Lesson 13

Reintroducing Inequalities

Goals

- Comprehend the terms "less than or equal to" and "greater than or equal to" (in spoken and written language) and the symbols ≤ and ≥ (in written language).
- Recognize that more than one value for a variable makes the same inequality true.
- Use substitution to determine whether a given value for a variable makes an inequality true, and justify (orally) the answer.

Learning Targets

- I can explain what the symbols ≤ and ≥ mean.
- I can represent an inequality on a number line.
- I understand what it means for a number to make an inequality true.

Lesson Narrative

In this lesson, students revisit inequalities of the form $x < c$ or $x > c$, and are introduced to the symbols ≥ to show "greater than or equal to," and ≤ to show "less than or equal to." Students begin by finding values that are a solution to a given inequality by determining if the value makes the inequality true. Next, they consider how to represent a context involving the phrase "at least" using algebraic notation and graphically on a number line. The necessity for both an equation and an inequality leads to the introduction of the symbols ≤ and ≥. Lastly, students use substitution to determine whether given values make different inequalities true or false. Students must attend to precision when they specify the meaning of the inequality symbols and explain which values make each inequality true or false.

Student Learning Goal

Let's work with inequalities.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- Collect and Display (Activity 1)
- MLR8 (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Take Turns

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Greater Than One

5 min

Activity Narrative

In this activity, students identify values that would satisfy the inequality $x > 1$. Students recall the meaning of the > symbol and the fact that an inequality has many solutions. They also recall that 1 is not a solution to $x > 1$.

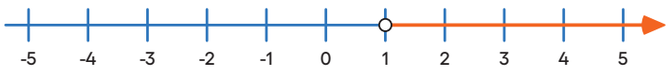
Launch

Since it may have been a while since students encountered this notation, remind them that $x > 1$ is read "x is greater than 1."

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The number line shows values of x that make the inequality $x > 1$ true.



Select **all** the values of x from this list that make the inequality $x > 1$ true.

A. 3

B. -3

C. 700

D. 1.05

E. 1

Activity Synthesis

Ask students to share a few more solutions to $x > 1$. After each student shares, ask the class whether they agree or disagree. Emphasize that this inequality has many solutions—in fact, any value greater than 1 is a solution.

To highlight the fact that "greater than 1" does not include 1, ask:

"What does the open circle at 1 mean?"

It means 1 is not included.

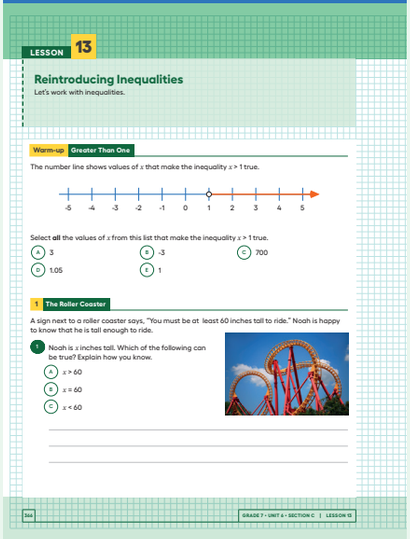
"Why isn't 1 a solution to the inequality x > 1?"

1 > 1 is not a true statement because 1 is equal to 1.

Building on Student Thinking

Some students may think 700 is not a solution to $x > 1$. Tell students that since there is an arrow at the end of the dark line, it includes all values that are greater than 1, even the ones not shown.

Student Workbook


LESSON 13

Reintroducing Inequalities

Let's work with inequalities.

Warm-up Greater Than One

The number line shows values of x that make the inequality $x > 1$ true.

-5 -4 -3 -2 -1 0 1 2 3 4 5

Select **all** the values of x from this list that make the inequality $x > 1$ true.

A 3 B -3 C 700

D 1.05 E 1

1 The Roller Coaster

A sign next to a roller coaster says, "You must be at least 60 inches tall to ride." Noah is happy to know that he is tall enough to ride.

1. Noah is x inches tall. Which of the following can be true? Explain how you know.

A $x > 60$

B $x = 60$

C $x < 60$

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 1, Launch)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Access for Perception.
Read all problems aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language, Attention

Building on Student Thinking

If students are having trouble interpreting the first three questions or articulating their responses, encourage them to make use of the number line that appears in the fourth question.

Activity 1
The Roller Coaster

15 min

Activity Narrative

In this activity, students express an inequality that includes the boundary value using only the symbols = and >. This generates a need for the symbol ≥, which is introduced in the *Activity Synthesis*. Students also recall the fact that a closed circle is used to graph an inequality that includes the boundary value.

Monitor for students who express the answer to the last question using words or using symbols. The responses to the last question will be used to introduce the new notation.

Students attend to precision when they carefully choose the symbols to express the possible values for the height in the last question.

Launch

Arrange students in groups of 2.

Allow students 5–10 minutes quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to express the possible values for Noah's friend's height. Display words and phrases, such as "at least," "greater than," and "greater than or equal to."

Student Task Statement

A sign next to a roller coaster says, "You must be at least 60 inches tall to ride." Noah is happy to know that he is tall enough to ride.

1. Noah is x inches tall. Which of the following can be true? Explain how you know.

   A. $x > 60$

   B. $x = 60$

   C. $x < 60$

   Sample response: "At least" means that Noah can be 60 inches or taller.

2. Noah's friend is 2 inches shorter than Noah. Can you tell if Noah's friend is tall enough to go on the ride? Explain or show your reasoning.

no

Sample reasoning: Since we don't know whether Noah's height is exactly 60, within 2 inches of 60, or more than 2 inches above 60, we can't know if 2 less than his height is at least 60.

3. List a possible height for Noah that would mean:

a. That his friend is tall enough to go on the ride.

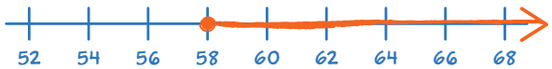
b. That his friend is not tall enough for the ride.

Sample responses:

a. Noah could be 63 inches tall, which means his friend is 61 inches tall and can ride.

b. Noah could be 61 inches tall, which means his friend is 59 inches tall and cannot ride.

4. On the number line, show all the possible heights that Noah's friend could be.



5. Noah's friend is y inches tall. Use y and any of the symbols <, =, > to express this height.

$y > 58$ or $y = 58$

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they used the number line to show the possible heights and how they expressed the possible heights using symbols. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. (For example, the display may have "height is greater than 60 or height is equal to 60" already on it and be updated with the more concise phrase "the height is greater than or equal to 60.")

The purpose of this discussion is to introduce the symbols ≥ and ≤. Ask selected students to share their response to the last question. They are likely to write something like "$y = 58$ or $y > 58$." The < and > symbols are not enough to capture what we need here with a single mathematical statement. Introduce the new symbols ≤ and ≥ that mean "less than or equal to" and "greater than or equal to." Invite students to use one of these new symbols to revise their answer to the last question. ($y \geq 58$ or $58 \leq y$).

Student Workbook


1 The Roller Coaster

2 Noah's friend is 2 inches shorter than Noah. Can you tell if Noah's friend is tall enough to go on the ride? Explain or show your reasoning.

3 List a possible height for Noah that would mean:

a. That his friend is tall enough to go on the ride.

b. That his friend is not tall enough for the ride.

4 On the number line, show all the possible heights that Noah's friend could be.

52 54 56 58 60 62 64 66 68

5 Noah's friend is y inches tall. Use y and any of the symbols <, =, > to express this height.

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Instructional Routines

MLR8: Discussion Supports

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Multilingual Learners (Activity 2, Task Statement)

MLR8: Discussion Supports. Students should take turns deciding whether a value makes the inequality true or false and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed _____, so I decided ..." When students disagree with each other, encourage them to challenge each other using the sentence frames: "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about inequalities.

Advances: Speaking, Conversing

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Consider breaking up the table one row at a time. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

Students who try to apply what they know about solving equations to solve the inequalities algebraically may come up with incorrect solutions. For instance, $100 < 4x$ may at first glance look equivalent to $x < 25$, since the "less than" sign appears. Students may incorrectly think that $-3x > -75$ is equivalent to $x > 25$. Ask these students, for example, what the solution to $100 = 4x$ means (25 is the value of x that makes $4x$ equal to 100). Then encourage these students to test values like 24 and 26 to see whether they are solutions to $100 < 4x$. This will be covered in greater detail in a later lesson, so this understanding does not need to be solidified at this time.

Activity 2

Is the Inequality True or False?

15 min

Activity Narrative

In this activity, students interpret the notation <, >, ≤, and ≥ to evaluate whether different values make an inequality true. It is not expected that students will solve the inequalities generally.

Monitor for students who use the following approaches:

- Substitute a value in for x, evaluate the expression, and think about whether the statement is true. For example, "Is -300 less than 75? Yes, so this is true."
- Draw a number line for each inequality and use it to reason about different values.

In this partner activity, students take turns deciding whether a value makes an inequality true. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Keep students in the same groups. Display the task for all to see. Tell students that for each number in the first row, they will decide if it makes the inequality in the first column true. If time allows, choose a student to be your partner and demonstrate how to do the activity, otherwise share these steps:

- One partner picks an inequality and a value.
- They decide whether the value makes the inequality true, and explain why they think it is true or false.
- The other partner listens and makes sure they agree with the reasoning.
- If they don't agree, the partners discuss until they come to an agreement.
- The students swap roles and repeat until the table is complete.

Student Task Statement

The table shows four inequalities and four values for x. Take turns with your partner to decide whether each value makes each inequality true, and complete the table with "true" or "false".

- For each decision you make, explain to your partner how you know it's true or false.
- For each decision that your partner makes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

x	0	100	-100	25
$x \le 25$	true	false	true	true
$100 < 4x$	false	true	false	false
$-3x > -75$	true	false	true	false
$10 \ge 35 - x$	false	true	false	true

Are You Ready for More?

Find an example of an inequality used in the real world and describe it using a number line.

Answers vary.

Activity Synthesis

Much discussion takes place between partners. The purpose of the discussion is to note the consequences of an inequality using ≤ versus <, and ≥ versus >.

Direct students' attention to $100 < 4x$ and $10 \geq 35 - x$. Substituting 25 for x in each of these inequalities gives $100 < 100$, which is false, and $10 \geq 10$, which is true. The key distinction is that ≥ and ≤ inequalities are considered true when both sides are equal, whereas < and > inequalities are considered false when both sides are equal. Emphasize that substituting a value in for x, and thinking about whether the resulting inequality is saying something true, is the most direct way to check whether the value is a solution.

- *"What was a strategy you used for determining whether a value makes an inequality true?"*

  "Were you surprised by or initially incorrect about any of the answers?"

Lesson Synthesis

The purpose of this discussion is to compare how the ≥ and ≤ symbols are the same and different from the > and < symbols. Begin by displaying the inequalities $x \geq 7$and $y \leq 3$ for all to see. For each inequality, discuss the following questions:

- *"What do we call this symbol?"*

  greater than or equal to; less than or equal to

- *"How do we read this inequality statement?"*

  x is greater than or equal to 7. y is less than or equal to 3.

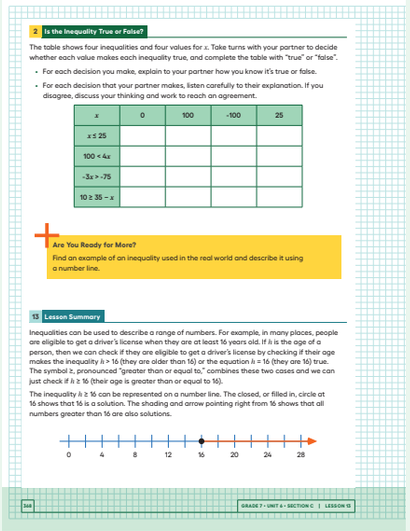
- *"How is this different from x > 7 or y < 3?"*

  With these inequalities, the value that is listed is not included as an acceptable value. For example, 7 > 7 is not true, while 7 ≥ 7 is true.

- *"On the number line, how would the graphs of x > 7 and x ≥ 7 look the same or different?"*

  Both would have a circle at 7 and have an arrow pointing to the right. The graph of x > 7 would have an open, or unfilled circle, to show that 7 is not a solution, while the graph of x ≥ 7 would have the circle filled in to show that 7 is a solution.

Student Workbook



2 Is the Inequality True or False?

The table shows four inequalities and four values for x. Take turns with your partner to decide whether each value makes each inequality true, and complete the table with "true" or "false".

- For each decision you make, explain to your partner how you know it's true or false.
- For each decision that your partner makes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

x	0	100	-100	25
$x \leq 25$				
$100 < 4x$				
$-3x > -75$				
$10 \geq 35 - x$				

Are You Ready for More?

Find an example of an inequality used in the real world and describe it using a number line.

13 Lesson Summary

Inequalities can be used to describe a range of numbers. For example, in many places, people are eligible to get a driver's license when they are at least 16 years old. If h is the age of a person, then we can check if they are eligible to get a driver's license by checking if their age makes the inequality $h > 16$ (they are older than 16) or the equation $h = 16$ (they are 16) true. The symbol ≥, pronounced "greater than or equal to," combines these two cases and we can just check if $h \geq 16$ (their age is greater than or equal to 16).

The inequality $h \geq 16$ can be represented on a number line. The closed, or filled in, circle at 16 shows that 16 is a solution. The shading and arrow pointing right from 16 shows that all numbers greater than 16 are also solutions.

0 4 8 12 16 20 24 28

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

Inequalities can be used to describe a range of numbers. For example, in many places, people are eligible to get a driver's license when they are at least 16 years old. If h is the age of a person, then we can check if they are eligible to get a driver's license by checking if their age makes the inequality $h > 16$ (they are older than 16) or the equation $h = 16$ (they are 16) true. The symbol ≥, pronounced "greater than or equal to," combines these two cases and we can just check if $h \geq 16$ (their age is greater than or equal to 16).

The inequality $h \geq 16$ can be represented on a number line. The closed, or filled in, circle at 16 shows that 16 is a solution. The shading and arrow pointing right from 16 shows that all numbers greater than 16 are also solutions.

Cool-down

What Is Different?

5 min

Student Task Statement

1. List some values for x that would make the inequality $-2x > 10$ true.

   Sample responses: -6, -7, -100, -5.001 (any number less than -5)

2. What is different about the values of x that make $-2x \geq 10$ true, compared to $-2x > 10$?

   Sample response: When x is -5, the inequality $-2x \geq 10$ is true, but the inequality $-2x > 10$. is false.

Lesson 13

Practice Problems

4 Problems

Problem 1

For each inequality, find two values for x that make the inequality true and two values that make it false.

a. $x + 3 > 70$

Sample response: true: $x = 70$ and $x = 100$, false: $x = 0$ and $x = -10$

b. $x + 3 < 70$

Sample response: true: $x = 60$ and $x = 0$, false: $x = 70$ and $x = 100$

c. $-5x < 2$

Sample response: true: $x = 1$ and $x = 2$, false: $x = -1$ and $x = -2$

d. $5x < 2$

Sample response: true: $x = 0$ and $x = -1$, false: $x = 1$ and $x = 100$

Problem 2

Here is an inequality: $-3x > 18$.

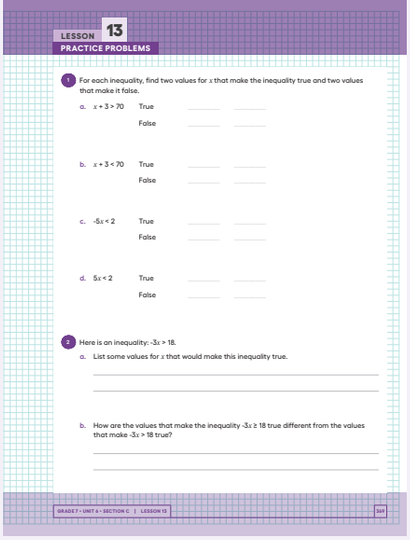
a. List some values for x that would make this inequality true.

any value less than -6

b. How are the values that make the inequality $-3x \geq 18$ true different from the values that make $-3x > 18$ true?

Sample response: The same values for x make both inequalities true, except for one value: -6. This value makes $-3x \geq 18$ true but not $-3x > 18$.

Student Workbook


LESSON 13
PRACTICE PROBLEMS

1 For each inequality, find two values for x that make the inequality true and two values that make it false.

a. $x + 3 > 70$ True ____ ____ False ____ ____

b. $x + 3 < 70$ True ____ ____ False ____ ____

c. $-5x < 2$ True ____ ____ False ____ ____

d. $5x < 2$ True ____ ____ False ____ ____

2 Here is an inequality: $-3x > 18$.

a. List some values for x that would make this inequality true.

b. How are the values that make the inequality $-3x \geq 18$ true different from the values that make $-3x > 18$ true?

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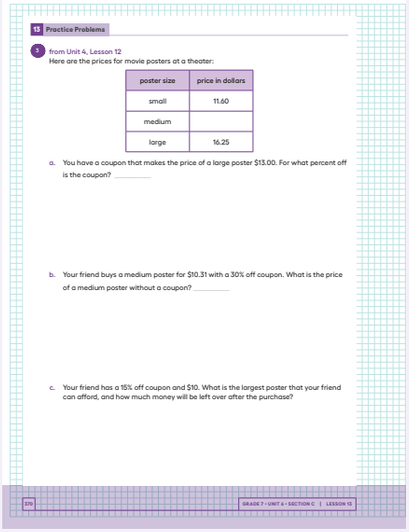
Student Workbook


13 Practice Problems

3 from Unit 4, Lesson 12
Here are the prices for movie posters at a theater:

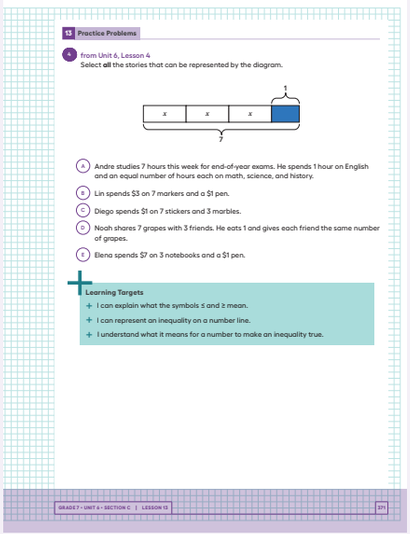
poster size	price in dollars
small	11.60
medium	
large	16.25

a. You have a coupon that makes the price of a large poster $13.00. For what percent off is the coupon? _____

b. Your friend buys a medium poster for $10.31 with a 30% off coupon. What is the price of a medium poster without a coupon? _____

c. Your friend has a 15% off coupon and $10. What is the largest poster that your friend can afford, and how much money will be left over after the purchase?

Student Workbook


13 Practice Problems

4 from Unit 6, Lesson 4
Select **all** the stories that can be represented by the diagram.

A. Andre studies 7 hours this week for end-of-year exams. He spends 1 hour on English and an equal number of hours each on math, science, and history.
B. Lin spends $3 on 7 markers and a $1 pen.
C. Diego spends $1 on 7 stickers and 3 marbles.
D. Noah shares 7 grapes with 3 friends. He eats 1 and gives each friend the same number of grapes.
E. Elena spends $7 on 3 notebooks and a $1 pen.

Learning Targets
+ I can explain what the symbols ≤ and ≥ mean.
+ I can represent an inequality on a number line.
+ I understand what it means for a number to make an inequality true.

Problem 3

from Unit 4, Lesson 12

Here are the prices for movie posters at a theater:

poster size	price in dollars
small	11.60
medium	
large	16.25

a. You have a coupon that makes the price of a large poster $13.00. For what percent off is the coupon?

20%

b. Your friend buys a medium poster for $10.31 with a 30% off coupon. What is the price of a medium poster without a coupon?

$14.73

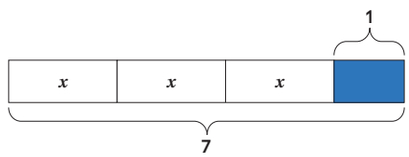
c. Your friend has a 15% off coupon and $10. What is the largest poster that your friend can afford, and how much money will be left over after the purchase?

small, $0.14

Problem 4

from Unit 6, Lesson 4

Select **all** the stories that can be represented by the diagram.



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