Lesson 3

What Are Probabilities?

Goals

- Generalize (orally) the relationship between the probability of an event and the number of possible outcomes in the sample space for an experiment in which each outcome in the sample space is equally likely.
- List (in writing) the sample space of a simple chance experiment.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.

Learning Targets

- I can use the sample space to calculate the probability of an event when all outcomes are equally likely.
- I can write out the sample space for a simple chance experiment.

Lesson Narrative

In this lesson students begin to assign probabilities to events happening from chance experiments. They understand that the greater the probability, the more likely the event will occur. They define an *outcome* as a possible result for a chance experiment. They learn that the **sample space** is the set of all possible outcomes, and they understand that a process is called **random** when the outcome of an experiment is based on chance. If there are n equally likely outcomes for a chance experiment, they reason abstractly to find that the **probability** of each of these outcomes is $\frac{1}{n}$.

Student Learning Goal

Let's find out what's possible.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

- Paper bags: Activity 2

Materials to Copy

- What's in the Bag Cutouts (1 copy for every 8 students): Activity 2

Required Preparation

Activity 2:
Print and cut up slips from the blackline master. Each set of slips should be put into a paper bag.

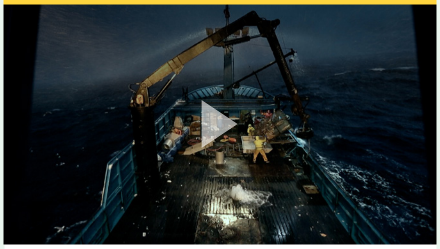
Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: 5 min Cool-down

Inspire Math

Crabbing video



Go Online
Before the lesson, show this video to **introduce** the real-world connection.

ilclass.com/l/614185

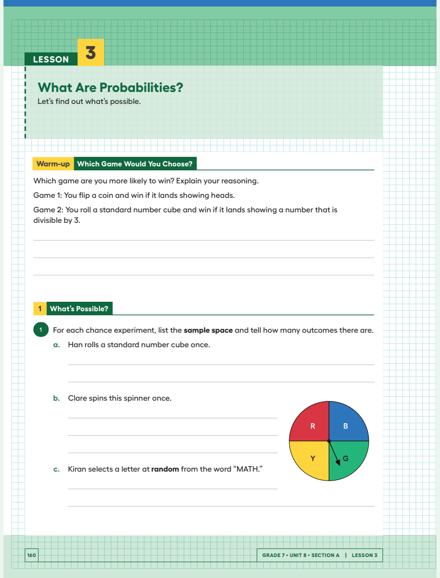
Please log in to the site before using the QR code or URL.

Building on Student Thinking

Some students may have trouble comparing $\frac{1}{2}$ and $\frac{2}{6}$. Ask students how they might write these values as decimals or to draw a shape and divide it into 6 equal regions, then think of what it would look like to shade half of the regions or $\frac{2}{6}$ of the regions.

Some students may struggle with the wording of the second game. Help them understand what it means for a number to be divisible by a certain number and consider providing them with a standard number cube to examine the possible values.

Student Workbook


LESSON 3

What Are Probabilities?

Let's find out what's possible.

Warm-up Which Game Would You Choose?

Which game are you more likely to win? Explain your reasoning.

Game 1: You flip a coin and win if it lands showing heads.

Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

1 What's Possible?

1 For each chance experiment, list the sample space and tell how many outcomes there are.

a. Han rolls a standard number cube once.

b. Clare spins this spinner once.

c. Kiran selects a letter at random from the word "MATH."

Warm-up

Which Game Would You Choose?

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to choose the more likely event based on their intuition about the possible outcomes of two chance experiments. The activities in this lesson that follow define probability and give ways to compute numerical values for the probability of chance events such as these.

Launch

Arrange students in groups of 2.

Give students 1 minute of quiet work time followed by time to share their response with a partner.

Follow with a whole-class discussion.

Student Task Statement

Which game are you more likely to win? Explain your reasoning.

Game 1: You flip a coin and win if it lands showing heads.

Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

Sample response: I would rather play Game 1 since I have 1 out of 2 ways to win, which is half of the time. In Game 2, I only have 2 out of 6 ways to win.

Activity Synthesis

Have partners share their answers and display the results for all to see. Select at least 1 student for each answer provided to give a reason for their choice.

If no student mentions it, explain that the number of possible outcomes that count as a win and the number of total possible outcomes are both important to determining the likelihood of an event. That is, although there are 2 ways to win with the standard number cube and only 1 way to win on the coin, the greater number of possible outcomes in the second game makes it less likely to provide a win.

Activity 1

What's Possible?

15 min

Activity Narrative

In this activity, students are introduced to the term "sample space." Students examine experiments to determine the set of outcomes in the sample space and then use the sample spaces to think about the likelihood of the events. Students reason quantitatively about each situation and represent the likelihood using numbers and the word probability.

Launch

Arrange students in groups of 2.

Define **random** as doing something so that the outcomes are based on chance. An example is putting the integers 1 through 20 on a spinner with each number in an equal sized section. Something that is not random might be answering a multiple choice question on a test for a subject being studied.

Explain that, for a chance experiment, each possible result is called an "outcome." The set of all possible outcomes is called the **sample space**. Spinning a spinner with equal sized sections marked 1 through 20 has a possible outcome of 20, but neither heads nor green is a possible outcome. The sample space is made up of all integers from 1 through 20.

Give students 10 minutes of partner work time and follow with a whole-class discussion.

Instructional Routines

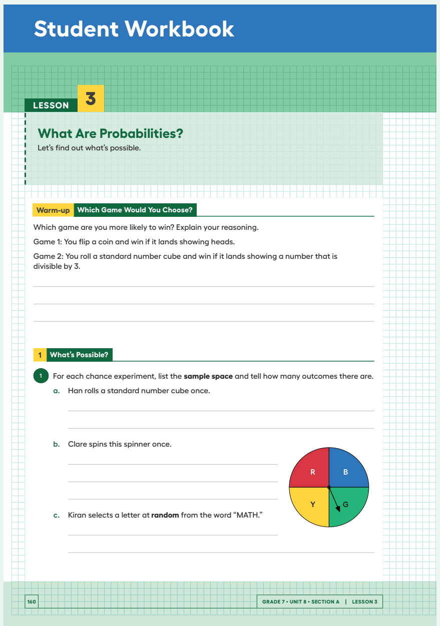
MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time.
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.


Student Workbook

LESSON 3

What Are Probabilities?

Let's find out what's possible.

Warm-up Which Game Would You Choose?

Which game are you more likely to win? Explain your reasoning.

Game 1: You flip a coin and win if it lands showing heads.

Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

1 What's Possible?

1 For each chance experiment, list the **sample space** and tell how many outcomes there are.

a. Han rolls a standard number cube once.

b. Clare spins this spinner once.

c. Kiran selects a letter at **random** from the word "MATH."

GRADE 7 • UNIT 8 • SECTION A | LESSON 3


Student Workbook

1 What's Possible?

d. Mai selects a letter at random from the alphabet.

e. Noah picks a card at random from a stack that has cards numbered 5 through 20.

2 Next, compare the likelihood of these outcomes. Be prepared to explain your reasoning.

a. Is Clare more likely to have the spinner stop on the red or blue section?

b. Is Kiran or Mai more likely to get the letter T?

c. Is Han or Noah more likely to get a number that is greater than 5?

3 Suppose you have a spinner that is evenly divided showing all the days of the week. You also have a bag of papers that list the months of the year. Are you more likely to spin the current day of the week or pull out the paper with the current month?

GRADE 7 • UNIT 8 • SECTION A | LESSON 3

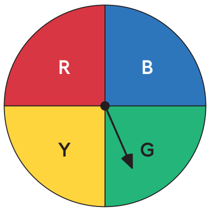
Student Task Statement

1. For each chance experiment, list the **sample space** and tell how many outcomes there are.

   a. Han rolls a standard number cube once.

      Sample space: 1, 2, 3, 4, 5, 6. There are 6 outcomes.

   b. Clare spins this spinner once.



      Sample space: Red, Blue, Green, Yellow. There are 4 outcomes.

   c. Kiran selects a letter at **random** from the word "MATH."

      Sample space: M, A, T, H. There are 4 outcomes.

   d. Mai selects a letter at random from the alphabet.

      Sample space: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. There are 26 outcomes.

   e. Noah picks a card at random from a stack that has cards numbered 5 through 20.

      Sample space: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20. There are 16 outcomes.

2. Next, compare the likelihood of these outcomes. Be prepared to explain your reasoning.

   a. Is Clare more likely to have the spinner stop on the red or blue section?

      Both are equally likely.

   b. Is Kiran or Mai more likely to get the letter T?

      Kiran is more likely to get a T since there are fewer possibilities in the sample space.

   c. Is Han or Noah more likely to get a number that is greater than 5?

      Noah is more likely to get a number that is greater than 5, because 15 out of the 16 outcomes are greater than 5 for Noah, but for Han, only 1 out of the 6 outcomes are greater than 5.

3. Suppose you have a spinner that is evenly divided showing all the days of the week. You also have a bag of papers that list the months of the year. Are you more likely to spin the current day of the week or pull out the paper with the current month?

   It is more likely to spin the current day, since there are only 7 possible days, but 12 possible months.

Are You Ready for More?

Are there any outcomes for two people in this activity that have the same likelihood? Explain or show your reasoning.

Sample response: It is equally likely that Clare will spin red and that Kiran will select an A. Since both outcomes only show up once and they both have sample spaces of 4 equally likely outcomes, these two events are equally likely.

Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the last question. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "How many outcomes are in each event?"
- "How did you use the number of outcomes to determine the likelihood?"
- "How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.
As time allows, invite students to compare their first and final drafts.
Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time,* explain that sometimes it is important to have an actual numerical value rather than a vague sense of likelihood. To answer how probable something is to happen, we assign a **probability**.

The probability of an event is a number that tells how likely it is to happen. Probabilities are values between 0 and 1 and can be expressed as a fraction, decimal, or percentage. Something that has a 50% chance of happening, like a coin landing heads up, can also be described by saying,

"The probability of a coin landing heads up is $\frac{1}{2}$" or "The probability of the coin landing heads up is 0.5."

When each outcome in the sample space is equally likely, we may calculate the probability of a desired event by dividing the number of outcomes for which the event occurs by the total number of outcomes in the sample space.

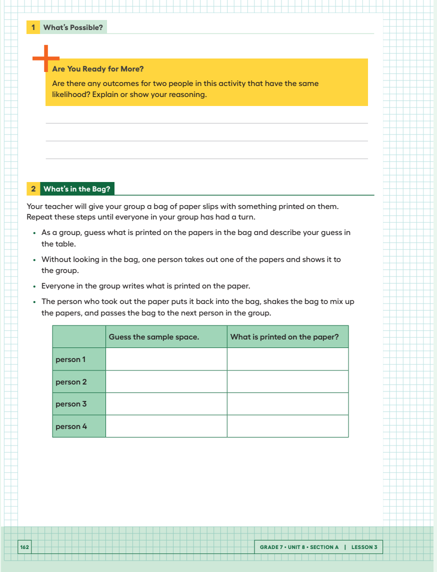
"What is the probability of spinning the current day from the last question? Explain your reasoning."

$\frac{1}{7}$, because today is 1 day out of the 7 possible days of the week

"What is the probability that Noah gets a number greater than 5? Explain your reasoning."

$\frac{15}{16}$, because there are 15 outcomes in the event and 16 outcomes in the sample space

Student Workbook


1 What's Possible?

Are You Ready for More?

Are there any outcomes for two people in this activity that have the same likelihood? Explain or show your reasoning.

2 What's in the Bag?

Your teacher will give your group a bag of paper slips with something printed on them. Repeat these steps until everyone in your group has had a turn.

- As a group, guess what is printed on the papers in the bag and describe your guess in the table.
- Without looking in the bag, one person takes out one of the papers and shows it to the group.
- Everyone in the group writes what is printed on the paper.
- The person who took out the paper puts it back into the bag, shakes the bag to mix up the papers, and passes the bag to the next person in the group.

	Guess the sample space.	What is printed on the paper?
person 1		
person 2		
person 3		
person 4		

162 GRADE 7 • UNIT 8 • SECTION A | LESSON 3

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Develop Language and Symbols.
Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of outcomes. Terms may include "random," "outcome," and "sample space."

Supports accessibility for: Conceptual Processing, Language

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, pause to check for understanding after 3–5 minutes of work time.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may think that the phrase "equally likely" means there is a 50% chance of it happening. Tell students that, in this context, each outcome is equally likely if the probability does not change if you change the question to a different outcome in the sample space. For example, "What is the probability you get a letter A from this bag?" has the same answer as the question, "What is the probability you get a letter B from this bag?"

Activity 2

What's in the Bag?

15 min

Activity Narrative

In this activity, students are introduced to the idea that not all sample spaces are obvious before actually doing the experiment. Therefore, it is not always possible to calculate the exact probabilities for events before doing or simulating the experiment. Students refine their guesses about the sample space by repeatedly drawing items from a bag and looking for patterns in this repetition.

Launch

Arrange students in groups of 4. Provide each group with a paper bag containing 1 set of slips cut from the blackline master.

Allow students 10 minutes for partner work and follow with a whole-class discussion.

Student Task Statement

Your teacher will give your group a bag of paper slips with something printed on them. Repeat these steps until everyone in your group has had a turn.

- As a group, guess what is printed on the papers in the bag and describe your guess in the table.
- Without looking in the bag, one person takes out one of the papers and shows it to the group.
- Everyone in the group writes what is printed on the paper.
- The person who took out the paper puts it back into the bag, shakes the bag to mix up the papers, and passes the bag to the next person in the group.

	Guess the sample space.	What is printed on the paper?
person 1		
person 2		
person 3		
person 4		

1. How is guessing the sample space the fourth time different from the first?

   Sample response: There is more information in round 4, so it narrows the possibilities.

2. Without looking in the bag, what could you do to get a better guess of the sample space?

   Sample response: Keep drawing out more papers.

3. Look at all the papers in the bag. Are any of your guesses correct?

   Sample response: No, it wasn't the whole alphabet, just A through O.

4. Are all of the possible outcomes equally likely? Explain.

   Yes, there is one of each letter.

5. Use the sample space to determine the **probability** that a fifth person would get the same outcome as person 1.

   $\frac{1}{15}$ since there is one out of 15 possible things to choose from.

Activity Synthesis

The purpose of this discussion is for students to understand that often, in the real-world, we do not know the entire sample space before doing the experiment. They will learn in later lessons how to estimate the probabilities for such experiments.

Consider asking some of the following questions:

- *"After the first paper is drawn, a group guesses, 'A bunch of letter Cs.' What might they have picked on their first paper that would lead to that guess? What could that group get on their second paper that would make them change their guess? Could they get something for the second paper that would make them sure their guess was right?"*

  They probably picked a letter C on their first draw. Any other letter on the second draw should make them change their guess. No, they might get another C that would make them think they are right, but with only two tries, there is still a good chance that other letters are in the bag.

- *"After the second paper is drawn, a group guesses that the sample space is 'All of the consonants.' What might they have picked in their first two papers that would lead to that guess? What could that group get on their third paper that would make them change their guess? Could they get something that would make them more sure of their guess?"*

  They probably picked two consonants on their first two draws. If they picked a vowel, they would have to change their guess. If they picked another consonant, they might feel better about the guess, but should still not be certain of it.

- *"How did you refine your predictions with each round?"*

  "If you had a new bag of papers and you took out papers 50 times and never got a 'Z,' would that mean there is no 'Z' in the bag?"

  Not necessarily, but it might make me wonder if it's not in there.

Student Workbook


2 What's in the Bag?

1 How is guessing the sample space the fourth time different from the first?

2 Without looking in the bag, what could you do to get a better guess of the sample space?

3 Look at all the papers in the bag. Are any of your guesses correct?

4 Are all of the possible outcomes equally likely? Explain.

5 Use the sample space to determine the **probability** that a fifth person would get the same outcome as person 1.

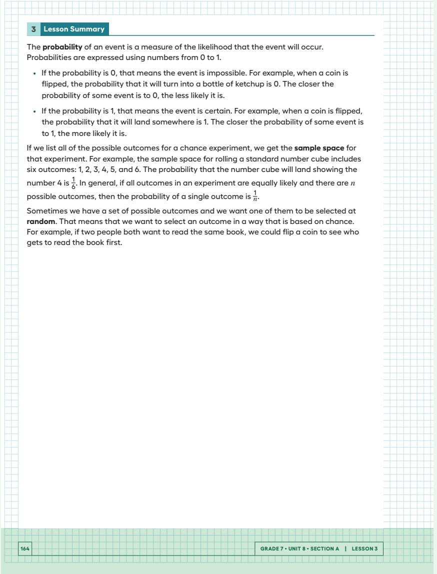
GRADE 7 • UNIT 8 • SECTION A | LESSON 3

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say …" Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Student Workbook



Lesson Synthesis

Consider asking some of the questions:

- *"If you choose one letter at random from the English alphabet, how many outcomes are there in the sample space? How many outcomes are in the event that a vowel (not including Y) is chosen?"*

  There are 26 outcomes in the sample space. There are 5 outcomes that are vowels: A, E, I, O, and U.

- *"What is the sample space of a chance experiment? How is the number of outcomes in the sample space related to the probability of an event if the outcomes in the sample space are equally likely?"*

  The sample space is the list of possible outcomes for an experiment. The number of outcomes in the sample space influences the denominator of the probability when written as a fraction.

- *"When there are 100 different outcomes in the sample space that are equally likely, what is the probability that a specific outcome will happen?"*

  $\frac{1}{100}$ or 1% or 0.01

Lesson Summary

The **probability** of an event is a measure of the likelihood that the event will occur. Probabilities are expressed using numbers from 0 to 1.

- If the probability is 0, that means the event is impossible. For example, when a coin is flipped, the probability that it will turn into a bottle of ketchup is 0. The closer the probability of some event is to 0, the less likely it is.
- If the probability is 1, that means the event is certain. For example, when a coin is flipped, the probability that it will land somewhere is 1. The closer the probability of some event is to 1, the more likely it is.

If we list all of the possible outcomes for a chance experiment, we get the **sample space** for that experiment. For example, the sample space for rolling a standard number cube includes six outcomes: 1, 2, 3, 4, 5, and 6. The probability that the number cube will land showing the number 4 is $\frac{1}{6}$. In general, if all outcomes in an experiment are equally likely and there are n possible outcomes, then the probability of a single outcome is $\frac{1}{n}$.

Sometimes we have a set of possible outcomes and we want one of them to be selected at **random.** That means that we want to select an outcome in a way that is based on chance. For example, if two people both want to read the same book, we could flip a coin to see who gets to read the book first.

Cool-down

Letter of the Day

5 min

Student Task Statement

A mother decides to teach her son about a letter each day of the week. She will choose a letter from the name of the day. For example, on Saturday she might teach about the letter S or the letter U, but not the letter M.

1. What letters are possible to teach using this method? (There are 15.)

   A, D, E, F, H, I, M, N, O, R, S, T, U, W, Y

2. What are 4 letters that can't be taught using this method?

   Any 4 of B, C, G, J, K, L, P, Q, V, X, Z

3. On TUESDAY, the mother writes the word on a piece of paper and cuts it up so that each letter is on a separate piece of paper. She mixes up the papers and picks one. What is the probability that she will choose the piece of paper with the letter Y? Explain your reasoning.

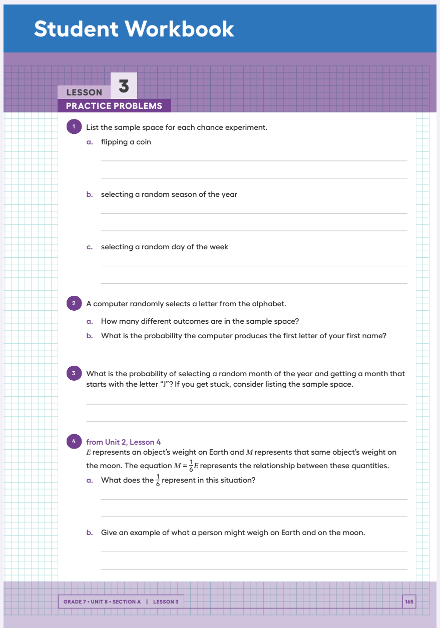
   $\frac{1}{7}$

   Sample reasoning: There are 7 outcomes in the sample space, all outcomes are equally likely, and there is only 1 outcome that corresponds to the letter Y.

Responding To Student Thinking

Points to Emphasize
If students struggle with finding the probability of a given situation, revisit strategies when opportunities arise over the next several lessons. For example, in the lesson referred to here, invite multiple students to share their thinking about how they determined the probability.

Unit 8, Lesson 4 Estimating Probabilities Through Repeated Experiments


Student Workbook

LESSON 3
PRACTICE PROBLEMS

1 List the sample space for each chance experiment.

a. flipping a coin

b. selecting a random season of the year

c. selecting a random day of the week

2 A computer randomly selects a letter from the alphabet.

a. How many different outcomes are in the sample space?

b. What is the probability the computer produces the first letter of your first name?

3 What is the probability of selecting a random month of the year and getting a month that starts with the letter "J"? If you get stuck, consider listing the sample space.

4 from Unit 2, Lesson 4
E represents an object's weight on Earth and M represents that same object's weight on the moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

a. What does the $\frac{1}{6}$ represent in this situation?

b. Give an example of what a person might weigh on Earth and on the moon.

Lesson 3

Practice Problems

6 Problems

Problem 1

List the sample space for each chance experiment.

a. flipping a coin

heads, tails

b. selecting a random season of the year

spring, summer, fall, winter

c. selecting a random day of the week

Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

Problem 2

A computer randomly selects a letter from the alphabet.

a. How many different outcomes are in the sample space?

26

b. What is the probability the computer produces the first letter of your first name?

$\frac{1}{26}$

Problem 3

What is the probability of selecting a random month of the year and getting a month that starts with the letter "J"? If you get stuck, consider listing the sample space.

$\frac{3}{12}$ (or equivalent)

Problem 4

from Unit 2, Lesson 4

E represents an object's weight on Earth and M represents that same object's weight on the moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

a. What does the $\frac{1}{6}$ represent in this situation?

Sample response: $\frac{1}{6}$ is the constant of proportionality relating an object's weight on Earth to its weight on the moon. Something that weighs 1 pound on the Earth weighs $\frac{1}{6}$ of a pound on the moon. Or, to find weight on the moon, multiply the weight on Earth by $\frac{1}{6}$. Or for every pound of weight on Earth, something has $\frac{1}{6}$ of a pound of weight on the moon.

b. Give an example of what a person might weigh on Earth and on the moon.

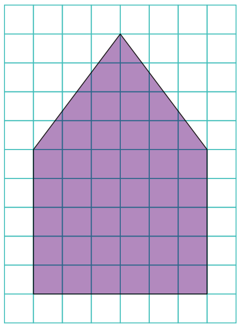
Sample response: A person who weighs 150 pounds on Earth weighs 25 pounds on the moon.

Problem 5

from Unit 7, Lesson 13

Here is a diagram of the base of a bird feeder which is in the shape of a pentagonal prism. Each small square on the grid is 1 in^2.

The distance between the two bases is 8 in. What will be the volume of the completed bird feeder?



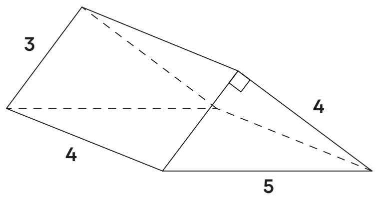
336 in^3

The area of the base is 42 in^2 because it is composed of a rectangle with area 30 in^2 and a triangle of area 12 in^2. Multiplying by the distance between the bases, we have $42 \cdot 8 = 336$.

Problem 6

from Unit 7, Lesson 14

Find the surface area of the triangular prism.



60 square units

Student Workbook