Lesson 5

Circumference and Wheels (Optional)

Goals

- Compare wheels of different sizes and explain (orally) why a larger wheel needs fewer rotations to travel the same distance.
- Generalize that the distance a wheel rolls in one rotation is equal to the circumference of the wheel.
- Write an equation to represent the proportional relationship between the number of rotations and the distance a wheel travels.

Learning Target

If I know the radius or diameter of a wheel, I can find the distance the wheel travels in some number of revolutions.

Lesson Narrative

This lesson is optional because it goes beyond grade-level expectations. The goal of this lesson is to extend students' understanding of circumference by exploring various proportional relationships related to how far a wheel travels when it rolls. This relationship is vital for how odometers and speedometers work in vehicles.

First, students notice that the circumference of a circle is the same as the distance a wheel rolls forward as it completes one rotation. Next, they see that there is a proportional relationship between the number of times a wheel rotates and the distance the wheel travels. Students use quantitative and abstract reasoning to represent this relationship with an equation. The last activity examines the relationship between the speed a vehicle is traveling and the number of rotations of the tires in a given amount of time.

Student Learning Goal

Let's explore how far different wheels roll.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Poll the Class

Required Materials

Materials to Gather

- Blank paper: Activity 1
- Cylindrical household items: Activity 1
- Receipt tape: Activity 1
- Rulers: Activity 1

Materials to Copy

- Units of Length Reference Sheet (1 copy for every 2 students): Activity 3

Required Preparation

Activity 1:

The circular objects that students measured earlier can be reused for this activity. Rather than wrapping something around each circle, students will roll the circle on a flat surface in order to measure its circumference. If reusing the same set of circular objects, make sure that the groups do not get the objects that they did in the previous activity.

Prepare to distribute blank paper that is long enough for students to trace one complete rotation of their cylindrical object. For objects with a diameter greater than 4 inches, receipt tape may be better.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	15 min	10 min	5 min

Warm-up

A Rope and a Wheel

5 min

Activity Narrative

In this *Warm-up*, students analyze a statement comparing the distance around a wheel with the length of a rope. They apply their understanding of the meaning and rough value of π to determine whether the rope is long enough to go around the wheel. As students analyze Han's statement, they critique the reasoning of others.

Launch

Give students 1 minute of quiet think time followed by 2 minutes of partner discussion.

Student Task Statement

Han says that you can wrap a 5-foot rope around a wheel with a 2-foot diameter because $\frac{5}{2}$ is less than pi. Do you agree with Han? Explain your reasoning.

Sample reasoning: No, the circumference of the wheel is 2π feet. Since π is a little bit larger than 3, this is more than 6 feet, and the 5-foot rope will not fit all the way around.

Activity Synthesis

The goal of this discussion is for students to articulate that Han's calculation is correct, but his conclusion is incorrect. Invite several students to share their reasoning. After each response, ask the class if they agree or disagree.

The key takeaways are:

- The circumference of the wheel is 2π feet.
- π is larger than 3, so the circumference of this wheel is more than 6 feet.
- Han is right that $\frac{5}{2} < \pi$, but this means that the rope will not make it all the way around.

If time permits, extend the discussion by asking:

"Would a 6-foot rope be long enough to go around the wheel?"

No, because $\frac{6}{2}$ is still less than π.

"What about a 7-foot rope?"

Yes, because the circumference of the wheel is 2π feet, and this is less than 7.

Students may observe that it is possible to wrap the rope around the wheel going around a diameter twice as opposed to going around the circumference.

Activity 1
Rolling, Rolling, Rolling

15 min

Activity Narrative

In this activity, students roll circular objects across a paper and mark how far they travel in one complete rotation. They divide the distance traveled by the diameter of the circle and see that the quotient is close to π. They relate this to what they previously learned about the relationship between the diameter of a circle and its circumference.

As students investigate the distance an object rolls, by drawing, measuring, calculating, and comparing, they persevere in making sense of circumference.

Monitor for students who recognize that the distance the object rolls is equal to the circumference of the circle.

Launch

Demonstrate drawing a diagonal line from one corner to the far corner on a piece of paper. Then, demonstrate rolling a circle along the line, marking the beginning and end of one complete rotation.

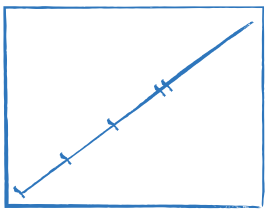
Arrange students in groups of 2. Distribute one circular object to each group. Provide access to blank paper (which should be long enough to complete one full rotation of each object) and rulers.

Student Task Statement

Your teacher will give you a circular object.

1. Follow these instructions to create the drawing:
   a. On a separate piece of paper, use a ruler to draw a diagonal line all the way across the page.
   b. Roll your object along the line and mark where it completes one rotation.
   c. Use your object to draw tick marks along the line that are spaced as far apart as the diameter of your object.

   A line with 3 equally spaced tick marks and a little more length after the third tick mark.



2. What do you notice?

   The distance the circle rolled is a little more than three times the diameter.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

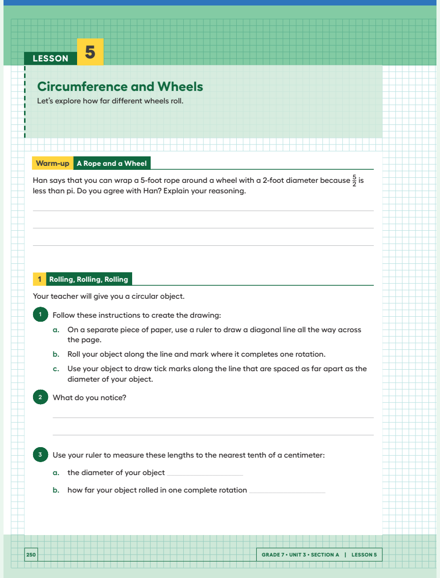
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Student Task)

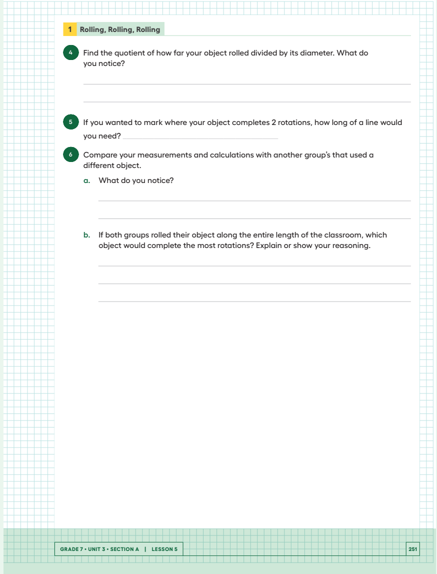
MLR8: Discussion Supports. Display sentence frames to support whole-class discussion: "I agree because …" or "I disagree because …"

Advances: Speaking, Conversing

Student Workbook



Student Workbook



1 Rolling, Rolling, Rolling

4 Find the quotient of how far your object rolled divided by its diameter. What do you notice?

5 If you wanted to mark where your object completes 2 rotations, how long of a line would you need?

6 Compare your measurements and calculations with another group's that used a different object.

a. What do you notice?

b. If both groups rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

3. Use your ruler to measure these lengths to the nearest tenth of a centimeter:

   a. the diameter of your object

      Sample response: 6.8 cm

   b. how far your object rolled in one complete rotation

      Sample response: 21.5 cm

4. Find the quotient of how far your object rolled divided by its diameter. What do you notice?

   Sample response: 3.1617 This is close to pi.

5. If you wanted to mark where your object completes 2 rotations, how long a line would you need?

   Sample response: One rotation was about 21.5 cm, so 2 rotations would require about 43 cm.

6. Compare your measurements and calculations with another group's that used a different object.

   a. What do you notice?

      Sample response: The measurements for diameter and distance rolled were both less than ours, but their quotient was also close to pi.

   b. If both groups rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

      Sample response: The smallest object would complete the most rotations, because it travels the shortest distance for one complete rotation. It has the smallest circumference.

Activity Synthesis

The goal of this discussion is for students to understand that the distance a wheel travels in one complete rotation is equal to the circumference of the wheel.

Poll the class on the quotients they found for the distance their object rolled divided by the object's diameter. Connect the different responses to the learning goals by asking questions such as:

- *"What do you notice about these values?"*

  They are close to π.

- *"Why does it make sense for this quotient to equal pi?"*

  The distance an object rolls in one complete rotation is equal to the object's circumference, and $C \div d = \pi$.

- *"Why aren't all our measurements exactly equal to pi?"*

  measurement error

Remind students of the activity from the other day when they measured the circumference of circular objects:

- *"When we measure the circumference by wrapping a measuring tape around the circle, the circle stays in place while the measuring tape goes around it. When we roll the circle, we can imagine the measuring tape unwinding while the circle moves."*

If desired, discuss which method of measuring the circumference was more precise (rolling the circle or wrapping a measuring tape around it)? Some reasons why measuring the circumference of the circle directly may be more precise include:

- When you roll the circular object, it is hard to keep it going in a straight line.
- It is difficult to mark one rotation precisely.

Activity 2

Rotations and Distance

15 min

Activity Narrative

In this activity, students investigate proportional relationships between the number of rotations a wheel makes and the distance that wheel travels. Students make repeated calculations with explicit numbers and then write an equation representing the proportional relationship. When students write an equation to represent the relationship and then use the equation to solve problems, they reason quantitatively and abstractly.

Launch

Instruct students to use 3.14 as the approximation for π in these problems. Keep students in the same groups. Give students 5–6 minutes of quiet work time, followed by partner and whole-class discussion.

If time is limited, consider assigning half of the class to focus on the problem about the car and the other half to focus on the problem about the bike.

Student Task Statement

1. A car wheel has a diameter of 20.8 inches.

   a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

   1 rotation: 20.8π inches or about 65.3 inches, 5 rotations: about 327 inches, 30 rotations: about 1,960 inches

   b. Write an equation relating the distance that the car travels in inches, c, to the number of wheel rotations, x.

   $c = 20.8\pi x$

   c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

   The car wheel makes about 970 rotations. One mile is 5,280 feet or 63,360 inches. Dividing this by 20.8π will give the number of wheel revolutions to go one mile.

2. A bike wheel has a radius of 13 inches.

   a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

   1 rotation: 26π inches or about 81.7 inches, 5 rotations: about 408 inches, 30 rotations: about 2,450 inches

Access for Multilingual Learners (Activity 2, Student Task)

MLR5: Co-Craft Questions.
Keep books or devices closed. Display only the problem stem and related image, without revealing the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task and invite additional connections.

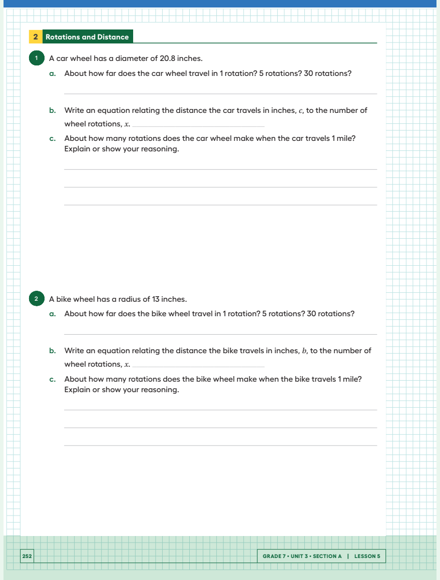
Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Provide a blank two-column table for students to process and organize information. Invite students to share their column labels (for example, "number of rotations" and "total distance") and how they organized the given information.

Supports accessibility for: Organization, Attention

Student Workbook


2 Rotations and Distance

1 A car wheel has a diameter of 20.8 inches.

a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

b. Write an equation relating the distance the car travels in inches, c, to the number of wheel rotations, x.

c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

2 A bike wheel has a radius of 13 inches.

a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

b. Write an equation relating the distance the bike travels in inches, b, to the number of wheel rotations, x.

c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

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Student Workbook

b. Write an equation relating the distance that the bike travels in inches, b, to the number of wheel rotations, x.

$b = 26\pi x$

c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

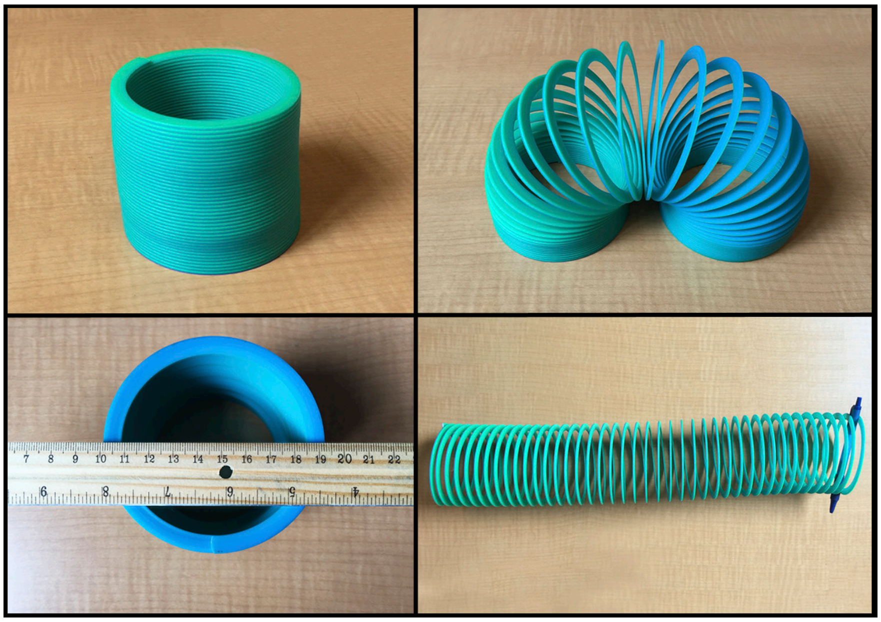
The bike wheel makes about 776 rotations. One mile is 5,280 feet or 63,360. Dividing this by 26π will give the number of wheel revolutions to go one mile.

Building on Student Thinking

Some students may struggle to convert between inches and miles for answering the last part of each question. Remind students that there are 5,280 feet in a mile. Ask students how many inches are in 1 foot. Make sure students arrive at a final answer of 63,360 inches in one mile before calculating the number of rotations made by each wheel.

Are You Ready for More?

Here are some pictures of a spring toy.



If you could stretch out the spring completely straight, how long would it be? Explain or show your reasoning.

We can compute the approximate length if we know the diameter of the circle and the total number of loops. Diameter is ≈ 9.5 cm. There are 42 loops. Therefore, $(9.5)(\pi)(42) \approx 1252.86$. The length is about 1,253 cm, or about 12.5 meters.

Activity Synthesis

The goal of this discussion is to make sense of the equations that represent the situations. When looking at the relationship between the number of rotations and the distance a wheel travels, the constant of proportionality is the circumference of the wheel.

Ask students:

- *"What is the constant of proportionality for each relationship? What does that tell you about the situation?"*

  65.3 and 81.7, the number of inches each wheel travels per rotation, which is also the circumference of each wheel.

- *"How do the two wheels compare? How can you see this in the equations?"*

  The bike wheel is larger. The constant of proportionality is larger in the equation representing the bike.

- *"Which wheel makes fewer rotations to travel one mile?"*

  The bike does. Its wheels are larger, so it moves farther in one rotation.

Activity 3: Optional

Rotations and Speed

15 min

Activity Narrative

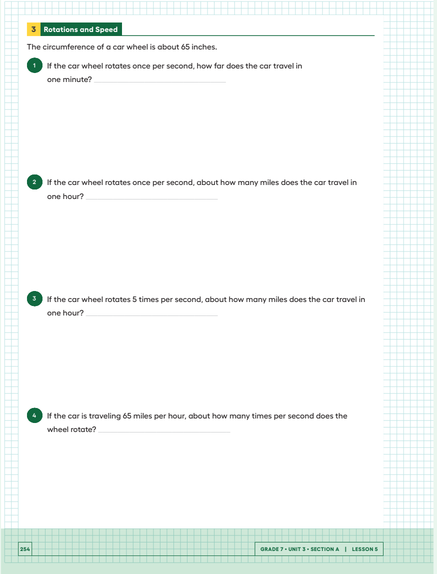
In this activity, students investigate the relationship between the rotational speed of a wheel (in rotations per second) and the linear speed of the vehicle (in miles per hour).

First, students must apply their recent learning to recognize that the circumference of the wheel equals the distance the car travels for one complete rotation of the wheel. Then, students work through a series of unit conversions to get from rotations per second to miles per hour. As students choose steps for converting the rates into different units, they reason quantitatively and abstractly.

Monitor for students who use the following strategies to solve the last problem:

- Reverse all the conversion steps they used to solve the previous problems, so 65 miles per hour is 343,200 feet per hour or 1,144 inches per second or 17.6 rotations per second
- Compare the new speed to their previous calculations for 1 rotation per second, such as
  - Compare 65 miles per hour to the approximation 3.7 miles per hour gets $65 \div 3.7 = 17.\overline{567}$
  - Convert 65 miles per hour to 343,200 feet per hour and then comparing to the previous rate of 19,500 feet per hour gets $343{,}200 \div 19{,}500 = 17.6$.

Student Workbook


3 Rotations and Speed

The circumference of a car wheel is about 65 inches.

1 If the car wheel rotates once per second, how far does the car travel in one minute?

2 If the car wheel rotates once per second, about how many miles does the car travel in one hour?

3 If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour?

4 If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate?

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Launch

Keep students in the same groups. A copy of the Units of Length Reference Sheet can be found in the student workbook.

Give students 4–5 minutes of quiet work time for the first three problems, followed by partner discussion. Then give students 2–3 minutes of partner work time to consider the last problem.

Student Task Statement

The circumference of a car wheel is about 65 inches.

1. If the car wheel rotates once per second, how far does the car travel in one minute? 3,900 inches (or 325 feet)

   There are 60 seconds in a minute, so if the car wheel rotates once per second, that's 60 rotations in a minute. At 65 inches per rotation that is $60 \cdot 65$, or 3,900 inches. This is equivalent to 325 feet because there are 12 inches in a foot, and $3{,}900 \div 12 = 325$.

2. If the car wheel rotates once per second, about how many miles does the car travel in one hour? about 3.7 miles per hour

   There are 60 minutes in an hour, and the car travels 325 feet in a minute, so in one hour the car would travel $60 \cdot 325$, or 19,500 feet. This is approximately 3.7 miles, because there are 5,280 feet in a mile, and $19{,}500 \div 5{,}280 \approx 3.7$.

3. If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour? about 18.5 miles per hour

   If the car wheel is rotating 5 times per second, then the car will travel 5 times farther than it did when it was rotating once per second. That's about 97,500 feet ($5 \cdot 19{,}500 = 97{,}500$) or 18.5 miles ($5 \cdot 3.7 = 18.5$).

4. If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate? about 17.6 rotations per second

   If the car travels 65 miles per hour that is 343,200 feet in an hour. Each rotation of the wheel per second amounts to 19,500 feet traveled in one hour. So to travel 343,200 feet in an hour the wheels will rotate $343200 \div 19500$, or about 17.6 times per second.

Building on Student Thinking

Some students may do the calculations in feet but not know how to convert their answers to miles. Remind them that there are 5,280 feet in 1 mile.

Activity Synthesis

The goal of this discussion is to highlight students' use of multi-step ratio reasoning in solving these problems.

Invite students to share their reasoning for the first two questions. Emphasize the different rates that come up while solving these problems, such as:

- 65 inches of distance traveled per rotation of the wheels
- 1 rotation of the wheel per second
- 65 inches of distance traveled per second

- 60 seconds per minute
- 3,900 inches of distance traveled per minute
- 12 inches per foot
- 325 feet of distance traveled per minute
- 60 minutes per hour
- 19,500 feet of distance traveled per hour
- 5,280 feet per mile
- about 3.7 miles of distance traveled per hour

If time permits, invite students to share their reasoning for the last problem. Consider displaying this table with no rates in the right column except for the bottom row.

1 rotation per second	
65 inches per second	
3,900 inches per minute	
325 feet per minute	
19,500 feet per hour	
3.7 miles per hour	65 miles per hour

As students share their reasoning, record equivalent rates in the right column working from the bottom of the chart to the top.

1 rotation per second	17.6 rotations per second
65 inches per second	1,144 inches per second
3,900 inches per minute	68,640 inches per minute
325 feet per minute	5,720 feet per minute
19,500 feet per hour	343,200 feet per hour
3.7 miles per hour	65 miles per hour

Lesson Synthesis

Share with students:

"Today we saw that the circumference of a circle is how far the circle rolls in one complete revolution. The distance a wheel travels is proportional to the number of rotations that the wheel makes, and the constant of proportionality equals the circumference of the wheel."

If desired, use this example to review these concepts:

"A wheel has a diameter of 7 inches and a circumference of 22 inches."

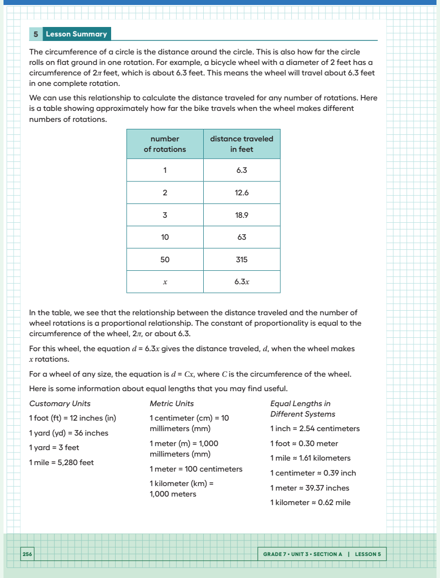
"How far does the wheel travel in 1 rotation? 2 rotations? 10 rotations?"

22 inches, 44 inches, 220 inches

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


5 Lesson Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 2 feet has a circumference of 2π feet, which is about 6.3 feet. This means the wheel will travel about 6.3 feet in one complete rotation.

We can use this relationship to calculate the distance traveled for any number of rotations. Here is a table showing approximately how far the bike travels when the wheel makes different numbers of rotations.

number of rotations	distance traveled in feet
1	6.3
2	12.6
3	18.9
10	63
50	315
x	$6.3x$

In the table, we see that the relationship between the distance traveled and the number of wheel rotations is a proportional relationship. The constant of proportionality is equal to the circumference of the wheel, 2π, or about 6.3.

For this wheel, the equation $d = 6.3x$ gives the distance traveled, d, when the wheel makes x rotations.

For a wheel of any size, the equation is $d = Cx$, where C is the circumference of the wheel.

Here is some information about equal lengths that you may find useful.

Customary Units
1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units
1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems
1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers
1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

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"What is an equation that represents the relationship between the number of rotations and the distance this wheel travels?"

$d = 22n$, where d represents distance traveled and n represents number of rotations

"Another wheel has a circumference of 18 inches. If both wheels travel the same distance, which wheel completes more rotations?"

The smaller wheel must complete more rotations to travel the same distance as the larger wheel.

Lesson Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 2 feet has a circumference of 2π feet, which is about 6.3 feet. This means that the wheel will travel about 6.3 feet in one complete rotation.

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For this wheel, the equation $d = 6.3x$ gives the distance traveled, d, when the wheel makes x rotations.

For a wheel of any size, the equation is $d = Cx$, where C is the circumference of the wheel.

Cool-down

Biking Distance

5 min

Student Task Statement

The wheels on Noah's bike have a circumference of about 5 feet.

1. How far does the bike travel as the wheel makes 15 complete rotations?

   75 feet, because $5 \cdot 15 = 75$

2. How many times do the wheels rotate if Noah rides 40 feet?

   8 rotations, because $40 \div 5 = 8$

Lesson **5**

Practice Problems

5 Problems

Problem 1

Find the distance each wheel travels.

a. The circumference of a wagon wheel is 25 inches. The wheel makes 4 complete rotations. 100 inches

b. The diameter of a bike wheel is 27 inches. The wheel makes 15 complete rotations. about 1,272 inches (or 106 feet)

c. The radius of a skateboard wheel is 2.6 centimeters. The wheel makes 100 complete rotations. about 1,633 centimeters (or 16.33 meters)

Problem 2

The wheels on Kiran's bike are 64 inches in circumference. How many times do the wheels rotate if Kiran rides 300 yards? about 169 times

(there are 36 inches in a yard so 10,800 inches in 300 yards and $10{,}800 \div 64 \approx 169$)

Problem 3

from Unit 3, Lesson 4

The numbers are measurements of diameter, radius, and circumference of Circles A and B. Circle A is smaller than Circle B. Which number belongs to which quantity?

2.5 5 7.6 15.2 15.7 47.7

diameter of Circle A: 5

radius of Circle A: 2.5

circumference of Circle A: 15.7

diameter of Circle B: 15.2

radius of Circle B: 7.6

circumference of Circle B: 47.7

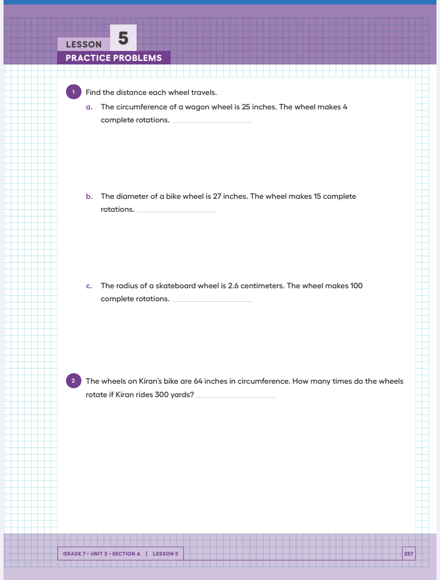
Problem 4

from Unit 3, Lesson 3

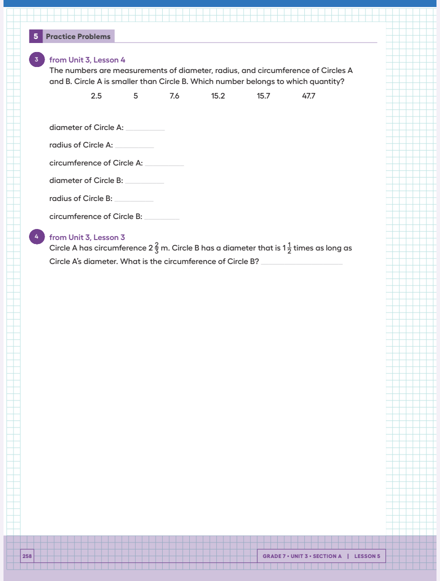
Circle A has circumference $2\frac{2}{3}$ m. Circle B has a diameter that is $1\frac{1}{2}$ times as long as Circle A's diameter. What is the circumference of Circle B? 4 m

(if the diameter of Circle B is $1\frac{1}{2}$ times larger than Circle A, its circumference must be as well, so we can rewrite to calculate: $\left(\frac{8}{3}\right)\left(\frac{3}{2}\right) = 4$)

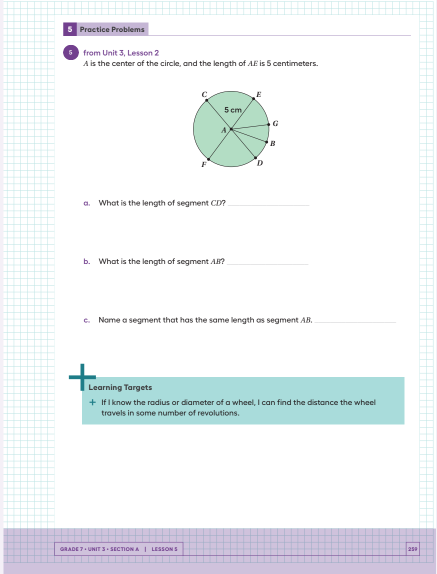
Student Workbook



Student Workbook



Student Workbook


5 Practice Problems

from Unit 3, Lesson 2

A is the center of the circle, and the length of AE is 5 centimeters.

a. What is the length of segment CD?

b. What is the length of segment AB?

c. Name a segment that has the same length as segment AB.

Learning Targets

+ If I know the radius or diameter of a wheel, I can find the distance the wheel travels in some number of revolutions.

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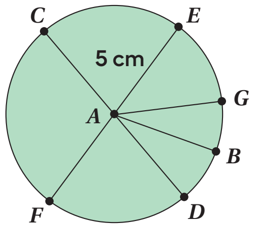
Problem 5

from Unit 3, Lesson 2

A is the center of the circle, and the length of AE is 5 centimeters.



a. What is the length of segment CD? 10 cm

b. What is the length of segment AB? 5 cm

c. Name a segment that has the same length as segment AB. *CA, AF, AD, AG, or AE*