Ratios and Rates with Fractions

Goals

Compare and contrast (orally and in writing) different strategies for solving a problem involving equivalent ratios with fractional quantities.

 Explain (orally and in writing) how to find and use a unit rate to solve a problem involving fractional quantities.

Learning Target

I can solve problems about ratios of fractions and decimals.

Lesson Narrative

In this lesson, students compute scale factors and unit rates associated with ratios of fractions. They solve problems involving constant speed where the distance traveled and elapsed time are both given as fractional values. As students make sense of these more complicated problems, they can apply representations and strategies that they learned previously for working with ratios.

The last activity is optional because it provides an opportunity for additional practice scaling a figure with fractional side lengths. This activity works best when each student has access to a device that can run the applet because they will benefit from seeing the relationship in a dynamic way.

Student Learning Goal

Let's calculate some rates with fractions.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR7: Compare and Connect (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)
- MLR6: Three Reads (Activity 3)

Instructional Routines

- Math Talk
- MLR5: Co-Craft Questions
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Preparation

Activity 3:

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline



Warm-up



Activity 1



Activity 2



Activity 3



Lesson Synthesis



5 min

Cool-down

Activity 1

Warm-up

Math Talk: Division



Activity Narrative

This Math Talk focuses on dividing by a fraction. It encourages students to think about the meaning of division and to rely on properties of operations to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students calculate unit rates for quantities with fractional values.

To apply reasoning from previous expressions to help evaluate the next expression, students need to look for and make use of structure.

Warm-up

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- · Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- · Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the Activity Synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

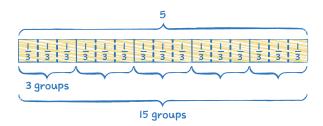
Find each answer mentally.

A. How many $\frac{1}{3}$ s are there in 5?

15

Sample reasoning:

• There are three $\frac{1}{3}$ s in 1. There are 5 times as many $\frac{1}{3}$ s in 5, and 5 · 3 = 15,



B. What is $2 \div \frac{1}{3}$?

Sample reasoning: There are three $\frac{1}{3}$ s in I. There are twice as many $\frac{1}{3}$ s in 2, and $2 \cdot 3 = 6$.

C.What is $\frac{1}{2} \div \frac{1}{3}$?

Sample reasoning: Multiplying a number by 3 gives the same answer as dividing it by $\frac{1}{3}$, so $\frac{1}{2} \cdot 3 = 1\frac{1}{2}$.

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.



Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.



Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Student Workbook

Students may get stuck trying to remember a procedure to divide fractions. Help students reason about the meaning of division by asking

"How many $\frac{1}{3}$ s are there in ____

LESSON 2 Ratios and Rates with Fractions (a) What is $2 + \frac{1}{3}$? © What is $\frac{1}{2} + \frac{1}{3}$? (D) What is $2\frac{1}{2} + \frac{1}{3}$?

GRADE 7 · UNIT 4 · SECTION A | LESSON 2

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

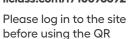
Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ , so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR7: Compare and Connect





code or URL.



Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion.

D.What is
$$2\frac{1}{2} \div \frac{1}{3}$$
?

 $7\frac{1}{2}$

Sample reasoning:

o $2\frac{1}{2} \cdot 3 = 7\frac{1}{2}$

o $2\frac{1}{2}$

3 groups

 $1\frac{1}{2}$ groups

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway is for students to have several methods for explaining why it makes sense that dividing by a fraction gives the same answer as multiplying by its reciprocal.

Activity 1

A Train Is Traveling at ...

10 min

Activity Narrative

In this activity students apply proportional reasoning to solve problems about the speed of a train. This activity serves as an intermediate step building towards unit rates involving compound fractions because the distance the train travels is fractional but the given time is a whole number. As students create representations to help them find equivalent ratios, they are making use of structure.

Monitor for students who:

- Divide $\frac{15}{2}$ by 6 to find the distance traveled in 1 minute, then multiply it by 100.
- Draw a double number line.
- Create a table of equivalent ratios.
- Write an equation to represent the relationship between distance traveled and elapsed time.

The key connection is how each method finds and uses the unit rate.

Launch

Give students 4–5 minutes of quiet work time. Encourage them to find more than one strategy if they have time. Follow with whole-class discussion about the various strategies they used.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

A train is traveling at a constant speed and goes $7\frac{1}{2}$ kilometers in 6 minutes. At that rate:

- 1. How far does the train go in 1 minute?
 - $\frac{5}{4}$ kilometers (or equivalent)
- 2. How far does the train go in 100 minutes?

125 kilometers

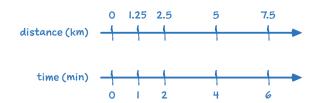
3. How long does it take the train to travel 100 kilometers?

80 minutes

4. Create a representation of your choice that shows the relationship between the elapsed time and distance traveled for this train.

Sample responses:

Double Number Line:



• Table:

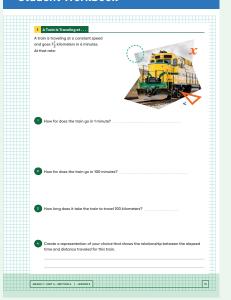
distance (km)	time (min)
7.5	6
<u>5</u> 4	ı
<u>500</u> 4	100

• Equation: d = 1.25t

Building on Student Thinking

Students might calculate the unit rate as $6 \div \frac{15}{2}$. Ask students what this number would mean in this problem? (This number means that it takes $\frac{4}{5}$ of a minute to travel 1 kilometer.) In this case, students should be encouraged to create a table or a double number line, since it will help them make sense of the meaning of the numbers.

Student Workbook



Lesson 2 Warm-up **Activity 1 Activity 2** Activity 3 Lesson Synthesis Cool-down

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.

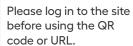
Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "First, I ______ because ..." "I noticed _____ so I ..." "Why did you ...?" "I agree/disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544





Access for Multilingual Learners (Activity 2, Launch)

MLR5: Co-Craft Questions.

Keep student workbooks or devices closed. Display only the problem stem without revealing the question, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Differentiate the degree of difficulty or complexity. Begin with more accessible values. For example, "Diego ran 3 miles in $\frac{1}{2}$ of an hour."

Supports accessibility for: Conceptual Processing, Memory

Activity Synthesis

The goal of this discussion is to review different strategies for solving problems involving constant rates. Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"How do these different representations show the same information?"

"How does the distance traveled in 1 minute show up in each representation?"

"Did anyone solve the problem the same way but would explain it differently?"

The key takeaway is that there are many different ways to represent the situation, but it is often helpful to find the unit rate.

Activity 2

Comparing Running Speeds

10 min

Activity Narrative

In this activity students solve a problem involving constant speed. Both the distance and the time are given as fractions, giving students the opportunity to divide a fraction by a fraction to calculate the unit rate. As students calculate equivalent rates and compare the rates, they are reasoning quantitatively and abstractly.

Monitor for students who use these different strategies to compare the two runners' speeds:

- Find the amount of time it would take each person to run the same distance.
- Find the distance each person could run in the same amount of time.
- Calculate a unit rate for each person. (This is a special case of one of the two previous bullets, depending on whether the unit rate represents the person's speed or pace.)

Launch 🙎

Arrange students in groups of 2.

Give students 3–4 minutes of quiet work time followed by time for partner discussion.

Student Task Statement

Lin ran $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Noah ran $8\frac{2}{3}$ miles in $\frac{4}{3}$ of an hour.

Who ran faster, Noah or Lin? Explain or show your reasoning.

Lin ran faster than Noah.

Sample reasoning:

- Lin ran $6\frac{7}{8}$ miles in one hour since $2\frac{3}{4} \div \frac{2}{5} = 6\frac{7}{8}$.
- Noah ran $6\frac{1}{2}$ miles in one hour since $8\frac{2}{3} \div \frac{4}{3} = 6\frac{1}{2}$.
- It took Lin $\frac{8}{55}$ of an hour to run a mile since $\frac{2}{5} \div 2\frac{3}{4} = \frac{8}{55}$. That is $8\frac{8}{11}$ minutes since $\frac{8}{55} \cdot 60 = 8\frac{8}{11}$.
- It took Noah $\frac{2}{13}$ of an hour to run a mile since $\frac{4}{3} \div 8\frac{2}{3} = \frac{2}{13}$. That is $9\frac{3}{13}$ minutes since $\frac{2}{13} \cdot 60 = 9\frac{3}{13}$.

Are You Ready for More?

Nothing can go faster than the speed of light, which is 299,792,458 meters per second. Which of these are possible?

- 1. Traveling a billion meters in 5 seconds.
 - possible (200,000,000 meters per second)
- 2. Traveling a meter in 2.5 nanoseconds. (A nanosecond is a billionth of a second.) impossible (400,000,000 meters per second)
- **3.** Traveling 1 parsec in 1 year. (A parsec is about 3.26 light years, and a light year is the distance light can travel in a year.)
 - impossible, because traveling I parsec in I year means traveling 3.26 times faster than the speed of light

Activity Synthesis

Invite students to share how they determined whether Noah or Lin ran faster. Ask students to describe similarities and differences between the different strategies and representations used.

To involve more students in the discussion, consider asking:

"Who can restate ______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Highlight any places in the solution process where students divided fractions. Ask students to describe how they found the quotient.

Building on Student Thinking

The Warm-up was intended to remind students of some strategies for dividing fractions by fractions, but students may need additional support working with the numbers in this task.

Students might have a hard time guessing their partner's question given only the answer. Ask their partners to share the process they used to calculate the solution. They might leave out numbers and first describe the general steps they took to find the answer. If their partner is still unable to guess the question, have them share the specific number they used. If they need additional support to guess the question, have their partner show them their work on paper (without sharing the question they answered), and see if this helps them figure out the question.

Student Workbook



Lesson 2 Warm-up Activity 1 Activity 2 **Activity 3** Lesson Synthesis Cool-down

Instructional Routines

MLR6: Three Reads ilclass.com/r/10695568





Access for Multilingual Learners (Activity 3, Launch)

MLR6: Three Reads.

Keep student workbooks or devices closed. Display only the problem stem, without revealing the questions. Say,

"We are going to read this situation 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Building on Student Thinking

Students might get stuck thinking the scaled copy needs to measure 11 inches by 9 inches. Ask students:

"Does the copy of the painting have to cover the entire notebook?"

"What are some other options if the image doesn't cover the entire notebook?"

"What if the image is bigger than the notebook cover? What if it is smaller?"

Activity 3: Optional

Scaling the Mona Lisa



Activity Narrative

There is a digital version of this activity.

In this activity, students determine a scale factor that will create a scaled copy that fits within given maximum dimensions. The ratio between the maximum length and width is not the same as the ratio between the original figure's length and width. Students must choose how much extra space to leave so that the scaled copy keeps the same aspect ratio as the original figure.

There is more than one reasonable solution to this problem, depending on what other aspects students choose to consider. For example, some students might try to find the largest possible scaled copy that will fit on the cover while others may choose to leave extra room on the cover for a title. Another possible approach would be to strategically choose a value that fits inside the maximum and can be calculated efficiently, such as 10 inches by 7 inches. As students reason about the situation and its constraints, they are making sense of the problem.

In the digital version of the activity, students use an applet to resize and reposition an illustration of the Mona Lisa. The applet allows students to visually compare the size of the painting with the size of the notebook. This activity works best when students have access to the applet because they will benefit from seeing the relationship in a dynamic way. If students don't have access, displaying the applet for all to see would be helpful during the *Launch*.

Launch 22

Consider displaying an image of the Mona Lisa painting. Invite students to share what they know about this painting or about its creator, Leonardo da Vinci.

Arrange students in groups of 2.

Give 3–5 minutes of quiet work time to do the problem.

Then ask them to take turns sharing with their partner the method used to calculate scale factor and discussing the reasonableness of their answers.

Lesson 2 Warm-up Activity 1 Activity 2 **Activity 3** Lesson Synthesis Cool-down

Student Task Statement

In real life, the Mona Lisa measures $2\frac{1}{2}$ feet by $1\frac{3}{4}$ feet. A company that makes office supplies wants to print a scaled copy of the Mona Lisa on the cover of a notebook that measures 11 inches by 9 inches.

1. What size should they use for the scaled copy of the Mona Lisa on the notebook cover?

Sample responses:

- 10 inches by 7 inches (Converting to inches by multiplying the number of feet by 12, the dimensions $2\frac{1}{2}$ feet by $1\frac{3}{4}$ feet are equivalent to 30 inches by 21 inches. Multiplying both of these by $\frac{1}{3}$, the dimensions of the scaled copy would be 10 inches by 7 inches. This size would fit on the notebook with some space around the edges.)
- II inches by 7.7 inches (The ratio of $2\frac{1}{2}$ and $1\frac{3}{4}$ is equivalent to 1 to 0.7, which is found by dividing $1\frac{3}{4}$ by $2\frac{1}{2}$. If we multiply both 1 and 0.7 by 11, the dimensions would be 11 inches by 7.7 inches.)
- 2. What is the scale factor from the real painting to its copy on the notebook cover?

Sample responses:

- The scale factor is $\frac{1}{3}$ for a copy that is 10 inches by 7 inches.
- The scale factor is $\frac{11}{30}$ for a copy that is 11 inches by 7.7 inches.
- **3.** Discuss your thinking with your partner. Did you use the same scale factor? If not, is one more reasonable than the other?

No written response required.

Activity Synthesis

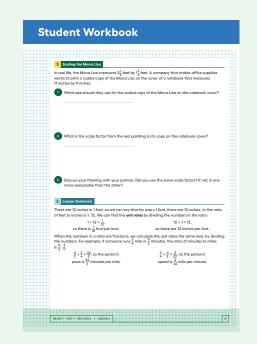
The purpose of this discussion is to highlight the different considerations that students took into account when deciding on an appropriate size for the scaled image. Invite students to share their reasoning for choosing the scale factor they did.

Consider asking students:

"Were there any similarities between the methods you and your partner used? Were there any differences?"

"Is it possible to scale down the Mona Lisa so that it perfectly covers the notebook? Why or why not?"

No, if you made it fit both the width and height, it would look squished vertically or stretched horizontally compared to the painting.



Warm-up

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Synthesis

Share with students,

"Today we worked with ratios and rates that involved fractions."

To review some of the different strategies, consider asking students:

"What are strategies we can use to find solutions to ratio problems that involve fractions?"

double number line, tables, calculating unit rate

"How are those strategies the same or different from ways we previously solved ratio problems that didn't involve fractions?"

They are structurally the same, but the arithmetic might take more time.

Lesson Summary

There are 12 inches in 1 foot, so we can say that for every 1 foot, there are 12 inches, or the ratio of feet to inches is 1:12. We can find the unit rates by dividing the numbers in the ratio:

$$1 \div 12 = \frac{1}{12}$$
, $12 \div 1 = 12$,

so there is $\frac{1}{12}$ foot per inch.

so there are 12 inches per foot.

When the numbers in a ratio are fractions, we calculate the unit rates the same way: by dividing the numbers. For example, if someone runs $\frac{3}{4}$ mile in $\frac{11}{2}$ minutes, the ratio of minutes to miles is $\frac{11}{2}$: $\frac{3}{4}$.

$$\frac{11}{2} \div \frac{3}{4} = \frac{22}{3}$$
, so the person's

$$\frac{3}{4} \div \frac{11}{2} = \frac{3}{22}$$
, so the person's

pace is
$$\frac{22}{3}$$
 minutes per mile.

pace is
$$\frac{22}{3}$$
 minutes per mile. speed is $\frac{3}{22}$ mile per minute.

Cool-down

Comparing Orange Juice Recipes

Student Task Statement

- Clare mixes $2\frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of orange juice concentrate.
- Han mixes $1\frac{2}{3}$ cups of water with $\frac{1}{4}$ cup of orange juice concentrate.

Whose orange juice mixture tastes stronger? Explain or show your reasoning.

Han's mixture tastes stronger

Sample reasoning: Clare uses $7\frac{1}{2}$ cups of water per cup of orange juice concentrate, because $2\frac{1}{2} \div \frac{1}{3} = 7\frac{1}{2}$. Han uses $6\frac{2}{3}$ cups of water per cup of orange juice concentrate, because $1\frac{2}{3} \div \frac{1}{4} = 6\frac{2}{3}$. Han's mixture has less water for the same amount of orange juice concentrate.

Practice Problems

7 Problems

Problem 1

A cyclist rode $3\frac{3}{4}$ miles in $\frac{3}{10}$ hour.

- a. How fast was she going in miles per hour?
 - $12\frac{1}{2}$ miles per hour (or equivalent)
- **b.** At that rate, how long will it take her to go $4\frac{1}{2}$ miles?
 - 0.36 hour (or 21.6 minutes)

Problem 2

A recipe for making incense sticks calls for 6 tablespoons of joss powder, $1\frac{1}{2}$ teaspoons of frankincense, and $\frac{3}{4}$ teaspoon of myrrh. To make incense sticks that would smell the same, how much of the other two ingredients would you need to mix with:

- a. 4 tablespoons of joss powder?
 - I teaspoon of frankincense and $\frac{1}{2}$ teaspoon of myrrh
- **b.** 9 tablespoons of joss powder?
 - $2\frac{1}{4}$ teaspoons of frankincense and $\frac{9}{8}$ teaspoons of myrrh
- c. 1 teaspoon of myrrh?
 - 8 tablespoons of joss powder and 2 teaspoons of frankincense

Problem 3

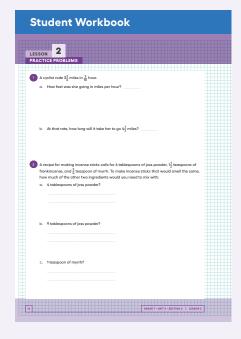
At a deli counter,

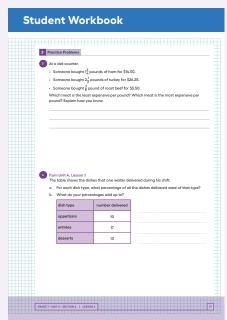
- Someone bought $1\frac{3}{4}$ pounds of ham for \$14.50.
- Someone bought $2\frac{1}{2}$ pounds of turkey for \$26.25.
- Someone bought $\frac{3}{8}$ pound of roast beef for \$5.50.

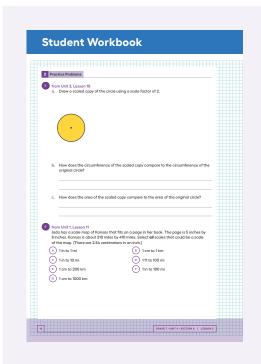
Which meat is the least expensive per pound? Which meat is the most expensive per pound? Explain how you know.

Ham is the least expensive, and roast beef is the most expensive.

Sample reasoning: Ham costs about \$8.29 per pound, because 14.50 ÷ $I_{\frac{3}{4}}^{\frac{3}{4}} = 8\frac{2}{7} \approx 8.29$. Roast beef costs about \$14.67 per pound, because $5.50 \div \frac{3}{8} = 14\frac{2}{3} \approx 14.67$. Turkey costs about \$10.50 per pound, because $26.25 \div 2\frac{1}{2} = 10.50$. While these prices per pound are not exact, they are far enough apart to put the costs in order with certainty.







Problem 4 from Unit 4, Lesson 1

The table shows the dishes that one waiter delivered during his shift.

a. For each dish type, what percentage of all the dishes delivered were of that type?

25% appetizers; 42.5% entrées; 32.5% desserts

b. What do your percentages add up to?

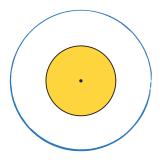
dish type	number delivered
appetizers	10
entrées	17
desserts	13

100%

Problem 5

from Unit 3, Lesson 10

a. Draw a scaled copy of the circle using a scale factor of 2.



b. How does the circumference of the scaled copy compare to the circumference of the original circle?

The circumference of the scaled copy is twice the circumference of the original.

c. How does the area of the scaled copy compare to the area of the original circle?

The area of the scaled copy is four times the area of the original.

Problem 6

from Unit 1, Lesson 11

Jada has a scale map of Kansas that fits on a page in her book. The page is 5 inches by 8 inches. Kansas is about 210 miles by 410 miles. Select **all** scales that could be a scale of the map. (There are 2.54 centimeters in an inch.)

A. 1 in to 1 mi

B. 1 cm to 1 km

C. 1 in to 10 mi

D. 1 ft to 100 mi

E. 1 cm to 200 km

F. 1 in to 100 mi

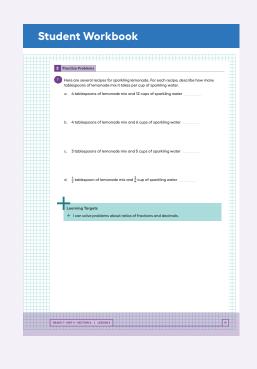
G. 1 cm to 1000 km

Problem 7

Here are several recipes for sparkling lemonade. For each recipe, describe how many tablespoons of lemonade mix it takes per cup of sparkling water.

- **a.** 4 tablespoons of lemonade mix and 12 cups of sparkling water $\frac{4}{12}$ or $\frac{1}{3}$ tablespoon lemonade mix per cup of sparkling water
- **b.** 4 tablespoons of lemonade mix and 6 cups of sparkling water $\frac{4}{6}$ or $\frac{2}{3}$ tablespoon lemonade mix per cup of sparkling water
- c. 3 tablespoons of lemonade mix and 5 cups of sparkling water

 \$\frac{3}{5}\$ tablespoon of lemonade mix per cup of sparkling water
- d. $\frac{1}{2}$ tablespoon of lemonade mix and $\frac{3}{4}$ cup of sparkling water $\frac{2}{3}$ tablespoon of lemonade mix per cup of sparkling water



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