Lesson 7

Comparing Relationships with Tables

Goals

- Calculate and compare the quotients of the values in each row of a given table.
- Generate a different recipe for lemonade and describe (orally) how it would taste in comparison to a given recipe.
- Justify (orally) whether the values in a given table could or could not represent a proportional relationship.

Learning Target

I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional.

Student Learning Goal

Let's explore how proportional relationships are different from other relationships.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR5: Co-Craft Questions

Required Materials

Materials to Gather

- Four-function calculators: Lesson
- Math Community Chart: Warm-up

Required Preparation

Lesson:
Calculators can optionally be made available to take the focus off computation.

Lesson Narrative

In this lesson, students examine tables that represent nonproportional relationships. They divide the pairs of values on each row to calculate the unit rate and see that the unit rates are not the same for every row. Next, they use this observation to distinguish between tables that can and cannot represent a proportional relationship.

As students look at data from a context and reason about whether it makes sense quantitatively for the data to represent a proportional relationship, they practice making viable arguments.

A note on terminology:

It is not possible to determine that a relationship is certainly proportional just from examining a table. There may be other pairs of values not shown on the table that are related by a different unit rate. Therefore, when a table shows the same unit rate for every row, we say the relationship could be proportional.

Math Community
The goal of today's exercise is to use the suggestions from the previous exercise to revise the "Norms" sections of the Math Community Chart and to invite students to reflect on one norm that will be a strength for them. Both activities begin to build shared accountability for and investment in the classroom norms

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down (5 min)

Inspire Math

Golden Temple video

Go Online
Before the lesson, show this video to reinforce the real-world connection.

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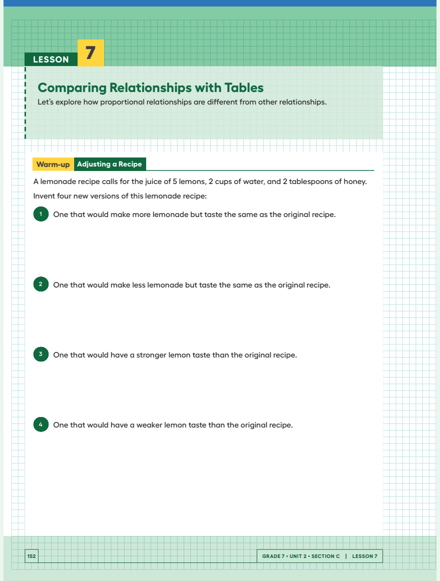
Access for Multilingual Learners (Warm-up, Launch)

MLR8: Discussion Supports.

Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.

Advances: Reading, Representing

Student Workbook


LESSON 7

Comparing Relationships with Tables

Let's explore how proportional relationships are different from other relationships.

Warm-up Adjusting a Recipe

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.

2. One that would make less lemonade but taste the same as the original recipe.

3. One that would have a stronger lemon taste than the original recipe.

4. One that would have a weaker lemon taste than the original recipe.

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Warm-up
Adjusting a Recipe

5 min

Activity Narrative

This activity encourages students to reason about equivalent ratios in a context. During the discussion, emphasize the use of ratios and proportions in determining the effect on the taste of the lemonade.

Launch

Arrange students in groups of 2.

Give students 2 minutes of quiet think time.

Optionally, instead of the abstract recipe description, you could bring in a clear glass, measuring implements, and the lemonade ingredients. Pour the ingredients in the glass and introduce the task that way.

Student Task Statement

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

The base recipe has a ratio of number of lemons to cups of water to tablespoons of honey of 5 : 2 : 2.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.

   Sample response: 10 lemons, 4 cups of water, 4 tablespoons honey

2. One that would make less lemonade but taste the same as the original recipe.

   Sample response: $2\frac{1}{2}$ lemons, 1 cup water, 1 tablespoon honey

3. One that would have a stronger lemon taste than the original recipe.

   Sample response: 8 lemons, 2 cups water, 2 tablespoons honey

4. One that would have a weaker lemon taste than the original recipe.

   Sample response: 2 lemons, 2 cups water, 2 tablespoons honey

Activity Synthesis

Invite students to share their versions of the recipe with the class and record them for all to see. After each explanation, ask the class if they agree or disagree and how the new lemonade would taste. After recording at least 3 responses for each, ask students to describe any patterns they notice how the recipe was adjusted. If students do not mention ratios in their descriptions, be sure to ask them how the ratios changed in their new recipe.

Math Community

After the *Warm-up*, display the Math Community Chart and a list of 2–5 revisions suggested by the class in the previous exercise for all to see. Remind students that norms are agreements that everyone in the class shares responsibility for, so everyone needs to understand and agree to work on upholding the norms. Briefly discuss any revisions and make changes to the "Norms" sections of the chart as the class agrees. Depending on the level of agreement or disagreement, it may not be possible to discuss all suggested revisions at this time. If that happens, plan to discuss the remaining suggestions over the next few lessons.

Tell students that the class now has an initial list of norms or "hopes" for how the classroom math community will work together throughout the school year. This list is just a start, and over the year it will be revised and improved as students in the class learn more about each other and about themselves and math learners.

Activity 1
Visiting the State Park

15 min

Activity Narrative

This activity provides the first example of a relationship that is not proportional. The second question focuses students' attention on the unit rates. If the relationship were proportional then regardless of the number of people in a vehicle, the cost per person would be the same. The question about the bus is to show students that they can't just scale up from 10. Students who write an equation also see that it is not of the form $y = kx$. In a later lesson students will learn that only equations of this form represent proportional relationships.

Monitor for students who approached this problem using different representations.

Launch

Keep students in the same groups. Give students 5 minutes of quiet work time, followed by partner and whole-class discussion.

Student Task Statement

Entrance to a state park costs $6 per vehicle, plus $2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in dollars
2	10
4	14
10	26

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

With 2 people, $5.00; with 4 people, $3.50; with 10 people, $2.60

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 1)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

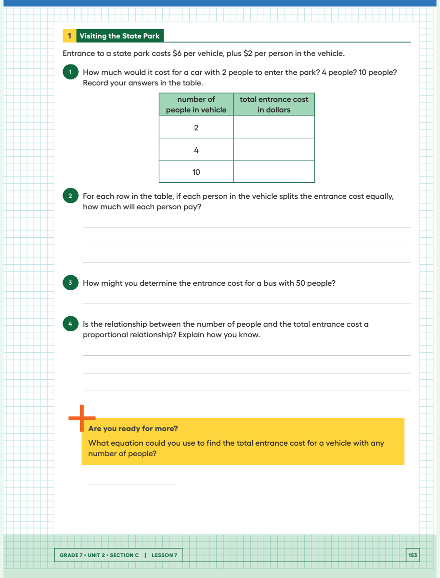
Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.

Represent the problem in multiple ways to support understanding of the situation. For example, by acting out or having students draw quick sketches of the different cases. In either case, be sure students have accounted for the cost of the vehicle in their calculations of the total entrance cost.

Supports accessibility for: Fine Motor Skills, Organization

Student Workbook


1 Visiting the State Park

Entrance to a state park costs $6 per vehicle, plus $2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in dollars
2	
4	
10	

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

3. How might you determine the entrance cost for a bus with 50 people?

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

Are you ready for more?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

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3. How might you determine the entrance cost for a bus with 50 people?

   $106. It still costs $6 for the vehicle, plus $2 for each of 50 people.

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

   No

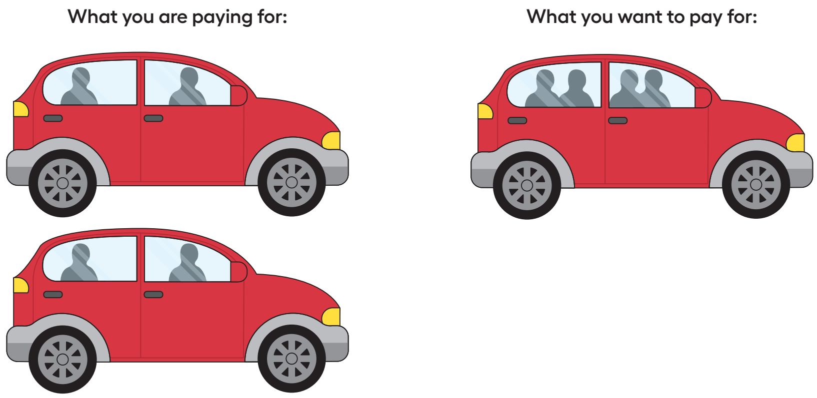
Sample reasonings:

- The ratios of people in the vehicle to total entrance cost are not equivalent.
- The cost per person is not the same for different numbers of people.
- There is not a consistent unit rate between number of people and total entrance cost.

Building on Student Thinking

Some students may not account for the cost of the vehicle. They will get the following table with incorrect values and will need to be prompted to include the cost of the vehicle.

number of people in vehicle	total entrance cost in dollars
2	10
4	20
10	50



Teachers will want to circulate around the room keeping an eye out for this mistake and address it as soon as possible so that students spend most of their work time analyzing the nonproportional relationship. These diagrams may be helpful in illustrating to them that their resulting prices include more than one vehicle. This gives them an opportunity to make sense of problems and persevere in solving them.

Are You Ready for More?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

$c = 6 + 2p$, where p is the number of people and c is the total entrance cost in dollars.

Activity Synthesis

The goal of this discussion is to highlight how we know that the relationship is not proportional. Select students to explain why they think the relationship is or is not proportional. Some reasons they could give include:

- The cost per person is different for different numbers of people in a vehicle. In other words, the quotients of the values in each row are not equal for all rows of the table.
- The ratios of people in the vehicle to total entrance cost are not equivalent. We can't multiply the entries in one row by a scale factor to get the entries in another row.
- We can't multiply the entries in the first column by the same number (constant of proportionality) to get the numbers in the second column.

Students who found an equation will also note that the equation is not of the same form as other equations, but they can't use this as a criterion until the class has established that only equations of this form represent proportional relationships. (This part of the discussion should come at the end of the next lesson, after students have analyzed lots of different equations.)

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

💬 ***"Yes, the relationship is proportional. It says it costs $2 per person, so the constant of proportionality is 2."***

Ask,

💬 ***"What parts of this response are unclear, incorrect, or incomplete?"***

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Activity 2
Running Laps

15 min

Activity Narrative

The purpose of this activity is to understand that discrete values in a table can be used to know for sure that a relationship is not proportional. However, they can't be used to know for sure that a relationship definitely is proportional. There could be other values in the relationship, which are not shown on the table, that don't fit the pattern.

This activity builds on previous ones involving constant speed but it analyzes pace (minutes per lap) rather than speed (laps per minute). Explaining why the information given in the table is enough to conclude that Han didn't run at a constant pace but is not enough to know for sure whether Clare ran at a constant pace requires students to make a viable argument.

Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Instructional Routines

MLR5: Co-Craft Questions

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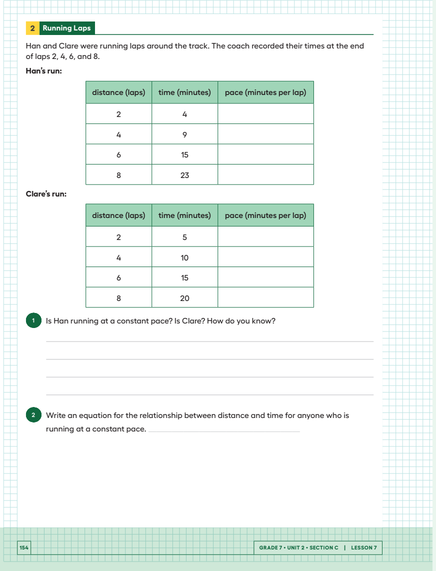
Building on Student Thinking

Students are likely to answer that Clare is running at a constant pace because the minutes per lap shown in the table are the same for each lap. Because we only have four data points in a table, spaced at 5-minute intervals, Clare could still be speeding up and slowing down between the recorded times. However, given the data, it is reasonable to assume Clare is running at a constant pace for the purpose of estimating times or distances.

Student Workbook


2 Running Laps

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	5	
4	10	
6	15	
8	20	

1 Is Han running at a constant pace? Is Clare? How do you know?

2 Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

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Launch

Keep students in the same groups of 2. Introduce the context of running laps. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

- Display only the problem stem and tables, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.
- Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, "What do these questions have in common? How are they different?" Listen for and amplify language related to the learning goal, such as "constant pace," "not constant," "proportional," or "constant of proportionality."
- Reveal the question(s) and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

"Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it."

"How do your questions relate to deciding whether a relationship is proportional?"

Give 4–5 minutes of quiet work time, followed by partner and whole-class discussion.

Student Task Statement

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	4	2
4	9	2.25
6	15	2.5
8	23	2.875

Clare's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	5	2.5
4	10	2.5
6	15	2.5
8	20	2.5

1. Is Han running at a constant pace? Is Clare? How do you know?

   Han is not running at a constant pace because the numbers in the third column are not the same. Clare appears to be running at a constant pace. At the times recorded in the table, the minutes per lap are the same, but that does not guarantee that Clare's pace is constant between the recorded times. For example, she might stand still for half a minute, then complete a lap in 2 minutes.

2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

   $t = 2.5d$, where t represents time in minutes and d represents distance in laps.

Activity Synthesis

Invite students to explain why they think each person is or is not running at a constant pace. Point out to students that although the data points in the table for Clare are pairs in a proportional relationship, these four pairs of values do not guarantee that Clare ran at a constant pace. She might have, but it is unknown if she was running at a constant pace between the times that the coach recorded.

Ask the following questions:

"Can you represent either relationship with an equation?"

The answer for Han is "no" and the answer for Clare is "yes, if she really ran at a constant pace between the points in time when the times were recorded."

"What equation can we write to represent Clare's pace in minutes per lap?"

$t = 2.5d$

"In the table for Clare's run, do the pairs of values still form a proportional relationship if we calculate laps per minute instead of minutes per lap?"

yes

"What equation can we write to represent that relationship in laps per minute?"

$d = 0.4t$

Lesson Synthesis

Share with students,

"Today we learned some ways to tell whether a table could represent a proportional relationship."

To review how to determine whether a table represents a proportional relationship, consider asking students:

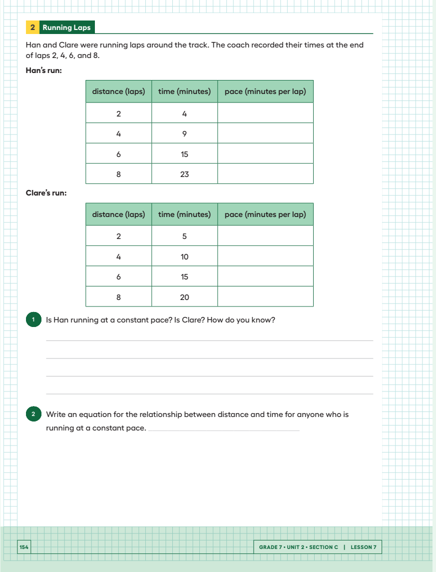
"What operation did we use to calculate the unit rate for each row of the table?"

division

"What do we know about the relationship if the quotient is the same for each row in the table?"

The table could represent a proportional relationship.

Student Workbook



2 Running Laps

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	pace (minutes per lap)
2	5	
4	10	
6	15	
8	20	

1. Is Han running at a constant pace? Is Clare? How do you know?

2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace. ______

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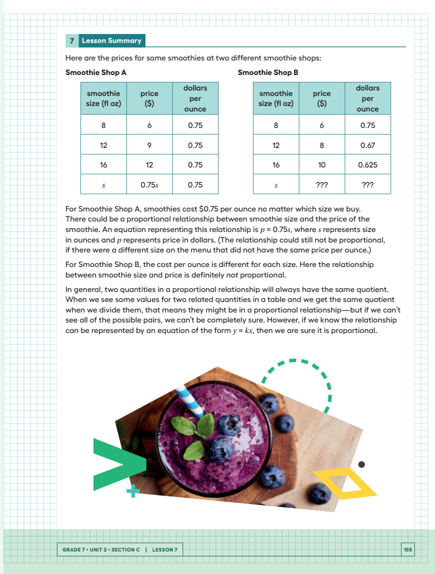
Student Workbook



"What do we know about the relationship if all the quotients are not the same?"

The table definitely does not represent a proportional relationship.

If desired, use this example to review these concepts:

number of people	cost in dollars	dollars per person
2	10	5
3	15	5
4	20	5
5	20	4

The table shows the cost to attend a choir concert. The first three rows have the same quotient, but the last row has a different quotient.

"Is the relationship between the number of people and the cost in dollars proportional? Why or why not?"

No, because there is not one constant of proportionality that relates the values on every row of the table.

Lesson Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (fl oz)	price ($)	dollars per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (fl oz)	price ($)	dollars per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

For Smoothie Shop A, smoothies cost $0.75 per ounce no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is $p = 0.75s$, where s represents size in ounces and p represents price in dollars. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.)

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be completely sure. However, if we know the relationship can be represented by an equation of the form $y = kx$, then we are sure it is proportional.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the question "What is one of our classroom norms that is a strength for you? Why?" Tell students that as a culmination to establishing the initial list of mathematical community norms, they are now asked to share one norm they think will be a strength for them. To help students understand what the question is asking, share a personal example. For example,

"I think that 'Ask clarifying questions' is a norm that is a strength for me because I am good at asking questions when I don't think I understand how someone else is thinking about a problem. Instead of just telling you what I think you should do, I make sure to ask questions until I understand what YOU are doing."

Display these prompts for all to see:

"One of our classroom norms that will be a strength for me is _____."

"I think this will be a strength for me because _____."

Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify which norms students feel more confident about and which norms were not listed as strengths by many students. In some cases, students may not think a norm is a strength because they are not sure what that norm looks like or sounds like. So, focus on identifying those norms in the class when they happen.

For example, during group work students ask a quiet group member which representation they prefer, and that student shares a third representation that the group had not even considered. Asking the quiet student illustrates a norm like "we invite others into the math." Pointing out that action when it happens helps students understand the norm and see how it can benefit the math thinking of the entire group. This understanding and appreciation can promote the use of that norm in the math community.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Apples and Pizza

5 min

Student Task Statement

1. Based on the information in the table, is the cost of the apples proportional to the weight of apples?

pounds of apples	cost of apples
2	\$3.76
3	\$5.64
4	\$7.52
5	\$9.40

Yes, because the cost per pound of apples is the same in each row, 1.88 dollars per pound.

2. Based on the information in the table, is the cost of the pizza proportional to the number of toppings?

number of toppings	cost of pizza
2	\$11.99
3	\$13.49
4	\$14.99
5	\$16.49

No, because the cost per topping is not the same in each row. (An equation is $C = 1.50T + 8.99$ but students do not need to provide an equation.)

3. Write an equation for the proportional relationship.

$c = 1.88p$, where c represents the cost of the apples and p represents the pounds of apples.

Lesson 7

Practice Problems

5 Problems

Problem 1

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

a. How loud a sound is.

distance from listener (ft)	sound level (dB)
5	85
25	79
35	73
40	67

Not proportional
The ratio of distance from listener to sound level is not always the same.

b. The cost of fountain drinks at Hot Dog Hut.

volume (fl oz)	cost ($)
16	1.49
20	1.59
30	1.89

Not proportional
The ratio of volume to cost is not always the same.

Problem 2

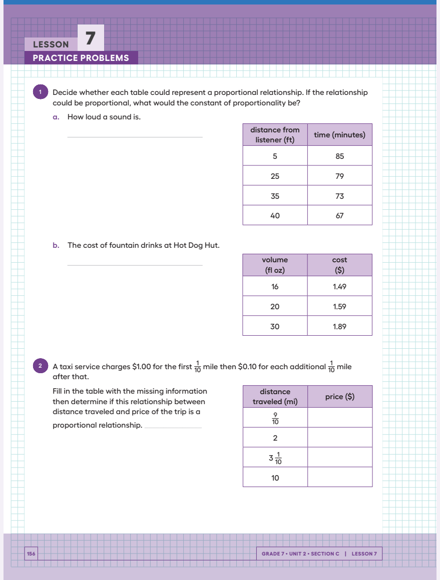
A taxi service charges $1.00 for the first $\frac{1}{10}$ mile then $0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

distance traveled (mi)	price ($)
$\frac{9}{10}$	1.80
2	2.90
$3\frac{1}{10}$	4.00
10	10.90

This is not a proportional relationship since the ratio of price to distance traveled is not always the same.

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1 Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

a. How loud a sound is.

distance from listener (ft)	time (minutes)
5	85
25	79
35	73
40	67

b. The cost of fountain drinks at Hot Dog Hut.

volume (fl oz)	cost ($)
16	1.49
20	1.59
30	1.89

2 A taxi service charges $1.00 for the first $\frac{1}{10}$ mile then $0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

distance traveled (mi)	price ($)
$\frac{9}{10}$	
2	
$3\frac{1}{10}$	
10	

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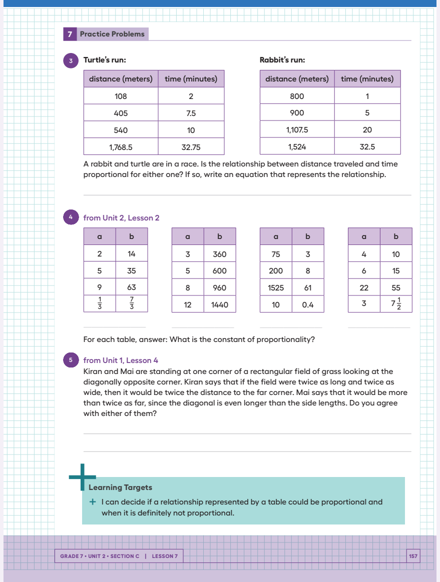
Student Workbook



Problem 3

Turtle's run:

distance (meters)	time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Rabbit's run:

distance (meters)	time (minutes)
800	1
900	5
1,107.5	20
1,524	32.5

A rabbit and turtle are in a race. Is the relationship between distance traveled and time proportional for either one? If so, write an equation that represents the relationship.

The distance might be proportional to the time for the turtle.
The equation would be $d = 54 \cdot t$, where d represents the distance traveled in meters and t is the time in minutes.

Problem 4

from Unit 2, Lesson 2

For each table, answer: What is the constant of proportionality?

a	b
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

7

a	b
3	360
5	600
8	960
12	1400

120

a	b
75	3
200	8
1525	61
10	0.4

$\frac{1}{25}$ (or equivalent)

a	b
4	10
6	15
22	55
3	$7\frac{1}{2}$

$2\frac{1}{2}$ (or equivalent)

Problem 5

from Unit 1, Lesson 4

Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

Kiran is correct.
If we scale the length and width of a rectangle by a factor of 2, then the diagonal will also scale by a factor of 2.