Measuring Long Distances over Uneven Terrain

Goals

- Choose a method and appropriate measuring tools to measure the length of a path.
- Compare and contrast (orally and in writing) methods for measuring long distances.
- Compare measurements of a path and represent (in writing) the difference between measurements as a percentage.

Lesson Narrative

In this optional lesson, students brainstorm ideas about how to measure long distances, possibly over uneven terrain. As students work in groups to try out the accuracy and effectiveness of different methods, they construct arguments and critique the reasoning of others. Some of the methods involve proportional reasoning. Students use appropriate tools when they plan and try out methods of measurement.

Student Learning Goal

Let's measure long distances over uneven terrain.

Access for Students with Diverse Abilities

• Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

• MLR1: Stronger and Clearer Each Time

Required Materials

Materials to Gather

- Measuring tapes: Activity 2, Activity 3
- Meter sticks: Activity 2, Activity 3
- Yardsticks: Activity 2, Activity 3

Activity 3:

Choose a path outside of the classroom that students can measure the length of. The length should be between 50 and 100 meters (long enough that it cannot be measured directly with a tape measure. Ideally it should include some curves or elevation changes (but not stairs). A long hallway would also be okay, especially if it goes around corners. This is not part of the 5K course, rather it is just a path to test measuring methods.

Lesson Timeline



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Activity 2



Activity 3

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence

Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity. Supports accessibility for: Social-Emotional Functioning, Attention



Activity 1

How Far Is It?



Activity Narrative

Students have experienced measuring short distances with a ruler or a measuring tape. In this activity, students start to think about how they can measure longer distances over uneven terrain. This activity is intended to set the stage for the upcoming activities, not to completely resolve the question. Students have an opportunity to think about the limitations of methods that may work for short distances but not for long distances. They also consider real-world situations that involve the measurement of long distances.

Launch

Arrange students in groups of 3–4. They will stay in these groups throughout this four-lesson section. Ask students how they have measured the length of objects in school (with a ruler, yardstick, or measuring tape). Where else in real life do people measure distances, especially longer ones? Brainstorm some situations together (distance driven in a car, length of a garden fence, length of a hiking trail, etc.). Give students 2–3 minutes of quiet work time, followed by small-group discussion.

Student Task Statement

How do people measure distances in different situations? What tools do they use? Come up with at least three different methods and situations where those methods are used.

Sample responses:

- Use a yardstick, a measuring tape, or ruler repeatedly, if necessary
- When driving, use the odometer to measure distance between departure and destination location
- Count steps and estimate how long one step is, for example, to measure the distance across the room
- Estimate an inch using a finger and iterate across the width of the table to measure table width
- Count the number of ceiling tiles or the number of windows across the room, estimate the width of tiles or windows and multiply the number of objects times the width of the object
- Use a GPS
- Use "rate times time," if they know their speed and how long it takes to get somewhere

Activity Synthesis

Invite students to share some ideas of how to measure with their group.

Activity 2

Planning a 5K Course



Activity Narrative

In the previous activity, students started to think about how to measure distances in different situations. The activity introduces the context of designing a course for a 5K fundraising walk. Students will continue working with this context in future lessons. In this activity, they come up with a method for measuring the walking distance of a path that is too long to measure with a measuring tape. As students select appropriate strategies to measure distances, they have the opportunity to consider the limitations and benefits of each tool when measuring.

Launch

Keep students in the same groups. Provide access to measuring tools, such as yardsticks, meter sticks, and tape measures. Ask students if they have ever been in or watched a walk-a-thon or a race. Explain that sometimes a race is done by repeating a shorter course several times, for example, a mile is about 4 laps around a track. For this activity, they should plan for a course that is about 500 meters long that walkers can go around multiple times.

Give students 5-6 minutes to work with their group.

Student Task Statement

Imagine the school will be holding a 5K fundraising walk on the school grounds. Your class is supposed to design the course for the walk.

- 1. What will you need to do to design the course for the walk?
 - Sample response: We need to find a course for one lap of the race, decide where the start and end is, measure it, and then figure out how many times one has to go around it to complete 5 km.
- 2. Come up with a method to measure the course.

Sample responses:

- · Use a measuring tape over and over again.
- Measure your stride length, and then count the number of steps.
- Find a map of the campus and use the scale on the map to compute the length of the course.

Activity Synthesis

Check in with each group about their method of measurement. When they have a plan that they understand, they can move to the next activity.

Instructional Routines

MLR8: Discussion Supports

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Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.

Display sentence frames to support students in preparing their plan:

"First, I will _____ because ..." and
"I will use _____ because ..."

Advances: Speaking, Writing



Instructional Routines

MLR1: Stronger and Clearer Each Time

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code or URL.



Access for Multilingual Learners (Activity 3, Narrative)

MLR1: Stronger and Clearer Each Time

This activity uses the Stronger and Clearer Each Time math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.



Activity 3

Comparing Methods



Activity Narrative

In this activity, students use the method they came up with in the previous activity to measure the length of a path chosen by the teacher.

In this group activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language they use to describe their thinking.

It is not important that students' results are very accurate. They will measure the distance again with a trundle wheel in a later lesson. The main point of this activity is to think about measurement methods and to discuss the advantages and disadvantages of different methods.

Launch

Keep students in the same groups. Provide access to measuring tools. Show students the path they should measure. Give students time to measure with their group. Each group can begin working on this activity as soon as they have finished the previous activity and checked in with the teacher.

If a group finishes this activity early, consider asking them to compare their results with additional groups.

Student Task Statement

Let's see how close different measuring methods are to each other. Your teacher will show you a path to measure.

- 1. Use your method to measure the length of the path at least two times.

 Answers vary.
- 2. Decide what distance you will report to the class.

Sample response: Report the average between the two measurements.

- **3.** Compare your results with another group's. Express the differences between the measurements in terms of percentages.
 - Sample response: If group A's measurement is 50 m and group B's measurement is 51 m, then group B's measurement is 2% larger than group A's since $51 \div 50 = 1.02$.
- **4.** Discuss the advantages and disadvantages of each group's method.

Sample responses:

- Method: Use a measuring tape over and over again. Advantage:
 Can be very accurate. Disadvantage: It takes two people and is quite cumbersome. If not done carefully, each time the tape moves, an error is introduced. So this is not very practical for long distances and if there are a lot of corners to go around.
- Method: Measure stride length, and then count the number of strides.
 Advantages: Very easy to do and very quick. Disadvantage: Not all strides are equal. The longer the distance, the more chances for errors there are.

Activity Synthesis

Use Stronger and Clearer Each Time to give students an opportunity to revise and refine their response to the last question, about the advantages and disadvantages of each group's method. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

 \bigcirc "____ makes sense, but what do you mean when you say ... ?"

"Can you describe that another way?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After Stronger and Clearer Each Time, invite the different groups to share their solutions. Ask them to:

- Compare how close their answers are.
- Compute the approximate relative error (difference/total length).
- Discuss the advantages and disadvantages of their methods and sources of discrepancies in their measurements, and how a small error can propagate.

The takeaway should include:

- Proportional reasoning can be used to find longer distances. If it is known that it takes 10 steps to walk 8 meters, then it will take 20 steps to walk 16 meters.
- Small errors can magnify over longer distances.
- Methods were either not very precise (prone to introduce error), or they were precise but cumbersome to implement.