Lesson 8

Reasoning about Solving Equations (Part 2)

Goals

- Compare and contrast (orally) different strategies for solving an equation of the form $p(x + q) = r$.
- Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form $p(x + q) = r$.
- Interpret a balanced hanger diagram with multiple groups, and justify (in writing) that there is more than one way to write an equation that represents the relationship shown.

Learning Targets

- I can explain how a balanced hanger and an equation represent the same situation.
- I can explain why some balanced hangers can be represented by two different equations, one with parentheses and one without.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can identify an equation that represents the weights on a balanced hanger diagram.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Action and Expression (Activity 2)

Access for Multilingual Learners

- Stronger and Clearer (Activity 1)
- Compare and Connect (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect

Lesson Narrative

This lesson continues the work of developing efficient equation solving strategies, justified by working with hanger diagrams. The goal of this lesson is for students to understand two different ways to solve an equation of the form $p(x + q) = r$ efficiently. After a *warm-up* to revisit the distributive property, the first activity asks students to explain why either of two equations could represent a diagram and reason about a solution. The next activity asks students to match equations to diagrams and then solve the equations. The goal is for students to see and understand two approaches to solving this type of equation.

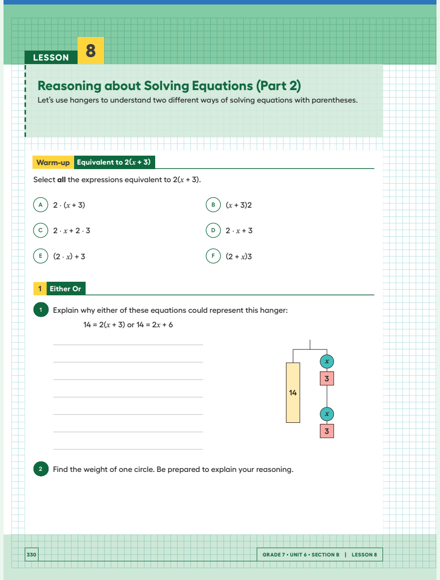
Student Learning Goal

Let's use hangers to understand two different ways of solving equations with parentheses.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)

Student Workbook


LESSON 8

Reasoning about Solving Equations (Part 2)

Let's use hangers to understand two different ways of solving equations with parentheses.

Warm-up Equivalent to $2(x + 3)$

Select **all** the expressions equivalent to $2(x + 3)$.

A. $2 \cdot (x + 3)$
B. $(x + 3)2$
C. $2 \cdot x + 2 \cdot 3$
D. $2 \cdot x + 3$
E. $(2 \cdot x) + 3$
F. $(2 + x)3$

1 Either Or

1. Explain why either of these equations could represent this hanger:

$14 = 2(x + 3)$ or $14 = 2x + 6$

2. Find the weight of one circle. Be prepared to explain your reasoning.

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Warm-up

Equivalent to $2(x + 3)$

5 min

Activity Narrative

In this activity, students identify expressions that are equivalent to a given expression, which involves applying the distributive property. In order to understand the two ways of solving an equation of the form $p(x + q) = r$ in upcoming lessons, it is helpful to have some fluency with the distributive property.

Launch

Arrange students in groups of 2.

Give 3 minutes of quiet work time and then invite students to share their responses with their partner, followed by a whole-class discussion.

Student Task Statement

Select **all** the expressions equivalent to $2(x + 3)$.

A. $2 \cdot (x + 3)$
B. $(x + 3)2$
C. $2 \cdot x + 2 \cdot 3$
D. $2 \cdot x + 3$
E. $(2 \cdot x) + 3$
6. $(2 + x)3$

Activity Synthesis

The purpose of this discussion is to recall that $2 \cdot (x + 3)$ is equivalent to $2 \cdot x + 2 \cdot 3$ because of the distributive property.

Possible discussion questions:

- *"What does it mean for expressions to be equivalent?"*

  They have the same value, no matter what the value of the variable is.

- *"Why is $2 \cdot (x + 3)$ equivalent to $2 \cdot x + 2 \cdot 3$?"*

  because of the distributive property

- *"Can you think of another expression that is equivalent to $2(x + 3)$?"*

  One example is $2x + 6$.

Activity 1

Either Or

15 min

Activity Narrative

In this activity, students are presented with a balanced hanger diagram and are asked to explain why each of two different equations could represent it. They are then asked to find the unknown weight. Note that no particular solution method is prescribed. Give students a chance to come up with a reasonable approach, and then use the Activity Synthesis to draw connections between the diagram and each of the two equations. Monitor for one student who structures the diagram like $2(x + 3) = 14$, and another like $2x + 6 = 14$.

When students articulate their reasoning, they have an opportunity to attend to precision in the language they use to describe their thinking. They might first propose less formal or imprecise language, and after sharing with a partner, revise their explanation to be clearer and stronger.

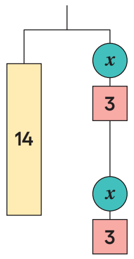
Launch

Keep students in the same groups.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Student Task Statement

1. Explain why either of these equations could represent this hanger:



$14 = 2(x + 3)$ or $14 = 2x + 6$

Sample response: The diagram shows 14 balanced with 2 groups of $x + 3$, and this corresponds to $14 = 2(x + 3)$. The diagram also shows 14 balanced with 2 x's and another 6 units of weight, which corresponds to $14 = 2x + 6$.

2. Find the weight of one circle. Be prepared to explain your reasoning.

4 units

Sample reasoning:

- Since 2 groups of $x + 3$ weighs 14 units, 1 group must weigh 7 units. If $x + 3 = 7$, then $x = 4$.
- Remove 6 units from each side, leaving $8 = 2x$. Therefore, $x = 4$.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners

This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, create a balanced hanger using concrete objects. Be sure to use individual pieces for each part of the diagram. Demonstrate moving pieces off of the hanger to create an equation. Invite students to show different ways to create the same equation.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Activity Synthesis

The purpose of this discussion is to understand viable alternatives for solving for an unknown weight by reasoning about a hanger diagram. Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to why either equation could represent this hanger. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

"The part I understood best was …"

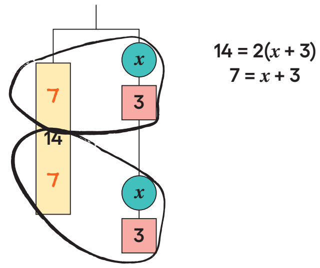
"Can you say more about … ?"

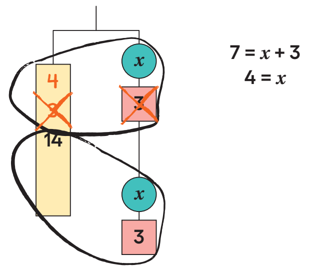
"How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time,* ask one student to present who divided by 2 first, and another student to present who subtracted 6 first.

If no one mentions one of these approaches, demonstrate it. Show how the hanger supports either approach. The finished work might look like this for the first equation:





For the second equation, rearrange the right side of the hanger, first, so that 2 x's are on the top and 6 units of weight are on the bottom. Then cross off 6 from each side and divide each side by 2. Show this side by side with "doing the same thing to each side" of the equation.

Activity 2

Use Hangers to Understand Equation Solving, Again

15 min

Activity Narrative

In this activity, students match hangers to equations, and then solve for an unknown weight first by reasoning about the diagram and then by reasoning about the equation. Monitor for students who:

- Rewrite $p(x + q) = r$ as $px + pq = r$ first.
- Divide both sides by p first.

Launch

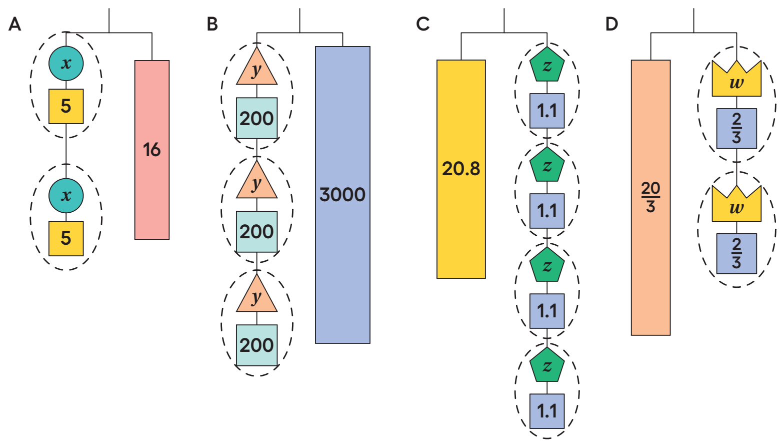
Keep students in the same groups.

Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Here are some balanced hanger diagrams. Each piece is labeled with its weight.



For each diagram:

1. Explain how to figure out the weight of a piece labeled with a variable by reasoning about the diagram.

   Sample responses for the first diagram:

   a. Split each side into two groups with $x + 5$ in each group on the left and 8 in each group on the right. From one of these groups, remove 5 units from each side. This shows that $x = 3$.

   b. Rearrange the left side so that there are 2 x's on top and 10 units on the bottom. Remove 10 units of weight from each side, leaving 2 x's on the left and 6 units on the right. Each x must weigh 3 units for the hanger to be in balance.

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 2, Synthesis)

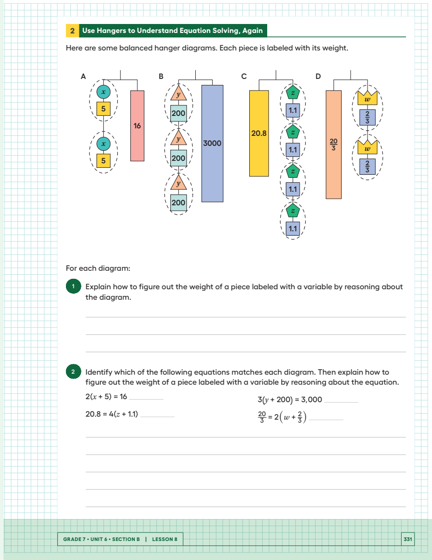
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, show only 2 hangers and 2 equations. If students finish early, assign the remaining hangers and equations.

Supports accessibility for: Organization, Attention

Student Workbook



2. Identify which of the following equations matches the diagram. Then explain how to figure out the weight of a piece labeled with a variable by reasoning about the equation.

$2(x + 5) = 16$

$3(y + 200) = 3{,}000$

$20.8 = 4(z + 1.1)$

$\frac{20}{3} = 2\left(w + \frac{2}{3}\right)$

Each equation corresponds to the diagram with the variable that matches. Sample responses for $2(x + 5) = 16$:

a. Divide each side by 2, leaving $x + 5 = 8$. Subtract 5 from each side, leaving $x = 3$.

b. Use the distributive property to write $2x + 10 = 16$. Subtract 10 from each side leaving $2x = 6$. Divide each side by 2 leaving $x = 3$.

Activity Synthesis

The purpose of this discussion is to understand different viable methods for solving an equation of the form $p(x + q) = r$.

Select one hanger diagram for which one student divided by p first and another student distributed p first. Display the two solution methods side by side, along with the hanger diagram. Invite students to briefly describe their solution, then use *Compare and Connect* to help students compare, contrast, and connect the different solutions. Here are some questions for discussion:

"What do the solutions have in common? How are they different?"

"Did anyone solve the problem the same way but would explain it differently?"

"Why do the different approaches lead to the same solutions?"

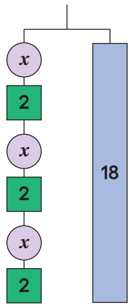
Lesson Synthesis

Display the equation $4(x + 7) = 40$. Ask one partner to solve by dividing first and the other to solve by distributing first. Then check that both students got the same solution and that it makes the equation true. If students get stuck, encourage them to draw a diagram to represent the equation.

Lesson Summary

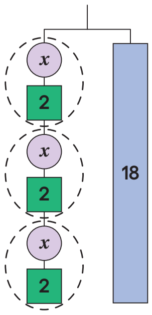
The balanced hanger diagram shows the amounts on the left equal the amounts on the right. The left side has 3 pieces that each have unknown weight x and 3 pieces that each weigh 2 units. So, the left side shows 3 x's plus 6 units. The right side shows 18 units. We could represent this diagram with an equation and solve the equation the same way we did before.

$$3x + 6 = 18$$
$$3x = 12$$
$$x = 4$$



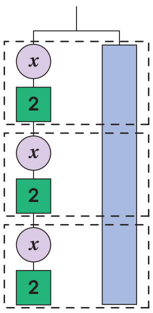
$3x + 6 = 18$
$3x = 12$
$x = 4$

Since there are 3 groups of $x + 2$ on the left, we could represent this hanger with a different equation: $3(x + 2) = 18$.



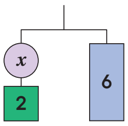
$3(x + 2) = 18$

The two sides of the hanger balance with these weights: 3 groups of $x + 2$ on one side, and 18, or 3 groups of 6, on the other side.



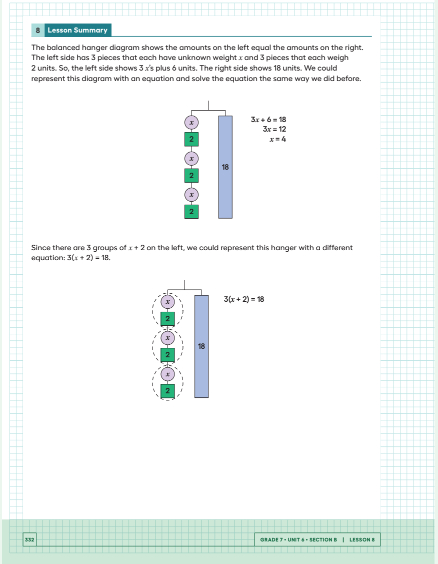
$\frac{1}{3} \cdot 3(x + 2) = \frac{1}{3} \cdot 18$

The two sides of the hanger will balance with $\frac{1}{3}$ of the weight on each side:

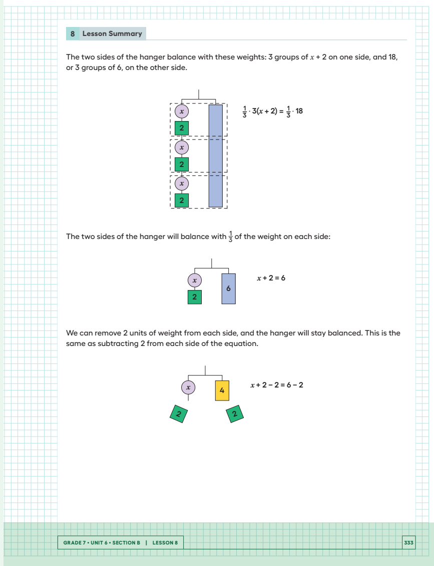


$x + 2 = 6$

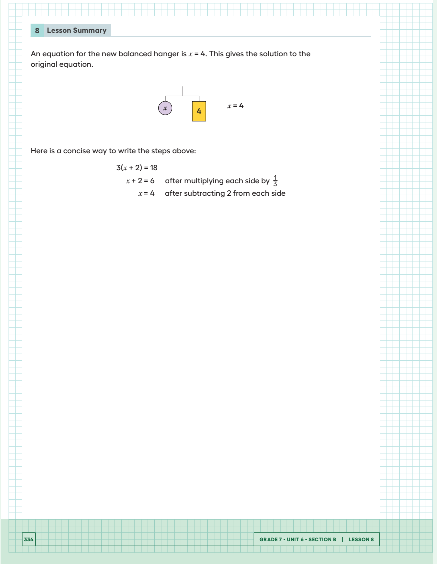
Student Workbook



Student Workbook



Student Workbook


8 Lesson Summary

An equation for the new balanced hanger is $x = 4$. This gives the solution to the original equation.

$x = 4$

Here is a concise way to write the steps above:

$3(x + 2) = 18$

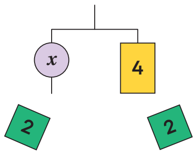
$x + 2 = 6$ after multiplying each side by $\frac{1}{3}$

$x = 4$ after subtracting 2 from each side

Responding To Student Thinking

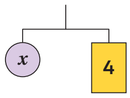
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

We can remove 2 units of weight from each side, and the hanger will stay balanced. This is the same as subtracting 2 from each side of the equation.



$x + 2 - 2 = 6 - 2$

An equation for the new balanced hanger is $x = 4$. This gives the solution to the original equation.



$x = 4$

Here is a concise way to write the steps above:

$3(x + 2) = 18$

$x + 2 = 6$ after multiplying each side by $\frac{1}{3}$

$x = 4$ after subtracting 2 from each side

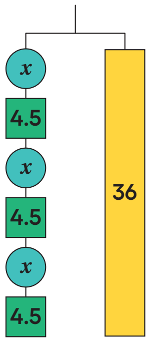
Cool-down

Solve Another Equation

5 min

Student Task Statement

Solve the equation $3(x + 4.5) = 36$. If you get stuck, use the diagram.



7.5

Sample reasoning:

- Divide each side by 3 leaving $x + 4.5 = 12$, then subtract 4.5 from each side.
- The distributive property gives $3x + 13.5 = 36$. Subtract 13.5 from each side leaving $3x = 22.5$. Divide each side by 3.

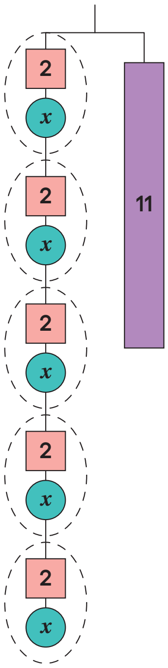
Lesson 8

Practice Problems

4 Problems

Problem 1

Here is a balanced hanger diagram:



a. Write an equation to represent the diagram.

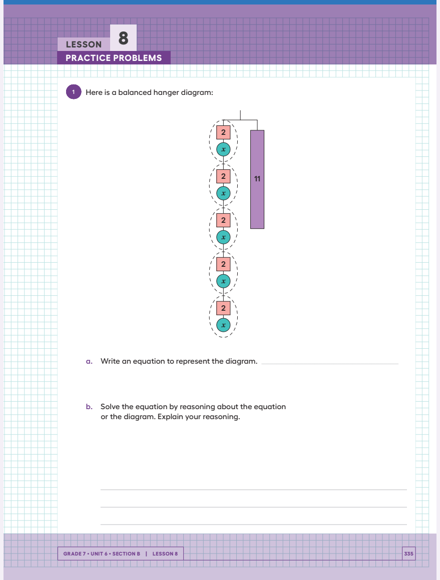
$5(x + 2) = 11$ (or equivalent)

b. Solve the equation by reasoning about the equation or the diagram. Explain your reasoning.

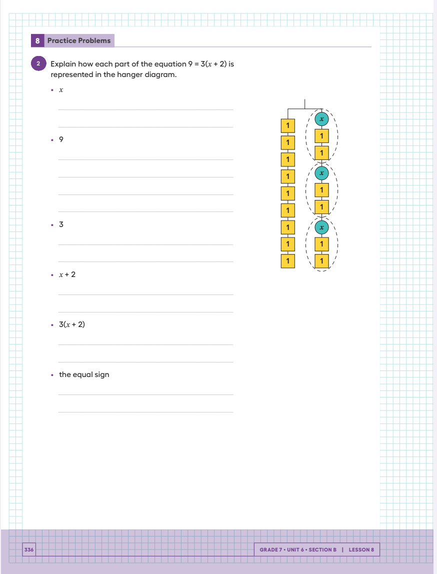
$x = 0.2$

Sample reasoning: Divide both sides of the diagram into 5 equal groups to get a circle labeled x and a rectangle labeled 2 in each group on the left, and a rectangle labeled 2.2 in each group on the right (or $x + 2 = 2.2$ with the equation). Then subtract 2 from each side to get a circle on the left and rectangle with 0.2 on the right (or $x = 0.2$ with the equation).

Student Workbook



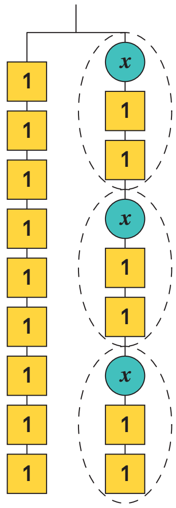
Student Workbook



Problem 2

Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger diagram.

- x
- 9
- 3
- $x + 2$
- $3(x + 2)$
- the equal sign



Sample response:

- The circle has an unknown weight, represented by the variable x.
- The left side has 9 squares, each weighing 1 unit.
- There are 3 identical groups on the right side.
- Each group on the right side is made up of one circle with weight x units and 2 squares of weight 1 unit each.
- The total weight of those 3 identical groups is the total weight of the right side.
- The equal sign is seen in the hanger being balanced.

Problem 3

from Unit 4, Lesson 11

Select the word from the following list that best describes each situation.

6 **A.** You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.

2 **B.** For every car sold, a car salesman is paid 6% of the car's price.

5 **C.** Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.

4 **D.** An antique furniture store pays $200 for a chair, adds 50% of that amount, and sells the chair for $300.

3 **E.** The normal price of a mattress is $600, but it is on sale for 10% off.

1 **F.** For any item you purchase in Texas, you pay an additional 6.25% of the item's price to the state government.

1. Tax
2. Commission
3. Discount
4. Markup
5. Tip or gratuity
6. Interest

Student Workbook


8 Practice Problems

3 from Unit 4, Lesson 11

Select the word from the following list that best describes each situation.

A You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.

B For every car sold, a car salesman is paid 6% of the car's price.

C Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.

D An antique furniture store pays $200 for a chair, adds 50% of that amount, and sells the chair for $300.

E The normal price of a mattress is $600, but it is on sale for 10% off.

F For any item you purchase in Texas, you pay an additional 6.25% of the item's price to the state government.

1 Tax
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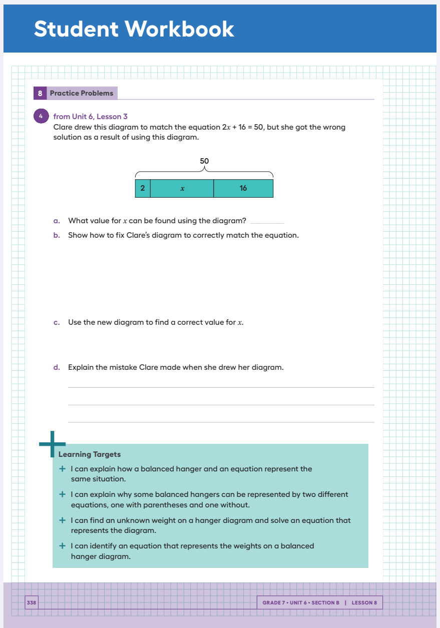

Student Workbook

8 Practice Problems

4 from Unit 6, Lesson 3

Clare drew this diagram to match the equation $2x + 16 = 50$, but she got the wrong solution as a result of using this diagram.

a. What value for x can be found using the diagram?

b. Show how to fix Clare's diagram to correctly match the equation.

c. Use the new diagram to find a correct value for x.

d. Explain the mistake Clare made when she drew her diagram.

Learning Targets

+ I can explain how a balanced hanger and an equation represent the same situation.
+ I can explain why some balanced hangers can be represented by two different equations, one with parentheses and one without.
+ I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
+ I can identify an equation that represents the weights on a balanced hanger diagram.

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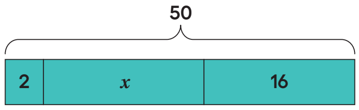
Problem 4

from Unit 6, Lesson 3

Clare drew this diagram to match the equation $2x + 16 = 50$, but she got the wrong solution as a result of using this diagram.



a. What value for x can be found using the diagram?

$x = 32$

Sample reasoning: Since the three parts 2, x, and 16 sum to 50 in the diagram, x can be found by subtracting 2 and 16 from 50.

b. Show how to fix Clare's diagram to correctly match the equation.

Sample response: Change the first part from 2 to x. Then the three parts of the diagram are x, x, and 16, for a total of $2x + 16$.

c. Use the new diagram to find a correct value for x.

$x = 17$

Sample reasoning: Since the corrected diagram shows that the number 50 is divided into parts of size x, x and 16, the two x's must together equal 16 less than 50, which is 34. This means that one x is 17.

d. Explain the mistake Clare made when she drew her diagram.

Sample response: Clare showed $2 + x$ instead of $2 \cdot x$. She might not understand that $2x$ means 2 multiplied by x, or she might not understand that the tape diagram shows parts adding up to a whole.