Lesson 10

Changing Scales in Scale Drawings

Goals

- Determine how much actual area is represented by one square unit in a scale drawing.
- Generalize (orally) that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases.
- Reproduce a scale drawing at a different scale and explain (orally) the solution method.

Learning Targets

- Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale.
- I can use a scale drawing to find actual areas.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Math Community Chart: Lesson
- Math Community Chart: Warm-up
- Geometry toolkits: Activity 2

Materials to Copy

- Same Plot, Different Drawings Cards (1 copy for every 24 students): Activity 1

Lesson Narrative

In this lesson, students are given a scale drawing and asked to recreate it at a different scale. They also examine how much area, on the actual object, is represented by 1 square unit on the scale drawing. For example, if the scale is 1 cm to 50 m, then 1 cm^2 represents $50 \cdot 50$, or 2,500 m^2.

Two possible strategies to reproduce a scale drawing at a different scale are:

- Calculating the actual lengths and then using the new scale to find lengths on the new scale drawing
- Relating the two scales and calculating the lengths for the new scale drawing using corresponding lengths on the given drawing

Throughout this lesson, students observe and explain structure, both when they reproduce a scale drawing at a different scale and when they study how the area of a scale drawing depends on the scale.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Lesson 10

Changing Scales in Scale Drawings

Lesson Narrative (continued)

Math Community

Today, students use sticky notes to document actions in the “Doing Math” sections of the Math Community Chart that they see or hear throughout the lesson. During the *Lesson Synthesis*, students share what they noticed, and then they suggest additions for the chart as part of the *Cool-down*. The work today continues to build a foundation for developing math community norms in a later exercise and is the start of students identifying strengths in the actions of their peers.

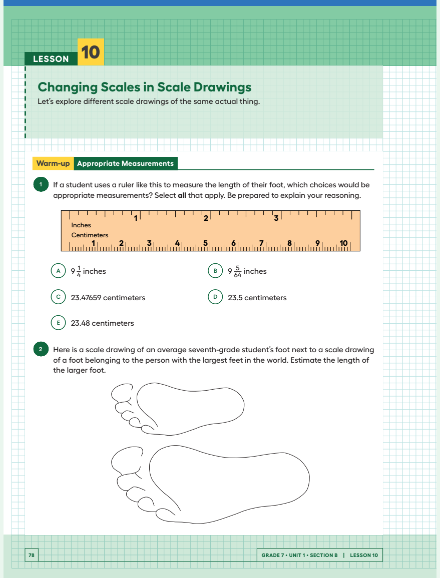
Student Learning Goal

Let’s explore different scale drawings of the same actual thing.

Building on Student Thinking

Some students may say the large foot is about $3\frac{1}{2}$ inches or about 9 centimeters long, because they assume the ruler shown in the first question is at the same scale as the feet shown in the second question. Explain that the images are drawn at different scales.

Student Workbook



Warm-up
Appropriate Measurements

5 min

Activity Narrative

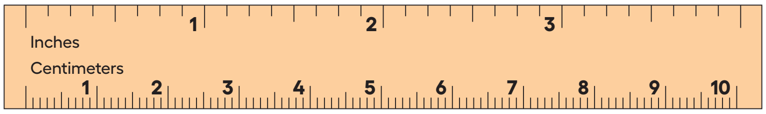
This *Warm-up* prompts students to attend to precision in measurements, which will be important in upcoming work.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time to estimate the length of their own foot in centimeters or inches, and a moment to share their estimate with a partner. Then, ask them to complete the task.

Student Task Statement

1. If a student uses a ruler like this to measure the length of their foot, which choices would be appropriate measurements? Select **all** that apply. Be prepared to explain your reasoning.



A. $9\frac{1}{4}$ inches

B. $9\frac{5}{64}$ inches

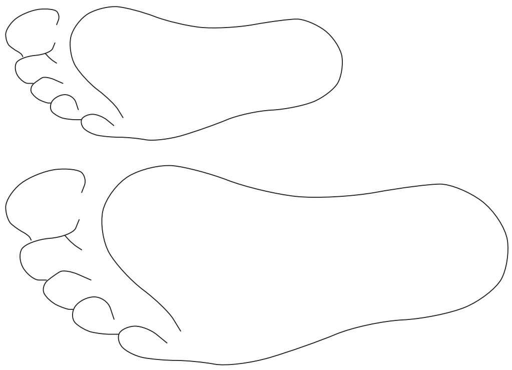
C. 23.47659 centimeters

D. 23.5 centimeters

E. 23.48 centimeters

Sample reasoning: Since the ruler is only marked in $\frac{1}{8}$ inches and $\frac{1}{10}$ centimeter, we could not get measurements as precise as B, C, or E.

2. Here is a scale drawing of an average seventh-grade student's foot next to a scale drawing of a foot belonging to the person with the largest feet in the world. Estimate the length of the larger foot.



The largest foot in the world is about 1.5 times as long as the average seventh grader's foot. Sample reasoning: A seventh grader's foot is about 10 inches long, so the largest foot is about 15 inches or 1 foot and 3 inches long.

Activity Synthesis

Select a few students to share the measurements they think would be appropriate based on the given ruler. Consider displaying the picture of the ruler for all to see and recording students' responses on it. After each response, poll the class on whether they agree or disagree.

If students consider B, C, or E to be an appropriate measurement, ask them to share how to get such a level of precision on the ruler. Make sure students understand that reporting measurements to the nearest $\frac{1}{64}$ of an inch or to the hundred-thousandths of a centimeter would not be appropriate (i.e., show that the ruler does not allow for these levels of precision).

Choice E of 23.48 cm may merit specific attention. With the ruler, it is possible to *guess* that the hundredths place is an 8. This may even be correct. The problem with reporting the measurement in this way is that someone who sees this might misinterpret it and imagine that an extremely accurate measuring device was used to measure the foot, rather than this ruler. The way a measurement is reported reflects how the measurement was taken.

Next, invite students to share their estimates for the length of the large foot. Since it is difficult to measure the length of these feet very precisely, these measurements should not be reported with a high level of precision; the nearest centimeter would be appropriate.

Math Community

After the *Warm-up*, display the revised Math Community Chart created from student responses in Exercise 3. Tell students that today they are going to monitor for two things:

- "Doing Math" actions from the chart that they see or hear happening.
- "Doing Math" actions that they see or hear that they think should be added to the chart.

Provide sticky notes for students to record what they see and hear during the lesson.

Activity 1
Same Plot, Different Drawings

15 min

Activity Narrative

In this activity, students create a scale drawing and then compare multiple scale drawings of the same thing at different scales. Each group member uses a different scale to calculate scaled lengths of the same plot of land, draw a scale drawing, and calculate its scaled area.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Narrative)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

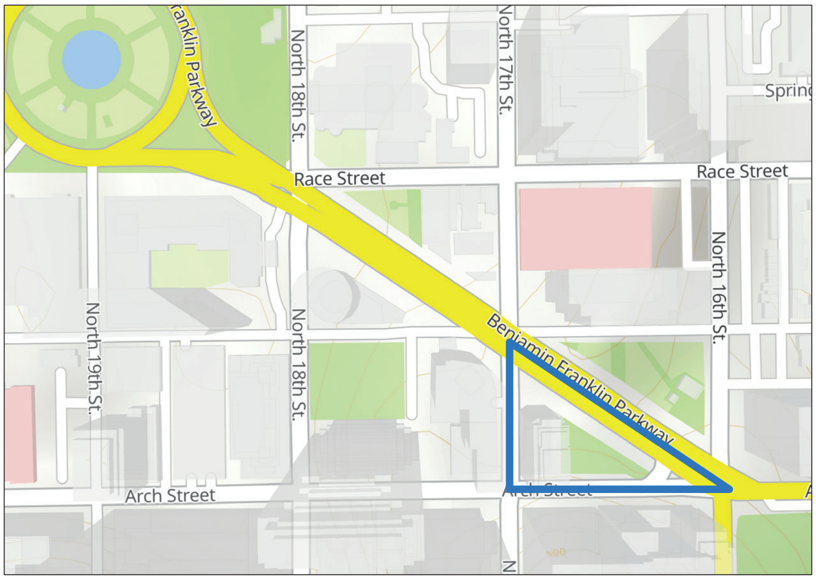
Next, each group member uses a different scale to calculate scaled lengths of the same plot of land, draw a scale drawing, and calculate its scaled area. The group then orders the different drawings and analyzes them. They think about how many square meters of actual area are represented by one square centimeter on each drawing. Students are likely to determine this value in two ways:

- By visualizing what a 1 × 1 centimeter square represents at a given scale (for example, at a scale of 1 cm to 5 m, each 1 cm^2 represents $5 \cdot 5$, or 25 m^2).
- By dividing the actual area represented by the scale drawing by the area of their scale drawing.

As students examine the relationships between scale, lengths in scale drawings, and area in scale drawings, they practice looking for and making use of mathematical structure.

Launch

Display this map of a neighborhood in Philadelphia for all to see. Tell students that they are going to create a new map of the triangular piece of land at a different scale.



Tell students that the actual base of the triangle is 120 meters and its actual height is 90 meters. Ask,

"What is the area of the plot of land?"

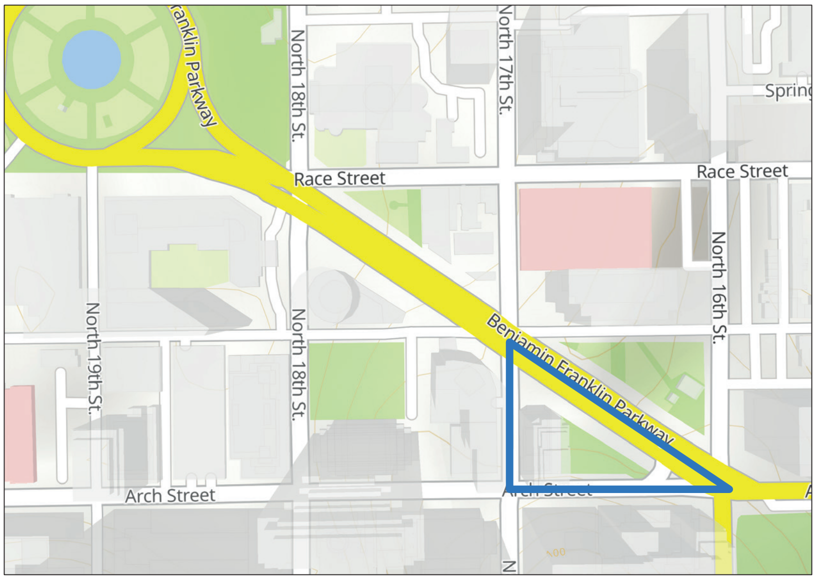
5,400 square meters—one half the base times the height of the triangle.

Arrange students in groups of 5–6 and provide access to centimeter graph paper. Assign each student in a group a different scale (from the blackline master) to use to create a scale drawing. Remind students to include the units in their measurements. Give students 4–5 minutes of quiet work time to answer the first 3 questions.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson. Give students 3–4 minutes to work on the last question in their groups.

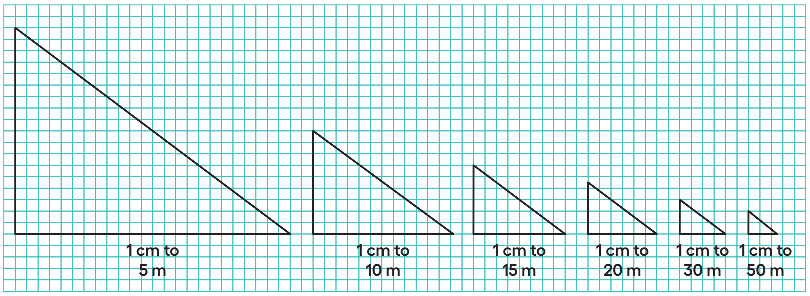
Student Task Statement

Here is a map showing a plot of land in the shape of a right triangle.



1. Your teacher will assign you a scale to use. On centimeter graph paper, make a scale drawing of the plot of land. Make sure to write your scale on your drawing.

   Right triangles of various sizes:



2. What is the area of the triangle you drew?

   Depends on the assigned scale.

   Explain or show your reasoning.

   Sample responses:

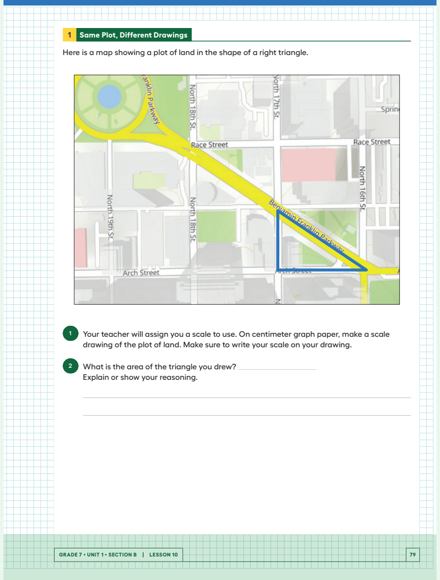
   - 216 cm², because $\frac{1}{2} \cdot 24 \cdot 18 = 216$
   - 54 cm², because $\frac{1}{2} \cdot 12 \cdot 9 = 54$
   - 24 cm², because $\frac{1}{2} \cdot 8 \cdot 6 = 24$
   - 13.5 cm², because $\frac{1}{2} \cdot 6 \cdot (4.5) = 13.5$
   - 6 cm², because $\frac{1}{2} \cdot 4 \cdot 3 = 6$
   - 2.16 cm², because $\frac{1}{2} \cdot (2.4) \cdot (1.8) = 2.16$

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook



Student Workbook


1 Same Plot, Different Drawings

3 How many square meters are represented by 1 square centimeter in your drawing?

4 After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?

Are You Ready for More?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1 Noah uses the scale 1 cm to 200 m.

2 Elena uses the scale 2 cm to 25 m.

3. How many square meters are represented by 1 square centimeter in your drawing?

Depends on the assigned scale. Sample responses:

- 25 m², because 5400 ÷ 216 = 25
- 100 m², because 5400 ÷ 54 = 100
- 225 m², because 5400 ÷ 24 = 225
- 400 m², because 5400 ÷ 13.5 = 400
- 900 m², because 5400 ÷ 6 = 900
- 2,500 m², because 5400 ÷ 2.16 = 2500

4. After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?

The smaller the number of meters represented by one centimeter, the larger the scale drawing is.

Are You Ready for More?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1. Noah uses the scale 1 cm to 200 m.

Noah's drawing will be smaller than all the other drawings. The scale that created the smallest drawing so far was 1 cm to 50 m. Each length in a drawing done at 1 cm to 200 m will be 4 times as small as in the 1 cm-to-50 m drawing because every centimeter represents 4 times as much length.

2. Elena uses the scale 2 cm to 25 m.

The scale 2 cm to 25 m is equivalent to 1 cm to 12.5 m, so Elena's drawing will be larger than the 1 cm to 15 m drawing but smaller than the 1 cm to 10 m drawing.

Activity Synthesis

The goal of this discussion is to compare the structure of larger and smaller scale drawings of the same thing, both in terms of length and area. Direct students' attention to the reference created using *Collect and Display*. Ask students to share what they noticed about the scale drawings placed in order. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

To highlight some of the patterns students may have noticed, consider asking questions such as:

"How does a change in the scale influence the size of the drawings?"

As the length being represented by 1 cm gets larger, the size of the drawing decreases.

"Look at the drawings with the scales '1 cm to 5 m' and '1 cm to 15 m.' How do the lengths in these two drawings compare? How do the areas compare?"

The lengths are three times as long in the drawing with the scale '1 cm to 5 m.' The area is 9 times as great.

"Look at the drawings with the scales '1 cm to 5 m' and '1 cm to 50 m.' How do the lengths in these two drawings compare? How do the areas compare?"

The lengths are ten times as long in the drawing with the scale '1 cm to 5 m.' The area is 100 times as great.

Help students to observe and formulate these patterns:

- As the number of meters represented by one centimeter increases, the lengths in the scale drawing decrease.
- As the number of meters represented by one centimeter increases, the area of the scale drawing also decreases, but it decreases by the square of the factor for the lengths (because finding the area means multiplying the length and width, both of which decrease by the same factor).

Activity 2

A New Drawing of the Playground

15 min

Activity Narrative

In this activity, students are given a scale drawing to reproduce at a different scale. This builds on previous work they did creating scale drawings given the actual dimensions and different scales.

There are two different types of reasoning students may apply. Monitor for students who:

- Use the given scale drawing to find the dimensions of the actual school playground and then use those measurements to find the dimensions of the new scale drawing.
- Calculate the measurements in the new drawing directly from the original drawing, by multiplying each length by $\frac{3}{2}$.

Plan to have students present in this order, from more concrete to more efficient. Some students may initially follow the first approach and then figure out the second approach after noticing regularity in their repeated calculations.

Launch

Tell students that they are going to reproduce a scale drawing using a different scale. The scale for the given drawing is 1 cm to 30 meters, and they are going to make a new scale drawing at a scale of 1 cm to 20 meters. Ask them if they think the new drawing will be larger or smaller than the given one.

Arrange students in groups of 3. Make sure students have access to their geometry toolkits. Give students 5 minutes of quiet work time, followed by 3–4 minutes of group discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Instructional Routines

5 Practices

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Building on Student Thinking

Some students may not know how to begin the task. Prompt them to start by calculating the actual length of each side of the playground.

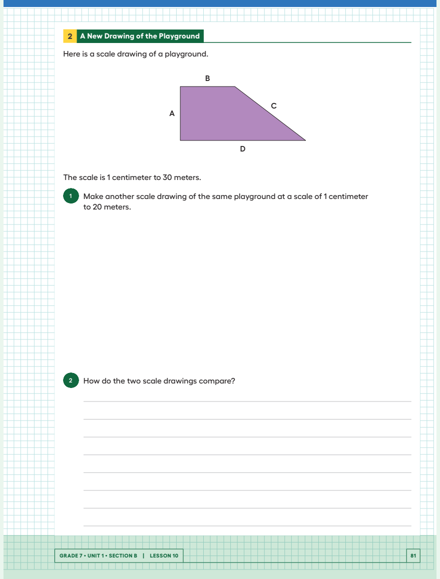
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language. Ask,

> *“Who can restate what ________ shared using mathematical language?”*

Advances: Listening, Speaking

Student Workbook


2 A New Drawing of the Playground

Here is a scale drawing of a playground.

The scale is 1 centimeter to 30 meters.

1 Make another scale drawing of the same playground at a scale of 1 centimeter to 20 meters.

2 How do the two scale drawings compare?

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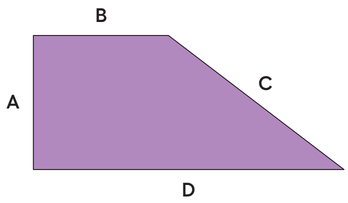
Student Task Statement

Here is a scale drawing of a playground.



The scale is 1 centimeter to 30 meters.

1. Make another scale drawing of the same playground at a scale of 1 centimeter to 20 meters.

   Scaled copy of the drawing where each edge is 1.5 times as long as in the drawing

2. How do the two scale drawings compare?

   The new drawing is larger. Sample reasoning: When 1 cm represents 20 m, it takes 1.5 cm to represent 30 m. So the length measurements on the 1 cm to 20 m scale are 1.5 times as long as they are with the 1 cm to 30 m scale. The area measurements are 2.25 $(1.5 \cdot 1.5)$ times as large.

Activity Synthesis

The purpose of this discussion is to make connections between scale drawings that are reproduced at a different scale and scaled copies. Invite previously selected students to share how they created the new scale drawing. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

“What is the same about these strategies? What is different?”

“Why do the different approaches lead to the same outcome?”

“Did anyone solve the problem the same way, but would explain it differently?”

The key takeaways are:

- The new scale drawing is a scaled copy of the original scale drawing. The shapes are the same (both represent the same playground), but the sizes are different.
- The scale factor from the original drawing to the new drawing is $\frac{3}{2}$ (or 1.5). Multiplying by 30 and then dividing by 20 gets the same result as just multiplying by $\frac{3}{2}$.
- The drawing with a scale of 1 cm to 20 m is larger than the drawing with a scale of 1 cm to 30 m. When 1 centimeter on the scale drawing represents a shorter actual distance, it takes more of those centimeters to describe the object.

If time permits, include area in the discussion of these scale drawings. Consider asking questions like:

“On the original map with the scale of 1 cm to 30 m, how much area does 1 square centimeter represent?”

900 cm^2

💬 *"On the new map with the scale of 1 cm to 20 m, how much area does one square centimeter represent?"*

400 cm^2

💬 *"How many times as large as the original map is the new map?"*

The side lengths are 1.5 times as long. The area is $1.5 \cdot 1.5$, or 2.25, times as large.

Lesson Synthesis

Share with students

💬 *"Today we reproduced a scale drawing at a different scale."*

If desired, use this example to review these concepts:

💬 *"You have a map that uses the scale 1 cm to 200 m. You draw a new map of the same place using the scale 1 cm to 20 m."*

Consider asking students:

💬 *"How does your new map compare to your original map?"*

It is larger. The lengths are 10 times as long, and the area is 100 times as large.

💬 *"How much actual area does 1 cm^2 on your new map represent?"*

400 m^2

💬 *"How much actual area did 1 cm^2 on your original map represent?"*

$40{,}000 \text{ m}^2$

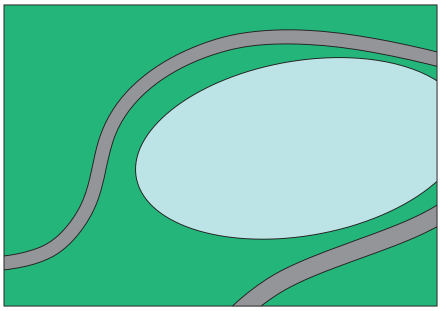
Math Community

Invite 2–3 students to share what "Doing Math" actions they noticed. Record and display their responses for all to see, such as by adding check marks to any already listed items or adding new items near the chart for the class to consider adding. Next, give students 1–2 minutes with a partner to discuss any changes or revisions they think the chart needs. Tell students they can suggest revisions during the *Cool-down*.

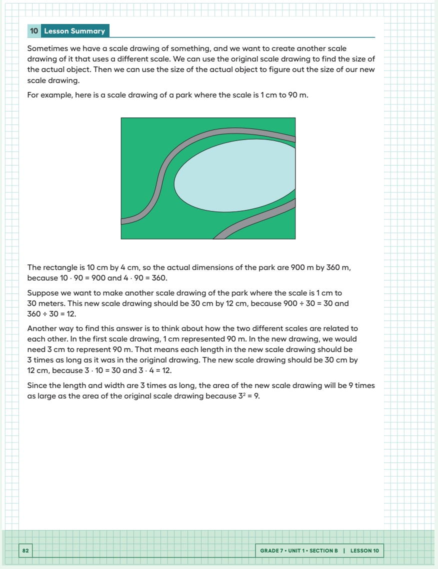
Lesson Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.



Student Workbook



10 Lesson Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.

The rectangle is 10 cm by 4 cm, so the actual dimensions of the park are 900 m by 360 m, because $10 \cdot 90 = 900$ and $4 \cdot 90 = 360$.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 meters. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \cdot 10 = 30$ and $3 \cdot 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing because $3^2 = 9$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

The rectangle is 10 cm by 4 cm, so the actual dimensions of the park are 900 m by 360 m, because $10 \cdot 90 = 900$ and $4 \cdot 90 = 360$.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 meters. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \cdot 10 = 30$ and $3 \cdot 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing because $3^2 = 9$.

Math Community

Before distributing the *Cool-downs,* display the Math Community Chart and the community building question "What additions or revisions would you make to the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the Math Community Chart before Exercise 5.

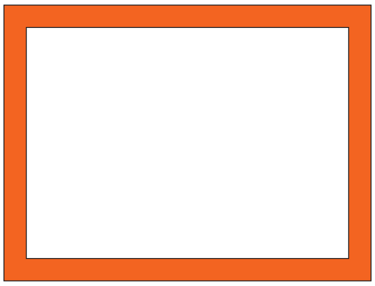
Cool-down

Window Frame

5 min

Student Task Statement

Here is a scale drawing of a window frame that uses a scale of 1 cm to 6 inches.



Create another scale drawing of the window frame that uses a scale of 1 cm to 12 inches.

Scaled copy of the drawing where each length is half as long as in the original.

Lesson **10**

Practice Problems

5 Problems

Problem 1

Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

a. How long and how wide is the actual swimming pool?

Answers depend on size of the printed image.

Sample response: The scale drawing is 10 cm long and 5 cm wide so the actual swimming pool is 10 m long and 5 m wide.

b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

It will be smaller. Each centimeter will represent a larger distance so it will take fewer centimeters to represent the width and length of the swimming pool.

c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

The length and width will each be half as long as the given scale drawing.

Problem 2

A map of a park has a scale of 1 inch to 1,000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.

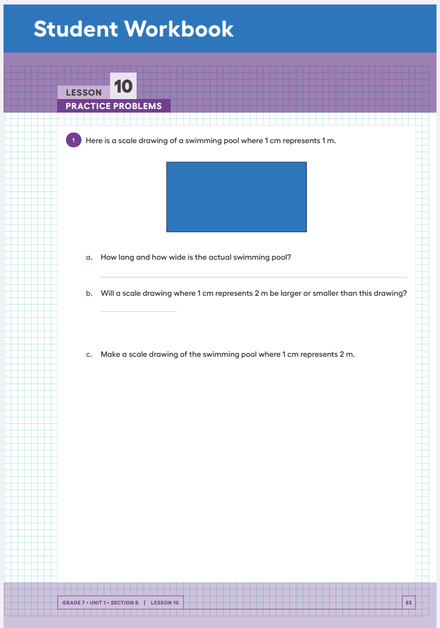
The map with a scale of 1 inch to 500 feet. It takes twice the number of units on this map to represent the same actual distance covered by the other map. For example, on the 1 inch to 1,000 feet map, it takes 1 inch to represent 1,000 feet in the actual park. On the 1 inch to 500 feet map, it takes 2 inches to represent the same 1,000 feet in the park.

Problem 3

On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your reasoning.

I disagree

Sample reasoning: At the scale of 1 inch to 12 feet, every 1 square inch represents 144 square feet, because $12 \cdot 12 = 144$. The actual area of the restaurant should be 8,640 square feet, because $60 \cdot 144 = 8,640$.



Student Workbook

LESSON 10

PRACTICE PROBLEMS

1 Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

a. How long and how wide is the actual swimming pool?

b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

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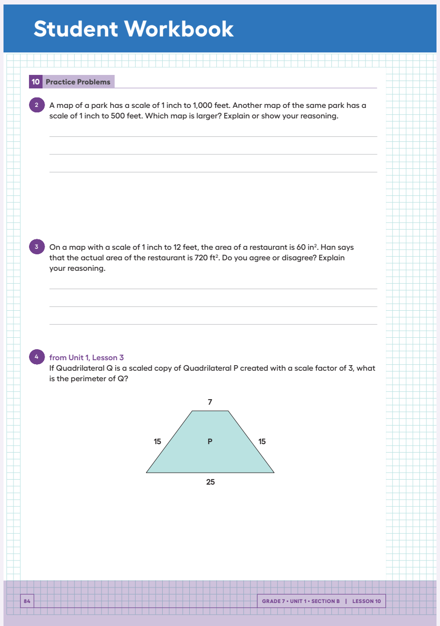

Student Workbook

10 Practice Problems

2 A map of a park has a scale of 1 inch to 1,000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.

3 On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your reasoning.

4 from Unit 1, Lesson 3

If Quadrilateral Q is a scaled copy of Quadrilateral P created with a scale factor of 3, what is the perimeter of Q?

7

15 P 15

25

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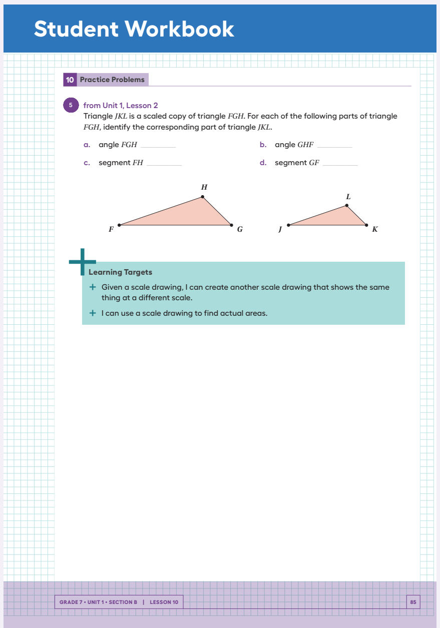
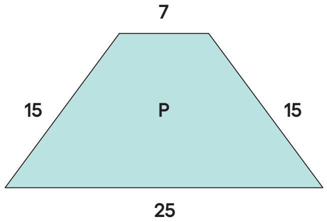


Problem 4

from Unit 1, Lesson 3

If Quadrilateral Q is a scaled copy of Quadrilateral P created with a scale factor of 3, what is the perimeter of Q?

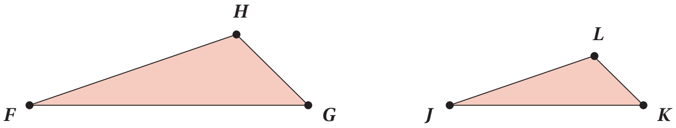
186



Problem 5

from Unit 1, Lesson 2

Triangle JKL is a scaled copy of triangle FGH. For each of the following parts of triangle FGH, identify the corresponding part of triangle JKL.



a. angle FGH

angle JKL

b. angle GHF

angle KLJ

c. segment FH

segment JL

d. segment GF

segment KJ