Lesson 6

Estimating Probabilities Using Simulation

Goals

- Comprehend that the term "simulation" (in written and spoken language) refers to a chance experiment used to represent a real-world situation.
- Describe (orally and in writing) a simple chance experiment that could be used to simulate a real-world event.
- Perform a simulation, and use the results to estimate the probability of a simple event in a real-world situation (using words and other representations).

Learning Target

I can simulate a real-world situation using a simple experiment that reflects the probability of the actual event.

Lesson Narrative

This lesson introduces the idea of *simulation*. Groups of students use and create different **simulations**, chance experiments that are designed to approximate the probability of a real-world event.

Students build on their work with calculating relative frequencies of experiments to calculate frequencies using a simulation. In this lesson, the outcomes students are tracking are from an experiment designed to represent the outcome of some other experiment that would be harder to study directly. Students see that a simulation depends on the experiment used in the simulation being a reasonable stand-in for the actual experiment of interest.

Student Learning Goal

Let's simulate real-world situations.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- Which Three Go Together?

Required Materials

Materials to Gather

- Number cubes: Activity 1
- Paper bags: Activity 1
- Paper clips: Activity 1

Materials to Copy

- Diego's Walk Cards (1 copy for every 18 students): Activity 1

Required Preparation

Activity 1:
Provide each group of 3 supplies for 1 type of simulation: choosing a situation slip from a bag, spinning a spinner, or rolling 2 number cubes. The supplies for the simulations include:

- a paper bag containing a set of slips cut from the blackline master
- a spinner cut from the blackline master, a pencil, and a paper clip
- 2 standard number cubes

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Warm-up
Which Three Go Together: Spinners

10 min

Activity Narrative

This *Warm-up* prompts students to compare four images of spinners. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

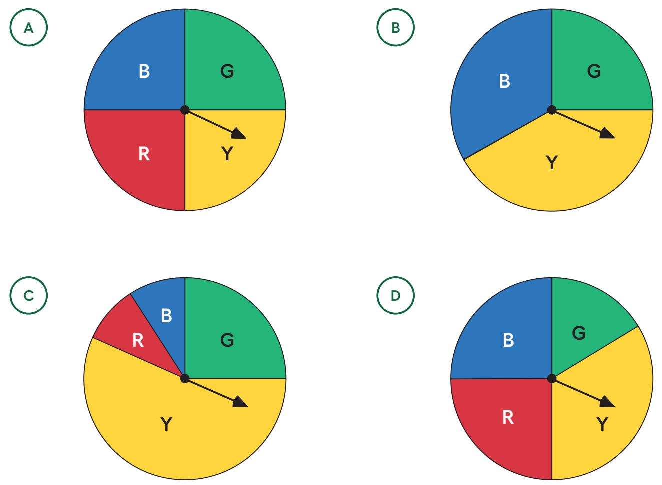
Arrange students in groups of 2–4. Display the image for all to see. Ask students to indicate when they have noticed which image does not belong and can explain why.

Give students 2 minutes of quiet think time and then time to share their thinking with their group.

After everyone has conferred in groups, ask the group to offer at least one reason each image doesn't belong. Follow with a whole-class discussion.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

- A, B, and C go together because the green region is one fourth of the circle.
- A, B, and D go together because each region is less than half of the circle
- A, C, and D go together because they have 4 regions.
- B, C, and D go together because the regions are not all the same size.

Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL.

Student Workbook

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "outcome," "region," or "sample space," and to clarify their reasoning as needed. Consider asking:

"How do you know ... ?"

"What do you mean by ... ?"

"Can you say that in another way?"

Activity 1

Diego's Walk

15 min

Activity Narrative

In this activity, students estimate the probability of a real-world event by simulating the experience with a chance experiment. They must make assumptions to simplify the situation and represent it as a simulation as well as analyze the results of the simulation to make a prediction. Students see that multiple simulation methods can result in similar estimates for the probability of the actual event.

The bag of papers and spinner are designed to have a probability of 0.7 to wait more than 1 minute. The number cubes have a probability of approximately 0.72 to wait more than 1 minute. To the extent that the students are estimating the probabilities, these are close enough to give similar results.

Launch

Arrange students in groups of 3. Prepare each group with supplies for 1 type of simulation: choosing a slip from a bag, spinning a spinner, or rolling 2 number cubes. The supplies for these simulations include:

- a bag containing a set of slips from the blackline master
- a spinner cut from the blackline master, a pencil, and a paper clip
- 2 standard number cubes

Set up the simulation by telling the students,

"Diego must cross a busy intersection at a crosswalk on his way to school. Some days he is able to cross immediately or wait only a short while. Other days, he must wait for more than 1 minute for the signal to indicate he may cross the street. We will simulate his luck at this intersection using different methods and estimate his probability of waiting more than 1 minute."

Tell students that a **simulation** is a chance experiment that can be used to estimate the probability of an event. They will use the materials they are given to simulate Diego's walk.

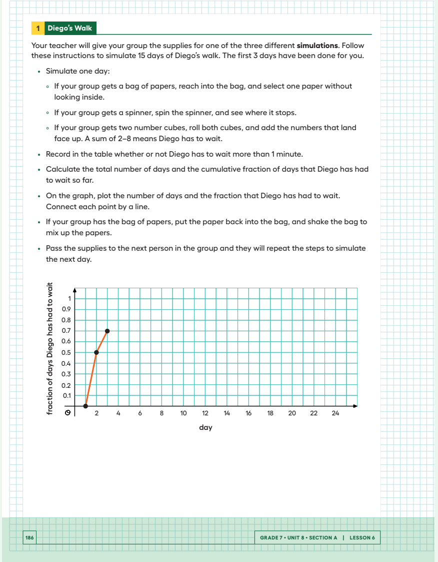
Give students 15 minutes for group work, and follow with a whole-class discussion.

Student Task Statement

Your teacher will give your group the supplies for one of the three different **simulations**. Follow these instructions to simulate 15 days of Diego's walk. The first 3 days have been done for you.

- Simulate one day:
  - If your group gets a bag of papers, reach into the bag, and select one paper without looking inside.
  - If your group gets a spinner, spin the spinner, and see where it stops.
  - If your group gets two number cubes, roll both cubes, and add the numbers that land face up. A sum of 2–8 means Diego has to wait.
- Record in the table whether or not Diego has to wait more than 1 minute.
- Calculate the total number of days and the cumulative fraction of days that Diego has had to wait so far.
- On the graph, plot the number of days and the fraction that Diego has had to wait. Connect each point by a line.
- If your group has the bag of papers, put the paper back into the bag, and shake the bag to mix up the papers.
- Pass the supplies to the next person in the group and they will repeat the steps to simulate the next day.

Student Workbook


1 Diego's Walk

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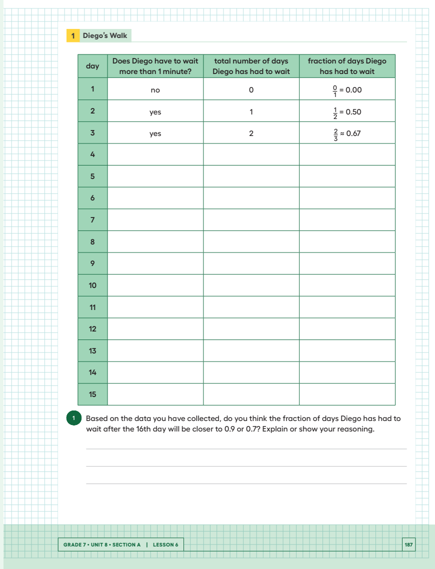
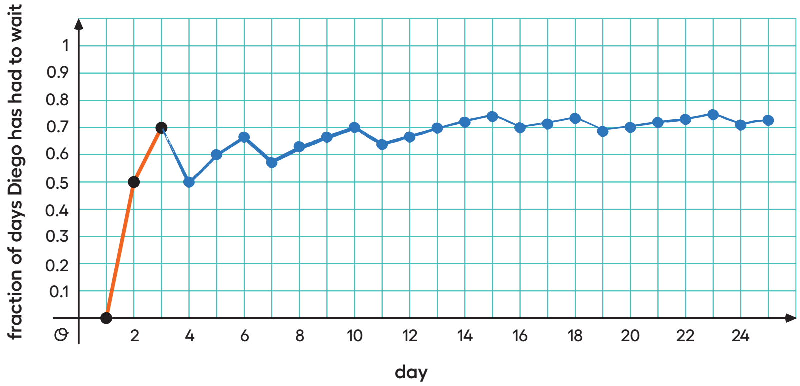
Student Workbook



1 Diego's Walk

day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
1	no	0	$\frac{0}{1} = 0.00$
2	yes	1	$\frac{1}{2} = 0.50$
3	yes	2	$\frac{2}{3} \approx 0.67$
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

1 Based on the data you have collected, do you think the fraction of days Diego has had to wait after the 16th day will be closer to 0.9 or 0.7? Explain or show your reasoning.



day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
1	no	0	$\frac{0}{1} = 0.00$
2	yes	1	$\frac{1}{2} = 0.50$
3	yes	2	$\frac{2}{3} \approx 0.67$
4			
5			
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11			
12			
13			
14			
15			

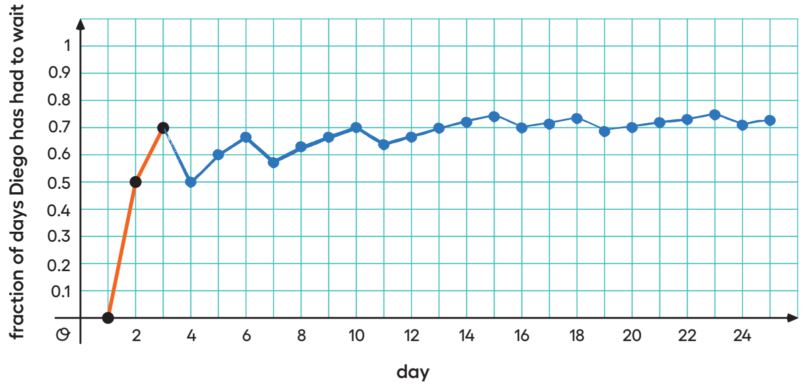
1. Based on the data you have collected, do you think the fraction of days Diego has had to wait after the 16th day will be closer to 0.9 or 0.7? Explain or show your reasoning.

Sample response: Probably closer to 0.7 since our fraction after 16 trials was $\frac{11}{16} \approx 0.69$, which is closer to 0.7 than 0.9.

2. Continue the simulation for 10 more days. Record your results in this table and on the graph from earlier.

day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

Sample response:



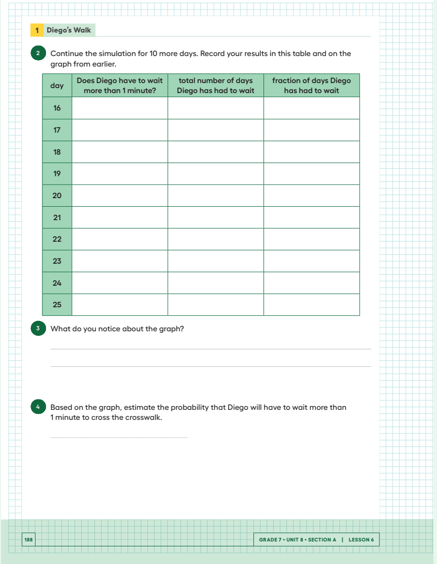
3. What do you notice about the graph?

Sample response: At the beginning, the graph jumps up and down a lot, but seems to be leveling out near the end.

4. Based on the graph, estimate the probability that Diego will have to wait more than 1 minute to cross the crosswalk.

Sample response: I estimate the probability to be 0.7, since the graph seems to be leveling out there.

Student Workbook


1 Diego's Walk

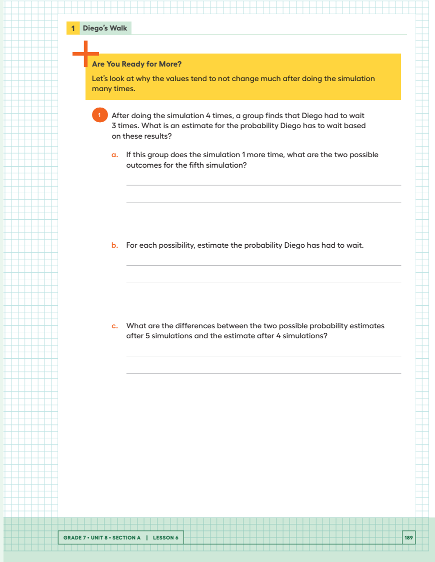
2 Continue the simulation for 10 more days. Record your results in this table and on the graph from earlier.

day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

3 What do you notice about the graph?

4 Based on the graph, estimate the probability that Diego will have to wait more than 1 minute to cross the crosswalk.

Student Workbook


1 Diego's Walk

Are You Ready for More?

Let's look at why the values tend to not change much after doing the simulation many times.

1 After doing the simulation 4 times, a group finds that Diego had to wait 3 times. What is an estimate for the probability Diego has to wait based on these results?

a. If this group does the simulation 1 more time, what are the two possible outcomes for the fifth simulation?

b. For each possibility, estimate the probability Diego has had to wait.

c. What are the differences between the two possible probability estimates after 5 simulations and the estimate after 4 simulations?

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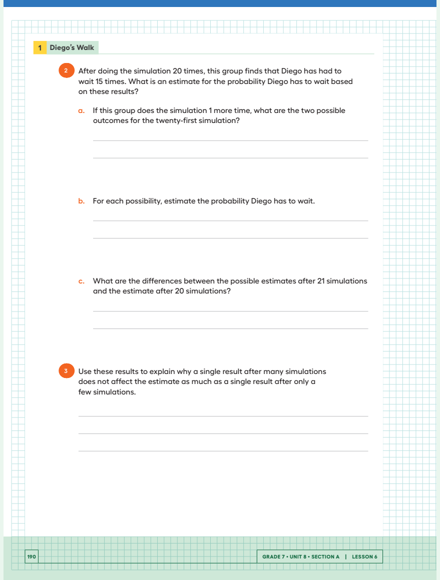
Student Workbook


1 Diego's Walk

2 After doing the simulation 20 times, this group finds that Diego has had to wait 15 times. What is an estimate for the probability Diego has to wait based on these results?

a. If this group does the simulation 1 more time, what are the two possible outcomes for the twenty-first simulation?

b. For each possibility, estimate the probability Diego has to wait.

c. What are the differences between the possible estimates after 21 simulations and the estimate after 20 simulations?

3 Use these results to explain why a single result after many simulations does not affect the estimate as much as a single result after only a few simulations.

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Are You Ready for More?

Let's look at why the values tend to not change much after doing the simulation many times.

1. After doing the simulation 4 times, a group finds that Diego had to wait 3 times. What is an estimate for the probability Diego has to wait based on these results?

   $\frac{3}{4}$

   a. If this group does the simulation 1 more time, what are the two possible outcomes for the fifth simulation?

   Either he has to wait or does not on the fifth day.

   b. For each possibility, estimate the probability Diego has had to wait.

   If he has to wait, the probability is $\frac{4}{5}$. If he does not have to wait, the probability is $\frac{3}{5}$.

   c. What are the differences between the two possible probability estimates after 5 simulations and the estimate after 4 simulations?

   Difference if Diego has to wait: $\frac{1}{20} = 0.05$, since $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$. Difference if Diego doesn't have to wait: $\frac{3}{20} = 0.15$, since $\frac{3}{4} - \frac{3}{5} = \frac{3}{20}$.

2. After doing the simulation 20 times, this group finds that Diego has had to wait 15 times. What is an estimate for the probability Diego has to wait based on these results?

   $\frac{15}{20}$, which is equal to $\frac{3}{4}$

   a. If this group does the simulation 1 more time, what are the two possible outcomes for the twenty-first simulation?

   Either he has to wait or does not on the twenty-first day.

   b. For each possibility, estimate the probability Diego has to wait.

   If he has to wait, the probability is $\frac{16}{21}$. If he does not have to wait, the probability is $\frac{15}{21}$.

   c. What are the differences between the possible estimates after 21 simulations and the estimate after 20 simulations?

   Difference if Diego has to wait: $\frac{1}{84} \approx 0.012$, since $\frac{16}{21} - \frac{15}{20} = \frac{1}{84}$. Difference if Diego doesn't have to wait: $\frac{1}{28} \approx 0.036$, since $\frac{15}{20} - \frac{15}{21} = \frac{1}{28}$.

3. Use these results to explain why a single result after many simulations does not affect the estimate as much as a single result after only a few simulations.

   Sample reasoning: When numbers are smaller, increasing by 1 is more important than when numbers are larger. For example, changing a denominator from 2 to 3 is more important than changing 20 to 21. This means that the results will not change as dramatically after a large number of simulations.

Activity Synthesis

The purpose of this discussion is for students to understand why simulations are useful in place of actual experiments.

Select at least 1 group for each of the simulation methods to display the materials they used to run their simulation and explain the steps involved in using their materials.

Ask students,

"Why do you think these simulations are more useful than actually doing the experiment many times?"

It would take a lot of time and work for Diego to walk to school more than usual, but it is easy to do the simulation many times quickly.

Select students to share what they noticed about the graph of the fraction of days Diego had to wait as the simulated days went on.

Activity 2
Designing Experiments

10 min

Activity Narrative

In this activity, students have the opportunity to design their own simulations that could be used to estimate probabilities of real-life events. Students attend to precision by assigning each possible outcome for the real-life experiment to a corresponding outcome in their simulation in such a way that the pair of outcomes have the same probability. In the discussion following the activity, students are asked to articulate how these simulations could be used to estimate probabilities of certain events.

Launch

Keep students in groups of 3.

Give students 5 minutes quiet work time to design their own experiments, followed by small-group discussion to compare answers for the situations and whole-class discussion.

As students work, monitor for students who are using the same chance events for multiple scenarios (for example, always using a spinner) and encourage them to think about other ways to simulate the event.

Student Task Statement

For each situation, describe a chance experiment that would fairly represent it.

Sample responses:

1. Six people are going out to lunch together. One of them will be selected at random to choose which restaurant to go to. Who gets to choose?

   Assign each person a number from 1 to 6, then roll a number cube. The person whose number is face up on the number cube is the one to choose.

2. After a robot stands up, it is equally likely to step forward with its left foot or its right foot. Which foot will it use for its first step?

   Flip a coin. If it is heads, it should step with the left foot. If it is tails, it should step with the right.

3. In a computer game, there are 3 tunnels. Each time the level loads, the computer randomly selects 1 of the tunnels to lead to the castle. Which tunnel is it?

   Label each tunnel "left," "right," and "middle." Make a spinner that has 3 equal sections with one of these labels in each section. Spin the spinner and the castle is behind the tunnel the spinner lands on.

4. Your school is taking 4 buses of students on a field trip. Will you be assigned to the same bus that your math teacher is riding on?

   Assign a color to each bus. Put 1 block of each color in a bag, then reach in and pull out 1 block. If the block is red, then you are on the same bus.

Activity Synthesis

The purpose of this discussion is for students to think more deeply about the connections between the real-life experiment and the simulation.

Select groups to share the simulations they designed for each of the situations.

Some questions for discussion:

- *"How could a standard number cube be used to simulate the situation with the buses?"*

  Each bus is assigned a number 1 through 4. If the cube lands on 5 or 6, roll again and do not count it in the simulation.

- *"If one of the buses is numbered with your math teacher's favorite number and you want to increase the probability of that bus being selected, how could you change the simulation to do this?"*

  Add more of the related outcome. For example, using the standard number cube as in the previous discussion question, the bus with the favorite number could be assigned numbers 4 and 5 while the other buses are still 1 through 3.

- *"Two of the tunnels in the video game lead to a swamp that ends the game. How could you use the simulation to estimate the probability of choosing one of those two tunnels?"*

  Since all of the tunnels are equally likely to lead to the swamp, it can be assumed that "left" and "right" lead to the swamp. Spin the spinner many times and use the fraction of times it ends on "left" or "right" to estimate the probability of ending the game. It should happen $\frac{2}{3}$, or about 67%, of the time.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem solving, chunk this task into more manageable parts. For example, invite students to select 2–3 of the situations to complete.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may think that the number of outcomes in the sample space must be the same in the simulation as in the real-life situation. Ask students how we could use the results from the roll of a standard number cube to represent a situation with only two equally likely outcomes (by making use of some extra options to count as "roll again").

Student Workbook


2 Designing Experiments

For each situation, describe a chance experiment that would fairly represent it.

1 Six people are going out to lunch together. One of them will be selected at random to choose which restaurant to go to. Who gets to choose?

2 After a robot stands up, it is equally likely to step forward with its left foot or its right foot. Which foot will it use for its first step?

3 In a computer game, there are 3 tunnels. Each time the level loads, the computer randomly selects 1 of the tunnels to lead to the castle. Which tunnel is it?

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support whole-class discussion: "I agree because …" and "I disagree because …"

Advances: Speaking, Conversing

“You and a friend are among the people going to lunch. How could you use the simulation you designed to estimate the probability that you or your friend will be the one to choose the restaurant?”

My friend and I will be represented by 1 and 2 on a number cube. Roll the number cube a lot of times and find the fraction of times 1 or 2 appear, then estimate the probability that one of us will be the ones selected.

Lesson Synthesis

Consider asking these discussion questions:

“What is a simulation?”

A related chance experiment that is easy to do and has similar probabilities to the original chance experiment of interest.

“Why might we want to run a simulation rather than the actual experiment?”

Simulations are easier and usually faster to do multiple times, so using them to get an estimate of the probability of an event is sometimes preferred.

”If we conduct a few trial simulations of a situation and record the fraction of outcomes for which a particular event occurs, how might we know that we have done enough simulations to have a good estimate of the probability of that event happening?”

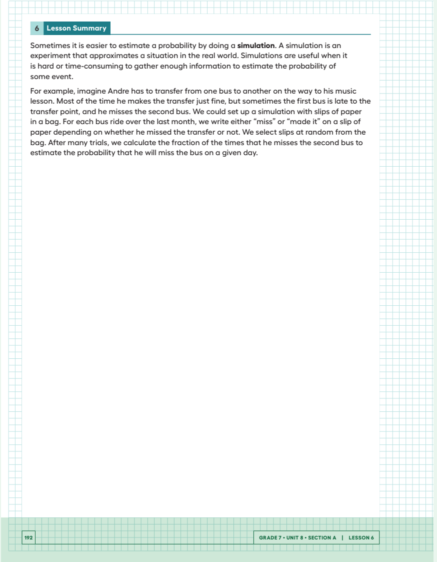
when the fractions seem to not be changing very much based on how accurate we want the estimate to be

Lesson Summary

Sometimes it is easier to estimate a probability by doing a **simulation**. A simulation is an experiment that approximates a situation in the real world. Simulations are useful when it is hard or time-consuming to gather enough information to estimate the probability of some event.

For example, imagine Andre has to transfer from one bus to another on the way to his music lesson. Most of the time he makes the transfer just fine, but sometimes the first bus is late to the transfer point, and he misses the second bus. We could set up a simulation with slips of paper in a bag. For each bus ride over the last month, we write either “miss” or “made it” on a slip of paper depending on whether he missed the transfer or not. We select slips at random from the bag. After many trials, we calculate the fraction of the times that he misses the second bus to estimate the probability that he will miss the bus on a given day.

Student Workbook



6 Lesson Summary

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Math Community

Before distributing the *Cool-downs,* display the Math Community Chart and these questions:

- "What norm(s) should stay the way they are?"
- "What norm(s) do you think should be made more clear? How?"
- "What norms are missing that you would add?"
- "What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

Video Game Weather

5 min

In this activity, students use their understanding of simulations to design a chance experiment that can be easily repeated while mimicking another situation with the same probability of a certain event.

Student Task Statement

In a video game, the chance of rain each day is always 30%. At the beginning of each day in the video game, the computer generates a random integer between 1 and 50. Explain how you could use this number to simulate the weather in the video game.

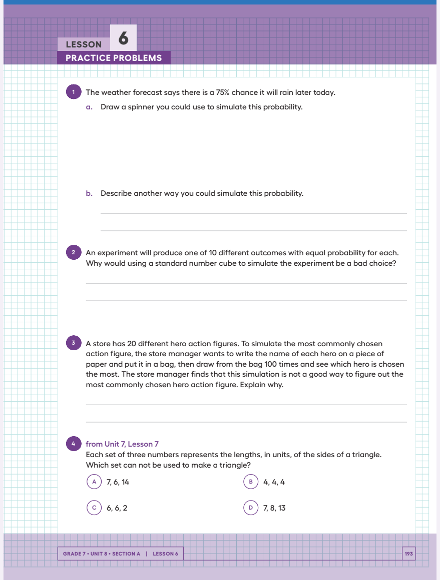
Sample response: If the number is between 1 and 15, the video game should create a rainy day. If the number is between 16 and 50, the video game should not create a rainy day.

Lesson 6

Practice Problems

5 Problems

Student Workbook


LESSON 6
PRACTICE PROBLEMS

1 The weather forecast says there is a 75% chance it will rain later today.

a. Draw a spinner you could use to simulate this probability.

b. Describe another way you could simulate this probability.

2 An experiment will produce one of 10 different outcomes with equal probability for each. Why would using a standard number cube to simulate the experiment be a bad choice?

3 A store has 20 different hero action figures. To simulate the most commonly chosen action figure, the store manager wants to write the name of each hero on a piece of paper and put it in a bag, then draw from the bag 100 times and see which hero is chosen the most. The store manager finds that this simulation is not a good way to figure out the most commonly chosen hero action figure. Explain why.

4 from Unit 7, Lesson 7
Each set of three numbers represents the lengths, in units, of the sides of a triangle. Which set can not be used to make a triangle?

A 7, 6, 14
B 4, 4, 4
C 6, 6, 2
D 7, 8, 13

Problem 1

The weather forecast says there is a 75% chance it will rain later today.

Sample responses:

a. Draw a spinner you could use to simulate this probability.

A circle is drawn with $\frac{3}{4}$ colored blue and labeled "rain," and the other $\frac{1}{4}$ is left white and labeled "no rain."

b. Describe another way you could simulate this probability.

Put 4 marbles in a bag, 3 blue and 1 white. The blue marbles represent rain.

Problem 2

An experiment will produce one of 10 different outcomes with equal probability for each. Why would using a standard number cube to simulate the experiment be a bad choice?

A standard number cube only has 6 outcomes, so it cannot produce all 10 possibilities from the experiment.

Problem 3

A store has 20 different hero action figures. To simulate the most commonly chosen action figure, the store manager wants to write the name of each hero on a piece of paper and put it in a bag, then draw from the bag 100 times and see which hero is chosen the most. The store manager finds that this simulation is not a good way to figure out the most commonly chosen hero action figure. Explain why.

Sample response: Selecting a hero action figure to buy is not a chance experiment. People will buy their favorites, so it is not done at random.

Problem 4

from Unit 7, Lesson 7

Each set of three numbers represents the lengths, in units, of the sides of a triangle. Which set can not be used to make a triangle?

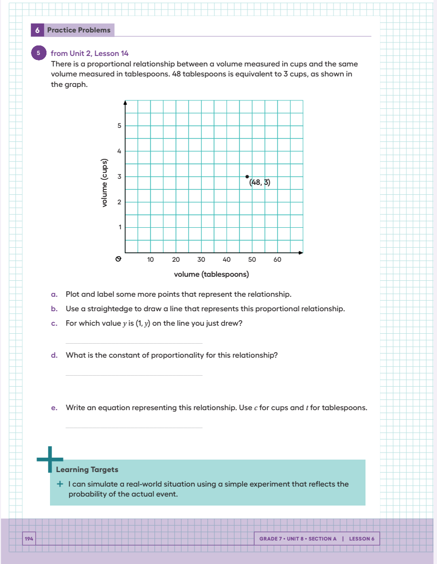
A. 7, 6, 14

B. 4, 4, 4

C. 6, 6, 2

D. 7, 8, 13

Student Workbook


6 Practice Problems

5 from Unit 2, Lesson 14

There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 48 tablespoons is equivalent to 3 cups, as shown in the graph.

a. Plot and label some more points that represent the relationship.

b. Use a straightedge to draw a line that represents this proportional relationship.

c. For which value y is $(1, y)$ on the line you just drew?

d. What is the constant of proportionality for this relationship?

e. Write an equation representing this relationship. Use c for cups and t for tablespoons.

Learning Targets

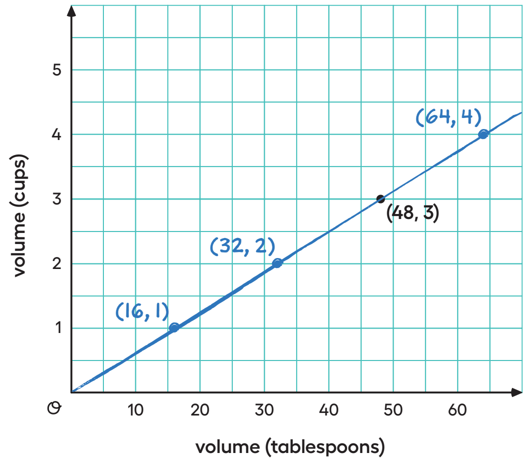
+ I can simulate a real-world situation using a simple experiment that reflects the probability of the actual event.

194 GRADE 7 • UNIT 8 • SECTION A | LESSON 6

Problem 5

from Unit 2, Lesson 14

There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 48 tablespoons is equivalent to 3 cups, as shown in the graph.



a. Plot and label some more points that represent the relationship.

See image.

b. Use a straightedge to draw a line that represents this proportional relationship.

See image.

c. For which value y is $(1, y)$ on the line you just drew?

$\frac{1}{16}$

d. What is the constant of proportionality for this relationship?

$\frac{1}{16}$ cup per tablespoon

e. Write an equation representing this relationship. Use c for cups and t for tablespoons.

$c = \frac{1}{16}t$ (or equivalent)