Lesson 14

Solving More Systems

Goals

- Calculate values that are a solution for a system of equations, and explain (orally) the solution method.
- Generalize (orally) a process for solving systems of equations using substitution.
- Justify (orally and in writing) that a particular system of equations has no solutions using the structure of the equations.

Learning Target

I can use the structure of equations to help me figure out how many solutions a system of equations has.

Lesson Narrative

In this lesson, students practice solving systems of equations including equations written in a form other than $y = mx + b$. They must make use of the structure of the equations to reason about other ways that substitution can be used to combine the given equations into a new equation involving only one variable.

Student Learning Goal

Let's solve systems of equations.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Math Talk: Solving Systems

5 min

Activity Narrative

This *Math Talk* focuses on systems of equations where one variable is already solved. It encourages students to think about what the equations mean and to rely on the structure of the equations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students solve additional systems.

To solve the systems, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.

Invite students to share their strategies, and record and display their responses for all to see.

Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each system mentally.

A. $\begin{cases} x = 8 \\ y = -11 \end{cases}$

(8, -11)

Sample reasoning: The equations tell us the answers.

B. $\begin{cases} x = 5 \\ y = x - 7 \end{cases}$

(5, -2)

Sample reasoning: The first equation tells us the correct value for x. Then I substituted it into the second equation to find y.

C. $\begin{cases} y = 3x - 2 \\ y = 4 \end{cases}$

(2, 4)

Sample reasoning: The second equation tells us the correct value for y. Then I substituted it in for y in the first equation and solved for x by adding 2, then dividing by 3.

D. $\begin{cases} y = 2x + 3 \\ y = \frac{1}{2}(4x + 3) \end{cases}$

no solution

Sample reasoning: I distributed the $\frac{1}{2}$ in the second equation to get $y = 2x + \frac{3}{2}$. Because the two lines have the same slope and different y-intercepts, they are parallel lines with no solution.

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

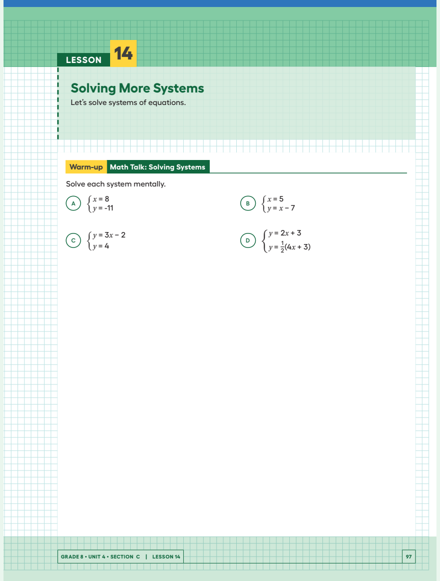
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 14
Solving More Systems
Let's solve systems of equations.
Warm-up Math Talk: Solving Systems
Solve each system mentally.

Access for Multilingual Learners (Warm-up, Activity Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ___'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Challenge Yourself

15 min

Activity Narrative

In this activity, students solve systems of linear equations that lend themselves to substitution. There are four kinds of systems presented: One kind has both equations given with the y value isolated on one side of the equation, another kind has one of the variables given as a constant, a third kind has one variable given as a multiple of the other, and the last kind has one equation given as a linear combination. This progression of systems nudges students toward the idea of substituting an expression in place of the variable it is equal to.

Notice which kinds of systems students think are the least and most difficult to solve.

In future grades, students will manipulate equations to isolate one of the variables in a linear system of equations. For now, students do not need to solve a system like $x + 2y = 7$ and $2x - 2y = 2$ using this substitution method.

Launch

Arrange students in groups of 2.

Give students 10 minutes of quiet work time. Encourage students to check in with their partner between questions.

Tell students that if there is disagreement, they should work to reach an agreement. Follow with a whole-class discussion.

Student Task Statement

Here are a lot of systems of equations:

$A \begin{cases} y = 4 \\ x = -5y + 6 \end{cases}$ $B \begin{cases} y = 7 \\ x = 3y - 4 \end{cases}$ $C \begin{cases} y = \frac{3}{2}x + 7 \\ x = -4 \end{cases}$

$D \begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$ $E \begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$ $F \begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$

$G \begin{cases} y = 3x \\ x = -2y + 56 \end{cases}$ $H \begin{cases} x = 2y - 15 \\ y = -2x \end{cases}$ $I \begin{cases} 3x + 4y = 10 \\ x = 2y \end{cases}$

$J \begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$ $K \begin{cases} y = -2x + 5 \\ 2x + 3y = 31 \end{cases}$ $L \begin{cases} x + y = 10 \\ x = 2y + 1 \end{cases}$

1. Without solving, identify 3 systems that you think would be the least difficult to solve and 3 systems that you think would be the most difficult to solve. Be prepared to explain your reasoning.

   Sample response: *A*, *B*, and *C* seem easy since one of the variable solutions is already given. *J*, *K*, and *L* seem the most difficult since there are multiple terms and the variables are on the same side of the equation in some of the equations.

2. Choose 4 systems to solve. At least one should be from your "least difficult" list, and one should be from your "most difficult" list.

   Any four of these solutions:

   A. $x = -14, y = 4$

   B. $x = 17, y = 7$

   C. $x = -4, y = 1$

   D. $x = 4, y = -2$

   E. $x = -5, y = 10$

   F. $x = 2, y = 4$

   G. $x = 8, y = 24$

   H. $x = -3, y = 6$

   I. $x = 2, y = 1$

   J. $x = 9, y = 29$

   K. $x = -4, y = 13$

   L. $x = 7, y = 3$

Building on Student Thinking

Some students may have trouble transitioning from systems where both equations are given with one variable isolated to other kinds of systems. Ask these students to look at a system where one of the variables is given as a constant. For example, ask them to look at equation B:

$$\begin{cases} y = 7 \\ x = 3y - 4 \end{cases}$$

Ask,

"If y is equal to 7, then what is 3y equal to?"

If a student continues to struggle, refer them back to this example and then ask,

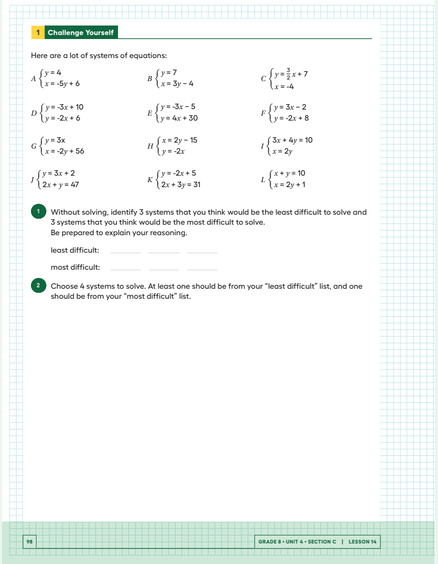
"In this new problem, do we know what expression y (or x) is equal to? Then whenever we see y (or x), what can we replace it with instead?"

Access for Students with Diverse Abilities (Activity 1, Activity Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code the expression and the variable that are connected through substitution.

Supports accessibility for: Visual-Spatial Processing

Student Workbook


1 Challenge Yourself

Here are a lot of systems of equations:

1. Without solving, identify 3 systems that you think would be the least difficult to solve and 3 systems that you think would be the most difficult to solve. Be prepared to explain your reasoning.

least difficult:

most difficult:

2. Choose 4 systems to solve. At least one should be from your "least difficult" list, and one should be from your "most difficult" list.

Activity Synthesis

This discussion has a two main takeaways. The first is to formalize the idea of substitution in a system of equations. Another is to recognize that systems where both equations are written with one variable isolated are actually special cases of substitution.

Invite students to share which systems they thought would be easiest to solve and which would be hardest. To involve more students in the conversation, consider asking:

- *"Did you change your mind about any of the systems being more or less difficult after you solved them?"*

  "What was similar in these problems? What was different?"

  The systems vary slightly in how they are presented, but all of the problems can be solved by replacing a variable with an expression it is equal to.

- *"Will your strategy work for the other systems in this list?"*

  Yes, substitution works in all the given problems.

Tell students that the key underlying concept for all of these problems is that it is often helpful to replace a variable with the expression it is equal to, and that this is called "substitution." Point out that even setting the expressions for y in the first two problems equal to each other is really substituting y in one equation with the expression it is equal to as given by the other equation. It may be helpful for students to hear language like, "Because y is equal to $-2x$, that means wherever I see y, I can substitute in $-2x$."

Activity 2

Five Does Not Equal Seven

15 min

Activity Narrative

In this activity, students are asked to make sense of Tyler's justification for the number of solutions to the system of equations. This activity continues the thread of reasoning about the structure of an equation, and the focus should be on what, specifically, in the equations students think Tyler sees that makes him believe that the system has no solutions.

Launch

Give students 2–3 minutes of quiet think time to read the problem and decide if they agree or disagree with Tyler. Use the remaining time for a whole-class discussion.

Student Task Statement

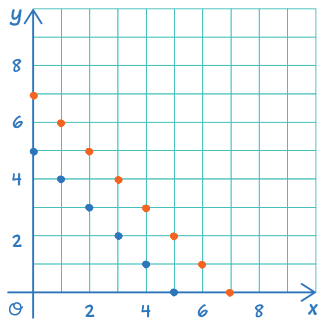
Tyler looks at this system of equations:

$$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$$

He says, "Just looking at the system, I can see it has no solution. If you add 2 numbers, that sum can't be equal to 2 different numbers."

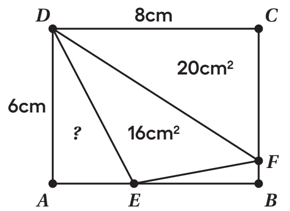
Do you agree with Tyler?

Sample response: I agree with Tyler because the sum of two numbers cannot be equal to both 5 and 7. On a graph, it's clear that the lines will never cross.



Are You Ready for More?

In rectangle $ABCD$, side AB is 8 centimeters and side BC is 6 centimeters. F is a point on BC and E is a point on AB. The area of triangle DFC is 20 square centimeters, and the area of triangle DEF is 16 square centimeters.
What is the area of triangle AED?



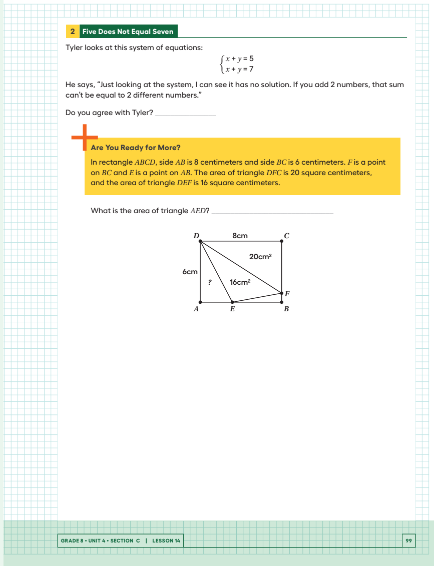
$\frac{48}{5}$ square centimeters (or equivalent)

Since the area of triangle DFC is 20 and $DC = 8$, $CF = 5$ and hence $FB = 1$. The area of rectangle $ABCD$ is 48 square centimeters. Summing the areas of the four triangles, we get $48 = 20 + 16 + \frac{1}{2} \cdot 6 \cdot AE + \frac{1}{2} \cdot BE$.

We also have $AE + EB = 8$. This is a system of equations where one solution is $AE = \frac{16}{5}$ leading to the area of triangle AED is $\frac{48}{5}$.

It may simplify the work to use a variable to represent the lengths of AE and EB.

Student Workbook



Activity Synthesis

The goal of this discussion is to look at one way to reason about the structure of a system of equations in order to determine the solution and then have students come up with their own reasoning about a different, but similar, system of equations.

Poll the class to see how many students agree with Tyler and how many students disagree with Tyler. If possible, invite students from each side to explain their reasoning. As students explain, it should come out that Tyler is correct and, if no student brings up the idea, make sure to point out that we can also visualize this by graphing the equations in the system and noting that the lines look parallel and will never cross.

In the previous activity, students noticed that if they knew what one variable was equal to, they could substitute that value or expression into another equation in the same problem. Point out that, in this problem, the expression $(x + y)$ is equal to 5 in the first equation. If the lines intersect for a solution, then we can replace $(x + y)$ with 5 in the second equation. This results in $5 = 7$ which is not true, so the lines must not intersect.

Display this system and ask students how many solutions they think it has and to give a signal when they think they know:

$$\begin{cases} 4x + 2y = 8 \\ 2x + y = 5 \end{cases}$$

Once the majority of the class signals they have an answer, invite several students to explain their thinking. There are multiple ways students might use to reason about the number of solutions this system has. During the discussion, encourage students to use the terms "coefficient" and "constant term" in their reasoning. Introduce these terms if needed to help students recall their meanings. Bring up these possibilities if no students do so in their explanations:

"Rewrite the second equation to isolate the y variable and substitute the new expression into the first equation in order to find that the system of equations has no solutions."

"Notice that both equations are lines with the same slope but different y-intercepts, which means that the system of equations has no solutions."

"Notice that 4x + 2y is double 2x + y, but 8 is not 5 doubled, so the system of equations must have no solution."

Lesson Synthesis

To emphasize the concepts from this lesson, consider displaying the three systems and asking these discussion questions:

$$\begin{cases} x = 2 \\ y = 3x - 1 \end{cases}$$

$$\begin{cases} x = 2y + 4 \\ x = 9 - 3y \end{cases}$$

$$\begin{cases} x = 2y + 3 \\ y = 2x - 9 \end{cases}$$

- *"What is the first step you would take to solve the first system?"*

  Because we already know the x value of the solution, we only need to find the y value. Substituting 2 in for x in the other equation should help us solve for the y value that makes both equations true when x is 2.

- *"What steps would you take to solve the second system?"*

  Because we know two expressions that are equal to x, we can set those expressions equal to one another. Therefore, we know that $2y + 4 =$
  $9 - 3y$,
  which can be solved using the techniques to solve equations with a single variable. when we know the value for y, we can find the value for x from either of the original equations from the system.

- *"For the third system, a student begins the substitution method by writing $y = 22y + 3 - 9$ then $y = 4y - 6$. What has this student done wrong?"*

  When substituting for x, the student did not multiply the entire expression by 2.

Lesson Summary

When we have a system of linear equations where one of the equations is of the form y = [stuff] or x = [stuff], we can solve it algebraically by using a technique called substitution. The basic idea is to replace a variable with an expression that it is equal to (so the expression is like a substitute for the variable). For example, let's start with the system:

$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

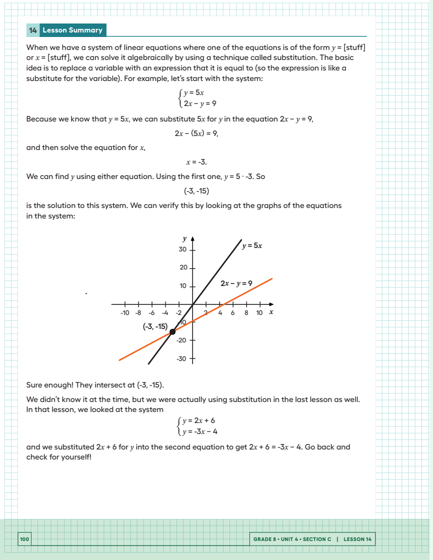
Because we know that $y = 5x$, we can substitute $5x$ for y in the equation $2x - y = 9$,

$$2x - (5x) = 9,$$

and then solve the equation for x,

$$x = -3.$$

Student Workbook



14 Lesson Summary

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$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

Because we know that $y = 5x$, we can substitute $5x$ for y in the equation $2x - y = 9$,

$$2x - (5x) = 9,$$

and then solve the equation for x,

$$x = -3.$$

We can find y using either equation. Using the first one, $y = 5 \cdot -3$. So

$$(-3, -15)$$

is the solution to this system. We can verify this by looking at the graphs of the equations in the system:

Sure enough! They intersect at (-3, -15).

We didn't know it at the time, but we were actually using substitution in the last lesson as well. In that lesson, we looked at the system

$$\begin{cases} y = 2x + 6 \\ y = -3x - 4 \end{cases}$$

and we substituted $2x + 6$ for y into the second equation to get $2x + 6 = -3x - 4$. Go back and check for yourself!

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Responding To Student Thinking

Points to Emphasize
If students struggle to solve a system of equations algebraically, these practice problems can be used to reinforce student understanding.
Unit 4, Lesson 14, Practice Problem 1

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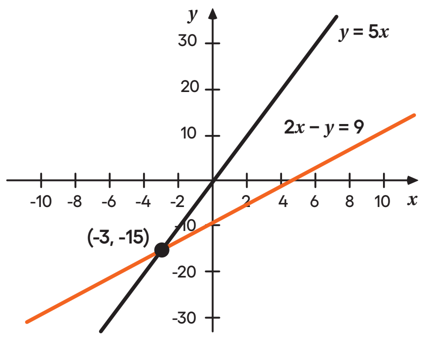

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$$\begin{cases} y = 2x + 6 \\ y = -3x - 4 \end{cases}$$

and we substituted $2x + 6$ for y into the second equation to get $2x + 6 = -3x - 4$. Go back and check for yourself!

Cool-down
Solve It

5 min

This *Cool-down* asks students to solve a system of equations presented in an algebraic form. Although no method is specified, the main ideas from this lesson, as well as a lack of coordinate plane, may lead students to use a substitution method, which is both efficient and effective on this system.

Student Task Statement

Solve this system of equations: $\begin{cases} y = 2x \\ x = -y + 6 \end{cases}$

(2, 4)

Sample reasoning: Use the substitution method to rewrite the system as the one variable equation $x = -(2x) + 6$, then solve.

Lesson 14

Practice Problems 6 Problems

Problem 1

Solve: $\begin{cases} y = 6x \\ 4x + y = 7 \end{cases}$

$\left(\frac{7}{10}, \frac{21}{5}\right)$

Problem 2

Solve: $\begin{cases} y = 3x \\ x = -2y + 70 \end{cases}$

$(10, 30)$

Problem 3

Which equation, together with $y = -1.5x + 3$, makes a system with one solution?

A. $y = -1.5x + 6$

B. $y = -1.5x$

C. $2y = -3x + 6$

D. $2y + 3x = 6$

E. $y = -2x + 3$

Problem 4

The system $x - 6y = 4$, $3x - 18y = 4$ has no solution.

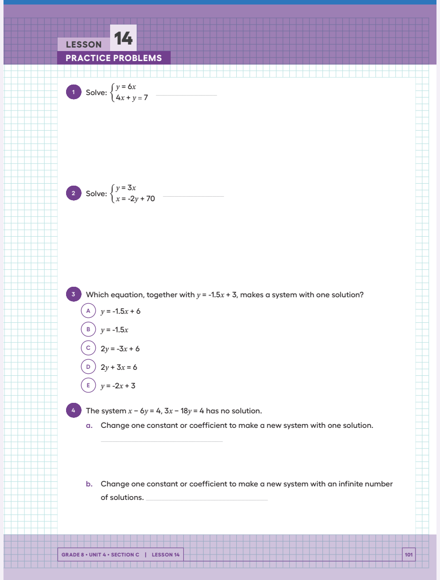
a. Change one constant or coefficient to make a new system with one solution.

Sample response: $2x - 6y = 4$

b. Change one constant or coefficient to make a new system with an infinite number of solutions.

Sample response: $3x - 18y = 12$

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 Solve: $\begin{cases} y = 6x \\ 4x + y = 7 \end{cases}$

2 Solve: $\begin{cases} y = 3x \\ x = -2y + 70 \end{cases}$

3 Which equation, together with $y = -1.5x + 3$, makes a system with one solution?

A $y = -1.5x + 6$

B $y = -1.5x$

C $2y = -3x + 6$

D $2y + 3x = 6$

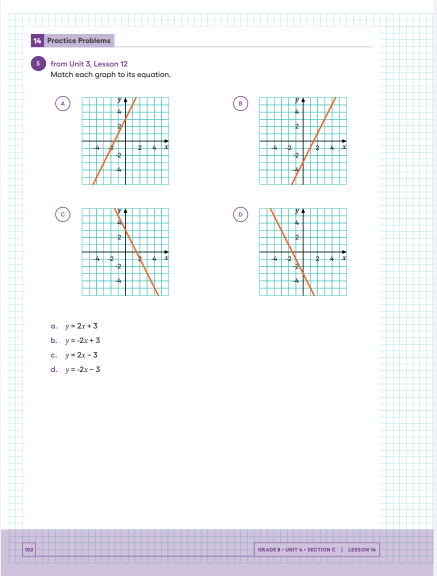
E $y = -2x + 3$

4 The system $x - 6y = 4$, $3x - 18y = 4$ has no solution.

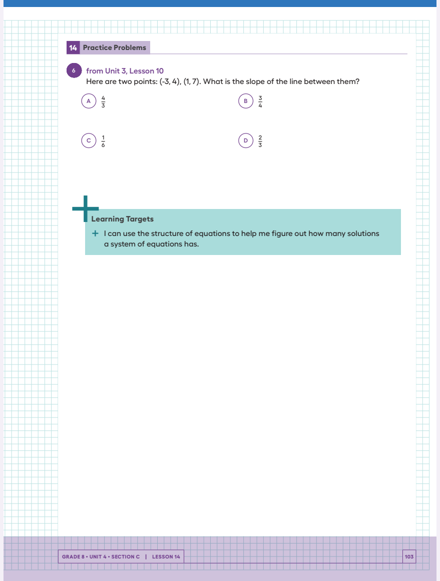
a. Change one constant or coefficient to make a new system with one solution.

b. Change one constant or coefficient to make a new system with an infinite number of solutions.

Student Workbook



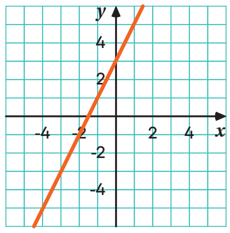
Student Workbook



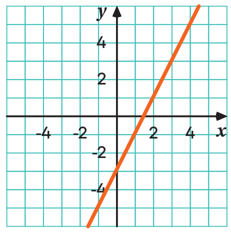
Problem 5 from Unit 3, Lesson 12

Match each graph to its equation.

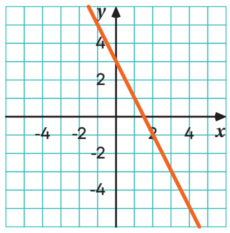
A



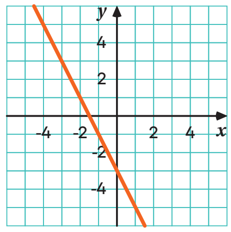
B



C



D



a. $y = 2x + 3$

A

b. $y = -2x + 3$

C

c. $y = 2x - 3$

B

d. $y = -2x - 3$

D

Problem 6 from Unit 3, Lesson 10

Here are two points: (-3, 4), (1, 7). What is the slope of the line between them?

A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$