Lesson 17

Modeling with Inequalities

Goals

- Critique (orally) the solution to an inequality, including whether fractional or negative values are reasonable.
- Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value. Ask questions to elicit that information.
- Write and solve an inequality of the form $px + q > r$ or $px + q < r$ to answer a question about a situation with a constraint.

Learning Target

I can use what I know about inequalities to solve real-world problems.

Lesson Narrative

In this lesson, students consolidate what they have learned over the last few lessons about writing and solving inequalities. They also consider whether fractional values or negative values make sense in context as solutions to an inequality. For instance, if they find that $x < 7$, but x represents the number of students who can go on a trip, then they should realize that x cannot be 3.25, nor can x be -2. The main activity uses the *Information Gap* routine. To obtain all the necessary information, students need to persevere in asking questions and to communicate precisely.

The last activity is optional because it provides an opportunity for additional practice with writing an inequality to represent a situation and interpreting the solution in context.

Student Learning Goal

Let's look at solutions to inequalities.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- Information Gap (Activity 1)
- MLR6 (Activity 2)

Instructional Routines

- MLR4: Information Gap Cards
- MLR6: Three Reads

Required Materials

Materials to Gather

- Math Community Chart: Activity 1

Materials to Copy

- Giving Advice Cards (1 copy for every 4 students): Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	20 min	15 min	10 min	5 min

Warm-up
Ordering Sandwiches

10 min

Activity Narrative

In this *Warm-up,* students interpret an inequality in a real-world situation and reason about the quantities in its solution. Some of the statements involve reasoning about how a sandwich shop sells its sandwiches, however, the focus of the discussion should be on the meaning of the solution. Students should reason that they cannot order more than 13.86 sandwiches, but can order any number of sandwiches less than 13.86.

The context in this problem provides an opportunity for students to think about aspects of mathematical modeling like discrete versus continuous solutions and rounding.

Launch

Arrange students in groups of 2.

Give students 2 minutes of quiet work time followed by 1 minute to compare their responses with a partner.

Follow with a whole-class discussion.

Student Task Statement

The stage manager of the school musical is trying to figure out how many sandwiches he can order with the $83 he collected from the cast and crew. Sandwiches cost $5.99 each, so he lets x represent the number of sandwiches he will order and writes $5.99x \leq 83$. He solves this to 2 decimal places, getting $x \leq 13.86$.

Determine whether each statement about this situation is true. Be prepared to explain your reasoning.

1. He can call the sandwich shop and order exactly 13.86 sandwiches.

   Probably not. Sample reasoning: It is unlikely a sandwich shop would sell precisely .86 of a sandwich.

2. He can round up and order 14 sandwiches.

   No. Sample reasoning: The solution of $x \leq 13.86$ means that 14 sandwiches would cost more than the $83 the group can spend.

3. He can order 12 sandwiches.

   Yes. Sample reasoning: The solution of $x \leq 13.86$ means that 12 sandwiches will cost less than (or equal to) $83.

4. He can order 9.5 sandwiches.

   Possibly. Sample reasoning: The sandwich shop may sell half sandwiches for half the price of a whole sandwich.

5. He can order 2 sandwiches.

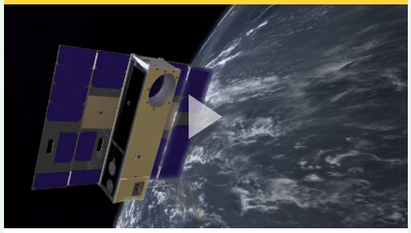
   Yes. Sample reasoning: At a glance, 2 sandwiches will cost much less than $83.

6. He can order -4 sandwiches.

   No. Sample reasoning: Though $-4 \leq 13.86$ is true and -4 is a numerical solution to the inequality, it does not make sense to order -4 sandwiches.

Inspire Math

CubeSats video



Go Online
Before the lesson, show this video to review the real-world connection.

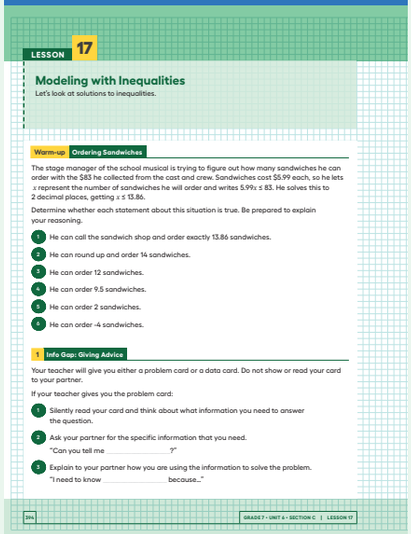
ilclass.com/l/614242

Please log in to the site before using the QR code or URL.

Building on Student Thinking

Some students may think of 13.86 sandwiches as 14 whole sandwiches because it rounds to that number, and 13.86 doesn't make sense to them in the context of sandwiches. Consider asking these students to use a calculator to find the cost of 14 sandwiches to show that it is not a solution to the inequality. Explain that although sandwich shops may not sell sandwiches in fractional pieces, the maximum amount that can be ordered is 13.86.

Student Workbook


LESSON 17

Modeling with Inequalities

Let's look at solutions to inequalities.

Warm-up Ordering Sandwiches

The stage manager of the school musical is trying to figure out how many sandwiches he can order with the $83 he collected from the cast and crew. Sandwiches cost $5.99 each, so he lets x represent the number of sandwiches he will order and writes $5.99x \leq 83$. He solves this to 2 decimal places, getting $x \leq 13.86$.

Determine whether each statement about this situation is true. Be prepared to explain your reasoning.

1. He can call the sandwich shop and order exactly 13.86 sandwiches.
2. He can round up and order 14 sandwiches.
3. He can order 12 sandwiches.
4. He can order 9.5 sandwiches.
5. He can order 2 sandwiches.
6. He can order -4 sandwiches.

1 Info Gap: Giving Advice

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
   "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem.
   "I need to know _____ because..."

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

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Access for Multilingual Learners (Activity 1, Task Statement)

This activity uses the Information Gap math language routine, which facilitates meaningful interactions by

positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity Synthesis

The purpose of this discussion is to highlight how the situation represented by a solution might further constrain the solution.

Poll the class about whether they think each statement is valid. Ask a student to explain why the invalid statements don't work. Record and display their responses for all to see.

For each statement, students should mention the following ideas:

1. Even though 13.86 makes the inequality true, most sandwich shops would not let you order 13.86 sandwiches.
2. He doesn't have enough money to order 14 sandwiches. He has to order a number of sandwiches that is less than or equal to 13.86.
3. He can order 12 sandwiches.
4. He may be able to order 9.5 sandwiches if the shop allows orders in $\frac{1}{2}$-sandwich increments.
5. He can order 2 sandwiches.
6. Even though -4 makes the inequality true, that value doesn't make sense in this context.

Activity 1

Info Gap: Giving Advice

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to set up and solve inequalities that represent real-life situations.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms, or invite a student to read them out loud. Tell students that during this activity, they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Launch

Tell students they will practice using their knowledge of inequalities to think about specific situations and interpret what their solutions mean in those situations. Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

Problem Card 1: 21 loads. Sample reasoning: The situation can be represented with $-1.65x + 50 \geq 15$, where x is the number of loads. The solution is $x \leq 21.2$ (rounded to the nearest tenth).

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

Problem Card 2: widths up to 25.5 centimeters. Sample reasoning: The situation can be represented with $14 + 2w \leq 65$, where w is the width of the frame. The solution to this is $w \leq 25.5$.

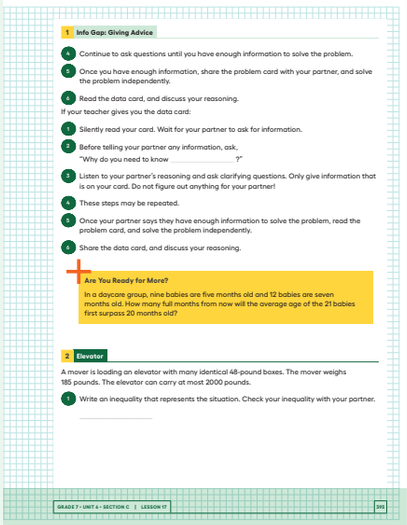
Access for Students with Diverse Abilitie (Activity 1, Task Statement)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity or provide students with a physical copy.
Supports accessibility for: Memory, Organization

Building on Student Thinking

If students do not know where to start, suggest that they first identify the quantity that should be variable and choose a letter to represent it. In Elena's problem, it may help to remind students that they know how to write a formula for the area of a rectangle.

Student Workbook



1 Info Gap: Giving Advice

4 Continue to ask questions until you have enough information to solve the problem.

5 Once you have enough information, share the problem card with your partner, and solve the problem independently.

6 Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1 Silently read your card. Wait for your partner to ask for information.

2 Before telling your partner any information, ask, "Why do you need to know _____?"

3 Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

4 These steps may be repeated.

5 Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.

6 Share the data card, and discuss your reasoning.

Are You Ready for More?

In a daycare group, nine babies are five months old and 12 babies are seven months old. How many full months from now will the average age of the 21 babies first surpass 20 months old?

2 Elevator

A mover is loading an elevator with many identical 48-pound boxes. The mover weighs 185 pounds. The elevator can carry at most 2000 pounds.

1 Write an inequality that represents the situation. Check your inequality with your partner.

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Instructional Routines

MLR6: Three Reads

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Are You Ready for More?

In a daycare group, nine babies are five months old and 12 babies are seven months old. How many full months from now will the average age of the 21 babies first surpass 20 months old?

14 months

Sample reasoning: Right now, the sum of all their ages is 129 months, because $9 \cdot 5 + 12 \cdot 7 = 129$. After x months, the sum of all the babies' ages will have increased by $21x$. For 21 babies to have an average age of 20 months, the sum of all their ages would need to be 420 months, because $\frac{420}{21} = 20$. Solving the inequality $129 + 21x > 420$ we get $x > 13.86$.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"In Noah's problem, is 1.5 loads of laundry a solution to the inequality?"*
- *"In Elena's problem, can the width of the frame be -10 centimeters? Can the width of the frame be 0 centimeters? How about 0.1 centimeter?"*
- *"Which situations have only whole-number solutions?"*
- *"In Noah's problem, should we round up or down?"*

Highlight for students that both the solution to the inequality and the situation itself go into writing a reasonable answer to the question.

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- *"I noticed our norm '_____' in action today, and it really helped me/my group because _____."*

Activity 2

Elevator

15 min

In this activity, students write and solve inequalities to answer the question. The context in this problem provides an opportunity for students to think about aspects of mathematical modeling like discrete versus continuous solutions and rounding.

Launch

Give students 4 minutes of quiet work time, followed by partner and whole-class discussion.

Student Task Statement

A mover is loading an elevator with many identical 48-pound boxes. The mover weighs 185 pounds. The elevator can carry at most 2000 pounds.

1. Write an inequality that represents the situation. Check your inequality with your partner.

   $48b + 185 \leq 2{,}000$ (or equivalent), where b is the number of identical boxes

2. Solve your inequality and explain what the solution means.

   $b \leq 37.8125$ (or equivalent)

   Sample reasoning: The mover can put up to 37 boxes on the elevator.

3. Graph the solution to your inequality on a number line.

   A number line shows a closed circle at 37 and an arrow to the left.

4. If the mover asked, "How many boxes can I load on this elevator at a time?" what would you tell them?

   Sample response: 37 or fewer boxes

Activity Synthesis

The purpose of this discussion is to clarify how to write a reasonable answer to a question based on both the solution to the inequality and the meaning of the situation.

Consider discussing:

- *"The number line shows that 5.5 is a solution to the inequality, but is it a solution to the problem in context?"*

  No, it is not a solution to the problem because the mover can't load half of a box.

- *"Do we want to change the number line somehow to show this?"*

  We could plot discrete points, or we could simply leave it as is, but just know that for a problem with this context, we're only going to use whole-number solutions.

- *"Which type of inequality would you use to describe answers using no more than or no less than?"*

  $\leq$ and $\geq$, respectively.

- *"How did you know which way to round?"*

  Round down, otherwise the values would go over the weight limit.

- *"What other limitations do the contexts place on the solutions?"*

  You can't have a negative number of boxes.

Lesson Synthesis

Share with students, "Today we wrote and solved inequalities. We practiced asking questions to get the needed information to solve a problem."

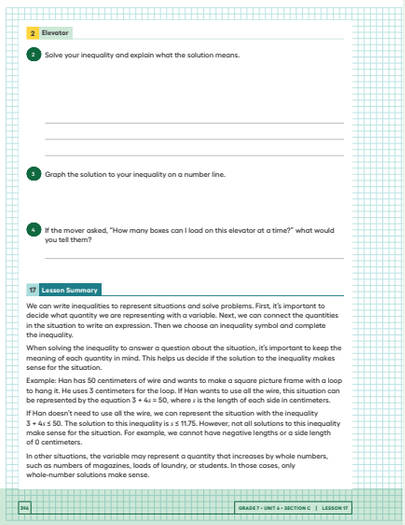
Invite students to reflect on how they can use inequalities to represent and solve real-world problems. Here are some questions to elicit student thinking:

- *"What are some things to keep in mind when writing an inequality to represent a situation?"*

Access for Multilingual Learners (Activity 2, Task Statement)

MLR6: Three Reads.
Keep books or devices closed. Display only the problem stem, without revealing the questions.
"We are going to read this 3 times."
After the 1st read:
"Tell your partner what this situation is about."
After the 2nd read:
"List the quantities. What can be counted or measured?"
For the 3rd read: Reveal and read the questions. Ask,
"What are some ways we might get started on this?"
Advances: Reading, Representing

Student Workbook


2 Elevator

2. Solve your inequality and explain what the solution means.

3. Graph the solution to your inequality on a number line.

4. If the mover asked, "How many boxes can I load on this elevator at a time?" what would you tell them?

17 Lesson Summary

We can write inequalities to represent situations and solve problems. First, it's important to decide what quantity we are representing with a variable. Next, we can connect the quantities in the situation to write an expression. Then we choose an inequality symbol and complete the inequality.

When solving the inequality to answer a question about the situation, it's important to keep the meaning of each quantity in mind. This helps us decide if the solution to the inequality makes sense for the situation.

Example: Han has 50 centimeters of wire and wants to make a square picture frame with a loop to hang it. He uses 3 centimeters for the loop. If Han wants to use all the wire, this situation can be represented by the equation $3 + 4s = 50$, where s is the length of each side in centimeters.

If Han doesn't need to use all the wire, we can represent the situation with the inequality $3 + 4s \leq 50$. The solution to this inequality is $s \leq 11.75$. However, not all solutions to this inequality make sense for the situation. For example, we cannot have negative lengths or a side length of 0 centimeters.

In other situations, the variable may represent a quantity that increases by whole numbers, such as numbers of magazines, loads of laundry, or students. In those cases, only whole-number solutions make sense.

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Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "I chose this inequality symbol because ..." "To solve the inequality, first I _____ because ..." "Why did you ... ?" or "What does _____ mean?"

Supports accessibility for: Language, Social-Emotional Functioning

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of interpreting and solving inequalities. If students struggle, plan to make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.
Grade 7, Unit 6, Section C Inequalities

"What are some things to keep in mind when solving an inequality?"

"What are some things to keep in mind when interpreting what the solution to an inequality tells you about the situation?"

Lesson Summary

We can write inequalities to represent situations and solve problems. First, it's important to decide what quantity we are representing with a variable. Next, we can connect the quantities in the situation to write an expression. Then we choose an inequality symbol and complete the inequality.

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In other situations, the variable may represent a quantity that increases by whole numbers, such as numbers of magazines, loads of laundry, or students. In those cases, only whole-number solutions make sense.

Cool-down

Playlist Timing

5 min

Student Task Statement

Elena is trying to create a playlist that lasts no more than 2 hours (120 minutes). She has already added songs that total 15 minutes. She reads that the average song length on her music streaming service is 3.5 minutes. Elena writes the inequality $3.5x + 15 \geq 120$ and solves it to find the solution $x \geq 30$.

1. Explain how you know Elena made a mistake based on her solution.

   Sample response: $x \geq 30$ means Elena can add more than 30 songs on the playlist. This doesn't make sense because there should be a maximum limit on songs rather than a minimum limit.

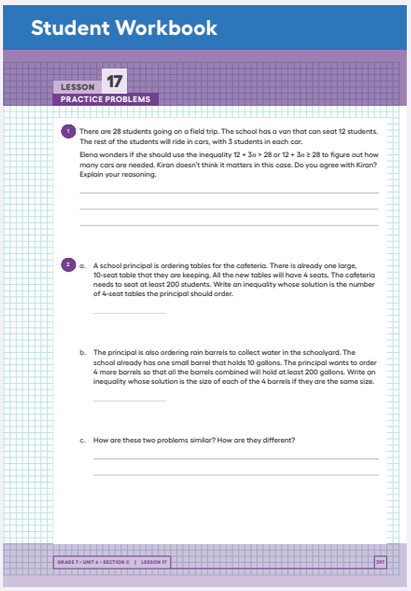
2. Fix Elena's inequality and explain what each part of the inequality means.

   The correct inequality is $3.5x + 15 \leq 120$. The number 3.5 represents the average length of each song. The variable x represents the number of songs that Elena adds. The 15 represents the 15 minutes of songs that are already on the playlist. The ≤ 120 represents that the total number of minutes has to be less than or equal to 120.

Lesson 17

Practice Problems

5 Problems



Problem 1

There are 28 students going on a field trip. The school has a van that can seat 12 students. The rest of the students will ride in cars, with 3 students in each car.

Elena wonders if she should use the inequality $12 + 3n > 28$ or $12 + 3n \geq 28$ to figure out how many cars are needed. Kiran doesn't think it matters in this case. Do you agree with Kiran? Explain your reasoning.

Yes, I agree with Kiran that it doesn't matter

Sample reasoning: In this case n represents a number of cars, so only whole number values of n make sense for the situation. (There can't be fractions of a car or a negative number of cars.) $12 + 3n = 28$ has the solution $n = \frac{16}{3}$, so the number of cars needed is 6.

Problem 2

a. A school principal is ordering tables for the cafeteria. There is already one large, 10-seat table that they are keeping. All the new tables will have 4 seats. The cafeteria needs to seat at least 200 students. Write an inequality whose solution is the number of 4-seat tables the principal should order.

$10 + 4n \geq 200$ (or equivalent)

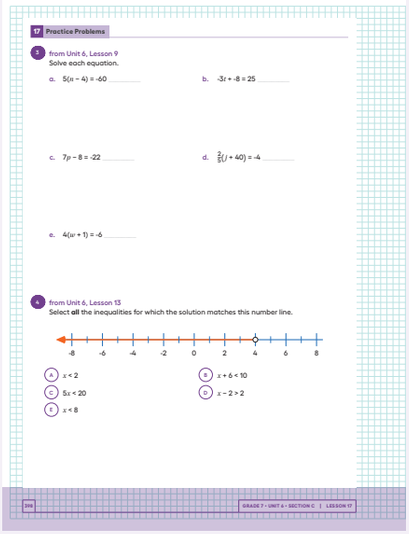
b. The principal is also ordering rain barrels to collect water in the schoolyard. The school already has one small barrel that holds 10 gallons. The principal wants to order 4 more barrels so that all the barrels combined will hold at least 200 gallons. Write an inequality whose solution is the size of each of the 4 barrels if they are the same size.

$10 + 4n \geq 200$ (or equivalent)

c. How are these two problems similar? How are they different?

Sample response: Solutions to the first inequality must be whole numbers greater or equal to 47.5 because a solution represents a number of tables. Solutions to the second inequality can be any number greater or equal to 47.5 because a solution represents the volume of a barrel, which can be a whole number or not.

Student Workbook


17 Practice Problems

3 from Unit 6, Lesson 9
Solve each equation.

a. $5(n - 4) = -60$ _____

b. $-3t + -8 = 25$ _____

c. $7p - 8 = -22$ _____

d. $\frac{2}{5}(j + 40) = -4$ _____

e. $4(w + 1) = -6$ _____

4 from Unit 6, Lesson 13
Select **all** the inequalities for which the solution matches this number line.

-8 -6 -4 -2 0 2 4 6 8

A. $x < 2$
B. $x + 6 < 10$
C. $5x < 20$
D. $x - 2 > 2$
E. $x < 8$

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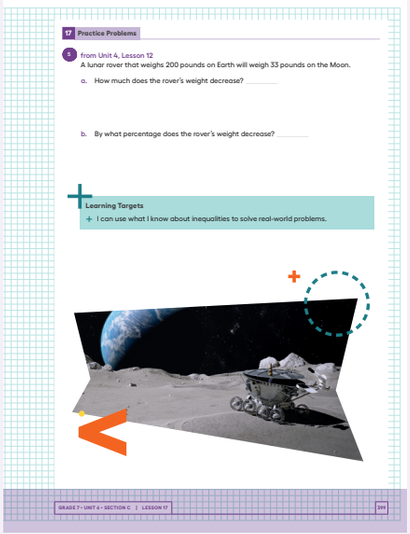
Student Workbook


17 Practice Problems

5 from Unit 4, Lesson 12
A lunar rover that weighs 200 pounds on Earth will weigh 33 pounds on the Moon.

a. How much does the rover's weight decrease? _____

b. By what percentage does the rover's weight decrease? _____

Learning Targets
+ I can use what I know about inequalities to solve real-world problems.

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Problem 3

from Unit 6, Lesson 9

Solve each equation.

a. $5(n - 4) = -60$

$n = -8$

b. $-3t + -8 = 25$

$t = -11$

c. $7p - 8 = -22$

$p = -2$

d. $\frac{2}{5}(j + 40) = -4$

$j = -50$

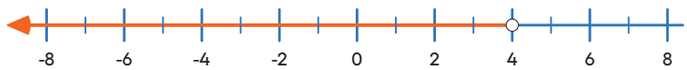
e. $4(w + 1) = -6$

$w = \frac{-10}{4}$ (or equivalent)

Problem 4

from Unit 6, Lesson 13

Select **all** the inequalities for which the solution matches this number line.



A. $x < 2$

B. $x + 6 < 10$

C. $5x < 20$

D. $x - 2 > 2$

E. $x < 8$

Problem 5

from Unit 4, Lesson 12

A lunar rover that weighs 200 pounds on Earth will weigh 33 pounds on the Moon.

a. How much does the rover's weight decrease?

167 pounds

b. By what percentage does the rover's weight decrease?

83.5% (167 ÷ 200 = 0.835)