# Lesson 6

## Similarity

### Goals

- Comprehend that the phrase "similar figures" (in written and spoken language) means there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other.
- Justify (orally) the similarity of two figures using a sequence of transformations that takes one figure to the other.

### Learning Targets

- I can apply a sequence of transformations to one figure to get a similar figure.
- I can use a sequence of transformations to explain why two figures are similar.

### Lesson Narrative

The purpose of this lesson is to define that two figures are **similar** if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. It is important to note that there are many different sequences that could show two figures are similar.

Students begin the lesson by studying pairs of triangles where some pairs are scaled copies of each other and some pairs are not. Next, in order to show that scaled copies are similar, students find and describe a sequence of transformations that takes one figure to the other using precise mathematical language. Students also practice sketching similar figures created using given transformations. An optional activity provides additional practice finding a sequence of transformations that show two figures are similar.

In future lessons, students will learn other methods for showing similarity, but in this lesson the focus is on the definition of similarity in terms of transformations.

### Student Learning Goal

Let's explore similar figures.

### Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

### Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Which Three Go Together?

### Required Materials

**Materials to Gather**

- Geometry toolkits: Activity 1, Activity 2

**Materials to Copy**

- Methods for Translations and Dilations Cards (1 copy for every 2 students): Activity 3

### Required Preparation

**Activity 1:**
Provide access to geometry toolkits. For the digital version of the activity, acquire devices that can run the applet.

**Activity 2:**
Provide access to geometry toolkits.

### Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Activity 3 | Lesson Synthesis | Assessment: Cool-down |
|---|---|---|---|---|---|
| 10 min | 20 min | 10 min | 10 min | 10 min | 5 min |

**Instructional Routines**

**Which Three Go Together?**

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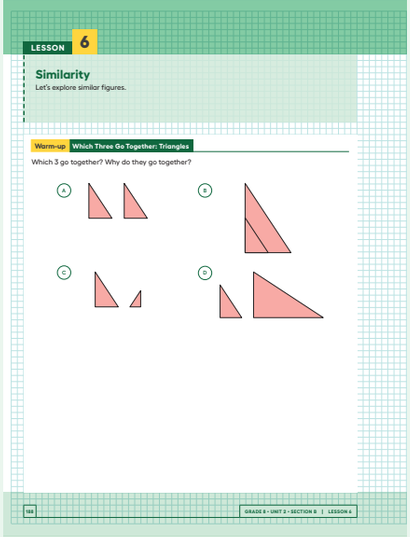
**Student Workbook**


LESSON 6

Similarity

Let's explore similar figures.

Warm-up Which Three Go Together: Triangles

Which 3 go together? Why do they go together?

A B C D

## Warm-up

## Which Three Go Together: Triangles

5 min

### Activity Narrative

This *Warm-up* prompts students to compare four images. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

### Launch

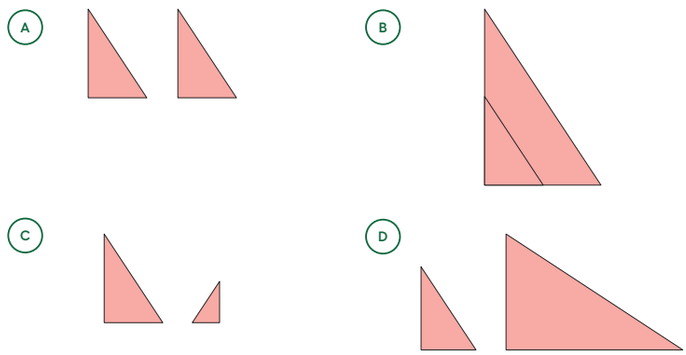
Arrange students in groups of 2–4. Display the images for all to see.

Give students 1 minute of quiet think time and ask them to indicate when they have noticed 3 images that go together and can explain why.

Next, tell students to share their response with their group, and then together find as many sets of three as they can.

### Student Task Statement

Which 3 go together? Why do they go together?




Sample responses:

A, B, and C go together because:

- Both triangles in each pair are scaled copies of each other.
- Both triangles in each pair are dilations of each other.

A, B, and D go together because:

- Both triangles in each pair are "facing" the same direction.

A, C, and D go together because:

- None of the triangles in each pair overlap each other.
- None of the triangles in each pair share a common vertex.

B, C, and D go together because:

- Both triangles in each pair are different sizes.

### Activity Synthesis

Invite each group to share 1 reason why a particular set of 3 go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "scaled copies" and "dilation," and to clarify their reasoning as needed. Consider asking:

- *"How do you know ...?"*
- *"What do you mean by ...?"*
- *"Can you say that in another way?"*

## Activity 1

## Similarity Transformations (Part 1)

20 min

### Activity Narrative

**There is a digital version of this activity.**

In this activity, students learn that two figures are similar when there is a sequence of translations, reflections, rotations and dilations that takes one figure to the other.

When two shapes are similar but not congruent, the sequence of steps showing the similarity usually has a single dilation and then the rest of the steps are rigid transformations. The dilation can come at any time and it does not matter which figure is the original. An important thing for students to notice in this activity is that there is more than one sequence of transformations that show two figures are similar. Monitor for students who insert a dilation at different places in the sequence. Also monitor for how students find the scale factor for the hexagons.

In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language they use to describe their thinking.

In the digital version of the activity, students use an applet to perform transformations of a figure on a grid. The applet allows students to test transformations dynamically and precisely. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

### Instructional Routines

**MLR1: Stronger and Clearer Each Time**

**ilclass.com/r/10695479**

Please log in to the site before using the QR code or URL.

### Access for Multilingual Learners (Activity 1)

**MLR1: Stronger and Clearer Each Time.**
This activity uses the Stronger and *Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

**Access for Students with Diverse Abilities (Activity 1, Launch)**

**Action and Expression: Provide Access for Physical Action.**

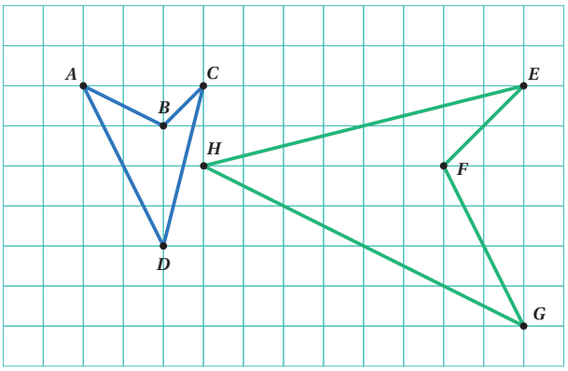
Provide access to tools and assistive technologies such as a device that can run the digital applet.

*Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization*

## Launch

Provide access to geometry toolkits. Arrange students in groups of 2.

Begin by reminding students how figures can be scaled copies of each other. Display the figure for all to see.




For example, if quadrilateral $EFGH$ is a scaled copy of quadrilateral $ABCD$ with scale factor 2, then quadrilateral $ABCD$ is also a scaled copy of quadrilateral $EFGH$ with scale factor $\frac{1}{2}$. The transformation that creates scaled copies is called a "dilation."

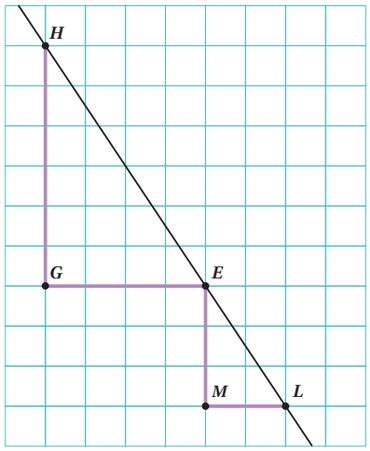
Explain that quadrilateral $ABCD$ and quadrilateral $EFGH$ are **similar** because one can fit exactly over the other after a series of rigid transformations and dilations.

There are many methods that can show similarity. Review at least 2 methods with the class to ensure students understand the precision of language necessary to communicate the details of each transformation. Encourage students to first identify the corresponding parts and then come up with a plan to take one figure to the other. Some sample methods:

1. Method 1 ($ABCD$ to $GFEH$: Dilate, translate, rotate, reflect)
   - **a.** Dilate using $D$ as the center with scale factor 2.
   - **b.** Translate $D$ to $H$.
   - **c.** Rotate clockwise by 90 degrees using $H$ as the center.
   - **d.** Reflect using the line that contains $H$ and $F$.
2. Method 2 ($GFEH$ to $ABCD$: Reflect, translate, rotate, dilate)
   - **a.** Reflect using the line that contains $H$ and $F$.
   - **b.** Translate $H$ to $D$.
   - **c.** Rotate counterclockwise by 90 degrees using $H$ as the center.
   - **d.** Dilate using $H$ as the center with scale factor of $\frac{1}{2}$.
3. Method 3 ($ABCD$ to $GFEH$: Translate, rotate, reflect, dilate)
   - **a.** Translate $B$ to $F$.
   - **b.** Rotate clockwise by 90 degrees using $F$ as the center.
   - **c.** Reflect using the line that contains $F$ and $H$.
   - **d.** Dilate using $F$ as the center with scale factor 2.

## Student Task Statement

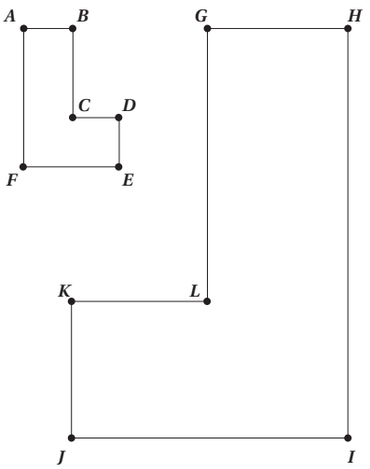
1. Triangle $EGH$ and triangle $LME$ are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.




Sample response:

- Begin with triangle *LME*.
- Translate *L* to *E*.
- Dilate using *E* as the center with scale factor 2.

2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.




Sample response:

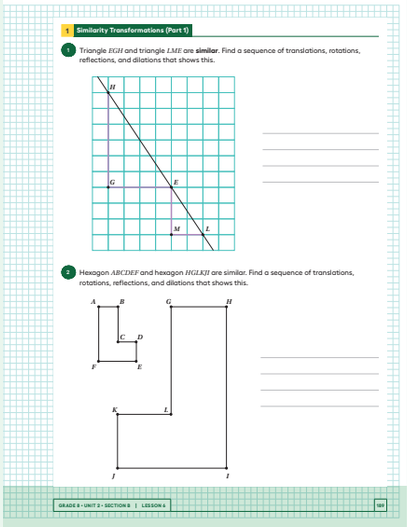
- Begin with figure *ABCDEF*.
- Reflect using the line that contains *A* and *F*.
- Translate *F* to *I*.
- Dilate using *I* as the center with scale factor 3.

### Building on Student Thinking

Some students may not recall the 3 types of rigid transformations. Prompt them to refer to the classroom display from a previous unit that provides an example of a rotation, a reflection, and a translation.

For the second problem, students may get stuck finding the scale factor. Tell them they can approximate by measuring sides of the 2 figures.

### Student Workbook


1 Similarity Transformations (Part 1)

1 Triangle $EGH$ and triangle $LME$ are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.

2 Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.

GRADE 8 • UNIT 2 • SECTION B | LESSON 6

**Student Workbook**


1 Similarity Transformations (Part 1)

Are You Ready for More?

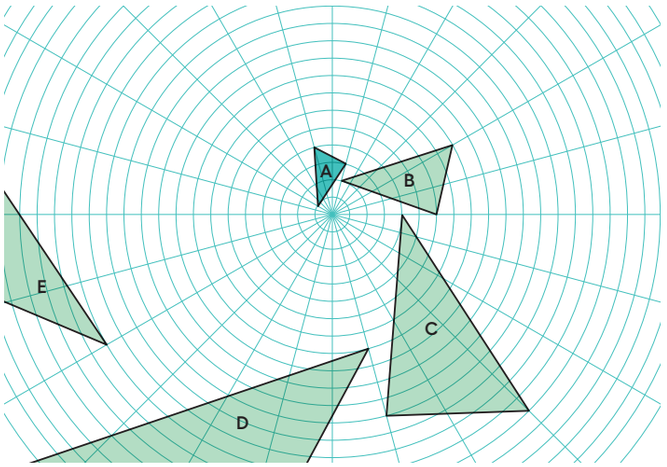
The same sequence of transformations that takes Triangle A to Triangle B, also takes Triangle B to Triangle C, and so on. Describe a possible sequence of transformations.



GRADE 8 • UNIT 2 • SECTION B | LESSON 6

## Are You Ready for More?

The same sequence of transformations that takes Triangle A to Triangle B, also takes Triangle B to Triangle C, and so on. Describe a possible sequence of transformations.




Sample response: Dilate from the center of the circular grid, with scale factor 2, then rotate clockwise 75 degrees.

## Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the first question. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "_____ makes sense, but what do you mean when you say …?"
- "Can you say more about …?"
- "What do you mean when you say …?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time*, invite select students to share their sequence for the first problem. Emphasize that there are multiple ways to show the two triangles are similar and any valid sequence is allowed. Ensure that students communicate each transformation in the sequence using precise mathematical language. Here are some questions for discussion:

- *"How have the methods shared by your classmates been alike or different?"*

  All methods required a dilation. Some methods went from triangle *HGE* to triangle *EML* and some went the other way.

- *"What pieces of information are necessary to completely describe a dilation?"*

  a scale factor, a center of dilation, and an object being dilated.

## Activity 2
## Similarity Transformations (Part 2)

10 min

### Activity Narrative

This activity helps students visualize what happens to figures under different kinds of transformations. By recognizing patterns in the image results after using certain transformations, students can make connections between the orientations of the original figure and the resulting images after transformation. They can also apply this to find transformations for other problems.

### Launch

Provide access to geometry toolkits.

Give students 1 minute of quiet time to work the first question and then pause for a brief whole-class discussion.

The figure in this task is intended to resemble a hand with all of the fingers together and the thumb sticking out. Encourage students to draw rough sketches for the resulting images. To be clear on the level of precision necessary, highlight 2–3 examples of student work that are not exact but capture the main features of the figure. Students may approximate side lengths and angles, but it should be clear where the corresponding parts of the image are and whether the image is larger or smaller than the original.

### Building on Student Thinking

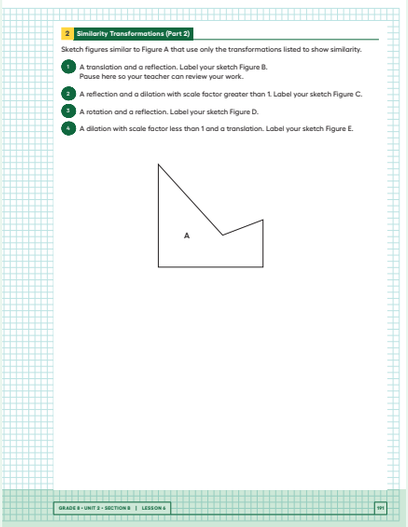
Some students may choose an exact scale factor or measure the exact angle sizes. Explain that precise measurements are not needed in this task—they are just sketching similar figures.

### Access for Students with Diverse Abilities (Activity 2, Student Task)

**Engagement: Develop Effort and Persistence.**
Chunk this task into more manageable parts. After the first problem, invite a few students to think aloud and share their sketches to guide the rest of the individual work time. Check in with students to provide feedback and encouragement after each chunk. Look for evidence that students understand the effect each type of transformation has on the sketch.

*Supports accessibility for: Attention, Social-Emotional Functioning*

### Student Workbook


2 Similarity Transformations (Part 2)

Sketch figures similar to Figure A that use only the transformations listed to show similarity.

1. A translation and a reflection. Label your sketch Figure B. Pause here so your teacher can review your work.
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.
3. A rotation and a reflection. Label your sketch Figure D.
4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

A

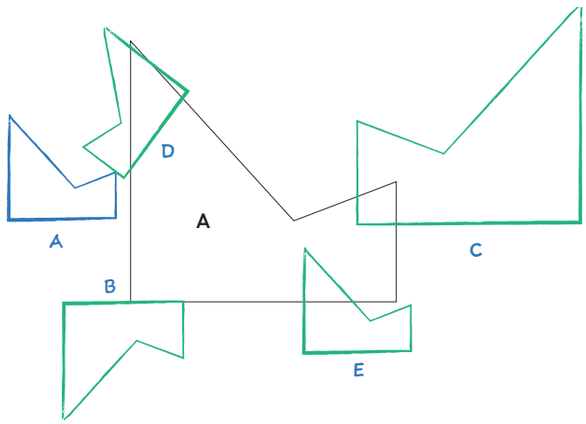
GRADE 8 • UNIT 2 • SECTION B | LESSON 6 191

### Student Task Statement

Sketch figures similar to Figure A that use only the transformations listed to show similarity.

1. A translation and a reflection. Label your sketch Figure B. Pause here so your teacher can review your work.
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.
3. A rotation and a reflection. Label your sketch Figure D.
4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

Sample responses:




### Activity Synthesis

The goal of this discussion is for students to notice things each answer has in common so that they can make connections to the types of transformations that might be useful in showing that 2 figures are similar.

Select students to display their answers for each of the remaining problems. Here are some questions for discussion:

- *"What do you notice about all of the dilations with a scale factor greater than 1?"*

  They all create an image larger than the original.

- *"What do you notice about all of the reflections?"*

  Reflections will make the resulting image look like the back of a right hand instead of the back of a left hand as in the original image.

- *"What do you notice about all of the rotations?"*

  Rotations will appear to "tilt" or "turn" the figure, unless the rotation is a multiple of 360 degrees, where it will look the same.

- *"What do you notice about all of the dilations with a scale factor smaller than 1?"*

  They all create an image smaller than the original.

- *"What do you notice about all of the translations?"*

  Translations will slide the figure in some direction, but the size and orientation remain the same.

## Activity 3: Optional
## Methods for Translations and Dilations

10 min

### Activity Narrative

The purpose of this optional activity is for students to practice showing that 2 shapes are similar. It is an opportunity for extra practice that not all classes or students may need. Some students will start with triangle $ABC$ and take this to triangle $DEF$ while others start with $DEF$ and take this to $ABC$.

Monitor for students who use different centers of dilation in their sequence, particularly as the first step in the sequence. Invite these students to share, highlighting the fact that 2 dilations with the same scale factor but different centers differ by a translation. Also monitor for students whose sequences of rigid motions and dilations are the same but in the opposite order, one set taking $ABC$ to $DEF$ and the opposite taking $DEF$ back to $ABC$.

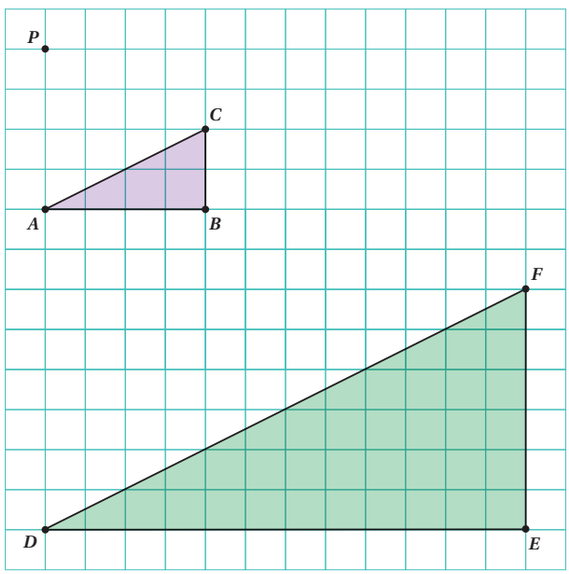
### Launch

Arrange students in groups of 2. Give each group 1 complete set of cards.

If necessary, remind students that 2 figures are similar if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to another. Explain that they need to find at least 1 way to show that triangle $ABC$ and triangle $DEF$ are similar using only the transformations they are given on their cards.

### Student Task Statement

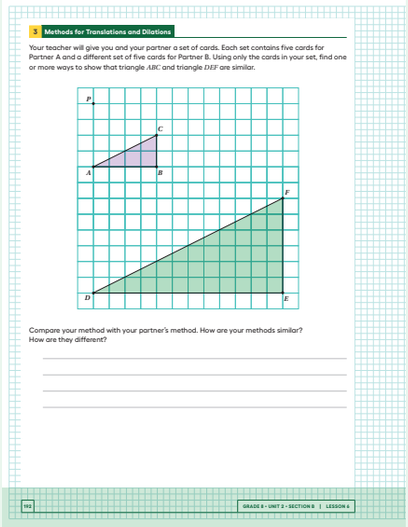
Your teacher will give you and your partner a set of cards. Each set contains five cards for Partner A and a different set of five cards for Partner B. Using only the cards in your set, find one or more ways to show that triangle $ABC$ and triangle $DEF$ are similar. Compare your method with your partner's method. How are your methods similar? How are they different?




### Building on Student Thinking

Some students may think that it is necessary to perform transformations in the same order or that one particular point needs to be the center. Prompt them to recall a previous activity where multiple methods were all valid. If partners choose the same methods, prompt them to try it another way that will have the same end result.

### Student Workbook


3 Methods for Translations and Dilations

Your teacher will give you and your partner a set of cards. Each set contains five cards for Partner A and a different set of five cards for Partner B. Using only the cards in your set, find one or more ways to show that triangle $ABC$ and triangle $DEF$ are similar.



Compare your method with your partner's method. How are your methods similar? How are they different?

Sample responses for Partner A:

- Dilate triangle $ABC$ using center point $P$ and scale factor 3.
- Dilate using center point $A$ and scale factor 3 followed by translating from $A$ to $D$.
- Translate from $A$ to $D$ followed by dilating using center $D$ and scale factor 3.

Sample responses for Partner B:

- Dilate triangle $DEF$ using center point $P$ and scale factor $\frac{1}{3}$.
- Dilate using center point $D$ and scale factor $\frac{1}{3}$ followed by translating from $D$ to $A$.
- Translate from $D$ to $A$ followed by dilating using center $A$ and scale factor $\frac{1}{3}$.

## Activity Synthesis

The goal of the discussion is to make sure students understand how similarity can be shown in different ways, and how the transformations in the differing methods are related.

Invite previously selected students to share. As students share their responses, discuss:

- *"What do you notice about the scale factors you and your partner used?"*

  The scale factors for the dilations are reciprocals regardless of when the dilations are done in the sequence.

- *"What do you notice about the translations you and your partner used?*

  The translations are inverses of each other: "Translate $A$ to $D$" instead of "Translate $D$ to $A$".

- *"What do you notice about the centers of dilation you and your partner used?"*

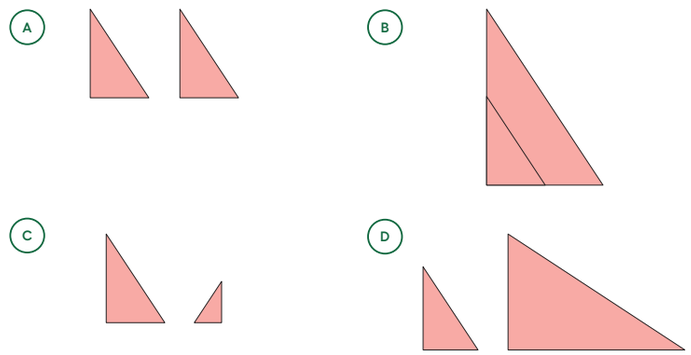
  Any point can be chosen as the center of dilation if the scale factor is known, because the position can be adjusted using a translation.

- *"What do you notice about the order of the transformations you and your partner used?"*

  The order of the transformation will affect the result, but many different orders can be used to get the same result.

## Lesson Synthesis

The goal of this discussion is to review the definition of similar figures. Display the figure from the *Warm-up* for all to see.




Discuss:

- *"Do any of these pairs show similar triangles? How do you know?"*

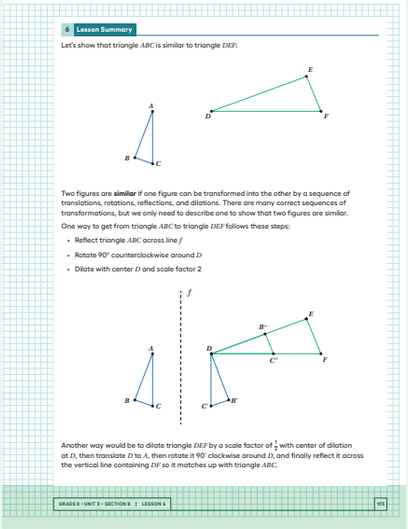
  The pair of triangles in C is similar because the larger triangle can be dilated and reflected to get the smaller triangle.

- *"Do any of these pairs show triangles that are not similar? How do you know?"*

  The pair of triangles in D is not similar because the lengths of corresponding sides are not all multiplied by the same scale factor.

Ask students how we can tell if any two figures are similar. (They are scaled copies of each other. One figure is a dilation of the other. There is a sequence of transformations that takes one figure to the other figure.) As students share, add the term "similar" along with a definition and examples to a new or existing classroom display, such as a word wall or anchor chart.

## Student Workbook


6 Lesson Summary

Let's show that triangle $ABC$ is similar to triangle $DEF$:

Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from triangle $ABC$ to triangle $DEF$ follows these steps:

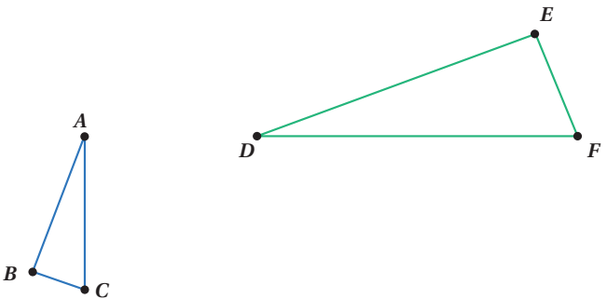
- Reflect triangle $ABC$ across line $f$
- Rotate 90° counterclockwise around $D$
- Dilate with center $D$ and scale factor 2

Another way would be to dilate triangle $DEF$ by a scale factor of $\frac{1}{2}$ with center of dilation at $D$, then translate $D$ to $A$, then rotate it 90° clockwise around $D$, and finally reflect it across the vertical line containing $DF$ so it matches up with triangle $ABC$.

GRADE 8 • UNIT 2 • SECTION B | LESSON 6 193

## Lesson Summary

Let's show that triangle $ABC$ is similar to triangle $DEF$:

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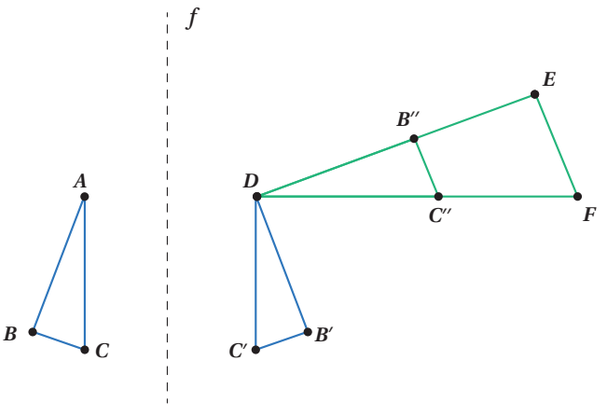



Another way would be to dilate triangle $DEF$ by a scale factor of $\frac{1}{2}$ with center of dilation at $D$, then translate $D$ to $A$, then rotate it 90° clockwise around $D$, and finally reflect it across the vertical line containing $DF$ so it matches up with triangle $ABC$.

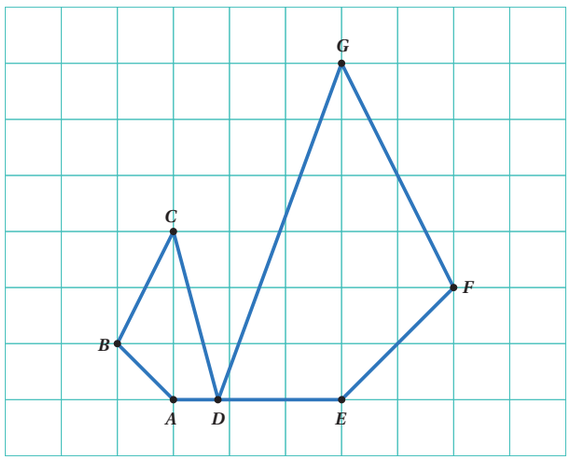
Cool-down

## Showing Similarity

5 min

### Student Task Statement

Elena gives the following sequence of transformations to show that the 2 figures are similar by transforming $ABCD$ into $EFGD$.




1. Dilate using center $D$ and scale factor 2.
2. Reflect using the horizontal line through $D$.

   Is Elena's method correct? If not, explain how you could fix it.

   Elena's method is not correct.

   Sample response: After dilating using $D$ as the center with a scale factor of 2, Elena can reflect over the vertical line through $D$ rather than the horizontal line through $D$.

### Responding To Student Thinking

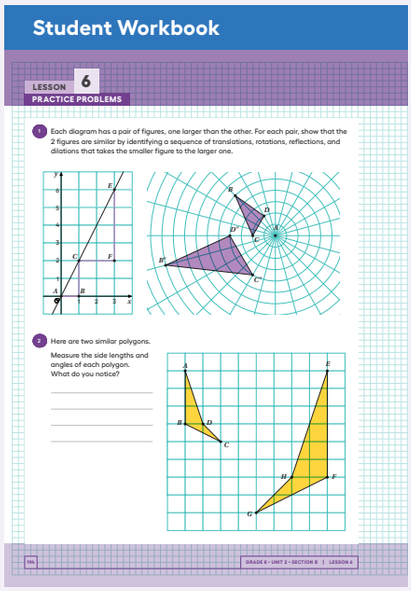
**Points to Emphasize**

If students struggle with identifying the correct sequence of transformations to show that the two figures are similar, look for opportunities over the next several lessons to invite students to describe the three types of rigid transformations and what information is needed to perform each one.

## Practice Problems

3 Problems

**Student Workbook**

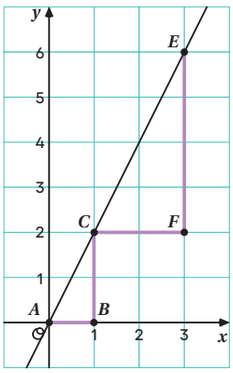

LESSON 6
PRACTICE PROBLEMS

1. Each diagram has a pair of figures, one larger than the other. For each pair, show that the 2 figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.

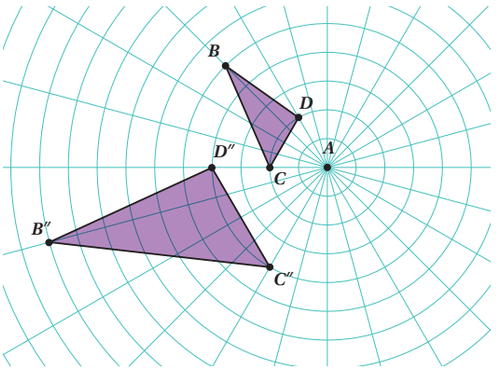
2. Here are two similar polygons.
Measure the side lengths and angles of each polygon.
What do you notice?

### Problem 1

Each diagram has a pair of figures, one larger than the other. For each pair, show that the 2 figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.




- Sample response: Translate $A$ to $C$, and then dilate with center $C$ by a factor of 2.

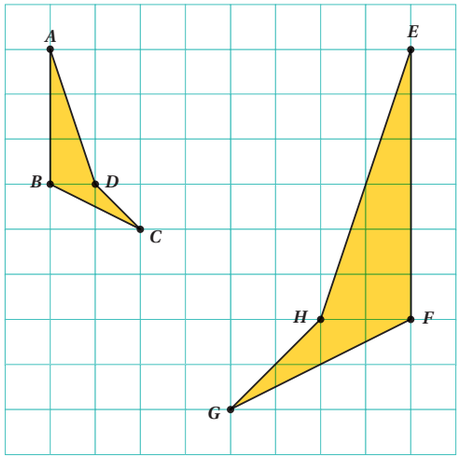



- Sample response: Rotate 60° counterclockwise with center $A$, and then dilate using a scale factor of 2 centered at $A$.

## Problem 2

Here are two similar polygons.

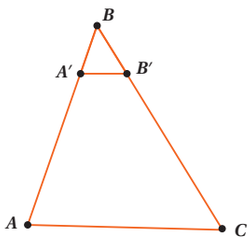
Measure the side lengths and angles of each polygon. What do you notice?



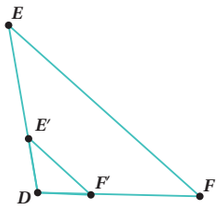

Sample response: Corresponding side lengths in the larger polygon are double the side lengths of the smaller polygon, while corresponding angles all have the same measure.

## Problem 3

Each figure shows a pair of similar triangles, one contained in the other. For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.

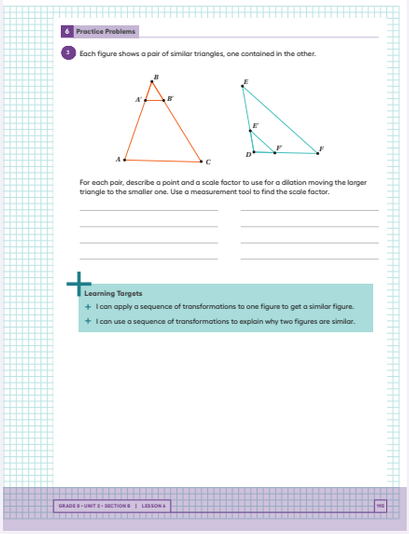



- Center of dilation: $B$, scale factor: $\frac{1}{4}$




- Center of dilation: $D$, scale factor: $\frac{1}{3}$

### Student Workbook


6 Practice Problems

3 Each figure shows a pair of similar triangles, one contained in the other.

For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.

Learning Targets

+ I can apply a sequence of transformations to one figure to get a similar figure.
+ I can use a sequence of transformations to explain why two figures are similar.