Two Equations for Each Relationship

Goals

- Use the word "reciprocal" to explain (orally and in writing) that there are two related constants of proportionality for a proportional relationship.
- Write two equations that represent the same proportional relationship, i.e., y = kx and $x = \left(\frac{1}{k}\right)y$, and explain (orally) what each equation means.

Learning Targets

- I can find two constants of proportionality for a proportional relationship.
- I can write two equations representing a proportional relationship described by a table or story.

Lesson Narrative

In this lesson, students practice writing two different equations for the same proportional relationship. This is accomplished by switching which quantity is regarded as being proportional to the other. Students see why the constants of proportionality associated with the two equations are reciprocals of each other.

For example, if a person runs at a constant speed and travels 12 miles in 2 hours, then the distance traveled is proportional to the time elapsed, with constant of proportionality 6, because distance = $6 \cdot \text{time}$. The time elapsed is proportional to distance traveled with constant of proportionality $\frac{1}{6}$, because time = $\frac{1}{6}$ · distance.

The activities in this lesson use familiar contexts, but not identical situations from previous lessons: measurement conversions and water flowing at a constant rate. Students are expected to use methods developed earlier: organize data in a table, write and solve an equation to determine the constant of proportionality, and generalize from repeated calculations to arrive at an equation. Students also practice reasoning quantitatively and abstractly as they write or use an equation and then interpret their answers in the context of the situation. The last activity is optional because it provides an opportunity for additional practice with a new context.

Student Learning Goal

Let's investigate equations that represent proportional relationships.

Lesson Timeline

Warm-up

15

Activity 1

15

Activity 2

10

Activity 3

Lesson Synthesis

10

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 3)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR5: Co-Craft Questions (Activity 3)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR5: Co-Craft Questions
- · Which Three Go Together?

Assessment

Cool-down

Lesson 5 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

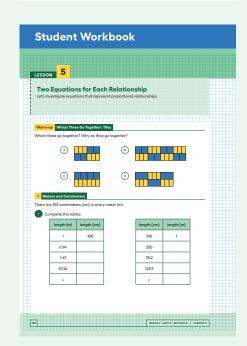
Instructional Routines

Which Three Go Together?

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Warm-up

Which Three Go Together: Tiles



Activity Narrative

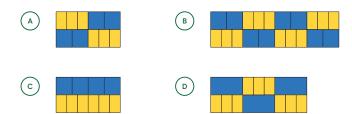
This Warm-up prompts students to compare four geometric patterns. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the images for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three images that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- The ratio of the number of blue tiles to the number of yellow tiles is 2:3.
- They have an even number of blue tiles and an even number of yellow tiles.
- You can use vertical cuts to make groups of 5 tiles (with 2 blue and 3 yellow in each group).

A, B, and D go together because:

- They have both colors on the top and on the bottom.
- They have some blue tiles on the bottom.
- They have some yellow tiles on the top.

A, C, and D go together because:

- The pattern ends (on the right) with blue on top and yellow on bottom.
- They have fewer than 15 total tiles.

B, C, and D go together because:

- The pattern starts (on the left) with blue on top and yellow on bottom.
- · Each image has 6 yellow tiles on the bottom.

Lesson 5 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "row," "group," "partition," "even," "odd," "horizontal," "vertical," "ratio," or "area," and to clarify their reasoning as needed. Consider asking:

○ "How do you know ... ?"

"What do you mean by ...?"

"Can you say that in another way?"

If time allows, invite 2–3 students to briefly share what they notice all of the figures have in common. For example:

- They are all rectangles composed of smaller rectangles.
- Half their area is blue and half is yellow.
- They all have an even number of total tiles.

The purpose of this concluding share out is to reinforce the importance of using precise terminology. For example, saying "the ratio of blue to yellow" is not specific enough. The ratio of blue area to yellow area is 1:1 for all of the figures, while the ratio of blue pieces to yellow pieces is either 2:3 or 1:3.

Activity 1

Meters and Centimeters

15 min

Activity Narrative

In this activity, students practice writing an equation to represent a proportional relationship given in a table. This activity revisits the idea that there are two reciprocal constants of proportionality between two related quantities. Students use repeated reasoning to arrive at the equations and to identify the constants of proportionality as reciprocals.

We return to the context of measurement conversion, again examining the same distances measured in two different units. Previously students compared centimeters with millimeters and saw that the constants of proportionality were 10 and $\frac{1}{10}$. Here, students compare meters with centimeters and see that the constants of proportionality are 100 and $\frac{1}{100}$. The similarities between this activity and the earlier one may cause this activity to go very quickly.

Launch

Remind students that in an earlier lesson, they examined the relationship between millimeters and centimeters. Tell them that today, they will examine the relationship between centimeters and meters.

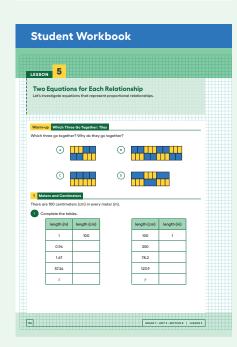
Warm-up

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation such as: "First, I ______ because ..." "I noticed _____, so I..." "Why did you ... ?" and "I agree/ disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

Student Task Statement

There are 100 centimeters (cm) in every meter (m).

1. Complete the tables.

length (m)	length (cm)
1	100
0.94	94
1.67	167
57.24	5,724
X	100 <i>x</i>

length (cm)	length (m)
100	1
250	2.5
78.2	0.782
123.9	1.239
у	0.01 <i>y</i>

2. For each table, find the constant of proportionality.

The constant of proportionality for the first table is 100, and for the second table it is 0.01 or $\frac{1}{100}$.

3. Describe the relationship between these two constants of proportionality.

Sample response: The constants of proportionality are reciprocals.

4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

$$y = 100x$$
 and $x = 0.01y$ or $x = \frac{1}{100}y$

Are You Ready for More?

1. How many cubic centimeters are there in 1 cubic meter?

2. How do you convert cubic centimeters to cubic meters?

Sample response: Multiply by $\frac{1}{1,000,000}$ (or 0.0000001).

3. How do you convert the other way?

Sample response: Multiply by 1,000,000.

Activity Synthesis

The purpose of this discussion is to highlight how the two equations illustrate the reciprocal relationship. Invite students to share how they found the equation for each table. Consider asking:

"Where does the constant of proportionality occur in each table and equation?"

In the table, the constant of proportionality is in the right column on the row that has I in the left column. In the equation, the constant of proportionality is right next to the variable that represents the quantity that was in the left column of the table. (2) "What is the relationship between the two constants of proportionality? How can you use the equations to see why this should be true?"

They are reciprocals. This makes sense because the variables are in opposite places.

Display and discuss this sequence of equivalent equations to help students see why the constants of proportionality are reciprocals:

v = 100

$$\left(\frac{1}{100}\right)y = \frac{1}{100} \cdot (100x)$$

$$\left(\frac{1}{100}\right)y = x$$

$$x = \left(\frac{1}{100}\right)y$$

This line of reasoning should be accessible to students, because it builds on grade 6 work with expressions and equations.

Ask students to interpret the meaning of the equations in the context:

- "What do the equations tell us about the conversion from meters to centimeters and back?"
 - Given the length in meters, to find the length in centimeters we can multiply the number of meters by 100.
 - Given the length in centimeters, to find the length in meters we can multiply the number of centimeters by $\frac{1}{100}$.

Activity 2

Filling a Water Cooler

15 min

Activity Narrative

In this activity, students make sense of the two rates associated with a given proportional relationship. Students are asked to identify the two equations that represent a situation, working with both the number of gallons per minute and the number of minutes per gallon. This activity is the first time that no table is given to help students make sense of the proportional relationship, though students may find it helpful to create a table.

Monitor for students who use different ways to decide if the cooler was filling faster before or after the flow rate was changed.

Launch

Give students 4–5 minutes quiet work time followed by partner and a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, give students one equation at a time and have them determine if it represents the relationship correctly or not by testing out example values.

Supports accessibility for: Organization, Attention

Building on Student Thinking

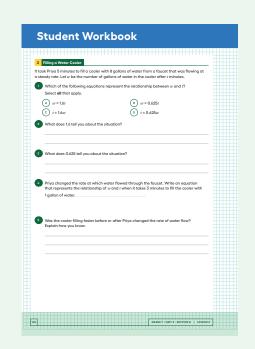
For the first question, if students struggle to identify the correct equations, encourage them to create two tables of values for the situation. Encourage them to create rows for both unit rates, in order to foster connections to prior learning.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question, about whether the cooler was filling faster before or after Priya changed the rate of water flow. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening



Student Task Statement

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t?

Select all that apply.

A.
$$w = 1.6t$$

B.
$$w = 0.625t$$

C.
$$t = 1.6w$$

D.
$$t = 0.625w$$

2. What does 1.6 tell you about the situation?

The water is flowing at 1.6 gallons per minute.

3. What does 0.625 tell you about the situation?

It takes 0.625 minutes for I gallon of water to flow out of the faucet (or into the cooler).

4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.

$$t = 3w \text{ or } w = \frac{1}{3}t$$

5. Was the cooler filling faster before or after Priya changed the rate of water flow? Explain how you know.

Before

Sample reasonings:

- Before the change, it took 0.625 minutes to get one gallon, but after the change, it took 3 minutes to get one gallon.
- Before the change, she got 1.6 gallons per minute, but after the change, she only got $\frac{1}{3}$ of a gallon per minute.

Activity Synthesis

The goal of this discussion is to connect the meaning of each constant of proportionality with the structure of each equation that represents the relationship. Invite students to share how they decided which equations represent the situation. Ask students to interpret what the equations tell us about the situation.

- To find the amount of water, w, we can multiply the elapsed time in minutes, t, by 1.6.
- To find the elapsed time, t, we can multiply the number of gallons of water, w, by 0.625.

If not mentioned by students, highlight the fact that 1.6 and 0.625 are reciprocals. Since these constants of proportionality are given as decimals in the equations, it may be harder for students to recognize this relationship. Consider asking half the class to calculate $1 \div 1.6$ while the other half calculates $1 \div 0.625$.

Next, invite students to share their responses to the last two questions about Priya changing the rate of water flow. Two possible approaches for the last question are:

- Comparing the new equation t = 3w to the previous equation t = 0.625w. In these equations, the constant of proportionality represents the numbers of minutes per gallon. Since 3 is greater than 0.625, that means it takes Priya more time to get the same amount of water after changing the rate of water flow.
- Comparing the new equation $w = \frac{1}{3}t$ to the previous equation w = 1.6t. In these equations, the constant of proportionality represents the number of gallons per minute. Since $\frac{1}{3}$ is less than 1.6, that means Priya gets less water in the same amount of time changing the rate of water flow.

It is not necessary to demonstrate every possible approach. The goal is for students to see how keeping in mind the meaning of the numbers and variables is helpful for making sense of the situation.

Activity 3: Optional

Feeding Shrimp

10 min

Activity Narrative

This activity provides an additional opportunity for students to represent a proportional relationship with two related equations in a new context. This situation builds on the earlier work students did with feeding a crowd, but includes more complicated calculations. Students interpret the meaning of the constants of proportionality in the context of the situation and use the equations to answer questions.

Launch 22

Arrange students in groups of 2. Introduce the context of feeding animals in an aquarium.

Give students 6 minutes of partner work time followed by wholeclass discussion.

Access for Multilingual Learners (Activity 3, Launch)

MLR5: Co-Craft Questions.

Keep books or devices closed.
Display only the problem stem,
without revealing the questions,
and ask students to record possible
mathematical questions that could
be asked about the situation. Invite
students to compare their questions
before revealing the task. Ask, "What
do these questions have in common?
How are they different?" Reveal the
intended questions for this task and
invite additional connections.

Advances: Reading, Writing

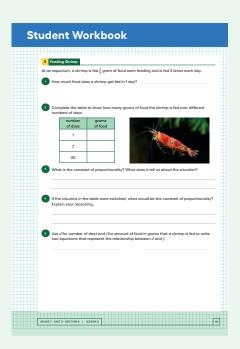
Warm-up

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

Differentiate the degree of difficulty or complexity. Begin with more accessible values. For example, start students with an integer amount of grams of food for each feeding and let them solve the problem. Then invite students to solve the original problem.

Supports accessibility for: Conceptual Processing, Memory



Student Wor	
3 Feeding Shrimp	
At this rate, how much food does	a shrimp get fed in 75 days?
•	
	d 75 grams of shrimp food last? Explain or show
your reasoning.	
5 Lesson Summary	
	illes per hour, his distance in miles, d , is proportional. We can write the equation $d = 10\tau$ to represent the
proportional relationship. With this equ	action, it is easy to find the distance Kiran rode when we
now how long it took, because we ca We can rewrite the equation:	i just multiply the time by 10.
re contentite the equation.	d = 10r
	$\left(\frac{1}{26}\right)d = t$
	$t = \left(\frac{1}{10}\right)d$
	at the amount of time Kiran rode is proportional to the
	of proportionality is $\frac{1}{10}$. That form of the equation is
	ce and want to find how long it took, because we can just
nultiply the distance by $\frac{1}{10}$.	
	roportional relationship, we can write the equation in this case, the number k is the corresponding constant
of proportionality. We can also write the equation $x = \frac{1}{k}y$ and say, "x is proportional to y." In this	
case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each equation can be useful, depending on the information we have and the avantity we are trying to figure out.	
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Student Task Statement

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

- 1. How much food does a shrimp get fed in 1 day?
 - 3/5 gram
- **2.** Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

number of days	grams of food
1	3 5
7	4 1/5
30	18

3. What is the constant of proportionality? What does it tell us about the situation?

3

It tells the feeding rate for the shrimp, in grams per day.

4. If the columns in the table were switched, what would be the constant of proportionality? Explain your reasoning.

5

Sample reasoning: because the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

5. Use d for number of days and f for amount of food in grams that a shrimp is fed to write two equations that represent the relationship between d and f.

$$f = \frac{3}{5} d$$
 and $d = \frac{5}{3} f$

6. At this rate, how much food does a shrimp get fed in 75 days?

45 grams

7. At this rate, how many days would 75 grams of shrimp food last? Explain or show your reasoning.

125 days

Sample reasoning: $\frac{5}{3}$ (75) = I25

Activity Synthesis

The goal of this discussion is to highlight the structure of the two equations that represent the proportional relationship, including the meaning of the two constants of proportionality. Invite students to share their answers. Ask students which equation was most useful to answer each of the last two questions and to explain their reasoning.

Lesson Synthesis

Share with students,

"Today we saw how to write two different equations for the same proportional relationship."

To review the reciprocal relationship between these equations, consider asking students:

"In the first activity, we examined the proportional relationship between meters and centimeters. What two equations did we write for this relationship?"

$$y = 100x$$
 and $x = \frac{1}{100}y$ or $x = 0.01y$

O "Why were we able to write two equations for the same relationship?"

One is for converting meters to centimeters, and the other is for converting centimeters to meters.

"In the second activity, we examined a proportional relationship where we knew how long it took to fill a water cooler with a certain amount of water. What two equations did we determine would represent this relationship?"

$$w = 1.6t$$
 and $t = 0.625w$

"What do these pairs of equations have in common?"

They all have a structure like y = kx, where k is the constant of proportionality. In each pair, the variables are reversed, and the constants of proportionality are reciprocals.

If desired, use this example to review these concepts:

 \bigcirc "One equation that represents a proportional relationship is $w = \frac{1}{4}n$. What is a different equation that represents this same relationship?"

$$n = 4w$$

Lesson Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d, is proportional to the number of hours, t, that he rode. We can write the equation d = 10t to represent the proportional relationship. With this equation, it is easy to find the distance Kiran rode when we know how long it took, because we can just multiply the time by 10.

We can rewrite the equation:

$$d = 10t$$
$$\left(\frac{1}{10}\right)d = t$$
$$t = \left(\frac{1}{10}\right)d$$

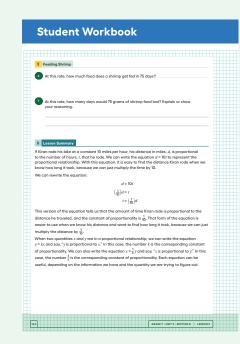
This version of the equation tells us that the amount of time Kiran rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form of the equation is easier to use when we know his distance and want to find how long it took, because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation y = kx and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality. We can also write the equation $x = \frac{1}{k}y$ and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each equation can be useful, depending on the information we have and the quantity we are trying to figure out.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.



Cool-down

Flight of the Albatross



Student Task Statement

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed. Using d for distance in kilometers and t for number of hours, an equation that represents this situation is d = 50t.

1. What are two constants of proportionality for the relationship between distance in kilometers and number of hours? What is the relationship between these two values?

$$50 \text{ and } \frac{1}{50}$$

Sample response: They are reciprocals of each other.

2. Write another equation that relates d and t in this context.

$$t = \frac{1}{50} d$$

Practice Problems

5 Problems

Problem 1

The table represents the relationship between a length measured in meters and the same length measured in kilometers.

- **a.** Complete the table.
- **b.** Write an equation for converting the number of meters to kilometers.

 Use *x* for the number of meters and *y* for the number of kilometers.

y = 0.001x (or equivalent)

meters	kilometers
1,000	1
3,500	3.5
500	0.5
75	0.075
1	0.001
X	0.00lx

Problem 2

Concrete building blocks weigh 28 pounds each. Using b for the number of concrete blocks and w for the weight, write two equations that relate the two variables. One equation should begin with w = and the other should begin with b =.

$$w = 28b$$

$$b = \frac{1}{28} w$$

Problem 3

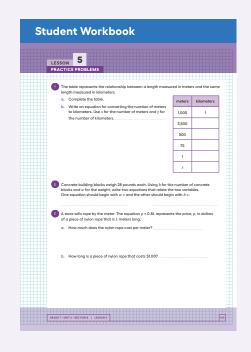
A store sells rope by the meter. The equation p = 0.8L represents the price, p, in dollars of a piece of nylon rope that is L meters long.

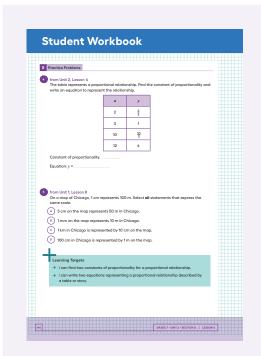
a. How much does the nylon rope cost per meter?

\$0.80

b. How long is a piece of nylon rope that costs \$1.00?

1.25 meters





Problem 4

From Unit 2, Lesson 4

The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

а	у
2	<u>2</u> 3
3	1
10	<u>10</u> 3
12	4

Constant of proportionality: $\frac{1}{3}$

Equation: $y = \frac{1}{3} \alpha$

Problem 5

from Unit 1, Lesson 8

On a map of Chicago, 1 cm represents 100 m. Select **all** statements that express the same scale.

- **A.** 5 cm on the map represents 50 m in Chicago.
- **B.** 1 mm on the map represents 10 m in Chicago.
- **C.** 1 km in Chicago is represented by 10 cm on the map.
- **D.** 100 cm in Chicago is represented by 1 m on the map.