Lesson 1

Relationships of Angles

Goals

- Comprehend that the word “degrees” (in spoken and written language) and the symbol ° (in written language) refer to the amount of turn between two different directions.
- Determine the angle measures of pattern blocks, and explain (orally) the reasoning.
- Recognize 180° and 360° angles, and identify when adjacent angles add up to these amounts.

Learning Targets

- I can find unknown angle measurements by reasoning about adjacent angles with known measures.
- I can recognize when an angle measures 90°, 180°, or 360°.

Lesson Narrative

In this lesson, students gain hands-on experience composing, decomposing, and measuring angles. Students connect vocabulary such as **right angles** and **straight angles** with their degree measures, as well as the description of going all the way around a point as 360 degrees. Students fit pattern blocks around a point using these relationships to determine the unknown angles on the remaining pattern blocks. Students use the structure of the blocks completely going around a point with no gaps to solve for the angles.

Student Learning Goal

Let’s examine some special angles.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Blank paper: Warm-up, Activity 1
- Scissors: Warm-up, Activity 1
- Pattern blocks: Activity 1, Activity 2
- Straightedges: Activity 1
- Protractors: Activity 3

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson:
Prepare one set of pattern blocks for each group of 3–4 students, include blocks consisting of at least 3 yellow hexagons and 6 of each of the other shapes.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	10 min	10 min	10 min	5 min

Warm-up
Visualizing Angles

5 min

Activity Narrative

The purpose of this *Warm-up* is to bring back to mind what students have learned previously about angle measures, as well as to discuss what aspects of each figure tell us about an angle.

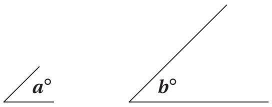
Listen for the language that students use as they compare the sizes of the angles in the first question and as they name the angles in the second question. Select students to share different ways of naming the same angle.

Launch

Give students 1 minute of quiet work time, followed by a whole-class discussion.

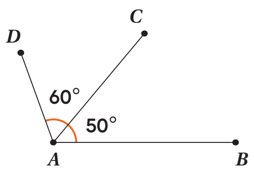
Student Task Statement

1. Which angle is bigger?



Sample response: Neither. Both angles have the same measure.

2. Identify an obtuse angle in the diagram.



Angle DAB (or angle BAD) is obtuse. It measures 110° because $60 + 50 = 110$.

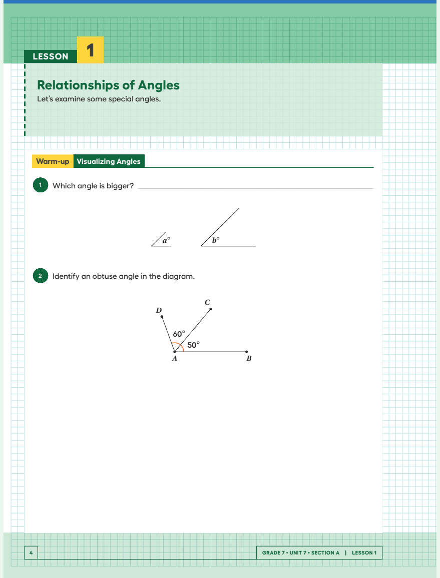
Activity Synthesis

The goal of this discussion is to ensure that students understand that angles measure the amount of turn between two different directions.

Ask students to share how they decided whether Angle a and Angle b are the same or different sizes. If students do not agree that the angles are the same size, display this applet for all to see:

The Geogebra applet 'Visualizing Angles' is available here: **ilclass.com/l/395006**

Student Workbook



Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display. This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Demonstrate dragging one angle onto the other angle.

Display the applet or figure in the second question, and ask previously identified students to share their responses. Make sure students understand that saying angle A is not specific enough when referring to this diagram, because there is more than one angle with its vertex at point A.

Explain to the students that by using three points to refer to an angle, with the middle point being the vertex of the angle, we can be sure that others will understand which angle we are talking about. Have students practice this way of referring to angles by asking questions such as:

- *"Which angle is bigger, angle DAC or angle CAB?"*

  Angle DAC is bigger because its measure is 60 degrees. It doesn't matter that segment BA is longer than segment DA.

- *"Which angle is bigger, angle CAB or angle BAC?"*

  They are both the same size, because they are two names for the same angle.

Also explain to students that in a diagram an arc is often placed between the two sides of the angle being referred to.

Tell students that angles DAC and CAB are known as **adjacent angles** because they are next to each other, sharing segment AC as one of their sides and A as their vertex.

Activity 1

Pattern Block Angles

15 min

Activity Narrative

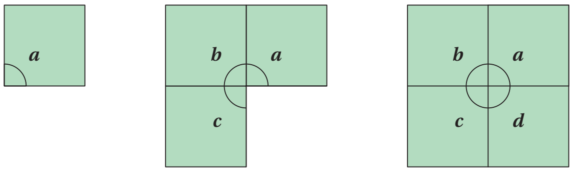
There is a digital version of this activity.

The purpose of this activity is to use the fact that the sum of the angles all the way around a point is 360 degrees to reason about the measure of other angles. In this activity, students use the structure of pattern blocks to explore configurations that make 360 degrees and to solve for angles of the individual blocks. For this activity, there are multiple configurations of blocks that will accomplish the task.

This activity works best when each student has access to the pattern blocks. If pattern blocks are not available, consider using the digital version of the activity. In the digital version, students use an applet to dynamically manipulate shapes to determine angle measures.

Launch

Arrange students in groups of 3–4. Display the figures in this image one at a time, or use actual pattern blocks to recreate these figures for all to see.



Ask these questions after each figure is displayed:

- *"What is the measure of Angle a? How do you know?"*

  90°, because it is a right angle.

- *"What is the total measure of $a + b + c$?"*

  270°, because $90 + 90 + 90 = 270$.

- *"What is the total measure of $a + b + c + d$?"*

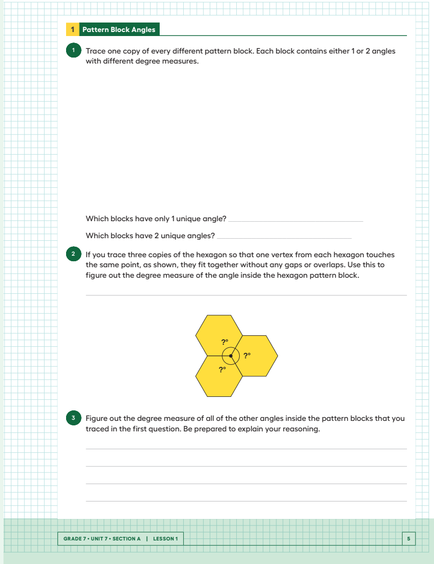
  360°, because $4 \cdot 90 = 360$.

Reinforce that 360° is once completely around a point by having students stand up, hold their arm out in front of them, and turna 360° around. Students who are familiar with activities like skateboarding or figure skating will already have a notion of 360° as a full rotation and 180° as half of a rotation.

Distribute pattern blocks.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe angle measures. Display words and phrases such as "the total is 360°," "the angles are all the same," "different," and "unique."

Student Workbook

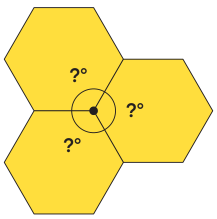


Student Task Statement

1. Trace one copy of every different pattern block. Each block contains either 1 or 2 angles with different degree measures. Which blocks have only 1 unique angle? Which blocks have 2 unique angles?

   The hexagon, triangle, and square are all blocks with one unique angle measure. The trapezoid and both rhombuses are blocks with two different angle measures.

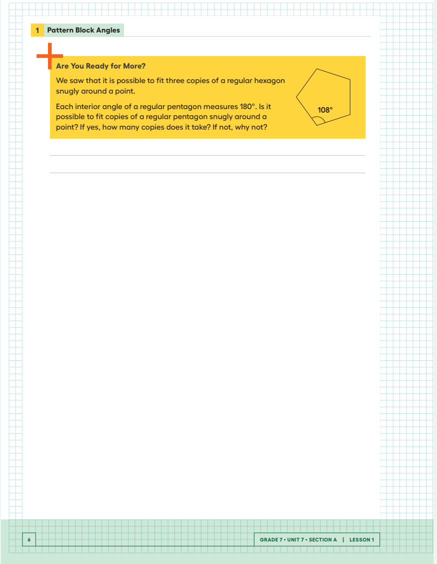
2. If you trace three copies of the hexagon so that one vertex from each hexagon touches the same point, as shown, they fit together without any gaps or overlaps. Use this to figure out the degree measure of the angle inside the hexagon pattern block.



   120°

   Sample reasoning: It takes 3 blocks to go around a point, and $360 \div 3 = 120$.

Student Workbook



3. Figure out the degree measure of all of the other angles inside the pattern blocks that you traced in the first question. Be prepared to explain yourreasoning.

   a. Green triangle: All 3 angles measure 60°. Sample reasoning: It takes 6 to go around a point, and $360 \div 6 = 60$.

   b. Tan rhombus: 2 angles measure 30°, and 2 angles measure 150°. Sample reasoning: It takes 2 small angles to equal the measure of a triangle and $60 \div 2 = 30$. The other angles measure 150°, since 5 of the smaller angles can fit together to equal this angle measure and $30 \cdot 5 = 150$.

   c. Blue rhombus: 2 angles measure 60°, and 2 angles measure 120°. Sample reasoning: The smaller angles are the same angle as in the triangles, and the larger angles are the same angle as in the hexagons.

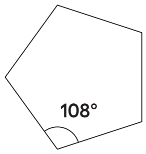
   d. Red trapezoid: 2 angles measure 60°, and 2 angles measure 120°. Sample reasoning: The smaller angles are the same angle as in the triangles, and the larger angles are the same angle as in the hexagon.

   e. Orange square: All 4 angles measure 90°. Sample reasoning: It takes 4 angles to go around a point, and $360 \div 4 = 90$.

Are You Ready for More?

We saw that it is possible to fit three copies of a regular hexagon snugly around a point.

Each interior angle of a regular pentagon measures 108°. Is it possible to fit copies of a regular pentagon snugly around a point? If yes, how many copies does it take? If not, why not?



No

Three copies give 324°, because $3 \cdot 108 = 324$. This is not enough—there would be a gap left over—because 360° are needed to get all the way around. Four copies give 432°, because $4 \cdot 108 = 432$. This is too much! The fourth copy would overlap the first, rather than fit snugly.

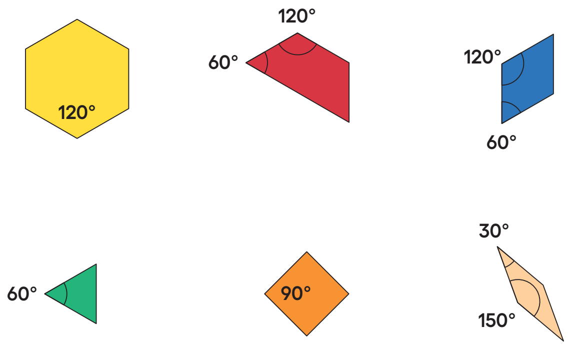
Activity Synthesis

Direct students' attention to the reference created using Collect and Display. Ask students to share their strategies for figuring out each angle measure for the pattern blocks. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. (For example, the display may have "The angles in the triangles are all the same" already on it and be updated with the more precise phrase "The triangles have 1 unique angle.")

As students describe the angle measures for each shape, write an equation to represent how their angles add up to 360 degrees. Listen carefully for how students describe their reasoning, and make your equation match the vocabulary that they use. For example, students might have reasoned about 6 green triangles by thinking $60 + 60 + 60 + 60 + 60 + 60 = 360$, or $6 \cdot 60 = 360$, or $360 \div 6 = 60$.

When an angle from one block is known, it can be used to help figure out angles for other blocks. For example, students may say that they knew the angles on the yellow hexagon measured 120° because they could fit two of the green triangles onto one corner of the hexagon, and 60 + 60 = 120, or $2 \cdot 60 = 120$. There are many different ways students could have reasoned about the angles on each block, and it is okay if they didn't think back to 360° for every angle.

Before moving on to the next activity, ensure that students know the measure of each interior angle of each shape in the set of pattern blocks. Display these measures for all to see throughout the remainder of the lesson.



Activity 2

More Pattern Block Angles

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use the pattern block angles to determine the measure of other given angles. Students also recognize that a straight angle can be considered an angle and not just a line. Students are asked to find different combinations of pattern blocks that form a straight angle to connect the algebraic representation of summing angles and the geometric representation of joining angles with the same vertex.

If students find only one combination of pattern blocks that form each angle, encourage them to look for more combinations.

As students work on the task, monitor for students who use different combinations of blocks to form a straight angle.

This activity works best when each student has access to the pattern blocks. If pattern blocks are not available, consider using the digital version of the activity. In the digital version, students use an applet to dynamically manipulate shapes to determine angle measures.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Begin with a physical demonstration of using pattern blocks to determine the measure of an angle to support connections between new situations and prior understandings. Consider using the prompts:

> *"What do we already know about the angles in this shape?"*

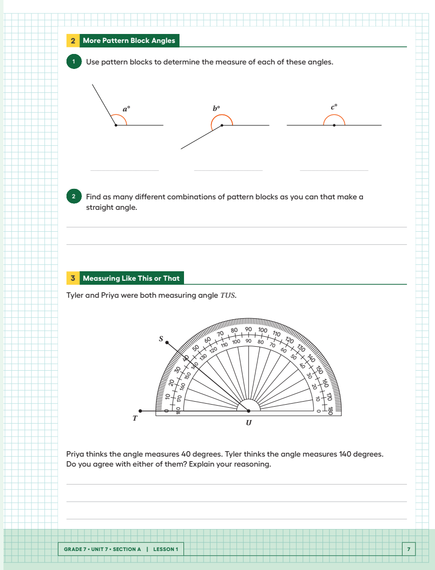
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Building on Student Thinking

If students struggle to identify Angle b, consider asking these questions to clarify their thinking:

> *"How does the little arc help us know what part of the angle we are interested in?"*
> *"How does Angle b compare to a straight angle? How does that help you figure out its measure?"*

Student Workbook


2 More Pattern Block Angles

1 Use pattern blocks to determine the measure of each of these angles.

2 Find as many different combinations of pattern blocks as you can that make a straight angle.

3 Measuring Like This or That

Tyler and Priya were both measuring angle TUS.

Priya thinks the angle measures 40 degrees. Tyler thinks the angle measures 140 degrees. Do you agree with either of them? Explain your reasoning.

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Launch

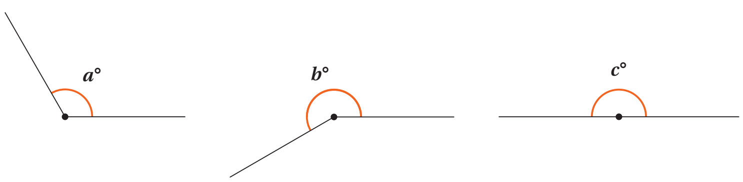
Arrange students in groups of 3–4. A device that can run the applet is needed for every 3–4 students.

Tell students:

"If an angle has a measure of 180°, then its sides form a straight line. An angle that forms a straight line is called a straight angle."

Student Task Statement

1. Use pattern blocks to determine the measure of each of these angles.



a. 120°

Sample response: It is the same size as one vertex of the yellow hexagon or two green triangles put together.

b. 210°

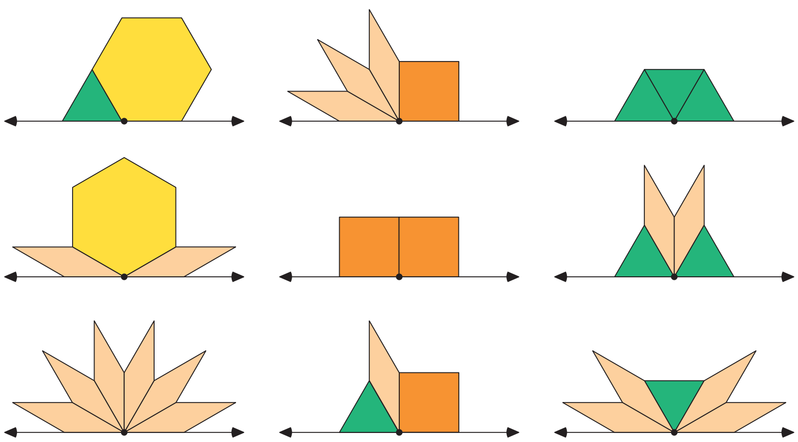
Sample response: It is the same size as one yellow hexagon and one orange square put together.

c. 180°

Sample response: It is the same size as three green triangles put together.

2. Find as many different combinations of pattern blocks as you can that make a straight angle.

Sample responses:



Activity Synthesis

The goal of this discussion is for students to be exposed to many different examples of angle measures that add up to 180 degrees.

First, instruct students to compare, with a partner, their answers to the first question and to share their reasoning until they reach an agreement. To help students see Angle c as a 180-degree angle and not just as a straight line, consider using only the smaller angle on the tan rhombus blocks to measure all three figures: Composing four tan rhombuses gives an angle measuring a degrees, seven rhombuses give an angle measuring b degrees, and six rhombuses give an angle measuring c degrees.

Next, select previously identified students to share their solutions to the second question. For each combination of blocks that is shared, invite other students in the class to write an equation, displayed for all to see, that reflects the reasoning.

Activity 3: Optional

Measuring Like This or That

10 min

Activity Narrative

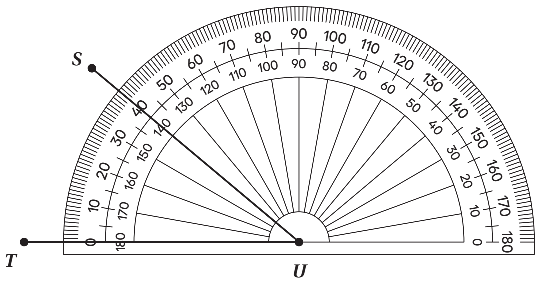
The purpose of this optional activity is to address the common error of reading a protractor from the wrong end. The problem gives students the opportunity to critique someone else's thinking and make an argument if they agree with either students' claim.

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time, followed by a partner and whole-class discussion.

Student Task Statement

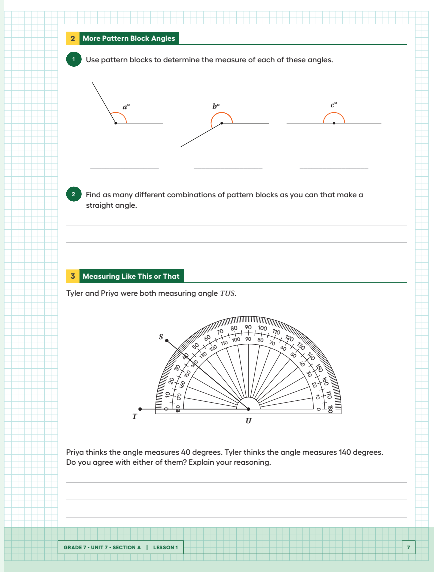
Tyler and Priya were both measuring angle TUS.



Priya thinks the angle measures 40 degrees. Tyler thinks the angle measures 140 degrees. Do you agree with either of them? Explain your reasoning.

Sample response: I agree with Priya, since the angle clearly measures less than 90 degrees. I think Tyler measured from the wrong end of the protractor.

Student Workbook


2 More Pattern Block Angles

1 Use pattern blocks to determine the measure of each of these angles.

2 Find as many different combinations of pattern blocks as you can that make a straight angle.

3 Measuring Like This or That

Tyler and Priya were both measuring angle TUS.

Priya thinks the angle measures 40 degrees. Tyler thinks the angle measures 140 degrees. Do you agree with either of them? Explain your reasoning.

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Activity Synthesis

Ask students to indicate whether they agree with Priya or Tyler. Invite students to explain their reasoning until the class comes to an agreement that the measurement of angle TUS is 40 degrees.

Ask students how Tyler could know that his answer of 140 degrees is unreasonable for the measure of angle TUS. Possible discussion points include:

“Is angle TUS acute, right, or obtuse?”

acute

“Where is there an angle that measures 140 degrees in this figure?”

adjacent to angle TUS, from side US to the other side of the protractor

Make sure that students understand that a protractor is often labeled with two sets of angle measures, and they need to consider which side of the protractor they are measuring from.

Lesson Synthesis

The purpose of this discussion is for students to articulate what they know about various types of angles. Consider adding to or updating the display created using the *Collect and Display* math language routine as students share their thinking.

Here are some questions to elicit student descriptions:

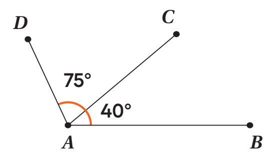
“What do you know about a right angle and a straight angle?”

A right angle is 90 degrees, and it is one of the corners of a square. A straight angle makes a straight line, and it is 180 degrees. Putting 2 right angles together makes a straight angle, and there are lots of other ways to make a straight angle.

“What does it look like when angles are adjacent?”

The two angles share a vertex and one side.

Display this image and ask students to name which angles are adjacent and what that tells them about angle BAD.

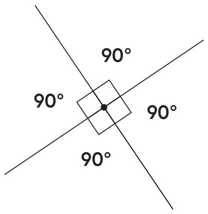


Angle BAC is adjacent to angle CAD, which means that angle BAD must be 115 degrees.

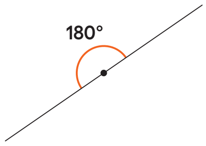
If time allows, invite students to name each of the angles, and use words from the display to describe their properties.

Lesson Summary

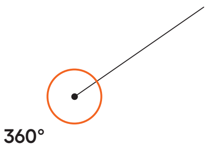
When two lines intersect and form four equal angles, we call each one a **right angle.** A right angle measures 90°. You can think of a right angle as a quarter turn in one direction or the other.



An angle in which the two sides form a straight line is called a **straight angle.** A straight angle measures 180°. A straight angle can be made by putting right angles together. You can think of a straight angle as a half turn, so that you are facing in the opposite direction after you are done.

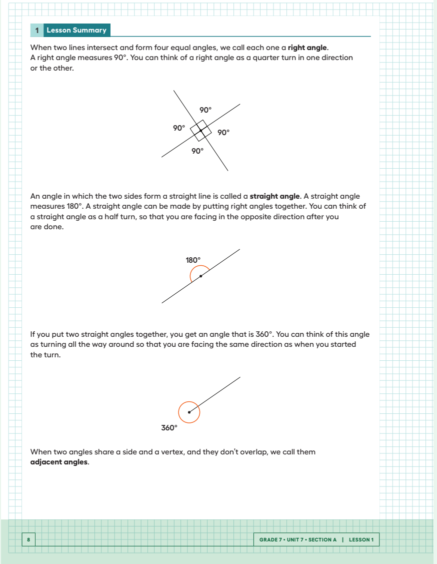


If you put two straight angles together, you get an angle that is 360°. You can think of this angle as turning all the way around so that you are facing the same direction as when you started the turn.



When two angles share a side and a vertex, and they don't overlap, we call them **adjacent angles.**

Student Workbook



1 Lesson Summary

When two lines intersect and form four equal angles, we call each one a **right angle**. A right angle measures 90°. You can think of a right angle as a quarter turn in one direction or the other.

90° 90° 90° 90°

An angle in which the two sides form a straight line is called a **straight angle**. A straight angle measures 180°. A straight angle can be made by putting right angles together. You can think of a straight angle as a half turn, so that you are facing in the opposite direction after you are done.

180°

If you put two straight angles together, you get an angle that is 360°. You can think of this angle as turning all the way around so that you are facing the same direction as when you started the turn.

360°

When two angles share a side and a vertex, and they don't overlap, we call them **adjacent angles**.

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Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

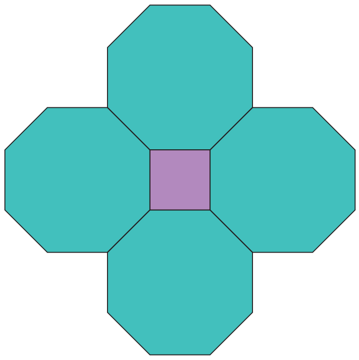
Cool-down

Identical Isosceles Triangles

5 min

Student Task Statement

This pattern is composed of a square and some regular octagons.



In this pattern, all of the angles inside the octagons have the same measure. The shape in the center is a square. Find the measure of one of the angles inside one of the octagons.

135°

Sample reasoning: The angles in the square are 90°. Since the angles around a point add up to 360°, then 2 octagon angles must be 360 – 90, or 270°. Since all of the octagon angles are the same, each angle is 270 ÷ 2 or 135°.

Lesson 1

Practice Problems

6 Problems

Problem 1

Here are questions about two types of angles.

a. Draw a right angle. How do you know it's a right angle? What is its measure in degrees?

90°

Responses vary. Sample responses: I used a protractor and measured; a square pattern block fits perfectly inside it; the corner of my notebook paper fits perfectly inside it.

b. Draw a straight angle. How do you know it's a straight angle? What is its measure in degrees?

180°

Responses vary. Sample response: I drew a straight line, and a straight angle is an angle formed by a straight line.

Problem 2

An equilateral triangle's angles each have a measure of 60 degrees.

a. Can you put copies of an equilateral triangle together to form a straight angle? Explain or show your reasoning.

Yes

3 triangles are needed because 180 ÷ 3 = 60.

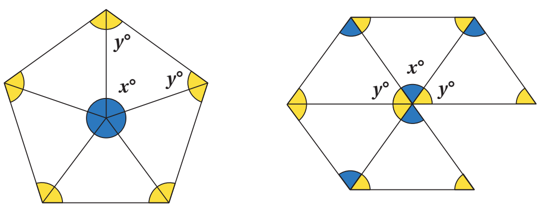
b. Can you put copies of an equilateral triangle together to form a right angle? Explain or show your reasoning.

No

One 60° angle is not enough, and two is too much.

Problem 3

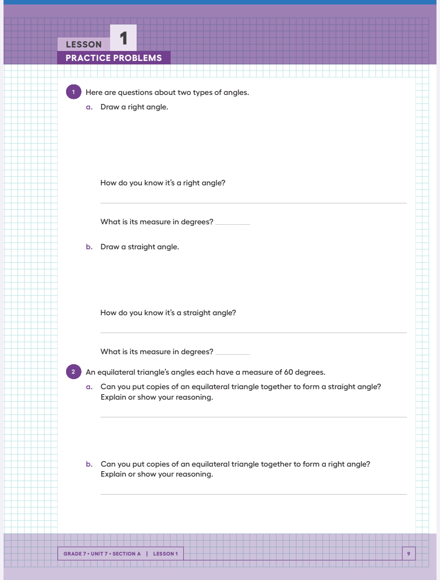
Here are two different patterns made out of the same five identical isosceles triangles. Without using a protractor, determine the measures of Angles x and y. Explain or show your reasoning.



$x = 72$ and $y = 54$

Since there are 5 copies of the angle that measures x around a single point in the first picture, we know that $5x = 360$, so $x = 72$. In the second picture, we know that two copies of y and one copy of x make a straight angle, so $2y + 72 = 180$. Since we already know x, we can figure out that $y = 54$.

Student Workbook



LESSON 1
PRACTICE PROBLEMS

1 Here are questions about two types of angles.

a. Draw a right angle.

How do you know it's a right angle?

What is its measure in degrees?

b. Draw a straight angle.

How do you know it's a straight angle?

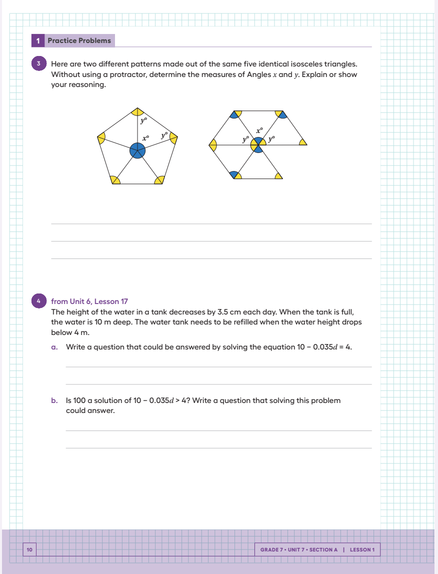
What is its measure in degrees?

2 An equilateral triangle's angles each have a measure of 60 degrees.

a. Can you put copies of an equilateral triangle together to form a straight angle? Explain or show your reasoning.

b. Can you put copies of an equilateral triangle together to form a right angle? Explain or show your reasoning.

Student Workbook



1 Practice Problems

3 Here are two different patterns made out of the same five identical isosceles triangles. Without using a protractor, determine the measures of Angles x and y. Explain or show your reasoning.

4 from Unit 6, Lesson 17

The height of the water in a tank decreases by 3.5 cm each day. When the tank is full, the water is 10 m deep. The water tank needs to be refilled when the water height drops below 4 m.

a. Write a question that could be answered by solving the equation $10 - 0.035d = 4$.

b. Is 100 a solution of $10 - 0.035d > 4$? Write a question that solving this problem could answer.

Student Workbook


1 Practice Problems

5 from Unit 6, Lesson 18

Use the distributive property to write an expression that is equivalent to each given expression.

a. $-3(2x-4)$

b. $0.1(-90+50a)$

c. $-7(-x-9)$

d. $\frac{4}{5}(10y+-x+-15)$

6 from Unit 2, Lesson 3

Lin's puppy is gaining weight at a rate of 0.125 pounds per day. Describe the weight gain in days per pound.

Learning Targets

+ I can find unknown angle measurements by reasoning about adjacent angles with known measures.
+ I can recognize when an angle measures 90°, 180°, or 360°.

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Problem 4

from Unit 6, Lesson 17

The height of the water in a tank decreases by 3.5 cm each day. When the tank is full, the water is 10 m deep. The water tank needs to be refilled when the water height drops below 4 m.

a. Write a question that could be answered by solving the equation $10 - 0.035d = 4$.

Answers vary.

Sample response: "How many days can pass before the water tank needs to be refilled?"

b. Is 100 a solution of $10 - 0.035d > 4$? Write a question that solving this problem could answer.

Answers vary.

Sample response: Yes. "Is there still enough water in the tank after 100 days?"

Problem 5

from Unit 6, Lesson 18

Use the distributive property to write an expression that is equivalent to each given expression.

a. $-3(2x-4)$

$-6x+12$

b. $0.1(-90+50a)$

$-9+5a$

c. $-7(-x-9)$

$7x+63$

d. $\frac{4}{5}(10y+-x+-15)$

$8y-\frac{4}{5}x-12$

Problem 6

from Unit 2, Lesson 3

Lin's puppy is gaining weight at a rate of 0.125 pounds per day. Describe the weight gain in days per pound.

8 days per pound