Lesson 1

Exponent Review

Goals

- Comprehend that repeated division by 2 is equivalent to repeated multiplication by one-half.
- Create an expression that represents repeated multiplication, and explain (orally) how the structure of the expression helps compare quantities.

Learning Targets

- I can use exponents to describe repeated multiplication.
- I understand the meaning of a term with an exponent.

Lesson Narrative

The purpose of this lesson is to review concepts regarding whole number exponents and to introduce the **base (of an exponent)** as the factor that is multiplied repeatedly. In a previous course, students learned that an **exponent** tells how many factors of the base to multiply.

In this lesson, students make sense of problems presented in a new way. They are introduced to the idea that repeated division by a number can be thought of as repeated multiplication by the reciprocal of that number, which plays a key role in later work with negative exponents.

Student Learning Goal

Let's review exponents.

Instructional Routines

- Poll the Class
- Which Three Go Together?

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)
- MLR7: Compare and Connect (Activity 2)

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Which Three Go Together: Fours

5 min

Activity Narrative

This *Warm-up* prompts students to compare four expressions. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terminology that students know and how they talk about **bases** and **exponents**.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. 2^4

B. 4^2

C. $4^4 - 240$

D. 4^4

Sample responses:

A, B, and C go together because:

- They all equal 16.

A, B, and D go together because:

- They all have a single term.
- They don't have any subtraction.

A, C, and D go together because:

- They all have a term with an exponent of 4.
- They all have a term that is raised to the power of 4.

B, C, and D go together because:

- They all have a 4 being raised to a power.
- They all have a term with a base of 4.

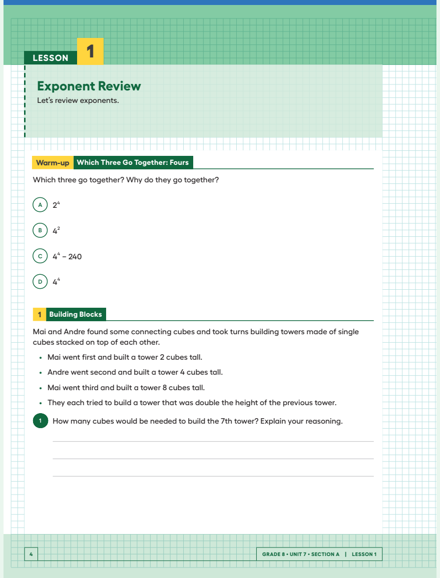
Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL.

Student Workbook



Instructional Routines

Poll the Class

ilclass.com/r/10694985
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.
Keep student workbooks or devices closed. Display only the problem stem and bullets, without revealing the questions.

"We are going to read this information 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the question(s). Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Develop Expression and Communication.
Give students access to connecting cubes or other similar stackable objects.
Supports accessibility for: Conceptual Processing, Organization

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology that they use, such as "power" or "exponent," and to clarify their reasoning as needed. For example, a student may say that A, B, and C all equal 16. Ask how they know this is the case.

If not mentioned by students, tell them that the **base (of an exponent)** tells what factor to multiply repeatedly. If necessary, remind students that the **exponent** tells how many factors to multiply. For example, in 2^4, the base is 2 and the exponent is 4, which means that there are 4 factors of 2 being multiplied together. So $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

Activity 1

Building Blocks

15 min

Activity Narrative

The purpose of this activity is to remind students about the need for exponential notation when thinking about problems involving repeated multiplication. Before students begin working, they are explicitly asked to make an estimate. Making a reasonable estimate and comparing a computed value to one's estimate is often an important aspect of making sense of problems.

Launch

Read, or invite a student to read, the first sentence and the bullets aloud. Make sure that students understand how each successive tower is constructed. If possible, show physical examples of the first few towers using connecting cubes, blocks, or any other similarly stackable object. Otherwise, display this image for all to see.

Before students do calculations or any other work, poll the class about which tower they think will require more than 1 million connecting cubes. Display the results of the poll for all to see.

If possible, display the "Doubling Coins" applet to help students visualize the growth. While the applet shows coins that double each day instead of cubes, emphasize how doubling can result in very large numbers, even after just 14 days.

The Geogebra applet "8.7.1.2 Doubling Coins" is available here:
ilclass.com/l/395907

Reveal to students that the 20th tower will require 1,048,576 cubes. Explain that this was determined by multiplying repeatedly by a factor of 2. If necessary, remind students that an expression with 20 factors that are 2 can also be written as 2^{20}.

Student Task Statement

Mai and Andre found some connecting cubes and took turns building towers made of single cubes stacked on top of each other.

- Mai went first and built a tower 2 cubes tall.
- Andre went second and built a tower 4 cubes tall.
- Mai went third and built a tower 8 cubes tall.
- They each tried to build a tower that was double the height of the previous tower.

1. How many cubes would be needed to build the 7th tower? Explain your reasoning.

   The 7th tower would need 128 cubes.

2. The number of cubes needed to build the 25th tower is very, very large. Write an expression to represent this number without computing its value.

   2^{25}

3. The 28th tower would require even more cubes than the 25th tower. How many times more cubes are needed to build the 28th tower compared to the 25th tower?

   Sample response: The 28th tower would require 8 times more cubes than the 25th tower because the number of cubes needed has doubled 3 times.

Activity Synthesis

The goal of this discussion is for students to understand exponential notation and use it to reason about a situation that involves repeated multiplication. Display the table for all to see.

expanded	exponent	value
2	2^1	2
$2 \cdot 2 \cdot 2 \cdot 2$	2^4	16
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6	64

Tell students that exponents allow us to perform operations and reason about numbers that are too large to calculate by hand. Explain that the “expanded” column shows the factors being multiplied, the “exponent” column shows how to write the repeated multiplication more succinctly with exponents, and the “value” column shows the decimal value. Discuss with students:

“How many times larger is 2^6 than 2^4?”

4 times larger

“How does the “expanded” column help explain this?”

The “expanded” column shows that 2^6 has 2 more factors of 2 compared to 2^4, and $2 \cdot 2 = 4$

“How many times larger is 2^{20} than 2^{16}?”

16 times larger because 2^{20} has 4 more factors that are 2, and $2 \cdot 2 \cdot 2 \cdot 2 = 16$

Building on Student Thinking

If some students interpret the last question to mean “How many more cubes does the 28th tower have compared to the 25th tower,” or if students think they need to know exactly how many cubes are used in each tower to be able to answer this question, consider asking:

“How many cubes are needed for the 5th tower?”

32 or 2^5 cubes

“How many cubes are needed for the 6th tower?”

64 or 2^6 cubes

“How many times bigger is 64 compared to 32? How many times bigger is 2^6 compared to 2^5?”

Either way the values are expressed, each new tower needs 2 times as many cubes as the one before it.

Student Workbook


1 Building Blocks

2. The number of cubes needed to build the 25th tower is very, very large. Write an expression to represent this number without computing its value.

3. The 28th tower would require even more cubes than the 25th tower. How many times more cubes are needed to build the 28th tower compared to the 25th tower?

2 Shrinking Tower

Imagine a tall tower that is different from any other tower. One day this tower is only half as tall as it was the day before!

- On the second day, the tower is $\frac{1}{4}$ of its original height.
- On the third day, the tower is $\frac{1}{8}$ of its original height.

1. What fraction of the original height is the tower after 6 days?

2. What fraction of the original height is the tower after 28 days? Write an expression to describe this without computing its value.

3. Will the tower ever disappear completely? If so, after how many days?

GRADE 8 • UNIT 7 • SECTION A | LESSON 1 5

Activity 2

Shrinking Tower

15 min

Activity Narrative

This activity prompts students to think about repeated division by 2 as being equivalent to repeated multiplication by $\frac{1}{2}$. This will lay the foundation for understanding negative exponents in later lessons.

Launch

Remind students that in the previous activity where Mai and Andre took turns building towers, the 20th tower would require over 1 million cubes. Ask students if they think this tower would be realistic to build.

No, it would not.

Then ask students to think about tall buildings or structures (silos, windmills, cell phone towers, etc.) in the local community. Ask students to imagine one of these tall structures suddenly being half as tall. Discuss whether this would be beneficial or detrimental to the community. For example, a cell phone tower that was suddenly half as tall might be beneficial since it is less of an eyesore, while a silo that stores grain suddenly being half its original height may be detrimental.

If necessary, explain to students that just as a tower made out of 1 million cubes would be unrealistic to build, the tower that gets shorter and shorter in the *Task Statement* is also unrealistic.

Arrange students in groups of 2. Allow 5 minutes of work time followed by a brief partner discussion. Conclude with a whole-class discussion.

Student Task Statement

Imagine a tall tower that is different from any other tower. One day this tower is only half as tall as it was the day before!

- On the second day, the tower is $\frac{1}{4}$ of its original height.
- On the third day, the tower is $\frac{1}{8}$ of its original height.

1. What fraction of the original height is the tower after 6 days? $\frac{1}{64}$
2. What fraction of the original height is the tower after 28 days?

   Write an expression to describe this without computing its value. $\frac{1}{2^{28}}$ or $\left(\frac{1}{2}\right)^{28}$
3. Will the tower ever disappear completely? If so, after how many days?

   The tower will never completely disappear because each time the tower becomes half as tall, there is still the other half of the height left.

Are You Ready for More?

A rancher is tracking the ancestry of his prize cattle. Each cow has 2 parents and each parent also has two parents.

1. Draw a family tree showing a cow, its parents, its grandparents, and its great-grandparents.

   A diagram with a tree structure that shows the cow on one level (0 generations back), its two parents on the next, its four grandparents on the next, and its eight great-grandparents last (3 generations back).

2. We say that the cow's eight great-grandparents are "three generations back" from the cow. At which generation back would a cow have 262,144 ancestors?

   18 generations, because $2^{18} = 262{,}144$

Activity Synthesis

The goal of this discussion is for students to understand that repeated division by 2 corresponds to repeated multiplication by $\frac{1}{2}$. Ask students to discuss their responses with their partner before discussing the following questions:

- *"What are some different ways we can describe what is happening to the height of this tower each day?"*

  The height is cut in half. The height is getting divided by 2.

- *"How can division by 2 be represented with multiplication? That is, if we want to cut something in half, what value can we multiply by?"*

  We can multiply by $\frac{1}{2}$ or 0.5.

- *"How can repeated division by 2 be represented with exponents?"*

  Since dividing by 2 is the same as multiplying by $\frac{1}{2}$, repeated division by 2 corresponds to repeated multiplication by $\frac{1}{2}$, which can be written as $\left(\frac{1}{2}\right)^n$ or $\frac{1}{2^n}$.

Lesson Synthesis

The goal of the discussion is to make sure that students understand that exponents indicate repeated multiplication. Here are some questions for discussion:

- *"What does it mean when we write 2^{42}?"*

  It means that 2 has been multiplied repeatedly 42 times. To expand this into factors would show 42 factors that are 2.

- *"How many times larger is 2^{45} than 2^{42}?"*

  2^{45} is 8 times larger than 2^{42} because it has 3 more factors that are 2, so it has been multiplied by 2 an extra 3 times.

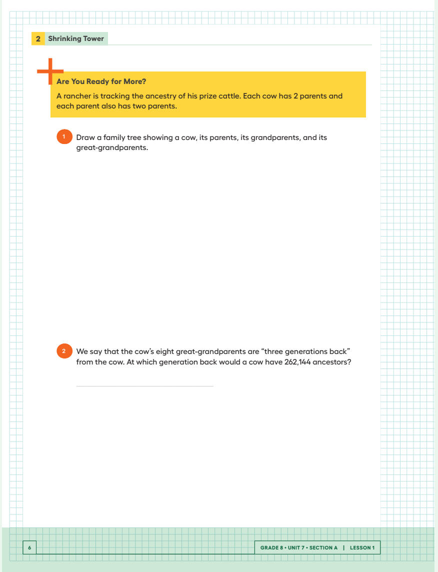
- *"What does it mean when I write $\left(\frac{1}{2}\right)^{42}$?"*

  It means that $\frac{1}{2}$ has been multiplied repeatedly 42 times. To expand this into factors would show 42 factors that are $\frac{1}{2}$.

- *"Which is greater, $\left(\frac{1}{2}\right)^{42}$ or $\left(\frac{1}{2}\right)^{45}$? Why?"*

  $\left(\frac{1}{2}\right)^{42}$ is greater because multiplying by $\frac{1}{2}$ results in a value closer to 0 and $\left(\frac{1}{2}\right)^{45}$ has been multiplied by $\frac{1}{2}$ three extra times.

Student Workbook



(Activity 2, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations for the fraction of the tower's original height on the 28th day: $\left(\frac{1}{2}\right)^{28}$ and $\frac{1}{2^{28}}$. Ask

"How are the two expressions the same?"
"How are they different?"

and

"How does the day show up in each expression?"

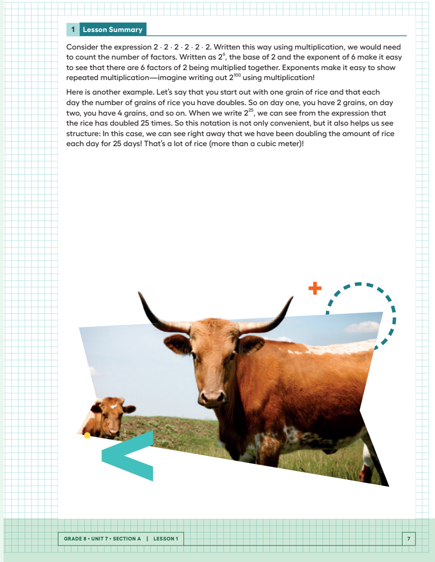
Advances: Representing, Conversing

Responding To Student Thinking

Press Pause
If most students struggle with expressing repeated multiplication with exponents, make time to revisit related work in the Section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 6, Section C Expressions with Exponent

Student Workbook



Lesson Summary

Consider the expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. Written this way using multiplication, we would need to count the number of factors. Written as 2^6, the base of 2 and the exponent of 6 make it easy to see that there are 6 factors of 2 being multiplied together. Exponents make it easy to show repeated multiplication — imagine writing out 2^{100} using multiplication!

Here is another example. Let's say that you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25}, we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: In this case, we can see right away that we have been doubling the amount of rice each day for 25 days! That's a lot of rice (more than a cubic meter)!

Cool-down

Exponent Check

5 min

Student Task Statement

1. What is the value of 3^4?

   81, because $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$.

2. How many times bigger is 3^{15} compared to 3^{12}?

   3^{15} is 27 times larger than 3^{12}, because 3^{15} has 3 more factors that are 3 and $3^3 = 27$

Lesson 1

Practice Problems

6 Problems

Problem 1

Write each expression using an exponent:

a. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ 7^5

b. $\left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$ $\left(\frac{4}{5}\right)^5$

c. $(9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$ $(9.3)^8$

d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.) 2^8

Problem 2

Evaluate each expression:

a. 2^5 32

b. 3^3 27

c. 4^3 64

d. 6^2 36

e. $\left(\frac{1}{2}\right)^4$ $\frac{1}{16}$

f. $\left(\frac{1}{3}\right)^2$ $\frac{1}{9}$

Problem 3

Clare made \$160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

a. How much money will Clare have in her account after 1 year? After 2 years? \$164.80, \$169.74

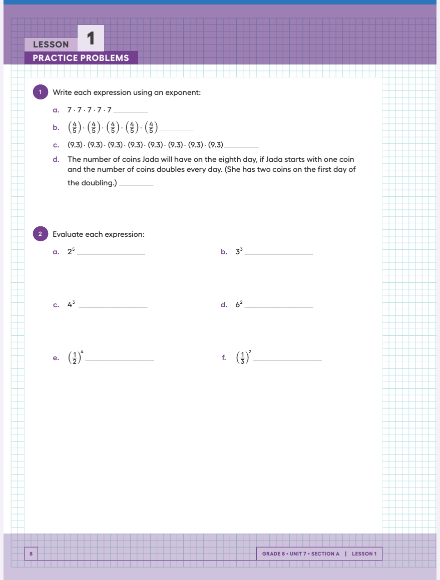
b. How much money will Clare have in her account after 5 years? Explain your reasoning.

\$185.48

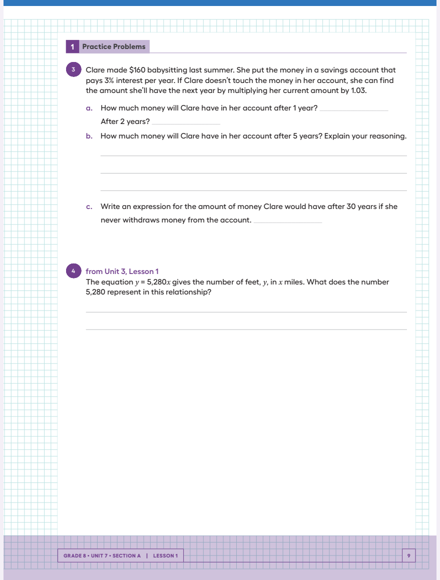
Sample reasoning: $160 \cdot 1.03^5$ (or multiply 160 by 1.03 five times)

c. Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account. $160 \cdot 1.03^{30}$

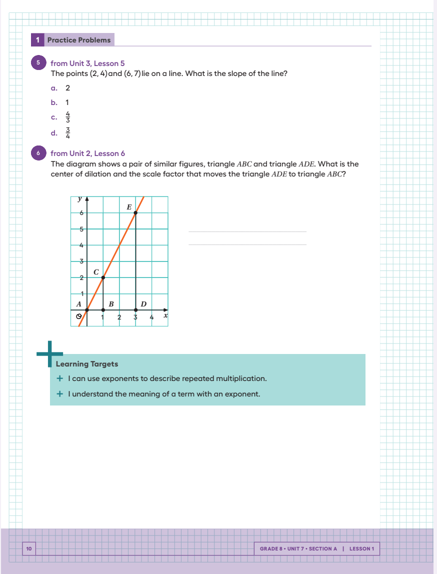
Student Workbook



Student Workbook



Student Workbook



Problem 4

from Unit 3, Lesson 1

The equation $y = 5{,}280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

Sample response: There are 5,280 feet in every mile. For example, each additional mile that someone travels is equivalent to traveling an additional 5,280 feet.

Problem 5

from Unit 3, Lesson 5

The points (2, 4) and (6, 7) lie on a line. What is the slope of the line?

A. 2

B. 1

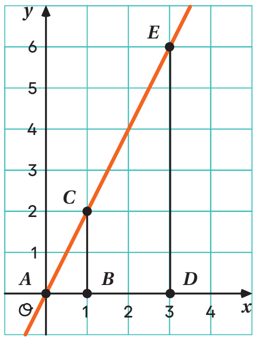
C. $\frac{4}{3}$

D. $\frac{3}{4}$

Problem 6

from Unit 2, Lesson 6

The diagram shows a pair of similar figures, triangle ABC and triangle ADE. What is the center of dilation and the scale factor that moves the triangle ADE to triangle ABC?



Center: A, scale factor: $\frac{1}{3}$