Lesson 7

Simulating Multi-step Experiments

Goals

- Coordinate (orally) a real-world situation and a chance event that could be used to simulate that situation.
- Perform a multi-step simulation, and use the results to estimate the probability of a compound event in a real-world situation (using words and other representations).

Learning Target

I can use a simulation to estimate the probability of a multi-step event.

Lesson Narrative

In this lesson, students see that compound events can be simulated by using multiple chance experiments. The simulation relies on student ability to model the situations mathematically so that the simulations match the probabilities of the original situations. For example, if we want to know the probability that it will rain each day of the week when there is a 50% chance of rain each day, we can flip a coin to represent each day. Therefore, if we toss a coin 70 times, we will have run this simulation only 10 times. Students construct arguments to explain why they match proposed simulations to situations.

An optional activity is available for practice with writing the probability of a multi-step situation. Students play a game and consider how changing the rules might affect the probability of winning the game.

Student Learning Goal

Let's simulate more complicated events.

Access for Students with Diverse Abilities

- Representation (Activity 2, Activity 3)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Warm-up)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- MLR5: Co-Craft Questions
- Take Turns

Required Materials

Materials to Gather

- Paper clips: Activity 1
- Paper bags: Activity 2
- Snap cubes: Activity 2

Materials to Copy

- Alpine Zoom Handout (1 copy for every 6 students): Activity 1

Required Preparation

Activity 2:

- Prepare a paper bag containing 3 snap cubes (2 black and 2 white) for every 3 students.

Activity 3:

- Make simulation tools (number cubes, bags with colored snap cubes, and so on) available.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	10 min	5 min

Warm-up
Ski Business

5 min

Activity Narrative

In this *Warm-up,* students consider a situation and propose questions related to the situation. While students may ask many different kinds of questions, the important discussion points are about understanding the context and the relationship to simulation and probability.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Introduce the context of businesses built around skiing. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

Alpine Zoom is a ski business. To make money over spring break, they need it to snow at least 4 out of the 10 days.

Sample responses:

- What is the probability it will snow each day?
- What is the probability the business will make money this spring break?
- Can we design a simulation to predict the weather for 10 days?

Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

- *"What do these questions have in common? How are they different?" Listen for and amplify language related to the learning goal, such as "probability" and "simulation."*

If it does not come up, ask how students might find the probability of snow in the next few days (look at a weather forecast).

Instructional Routines

MLR5: Co-Craft Questions

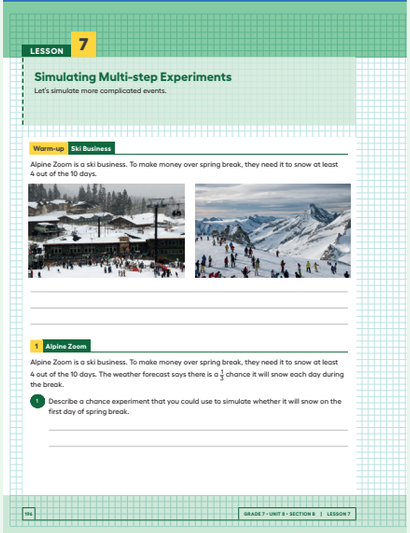
ilclass.com/r/10695544

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Warm-up)

MLR5: Co-Craft Questions. This activity uses *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Student Workbook


LESSON 7

Simulating Multi-step Experiments

Let's simulate more complicated events.

Warm-up Ski Business

Alpine Zoom is a ski business. To make money over spring break, they need it to snow at least 4 out of the 10 days.

1 Alpine Zoom

Alpine Zoom is a ski business. To make money over spring break, they need it to snow at least 4 out of the 10 days. The weather forecast says there is a $\frac{1}{3}$ chance it will snow each day during the break.

1. Describe a chance experiment that you could use to simulate whether it will snow on the first day of spring break.

Activity 1

Alpine Zoom

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students continue to model real-life situations with simulations, but now the situations have more than one part. Finding the exact probability for these situations is advanced, but simulations are not difficult to run and an estimate of the probability can be found using the long-run results from simulations. Provide access to a variety of simulation tools such as number cubes, paper and bags, and protractors for spinners for students to use. If other simulation tools are not available, you will need the blackline master.

In the digital version of the activity, students use an applet to spin a spinner to simulate the situation. The applet allows students to spin a spinner and record the outcome of the spin. The digital version may be preferable if students would benefit from running and understanding a simulation rather than designing and using their own simulation, or if there is not enough time to create and run the simulations by hand.

Launch

Arrange students in groups of 3. After students have had a chance to think about an experiment themselves, select groups to share their responses.

If possible, allow them to use the simulation they have suggested. If the simulation is not readily available, provide each group with a spinner from the blackline master.

Give students 5 minutes for partner discussion, 5 minutes to run the simulation, then 5 minutes for a whole-class discussion.

Student Task Statement

Alpine Zoom is a ski business. To make money over spring break, they need it to snow at least 4 out of the 10 days. The weather forecast says there is a $\frac{1}{3}$ chance it will snow each day during the break.

1. Describe a chance experiment that you could use to simulate whether it will snow on the first day of spring break.

   Sample response: Roll a number cube. If it lands on a 5 or 6, it will snow on the first day of break. If it lands on anything else, it will not snow.

2. How could this chance experiment be used to estimate a probability that Alpine Zoom will make money?

   Sample response: Do the chance experiment 10 times and write down whether it snows each day. If it snows on at least 4 days, then the company will make money.

3. Simulate the weather for 10 days to see if Alpine Zoom will make money over spring break. Record your results in the first row of the table.

Sample response:

	day 1	day 2	day 3	day 4	day 5	day 6
simulation 1	snow	no	no	no	snow	no
simulation 2	no	no	no	snow	snow	
simulation 3	snow	no	snow	no	no	
simulation 4	no	no	no	no	no	
simulation 5	no	no	snow	no	no	

	day 7	day 8	day 9	day 10	Did they make money?
simulation 1	snow	no	no	snow	yes
simulation 2					
simulation 3					
simulation 4					
simulation 5					

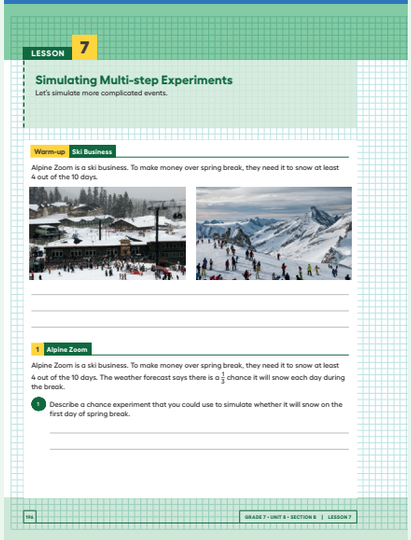
4. Repeat the previous step to simulate the entire break 4 more times. Record your results in the other rows of the table.
5. Based on your group's simulations, estimate the probability that Alpine Zoom will make money.

   Sample response: $\frac{2}{5}$

Building on Student Thinking

Students may be confused by the phrase "at least 4 days." Explain that in this context, it means 4 or more.

Student Workbook


LESSON 7

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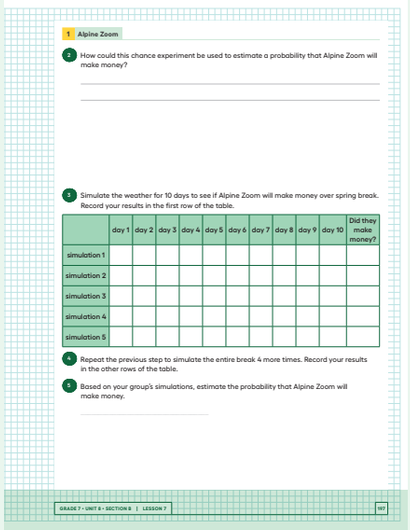
Student Workbook


1 Alpine Zoom

2. How could this chance experiment be used to estimate a probability that Alpine Zoom will make money?

3. Simulate the weather for 10 days to see if Alpine Zoom will make money over spring break. Record your results in the first row of the table.

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10	Did they make money?
simulation 1											
simulation 2											
simulation 3											
simulation 4											
simulation 5											

4. Repeat the previous step to simulate the entire break 4 more times. Record your results in the other rows of the table.
5. Based on your group's simulations, estimate the probability that Alpine Zoom will make money.

Activity Synthesis

The purpose of this discussion is for students to understand the connection between the results of their simulation and the real-life situation.

Ask each group for the number of times Alpine Zoom made money in their simulations.

Consider asking these discussion questions:

- *"Using the class's data, estimate the probability that Alpine Zoom will make money."*

  Theoretically, this should be close to 45%.

- *"Do you anticipate Alpine Zoom will make money this spring break?"*

  It's not likely, but it's possible.

- *"Over the next 10 years, if the weather patterns continue to be the same, do you anticipate Alpine Zoom will make money over that time or not?"*

  This is even less likely. There is less than a 50% chance it will make money each season, so over 10 years, it will probably lose money more than it will make.

- *"Is this a business you would invest in? Explain your reasoning."*

  I would not invest in it because it is unlikely to make money over the years.

Activity 2: Optional

Kiran's Game

15 min

Activity Narrative

In this activity, students practice doing many trials of multi-step situations to estimate the probability of an event. In the discussion following the activity, students construct arguments about how changes to the game might affect the probability of winning.

Launch

Arrange students in groups of 3. Provide each group with a paper bag containing 2 black blocks and 2 white blocks inside. If black and white blocks are not available, instruct students on their color equivalents.

Give students 5 minutes to run the simulation, 5 minutes for partner discussion, then have a whole-class discussion.

Student Task Statement

Kiran invents a game that uses a board with alternating black and white squares. A playing piece starts on a white square and must advance 4 squares to the other side of the board within 5 turns to win the game.

For each turn, the player draws a block from a bag containing 2 black blocks and 2 white blocks. If the block color matches the color of the next square on the board, the playing piece moves onto it. If it does not match, the playing piece stays on its current square.

1. Take turns playing the game until each person in your group has played the game twice.

   No response needed.

2. Use the results from all the games your group played to estimate the probability of winning Kiran's game.

   Because nobody won in all 6 games played, the probability of winning should be low. Because it's possible to win, though, I don't think the probability should be 0. The probability is probably between 0 and $\frac{1}{6}$.

3. Do you think your estimate of the probability of winning is a good estimate? How could it be improved?

   I don't think this is a very good estimate of the probability of winning because we only played 6 times, and the chances of winning are so low. It could be improved by playing the game a lot more times.

Are You Ready for More?

How would each of these changes, on its own, affect the probability of winning the game?

1. Change the rules so that the playing piece must move 7 spaces within 8 moves.

   It would be harder to win. The player can still only get 1 wrong block, but now they must get it right 7 times instead of only 4.

2. Change the board so that all the spaces are black.

   This would not affect the chances of winning. There is still a probability of 0.5 to move each time.

3. Change the blocks in the bag to 3 black blocks and 1 white block.

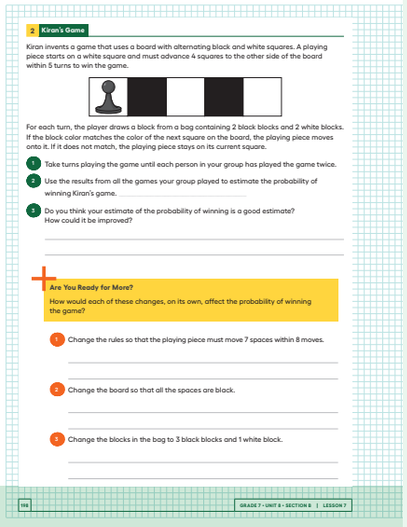
   This would make moving onto black squares easier, but harder to move on to white squares. This game is slightly more difficult to win than the original.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Provide students with a graphic organizer, such as a two-column table, to record the results of the simulation and probability of winning.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook


2 Kiran's Game

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For each turn, the player draws a block from a bag containing 2 black blocks and 2 white blocks. If the block color matches the color of the next square on the board, the playing piece moves onto it. If it does not match, the playing piece stays on its current square.

1. Take turns playing the game until each person in your group has played the game twice.
2. Use the results from all the games your group played to estimate the probability of winning Kiran's game.
3. Do you think your estimate of the probability of winning is a good estimate? How could it be improved?

Are You Ready for More?

How would each of these changes, on its own, affect the probability of winning the game?

1. Change the rules so that the playing piece must move 7 spaces within 8 moves.
2. Change the board so that all the spaces are black.
3. Change the blocks in the bag to 3 black blocks and 1 white block.

Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.

Activity Synthesis

The purpose of the discussion is for students to think about how changing the rules of the game might change the probability of winning.

Collect the data from the class for the number of wins and display the results for all to see.

Consider asking these discussion questions:

- *"Based on the class's data, estimate the probability of winning the game."*

  The theoretical probability of winning is $\frac{3}{16} \approx 0.19$.

- *"Does the game seem too easy or hard to win? If so, how could Kiran change the game slightly to make it harder or easier?"*

  If it is too hard, move the piece closer to the end or allow more than 5 moves to win.

- *"The bag contains 2 black and 2 white blocks. If the bag had 4 blocks of each color, would that make it easier or harder to win?"*

  Neither. It would be the same difficulty since there is still an equal chance to get each color.

- *"Do you think the estimate from the class's data is a better estimate than the one you got on your own?"*

  Since there is more data from the entire class, it should be a better estimate than one individual's.

Activity 3

Simulation Nation

10 min

Activity Narrative

In this activity, students practice what they have learned about simulations by matching real-life scenarios to simulations. In the discussion, students are asked to explain their reasoning for their choices and think about other valid choices that could be made.

Launch

Keep students in groups of 3.

Give students 5 minutes of small-group time to take turns matching the items and discussing their reasoning.

Then follow with a whole-class discussion.

Student Task Statement

Match each situation to a **simulation**.

Situations:

Simulation 2 **A.** In a small lake, 25% of the fish are female. You capture a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability that at least 3 of the 5 fish are female?

Simulation 4 **B.** Elena makes about 80% of her free throws. Based on her past successes with free throws, what is the probability that she will make exactly 4 out of 5 free throws in her next basketball game?

Simulation 1 **C.** On a game show, a contestant must pick one of three doors. In the first round, the winning door has a vacation. In the second round, the winning door has a car. What is the probability of winning a vacation and a car?

Simulation 3 **D.** Your choir is singing in 4 concerts. You and one of your classmates both learned the solo. Before each concert, there is an equal chance the choir director will select you or the other student to sing the solo. What is the probability that you will be selected to sing the solo in exactly 3 of the 4 concerts?

Simulations:

1. Toss a standard number cube 2 times and record the outcomes. To estimate the probability, repeat this process many times and find the proportion of the simulations in which a 1 or 2 appears both times.
2. Make a spinner with four equal sections labeled 1, 2, 3, and 4. To estimate the probability, spin the spinner 5 times and record the outcomes. Repeat this process many times and find the proportion of the simulations in which a 4 appears 3 or more times.
3. Toss a fair coin 4 times and record the outcomes. To estimate the probability, repeat this process many times, and find the proportion of the simulations in which exactly 3 heads appear.
4. Place 8 blue chips and 2 red chips in a bag. Shake the bag, select a chip, record its color, and then return the chip to the bag. Repeat the process 4 more times to obtain a simulated outcome. To estimate the probability, repeat this process many times and find the proportion of the simulations in which exactly 4 blues are selected.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed _____, so I matched ..." Encourage students to challenge each other when they disagree.

Advances: Speaking, Listening

Building on Student Thinking

Students may not see the connection between the standard number cube and the situation with 3 doors. Remind students it is important that the probabilities match, but not necessarily the outcomes. Since the simulation matches 2 of the outcomes to one door, the probabilities will match.

Student Workbook


3 Simulation Notion

Match each situation to a **simulation**.

Situations:

A. In a small lake, 25% of the fish are female. You capture a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability that at least 3 of the 5 fish are female?

B. Elena makes about 80% of her free throws. Based on her past successes with free throws, what is the probability that she will make exactly 4 out of 5 free throws in her next basketball game?

C. On a game show, a contestant must pick one of three doors. In the first round, the winning door has a vacation. In the second round, the winning door has a car. What is the probability of winning a vacation and a car?

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Simulations:

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GRADE 7 • UNIT 8 • SECTION B | LESSON 7 189

Access for Students with Diverse Abilities (Activity 3, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code matching situations and simulations.

Supports accessibility for: Visual-Spatial Processing

Activity Synthesis

The purpose of this discussion is for students to articulate the reasons they chose to match the items they did.

For each situation, select students to explain why the simulation should go with it. Although some students may have just looked at a portion of the situation and simulation, encourage them to explain all of the parts of the simulation. For example, in the situation involving fish, 25% is mentioned and the spinner is the only option that also has a 25% chance associated with it. Prompt students for more details by asking,

- *"Why do we need to spin the spinner 5 times?"*

  A fish is selected from the lake 5 times.

- *"Why does the number need to show up 3 or more times?"*

  We want a probability that three or more fish are female.

- *"What do the numbers 1 through 4 represent when doing a trial with the spinner?"*

  Each section represents a $\frac{1}{4}$ probability. The section labeled '4' is the 25% chance that a fish will be female, while sections labeled 1–3 are the 75% chance that a fish will not be female.

- *"Could the spinner have 8 sections? If so, how would you label the sections? What would each label represent?"*

  Yes. Label sections 1–8, where sections 7–8 represent the 25% chance that a fish will be female, while sections labeled 1–6 are the 75% chance that a fish will not be female.

For each of the scenarios, ask students if any part of it could be changed and still result in the simulation working. For example, there could be 4 blue chips and 1 red chip in the bag for Simulation D. For Simulation C, we could count the fraction of times when 3 tails appear rather than heads.

Lesson Synthesis

Consider asking these discussion questions:

- *"How are the simulations in this lesson different from the simulations in the previous lesson?"*

  These have multiple parts for each experiment. Also, it would be difficult to compute the exact probability, so simulations seem more necessary.

- *"The chance that it will be cloudy on a single day is simulated by rolling a standard number cube twice. How many times will the number cube need to be rolled to simulate a week?"*

  14 times. It is rolled twice for each day, and there are 7 days in a week, so 14 rolls are needed.

- *"Each day, a student reaches into a bowl of fruit and randomly selects one for their lunch that day. To simulate the situation, he creates a spinner with 4 equal sections labeled "apple," "orange," "watermelon," and "peach." Why might this simulation not represent the situation very well?"*

  Usually watermelons are much larger than the other 3 fruits listed, so there is probably not an equal chance of that being selected, so the spinner should probably have a larger wedge for watermelons.

Lesson Summary

The more complex a situation is, the harder it can be to estimate the probability of a particular event happening. Well-designed simulations are a way to estimate a probability in a complex situation, especially when it would be difficult or impossible to determine the probability from reasoning alone.

To design a good simulation, we need to know something about the situation. For example, if we want to estimate the probability that it will rain every day for the next three days, we could look up the weather forecast for the next three days. Here is a table showing a weather forecast:

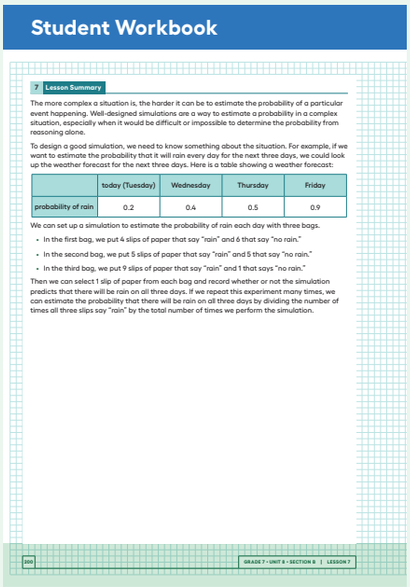
	today (Tuesday)	Wednesday	Thursday	Friday
probability of rain	0.2	0.4	0.5	0.9

We can set up a simulation to estimate the probability of rain each day with three bags.

- In the first bag, we put 4 slips of paper that say “rain” and 6 that say “no rain.”
- In the second bag, we put 5 slips of paper that say “rain” and 5 that say “no rain.”
- In the third bag, we put 9 slips of paper that say “rain” and 1 that says “no rain.”

Then we can select 1 slip of paper from each bag and record whether or not the simulation predicts that there will be rain on all three days. If we repeat this experiment many times, we can estimate the probability that there will be rain on all three days by dividing the number of times all three slips say “rain” by the total number of times we perform the simulation.

Student Workbook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Then we can select 1 slip of paper from each bag and record whether or not the simulation predicts that there will be rain on all three days. If we repeat this experiment many times, we can estimate the probability that there will be rain on all three days by dividing the number of times all three slips say "rain" by the total number of times we perform the simulation.

100 GRADE 7 • UNIT 8 • SECTION B | LESSON 7

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Battery Life

5 min

Student Task Statement

The probability of a certain brand of battery going dead within 15 hours is $\frac{1}{3}$. Noah has a toy that requires 4 of these batteries. He wants to estimate the probability that at least one battery will die before 15 hours are up.

1. Noah will simulate the situation by putting marbles in a bag. Drawing one marble from the bag will represent the outcome of one of the batteries in the toy after 15 hours. Red marbles represent a battery that dies before 15 hours are up, and green marbles represent a battery that lasts longer.

   How many marbles of each color should he put in the bag? Explain your reasoning.

   1 red marble and 2 green marbles (or some multiple of these)

   Sample reasoning: Based on the probability of each battery dying, $\frac{1}{3}$ of the marbles should be red.

2. After doing the simulation 5 times, Noah has these results.

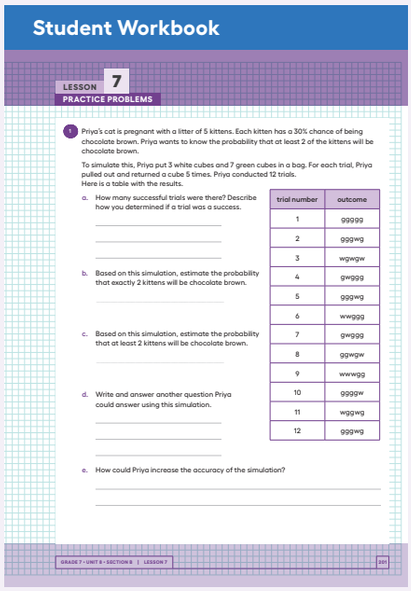
trial	result
1	GGRG
2	GRGR
3	GGGG
4	RGGG
5	GGGR

   What should he use as an estimate of the probability that at least one battery will die within 15 hours?

   $\frac{4}{5}$ or equivalent

Practice Problems

4 Problems


Student Workbook

LESSON 7
PRACTICE PROBLEMS

1 Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least 2 of the kittens will be chocolate brown.

To simulate this, Priya put 3 white cubes and 7 green cubes in a bag. For each trial, Priya pulled out and returned a cube 5 times. Priya conducted 12 trials.
Here is a table with the results.

a. How many successful trials were there? Describe how you determined if a trial was a success.

b. Based on this simulation, estimate the probability that exactly 2 kittens will be chocolate brown.

c. Based on this simulation, estimate the probability that at least 2 kittens will be chocolate brown.

d. Write and answer another question Priya could answer using this simulation.

e. How could Priya increase the accuracy of the simulation?

trial number	outcome
1	ggggg
2	gggwg
3	wgwgw
4	gwggg
5	gggwg
6	wwggg
7	gwggg
8	ggwgw
9	wwwgg
10	ggggw
11	wggwg
12	gggwg

GRADE 7 • UNIT 6 • SECTION B | LESSON 7 201

Problem 1

Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least 2 of the kittens will be chocolate brown.

To simulate this, Priya put 3 white cubes and 7 green cubes in a bag. For each trial, Priya pulled out and returned a cube 5 times. Priya conducted 12 trials.

Here is a table with the results.

trial number	outcome
1	ggggg
2	gggwg
3	wgwgw
4	gwggg
5	gggwg
6	wwggg
7	gwggg
8	ggwgw
9	wwwgg
10	ggggw
11	wggwg
12	gggwg

a. How many successful trials were there? Describe how you determined if a trial was a success.

5 of the 12 trials were successful. Drawing 2 or more white blocks (w's) counted as a success.

b. Based on this simulation, estimate the probability that exactly 2 kittens will be chocolate brown.

$\frac{3}{12}$ (or equivalent, or nearby approximation)

c. Based on this simulation, estimate the probability that at least 2 kittens will be chocolate brown.

$\frac{5}{12}$ (or equivalent, or nearby approximation)

d. Write and answer another question Priya could answer using this simulation.

Sample response: What is the probability that none of the kittens will be chocolate brown? $\frac{1}{12}$ (or equivalent, or nearby approximation)

e. How could Priya increase the accuracy of the simulation?

Priya could conduct more trials to increase the accuracy of the simulation.

Problem 2

A team has a 75% chance to win each of the 3 games they will play this week. Clare simulates the week of games by putting 4 pieces of paper in a bag, 3 labeled "win" and 1 labeled "lose." She draws a paper, writes down the result, then replaces the paper and repeats the process two more times. Clare gets the following result: win, win, lose. What can Clare do to estimate the probability the team will win at least 2 games?

Sample response: She needs to repeat the process many more times to get a good estimate of the probability. She has only done it once right now. After she has repeated the simulation of the week many times, she could count the fraction of simulated weeks that included at least 2 wins and use that as an estimate for the probability.

Problem 3

from Unit 8, Lesson 5

a. List the sample space for selecting a letter at random from the word "PINEAPPLE."

P, I, N, E, A, L (or equivalent)

b. A letter is randomly selected from the word "PINEAPPLE." Which is more likely, selecting "E" or selecting "P"? Explain your reasoning.

Selecting the letter "P" is more likely because there are 3 Ps in the word "pineapple," and there are only 2 Es.

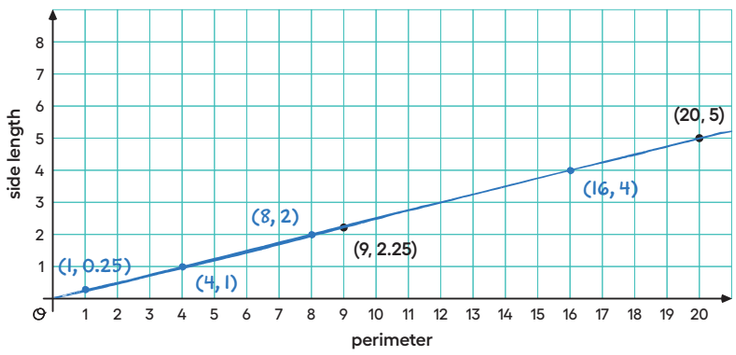
Problem 4

from Unit 2, Lesson 11

On a graph of side length of a square vs. its perimeter, a few points are plotted.

a. Add at least two more ordered pairs to the graph.

Sample response:

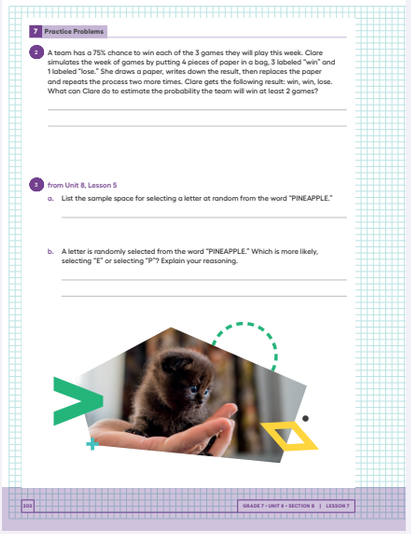


b. Is there a proportional relationship between the perimeter and side length? Explain how you know.

There is a proportional relationship between side length and perimeter.

Sample reasoning: When graphed, the ordered pairs lie on a line that passes through the origin.

Student Workbook



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