Lesson 12

Solving Multi-step Percentage Problems

Goals

- Determine what information is needed to solve a problem involving sales tax and discounts. Ask questions to elicit that information.
- Explain (orally) how to calculate the percentage given the dollar amounts before and after a sales tax, tip, or discount.

Learning Target

I can solve problems that involve multiple percentages.

Lesson Narrative

In this lesson, students consolidate what they have learned over the last few lessons. They solve a variety of multi-step percentage problems involving taxes, tips, discounts, and commissions, including problems involving fractional percentages. The main activity uses the *Information Gap* routine. To obtain all the necessary information, students need to persevere in asking questions and to communicate precisely.

The last activity is optional because it provides an opportunity for additional practice with finding the percentage of the increase or decrease.

Student Learning Goal

Let's solve more problems about sales tax and discounts.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR4: Information Gap (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR4: Information Gap Cards
- Which Three Go Together?

Required Materials

Materials to Gather

- Four-function calculators: Activity 1

Materials to Copy

- Sporting Goods Cards (1 copy for every 2 students): Activity 1

Lesson Timeline

5 min	20 min	10 min	10 min	Assessment: 5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Warm-up

Which Three Go Together: Equations

5 min

Activity Narrative

This *Warm-up* prompts students to compare four equations. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the equations for all to see.

Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three equations that go together and can explain why.

Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $1.08 \cdot 25 = 27$

B. $1.08 \cdot 25 = x$

C. $1.08x = 27$

D. $\left(1 + \frac{x}{100}\right) \cdot 25 = 27$

Sample responses:

A, B, and C go together because:

- They have the number 1.08.
- The percentage of the increase is known.

A, B, and D go together because:

- They have the number 25.
- The original value before the increase is known.

A, C, and D go together because:

- They have the number 27.
- The new value after the increase is known.

B, C, and D go together because:

- They have a variable.
- A value is unknown.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

Instructional Routines

Which Three Go Together?

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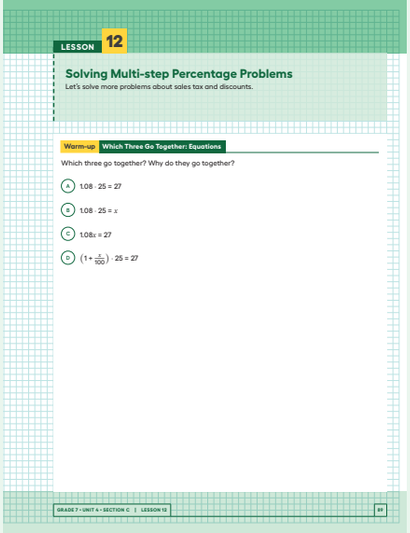
Student Workbook


LESSON 12

Solving Multi-step Percentage Problems

Let's solve more problems about sales tax and discounts.

Warm-up Which Three Go Together: Equations

Which three go together? Why do they go together?

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B. $1.08 \cdot 25 = x$

C. $1.08x = 27$

D. $\left(1 + \frac{x}{100}\right) \cdot 25 = 27$

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Instructional Routines

MLR4: Information Gap Cards

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Access for Multilingual Learners (Activity 1)

MLR4: Information Gap

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Students might fail to notice that Elena and Andre buy multiple cans of tennis balls and packages of socks, respectively. Ask students to figure out how much 2 packages of socks (or 3 cans of tennis balls) will cost.

If students automatically give the 15% discount on all of Elena's purchases, ask students which of Elena's items fall under the discount.

Students might apply the discount after adding in the sales tax. Remind students that the discount is applied to the subtotal before the tax is calculated.

Some students may include the sales tax when calculating the percentage of Andre's savings. Remind them that the problem specifies "before tax."

During the discussion, prompt students to explain the meaning of any terminology they use, such as "variable," "coefficient," "product," "factor," and "percent increase," and to clarify their reasoning as needed. Consider asking:

- *"How do you know … ?"*
- *"What do you mean by … ?"*
- *"Can you say that in another way?"*

If time allows, invite 2–3 students to briefly share what they notice that all of the equations have in common. (For example, they could all represent an 8% increase from 25 to 27.) The purpose of this concluding share out is to remind students how an equation can represent a situation involving percent increase or decrease, which will be helpful for this lesson.

Activity 1
Info Gap: Sporting Goods

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to determine the total savings after various discounts are applied to different items.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch

Tell students they will calculate the discounts on various sports equipment. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it. There is an extra set of cards available for demonstration purposes.

Arrange students in groups of 2 or 4. If students are new to the *Information Gap* routine, allowing them to work in groups of 2 for each role supports communication and understanding. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.

2. Ask your partner for the specific information that you need. "Can you tell me _____?"

3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."

4. Continue to ask questions until you have enough information to solve the problem.

5. Once you have enough information, share the problem card with your partner, and solve the problem independently.

6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

These steps may be repeated.

1. Silently read your card. Wait for your partner to ask for information.

2. Before telling your partner any information, ask, "Why do you need to know _____?"

3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.

5. Share the data card, and discuss your reasoning.

1. Elena will pay $52.68. (Elena spent 0.85 • 43, or $36.55, on the tennis racket and 3 • 4, or $12, on the cans of tennis balls. The total price before tax was $48.55, because 36.55 + 12 = 48.55. The total cost including tax is 1.085 • 48.55, or about $52.68.)
2. Andre saved about 8.7%. (The regular price for the baseball glove and two packages of socks is $46, because 34 + 6 + 6 = 46. Andre saved $4: each package of socks was discounted by $2, he bought 2 packages, and 2 • 2 = 4. This savings is about 8.7% of the regular price, because 4 ÷ 46 ≈ 0.087.)

Student Workbook


1 Info Gap: Sporting Goods

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

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3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ...."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

These steps may be repeated.

1. Silently read your card. Wait for your partner to ask for information.
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4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

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Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"What information did you and your partner have to figure out?"*

  "How did you determine the cost of Elena's tennis racket?"

  I multiplied the original cost by 0.85; I multiplied by 0.15 and subtracted it from the original cost.

- *"How did you determine the total cost after tax for Elena's purchases?"*

  I multiplied the total by 1.085; I multiplied by 0.085 and added it to the original cost.

- *"What different calculations did you have to make for Andre and Elena's situations?"*

  "Was there information given that you did not need to use?"

Highlight any strategies students used for keeping track of the information they received and how they decided what information they still needed.

Activity 2: Optional

What Is the Percentage?

10 min

Activity Narrative

In this activity, students continue to practice finding percentages from dollar amounts including commission, tip, and markdown. Some of the questions require multiple steps to solve for the percentage needed. In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time followed by time for partner discussion. Then hold a whole-class discussion.

Student Task Statement

1. A salesperson sold a car for $18,250, and their commission is $693.50. What percentage of the sale price is their commission?

   3.8% (693.50 ÷ 18,250 = 0.038)

2. The bill for a meal was $33.75. The customer left $40.00. What percentage of the bill was the tip?

   about 18.52% (The tip is $6.25 and 6.25 ÷ 33.75 ≈ 0.1852.)

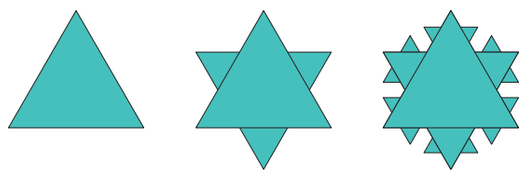
3. The original price of a bicycle was $375. Now it is on sale for $295. What percentage of the original price was the markdown?

   about 21.33% (The price was marked down by $80, and 80 ÷ 375 ≈ 0.2133.)

Are You Ready for More?

To make a Koch snowflake,

- Start with an equilateral triangle. This is step 1.
- Divide each side into 3 equal pieces. Construct a smaller equilateral triangle on the middle third. This is step 2.
- Do the same to each of the newly created sides. This is step 3.
- Keep repeating this process.



step 1 step 2 step 3

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 1)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Provide a copy of the chart from the previous lesson for students to use as a reference of monetary contexts that involve percentages.

Supports accessibility for: Memory, Organization

Student Workbook

2 What Is the Percentage?

1 A salesperson sold a car for $18,250, and their commission is $693.50. What percentage of the sale price is their commission?

2 The bill for a meal was $33.75. The customer left $40.00. What percentage of the bill was the tip?

3 The original price of a bicycle was $375. Now it is on sale for $295. What percentage of the original price was the markdown?

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By what percentage does the perimeter increase at step 2? Step 3? Step 10?

The perimeter increases by a factor of $\frac{4}{3}$ at each step, so the percent increase at each step is $33\frac{1}{3}\%$.

- From step 1 to step 2, the perimeter has increased by about 33.3%.
- From step 1 to step 3, the perimeter has increased by about 77.8% since $\frac{4}{3} \cdot \frac{4}{3} = \frac{16}{9} \approx 1.778$ and $1.778 - 1 = 0.778$.
- From step 1 to step 10, the perimeter has increased by about 1,675.8% since $\left(\frac{4}{3}\right)^{10} = \frac{1{,}048{,}576}{59{,}049} \approx 17.758$ and $17.758 - 1 = 16.758$.

Activity Synthesis

The purpose of this discussion is for students to highlight the importance of interpreting results in the context of the situation to make sure that they answer the question that was asked. Invite students to share their different methods for solving the last two questions. Sequence them so that strategies using diagrams come first, and those using equations come second.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"*The markdown is 78.6% because 295 ÷ 375 = 0.786̄.*"

Ask,

"*What parts of this response are unclear, incorrect, or incomplete?*"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

The key takeaway is that $78.\overline{6}$ represents the percentage that the sale price is of the original price. The question asks for the percentage that the markdown is of the original price, which is $100 - 78.\overline{6}$, or $21.\overline{3}$.

Lesson Synthesis

Share with students,

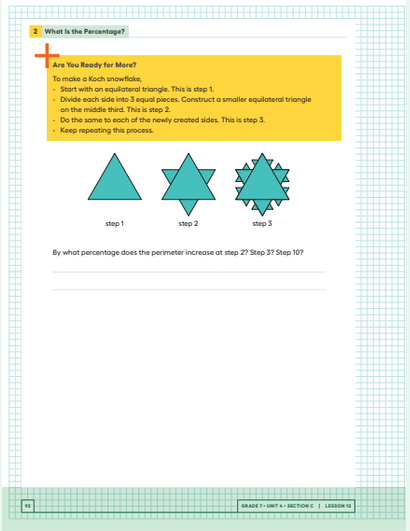
"*Today we solved problems that involved multiple steps with different percentages. We practiced asking questions to get the needed information to solve a problem.*"

To review these concepts, consider asking students:

"*What are some types of situations that we have seen where there is a percent increase?*"

tax, tip, markup

Student Workbook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- Keep repeating this process.

step 1 step 2 step 3

By what percentage does the perimeter increase at step 2? Step 3? Step 10?

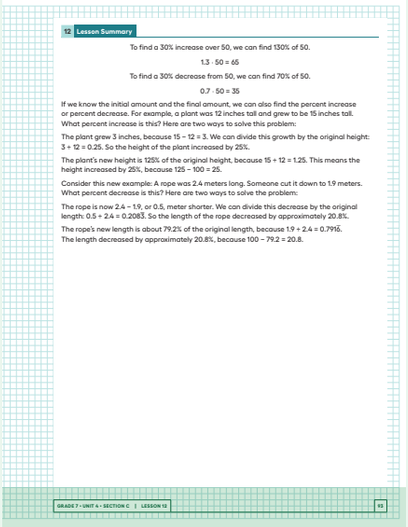
Responding To Student Thinking

Press Pause

By this point in the unit, there should be some student mastery of solving problems involving percent increase or decrease. If students struggle, plan to make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 4, Lesson 11 Percentage Contexts

Student Workbook



12 Lesson Summary

To find a 30% increase over 50, we can find 130% of 50.

$1.3 \cdot 50 = 65$

To find a 30% decrease from 50, we can find 70% of 50.

$0.7 \cdot 50 = 35$

If we know the initial amount and the final amount, we can also find the percent increase or percent decrease. For example, a plant was 12 inches tall and grew to be 15 inches tall. What percent increase is this? Here are two ways to solve this problem:

The plant grew 3 inches, because 15 – 12 = 3. We can divide this growth by the original height: 3 ÷ 12 = 0.25. So the height of the plant increased by 25%.

The plant's new height is 125% of the original height, because 15 ÷ 12 = 1.25. This means the height increased by 25%, because 125 – 100 = 25.

Consider this new example: A rope was 2.4 meters long. Someone cut it down to 1.9 meters. What percent decrease is this? Here are two ways to solve the problem:

The rope is now 2.4 – 1.9, or 0.5, meter shorter. We can divide this decrease by the original length: $0.5 \div 2.4 = 0.208\overline{3}$. So the length of the rope decreased by approximately 20.8%.

The rope's new length is about 79.2% of the original length, because $1.9 \div 2.4 = 0.791\overline{6}$. The length decreased by approximately 20.8%, because 100 – 79.2 = 20.8.

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"What are some types of situations that we have seen where there is a percent decrease?"

discount, commission

"If a situation involves a 10% decrease followed by a 5% increase, how can you find the new amount?

Multiply the original amount by 0.9, and multiply that result by 1.05.

Lesson Summary

To find a 30% increase over 50, we can find 130% of 50.

$$1.3 \cdot 50 = 65$$

To find a 30% decrease from 50, we can find 70% of 50.

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If we know the initial amount and the final amount, we can also find the percent increase or percent decrease. For example, a plant was 12 inches tall and grew to be 15 inches tall. What percent increase is this? Here are two ways to solve this problem:

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Cool-down

Shoes on Sale

5 min

Students who answer $72.24 may have combined the two percentages and calculated a 14% discount from the original price, $0.86 \cdot 85 = 72.24$. The sales tax should be 6% of the sale price, not 6% of the normal price.

Student Task Statement

A pair of shoes normally costs $85. They are on sale for 20% off. A sales tax of 6% is added to the sale price.

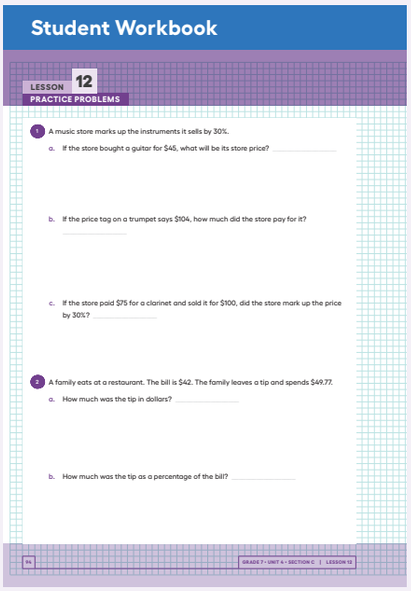
How much will the shoes cost after the discount and the tax?

$72.08, because $0.8 \cdot 85 = 68$ and $1.06 \cdot 68 = 72.08$

Lesson 12

Practice Problems

5 Problems



Problem 1

A music store marks up the instruments it sells by 30%.

a. If the store bought a guitar for $45, what will be its store price? $58.50

b. If the price tag on a trumpet says $104, how much did the store pay for it?

$80.00

c. If the store paid $75 for a clarinet and sold it for $100, did the store mark up the price by 30%?

no

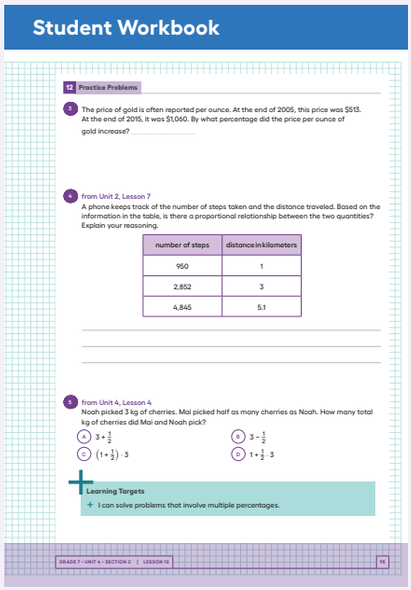
The store marked the price up by $\frac{1}{3}$, or about 33.3% (rounded to the nearest tenth of a percent). The store needed to sell it for $97.50 to have a 30% markup.

Problem 2

A family eats at a restaurant. The bill is $42. The family leaves a tip and spends $49.77.

a. How much was the tip in dollars? $7.77

b. How much was the tip as a percentage of the bill? 18.5%


Student Workbook

12 Practice Problems

3 The price of gold is often reported per ounce. At the end of 2005, this price was $513. At the end of 2015, it was $1,060. By what percentage did the price per ounce of gold increase?

4 from Unit 2, Lesson 7
A phone keeps track of the number of steps taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your reasoning.

number of steps	distance in kilometers
950	1
2,852	3
4,845	5.1

5 from Unit 4, Lesson 4
Noah picked 3 kg of cherries. Mai picked half as many cherries as Noah. How many total kg of cherries did Mai and Noah pick?

A $3+\frac{1}{2}$ B $3-\frac{1}{2}$
C $(1+\frac{1}{2})\cdot 3$ D $1+\frac{1}{2}\cdot 3$

Learning Targets
+ I can solve problems that involve multiple percentages.

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Problem 3

The price of gold is often reported per ounce. At the end of 2005, this price was $513. At the end of 2015, it was $1,060. By what percentage did the price per ounce of gold increase?

about 107% (1,060 − 513 = 547 and 547 ÷ 513 ≈ 1.07)

Problem 4

from Unit 2, Lesson 7

A phone keeps track of the number of steps taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your reasoning.

number of steps	distance in kilometers
950	1
2,852	3
4,845	5.1

No, there is not a proportional relationship.

Sample reasoning: Since the first row shows that there are 950 steps in 1 kilometer, there should be 2,850 steps in 3 kilometers (since $950 \cdot 3 = 2{,}850$), but the table shows 2,852 steps.

Problem 5

from Unit 4, Lesson 4

Noah picked 3 kg of cherries. Mai picked half as many cherries as Noah. How many total kg of cherries did Mai and Noah pick?

A. $3+\frac{1}{2}$

B. $3-\frac{1}{2}$

C. $\left(1+\frac{1}{2}\right)\cdot 3$

D. $1+\frac{1}{2}\cdot 3$