Lesson 6

Distinguishing between Two Types of Situations

Goals

- Categorize equations of the forms $px + q = r$ and $p(x + q) = r$, and describe (orally) the categories.
- Interpret a verbal description of a situation (in written language), and write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent it.

Learning Targets

- I understand the similarities and differences between the two main types of equations we are studying in this unit.
- When I have a situation or a tape diagram, I can represent it with an equation.

Lesson Narrative

The purpose of this lesson is to distinguish between equations of the form $px + q = r$ and $p(x + q) = r$. Corresponding tape diagrams are used as tools in this work, along with situations that these equations can represent. First, students sort equations into categories of their choosing. The main categories to highlight distinguish between the two main types of equations being studied. Then students consider two stories and corresponding diagrams and write equations to represent them. They use these representations to find an unknown value in the story.

Student Learning Goal

Let's think about equations with and without parentheses and the kinds of situations they describe.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Instructional Routines

- Card Sort
- Take Turns
- Which Three Go Together?

Required Materials

Materials to Copy

- Categories of Equations Cards (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

- Print and cut up copies of the blackline master ahead of time. You will need 1 set for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

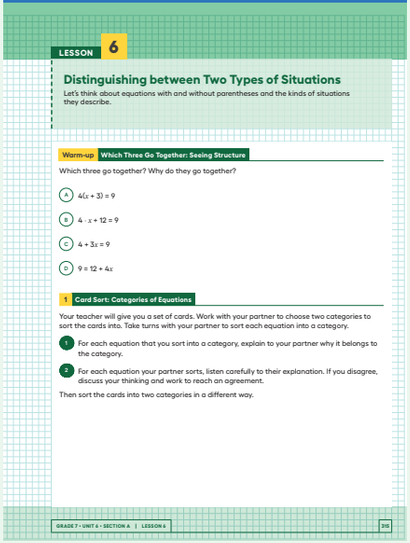
Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

Please log in to the site before using the QR code or URL.

Student Workbook



LESSON 6

Distinguishing between Two Types of Situations

Let's think about equations with and without parentheses and the kinds of situations they describe.

Warm-up Which Three Go Together: Seeing Structure

Which three go together? Why do they go together?

A. $4(x + 3) = 9$

B. $4 \cdot x + 12 = 9$

C. $4 + 3x = 9$

D. $9 = 12 + 4x$

1 Card Sort: Categories of Equations

Your teacher will give you a set of cards. Work with your partner to choose two categories to sort the cards into. Take turns with your partner to sort each equation into a category.

1. For each equation that you sort into a category, explain to your partner why it belongs to the category.
2. For each equation your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Then sort the cards into two categories in a different way.

GRADE 7 • UNIT 6 • SECTION A | LESSON 6 310

Warm-up

Which Three Go Together: Seeing Structure

5 min

Activity Narrative

This *Warm-up* prompts students to compare four equations. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the equations in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the equations for all to see.

Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three equations that go together and can explain why.

Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $4(x + 3) = 9$

B. $4 \cdot x + 12 = 9$

C. $4 + 3x = 9$

D. $9 = 12 + 4x$

Sample responses:

- A, B, and C go together because they all have the total, 9, on the right side of the equal sign.
- A, B, and D go together because they are all equivalent (and they all have negative solutions).
- A, C, and D go together because they all use "next to" notation for multiplication.
- B, C, and D go together because they have two terms that are added (there are no parentheses).

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "times," "plus," "minus," "distribute," "dot," or "same answer," and to clarify their reasoning as needed.

Consider asking:

- *"How do you know … ?"*
- *"What do you mean by … ?"*
- *"Can you say that in another way?"*

Activity 1
Card Sort: Categories of Equations

15 min

Activity Narrative

In this partner activity, students take turns sorting equations into categories of their own choosing. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Tell students that the cards contain equations. They will work with their partner to create categories, and then take turns sorting the cards into the categories. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group one set of slips cut from the blackline master.

Student Task Statement

Your teacher will give you a set of cards. Work with your partner to choose two categories to sort the cards into. Take turns with your partner to sort each equation into a category.

1. For each equation that you sort into a category, explain to your partner why it belongs to the category.
2. For each equation your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Then sort the cards into two categories in a different way.

Categories vary.

Activity Synthesis

Once all groups have completed the *Card Sort*, discuss:

- *"How do you describe the categories you chose?"*
- *"Did you need to make adjustments in your categories? What adjustments were made and why?"*
- *"Were any equations difficult to categorize? What made them difficult?"*

The purpose of this discussion is to uncover that equations of the form $px + q$ have a different structure than those of the form $p(x + q)$. Select 2–3 groups to share their categories and how they decided to sort their equations. Discuss as many different sets of categories as the time allows.

Instructional Routines

Card Sort

ilclass.com/r/10783726

Please log in to the site before using the QR code or URL

Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

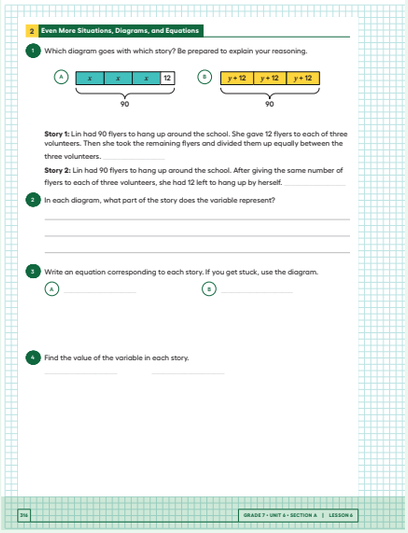
Access for Students with Diverse Abilities (Activity 2, Task Statement)

Representation: Develop Language and Symbols.

Support understanding of the problem by inviting students to act out each story. For example, have one student hand copies of a flyer to three other students while saying a phrase like "twelve for you." Consider clarifying the meaning of the word "flyer" in this context.

Supports accessibility for: Conceptual Processing, Language

Student Workbook


2 Even More Situations, Diagrams, and Equations

1 Which diagram goes with which story? Be prepared to explain your reasoning.

A: x x x 12 — 90 B: y + 12 y + 12 y + 12 — 90

Story 1: Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Story 2: Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

2 In each diagram, what part of the story does the variable represent?

3 Write an equation corresponding to each story. If you get stuck, use the diagram.

A B

4 Find the value of the variable in each story.

214 GRADE 7 • UNIT 6 • SECTION A | LESSON 6

Activity 2

Even More Situations, Diagrams, and Equations

15 min

Activity Narrative

In this activity, students match two tape diagrams with two situations, explain what the variable represents, write equations, and then solve them. This activity is an opportunity to put together the learning of the past several lessons: correspondences between tape diagrams, equations, and stories, and using representations to reason about a solution. The focus of this activity is still contrasting the two main types of equations that students encounter in this unit. In order to coordinate the tape diagrams with the situations and write equations, students need to notice and make use of structure.

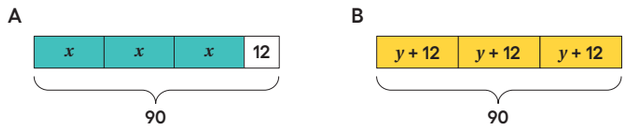
Launch

Keep students in the same groups.

Give 5 minutes of quiet work time followed by sharing with a partner and a whole-class discussion.

Student Task Statement

1. Which diagram goes with which story? Be prepared to explain your reasoning.



Story 1: Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Story 2: Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

Diagram A goes with Story 2, and Diagram B goes with Story 1.

2. In each diagram, what part of the story does the variable represent?

In Diagram A, x represents the number of flyers Lin gave to each volunteer. In Diagram B, y represents the remaining flyers Lin gave to each volunteer (after giving each of them 12 to start).

3. Write an equation corresponding to each story. If you get stuck, use the diagram.

Story 1: $3(y + 12) = 90$ (or equivalent), Story 2: $3x + 12 = 90$ (or equivalent)

4. Find the value of the variable in the story.

Story 1: $y = 18$

Story 2: $x = 26$.

Activity Synthesis

The purpose of this discussion is to understand why a particular diagram and equation represents a situation, and why the solution is true. For each story, select 1 or more groups to present the matching diagram, their equation, and their solution method. Possible questions to ask:

- *"How were the diagrams alike? How were they different?"*

  They have the same numbers and a variable. One has 3 equal groups and an extra bit, the other just has 3 equal groups, but each group is a sum.

- *"How were the stories alike? How were they different?"*

  They were both about distributing 90 flyers. In one story, Lin makes a series of moves to each volunteer. In the other story, she gives each volunteer the same amount, but then there are some left over.

- *"What parts of the story made you think that one diagram represented it?"*
- *"Explain how you reasoned about the story, diagram, or equation to find the value of the variable."*

Lesson Synthesis

Display the two equations from the last activity for all to see:

$$3x + 12 = 90$$

$$3(y + 12) = 90$$

Tell students,

- *"These equations have lots of things in common. They each have a 3, a variable, a 12, a 90, an equal sign, multiplication, and addition. How are these equations different?"*

Ask students to think about this question quietly for a moment and share with a partner, then ask a few students to share with the whole class.

Highlight any responses that speak in general terms about the structure of the equations. Examples:

- One equation is the sum of a product and a number and the other is the product of a sum.
- If we substituted a value for the variable in one equation, we would multiply it by 3 first and then add 12. For the other, we would add 12 first and then multiply by 3.
- One has three equal groups and an extra bit, and the other just has 3 equal groups, but the groups are each the result of adding 12 to an unknown.

Lesson Summary

In this lesson, we encountered two main types of situations that can be represented with an equation. Here is an example of each type:

1. After adding 8 students to each of 6 same-sized teams, there were 72 students altogether.
2. After adding an 8-pound box of tennis rackets to a crate with 6 identical boxes of table tennis paddles, the crate weighed 72 pounds.

Student Workbook

6 Lesson Summary

In this lesson, we encountered two main types of situations that can be represented with an equation. Here is an example of each type:

1. After adding 8 students to each of 6 same-sized teams, there were 72 students altogether.
2. After adding an 8-pound box of tennis rackets to a crate with 6 identical boxes of table tennis paddles, the crate weighed 72 pounds.

The first situation has all equal parts, since additions are made to *each* team. An equation that represents this situation is $6(x + 8) = 72$, where x represents the original number of students on each team. Eight students were added to each group, there are 6 groups, and there are a total of 72 students.

In the second situation, there are 6 equal parts added to one other part. An equation that represents this situation is $6x + 8 = 72$, where x represents the weight of each box of table tennis paddles. There are 6 boxes of table tennis paddles, an additional box that weighs 8 pounds, and the crate weighs 72 pounds altogether.

In the first situation, there were 6 equal groups, and 8 students added to each group.

$6(x + 8) = 72$.

In the second situation, there were 6 equal groups, but 8 more pounds in addition to that.

$6x + 8 = 72$.

GRADE 7 • UNIT 6 • SECTION A | LESSON 6 107

Responding To Student Thinking

Points to Emphasize

If most students struggle with interpreting a situation and writing an equation to match, plan to focus on strategies when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share how they represented situations with equations in the following lesson:

Grade 7, Unit 6, Lesson 11 Using Equations to Solve Problems

The first situation has all equal parts, since additions are made to *each* team. An equation that represents this situation is $6(x + 8) = 72$, where x represents the original number of students on each team. Eight students were added to each group, there are 6 groups, and there are a total of 72 students.

In the second situation, there are 6 equal parts added to one other part. An equation that represents this situation is $6x + 8 = 72$, where x represents the weight of each box of table tennis paddles. There are 6 boxes of table tennis paddles, an additional box that weighs 8 pounds, and the crate weighs 72 pounds altogether.

In the first situation, there were 6 equal groups, and 8 students added to each group. $6(x + 8) = 72$.

In the second situation, there were 6 equal groups, but 8 more pounds in addition to that. $6x + 8 = 72$.

Cool-down

After-School Tutoring

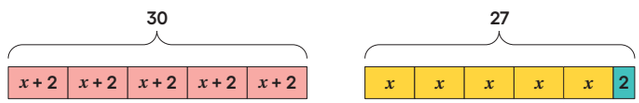
5 min

Student Task Statement

Write an equation for each story. Then find the number of problems originally assigned by each teacher. If you get stuck, try drawing a diagram to represent the story.

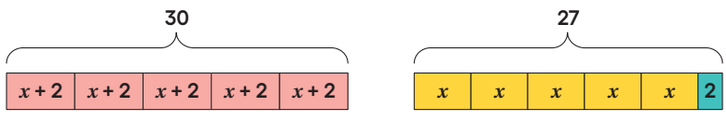
1. Five students came for after-school tutoring. Lin's teacher assigned each of them the same number of problems to complete. Then he assigned each student 2 more problems. In all, 30 problems were assigned.

   $5(x + 2) = 30$ (or equivalent), solution: $x = 4$; The teacher originally assigned 4 problems to each student.



2. Five students came for after-school tutoring. Priya's teacher assigned each of them the same number of problems to complete. Then she assigned 2 more problems to one of the students. In all, 27 problems were assigned.

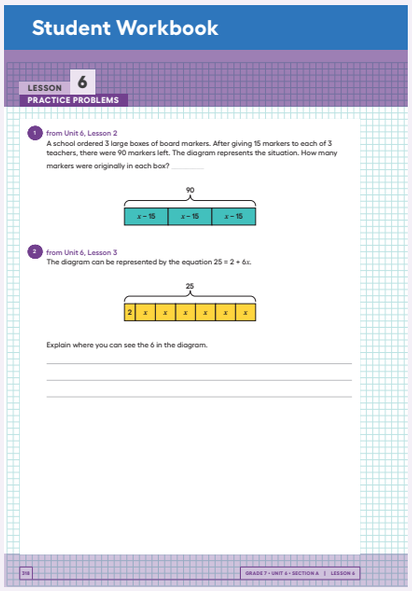
   $5x + 2 = 27$ (or equivalent), solution: $x = 5$; The teacher originally assigned 5 problems to each student.



Lesson 6

Practice Problems

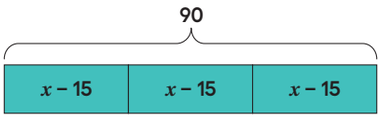
4 Problems



Problem 1

from Unit 6, Lesson 2

A school ordered 3 large boxes of board markers. After giving 15 markers to each of 3 teachers, there were 90 markers left. The diagram represents the situation. How many markers were originally in each box?

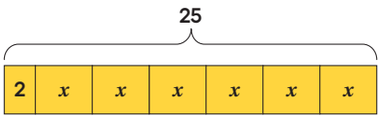


45 markers

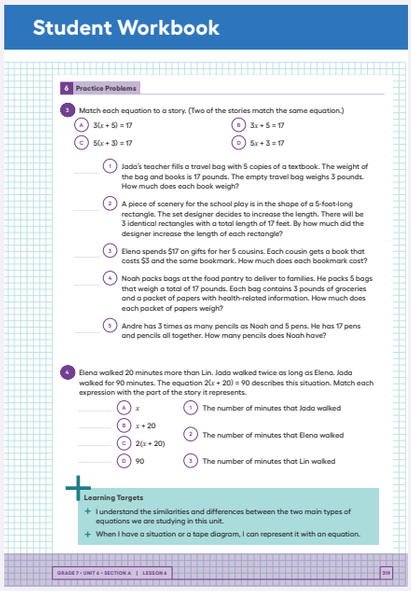
Problem 2

from Unit 6, Lesson 3

The diagram can be represented by the equation $25 = 2 + 6x$. Explain where you can see the 6 in the diagram.



Sample response: There are 6 equal parts labeled x.


Student Workbook

6 Practice Problems

3 Match each equation to a story. (Two of the stories match the same equation.)

A 3(x + 5) = 17
B 3x + 5 = 17
C 5(x + 3) = 17
D 5x + 3 = 17

1 Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?

2 A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The set designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?

3 Elena spends $17 on gifts for her 5 cousins. Each cousin gets a book that costs $3 and the same bookmark. How much does each bookmark cost?

4 Noah packs bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?

5 Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

4 Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation 2(x + 20) = 90 describes this situation. Match each expression with the part of the story it represents.

A x
B x + 20
C 2(x + 20)
D 90

1 The number of minutes that Jada walked
2 The number of minutes that Elena walked
3 The number of minutes that Lin walked

Learning Targets
+ I understand the similarities and differences between the two main types of equations we are studying in this unit.
+ When I have a situation or a tape diagram, I can represent it with an equation.

GRADE 7 • UNIT 6 • SECTION A | LESSON 6 209

Problem 3

Match each equation to a story. (Two of the stories match the same equation.)

A. $3(x + 5) = 17$ **B.** $3x + 5 = 17$

C. $5(x + 3) = 17$ **D.** $5x + 3 = 17$

D **1.** Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?

A **2.** A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The set designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?

C **3.** Elena spends $17 on gifts for her 5 cousins. Each cousin gets a book that costs $3 and the same bookmark. How much does each bookmark cost

C **4.** Noah packs bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?

B **5.** Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

Problem 4

Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation $2(x + 20) = 90$ describes this situation. Match each expression with the part of the story it represents.

3 **A.** x

2 **B.** $x + 20$

1 **C.** $2(x + 20)$

1 **D.** 90

1. The number of minutes that Jada walked

2. The number of minutes that Elena walked

3. The number of minutes that Lin walked