Lesson 3

Representing Proportional Relationships

Goals

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Determine what information is needed to create graphs that represent proportional relationships. Ask questions to elicit that information.

Learning Target

I can scale and label coordinate axes in order to graph a proportional relationship.

Lesson Narrative

In this lesson students label and choose a scale for empty pairs of axes in order to graph a proportional relationship. First, students create a graphical representation of a proportional relationship when given a table and a description to start from. They learn that the **rate of change** in a proportional relationship is the same as the constant of proportionality: the amount one variable changes by when the other variable increases by 1. Next, students use the *Info Gap* structure to graph a proportional relationship on an empty pair of axes that includes a specific point. Students will need to request information about the proportional relationship as well as calculate the specific point. The focus is on the graphs students create and their decisions on how to scale the axes in an appropriate manner for the situation.

Student Learning Goal

Let's graph proportional relationships.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Cool-down (Assessment)
10 min	25 min	10 min	5 min

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Copy

- Graphing Proportional Relationships Cards (1 copy for every 2 students): Activity 1

Warm-up
A Car Wash

10 min

Activity Narrative

This activity gives students a chance to choose an appropriate scale when graphing a proportional relationship on a given set of blank axes. Monitor for students who create particularly clear graphs using situation appropriate scales. For example, since the problem is about a car wash, the scale for the axis showing the number of cars does not need to extend into the thousands.

Launch

Arrange students in groups of 2. Provide access to straightedges.

Ask students,

"What are some different ways the communities you are a part of raise money for a cause?"

Walk-a-thon, put on an event and sell tickets, car wash, hold a raffle, sell coupon books.

After a brief quiet think time, invite students to share their experiences.

Explain that an Origami Club wants to take a trip to see an origami exhibit at an art museum. Then read, or have a student read the Description in the *Student Task Statement* out loud. Explain that the same information is also shown in the table. Give students 3–4 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are two ways to represent a situation.

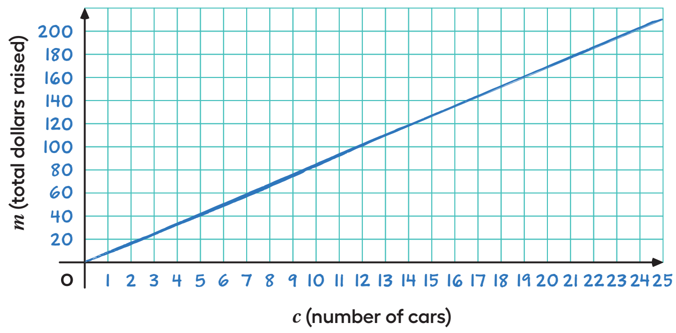
Description:

The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of $93.50. After 23 cars, they raised a total of $195.50.

Table:

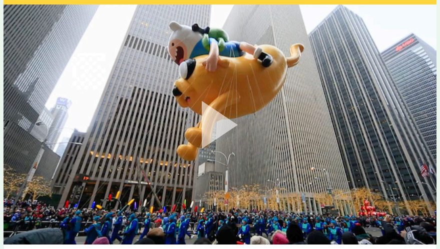
number of cars	amount raised in dollars
11	93.50
23	195.50

Create a graph that represents this situation.



Inspire Math

Helium video

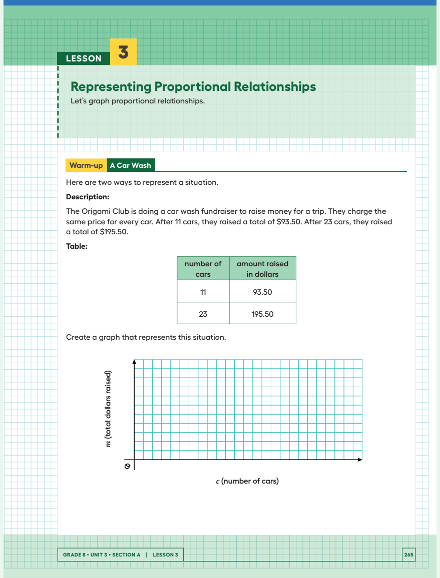


Go Online
Before the lesson, show this video to **introduce** the real-world connection.

ilclass.com/r/614144

Please log in to the site before using the QR code or URL.

Student Workbook



Instructional Routines

MLR4: Information Gap Cards

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Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards. This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity Synthesis

The purpose of this discussion is to introduce students to the term "rate of change." Begin by inviting 2–3 students to share the graphs they created. Emphasize how different scales can be used, but in order to be helpful, the scale for the number of cars, c, on the horizontal axis should extend to at least 23 and the scale for the amount raised in dollars, m, on the vertical axis should extend to at least 200.

Next, tell students that an equation that represents this situation is $m = 8.5c$, where c is the number of cars, and m is the total dollars raised. Display this equation for all to see, then discuss:

- *"What is the constant of proportionality and what does it mean?"*

  The constant of proportionality is 8.5 and it means that each car washed raised \$8.50.

- *"How can you see the constant of proportionality in the graph and the table?"*

  Graph: The slope of the line is equivalent to 8.5. Table: For any given row, the amount raised in dollars divided by the number of cars washed equals 8.5.

- *"Which representation do you think is more useful when calculating the constant of proportionality? Why?"*

Explain that the constant of proportionality can be thought of as the **rate of change**: the amount one variable changes by when the other variable increases by 1. In the case of the Origami Club's car wash, the rate of change of m, the amount they raise in dollars, with respect to c, the number of cars they wash, is 8.50 dollars per car.

Activity 1

Info Gap: Graphing Proportional Relationships

25 min

Activity Narrative

In this activity, students graph a proportional relationship but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch

Tell students they will be graphing some proportional relationships. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to 1 student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Problem Card 1: 76.5 grams of honey are needed for 17 cups of flour.

Graphs vary.

Possible scale: 0–28 on the cups of flour axis, 0–140 on the grams of honey axis.

Problem Card 2: 57.5 grams of salt are needed for 23 cups of flour.

Graphs vary.

Possible scale: 0–28 on the cups of flour axis, 0–70 on the grams of salt axis.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.

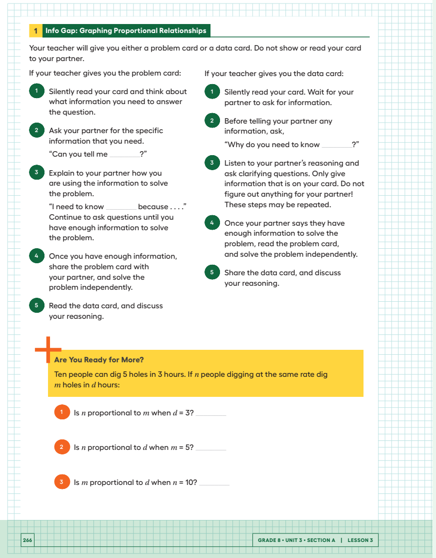
Supports accessibility for: Memory, Organization

Building on Student Thinking

If students are unsure how to scale the axes on their graphs, consider asking:

"What are the largest values that need to be shown on the graph?"
"How many grid lines are there?"

Student Workbook



1 Info Gap: Graphing Proportional Relationships

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If your teacher gives you the problem card:

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2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
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Are You Ready for More?

Ten people can dig 5 holes in 3 hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$? _____
2. Is n proportional to d when $m = 5$? _____
3. Is m proportional to d when $n = 10$? _____

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Are You Ready for More?

Ten people can dig 5 holes in 3 hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$?

   Yes, because if 10 people can dig 5 holes in 3 hours, then 2 people can dig 1 hole in 3 hours, so $n = 2m$.

2. Is n proportional to d when $m = 5$?

   No, because if the number of people doubles, then the time it takes to dig the holes halves, so n is not a constant times d.

3. Is m proportional to d when $n = 10$?

   Yes, because if 10 people can dig 5 holes in 3 hours, then they can dig $\frac{5}{3}$ holes in 1 hour, so $m = \frac{5}{3}d$.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"How did you decide what to label the two axes?"

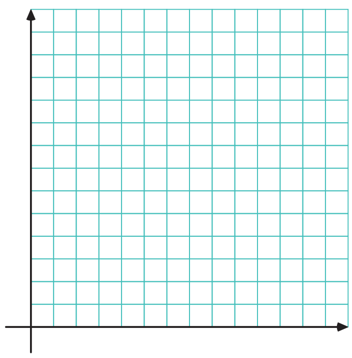
"How did you decide to scale the horizontal axis? The vertical axis?"

"Where can you see the rate of change of grams of honey per cups of flour on the graph?"

"Where can you see the rate of change of grams of salt per cups of flour on the graph?"

Lesson Synthesis

The goal of this discussion is for students to consider the importance of choosing a scale when creating graphs. Display this image or a similar blank coordinate plane and tell students that the proportional relationship $y = 5.5x$ includes the point (18, 99) on its graph.



Ask what an appropriate scale might be to show this point on a pair of axes with a 10 by 10 grid.

Each horizontal grid line could represent 2 units and each vertical grid line could represent 10 or 20 units.

Next ask students,

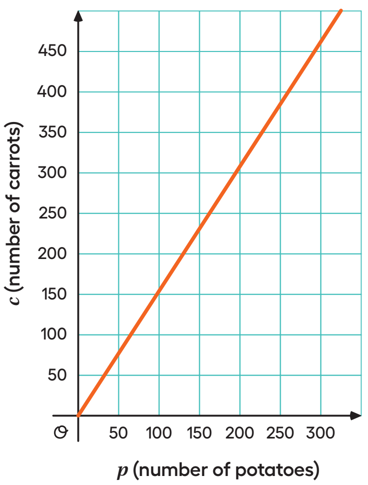
"What are some important ideas to remember when analyzing or creating a graph in the future?"

Consider creating a classroom display with their responses. Important ideas to highlight include:

- When studying a graph, pay attention to the label on each axis.
  For example, the placement of the variables may mean a graph is showing the pace of a bug instead of the speed of a bug.
- When studying a graph, pay attention to the scale. For example, one graph may appear steeper than another graph, but it is the actual value of the slopes that matters.
- When creating a graph, consider the question being asked and the information given when determining the scale. For example, make sure that the scale chosen extends far enough to show the necessary data.

Lesson Summary

Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are p potatoes and c carrots, then $c = \frac{3}{2}p$.

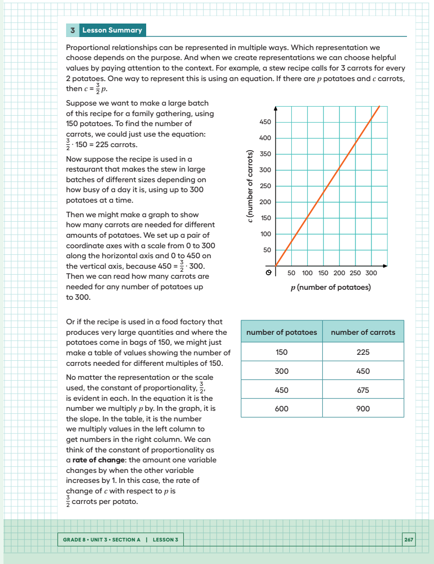


Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots, we could just use the equation: $\frac{3}{2} \cdot 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy of a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \cdot 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.

Student Workbook



3 Lesson Summary

Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are p potatoes and c carrots, then $c = \frac{3}{2}p$.

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots, we could just use the equation: $\frac{3}{2} \cdot 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy of a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \cdot 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.

Or if the recipe is used in a food factory that produces very large quantities and where the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiples of 150.

number of potatoes	number of carrots
150	225
300	450
450	675
600	900

No matter the representation or the scale used, the constant of proportionality, $\frac{3}{2}$, is evident in each. In the equation it is the number we multiply p by. In the graph, it is the slope. In the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change**: the amount one variable changes by when the other variable increases by 1. In this case, the rate of change of c with respect to p is $\frac{3}{2}$ carrots per potato.

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Responding To Student Thinking

Points to Emphasize
If most students struggle with scaling their own axes, in the second question of the activity referred to here, visit groups and support their understanding of how to determine a scale when creating a graph.

Unit 3, Lesson 4, Warm-up, What's the Relationship?

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Cool-down

Graph the Relationship

5 min

Student Task Statement

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

salt (grams)	honey (grams)
10	14
25	35

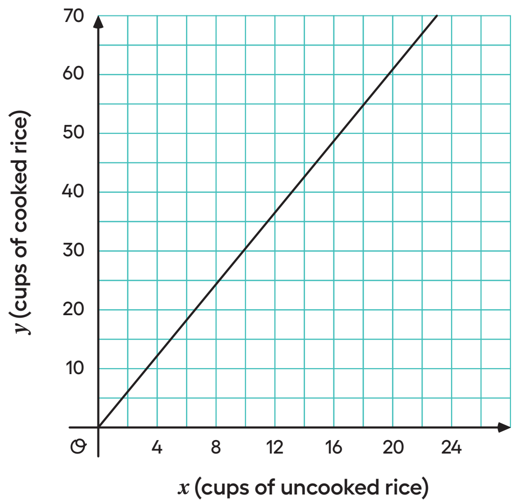
Possible graph: Axes labeled from 0 to 140, with grams of salt on the horizontal axis and grams of honey on the vertical. Coordinate points may include (0, 0), (10, 14), and (70, 98).

Practice Problems

4 Problems

Problem 1

This graph describes the relationship between the volume of uncooked rice and the volume of the rice after it is cooked.



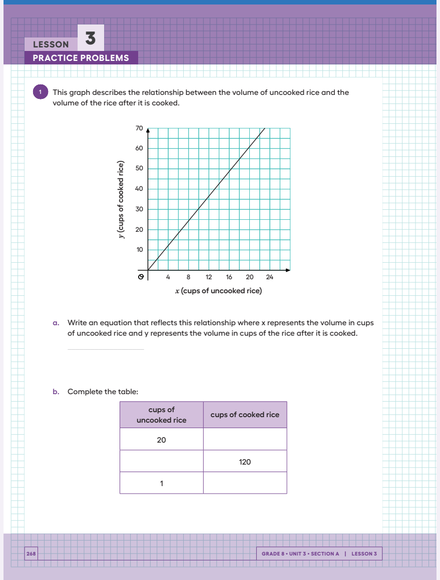
a. Write an equation that reflects this relationship where x represents the volume in cups of uncooked rice and y represents the volume in cups of the rice after it is cooked.

$y = 3x$ or $\frac{y}{x} = \frac{60}{20}$ (or equivalent)

b. Complete the table:

cups of uncooked rice	cups of cooked rice
20	60
$33\frac{1}{3}$	120
1	3

Student Workbook


LESSON 3

PRACTICE PROBLEMS

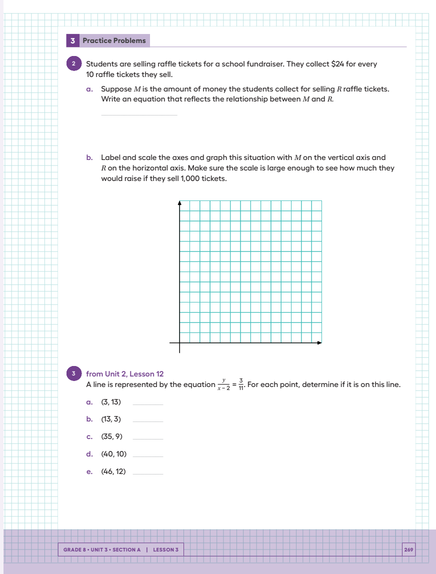
1 This graph describes the relationship between the volume of uncooked rice and the volume of the rice after it is cooked.

a. Write an equation that reflects this relationship where x represents the volume in cups of uncooked rice and y represents the volume in cups of the rice after it is cooked.

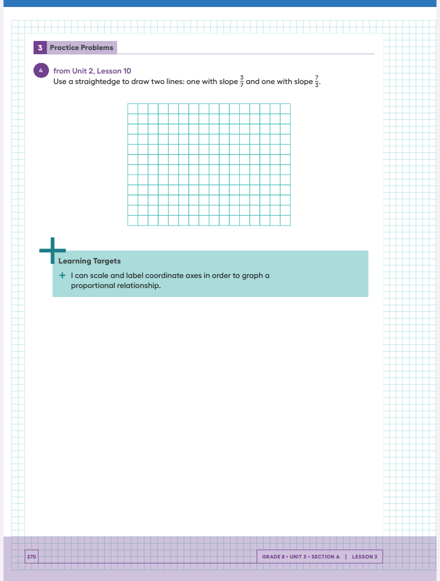
b. Complete the table:

cups of uncooked rice	cups of cooked rice
20	
	120
1	

Student Workbook



Student Workbook



Problem 2

Students are selling raffle tickets for a school fundraiser. They collect $24 for every 10 raffle tickets they sell.

a. Suppose M is the amount of money the students collect for selling R raffle tickets. Write an equation that reflects the relationship between M and R.

$M = \frac{12}{5}R$ (or equivalent)

b. Label and scale the axes and graph this situation with M on the vertical axis and R on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1,000 tickets.

on coordinate axes with R on the horizontal axis and M on the vertical axis, a ray through (0, 0) and (10, 24) (or equivalent)

Problem 3

from Unit 2, Lesson 12

A line is represented by the equation $\frac{y}{x-2} = \frac{3}{11}$. For each point, determine if it is on this line.

a. (3, 13)

No

b. (13, 3)

Yes

c. (35, 9)

Yes

d. (40, 10)

No

e. (46, 12)

Yes

Problem 4

from Unit 2, Lesson 10

Use a straightedge to draw two lines: one with slope $\frac{3}{7}$ and one with slope $\frac{7}{3}$.