Lesson 9

Multi-step Experiments

Goals

- Choose a method for representing the sample space of a compound event, and justify (orally) the choice.
- Use the sample space to determine the probability of a compound event, and explain (orally, in writing, and using other representations) the reasoning.

Learning Target

I can use the sample space to calculate the probability of an event in a multi-step experiment.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 3)

Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder
- Poll the Class

Lesson Narrative

In this lesson, students continue writing out the sample spaces for chance experiments that have multiple steps and also begin using those sample spaces to calculate the probability of certain events. Students must reason quantitatively and abstractly to make sense of the situations and represent the sample spaces mathematically.

An optional activity provides additional practice writing probabilities of events based on the sample space. The situation in this activity involves drawing cards without replacement, so the structures are not as symmetric as other examples.

Student Learning Goal

Let's look at probabilities of experiments that have multiple steps.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	20 min	15 min	10 min	5 min

Warm-up
Notice and Wonder: Spinning

5 min

Activity Narrative

The purpose of this *Warm-up* is to get students thinking about probabilities and spinners, which will be useful when students compute these values in a later activity. While students may notice and wonder many things about these spinners, the probabilities are the important discussion points.

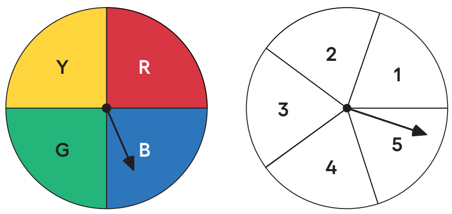
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- The first spinner has 4 sections and the other has 5.
- The first spinner has colors and letters, the second one has numbers.
- Within each spinner, the sections are equally sized.

Students may wonder:

- What is the probability of getting something like blue and 5 if both spinners are spun?
- Do you choose which one to spin or do you spin both?
- If you spin both, how many different outcomes will there be?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If probability does not come up during the conversation, ask students to discuss this idea.

Instructional Routines

Notice and Wonder

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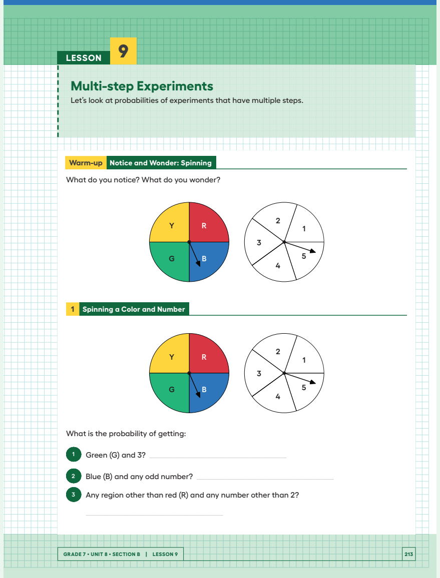
Student Workbook


LESSON 9

Multi-step Experiments

Let's look at probabilities of experiments that have multiple steps.

Warm-up Notice and Wonder: Spinning

What do you notice? What do you wonder?

1 Spinning a Color and Number

What is the probability of getting:

1. Green (G) and 3?
2. Blue (B) and any odd number?
3. Any region other than red (R) and any number other than 2?

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Develop Expression and Communication. Give students access to physical spinners to manipulate.

Supports accessibility for: Conceptual Processing, Organization

Activity 1

Spinning a Color and Number

10 min

Activity Narrative

In this activity, students are reminded how to calculate probability based on the number of outcomes in the sample space, then apply that to multi-step experiments. The events are described in everyday language, so students need to reason abstractly to identify the outcomes described. This lesson begins with students returning to a problem they have previously seen when writing out the sample space. This will save students some time if they can recall or refer back to the initial problem.

Launch

Tell students:

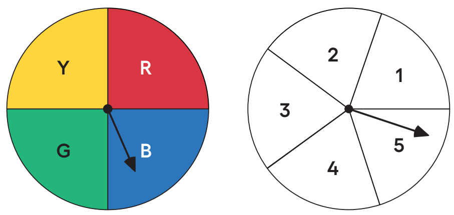
"For sample spaces where each outcome is equally likely, recall that the probability of an event can be computed by counting the number of outcomes in the event and dividing that number by the total number of outcomes in the sample space."

For example, there are 12 possible outcomes when flipping a coin and rolling a number cube. If we want the probability of getting heads and rolling an even number, we count that there are 3 ways to do this (H2, H4, and H6) out of the 12 outcomes in the sample space. So the probability of getting heads and an even number should be ($\frac{3}{12}$ or $\frac{1}{4}$, or 0.25).

Remind students that they have already drawn out the sample space for this chance experiment in a previous activity, and they may use that to help answer the questions.

Give students 5 minutes quiet work time followed by partner and whole-class discussion.

Student Task Statement



What is the probability of getting:

1. Green (G) and 3?

   $\frac{1}{20}$, since there is only 1 outcome that has green and 3, and there are 20 equally likely outcomes in the sample space.

2. Blue (B) and any odd number?

   $\frac{3}{20}$, since there are 3 outcomes that have blue and an odd number.

3. Any region other than red (R) and any number other than 2?

   $\frac{12}{20}$, since there are 12 outcomes that have any region besides red and any number besides 2.

Activity Synthesis

The purpose of this discussion is for students to explain their interpretations of the questions and share methods for solving.

Some questions for discussion:

- *"How did you calculate the number of outcomes in the sample space?"*

  counting the items in the tree, table, or list, or using the multiplication idea from an earlier lesson

- *"For each problem, how many outcomes were in the event that was described?"*

  There is only 1 way to get G and 3. There are 3 ways to get B and an odd number: B1, B3, B5. There are 12 outcomes in the last event because there are 3 regions that are not R and 4 regions that are not 2, and $3 \cdot 4 = 12$.

Activity 2

Cubes and Coins

20 min

Activity Narrative

In this activity, students continue to compute probabilities for multi-step experiments using the number of outcomes in the sample space. They are assigned either a list, table, or tree to examine the sample space for a chance experiment, then they use their understanding of the sample space to write probabilities for various events happening. Students are also reminded that some events have a probability of 0, which represents an event that is impossible. In the discussion following the activity, students are asked to think about the probabilities of two events that make up the entire sample space and have no outcomes common to both events.

Launch

Keep students in groups of 2.

Assign each group a representation for writing out the sample space: a tree, a table, or a list. Tell students that they should write out the sample space for the first problem using the representation they were assigned. (This was done for them in a previous lesson, and they are allowed to use those as a guide if they wish.)

Tell students that they should work on the first problem only and then pause for a discussion before proceeding to the next problems.

Give students 2 minutes of partner work time for the first problem followed by a pause for a whole-class discussion centered around the different representations for sample space.

Instructional Routines

Poll the Class

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After all groups have completed the first question, select at least one group for each representation, and have them explain how they arrived at their answer. As the groups explain, display the appropriate representations for all to see. Ask each of the groups how they counted the number of outcomes in the sample space as well as the number of outcomes in the event using their representation.

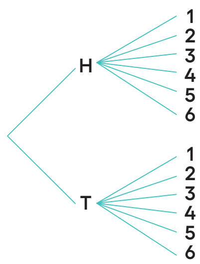
List:

Heads 1, heads 2, heads 3, heads 4, heads 5, heads 6, tails 1, tails 2, tails 3, tails 4, tails 5, tails 6

Table:

1	2	3	4	5	6
H1	H2	H3	H4	H5	H6
T1	T2	T3	T4	T5	T6

Tree:



After students have had a chance to explain how they used the representations, ask students to give some pros and cons for using each of the representations. For example, the list method may be easy to write out and interpret but could be very long and is not the easiest method for keeping track of which outcomes have been written and which still need to be included.

Allow the groups to continue with the remaining problems, telling them they may use any method they choose to work with the sample space for these problems.

Give students 10 minutes of partner work time followed by a whole-class discussion about the activity as a whole.

Student Task Statement

1. Your teacher will assign you to use either a list, table, or tree. Be prepared to explain your reasoning.

   A number cube is rolled and a coin is flipped.

   a. What is the probability of getting tails and a 6?

   $\frac{1}{12}$

   There is only 1 outcome in the sample space that has tails and a 6, and the sample space contains 12 equally likely outcomes.

   b. What is the probability of getting heads and an odd number?

   $\frac{3}{12}$

   There are 3 outcomes in the sample space that have heads and an odd number.

   Pause here so your teacher can review your work.

2. You may use any method you wish to answer these questions. Suppose you roll two number cubes. What is the probability of getting:

   a. Both cubes showing the same number?

   $\frac{6}{36}$

   There are 6 outcomes where the same number is showing, and the sample space contains 36 equally likely outcomes.

   b. Exactly one cube showing an even number?

   $\frac{18}{36}$

   There are 18 outcomes where exactly one of the cubes shows an even number.

   c. At least one cube showing an even number?

   $\frac{27}{36}$

   There are 27 outcomes where at least one of the cubes shows an even number.

   d. Two values that have a sum of 8?

   $\frac{5}{36}$

   There are 8 outcomes where the sum is 8 (2 and 6, 3 and 5, 4 and 4, 5 and 3, 6 and 2).

   e. Two values that have a sum of 13?

   0

   It is impossible to get two values whose sum is 13.

3. Jada flips three quarters. What is the probability that all three will land showing the same side?

   $\frac{2}{8}$

   There are 2 ways to get the coins showing the same side (all heads or all tails), and there are 8 equally likely outcomes in the sample space.

Building on Student Thinking

Some students may not recognize that rolling a 2 then a 3 is different from rolling a 3 then a 2. Ask students to imagine the number cubes are different colors to help see that there are actually 2 different ways to get these results.

Similarly, some students may think that HHT counts the same as HTH and THH. Ask the student to think about the coins being flipped one at a time rather than all tossed at once. Drawing an entire tree and seeing all the branches may further help.

Student Workbook


2 Cubes and Coins

1 Your teacher will assign you to use either a list, table, or tree. Be prepared to explain your reasoning.
A number cube is rolled and a coin is flipped.
a. What is the probability of getting tails and a 6?
b. What is the probability of getting heads and an odd number?
Pause here so your teacher can review your work.

2 You may use any method you wish to answer these questions. Suppose you roll two number cubes. What is the probability of getting:
a. Both cubes showing the same number?
b. Exactly one cube showing an even number?
c. At least one cube showing an even number?
d. Two values that have a sum of 8?
e. Two values that have a sum of 13?

3 Jada flips three quarters. What is the probability that all three will land showing the same side?

Instructional Routines

MLR5: Co-Craft Questions

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Activity Synthesis

The purpose of the discussion is for students to explain their methods for solving the problems and to discuss how writing out the sample space aided in their solutions.

Poll the class on how they computed the number of outcomes in the sample space and the number of outcomes in the event for the second set of questions given these options: list, table, tree, computed outcomes without writing them all out, or another method.

Consider these questions for discussion:

- *"Which representation did you use for each of the problems?"*

  "Do you think you will always try to use the same representation, or can you think of situations when one representation might be better than another?"

  "Do you have a method for finding the number of outcomes in the sample space or event that is more efficient than just counting them?"

  The number of outcomes in the sample space for the number cubes can be found using $6 \cdot 6 = 36$. To find the number of outcomes with at least 1 even number, I know there are 6 for each time an even is rolled first and only 3 for each time an odd number is rolled first, so the number of outcomes is $3 \cdot 6 + 3 \cdot 3 = 27$.

- *"One of the events has a probability of 0. What does this mean?"*

  It is impossible.

- *"What is the probability of an event that is certain?"*

  1

- *"Jada was concerned with having all three coins show the same side. What would be the probability of having at least one coin not match the others?"*

  68. There are 6 outcomes where at least one coin does not match: HHT, TTH, HTH, THT, HTT, THH.

- *"How are the probability of having at least 1 coin not matching the other and the probability of having the coins match related? How does it address Jada's concern?"*

  Because every outcome in the sample space has the coins either matching or not, and because there is no outcome that applies to both events, together the sum of their probabilities must be 100%, or 1.

Activity 3: Optional

Pick a Card

15 min

Activity Narrative

In this activity, students see an experiment that has two steps where the result of the first step influences the possibilities for the second step. Often this process is referred to as doing something "without replacement." At this stage, students should approach these experiments in a very similar way to all of the other probability questions they have encountered, but they must be very careful about the number of outcomes in the sample space.

Launch

Keep students in groups of 2.

Give students 5–7 minutes of quiet work time followed by partner and whole-class discussion.

Identify students who are not noticing that it is impossible to draw the same color twice based on the instructions. Refocus these students by asking them to imagine drawing a red card on the first pick and thinking about what's possible to get for the second card.

Student Task Statement

Imagine there are 5 cards. On one side they look the same and on the other side they are colored red, yellow, green, white, and black. You mix up the cards and select one of them without looking. Then, without putting that card back, you mix up the remaining cards and select another one.

1. What are the outcomes in the sample space? How many outcomes are there?

   Sample space: RY, RG, RW, RB, YR, YG, YW, YB, GR, GY, GW, GB, WR, WY, WG, WB, BR, BY, BG, BW. There are 20 different outcomes.

2. What structure did you use to write all of the outcomes (list, table, tree, something else)? Explain why you chose that structure.

   Sample response: I used a tree since it was easier to keep track of how the first card selected would affect what was possible for the second card.

3. What is the probability that:

   a. You get a white card and a red card (in either order)?

      $\frac{2}{20} = \frac{1}{10}$, because there are 2 outcomes that have those 2 cards (RW and WR), and the outcomes in the sample space are equally likely

   b. You get a black card (either time)?

      $\frac{8}{20} = \frac{2}{5}$, because there are 8 outcomes that have a black card (RB, YB, GB, WB, BR, BY, BG, BW)

   c. You do not get a black card (either time)?

      $\frac{12}{20} = \frac{3}{5}$, because there are 12 outcomes that do not have a black card

   d. You get a blue card?

      0, because there are no blue cards in the deck

   e. You get 2 cards of the same color?

      0, because there is only 1 card of each color, and the same card cannot be chosen twice if the first one is not put back

   f. You get 2 cards of different colors?

      1, because all of the possible outcomes have two different colors

Access for Students with Diverse Abilities (Activity 3, Student Task)

Representation: Access for Perception.
Use colored cards to demonstrate the situation.

Supports accessibility for: Conceptual Processing, Language, Memory

Building on Student Thinking

Students may misread the problem and think that they replace the card before picking the next one. Ask these students to read the problem more carefully and ask them,

"What is possible to get when you draw the second card while you already have a red card in your hand?"

Student Workbook


3 Pick a Card

Imagine there are 5 cards. On one side they look the same and on the other side they are colored red, yellow, green, white, and black. You mix up the cards and select one of them without looking. Then, without putting that card back, you mix up the remaining cards and select another one.

1. What are the outcomes in the sample space? How many outcomes are there?

2. What structure did you use to write all of the outcomes (list, table, tree, something else)? Explain why you chose that structure.

3. What is the probability that:

   a. You get a white card and a red card (in either order)? ______
   b. You get a black card (either time)? ______
   c. You do not get a black card (either time) ? ______
   d. You get a blue card? ______
   e. You get 2 cards of the same color? ______
   f. You get 2 cards of different colors? ______

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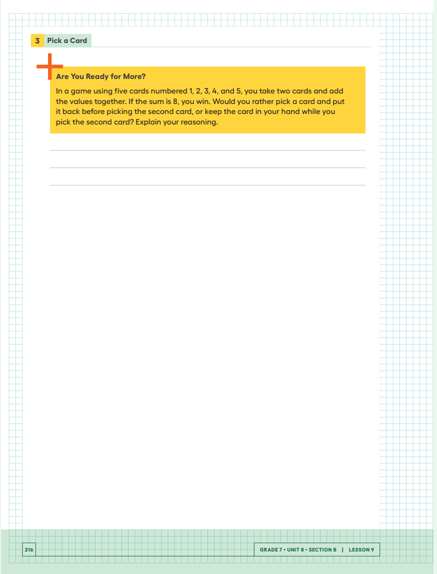
Student Workbook


3 Pick a Card

Are You Ready for More?

In a game using five cards numbered 1, 2, 3, 4, and 5, you take two cards and add the values together. If the sum is 8, you win. Would you rather pick a card and put it back before picking the second card, or keep the card in your hand while you pick the second card? Explain your reasoning.

Are You Ready for More?

In a game using five cards numbered 1, 2, 3, 4, and 5, you take two cards and add the values together. If the sum is 8, you win. Would you rather pick a card and put it back before picking the second card, or keep the card in your hand while you pick the second card? Explain your reasoning.

I am more likely to win if I put the card back.

Sample reasoning: If I put it back, I can win with these outcomes: 3,5; 4,4; 5,3. Since this way has 25 equally likely outcomes in the sample space, the probability of winning is $\frac{3}{25} = 0.12$. If I do not put it back, I can win with these outcomes: 3,5 or 5,3. Since this way has 20 equally likely outcomes in the sample space, the probability of winning is $\frac{2}{20} = 0.1$.

Activity Synthesis

The purpose of the discussion is for students to compare the same context with replacement and without replacement.

Consider asking these questions for discussion:

- *"How would your calculations change if the experiment required replacing the first card before picking a second card?"*

  There would be 25 outcomes in the sample space. The probability of getting the same color twice would be $\frac{5}{25}$. The probability of getting different colors would be $\frac{20}{25}$. The probability of getting red and white would be $\frac{2}{25}$. The probability of getting a black card would be $\frac{9}{25}$, and not getting a black card would be $\frac{16}{25}$. It would still be impossible to get a blue card, so its probability would be 0.

- *"What do you notice about the sum of the probability of getting a black card and the probability of not getting a black card?"*

  They have a sum of 1.

- *"Why might these outcomes have probabilities with this relationship?"*

  Since the player either gets a black card or not, together their probabilities should be 1, or 100%.

Lesson Synthesis

These discussion questions will help students reflect on their learning:

- *"When the outcomes in the sample space are equally likely, how is the size of the sample space used to calculate the probability of an event?"*

  When the sample space has size n, the probability of a single outcome should be $\frac{1}{n}$.

- *"Now that you've had plenty of practice, do you have a favorite method for writing out the sample space?"*

  I like using the tree.

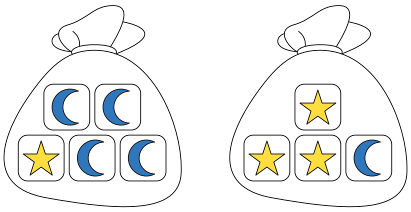
- *"What are some disadvantages of using lists, tables, and trees?"*

  Lists are hard for large sample spaces, tables are not useful if there are more than one part, and trees can get large.

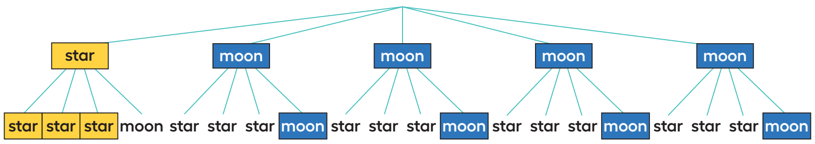
Lesson Summary

Suppose we have two bags. One contains 1 star block and 4 moon blocks. The other contains 3 star blocks and 1 moon block.

If we select 1 block at random from each, what is the probability that we will get 2 star blocks or 2 moon blocks?



To answer this question, we can draw a tree diagram to see all of the possible outcomes.



There are $5 \cdot 4 = 20$ possible outcomes. Of these, 3 of them are both stars, and 4 are both moons. So the probability of getting 2 star blocks or 2 moon blocks is $\frac{7}{20}$.

In general, if all outcomes in an experiment are equally likely, then the probability of an event is the fraction of outcomes in the sample space for which the event occurs.

Cool-down

A Number Cube and 10 Cards

5 min

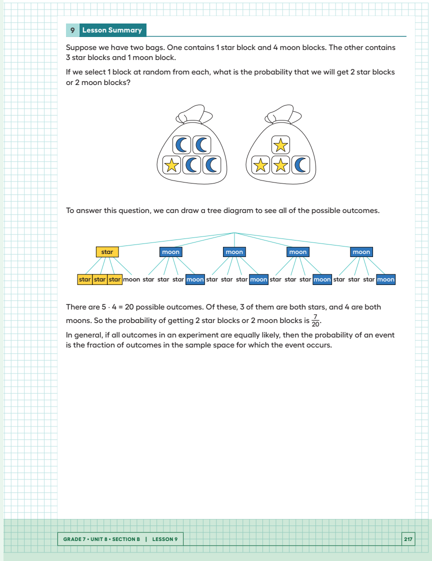
Student Task Statement

Lin plays a game that involves a standard number cube and a deck of ten cards numbered 1 through 10. If both the cube and card have the same number, Lin gets another turn. Otherwise, play continues with the next player.

What is the probability that Lin gets another turn?

$\frac{6}{60}$ (or equivalent), since there are 6 outcomes for which the numbers match and 60 equally likely outcomes in the sample space $(6 \cdot 10 = 60)$

Student Workbook


Lesson Summary

Suppose we have two bags. One contains 1 star block and 4 moon blocks. The other contains 3 star blocks and 1 moon block.

If we select 1 block at random from each, what is the probability that we will get 2 star blocks or 2 moon blocks?

To answer this question, we can draw a tree diagram to see all of the possible outcomes.

There are $5 \cdot 4 = 20$ possible outcomes. Of these, 3 of them are both stars, and 4 are both moons. So the probability of getting 2 star blocks or 2 moon blocks is $\frac{7}{20}$.

In general, if all outcomes in an experiment are equally likely, then the probability of an event is the fraction of outcomes in the sample space for which the event occurs.

Responding To Student Thinking

Points to Emphasize

If students struggle with determining the probability of a compound event, use the practice problems as opportunities to revisit methods for representing the sample space such as using lists, tables, and trees. For example, review these methods when students find the propability in the practice problem referred to here:

Unit 8, Lesson 9, Practice Problem 1

Student Workbook


LESSON 9
PRACTICE PROBLEMS

1. A vending machine has 5 colors (white, red, green, blue, and yellow) of gumballs and an equal chance of dispensing each. A second machine has 4 different animal-shaped toys (lion, elephant, horse, and alligator) and an equal chance of dispensing each. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion toy?

2. The numbers 1 through 10 are put in one bag. The numbers 5 through 14 are put in another bag. When one number is chosen from each bag, what is the probability of getting the same number? Explain your reasoning.

3. When rolling 3 standard number cubes, the probability of getting all three numbers to match is $\frac{6}{216}$. What is the probability that the three numbers do not all match? Explain your reasoning.

4. from Unit 8, Lesson 8
For each event, write the sample space and tell how many outcomes there are.
a. Roll a standard number cube. Then flip a quarter.
b. Select a month. Then select 2020 or 2025.

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Lesson 9

Practice Problems

5 Problems

Problem 1

A vending machine has 5 colors (white, red, green, blue, and yellow) of gumballs and an equal chance of dispensing each. A second machine has 4 different animal-shaped toys (lion, elephant, horse, and alligator) and an equal chance of dispensing each. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion toy?

$\frac{1}{20}$

Problem 2

The numbers 1 through 10 are put in one bag. The numbers 5 through 14 are put in another bag. When one number is chosen from each bag, what is the probability of getting the same number? Explain your reasoning.

$\frac{6}{100}$, (or equivalent)

Sample reasoning: It is possible to get two 5s, 6s, 7s, 8s, 9s, or 10s, and there are 100 possible outcomes in the sample space (10, 10).

Problem 3

When rolling 3 standard number cubes, the probability of getting all three numbers to match is $\frac{6}{216}$. What is the probability that the three numbers do not all match? Explain your reasoning.

$\frac{210}{216}$, (or equivalent)

Sample reasoning: The three numbers either all match or do not all match, so together these probabilities must add to 1.

Problem 4

from Unit 8, Lesson 8

For each event, write the sample space and tell how many outcomes there are.

a. Roll a standard number cube. Then flip a quarter.

12 outcomes: 1h, 1t, 2h, 2t, 3h, 3t, 4h, 4t, 5h, 5t, 6h, 6t

b. Select a month. Then select 2020 or 2025.

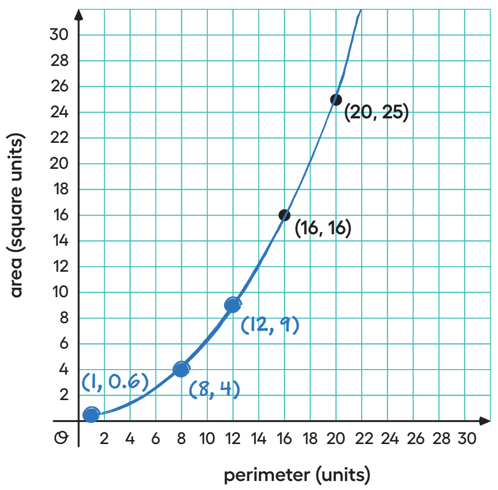
24 outcomes: Jan 2020, Jan 2025, Feb 2020, Feb 2025, Mar 2020, Mar 2025, Apr 2020, Apr 2025, May 2020, May 2025, June 2020, June 2025, July 2020, July 2025, Aug 2020, Aug 2025, Sep 2020, Sep 2025, Oct 2020, Oct 2025, Nov 2020, Nov 2025, Dec 2020, Dec 2025

Problem 5

from Unit 2, Lesson 11

On a graph of the area of a square vs. its perimeter, a few points are plotted.

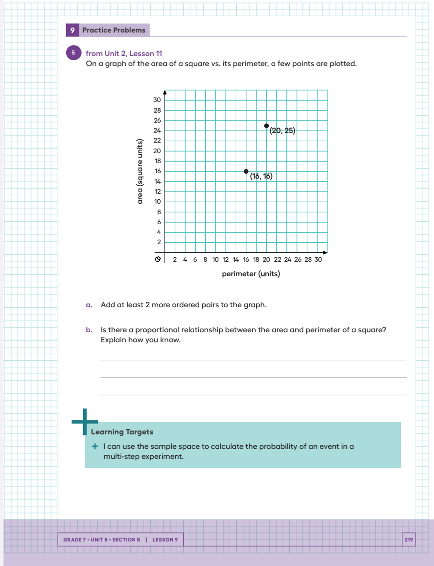
a. Add at least 2 more ordered pairs to the graph.



b. Is there a proportional relationship between the area and perimeter of a square? Explain how you know.

There is not a proportional relationship between area and perimeter. Sample reasoning: When graphed, the ordered pairs do not lie on a line that passes through the origin.

Student Workbook


Practice Problems

from Unit 2, Lesson 11

On a graph of the area of a square vs. its perimeter, a few points are plotted.

a. Add at least 2 more ordered pairs to the graph.

b. Is there a proportional relationship between the area and perimeter of a square? Explain how you know.

Learning Targets

I can use the sample space to calculate the probability of an event in a multi-step experiment.