Lesson 14

Alternate Interior Angles

Goals

- Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.
- Justify (orally and in writing) that alternate interior angles made by a transversal connecting two parallel lines are congruent using properties of rigid motions.

Learning Target

If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements.

Student Learning Goal

Let's explore why some angles are always equal.

Lesson Narrative

In this lesson, students justify that **alternate interior angles** are congruent, and use this and the vertical angle theorem, previously justified, to solve problems.

Thus far in this unit, students have studied different types of rigid motions, using them to examine and build different figures. This work continues here, with an emphasis on examining angles. In a previous lesson, 180-degree rotations were employed to show that vertical angles, made by intersecting lines, are congruent. The *Warm-up* recalls previous facts about angles made by intersecting lines, including both vertical and adjacent angles. Next a third line is added, parallel to one of the two intersecting lines. There are now 8 angles, 4 each at the two intersection points of the lines. At each vertex, vertical and adjacent angles can be used to calculate all angle measures once one angle is known. But how do the angle measures compare at the two vertices? It turns out that each angle at one vertex is congruent to the corresponding angle (via translation) at the other vertex and this can be seen using rigid motions. Translations and 180-degree rotations are particularly effective at revealing the relationships between the angle measures.

Students will notice as they calculate angles that they are repeatedly using vertical and adjacent angles and that often they have a choice which method to apply. They will also notice that the angles made by parallel lines cut by a **transversal** are the same and this observation is the key structure in this lesson

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2

Lesson Timeline

5 min	15 min	10 min	10 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Angle Pairs

5 min

Activity Narrative

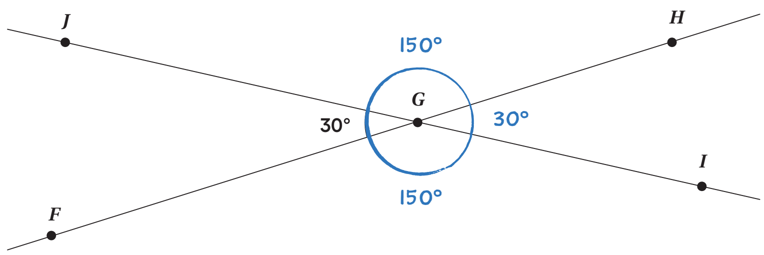
The purpose of this activity is for students to recall prior work with supplementary angles and to connect vertical angles and 180-degree rotations of intersecting lines. As students find the angle measures, listen to their conversations, specifically for the use of vocabulary such as "supplementary angles," "vertical angles," and "rotations."

Launch

Provide access to geometry toolkits, including protractors and tracing paper. If needed, display the image from the problem and invite a student to state the name of the 30° angle (JGF). Consider tracing the segments from J to G, then G to F, as the angle is being named to help students visualize the naming convention for angles where the middle letter denotes the angle's vertex.

Student Task Statement

1. Find the measure of angle JGH. Explain or show your reasoning.



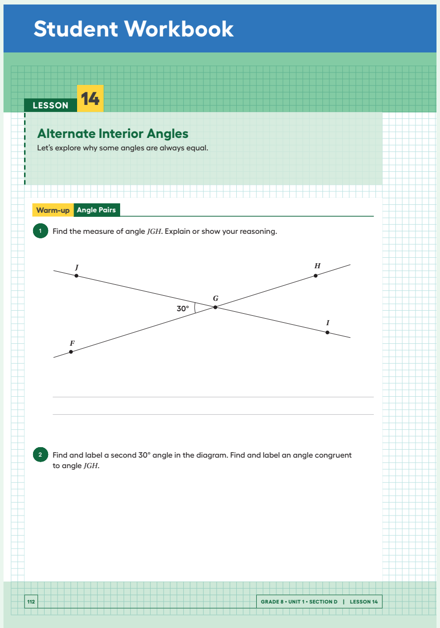
150°

Sample response: In the diagram, the given 30° angle and angle JGH are supplementary, so they add up to 180.

2. Find and label a second 30° angle in the diagram. Find and label an angle congruent to angle JGH.

See image

Student Workbook


LESSON 14

Alternate Interior Angles

Let's explore why some angles are always equal.

Warm-up Angle Pairs

1 Find the measure of angle JGH. Explain or show your reasoning.

2 Find and label a second 30° angle in the diagram. Find and label an angle congruent to angle JGH.

GRADE 8 • UNIT 1 • SECTION D | LESSON 14

Access for Multilingual Learners (Activity 1, Launch)

MLR2: Collect and Display.
Collect the language students use to find angle congruence. Display words and phrases such as "congruent," "translate," and "rotate." During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Activity Synthesis

Display the image for all to see. Invite students to share their responses, adding onto the image as needed to help make clear student thinking.
If no students use supplementary angles or the property that a straight line is 180°, ask students how they could determine the measure of angle JGH without a protractor. Highlight 2 supplementary angles, such as JGF and JGH, and write the term "supplementary" on the display near those angles. Highlight two vertical angles such as JGF and HGI and write the term "vertical angles" on the display.

Activity 1

Cutting Parallel Lines with a Transversal

15 min

Activity Narrative

In this task, students explore the relationship between angles formed when two parallel lines are cut by a transversal line. Students investigate whether knowing the measure of one angle is sufficient to figure out all the angle measures in the diagram.

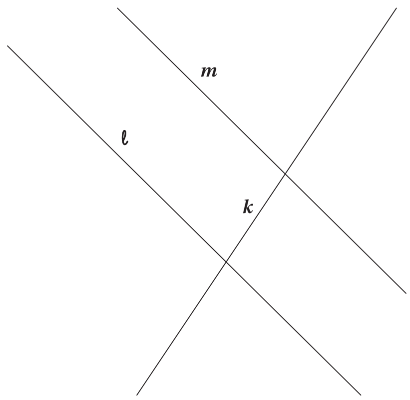
Monitor for students who use these different strategies to find the angle measures:

- Measure every angle using a protractor
- Measure some angles and use tracing paper to identify congruent angles
- Use what they know about vertical and supplementary angles to identify some angles

Select these students to share during the whole-class discussion.

Launch

A **transversal** (or *transversal line*) for a pair of parallel lines is a line that meets each of the parallel lines at exactly one point. Introduce this idea and provide a picture such as this picture where line k is a transversal for parallel lines ℓ and m:

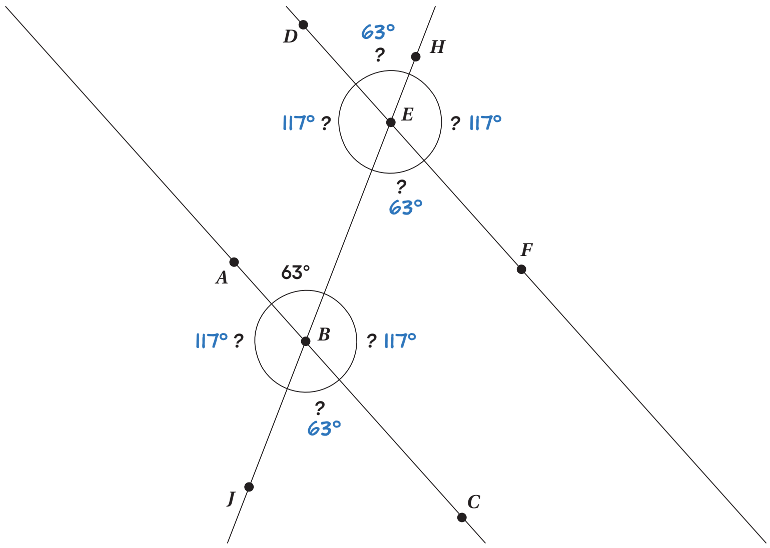


Arrange students in groups of 2. Provide access to geometry toolkits.

Give students 2–3 minutes of quiet think time, then 5–8 minutes of partner work time, followed by a whole-class discussion.

Student Task Statement

Lines AC and DF are parallel. They are cut by **transversal** HJ.



1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.

   Sample reasoning:

   - Tracing paper helped find the three 117-degree angles. Each of the other four angles is supplementary to a 117-degree angle, so they are all 63-degree angles.
   - Using pairs of vertical angles shows that angle CBJ is a 63-degree angle. The other angles at vertex B can be found using supplementary angles. The angles at vertex E can be found the same way after using tracing paper to find one of them.

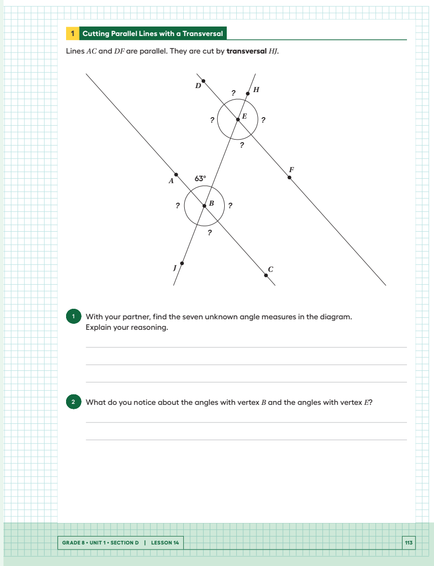
2. What do you notice about the angles with vertex B and the angles with vertex E?

   Sample response: The angles in the same place relative to the transversal have the same measure.

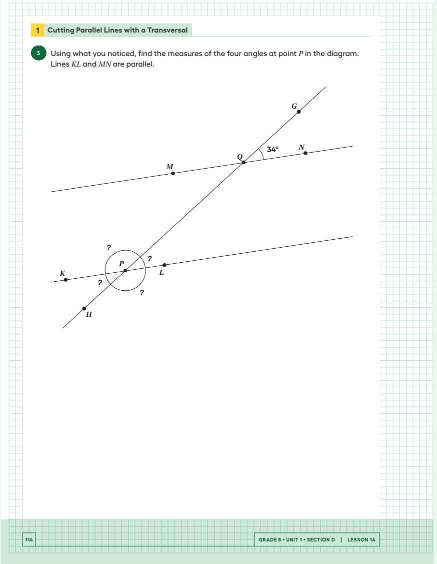
Building on Student Thinking

Students may fill in congruent angle measurements based on the argument that they look the same size. Ask students how they can be certain that the angles don't differ in measure by 1 degree. Encourage them to explain how we can know for sure that the angles are exactly the same measure.

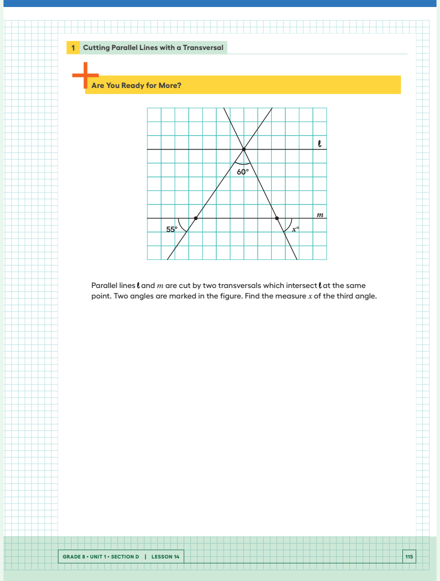
Student Workbook



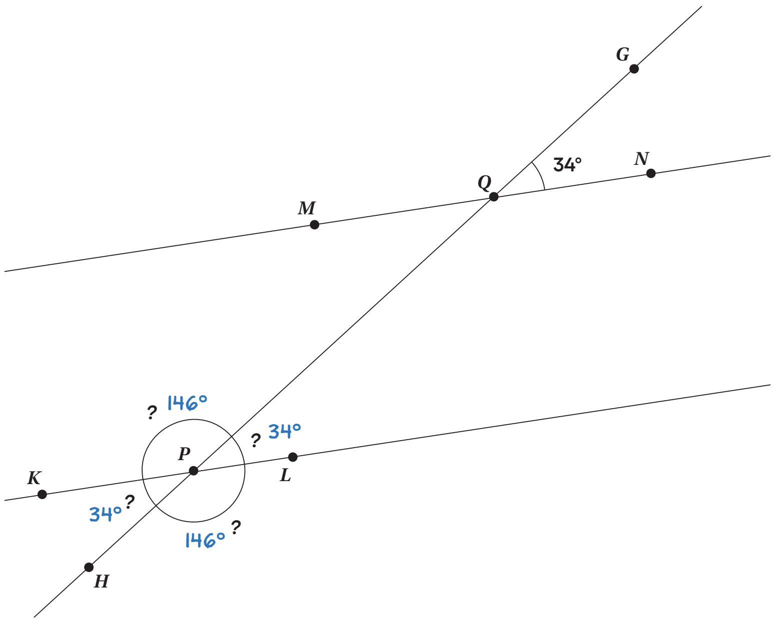
Student Workbook



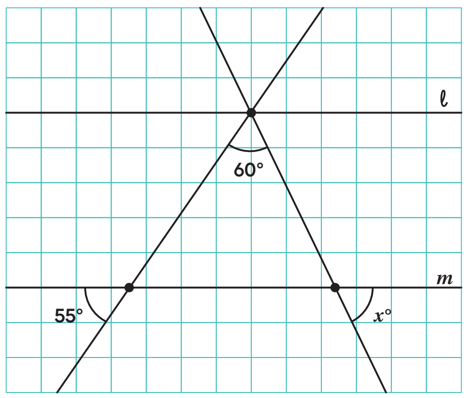
Student Workbook



3. Using what you noticed, find the measures of the four angles at point P in the diagram. Lines KL and MN are parallel.



Are You Ready for More?



Parallel lines ℓ and m are cut by two transversals which intersect ℓ at the same point. Two angles are marked in the figure. Find the measure x of the third angle.

$x = 65$

Sample response: Using tracing paper or the pattern from the activity, the angles around the point on line ℓ are corresponding angles to the marked angles on line m. We see that the angles marked 55°, 60°, and x° make a straight angle, so $55 + 60 + x = 180$.

Activity Synthesis

The goal of this discussion is for students to describe the relationships they notice between the angles formed when two parallel lines are cut by a transversal.

Display the image from the problem for all to see and invite groups to share what they noticed. Encourage students to use precise vocabulary, such as supplementary and vertical angles, when describing how they figured out the different angle measurements. After students point out the matching angles at the two vertices, define the term **alternate interior angles**: Alternate interior angles are created when two parallel lines are crossed by another line called a transversal. Alternate interior angles are inside the parallel lines and on opposite sides of the transversal.

Ask a few students to identify the pairs of alternate interior angles from the activity.

Activity 2
Alternate Interior Angles Are Congruent

10 min

Activity Narrative

The goal of this task is for students to connect their work with rigid transformations with the property that alternate interior angles are congruent. In this activity, students use properties of 180-degree rotations to find corresponding angles in the figure. As students describe their rigid transformations to their partner and listen to their partner's reasoning, they construct and critique the argument that these angles are congruent.

Listen for different strategies students use to show that the angles are congruent and select these students to share their strategies during the discussion.

Approaches might include:

- A 180-degree rotation about M
- First translating P to Q and then applying a 180-degree rotation with center Q

Launch

Arrange students in groups of 2.

Give 2–3 minutes of quiet work time, followed by 2–3 minutes of partner discussion, then a whole-class discussion.

Provide access to geometry toolkits. Tell students that in this activity, we will try to figure out why we saw all the matching angles we did in the last activity.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as "First, I ___ because …", "I noticed ___ so I …", "I agree/disagree because … ," or "Why did you …?"

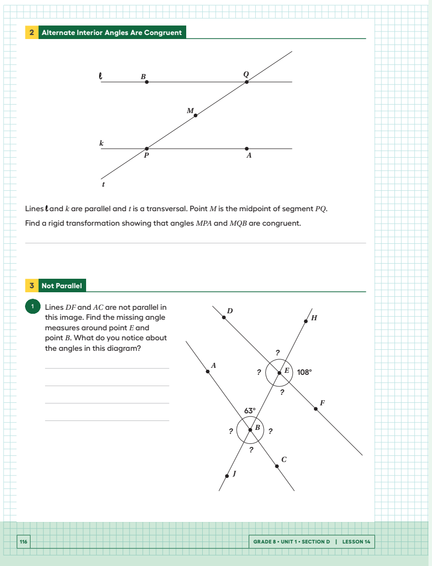
Supports accessibility for: Language, Organization

Access for Multilingual Learners (Activity 2, Synthesis)

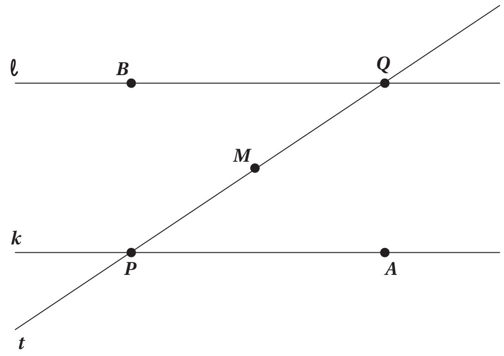
MLR8: Discussion Supports.
For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

Student Workbook



Student Task Statement



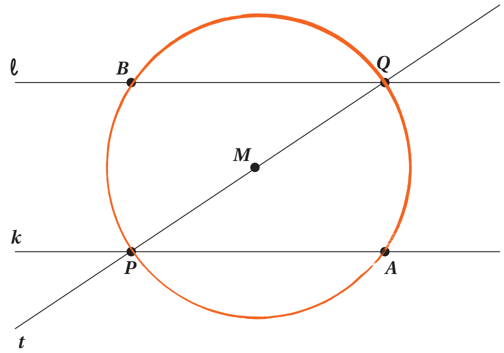
Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.
Find a rigid transformation showing that angles MPA and MQB are congruent.

Rotate the picture 180° with center M.

Activity Synthesis

Invite previously selected students to share their explanations. Ask students to describe and demonstrate the transformations they used to show that alternate interior angles are congruent. If any students connect this diagram to earlier work with 180-degree rotations or their justifications that vertical angles are congruent, invite them to share their observations.

Consider displaying this image for all to see as students share their thinking:



If any students use a translation to take P to Q or vice versa then claim that vertical angles are congruent, encourage more precision in their description by asking what rigid transformation tells them that vertical angles are congruent.

Activity 3: Optional

Not Parallel

10 min

Activity Narrative

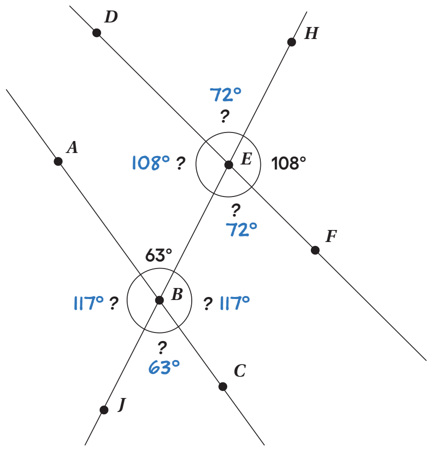
This activity is optional because it provides additional opportunities for students to find supplementary and vertical angles on a diagram. Students also compare their reasoning about alternate interior angles and transversals with a diagram that does not include parallel lines. This allows students to conclude that when two lines which are not parallel are cut by a third line, there are no alternate interior angles formed.

Launch

Provide access to geometry toolkits.

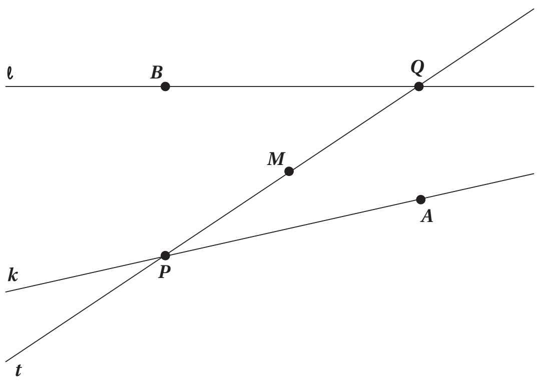
Student Task Statement

1. Lines DF and AC are not parallel in this image. Find the missing angle measures around point E and point B.
What do you notice about the angles in this diagram?



Sample response: The two pairs of vertical angles at each vertex are congruent. Also adjacent angles at each vertex are supplementary. The angle measures at B and E are different.

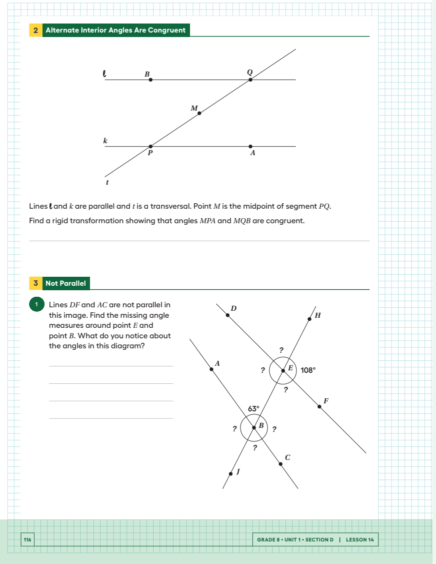
2. Point M is the midpoint of line segment QP. Can you find a rigid transformation that shows angle BQM is congruent to angle MPA? Explain your reasoning.



no

Sample response: If ℓ and m are not parallel, a 180-degree rotation around M takes P to Q, but it does not take m to ℓ because m is not parallel to ℓ. There is not a rigid transformation that takes angle BQM to angle MPA because they are not congruent.

Student Workbook



2 Alternate Interior Angles Are Congruent

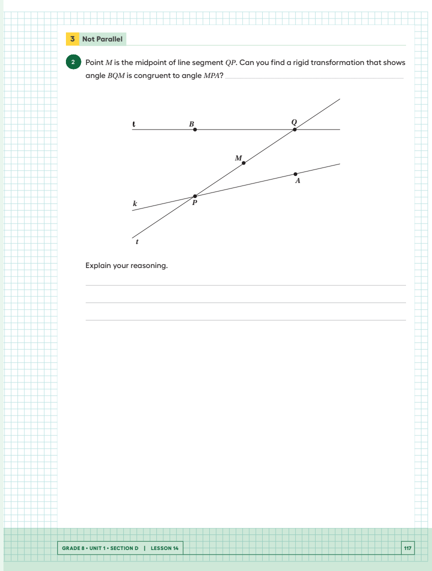
Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.
Find a rigid transformation showing that angles MPA and MQB are congruent.

3 Not Parallel

1. Lines DF and AC are not parallel in this image. Find the missing angle measures around point E and point B. What do you notice about the angles in this diagram?

116 | GRADE 8 • UNIT 1 • SECTION D | LESSON 14

Student Workbook



3 Not Parallel

2. Point M is the midpoint of line segment QP. Can you find a rigid transformation that shows angle BQM is congruent to angle MPA?

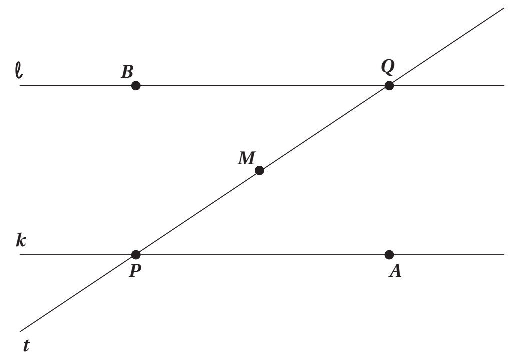
Explain your reasoning.

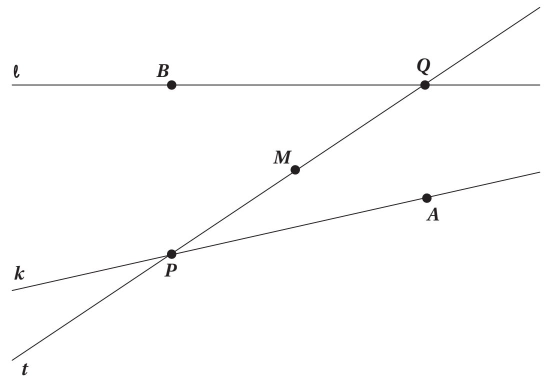
GRADE 8 • UNIT 1 • SECTION D | LESSON 14 | 117

Activity Synthesis

The goal of this discussion is for students to articulate that for pairs of alternate interior angles to be formed, a transversal must cut two parallel lines. If the lines are not parallel, then we cannot use rigid transformations to show these pairs of angles are congruent.

Display both of these images for all to see:





Here are some questions for discussion:

- *"What differences do you see between these diagrams?"*

  One diagram has a set of parallel lines, the other does not have any. One diagram has the same angle measures around point Q and P, but the other has different angle measures around Q and P.

- *"Which of these diagrams has a transversal and alternate interior angles?"*

  The diagram on the left with the parallel lines.

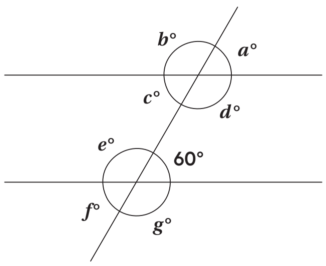
- *"If you know the angle measure of BQM, what other angles can you determine on each diagram?"*

  On the left diagram, you can determine all of the angles around point Q and then use alternate interior angles to determine the angles around point P. On the right diagram, you can only know the angles around point Q.

Lesson Synthesis

The goal of this discussion is for students to articulate which angles are congruent to one another and give an example of a rigid transformation that explains why.

Display the image of two parallel lines cut by a transversal. Tell students that in cases like this, translations and rotations can be particularly useful in figuring out angle measurements since they move angles to new positions, but the angle measure does not change.



Invite students to identify pairs of alternate interior, vertical, and supplementary angles in the image.

Here are some questions for discussion:

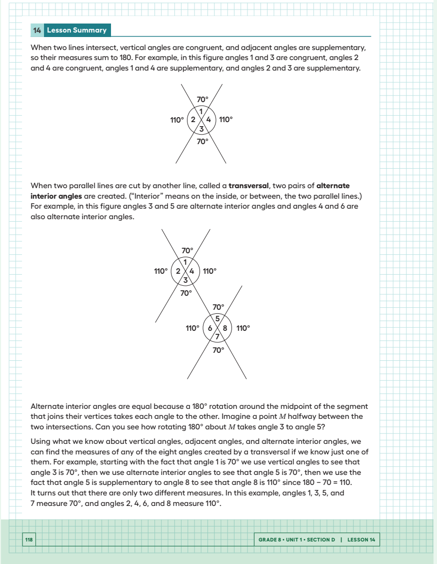
- What is the value of c, and how do you know?

  $c = 60$ because it is the measure of an angle forming an alternate interior angle with the given 60-degree angle.

- How do you know the values of e and d?

  e and d both equal 120 because they are also alternate interior angles, each supplementary to a 60-degree angle.

Student Workbook


14 Lesson Summary

When two lines intersect, vertical angles are congruent, and adjacent angles are supplementary, so their measures sum to 180. For example, in this figure angles 1 and 3 are congruent, angles 2 and 4 are congruent, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.

When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.

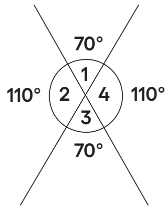
Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections. Can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since 180 − 70 = 110. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.

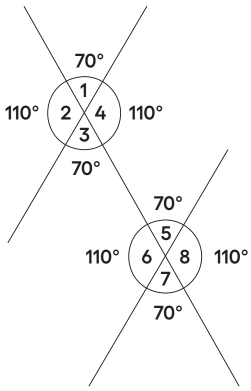
118 GRADE 8 • UNIT 1 • SECTION D | LESSON 14

Lesson Summary

When two lines intersect, vertical angles are congruent, and adjacent angles are supplementary, so their measures sum to 180. For example, in this figure angles 1 and 3 are congruent, angles 2 and 4 are congruent, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections. Can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since 180 – 70 = 110. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.

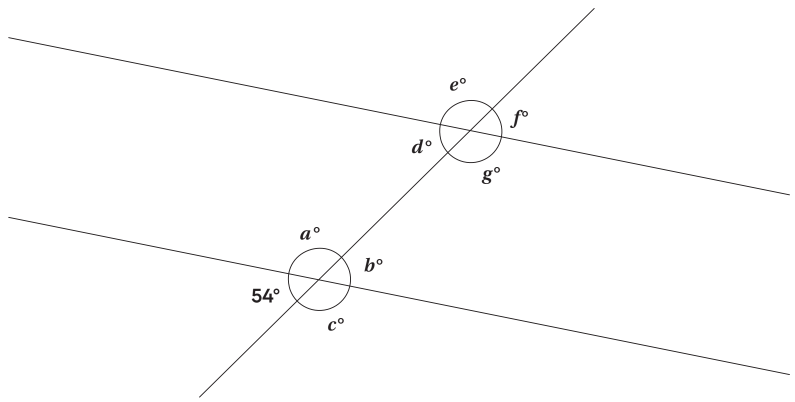
Cool-down

All The Rest

5 min

Student Task Statement

The diagram shows two parallel lines cut by a transversal. One angle measure is shown.



Find the values of a, b, c, d, e, f, and g.

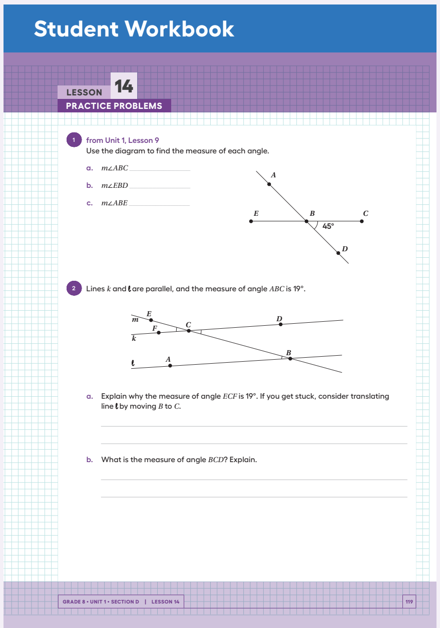
- a: 126
- b: 54
- c: 126
- d: 54
- e: 126
- f: 54
- g: 126

Responding To Student Thinking

Points to Emphasize

If students struggle with this *Cool-down*, as opportunities arise in this section, focus on identifying congruent angles for a set of parallel lines cut by a transversal. For example, in the activity referred to here, highlight one angle in the diagram and ask students to identify all of the congruent angles.

Unit 1, Lesson 16, Warm-up
All the Angles


Student Workbook

LESSON 14

PRACTICE PROBLEMS

1 from Unit 1, Lesson 9

Use the diagram to find the measure of each angle.

a. $m\angle ABC$

b. $m\angle EBD$

c. $m\angle ABE$

2 Lines k and ℓ are parallel, and the measure of angle ABC is 19°.

a. Explain why the measure of angle ECF is 19°. If you get stuck, consider translating line ℓ by moving B to C.

b. What is the measure of angle BCD? Explain.

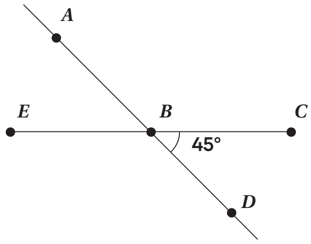
Practice Problems

5 Problems

Problem 1

from Unit 1, Lesson 9

Use the diagram to find the measure of each angle.



a. $m\angle ABC$

135 degrees

b. $m\angle EBD$

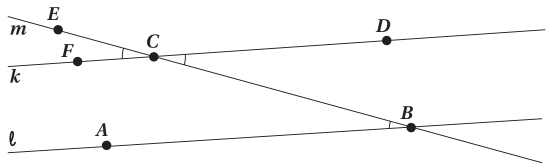
135 degrees

c. $m\angle ABE$

45 degrees

Problem 2

Lines k and ℓ are parallel, and the measure of angle ABC is 19°.



a. Explain why the measure of angle ECF is 19°. If you get stuck, consider translating line ℓ by moving B to C.

Sample reasoning: If ℓ is translated so that B goes to C, then ℓ goes to k because k is parallel to ℓ. Angle ABC matches up with angle FCE after this translation, so FCE (and ECF) is also a 19° angle.

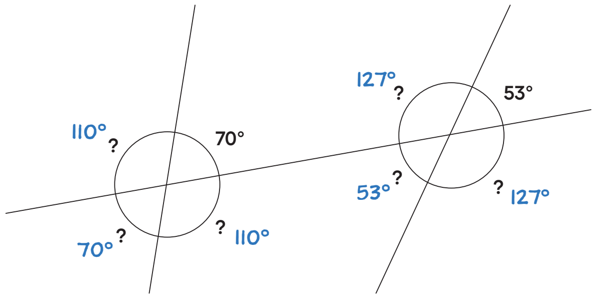
b. What is the measure of angle BCD? Explain.

19°

Sample reasoning: Angles ECF and BCD are congruent because they are vertical angles. Since angle ECF is a 19° angle, so is angle BCD.

Problem 3

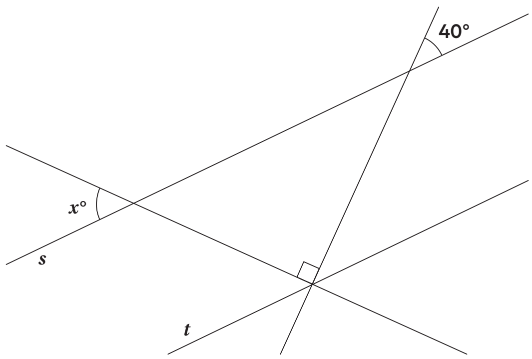
The diagram shows three lines with some marked angle measures:



Find the missing angle measures marked with question marks.

Problem 4

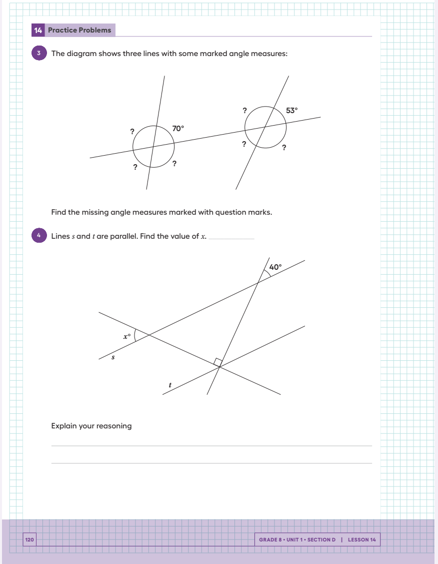
Lines s and t are parallel. Find the value of x. Explain your reasoning.



$x = 50$

Sample reasoning: The measure of the given angle is 40 degrees and the corresponding angle on line t also measures 40 degrees. This angle is adjacent to the indicated 90-degree angle, on its right side. Similarly, the angle that measures $x°$ corresponds to the angle that is adjacent to the indicated 90-degree angle, on its left side. This gives the equation $40 + 90 + x = 180$. x is 50 degrees, because $180 - (90 + 40) = 50$.

Student Workbook


14 Practice Problems

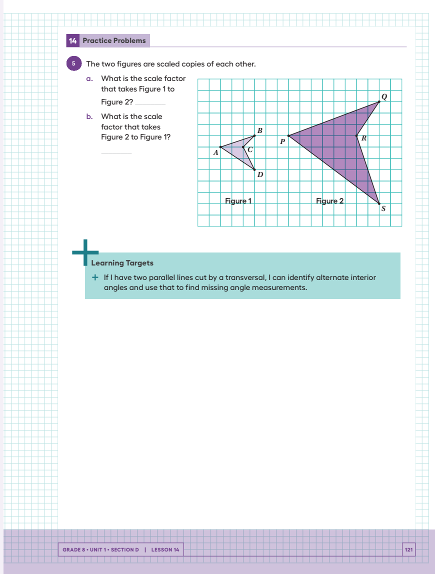
3 The diagram shows three lines with some marked angle measures:

Find the missing angle measures marked with question marks.

4 Lines s and t are parallel. Find the value of x.

Explain your reasoning

Student Workbook



14 Practice Problems

5 The two figures are scaled copies of each other.

a. What is the scale factor that takes Figure 1 to Figure 2?

b. What is the scale factor that takes Figure 2 to Figure 1?

Learning Targets

+ If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements.

GRADE 8 • UNIT 1 • SECTION D | LESSON 14

121

Problem 5

The two figures are scaled copies of each other.

a. What is the scale factor that takes Figure 1 to Figure 2?

3

b. What is the scale factor that takes Figure 2 to Figure 1?

$\frac{1}{3}$