Lesson 9

How Much in Each Group? (Part 2) (Optional)

Goals

- Interpret a situation (presented in written language or using other representations) involving equal-sized groups, and generate mathematical questions that could be asked about it.
- Solve a problem involving division of fractions, and present the solution method (orally, in writing, and using other representations).

Learning Target

I can find the amount in one group in different real-world situations.

Lesson Narrative

In this lesson, students practice finding the amount in one group given the amount in a different number of groups. They are also prompted to interpret, represent, and solve different kinds of division problems. In one activity, students are not explicitly told whether the situations involve finding the number of groups or finding the amount in each group. They decide on an interpretation that would enable them to solve the problem. Students are also required to identify relevant information (from a video, a picture, or written statements) that would help them answer questions. In an optional activity, they invent a situation that can be represented by a given division expression.

Because the problems in this lesson are less straightforward, students will need to make sense of the problems and persevere to solve them. As students move back and forth between the contexts and the abstract equations and diagrams that represent them, they reason abstractly and quantitatively.

Student Learning Goal

Let's practice dividing fractions in different situations.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 3)
- MLR5: Co-Craft Questions (Activity 1)

Instructional Routines

- 5 Practices
- Math Talk

Required Materials

Materials to Gather

- Geometry toolkits: Activity 3
- Tools for creating a visual display: Activity 3, Activity 4

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	15 min	15 min	15 min	10 min	5 min
					Cool-down

Inspire Math

A Fine Line video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614208

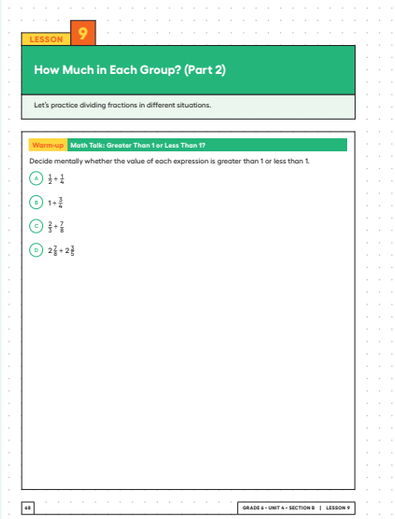
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



LESSON 9

How Much in Each Group? (Part 2)

Let's practice dividing fractions in different situations.

Warm-up Math Talk: Greater Than 1 or Less Than 1?

Decide mentally whether the value of each expression is greater than 1 or less than 1.

A. $\frac{1}{2} \div \frac{1}{4}$

B. $1 \div \frac{3}{4}$

C. $\frac{2}{3} \div \frac{7}{8}$

D. $2\frac{2}{8} \div 2\frac{3}{8}$

Warm-up

Math Talk: Greater Than 1 or Less Than 1?Activity Narrative

5 min

This *Math Talk* focuses on estimating and benchmarking quotients of fractions. It encourages students to think about the relative size of the dividend and divisor in a division expression and to rely on their understanding of division to mentally solve problems. The reasoning elicited here will be helpful later in the lesson when students solve problems that involve dividing fractions.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Before moving to the next problem, use the questions in the activity synthesis to involve more students in the conversation.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether the value of each expression is greater than 1 or less than 1.

A. $\frac{1}{2} \div \frac{1}{4}$ Greater than 1.

Sample reasoning:

- There are two $\frac{1}{4}$s in $\frac{1}{2}$.
- $2 \cdot \frac{1}{4} = \frac{1}{2}$

B. $1 \div \frac{3}{4}$ Greater than 1

Sample reasoning:

- The division can mean "How many $\frac{3}{4}$s are in 1?" and there is a little more than 1 group of $\frac{3}{4}$ in 1.
- $\frac{3}{4} \cdot \frac{4}{3} = 1$, and $\frac{4}{3}$ is greater than 1.

C. $\frac{2}{3} \div \frac{7}{8}$ Less than 1

Sample reasoning:

- The divisor $\frac{7}{8}$ is greater than the dividend $\frac{2}{3}$, so the quotient is not quite 1 whole.
- There is less than 1 group of $\frac{7}{8}$ in $\frac{2}{3}$.

D. $2\frac{7}{8} \div 2\frac{3}{5}$ Greater than 1

Sample reasoning:

- $\frac{7}{8}$ is closer to 1 than $\frac{3}{5}$ is, so $2\frac{7}{8}$ is greater than $2\frac{3}{5}$. Since the dividend is greater than the divisor, the quotient is greater than 1.
- $\frac{7}{8}$ is greater than $\frac{3}{4}$ while $\frac{3}{5}$ is less than $\frac{3}{4}$, which means that $\frac{7}{8}$ is greater than $\frac{3}{5}$. It also means that $2\frac{7}{8}$ is greater than $2\frac{3}{5}$ and there is more than one group of $2\frac{3}{5}$ in $2\frac{7}{8}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate ___'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to ___'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Highlight that we can estimate the reasonableness of our answers to division questions by thinking about how the dividend compares to the divisor. We can also use the relationship between multiplication and division to check our answers. For instance, $\frac{1}{2} \div \frac{1}{4} = ?$ corresponds to $? \cdot \frac{1}{4} = \frac{1}{2}$, so we can multiply the quotient and $\frac{1}{4}$ to see if it gives the product of $\frac{1}{2}$.

If the idea of estimating quotients by using benchmark fractions does not come up, discuss it with students. For instance, if students are unsure how $\frac{7}{8}$ and $\frac{3}{5}$ compare, prompt them to think of a familiar fraction that is close to both numbers and to compare each number to that benchmark instead.

Ask students:

- *"How does $\frac{7}{8}$ compare to $\frac{3}{4}$?" and "How does $\frac{3}{5}$ compare to $\frac{3}{4}$?"*

Building on Student Thinking

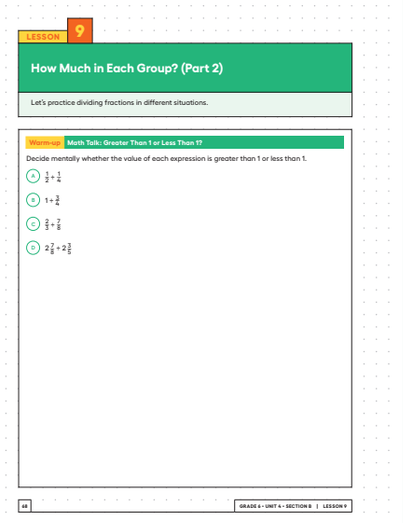
Some students may think that a diagram is needed to help them reason about each division. Remind them that in earlier lessons they interpreted an expression such as $10 \div 4$ to mean "How many 4s are in 10?" Ask if the same interpretation could apply here. Also encourage students to recall how the size of the divisor affects the quotient.

Access for Multilingual Learners (Warm-Up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ___ because ..." or "I noticed ___ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Student Workbook


LESSON 9

How Much in Each Group? (Part 2)

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C. $\frac{2}{3} \div \frac{7}{8}$

D. $2\frac{7}{8} \div 2\frac{3}{5}$

Instructional Routines

5 Practices

ilclass.com/r/10690701

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.
Keep student workbooks or devices closed. Show the video. Then display only the images of the water containers, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.
Students may benefit from watching the video more than once. Consider giving students explicit guidance about which aspects of the video to focus on each time.

Supports accessibility for: Language, Attention

Activity 1

Two Water Containers

20 min

Activity Narrative

This activity prompts students to solve a problem involving division of fractions in a less-scaffolded way. Students can see two relevant numbers to work with but need to interpret the context, the visual information, and the written question to decide whether the missing value is the size of one group, the number of groups, or the given amount. As they coordinate various pieces of information, students practice making sense of a problem and persevere in solving it.

Monitor for the different strategies that students use to answer the question about the amount in the dispenser. Here are some approaches that students may take, from more concrete to more abstract:

- Annotating on the images to make sense of the quantities.
- Drawing a tape diagram or another type of diagram to represent the partially-filled water container.
- Reasoning verbally about the relationship between known and unknown quantities.
- Writing equations to describe the relationships between quantities.

Launch

Tell students that they will solve a problem involving two water containers—a measuring cup and a water dispenser. Show the short video to introduce the context.

Video "Water in Containers" available here:
player.vimeo.com/video/304137827

Arrange students in groups of 2.

Give students a minute to read and discuss the first question.

Then give students 3–4 minutes of quiet time to answer the question about how much water can fit in the dispenser, followed by time to share their responses.

Ask students to discuss any disagreements they might have about their interpretation of the problem, their solving process, and the answer before continuing with the rest of the activity.
Identify students who used each strategy described in the *Activity Narrative*, and ask them to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

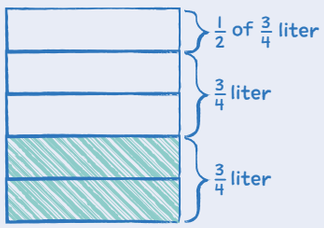
1. After looking at these pictures, Lin says, "I see the fraction $\frac{2}{5}$." Jada says, "I see the fraction $\frac{3}{4}$." What quantities are Lin and Jada referring to?

   Lin is seeing that $\frac{2}{5}$ of the water dispenser is filled. Jada is seeing that the amount of water is $\frac{3}{4}$ liter.

2. How many liters of water fit in the water dispenser? Show or explain your reasoning. If you get stuck, consider drawing a diagram.

   $\frac{15}{8}$ or $1\frac{7}{8}$ liters

   Sample reasoning:

   - 

   - If $\frac{3}{4}$ liter fits into $\frac{2}{5}$ of the dispenser, then half of $\frac{3}{4}$ or $\frac{3}{8}$ fits into $\frac{1}{5}$ of the dispenser. This means the whole dispenser can fit $5 \cdot \frac{3}{8}$ or $\frac{15}{8}$ liters.

3. To represent the question, Lin writes $\frac{2}{5} \cdot ? = \frac{3}{4}$. Explain why this equation represents the question and the situation.

   Sample response: The question is asking: "If $\frac{2}{5}$ of a dispenser is $\frac{3}{4}$ liter, how much can fit in the whole dispenser?" or "$\frac{2}{5}$ times what number equals $\frac{3}{4}$?" This can be written as $\frac{2}{5} \cdot ? = \frac{3}{4}$.

4. Write a division equation that represents the question.

   $\frac{3}{4} \div \frac{2}{5} = ?$

Activity Synthesis

One goal of the discussion is to draw students' attention to any structure in the different reasoning strategies. Another goal is to clarify why the equations $\frac{2}{5} \cdot ? = \frac{3}{4}$ and $\frac{3}{4} \div \frac{2}{5} = ?$ can represent the situation.

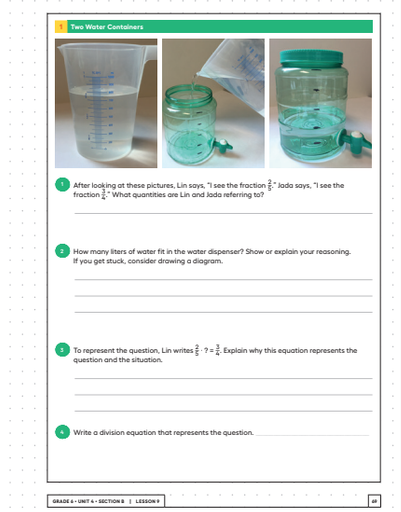
Invite previously selected students to share their solutions and reasoning. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Building on Student Thinking

Students may not immediately see that to answer the question "How many liters of water fit in the dispenser?" requires relating the amount in liters (as shown in the measuring cup) to the fraction of the dispenser that is filled with water. Consider showing the video again and following up with questions such as:

"What do you know about the water in the measuring cup?"
"What do you know about the water in the dispenser?"
"Do we know how many liters of water are in the dispenser?"

Student Workbook


1 Two Water Containers

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2. How many liters of water fit in the water dispenser? Show or explain your reasoning. If you get stuck, consider drawing a diagram.

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4. Write a division equation that represents the question.

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames, such as the following, to support student conversation: "This method works/doesn't work because ..." "First, I ___ because ..." "Another strategy would be ___ because ..."

Supports accessibility for: Language, Social-Emotional Functioning

Connect the different responses to the learning goals by asking questions such as:

- *"Many of the strategies involved dividing $\frac{3}{4}$ liter by 2. Where can we see that step in each strategy?"*

  "What information did that step give us?"

  The amount of water in one-fifth of a dispenser, which is $\frac{3}{8}$ liter

- *"Another common step was to multiply $\frac{3}{8}$ liter by 5. Where can we see that step in each strategy?"*

  "What did that step do?"

  It gave the amount in five-fifths of the dispenser, or the whole dispenser.

Next, invite students to share their explanations on why Elena's multiplication equation represents the question and situation. Highlight explanations that connect $\frac{2}{5} \cdot ? = \frac{3}{4}$ to the question "$\frac{2}{5}$ of what number equals $\frac{3}{4}$?" which is what the question is asking.

- *"What division equation can we write to represent the question?"*

  $\frac{3}{4} \div \frac{2}{5} = ?$

- *"How can we check if $\frac{15}{8}$ or $1\frac{7}{8}$ liters is the quotient and the amount that can fit in a dispenser?"*

  We can find $\frac{2}{5} \cdot \frac{15}{8}$ and see if it gives $\frac{3}{4}$.

Activity 2
Amount in One Group

15 min

Activity Narrative

In this activity, students practice reasoning about the amount in one group in division situations. They continue to write equations and draw diagrams to support their reasoning. In two problems (odd-numbered), the given number of groups is greater than 1. In the other two problems (even-numbered), a fraction of a group is given. Though this does not affect the structure of the equations that students write, students need to take care to reflect this information correctly in their diagrams. In doing so, students practice reasoning abstractly and quantitatively.

Launch

Arrange students in groups of 2. Ask each group to choose two questions to answer—an even-numbered one and an odd-numbered one.

Give students 7–8 minutes of quiet work time, and a few minutes to share their work with their partner.

Provide access to geometry toolkits and tools for creating a visual display. If time permits, ask each group to create a visual display of their solution and reasoning for one question (either of their choice or as assigned). Emphasize that they should organize their reasoning so it can be followed by others.

Student Task Statement

For each situation you choose:

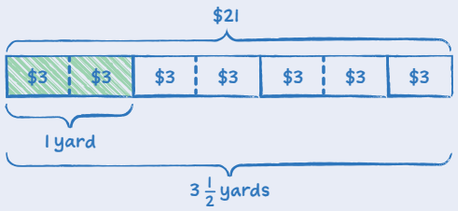
- Answer the question and show your reasoning. You can draw a tape diagram if you find it helpful.
- Write a multiplication equation and a division equation to represent the question.

1. Jada bought $3\frac{1}{2}$ yards of fabric for $21. How much did each yard cost?

Each yard costs $6.

Multiplication equation: $3\frac{1}{2} \cdot ? = 21$

Division equation: $21 \div 3\frac{1}{2} = ?$



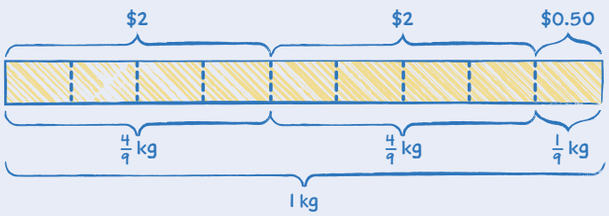
Sample reasoning: There are seven $\frac{1}{2}$-yard lengths in $3\frac{1}{2}$ yards. If seven $\frac{1}{2}$-yard lengths cost $21, then each $\frac{1}{2}$ yard costs $21 ÷ 7 (or $3). This means each yard is $6.

2. $\frac{4}{9}$ kilogram of baking soda costs $2. How much does 1 kilogram of baking soda cost?

Each kilogram of baking soda costs $4.50.

Multiplication equation: $\frac{4}{9} \cdot ? = 2$

Division equation: $2 \div \frac{4}{9} = ?$



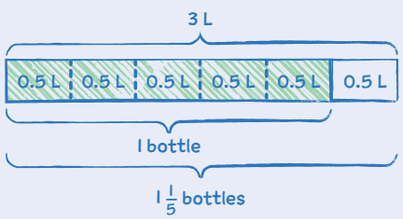
Sample reasoning: Because $\frac{4}{9}$ kilogram costs $2, $\frac{1}{9}$ kilogram costs one fourth of $2, which is $0.50, and 1 kilogram costs 9 times $0.50, which is $4.50.

3. Diego can fill $1\frac{1}{5}$ bottles with 3 liters of water. How many liters of water fill 1 bottle?

$\frac{5}{2}$ or $2\frac{1}{2}$ liters fill 1 bottle.

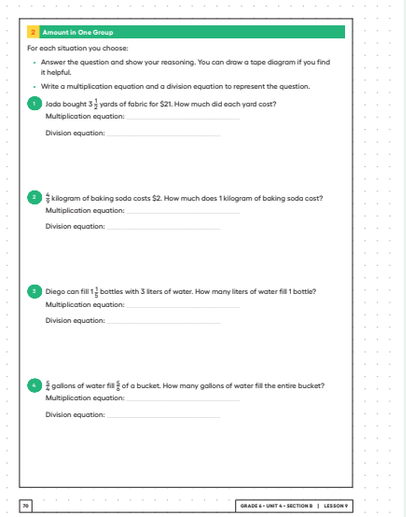
Multiplication equation: $1\frac{1}{5} \cdot ? = 3$

Division equation: $3 \div 1\frac{1}{5} = ?$.



Sample reasoning: If 3 liters fill $1\frac{1}{5}$ or $\frac{6}{5}$ bottles, then $3 ÷ 6$ or $\frac{1}{2}$ liter fills $\frac{1}{5}$ bottle and $5 \cdot \frac{1}{2}$ (or $2\frac{1}{2}$) liters fill 1 bottle.

Student Workbook


2 Amount in One Group

For each situation you choose:

- Answer the question and show your reasoning. You can draw a tape diagram if you find it helpful.
- Write a multiplication equation and a division equation to represent the question.

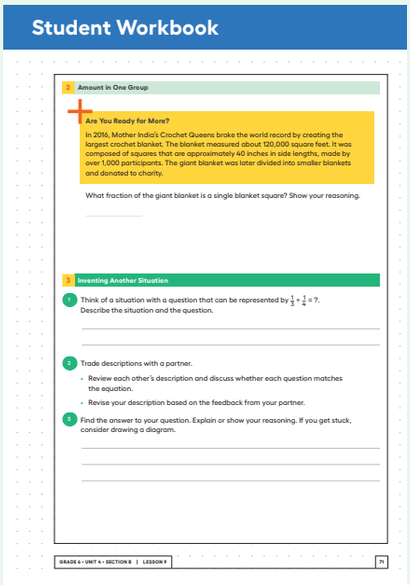
1. Jada bought $3\frac{1}{2}$ yards of fabric for $21. How much did each yard cost?
Multiplication equation: _____
Division equation: _____

2. $\frac{4}{9}$ kilogram of baking soda costs $2. How much does 1 kilogram of baking soda cost?
Multiplication equation: _____
Division equation: _____

3. Diego can fill $1\frac{1}{5}$ bottles with 3 liters of water. How many liters of water fill 1 bottle?
Multiplication equation: _____
Division equation: _____

4. $\frac{5}{4}$ gallons of water fill $\frac{5}{8}$ of a bucket. How many gallons of water fill the entire bucket?
Multiplication equation: _____
Division equation: _____

Student Workbook



2 Amount in One Group

Are You Ready for More?

In 2016, Mother India's Crochet Queens broke the world record by creating the largest crochet blanket. The blanket measured about 120,000 square feet. It was composed of squares that are approximately 40 inches in side lengths, made by over 1,000 participants. The giant blanket was later divided into smaller blankets and donated to charity.

What fraction of the giant blanket is a single blanket square? Show your reasoning.

3 Inventing Another Situation

1. Think of a situation with a question that can be represented by $\frac{1}{3} \div \frac{1}{4} = ?$. Describe the situation and the question.
2. Trade descriptions with a partner.
   - Review each other's description and discuss whether each question matches the equation.
   - Revise your description based on the feedback from your partner.
3. Find the answer to your question. Explain or show your reasoning. If you get stuck, consider drawing a diagram.

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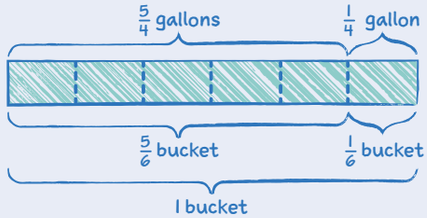
4. $\frac{5}{4}$ gallons of water fill $\frac{5}{6}$ of a bucket. How many gallons of water fill the entire bucket?

$1\frac{1}{2}$ gallons fill the entire bucket.

Sample reasoning: $\frac{5}{4}$ gallons fill $\frac{5}{6}$ bucket, so each $\frac{1}{4}$ gallon fills $\frac{1}{6}$ bucket.

One whole bucket is $\frac{4}{4}$ bucket; it would take $6 \cdot \frac{1}{4}$ or $\frac{6}{4}$ or $1\frac{1}{2}$ gallons to fill it.



Multiplication equation: $\frac{5}{6} \cdot ? = \frac{5}{4}$. Division equation: $\frac{5}{4} \div \frac{5}{6} = ?$

Are You Ready for More?

In 2016, Mother India's Crochet Queens broke the world record by creating the largest crochet blanket. The blanket measured about 120,000 square feet. It was composed of squares that are approximately 40 inches in side lengths, made by over 1,000 participants. The giant blanket was later divided into smaller blankets and donated to charity.

What fraction of the giant blanket is a single blanket square? Show your reasoning.

$\frac{1}{10,800}$

Sample reasoning: The area of 1 blanket square is $40 \cdot 40$ (or 1,600) square inches. There are 144 square inches in 1 square foot, so the area of the giant blanket is $120,000 \cdot 144$ (or 17,280,000) square inches. $17,280,000 \div 1,600 = 10,800$, so there are 10,800 blanket squares in the giant blanket. One blanket square is $\frac{1}{10,800}$ of it.

Activity Synthesis

For each question, invite previously selected 1–2 groups to share their responses. Display students' diagrams and reasoning for all to see. Ask the class to observe how the presented strategies compare to one another and to their own. Discuss students' observations.

If doing a gallery walk, arrange for groups who are assigned the same question to present their visual displays near one another.

Give students a few minutes to visit the displays and to see how others reasoned about the same two questions that they chose.

Ask students to observe how their strategies are the same as or different from others'. Invite students to share their observations afterward.

Highlight that we can find the size of 1 group when given the amount in a larger number of groups (as in the odd-numbered questions), or when given the amount in a fraction of a group (as in the even-numbered questions).

Activity 3

Inventing Another Situation: Optional

20 min

Activity Narrative

This open-ended activity gives students a chance to choose a situation and a question that can be represented by a division expression, find the value of that expression, and make sense of the value in context.

By now students will have seen a variety of situations in which a division means finding "How many groups of this in that?" or finding "How much in each group?" and can refer to these two interpretations of division to get started. As students work, monitor for the interpretations chosen, the range of attributes involved (such as length, volume, or weight), and the different types of diagrams used. Select a couple of students who interpret the expression in different ways, and ask them to share later.

Launch

Arrange students in groups of 2. Display the equation $5 \div 4 = ?$ for all to see.

Give students a minute to think of a story or a situation with a question that the equation could represent.

Follow that by giving students a minute to share their story with their partner. Select 2 students who interpreted the division differently to share their stories: one about dividing 5 into 4 groups and another about dividing 5 into groups of 4. Ask students what the quotient $1\frac{1}{4}$ means in each case.

Display another equation for all to see: $5 \div \frac{1}{4} = ?$.

Give partners 2–3 minutes to collaborate on a new story with a question for this equation.

Remind them that just as in the case of $5 \div 4 = ?$, there are two types of questions that could be asked in this case. Select a couple of partners that chose different interpretations of division to share their stories. Ask the class what the quotient is and what it means in each situation.

Tell students that they will now invent a new situation with a question for the equation $\frac{1}{3} \div \frac{1}{4} = ?$, trade descriptions with a partner for feedback, and then answer the question they wrote.

Access for Multilingual Learners (Activity 3, Launch)

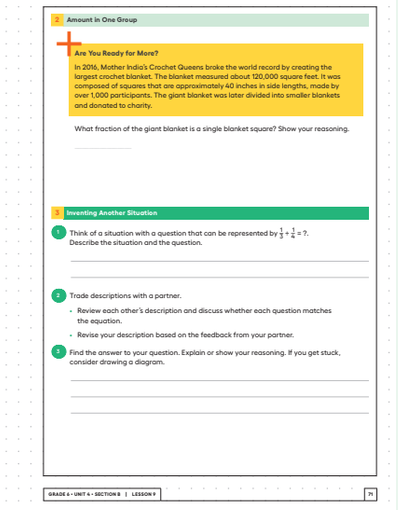
MLR8: Discussion Supports. Display sentence frames to support peer feedback. For example: "The description does (or does not) match the equation because ..." "To improve the ___, you could ..."

Advances: Speaking, Representing

Building on Student Thinking

If students are unsure how to invent a situation with two fractions as the known quantities, consider asking them to first try inventing a situation with the dividend being a whole number. For example, ask what $3 \div \frac{1}{4}$ could represent and then consider what $\frac{1}{3} \div \frac{1}{4}$ could represent.

Student Workbook



2 Amount in One Group

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2. Trade descriptions with a partner.
   - Review each other's description and discuss whether each question matches the equation.
   - Revise your description based on the feedback from your partner.
3. Find the answer to your question. Explain or show your reasoning. If you get stuck, consider drawing a diagram.

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Student Task Statement

1. Think of a situation with a question that can be represented by $\frac{1}{3} \div \frac{1}{4} = ?$. Describe the situation and the question.
   - Sample response for "$\frac{1}{4}$ of what number is $\frac{1}{3}$?" interpretation:

     Lin filled $\frac{1}{4}$ of a bottle with $\frac{1}{3}$ liter of water. How much water is needed to fill the whole bottle?
   - Sample response for "How many $\frac{1}{4}$s are in $\frac{1}{3}$?" interpretation:

     Kiran fills a small bottle with $\frac{1}{4}$ liter of water. How many bottles can he fill with $\frac{1}{3}$ liter of water?
2. Trade descriptions with a partner.
   - Review each other's description and discuss whether each question matches the equation.
   - Revise your description based on the feedback from your partner.

   No answer required.
3. Find the answer to your question. Explain or show your reasoning. If you get stuck, consider drawing a diagram.
   - Sample response for "$\frac{1}{4}$ of what number is $\frac{1}{3}$?" interpretation:

     $\frac{4}{3}$ (or $1\frac{1}{3}$) liters
   - Sample reasoning: If $\frac{1}{3}$ liter fills a fourth of a bottle, then 4 times $\frac{1}{3}$ fills the whole bottle.
   - Sample response for "How many $\frac{1}{4}$s are in $\frac{1}{3}$?" interpretation:

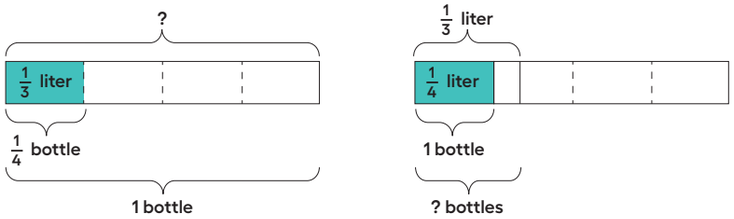
     $\frac{4}{3}$ (or $1\frac{1}{3}$) bottles
   - Sample reasoning: If $\frac{1}{4}$ liter fills 1 bottle, then 1 liter fills 4 bottles and $\frac{1}{3}$ liter fills a third of 4 bottles, which is $\frac{4}{3}$ (or $1\frac{1}{3}$) bottles.

Activity Synthesis

Invite two previously selected students to each share the situation they invented, the question, and their reasoning strategy. Record or display their work for all to see to highlight the two interpretations of $\frac{1}{3} \div \frac{1}{4} = ?$. Or consider using the following examples to illustrate them:

- Finding the size of 1 group: If $\frac{1}{3}$ liter fills $\frac{1}{4}$ of a bottle, how many liters fill 1 bottle?
- Finding the number of groups: If 1 bottle contains $\frac{1}{4}$ liter of water, how many bottles can be filled with $\frac{1}{3}$ liter of water?

In this case, students will likely find the first interpretation of division easier to represent and to solve using a diagram.



The first diagram shows the content of $\frac{1}{4}$ of a bottle, which is $\frac{1}{3}$ liter, being multiplied by 4 to get the content of 1 bottle, which is $\frac{4}{3}$ liters.

The second diagram shows that $\frac{1}{4}$ liter fills 1 bottle, so 1 liter fills 4 bottles, and $\frac{1}{3}$ liter fills a third of the 4 bottles, which is $\frac{4}{3}$ bottles. (Students might also think of $\frac{1}{4}$ as $\frac{3}{12}$, and $\frac{1}{3}$ as $\frac{4}{12}$, and then reason that there are $1\frac{1}{3}$ groups of $\frac{3}{12}$ in $\frac{4}{12}$.)

Lesson Synthesis

A key takeaway from the lesson is that sometimes it is not immediately obvious whether a division situation involves finding the number of groups or the size of 1 group. There may be two "1 wholes" to keep track of (1 whole group and 1 whole unit of a quantity) and two possible questions that could be asked. We need to analyze the situation carefully to reason correctly.

Consider displaying a new problem: "How long is a whole trip if $\frac{2}{3}$ of the trip is $\frac{4}{5}$ mile?"

Give students a minute of quiet time to consider what we can tell about the groups and what information is unknown. Discuss questions such as:

- *"What do we know about the groups in this case?"*

  One full trip is 1 group. Two-thirds of a group is $\frac{4}{5}$ mile.

- *"What multiplication and division equations can we write to represent this situation?"*

  $\frac{2}{3} \cdot ? = \frac{4}{5}$ and $\frac{4}{5} \div \frac{2}{3} = ?$

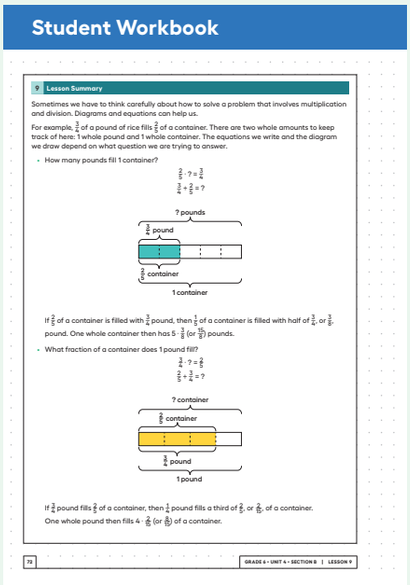
- *"How might we set up a tape diagram to help us answer the question?"*

  We can set up two segments to represent $\frac{2}{3}$ of a trip and $\frac{4}{5}$ mile, so we can see that each $\frac{1}{3}$ of a trip is $\frac{2}{5}$ mile and triple it for the whole trip.

- *"How can we tell if our answer is reasonable?"*

  We can check if $\frac{2}{3}$ of the answer is $\frac{4}{5}$, or multiply the answer by $\frac{2}{3}$ to see if it gives $\frac{4}{5}$.

Student Workbook



Lesson Summary

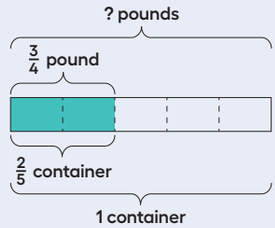
Sometimes we have to think carefully about how to solve a problem that involves multiplication and division. Diagrams and equations can help us.

For example, $\frac{3}{4}$ of a pound of rice fills $\frac{2}{5}$ of a container. There are two whole amounts to keep track of here: 1 whole pound and 1 whole container. The equations we write and the diagram we draw depend on what question we are trying to answer.

- How many pounds fill 1 container?

$$\frac{2}{5} \cdot ? = \frac{3}{4}$$

$$\frac{3}{4} \div \frac{2}{5} = ?$$

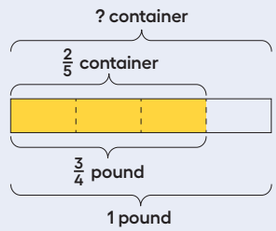


If $\frac{2}{5}$ of a container is filled with $\frac{3}{4}$ pound, then $\frac{1}{5}$ of a container is filled with half of $\frac{3}{4}$, or $\frac{3}{8}$, pound. One whole container then has $5 \cdot \frac{3}{8}$ (or $\frac{15}{8}$) pounds.

- What fraction of a container does 1 pound fill?

$$\frac{3}{4} \cdot ? = \frac{2}{5}$$

$$\frac{2}{5} \div \frac{3}{4} = ?$$



If $\frac{3}{4}$ pound fills $\frac{2}{5}$ of a container, then $\frac{1}{4}$ pound fills a third of $\frac{2}{5}$, or $\frac{2}{15}$, of a container. One whole pound then fills $4 \cdot \frac{2}{15}$ (or $\frac{8}{15}$) of a container.

Cool-down

Refilling a Soap Dispenser

5 min

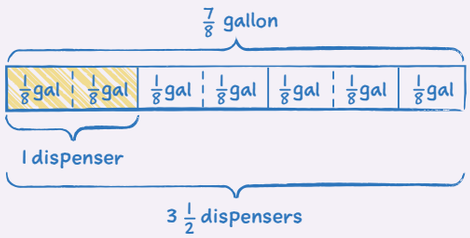
Student Task Statement

Noah is filling soap dispensers with liquid soap. He fills $3\frac{1}{2}$ dispensers with $\frac{7}{8}$ gallon of liquid soap.

How much liquid soap fills 1 dispenser? Show your reasoning.

$\frac{2}{8}$ or $\frac{1}{4}$ gallon

Sample reasoning:

- If $\frac{7}{8}$ gallon fills $3\frac{1}{2}$ dispensers, then $\frac{1}{8}$ gallon fills $\frac{1}{2}$ dispenser, and $\frac{2}{8}$ gallon fills 1 dispenser.
- 

Responding To Student Thinking

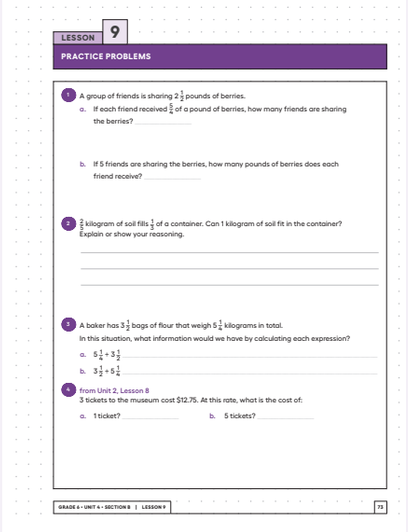
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 9

Practice Problems

6 Problems

Student Workbook



LESSON 9

PRACTICE PROBLEMS

1. A group of friends is sharing $2\frac{1}{2}$ pounds of berries.
   a. If each friend received $\frac{5}{4}$ of a pound of berries, how many friends are sharing the berries? _____
   b. If 5 friends are sharing the berries, how many pounds of berries does each friend receive? _____
2. $\frac{2}{5}$ kilogram of soil fills $\frac{1}{3}$ of a container. Can 1 kilogram of soil fit in the container? Explain or show your reasoning.
3. A baker has $3\frac{1}{2}$ bags of flour that weigh $5\frac{1}{4}$ kilograms in total. In this situation, what information would we have by calculating each expression?
   a. $5\frac{1}{4} \div 3\frac{1}{2}$ _____
   b. $3\frac{1}{2} \div 5\frac{1}{4}$ _____
4. from Unit 2, Lesson 8
   3 tickets to the museum cost $12.75. At this rate, what is the cost of:
   a. 1 ticket? _____ b. 5 tickets? _____

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Problem 1

A group of friends is sharing $2\frac{1}{2}$ pounds of berries.

a. If each friend received $\frac{5}{4}$ of a pound of berries, how many friends are sharing the berries?

2 friends ($2\frac{1}{2} \div \frac{5}{4} = 2$)

b. If 5 friends are sharing the berries, how many pounds of berries does each friend receive?

$\frac{1}{2}$ pound ($2\frac{1}{2} \div 5 = \frac{1}{2}$)

Problem 2

$\frac{2}{5}$ kilogram of soil fills $\frac{1}{3}$ of a container. Can 1 kilogram of soil fit in the container? Explain or show your reasoning.

Yes

Sample reasoning:

- The container can fit $3 \cdot \frac{2}{5}$ (or $\frac{6}{5}$) kilograms of soil, which is greater than 1 kilogram.
- $\frac{2}{5} \div \frac{1}{3}$ gives a quotient greater than 1, which means that the container can fit more than 1 kilogram.

Problem 3

A baker has $3\frac{1}{2}$ bags of flour that weigh $5\frac{1}{4}$ kilograms in total.

In this situation, what information would we have by calculating each expression?

a. $5\frac{1}{4} \div 3\frac{1}{2}$

The weight (in kilograms) of 1 bag of flour

b. $3\frac{1}{2} \div 5\frac{1}{4}$

The fraction of the bag that 1 kilogram of flour would take up

Problem 4

from Unit 2, Lesson 8

3 tickets to the museum cost $12.75. At this rate, what is the cost of:

a. 1 ticket?

$4.25 because 12.75 divided by 3 is 4.25

b. 5 tickets?

$21.25 because the unit rate is 4.25, and (4 . 25) · 5 = 21.25

Problem 5

from Unit 2, Lesson 9

Elena went 60 meters in 15 seconds. Noah went 50 meters in 10 seconds. Elena and Noah both moved at a constant speed.

a. How far did Elena go in 1 second?

Elena went 4 meters in 1 second because 60 ÷ 15 = 4.

b. How far did Noah go in 1 second?

Noah went 5 meters in 1 second because 50 ÷ 10 = 5.

c. Who went faster? Explain or show your reasoning.

Noah went faster; he ran more distance in 1 second. After the distances traveled in 1 second are computed, they can be compared directly.

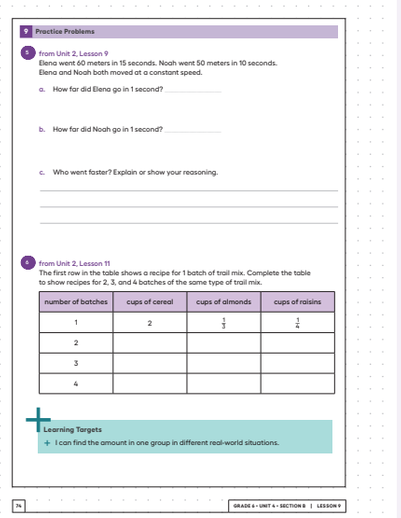
Problem 6

from Unit 2, Lesson 11

The first row in the table shows a recipe for 1 batch of trail mix. Complete the table to show recipes for 2, 3, and 4 batches of the same type of trail mix.

number of batches	cups of cereal	cups of almonds	cups of raisins
1	2	$\frac{1}{3}$	$\frac{1}{4}$
2	4	$\frac{2}{3}$	$\frac{1}{2}$
3	6	1 (or equivalent)	$\frac{3}{4}$
4	8	$\frac{4}{3}$	1 (or equivalent)

Student Workbook


9 Practice Problems

5 from Unit 2, Lesson 9
Elena went 60 meters in 15 seconds. Noah went 50 meters in 10 seconds. Elena and Noah both moved at a constant speed.
a. How far did Elena go in 1 second?
b. How far did Noah go in 1 second?
c. Who went faster? Explain or show your reasoning.

6 from Unit 2, Lesson 11
The first row in the table shows a recipe for 1 batch of trail mix. Complete the table to show recipes for 2, 3, and 4 batches of the same type of trail mix.

number of batches	cups of cereal	cups of almonds	cups of raisins
1	2	$\frac{1}{3}$	$\frac{1}{4}$
2			
3			
4			

Learning Targets
+ I can find the amount in one group in different real-world situations.

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