Lesson 3

Equations for Functions

Goals

- Calculate the output of a function for a given input using an equation in two variables, and interpret (orally and in writing) the output in context.
- Determine (orally and in writing) the independent and dependent variables of a function, and explain (orally) the reasoning.

Learning Targets

- I can find the output of a function when I know the input.
- I can name the independent and dependent variables for a given function and represent the function with an equation.

Lesson Narrative

So far students have used input-output diagrams, tables, and descriptions of the rules to represent functions. In this lesson, students begin to make connections between these representations of functions and algebraic equations, continuing to build their skills reasoning abstractly and quantitatively about functions.

This lesson also introduces the use of independent and dependent variables in the context of functions. Students learn that the **independent variable** represents the input of a function, while the **dependent variable** represents the output. For an equation that relates two quantities, it is sometimes possible to write either of the variables as a function of the other. For example, in the activity "Dimes and Quarters," either the number of quarters or the number of dimes could be used as the independent variable for the situation. If we know the number of quarters and have questions about the number of dimes, then this would be a reason to choose the number of quarters as the independent variable.

Student Learning Goal

Let's find outputs from equations.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

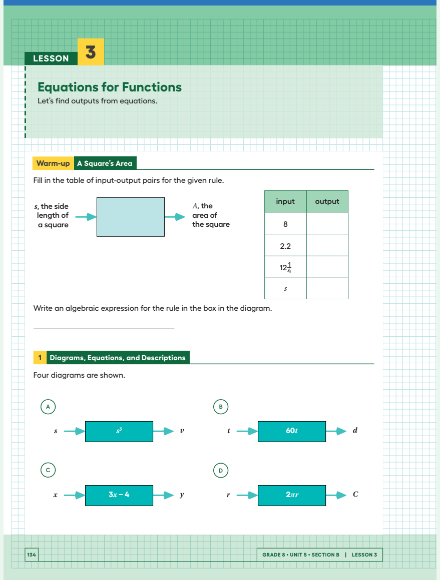
- MLR6: Three Reads (Activity 2)

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Student Workbook



Warm-up
A Square's Area

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to use repeated reasoning to write an algebraic expression to represent a rule of a function.

Launch

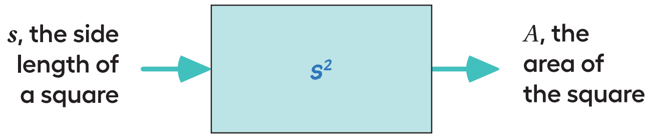
Arrange students in groups of 2.

Give students 1–2 minutes of quiet work time and then time to share their algebraic expression with their partner.

Follow with a whole-class discussion.

Student Task Statement

Fill in the table of input-output pairs for the given rule.



input	output
8	64
2.2	4.84
$12\frac{1}{4}$	$150\frac{1}{16}$
s	s^2 or A

Write an algebraic expression for the rule in the box in the diagram.

Activity Synthesis

The purpose of this discussion is for students to connect a rule to a table of input-output pairs and an algebraic expression. Students are also reintroduced to the terms “independent variables” and “dependent variables” in the context of the inputs and outputs of functions.

Select students to share how they found each of the outputs. After each response, ask the class if they agree or disagree. Record and display responses for all to see. If both responses are not mentioned by students for the last row, tell students that we can either put s^2 or A there. Tell students we can write the equation $A = s^2$ to represent the rule of this function.

End the discussion by telling students that while we’ve used the terms “input” and “output” so far to talk about specific values, when a letter is used to represent any possible input we call it the **independent variable**, and the letter used to represent all the possible outputs is the **dependent variable.** Students may recall these terms from earlier grades. In this case, s is the independent variable and A the dependent variable, and we say “A depends on s.”

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “____ corresponds to ____ because …” “First I ____ because …” and “How do you know …?”

Supports accessibility for: Language, Social-Emotional Functioning

Activity 1

Diagrams, Equations, and Descriptions

15 min

Activity Narrative

The purpose of this activity is for students to make connections between different representations of functions and start transitioning from input-output diagrams to other representations of functions. Students match input-output diagrams to descriptions and come up with equations for each of those matches. Students then calculate an output given a specific input and determine the independent and dependent variables.

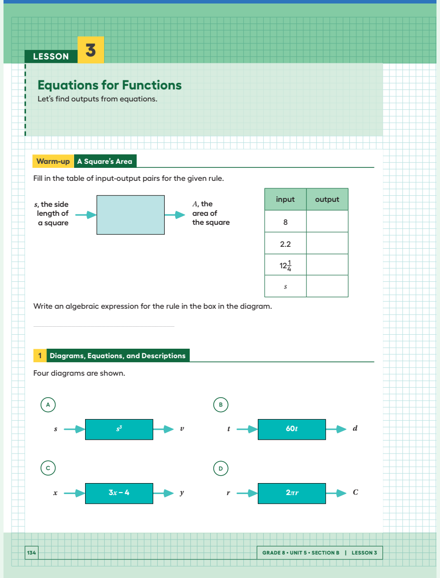
Launch

Arrange students in groups of 2.

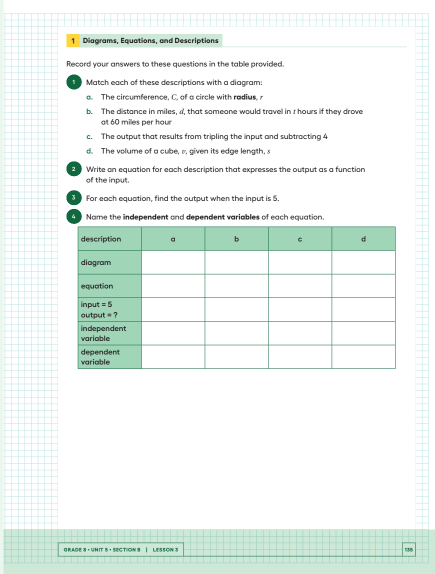
Give students 3–5 minutes of quiet work time and time to share their responses with their partner and come to agreement on their answers.

Follow with whole-class discussion.

Student Workbook

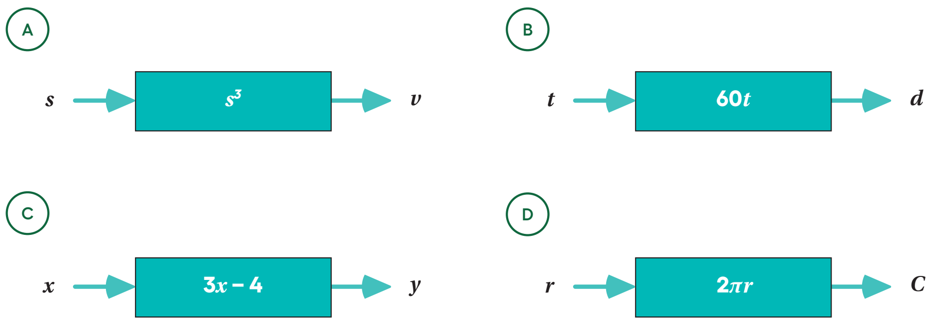


Student Workbook



Student Task Statement

Four diagrams are shown.



Record your answers to these questions in the table provided.

1. Match each of these descriptions with a diagram:
   - **a.** The circumference, C, of a circle with **radius**, r
   - **b.** The distance in miles, d, that someone would travel in t hours if they drove at 60 miles per hour
   - **c.** The output that results from tripling the input and subtracting 4
   - **d.** The volume of a cube, v, given its edge length, s
2. Write an equation for each description that expresses the output as a function of the input.
3. For each equation, find the output when the input is 5.
4. Name the **independent** and **dependent variables** of each equation.

description	a	b	c	d
diagram	D	B	C	A
equation	$C = 2\pi r$	$d = 60t$	$y = 3x - 4$	$v = s^3$
input = 5 output = ?	$10\pi \approx 31.4$	300	11	125
independent variable	r	t	x	s
dependent variable	C	d	y	v

Are You Ready for More?

Choose a 3-digit number as an input.

Apply the following rule to it, one step at a time:

- Multiply your number by 7.
- Add 1 to the result.
- Multiply the result by 11.
- Subtract 5 from the result.
- Multiply the result by 13.
- Subtract 78 from the result to get the output.

Can you describe a simpler way to describe this rule? Why does this work?

If we apply the steps to a generic 3-digit number x, the result is

$$13(11(7x+1)-5)-78 = 1{,}001x.$$

For any 3-digit number x, the number $1{,}001x$ is just that number repeated twice. This works since $1{,}001x = 1{,}000x + x$. For example,

$$\begin{aligned} 1{,}001 \cdot 314 &= 1{,}000 \cdot 314 + 1 \cdot 314 \\ &= 314{,}000 + 314 \\ &= 314{,}314 \end{aligned}$$

Activity Synthesis

The goal of this discussion is for students to describe the connections they see between the different entries for the four descriptions. This discussion should also highlight the naming of independent and dependent variables for the four functions using the letters shown in the input-output diagrams.

Display the table for all to see, and select different groups to share the answers for a column in the table. As groups share their answers, ask:

"How did you know that this diagram matched with this description?"

We remembered the formula for the circumference of a circle, so we knew the first description went with diagram D.

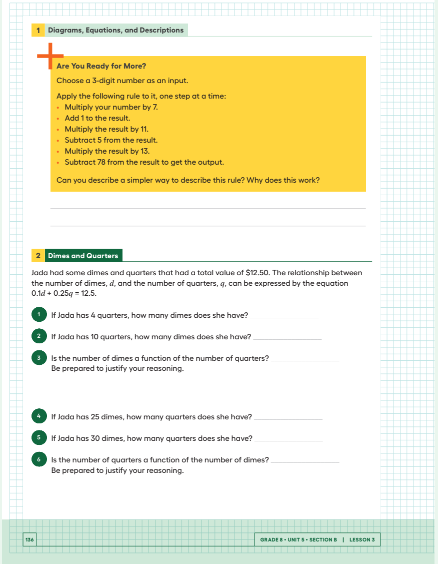
"Where in the equation do you see the rule that is in the diagram?"

The equation is the output set equal to the rule describing what happens to the input in the diagram.

"Explain why you chose those quantities for your independent and dependent variables."

We know the independent variable is used to calculate the dependent variable, so we matched them up with the input and output shown in the diagram.

Student Workbook


1 Diagrams, Equations, and Descriptions

Are You Ready for More?

Choose a 3-digit number as an input.

Apply the following rule to it, one step at a time:

- Multiply your number by 7.
- Add 1 to the result.
- Multiply the result by 11.
- Subtract 5 from the result.
- Multiply the result by 13.
- Subtract 78 from the result to get the output.

Can you describe a simpler way to describe this rule? Why does this work?

2 Dimes and Quarters

Jada had some dimes and quarters that had a total value of $12.50. The relationship between the number of dimes, d, and the number of quarters, q, can be expressed by the equation $0.1d + 0.25q = 12.5$.

1. If Jada has 4 quarters, how many dimes does she have? ______
2. If Jada has 10 quarters, how many dimes does she have? ______
3. Is the number of dimes a function of the number of quarters? ______ Be prepared to justify your reasoning.
4. If Jada has 25 dimes, how many quarters does she have? ______
5. If Jada has 30 dimes, how many quarters does she have? ______
6. Is the number of quarters a function of the number of dimes? ______ Be prepared to justify your reasoning.

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Activity 2

Dimes and Quarters

15 min

Activity Narrative

The purpose of this activity is for students to work with a function where either variable could be the independent variable. Knowing the total value for an unknown number of dimes and quarters, students are first asked to consider if the number of dimes could be a function of the number of quarters and then asked if the reverse is also true. Since this isn't always the case when students are working with functions, the discussion should touch on reasons for choosing one variable versus the other, which can depend on the types of questions one wants to answer.

Launch

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time followed by partner discussion for students to compare their answers and resolve any differences.

Follow with a whole-class discussion.

Select students who efficiently rewrite the original equation in the third problem and the last problem to share during the discussion.

Student Task Statement

Jada had some dimes and quarters that had a total value of \$12.50. The relationship between the number of dimes, d, and the number of quarters, q, can be expressed by the equation $0.1d + 0.25q = 12.5$.

1. If Jada has 4 quarters, how many dimes does she have?

   115

   If $q = 4$, then the equation tells us that $0.1d + (0.25) \cdot 4 = 12.5$. Subtracting 1 from each side gives $0.1d = 11.5$, so $d = 115$.

2. If Jada has 10 quarters, how many dimes does she have?

   100

   If $q = 10$, then the equation tells us that $0.1d + (0.25) \cdot 10 = 12.5$. Subtracting 2.5 from each side gives $0.1d = 10$, so $d = 100$.

3. Is the number of dimes a function of the number of quarters? Be prepared to justify your reasoning.

   Yes

   Sample reasoning: If you know the number of quarters, then you can determine the number of dimes from the equation by substituting the value for q and calculating the value for d.

4. If Jada has 25 dimes, how many quarters does she have?

   40

   If $d = 25$, then the equation tells us that $0.1(25) + (0.25)q = 12.5$. Subtracting 2.5 from both sides gives $0.25q = 10$, so $q = 40$.

Access for Multilingual Learners (Activity 2, Launch)

MLR6: Three Reads.
Keep student workbooks or devices closed. Display only the problem stem, without revealing the questions. Say,

"We are going to read this question 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

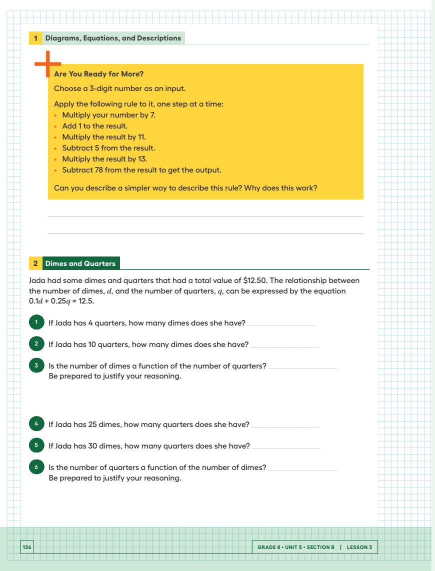
Building on Student Thinking

If students are not sure how to determine the number of dimes if there are 4 quarters, ask:

"Tell me what you know about the variables in the equation $0.1d + 0.25q = 12.5$."

"What if you knew the number of quarters was 0. How could you calculate the number of dimes?"

Student Workbook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Are You Ready for More?

Choose a 3-digit number as an input.

Apply the following rule to it, one step at a time:
- Multiply your number by 7.
- Add 1 to the result.
- Multiply the result by 11.
- Subtract 5 from the result.
- Multiply the result by 13.
- Subtract 78 from the result to get the output.

Can you describe a simpler way to describe this rule? Why does this work?

2 Dimes and Quarters

Jada had some dimes and quarters that had a total value of \$12.50. The relationship between the number of dimes, d, and the number of quarters, q, can be expressed by the equation $0.1d + 0.25q = 12.5$.

1. If Jada has 4 quarters, how many dimes does she have? ______
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5. If Jada has 30 dimes, how many quarters does she have? ______
6. Is the number of quarters a function of the number of dimes? ______ Be prepared to justify your reasoning.

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5. If Jada has 30 dimes, how many quarters does she have?

   38

   If $d = 30$, then the equation tells us that $0.1(30) + (0.25)q = 12.5$. Subtracting 3 from both sides gives $0.25q = 9.5$, so $q = 38$.

6. Is the number of quarters a function of the number of dimes? Be prepared to justify your reasoning.

   Yes

   Sample reasoning: If you know the number of dimes, then you can determine the number of quarters from the equation by substituting the value for d and calculating the value for q.

Activity Synthesis

The goal of this discussion is to clarify the confusion that can happen with some relationships—which variable is independent and which variable is dependent?

Begin the discussion with a quick show of hands from anyone that responded yes to the third question and the final question. Invite 2–3 students who responded yes to share their responses. Highlight students' use of the original equation to calculate the value of the other variable.

Tell students that when we have an equation like $0.1d + 0.25q = 12.5$, we can choose either d or q to be the independent variable. That means we are viewing one as depending on the other.

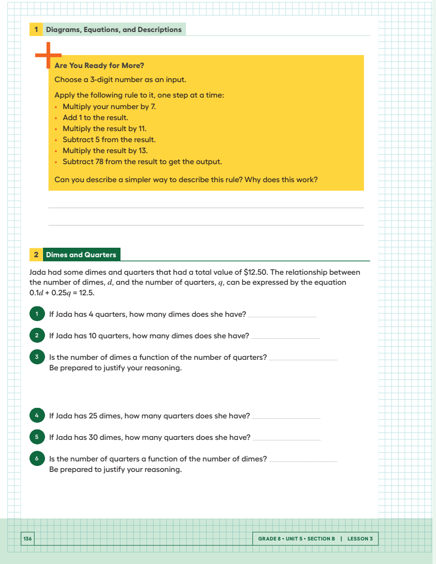
- If we know the number of quarters and want to answer a question about the number of dimes, then we are thinking of d as a function of q, and we can substitute a value in for q and solve for d.
- If we know the number of dimes and want to answer a question about the number of quarters, then we are thinking of q as a function of d, and we can substitute a value in for d and solve for q.

Display the diagrams for all to see, which show what the expressions would be if we rewrote the original equation in the ways described:

$q \rightarrow \boxed{125 - 2.5q} \rightarrow d$ $\qquad$ $d \rightarrow \boxed{50 - 0.4d} \rightarrow q$

Ensure students understand that this type of rearranging with equations doesn't always make sense because sometimes only one variable is a function of the other, and sometimes neither is a function of the other. The *Lesson Synthesis* highlights an example from earlier where swapping the independent and dependent variables does not work. We will continue to explore when these different things happen in future lessons.

Student Workbook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Are You Ready for More?

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Can you describe a simpler way to describe this rule? Why does this work?

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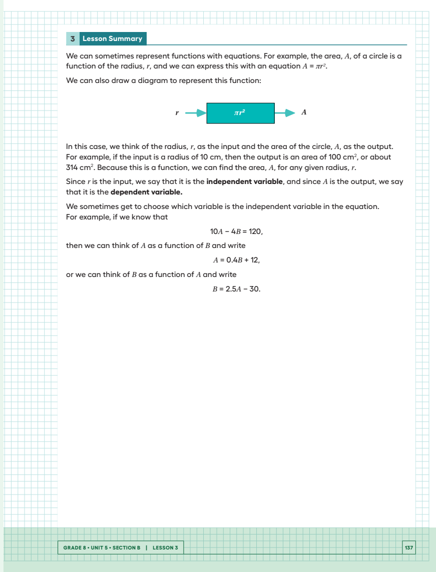
Student Workbook



Lesson Synthesis

The purpose of this discussion is for students to consider a situation where the independent and dependent variables cannot be "swapped."

Tell students that when we can represent a function with an equation and use variables, we name the input as the independent variable and the output as the dependent variable. Sometimes we can choose, depending on the situation, which variable is the independent variable and which is the dependent variable and write the equation accordingly. Sometimes we cannot choose.

Ask students to consider a situation from earlier: squaring a number. Here are some questions for discussion:

- *"Can we think of squaring a number as a function? Why or why not?"*

  Yes, the input would be a number and the output would be the square of that number. Each number has one and only one value when squared.

- *"What could the equation representing this function look like?"*

  $n^2 = S$

- *"For this equation, what is the independent variable? The dependent?"*

  The independent variable is n, and using it, I can calculate the dependent variable of S.

- *"Can we think of this relationship the other way as a function? That is, if we have a number, do we know what number was squared to get it?"*

  No, because knowing the square of a number does not mean we know what number was squared. For example, if $S = 16$, then n could be 4 or -4. Since there are two possible outputs, this is not an example of a function.

Lesson Summary

We can sometimes represent functions with equations. For example, the area, A, of a circle is a function of the radius, r, and we can express this with an equation $A = \pi r^2$.

We can also draw a diagram to represent this function:

In this case, we think of the radius, r, as the input and the area of the circle, A, as the output. For example, if the input is a radius of 10 cm, then the output is an area of 100π cm^2, or about 314 cm^2. Because this is a function, we can find the area, A, for any given radius, r.

Since r is the input, we say that it is the **independent variable**, and since A is the output, we say that it is the **dependent variable**.

We sometimes get to choose which variable is the independent variable in the equation. For example, if we know that

$$10A - 4B = 120,$$

then we can think of A as a function of B and write

$$A = 0.4B + 12,$$

or we can think of B as a function of A and write

$$B = 2.5A - 30.$$

Cool-down

The Value of Some Quarters

5 min

Student Task Statement

The value v of your quarters (in cents) is a function of n, the number of quarters you have.

1. Draw an input-output diagram to represent this function.

   See diagram:

2. Write an equation that represents this function.

   $v = 25n$

   This reflects the statement that the value (in cents) of my collection of quarters is always 25 times the number of quarters I have.

3. Find the output when the input is 10.

   When the input is 10, the output is 250 (since $250 = 25 \cdot 10$).

4. Identify the independent and dependent variables.

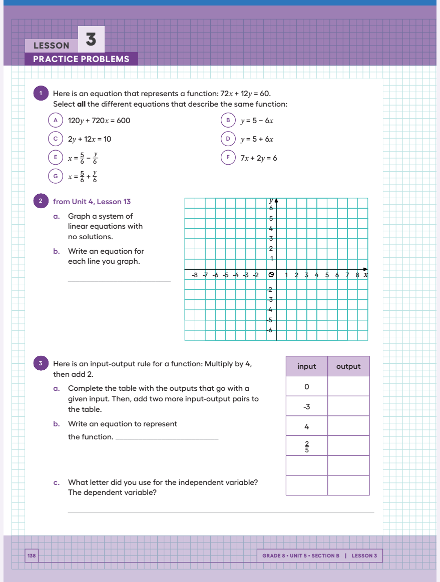
   n is the independent variable, and v is the dependent variable.

Responding To Student Thinking

Points to Emphasize
If most students struggle with writing an equation in the form $y = mx + b$, revisit strategies for doing so as opportunities arise over the next several lessons. For example, at the start of the *Launch* of the activity referred to here, revisit this *Cool-down* and invite students to share how they wrote the equation relating the value of quarters to the number of quarters.

Grade 8, Unit 5, Lesson 4, Activity 1
Equations and Graphs of Functions

Student Workbook



LESSON 3
PRACTICE PROBLEMS

1. Here is an equation that represents a function: $72x + 12y = 60$.
Select **all** the different equations that describe the same function:

- A. $120y + 720x = 600$
- B. $y = 5 - 6x$
- C. $2y + 12x = 10$
- D. $y = 5 + 6x$
- E. $x = \frac{5}{6} - \frac{y}{6}$
- F. $7x + 2y = 6$
- G. $x = \frac{5}{6} + \frac{y}{6}$

2. from Unit 4, Lesson 13
   a. Graph a system of linear equations with no solutions.
   b. Write an equation for each line you graph.

3. Here is an input-output rule for a function: Multiply by 4, then add 2.
   a. Complete the table with the outputs that go with a given input. Then, add two more input-output pairs to the table.
   b. Write an equation to represent the function.
   c. What letter did you use for the independent variable? The dependent variable?

input	output
0	
-3	
4	
$\frac{2}{5}$	

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Lesson 3

Practice Problems

4 Problems

Problem 1

Here is an equation that represents a function: $72x + 12y = 60$.

Select **all** the different equations that describe the same function:

- **A.** $120y + 720x = 600$
- **B.** $y = 5 - 6x$
- **C.** $2y + 12x = 10$
- **D.** $y = 5 + 6x$
- **E.** $x = \frac{5}{6} - \frac{y}{6}$
- **F.** $7x + 2y = 6$
- **G.** $x = \frac{5}{6} + \frac{y}{6}$

Problem 2

from Unit 4, Lesson 13

a. Graph a system of linear equations with no solutions.

b. Write an equation for each line you graph.

Answers vary.

The graph could be any two lines that are parallel. The slope of each equation should be the same.

Problem 3

Here is an input-output rule for a function: Multiply by 4, then add 2.

a. Complete the table with the outputs that go with a given input. Then, add two more input-output pairs to the table.

Sample response:

input	output
0	2
-3	-10
4	18
$\frac{2}{5}$	3.6
Answer varies.	(input · 4) + 2
Answer varies.	

b. Write an equation to represent the function.

Sample response: $y = 4x + 2$

c. What letter did you use for the independent variable? The dependent variable?

Sample response: I used x as the independent variable and y as the dependent variable.

Problem 4

from Unit 4, Lesson 6

Solve each equation, and check your answer.

$2x + 4(3 - 2x) = \frac{3(2x + 2)}{6} + 4$

$x = 1$

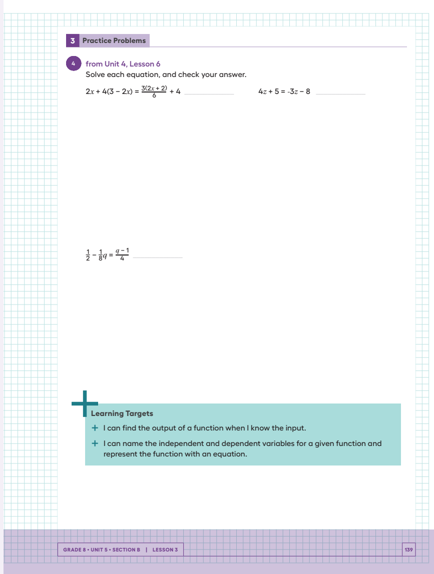
$4z + 5 = -3z - 8$

$z = \frac{-13}{7}$

$\frac{1}{2} - \frac{1}{8}q = \frac{q - 1}{4}$

$q = 2$

Student Workbook


3 Practice Problems

from Unit 4, Lesson 6

Solve each equation, and check your answer.

$2x + 4(3 - 2x) = \frac{3(2x + 2)}{6} + 4$

$4z + 5 = -3z - 8$

$\frac{1}{2} - \frac{1}{8}q = \frac{q - 1}{4}$

Learning Targets

+ I can find the output of a function when I know the input.

+ I can name the independent and dependent variables for a given function and represent the function with an equation.