Lesson 12

Representing Percentages in Different Ways

Goals

- Choose and create representations to solve problems such as A of B is ? and A of ? is C.
- Interpret diagrams that represent situations involving percentages and explain (orally) how the quantities are related.

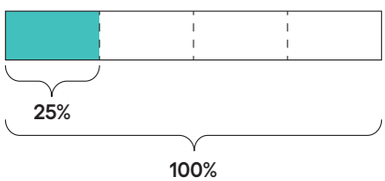
Learning Target

I can use tape diagrams to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?"

Lesson Narrative

In this lesson, students use two more familiar representations, tape diagrams and tables, to reason about percentages.

In prior grades, students used tape diagrams to reason about fractions and multiplicative comparisons. Here, tape diagrams can serve similar purposes, enabling students to see connections between percentages and fractions. For example, this tape diagram shows that 25% of a whole is the same as $\frac{1}{4}$ of that whole since 25% of the whole is one part when 100% of the whole is divided into four equal parts.



Students observe that when reasoning about percentages, it is important to indicate the 100%, just as it is important to indicate the whole when working with fractions. This is an opportunity to practice attending to precision.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

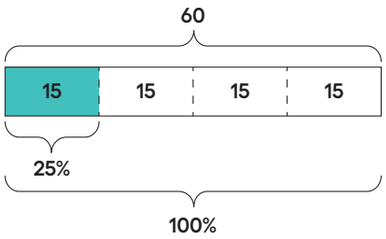
Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	5 min

Representing Percentages in Different Ways

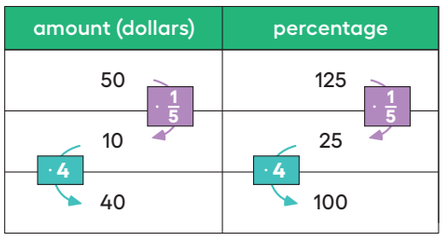
Lesson Narrative (continued)

If we are finding 25% of 60, then we can assign 100% to 60 and represent this on the tape diagram, which can then help us reason about 25% of 60.



A table can also be used to find 25% of 60. The structure of both a double number line diagram and a table encourages students to reason about equivalent ratios. Students may find the latter to be more flexible and efficient than the former, however, especially when dealing with percentages greater than 100%, or when finding the value for 100% when a percentage is known.

For instance, if 125% of a fundraising goal is $50, what is the fundraising goal? Here is one way to find 100% of the goal:



Student Learning Goal

Let's use tape diagrams to understand percentages.

Warm-up

Notice and Wonder: Tape Diagram

10 min

Activity Narrative

The purpose of this *Warm-up* is to elicit two ideas: that we can reason about percentages of a number in terms of fractions of a whole, and that tape diagrams can support this reasoning.

The given tape diagram represents the relationship between the baby weight and adult weight of Jada's puppy, which students previously represented with a double number line diagram. This is done to encourage students to think about the structure of each representation and how the same relationship and quantities are presented differently in the two diagrams. The thinking here will support students in choosing representations to make sense of percentage problems.

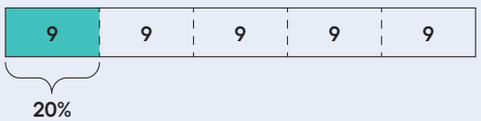
While students may notice and wonder many things about the diagram, the important discussion points are what the number of parts, the size of the parts, and the entire diagram might tell us about the situation represented.

Launch

Arrange students in groups of 2. Display the tape diagram for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder about. Give students 1 minute of quiet think time, and then 1 minute to discuss the things that they notice and wonder about with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- The diagram looks like the ones used to show fractions.
- The diagram is partitioned into 5 parts and one part is shaded.
- All the parts are labeled with the number 9.
- The shaded part is 20% of something.
- The entire tape represents 5 · 9.

Students may wonder:

- Why is only one part shaded?
- What situation does the diagram represent?
- Does the diagram represent Jada's puppy weight?

Instructional Routines

Notice and Wonder

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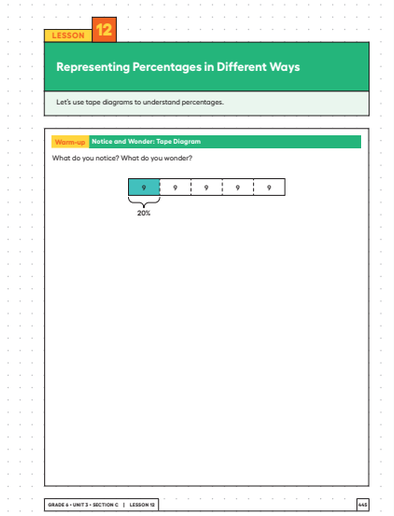
Student Workbook


LESSON 12

Representing Percentages in Different Ways

Let's use tape diagrams to understand percentages.

Warm-up Notice and Wonder: Tape Diagram

What do you notice? What do you wonder?

9 9 9 9 9

20%

Activity Synthesis

Ask students to share the things that they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

- *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If no students observed that (or wondered if) the tape diagram could represent the situation about Jada's puppy in a recent activity, ask students:

- *"Could this diagram represent the weight of Jada's puppy and its adult weight? How do you know?"*

Then, ask students:

- *"How can we see 100% in the diagram?"*

  It's the entire length of the diagram.

- *"What does it represent in this situation?"*

  the adult weight of Jada's puppy

- *"The adult weight of the puppy will be 45 pounds. How can you see that in the diagram?"*

  There are 5 groups of 9 in the diagram, and $5 \cdot 9 = 45$.

Display the double number line diagram from "Puppies Grow Up."

- *"Compare the way the tape diagram and the double number line diagram represent the same situation. How are they alike?"*

  They both show that 9 pounds is 20% of something. They both use the length of a diagram to represent weight and percentage.

- *"How are the diagrams different?"*

  The 100% is not shown on the double number line diagram. It doesn't show how many groups of 20% are in 100%. In the tape diagram, we see that there are 5 parts that are the same size. Each part represents 20% or a fifth of the tape, so the entire length of the diagram is 100%.

Point out to students that a percentage can sometimes be thought of as a fraction of a whole. While both representations we've used so far can help us solve problems, a tape diagram can show a number as a fraction of 100%, which can help us make sense of a situation.

Activity 1
Revisiting Jada's Puppy

15 min

Activity Narrative

In this activity, students continue to explore different representations—now including tape diagrams and tables—for solving problems involving percentages. The goal is for students to see that a relationship can be represented in different ways, make connections, and choose a representation based on the situation and what makes sense to them. Two or more partially completed representations are given to encourage their use by students.

The situations here involve multiplicative comparisons of two quantities. In the first situation, students need to find the value for 100% when the value for 10% is known. In the second, they find a quantity that is 150% as much as another quantity. Students must interpret the quantities being given and sought as well as what 100% represents. They also need to decide how to represent these quantities. In doing so, students practice reasoning quantitatively and abstractly.

When students work on the second question, monitor for those who make the value for 100% explicit in their chosen representation (if any is used). Also monitor for students who find the weight of the bunny in the following ways:

- Compute 10% of the weight of Jada's puppy and multiply it by 15.
- Find 50% of the weight of Jada's puppy and triple it.
- Multiply the weight of Jada's puppy by 1.5 directly because 150 is 1.5 times 100.

In both situations here, the quantities can be related in terms of fractions. For example, we can think of the weight of Jada's puppy as $\frac{1}{10}$ of the weight of another dog and the weight of the bunny as $\frac{3}{2}$ of the weight of Jada's puppy. The latter can help students relate percentages greater than 100 and fractions greater than 1. Monitor for students who make connections to fractions. Reasoning in terms of fractions can serve as a useful preview for the work on benchmark percentages in an upcoming lesson.

Launch

Keep students in groups of 2. Give students 2 minutes of quiet think time for the first question and then time for a partner discussion. Ask partners to share their responses and reasoning. Discuss how students knew that 100% represents the weight of the other dog.

Next, instruct students to read the stem of the second question. Ask them to discuss with their partner what 100% represents in that situation. Then, give students 2–3 minutes of quiet time to complete the second question. Tell students that three representations are started for them. If they wish, they can choose to mark up the double number line diagram, extend or partition the tape diagram, or complete the table to support their thinking.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially those of students who haven't shared recently.

Instructional Routines

MLR7: Compare and Connect

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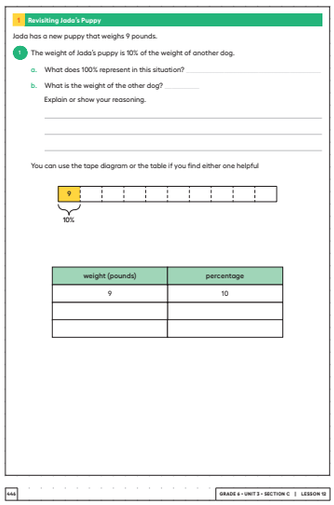
Access for Multilingual Learners (Activity 1)

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Student Workbook


1 Revisiting Jada's Puppy

Jada has a new puppy that weighs 9 pounds.

1. The weight of Jada's puppy is 10% of the weight of another dog.
   a. What does 100% represent in this situation?
   b. What is the weight of the other dog?
   Explain or show your reasoning.

You can use the tape diagram or the table if you find either one helpful

9

10%

weight (pounds)	percentage
9	10

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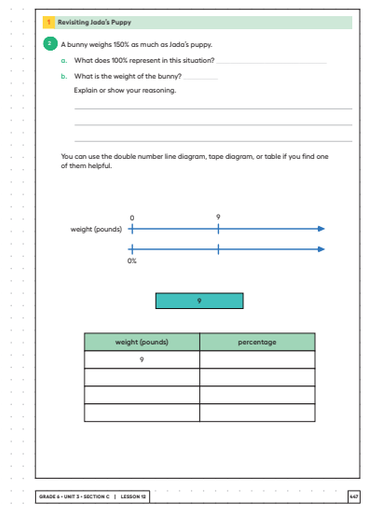
Student Workbook


1 Revisiting Jada's Puppy

2. A bunny weighs 150% as much as Jada's puppy.
   a. What does 100% represent in this situation?
   b. What is the weight of the bunny?
   Explain or show your reasoning.

You can use the double number line diagram, tape diagram, or table if you find one of them helpful.

weight (pounds) 0 9

0%

9

weight (pounds)	percentage
9	

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Student Task Statement

Jada has a new puppy that weighs 9 pounds.

1. The weight of Jada's puppy is 10% of the weight of another dog.

a. What does 100% represent in this situation?

100% represents the weight of the other dog.

b. What is the weight of the other dog? Explain or show your reasoning.

You can use the tape diagram or the table if you find either one helpful.

90 pounds.

Sample reasoning:

- The weight of the other dog is 10 times the weight of Jada's puppy.
  $10 \cdot 9 = 90$
- 

weight (pounds)	percentage
9 (×10) 90	10 (×10) 100

2. A bunny weighs 150% as much as Jada's puppy.

a. What does 100% represent in this situation?

100% represents the weight of Jada's puppy.

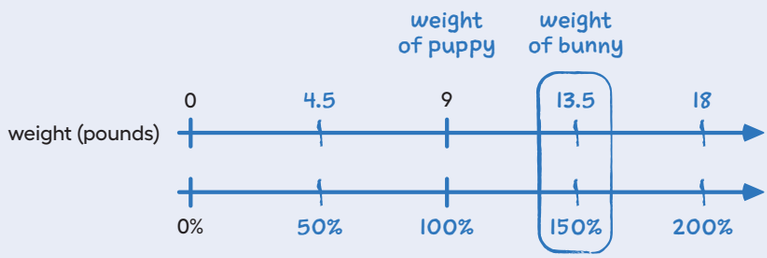
b. What is the weight of the bunny?

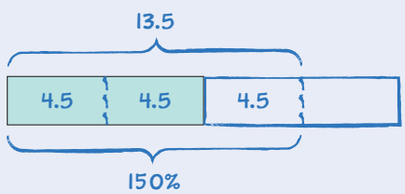
13.5 pounds

Explain or show your reasoning.

You can use the double number line diagram, tape diagram, or table if you find one of them helpful.

Sample reasoning:

- 

weight (pounds)	percentage
9 (×3/2) 13.5	100 (×3/2) 150
- 

Activity Synthesis

One goal of this discussion is to spotlight various ways to reason about 150% of a quantity and how the same multiplicative relationships are shown in different representations. Another goal is to reiterate that we can think about solving percentage problems in terms of finding equivalent ratios.

Display 2–3 approaches from previously selected students for all to see. Invite students to briefly describe their reasoning. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches and representations. Here are some questions for discussion:

- *"How are the approaches the same?"*

  They all started with thinking about the 9 pounds as 100%. Many of them involved dividing 9 and 100% by 2 and multiplying the result by 3. They involved finding 50%, or 10%, first before finding 150%.

- *"How are they different?"*

  They used different representations. The 9 pounds and 100% are described differently in each representation. Some approaches are more convenient or quicker than others.

Highlight that all the strategies involve finding a pounds-to-percentage ratio of _____ : 150 that is equivalent to 9 : 100. It makes sense to multiply 9 by $\frac{1}{2}$ (or divide 9 by 2) and then multiply by 3 because multiplying 100 by $\frac{1}{2}$ and then multiplying the result by 3 gives 150.

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight similar features in each table and diagram, such as how 100% is represented.

Supports accessibility for: Visual-Spatial Processing

Activity 2
Staying Hydrated

10 min

Activity Narrative

In this activity, students reason about percentages that describe both multiplicative comparison and parts of a whole. In the first situation, they reason about a quantity described as "120% as much as" another quantity. In the second situation, they find both B and C, where A% of B is C.

Students can use any representation or strategy. As they work, monitor for different approaches being used and connections that can be made across representations and strategies. For instance, students using a table, a double number line diagram, and a tape diagram to answer the first question may all start by reasoning about 20% of 5 cups before finding 120%. Note how they go about finding the 20% and the 120% in each case.

Regardless of how they reason, students need to first interpret the quantities and recognize the value that corresponds to 100% in each situation. Where and how 100% can be seen across representations and strategies is another connection worth making.

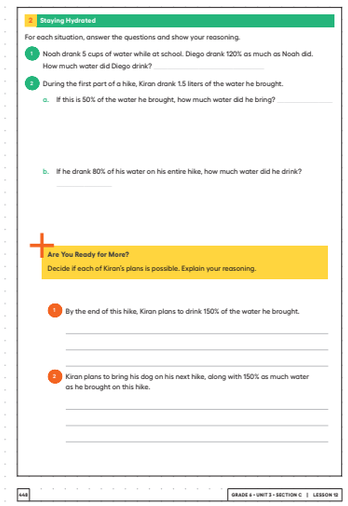
Some students may bypass using representations. For instance, they may reason about 50% and 80% of a quantity in terms of the fractions $\frac{1}{2}$ and $\frac{8}{10}$ (or $\frac{4}{5}$), and multiply or divide the given values by these fractions. To find 150% of Kiran's water (in the *Are You Ready for More?*) they may calculate (1.5) · 2. Consider asking these students to explain how their reasoning could be shown on one of the representations.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Encourage students to represent the problem through different modalities, such as by using drawings or physical objects, to better understand the situation. For example, ask them to represent 5 cups of water and 120% as much as that amount in their own way.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Workbook


2 Staying Hydrated

For each situation, answer the questions and show your reasoning.

1. Noah drank 5 cups of water while at school. Diego drank 120% as much as Noah did. How much water did Diego drink?

2. During the first part of a hike, Kiran drank 1.5 liters of the water he brought.
   a. If this is 50% of the water he brought, how much water did he bring?
   b. If he drank 80% of his water on his entire hike, how much water did he drink?

Are You Ready for More?
Decide if each of Kiran's plans is possible. Explain your reasoning.

1. By the end of this hike, Kiran plans to drink 150% of the water he brought.
2. Kiran plans to bring his dog on his next hike, along with 150% as much water as he brought on this hike.

Launch

Keep students in groups of 2. Give them 3–4 minutes of quiet work time and then time to share their responses and reasoning with their partner. Follow with a whole-class discussion.

Student Task Statement

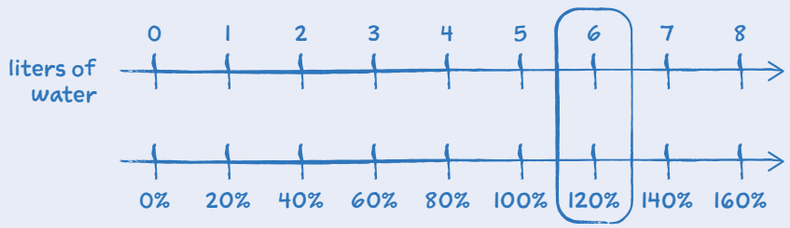
For each situation, answer the questions and show your reasoning.

1. Noah drank 5 cups of water while at school. Diego drank 120% as much as Noah did. How much water did Diego drink?

   6 cups

   Sample reasoning:

   - The amount Noah drank, 5 cups, is 100%, so 1 cup is 20%. 120 is 6 · 20, so Diego drank 6 · 1 or 6 cups.



2. During the first part of a hike, Kiran drank 1.5 liters of the water he brought.

a. If this is 50% of the water he brought, how much water did he bring?

3 liters

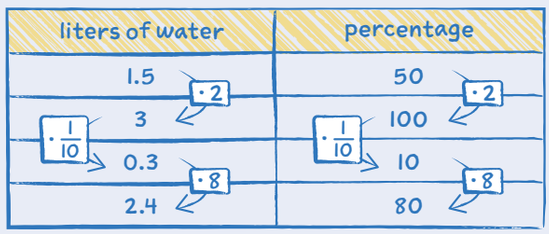
Sample reasoning:

- 50% is half of the water he brought, so he brought $2 \cdot (1.5)$ or 3 liters.

b. If he drank 80% of his water on his entire hike, how much water did he drink?

2.4 liters

Sample reasoning:



Are You Ready for More?

Decide if each of Kiran's plans is possible. Explain your reasoning.

1. By the end of this hike, Kiran plans to drink 150% of the water he brought.

This is not possible

Sample reasoning: This means he will drink more water than he brought. He can only do so if he drinks someone else's water!

2. Kiran plans to bring his dog on his next hike, along with 150% as much water as he brought on this hike.

This is possible

Sample reasoning: This means he will bring $1\frac{1}{2}$ times as much water next time.

Building on Student Thinking

When answering the second set of questions about Kiran's water consumption, some students may rush to find 50% of 1.5 liters and 80% of 1.5 liters. Ask them to re-interpret each question and clarify what the 1.5 liters represents as well as what each question is asking. Encourage them to draw a tape diagram to represent the information in each question—the values that are given and the value that is unknown.

Activity Synthesis

Focus the discussion on how students interpreted the quantities in each situation and how they reasoned accordingly. For instance, ask students how they knew which quantity corresponds to 100% in each situation and how they represented it.

At this point it is not necessary for students to formally conceptualize the two ways that percentages are used (to describe parts of whole and to describe comparative relationships). Drawing their attention to concrete and contextualized examples of both, however, serves to build this understanding intuitively.

If time permits, discuss students' responses to the last question. Select a student who used each representation—a double number line diagram, a table, and a tape diagram—to share their reasoning. As students explain, display those representations for all to see. If any students reasoned without these representations, select one to share their thinking as well. Point out connections across strategies and highlight that the percentages and their corresponding values in each case represent equivalent ratios.

Lesson Synthesis

One key insight from this lesson is that it is important to consider which quantity corresponds to 100% in situations involving percentages. Another insight is that double number line diagrams, tape diagrams, and tables can all help us represent and make sense of percentages.

Consider selecting a problem from the lesson or posing a new problem to discuss. For example, "Suppose we know that 140% of some number is 28. How can we use a double number line diagram, a tape diagram, and a table to find that number?"

Display a blank double number line diagram, a blank tape diagram, and a blank table. Complete each representation as students answer questions such as:

💬 *"In this situation, what value corresponds to 100%?"*

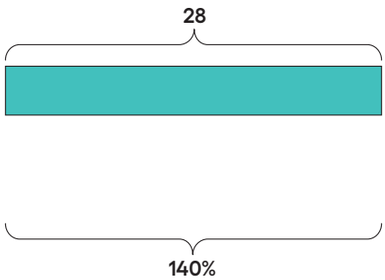
We don't know. 100% of the number is what we want to find.

💬 *"What information do we know?"*

140% of the number is 28.

"How do we show it on the double number line diagram? On the tape diagram? In a table?"

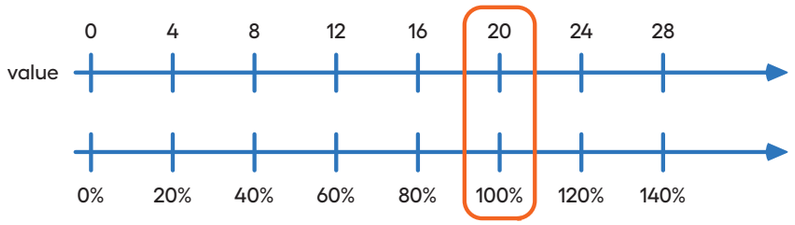
Label diagrams and table.



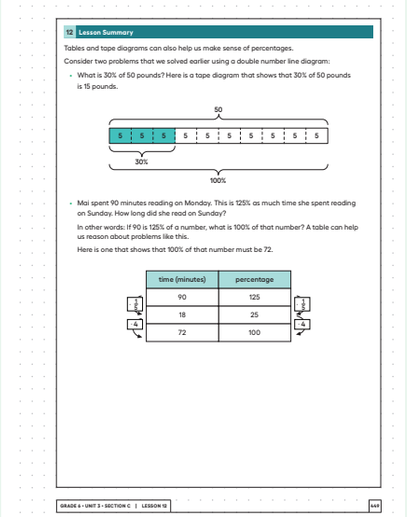
value	percentage
28	140

"How would we find the value of 100% using the double number line diagram?"

Partition the number lines and tape diagram into 7 or 14 equal parts, label the parts with increments of 4 and 20% or 2 and 10%, and find the value that aligns with 100%. Here is one way.



Student Workbook


12 Lesson Summary

Tables and tape diagrams can also help us make sense of percentages.

Consider two problems that we solved earlier using a double number line diagram:

- What is 30% of 50 pounds? Here is a tape diagram that shows that 30% of 50 pounds is 15 pounds.

50
5 5 5 5 5 5 5 5 5 5
30%
100%

- Mai spent 90 minutes reading on Monday. This is 125% as much time she spent reading on Sunday. How long did she read on Sunday?

In other words: If 90 is 125% of a number, what is 100% of that number? A table can help us reason about problems like this.

Here is one that shows that 100% of that number must be 72.

time (minutes)	percentage
90	125
18	25
72	100

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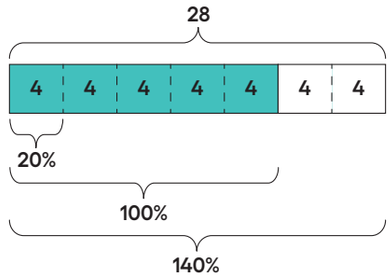
"How would we find the value of 100% using a tape diagram?"

Partition and label the tape as done with the double number lines. Here is one way.



"How would we find the value of 100% using a table?"

Here is one way:

value	percentage
28	140
4	20
20	100

(×1/7 from 28 to 4; ×1/7 from 140 to 20; ×5 from 4 to 20; ×5 from 20 to 100)

"What is the same about the ways we used the three representations?"

We paired 28 and 140%, divided those values by 7 or 14, and then multiplied them by 5 or 10.

"What is different?"

We had to partition and label the double number line and tape diagrams to show parts of 20% or 10%, but we only needed to write numbers when using the table.

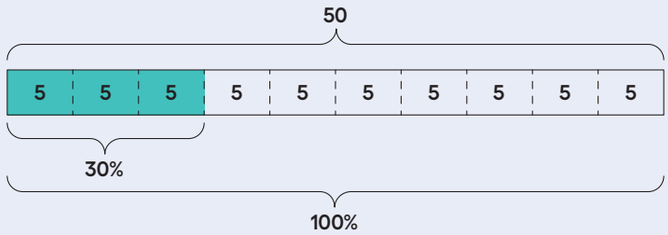
If time permits, consider asking students which strategy they prefer and why.

Lesson Summary

Tables and tape diagrams can also help us make sense of percentages.

Consider two problems that we solved earlier using a double number line diagram:

- What is 30% of 50 pounds? Here is a tape diagram that shows that 30% of 50 pounds is 15 pounds.



- Mai spent 90 minutes reading on Monday. This is 125% as much time she spent reading on Sunday. How long did she read on Sunday?

In other words: If 90 is 125% of a number, what is 100% of that number? A table can help us reason about problems like this.

Here is one that shows that 100% of that number must be 72.

time (minutes)	percentage
90	125
18	25
72	100

$\cdot \frac{1}{5}$ $\cdot 4$ $\cdot \frac{1}{5}$ $\cdot 4$

Cool-down

Small and Large

5 min

Student Task Statement

A small tank holds 36 liters of water. This is 75% of the water that a large tank holds.

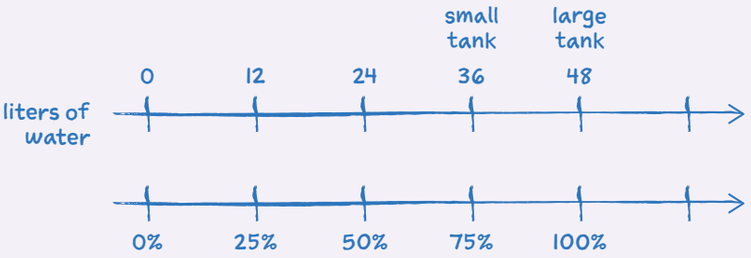
How much does the large tank hold?

48 units

Show your reasoning.

Sample reasoning:

- If 36 liters is 75% of the water in the large tank, then 12 liters is 25% and 4 · 12 or 48 liters is 100%.
- 

- 

Responding To Student Thinking

Points to Emphasize

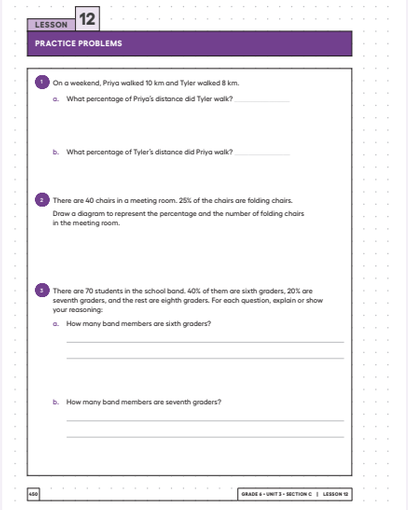
If students struggle with finding a percentage of a number, highlight ways to use representations to make sense of percentages. Do this whenever opportunities arise over the next several lessons. For example, in this activity, make sure to invite multiple students to draw connections between benchmark percentages of different quantities: Unit 3, Lesson 13, Activity 1 Liters, Meters, and Hours

Lesson 12

Practice Problems

6 Problems

Student Workbook


LESSON 12

PRACTICE PROBLEMS

1. On a weekend, Priya walked 10 km and Tyler walked 8 km.
   a. What percentage of Priya's distance did Tyler walk? _____
   b. What percentage of Tyler's distance did Priya walk? _____
2. There are 40 chairs in a meeting room. 25% of the chairs are folding chairs. Draw a diagram to represent the percentage and the number of folding chairs in the meeting room.
3. There are 70 students in the school band. 40% of them are sixth graders, 20% are seventh graders, and the rest are eighth graders. For each question, explain or show your reasoning:
   a. How many band members are sixth graders?
   b. How many band members are seventh graders?

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Problem 1

On a weekend, Priya walked 10 km and Tyler walked 8 km.

a. What percentage of Priya's distance did Tyler walk?

80%

b. What percentage of Tyler's distance did Priya walk?

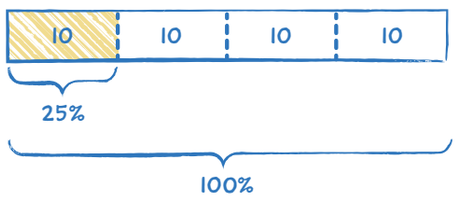
125%

Problem 2

There are 40 chairs in a meeting room. 25% of the chairs are folding chairs.

Draw a diagram to represent the percentage and the number of folding chairs in the meeting room.

Sample response: Each part in the tape diagram represents 25%. There are 4 groups of 25% in 100% and there are 4 groups of 10 in 40, so 25% of 40 chairs is 10 chairs.



Problem 3

There are 70 students in the school band. 40% of them are sixth graders, 20% are seventh graders, and the rest are eighth graders. For each question, explain or show your reasoning:

a. How many band members are sixth graders?

28 students

Sample reasoning: 10% of 70 is 7, so 40% of 70 is 4 · 7, which is 28.

b. How many band members are seventh graders?

14 students

Sample reasoning: 20% is half of 40% and 14 is half of 28.

c. The school's drama club has 110% as many students as the band. How many students are in the drama club?

77 students

Sample reasoning: 10% of 70 is 7, so 110% of 70 is 7 more students than 70, which is 77.

Problem 4

from Unit 3, Lesson 11

Jada has a monthly budget for her cell phone bill. Last month she spent 120% of her budget, and the bill was $60. What is Jada's monthly budget? Explain or show your reasoning.

$50

Sample reasoning: If 120% is 60, then 20% is 10 (multiplying each 120% and 60 by $\frac{1}{6}$ gives 20% and 10). If 20% is 10, then 100% is 5 times 10, which is 50.

Problem 5

from Unit 3, Lesson 7

Which is a better deal, 5 tickets for $12.50 or 8 tickets for $20.16? Explain your reasoning.

5 tickets for $12.50 is a better deal

Sample reasoning: Five tickets for $12.50 equals a unit rate of $2.50 per ticket (12.50 ÷ 5 = 2.50), and 8 tickets for $20.16 equals a unit rate of $2.52 per ticket (12.50 ÷ 8 = 2.52).

Problem 6

from Unit 3, Lesson 9

An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

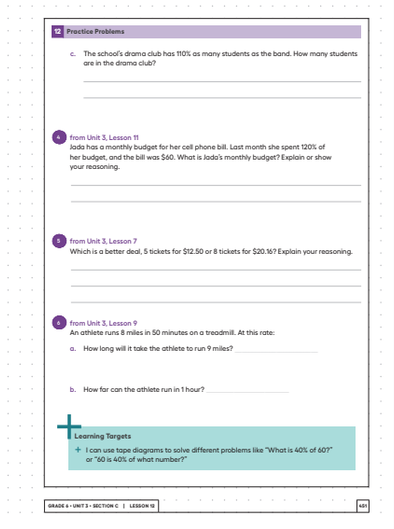
a. How long will it take the athlete to run 9 miles?

56.25 minutes (or equivalent)

b. How far can the athlete run in 1 hour?

9.6 miles (or equivalent)

Student Workbook



12 Practice Problems

c. The school's drama club has 110% as many students as the band. How many students are in the drama club?

4 from Unit 3, Lesson 11
Jada has a monthly budget for her cell phone bill. Last month she spent 120% of her budget, and the bill was $60. What is Jada's monthly budget? Explain or show your reasoning.

5 from Unit 3, Lesson 7
Which is a better deal, 5 tickets for $12.50 or 8 tickets for $20.16? Explain your reasoning.

6 from Unit 3, Lesson 9
An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

a. How long will it take the athlete to run 9 miles?

b. How far can the athlete run in 1 hour?

Learning Targets
+ I can use tape diagrams to solve different problems like "What is 40% of 60?" or "60 is 40% of what number?"

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