Lesson 18

Surface Area of a Cube

Goals

- Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as $6 \cdot s^2$.
- Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents.
- Write expressions, with or without exponents, to represent the surface area of a given cube.

Learning Targets

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson Narrative

In this lesson, students practice using the exponents "2" and "3" to express products that describe surface areas and volumes of cubes. They also use exponents to write square units and cubic units.

Students begin by writing numerical expressions to represent the surface areas and volumes of cubes with whole-number side lengths. Then, they generalize their observations to write variable expressions for the surface area and volume of any cube.

As they write numerical expressions, students practice looking for and making use of structure. They also practice looking for and expressing regularity in repeated reasoning to write formulas for the surface area and the volume of a cube.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- 5 Practices
- Math Talk
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Geometry toolkits: Activity 1, Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Lesson 18

Surface Area of a Cube

Math Community

The goal of today's exercise is to use the suggestions from the previous exercise to revise the "Norms" sections of the Math Community Chart and to invite students to reflect on one norm that will be a strength for them. Both activities begin to build shared accountability for and investment in the classroom norms.

Student Learning Goal

Let's write a formula to find the surface area of a cube.

Instructional Routines

Math Talk

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MLR8: Discussion Supports

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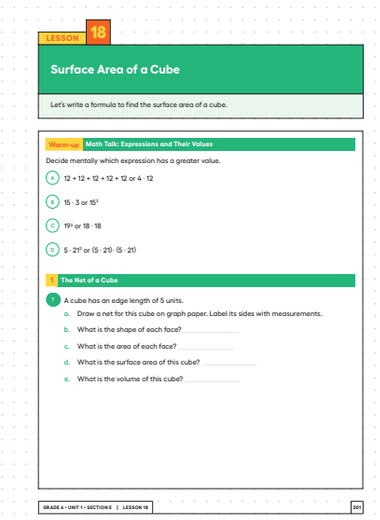
Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Building on Student Thinking

When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. Remind them about the meaning of the exponent notation. For example, show that $5 \cdot 3 = 15$, which is much smaller than $5 \cdot 5 \cdot 5$, which equals 125

Student Workbook


LESSON 18

Surface Area of a Cube

Let's write a formula to find the surface area of a cube.

Warm-up Math Talk: Expressions and Their Values

Decide mentally which expression has a greater value.

A. $12 + 12 + 12 + 12 + 12$ or $4 \cdot 12$

B. $15 \cdot 3$ or 15^3

C. 19^2 or $18 \cdot 18$

D. $5 \cdot 21^2$ or $(5 \cdot 21) \cdot (5 \cdot 21)$

1 The Net of a Cube

1. A cube has an edge length of 5 units.
   a. Draw a net for this cube on graph paper. Label its sides with measurements.
   b. What is the shape of each face?
   c. What is the area of each face?
   d. What is the surface area of this cube?
   e. What is the volume of this cube?

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Warm-up

Math Talk: Expressions and Their Values

5 min

Activity Narrative

This *Math Talk* focuses on the meaning of numbers and symbols in expressions. It encourages students to relate repeated addition to multiplication and to relate repeated multiplication to exponents. The numbers are selected to discourage students from computing the values of the expressions. Instead, they prompt students to rely on what they know about operations and exponents to mentally make comparisons. The understanding elicited here will be helpful later in the lesson when students write expressions to represent the surface area and volume of cubes.

To decide, without calculations, which of two expressions has the greater value, students need to look for and make use of structure. In explaining their reasoning, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally which expression has a greater value.

A. $12 + 12 + 12 + 12 + 12$ or $4 \cdot 12$

$12 + 12 + 12 + 12 + 12$
Sample reasoning: This expression is equivalent to $5 \cdot 12$, which is greater than $4 \cdot 12$

B. $15 \cdot 3$ or 15^3

15^3
Sample reasoning: 15^3 is $15 \cdot 15 \cdot 15$, so it is greater than $15 \cdot 3$, which is $15 + 15 + 15$

C. 19^2 or $18 \cdot 18$

19^2
Sample reasoning: 19^2 is $19 \cdot 19$, which is greater than $18 \cdot 18$

D. $5 \cdot 21^2$ or $(5 \cdot 21) \cdot (5 \cdot 21)$

$(5 \cdot 21) \cdot (5 \cdot 21)$
Sample reasoning: $5 \cdot 21$ is more than 100, so multiplying this number by itself will give a number greater than 10,000. Squaring 21 gives a number that is a little more than 400. Multiplying that number by 5 gives a product that is more than 2,000 but not anywhere near 10,000

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

To support students in upcoming work, highlight the following ideas if they are not already mentioned by students:

- We can express repeated addition with multiplication. $5 \cdot 12$ is a more concise way to write $12 + 12 + 12 + 12 + 12$
- We can express repeated multiplication with an exponent. 15^3 is a more concise way to write $15 \cdot 15 \cdot 15$
- The parentheses in the last expression tells us that it is the value of $5 \cdot 21$, not just one of the numbers, that is being squared.

Math Community

Display the Math Community Chart and a list of 2–5 revisions suggested by the class in the previous exercise for all to see. Remind students that norms are agreements that everyone in the class shares responsibility for, so everyone needs to understand and agree to work on upholding the norms. Briefly discuss any revisions and make changes to the "Norms" sections of the chart as the class agrees. Depending on the level of agreement or disagreement, it may not be possible to discuss all suggested revisions at this time. If that happens, plan to discuss the remaining suggestions over the next few lessons.

Tell students that the class now has an initial list of norms or "hopes" for how the classroom math community will work together throughout the school year. This list is just a start, and over the year it will be revised and improved as students in the class learn more about each other and about themselves and math learners.

Activity 1

The Net of a Cube

20 min

Activity Narrative

This activity serves two goals. One goal is to allow students to practice drawing a net and finding the surface area and volume of a cube. The other goal is to encourage students to write expressions to represent the surface area and volume of a cube.

In the first set of questions, the edge length of the cube is a small number (5), enabling students to compute the surface area and volume numerically. Use students' work here to check that they are drawing a net correctly. In the next set of questions, the edge length of the cube is a larger number (17), making computation more cumbersome and prompting students to build expressions rather than evaluating them.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Instructional Routines

5 Practices

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.
Activate or supply background knowledge. To help students recall the meanings of the terms "net," "surface area," and "volume," ask, "How is a net related to a three-dimensional shape?", "What parts of a shape contribute to its surface area?", or "How are volume and surface area different?"
Supports accessibility for: Memory, Language

Building on Student Thinking

Students might think the surface area is $(17 \cdot 17)^6$. Prompt students to write down how they would compute surface area step by step, before trying to encapsulate their steps in an expression. Dissuade students from using calculators in the last two problems and assure them that building an expression does not require extensive computation. Students may think that refraining from using a calculator means performing all calculations—including those of larger numbers—on paper or mentally, especially if they are unclear about the meaning of the term "expression." Ask them to refer to the expressions in the *Warm-up*, or share examples of expressions in a few different forms, to help them see how surface area and volume can be expressed without computation.

As students work on the questions about a cube with a 17-inch edge length, monitor for students who write different expressions for the second question. Here are some expressions they may write, from longer (more expanded) to shorter (more succinct):

- Products, such as $17 \cdot 17$, or $17 \cdot 17 \cdot 17$
- Sums of products, such as $(17 \cdot 17) + (17 \cdot 17) + \ldots$
- Combination of like terms, such as $6 \cdot (17 \cdot 17)$
- Exponents, such as $17^2 + 17^2 + \ldots$, or 17^3
- Completed calculations, such as 1,734 or 4,913

A note about notation:

In a later unit, students will learn that $5 \cdot x$ means the same as $5x$. At this point, expect them to write $6 \cdot 17^2$ instead of $6(17^2)$. It is not critical that they understand that a number placed next to a variable (or a number placed next to an expression in parentheses) are being multiplied.

Launch

Arrange students in groups of 2. Give students access to their geometry toolkits. Tell students to use graph paper to draw a net of the first cube and to try to answer the questions without using a calculator. Give students 8–10 minutes of quiet work time followed by 1–2 minutes to share their responses with their partner.

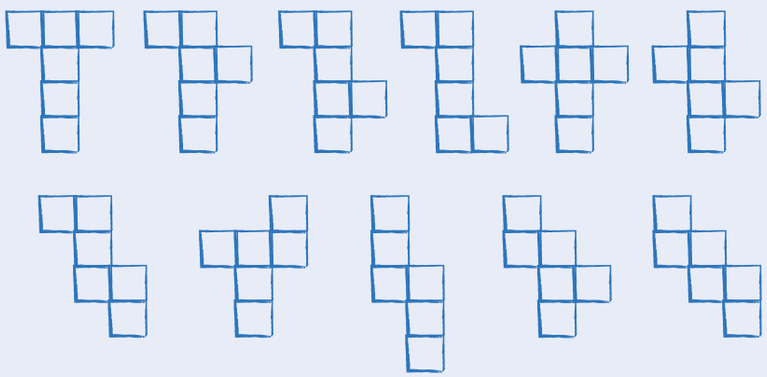
Select students with different expressions, such as those described in the activity narrative, to share later.

Student Task Statement

1. A cube has an edge length of 5 units.

a. Draw a net for this cube on graph paper. Label its sides with measurements.

Eleven unique nets are possible.



b. What is the shape of each face?

A square

c. What is the area of each face?

25 square units

d. What is the surface area of this cube?

150 square units

e. What is the volume of this cube?

125 cubic units

2. A second cube has an edge length of 17 units.

a. Sketch a net for this cube. Label its sides with measurements.

Drawing should be one of the 11 nets shown in the previous problem.

b. Explain why the area of each face of this cube is 17^2 square units.

Sample response: The side length of each square face is 17 units, so its area is $17 \cdot 17$, or 17^2 square units.

c. Write an expression for the surface area, in square units.

$6 \cdot 17^2$ (or equivalent)

d. Write an expression for the volume, in cubic units.

17^3 (or equivalent)

Activity Synthesis

The purpose of this discussion is to emphasize that exponents can be used to concisely express calculations for surface area and volume. Briefly discuss the answers to the questions about the cube with a 5-inch edge length. Then, focus the discussion on the larger cube with a 17-inch edge length.

Invite previously selected students to share their expressions for the last two questions. Sequence the responses in the following order to help students see how the expressions $6 \cdot 17^2$ and 17^3 come about. If any expressions are missing but are needed to illustrate the idea of writing succinct expressions, add them to the lists. Refer to parts of expressions using terms such as "sum," "product," and "factor" to support students in using mathematical terms when working with expressions.

Surface area:

- $(17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17)$
- $17^2 + 17^2 + 17^2 + 17^2 + 17^2 + 17^2$
- $6 \cdot (17 \cdot 17)$
- $6 \cdot (17^2)$
- $6 \cdot (289)$
- 1,734

Volume:

- $17 \cdot 17 \cdot 17$
- 17^3
- 4,913

Connect the expressions to the learning goals by asking questions such as:

"In each expression, where do you see the area of one face of the cube?"

$17 \cdot 17$, 17^2, 289

Instructional Routines

5 Practices

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- *"In one expression, the number 17 is written twelve times. In another expression, it is written six times. How do they both represent the same surface area?"*

  They both show the sum of the areas of the six square faces. The second expression uses the exponent 2 to represent the product of two 17s.

- *"Some expressions show addition but others don't. How do they all represent the surface area?"*

  Multiplication is used for repeated addition. Adding six of the same expression is the same as multiplying it by 6

- *"Which expression doesn't require a lot of computation and is the most efficient way to represent the surface area of the cube?"*

  $6 \cdot 17^2$

- *"Which expression doesn't require a lot of computation and is the most efficient way to represent the volume of the cube?"*

  17^3

To further encourage students to see regularity in the expressions, consider offering another example, for instance:

- *"Suppose the edge length of a cube is 38 cm. How can we express its surface area and volume?"*

  $6 \cdot (38^2)$ cm^2 and 38^3 cm^3, respectively

Using a large number for the edge length will discourage calculation and prompt students to focus on building an expression and using exponents.

If time permits, consider directing students' attention to the units of measurements. Remind students that, rather than writing $6 \cdot (17^2)$ square units, we can write $6 \cdot (17^2)$ units^2, and instead of 17^3 cubic units, we can write 17^3 units^3. Unit notations will appear again later in the course, so it can also be reinforced later.

Activity 2

Every Cube in the Whole World

10 min

Activity Narrative

In this activity, students develop the formulas for the surface area and the volume of a cube in terms of a variable edge length s.

Students should be encouraged to refer to their work in the preceding activity as much as possible and to generalize from it. As before, monitor for different ways of writing expressions for surface area and volume. The expressions are likely to be in the following forms, listed from longer (more expanded) to shorter (more succinct):

- Products, such as $s \cdot s$ or $s \cdot s \cdot s$
- Sums of products, such as $(s \cdot s) + (s \cdot s) + \ldots$
- Combination of like terms, such as $6 \cdot (s \cdot s)$
- Expressions with exponents, such as $s^2 + s^2 + \ldots$, or s^3

As they write and discuss expressions for volume and surface area, students practice looking for and making use of structure and seeing regularity through repeated reasoning.

Launch

Give students access to their geometry toolkits and 7–8 minutes of quiet think time. Tell students that they will be answering the same questions as before, but with a variable for the side length. Encourage them to use the work they did earlier to help them here.

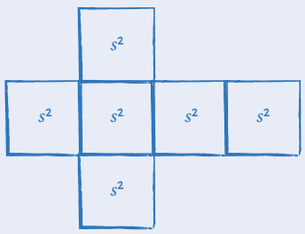
Select students who wrote different expressions to share their work later.

Student Task Statement

A cube has an edge length of s.

1. Draw a net for the cube.

   Sample response: (Each face is a square whose side lengths are s.)



2. Write an expression for the area of each face. Label each face with its area.

   The area of each face is s^2.

3. Write an expression for the surface area.

   The surface area is $6 \cdot s^2$.

4. Write an expression for the volume.

   The volume is s^3.

Activity Synthesis

To support students in seeing structure and generalizing, discuss the problems in as similar a fashion as was done in the earlier activity involving a cube with an edge length of 17 units.

Ask previously selected students to share their responses with the class. If possible, sequence the responses in the following order (as shown in the activity narrative) to help students see how the expressions $6 \cdot s^2$ and s^3 come about. If any expressions are missing but are needed to illustrate the idea of writing succinct expressions, add them to the lists. Refer to parts of expressions using terms such as "sum," "product," and "factor" to support students in using mathematical terms when working with expressions.

Surface area:

- $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$
- $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$
- $6(s \cdot s)$
- $6 \cdot (s^2)$ or $6 \cdot s^2$

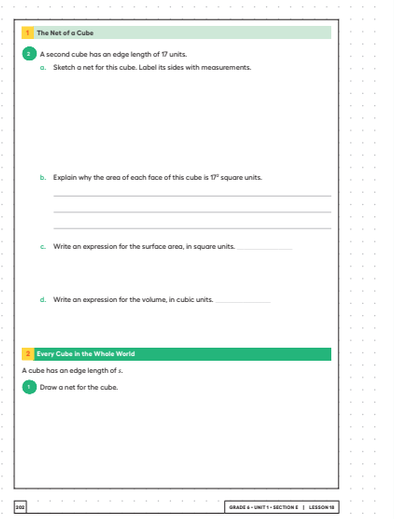
Building on Student Thinking

If students are unclear or unsure about using the variable s, explain that we are looking for an expression that would work for any edge length, and that a variable, such as s, can represent any number. The s could be replaced with any edge length in finding surface area and volume.
To connect students' work to earlier examples, point to the cube with edge length 17 units from the previous activity. Ask:

"If you wrote the surface area as $6 \cdot 17^2$ before, what should it be now?"

As students work, encourage those who may be more comfortable using multiplication symbols to instead use exponents whenever possible.

Student Workbook


1 The Net of a Cube

2 A second cube has an edge length of 17 units.

a. Sketch a net for this cube. Label its sides with measurements.

b. Explain why the area of each face of this cube is 17^2 square units.

c. Write an expression for the surface area, in square units.

d. Write an expression for the volume, in cubic units.

2 Every Cube in the Whole World

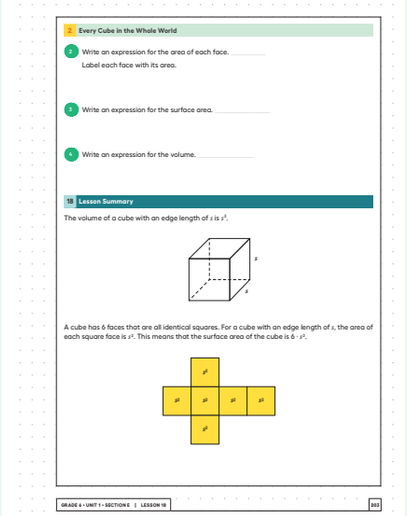
A cube has an edge length of s.

1 Draw a net for the cube.

A note about materials for an upcoming unit:

For the first lesson on the unit on ratios, students will need to bring in a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Inform or remind students about this.

Student Workbook


2 Every Cube in the Whole World

2. Write an expression for the area of each face. Label each face with its area.

3. Write an expression for the surface area.

4. Write an expression for the volume.

18 Lesson Summary

The volume of a cube with an edge length of s is s^3.

A cube has 6 faces that are all identical squares. For a cube with an edge length of s, the area of each square face is s^2. This means that the surface area of the cube is $6 \cdot s^2$.

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Volume

- $s \cdot s \cdot s$
- s^3

If students have trouble understanding where the most concise expression of surface area comes from, refer back to the example involving a numerical side length (a cube with an edge length of 17 units).

Present the surface area as $6 \cdot s^2$.

Lesson Synthesis

In this lesson, students wrote expressions for the volume and surface area of a cube. Consider asking students:

- *"A cube has an edge length of s. Why does the expression s^3 describe its volume?"*

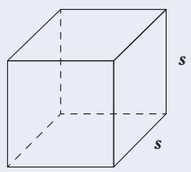
  The volume is $s \cdot s \cdot s$, which can be written as s^3

- *"Why does the expression $6 \cdot s^2$ describe its surface area?"*

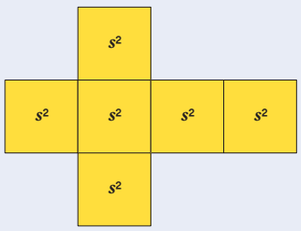
  A cube has 6 faces that are all identical squares. Each face has an area of $s \cdot s$ or s^2, so the total area for all the faces is 6 times s^2.

Lesson Summary

The volume of a cube with an edge length of s is s^3.



A cube has 6 faces that are all identical squares. For a cube with an edge length of s, the area of each square face is s^2. This means that the surface area of the cube is $6 \cdot s^2$.



Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the question "What is one of our classroom norms that is a strength for you? Why?" Tell students that as a culmination to establishing the initial list of mathematical community norms, they are now asked to share one norm they think will be a strength for them. To help students understand what the question is asking, share a personal example. For example,

> *"I think that 'Ask clarifying questions' is a norm that is a strength for me because I am good at asking questions when I don't think I understand how someone else is thinking about a problem. Instead of just telling you what I think you should do, I make sure to ask questions until I understand what YOU are doing."*

Display these prompts for all to see:

- One of our classroom norms that will be a strength for me is _____.
- I think this will be a strength for me because _____.

Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify which norms students feel more confident about and which norms were not listed as strengths by many students. In some cases, students may not think a norm is a strength because they are not sure what that norm looks like or sounds like. So, focus on identifying those norms in the class when they happen.

For example, during group work students ask a quiet group member which representation they prefer, and that student shares a third representation that the group had not even considered. Asking the quiet student illustrates a norm like "we invite others into the math." Pointing out that action when it happens helps students understand the norm and see how it can benefit the math thinking of the entire group. This understanding and appreciation can promote the use of that norm in the math community.

Cool-down

From Volume to Surface Area

5 min

Student Task Statement

1. A cube has an edge length of 11 inches. Write an expression for its volume and an expression for its surface area.

   Volume: 11^3 or $11 \cdot 11 \cdot 11$

   Surface area: $6 \cdot (11 \cdot 11)$ (or equivalent)

2. A cube has a volume of 7^3 cubic centimeters. What is its surface area?

   294 square centimeters

   $6 \cdot 7^2 = 294$

Responding To Student Thinking

Points to Emphasize

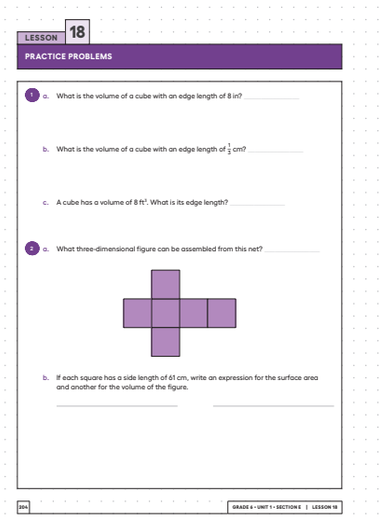
If students struggle with interpreting or writing expressions for the surface area or the volume of a cube, as opportunities arise, highlight the distinctions between these geometric attributes and ways to quantify them. For example, encourage students to draw a sketch, label the edge lengths, and write expressions for the volume and surface area of each cube described in this practice problem: Unit 1, Lesson 18, Practice Problem 1

Lesson 18

Practice Problems

6 Problems

Student Workbook


LESSON 18

PRACTICE PROBLEMS

1 a. What is the volume of a cube with an edge length of 8 in? _____

b. What is the volume of a cube with an edge length of $\frac{1}{3}$ cm? _____

c. A cube has a volume of 8 ft³. What is its edge length? _____

2 a. What three-dimensional figure can be assembled from this net? _____

b. If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

_____ _____

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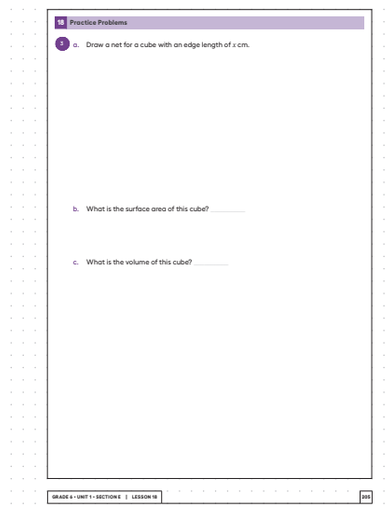
Student Workbook


18 Practice Problems

3 a. Draw a net for a cube with an edge length of x cm.

b. What is the surface area of this cube? _____

c. What is the volume of this cube? _____

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Problem 1

a. What is the volume of a cube with an edge length of 8 in?

512 cu in ($8 \cdot 8 \cdot 8 = 512$)

b. What is the volume of a cube with an edge length of $\frac{1}{3}$ cm?

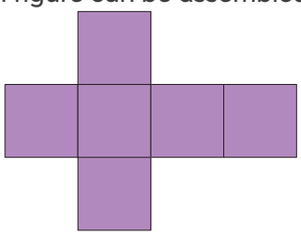
$\frac{1}{27}$ cu cm ($\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$)

c. A cube has a volume of 8 ft³. What is its edge length?

2 ft ($2 \cdot 2 \cdot 2 = 8$)

Problem 2

a. What three-dimensional figure can be assembled from this net?



Cube

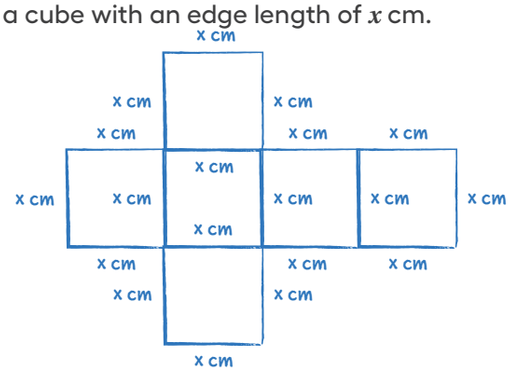
b. If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

Surface area: $6 \cdot 61^2$ sq cm

Volume: 61^3 cu cm

Problem 3

a. Draw a net for a cube with an edge length of x cm.



b. What is the surface area of this cube?

$6x^2$ sq cm (or equivalent)

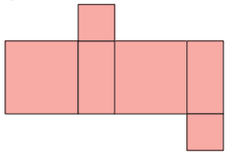
c. What is the volume of this cube?

$x \cdot x \cdot x$ cu cm (or equivalent)

Problem 4

from Unit 1, Lesson 14

Here is a net for a rectangular prism that was not drawn accurately.

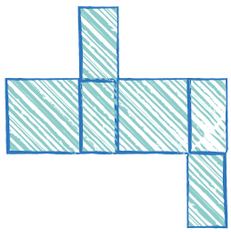


a. Explain what is wrong with the net.

When the shape is folded, the two small squares are not the right size to close the three-dimensional figure. The small squares can be replaced with rectangles as in the picture, or the large squares can be the same size and shape as the two (non-square) rectangles in the net.

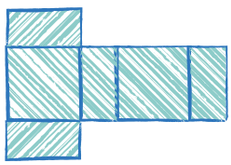
b. Draw a net that can be assembled into a rectangular prism.

Sample response:

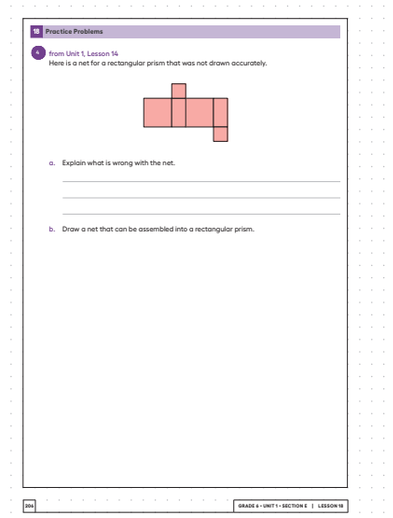


c. Create another net for the same prism.

Sample response:



Student Workbook


18 Practice Problems

4. from Unit 1, Lesson 14
Here is a net for a rectangular prism that was not drawn accurately.

a. Explain what is wrong with the net.

b. Draw a net that can be assembled into a rectangular prism.

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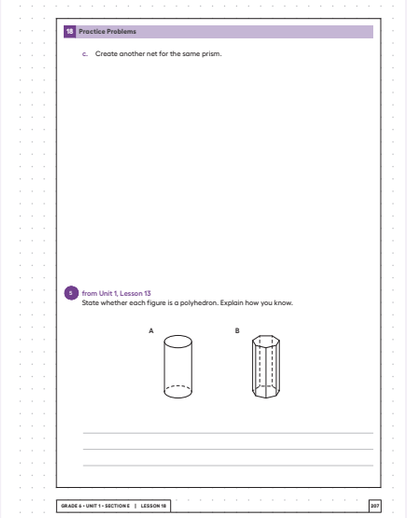
Student Workbook


18 Practice Problems

c. Create another net for the same prism.

5 from Unit 1, Lesson 13
State whether each figure is a polyhedron. Explain how you know.

A B

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Problem 5

from Unit 1, Lesson 13

State whether each figure is a polyhedron. Explain how you know.

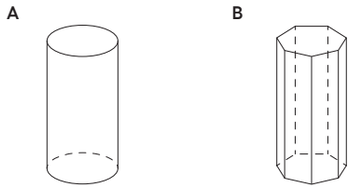


Figure A is not a polyhedron

Sample reasoning: It has a curved surface and there are faces that are not polygons.

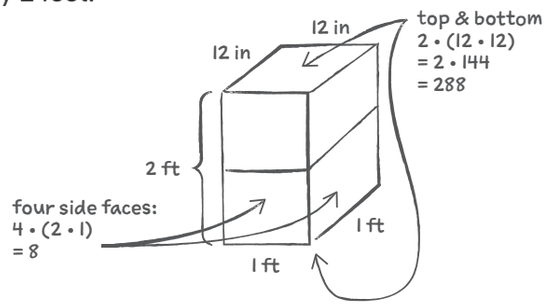
Figure B is a polyhedron

Sample reasoning: It is composed of polygons and each side of every polygon joins a side of another polygon.

Problem 6

from Unit 1, Lesson 12

Here is Elena's work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

Disagree

Sample reasoning: Elena calculated the area of the top and bottom faces in square inches but the area of the side faces in square feet. The combined area of the top and bottom faces is 2 square feet, so the correct surface area is 10 square feet.