Lesson 8

Relating Area to Circumference

Goals

- Generalize a process for finding the area of a circle, and justify (orally) why this can be abstracted as πr^2.
- Show how a circle can be decomposed and rearranged to approximate a polygon, and justify (orally and in writing) that the area of this polygon is equal to half of the circle's circumference multiplied by its radius.

Learning Targets

- I can explain how the area of a circle and its circumference are related to each other.
- I know the formula for area of a circle.

Lesson Narrative

In this lesson, students see that the area of a circle can be found by multiplying πr^2. They look at informal dissection arguments to derive that relationship.

First, students cut and rearrange a circle into a shape that approximates a parallelogram. The next activity is optional because it shows a different way to cut and rearrange a circle into a shape that approximates a triangle. In each case, the area of the polygon is equal to $\frac{1}{2} \cdot$ circumference $\cdot$ radius. Using algebraic reasoning, students construct and critique arguments that this is equivalent to $\pi r \cdot r$ or πr^2. The little "2" up in the air is pronounced **squared** and means that the value of r is multiplied by itself. Finally, students apply the formula $A = \pi r^2$ to solve problems.

Student Learning Goal

Let's rearrange circles to calculate their areas.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 3)

Instructional Routines

- MLR3: Critique, Correct, Clarify

Required Materials

Materials to Gather

- Blank paper: Activity 1
- Glue or glue sticks: Activity 1
- Markers: Activity 1
- Scissors: Activity 1

Materials to Copy

- Making a Polygon out of a Circle Cutouts (1 copy for every 12 students): Activity 1

Required Preparation

Activity 1:
Consider copying the blackline master onto different colors of paper. If different colors of copy paper are not available, the blackline master can be copied onto white paper and then construction paper can be used for the other sheet that the pieces are glued onto.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	10 min	5 min

Warm-up

Irrigating a Field

5 min

Activity Narrative

In this activity, students estimate the area of a circle by comparing it to a surrounding square. Students should recognize that the area of the circle is less than 640,000 m^2, which is the area of the surrounding square.

Students who completed the optional activity "Covering a Circle" may recall that the area of a circle with radius r is a little more than $3r^2$. They can use this relationship to determine that the circle's area is slightly greater than 480,000 m^2.

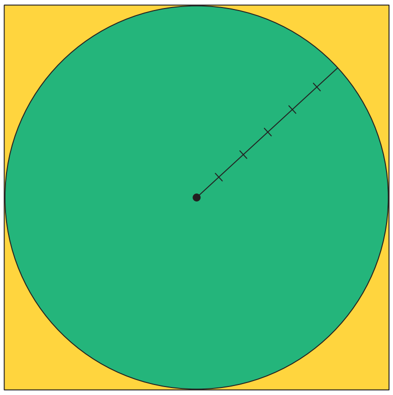
Launch

Explain that some farms have circular fields because they use center-pivot irrigation. If desired, display these images to familiarize students with the context.

Ask students to estimate the circular growing area (green region) in the image in their books or devices. Give students 1–2 minutes of quiet think time followed by whole-group discussion.

Student Task Statement

A circular field is set into a square with an 800-m side length.



What is the field's area? Record an estimate that is:

too low	about right	too high
320,000 m^2 to 400,000 m^2	480,000 m^2 to 520,000 m^2	640,000 m^2 or more

Sample reasoning:

- The square around the field has an area of $800 \cdot 800$, or 640,000 m^2. The circular field is a little less than that, around 80% or so.
- The radius of the field is 400 m. A square with side lengths of 400 m has an area of 160,000 m^2. It takes a little more than 3 of these squares to cover the circle, and $3 \cdot 160{,}000 = 480{,}000$.

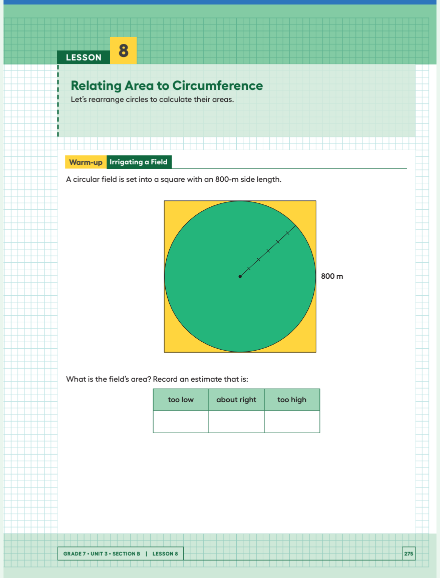
Building on Student Thinking

Students might think the answer should be 640,000 m^2 because that is the area of the square, not realizing that they are being asked to find the area of a circle. Ask them what shape is the region where the plants are growing.

Some students might incorrectly calculate the area of the square to be 6,400 m^2 and therefore estimate that the circle would be about 5,000 m^2.

Some students might try to use what they learned in the previous lessons about the relationship between the area of a circle and the area of a square with side length equal to the circle's radius. Point out that the question is asking for an estimate and answer choices all differ by a factor of 10.

Student Workbook


LESSON 8

Relating Area to Circumference

Let's rearrange circles to calculate their areas.

Warm-up Irrigating a Field

A circular field is set into a square with an 800-m side length.

800 m

What is the field's area? Record an estimate that is:

too low	about right	too high

Activity Synthesis

Invite students to share their estimation strategies. To involve more students in the conversation, consider asking:

"Who can restate _________'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _________'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Making a Polygon out of a Circle

20 min

Activity Narrative

In this activity, students cut and rearrange parts of a circle to approximate a parallelogram. Then they use what they know about finding the area of a parallelogram to develop a formula for the area of a circle.

Students see that the area of the parallelogram would be calculated by multiplying half of the circle's circumference times its radius. Since students are not familiar with the process of writing proofs, it is necessary to walk them through writing the justification that uses the formula for the area of the parallelogram to develop the formula for the area of the circle. As students successively cut the circle pieces in half, rearrange them, and notice that the result more closely resembles a parallelogram, they look for regularity in repeated reasoning. As students use what they know about the area of a parallelogram to formulate the area of the circle, they make use of structure.

Launch

Arrange students in groups of 2. Distribute one pair of circles from the blackline master to each group. Also provide each group with a sheet of paper that is a different color, a pair of scissors, and glue or tape.

Remind students that in the past they decomposed and rearranged a shape to figure out its area. Demonstrate how to do the first 3 steps of the activity, and invite students to follow along with your example. Ask how the area of the new shape differs from that of the circle. Solicit some ideas on what the new shape resembles and how the area of such a shape could be approximated. Without resolving this, ask students to continue the process.

Monitor how students are handling the process of cutting and rearranging the pieces. If there is a group that is doing well and has eighths that are neat and large enough, consider asking them to repeat the process once more to make sixteenths before gluing the pieces onto their other paper.

Student Task Statement

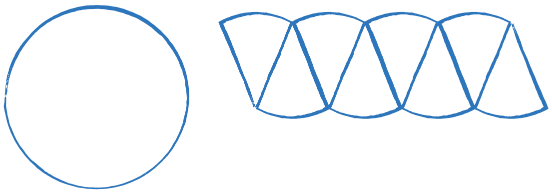
Your teacher will give you a page with two circles on it and a piece of paper that is a different color.

Follow these instructions to create a visual display:

1. Cut out both circles, cutting around the thick outline.
2. Fold and cut *one* of the circles into fourths.
3. Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom.

   Pause here so your teacher can review your work.
4. Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
   Pause here so your teacher can review your work.
5. Glue the remaining circle and the new shape onto a piece of paper that is a different color.

   1–5. The shapes shown here.



After you finish gluing your shapes, answer the following questions.

1. How do the areas of the two shapes compare?

   They are equal.
2. What polygon does the shape made of the circle pieces most resemble?

   It is like a parallelogram but with a squiggly top and bottom.
3. How could you find the area of this polygon?

   Multiply base times height.

Building on Student Thinking

Students might not fold the wedges accurately or make a straight cut. Remind them that the halves must be equal.

Activity Synthesis

The purpose of this discussion is for students to make sense of an informal derivation for the area of a circle by comparing it to the area of a parallelogram.

Ask students:

- *"What polygon does the shape made of the circle pieces most resemble?"*

  A parallelogram or rectangle
- *"How does the area of this new shape compare to the area of the original circle? Why?"*

  They are equal because decomposing and rearranging a shape doesn't change its area.

Access for Multilingual Learners

MLR2: Collect and Display.
Collect the language that students use to describe the rearranged shape and its area. Display words and phrases such as: "less bumpy," "more straight," "like a parallelogram," "same area," "rearranged," "the base is half of the circle," "the height is the radius," etc. During the synthesis, invite students to borrow language from the display and update it as needed. For example, a student may rephrase "the base is half of the circle" as "the base of the parallelogram is half the length of the circumference of the circle."

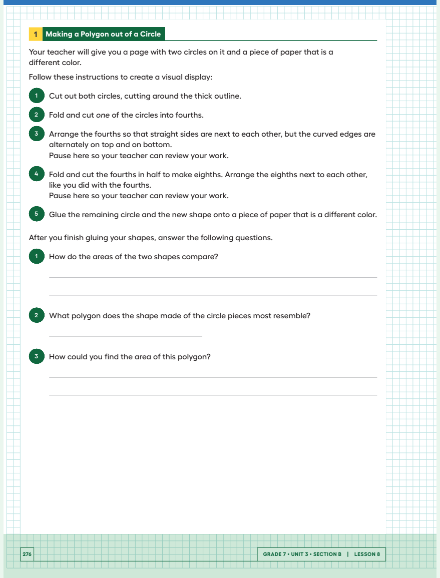
Advances: Conversing, Reading

Access for Students with Diverse Abilities

Engagement: Develop Effort and Persistence.
Provide a checklist or reminders that focus on increasing the length of on-task orientation in the face of distractions. For example, provide students with a task checklist that makes all the required components of the visual display explicit.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook



1 Making a Polygon out of a Circle

Your teacher will give you a page with two circles on it and a piece of paper that is a different color.

Follow these instructions to create a visual display:

1. Cut out both circles, cutting around the thick outline.
2. Fold and cut *one* of the circles into fourths.
3. Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom.
   Pause here so your teacher can review your work.
4. Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
   Pause here so your teacher can review your work.
5. Glue the remaining circle and the new shape onto a piece of paper that is a different color.

After you finish gluing your shapes, answer the following questions.

1. How do the areas of the two shapes compare?
2. What polygon does the shape made of the circle pieces most resemble?
3. How could you find the area of this polygon?

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"If we continued cutting the wedges in half, how would that affect the new shape?"

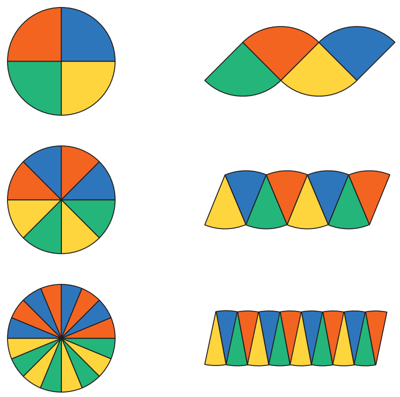
It would look even more like a parallelogram or rectangle. The bumpy top and bottom straighten out, and the slanted height becomes more vertical.

Consider displaying this applet for all to see during the discussion.

The Geogebra applet 'Making a Polygon out of a Circle' is available on Imagine Learning Classroom for this lesson.

This applet was created in GeoGebra by Malin Christersson.

Or display this image:



Guide students through a process for determining the area of the rearranged shape, by asking questions like the following and demonstrating how students can record the answers on their visual display:

"How do we calculate the area of a parallelogram?"

base times height

- Write "Area = base · height" where there is some open space on the visual display.

"How long is the base of the parallelogram?"

approximately equal to half of the circle's circumference

- Write $\frac{1}{2}$ Circumference near the base of the parallelogram.

"How do we calculate the circumference of a circle?"

pi times diameter

- Write Circumference = πd around the circle.

"What measurement is equal to half of the diameter?"

the radius

- Add "= πr" after $\frac{1}{2}$ Circumference near the base of the parallelogram.

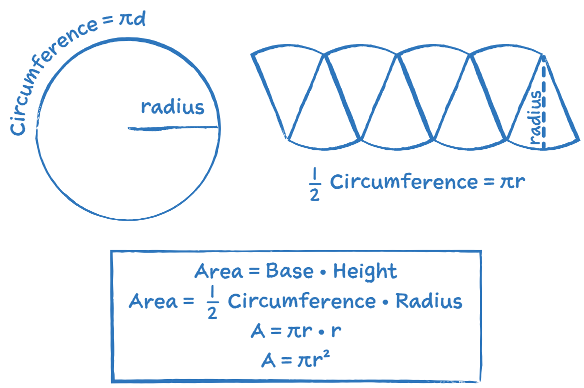
"How long is the height of the parallelogram?"

approximately equal to the radius of the circle

- Draw a radius on the circle and label it "radius." Draw a height for the parallelogram and label it "radius."

"Let's rewrite the area of the parallelogram using these equivalences. For each step, tell me how we know it is true."

- In the open space, underneath "Area = base · height" write the following equations:
  - Area = $\frac{1}{2}$Circumference · Radius
  - A = $\pi r \cdot r$
  - A = πr^2



Activity 2: Optional

Making Another Polygon out of a Circle

10 min

Activity Narrative

The purpose of this activity is for students to consider a different way to cut and reassemble a circle into something resembling a polygon in order to calculate its area. This time the polygon is a triangle, but the area of the circle can still be found by multiplying $\frac{1}{2}$ times the circumference times the radius.

In this activity, students write their own justification for the area of a circle. Monitor for students who have different explanations, including expressions such as:

- Area = $\frac{1}{2} \cdot$ base · height
- Area = $\frac{1}{2} \cdot$ circumference · radius
- Area = $\frac{1}{2} \cdot (\pi d) \cdot r$
- Area = $\frac{1}{2} \cdot (2\pi r) \cdot r$
- Area = $\pi r \cdot r$
- Area = πr^2

As students create their own derivations for the area of a circle and compare them with others', they construct arguments and critique reasoning.

A note about precision and approximation:

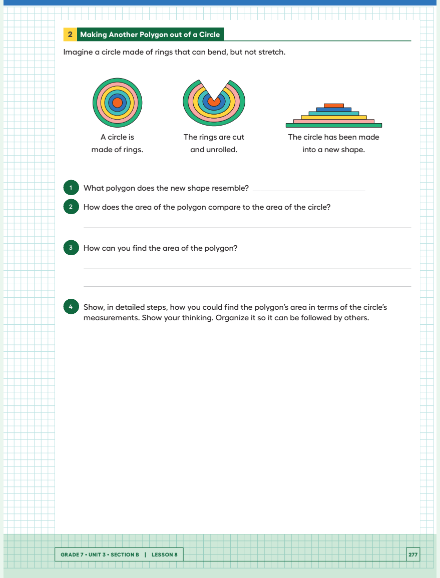
If the bands making up the circle really did not stretch, then they would not form rectangles when they are unwound because the circumference of the inner circle is not the same as the circumference of the outer circle in each band. A rectangle is an appropriate approximation for the shape in terms of calculating its area.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.
Provide access to the digital applet. Ask students to identify correspondences between the measurements of the figures before and after the transformation.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook



Launch

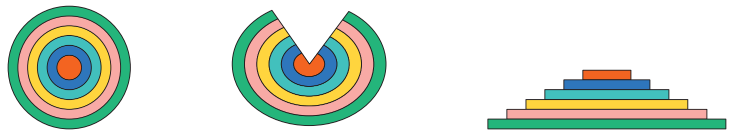
Consider displaying the applet for all to see, to help students visualize the process of cutting and unrolling the circle. The Geogebra applet 'Making Another Polygon out of a Circle' is available on Imagine Learning Classroom for this lesson.

Give students quiet work time followed by partner and whole-class discussion.

Select work from students who have a clear and organized explanation, such as some of the expressions shown in the activity narrative, to share later.

Student Task Statement

Imagine a circle made of rings that can bend, but not stretch.



A circle is made of rings. | The rings are cut and unrolled. | The circle has been made into a new shape.

1. What polygon does the new shape resemble?

   a triangle

2. How does the area of the polygon compare to the area of the circle?

   The areas of the shapes are equal.

3. How can you find the area of the polygon?

   It looks like a triangle, so the area can be found with the formula
   Area $= \frac{1}{2} \cdot$ base $\cdot$ height.

4. Show, in detailed steps, how you could find the polygon's area in terms of the circle's measurements. Show your thinking. Organize it so it can be followed by others.

   The base of the "triangle" has length equal to the circumference of the circle, while its height is the radius of the circle. So:

   - Area $= \frac{1}{2} \cdot$ circumference $\cdot$ radius
   - Area $= \frac{1}{2} \cdot \pi d \cdot r$
   - Area $= \pi r \cdot r$
   - Area $= \pi r^2$

Building on Student Thinking

If students struggle to imagine the circle and how it is cut and rearranged, suggest a familiar material for the rings that bends but does not stretch (for example, a cord or chain).

Activity Synthesis

The purpose of this discussion is for students to collaboratively create an informal derivation for the area of a circle based on comparing it to the area of a triangle.

Ask students:

"What polygon does the unrolled shape most resemble?"

a triangle

"How do we calculate the area of a triangle?"

$\frac{1}{2}$ base times height

Next, ask students to trade papers with a partner and discuss each other's work for finding the area. Display these prompts for all to see:

"I agree with this step because ..."

"I disagree with this step because ..."

"Can you explain how ... ?"

"Can you explain why ... ?"

"You should expand on ..."

If time permits, ask students to revise their explanations based on their partner's feedback.

Display 1–2 approaches from previously selected students for all to see. Connect the responses to the learning goals by asking questions such as:

"Where does the r^2 expression come from?"

"What kinds of details or language helped you understand the display?"

"Are there any details or language that you have questions about?"

The key takeaway is that the area of a circle can be found by multiplying πr^2.

Activity 3

Objects for a Powwow

10 min

Activity Narrative

In this activity, students calculate the area of circles given their radius or diameter. For the problem where the radius is given, students practice applying the formula $A = \pi r^2$. For the problem where the diameter is given, students must think about what extra steps are required. This is similar to the thinking they did previously when they were given a circle's radius and asked to calculate its circumference.

Because of the properties of multiplication, there is some flexibility when it comes to strategies for calculating circumference given the radius, such as:

- Multiplying the radius by 2 to find the diameter, then multiplying the diameter by π.
- Multiplying the radius by 2π to directly calculate the circumference.
- Multiplying the radius by π first and then multiplying that product by 2.

Instructional Routines

MLR3: Critique, Correct, Clarify

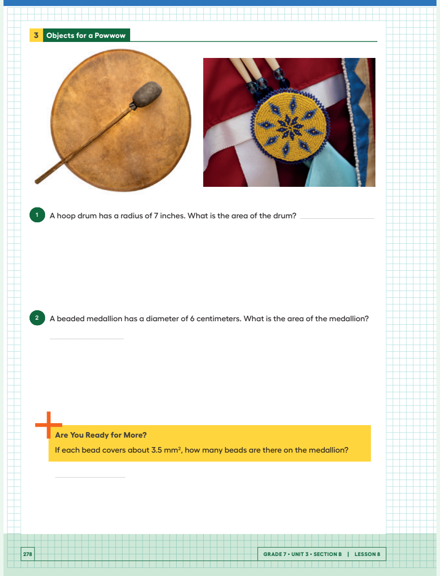
ilclass.com/r/10695504

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 3)

MLR3: Critique, Correct, Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Student Workbook


3 Objects for a Powwow

1 A hoop drum has a radius of 7 inches. What is the area of the drum?

2 A beaded medallion has a diameter of 6 centimeters. What is the area of the medallion?

Are You Ready for More?

If each bead covers about 3.5 mm², how many beads are there on the medallion?

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However, because the formula for area of a circle involves squaring the radius, there is less flexibility possible for calculating the area given the diameter. Students are most likely to be successful using the strategy:

- Dividing the diameter by 2 to find the radius, then using the formula $A = \pi r^2$.

During the whole-class discussion, students critique a response that is intentionally incorrect and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Introduce the context of this activity by asking students,

"What are some special objects that you are used to seeing at a party or celebration?"

Then explain that Lakota people from around South Dakota hold large celebrations called wacipis, or powwows. The wacipi involves song, dance, and colorful outfits.

Display the image of the hoop drum and beaded medallion for all to see. Explain that these are some examples of the special objects you could see at a powwow. If desired, explain that the circle shape is an important symbol in the Lakota culture.

Give students 4–5 minutes of quiet work time, followed by whole-class discussion.

Student Task Statement

1. A hoop drum has a radius of 7 inches. What is the area of the drum?

   153.9 in², because $\pi \cdot 7^2 \approx 153.9$

2. A beaded medallion has a diameter of 6 centimeters. What is the area of the medallion?

   28.3 cm², because $\pi \cdot 3^2 \approx 28.3$

Building on Student Thinking

Some students may use 6 centimeters as if it were the radius of the medallion, rather than the diameter. Point out that the first problem gives the radius of the hoop drum, while the second question gives the diameter of the medallion. Prompt them to consider how they could determine the radius of the medallion.

Are You Ready for More?

If each bead covers about 3.5 mm^2, how many beads are there on the medallion?

about 808 or 809 beads

Sample reasoning:

- The radius of the medallion is 30 mm. The area of the medallion in square millimeters is $\pi \cdot 30^2$, or about 2,827 mm^2. Since each bead covers about 3.5 mm^2, the total number of beads is $2{,}827 \div 3.5$, or about 808.
- The area of the medallion in square centimeters is about 28.3 cm^2. In square millimeters this is 2,830 mm^2. The total false of beads is $2{,}830 \div 3.5$, or about 809.

Activity Synthesis

The purpose of this discussion is to highlight students' strategies for dealing with being given the diameter of the beaded medallion, rather than its radius.

First, invite students to share their reasoning about the area of the hoop drum. Once the class is in agreement on the answer, move the discussion on by asking:

"What was different about the second problem compared to the first problem?"

While there are several differences students may notice, the important discussion point is that the diameter was given, rather than the radius.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response for finding the area of the medallion by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"To find the area of the medallion, I multiplied π times 62. Then I realized that 6 was the diameter and not the radius, so I divided my answer by 2."

Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

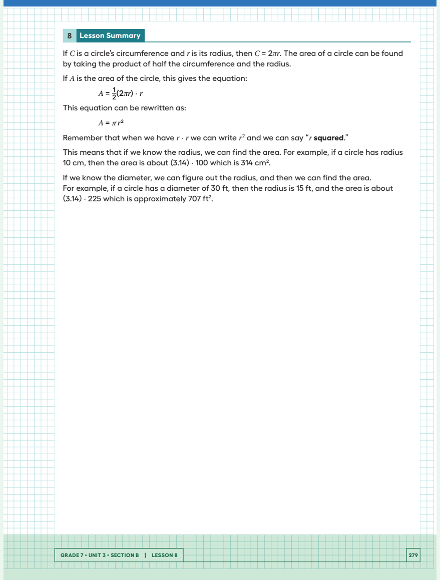
The key takeaway is that it is important to divide the diameter by 2 first to find the radius before applying the formula $A = \pi r^2$.

Responding To Student Thinking

Points to Emphasize
If students struggle with recognizing the relationships between the radius, diameter, circumference, and area of a circle, focus on this when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about the measurements of the circles in this activity:

Unit 3, Lesson 10, Activity 1 Card Sort: Circle Problems

Student Workbook



Lesson Synthesis

Share with students:

"Today we determined a formula for finding the area of any circle."

To review the formula for finding the area of a circle, consider asking students:

"What is a formula for finding the area of a circle?"

$A = \pi r^2$

"In the formula $A = \pi r^2$, what does the little 2 mean? Where did it come from?"

squared, r^2 represents $r \cdot r$.

"How can you find the area of a circle with a radius of 10?"

Multiply π times 100, because $10^2 = 100$.

"How can you find the area of a circle with a diameter of 10?"

Multiply π times 25, because $10 \div 2 = 5$, and $5^2 = 25$.

Lesson Summary

If C is a circle's circumference and r is its radius, then $C = 2\pi r$. The area of a circle can be found by taking the product of half the circumference and the radius.

If A is the area of the circle, this gives the equation:

$A = \frac{1}{2}(2\pi r) \cdot r$

This equation can be rewritten as:

$A = \pi r^2$

Remember that when we have $r \cdot r$ we can write r^2, and we can say "r **squared**."

This means that if we know the radius, we can find the area. For example, if a circle has a radius of 10 cm, then its area is about (3.14) · 100 which is 314 cm^2.

If we know the diameter, we can figure out the radius, and then we can find the area. For example, if a circle has a diameter of 30 ft, then the radius is 15 ft, and the area is about (3.14) · 225 which is approximately 707 ft^2.

Cool-down

A Circumference of 44

5 min

Student Task Statement

A circle's circumference is approximately 44 cm. Complete each statement using one of these values:

7, 11, 14, 22, 88, 138, 154, 196, 380, 616.

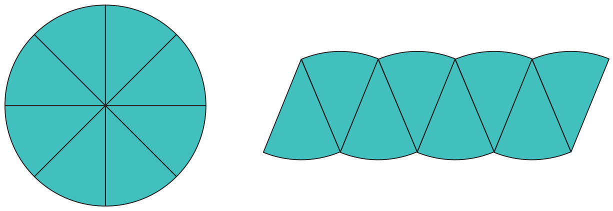
1. The circle's diameter is approximately 14 cm
2. The circle's radius is approximately 7 cm
3. The circle's area is approximately 154 cm^2

Practice Problems

7 Problems

Problem 1

The picture shows a circle divided into 8 equal wedges, which are rearranged.



The radius of the circle is r, and its circumference is $2\pi r$. How does the picture help to explain why the area of the circle is πr^2?

The rearranged shape looks more and more like a rectangle as the circle is cut into more pieces. The length of the rectangle is about half of the circumference of the circle, or πr. Its height is roughly the radius r. So the area of the rectangle (and of the circle) is πr^2.

Problem 2

A paper plate has a radius of 4.5 inches. What is the area of the plate?

about 64 in^2

Problem 3

Jada paints a circular table that has a diameter of 37 inches. What is the area of the table?

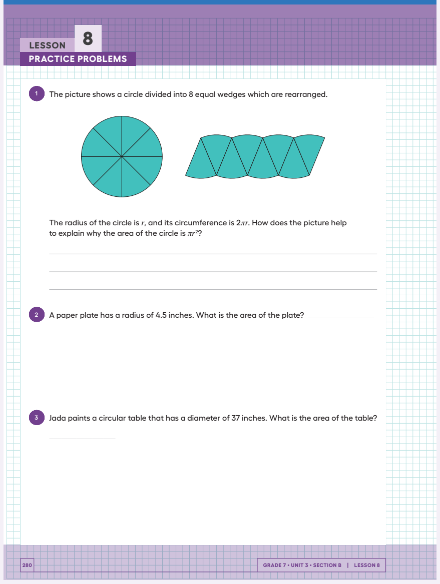
about 1,075 in^2

Problem 4

A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle.

The radius is approximately 12 cm.
The diameter is approximately 24 cm.
The area is approximately 460 cm^2.

Student Workbook



LESSON 8
PRACTICE PROBLEMS

1 The picture shows a circle divided into 8 equal wedges which are rearranged.

The radius of the circle is r, and its circumference is $2\pi r$. How does the picture help to explain why the area of the circle is πr^2?

2 A paper plate has a radius of 4.5 inches. What is the area of the plate?

3 Jada paints a circular table that has a diameter of 37 inches. What is the area of the table?

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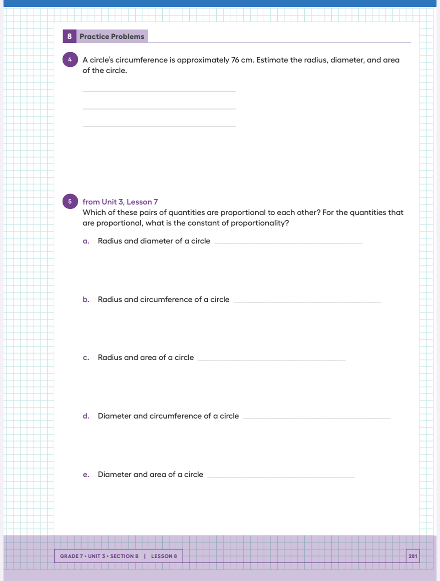
Student Workbook



8 Practice Problems

4 A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle.

5 from Unit 3, Lesson 7
Which of these pairs of quantities are proportional to each other? For the quantities that are proportional, what is the constant of proportionality?

a. Radius and diameter of a circle
b. Radius and circumference of a circle
c. Radius and area of a circle
d. Diameter and circumference of a circle
e. Diameter and area of a circle

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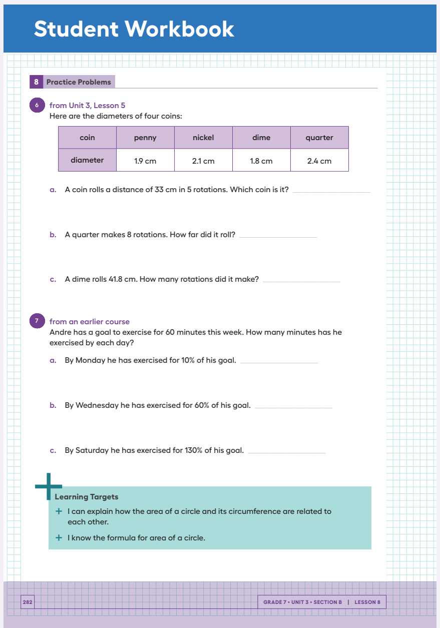
Student Workbook

8 Practice Problems

6 from Unit 3, Lesson 5
Here are the diameters of four coins:

coin	penny	nickel	dime	quarter
diameter	1.9 cm	2.1 cm	1.8 cm	2.4 cm

a. A coin rolls a distance of 33 cm in 5 rotations. Which coin is it?

b. A quarter makes 8 rotations. How far did it roll?

c. A dime rolls 41.8 cm. How many rotations did it make?

7 from an earlier course
Andre has a goal to exercise for 60 minutes this week. How many minutes has he exercised by each day?

a. By Monday he has exercised for 10% of his goal.

b. By Wednesday he has exercised for 60% of his goal.

c. By Saturday he has exercised for 130% of his goal.

Learning Targets
+ I can explain how the area of a circle and its circumference are related to each other.
+ I know the formula for area of a circle.

Problem 5 from Unit 3, Lesson 7

Which of these pairs of quantities are proportional to each other? For the quantities that are proportional, what is the constant of proportionality?

a. Radius and diameter of a circle

Yes, the diameter is twice the radius so the constant of proportionality is either 2 or $\frac{1}{2}$.

b. Radius and circumference of a circle

Yes, the circumference is 2π times the radius so the constant of proportionality is either 2π or $\frac{1}{2\pi}$.

c. Radius and area of a circle

no

d. Diameter and circumference of a circle

Yes, the circumference is π times the diameter so the constant of proportionality is either π or $\frac{1}{\pi}$.

e. Diameter and area of a circle

no

Problem 6 from Unit 3, Lesson 5

Here are the diameters of four coins:

coin	penny	penny	dime	quarter
diameter	1.9 cm	2.1 cm	1.8 cm	2.4 cm

a. A coin rolls a distance of 33 cm in 5 rotations. Which coin is it?

nickel because $33 \div 5 \div \pi \approx 2.1$

b. A quarter makes 8 rotations. How far did it roll?

about 60.3 cm because $2.4 \cdot \pi \cdot 8 \approx 60.3$

c. A dime rolls 41.8 cm. How many rotations did it make?

about 7 because $41.8 \div \pi \div 1.8 \approx 7$

Problem 7 from an earlier course

Andre has a goal to exercise for 60 minutes this week. How many minutes has he exercised by each day?

a. By Monday he has exercised for 10% of his goal.

6 minutes

b. By Wednesday he has exercised for 60% of his goal.

36 minutes

c. By Saturday he has exercised for 130% of his goal.

78 minutes