the city with 400,000 cars wanted to absorb as much carbon dioxide as was released by all these cars each year, how many parks and clam beds of these sizes would they need?

5. College students in the Netherlands developed a car that releases -2 kilograms of carbon dioxide per year. If half of the cars in the city were this new type of car, how much carbon dioxide would be released by all the cars in the city in one year?

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Student Task Statement

The table shows how much carbon dioxide, on average, is released by each of these things in one year.

object	average amount of carbon dioxide released in a year (kilograms)
campfire	9
car	4,500
semi-truck	200,000
oak tree	-22
pine tree	-10
oyster	-0.004
clam	-0.003

1. If there are 400,000 cars in a city, about how much carbon dioxide is released by these cars in one year?

1.8 billion kilograms, because $4{,}500 \cdot 400{,}000 = 1{,}800{,}000{,}000$

2. If there are 10,000 clams in a clam bed, about how much carbon is released by this clam bed in one year?

-30 kilograms, because $-0.003 \cdot 10{,}000 = -30$

3. If a park has 25 pine trees and 10 oak trees, about how much carbon dioxide is released by these trees in one year?

-470 kilograms, because $-10 \cdot 25 + -22 \cdot 10 = -470$

4. If the city with 400,000 cars wanted to absorb as much carbon dioxide as was released by all these cars each year, how many parks and clam beds of these sizes would they need?

Sample response: 3.6 million parks and 3.6 million clam beds, because $-470 + -30 = -500$ and $1{,}800{,}000{,}000 \div 500 = 3{,}600{,}000$

5. College students in the Netherlands developed a car that releases -2 kilograms of carbon dioxide per year. If half of the cars in the city were this new type of car, how much carbon dioxide would be released by all the cars in the city in one year?

899,600,000 kilograms, because $4{,}500 \cdot 200{,}000 = 900{,}000{,}000$ from the old cars and $-2 \cdot 200{,}000 = -400{,}000$ from the new cars make a total of $900{,}000{,}000 + -400{,}000 = 899{,}600{,}000$

Activity Synthesis

The purpose of this discussion is to reinforce these key ideas:

- The product of two numbers with different signs is negative.
- The sum of two numbers with different signs will be the same sign as the number with the largest magnitude (or the sum will be 0 if the two numbers have the same magnitude).

Invite students to share their responses and reasoning to the fourth question asking how many parks and clam beds a city would need to absorb the carbon dioxide released by 400,000 cars. As students share their combinations of parks and clam beds, consider discussing the following questions:

- *"How did you calculate the amount of carbon dioxide released by the parks?"*

  I multiplied the average amount of carbon dioxide released by one park and multiplied it by the total number of parks.

- *"Why is the carbon dioxide released by the parks a negative number? How does that make sense in this situation?"*

  A positive number multiplied by a negative number results in a negative number. This makes sense because trees absorb carbon dioxide.

- *"How do we know when the number of parks and clam beds will be enough to absorb the carbon dioxide released by the cars?"*

  The sum of all the numbers will be 0.

- *"How can a set of numbers have a sum of 0?"*

  The total magnitude of the positive numbers is the same as the total magnitude of the negative numbers.

Lesson Synthesis

Share with students,

- *"Today we worked with situations that involved multiplying signed numbers."*

To review what students learned about multiplying a negative numbers times a positive number, consider asking:

- *"How can we represent how fast something is moving to the left or right from a starting point?"*

  We can pick one direction to represent with a positive velocity, and then the other direction has a negative velocity.

- *"If an object moves at a velocity of 4 meters per second for 5 seconds, what is its finish point? Why?"*

  20 meters in the positive direction from the starting point, because $4 \cdot 5 = 20$

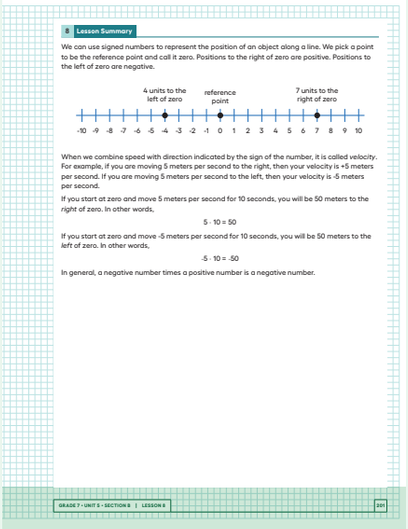
- *"If an object moves at a velocity of -4 meters per second for 5 seconds, what is its finish point? Why?"*

  20 meters in the negative direction from the starting point, because $-4 \cdot 5 = -20$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


Lesson Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.

4 units to the left of zero | reference point | 7 units to the right of zero

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

When we combine speed with direction indicated by the sign of the number, it is called velocity. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be 50 meters to the *right* of zero. In other words,

$5 \cdot 10 = 50$

If you start at zero and move -5 meters per second for 10 seconds, you will be 50 meters to the *left* of zero. In other words,

$-5 \cdot 10 = -50$

In general, a negative number times a positive number is a negative number.

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> *"If an object releases 6 kilograms of carbon dioxide for 3 years, how much total carbon dioxide is released? Why"*

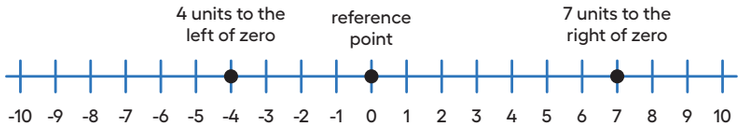
18 kilograms, because $6 \cdot 3 = 18$

> *"If an object absorbs 6 kilograms of carbon dioxide for 3 years, how much total carbon dioxide is released? Why"*

-18 kilograms, because $-6 \cdot 3 = -18$

Lesson Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.



When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be 50 meters to the *right* of zero. In other words,

$$5 \cdot 10 = 50$$

If you start at zero and move -5 meters per second for 10 seconds, you will be 50 meters to the *left* of zero. In other words,

$$-5 \cdot 10 = -50$$

In general, a negative number times a positive number is a negative number.

Cool-down

Multiplication Equations

5 min

Student Task Statement

Two runners start at the same point. For each runner, write a multiplication equation that describes their journey.

1. Lin runs for 25 seconds at 8 meters per second. What is her finish point?

   Sample response: $8 \cdot 25 = 200$

2. Diego runs for 30 seconds at -9 meters per second. What is his finish point?

   Sample response: $-9 \cdot 30 = -270$

Lesson 8

Practice Problems

6 Problems

Problem 1

A number line can represent positions that are north and south of a truck stop on a highway. Decide whether you want positive positions to be north or south of the truck stop. Then plot the following positions on a number line.

a. The truck stop

b. 5 miles north of the truck stop

c. 3.5 miles south of the truck stop

Either choice is fine as long as students are consistent in the next part.

Problem 2

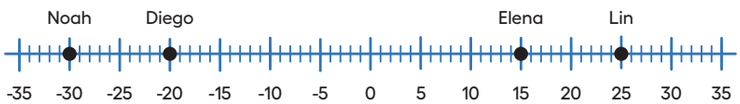
a. How could you distinguish between traveling west at 5 miles per hour and traveling east at 5 miles per hour without using the words "east" and "west"?

Sample response: By giving the velocities opposite signs.

b. Four people are cycling. They each start at the same point. (0 represents their starting point.) Plot their finish points after 5 seconds of cycling on a number line.

- Lin cycles at 5 meters per second.
- Diego cycles at -4 meters per second.
- Elena cycles at 3 meters per second.
- Noah cycles at -6 meters per second.

Sample response:



Problem 3

from Unit 5, Lesson 6

Find the value of each expression.

a. 16.2 + -8.4

7.8

b. $\frac{2}{5} - \frac{3}{5}$

$\frac{-1}{5}$ (or equivalent)

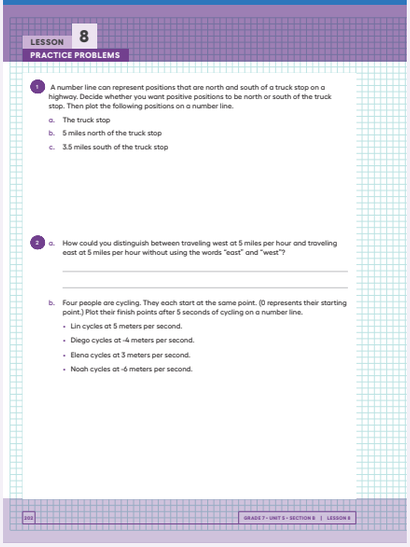
c. -9.2 + -7

-16.2

d. $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$

$-3\frac{1}{8}$ (or equivalent)

Student Workbook


LESSON 8
PRACTICE PROBLEMS

1 A number line can represent positions that are north and south of a truck stop on a highway. Decide whether you want positive positions to be north or south of the truck stop. Then plot the following positions on a number line.
a. The truck stop
b. 5 miles north of the truck stop
c. 3.5 miles south of the truck stop

2 a. How could you distinguish between traveling west at 5 miles per hour and traveling east at 5 miles per hour without using the words "east" and "west"?

b. Four people are cycling. They each start at the same point. (0 represents their starting point.) Plot their finish points after 5 seconds of cycling on a number line.
- Lin cycles at 5 meters per second.
- Diego cycles at -4 meters per second.
- Elena cycles at 3 meters per second.
- Noah cycles at -6 meters per second.

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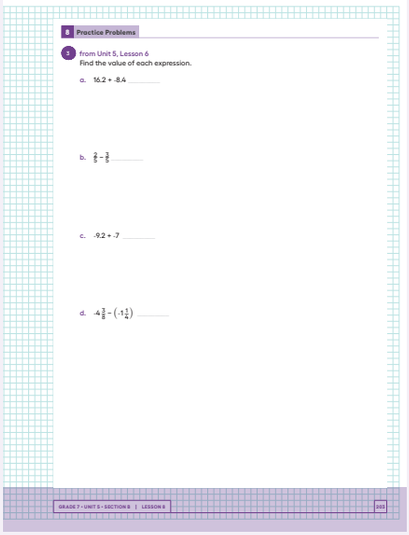
Student Workbook


8 Practice Problems

3 from Unit 5, Lesson 6
Find the value of each expression.
a. 16.2 + -8.4
b. $\frac{2}{5} - \frac{3}{5}$
c. -9.2 + -7
d. $-4\frac{3}{8} - \left(-1\frac{1}{4}\right)$

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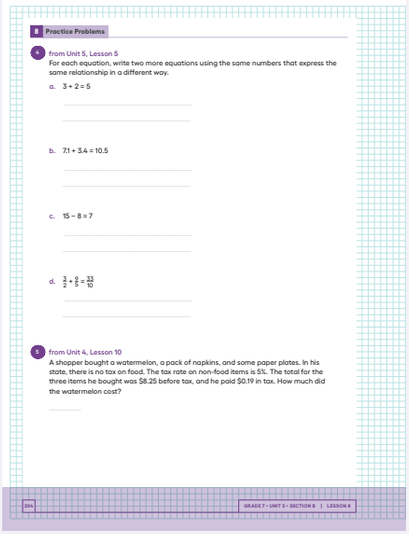
Student Workbook


8 Practice Problems

5 from Unit 5, Lesson 5
For each equation, write two more equations using the same numbers that express the same relationship in a different way.

a. 3 + 2 = 5

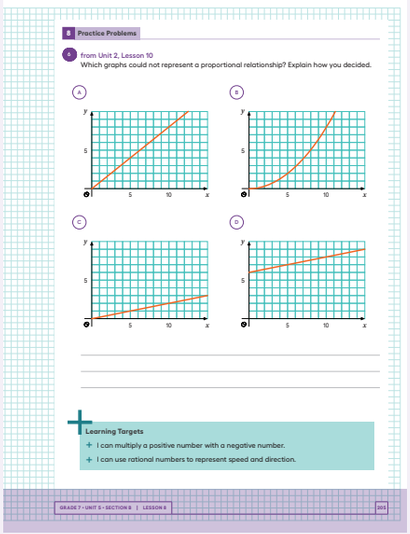
b. 7.1 + 3.4 = 10.5

c. 15 − 8 = 7

d. $\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$

6 from Unit 4, Lesson 10
A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was $8.25 before tax, and he paid $0.19 in tax. How much did the watermelon cost?

Student Workbook


8 Practice Problems

6 from Unit 2, Lesson 10
Which graphs could not represent a proportional relationship? Explain how you decided.

A B C D

Learning Targets
+ I can multiply a positive number with a negative number.
+ I can use rational numbers to represent speed and direction.

Problem 4

from Unit 5, Lesson 5

For each equation, write two more equations using the same numbers that express the same relationship in a different way.

For each question, students should have 2 of the 3 equations listed.

a. 3 + 2 = 5

5 − 3 = 2; 5 − 2 = 3; 2 + 3 = 5

b. 7.1 + 3.4 = 10.5

10.5 − 3.4 = 7.1; 10.5 − 7.1 = 3.4; 3.4 + 7.1 = 10.5

c. 15 − 8 = 7

15 − 7 = 8; 8 + 7 = 15; 7 + 8 = 15

d. $\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$

$\frac{33}{10} - \frac{3}{2} = \frac{9}{5}$, $\frac{33}{10} - \frac{9}{5} = \frac{3}{2}$, $\frac{9}{5} + \frac{3}{2} = \frac{33}{10}$

Problem 5

from Unit 4, Lesson 10

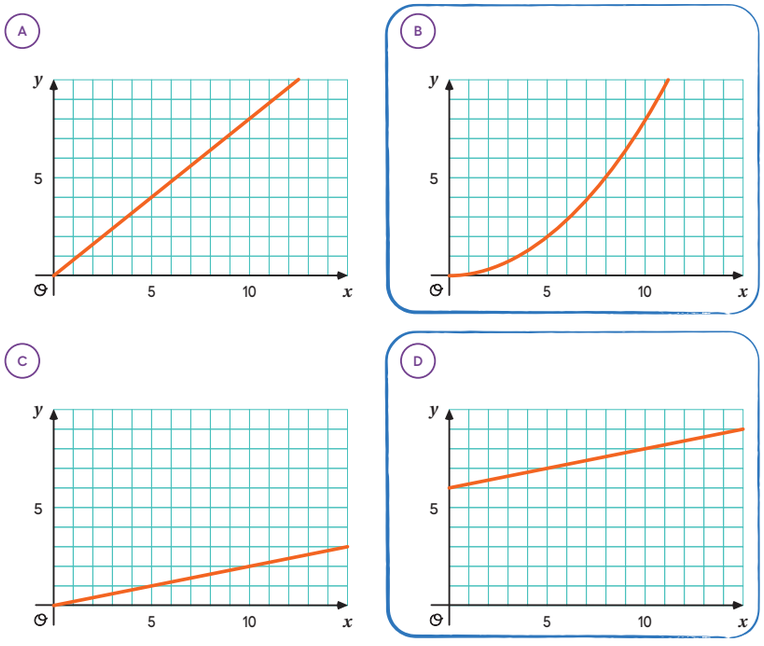
A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was $8.25 before tax, and he paid $0.19 in tax. How much did the watermelon cost?

$4.45, because the non-food items cost 0.19 ÷ 0.05 = 3.8 and the watermelon cost 8.25 − 3.8 = 4.45

Problem 6

from Unit 2, Lesson 10

Which graphs could not represent a proportional relationship? Explain how you decided.



Sample reasoning: B is not a straight line, and D does not go through the origin.