# **Making Scaled Copies**

# Goals

- Critique (orally and in writing) different strategies (expressed in words and through other representations) for creating scaled copies of a figure.
- Draw a scaled copy of a given figure using a given scale factor.
- Generalize (orally and in writing) that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive.

# **Learning Targets**

- I can draw a scaled copy of a figure using a given scale factor.
- I know what operation to use on the side lengths of a figure to produce a scaled copy.

# **Lesson Narrative**

In this lesson, students draw scaled copies of simple shapes, applying what they have learned about corresponding parts and scale factor. They strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process and have opportunities to use tools like tracing paper or index cards strategically.

As students draw scaled copies and analyze scaled relationships more closely, encourage them to continue using the terms scale factor and corresponding in their reasoning.

#### **Math Community**

In this lesson, students review the themes that arose when they shared their initial thoughts in Exercise 1 about what they think it should look like and sound like to do math together as a community. Students then have a chance to both affirm and add to the ideas that were generated.

# **Student Learning Goal**

Let's draw scaled copies.

### **Lesson Timeline**

Warm-up

10

**Activity 1** 

10

**Activity 2** 

10

**Activity 3** 

#### **Access for Students with Diverse Abilities**

- Action and Expression (Warm-up)
- Engagement (Activity 1, Activity 2)

### **Access for Multilingual Learners**

- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1, Activity 3)

#### **Instructional Routines**

- 5 Practices
- Math Talk
- MLR2: Collect and Display
- MLR8: Discussion Supports

#### **Required Materials**

#### **Materials to Gather**

- · Math Community Chart: Warm-up
- · Sticky notes: Warm-up
- · Geometry toolkits: Activity 1

#### **Required Preparation**

#### **Activity 1:**

For the digital version of the activity, acquire devices that can run the applet.



Cool-down

10

**Lesson Synthesis** 

## **Inspire Math**

#### **Movie Monsters video**



#### Go Online

Before the lesson, show this video to introduce the real-world connection.

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#### **Instructional Routines**

#### **Math Talk**

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## **Instructional Routines**

# MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Warm-up, Launch)

# Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

#### Warm-up

# **Math Talk: Missing Operation**



### **Activity Narrative**

This Math Talk focuses on incomplete equations that have only an initial value and an answer. It encourages students to think about operations and to rely on what they know about relationships between numbers to make each equation true. During the whole-class discussion, students see that for any pair of values, there is both an additive and a multiplicative relationship between them. The understanding elicited here will be helpful later in the lesson when students analyze copies of figures and identify what operation relates the corresponding side lengths.

## Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

#### **Student Task Statement**

Complete each equation to make it true.

**A.**5 \_\_\_\_\_ = 10

Sample responses:

- · 5 + 5 = 10
- o 5 · 2 = 10
- **B.** 3 \_\_\_\_ = 15

Sample responses:

- o 3 + I2 = I5
- o 3 · 5 = 15
- **C.**14 \_\_\_\_ = 21

Sample responses:

- o 14 + 7 = 21
- 0 14 + 6 + 1 = 21
- $0.14 \div 2 \cdot 3 = 21$
- o 14 · 1.5 = 21
- **D.**30 = 6

Sample responses:

- 0.30 24 = 6
- o 30 ÷ 5 = 6
- $0.30 \cdot \frac{1}{5} = 6$

## **Activity Synthesis**

Make sure that at least one strategy for each problem uses multiplication before moving to the next problem. If needed, ask

"How could we use multiplication to complete this equation?"

To involve more students in the conversation, consider asking:

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to \_\_\_\_\_\_'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway is that for any such problem, there is a solution that uses addition (or subtraction) as well as a solution that uses multiplication (or division).

#### **Math Community**

After the Warm-up, display the class Math Community Chart for all to see and explain that the listed "Doing Math" actions come from the sticky notes students wrote in the first exercise. Give students 1 minute to review the chart. Then invite students to identify something on the chart they agree with and hope for the class or something they feel is missing from the chart and would like to add. Record any additions on the chart. Tell students that the chart will continue to grow and that they can suggest other additions that they think of throughout today's lesson during the Cool-down.

#### **Activity 1: Optional**

#### **Drawing Scaled Copies**

10 min

#### **Activity Narrative**

### There is a digital version of this activity.

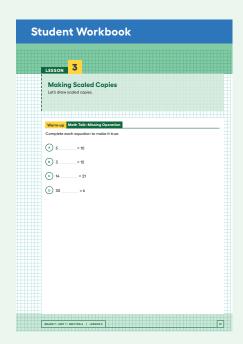
The purpose of this activity is to give students more practice drawing scaled copies of simple geometric figures on a grid. When trying to scale non-horizontal and non-vertical segments, students may think of using tracing paper or a ruler to measure lengths and a protractor to measure angles. Make sure they have a chance to see how the structure of the grid can be useful for scaling the lengths of non-vertical and non-horizontal segments.

To create scaled copies, students need to attend to all parts of the original figure, or else the copy will not be scaled correctly. Use of the grid for scaling non-horizontal and non-vertical segments is a good example of using tools strategically.

As students work, monitor for students who find a way to scale segment lengths properly but neglect to consider the size of corresponding angles (especially in making a copy of Figure B or Figure D).

#### **Building on Student Thinking**

Students may attempt to solve each problem instead of reasoning about the numbers and operations. If a student is calculating an exact solution to each problem, ask the student to look closely at the characteristics of the numbers and how an operation would affect those numbers.



# Access for Multilingual Learners (Warm-up, Synthesis)

#### MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_\_ because ..." or "I noticed \_\_\_\_\_ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

#### **Instructional Routines**

# MLR8: Discussion Supports

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# Access for Students with Diverse Abilities (Activity 1, Launch)

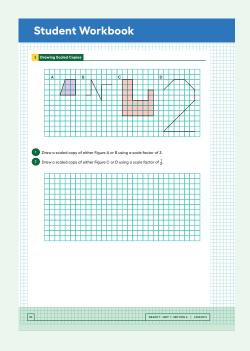
# Engagement: Provide Access by Recruiting Interest.

Leverage choice around perceived challenge. Invite students to select Figures A and D to practice this concept or Figures B and C if they feel confident with drawing scaled copies.

Supports accessibility for: Organization, Social-Emotional Functioning

### **Building on Student Thinking**

Some students may think that Figure C cannot be scaled by a factor of  $\frac{1}{2}$  because some vertices will not land on intersections of grid lines. Clarify that the grid helps us see lengths in whole units but segments we draw on them are not limited to whole units in length.

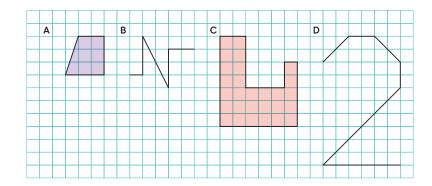


In the digital version of the activity, students use an applet to draw scaled copies of figures on a grid. The applet allows students to add, remove, adjust, and label points and line segments. The digital version may help students draw quickly and accurately so they can focus more on the mathematical analysis of scale factor.

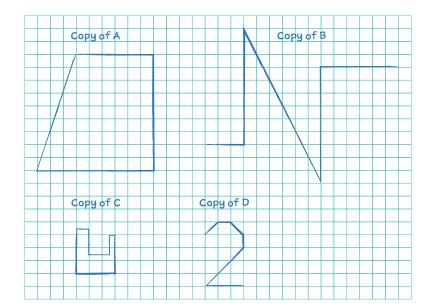
## Launch

Give students 3 minutes of quiet time to draw and another 3 minutes to share their drawings with a partner, check each other's work, and make revisions. Provide access to their geometry toolkits.

### **Student Task Statement**



- 1. Draw a scaled copy of either Figure A or B using a scale factor of 3.
- **2.** Draw a scaled copy of either Figure C or D using a scale factor of  $\frac{1}{2}$ .



## **Activity Synthesis**

Invite students to share their strategies of how they used the grid (or other tools) to make sure their drawings were scaled copies. Consider asking questions like:

- How did you know how long to make each side in your scaled copy?
- · How did you know how big to make each angle in your scaled copy?
- If you made a mistake while drawing your scaled copy, how could you tell?

Model, prompt, and listen for the language students are using to distinguish between scaled and not scaled figures. Emphasize the usefulness of the grid in drawing and checking right angles, and for drawing and checking lengths of segments. All correct answers will be the same size and shape, but they could be drawn in different positions on the grid.

# **Activity 2**

# Which Operations? (Part 1)

10 min

## **Activity Narrative**

The purpose of this activity is to contrast the effects of multiplying side lengths versus adding to side lengths when creating copies of a polygon. To find the corresponding side lengths on a scaled copy, the side lengths of a figure are all *multiplied* (or divided) by the same number. However, students often mistakenly think that adding or subtracting the same number to all the side lengths will also create a scaled copy. When students recognize that there is a multiplicative relationship between the side lengths rather than an additive one, they are looking for and making use of structure.

Monitor for students who:

- Notice that Diego's copy is no longer a polygon while Jada's still is.
- Notice that all the corresponding angles have equal measures (either 90 or 270 degrees).
- Notice that the relationships between side lengths in Diego's copy have changed (for example, Side 1 is no longer twice as long as Side 2) while in Jada's copy these relationships stayed the same as in the original.
- Describe Jada's copy as having all side lengths divided by 3.
- Describe Jada's copy as having all side lengths a third as long as the original lengths.
- Describe Jada's copy as having a scale factor of  $\frac{1}{3}$ .

### Launch

Give students 2-3 minutes of quiet think time.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Give students 2 minutes to share their thinking with a partner.

# Access for Multilingual Learners (Activity 1, Synthesis)

#### MLR8: Discussion Supports.

For each strategy that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

# Access for Multilingual Learners (Activity 2, Narrative)

#### **MLR2: Collect and Display**

This activity uses the *Collect and Display* math language routine to advance students in developing their mathematical language.

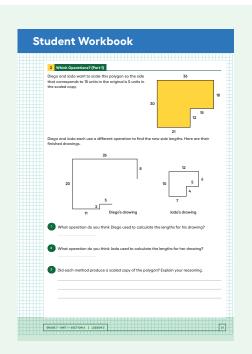
#### **Instructional Routines**

# MLR2: Collect and Display

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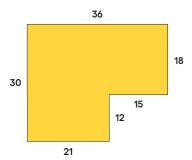
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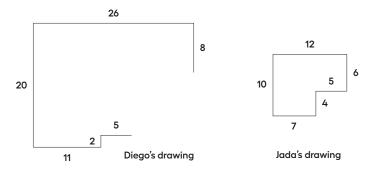


# **Student Task Statement**

Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.



Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.



**1.** What operation do you think Diego used to calculate the lengths for his drawing?

Since we can get from 15 to 5 by subtracting 10, Diego may have subtracted 10 units from the length of every side. Subtracting 10 from each side length in the original gives Diego's picture

**2.** What operation do you think Jada used to calculate the lengths for her drawing?

Jada went from 15 to 5 by multiplying by  $\frac{1}{3}$  or dividing by 3. Multiplying each side by  $\frac{1}{3}$  in the original gives Jada's picture.

**3.** Did each method produce a scaled copy of the polygon? Explain your reasoning.

No, only Jada's method produces a scaled copy.

Sample reasoning: Subtracting IO from each length did not work because now the figure is no longer a polygon. There is a big gap between the two sides that should meet. To create a scaled copy, every length needs to be multiplied (or divided) by the same number.

## **Activity Synthesis**

Invite previously selected students to share their answers and reasoning. Sequence their explanations from most general to most technical. Before moving to the next activity, consider asking questions like these:

- "What is the scale factor used to create Jada's drawing? What about for Diego's drawing?"
  - $\frac{1}{3}$  for Jada's. There isn't one for Diego's, because it is not a scaled copy.
- "What can you say about the corresponding angles in Jada and Diego's drawings?"

They are all equal, even though one is a scaled copy and one is not.

"Subtraction of side lengths does not (usually) produce scaled copies. Do you think addition would work?"

Answers vary.

Note: There are rare cases when adding or subtracting the same length from each side of a polygon (and keeping the angles the same) *will* produce a scaled copy, namely if all side lengths are the same. If not mentioned by students, it is not important to discuss this at this point.

## **Activity 3**

#### Which Operations? (Part 2)

10 min

# **Activity Narrative**

This activity reinforces the idea that scaling involves multiplication. In the previous activity, students saw that subtracting the same value from all side lengths of a polygon did not produce a (smaller) scaled copy. This activity makes the case that adding the same value to all lengths also does not produce a (larger) scaled copy. As students explain why Andre's method would not create a scaled copy, they are constructing arguments.

This activity gives students a chance to draw a scaled copy without a grid and to use paper as a measuring tool. To create a copy using a scale factor of 2, students need to mark the length of each original segment and transfer it twice onto their drawing surface, reinforcing—in a tactile way—the meaning of scale factor. The angles in the polygon are right angles (and a 270 degree angle in one case) and can be made using the corner of an index card.

Monitor for students who use these strategies to create the new drawing:

- Add 4 units to all the sides and draw a shape that is not a scaled copy (either by changing the angles or creating a figure that isn't closed).
- Sketch a figure that looks like a scaled copy and label it with side lengths that are 4 units longer, without attending to whether the numbers match the actual lengths.
- Show that adding 4 units would not result in a consistent scale factor between pairs of corresponding side lengths.
- Use a ruler to measure the side lengths of the original shape and multiply these measurements by 2.

# Access for Students with Diverse Abilities (Activity 2, Synthesis)

# Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "I noticed \_\_\_\_\_\_ so I ..." "Both Diego's drawing and Jada's drawing are alike because ..." "Diego's drawing and Jada's drawing and Jada's drawing are different because ..." and "Is it always true that ...?"

Supports accessibility for: Language, Social-Emotional Functioning

#### **Instructional Routines**

#### **5 Practices**

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## Instructional Routines

# MLR8: Discussion Supports

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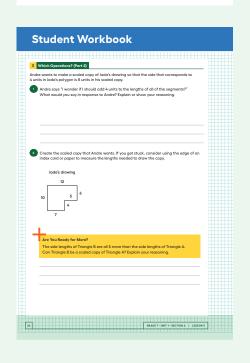
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#### **Building on Student Thinking**

Some students might not be convinced that making each segment 4 units longer will not work. To show that adding 4 units would work, they might simply redraw the polygon and write side lengths that are 4 units longer, regardless of whether the numbers match the actual lengths. Urge them to check the side lengths by measuring. Tell them (or show, if needed) how the 4-unit length in Jada's drawing could be used as a measuring unit and added to all sides.

Other students might add 4 units to all sides and manage to make a polygon but change the angles along the way. If students do so to make the case that the copy will not be scaled, consider sharing their illustrations with the class, as these can help to counter the idea that "scaling involves adding." If, however, students do this to show that adding 4 units all around does work, address the misconception. Ask them to recall the size of corresponding angles in scaled copies, or remind them that angles in a scaled copy are the same size as their counterparts in the original figure.



- Use an index card to measure the side lengths of the original shape and draw lengths that are twice as long.
- Use an index card to measure and recreate the right angles in the figure.

Plan to have students present in this order to support moving them from additive to multiplicative reasoning about the side lengths.

The routine of Anticipate, Monitor, Select, Sequence, Connect (5 Practices) requires a balance of planning and flexibility. The anticipated approaches might not surface in every class, and there may be reason to change the order in which strategies are presented. While monitoring, keep in mind the learning goal and adjust the order to ensure all students have access to the first idea presented (whether that be a common misconception or a different approach).

## Launch

Ask students to read the task statement and then check that they understand which side of the polygon Andre would like to be 8 units long on his drawing. Provide access to index cards, so that students can use it as a measuring tool. Consider not explicitly directing students as to its use to give them a chance to use tools strategically.

Some students may not realize how to use an index card or a sheet of paper to measure lengths. Before demonstrating, encourage them to think about how a length in the given polygon could be copied onto an index card and used as an increment for measuring. If needed, show how to mark the 4-unit length along the edge of a card and to use the mark to determine the needed lengths for the copy.

Give students 5–6 minutes of quiet work time, and then 2 minutes to share their work with a partner. Select students with different strategies, such as those described in the activity narrative, to share later.

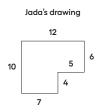
## **Student Task Statement**

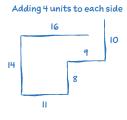
Andre wants to make a scaled copy of Jada's drawing so that the side that corresponds to 4 units in Jada's polygon is 8 units in his scaled copy.

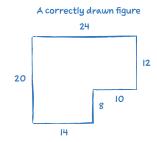
**1.** Andre says "I wonder if I should add 4 units to the lengths of all of the segments?" What would you say in response to Andre? Explain or show your reasoning.

Sample reasoning: Adding 4 units would not work because the shape of the copy would be different than the shape of the original. For example, in the original drawing, the top horizontal segment is 12 units and the two bottom horizontal segments (5 units and 7 units) also add up to 12 units. If we add 4 units to each segment, the top horizontal segment will be 16 units long, and the two bottom horizontal segments will be 9 units and 11 units, or a total of 20 units. There will be a gap where two segments should meet, or if we make the two ends meet, the angles will no longer be right angles. See the figure on the left.

2. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.







# Are You Ready for More?

The side lengths of Triangle B are all 5 more than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your reasoning.

Yes, if Triangle A is equilateral, then its side lengths are all the same. Adding 5 to each side, the lengths will still be the same and so Triangle B will also be equilateral.

If Triangle A is not equilateral, then Triangle B will not be a scaled copy of Triangle A. To see why, notice that adding 5 to a side length of 5 doubles the side length. Adding 5 to a side length that is greater than 5 changes the side by a scale factor less than 2. Adding 5 to a side length less than 5 changes the side length by a scale factor greater than 2. So if one side length of Triangle A is 5, all side lengths have to be 5 or else Triangle B will not be a scaled copy of Triangle A. This reasoning works for side lengths other than 5.

#### **Activity Synthesis**

The purpose of the activity is to explicitly call out a potential misunderstanding about how scale factors work, emphasizing that scale factors work by multiplying existing side lengths by a common factor, rather than by adding a common length to each.

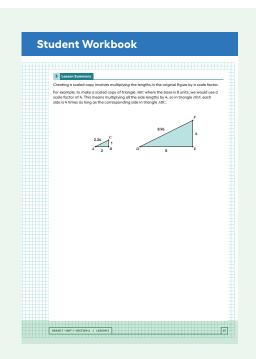
Invite previously selected students to share their explanations or illustrations that adding 4 units to the length of each segment would not work (for example, the copy is no longer a polygon, or the copy has angles that are different from those in the original figure). Then, select a couple of other students to show their scaled copies and share how they created the copies. Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

# Access for Multilingual Learners (Activity 3, Synthesis)

#### **MLR8: Discussion Supports.**

Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking



As students share their approaches, consider asking:

"What scale factor did you use to create your copy? Why?"

"How did you measure the side lengths for the copy?"

"How did you measure the angles for the copy?"

After the pre-selected students have finished sharing with the whole class, connect the different responses to the learning goals by asking questions such as:

O "Did this approach create a scaled copy? Why or why not?"

"How did the scale factor show up in each method?"

"What worked well in \_\_\_\_\_'s approach? What did not work well?"

"What role does multiplication play in each approach?"

## **Lesson Synthesis**

Share with students

☐ "Today we saw that making scaled copies involves multiplication."

To reinforce that the relationship between side lengths is multiplicative, consider asking students:

"How do we draw a scaled copy of a figure?"

To draw a scaled copy of a figure, we need to multiply all of the lengths by the scale factor.

Can we create scaled copies by adding or subtracting the same value from all lengths? Why or why not?"

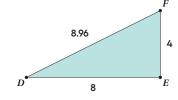
No, adding or subtracting the same value to all lengths will usually not create a scaled copy.

### **Lesson Summary**

Creating a scaled copy involves *multiplying* the lengths in the original figure by a scale factor.

For example, to make a scaled copy of triangle ABC where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle DEF, each side is 4 times as long as the corresponding side in triangle ABC.





## Launch

## **Math Community**

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "Is there anything that you would like to add to the student 'Doing Math' section of the chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use the themes to add to or revise the student section of the Math Community Chart before Exercise 3.

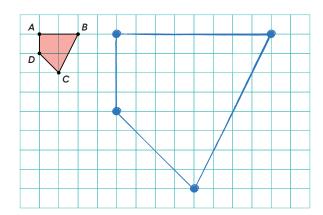
## Cool-down

### **More Scaled Copies**

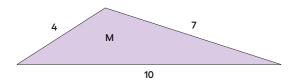
5 min

# **Student Task Statement**

**1.** Create a scaled copy of *ABCD* using a scale factor of 4.



2. Triangle Z is a scaled copy of Triangle M.



Select all the sets of values that could be the side lengths of Triangle Z.

**A.**8, 11, and 14.

B. 10, 17.5, and 25.

C. 6, 9, and 11.

**D.**6, 10.5, and 15.

**E.** 8, 14, and 20.

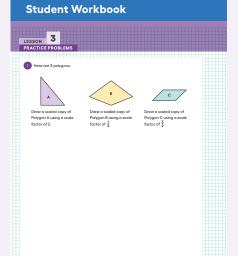
## **Responding To Student Thinking**

#### Points to Emphasize

If students struggle with finding all the possible measurements of a similar triangle, focus on scaled lengths when opportunities arise over the next several lessons. For example, in this activity, have students highlight how all distances in a scaled copy (not just the side lengths of the figure) are related by the same scale factor:

Unit 1, Lesson 4, Activity 1 Measuring the Three Quadrilaterals

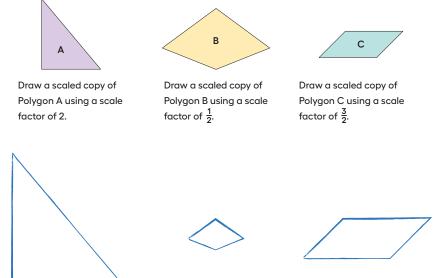
## **Practice Problems**



Student Workbook

# **Problem 1**

Here are 3 polygons.



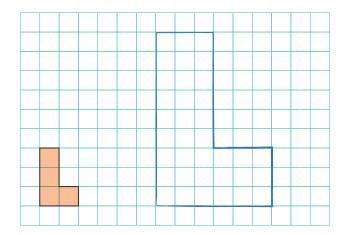
## Problem 2

Quadrilateral A has side lengths 6, 9, 9, and 12. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side of length 2. What is the perimeter of Quadrilateral B?

The scale factor is  $\frac{1}{3}$ , so the side lengths of Quadrilateral B are 2, 3, 3, and 4. The sum of these four numbers is 12, so the perimeter of Quadrilateral B is 12.

### **Problem 3**

**a.** Draw a scaled copy of this polygon so that the scaled copy has a perimeter of 30 units.



**b.** What is the scale factor? Explain how you know.

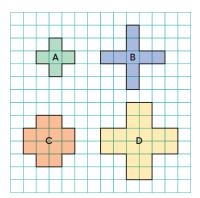
The perimeter of the original polygon is 10 units. Since the perimeter of a scaled copy is multiplied by the scale factor, a scale factor of 3 needs to be applied to get a copy with a perimeter of 30.

Problem 4

from Unit 1, Lesson 1

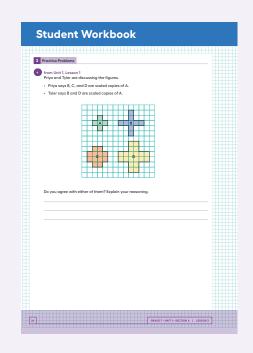
Priya and Tyler are discussing the figures.

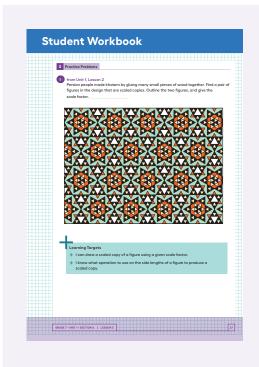
- Priya says B, C, and D are scaled copies of A.
- Tyler says B and D are scaled copies of A.



Do you agree with either of them? Explain your reasoning.

Sample response: I agree with neither one. Only D is a scaled copy of A. In D, the length of each segment of the plus sign is twice the length of its corresponding segment in A. In B and C, some segments are double their corresponding lengths in A but some are not.

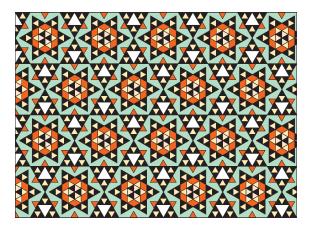




# Problem 5

from Unit 1, Lesson 2

Persian people made khatam by gluing many small pieces of wood together. Find a pair of figures in the design that are scaled copies. Outline the two figures, and give the scale factor.



# Sample responses:

- A pair of equilateral triangles with a scale factor of 1, 2, 3, 4,  $\frac{3}{2}$ , or  $\frac{4}{3}$
- · A pair of hexagons with a scale factor of 2 or 4
- A pair of six-pointed stars with a scale factor of 2

LESSON 3 • PRACTICE PROBLEMS