Lesson 11

Filling Containers

Goals

- Create a graph of a function from collected data, and interpret (in writing) a point on the graph.
- Draw a container for which the height of water as a function of volume would be represented as a piecewise linear function, and explain (orally) the reasoning.
- Interpret (orally and in writing) a graph of heights of certain cylinders as a function of volume, and compare the rates of change of the functions.

Learning Targets

- I can collect data about a function and represent it as a graph.
- I can describe the graph of a function in words.

Lesson Narrative

The goal of this lesson is to start interweaving the development of the function concept with the development of formulas for volumes. This work will start with cylinders and progress to cones and spheres in later lessons.

Because students have not yet learned these formulas, the context of filling a cylindrical container with water is useful for developing the abstract concept of functions. It makes physical sense that the height of the water is a function of its volume even if we cannot write down an equation for the function. At the same time, considering how changing the diameter of the cylinder changes the graph of the function helps students develop a geometric understanding of how the volume is related to the height and the diameter.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Which Three Go Together?

Required Materials

Materials to Gather

- Graduated cylinders: Activity 1

Required Preparation

Activity 1:
Each group of 3–4 students needs 1 graduated cylinder and water.

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	5 min

Lesson 11

Filling Containers

Lesson Narrative (continued)

In this lesson, students fill a graduated cylinder with different amounts of water and draw the graph of the height as a function of the volume. They next consider how their data and graph would change if their cylinder had a different diameter. The following activity turns the situation around: When given a graph showing the height of water in a container as a function of the volume of water in the container, can students create a sketch of what the container must look like?

Student Learning Goal

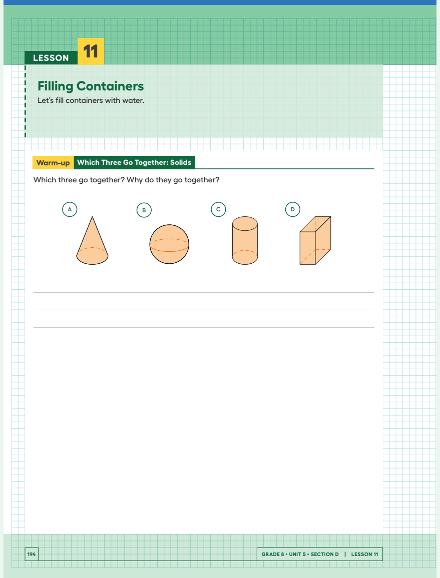
Let's fill containers with water.

Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 11
Filling Containers
Let's fill containers with water.
Warm-up Which Three Go Together: Solids
Which three go together? Why do they go together?

Warm-up
Which Three Go Together: Solids

5 min

Activity Narrative

This *Warm-up* prompts students to compare four different objects and think about how they are similar and different from objects they have encountered in previous activities and grade levels. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

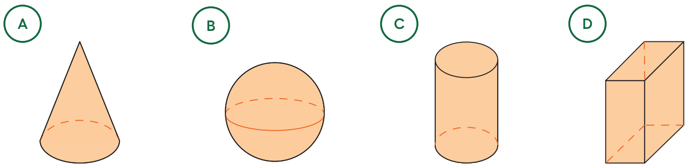
Launch

Arrange students in groups of 2–4. Display the objects for all to see. If time allows, after 30 seconds of quiet think time, invite 2–3 students to briefly share what they notice all of the figures have in common (for example, they all look like three-dimensional objects). The purpose of this initial share out is to support all students in naming common attributes before they work to identify more specific features that a group of only three out of the four have.

Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three shapes that go together and can explain why. Next, tell each student to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- All have curved surfaces.
- All can roll if laid sideways.
- All look like a circle if held a certain way.

A, B, and D go together because:

- They do not have exactly 2 bases.

A, C, and D go together because:

- All have at least 1 flat surface.
- All could be modeled using paper.

B, C, and D go together because:

- If turned upside down, they look the same.
- They have no sharp points.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any geometric terminology they use, such as "side," "edge," "radius," or "surface area," and to clarify their reasoning as needed. Consider asking:

"How do you know …?"

"What do you mean by …?"

"Can you say that in another way?"

Activity 1

Height and Volume

20 min

Activity Narrative

There is a digital version of this activity.

In this activity, students investigate how the height of water in a graduated cylinder is a function of the volume of water in the graduated cylinder. Students make predictions about how the graph will look and then test their prediction by filling the graduated cylinder with different amounts of water, gathering and graphing the data.

In the digital version of the activity, students use an applet to do the experiment. The applet allows students to fill a digital cylinder that displays the volume of water added and the height of the water. The applet also lets students change the radius of the cylinder. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Launch

Arrange students in groups of 3–4. Be sure students know how to measure using a graduated cylinder. If needed, display a graduated cylinder filled to a specific measurement for all to see, and demonstrate to students how to read the measurement. Give each group access to a graduated cylinder and water.

If possible, consider assigning different groups graduated cylinders with different radii to compare and contrast their graphs during the *Activity Synthesis*.

Give groups 8–10 minutes to work on the task, and follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

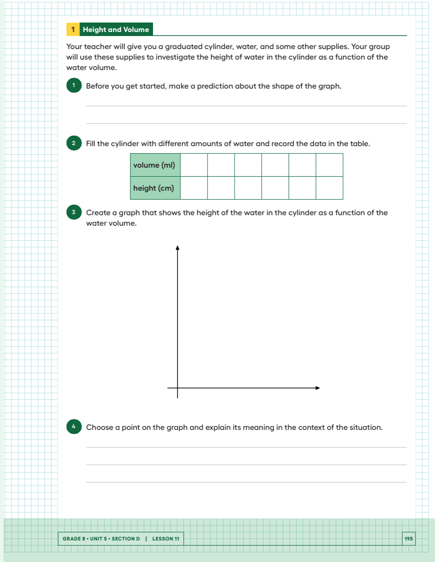
Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies, such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Access for Multilingual Students (Activity 1, Launch)

MLR8: Discussion Supports.
Display sentence frames to support small-group discussion. Examples: "I think ____, because ____" or "I (agree/disagree) because____."

Advances: Speaking, Conversing

Student Workbook


1 Height and Volume

Your teacher will give you a graduated cylinder, water, and some other supplies. Your group will use these supplies to investigate the height of water in the cylinder as a function of the water volume.

1 Before you get started, make a prediction about the shape of the graph.

2 Fill the cylinder with different amounts of water and record the data in the table.

volume (ml)						
height (cm)						

3 Create a graph that shows the height of the water in the cylinder as a function of the water volume.

4 Choose a point on the graph and explain its meaning in the context of the situation.

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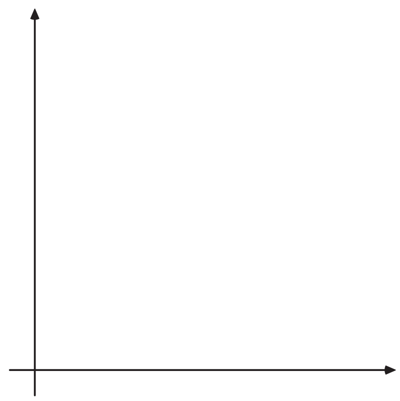
Student Task Statement

Your teacher will give you a graduated cylinder, water, and some other supplies. Your group will use these supplies to investigate the height of water in the cylinder as a function of the water volume.

1. Before you get started, make a prediction about the shape of the graph.
2. Fill the cylinder with different amounts of water and record the data in the table.

volume (ml)						
height (cm)						

3. Create a graph that shows the height of the water in the cylinder as a function of the water volume.



4. Choose a point on the graph and explain its meaning in the context of the situation.

Answers vary according to the shape of the cylinder and the specific measurements taken.

Sample response: For the last part about interpreting points, the point (150, 3) on the graph represents that when 150 milliliters of water is poured into the cylinder, the height of the water would be 3 centimeters

Activity Synthesis

The goal of this discussion is for students to understand how the experiment can be modeled with a linear function.

Invite 2–3 groups to share their graphs, and display these for all to see. Consider asking students the following questions:

"What do you notice about the shape of your graph?"

"What is the independent variable of your graph? Dependent variable?"

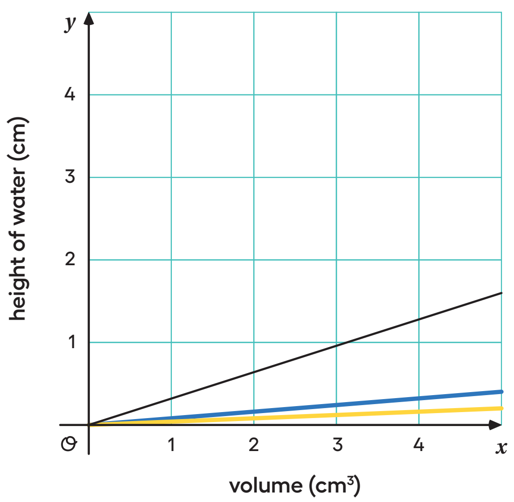
Volume in cubic centimeters is the independent variable, height of the water in centimeters is the dependent variable.

"How does this graph differ from what you predicted the shape would be?"

I thought the graph would curve because of the round shape of the cylinder, but it's straight.

"For the last question, what point did you choose, and what does that point mean in the context of this activity?"

Conclude the discussion by asking students to predict how the graph would change if their cylinder had double the diameter. Alternatively, if it was possible to work with graduated cylinders with different diameters, display those graphs now. After a few responses, display this graph for all to see:



Explain that each line represents the graph of a cylinder with a different radius. One cylinder has a radius of 1 cm, another has a radius of 2 cm, and another has a radius of 3 cm. Ask students to consider which line must represent which cylinder. Ask,

"How did the slope of each graph change as the radius increased?"

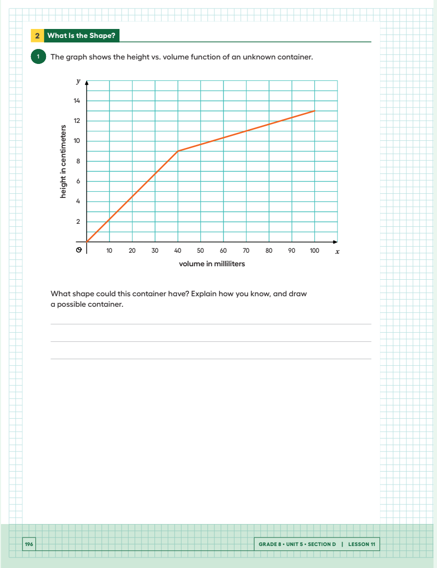
The larger the radius, the less steep the slope. This is because for a cylinder with a larger base, the same volume of water will not fill as high up the side of the cylinder.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Represent the same information through different modalities by using a range of different-sized containers for students to test to determine if their volume and height could be represented by the graphs.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Workbook



Activity 2
What Is the Shape?

10 min

Activity Narrative

Previously, students were given a container and asked to draw the graph of the height as a function of the volume. In this activity, students are given the graph and asked to draw a sketch of the container that could have generated that height function. The goal of this activity is for students to practice connecting words to graphs of functions.

Launch

Arrange students in groups of 2.

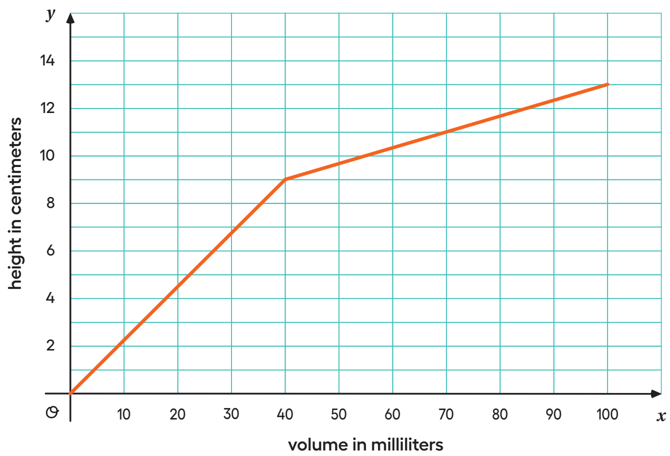
Give students 3–5 minutes of quiet work time and then time to share their drawings with their partner.

Follow with a whole-class discussion.

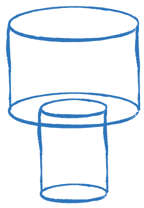
If time is short, consider asking half of the class to work on the first question and the other half to work on the second question, then complete the last question together as part of the *Activity Synthesis*.

Student Task Statement

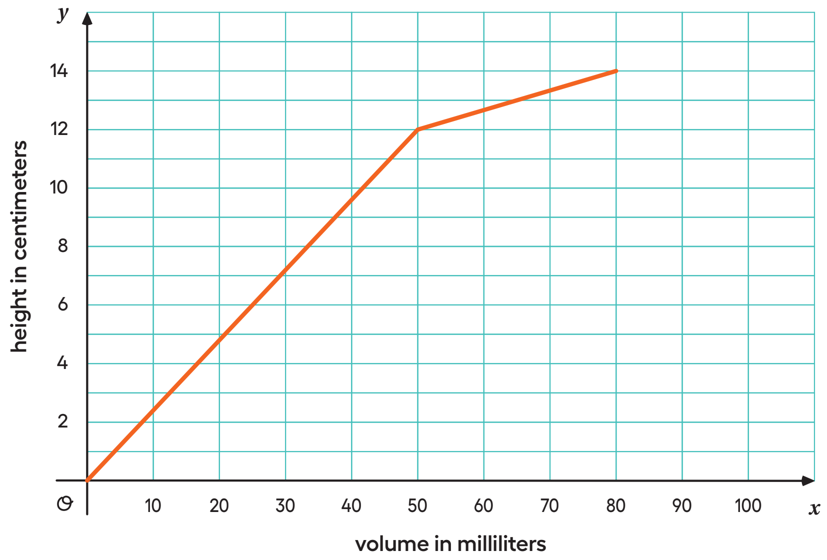
1. The graph shows the height vs. volume function of an unknown container. What shape could this container have? Explain how you know, and draw a possible container.



Sample response: It could have a shape in the form of two cylinders stacked on top of each other, with the upper cylinder having a greater radius. The height grows linearly with the volume in each cylinder, but as the water level rises into the second container, the height will begin to grow less quickly (since it takes more volume to achieve the same increase in height).



2. The graph shows the height vs. volume function of a different unknown container. What shape could this container have? Explain how you know, and draw a possible container.



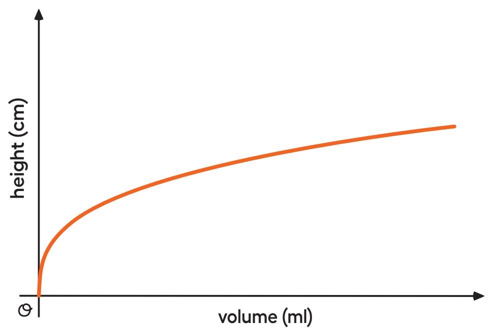
Sample response: It could be 3 cylinders stacked on top of each other. The bottom cylinder should be the tallest. The middle cylinder should be shorter and have a smaller radius than the bottom. The top cylinder should be the shortest but have the largest radius.

3. How are the two containers similar? How are they different?

Sample response: Both containers are made up of cylinders stacked on top of each other. The containers are different because the first container is made up of two parts, while the second is made up of three parts.

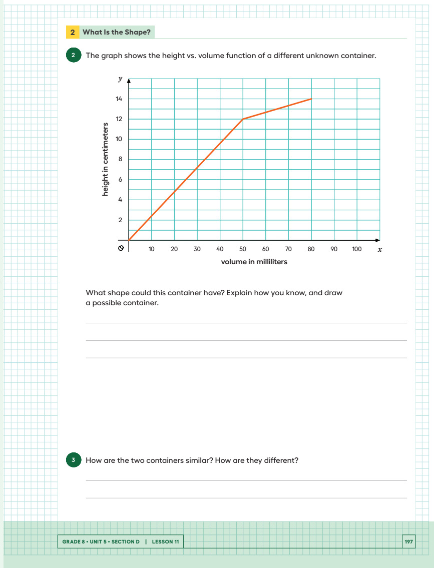
Are You Ready for More?

The graph shows the height vs. volume function of an unknown container. What shape could this container have? Explain how you know, and draw a possible container.

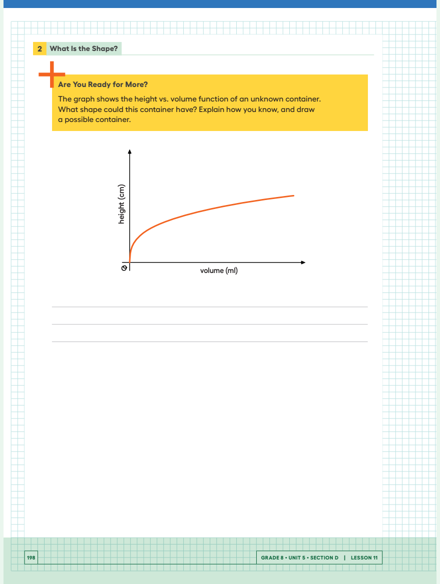


This graph in particular is made using the shape of a cone with its vertex at the bottom. Any shape that is very thin at the bottom and gradually gets wider as it goes up would be a reasonable answer.

Student Workbook



Student Workbook

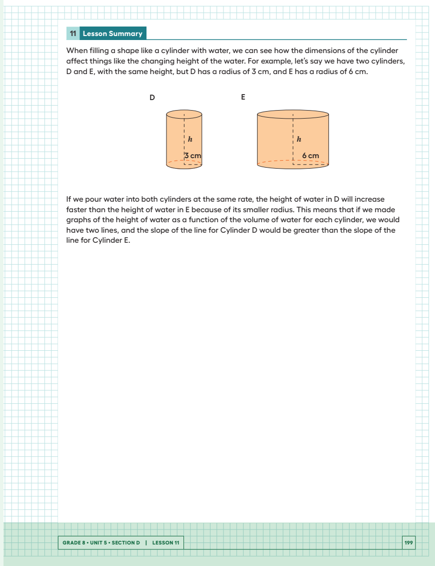


Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to “How are the two containers similar? How are they different?” Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Workbook


11 Lesson Summary

When filling a shape like a cylinder with water, we can see how the dimensions of the cylinder affect things like the changing height of the water. For example, let's say we have two cylinders, D and E, with the same height, but D has a radius of 3 cm, and E has a radius of 6 cm.

If we pour water into both cylinders at the same rate, the height of water in D will increase faster than the height of water in E because of its smaller radius. This means that if we made graphs of the height of water as a function of the volume of water for each cylinder, we would have two lines, and the slope of the line for Cylinder D would be greater than the slope of the line for Cylinder E.

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Activity Synthesis

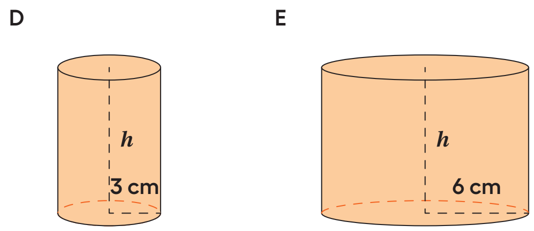
Select students to share the different containers they drew. Display their drawings and the graph for all to see. Ask students to explain how they came up with their drawing and refer to parts in the graph that determined the shape of their container.

Lesson Synthesis

To support students working flexibly between graphs and descriptions of functions, invite students make their own graph showing the height and volume of a container. Tell students to use 2–5 lines for their container. Once the graphs are made, ask students to swap with a partner and try to draw the shape of their partner’s container. Conclude by inviting 2–3 groups to share their graphs and container drawings.

Lesson Summary

When filling a shape like a cylinder with water, we can see how the dimensions of the cylinder affect things like the changing height of the water. For example, let’s say we have two cylinders, D and E, with the same height, but D has a radius of 3 cm, and E has a radius of 6 cm.



If we pour water into both cylinders at the same rate, the height of water in D will increase faster than the height of water in E because of its smaller radius. This means that if we made graphs of the height of water as a function of the volume of water for each cylinder, we would have two lines, and the slope of the line for Cylinder D would be greater than the slope of the line for Cylinder E.

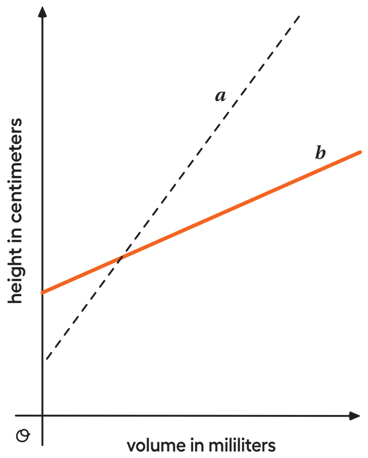
Cool-down

Which Cylinder?

5 min

Student Task Statement

Two cylinders, a and b, each started with different amounts of water. The graph shows how the height of the water changed as the volume of water increased in each cylinder. Which cylinder has the larger radius? Explain how you know.



Cylinder b

Sample reasoning: A cylinder with a large radius would have a smaller change in height (slope) for the same volume of water added when compared to a cylinder with a smaller radius. Since the line for b has the smaller slope, it must be the cylinder with the larger radius.

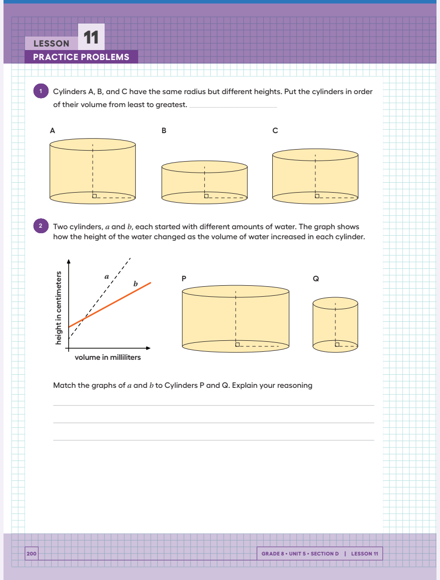
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

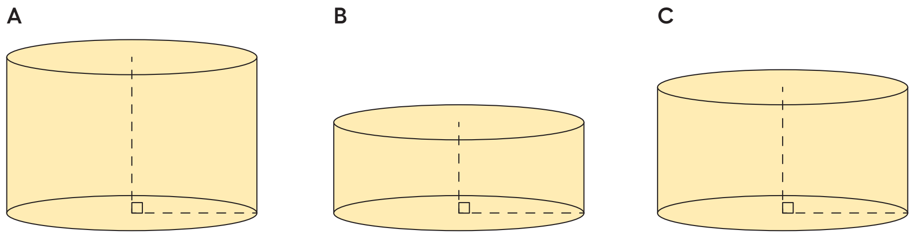
4 Problems

Student Workbook


LESSON 11
PRACTICE PRODUCTS
1 Cylinders A, B, and C have the same radius but different heights. Put the cylinders in order of their volume from least to greatest.
2 Two cylinders, a and b, each started with different amounts of water. The graph shows how the height of the water changed as the volume of water increased in each cylinder.
Match the graphs of a and b to Cylinders P and Q. Explain your reasoning

Problem 1

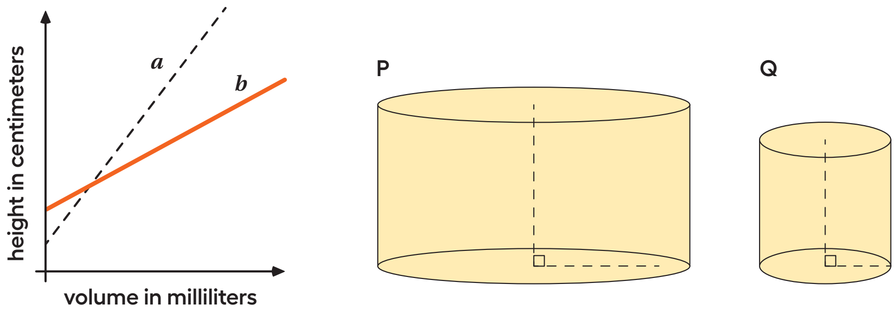
Cylinders A, B, and C have the same radius but different heights. Put the cylinders in order of their volume from least to greatest.



Cylinder B, Cylinder C, Cylinder A

Problem 2

Two cylinders, a and b, each started with different amounts of water. The graph shows how the height of the water changed as the volume of water increased in each cylinder. Match the graphs of a and b to Cylinders P and Q. Explain your reasoning.

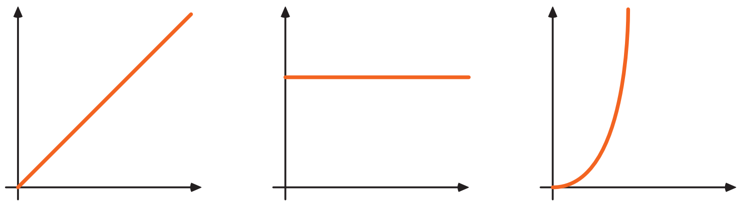


Line a matches Cylinder Q and line b matches Cylinder P.

Sample reasoning: A cylinder with a large radius would have a smaller rate of change (slope) for the same volume of water added when compared to a cylinder with a smaller radius. Since line b has the smaller slope, it must be the cylinder with the larger radius.

Problem 3

Which of the following graphs could represent the volume of water in a cylinder as a function of its height? Explain your reasoning.



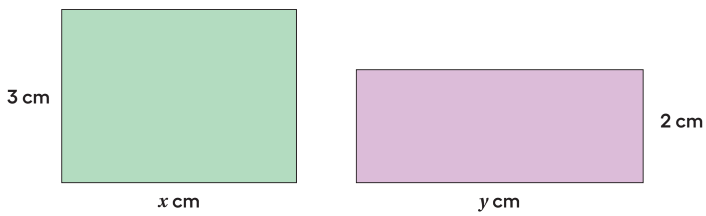
The linear, increasing graph

Sample reasoning: As the height of water in a cylinder increases, the volume increases by the same scale factor.

Problem 4

from Unit 5, Lesson 3

Together, the areas of the rectangles sum to 30 square centimeters.



a. Write an equation showing the relationship between x and y.

$3x + 2y = 30$ (or equivalent)

b. Fill in the table with the missing values.

x	3	$6\frac{2}{3}$	8	$3\frac{1}{3}$	12
y	10.5	5	3	10	not possible

Student Workbook