Lesson 9

When Are They the Same?

Goals

- Create an equation in one variable to represent a situation in which two conditions are equal.
- Interpret the solution of an equation in one variable in context.

Learning Target

I can use an expression to find when two things, like height, are the same in a real-world situation.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR6: Three Reads (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR6: Three Reads

Lesson Narrative

In this lesson students apply their knowledge of solving equations by considering real world situations. Using the given expressions for each situation, students are asked to determine when certain quantities are the same. Students should notice that they can set expressions equal to each other to solve for the variable of interest.

The work in this lesson introduces students to a simple form of solving systems of equations in which equations solved for the same variable can be set equal to one another as a basic form of substitution.

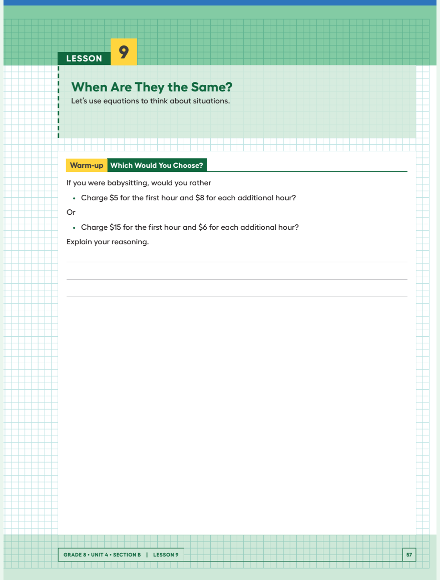
Student Learning Goal

Let's use equations to think about situations.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	15 min	10 min	5 min

Student Workbook


LESSON 9

When Are They the Same?

Let's use equations to think about situations.

Warm-up Which Would You Choose?

If you were babysitting, would you rather

- Charge $5 for the first hour and $8 for each additional hour?

Or

- Charge $15 for the first hour and $6 for each additional hour?

Explain your reasoning.

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Warm-up

Which Would You Choose?

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to reason about two situations that can be represented with linear equations. Because the number of babysitting hours determines which situation would be most profitable, there is no one correct answer to the question. Students are asked to explain their reasoning.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

If you were babysitting, would you rather

- Charge $5 for the first hour and $8 for each additional hour?

Or

- Charge $15 for the first hour and $6 for each additional hour?

Explain your reasoning.

Sample responses:

- I would choose to charge $15 for the first hour and $6 for each additional hour if I only babysit for up to 5 hours.
- It doesn't matter which I choose if I babysit for 6 hours (the first hour plus 5 additional hours) because the amount I will earn is the same.
- I would choose $5 for the first hour and $8 for each additional hour if I babysit for more than 6 hours.

Activity Synthesis

Survey the class to determine which situation they would choose. Invite students from each side to explain their reasoning. Record and display these ideas for all to see. If no one reasoned about babysitting for less than 5 hours, and therefore chose the second option, mention this idea to students.

If students do not use linear equations or graphs to choose a situation, and there is time, ask students for the equation and graph that could be used to model each situation.

Activity 1
Water Tanks

10 min

Activity Narrative

The goal of this activity is for students to solve an equation in a real-world context while previewing some future work solving systems of equations. Here, students first make sense of the situation using a table of values describing the water heights of two tanks and then use the table to estimate when the water heights are equal. A key point in this activity is the next step: taking two expressions representing the water heights in two different tanks for a given time and recognizing that the equation created by setting the two expressions equal to one another has a solution that is the value for time, t, when the water heights are equal.

Launch

Give students 2–3 minutes to read the context and answer the first problem. Select students to share their answer with the class, choosing students with different representations of the situation, if possible. Give 3–4 minutes for the remaining problems, followed by a whole-class discussion.

To introduce the context, tell students that many areas have water towers or tanks that are used to increase the water pressure of plumbing in the area. In some large cities, they can be seen on rooftops, and in other areas they stand alone, often with the town's name written on the side.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe what is happening in each tank. Display words and phrases such as "rate of change," "initial amount," and "differences in rates."

Instructional Routines

MLR2: Collect and Display

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Please log in to the site before using the QR code or URL.

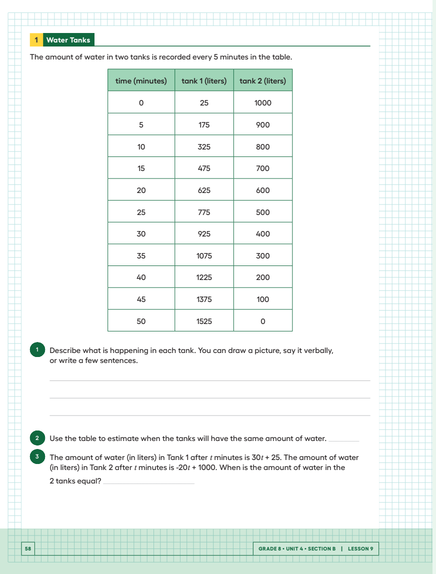
Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Represent the same information through different modalities by using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help illustrate the information provided.

Student Workbook


1 Water Tanks

The amount of water in two tanks is recorded every 5 minutes in the table.

time (minutes)	tank 1 (liters)	tank 2 (liters)
0	25	1000
5	175	900
10	325	800
15	475	700
20	625	600
25	775	500
30	925	400
35	1075	300
40	1225	200
45	1375	100
50	1525	0

1. Describe what is happening in each tank. You can draw a picture, say it verbally, or write a few sentences.
2. Use the table to estimate when the tanks will have the same amount of water.
3. The amount of water (in liters) in Tank 1 after t minutes is $30t + 25$. The amount of water (in liters) in Tank 2 after t minutes is $-20t + 1000$. When is the amount of water in the 2 tanks equal?

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Student Task Statement

The amount of water in two tanks is recorded every 5 minutes in the table.

time (minutes)	tank 1 (liters)	tank 2 (liters)
0	25	1000
5	175	900
10	325	800
15	475	700
20	625	600
25	775	500
30	925	400
35	1075	300
40	1225	200
45	1375	100
50	1525	0

1. Describe what is happening in each tank. You can draw a picture, say it verbally, or write a few sentences.

   Sample response: The water in Tank 1 is increasing while the water in Tank 2 is decreasing.

2. Use the table to estimate when the tanks will have the same amount of water.

   Sample response: The tanks will have the same amount of water between 15 and 20 minutes, but closer to 20 minutes.

3. The amount of water (in liters) in Tank 1 after t minutes is $30t + 25$.
   The amount of water (in liters) in Tank 2 after t minutes is $-20t + 1000$.
   When is the amount of water in the 2 tanks equal?

   19 and a half minutes after the start

   Sample reasoning: When the amounts of water in the two tanks are equal, $30t + 25 = -20t + 1000$. This happens when $t = 19.5$.

Activity Synthesis

The purpose of this discussion is to elicit student thinking about why setting the two expressions in the task statement equal to one another is both possible and a way to solve the final problem.

Direct students' attention to the reference created using *Collect and Display*. Ask students to explain how the expressions in the third question are related to what they noticed was happening. Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond.

Consider asking:

- *"What does t represent in the first expression? The second?"*

  In each expression, t is the time in minutes since the tank's water level started being recorded.

- *"After we substitute a time in for t and simplify one of the expressions to be a single number, what does that number represent? What units does it have?"*

  The number represents the amount of liters in the water tank.

- *"How accurate was your estimate about the water heights using the table?"*

  My estimate was within a few minutes of the actual answer.

- *"How do you know from the expressions that there is only 1 value for t that makes $30t + 25 = -20t + 1000$ true?"*

  The coefficients of t on each side are different.

- *"If you didn't know which expression in the last problem belonged to which tank, how could you figure it out?"*

  One of the expressions is increasing as t increased, which means it must be Tank 1. The other is decreasing as t increases, so it must be Tank 2.

- *"How did you find the time at which the two water heights were equal using the expressions?"*

  Because each expression gives the height for a specific time, t, and we want to know when the heights are equal, I set the two expressions equal to each other and then solved for the t-value that made the new equation true.

Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 2)

MLR6: Three Reads
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Activity 2

Elevators

15 min

Activity Narrative

In this activity, students work with two expressions that represent the travel time of an elevator to a specific height. As with the previous activity, the goal is for students to work within a real-world context to understand taking two separate expressions and setting them equal to one another as a way to determine more information about the context.

Launch

Tell students to close their books or devices (or to keep them closed).

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the image, without revealing the questions.

For the first read, read the problem aloud, and then ask,

"What is this situation about?"

elevators going up and down

Listen for and clarify any questions about the context.

After the second read, ask students to list any quantities that can be counted or measured.

the equations could be used to find the speed and initial heights

After the third read, reveal the question:

"When do the elevators pass one another?"

and ask,

"What are some ways that we might get started on this?"

Invite students to name some possible starting points, referring to quantities from the second read.

Set the equations equal to one another to get the height when they pass, and then substitute that into the equation to get the time.

You may wish to share with the class that programming elevators in buildings to best meet the demands of the people in the building can be a complicated task depending on the number of floors in a building, the number of people, and the number of elevators. For example, many large buildings in cities have elevators programmed to stay near the ground floor in the morning when employees are arriving and then to stay on higher floors in the afternoon when employees leave work.

Arrange students in groups of 2.

Give 2–3 minutes of quiet work time for the first 2 questions, and then ask students to pause and discuss their solutions with their partner.

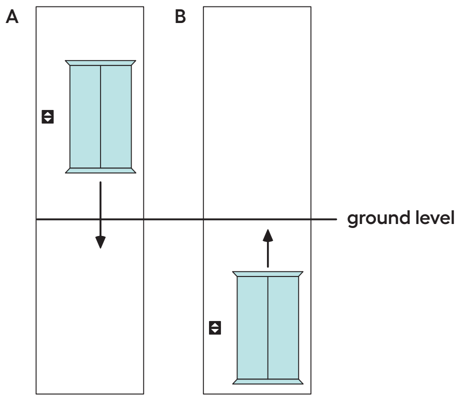
Give 3–4 minutes for partners to work on the remaining questions, and follow that with a whole-class discussion.

Student Task Statement

A building has two elevators that both go above and below ground.

At a certain time of day, the travel time, in seconds, that it takes Elevator A to reach height h in meters is given by the equation $t = 0.8h + 16$ seconds.

The travel time for Elevator B is given by the equation $t = -0.8h + 12$.



1. What is the height of each elevator at this time?

   Elevator A is 20 meters below ground. At $t = 0$, the initial height of Elevator A can be found by solving the equation $0 = 0.8h + 16$. Elevator B is 15 meters above ground. At $t = 0$, the initial height of Elevator B can be found by solving the equation $0 = -0.8h + 12$.

2. How long does it take each elevator to reach ground level at this time?

   The elevator reaches the ground when $h = 0$. Elevator A reaches ground level after 16 seconds because $0.8(0) + 16 = 16$. Elevator B reaches ground level after 12 seconds because $-0.8(0) + 12 = 12$.

3. If the two elevators travel toward one another, at what height do they pass each other? How long does it take?

   At 14 seconds both elevators are 2.5 meters below ground. The solution can be found by solving the equation $0.8h + 16 = -0.8h + 12$.

Are You Ready for More?

1. In a two-digit number, the ones digit is twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the number.

   $10x + 2x + 36 = 10(2x) + x$ or equivalent, where x represents the tens digit. $x = 4$ so the number is 48

2. The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 less than the original number. Find the number.

   $10x + (11 - x) - 45 = 10(11 - x) + x$ or equivalent, where x represents the tens digit. $x = 8$ so the number is 83

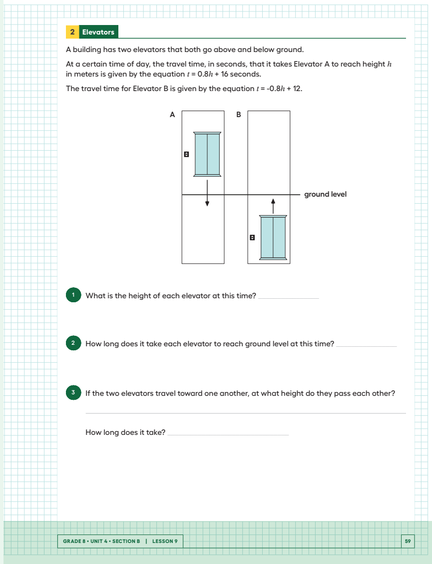
3. The sum of the digits in a two-digit number is 8. The value of the number is 4 less than 5 times the ones digit. Find the number.

   $10x + (8 - x) = 5(8 - x) - 4$ or equivalent, where x represents the tens digit. $x = 2$ so the number is 26

Building on Student Thinking

Students may mix up height and time while working with these expressions. For example, they may think that at $h = 0$, the height of the elevators is 16 meters and 12 meters, respectively, instead of the correct interpretation that the elevators reach a height of 0 meters at 16 seconds and 12 seconds, respectively. Ask them to explain in their own words what h represents and then what $0.8h + 16$ represents, including using units, to help their understanding.

Student Workbook


2 Elevators

A building has two elevators that both go above and below ground.

At a certain time of day, the travel time, in seconds, that it takes Elevator A to reach height h in meters is given by the equation $t = 0.8h + 16$ seconds.

The travel time for Elevator B is given by the equation $t = -0.8h + 12$.

1 What is the height of each elevator at this time?

2 How long does it take each elevator to reach ground level at this time?

3 If the two elevators travel toward one another, at what height do they pass each other?

How long does it take?

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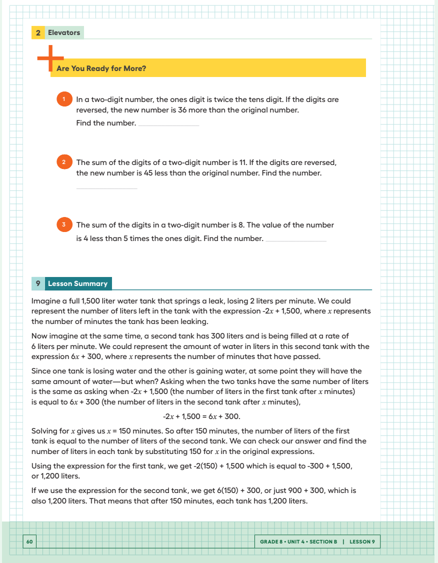
Student Workbook


2 Elevators

Are You Ready for More?

1 In a two-digit number, the ones digit is twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the number.

2 The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 less than the original number. Find the number.

3 The sum of the digits in a two-digit number is 8. The value of the number is 4 less than 5 times the ones digit. Find the number.

9 Lesson Summary

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Access for Students with Diverse Abilities (Activity 2, Activity Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the different elevators to match the equations representing them.

Supports accessibility for: Visual-Spatial Processing

Activity Synthesis

This discussion should focus on the act of setting the two expressions equal and what that means in the context of the situation.

Consider asking:

- *"If someone thought that the height of Elevator A before we started timing was 16 meters because they substituted 0 for the variable of the expression $0.8h + 16$ and got 16, how would you help them correct their answer?"*

  I would remind them that h is height and $0.8h + 16$ is the time, so when we start timing at 0 that means $0.8h + 16 = 0$, not that $h = 0$.

- *"Which of the elevators in the image is A and which is B? How do you know?"*

  A is the elevator on the right since at time 0 the height is negative, while B is the elevator on the left since at time 0 the height is positive.

- *"How did you know there would be only 1 time when the elevators are at the same height?"*

  After setting up the equation, the coefficients on each side of the equation are different.

- *"Describe a situation in which the 2 elevators are never at the same height at the same time. What would their equations look like?"*

  If Elevator B is a little higher than Elevator A and they both move at the same rate. For example $t = 0.8h + 16$ and $t = 0.8h + 14$.

Lesson Synthesis

Arrange students in groups of 2. Ask partners to think of other situations in which two quantities are changing and they want to know when the quantities are equal. Give groups time to discuss and write down a few sentences explaining their situation. Invite groups to share their situation with the class. (For example, in a race where participants walk at steady rates but the slower person has a head start, when will they meet?) Consider allowing groups to share their situation by drawing a picture, making a graph, explaining in words, or acting it out.

Then ask students if they could modify their situations so that there are either infinitely many or no solutions. (For example, in the same race, the person with the head start walks at the same pace as the person who didn't get a head start.)

Lesson Summary

Imagine a full 1,500 liter water tank that springs a leak, losing 2 liters per minute. We could represent the number of liters left in the tank with the expression $-2x + 1{,}500$, where x represents the number of minutes the tank has been leaking.

Now imagine at the same time, a second tank has 300 liters and is being filled at a rate of 6 liters per minute. We could represent the amount of water in liters in this second tank with the expression $6x + 300$, where x represents the number of minutes that have passed.

Since one tank is losing water and the other is gaining water, at some point they will have the same amount of water—but when? Asking when the two tanks have the same number of liters is the same as asking when $-2x + 1{,}500$ (the number of liters in the first tank after x minutes) is equal to $6x + 300$ (the number of liters in the second tank after x minutes),

$$-2x + 1{,}500 = 6x + 300.$$

Solving for x gives us $x = 150$ minutes. So after 150 minutes, the number of liters of the first tank is equal to the number of liters of the second tank. We can check our answer and find the number of liters in each tank by substituting 150 for x in the original expressions.

Using the expression for the first tank, we get $-2(150) + 1{,}500$ which is equal to $-300 + 1{,}500$, or 1,200 liters.

If we use the expression for the second tank, we get $6(150) + 300$, or just $900 + 300$, which is also 1,200 liters. That means that after 150 minutes, each tank has 1,200 liters.

Responding To Student Thinking

Points to Emphasize

If students struggle to interpret the equation in context, revisit this situation. Look at the equation term by term, and then connect it to the meaning of equivalence. After the *Warm-up* for the lesson, it may be helpful to graph each side of the equation to understand how the terms affect the graph:

Unit 4, Lesson 10 On or Off the Line?

Cool-down

Printers and Ink

5 min

Student Task Statement

To own and operate a home printer, it costs \$100 for the printer and an additional \$0.05 per page for ink. To print out pages at an office store, it costs \$0.25 per page. Let p represent number of pages.

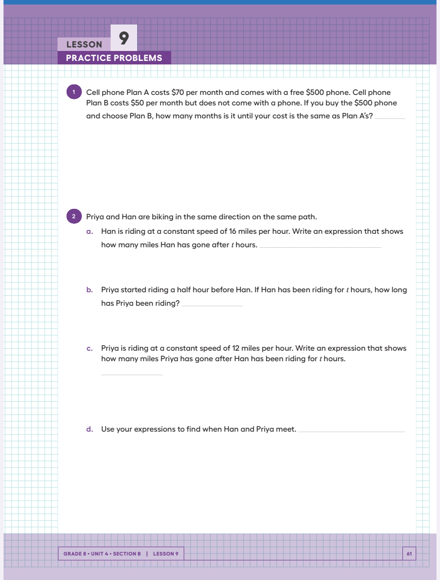
1. What does the equation $100 + 0.05p = 0.25p$ represent?

   The equation represents when the cost for owning and operating a home printer is equal to the cost for printing at an office store.

2. The solution to that equation is $p = 500$. What does the solution mean?

   The solution of $p = 500$ means that the costs are equal for printing 500 pages.

Student Workbook


LESSON 9
PRACTICE PROBLEMS

1. Cell phone Plan A costs $70 per month and comes with a free $500 phone. Cell phone Plan B costs $50 per month but does not come with a phone. If you buy the $500 phone and choose Plan B, how many months is it until your cost is the same as Plan A's?

2. Priya and Han are biking in the same direction on the same path.

a. Han is riding at a constant speed of 16 miles per hour. Write an expression that shows how many miles Han has gone after t hours.

b. Priya started riding a half hour before Han. If Han has been riding for t hours, how long has Priya been riding?

c. Priya is riding at a constant speed of 12 miles per hour. Write an expression that shows how many miles Priya has gone after Han has been riding for t hours.

d. Use your expressions to find when Han and Priya meet.

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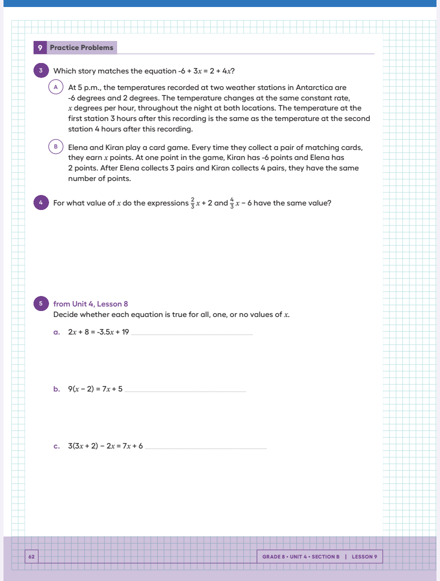
Student Workbook


9 Practice Problems

3. Which story matches the equation $-6 + 3x = 2 + 4x$?

A. At 5 p.m., the temperatures recorded at two weather stations in Antarctica are -6 degrees and 2 degrees. The temperature changes at the same constant rate, x degrees per hour, throughout the night at both locations. The temperature at the first station 3 hours after this recording is the same as the temperature at the second station 4 hours after this recording.

B. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn x points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

4. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

5. from Unit 4, Lesson 8
Decide whether each equation is true for all, one, or no values of x.

a. $2x + 8 = -3.5x + 19$

b. $9(x - 2) = 7x + 5$

c. $3(3x + 2) - 2x = 7x + 6$

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Lesson **9**

Practice Problems

6 Problems

Problem 1

Cell phone Plan A costs $70 per month and comes with a free $500 phone. Cell phone Plan B costs $50 per month but does not come with a phone. If you buy the $500 phone and choose Plan B, how many months is it until your cost is the same as Plan A's? 25 months

Problem 2

Priya and Han are biking in the same direction on the same path.

a. Han is riding at a constant speed of 16 miles per hour. Write an expression that shows how many miles Han has gone after t hours. $16t$ miles

b. Priya started riding a half hour before Han. If Han has been riding for t hours, how long has Priya been riding? $t + \frac{1}{2}$ hours

c. Priya is riding at a constant speed of 12 miles per hour. Write an expression that shows how many miles Priya has gone after Han has been riding for t hours. $12\left(t + \frac{1}{2}\right)$ miles

d. Use your expressions to find when Han and Priya meet. $t = \frac{3}{2}$ hours

Sample reasoning: To find when Han and Priya meet, set the two expressions equal to one another: $16t = 12\left(t + \frac{1}{2}\right)$. They meet after Han rides for one and a half hours and Priya rides for two hours.

Problem 3

Which story matches the equation $-6 + 3x = 2 + 4x$?

A. At 5 p.m., the temperatures recorded at two weather stations in Antarctica are -6 degrees and 2 degrees. The temperature changes at the same constant rate, x degrees per hour, throughout the night at both locations. The temperature at the first station 3 hours after this recording is the same as the temperature at the second station 4 hours after this recording.

B. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn x points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

Problem 4

For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

$x = 12$

Problem 5

from Unit 4, Lesson 8

Decide whether each equation is true for all, one, or no values of x.

a. $2x + 8 = -3.5x + 19$ True for one value of x.

b. $9(x - 2) = 7x + 5$ True for one value of x.

c. $3(3x + 2) - 2x = 7x + 6$ True for all values of x.

Problem 6

from Unit 4, Lesson 6

Solve each equation. Explain your reasoning.

$3d + 16 = -2(5 - 3d)$

$d = \frac{26}{3}$

Sample reasoning: Distribute on the right side of the equation, add 10 to each side, subtract $3d$ from each side, then divide each side by 3.

$2k - 3(4 - k) = 3k + 4$

$k = 8$

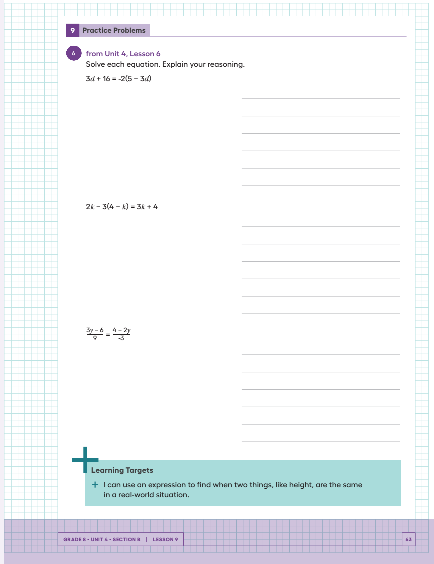
Sample reasoning: Distribute and combine like terms on the left side, subtract $3k$ on each side, add 12 to each side, and then divide each side by 2.

$\frac{3y - 6}{9} = \frac{4 - 2y}{-3}$

$y = 2$

Sample reasoning: Multiply each side by 9, distribute -3 on the right side, subtract $3y$ on each side, add 12 to each side, and then divide each side by 3.

Student Workbook


Practice Problems

from Unit 4, Lesson 6

Solve each equation. Explain your reasoning.

$3d + 16 = -2(5 - 3d)$

$2k - 3(4 - k) = 3k + 4$

$\frac{3y - 6}{9} = \frac{4 - 2y}{-3}$

Learning Targets

+ I can use an expression to find when two things, like height, are the same in a real-world situation.