Lesson 13

Two Graphs for Each Relationship

Goals

- Coordinate (orally and in writing) tables, graphs, and equations that represent the same proportional relationship.
- Interpret two different graphs that represent the same proportional relationship, but have reversed which quantity is represented on each axis.
- Write an equation to represent a proportional relationship given only one pair of values or one point on the graph.

Learning Targets

- I can interpret a graph of a proportional relationship using the situation.
- I can write an equation representing a proportional relationship from a graph.

Lesson Narrative

In this lesson, students make connections between tables, equations, and graphs that represent the same proportional relationship. They start with an activity designed to help them see the different ways in which the structure of the graph and the equation are connected. For example, the coordinates for any point (a, b) on the graph can be divided to find the constant of proportionality $k = \frac{b}{a}$ in the equation. Also, the point $(1, k)$ on the graph gives the constant of proportionality, k.

This work prepares students for the next activity where they see two ways to graph a proportional relationship, depending on which quantity goes on which axis. It builds on previous similar work with tables and equations, giving students an opportunity to remember the fact that the constants of proportionality in the two ways are reciprocals.

One of the activities in this lesson works best when each student has access to devices that can run the applet, because students will benefit from seeing the relationship in a dynamic way.

Student Learning Goal

Let's use tables, equations, and graphs to answer questions about proportional relationships.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	5 min

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR7: Compare and Connect
- Which Three Go Together?

Required Materials

Materials to Gather

- Rulers: Activity 1

Materials to Copy

- Tables, Graphs, and Equations Handout (1 copy for every 3 students): Activity 1

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Warm-up
Which Three Go Together: Graphs

5 min

Activity Narrative

This *Warm-up* prompts students to compare four graphs. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

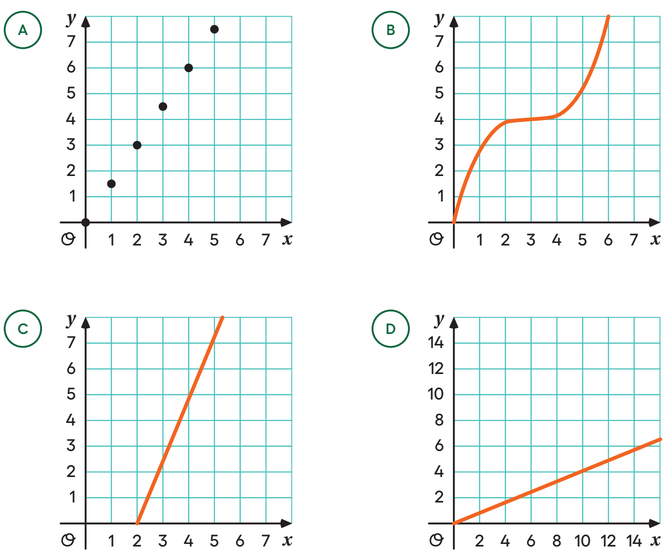
Launch

Arrange students in groups of 2–4. Display the graphs for all to see.

Give students 1 minute of quiet think time and ask them to indicate when they have noticed three graphs that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- The axes are counting by 1s.
- They are steep. They go up faster than they go over.

A, B, and D go together because:

- They pass through the origin (0, 0).

A, C, and D go together because:

- They are straight lines or lie on a straight line.
- B, C, and D go together because:
- They are solid (not dotted).

Instructional Routines

Which Three Go Together?

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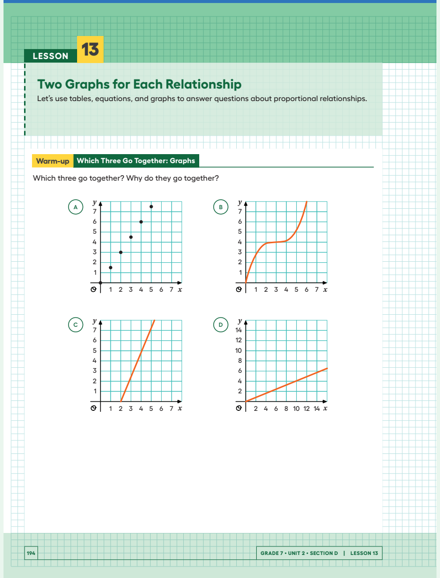
Student Workbook



Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Instructional Routines

MLR7: Compare and Connect

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Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "straight line," "solid line," "steep," "shallow," and "origin," and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

If time allows, invite 2–3 students to briefly share what they notice all of the figures have in common (for example, they are all graphs in the first quadrant, they all go up from left to right, the axes are labeled with numbers but not with quantities). The purpose of this concluding share out is to provide more opportunities for students to use terminology to describe aspects of graphs.

Activity 1

Tables, Graphs, and Equations

20 min

Activity Narrative

There is a digital version of this activity.

In this activity students identify correspondences between different representations of a proportional relationship, including a table, a graph, and an equation. Students are guided to notice:

- Any pair of positive values (a, b) determine a proportional relationship.
- Given a point (a, b) other than the origin on the graph of a line through the origin, the constant of proportionality is always $\frac{b}{a}$ (for all points other than the origin).
- In an equation $y = \frac{b}{a}x$ that represents the relationship, the constant of proportionality appears as the coefficient of x.
- The constant of proportionality is the y-coordinate when x is 1, that is, $\left(1, \frac{b}{a}\right)$ is a point on the graph.

As students recognize these correspondences between representations, they are making use of structure.

There are three different starting points given, and each student is assigned to work with one of them. After creating and analyzing their representations, students compare their work with others who had a different starting point. Monitor for students who notice these connections across representations while working with each of the three different given points (a, b).

In the digital version of the activity, students use an applet to manipulate the graph of a proportional relationship. The applet allows students to see how the equation and the values in the table change when the point is moved. This activity works best when each student has access to the applet, because students will benefit from seeing the relationship in a dynamic way. If students don't have individual access, projecting the applet would be helpful during the *Activity Synthesis*.

Launch

Arrange students in groups of 3. Assign each student in each group a letter: A, B, or C. Provide access to rulers.

Give students 5–7 minutes of quiet work time followed by small group discussion.

As students complete the third question, give them the table that goes with their assigned point, from the blackline master. They should use the table to check their y-values for the x-values 2 and 6 and then continue answering the questions.

Select work from students with different starting points to share later.

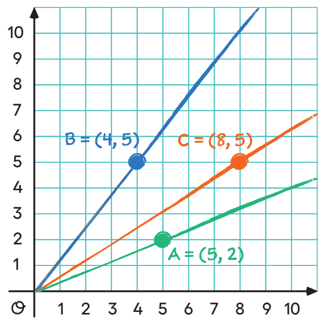
Student Task Statement

Answers depend on which point the student was assigned. Any equivalent answer is acceptable.

Your teacher will assign you *one* of these three points:

$$A = (5, 2),\ B = (4, 5),\ C = (8, 5).$$

1. On the coordinate plane, plot and label only your assigned point.



2. Graph the proportional relationship that is defined by your point. That is, use a ruler to draw a line that starts at the origin, goes through your point, and continues to the edge of the grid.

See graph

3. Use your graph to find the y-value that goes with each of these x-values.

x	y
2	
6	

Your teacher will give you a completed table. Use it to check your values.

for point A:

x	y
2	0.8
6	2.4

for point B:

x	y
2	2.5
6	7.5

for point C:

x	y
2	1.25
6	3.75

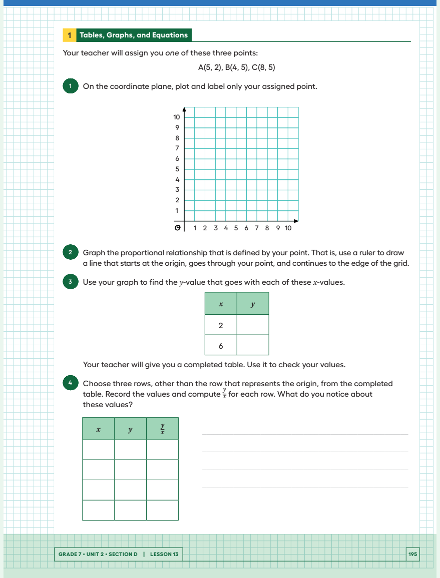
Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

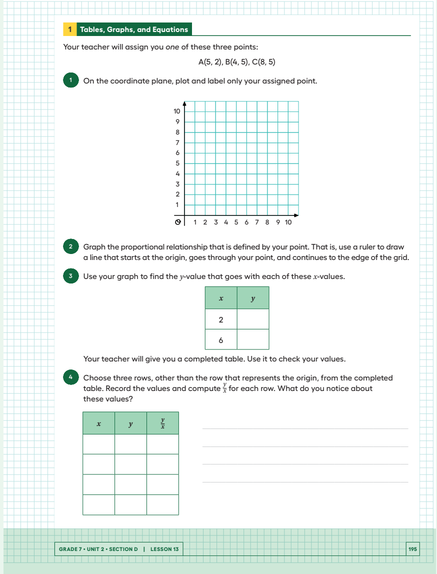
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization, Attention

Student Workbook



Student Workbook



1 Tables, Graphs, and Equations

Your teacher will assign you *one* of these three points:

A(5, 2), B(4, 5), C(8, 5)

1. On the coordinate plane, plot and label only your assigned point.

2. Graph the proportional relationship that is defined by your point. That is, use a ruler to draw a line that starts at the origin, goes through your point, and continues to the edge of the grid.
3. Use your graph to find the y-value that goes with each of these x-values.

x	y
2	
6	

Your teacher will give you a completed table. Use it to check your values.

4. Choose three rows, other than the row that represents the origin, from the completed table. Record the values and compute $\frac{y}{x}$ for each row. What do you notice about these values?

x	y	$\frac{y}{x}$

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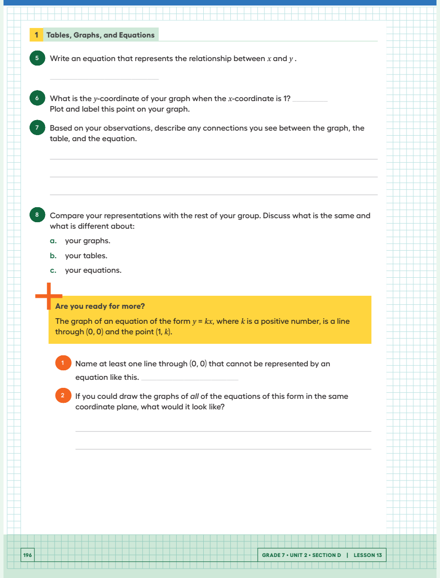
Student Workbook



1 Tables, Graphs, and Equations

5. Write an equation that represents the relationship between x and y.
6. What is the y-coordinate of your graph when the x-coordinate is 1? ______ Plot and label this point on your graph.
7. Based on your observations, describe any connections you see between the graph, the table, and the equation.
8. Compare your representations with the rest of your group. Discuss what is the same and what is different about:
   a. your graphs.
   b. your tables.
   c. your equations.

Are you ready for more?

The graph of an equation of the form $y = kx$, where k is a positive number, is a line through (0, 0) and the point $(1, k)$.

1. Name at least one line through (0, 0) that cannot be represented by an equation like this. ______
2. If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

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4. Choose three rows, other than the row that represents the origin, from the completed table. Record the values and compute $\frac{y}{x}$ for each row. What do you notice about these values?

x	y	$\frac{y}{x}$

- for point A: for all rows, $\frac{y}{x} = 0.4$
- for point B: for all rows, $\frac{y}{x} = 1.25$
- for point C: for all rows, $\frac{y}{x} = 0.625$

5. Write an equation that represents the relationship between x and y.

for point A:

$y = \frac{2}{5}x$

for point B:

$y = \frac{5}{4}x$

for point C:

$y = \frac{5}{8}x$

6. What is the y-coordinate of your graph when the x-coordinate is 1? Plot and label this point on your graph.

for point A:

$\left(1, \frac{2}{5}\right)$

for point B:

$\left(1, \frac{5}{4}\right)$

for point C:

$\left(1, \frac{5}{8}\right)$

7. Based on your observations, describe any connections you see between the graph, the table, and the equation.

Sample response: The point $(1, k)$ appears on both the graph and in the table, and the equation is $y = kx$.

8. Compare your representations with the rest of your group. Discuss what is the same and what is different about:

a. your graphs.

b. your tables.

c. your equations.

Sample responses:

- All of the graphs are lines through the origin, but they have different steepnesses.
- When the y-coordinates are written as fractions, there are consistencies among them, such as all having the same denominator.
- In each table, all of the $\frac{y}{x}$ values are equal.
- The equations all include a y and an x. They all include a different number, but the number corresponds to a value in its table.

Are You Ready for More?

The graph of an equation of the form $y = kx$, where k is a positive number, is a line through (0, 0) and the point (1, k).

1. Name at least one line through (0, 0) that cannot be represented by an equation like this.

The x- and y-axes are both examples. Any line through (0, 0) and (1, k) where k is negative is also an example.

2. If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

It would look like you completely shaded in the first and third quadrants of the coordinate plane.

Activity Synthesis

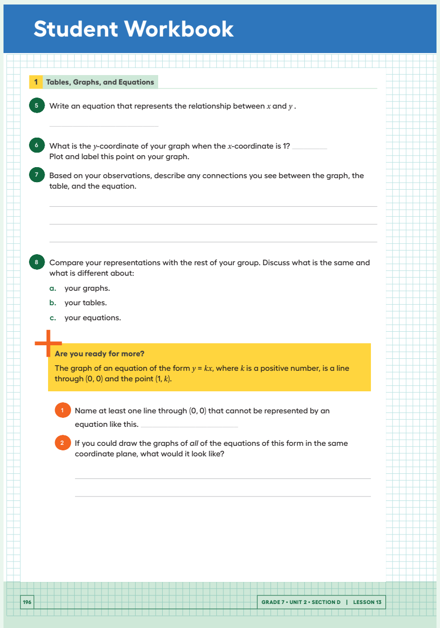
The goal of this discussion is to highlight connections between different representations of a proportional relationship. Display an example of the table, equation, and graph for each of the three starting points for all to see.

Use *Compare and Connect* to help students compare, contrast, and connect the different representations. Here are some questions for discussion:

"How do these different representations show the same information?"

"Are there any benefits or drawbacks to one representation compared to another?"

"How does the constant of proportionality show up in each representation?"


Student Workbook

1 Tables, Graphs, and Equations

5 Write an equation that represents the relationship between x and y .

6 What is the y-coordinate of your graph when the x-coordinate is 1? ______
Plot and label this point on your graph.

7 Based on your observations, describe any connections you see between the graph, the table, and the equation.

8 Compare your representations with the rest of your group. Discuss what is the same and what is different about:

a. your graphs.

b. your tables.

c. your equations.

Are you ready for more?

The graph of an equation of the form $y = kx$, where k is a positive number, is a line through (0, 0) and the point (1, k).

1 Name at least one line through (0, 0) that cannot be represented by an equation like this. ______

2 If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

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Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, use a different color for each person to highlight the connection between the graph, equation, and constant of proportionality.

Supports accessibility for: Visual-Spatial Processing

Ensure that all the important connections are highlighted:

- A graph of a line through the origin and passing through the first quadrant represents a proportional relationship.
- The value of $\frac{b}{a}$ computed from any point (a, b) on that line (other than the origin) is the constant of proportionality.
- An equation of the relationship is given by $y = kx$ where k is $\frac{b}{a}$ for any point (a, b) on the graph other than the origin.

Activity 2

Balloon Animal Contest

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to help students see that a proportional relationship between two quantities can be represented by two different graphs, depending on which quantity is graphed on which axis. This builds on previous activities where students saw that a proportional relationship is associated with two different, reciprocal rates. In this situation, the two rates are the number of balloon animals per minute and the number of minutes per balloon animal. Students attend to precision as they specify the meaning of each constant of proportionality.

In the digital version of the activity, students use an applet to graph and compare proportional relationships on two sets of axes. The applet allows students to add, remove, adjust, and label points and lines. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Launch

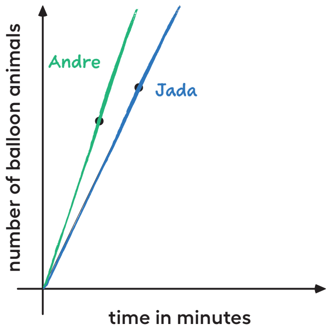
Keep students in the same groups. Consider displaying an image of balloon animals or a video of someone making a balloon animal to familiarize students with the context.

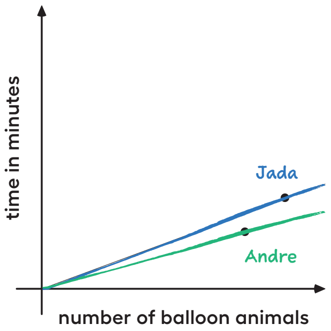
Student Task Statement

Andre and Jada had a contest making balloon animals.

- Andre made 10 balloon animals in 3 minutes.
- Jada made 12 balloon animals in 5 minutes.

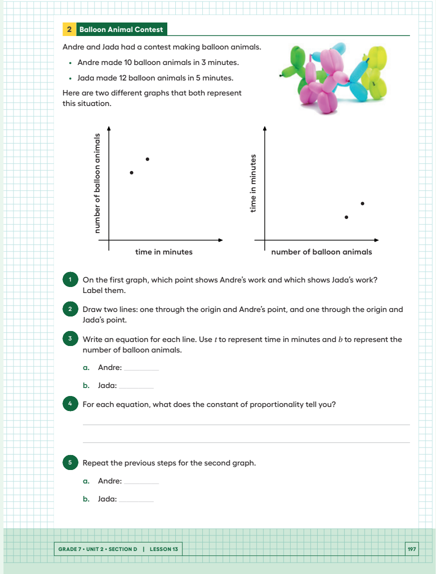
Here are two different graphs that both represent this situation.





1. On the first graph, which point shows Andre's work and which shows Jada's work? Label them.
2. Draw two lines: one through the origin and Andre's point, and one through the origin and Jada's point.
3. Write an equation for each line. Use t to represent time in minutes and b to represent the number of balloon animals.
   a. Andre: $b = \frac{10}{3}t$
   b. Jada: $b = \frac{12}{5}t$ or $b = 2.4t$
4. For each equation, what does the constant of proportionality tell you?
   Andre made $\frac{10}{3}$ (or $3\frac{1}{3}$ or approximately 3.33) balloon animals per minute.
   Jada made $\frac{12}{5}$ (or $2\frac{2}{5}$ or 2.4) balloon animals per minute.
5. Repeat the previous steps for the second graph.
   a. Andre: $t = \frac{3}{10}b$ or $t = 0.3$
   Andre takes $\frac{3}{10}$, or 0.3, minute per balloon animal.
   (Possibly: Andre takes 18 seconds per balloon animal.)
   b. Jada: $t = \frac{5}{12}b$
   Jada takes $\frac{5}{12}$, or approximately 0.42, minute per balloon animal.
   (Possibly: Jada takes 25 seconds per balloon animal.)

Student Workbook



2 Balloon Animal Contest

Andre and Jada had a contest making balloon animals.

- Andre made 10 balloon animals in 3 minutes.
- Jada made 12 balloon animals in 5 minutes.

Here are two different graphs that both represent this situation.

number of balloon animals / time in minutes

time in minutes / number of balloon animals

1. On the first graph, which point shows Andre's work and which shows Jada's work? Label them.
2. Draw two lines: one through the origin and Andre's point, and one through the origin and Jada's point.
3. Write an equation for each line. Use t to represent time in minutes and b to represent the number of balloon animals.
   a. Andre: ______
   b. Jada: ______
4. For each equation, what does the constant of proportionality tell you?
5. Repeat the previous steps for the second graph.
   a. Andre: ______
   b. Jada: ______

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Activity Synthesis

First, invite a few students to share their reasoning for how they determined which points represented Andre's work and which represented Jada's. Ensure that students are in agreement before continuing.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the question about Andre's rate by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"Andre was making balloon animals at a rate of $\frac{3}{10}$."

- Ask, "What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

The key takeaways are:

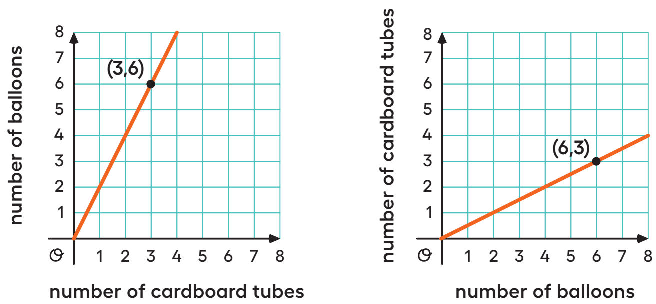
- There are multiple ways to express the constant of proportionality between two quantities. The important thing is that we communicate the meaning of the number clearly.
- $\frac{10}{3}$, $3\frac{1}{3}$, $\frac{3}{10}$, and 0.3 are all possible correct answers:
    - $\frac{10}{3}$ or $3\frac{1}{3}$ represents the number of balloon animals per minute that Andre made.
    - $\frac{3}{10}$ or 0.3 represents the number of minutes per balloon animal that Andre made.
- $\frac{3}{10}$ and $\frac{10}{3}$ are reciprocals of each other.

Lesson Synthesis

Share with students,

"Today we looked at different representations of the same proportional relationship. We found connections between the table, graph, and equation. We also looked at a pair of graphs that showed the same proportional relationship but swapped which quantity was on each axis."

If desired, use this example to review these concepts.



"A person uses balloons and cardboard tubes to make toy barbells."

"What is the constant of proportionality for the relationship shown on the first graph? How do you know?"

2

The y-value is 2 when the x-value is 1. You can also divide 6 by 3 to get 2.

"What equation could represent this relationship?"

$b = 2c$ (or equivalent)

"What is the constant of proportionality for the relationship shown on the second graph? How do you know?"

$\frac{1}{2}$

You can see that the y-value is $\frac{1}{2}$ when the x-value is 1.
You can also divide 3 by 6 to get 0.5.

"What equation could represent this relationship?"

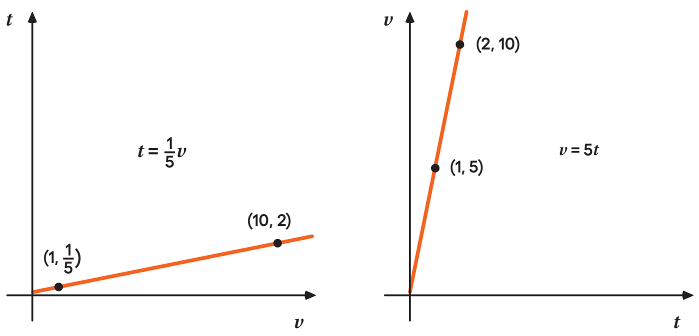
$c = \frac{1}{2}b$ (or equivalent)

Lesson Summary

Imagine that a faucet is leaking at a constant rate and that every 2 minutes, 10 milliliters of water leaks from the faucet. There is a proportional relationship between the volume of water and elapsed time.

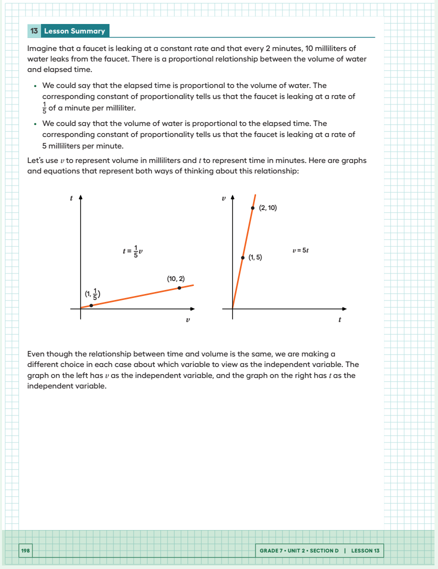
- We could say that the elapsed time is proportional to the volume of water. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of $\frac{1}{5}$ of a minute per milliliter.
- We could say that the volume of water is proportional to the elapsed time. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of 5 milliliters per minute.

Let's use v to represent volume in milliliters and t to represent time in minutes. Here are graphs and equations that represent both ways of thinking about this relationship:



Even though the relationship between time and volume is the same, we are making a different choice in each case about which variable to view as the independent variable. The graph on the left has v as the independent variable, and the graph on the right has t as the independent variable.

Student Workbook


13 Lesson Summary

Imagine that a faucet is leaking at a constant rate and that every 2 minutes, 10 milliliters of water leaks from the faucet. There is a proportional relationship between the volume of water and elapsed time.

- We could say that the elapsed time is proportional to the volume of water. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of $\frac{1}{5}$ of a minute per milliliter.
- We could say that the volume of water is proportional to the elapsed time. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of 5 milliliters per minute.

Let's use v to represent volume in milliliters and t to represent time in minutes. Here are graphs and equations that represent both ways of thinking about this relationship:

Even though the relationship between time and volume is the same, we are making a different choice in each case about which variable to view as the independent variable. The graph on the left has v as the independent variable, and the graph on the right has t as the independent variable.

Responding To Student Thinking

Press Pause

By this point in the unit, there should be some student mastery of creating graphs of a proportional relationship. If students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 2, Section D Representing Proportional Relationships with Graphs

Cool-down

Stickers for Sale

5 min

Student Task Statement

Elena went to a store where you can buy individual stickers. All the large stickers cost the same price. Elena bought 10 large stickers for \$2.50.

1. How much do large stickers cost per sticker?

   \$0.25, because $2.50 \div 10 = 0.25$

2. How many large stickers can you buy per dollar?

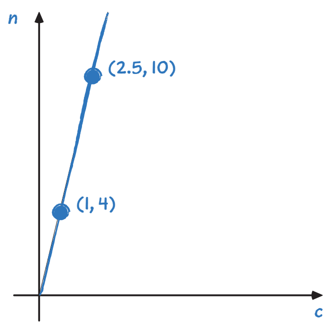
   4 stickers, because $10 \div 2.5 = 4$

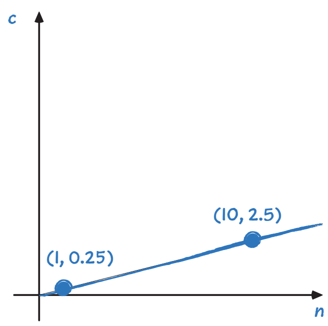
3. Write two different equations that represent this situation. Use n for number of stickers and c for cost in dollars.

   $n = 4c$ and $c = 0.25n$ (or equivalent)

4. Choose one of your equations, and sketch its graph. Be sure to label the axes.

   Students are only asked to create one of these graphs. It is not necessary that they plot and label any points, but it could be a helpful step in creating a reasonably accurate graph.





Lesson 13

Practice Problems

4 Problems

Problem 1

At the supermarket you can fill your own honey bear container. A customer buys 12 ounces of honey for $5.40.

a. How much does honey cost per ounce?

$0.45 per ounce

b. How much honey can you buy per dollar?

About 2.2 ounces

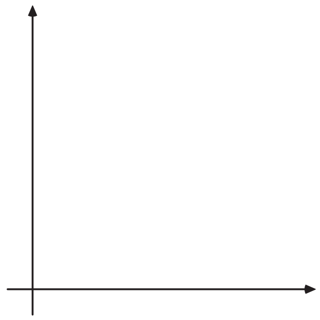
c. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.

- $c = 0.45h$
- $h = 2.2c$

d. Choose one of your equations, and sketch its graph. Be sure to label the axes.

Students should have one of two linear graphs going through the origin.

- **Graph 1: $c = 0.45h$, horizontal axis label: "h, honey (ounces)," vertical axis label: "c, cost (dollars)."**
- **Graph 2: $h = 2.2c$, horizontal axis label: "c, cost (dollars)," vertical axis label: "h, honey (ounces)."**



Problem 2

The point $\left(3, \frac{6}{5}\right)$ lies on the graph representing a proportional relationship. Which of the following points also lie on the same graph? Select **all** that apply.

A. (1, 0.4)

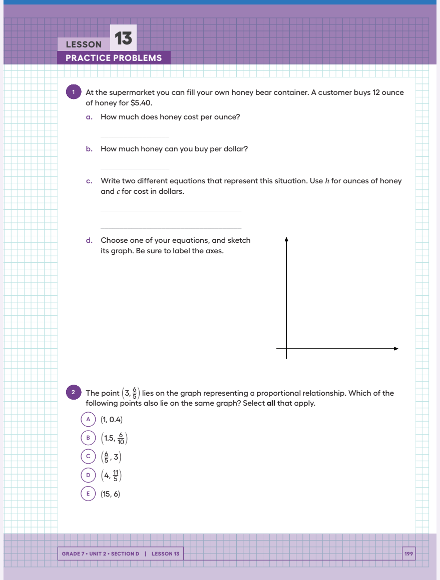
B. $\left(1.5, \frac{6}{10}\right)$

C. $\left(\frac{6}{5}, 3\right)$

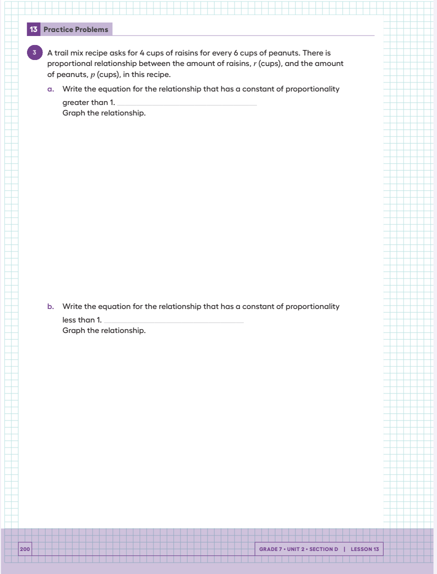
D. $\left(4, \frac{11}{5}\right)$

E. (15, 6)

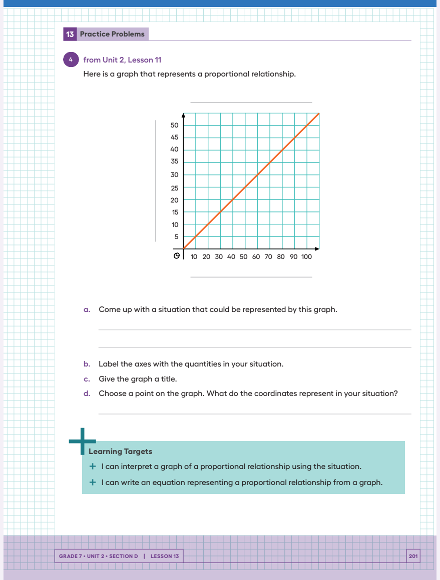
Student Workbook



Student Workbook



Student Workbook



Problem 3

A trail mix recipe asks for 4 cups of raisins for every 6 cups of peanuts. There is proportional relationship between the amount of raisins, r (cups), and the amount of peanuts, p (cups), in this recipe.

a. Write the equation for the relationship that has a constant of proportionality greater than 1.

$p = \frac{6}{4} r$

Graph the relationship. Students should graph $p = \frac{6}{4} r$, label the horizontal axis with "r" or "raisins (cups)" and the vertical axis with p or "peanuts (cups)". Since this is a proportional relationship, the graph should be linear and go through the origin.

b. Write the equation for the relationship that has a constant of proportionality less than 1.

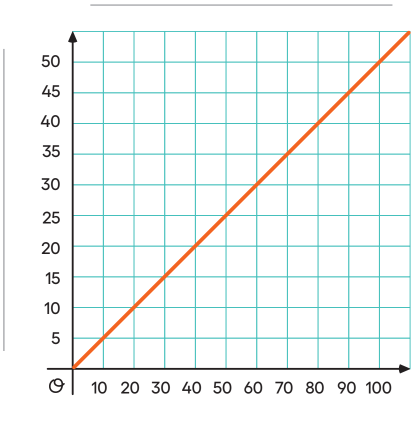
$r = \frac{4}{6} p$

Graph the relationship. Students should graph $r = \frac{4}{6} p$, label the horizontal axis with "p" or "peanuts (cups)" and the vertical axis with "r" or "raisins (cups)." Since this is a proportional relationship, the graph should be linear and go through the origin. The slope of this graph should be less steep than the previous graph.

Problem 4

from Unit 2, Lesson 11

Here is a graph that represents a proportional relationship.



a. Come up with a situation that could be represented by this graph.

Sample response: For every 2 gallons of gray paint created, 1 gallon of black paint is used.

b. Label the axes with the quantities in your situation.

Sample response:
- Horizontal axis: gray paint (gallons).
- Vertical axis: black paint (gallons).

c. Give the graph a title.
Sample response: Title: Amount of Black Paint Needed to Create Gray Paint

d. Choose a point on the graph. What do the coordinates represent in your situation?
Sample response: The point (60, 30) means, in order to make 60 gallons of gray paint, 30 gallons of black paint are needed.