

Using Equations to Solve Problems

Goals

- Interpret and coordinate tape diagrams, equations, and verbal descriptions for situations involving signed numbers.
- Solve an equation of the form $px + q = r$ or $p(x + q) = r$ to determine an unknown quantity in a situation, and present the solution method (orally, in writing, and through other representations).
- Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent a situation involving signed numbers.

Learning Target

I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Instructional Routines

- 5 Practices

Required Materials

Materials to Gather

- Sticky notes: Activity 2
- Tools for creating a visual display: Activity 2

Required Preparation

Lesson:

- Decide if students will conduct group presentations or a Gallery Walk for the last activity. If so, prepare tools for creating a visual display and about 3 sticky notes per student. If not, these materials are not necessary.

Lesson Narrative

This lesson brings together the skills and concepts that have been studied in the unit so far. Students solve problems that can be represented by equations of the form $p(x + q) = r$ and $px + q = r$. A bit of scaffolding is offered in the first activity to reactivate students' understanding of tape diagrams, but after that, no scaffolding is offered so that students can make sense of problems and choose representations on their own. The last activity is optional because no new concepts are covered. It does, however, offer additional practice writing and solving equations of the form $px + q = r$ and $px + q = r$ to solve real-world problems.

Student Learning Goal

Let's use tape diagrams, equations, and reasoning to solve problems.

Lesson Timeline

5
min

Warm-up

20
min

Activity 1

10
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Warm-up

Remember Tape Diagrams

5 min

Activity Narrative

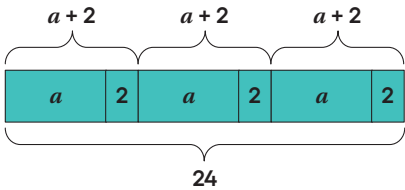
In this *Warm-up*, students write a story and an equation that could be represented by a tape diagram. The purpose is to reactivate students’ understanding of tape diagrams to make it more likely that tape diagrams are accessible as a representation for them to choose in this lesson. The diagram was deliberately constructed to encourage some students to write an equation like $24 = 3(a + 2)$ and others like $24 = 3a + 6$. Monitor for one student who writes each type of equation.

Launch

Arrange students in groups of 2.

Give 5 minutes of quiet think time and time to share their work with a partner followed by a whole-class discussion.

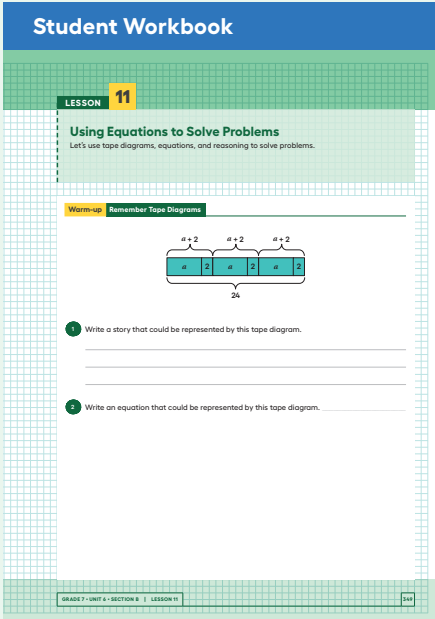
Student Task Statement



- 1. Write a story that could be represented by this tape diagram.
Sample response: A baker put a cookies in each of 3 boxes. Then he put 2 more cookies in each box, so there were a total of 24 total in the 3 boxes.
- 2. Write an equation that could be represented by this tape diagram.
 $24 = 3(a + 2)$ or $24 = 3a + 6$ (or equivalent)

Activity Synthesis

The purpose of this discussion is to help students see that an equation representing the diagram can be written in either form, $3(a + 2) = 24$ or $3a + 6 = 24$. After students have had a chance to share their work with their partner, select a few students to share their stories. Then select one student to share each type of equation and explain its structure.



Instructional Routines

5 Practices

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Access for Students with Diverse Abilities
(Activity 1, Task Statement)

Action and Expression: Internalize
Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for:
Organization, Attention

Student Workbook

At the Community Festival

Tyler makes invitations to the community festival. He has already made some of the invitations, and he wants to finish the rest of them within a week. He plans to make the same number of invitations each day. Tyler draws a diagram to represent the situation.

a. Explain how each part of the situation is represented in Tyler's diagram:
How many total invitations Tyler is trying to make

How many invitations he has made already

How many days he has to finish the invitations

b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.

Activity 1

At the Community Festival

20
min

Activity Narrative

In this activity, students use a tape diagram to help them reason about a situation, write an equation that represents it, and solve the equation. Students can use both the diagram and the solution strategy of doing the same to each side and undoing that they saw in the past few lessons. The first two questions provide more scaffolding and the last question provides none.

When students work on the last question, monitor for students who:

- Reason numerically without any diagrams or representations.
- Create a tape diagram and use it to reason numerically.
- Write an equation like $6(x - 1.5) = 46.5$ and solve it by using the distributive property to find the total amount saved, $6 \cdot 1.50$.
- Write an equation and solve it by first dividing by 6 to find the cost of each discounted ticket.

For the last question, students need to decide how to represent the situation and use their representation to reason about a solution, which requires reasoning abstractly and quantitatively.

Launch

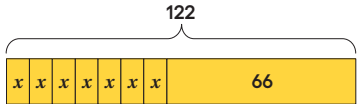
Keep students in the same groups.

Give students 5–10 minutes of quiet work time and partner discussions followed by a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

1. Tyler makes invitations to the community festival. He has already made some of the invitations, and he wants to finish the rest of them within a week. He plans to make the same number of invitations each day. Tyler draws a diagram to represent the situation.

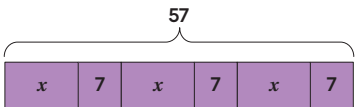


- a. Explain how each part of the situation is represented in Tyler's diagram:
- How many total invitations Tyler is trying to make
- How many invitations he has made already
- How many days he has to finish the invitations

Sample response: The whole tape labeled 122 shows the total number of invitations Tyler plans to make. The part labeled 66 shows the number of invitations he has made already. The 7 boxes show the number of days he has left to finish the invitation. Each of those boxes is labeled x because we don't know how many invitations Tyler will make each of those days.

- b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.
- 8 invitations
- Sample reasoning: Subtract 66 from 122 and divide the result, 56, by 7.
- c. Write an equation that represents the situation. Explain how each part of the situation is represented in your equation.
- $7x + 66 = 122$ (or equivalent)
- Sample reasoning: Tyler makes the same amount x each of 7 days, so $7x$ represents the number of invitations made in 7 days. He already made 66, so add those on, for a total of 122.
- d. Show how to solve your equation.
- Sample response: $7x + 66 = 122$, $7x + 66 + (-66) = 122 + (-66)$, $7x = 56$, $7x \div 7 = 56 \div 7$, $x = 8$.

2. Noah and his sister make prize bags for a game at the festival. Noah puts 7 pencil erasers in each bag. His sister puts an equal number of stickers in each bag. After filling 3 of the bags, they have used a total of 57 items.



- a. Explain how the diagram represents the situation.
- Sample response: There are 3 groups of x stickers and 3 groups of 7 erasers. All together there are 57 items.
- b. Noah writes the equation $3(x + 7) = 57$ to represent the situation. Do you agree with him? Explain your reasoning.
- Sample response: Yes, Noah's equation says that 3 groups of $x + 7$ gives a total of 57 items.
- c. How many stickers does Noah's sister put in each prize bag? Explain or show your reasoning.

- 12 stickers
- Sample reasoning: Each group of $x + 7$ represents $57 \cdot \frac{1}{3}$, or 19 items, so $x = 12$. Another way to find x is to subtract 21 from 57 and then $3x = 57 - 21 = 36$ so $x = 12$. The first strategy represents first multiplying each side of the equation by $\frac{1}{3}$ (or dividing each side by 3), and the second represents using the distributive property to write $3(x + 7)$ as $3x + 21$.
3. A family of 6 goes to the festival. They have a coupon for \$1.50 off each ticket. If they pay \$46.50 for all their tickets, how much does each ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

- \$9.25
- Sample reasoning: Divide $46.50 \div 6 = 7.75$ to find what they paid for each ticket and then add \$1.50. Another way is to reason that they saved a total of $6 \cdot 1.50 = 9$, so add the 9 back to 46.50 to find the price of 6 tickets without the coupon, and then divide by 6. These two strategies connect to writing the equation $6(t - 1.50) = 46.50$, and solving it either by dividing by 6 first, or by using the distributive property to write $6t - 9 = 46.50$, $6t = 55.50$, $t = 9.25$.

Student Workbook

1 At the Community Festival

c. Write an equation that represents the situation. Explain how each part of the situation is represented in your equation.

d. Show how to solve your equation.

2 Noah and his sister make prize bags for a game at the festival. Noah puts 7 pencil erasers in each bag. His sister puts an equal number of stickers in each bag. After filling 3 of the bags, they have used a total of 57 items.

a. Explain how the diagram represents the situation.

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Student Workbook

1 At the Community Festival

b. Noah writes the equation $3(x + 7) = 57$ to represent the situation. Do you agree with him? Explain your reasoning.

c. How many stickers does Noah's sister put in each prize bag? Explain or show your reasoning.

3 A family of 6 goes to the festival. They have a coupon for \$1.50 off each ticket. If they pay \$46.50 for all their tickets, how much does each ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

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Building on Student Thinking

The phrases “9 times as far” and “9 times as many” may lead students to think about multiplying by 9 instead of dividing (or multiplying by $\frac{1}{9}$). Encourage students to act out the situations or draw diagrams to help reason about the relationship between the quantities. Remind them to pay careful attention to what or who a comparison refers to.

Student Workbook

Running Around

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.

a. How many miles did Priya run last week?

b. Elena wrote the equation $\frac{1}{9}(x - 7) = 4$ to represent the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?

2. One day last week, 8 teachers joined $\frac{1}{2}$ of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?

3. Priya and Han plan a fundraiser for the running club. They begin with a balance of 80 dollars because of expenses. In the first hour of the fundraiser, they collect equal donations from 9 people, which brings their balance to -44. How much did each person give?

4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at an average rate of 50 feet per minute, they reach an elevation of -42 feet. How long did the descent take?

Activity Synthesis

Invite previously selected students to share their strategies for the last problem. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students’ work for all to see.

Connect the different responses to the learning goals by asking questions, such as:

“How are the approaches different, and what do they have in common?”

“Where do you see the 6 people, the \$1.50 coupon, and the \$46.50 total in each approach?”

“Which approach is the fastest? What are some reasons a person might use an approach that takes longer?”

Activity 2

Running Around

10 min

Activity Narrative

In this activity, students solve one or more word problems using strategies of their choice. The problems increase in difficulty. It is suggested that students create a visual display of one of the problems and do a Gallery Walk or presentation, but if time is short, you may choose to just have students work in their workbooks or devices.

Since these problems are relatively unscaffolded, students make sense of the problems and persevere in problem solving.

Launch

Keep students in the same groups. Instruct all groups to solve the first problem. If time allows, then assign one of the other problems for each group to solve. If opting to have students do presentations or a Gallery Walk, distribute tools for making a visual display.

Give students 5–6 minutes quiet work time and a partner discussion followed by a whole-class discussion or gallery walk.

Student Task Statement

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.

a. How many miles did Priya run last week?

43 miles

- b. Elena wrote the equation $\frac{1}{9}(x - 7) = 4$ to represent the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?

Sample response: Since Priya ran 9 times as far as Elena, I multiplied 9 by 4 to get 36 miles. This is the number of miles Priya ran this week, so 36 is 7 less than what she ran last week. I added 36 and 7 to get 43 miles. Elena solved an equation and I worked backwards with the information in the problem (or used a diagram). But we both took the same steps, multiply by 9 and add 7.

2. One day last week, 6 teachers joined $\frac{5}{7}$ of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?

35 members **Sample reasoning:** Solve the equation $\frac{5}{7}x + 6 = 31$ by subtracting 6 from each side and then dividing both sides by $\frac{5}{7}$.

3. Priya and Han plan a fundraiser for the running club. They begin with a balance of -80 dollars because of expenses. In the first hour of the fundraiser, they collect equal donations from 9 people, which brings their balance to -44. How much did each person give?

\$4 **Sample reasoning:** Solve $-80 + 9x = -44$ by adding 80 to each side and then dividing both sides by 9.

4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at an average rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

12 minutes **Sample reasoning:** Solve $128 - 50x = -472$ by subtracting 128 from both sides and then dividing both sides by -50.)

Are You Ready for More?

A musician performed at three local festivals. At the first festival, he doubled his money and spent \$30. At the second, he tripled his money and spent \$54. At the third, he quadrupled his money and spent \$72. In the end, the musician had \$48 left. How much did he have before performing at the festivals?

\$29

Arithmetic solution: Work backwards: $48 + 72 = 120$, $120 \div 4 = 30$, $30 + 54 = 84$, $84 \div 3 = 28$, $28 + 30 = 58$, $58 \div 2 = 29$.

Algebraic solution: Let m represent the original amount of money. After first festival: $2x - 30$. After second festival: $3(2x - 30) - 54 = 6x - 144$. After third festival, $4(6x - 144) - 72 = 24x - 648$. Solve the equation $24x - 648 = 48$ to find how much of the money he started with is left. Solution is $696 \div 24 = 29$.

Activity Synthesis

The purpose of this discussion is to compare and contrast different solution methods.

If students created a visual display and you opt to conduct a Gallery Walk, ask students to post their solutions. Distribute sticky notes and ask students to read others' solutions, using the sticky notes to leave questions or comments.

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Representation: Access for Perception.

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language

Student Workbook

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4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at an average rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

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Student Workbook

Running Around

Are You Ready for More?

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Lesson Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next 6 days, she ran the same distance each day. Clare ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where x represents the distance, in miles, Clare ran on each of the 6 days. Solving the equation gives the solution to the problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

Clare ran 3 miles each day.



111

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Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Give students a moment to review any questions or comments left on their displays.

Invite any students who chose to draw a diagram to share. Ask the class if they agree or disagree with each diagram and encourage them to suggest any revisions. Next, invite students who did not try to draw a diagram to share strategies. Ask students about any difficulties they had creating the expressions or equations. Did the phrase “9 times as many” suggest an incorrect expression? If yes, how did they catch and correct for this error?

Lesson Synthesis

Ask students to reflect on the work done in this unit so far. Ask:

“What strategies have you learned?”

“What kinds of problems can you solve that you weren’t able to, previously?”

Ask students to write down or share with a partner one new thing they have learned and one thing they still have questions or confusion about.

Lesson Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next 6 days, she ran the same distance each day. Clare ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where x represents the distance, in miles, Clare ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

Clare ran 3 miles each day.

Cool-down

The Basketball Game

5 min

Student Task Statement

Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score? Explain or show your reasoning.

14 points. Sample reasoning:

- Equation: $2(x - 9) = 10$, where x is the number of points scored by Andre. $x - 9 = 5$, $x = 14$.
- Reasoning: Diego scored half as many points as Noah, so he scored 5 points. Andre scored 9 points more than Diego, or 14 points.
- Diagram: One possibility is two boxes each with $x - 9$ showing a total of 10. Each box represents 5 points, so x is 14.

Practice Problems

6 Problems

Problem 1

from Unit 5, Lesson 9

Find the value of each variable.

a. $a \cdot 3 = -30$

$a = -10$

b. $-9 \cdot b = 45$

$b = -5$

c. $-89 \cdot 12 = c$

$c = -1,068$

d. $d \cdot 88 = -88,000$

$d = -1,000$

Problem 2

Match each story to an equation and its solution.

Equations:

• $5x - 7 = 3$

• $7 = 3(5 + x)$

• $3x + 5 = -7$

• $\frac{1}{3}(x + 7) = 5$

Solutions:

• $x = -4$

• $x = \frac{-8}{3}$

• $x = 2$

• $x = 8$

Stories:

- a. The temperature is -7 . Since midnight the temperature tripled and then rose 5 degrees. What was the temperature at midnight?

Equation: $3x + 5 = -7$

Solution: $x = -4$

- b. Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?

Equation: $\frac{1}{3}(x + 7) = 5$

Solution: $x = 8$

- c. A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now they can build 3 guitars in 7 hours. By how much did they reduce the time it takes to build each guitar?

Equation: $7 = 3(5 + x)$

Solution: $x = \frac{-8}{3}$

- d. A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

Equation: $5x - 7 = 3$

Solution: $x = 2$

Student Workbook

LESSON 11

PRACTICE PROBLEMS

1 from Unit 5, Lesson 9

Find the value of each variable.

a. $a \cdot 3 = -30$

b. $-9 \cdot b = 45$

c. $-89 \cdot 12 = c$

d. $88 \cdot d = -88,000$

2 Match each story to an equation and its solution.

Equations:

• $5x - 7 = 3$

• $7 = 3(5 + x)$

• $3x + 5 = -7$

• $\frac{1}{3}(x + 7) = 5$

Solutions:

• $x = -4$

• $x = \frac{-8}{3}$

• $x = 2$

• $x = 8$

Stories:

- a. The temperature is -7 . Since midnight the temperature tripled and then rose 5 degrees. What was the temperature at midnight?
- b. Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?
- c. A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now they can build 3 guitars in 7 hours. By how much did they reduce the time it takes to build each guitar?
- d. A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

- 1 A baby giraffe weighed 152 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

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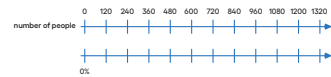
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Student Workbook

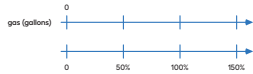
Practice Problems

6 Six teams are out on the fields practicing soccer. The teams all have the same number of players. A coach asks 2 players from each team to help move some equipment. Now there are 78 players on the fields. Write and solve an equation whose solution is the number of players on each team.

7 from Unit 4, Lesson 7
A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?



8 from Unit 4, Lesson 7
The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does the tank of the passenger car hold?



Learning Targets
+ I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation.

Problem 3

A baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

58 pounds

Sample reasoning: He gained $538 - 132$, or 406 pounds, over 7 months. $406 \div 7 = 58$. (Or solve $132 + 7x = 538$.)

Problem 4

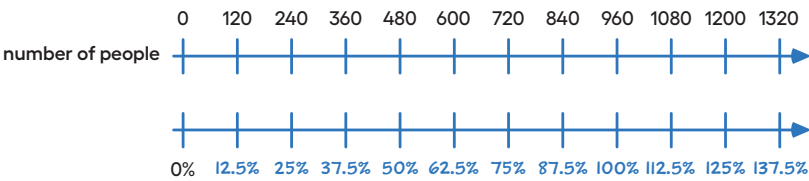
Six teams are out on the fields practicing soccer. The teams all have the same number of players. A coach asks 2 players from each team to help move some equipment. Now there are 78 players on the fields. Write and solve an equation whose solution is the number of players on each team.

Sample response: $6(x - 2) = 78$ or $6x - 12 = 78$ (or equivalent), $x = 15$

Problem 5

from Unit 4, Lesson 7

A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?

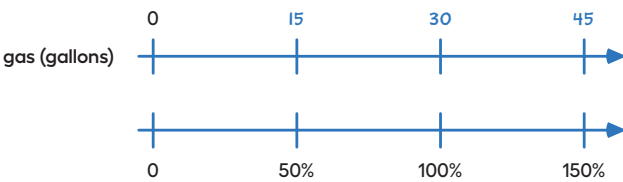


The town's population has grown by 25%.

Problem 6

from Unit 4, Lesson 7

The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does the tank of the passenger car hold?



15 gallons

Sample reasoning: 50% less than 30 is 15. (If the double number line is used, the tick marks on the top are labeled 0, 15, 30, 45.)

