Lesson 17

Building Prisms (Optional)

Goals

- Compare and contrast (orally) triangular prisms, including comparisons of their height, cross sections, surface area, and volume.
- Compose two triangular prisms into a new prism, and describe (orally and in writing) the composite shape.
- Draw and assemble a net of a triangular prism, given two side lengths of the prism's base and one angle measurement.

Learning Target

I can build a triangular prism from scratch.

Lesson Narrative

This lesson is optional. In this culminating lesson, students use what they have learned in this unit to build a triangular prism, given some measurements for the angles and sides of the triangular base. There are 4 possible solutions.

This lesson is organized into three activities. First, students draw triangles that could be the base of the prism, given the conditions. They select one of the 4 possible solutions and calculate its area. Then, students create and assemble a net for the prism. They calculate its volume and surface area. In the last activity, students experiment with different ways two prisms could be put together to make one larger prism. They analyze how different configurations affect the volume and surface area of the composed prism.

Student Learning Goal

Let's build a triangular prism from scratch.

Access for Students with Diverse Abilities

- Engagement (Activity 3)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Compasses: Activity 1
- Geometry toolkits: Activity 1, Activity 2
- Rulers marked with centimeters: Activity 1, Activity 2

Required Preparation

Activity 2:
Rulers should be part of the geometry toolkit, but make sure that the rulers provided have markings in centimeters.

Lesson Timeline

5 min	10 min	10 min	10 min	10 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis

Inspire Math

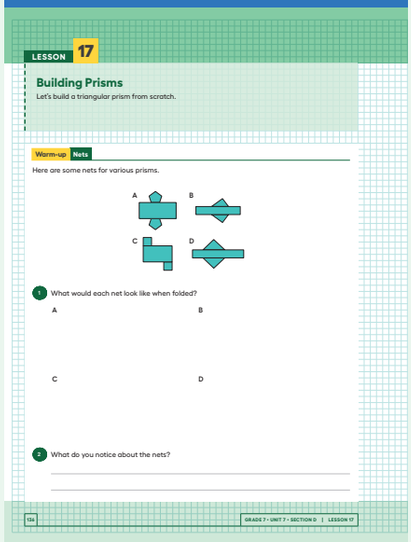
3-D Printed Houses video

Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614243

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 17
Building Prisms
Let's build a triangular prism from scratch.
Warm-up Nets
Here are some nets for various prisms.
A B C D
1 What would each net look like when folded?
A B C D
2 What do you notice about the nets?
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Warm-up

Nets

5 min

Activity Narrative

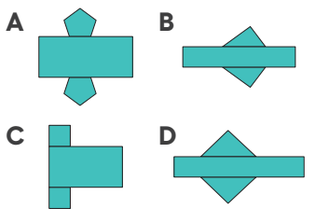
The purpose of this *Warm-up* is for students to reason about prisms formed from various nets. During the partner and whole-group discussions, listen for how students name each prism: pentagonal prism, triangular prism, square prism (but not a cube). Select students who correctly name each prism, and ask them to share during the whole-class discussion.

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time to look at the nets, followed by 2 minutes to describe, with a partner, each net.

Student Task Statement

Here are some nets for various prisms.



1. What would each net look like when folded?

   A: pentagonal prism

   B: triangular prism where the base is a right triangle

   C: square prism (but not a cube),

   D: triangular prism where the base is an isosceles triangle.

2. What do you notice about the nets?

   Sample responses:
   - They all have a long rectangle in the middle.
   - The bases on the bottom are upside down compared to the bases on the top.
   - There's one base on each long side of the rectangle.

Activity Synthesis

The goal of this discussion is to identify the prisms from their nets and to share key features of nets of prisms. Ask selected students to describe the object formed by each net. Record and display their responses for all to see. If a student's description does not include the name of the prism, ask other students to name the object and to explain how they know.

Ask students to share what they notice about all of the nets. Record and display their responses for all to see. While students may notice many things, important ideas to highlight during the discussion are:

- They all have a long rectangle in the middle.
- The bases on top and bottom are upside down.
- There's one base on each side of the rectangle.

Activity 1: Optional
Making the Base

10 min

Activity Narrative

This activity reviews the work that students did previously when they drew shapes with given conditions. Students draw as many different triangles as they can that could be the base of the triangular prism, given two side lengths and one angle measurement for the triangle.

In preparation for calculating surface area and volume in the next activity, students select one of their triangles and find its area. This will require them to draw and measure the height of the triangle. As needed, remind students that the height must be perpendicular to whichever side they are using as the base of their triangle. Also, prompt students to measure the height as precisely as possible, because it will influence the accuracy of their later calculations.

Launch

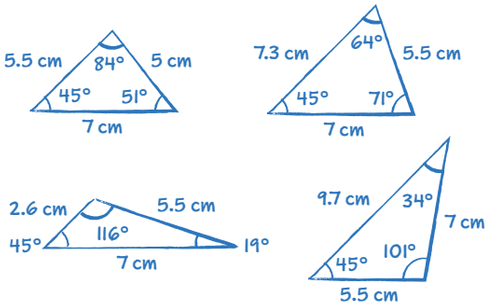
Provide access to geometry toolkits and compasses.

Student Task Statement

The base of a triangular prism has one side that is 7 cm long, one side that is 5.5 cm long, and one angle that measures 45°.

1. Draw as many different triangles as you can with these given measurements.

There are four possible triangles.



2. Select one of the triangles you have drawn. Measure and calculate to approximate its area. Explain or show your reasoning.

Sample responses:

- The area is approximately 13.6 cm^2 for the triangle with the third side of 5.0 cm.
- The area is approximately 18.2 cm^2 for the triangle with the third side of 7.3 cm.
- The area is approximately 6.3 cm^2 for the triangle with the third side of 2.6 cm.
- The area is approximately 18.9 cm^2 for the triangle with the third side of 9.7 cm.

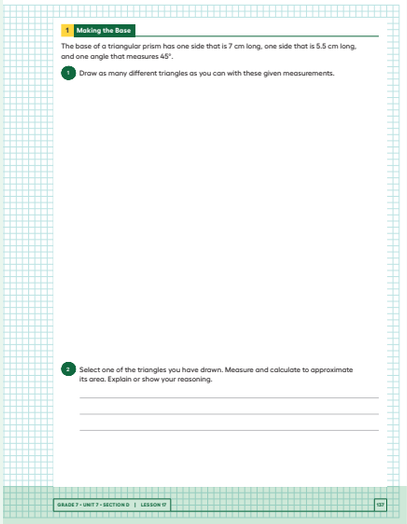
Building on Student Thinking

If students try to multiply two side lengths of the triangle to calculate the area, consider asking:

"Can you explain the parts of a triangle you are using to calculate the area?"

"How could you use an index card or other tool to make sure that the height and base are perpendicular?"

Student Workbook


1 Making the Base

The base of a triangular prism has one side that is 7 cm long, one side that is 5.5 cm long, and one angle that measures 45°.

1 Draw as many different triangles as you can with these given measurements.

2 Select one of the triangles you have drawn. Measure and calculate to approximate its area. Explain or show your reasoning.

Activity Synthesis

The goal is to show the four possible triangles and make sure each student has calculated the area of their triangle correctly because this will affect their calculations in the next activity.

Ask students to share triangles they drew so that everyone has an opportunity to see all four triangles. If any of the four triangles are not presented by students, demonstrate how to construct it. Ensure that the class agrees that 4 unique triangles have the given measurements. The third side length of the triangle could be 5.0 cm, 7.3 cm, 2.6 cm, or 9.7 cm.

Make sure that students have calculated the area of their selected triangle correctly, because this will affect their volume and surface area calculations in the next activity.

If the third side of the triangle is	then the area of the triangle should be about	possible strategies
5.0 cm	13.7 cm^2	$\frac{1}{2} \cdot 7 \cdot (3.9)$ or $\frac{1}{2} \cdot (5.5) \cdot (5.0)$
7.3 cm	18.2 cm^2	$\frac{1}{2} \cdot 7 \cdot (5.2)$ or $\frac{1}{2} \cdot (5.5) \cdot (6.6)$ or $\frac{1}{2} \cdot (7.3) \cdot (5.0)$
2.6 cm	6.3 cm^2	$\frac{1}{2} \cdot 7 \cdot (1.8)$ or $\frac{1}{2} \cdot (5.5) \cdot (2.3)$ or $\frac{1}{2} \cdot (2.6) \cdot (5.0)$
9.7	18.9 cm^2	$\frac{1}{2} \cdot 7 \cdot (5.4)$ or $\frac{1}{2} \cdot (5.5) \cdot (6.9)$ or $\frac{1}{2} \cdot (9.7) \cdot (3.9)$

Activity 2: Optional
Making the Prism

10 min

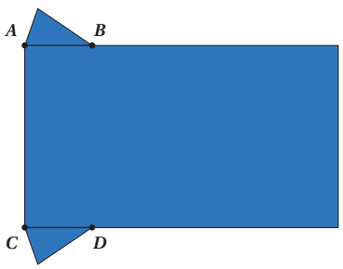
Activity Narrative

In this activity, students take the triangle they selected in the previous activity and use it as the base of their triangular prism.

Launch

Demonstrate the positions at which triangles should attach to the rectangle to form a net, by displaying an example and describing the important parts:

- The triangles must have a vertex at A and C.
- The triangles must be identical copies with one "upside down" from the other.
- Corresponding sides of each triangle must be along the side of the rectangle.



After students have drawn their net and before they cut it out and assemble it, make sure they have correctly positioned their bases—opposite from each other on the top and bottom of the rectangle and reflected. It will also make assembling the net easier for students if they draw lines subdividing the large rectangle into the individual rectangular faces and draw tabs where the faces will be glued or taped together.

Student Task Statement

Look at the Making the Prism resource at the end of this activity. Follow these instructions to complete the net and assemble the triangular prism:

1. Draw an identical copy of the triangle you selected in the previous activity along the top of the rectangle, with one vertex on point A.
2. Draw another copy of your triangle, flipped upside down, along the bottom of the rectangle, with one vertex on point C.
3. Determine how long the rectangle needs to be to wrap all the way around your triangular bases. Pause here so your teacher can review your work.
4. Cut out and assemble your net.

Student Workbook


2 Making the Prism

Your teacher will give you an incomplete net. Follow these instructions to complete the net and assemble the triangular prism:

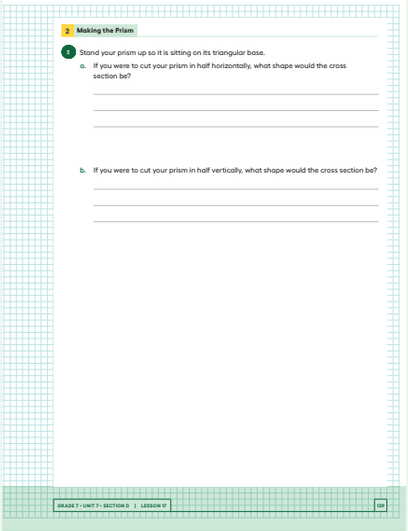
1. Draw an identical copy of the triangle you selected in the previous activity along the top of the rectangle, with one vertex on point A.
2. Draw another copy of your triangle, flipped upside down, along the bottom of the rectangle, with one vertex on point C.
3. Determine how long the rectangle needs to be to wrap all the way around your triangular bases. Pause here so your teacher can review your work.
4. Cut out and assemble your net.

After you finish assembling your triangular prism, answer these questions. Explain or show your reasoning.

1. What is the volume of your prism?

2. What is the surface area of your prism?

Student Workbook


2 Making the Prism

3 Stand your prism up so it is sitting on its triangular base.

a. If you were to cut your prism in half horizontally, what shape would the cross section be?

b. If you were to cut your prism in half vertically, what shape would the cross section be?

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After you finish assembling your triangular prism, answer these questions. Explain or show your reasoning.

1. What is the volume of your prism?

2. What is the surface area of your prism?

Sample responses:

If the third side of the triangle is	then the volume of the solid should be about	and the surface area of the solid should be about
5.0 cm	137 cm^3	202.2 cm^2
7.3 cm	182 cm^3	234.4 cm^2
2.6 cm	63 cm^3	163.6 cm^2
9.7	189 cm^3	259.8 cm^2

3. Stand your prism up so it is sitting on its triangular base.

a. If you were to cut your prism in half horizontally, what shape would the cross section be?

A triangle that is identical to the one I drew as my base.

b. If you were to cut your prism in half vertically, what shape would the cross section be?

A rectangle with one side the same length as the height of the prism.

Activity Synthesis

The goal is to make sure that students understand how the changes in the base triangle affect the prism's surface area and volume. Select students to share their answers for the cross sections. For cross sections taken in these two ways, all triangular prisms should have the same shapes as answers, although the actual size of the cross section will differ based on the size of the base triangle and the height of the prism.

The volume and surface areas will depend on the triangle they have chosen to use as their base.

Select students to share their methods for computing volume and surface area. The base area is important in the calculation of each, so students should use the values they computed in the previous activity.

Activity 3: Optional
Combining Prisms

10 min

Activity Narrative

Students combine their solid with a partner's and examine the new solid's properties.

Launch

Arrange students in groups of 2.

Student Task Statement

1. Compare your prism with your partner's prism. What is the same? What is different?

   Sample response: Both prisms have the same height. At least two side lengths of the base are the same. It is possible that both prisms are identical.

2. Find a way you can put your prism and your partner's prism together to make one new, larger prism. Describe your new prism.

   Sample responses: A taller triangular prism, a prism with a parallelogram base, a wider triangular prism.

3. Draw the base of your new prism and label the lengths of the sides.

   Answers vary.

4. As you answer these questions about your new prism, look for ways that you can use your calculations from the previous activity to help you. Explain or show your reasoning.

   a. What is the area of its base?

      If the students glue two sides together, the area of the base should be the sum of the two individual triangle base areas. If the students glue identical bases together, the area of the base is the same as the original area of one of the bases.

   b. What is its height?

      Sample responses: 10 cm, 20 cm. The height of the prism will depend on the faces used to glue the parts together.

   c. What is its volume?

      The volume of the combined object is the sum of the volume of the two individual triangular prisms.

   d. What is its surface area?

      The surface area of the combined object is the sum of the two individual objects minus twice the area of the shared face.

Access for Students with Diverse Abilities (Activity 3, Launch)

Engagement: Develop Effort and Persistence.
Provide prompts or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide students with the printed *Student Task Statement* to use as a checklist for task completion.
Supports accessibility for: Attention, Social-Emotional Functioning

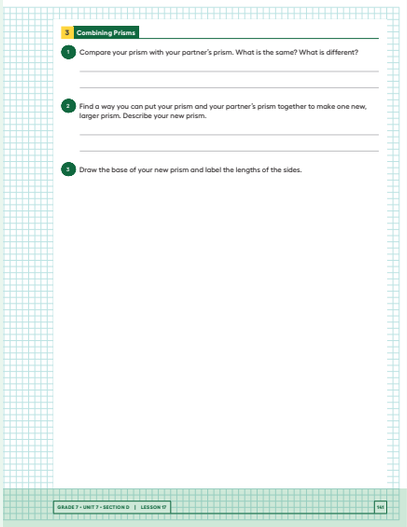
Building on Student Thinking

If students struggle with calculating the area of the base of the new prism, consider asking:

"What shapes is your new base made of?"

"How can you use what you know about the original bases to help you calculate the area of the new base?"

Student Workbook


3 Combining Prisms

1. Compare your prism with your partner's prism. What is the same? What is different?

2. Find a way you can put your prism and your partner's prism together to make one new, larger prism. Describe your new prism.

3. Draw the base of your new prism and label the lengths of the sides.

Student Workbook


3 Combining Prisms

Are You Ready for More?

How many identical copies of your prism would it take you to put together a new larger prism in which every dimension was twice as long?

Learning Targets

+ I can build a triangular prism from scratch.

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Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports. Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "Putting the small sides together makes the biggest prism" as "Putting the triangular faces together makes the longest prism."

Advances: Speaking, Representing

Are You Ready for More?

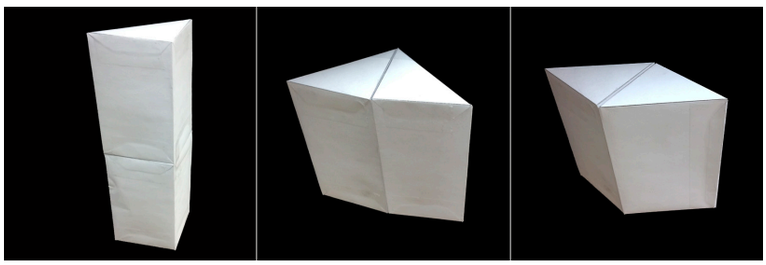
How many identical copies of your prism would it take you to put together a new larger prism in which every dimension was twice as long?

You would need 8 identical prisms (or 7 copies in addition to your original).

Activity Synthesis

The purpose of this discussion is to explore which attributes of the larger prisms were easiest to determine based on the original prisms and which were hardest.

Using two prisms that are identical, demonstrate putting them together against a matching side in various ways.



For each configuration, discuss with students:

- *"How does the area of the new prism compare with the area of the original prisms?"*
- *"How does the height of the new prism compare with the height of the original prisms?"*
- *"How does the volume of the new prism compare with the volume of the original prisms?"*
- *"How does the surface area of the new prism compare with the surface area of the original prisms?"*

In the first configuration,

- The area of the base is the same as in the original prism.
- The height is twice the height of the original prism.
- The volume is twice the volume of the original prism.
- The surface area is less than twice the surface area of the original prism (because of the sides that are put together).

In the second and third configurations,

- The area of the base is twice the original.
- The height is the same as the original.
- The volume is twice the original.
- The surface area is less than twice the original.

How could you put these two prisms together to make the largest surface area possible for the new prism? The smallest surface area possible?

If time allows, this activity can be extended to review relationships between angles as well.

- Take two of the prisms and put them together so that their 45 angles are adjacent.
- Ask students what type of angle is created and what the relationship between the two angles creating that angle must be.

  a right angle, complementary angles
- Take three identical prisms and put them together so that a different angle from each prism is adjacent. Ask students what type of angle is created.

  a straight angle

Ask students to identify pairs of angles that are supplementary.

Lesson Synthesis

Ask students to reflect on what they have learned in this unit, either in writing or by talking to a partner. Here are some suggested prompts:

- *"What is something you learned in this unit that surprised you?"*
- *"What is a new mathematical word you learned in this unit, and what does it mean?"*
- *"What is an idea that you learned about in this unit that is useful in the real world?"*
- *"Describe something that you were confused about at first, but that you understand now."*
- *"Describe something that you found challenging, but that you understood with some effort."*
- *"What is something from this unit that you are still wondering about?"*
- *"What was your favorite activity, and what did you learn from it?"*