Lesson 11

Line Designs (Optional)

Goals

- Create a graph of a line using a verbal description of its features.
- Describe (orally) the graph of a line using formal or informal language precisely enough to identify a unique line.

Learning Target

I can describe a line precisely enough that another student can draw it.

Lesson Narrative

This lesson is optional because it provides students with additional opportunities to practice drawing and describing lines and calculating the slope of lines given two points.

Students begin the lesson by observing two graphs and describing the similarities and differences between them. Students are encouraged to use precise language to describe characteristics of each graph, such as using slope and vertical and horizontal intercepts.

Next, students use the *Info Gap* structure to describe a collection of lines while their partner attempts to recreate the design. Students find there are many methods for describing the location of lines, but the focus in this lesson is on using slope and the coordinates of a point on the line.

Last, students recognize the importance of performing calculations in a consistent order when finding the horizontal or vertical change between a pair of coordinate points.

Student Learning Goal

Let's describe lines.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

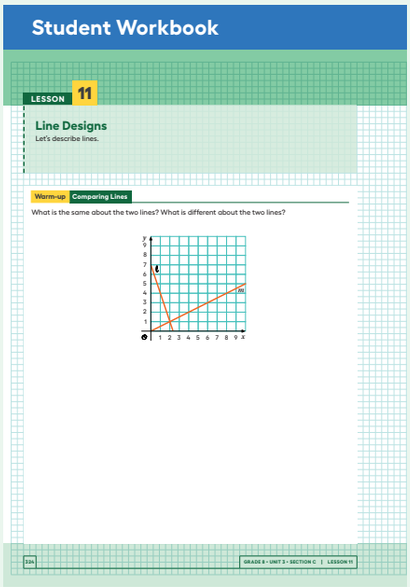
- Straightedges: Activity 1, Activity 2

Materials to Copy

- Making Designs Cards (1 copy for every 2 students): Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min



Warm-up

Comparing Lines

5 min

Activity Narrative

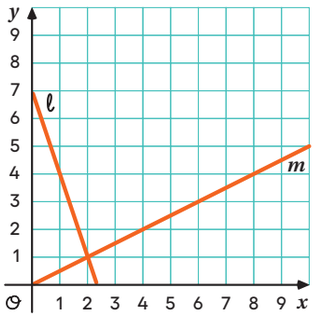
This activity encourages students to think about the language they use to describe lines. The terms elicited here will be helpful later in the lesson when students attempt to describe lines and draw lines being described to them in a later activity. When describing similarity and differences between two lines, students need to be precise in their word choice and use of language.

Launch

Give students 2 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

What is the same about the two lines? What is different about the two lines?



Sample responses:

- Both lines pass through the point (2, 1).
- Both lines show a linear relationship.
- Line m shows a proportional relationships and line ℓ does not.
- Line m has a positive slope and line ℓ has a negative slope.
- Line m has a vertical intercept of 0 and line ℓ has a vertical intercept of 7.
- Line ℓ does not go through the origin.

Activity Synthesis

The goal of this discussion is for students to hear how different terms can be used to describe two lines. Display the graph from the task for all to see.

Invite students to share similarities and differences between the two lines. When possible, record the observations on the graph. For example, if a student notices that the two lines have different horizontal intercepts, mark and label both of those points on the graph. Emphasize language about the slope of each line, intercepts, and coordinates of specific points.

If time allows, ask students to draw additional lines. For example, ask students to draw a line with the same vertical intercept as line ℓ but a different slope, or to draw a different line that also passes through the point (2, 1).

Activity 1
Making Designs

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to recreate a design made out of different lines. Thanks to Henri Picciotto for permission to use these designs.

The goal is for students to recognize the information that determines the location of a line in the coordinate plane. Acknowledge students who use the following strategies, but keep the discussion focused on describing each line using slope and intercepts or coordinates of points on the line.

- Describing one line as a vertical translation up or down of another line
- Describing a line as parallel to another line and containing a particular point
- Giving an equation to describe the line

Students are not expected to communicate by saying the equations of the lines, though there is nothing stopping them from doing so.

Launch

Tell students they will describe some lines to a partner to try and get them to recreate a design. Consider selecting a student as a partner to demonstrate the protocol to the class before distributing the designs and blank graphs.

Arrange students in groups of 2, in such a way that the partner drawing the design cannot peek at the design anywhere else in the room. Provide access to straightedges.

In each group, give a design card to one student and a blank coordinate plane to the other student. Once the first design has been successfully created, provide cards for the second design and instruct students to switch roles.

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards.

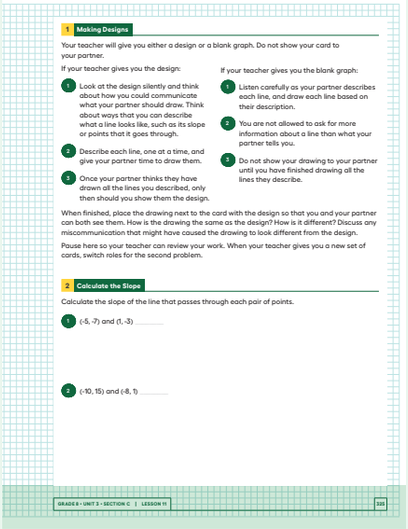
This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate precisely.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy. *Supports accessibility for: Memory, Organization*

Student Workbook



1 Making Designs

Your teacher will give you either a design or a blank graph. Do not show your card to your partner.

If your teacher gives you the design:

1. Look at the design silently and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or points that it goes through.
2. Describe each line, one at a time, and give your partner time to draw them.
3. Once your partner thinks they have drawn all the lines you described, only then should you show them the design.

If your teacher gives you the blank graph:

1. Listen carefully as your partner describes each line, and draw each line based on their description.
2. You are not allowed to ask for more information about a line than what your partner tells you.
3. Do not show your drawing to your partner until you have finished drawing all the lines they describe.

When finished, place the drawing next to the card with the design so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

2 Calculate the Slope

Calculate the slope of the line that passes through each pair of points.

1. (-5, -7) and (1, -3)
2. (-10, 15) and (-8, 1)

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Student Task Statement

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Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

Student designs should match those in the blackline master.

Activity Synthesis

Here are some questions for discussion:

- *"What details were important to pay attention to?"*

  the slope of the line, whether the slope was positive or negative, the vertical and horizontal intercepts

- *"How did you use coordinates to help communicate where the line was?"*

  Coordinates could tell a location that the line had to go through.

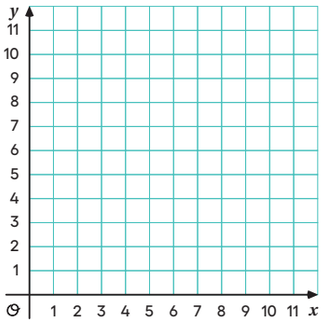
- *"How did you use slope to communicate how to draw the line?"*

  Slope told my partner how steep to draw the line and if it was going uphill or downhill.

- *"Were there any cases where your partner did not give enough information to know where to draw the line? What more information did you need?"*

  Answers vary.

Then display this blank coordinate plane, or one similar, for all to see.



Plot and label the point (2, 5) on the coordinate axes. Ask students to explain how to draw a line with a slope of -2 that goes through the point on the display. Demonstrate how to use the plotted point as one vertex of a slope triangle with a vertical length of 2 units and a horizontal length of 1 unit, and then draw in the third side of the triangle using a straightedge or ruler. Discuss:

- *"Can the point and slope used describe any other line on the coordinate plane?"*

  No

- *"Can this same line be described in a different way?"*

  Yes

- *"What are some other ways?"*

  a line with slope -2 that goes through the point (3, 3) or any other point on the line, a line with slope -2 with a vertical intercept of 9, the line $y = 9 - 2x$ (or equivalent)

Activity 2

Calculate the Slope

10 min

Activity Narrative

This activity provides additional practice for students to calculate the slope of a line given two points.

Launch

Arrange students in groups of 2–3. Explain that Jada calculated the slope of the line that passes through the points G and H. Display the coordinates of the two points and Jada's work for all to see:

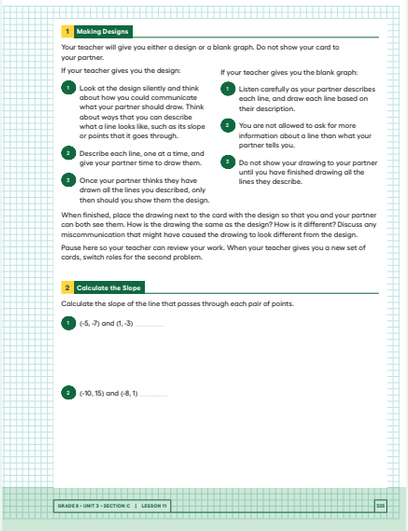
$G = (-1, 4)$ and $H = (7, -2)$

vertical change $= 4 - (-2) = 6$

horizontal change $= 7 - (-1) = 8$

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{6}{8}, \text{ or } \frac{3}{4}$$

Student Workbook



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2 Calculate the Slope

Calculate the slope of the line that passes through each pair of points.

1. (-5, -7) and (1, -3) _____
2. (-10, 15) and (-8, 1) _____

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Ask students whether the graph of this line will go uphill or downhill, from left to right. Since Jada's work shows a positive slope, students should say the line will go uphill. Then draw a sketch of the location of the 2 points in the coordinate plane and draw the line that passes through them. Tell students that the line looks like it is going downhill, which would mean a negative slope. Give students 2 minutes to discuss in their groups why Jada's work doesn't match the sketch.

Invite groups to share their reasoning. Emphasize that when calculating the difference between two values, it is important to subtract x-values and y-values in the same order. Have students calculate the correct slope of the line ($-\frac{6}{8}$, or $-\frac{3}{4}$).

If time allows, ask students to calculate the slope of the same two points but by subtracting the coordinates of G from those of H. Emphasize that the resulting slope is the same.

Student Task Statement

Calculate the slope of the line that passes through each pair of points.

1. (-5, -7) and (1, -3)

$\frac{2}{3}$ (or equivalent)

2. (-10, 15) and (-8, 1)

-7 (or equivalent)

Activity Synthesis

The goal of this discussion is for students to understand that when calculating slope, the order of the points does not matter as long as the same order is used for calculating both the vertical and horizontal change.

Invite previously selected students to share their strategy for calculating the slope of one of the pairs of points. Include one student who subtracted coordinate values of the first point from the second, and another student who subtracted the values of the second point from the first. Here are some questions for discussion:

- *"What do you notice about both approaches?"*

  Both result in the same value for slope. They subtracted their numbers in the opposite order. They kept the same order when calculating both the vertical and horizontal change.

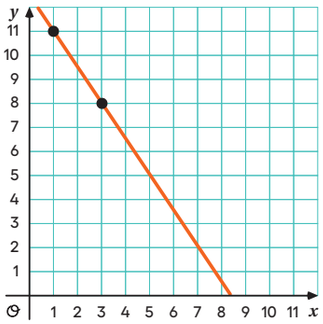
- *"Are there any advantages to picking one point to use first or second?"*

  Subtracting the numbers in a certain order may result in "easier" calculations.

If time allows, invite a student who drew a sketch of the two points to share their reasoning. While students can use any strategy to calculate slope, emphasize the sketch as a good way to check that the sign of the slope matches the direction of the line.

Lesson Synthesis

The goal of this discussion is for students to connect how the graph of a line in the coordinate plane can be described using slope and intercepts or coordinates of points on the line. Begin by displaying this graph for all to see:



Explain to students that they are trying to describe this line to someone without just showing them a picture. Discuss:

- *"What information is needed to communicate to the other person?"*

  the slope of the line and one point that the line passes through, or 2 points that the line passes through

- *"What are the coordinates of some points on this line?"*

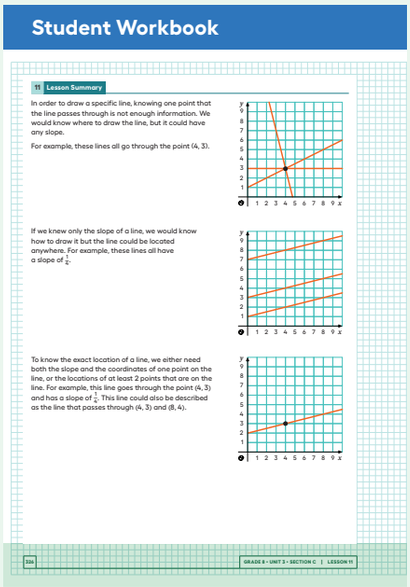
  (1, 11) and (3, 8)

- *"How can the slope of this line be calculated?"*

  A slope triangle can be drawn and the vertical distance can be divided by the horizontal distance.

Ask students to calculate the slope of this line. ($-\frac{3}{2}$ or equivalent) Then invite students to share possible ways to describe the line. For example, students may describe the line as having a slope of $-\frac{3}{2}$ and passing through the point (3, 8). Record the descriptions for all to see.

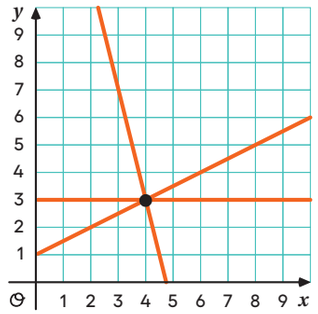
Student Workbook



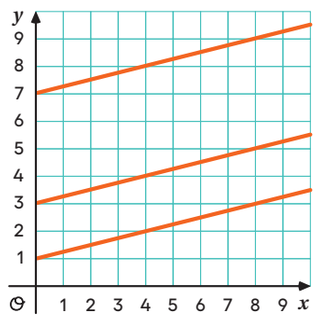
Lesson Summary

In order to draw a specific line, knowing one point that the line passes through is not enough information. We would know where to draw the line, but it could have any slope.

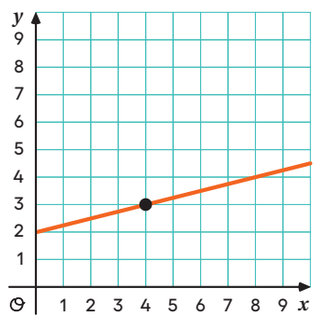
For example, these lines all go through the point (4, 3).



If we knew only the slope of a line, we would know how to draw it but the line could be located anywhere. For example, these lines all have a slope of $\frac{1}{4}$.



To know the exact location of a line, we either need both the slope and the coordinates of one point on the line, or the locations of at least 2 points that are on the line. For example, this line goes through the point (4, 3) and has a slope of $\frac{1}{4}$. This line could also be described as the line that passes through (4, 3) and (8, 4).



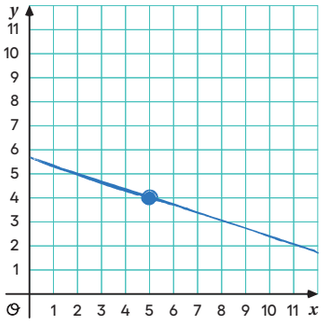
Cool-down

Another Way

5 min

Student Task Statement

1. Draw the line that has a slope of $-\frac{1}{3}$ and passes through the point (5, 4).



2. Describe this same line in a different way.

Sample responses:

- a line with slope $-\frac{1}{3}$ and passing through the point (8, 3)
- a line with slope $-\frac{1}{3}$ and vertical intercept at $\left(5\frac{2}{3}, 0\right)$
- the line of $y = 5\frac{2}{3} - \frac{1}{3}x$
- the line that goes through the points (2, 5) and (5, 4)

Responding To Student Thinking

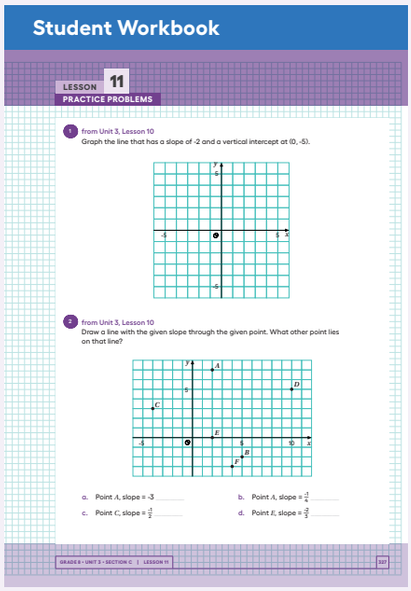
Press Pause

By this point in the unit, there should be some student mastery of describing and drawing lines using slope and a point on the line. If most students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 3, Section C Finding Slopes

Practice Problems

3 Problems


Student Workbook

LESSON 11
PRACTICE PROBLEMS

1 from Unit 3, Lesson 10
Graph the line that has a slope of -2 and a vertical intercept at (0, -5).

2 from Unit 3, Lesson 10
Draw a line with the given slope through the given point. What other point lies on that line?

a. Point A, slope = -3
b. Point A, slope = $\frac{-1}{4}$
c. Point C, slope = $\frac{-1}{2}$
d. Point E, slope = $\frac{-2}{3}$

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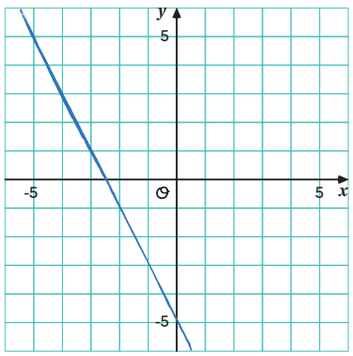
Problem 1

from Unit 3, Lesson 10

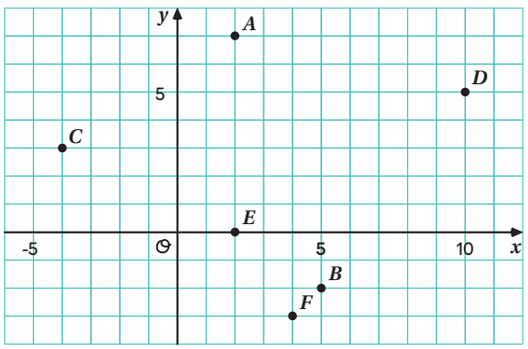
Graph the line that has a slope of -2 and a vertical intercept at (0, -5).



Problem 2

from Unit 3, Lesson 10

Draw a line with the given slope through the given point. What other point lies on that line?



a. Point A, slope = -3

Point *B*

b. Point A, slope = $\frac{-1}{4}$

Point *D*

c. Point C, slope = $\frac{-1}{2}$

Point *E*

d. Point E, slope = $\frac{-2}{3}$

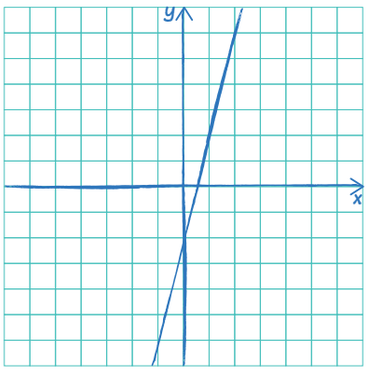
Point *B*

Problem 3

from Unit 3, Lesson 8

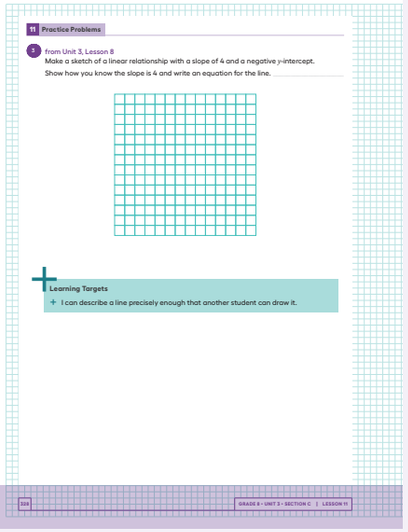
Make a sketch of a linear relationship with a slope of 4 and a negative y-intercept. Show how you know the slope is 4 and write an equation for the line.

Sample response:



The equation is $y = 4x - 2$ (or equivalent). I can tell the slope is 4 by looking at the points $(0, -2)$ and $(1, 2)$ since $\frac{2-2}{1-0} = \frac{4}{1} = 4$.

Student Workbook


11 Practice Problems

3 from Unit 3, Lesson 8
Make a sketch of a linear relationship with a slope of 4 and a negative y-intercept.
Show how you know the slope is 4 and write an equation for the line.

Learning Targets
+ I can describe a line precisely enough that another student can draw it.