Lesson 16

Decimal Representations of Rational Numbers

Goal

Represent rational numbers as equivalent decimals and fractions, and explain (orally) the solution method.

Learning Targets

- I can write a fraction as a repeating decimal.
- I understand that every number has a decimal expansion.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

Lesson Narrative

This lesson explores different representations of rational numbers. Students begin by considering an image of three number lines, each showing a more precise location of 0.375.

Next, students rewrite rational numbers written as decimals, square roots, and cube roots as numbers in fraction form. They see that it is not the symbols used to write a number that make it rational, but rather the fact that it can be rewritten in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Reversing their thinking, students then find the decimal representation of fractions with finite decimal expansions.

In the last activity students use long division with repeated reasoning to find that $\frac{2}{11} = 0.1818 \ldots$ Students plot this decimal expansion on a series of zooming number lines and observe an alternating pattern between the intervals that will continue forever.

This zooming number line representation supports students' understanding of place value and helps to form mental images of the two different ways a decimal expansion can represent a rational number—both as a terminating decimal when the decimal expansion eventually lands exactly on a tick mark and as a **repeating decimal** when the decimal expansion repeats forever but in a predictable way.

In the next lesson students continue to work with rational numbers that have infinite decimal expansions and contrast the decimal representations of irrational numbers.

Student Learning Goal

Let's learn more about how rational numbers can be represented.

Lesson Timeline

5 min	10 min	10 min	10 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Notice and Wonder: Number Lines

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to investigate a representation of rational numbers using zooming number lines, which will be useful when students work with a similar representation in upcoming activities. While students may notice and wonder many things about this image, the fact that each number line zooms in on a specific interval of the previous number line is the important discussion point.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is how each number line is partitioned into 10 equal intervals.

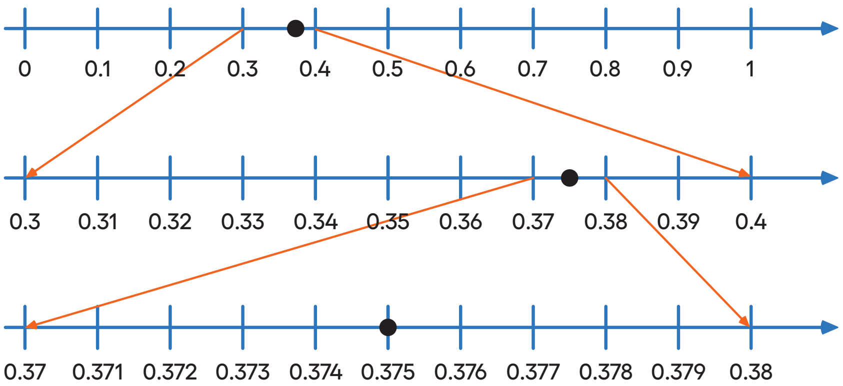
Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

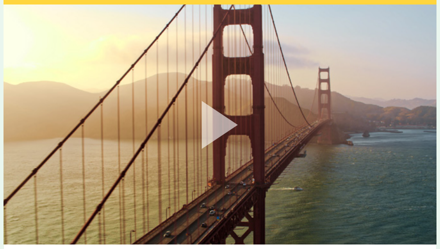
- Each number line zooms in on a specific interval from the previous number line.
- The first number line only shows tenths, the second shows hundredths, and the third shows thousandths.
- These look like the number lines from an earlier activity with scientific notation and very small numbers.

Students may wonder:

- Why does the dot look like it is moving around?
- What number does the dot represent?
- Do all three dots represent the same number?
- Are all three of those points 0.375?

Inspire Math

Golden Gate Bridge video



Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614248

Please log in to the site before using the QR code or URL.

Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

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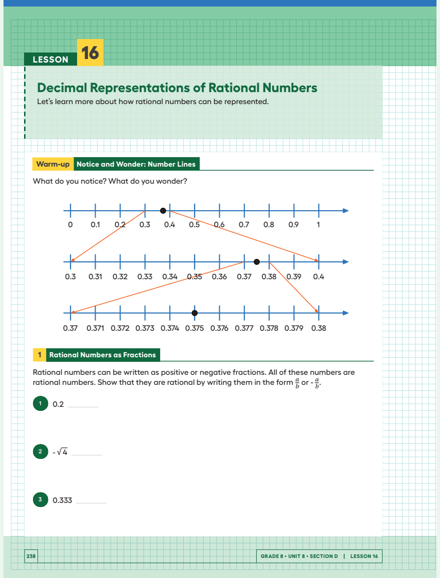
Student Workbook



Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.
Provide choice. Invite students to decide the order they complete the set of problems in each question.
Supports accessibility for: Organization, Attention

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the idea of what number the dot represents does not come up during the conversation, ask students to discuss this idea, but do not tell them the answer—it will be discussed in a following activity.

Activity 1

Rational Numbers as Fractions

10 min

Activity Narrative

In this activity, students review the idea that a rational number can be rewritten as a positive or negative fraction, and they rewrite rational numbers with finite decimal expansions in fraction form. Students will study rational numbers with infinite decimal forms in a following activity.

For the problems with roots, values were purposefully chosen to emphasize that not all numbers written using square root or cube root notation are irrational.

Monitor for students who write these different representations of 0.2 for the first problem:

$\frac{1}{5}$

$\frac{2}{10}$

The focus here is to see that there are many ways to express a rational number.

Launch

Remind students that a rational number is a number that can be written as a positive or negative fraction $\frac{a}{b}$, where a and b are integers (with $b \neq 0$). In fact, there are many equivalent fractions that represent a single rational number. For example, 5 is equivalent to $\frac{10}{2}$ and $\frac{15}{3}$.

Give students 3 minutes of quiet work time, and follow with a whole-class discussion. Select students who used each representation described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially of students who haven't shared recently.

Student Task Statement

Rational numbers can be written as positive or negative fractions. All of these numbers are rational numbers. Show that they are rational by writing them in the form $\frac{a}{b}$ or $-\frac{a}{b}$.

1. 0.2

   $\frac{2}{10}$ (or equivalent)

2. $-\sqrt{4}$

   $-\frac{2}{1}$ (or equivalent)

3. 0.333

   $\frac{333}{1,000}$

4. $\sqrt[3]{1,000}$

   $\frac{10}{1}$ (or equivalent)

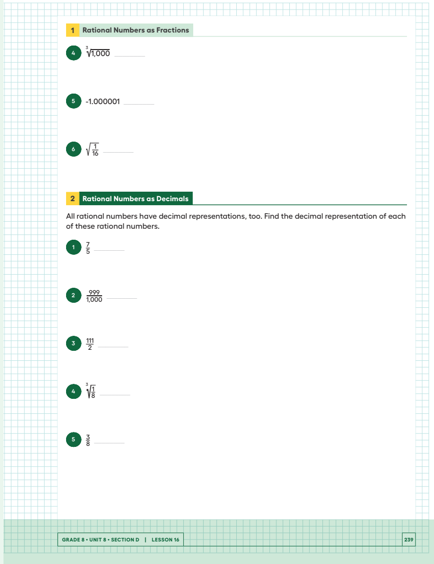
5. -1.000001

   $-\frac{1,000,001}{1,000,000}$

6. $\sqrt{\frac{1}{16}}$

   $\frac{1}{4}$

Student Workbook



Activity Synthesis

The goal of this discussion is to highlight how rational numbers can be represented in different ways. Begin by asking students what they noticed about the values used in the *Task Statement* (Some are positive and some are negative. Some are written as decimals and some written as square roots or cube roots.) Explain that while these numbers are all represented in different ways, they are all rational numbers because they can all be expressed as a positive or negative fraction.

Display answers for 0.2 from previously selected students for all to see.

Use *Compare and Connect* to help students compare, contrast, and connect the different approaches and representations. Draw a number line with the numbers 0, 1, and 2 with plenty of space between the integers for all to see. Here are some questions for discussion:

"How do we plot $\frac{1}{5}$? on this number line?"

Subdivide the segment from 0 to 1 into 5 equal pieces and $\frac{1}{5}$ is at the first mark after 0.

Then label the point $\frac{1}{5}$.

"How can we see that this is the same as $\frac{2}{10}$?"

Subdivide each fifth into two equal pieces—now each piece is $\frac{1}{10}$.

Then label the point $\frac{2}{10}$ and 0.2.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Provide Access by Recruiting Interest.
Provide choice. Invite students to decide the order they complete the set of problems.
Supports accessibility for: Organization, Attention

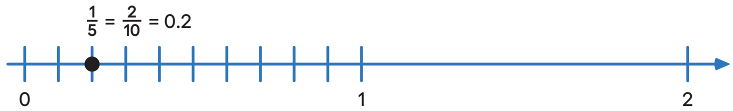
Building on Student Thinking

If students write that $0.333 = \frac{1}{3}$, consider asking:

"How did you determine that?"
"How would you read that decimal out loud?"

"How do these different representations show the same information?"

One representation is partitioned into twice as many parts, and we count over twice as many of them.



Emphasize the idea that any rational number can be plotted on the number line this way when written as a fraction $\frac{a}{b}$ — by subdividing the interval into b parts and counting over a of them.

Activity 2

Rational Numbers as Decimals

10 min

Activity Narrative

In this activity, students rewrite fractions with finite decimal expansions as decimals. This activity deepens students' understanding of what rational numbers are and how different representations highlight different features of a rational number.

Launch

Arrange students in groups of 2. Do not provide access to calculators.

Give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

All rational numbers have decimal representations, too. Find the decimal representation of each of these rational numbers.

1. $\frac{7}{5}$

   1.4

2. $\frac{999}{1,000}$

   0.999

3. $\frac{111}{2}$

   55.5

4. $\sqrt[3]{\frac{1}{8}}$

   0.5

5. $\frac{3}{8}$

   0.375

Activity Synthesis

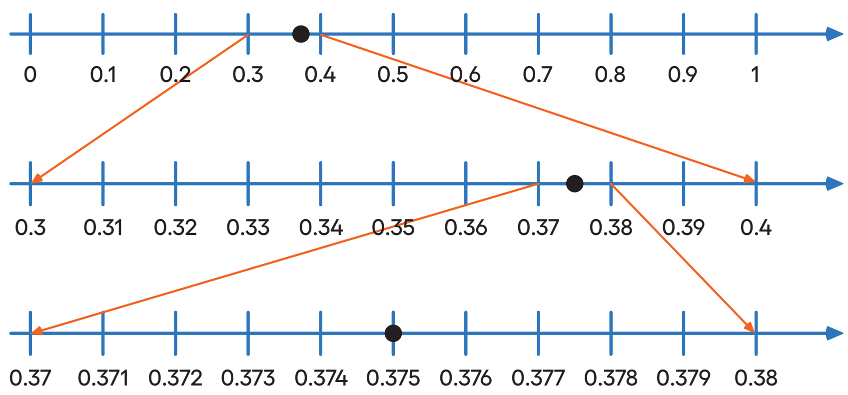
The goal of this discussion is to highlight that all rational numbers have a decimal representation in addition to a fractional representation.

Begin by inviting students to share how they rewrote $\frac{3}{8}$ as a decimal representation (0.375). Demonstrate the steps for long division, or have a student demonstrate the steps for all to see.

Ask students how they would go about plotting the fraction $\frac{3}{8}$ on a number line. (Divide the interval between 0 and 1 into 8 equal parts and count 3 parts to the right from 0.

Then ask students how they would go about plotting the decimal 0.375 on a number line. Explain that while 0.375 can be plotted at the same location as $\frac{3}{8}$, the same strategy does not make sense when plotting a number represented as a decimal since there is no denominator. In a decimal such as 0.375, the digits are referring to place value instead of equal parts.

Then display the number lines from the *Warm-up*.



Tell students that the three points on each number line all represent the same number: 0.375. Explain how the first number line shows the value of the number between 0.3 and 0.4, but closer to 0.4. In order to get a more precise idea of the number's location, the second number line is "zoomed in" to the interval between 0.3 and 0.4, and now we can tell that the number is between 0.37 and 0.38. By zooming in one more time, we see that the number lands exactly on one of the tick marks: 0.375.

If time allows, repeat this explanation with $\frac{999}{1,000}$.

Conclude the discussion by telling students that all rational numbers have a decimal representation. Rational numbers that eventually come to fall exactly on one of the tick marks—such as 0.375 did on the third number line—have a finite decimal expansion and are sometimes called a terminating decimal. Eventually all the digits after a certain number are just 0, so typically they are not written. For example, we can write $\frac{3}{8}$ as 0.375, 0.3750, or 0.3750000000.

Students will investigate repeating decimals in a following lesson, so there is no need to address them here.

Activity 3

Zooming In on $\frac{2}{11}$

10 min

Activity Narrative

In this activity, students rewrite a fraction with an infinite decimal expansion as a decimal. Students use repeated reasoning with division to justify to themselves that 0.1818 ... repeats the digits 1 and 8 forever. Students learn that this is an example of a **repeating decimal** because it has digits that keep going in the same pattern over and over.

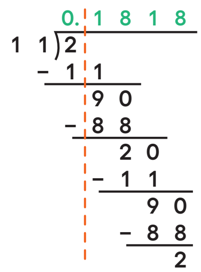
In later activities, students will deepen their understanding of rational numbers with infinite decimal expansions by learning how to rewrite numbers in that form as fractions.

Launch

Arrange students in groups of 2. Since this activity uses long division, do not provide access to calculators.

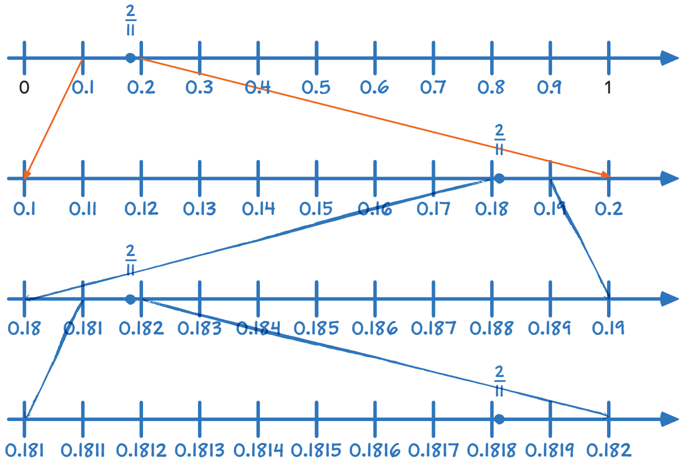
Begin by asking students what strategy could be used to determine the value of $\frac{2}{11}$ as a decimal (long division). Perform the first step of the long division for all to see.

Give students 1 minute to complete 3 more steps with a partner.



Then give students 3–4 minutes of quiet work time to complete the activity, and follow with a whole-class discussion.

Student Task Statement



1. On the topmost number line, label the tick marks. Next, find the first decimal place of $\frac{2}{11}$ using long division and estimate where $\frac{2}{11}$ should be placed on the top number line.

   See image.

2. Label the tick marks of the second number line. Find the next decimal place of $\frac{2}{11}$ by continuing the long division and estimate where $\frac{2}{11}$ should be placed on the second number line. Add arrows from the second to the third number line to zoom in on the location of $\frac{2}{11}$.

   See image.

3. Label the tick marks of the remaining number lines. Continue using long division to calculate the next two decimal places, and plot them on the remaining number lines.

   See image.

4. What do you think the decimal expansion of $\frac{2}{11}$ is?

   Sample response: I think the decimal expansion of $\frac{2}{11}$ is 0.1818 …

Are You Ready for More?

Let $x = \frac{25}{11} = 2.272727 ...$ and let $y = \frac{58}{33} = 1.75757575 ...$

For each of the following questions, first decide whether the fraction or decimal representations of the numbers are more helpful to answer the question, and then find the answer.

1. Which of x or y is closer to 2?

   y is closer to 2.

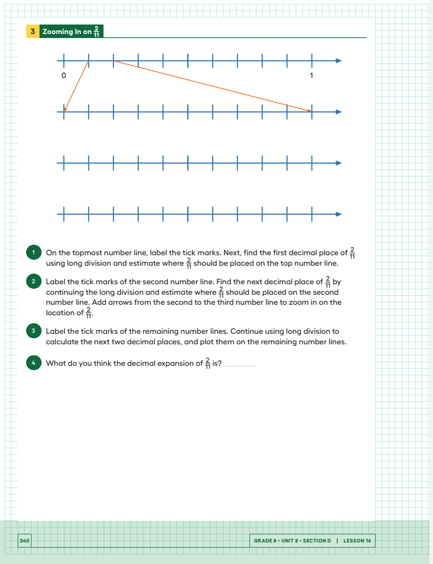
   From the decimal expansion, we can see that y is less than.
   25 units away from 2, but x is more than .25 units away. We could also see this from the fraction representation, though it may be slightly more time-consuming.

2. Find x^2.

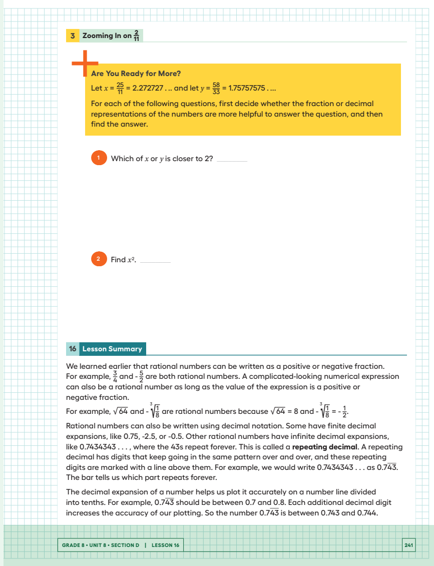
   $x^2 = \frac{625}{121}$

   This is an easy calculation from the fraction representation, but would be very challenging from the decimal representation: The decimal expansion for $\frac{625}{121}$ is $5.\overline{1652892561983471074380}$.

Student Workbook

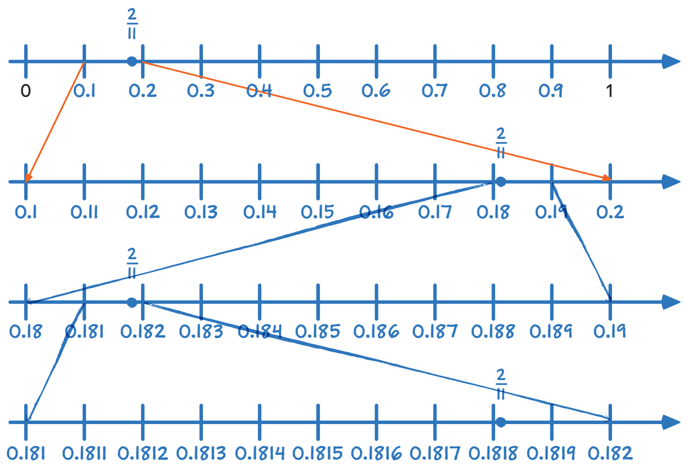


Student Workbook



Activity Synthesis

The purpose of this discussion is to explicitly state the repeated reasoning and successive approximation used to calculate each digit after the decimal point for $\frac{2}{11}$. Invite one or two students to share their number lines or display this image for all to see.



Ask students to share things they notice and wonder (I notice that the point for $\frac{2}{11}$ keeps jumping from left to right. I notice that the pattern seems to be repeating itself. I wonder if this pattern will continue forever.) Now ask students to share what they think the decimal representation of $\frac{2}{11}$ is, and record responses for all to see.

Tell students that sometimes the decimal representation of a rational number repeats like this forever. Unlike 0.375 where the decimal expansion eventually falls exactly on a tick mark since all the digits after the 5 are 0, the decimal expansion of $\frac{2}{11}$ will never land exactly on a tick mark when we zoom in by powers of 10, but will continue this pattern forever. Fractions like $\frac{2}{11}$ have an infinite decimal expansion and are sometimes called **repeating decimals**.

Tell students that there is a special notation to represent repeating decimals, and display the notation for all to see:

$0.18181818181818181818 \ldots = 0.\overline{18}$

Emphasize that even though $0.\overline{18}$ goes on forever, it is still a rational number because it repeats and is the decimal representation of the fraction $\frac{2}{11}$.

Lesson Synthesis

The purpose of this discussion is to emphasize that even though rational numbers are defined as a number that can be written as a positive or negative fraction, there are many different ways to represent them. Here are some questions for discussion:

- *"What is a rational number?"*

  A number that can be written as a positive or negative fraction.

- *"What are some examples of rational numbers?"*

  $\frac{2}{3}$, 5.82, $\sqrt[3]{27}$. Encourage different representations.

- *"What do we know about the decimal expansions of rational numbers?"*

  Sometimes the decimal expansion is finite (it terminates), and sometimes it is infinite (it repeats forever).

- *"What are some rational numbers with a finite decimal expansion?"*

  $\frac{1}{2}, \frac{4}{5}, \frac{99}{100}$, 0.84, 3, $\sqrt{25}$

- *"What are some rational numbers with an infinite decimal expansion?"*

  $\frac{1}{3}, \frac{5}{9}, 0.\overline{2}, 0.\overline{156}$

Students will learn more about rational numbers with infinite decimal expansions in an upcoming lesson, so it is not necessary for students to have many examples of rational numbers of this type at this time.

Lesson Summary

We learned earlier that rational numbers can be written as a positive or negative fraction. For example, $\frac{3}{4}$ and $-\frac{5}{2}$ are both rational numbers. A complicated-looking numerical expression can also be a rational number as long as the value of the expression is a positive or negative fraction. For example, $\sqrt{64}$ and $-\sqrt[3]{\frac{1}{8}}$ are rational numbers because $\sqrt{64} = 8$ and $-\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$.

Rational numbers can also be written using decimal notation. Some have finite decimal expansions, like 0.75, -2.5, or -0.5. Other rational numbers have infinite decimal expansions, like 0.7434343 ..., where the 43s repeat forever. This is called a **repeating decimal**. A repeating decimal has digits that keep going in the same pattern over and over, and these repeating digits are marked with a line above them. For example, we would write 0.7434343 ... as $0.7\overline{43}$. The bar tells us which part repeats forever.

The decimal expansion of a number helps us plot it accurately on a number line divided into tenths. For example, $0.7\overline{43}$ should be between 0.7 and 0.8. Each additional decimal digit increases the accuracy of our plotting. So the number $0.7\overline{43}$ is between 0.743 and 0.744.

Cool-down

An Unknown Rational Number

5 min

Student Task Statement

Explain how you know that -3.4 is a rational number.

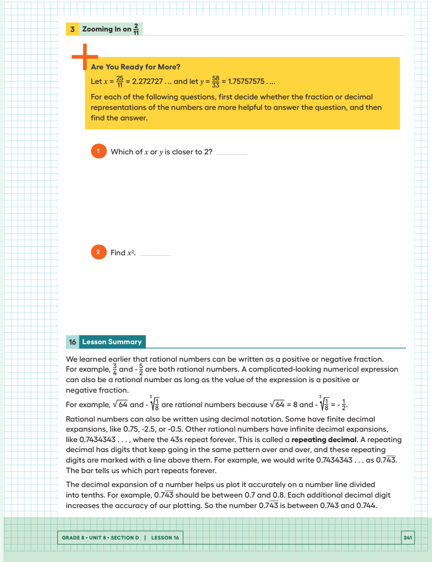
Sample response: $-3.4 = -\frac{34}{10}$, so it can be written as a negative fraction and is therefore a rational number.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



3 Zooming In on $\frac{2}{11}$

Are You Ready for More?

Let $x = \frac{25}{11} = 2.272727...$ and let $y = \frac{58}{33} = 1.75757575...$

For each of the following questions, first decide whether the fraction or decimal representations of the numbers are more helpful to answer the question, and then find the answer.

1. Which of x or y is closer to 2? ______

2. Find x^2. ______

16 Lesson Summary

We learned earlier that rational numbers can be written as a positive or negative fraction. For example, $\frac{3}{4}$ and $-\frac{5}{2}$ are both rational numbers. A complicated-looking numerical expression can also be a rational number as long as the value of the expression is a positive or negative fraction.

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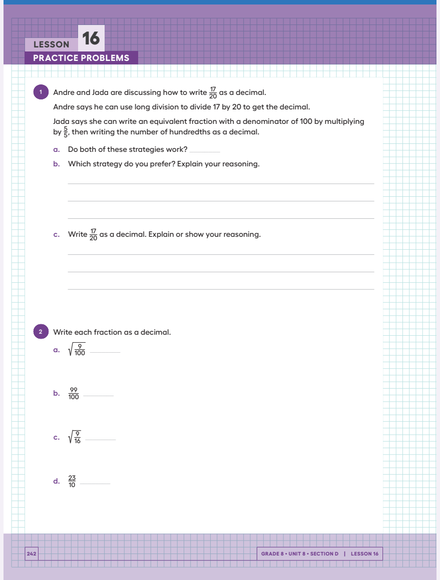
Practice Problems

6 Problems

Student Workbook


LESSON 16
PRACTICE PROBLEMS

1 Andre and Jada are discussing how to write $\frac{17}{20}$ as a decimal.
Andre says he can use long division to divide 17 by 20 to get the decimal.
Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, then writing the number of hundredths as a decimal.
a. Do both of these strategies work?
b. Which strategy do you prefer? Explain your reasoning.
c. Write $\frac{17}{20}$ as a decimal. Explain or show your reasoning.

2 Write each fraction as a decimal.
a. $\sqrt{\frac{9}{100}}$
b. $\frac{99}{100}$
c. $\sqrt{\frac{9}{16}}$
d. $\frac{23}{10}$

Problem 1

Andre and Jada are discussing how to write $\frac{17}{20}$ as a decimal.

Andre says he can use long division to divide 17 by 20 to get the decimal.

Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, then writing the number of hundredths as a decimal.

a. Do both of these strategies work?

Yes

b. Which strategy do you prefer? Explain your reasoning.

Sample responses:

- I prefer Jada's method because I can calculate it mentally.
- I prefer Andre's method because it always works, even if the denominator is not a factor of 100.

c. Write $\frac{17}{20}$ as a decimal. Explain or show your reasoning.

0.85

Sample reasoning: $\frac{17}{20} \cdot \frac{5}{5} = \frac{85}{100}$, so $\frac{17}{20}$ equals 0.85.

Problem 2

Write each fraction as a decimal.

a. $\sqrt{\frac{9}{100}}$

0.3

b. $\frac{99}{100}$

0.99

c. $\sqrt{\frac{9}{16}}$

0.75

d. $\frac{23}{10}$

2.3

Problem 3

Write each decimal as a fraction.

a. $\sqrt{0.81}$

$\frac{9}{10}$

b. 0.0276

$\frac{276}{10,000}$ (or equivalent)

c. $\sqrt{0.04}$

$\frac{1}{5}$ (or equivalent)

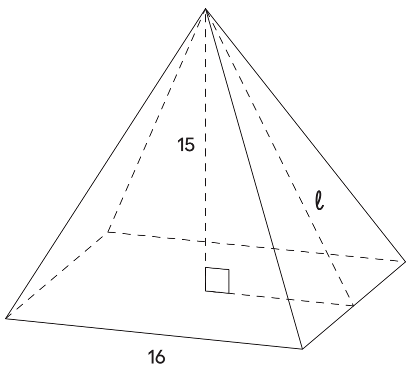
d. 10.01

$\frac{1,001}{100}$ (or equivalent)

Problem 4

from Unit 8, Lesson 11

Here is a right square pyramid.



a. What is the measurement of the slant height ℓ of the triangular face of the pyramid? If you get stuck, use a cross section of the pyramid.

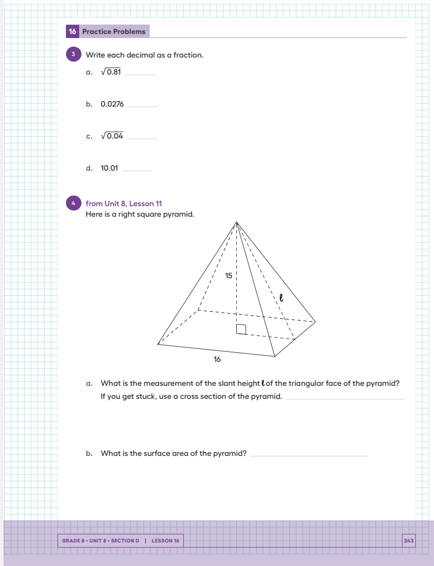
17 units, because $15^2 + 8^2 = 289$, and $\sqrt{289} = 17$

b. What is the surface area of the pyramid?

800 square units

Sample reasoning: The pyramid is made from a square and four triangles. The square's area, in square units, is $16^2 = 256$. Each triangle's area, in square units, is $\frac{1}{2} \cdot 16 \cdot 17 = 136$. The surface area, in square units, is $256 + 4 \cdot 136 = 800$.

Student Workbook



16 Practice Problems

3 Write each decimal as a fraction.

a. $\sqrt{0.81}$

b. 0.0276

c. $\sqrt{0.04}$

d. 10.01

4 from Unit 8, Lesson 11

Here is a right square pyramid.

a. What is the measurement of the slant height ℓ of the triangular face of the pyramid? If you get stuck, use a cross section of the pyramid.

b. What is the surface area of the pyramid?

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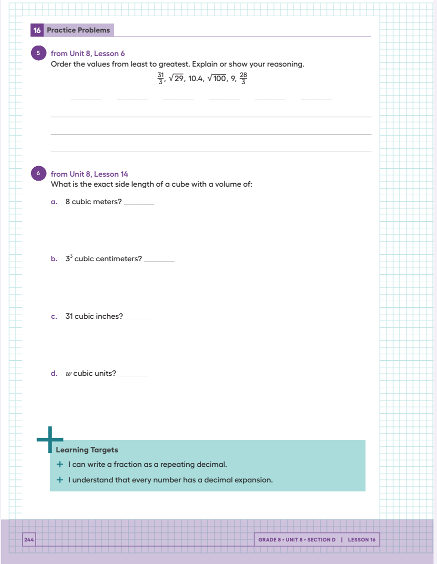
Student Workbook


16 Practice Problems

5 from Unit 8, Lesson 6
Order the values from least to greatest. Explain or show your reasoning.
$\frac{31}{3}$, $\sqrt{29}$, 10.4, $\sqrt{100}$, 9, $\frac{28}{3}$

6 from Unit 8, Lesson 14
What is the exact side length of a cube with a volume of:
a. 8 cubic meters?
b. 3^3 cubic centimeters?
c. 31 cubic inches?
d. w cubic units?

Learning Targets
+ I can write a fraction as a repeating decimal.
+ I understand that every number has a decimal expansion.

Problem 5

from Unit 8, Lesson 6

Order the values from least to greatest. Explain or show your reasoning.

$\frac{31}{3}$, $\sqrt{29}$, 10.4, $\sqrt{100}$, 9, $\frac{28}{3}$

$\sqrt{29}$ 9 $\frac{28}{3}$ $\sqrt{100}$ $\frac{31}{3}$ 10.4

Sample reasoning: Estimate $\sqrt{29}$ to be between 5 and 6 because $5^2 < 29 < 6^2$. The fraction $\frac{28}{3} = 9\frac{1}{3}$. The value $\sqrt{100}$ is equal to 10 because $10^2 = 100$. The fraction $\frac{31}{3} = 10\frac{1}{3}$, which is less than 10.4.

Problem 6

from Unit 8, Lesson 14

What is the exact side length of a cube with a volume of:

a. 8 cubic meters?

8 meters

b. 3^3 cubic centimeters?

3 centimeters

c. 31 cubic inches?

$\sqrt[3]{31}$ inches

d. w cubic units?

$\sqrt[3]{w}$ units