Parallel Lines and the Angles in a Triangle

Goals

- Create diagrams using 180-degree rotations of triangles to justify (orally and in writing) that the measure of angles in a triangle sum up to 180 degrees.
- Generalize that the sum of angles in a triangle is 180 degrees using rigid transformations or the congruence of alternate interior angles of parallel lines cut by a transversal.

Learning Target

I can explain using pictures why the sum of the angles in any triangle is 180 degrees.

Student Learning Goal

Let's see why the angles in a triangle add to 180 degrees.

Lesson Narrative

In this lesson, students apply properties of rigid transformations to show that the sum of the interior angles of any triangle is 180 degrees.

Students first explore the sum of the angles in a triangle on a grid line using rigid transformations that connect to their earlier work with parallel lines and alternate interior angles of a triangle. The argument that the angles sum to 180° depends on the gridlines to show the corresponding angles form a straight angle.

Then, students use a similar argument to show that the angles sum to 180 degrees for any triangle. Students use the structure of parallel lines, corresponding parts of congruent triangles, and straight angles to generalize for any triangle.

In the optional activity, students use the same structure of straight angles, parallel lines, and corresponding parts of congruent triangles to find the angle measures in a composite figure.

Lesson Timeline

5 min	15 min	15 min	5 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Assessment

Warm-up
All the Angles

5 min

Activity Narrative

The purpose of this activity is for students to make sense of a diagram they will use in a later activity to justify that the sum of the interior angles of a triangle is 180 degrees.

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of an image and practice generating mathematical questions.
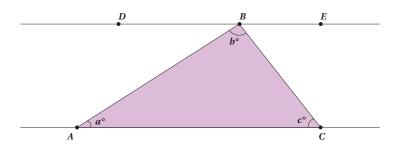
Launch 👥

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Display the image of a triangle with two parallel lines for all to see. Use *Co-Craft Questions* to orient students to the image and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Here is triangle ABC. Line DE is parallel to line AC.

Sample responses:

○ What is the measure of angle DBA?

○ What is the measure of angle EBC?

○ What rigid transformation takes angle DBA to angle BAC?

○ What is the sum of $a + b + c$?

Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

💬 *"What do these questions have in common? How are they different?"*

Listen for and amplify language related to the learning goal, such as using transformations to show the angle sum of triangles.

Inspire Math

Video Games video

Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614231

Please log in to the site before using the QR code or URL.
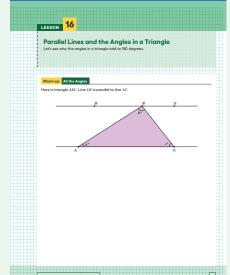

Instructional Routines

MLR5: Co-Craft Questions

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Please log in to the site before using the QR code or URL.

Student Workbook

Building on Student Thinking
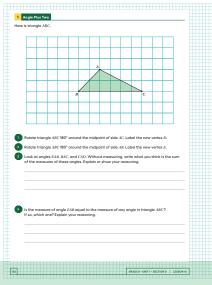
Some students may have trouble with the rotations. If they struggle, remind them of similar work they did in a previous lesson. Help them with the first rotation, and allow them to do the second rotation on their own.

Student Workbook

Activity 1

Angle Plus Two

15 min

Activity Narrative

The purpose of this activity is for students to use 180-degree rotations to create a diagram of 3 congruent triangles. As students create the diagram, they prepare informal arguments for congruent angles in the figure. Using these informal arguments, they find the sum of the interior angle measures of the original triangle.

A similar diagram will be used to generalize the sum of interior angles of a triangle in a later activity.
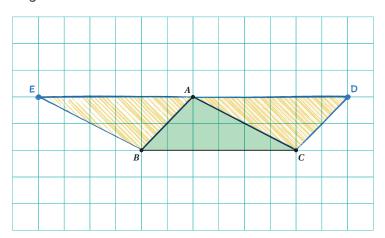
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Arrange students in groups of 2–3. Provide access to geometry toolkits.

Give 5 minutes individual work time, then 2–3 minutes to share their reasoning with their groups. Follow with a whole-class discussion.

Student Task Statement

Here is triangle ABC.

1. Rotate triangle ABC 180° around the midpoint of side AC. Label the new vertex D.

2. Rotate triangle ABC 180° around the midpoint of side AB. Label the new vertex E.

   See image

3. Look at angles EAB, BAC, and CAD. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.

   Sample response: They look like they will add to 180° because they appear to form a straight angle and there are 180° in a straight angle.

4. Is the measure of angle EAB equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.

   Yes, angle ABC

   Sample reasoning: When triangle ABC is rotated 180 degrees with center the midpoint of segment AB, $\angle ABC$ goes to $\angle EAB$.

5. Is the measure of angle CAD equal to the measure of any angle in triangle ABC? If so, which one? Explain your reasoning.

Yes, angle ACB

Sample reasoning: When triangle ABC is rotated 180 degrees with center the midpoint of segment AC, $\angle ACB$ goes to $\angle CAD$.

6. What is the sum of the measures of angles ABC, BAC, and ACB? Explain your reasoning.

Sample response: The total measure of these angles should be 180° because it is the same as the total measure of angles EAB, BAC, and CAD and these angles add up to 180°.

Activity Synthesis

The goal of this discussion is for students to explain how rigid transformations connect with the sum of interior angles of a triangle.
Here are some questions for discussion:

💬 *"How can you tell that line DE is parallel to line BC?"*

They were made using a 180-degree rotation, they are both on grid lines.

💬 *"How can you tell that the 3 angles around point A make a straight angle?"*

They all line up along a grid line, so they form a straight line.

💬 *If there were no grid lines, could we know that the three angles around point A would make a straight angle?*

Since we used a 180-degree rotation of the same line segment but around 2 different points, EA and DA have to be parallel to each other. Since they share point A, they have to be on the same line.

💬 *How does this tell us about the sum of the angles for triangle ABC?*

A straight angle is 180 degrees, and the three angles around point A that make a straight angle are congruent to the three angles in triangle ABC, so they must also total to 180 degrees.

Students will have more time to make the connection about parallel lines and a similar figure without grid lines, so it is okay if they cannot explain why they create a straight angle without using grid lines at this time.
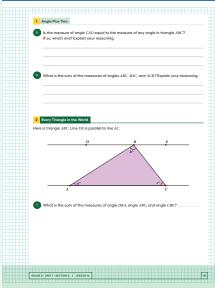

Student Workbook

Building on Student Thinking

Some students may say that a, b, and c are the three angles in a triangle, so they add up to 180. Make sure that these students understand that the goal of this activity is to explain why this must be true. Encourage them to use their answer to the first question and think about what they know about different angles in the diagram.

For the last question students may not understand why their work in the previous question only shows $a + b + c = 180$ for one particular triangle. Consider drawing a different triangle (without the parallel line to one of the bases), labeling the three angle measures a, b, c, and asking the student why $a + b + c = 180$ for *this* triangle.

Activity 2

Every Triangle in the World

15 min

Activity Narrative

In this activity, students will continue to explore the interior angles of a triangle. The purpose of this activity is to provide a complete argument, not depending on the grid, of why the sum of the three angles in a triangle is 180°.

Students may use the structure of parallel lines cut with a transversal to identify alternate interior angles on the figure or use a 180-degree rotation about the midpoint of triangle side lengths to identify corresponding parts of the figure. In either case, this argument works for any triangle in general, since no angle measures are given or needed.
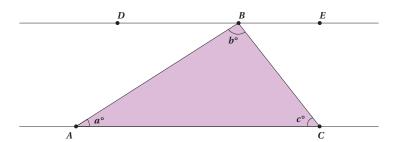
Launch 👥👤

Arrange students in groups of 2–3. Tell students they'll be working on this activity without the geometry toolkit.

Tell students that this is the same image from the *Warm-up*. Display the question "What is the sum of the measures of angle DBA, angle ABC, and angle CBE?" Ask students how this question is similar and different to the questions they came up with.

After students have shared the similarities and differences, give 1–2 minutes for groups to come up with a sum, then share with the whole class. Give another 3–5 minutes for groups to finish the rest of the questions.

Student Task Statement

Here is triangle ABC. Line DE is parallel to line AC.

1. What is the sum of the measures of angle DBA, angle ABC, and angle CBE?
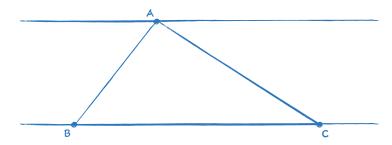
   180°

2. Use your answer to explain why $a + b + c = 180$.

Sample response: Angles *DBA* and *CAB* are congruent because these are alternate interior angles for the parallel lines *AC* and *DE* with transversal *AB*. Angles *EBC* and *BCA* are congruent because these are alternate interior angles for the parallel lines *AC* and *DE* with transversal *BC*. Angles *DBA*, *ABC*, and *CBE* make a line, and so their angle measures add up to 180°. Then $a + b + c = 180$.

3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is 180°.

Sample response:

For any triangle, draw a line parallel to one side, containing the opposite vertex.

With this picture, use the same argument to show that the sum of the three angles of the triangle is 180°. This works for every triangle.
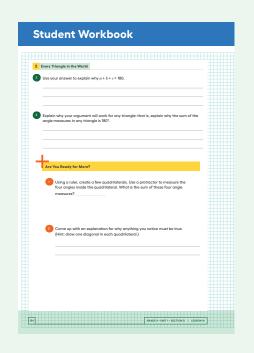
Are You Ready for More?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?

The sum of the interior angle measures in any quadrilateral is 360°. Note that since protractors are imprecise, physical measurements may range by a few degrees away from this.

2. Come up with an explanation for why anything you notice must be true. (Hint: draw one diagonal in each quadrilateral.)
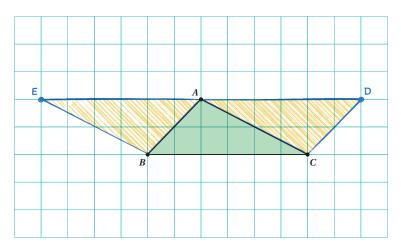
In any quadrilateral, a diagonal that partitions the quadrilateral into 2 triangles can be drawn. The sum of the angle measures in each triangle is 180°, and the 6 angles in the two triangles comprise all of the angles in the quadrilateral.
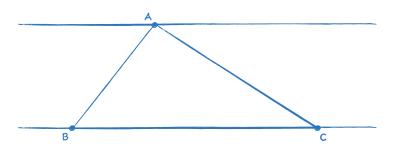
Activity Synthesis

The goal of this activity is for students to explain that the sum of the interior angles of any triangle must be 180 degrees.

Display both of these images for all to see:

Ask students how they are similar and how they are different.

Similarities students may notice:

- Triangle ABC is in both images.
- Triangle ABC is between 2 parallel lines.
- The 3 angles around point A are a straight angle.
- The sum of the angles inside triangle ABC is 180 degrees.

Differences students may notice:

- There are grid lines on the first image but not the second.
- We can measure the height of the triangles in the first image.
- The first image has 3 triangles but the second only has 1 triangle.
- The first image has parallel segments but the second has parallel lines.

Tell students that the first image helps us show that the sum of the angles in that particular triangle is 180 degrees, but we can create a diagram like the second one for any triangle. That means we can know that the sum of the angles in any triangle is 180 degrees.

Activity 3: Optional
Four Triangles Revisited

5 min

Activity Narrative

This activity is optional because it provides additional opportunity for students to identify corresponding parts of figures using rigid transformations. This image will also be helpful in a later unit as students investigate the Pythagorean theorem.

In this activity, students use the structure of rigid transformations to find angle measures. Some strategies that students may use include:
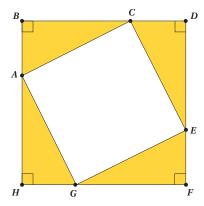
- Rigid transformations take one triangle to a congruent triangle.
- Congruent triangles have congruent corresponding angles.
- The measures of the angles that make up a straight line add up to 180 degrees.
- Parallel lines cut by a transversal have congruent alternate interior angles.

Launch

Provide 3 minutes of individual work time followed by a whole-class discussion.

Student Task Statement

This diagram shows a square $BDFH$ that has been made by images of triangle ABC under rigid transformations.

Given that angle BAC measures 53°, find as many other angle measures as you can.

All other angles can be determined from the one given measure.

Angles **ECD, GEF,** and **AGH** all measure 53° because the measure of ∠*BAC* is 53°. These three angles correspond to ∠*BAC* under a rigid motion of triangle *ABC*. To find angles **ACB, CED, EGF,** and **GAH,** notice that these are all congruent because they are all correspond to ∠*ACB* of triangle *ABC* under a rigid motion. Angle **ACB** measures 37° because the angles in a triangle add to 180°. One angle in triangle *ABC* measures 53°, and another measures 90°, so the third angle measures 180° − 53° − 90° = 37°.

Angles **ACE, CEG, EGA,** and **GAC** all measure 90°. Here is an argument for ∠*ACE*: We know that angle *ACB* measures 37° and angle *ECD* measures 53°. So angle **ACE** must measure 90° because it makes a line together with ∠*ACB* and ∠*ECD*.
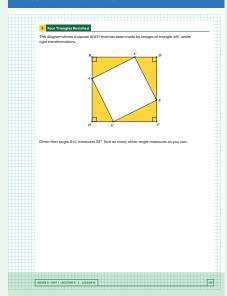
Student Workbook

Student Workbook

Activity Synthesis

Display the image of the 4 triangles for all to see. Invite students to share how they calculated one of the other unknown angles in the image, adding to the image until all the unknown angles are filled in.

If no student points it out, highlight that angles ACE, CEG, EGA, and GAC are all right angles.
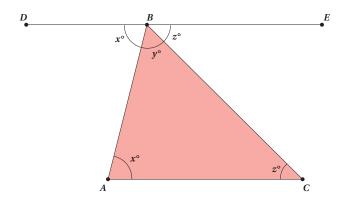
Lesson Synthesis

Display a triangle for all to see. Invite students to explain step by step the process to explain that the sum of the angles in the triangle is 180 degrees. Steps students may describe:

- Add a line through one vertex parallel to the opposite side of the triangle.

- The three angles on the new line form a straight angle, so they sum to 180 degrees.

- Since the lines are parallel, we can use alternate interior angles to show that one angle on the line is congruent to one angle in the triangle, and another angle on the line is congruent to the other angle in the triangle.

- Since the three angles in the triangle are congruent to the three angles that make a straight angle, the angles in the triangle must add up to 180 degrees.

Consider adding the illustration to the display for the unit with the statement: "The sum of the angles in a triangle is 180 degrees."

Lesson Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to 180°. Here is triangle ABC. Line DE is parallel to AC and contains B.
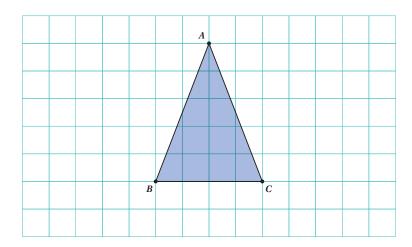

A 180° rotation of triangle ABC around the midpoint of AB interchanges angles A and DBA so they have the same measure (in the picture these angles are marked as $x°$). A 180° rotation of triangle ABC around the midpoint of BC interchanges angles C and CBE so they have the same measure (in the picture, these angles are marked as $z°$). Also, DBE is a straight line because 180° rotations take lines to parallel lines. So the three angles with vertex B make a line and they add up to 180° ($x + y + z = 180$). But x, y, z are the measures of the three angles in triangle ABC so the sum of the angles in a triangle is always 180°!

Cool-down

Angle Sum

5 min

Student Task Statement

What is the sum of the angle measures of triangle ABC? How do you know?

180°

Sample response: Since the base and vertex lie on grid lines, we can see the line parallel to BC through A is a straight line. The three angles around point A add up to a straight angle. Using alternate interior angles, two angles are congruent to angle B and angle C, and the third angle is the same as angle A. So angles A, B, and C add up to 180°.
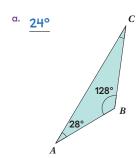
Practice Problems   5 Problems

Problem 1
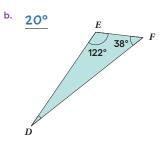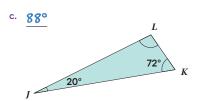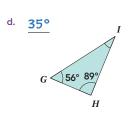
For each triangle, find the measure of the missing angle.

a. **24°**

b. **20°**

c. **88°**

d. **35°**

Problem 2

Is there a triangle with *two* right angles? Explain your reasoning.
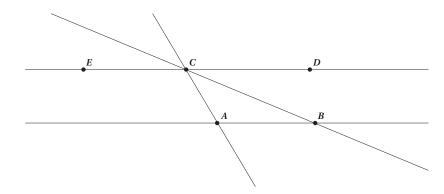
no

Sample reasoning: The three angles in a triangle add up to 180°. Two right angles would already make 180°, and so the third angle of the triangle would have to be 0°, which is not possible.

Problem 3

In this diagram, lines AB and CD are parallel. Angle ABC measures 35° and angle BAC measures 115°.

a. What is $m\angle ACE$?

115°

b. What is $m\angle DCB$?
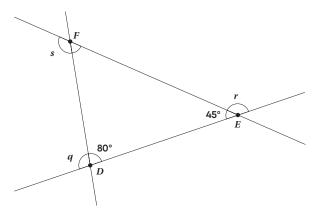
35°

c. What is $m\angle ACB$?

30°

Problem 4

Here is a diagram of triangle DEF.

a. Find the measures of angles q, r, and s.

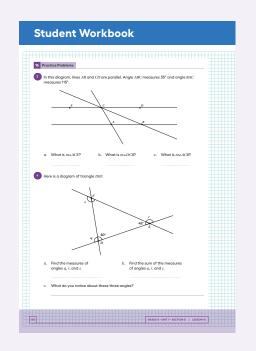$q = 100, r = 135, s = 125$

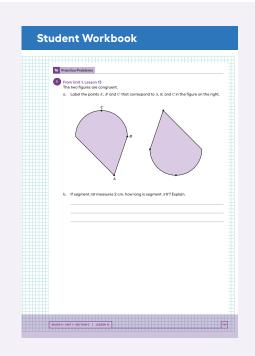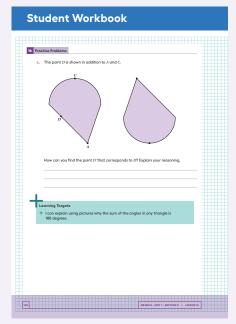**b.** Find the sum of the measures of angles q, r, and s.

$q + r + s = 360$

c. What do you notice about these three angles?

Sample response: The three angles together make one full revolution of a circle, or 360 degrees.
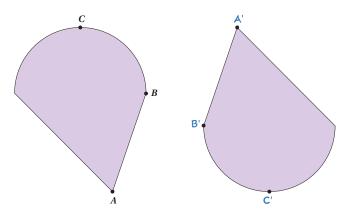
Student Workbook

Student Workbook

The two figures are congruent.

a. Label the points A', B' and C' that correspond to A, B, and C in the figure on the right.
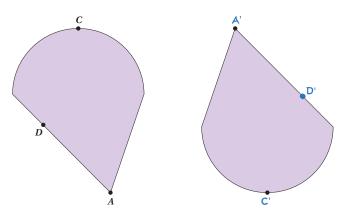

b. If segment AB measures 2 cm, how long is segment $A'B'$? Explain.

2 cm

Sample reasoning: The shapes are congruent, and the corresponding segments of congruent shapes are congruent.

c. The point D is shown in addition to A and C. How can you find the point D' that corresponds to D? Explain your reasoning.

Sample reasoning: As the figures are congruent, the point D' will be on the corresponding side and will be the same distance from C' that D is from C. D can be found by looking for the point on the segment going down and to the right from A' that is the appropriate distance from C'.