Lesson 9

Moves in Parallel

Goals

- Draw and label rigid transformations of lines and parallel lines and explain (orally and in writing) the relationship between the original and its image under the transformation.
- Generalize that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.
- Justify (orally) that vertical angles are congruent using informal arguments about 180-degree rotations of lines.

Learning Targets

- I can describe the effects of a rigid transformation on a pair of parallel lines.
- If I have a pair of vertical angles and know the angle measure of one of them, I can find the angle measure of the other.

Lesson Narrative

The focus of this lesson is for students to observe the effects of rigid transformations on lines and parallel lines. Parallel lines do not meet and are the same distance apart along their entire length, and a rigid transformation does not change these features. Students use the structure of parallel lines to conclude that the image of a set of parallel lines is also a set of parallel lines under a rigid transformation.

Students also investigate 180-degree rotations of points on a line and a set of intersecting lines. Students use arguments about 180-degree rotations to justify that **vertical angles** have the same measure.

Student Learning Goal

Let's transform some lines.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Required Materials

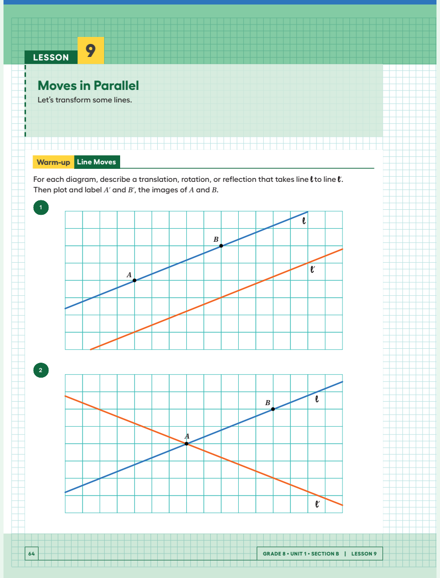
Materials to Gather

- Tracing paper: Warm-up, Activity 1
- Geometry toolkits: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Student Workbook



Warm-up

Line Moves

5 min

Activity Narrative

In this *Warm-up,* students practice applying rigid transformations to lines. Each image in this activity has the same starting line and students name the translation, rotation, or reflection that takes this line to the second marked line. Because of their infinite and symmetric nature, different transformations of lines look the same unless specific points are marked, so 1–2 points on each line are marked.

While students have experience transforming a variety of figures, this activity provides the opportunity to use precise language when describing transformations of lines while exploring how sometimes different transformations can result in the same final figures. During the activity, encourage students to look for more than one way to transform the original line.

Launch

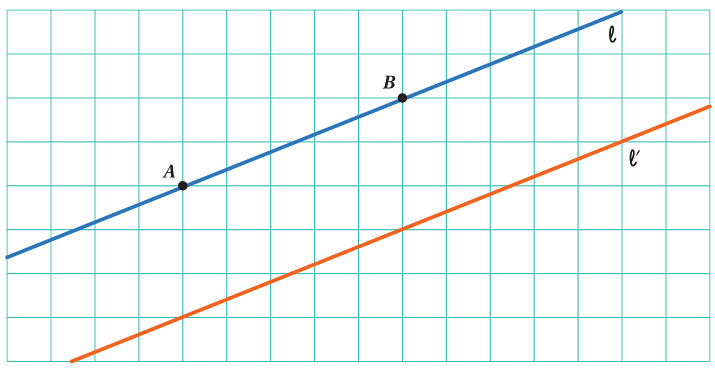
Provide access to tracing paper.

Give students 2 minutes of quiet work time followed by whole-class discussion.

Student Task Statement

For each diagram, describe a translation, rotation, or reflection that takes line ℓ to line ℓ'. Then plot and label A' and B', the images of A and B.

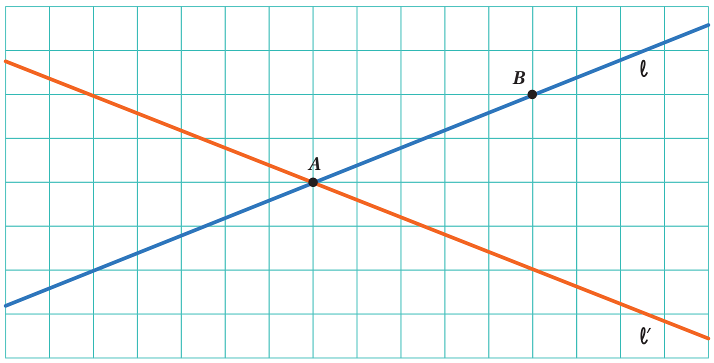
1.



Sample responses:

- Translation in many possible directions, for example, down 3 units
- Reflection over a line parallel to ℓ halfway between ℓ and ℓ'
- Rotation using a point halfway between ℓ and ℓ' as the center of rotation and an angle of 180°

2.



- Reflection across the vertical line through point A
- Reflection across the horizontal line through point A
- Counterclockwise rotation about point A by the obtuse angle whose vertex is at A
- Clockwise rotation about point A by the acute angle whose vertex is at A

Activity Synthesis

Invite students to share the transformations they chose for each problem. Each diagram has more than one possible transformation that would result in the final figure.

If students only found one, pause for 2–3 minutes and encourage students to see if they can find another.

For the first diagram, look for a single translation, single rotation, and single reflection that work. For the second diagram, look for a single rotation and a single reflection.

"Will a translation work for the second diagram? Explain your reasoning."

A translation will not work. Since translations do not incorporate a turn, translations of a line are parallel to the original line or are the same line.

Activity 1
Parallel Lines

15 min

Activity Narrative

In this activity, students investigate what happens to parallel lines under rigid transformations by performing three different transformations on a set of parallel lines.

Students recall that parallel lines do not meet, and they remain the same distance apart. Students use the structure of rigid transformations to observe that the image of a set of parallel lines after a rigid transformation is another set of parallel lines.

Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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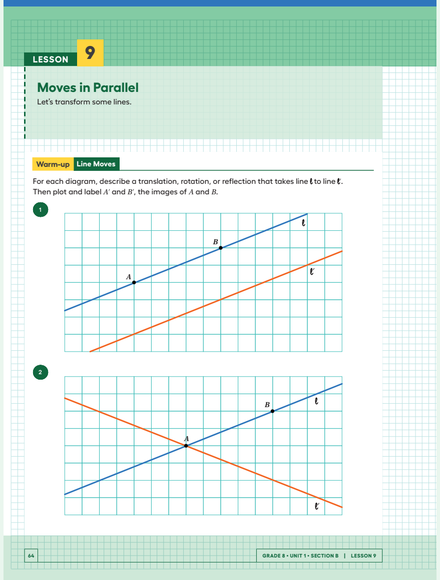
Student Workbook

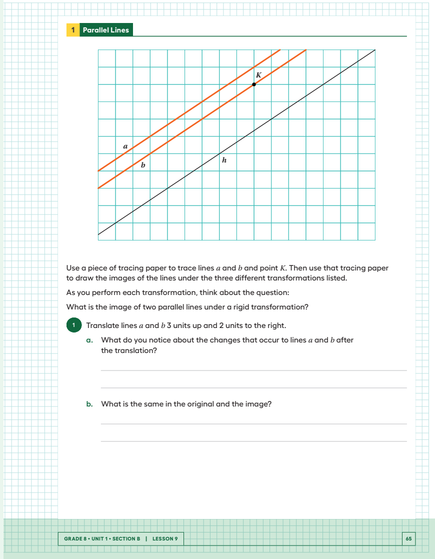


Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Internalize Self Regulation.
Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to one or two of the 3 problems, based on which transformation they feel most confident performing.

Supports accessibility for: Organization, Attention

Student Workbook


1 Parallel Lines

Use a piece of tracing paper to trace lines a and b and point K. Then use that tracing paper to draw the images of the lines under the three different transformations listed.

As you perform each transformation, think about the question:

What is the image of two parallel lines under a rigid transformation?

1. Translate lines a and b 3 units up and 2 units to the right.
   a. What do you notice about the changes that occur to lines a and b after the translation?
   b. What is the same in the original and the image?

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Building on Student Thinking

Students may not perform the transformations on top of the original image. Ask these students to place the traced lines over the original and perform each transformation from there.

Launch

Display this question for all to see, then read it aloud for the class:

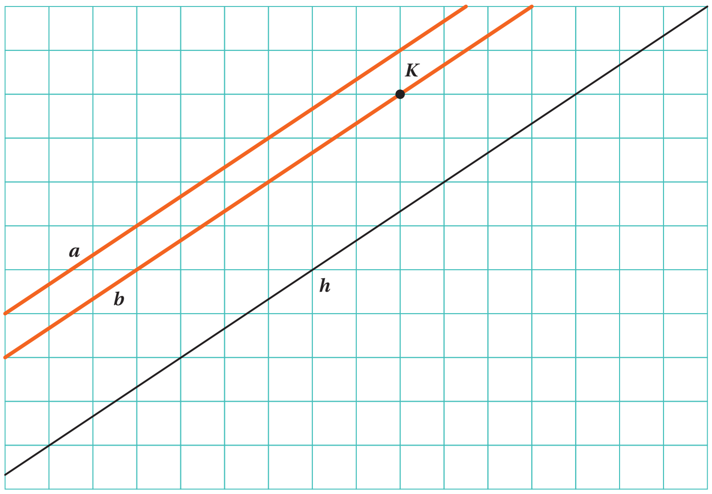
"What happens to parallel lines when we perform rigid transformations on them?"

Tell students they will investigate this question, and leave it displayed throughout the activity.

Arrange students in groups of 3. Provide access to tracing paper. Assign one problem to each member of the group.

Give 3–5 minutes of quiet think time, then have students share their findings with their groups.

Student Task Statement



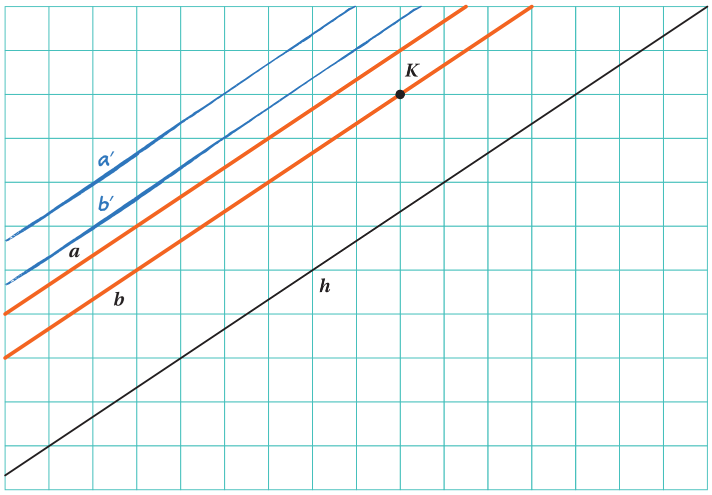
Use a piece of tracing paper to trace lines a and b and point K. Then use that tracing paper to draw the images of the lines under the three different transformations listed.

As you perform each transformation, think about the question:

What is the image of two parallel lines under a rigid transformation?

1. Translate lines a and b 3 units up and 2 units to the right.

   Translation: 3 grid square units up and 2 grid square units to the right



a. What do you notice about the changes that occur to lines a and b after the translation?

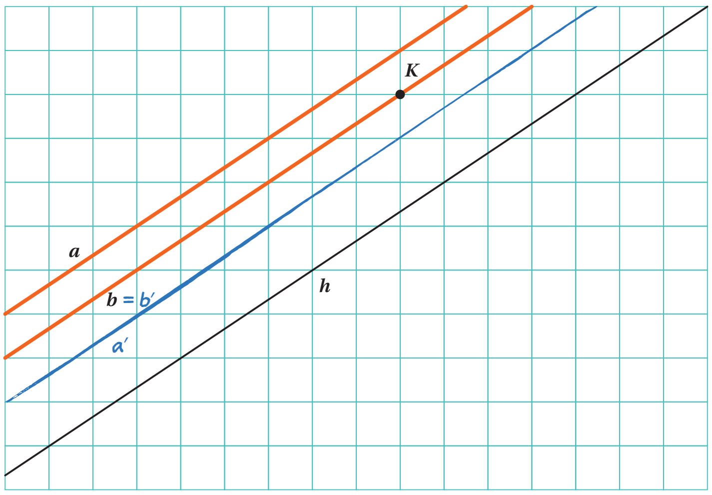
Sample response: All 4 lines, a, b, a', and b' are parallel. The lines a' and b' look like a and b but shifted upward.

b. What is the same in the original and the image?

Sample response: The pair of lines remain parallel. The distance between the lines did not change.

2. Rotate lines a and b counterclockwise 180° using K as the center of rotation.

Rotation around K



a. What do you notice about the changes that occur to lines a and b after the rotation?

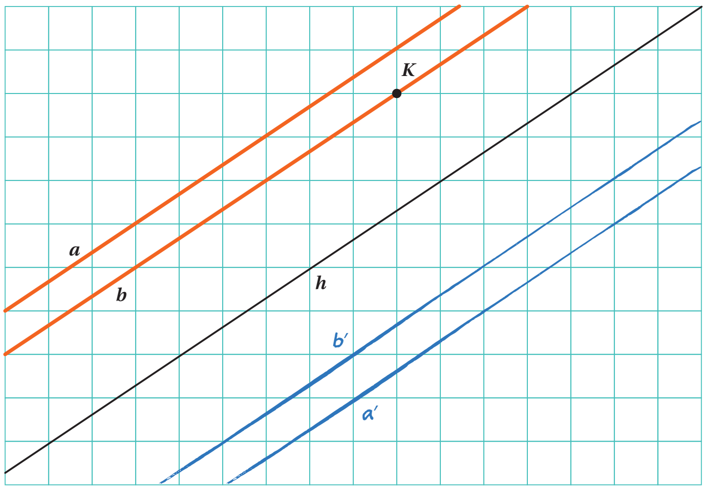
Sample response: The new pair of lines a' and b' are parallel to the original lines a and b. Lines b and b' are the same line.

b. What is the same in the original and the image?

Sample response: The lines a' and b' are still parallel and they are the same distance apart as a and b.

3. Reflect lines a and b across line h.

Reflection over line h.



Student Workbook

a. What do you notice about the changes that occur to lines a and b after the reflection?

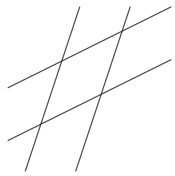
Sample response: Line a is above line b whereas line b' is above line a'.

b. What is the same in the original and the image?

Sample response: Lines a' and b' are still parallel and are the same distance apart as lines a and b. All four lines are parallel to one another.

Are You Ready for More?

When you rotate two parallel lines, sometimes the two original lines intersect their images and form a quadrilateral. What is the most specific thing you can say about this quadrilateral? Can it be a square? A rhombus? A rectangle that isn't a square? Explain your reasoning.



Sample response: The quadrilateral is always a rhombus. It can be a square if the two pairs of parallel lines are perpendicular. It can not be a rectangle that is not a square because the distance between the two sets of parallel lines is the same.

Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to "What happens to parallel lines when we perform rigid transformations on them?" In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

"___ makes sense, but what do you mean when you say … ?"

"Can you describe that another way?"

"How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

Provide these sentence frames to help students organize their thoughts in a clear, precise way:

"When you perform a translation on a set of parallel lines, then …"

"I know that ___ is true because …"

"Performing a rigid transformation on a set of parallel lines results in ___ because …"

Here is an example of a second draft:

> Performing a rigid transformation on a set of parallel lines results in another set of parallel lines because the distances and angles will stay the same between the original set and its image. The new lines won't always be parallel to the original lines, since they could have been rotated. A reflection will sometimes result in a set of lines parallel to the first one, and a translation will always result in a set of lines to the first one, since angles stay the same in a translation.

As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Activity 2

Let's Do Some 180s

15 min

Activity Narrative

In this activity, students explore a particular application of 180-degree rotations in order to take a deeper look at vertical angles. Students use the structure of rigid transformations and 180-degree rotations to informally demonstrate that vertical angles have the same angle measure.

Students rotate a line with marked points 180 degrees about a point on the line. Then students rotate an angle 180 degrees about a point on the line to draw conclusions about lengths and angles. Finally, students consider the intersection of two lines, the angles formed, and how the measurements of those angles can be deduced using a 180-degree rotation about the intersection of the lines. Students use arguments about rigid transformations and their observations about a 180-degree rotation to justify that vertical angles have the same measure.

Launch

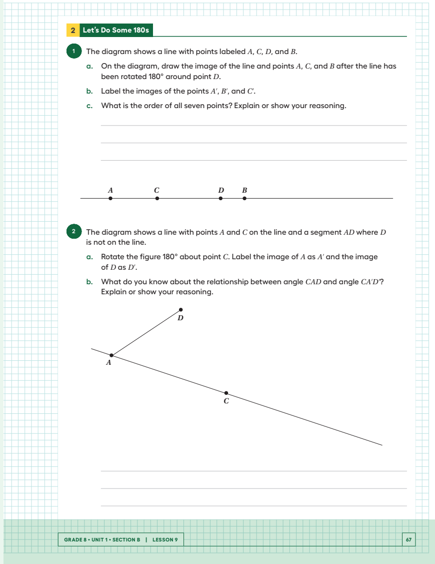
Before students read the activity, draw a line ℓ with a marked point D for all to see. Ask students to picture what the figure rotated 180° around point D looks like.

After a minute of quiet think time, invite students to share what they think the transformed figure would look like.

If no students suggest it, remind students that a rotation requires rotating the entire figure. Make sure all students agree that ℓ' looks "the same" as the original. If not brought up in students' explanations, ask for suggestions of features that would make it possible to quickly tell the difference between the ℓ' and ℓ, such as another point or if the line were different colors on each side of point D.

Provide access to tracing paper.

Student Workbook


2 Let's Do Some 180s

1. The diagram shows a line with points labeled A, C, D, and B.
   a. On the diagram, draw the image of the line and points A, C, and B after the line has been rotated 180° around point D.
   b. Label the images of the points A', B', and C'.
   c. What is the order of all seven points? Explain or show your reasoning.

2. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.
   a. Rotate the figure 180° about point C. Label the image of A as A' and the image of D as D'.
   b. What do you know about the relationship between angle CAD and angle $CA'D'$? Explain or show your reasoning.

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Student Task Statement

1. The diagram shows a line with points labeled A, C, D, and B.

   a. On the diagram, draw the image of the line and points A, C, and B after the line has been rotated 180° around point D.

   b. Label the images of the points A', B', and C'.

   c. What is the order of all seven points? Explain or show your reasoning.

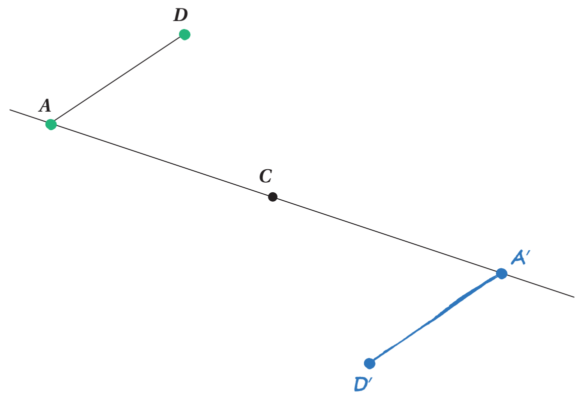
A, C, B', D, B, C', A'

Sample reasoning: The distance between each point and the center of rotation, D, is the same as the distance between the point's image and D. Since A is the farthest point from D, A' will be the farthest in the opposite direction.

2. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.

   a. Rotate the figure 180° about point C. Label the image of A as A' and the image of D as D'.

   b. What do you know about the relationship between angle CAD and angle $CA'D'$? Explain or show your reasoning.

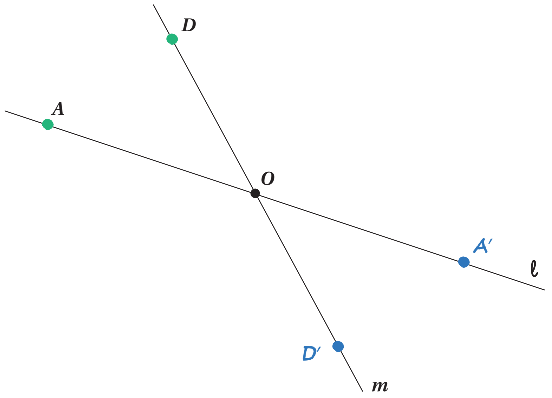


Sample response: The lengths of segment CA and segment CA' are the same, the lengths of segment AD and segment $A'D'$ are the same, and angles CAD and $CA'D'$ have the same measure because both distances and angle measures are preserved under rigid transformations.

3. The diagram shows two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.

a. Rotate the figure 180° around O. Label the image of A as A' and the image of D as D'.

b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.



Sample responses:

- Angles AOD and $A'OD'$ have the same measure.
- Angles DOA' and $D'OA$ have the same measure.
- Rotation is a rigid transformation, which preserves angle measures.

Activity Synthesis

The focus of the discussion should start with the relationships students find between the lengths of segments and angle measures, then move to the final question where students justify that vertical angles are congruent using rigid transformations. Questions to connect the discussion include:

"What relationships between lengths did we find after performing transformations?"

They are the same.

"What relationships between angle measures did we find after performing transformations?"

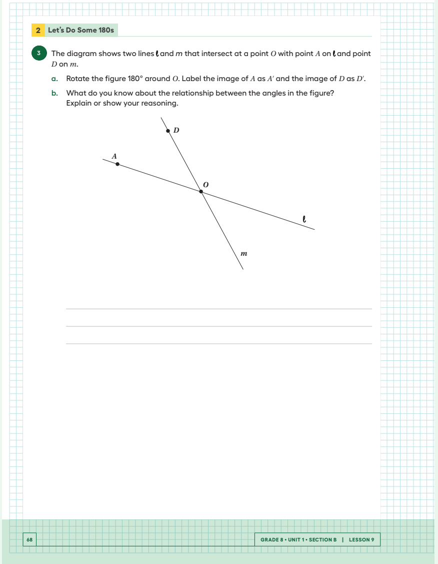
They are the same.

"How did you use transformations to justify that vertical angles are congruent?"

Vertical angles are an example of rotating 2 lines 180 degrees. Since the opposite angles are the images of each other, we know that the opposite angles must be the same.

If time permits, consider discussing connections to work in an earlier course with vertical angles, specifically that vertical angles form pairs of supplementary angles. Pairs of vertical angles have the same measure because they are both supplementary to the same angle. The argument using 180° rotations is different because no reference needs to be made to the supplementary angle. The 180° rotation shows that both pairs of vertical angles have the same measure directly by mapping them to each other.

Student Workbook


2 Let's Do Some 180s

3 The diagram shows two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.

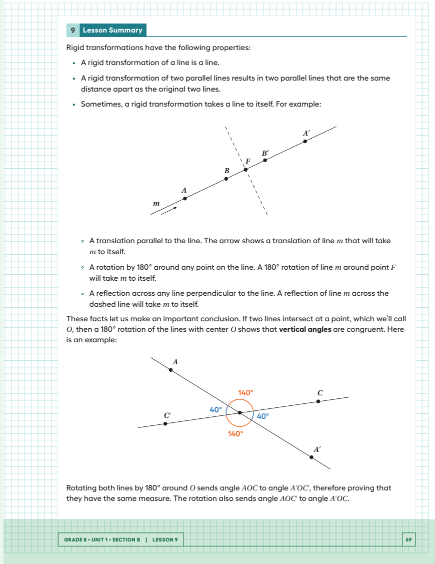
a. Rotate the figure 180° around O. Label the image of A as A' and the image of D as D'.

b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.

Building on Student Thinking

In the second question, students may not understand that rotating the figure includes both segment CA and segment AD since they have been working with rotating one segment at a time. Explain to these students that the figure refers to both of the segments. Encourage them to use tracing paper to help them visualize the rotation.

Student Workbook



Lesson Synthesis

The focus of this lesson is for students to articulate what happens when a rigid transformation is performed on parallel lines. In addition, students justify that vertical angles are congruent using properties of rigid transformations.

Here are some questions for discussion:

"When we perform a rigid transformation on parallel lines, what do we know about its image?"

The image of parallel lines is also a set of parallel lines. The parallel lines in the image will be the same distance apart as the original.

"When we rotate a line 180° around a point on the line, where does the line land?"

The image lines up with the original line, so they look the same. If there are other points on the line, they will be on the opposite side of the center of rotation.

"What do we know about rotations that helps justify that vertical angles are congruent?"

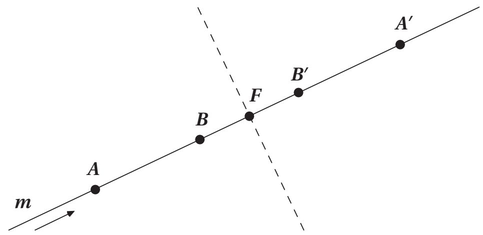
Rotating two intersecting lines about the point of intersection by 180° moves each angle to the angle that is vertical to it. Since angle measures stay the same under a rotation, the vertical angles must have the same measure.

In general, rigid transformations help us see that when we transform lines it might change the orientation, but the lines retain their original properties.

Lesson Summary

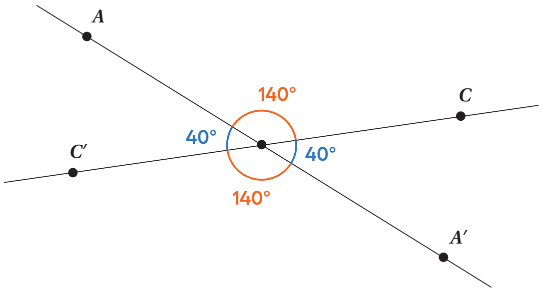
Rigid transformations have the following properties:

- A rigid transformation of a line is a line.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.
- Sometimes, a rigid transformation takes a line to itself. For example:



- A translation parallel to the line. The arrow shows a translation of line m that will take m to itself.
- A rotation by 180° around any point on the line. A 180° rotation of line m around point F will take m to itself.
- A reflection across any line perpendicular to the line. A reflection of line m across the dashed line will take m to itself.

These facts let us make an important conclusion. If two lines intersect at a point, which we'll call O, then a 180° rotation of the lines with center O shows that **vertical angles** are congruent. Here is an example:



Rotating both lines by 180° around O sends angle AOC to angle $A'OC'$, therefore proving that they have the same measure. The rotation also sends angle AOC' to angle $A'OC$.

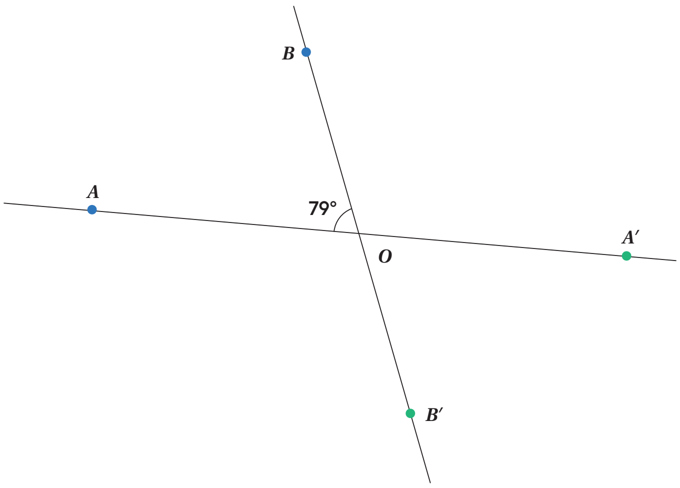
Cool-down

Finding Missing Measurements

5 min

Student Task Statement

Points A' and B' are the images of A and B after a 180° rotation around point O.



Answer each question and explain your reasoning *without* measuring segments or angles.

1. Name a segment whose length is the same as segment AO.

   Segment $A'O$, because A' is the image of A after a 180° rotation with center at O. This rotation preserves distances and takes segment AO to segment $A'O$.

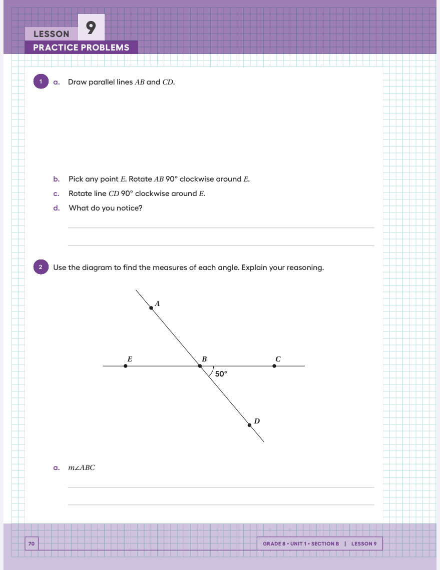
2. What is the measure of angle $A'OB'$?

   79°, the same measure as $\angle AOB$, because the 180° rotation with center at O takes $\angle AOB$ to $\angle A'OB'$. The rotation preserves angle measures.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Lesson 9

Practice Problems

5 Problems

Problem 1

a. Draw parallel lines AB and CD.

Answers vary

b. Pick any point E. Rotate AB 90° clockwise around E.

Sample response: The new line should be perpendicular to AB.

c. Rotate line CD 90° clockwise around E.

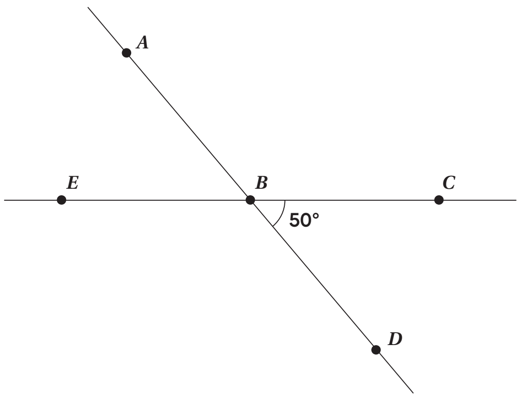
Sample response: The new line should be perpendicular to CD and parallel to $A'B'$.

d. What do you notice?

Sample response: The two new rotated lines are parallel.

Problem 2

Use the diagram to find the measures of each angle. Explain your reasoning.



a. $m\angle ABC$

130 degrees

Sample reasoning: Angle ABC and angle CBD make a line, so they add up to 180 degrees.

b. $m\angle EBD$

130 degrees

Sample reasoning: Angle EBD and angle CBD make a line, so they add up to 180 degrees.

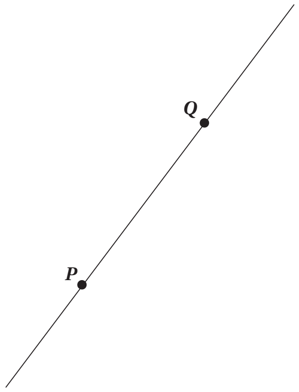
c. $m\angle ABE$

50 degrees

Sample reasoning: Angle ABE and angle ABC make a line, so they add up to 180 degrees.

Problem 3

Points P and Q are plotted on a line.



a. Find a point R so that a 180° rotation with center R sends P to Q and Q to P.

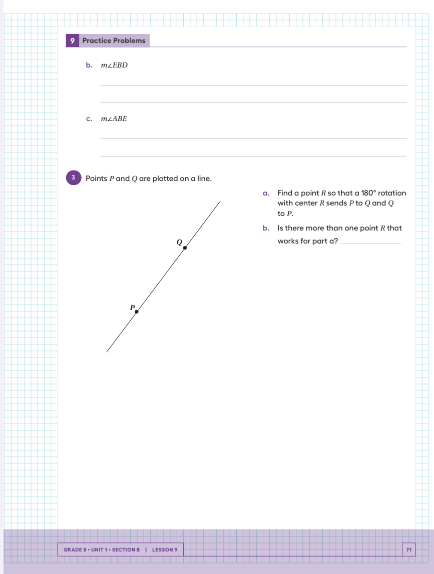
Sample response: If R is the midpoint of segment PQ, then a rotation of 180° with center R sends P to Q and Q to P.

b. Is there more than one point R that works for part a?

no

Sample reasoning: The midpoint of PQ is the only point that works. 180° rotations with any other center do not send P to Q or Q to P.

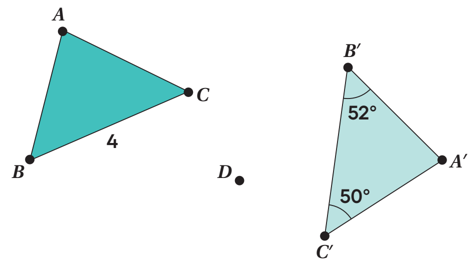
Student Workbook



Problem 4

from Unit 1, Lesson 7

In the picture triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is D.



a. What is the length of side $B'C'$? Explain how you know.

4 units
Sample reasoning: Rotations preserve side lengths, and side $B'C'$ corresponds to side BC under this rotation.

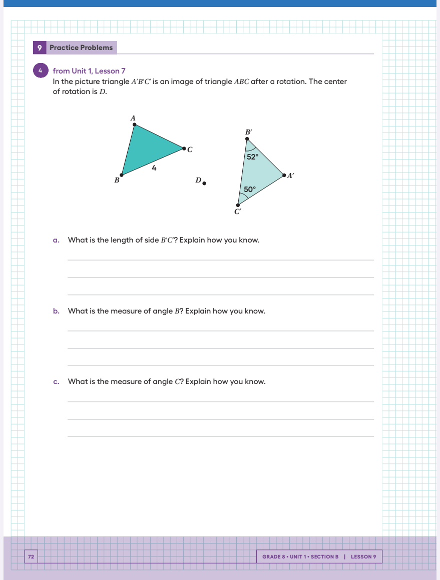
b. What is the measure of angle B? Explain how you know.

52 degrees
Sample reasoning: Rotations preserve angle measures, and angles B and B' correspond to each other under this rotation.

c. What is the measure of angle C? Explain how you know.

50 degrees
Sample reasoning: Rotations preserve angle measures, and angles C and C' correspond to each other under this rotation.

Student Workbook



Problem 5

from Unit 1, Lesson 6

The point (-4, 1) is rotated 180° counterclockwise using center (0, 0). What are the coordinates of the image?

A. (-1, -4)

B. (-1, 4)

C. (4, 1)

D. (4, -1)