# Lesson 4

## Tables, Equations, and Graphs of Functions

### Goals

- Determine whether a graph represents a function, and explain (orally) the reasoning.
- Identify the graph of an equation that represents a function, and explain (orally and in writing) the reasoning.
- Interpret (orally and in writing) points on a graph, including a graph of a function and a graph that does not represent a function.

### Learning Targets

- I can identify graphs that do, and do not, represent functions.
- I can use a graph of a function to find the output for a given input and to find the input(s) for a given output.

**Access for Students with Diverse Abilities**

- Representation (Activity 1)

**Access for Multilingual Learners**

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)

**Instructional Routines**

- MLR5: Co-Craft Questions
- Notice and Wonder

### Lesson Narrative

In this lesson, students work with graphs of functions in addition to the tables, equations, and descriptions used previously. They learn the conventions of graphing the independent variable (input) on the horizontal axis and the dependent variable (output) on the vertical axis and that each coordinate point represents an input-output pair of the function.

By matching contexts and graphs and reading information about functions from graphs and tables, students become familiar with the different representations and draw connections between them.

### Student Learning Goal

Let's connect equations and graphs of functions.

### Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Cool-down (Assessment) |
|---|---|---|---|---|
| 5 min | 15 min | 15 min | 10 min | 5 min |

## Warm-up
## Notice and Wonder: Doubling Back

5 min

### Activity Narrative

The purpose of this *Warm-up* is to prepare students to consider which graphs do and do not represent functions, which will be useful when students practice interpreting graphs of functions or make sense of why a specific graph could not represent a function. While students may notice and wonder many things about this graph, the interpretation of the context the graph represents is the important discussion point.

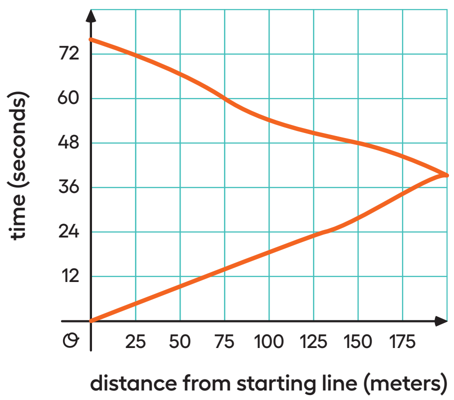
As students notice and wonder, they have the opportunity to reason abstractly and quantitatively if they consider the situation that the graph represents. This *Warm-up* also prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved.

### Launch

Arrange students in groups of 2. Display the graph for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice and wonder.

### Student Task Statement

What do you notice? What do you wonder?




Students may notice:

- The graph looks like a sideways V.
- The graph doubles back on itself.
- The graph is made of pieces that are almost, but not quite, straight.
- Distances go between 0 and 200 meters.
- Times go from 0 to a little more than 72 seconds.
- The horizontal axis tells us the distance from the starting line in meters.
- The vertical axis tells us the time in seconds.
- The person gets farther from the starting line and then comes back.

**Inspire Math**

**Drums video**

**Go Online**
Before the lesson, show this video to introduce the real-world connection.

**ilclass.com/l/614161**

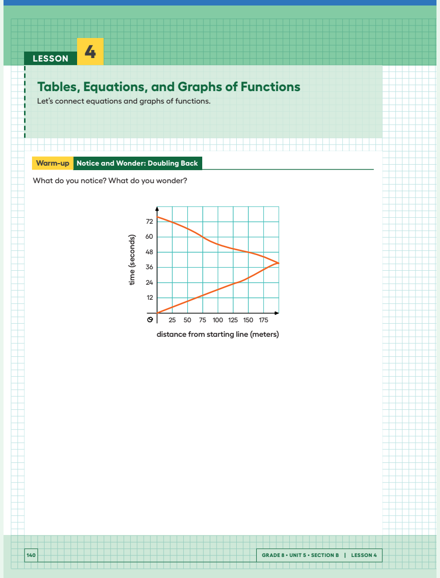
Please log in to the site before using the QR code or URL.

**Instructional Routines**

**Notice and Wonder**

**ilclass.com/r/10694948**
Please log in to the site before using the QR code or URL.

**Student Workbook**



- The person was 75 meters from the starting line at about 14 seconds and 60 seconds.
- The person got back to the starting line in about 75 seconds.
- The furthest distance the person got from the start line was 200 meters.

Students may wonder:

- Is this about one person or more than one person?
- Why does the graph double back?
- Who is the graph about?
- Why are they coming back?
- Are they running or walking?
- Did they take the same exact time to go out as they did to come back?
- What is the title of this graph?

### Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the graph. Next, ask students,

*"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If time allows and the situation the graph represents does not come up during the conversation, ask students to briefly discuss this idea.

## Activity 1

## Equations and Graphs of Functions

15 min

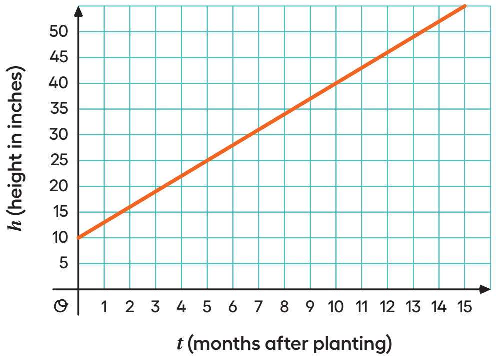
### Activity Narrative

The purpose of this activity is for students to connect different function representations and learn the conventions used to label a graph of a function. Students first match function contexts and equations to graphs. They next label the axes and calculate input-output pairs for each function. The focus of the discussion should be on what quantities students used to label the axes and recognizing the placement of the independent or dependent variables on the axes.

Monitor for students who recognize that there is one graph that is not linear and match that graph with the equation that is not linear.

## Launch

Arrange students in groups of 2. Display the graph for all to see. Ask students to consider what the graph might represent.




After brief quiet think time, select 1–2 students to share their ideas (for example, something starts at 10 inches and grows 15 inches for every 5 months that pass).

Remind students that axes labels help us determine what quantities are represented and they should always be included. Let them know that in this activity, the graphs of three functions have been started, but the labels are missing and part of their task is to figure out what those labels are meant to be.

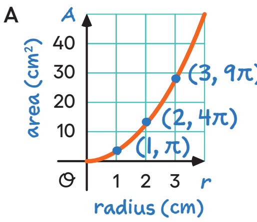
Give students 3–5 minutes of quiet work time and then time to share responses with their partner.

Encourage students to compare their explanations for the last three problems and resolve any differences. Follow with a whole-class discussion.

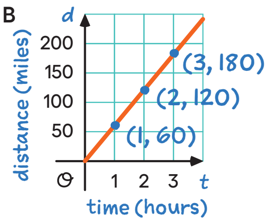
### Student Task Statement

The graphs of three functions are shown.

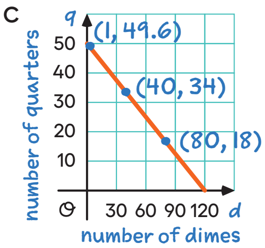
A




B




C




1. Match one of these equations to each of the graphs.

   a. $d = 60t$, where $d$ is the distance in miles that someone would travel in $t$ hours if they drove at 60 miles per hour.

   B. This graph represents a proportional relationship with a constant positive slope.

   b. $q = 50 - 0.4d$, where $q$ is the number of quarters and $d$ is the number of dimes in a pile of coins worth $12.50.

   C. This is the only one of the graphs, and the only equation, with a negative slope and vertical intercept of 50.

   c. $A = \pi r^2$, where $A$ is the area in square centimeters of a circle with radius $r$ centimeters.

   A. This is the only nonlinear relationship.

### Access for Students with Diverse Abilities (Activity 1, Launch)

**Representation: Internalize Comprehension.**
Use color coding and annotations to highlight connections between representations in a problem.
For example, color code inputs and outputs when calculating values, labeling axes, and plotting points.

*Supports accessibility for: Visual-Spatial Processing*

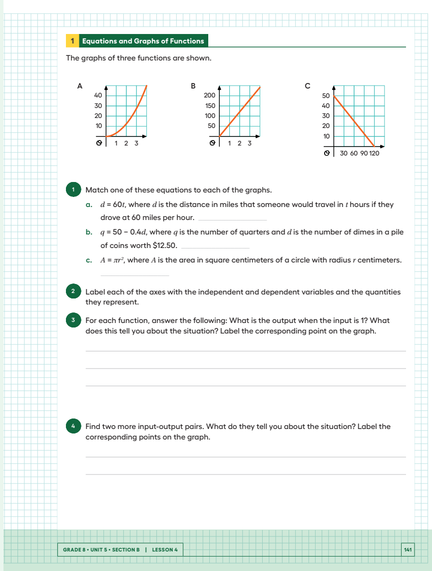
### Access for Multilingual Learners (Activity 1, Student Task)

**MLR1: Stronger and Clearer Each Time.**
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to

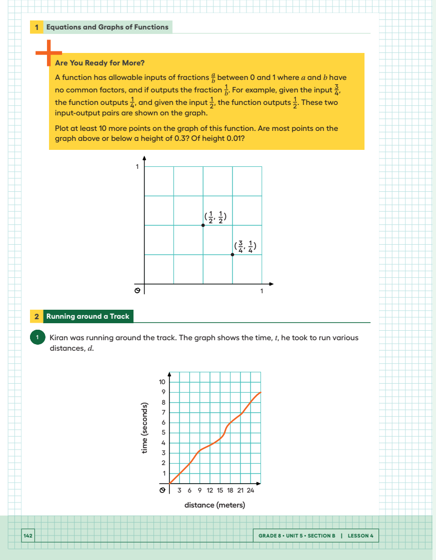
*“Match one of these equations to each of the graphs.”*

Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
*Advances: Writing, Speaking, Listening*

### Student Workbook



**Student Workbook**



1 Equations and Graphs of Functions

**Are You Ready for More?**

A function has allowable inputs of fractions $\frac{a}{b}$ between 0 and 1 where $a$ and $b$ have no common factors, and if outputs the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$, the function outputs $\frac{1}{4}$, and given the input $\frac{1}{2}$, the function outputs $\frac{1}{2}$. These two input-output pairs are shown on the graph.

Plot at least 10 more points on the graph of this function. Are most points on the graph above or below a height of 0.3? Of height 0.01?



2 Running around a Track

1 Kiran was running around the track. The graph shows the time, $t$, he took to run various distances, $d$.



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**2.** Label each of the axes with the independent and dependent variables and the quantities they represent.

See answer to part 4.

**3.** For each function, answer the following: What is the output when the input is 1? What does this tell you about the situation? Label the corresponding point on the graph.

In Figure A, we have point $(1, \pi)$, representing that a circle of radius 1 cm has area $\pi$ cm$^2$.
In Figure B, we have point (1, 60), representing that after traveling for 1 hour at 60 miles per hour, they would travel 60 miles.
In Figure C, we have point (1, 49.6). This does not have a concrete interpretation in terms of the context, as it says that if there were only one dime, there would be 49.6 quarters.

See answer to part 4 for the graphs.

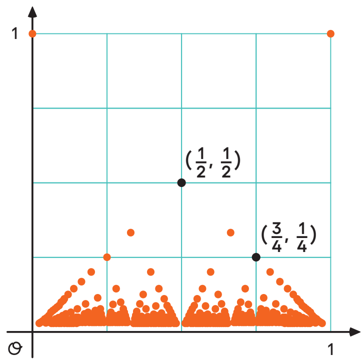
**4.** Find two more input-output pairs. What do they tell you about the situation? Label the corresponding points on the graph.

In Figure A, we mark the points $(2, 4\pi)$ and $(3, 9\pi)$, representing that circles of radius 2 cm and 3 cm have respective areas $4\pi$ cm$^2$ and $9\pi$ cm$^2$.
In Figure B, we mark the points (2, 120) and (3, 180), representing that after traveling for 2 and 3 hours at 60 miles per hour, they would travel 120 and 180 miles, respectively.
In Figure C, we mark the points (40, 34) and (80, 18), representing that if there were 40 and 80 dimes in the pile, there would be 34 and 18 quarters, respectively.

## Are You Ready for More?

A function has allowable inputs of fractions $\frac{a}{b}$ between 0 and 1 where $a$ and $b$ have no common factors, and if outputs the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$, the function outputs $\frac{1}{4}$, and given the input $\frac{1}{2}$, the function outputs $\frac{1}{2}$. These two input-output pairs are shown on the graph.

Plot at least 10 more points on the graph of this function. Are most points on the graph above or below a height of 0.3? Of height 0.01?




This is a very complicated graph! Here is a computer-generated plot of several hundred inputs with denominators $b < 50$. Only very few inputs have height above 0.3. The only ones above are $\frac{0}{1}$, $\frac{1}{1}$, and $\frac{1}{2}$. Every other fraction between 0 and 1 has a denominator of $b \geq 4$, so $\frac{1}{b} \leq \frac{1}{4}$. What's less obvious is that the same is true for height 0.01. Having an output less than 0.01 is the same as having $b > 100$. Since more fractions have $b > 100$ than $b < 100$, there are more points on the graph with height under 0.01 than over.

For more information, do some research on the Thomae function.

### Activity Synthesis

The purpose of this discussion is for students to understand the conventions of constructing a graph of a function and where input and outputs are found on a graph. Select previously identified students to share how they figured out $A = \pi r^2$ matched the nonlinear graph.

Ask students:

- *"Where are the independent variables labeled on the graphs?"*

  the horizontal axis

- *"Where are the dependent variables labeled on the graphs?"*

  the vertical axis

Tell students that by convention, the independent variable is on the horizontal axis, and the dependent variable is on the vertical axis. This means that when we write coordinate pairs, they are in the form of (input, output). For some functions, like the one with quarters and dimes, we can choose which is the independent variable and which is the dependent variable, which means the graph could be constructed either way based on our decision.

Conclude the discussion by asking students to share their explanations for the point (1, 49.6) for Figure C. (There is no such thing as 0.6 of a quarter.) Remind students that sometimes we have to restrict inputs to only those that make sense. Since it's not possible to have 49.6 quarters, an input of 1 dime does not make sense. Similarly, 2, 3, or 4 dimes result in numbers of quarters that do not make sense. 0 dimes or 5 dimes, however, do produce outputs that make sense. Sometimes it is easier to sketch a graph of the line even when graphing discrete points would be more accurate for the context. Keeping the context of a function in mind is important when making sense of the input-output pairs associated with the function.

## Activity 2
## Running around a Track

15 min

### Activity Narrative

The purpose of this activity is for students to interpret coordinates on graphs of functions and nonfunctions as well as understand that context does not dictate the independent and dependent variables.

In the first problem, time is a function of distance since each input of meters ran has one and only one output of seconds past. The graph and table help determine how long it takes for Kiran to run a specific distance. In the second problem, time is not a function of Priya's distance from the starting line since this graph includes her distance from the starting line as she returns to the starting line. This results in a graph in which each input does not give exactly one output.

#### Instructional Routines

**MLR5: Co-Craft Questions**

**ilclass.com/r/10695544**
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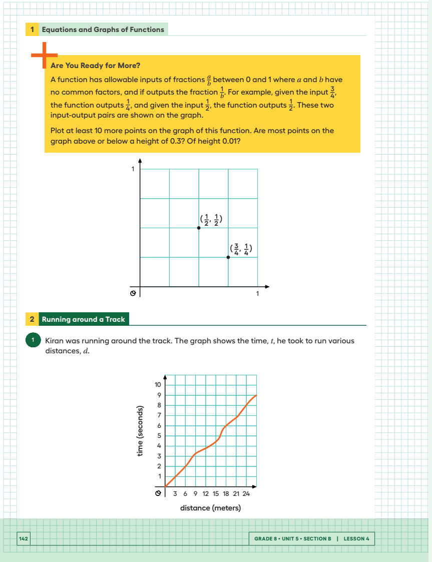
#### Access for Multilingual Learners (Activity 2)

**MLR5: Co-Craft Questions**
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

### Student Workbook


1 Equations and Graphs of Functions

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A function has allowable inputs of fractions $\frac{a}{b}$ between 0 and 1 where $a$ and $b$ have no common factors, and if outputs the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$, the function outputs $\frac{1}{4}$, and given the input $\frac{1}{2}$, the function outputs $\frac{1}{2}$. These two input-output pairs are shown on the graph.

Plot at least 10 more points on the graph of this function. Are most points on the graph above or below a height of 0.3? Of height 0.01?

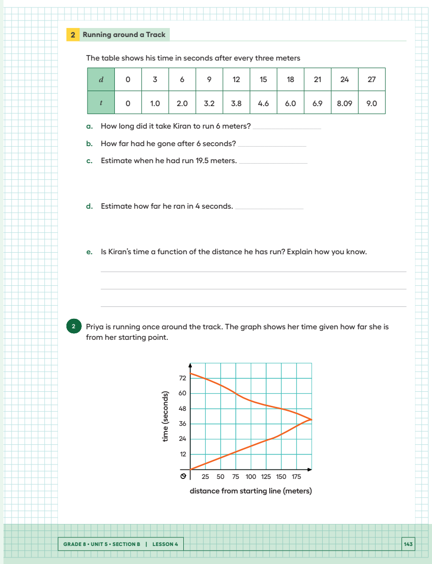


2 Running around a Track

1. Kiran was running around the track. The graph shows the time, $t$, he took to run various distances, $d$.



### Student Workbook


2 Running around a Track

The table shows his time in seconds after every three meters

| $d$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0 | 1.0 | 2.0 | 3.2 | 3.8 | 4.6 | 6.0 | 6.9 | 8.09 | 9.0 |

a. How long did it take Kiran to run 6 meters?

b. How far had he gone after 6 seconds?

c. Estimate when he had run 19.5 meters.

d. Estimate how far he ran in 4 seconds.

e. Is Kiran's time a function of the distance he has run? Explain how you know.

2. Priya is running once around the track. The graph shows her time given how far she is from her starting point.



## Launch 

Arrange students in groups of 2. Introduce the context of running around a track. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

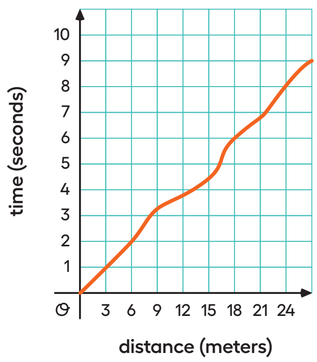
Display only the first problem stem and related image, without revealing the questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

- Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, "What do these questions have in common? How are they different?" Listen for and amplify language related to the learning goal, such as "increasing," "the input is distance," "the output is time," and "the time is increasing from 0 to 27 meters."
- Reveal the set of questions for the first problem, and give students 1–2 minutes to compare it to their own question and to those of their classmates. Invite students to identify similarities and differences with their partner before beginning the activity.

## Student Task Statement

1. Kiran was running around the track. The graph shows the time, $t$, he took to run various distances, $d$.




The table shows his time in seconds after every three meters.

| $d$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0 | 1.0 | 2.0 | 3.2 | 3.8 | 4.6 | 6.0 | 6.9 | 8.09 | 9.0 |

**a.** How long did it take Kiran to run 6 meters?

2 seconds, because in the table, when $d = 6$, we have $t = 2$

**b.** How far had he gone after 6 seconds?

18 meters, because in the table, when $t = 6$, we have $d = 18$

**c.** Estimate when he had run 19.5 meters.

Answers vary, but 6.45 seconds is a reasonable estimate.

It took Kiran 6 seconds to run 18 meters and 6.9 seconds to run 21 meters. Since 19.5 is halfway between 18 and 21, it is reasonable to estimate halfway between 6 seconds and 6.9 seconds. This estimate is further supported by the graph.

**d.** Estimate how far he ran in 4 seconds.

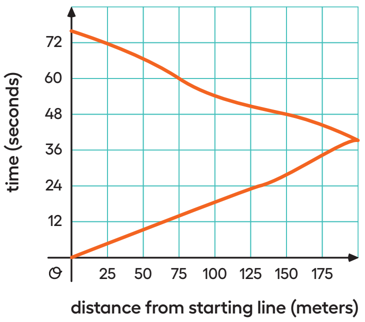
Answers vary, but 12.75 meters is a reasonable estimate.

He runs 12 meters in 3.8 seconds and 15 meters in 4.6 seconds. Since 4 is a quarter of the way from 3.8 to 4.6, a reasonable estimate for distance would be a quarter of the way from 12 to 15, which is 12.75. This estimate is further supported by the graph.

**e.** Is Kiran's time a function of the distance he has run? Explain how you know.

Sample reasoning: Kiran's time is a function of the distance he has traveled. By reading the graph, we can use the distance he has traveled to find the time it took him to travel it.

**2.** Priya is running once around the track. The graph shows her time given how far she is from her starting point.




**a.** What was her farthest distance from the starting line?

200 meters, reflected by the rightmost point on the provided graph

**b.** Estimate how long it took her to run around the track.

75 seconds. From the top-left point on the graph, we can see that the first time Priya returns to the starting line after her initial departure occurs at about 75 seconds.

**c.** Estimate when she was 100 meters from the starting line.

18 seconds and 54 seconds. There are two points on the graph representing a distance of 100 meters from the starting line—at heights 18 and 54.

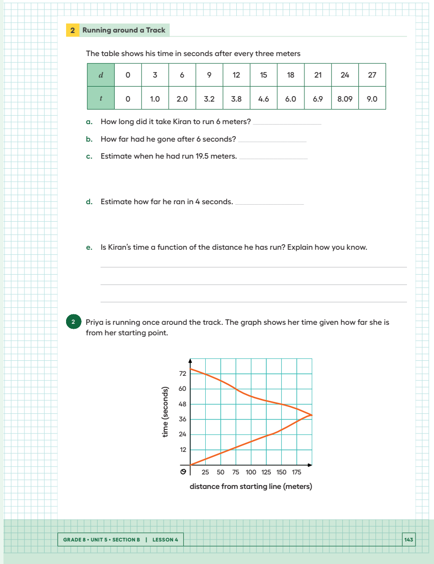
**d.** Estimate how far she was from the starting line after 60 seconds.

75 meters. There is only one point on the graph corresponding to a time of 60 seconds, and it occurs at a distance of 75 meters from the starting line.

**e.** Is Priya's time a function of her distance from her starting point? Explain how you know.

No. Sample reasoning: The time is not determined by the distance from the starting line, as the example of 100 meters above shows. There are two different times corresponding to the distance of 100 meters.

**Student Workbook**



2 Running around a Track

The table shows his time in seconds after every three meters

| $d$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0 | 1.0 | 2.0 | 3.2 | 3.8 | 4.6 | 6.0 | 6.9 | 8.09 | 9.0 |

a. How long did it take Kiran to run 6 meters?

b. How far had he gone after 6 seconds?

c. Estimate when he had run 19.5 meters.

d. Estimate how far he ran in 4 seconds.

e. Is Kiran's time a function of the distance he has run? Explain how you know.

2 Priya is running once around the track. The graph shows her time given how far she is from her starting point.

time (seconds) / distance from starting line (meters)

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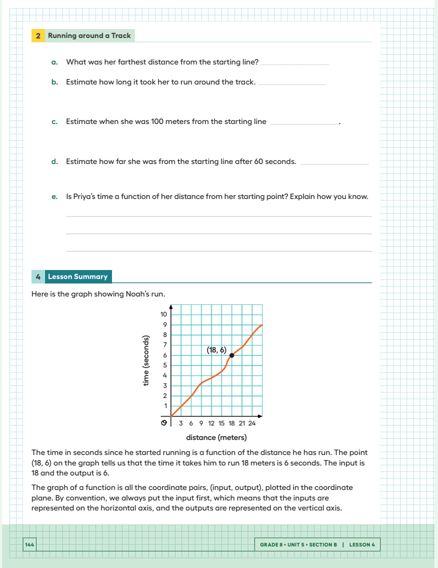
**Student Workbook**



2 Running around a Track

a. What was her farthest distance from the starting line?

b. Estimate how long it took her to run around the track.

c. Estimate when she was 100 meters from the starting line.

d. Estimate how far she was from the starting line after 60 seconds.

e. Is Priya's time a function of her distance from her starting point? Explain how you know.

4 Lesson Summary

Here is the graph showing Noah's run.

time (seconds) / distance (meters) / (18, 6)

The time in seconds since he started running is a function of the distance he has run. The point (18, 6) on the graph tells us that the time it takes him to run 18 meters is 6 seconds. The input is 18 and the output is 6.

The graph of a function is all the coordinate pairs, (input, output), plotted in the coordinate plane. By convention, we always put the input first, which means that the inputs are represented on the horizontal axis, and the outputs are represented on the vertical axis.

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### Activity Synthesis

The purpose of this discussion is for students to understand that independent and dependent variables are not determined by the context (and specifically that time is not always a function of distance). Select students to share their strategies for calculating the answers for the first set of problems. For each problem, ask students whether the graph or table was more useful. Further the discussion by asking:

- *"When are tables useful for answering questions?"*

  We can get exact values from the numbers in the table, while with the graph, we can only approximate.

- *"When are graphs more useful for answering questions?"*

  A line graph shows more input-output pairs than a table can list easily.

- *"Why does it make sense to have time be a function of distance in this problem?"*

  The farther Kiran runs, the longer it will take, so it makes sense to represent time as a function of distance.

- *"Does time always have to be a function of distance?"*

  No, this graph could be made the other way with time on the horizontal axis and distance on the vertical axis, and then it would show distance as a function of time, which makes sense since the longer Kiran runs, the further he will travel.

For the second graph, ask students to indicate if they think it represents a function or not. If there are students who say yes and no, invite students from each side to share their reasoning and try to persuade the rest of the class to their side. If all students are not persuaded that the graph is not a function, remind them that functions can only have one output for each input, yet the answer to the problem statement

*"Estimate when she was 100 meters from her starting point."*

has two possible responses, 18 seconds or 54 seconds. Since that question has two responses, the graph cannot represent function.

### Lesson Synthesis

Conclude the lesson by prompting students to think about how different representations of functions present the input and output of the function in different ways. Tell students to imagine we have a function with independent variable $x$ and dependent variable $y$. Here are some questions for discussion:

- *"How do we find input-output pairs from a graph of the function?"*

  Any coordinate on the graph gives an input-output pair where the input is the $x$-value and the output is the $y$-value.

- *"What is something you won't see on the graph of the function?"*

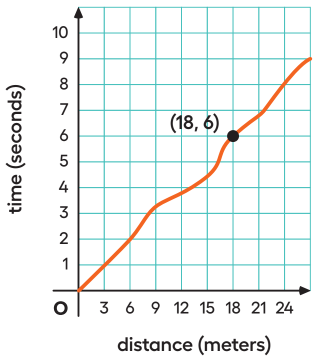
  The graph will never "double-back," or have two $y$-values for the same $x$-value, because each input will have only one output.

- *If the graph of the function contains the point (18, 6), what else do we know about the function?"*

  If we input 18 into the function, we will get 6 as an output. An equation for the function could be $y = \frac{1}{3}x$, but we would need to know more points on the graph to be sure.

## Lesson Summary

Here is the graph showing Noah's run.




The time in seconds since he started running is a function of the distance he has run. The point (18, 6) on the graph tells us that the time it takes him to run 18 meters is 6 seconds. The input is 18 and the output is 6.

The graph of a function is all the coordinate pairs, (input, output), plotted in the coordinate plane. By convention, we always put the input first, which means that the inputs are represented on the horizontal axis, and the outputs are represented on the vertical axis.

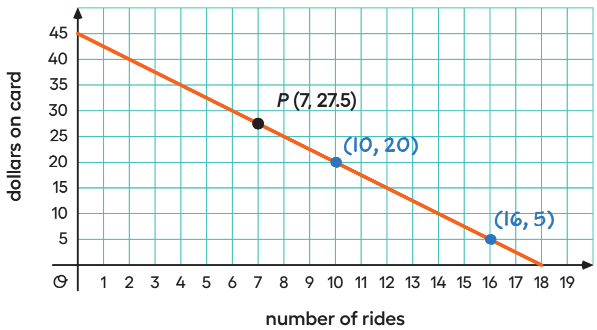
## Cool-down

## Subway Fare Card

5 min

### Student Task Statement

Here is the graph of a function showing the amount of money remaining on a subway fare card as a function of the number of rides taken.




1. What is the output of the function when the input is 10? On the graph, plot this point and label its coordinates. 20

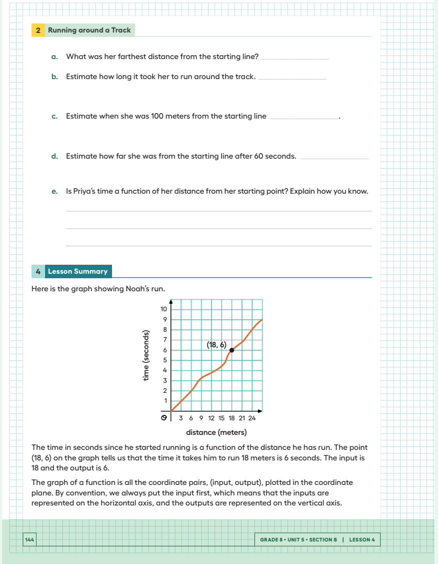
   See graph

2. What is the input to the function when the output is 5? On the graph, plot this point and label its coordinates. 16

3. What does point $P$ tell you about the situation?

   After taking 7 rides, there will be $27.50 remaining on the card.

## Student Workbook



**2 Running around a Track**

a. What was her farthest distance from the starting line? ______

b. Estimate how long it took her to run around the track. ______

c. Estimate when she was 100 meters from the starting line ______.

d. Estimate how far she was from the starting line after 60 seconds. ______

e. Is Priya's time a function of her distance from her starting point? Explain how you know.

**4 Lesson Summary**

Here is the graph showing Noah's run.

The time in seconds since he started running is a function of the distance he has run. The point (18, 6) on the graph tells us that the time it takes him to run 18 meters is 6 seconds. The input is 18 and the output is 6.

The graph of a function is all the coordinate pairs, (input, output), plotted in the coordinate plane. By convention, we always put the input first, which means that the inputs are represented on the horizontal axis, and the outputs are represented on the vertical axis.

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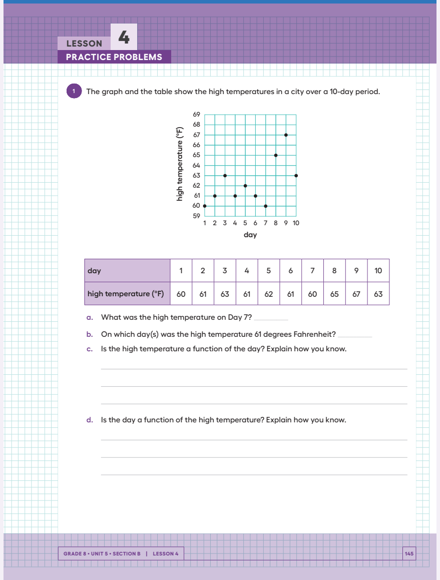
## Responding To Student Thinking

**More Chances**
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

**Student Workbook**


LESSON 4
PRACTICE PROBLEMS

1 The graph and the table show the high temperatures in a city over a 10-day period.

| day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| high temperature (°F) | 60 | 61 | 63 | 61 | 62 | 61 | 60 | 65 | 67 | 63 |

a. What was the high temperature on Day 7?
b. On which day(s) was the high temperature 61 degrees Fahrenheit?
c. Is the high temperature a function of the day? Explain how you know.
d. Is the day a function of the high temperature? Explain how you know.

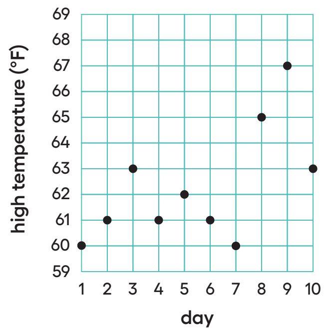
# Lesson 4

## Practice Problems

4 Problems

### Problem 1

The graph and the table show the high temperatures in a city over a 10-day period.




| day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| high temperature (°F) | 60 | 61 | 63 | 61 | 62 | 61 | 60 | 65 | 67 | 63 |

**a.** What was the high temperature on Day 7?

60 °F

**b.** On which day(s) was the high temperature 61 degrees Fahrenheit?

Days 2, 4, and 6

**c.** Is the high temperature a function of the day? Explain how you know.

Yes

Sample reasoning: There are no different outputs for the same input. That is, there is no day with two different high temperatures.

**d.** Is the day a function of the high temperature? Explain how you know.

No

Sample reasoning: Day is not a function of temperature as there are multiple days that have the same high temperature. There are different outputs for the same input.

## Problem 2

The amount Lin's sister earns at her part-time job is proportional to the number of hours she works. She earns $9.60 per hour.

**a.** Write an equation in the form $y = kx$ to describe this situation, where $x$ represents the number of hours worked and $y$ represents the amount earned in dollars.

$y = 9.6x$

**b.** Is $y$ a function of $x$? Explain how you know.

Yes

Sample reasoning: There is only one output for each input.

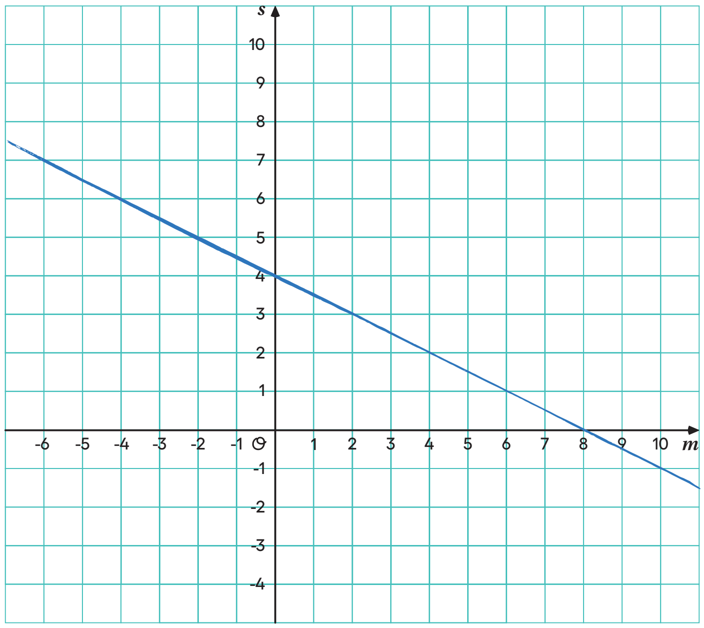
**c.** Write an equation describing $x$ as a function of $y$.

$x = \frac{1}{9.6}y$

## Problem 3

Use the equation $2m + 4s = 16$ to complete the table, then graph the line using $s$ as the dependent variable.

| $m$ | 0 | 2 | -2 | 8 |
|---|---|---|---|---|
| $s$ | 4 | 3 | 5 | 0 |



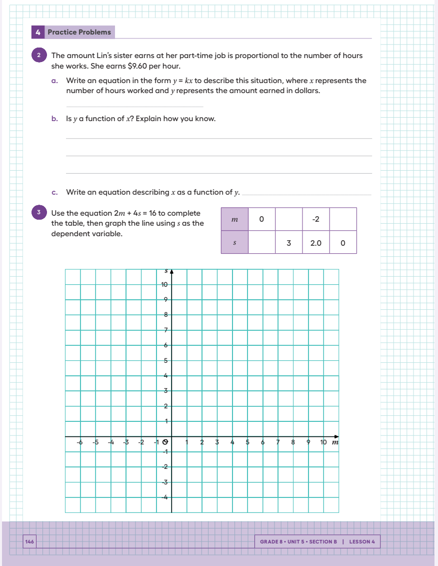

## Problem 4

from Unit 4, Lesson 13

Solve the system of equations: $\begin{cases} y = 7x + 10 \\ y = -4x - 23 \end{cases}$

(-3, -11)

**Student Workbook**



**Student Workbook**