Lesson 5

More Graphs of Functions

Goals

- Describe (orally and in writing) a graph of a function as "increasing" or "decreasing" over an interval, and explain (orally) the reasoning.
- Interpret (orally and in writing) a graph of temperature as a function of time, using language such as "input" and "output."

Learning Target

I can explain the story told by the graph of a function.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- 5 Practices
- Which Three Go Together?

Lesson Narrative

In this lesson, students begin to analyze graphs of functions and use them to answer questions about a context. Students also look at what happens over intervals of input values and learn that graphs can be viewed as dynamic objects that tell stories.

In the temperature activity, students connect specific features of the graph with specific features of the contextual situation, for example, the highest point on the graph with the highest temperature of the day and when it was attained. In the activity about garbage production, students investigate what happens over ranges of input values. The graph tells us how much garbage was produced at certain times, and we can also determine if the amount of garbage was increasing or decreasing over a specific time.

As students learn to interpret graphs in terms of a context and use them to answer questions, they learn an important skill in mathematical modeling.

Student Learning Goal

Let's interpret graphs of functions.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up

Which Three Go Together: Graphs

5 min

Activity Narrative

This *Warm-up* prompts students to compare four graphs. It gives students a reason to use language precisely. The activity also enables the teacher to hear the terminologies students know and how they talk about characteristics of graphs.

Launch

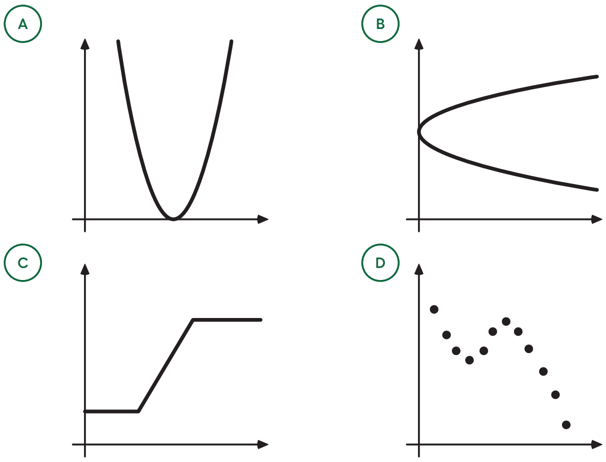
Arrange students in groups of 2–4. Display the graphs for all to see.

Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three graphs that go together and can explain why.

Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- These graphs are continuous.
- These graphs could be drawn without picking up the pencil.

A, B, and D go together because:

- These graphs all curve.
- These graphs have no straight line segments.

A, C, and D go together because:

- These graphs are all functions.
- For each input of these functions, there is one and only one output.

B, C, and D go together because:

- These graphs do not touch the horizontal axis.

Instructional Routines

Which Three Go Together?

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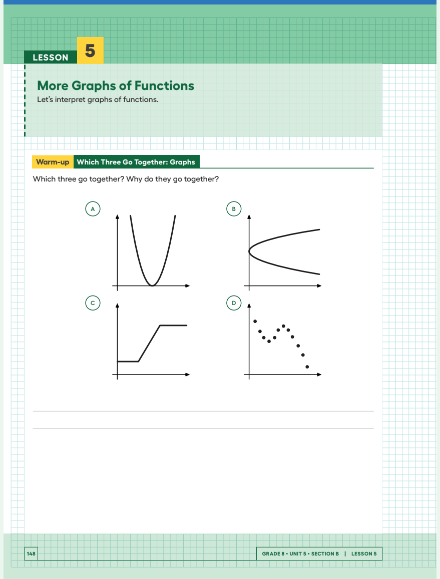
Student Workbook



Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "continuous," "discrete," "segment," and to clarify their reasoning as needed. Consider asking:

"How do you know ...?"

"What do you mean by ...?"

"Can you say that in another way?"

During the discussion, avoid introducing the traditional names of x and y for the axes unless students use them first. More formal vocabulary will be developed in later activities, lessons, and grades, and much of the motivation of this added vocabulary is to improve upon the somewhat clunky language we are led to use without it.

Activity 1

Time and Temperature

15 min

Activity Narrative

The purpose of this activity is for students to begin using a graph of a functional relationship between two quantities to make quantitative observations about their relationship. For some questions, students must identify specific input-output pairs, while in others, they can use the shape of the graph. For example, when asked for which time the temperature was warmer, students need only compare the relative height of the graph at the two different times. Similarly, students can identify another time the temperature was the same as at 4:00 p.m. without actually knowing the temperature at 4:00 p.m.

Launch

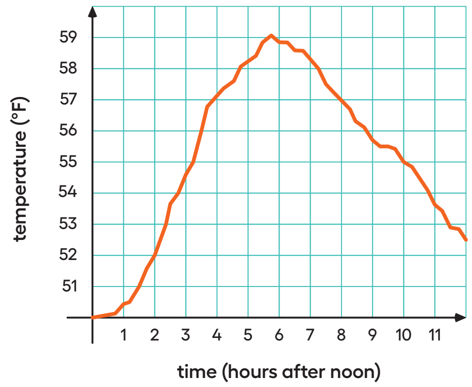
Arrange students in groups of 2.

Give students 4–6 minutes of quiet work time and then time to share their responses with their partner.

Select students who reason about the graph without identifying specific values to share during the *Activity Synthesis*. For example, a student may identify that the temperature was highest at about 5:45 p.m. by finding the highest point on the graph, without stating that that highest temperature was approximately 59 °F.

Student Task Statement

The graph shows the temperature between noon and midnight in one day in City A.



1. Is it warmer at 3:00 p.m. or 9:00 p.m.?

   9:00 p.m.

2. Approximately when is the temperature highest?

   5:45 p.m.

3. Find another time that the temperature is the same as it is at 4:00 p.m.

   At 8:00 p.m. the temperature in the city is the same as at 4:00 p.m.

4. Does the temperature change more between 1:00 p.m. and 3:00 p.m. or between 3:00 p.m. and 5:00 p.m.?

   The temperature changes approximately 4 degrees between 1:00 p.m. and 3:00 p.m. but only approximately 3.5 degrees between 3:00 p.m. and 5:00 p.m.

5. Does this graph show that temperature is a function of time, or time is a function of temperature?

   Temperature is a function of time.

6. When the input for the function is 8, what is the output? What does that tell you about the time and temperature?

   57; At 8:00 p.m., the temperature is 57 °F.

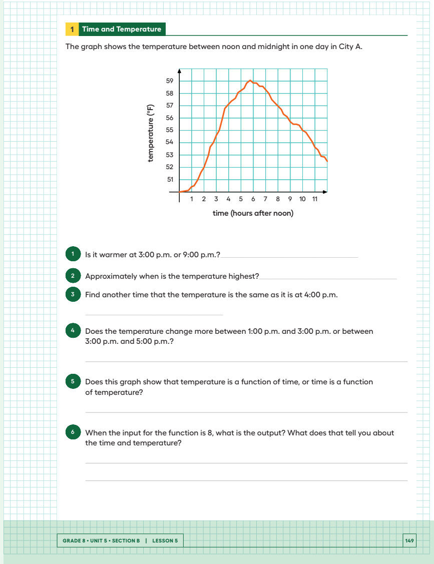
Activity Synthesis

Display the graph for all to see during the discussion. Invite 1–2 previously identified students to share how they found their answers on the displayed graph for the first four questions. If not mentioned by students, demonstrate how to find the solution to the fourth problem by either identifying the temperature values at each time and subtracting or by measuring the vertical change for each time interval.

For the final question, ask students to plot the point on their graphs if they did not already do so and describe what the point means in the context.

If time allows, give 1 minute quite think time for each group to come up with their own question that someone else could answer using the graph. Invite each group to share their question, and ask a different group to give the answer.

Student Workbook


1 Time and Temperature

The graph shows the temperature between noon and midnight in one day in City A.

1 Is it warmer at 3:00 p.m. or 9:00 p.m.?

2 Approximately when is the temperature highest?

3 Find another time that the temperature is the same as it is at 4:00 p.m.

4 Does the temperature change more between 1:00 p.m. and 3:00 p.m. or between 3:00 p.m. and 5:00 p.m.?

5 Does this graph show that temperature is a function of time, or time is a function of temperature?

6 When the input for the function is 8, what is the output? What does that tell you about the time and temperature?

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Develop Language and Symbols.
Maintain a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of the new vocabulary.
For example, the following words may be suggested: "increase," "decrease," "steep," "independent," "dependent," "function," and "variable."

Supports accessibility for: Language, Memory

Activity 2
Garbage

15 min

Activity Narrative

The purpose of this activity is for students to identify where a function is increasing or decreasing from a graphical representation. In the previous activity, students focused more on single points. In this activity they focus on collections of points within time intervals and what the overall shape of the graph says about the relationship between the two quantities.

Monitor for groups who use different strategies for identifying increasing or decreasing intervals. Some strategies, ordered here from simplest to most involved, might be:

- Using a finger to show that, as they move from left to right on the graph, the function trends upward or downward.
- Drawing line segments between discrete points and observing whether the line segment slants up or down as they move from left to right on the graph.
- Finding the amount of garbage that corresponds to different years and comparing their values using subtraction.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Arrange students in groups of 2, and display the first graph for all to see. If students are not familiar with time plots, explain that each point represents the value for one year, starting with the point for 1991. Ask students,

"Does the graph show the amount of garbage produced as a function of the year, or the year as a function of the amount of garbage produced?"

amount of garbage produced as a function of the year

Give groups 1 minute to think of a question that the information in the graph can answer. For example,

"About how much garbage was produced in 2010?"

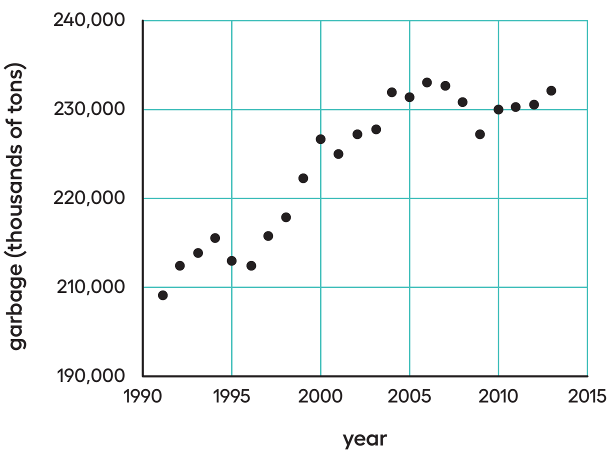
about 250,000 thousand or 250 billion tons

Invite 2–3 groups to each share their question, and ask a different group to give the answer.

Select students with different strategies for identifying intervals of increasing and decreasing, such as those described in the *Activity Narrative*, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

1. The graph shows the amount of garbage produced in the United States each year between 1991 and 2013.



   a. Did the amount of garbage increase or decrease between 1999 and 2000?

   Increase

   Based on the graph, from 1999 to 2000 the amount of garbage produced increased from about 235,000 to 245,000 thousand tons.

   b. Did the amount of garbage increase or decrease between 2005 and 2009?

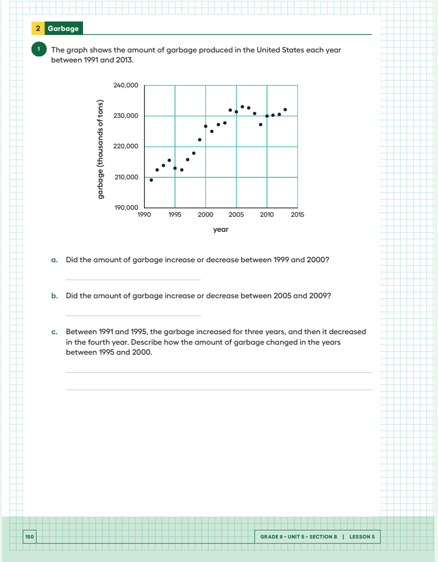
   Decrease

   Based on the graph, from 2005 to 2009 the amount of garbage produced decreased from about 255,000 to 245,000 thousand tons.

   c. Between 1991 and 1995, the garbage increased for three years, and then it decreased in the fourth year. Describe how the amount of garbage changed in the years between 1995 and 2000.

   The amount of garbage decreased for one year, then increased for four years in a row.

Student Workbook


2 Garbage

1 The graph shows the amount of garbage produced in the United States each year between 1991 and 2013.

a. Did the amount of garbage increase or decrease between 1999 and 2000?

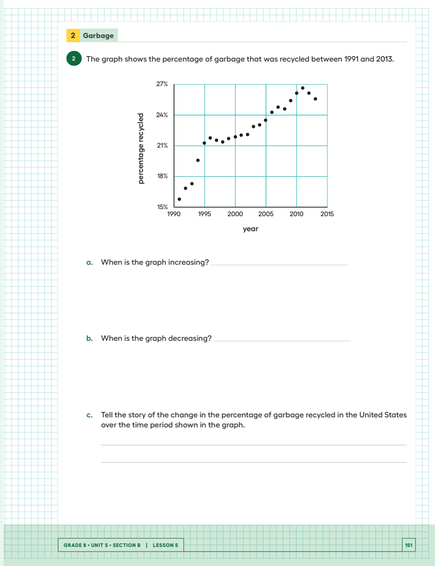
b. Did the amount of garbage increase or decrease between 2005 and 2009?

c. Between 1991 and 1995, the garbage increased for three years, and then it decreased in the fourth year. Describe how the amount of garbage changed in the years between 1995 and 2000.

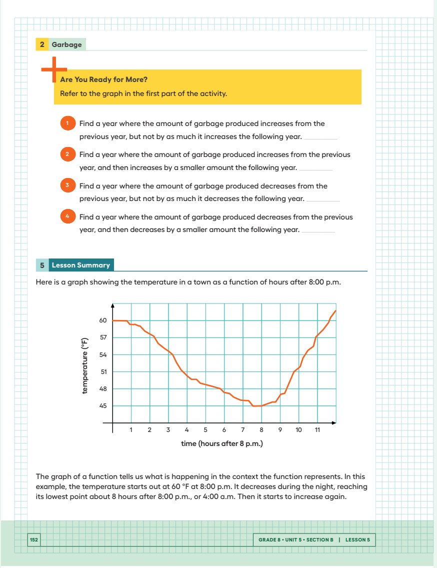
Building on Student Thinking

Students may not answer with a range of dates. They might instead list each year it increased. A list is acceptable, but be sure students see the connection between, for example, the list “1996, 1997, and 1998” and the same years stated as “from 1996 to 1998.”

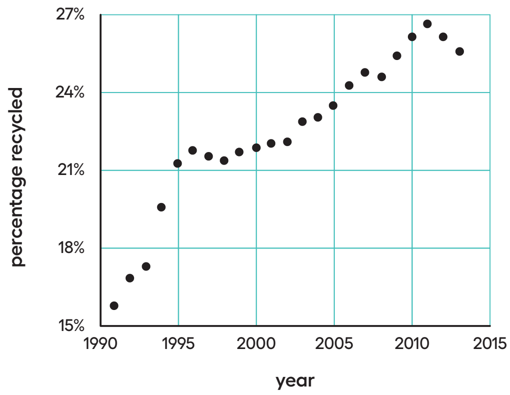
Student Workbook



Student Workbook



2. The graph shows the percentage of garbage that was recycled between 1991 and 2013.



a. When is the graph increasing?

The graph is increasing from 1991 to 1996 and from 1998 to 2011.

b. When is the graph decreasing?

The graph is decreasing from 1996 to 1998 and from 2011 to 2013.

c. Tell the story of the change in the percentage of garbage recycled in the United States over the time period shown in the graph.

Sample response: The percentage of garbage recycled generally increases from 1991 until a peak at 2011, and then begins to decrease. There are brief periods of decrease from 1996 to 1998 and from 2007 to 2008.

Are You Ready for More?

Refer to the graph in the first part of the activity.

1. Find a year where the amount of garbage produced increases from the previous year, but not by as much it increases the following year.

   2003

2. Find a year where the amount of garbage produced increases from the previous year, and then increases by a smaller amount the following year.

   1997

3. Find a year where the amount of garbage produced decreases from the previous year, but not by as much it decreases the following year.

   2008

4. Find a year where the amount of garbage produced decreases from the previous year, and then decreases by a smaller amount the following year.

   1995

Activity Synthesis

The purpose of this discussion is for students to use and listen to the language used to describe intervals of increasing and decreasing for the graphs.

While discussing each graph, display for all to see. For the first graph, invite previously selected students to share how they identified when the amount of garbage produced was increasing or decreasing. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"Which method seems most efficient?"*

  Drawing in a line segment between the points at the start and end of the time period makes it quick to tell if it is an increase or a decrease.

- *"Which method seems the most precise?"*

  Estimating the output value of the points and using subtraction to find the difference between the start and end of the time periods.

Conclude the discussion by displaying the second graph and asking, "How might you generally describe this graph to someone who couldn't see it?" (The percentage of garbage that was recycled increased overall from 1990 to 2011 but began decreasing from 2011 to 2013.) Invite students to share their descriptions with their partner, and then select 2–3 students to share with the class.

Lesson Synthesis

Use this discussion to reinforce the ideas that the graph of a function tells a story about the context it represents and that specific points on the graph connect to specific features of the situation.

Consider asking these questions about the graphs from the activities while displaying the graphs:

- *"On the temperature graph, how do we find the time when it is the coolest?"*

  We can find the point on the graph that is the lowest.

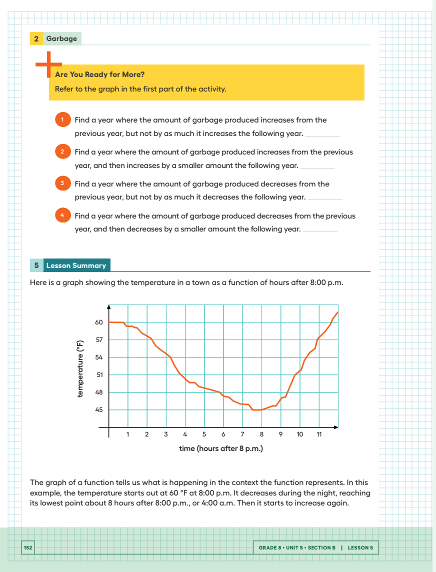
- *"On the temperature graph, how do we find the difference between the hottest and the coolest temperatures?"*

  We can find the lowest and the highest points on the graph and count the difference between them using the grid, or we can find the hottest temperature and subtract from it the coolest temperature. Either way the difference is about 9 degrees Fahrenheit.

- *"Looking at the garbage production graph, how does the production before 2005 compare with the production after 2005?"*

  From 1990 to 2005, production increases. After 2005, production levels off and seems to decrease slightly.

Student Workbook


2 Garbage

Are You Ready for More?

Refer to the graph in the first part of the activity.

1. Find a year where the amount of garbage produced increases from the previous year, but not by as much it increases the following year. ________
2. Find a year where the amount of garbage produced increases from the previous year, and then increases by a smaller amount the following year. ________
3. Find a year where the amount of garbage produced decreases from the previous year, but not by as much it decreases the following year. ________
4. Find a year where the amount of garbage produced decreases from the previous year, and then decreases by a smaller amount the following year. ________

5 Lesson Summary

Here is a graph showing the temperature in a town as a function of hours after 8:00 p.m.

The graph of a function tells us what is happening in the context the function represents. In this example, the temperature starts out at 60 °F at 8:00 p.m. It decreases during the night, reaching its lowest point about 8 hours after 8:00 p.m., or 4:00 a.m. Then it starts to increase again.

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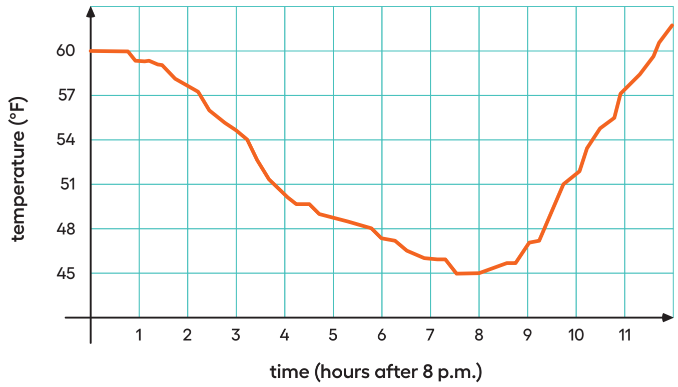
Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

Here is a graph showing the temperature in a town as a function of hours after 8:00 p.m.



The graph of a function tells us what is happening in the context the function represents. In this example, the temperature starts out at 60 °F at 8:00 p.m. It decreases during the night, reaching its lowest point about 8 hours after 8:00 p.m., or 4:00 a.m. Then it starts to increase again.

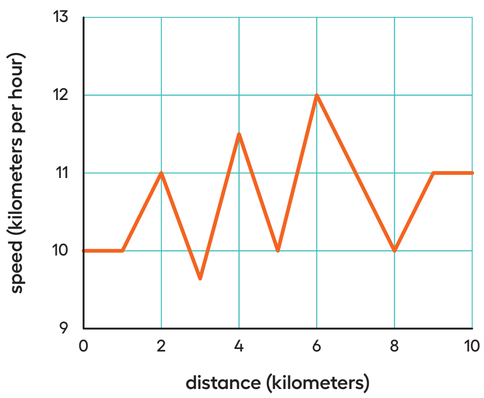
Cool-down

Diego's 10K Race

5 min

Student Task Statement

Diego runs a 10-kilometer race and keeps track of his speed.



1. What was Diego's speed at the 5-kilometer mark in the race?

   10 kilometers per hour

2. According to the graph, where was Diego when he was going the slowest during the race?

   3 kilometers into the race

3. Describe what happened to Diego's speed in the second half of the race (from 5 kilometers to 10 kilometers).

   Sample response: From 5 kilometers to 6 kilometers, Diego went faster, but he slowed down from 6 kilometers to 8 kilometers. He sped up again from 8 kilometers to 9 kilometers and finished the last kilometer at the same speed.

Lesson 5

Practice Problems

3 Problems

Problem 1

from Unit 4, Lesson 13

The solution to a system of equations is (6, -3). Choose two equations that might make up the system.

A. $y = -3x + 6$

B. $y = 2x - 9$

C. $y = -5x + 27$

D. $y = 2x - 15$

E. $y = -4x + 27$

Problem 2

from Unit 5, Lesson 3

A car is traveling on a small highway and is either going 55 miles per hour or 35 miles per hour, depending on the speed limit, until it reaches its destination 200 miles away. Let x represent the amount of time in hours that the car is going 55 miles per hour, and let y be the time in hours that the car is going 35 miles per hour. An equation describing the relationship is

$$55x + 35y = 200.$$

a. If the car spends 2.5 hours going 35 miles per hour on the trip, how long does it spend going 55 miles per hour?

About 2.05 hours

b. If the car spends 3 hours going 55 miles per hour on the trip, how long does it spend going 35 miles per hour?

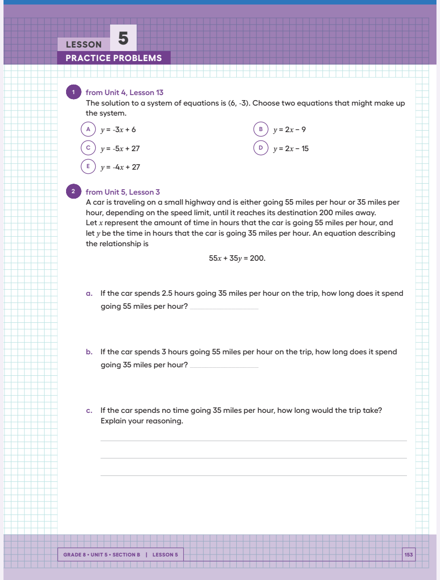
1 hour

c. If the car spends no time going 35 miles per hour, how long would the trip take? Explain your reasoning.

About 3.64 hours

Sample reasoning: If the car spent no time going 35 miles per hour, then it spent the entire trip going 55 mph, and the trip would be completed in about 3.64 hours.

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1 from Unit 4, Lesson 13
The solution to a system of equations is (6, -3). Choose two equations that might make up the system.
A $y = -3x + 6$
B $y = 2x - 9$
C $y = -5x + 27$
D $y = 2x - 15$
E $y = -4x + 27$

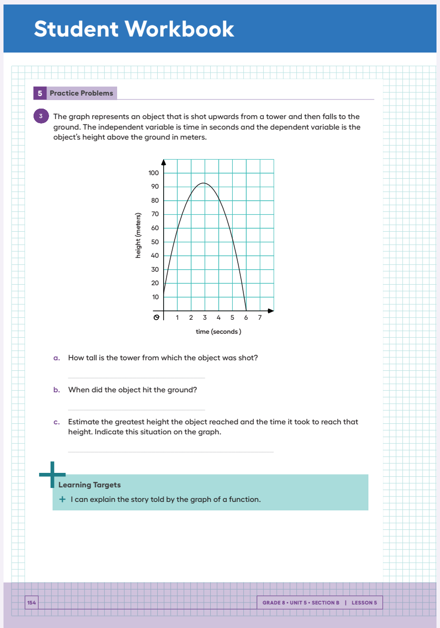
2 from Unit 5, Lesson 3
A car is traveling on a small highway and is either going 55 miles per hour or 35 miles per hour, depending on the speed limit, until it reaches its destination 200 miles away. Let x represent the amount of time in hours that the car is going 55 miles per hour, and let y be the time in hours that the car is going 35 miles per hour. An equation describing the relationship is

$55x + 35y = 200.$

a. If the car spends 2.5 hours going 35 miles per hour on the trip, how long does it spend going 55 miles per hour?

b. If the car spends 3 hours going 55 miles per hour on the trip, how long does it spend going 35 miles per hour?

c. If the car spends no time going 35 miles per hour, how long would the trip take? Explain your reasoning.


Student Workbook

5 Practice Problems

3 The graph represents an object that is shot upwards from a tower and then falls to the ground. The independent variable is time in seconds and the dependent variable is the object's height above the ground in meters.

a. How tall is the tower from which the object was shot?

b. When did the object hit the ground?

c. Estimate the greatest height the object reached and the time it took to reach that height. Indicate this situation on the graph.

Learning Targets
+ I can explain the story told by the graph of a function.

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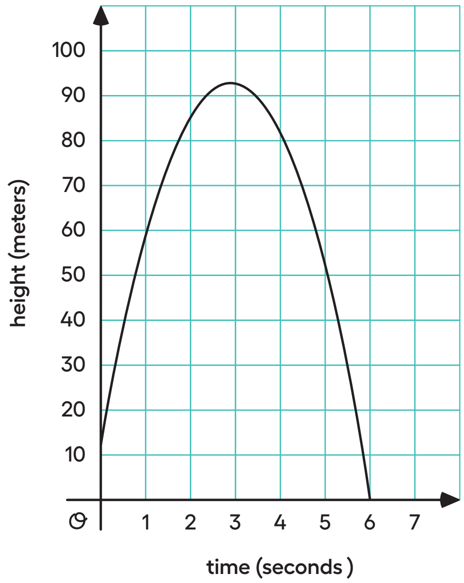
Problem 3

The graph represents an object that is shot upwards from a tower and then falls to the ground. The independent variable is time in seconds and the dependent variable is the object's height above the ground in meters.



a. How tall is the tower from which the object was shot?

about 10 meters

b. When did the object hit the ground?

6 seconds after it was shot

c. Estimate the greatest height the object reached and the time it took to reach that height. Indicate this situation on the graph.

approximately 93 meters high at 2.9 seconds

A point should be plotted at (2.9, 93).