More than That, Less than That

Goals

- Apply the distributive property to generate algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value, and explain (orally) the reasoning.
- Coordinate tables, equations, tape diagrams, and verbal descriptions that represent a relationship involving adding or subtracting a fraction of the initial value.
- Generalize a process for finding the value that is "half as much again," and justify (orally and in writing) why this can be abstracted as $\frac{3}{2}x$ or equivalent.

Learning Targets

- I can use the distributive property to rewrite an expression like $x + \frac{1}{2}x$ as $(1+\frac{1}{2})x$.
- I understand that "half as much again" and "multiply by $\frac{3}{2}$ " mean the same thing.

Student Learning Goal

Let's use fractions to describe increases and decreases.

Lesson Narrative

In this lesson, students see how to use the distributive property to write a compact expression for situations where one quantity is described in relation to another quantity in language such as "half as much again" and "one third more than." If y is half as much again as x, then $y = x + \frac{1}{2}x$. Using the distributive property, this can be written as $y = (1\frac{1}{2})x$. Students apply this sort of reasoning to various situations and other fractional amounts of increase or decrease.

As students investigate multiple examples to recognize the relationship $y = \frac{3}{2}x$, they are making use of repeated reasoning. As students look for opportunities to use the distributive property to write equations in a simpler way, they are making use of structure.

The last activity is optional because it provides an opportunity for additional practice relating descriptions, tape diagrams, and equations.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)
- Representation (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 3)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- · Card Sort
- · MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- · Notice and Wonder
- Take Turns

Required Materials

Materials To Copy

 Fractional Relationships Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up

10

Activity 1

10

Activity 2

10

Activity 3

10

Lesson Synthesis



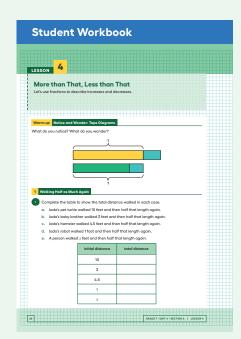
Cool-down

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

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Warm-up

Notice and Wonder: Tape Diagrams



Activity Narrative

The purpose of this *Warm-up* is to interpret representations that show an increase or decrease that is a fraction of the original value. This will be useful when students represent fractional increases or decreases in a later activity. While students may notice and wonder many things about these images, the important discussion points are the estimation of the size of the blue rectangle and how to distinguish whether the blue rectangle represents an increase or decrease.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is similarities and differences between the two tape diagrams.

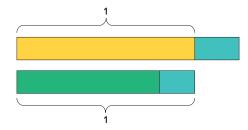
Launch 22

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- In each diagram, I am putting together two rectangles of different lengths.
- Each diagram has a blue rectangle and one other rectangle.
- The blue rectangles in each diagram look to be the same size.
- The yellow rectangle is the same length as the green and blue rectangles put together.
- The green rectangle looks to be three times as long as the blue rectangle.
- The yellow rectangle looks to be four times as long as the blue rectangle.
- The green rectangle looks to be three-fourths as long as the yellow rectangle.

Students may wonder:

- How many blue rectangles would it take to cover the green or yellow rectangle?
- · What fraction does the blue rectangle represent?
- · How long is the total length of the first diagram?
- · What is the missing length in the second diagram?
- · What situations do these diagrams represent?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the total length of the first diagram does not come up during the conversation, ask students to discuss this idea.

Activity 1

Walking Half as Much Again

10 min

Activity Narrative

In this activity, students find patterns in situations to connect to the distributive property. These patterns build understanding of the equations x + 0.5x = (1 + 0.5)x = 1.5x and $x + \frac{1}{2}x = \left(1 + \frac{1}{2}\right)x = 1\frac{1}{2}x$. Students should see that the expressions are all representations of the same thing. Students learn that multiplying a number by $\frac{1}{2}$ and adding that product to the original number is the same as multiplying by 1.5. This idea will be extended to percentages later.

Monitor for students who use these different strategies to make sense of the problem:

- Create a drawing or diagram.
- Write addition expressions, like 10 + 5 and 3 + 1.5.
- Write multiplication expressions, like 1.5 \cdot 10 and 1.5 \cdot 3.
- Identify the table as representing a proportional relationship and the constant of proportionality as $\frac{3}{2}$ (or equivalent).

Plan to have students present in this order to support moving them from concrete to more abstract representations.

As students calculate the total distance for a variety of initial distances and then the variable x, they make use of repeated reasoning.

Launch

Demonstrate, or ask students to demonstrate, walking a certain distance and then walking "half as much again".

Give students 5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Instructional Routines

5 Practices

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Instructional Routines

MLR:2 Collect and Display

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Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.

Collect the language students use to describe the relationship between initial distance and total distance. Display words and phrases, such as "adding one half," "50% more," "one and a half times," "distributive property," "initial amount," "final amount." During the Activity Synthesis, invite students to suggest ways to update the display:

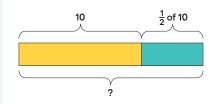
"What are some other words or phrases we should include?"

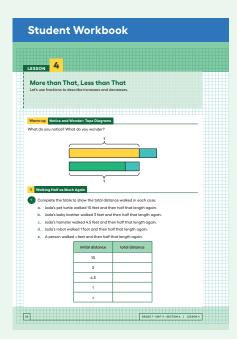
Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Building on Student Thinking

Some students may need support making sense of the situation. Ask them how they could draw a tape diagram to represent the first situation. For example, they could draw something like this.







Student Task Statement

- 1. Complete the table to show the total distance walked in each case.
 - a. Jada's pet turtle walked 10 feet and then half that length again.
 - b. Jada's baby brother walked 3 feet and then half that length again.
 - c. Jada's hamster walked 4.5 feet and then half that length again.
 - d. Jada's robot walked 1 foot and then half that length again.
 - **e.** A person walked x feet and then half that length again.

initial distance	total distance
10	15
3	4.5
4.5	6.7 5
1	1.5
x	$x + \frac{1}{2}x$

2. Explain how you computed the total distances for these cases.

Sample response: I took half of the initial distance walked and added it to the initial distance walked to get the total distance walked.

- **3.** Two students each wrote an equation to represent the relationship between the initial distance walked (x) and the total distance walked (y).
- Mai wrote $y = x + \frac{1}{2}x$.
- Kiran wrote $y = \frac{3}{2}x$.

Do you agree with either of them? Explain your reasoning.

They are both correct.

Sample reasonings:

- The quotient of total distance walked and the corresponding initial distance walked is constant (it's 1.5).
- The total distance walked is always 1.5 times the initial distance walked.
- I agree with Kiran because $10 \cdot 1.5 = 15$ and that works for every other entry in the table.

Are You Ready for More?

Zeno jumped 8 meters. Then he jumped half as far again (4 meters). Then he jumped half as far again (2 meters). So after 3 jumps, he was 8 + 4 + 2 = 14 meters from his starting place.

1. Zeno kept jumping half as far again. How far would he be after 4 jumps? 5 jumps? 6 jumps?

15 meters; $15\frac{1}{2}$ meters; $15\frac{3}{4}$ meters

2. Before he started jumping, Zeno put a mark on the floor that was exactly 16 meters from his starting place. How close can Zeno get to the mark if he keeps jumping half as far again?

He can get as close as he wants, because he can always cut the distance between his current position and the mark in half.

3. If you enjoyed thinking about this problem, consider researching Zeno's Paradox.

No response expected.

Activity Synthesis

The purpose of this discussion is to show where Kiran's expression comes from. Invite previously selected students to share how they computed the total distance. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions, such as:

"How does 'half that length again' show up in each method?"

"Why do the different approaches lead to the same outcome?"

"Did anyone solve the problem the same way but would explain it differently?"

The key takeaway is that both $x + \frac{1}{2}x$ and $\frac{3}{2}x$ represent the situation. The expressions are equivalent due to the distributive property.

Activity 2

Card Sort: Fractional Relationships

10 min

Activity Narrative

In this partner activity, students take turns matching situations, tape diagrams, and equations. They interpret wording like " $\frac{1}{3}$ more than that" and figure out how that relationship can be expressed using different mathematical representations. Students may initially think that the wording means $y = \frac{1}{3}x$; however, the tape diagram will help them realize that this is not true.

As students analyze different representations, they practice reasoning quantitatively and abstractly. In making connections across representations, they practice looking for and making use of structure

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "First, I _____ because ..." "I noticed ____ so I ..." "Why did you ...?" "I agree/disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning

Instructional Routines

Card Sort

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Instructional Routines

MLR8: Discussion Supports

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Instructional Routines

Take Turns

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.

Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed ______, so I matched ..." When students disagree, encourage them to challenge each other using these sentence frames: "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about the different representations.

Building on Student Thinking

Students may match the equation $y=\frac{2}{3}x$ with the situation "Elena biked x miles, and Noah biked $\frac{2}{3}$ more than that." Ask them who biked farther, Noah or Elena? The equation they choose must result in Noah biking a greater distance than Elena.

Instructional Routines

MLR7: Compare and Connect

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Launch

Tell students that the cards contain situations, tape diagrams, and equations and that they will take turns matching the cards. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of slips cut from the blackline master.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to match a description with an equation and a tape diagram.

- **1.** For each match that you find, explain to your partner how you know it's amatch.
- **2.** For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

The blackline master shows the correct matches.

Activity Synthesis

The purpose of this discussion is to make connections between the numbers in the description and the numbers in the equation. Select 2–3 groups to share one of their sets of cards and how they matched the description with a tape diagram and equation. Discuss as many different sets of cards as the time allows.

Consider asking:

"Which matches were tricky? Explain why."

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

"How is the number in the equation related to the number in the description?"

"What is different between a fractional amount more than the original and a fractional amount less than the original?"

Activity 3: Optional

More and Less

10 min

Activity Narrative

This activity gives students more practice connecting descriptions, tape diagrams, and equations for situations that involve an increase or decrease expressed as a fraction of the initial value. First students match descriptions and tape diagrams. Then they write equations that represent the situations. Finally, they apply their understanding to create their own situations for the tape diagrams that did not have a match.

Monitor for different ways students write the equations, including addition or subtraction and multiplication.



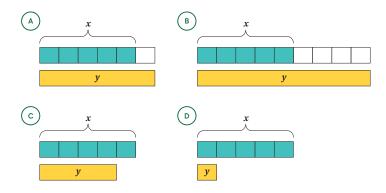
Arrange students in groups of 2.

Give students 1–2 minutes of quiet work time followed by time for partner discussion on the first problem. Then give students 4–5 minutes to complete the rest of the problems.

Follow with whole-class discussion.

Student Task Statement

1. Match each situation with a diagram. A diagram may not have a match.



- 1. Han slept for x hours. Mai slept $\frac{1}{5}$ less than that. Diagram C
- **2.** Mai biked x miles. Han biked $\frac{4}{5}$ more than that. Diagram B
- **3.** Han bought x pounds of books. Mai bought $\frac{4}{5}$ of that. Diagram C
- **2.** For each diagram, write an equation that represents the relationship between *x* and *y*.
 - **1.** Diagram A: $y = \frac{6}{5}x$ (or equivalent)
 - **2.** Diagram B: $y = \frac{9}{5}x$ (or equivalent)
 - **3.** Diagram C: $y = \frac{4}{5}x$ (or equivalent)
 - **4.** Diagram D: $y = \frac{1}{5}x$ (or equivalent)
- **3.** Write a story for one of the diagrams that doesn't have a match.

Sample responses:

- For Diagram A, Han ate x ounces of blueberries. Mai ate $\frac{1}{5}$ more than that.
- For Diagram D, Han has x quarters. Mai has $\frac{4}{5}$ less than that.

Activity Synthesis

The purpose of this discussion is to emphasize the connection between the numbers in the description and the numbers in the equation. First, ask students to share the equations they wrote for each diagram. To discuss the structure of the equations used, consider asking:

"How is the number in the equation related to the number in the description?"

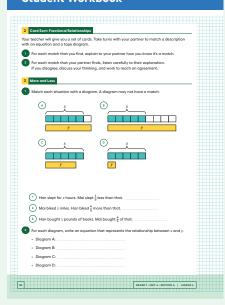
"What is different between a fractional amount more than the original versus less than the original?"

If time permits, invite a few students to share the stories they created for the leftover diagrams.

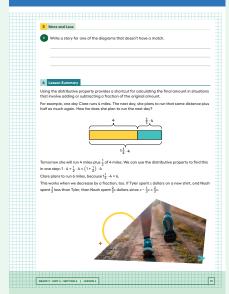
Building on Student Thinking

Students may match the equation $y=\frac{2}{3}x$ with the situation "Mai biked x miles, and Han biked $\frac{2}{3}$ more than that." Ask them who biked farther, Han or Mai? The equation they choose must result in Han biking a greater distance than Mai.

Student Workbook



Student Workbook



Warm-up

Access for Students with Diverse Abilities (Activity 3, Synthesis)

Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, color code the tape diagram to show how it relates to the corresponding equation.

Supports accessibility for: Visual-Spatial Processing

Access for Multilingual Learners (Activity 3, Synthesis)

MLR7: Compare and Connect.

Lead a discussion comparing, contrasting, and connecting the different representations for each situation. Ask,

"How are the equations the same?"

"How are they different?"

"Are there any benefits or drawbacks to one representation compared to another?"

Advances: Representing, Conversing

Lesson Synthesis

Share with students,

"Today we looked at situations where a quantity increased or decreased by a fraction of the starting amount. We used the distributive property to write expressions that represented these situations."

To review the role of the distributive property in making calculations more efficient, consider prompting students:

 \bigcirc "Give examples of equivalent expressions that represent an amount, x, plus a fraction of that amount."

$$x + \frac{1}{2}x = \frac{1}{2}x$$
; $x + \frac{2}{3}x = \frac{5}{3}x$

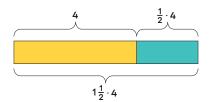
Give examples of equivalent expressions that represent an amount, *x* , minus a fraction of that amount."

$$x - \frac{1}{4}x = \frac{3}{4}x$$
; $x - \frac{2}{3}x = \frac{1}{3}x$

Lesson Summary

Using the distributive property provides a shortcut for calculating the final amount in situations that involve adding or subtracting a fraction of the original amount.

For example, one day Clare runs 4 miles. The next day, she plans to run that same distance plus half as much again. How far does she plan to run the next day?



Tomorrow she will run 4 miles plus $\frac{1}{2}$ of 4 miles. We can use the distributive property to find this in one step: $1 \cdot 4 + \frac{1}{2} \cdot 4 = \left(1 + \frac{1}{2}\right) \cdot 4$

Clare plans to run 6 miles, because $1\frac{1}{2} \cdot 4 = 6$.

This works when we decrease by a fraction, too. If Tyler spent x dollars on a new shirt, and Noah spent $\frac{1}{3}$ less than Tyler, then Noah spent $\frac{2}{3}x$ dollars since $x - \frac{1}{3}x = \frac{2}{3}x$.

Cool-down

Swimming and Skating

5 min

If students only write expressions, like $\frac{1}{4}x$ and $\frac{8}{5}x$, instead of equations, then they have shown they achieved the goal of the lesson. It's not necessary or desirable to hold students accountable for correctly interpreting expressions versus equations at this time.

Warm-up

Student Task Statement

Accept all equivalent forms of each response.

1. Tyler swam for x minutes, and Han swam for $\frac{3}{4}$ less than that. Write an equation to represent the relationship between the amount of time that Tyler spent swimming (x) and the amount of time that Han spent swimming (y).

$$y = \frac{1}{4}x$$

(Han swam $\frac{3}{4}x$ minutes less than Tyler swam. Han swam $\frac{1}{4}x$ minutes because $x - \frac{3}{4}x = \frac{1}{4}x$.)

2. Mai skated x miles, and Clare skated $\frac{3}{5}$ farther than that. Write an equation to represent the relationship between the distance that Mai skated (x) and the distance that Clare skated (y).

$$y = \frac{8}{5}x$$

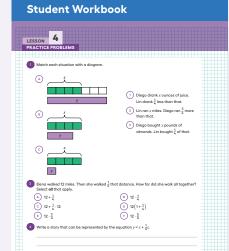
(Clare skated $\frac{3}{5}x$ miles farther than the number of miles Mai skated. Clare skated $\frac{8}{5}x$ miles because $x + \frac{3}{5}x = \frac{8}{5}x$.)

Responding To Student Thinking

Points to Emphasize

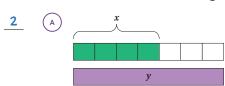
If students struggle with setting up the equations, plan to review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the tape diagrams in this activity:

Grade 7, Unit 4, Lesson 5, Activity 2 More and Less with Decimals 4

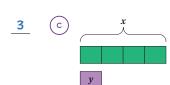


Problem 1

Match each situation with a diagram.



- **1.** Diego drank x ounces of juice. Lin drank $\frac{1}{4}$ less than that.
- **2.** Lin ran x miles. Diego ran $\frac{3}{4}$ more



3. Diego bought x pounds of almonds. Lin bought $\frac{1}{4}$ of that.

Problem 2

Elena walked 12 miles. Then she walked $\frac{1}{4}$ that distance. How far did she walk all together? Select **all** that apply.

A. 12 +
$$\frac{1}{4}$$

B.
$$12 \cdot \frac{1}{4}$$

C. 12 +
$$\frac{1}{4}$$
 · 12

D.
$$12(1+\frac{1}{4})$$

E.
$$12 \cdot \frac{3}{4}$$

F.
$$12 \cdot \frac{5}{4}$$

Problem 3

Write a story that can be represented by the equation $y = x + \frac{1}{4}x$.

Sample response: Andre slept x hours. Diego slept $\frac{1}{4}$ more than that.

Problem 4

from Unit 4, Lesson 1

Select **all** ratios that are equivalent to 4:5.

A. 2:2.5

B. 2:3

C. 3:3.75

D. 7:8

E. 8:10

F. 14:27.5

Problem 5

from Unit 3, Lesson 10

Jada is making circular birthday invitations for her friends. The diameter of the circle is 12 cm. She bought 180 cm of ribbon to glue around the edge of each invitation. How many invitations can she make?

4 cards (each card needs 12 π , or about 37.7, cm of ribbon, and 180 ÷ 37.7 \approx 4.77)

Problem 6

from an earlier course

- **a.** Find 10% of 85. **8.5**
- **b.** Find 1% of 85. 0.85
- **c.** Find 31% of 85. **26.35**

