Lesson 15

Adding and Subtracting with Scientific Notation

Goal

Generalize (orally and in writing) a process of adding and subtracting numbers in scientific notation and interpret results in context.

Learning Target

I can add and subtract numbers given in scientific notation.

Lesson Narrative

Students add and subtract with numbers expressed in scientific notation. They begin by studying expressions and determining the number of non-zero digits when evaluating each expression. This encourages students to make sense of how operations with scientific notation are alike and different.

Next, students critique three different strategies for adding and subtracting numbers in scientific notation. Students see that two of the strategies — writing the expression in decimal notation first, and rewriting one of the expressions so that they are both a multiple of the same power of 10 — can both be used to perform these types of calculations.

Students continue to make sense of addition and subtraction with scientific notation and interpreting their results in context when comparing the width of 5 planets side-by-side to the diameter of the sun.

Student Learning Goal

Let's add and subtract using scientific notation to answer questions about animals and the solar system.

Instructional Routines

- Math Talk
- MLR5: Co-Craft Questions

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	15 min	10 min	5 min

Instructional Routines

Card Sort

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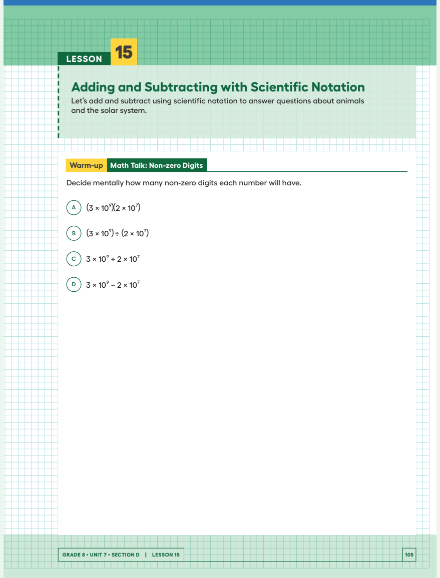
Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook


LESSON 15

Adding and Subtracting with Scientific Notation

Let's add and subtract using scientific notation to answer questions about animals and the solar system.

Warm-up Math Talk: Non-zero Digits

Decide mentally how many non-zero digits each number will have.

A $(3 \times 10^9)(2 \times 10^7)$

B $(3 \times 10^9) \div (2 \times 10^7)$

C $3 \times 10^9 + 2 \times 10^7$

D $3 \times 10^9 - 2 \times 10^7$

Warm-up

Math Talk: Non-zero Digits

5 min

Activity Narrative

This *Math Talk* focuses on operations with numbers written in scientific notation. It encourages students to think about powers of 10 and to rely on what they know about how the exponent of a power of 10 is related to the number of zeros to mentally solve problems. The strategies and understandings elicited here will be helpful later in the lesson when students compute with numbers in scientific notation.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally how many non-zero digits each number will have.

a. $(3 \times 10^9)(2 \times 10^7)$

One non-zero digit

Sample reasoning: Multiplying 3 and 2 gives us 6, and the rest will just add a bunch of zeros.

b. $(3 \times 10^9) \div (2 \times 10^7)$

Two non-zero digits

Sample reasoning: Dividing 3 by 2 gives us 1.5, and the rest will just move the decimal place and add a bunch of zeros.

c. $3 \times 10^9 + 2 \times 10^7$

Two non-zero digits

Sample reasoning: The 3 and the 2 have different place values and so when the values are added together, the 3 and 2 will remain as 2 separate digits and combine to make one digit of 5.

d. $3 \times 10^9 - 2 \times 10^7$

Three non-zero digits

Sample reasoning: The 3 and the 2 have different place values that differ by a factor of $, meaning they will be 2 places away from each other. When subtracted, we will have to "borrow" 2 times, resulting in 3 non-zero numbers.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ____'s strategy?"

"Do you agree or disagree? Why?" "What connections to previous problems do you see?"

Activity 1

Measuring the Planets

15 min

Activity Narrative

Students attend to precision when adding numbers in scientific notation, taking care that the numbers are first written as a decimal or with powers of 10 with the same exponent. Students critique the reasoning of Diego, Clare, and Kiran as they make sense of adding numbers in scientific notation.

Launch

Arrange students in groups of 2. Display this statement from the *Task Statement* for all to see:

"If Neptune and Saturn were side by side, would they be wider than Jupiter?"

Ask students what information they would need to know to answer this question and what type of calculations they would need to do. (They would need to know the diameters of all 3 planets. They would need to add the diameters of Neptune and Saturn and compare it to the diameter of Jupiter.)

Explain that Diego, Kiran, and Clare were wondering the same thing, and each tried solving the problem a different way. Give students 7–8 minutes to work with a partner, and follow that with a whole-class discussion.

Student Task Statement

Diego, Kiran, and Clare were wondering:

"If Neptune and Saturn were side by side, would they be wider than Jupiter?"

1. They start by trying to add 4.9×10^4 km and 1.2×10^5 km, the diameters of Neptune and Saturn. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.

   a. Diego says, "When we add the distances, we will get $4.9 + 1.2 = 6.1$. The exponent will be 9. So the two planets are 6.1×10^9 km side by side."

   I disagree with Diego

   Sample reasoning: The decimal place values do not match. It would be like writing $47 + 1.2 = 59$, which is not correct.

Access for Multilingual Learners (Activity 1, Synthesis)

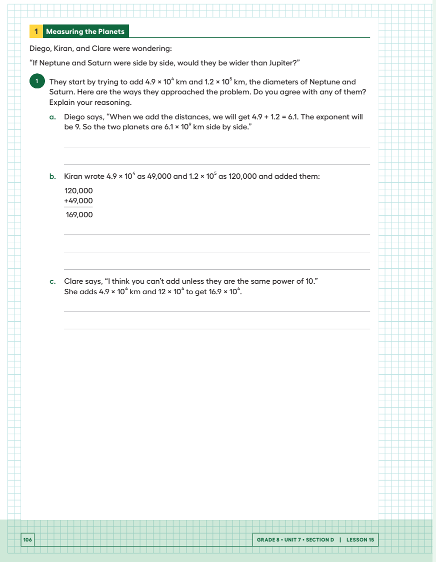
MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.
Invite students to explain their thinking orally, using numbers, pictures, or diagrams.
Supports accessibility for: Language, Fine Motor Skills

Student Workbook


1 Measuring the Planets

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1. They start by trying to add 4.9×10^4 km and 1.2×10^5 km, the diameters of Neptune and Saturn. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.

   a. Diego says, "When we add the distances, we will get $4.9 + 1.2 = 6.1$. The exponent will be 9. So the two planets are 6.1×10^9 km side by side."

   b. Kiran wrote 4.9×10^4 as 49,000 and 1.2×10^5 as 120,000 and added them:

   120,000
   +49,000
   169,000

   c. Clare says, "I think you can't add unless they are the same power of 10."
   She adds 4.9×10^4 km and 12×10^4 to get 16.9×10^4.

b. Kiran wrote 4.9×10^4 as 47,000 and 1.2×10^5 as 120,000 and added them:

$$\begin{array}{r} 120{,}000 \\ +49{,}000 \\ \hline 169{,}000 \end{array}$$

I agree with Kiran

Sample reasoning: The two distances are added correctly. $4.9 \times 10^4 = 49,000$ and $1.2 \times 10^5 = 120,000$, and they are added correctly.

c. Clare says, "I think you can't add unless they are the same power of 10." She adds 4.9×10^4 km and 12×10^4 to get 16.9×10^4.
I agree with Clare.

Sample reasoning: The two distances are added correctly. Since both terms are a multiple of 10^4, they have the same place value and can be added together.

Activity Synthesis

The purpose of this discussion is to highlight common misconceptions when adding or subtracting numbers in scientific notation and to introduce students to two possible strategies. Begin by asking students to share which students they agreed with and why. Here are some questions for discussion:

"How are Clare's and Kiran's approaches alike?"

They both reached the same sum by attending to place value.

"How are their approaches different?"

Kiran wrote the numbers as decimals and added them. Clare wrote the numbers with the same power of 10 and added them. Kiran's method might not work very well if the numbers are very large or very small. The decimal form of those numbers would be unwieldy.

"Why must the terms have the same power of 10 to be added?"

We can only add digits that are of the same place value. If the powers of 10 are different, the place values of the digits in the first factors of the two expressions would be different. For example, the 4 in 4.9×10^4 means 4 ten-thousands and the 1 in 1.2×10^5 means 1 hundred-thousand, so we cannot add 4.9 and 1.2.

"How might Clare have reasoned that 1.2×10^5 can be written as 12×10^4?"

Changing 1.2 into 12 requires multiplying by 10. To keep the value of the expression the same, we must divide it by 10, which decreases the exponent by 1, from 10^5 to 10^4.

"If the diameter of Jupiter is 1.43×10^5, are Neptune and Saturn side-by-side wider than Jupiter?"

Yes, since a width of 1.69×10^5 is greater than a width of 1.43×10^5.

Activity 2
A Celestial Dance

15 min

Activity Narrative

In this activity, students add quantities written in scientific notation in order to answer questions in context. To add numbers in scientific notation, students must reason abstractly and quantitatively by aligning place value and then interpreting their results in the context of the situation.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Arrange students in groups of 2. Introduce the table from the *Task Statement*. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

Display only the table, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation, before comparing questions with a partner.

- Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Reveal the questions and give students 1–2 minutes to compare them to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

"Which of your questions is most similar to or different from the ones provided? Why?"

"Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it."

Then give students 6–7 minutes to work with a partner, and follow that with a whole-class discussion.

Instructional Routines

MLR5: Co-Craft Questions

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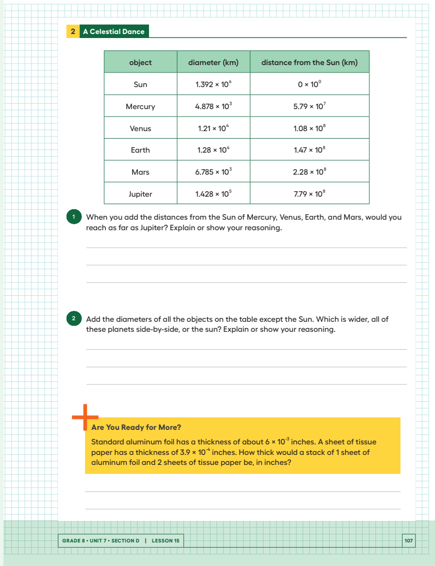
Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Student Workbook



Student Task Statement

object	diameter (km)	distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

1. When you add the distances from the Sun of Mercury, Venus, Earth, and Mars, would you reach as far as Jupiter? Explain or show your reasoning.

   The sum of the distances is not enough to reach Jupiter. Sample reasoning: $(5.79 \times 10^7) + (1.08 \times 10^8) + (1.47 \times 10^8) + (2.28 \times 10^8) =$ $(0.579 + 1.08 + 1.47 + 2.28) \times 10^8 = 5.409 \times 10^8$ km, which is less than 7.79×10^8 km.

2. Add the diameters of all the objects on the table except the Sun. Which is wider, all of these planets side-by-side, or the sun? Explain or show your reasoning.

   The 5 planets shown on the table side-by-side are not as wide as the Sun. Sample reasoning: $(4.878 \times 10^3) + (1.21 \times 10^4) + (1.28 \times 10^4) + (6.785 \times 10^3)$ $+ (1.428 \times 10^5) = (0.4878 + 1.21 + 1.28 + 0.6785 + 14.28) \times 10^4 = 17.9363 \times 10^4$ $= 1.79363 \times 10^5$ km, which is less than 1.392×10^6 km.

Are You Ready for More?

Standard aluminum foil has a thickness of about 6×10^{-3} inches. A sheet of tissue paper has a thickness of 3.9×10^{-4} inches. How thick would a stack of 1 sheet of aluminum foil and 2 sheets of tissue paper be, in inches?

0.00678, or 6.78×10^{-3} (or equivalent)

Activity Synthesis

The goal of this discussion is to highlight that values given in scientific notation can be added by carefully aligning the place values of all of the addends. Display the table from the *Task Statement* for all to see and consider discussing the following questions:

- *"Can we easily add the diameters of Mercury and Mars? Why?"*

  Yes, because both measurements are written with the same power of 10, so we can add the first factors together.

- *"What are some other planet diameters that we can add together without any additional steps?"*

  Venus and Earth because their diameters are both written with the same power of 10.

Activity 3: Optional
A Massive Farm

15 min

Activity Narrative

In this optional activity, students work with positive and negative exponents simultaneously. This activity may be useful if students need more experience with negative exponents or additional practice adding quantities expressed using scientific notation.

Launch

Arrange students in groups of 2. Tell students to explain their thinking to their partner and work to reach agreement. Give students 10-12 minutes to work, and follow that with a whole-class discussion.

Student Task Statement

The table shows the average mass of one individual creature and an estimated total number of those creatures on Earth. Use the table to answer each question, and explain or show your reasoning.

creature	total number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. On a farm there was a cow. And on the farm there were 2 sheep. There were also 3 chickens. What is the total mass of the 1 cow, the 2 sheep, the 3 chickens, and the 1 farmer on the farm?

   588 kg

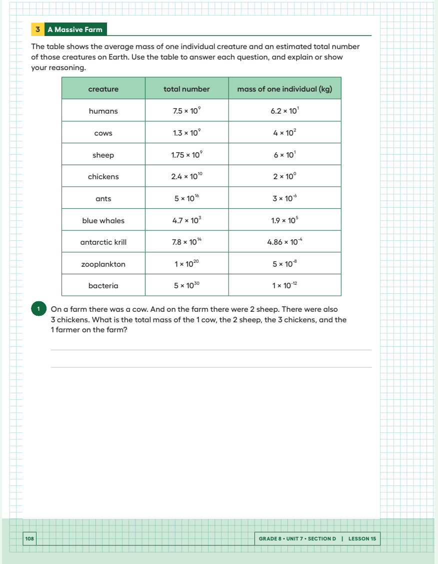
   Sample reasoning: The sum of the masses of 1 cow, 2 sheep, 3 chickens, and 1 farmer is $4 \times 10^2 + 2(6 \times 10^1) + 3(2 \times 10^0) + 6.2 \times 10^1 = 400 + 120 + 6 + 62 = 588$ kg.

2. What is the total mass of a human, a blue whale, and 6 ants all together?

   190,062.000018 kg

   Sample reasoning: The sum of the masses of 1 human, 1 blue whale, and 6 ants is $6.2 \times 10^1 + 1.9 \times 10^5 + 6(3 \times 10^{-6}) = 62 + 190{,}000 + 18 \times 10^{-6} = 190{,}062.000018$ kg.

Student Workbook



Student Workbook

3. Which is greater, the number of bacteria, or the number of all the other animals in the table put together?

The number of bacteria is greater.

Sample reasoning:

- There are 8 entries in the list that are not bacteria, and the second-highest number of individuals is 1×10^{20}, so the sum of all the individuals must be less than 8×10^{20}, which is still much less than the number of bacteria.
- Adding all the non-bacteria individuals gives 1.0005078×10^{20}. There are roughly 50 billion times as many bacteria.

Activity Synthesis

The goal of this discussion is to highlight the difference between multiplying or dividing numbers in scientific notation and adding and subtracting numbers in scientific notation. Here are some questions for discussion:

"When using scientific notation, what are some differences between the strategies for multiplying or dividing numbers and the strategies for adding and subtracting?"

When multiplying or dividing, the numbers don't have to have the same power of 10 like they do when adding or subtracting.

"Which operation do you think is the easiest? Hardest?"

Answers vary.

Lesson Synthesis

The purpose of this discussion is to reflect on the different methods for adding and subtracting numbers given in scientific notation. Here are some questions for discussion:

"Which method did you prefer to make sense of adding two numbers in scientific notation — Diego's (writing the number as a decimal) or Clare's (writing both numbers as a multiple of the same power of 10)?"

Answers vary.

"When using scientific notation, how is adding and subtracting different from multiplying and dividing? Which do you think is easier? Why do you think that is?"

When adding or subtracting, the powers of 10 need to match, but they don't when multiplying or dividing. Multiplying and dividing with scientific notation is easier, because it is possible to use exponent rules to help do calculations. Adding and subtracting is easier because once you write the numbers using the same power of 10, then adding and subtracting is the same as adding numbers that are not written in scientific notation

"Is there anything that you found surprising or interesting in the problems you did?"

Answers vary.

Lesson Summary

When adding decimal numbers, we need to pay close attention to place value. For example, when we calculate 13.25 + 6.7, we need to make sure to add hundredths to hundredths (5 and 0), tenths to tenths (2 and 7), ones to ones (3 and 6), and tens to tens (1 and 0).

$$\begin{array}{r} 13.25 \\ +\ 6.7 \\ \hline 19.95 \end{array}$$

We need to take the same care when we add or subtract numbers in scientific notation. For example, suppose we want to find how much further Earth is from the Sun than Mercury is from the Sun. Earth is about 1.5×10^8 km from the Sun, while Mercury is about 5.8×10^7 km. In order to find

$$1.5 \times 10^8 - 5.8 \times 10^7$$

we can rewrite this as

$$1.5 \times 10^8 - 0.58 \times 10^8.$$

Now that both numbers are written in terms of 10^8, we can subtract 0.58 from 1.5 to get

$$0.92 \times 10^8 .$$

Rewriting this in scientific notation, Earth is 9.2×10^7 km further from the Sun than Mercury is from the Sun.

Cool-down

Adding with Scientific Notation

5 min

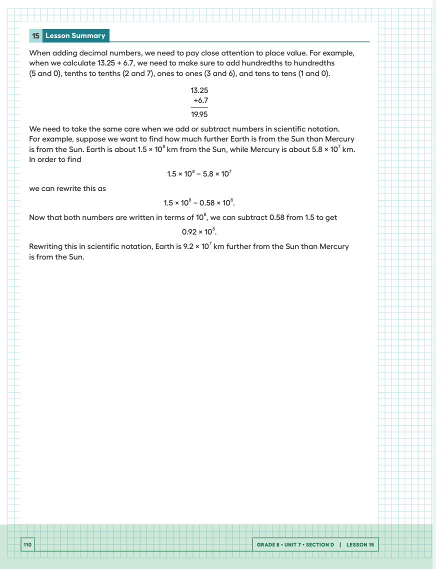
Student Task Statement

Elena wants to add $(2.3 \times 10^5) + (3.6 \times 10^6)$ and writes $(2.3 \times 10^5) + (3.6 \times 10^6) = 5.9 \times 10^6$.

Explain to Elena what her mistake was and what the correct solution is.

Sample response: Elena added 2.3 and 3.6 without realizing that 3.6×10^6 is over 10 times as large as 2.3×10^5. Instead, she should have first rewritten one of the expressions so that both were multiplied by the same power of 10. For example, rewriting 3.6×10^6 as a multiple of 10^5 gives $36 \times 10^5 + 2.3 \times 10^5 = (36 + 2.3) \times 10^5 = 38.3 \times 10^5 = 3.83 \times 10^6$.

Student Workbook



Responding To Student Thinking

Press Pause

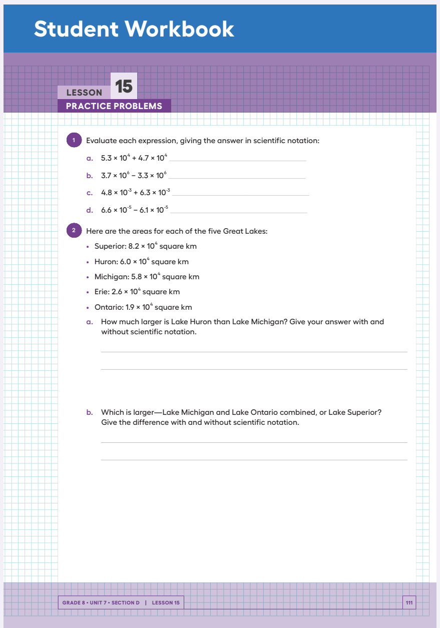
If most students struggle with adding or subtracting numbers written in scientific notation, make time to revisit examples of how only numbers with the same place value can be added together. For example, in practice problems like this:

Unit 7, Lesson 15, Practice Problem 1

Lesson 15

Practice Problems

5 Problems


Student Workbook

LESSON 15

PRACTICE PROBLEMS

1 Evaluate each expression, giving the answer in scientific notation:

a. $5.3 \times 10^4 + 4.7 \times 10^4$

b. $3.7 \times 10^6 - 3.3 \times 10^6$

c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$

d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$

2 Here are the areas for each of the five Great Lakes:

- Superior: 8.2×10^4 square km
- Huron: 6.0×10^4 square km
- Michigan: 5.8×10^4 square km
- Erie: 2.6×10^4 square km
- Ontario: 1.9×10^4 square km

a. How much larger is Lake Huron than Lake Michigan? Give your answer with and without scientific notation.

b. Which is larger—Lake Michigan and Lake Ontario combined, or Lake Superior? Give the difference with and without scientific notation.

Problem 1

Evaluate each expression, giving the answer in scientific notation:

a. $5.3 \times 10^4 + 4.7 \times 10^4$ 1×10^5

b. $3.7 \times 10^6 - 3.3 \times 10^6$ 4×10^5

c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$ 1.11×10^{-2}

d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$ 5×10^{-6}

Problem 2

Here are the areas for each of the five Great Lakes:

- Superior: 8.2×10^4 square km
- Huron: 6.0×10^4 square km
- Michigan: 5.8×10^4 square km
- Erie: 2.6×10^4 square km
- Ontario: 1.9×10^4 square km

a. How much larger is Lake Huron than Lake Michigan? Give your answer with and without scientific notation.

2×10^3 (or 2,000) square km

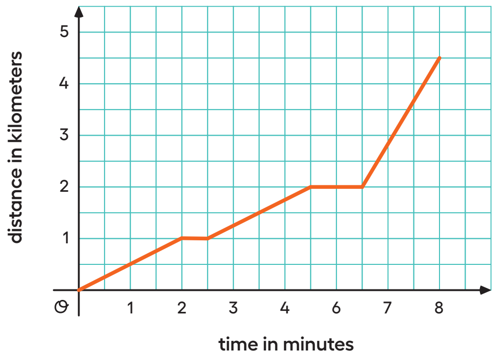
b. Which is larger—Lake Michigan and Lake Ontario combined, or Lake Superior? Give the difference with and without scientific notation.

Lake Superior is larger by 5×10^3 (or 5,000) square km.

Problem 3

from Unit 5, Lesson 10

a. Write a scenario that describes what is happening in the graph.



Sample response: A person is driving. The distance measures distance away from the person's house.

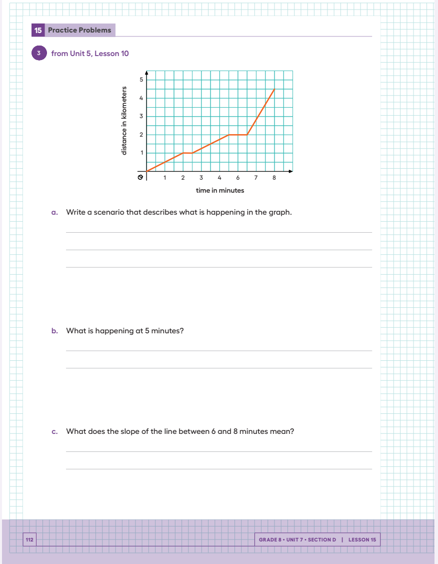
b. What is happening at 5 minutes?

Sample response: The person is stopped 2 km from home.

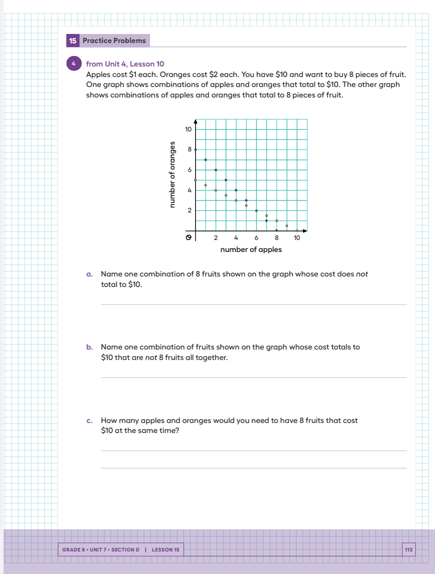
c. What does the slope of the line between 6 and 8 minutes mean?

Sample response: The slope between 6 and 8 minutes indicates the speed the person is driving (1 km per minute), which is faster than any of the person's speeds between 0 and 6 minutes.

Student Workbook



Student Workbook



15 Practice Problems

4 from Unit 4, Lesson 10

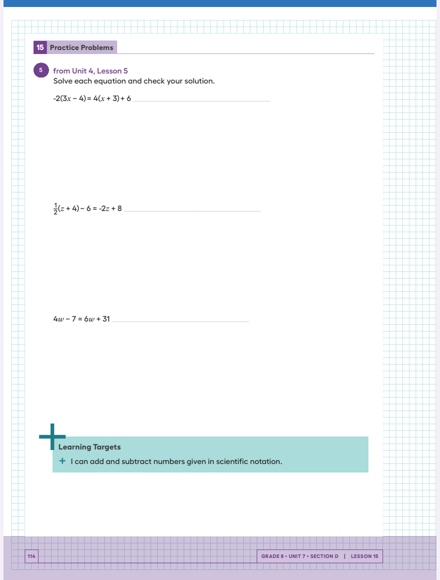
Apples cost $1 each. Oranges cost $2 each. You have $10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to $10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.

a. Name one combination of 8 fruits shown on the graph whose cost does *not* total to $10.

b. Name one combination of fruits shown on the graph whose cost totals to $10 that are *not* 8 fruits all together.

c. How many apples and oranges would you need to have 8 fruits that cost $10 at the same time?

Student Workbook



15 Practice Problems

5 from Unit 4, Lesson 5

Solve each equation and check your solution.

$-2(3x-4)=4(x+3)+6$

$\frac{1}{2}(z+4)-6=-2z+8$

$4w-7=6w+31$

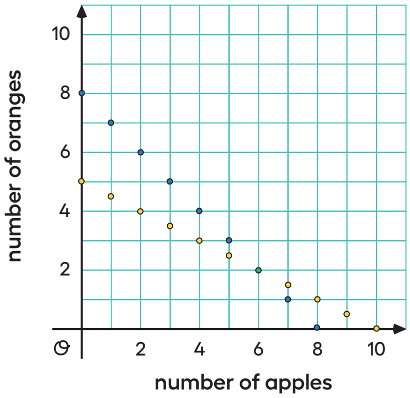
Learning Targets

+ I can add and subtract numbers given in scientific notation.

Problem 4

from Unit 4, Lesson 10

Apples cost $1 each. Oranges cost $2 each. You have $10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to $10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.



a. Name one combination of 8 fruits shown on the graph whose cost does *not* total to $10.

Sample response: 4 apples, 4 oranges

b. Name one combination of fruits shown on the graph whose cost totals to $10 that are *not* 8 fruits all together.

Sample response: 2 apples, 4 oranges

c. How many apples and oranges would you need to have 8 fruits that cost $10 at the same time?

6 apples and 2 oranges

Problem 5

from Unit 4, Lesson 5

Solve each equation and check your solution.

$-2(3x-4)=4(x+3)+6$ $x=-1$

$\frac{1}{2}(z+4)-6=-2z+8$ $z=\frac{24}{5}$

$4w-7=6w+31$ $w=-19$