Lesson 9

Multiplying Rational Numbers (Part 2)

Goals

- Generalize (orally) that the product of two negative numbers is positive.
- Interpret signed numbers used to represent elapsed time before or after a chosen reference point.
- Use patterns to find the product of signed numbers, and explain (orally and using other representations) the reasoning.

Learning Targets

- I can explain what it means when time is represented with a negative number in a situation about speed and direction.
- I can multiply two negative numbers.

Student Learning Goal

Let's multiply signed numbers

Lesson Narrative

In this lesson, students explore multiplying a negative number by a negative number. To make sense of why such a product is positive, they expand the context of constant velocity to include times before a chosen reference point.

First, students learn to interpret negative values for time as times before a chosen starting point. Assuming that the object was already moving at a constant velocity, they calculate its position at various negative times. An object that is moving with a constant negative velocity will be at a negative position after crossing the reference point, but before crossing this point its position was positive. That is, **(negative velocity) · (negative time) = (positive position).** As students make sense of situations that involve negative velocity or negative time and represent these situations with equations, they are reasoning abstractly and quantitatively.

The following two activities are optional, so teachers can choose what best fits the needs of their students. Both of the activities are designed to reveal patterns that reinforce what students have learned about multiplying signed numbers. The first optional activity uses the structure of the coordinate grid. The second optional activity expands the standard multiplication table to include negative factors. Teachers may select one activity for the whole class to work on or they may let each student choose.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2, Activity 3)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Required Materials

Materials to Copy

- Rational Numbers Multiplication Grid Handout (1 copy for every student): Activity 3

Required Preparation

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Activity 3:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	10 min	5 min

Multiplying Rational Numbers (Part 2)

Lesson Narrative (continued)

Some of the activities in this lesson work best when each student has access to a device that can run the applet because they will benefit from seeing the relationship in a dynamic way.

Warm-up

Before and After

5 min

Activity Narrative

In this *Warm-up*, students interpret negative time in the context of a person walking. This activity primes students to think about opposites and constant speeds, which will be useful in following activities.

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time followed by time for partner discussion.

Student Task Statement

1. Where was the person 5 seconds *after* this picture was taken?

   Students should mark a position to the right of the person's current position.

2. Where was the person 5 seconds *before* this picture was taken?

   Students should mark a position to the left of the person's current position. The marks on the left and right should be about equally distant from the person's current position.

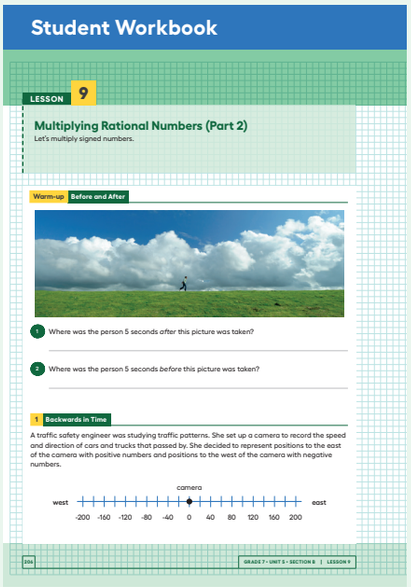
Activity Synthesis

The purpose of this discussion is to get students thinking about constant speeds and negative times. Begin by displaying the image from the *Task Statement* for all to see. Invite students to share where they think the person was 5 seconds earlier and will be 5 seconds later.

If necessary, point out that if we assume the person is walking at a constant speed, their before and after locations should be equally far from their current position in the image. Ask students how they might represent time in this situation.

5 seconds before the picture was taken could be -5, while 5 seconds after the picture was taken would be +5. The picture was taken when the time was 0.

Student Workbook


LESSON 9

Multiplying Rational Numbers (Part 2)

Let's multiply signed numbers.

Warm-up Before and After

1 Where was the person 5 seconds *after* this picture was taken?

2 Where was the person 5 seconds *before* this picture was taken?

1 Backwards in Time

A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.

camera

west east

-200 -160 -120 -80 -40 0 40 80 120 160 200

Access for Multilingual Learners (Activity 1, Student Task)

MLR8: Discussion Supports.
Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

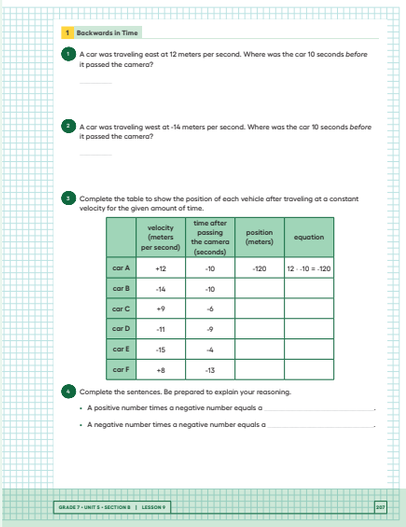
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Access for Perception.
Begin by showing a video of a car or person moving forwards, and then play the same footage in reverse to support understanding of the context.

Supports accessibility for: Conceptual Processing, Language

Student Workbook



1 Backwards in Time

1. A car was traveling east at 12 meters per second. Where was the car 10 seconds *before* it passed the camera?

2. A car was traveling west at -14 meters per second. Where was the car 10 seconds *before* it passed the camera?

3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	-10	-120	12 · -10 = -120
car B	-14	-10		
car C	+9	-6		
car D	-11	-9		
car E	-15	-4		
car F	+8	-13		

4. Complete the sentences. Be prepared to explain your reasoning.
   - A positive number times a negative number equals a _____.
   - A negative number times a negative number equals a _____.

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Activity 1
Backwards in Time

15 min

Activity Narrative

In this activity, students consider a context of vehicles traveling past a traffic camera at varied constant velocities and in different directions. Students reason about what it means to have a negative time (the vehicle has not yet passed the traffic camera) or a negative velocity (the vehicle is traveling from east to west) in this context. They engage in reasoning abstractly and quantitatively as they represent velocity, distance, and time relationships using an equation, then interpret the results in terms of the situation.

Students also use repeated reasoning to describe a pattern for identifying the sign of the product of two negative numbers.

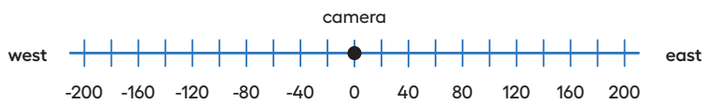
Launch

Keep students in the same groups. Remind the students of movement east or west as positive or negative velocity.

This activity is the same context as one in the previous lesson, and the questions are related. So students should be able to get to work rather quickly. However, each question requires some careful thought, and one question builds on the other. Consider suggesting that students check in with their partner frequently and explain their thinking. Additionally, you might consider asking students to pause after each question for a quick whole-class discussion before continuing to the next question.

Student Task Statement

A traffic safety engineer was studying traffic patterns. She set up a camera to record the speed and direction of cars and trucks that passed by. She decided to represent positions to the east of the camera with positive numbers and positions to the west of the camera with negative numbers.



1. A car was traveling east at 12 meters per second. Where was the car 10 seconds *before* it passed the camera?

   the position -120 meters, which is west of the camera

2. A car was traveling west at -14 meters per second. Where was the car 10 seconds *before* it passed the camera?

   the position +140 meters, which is east of the camera

3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	-10	-120	12 · -10 = -120
car B	-14	-10	+140	-14 · -10 = 140
car C	+9	-6	-54	9 · (-6) = -54
car D	-11	-9	+99	-11 · (-9) = 99
car E	-15	-4	+60	-15 · (-4) = 60
car F	+8	-13	-104	8 · (-13) = -104

4. Complete the sentences. Be prepared to explain your reasoning.

- A positive number times a negative number equals a negative number.
- A negative number times a negative number equals a positive number.

Activity Synthesis

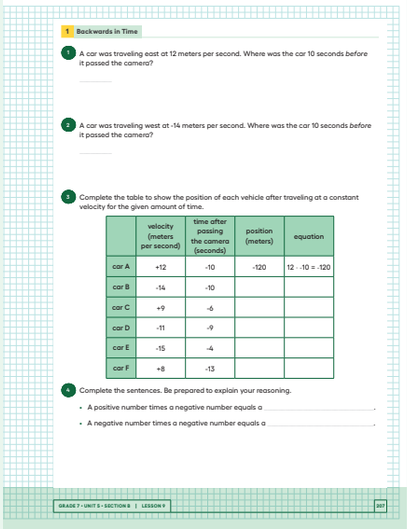
The key thing for students to understand from this discussion is that a negative number multiplied by another negative number results in a positive number. Begin by displaying the number line and table from the *Task Statement* for all to see. Invite students to share their responses, and record them in the table.

Then select a car whose equation has a positive and a negative factor, and demonstrate its path on the number line. For example, explain that Car A is traveling from west to east since its velocity is positive. Since its time is represented by a negative number, the car has not yet passed the camera and, therefore, must be located on the left side of the camera, making its position a negative number.

Then select a car whose equation has two negative factors. For example, Car D has a negative velocity, which means that it is traveling from east to west. It also has a negative time, which means it has not yet passed the camera. Car D must be located to the right of the camera, making its position a positive number.

Ask students to complete the sentences in the last problem. Consider posting these in the classroom for all to see for the next few lessons.

Student Workbook


1 Backwards in Time

1. A car was traveling east at 12 meters per second. Where was the car 10 seconds *before* it passed the camera?

2. A car was traveling west at -14 meters per second. Where was the car 10 seconds *before* it passed the camera?

3. Complete the table to show the position of each vehicle after traveling at a constant velocity for the given amount of time.

	velocity (meters per second)	time after passing the camera (seconds)	position (meters)	equation
car A	+12	-10	-120	12 · -10 = -120
car B	-14	-10		
car C	+9	-6		
car D	-11	-9		
car E	-15	-4		
car F	+8	-13		

4. Complete the sentences. Be prepared to explain your reasoning.
- A positive number times a negative number equals a _____.
- A negative number times a negative number equals a _____.

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Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as the digital applet.

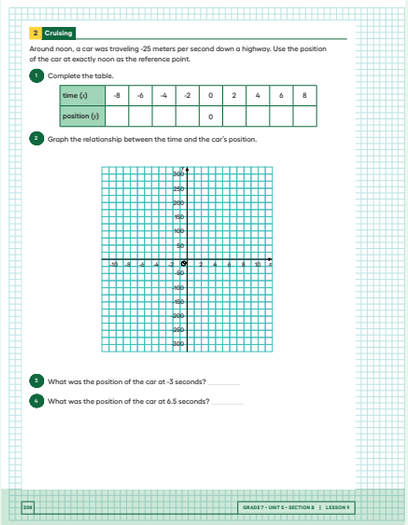
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Access for Multilingual Learners (Activity 2, Student Task)

MLR2: Collect and Display.
Collect the language that students use to describe where the car is at -3 seconds and at 6.5 seconds. Display words and phrases, such as "-3 seconds on the table is between -4 and -2" or "I found 6.5 on the horizontal axis." During the *Activity Synthesis*, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student Workbook


2 Cruising

Around noon, a car was traveling -25 meters per second down a highway. Use the position of the car at exactly noon as the reference point.

1. Complete the table.

time (x)	-8	-6	-4	-2	0	2	4	6	8
position (y)					0				

2. Graph the relationship between the time and the car's position.

3. What was the position of the car at -3 seconds? _____

4. What was the position of the car at 6.5 seconds? _____

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Activity 2: Optional
Cruising

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students find the position of a car traveling at a constant velocity at different positive and negative times and plot points representing the car's position in the coordinate plane. They see that just as with constant speed, the graph goes through (0, 0), but because the velocity is negative, it slants downward from left to right instead of passing through the first quadrant.

In the digital version of the activity, students use an applet to plot points in the coordinate plane. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Launch

Arrange students in groups of 2. Give them 6 minutes of quiet work time followed by time for partner discussion. Then follow with a whole-class discussion.

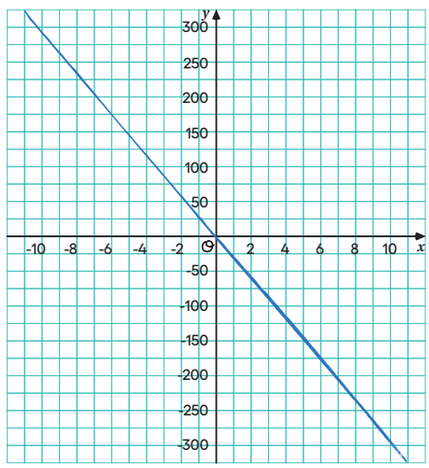
Student Task Statement

Around noon, a car was traveling -25 meters per second down a highway. Use the position of the car at exactly noon as the reference point.

1. Complete the table.

time (x)	-8	-6	-4	-2	0	2	4	6	8
position (y)	200	150	100	50	0	-50	-100	-150	-200

2. Graph the relationship between the time and the car's position.



3. What was the position of the car at -3 seconds?

75 meters

4. What was the position of the car at 6.5 seconds?

-162.5 meters

Are You Ready for More?

Find the value of these expressions without using a calculator.

$(-1)^2$ 1 $(-1)^3$ -1

$(-1)^4$ 1 $(-1)^{99}$ -1

Activity Synthesis

The purpose of this discussion is for students to write an equation describing the position of the car at a given time and to compare this relationship to a proportional relationship. Consider discussing the following questions:

- *"How are this graph and the graph of a proportional relationship the same or different?"*

  Both graphs go through the point (0, 0). Graphs of proportional relationships slant upwards, while this graph slants downward.

- *"How can we write an equation to represent this situation if y is the car's position and x is the time in seconds?"*

  $y = -25x$ (print) or $y = -32$ (digital)

Activity 3: Optional

Rational Numbers Multiplication Grid

10 min

Activity Narrative

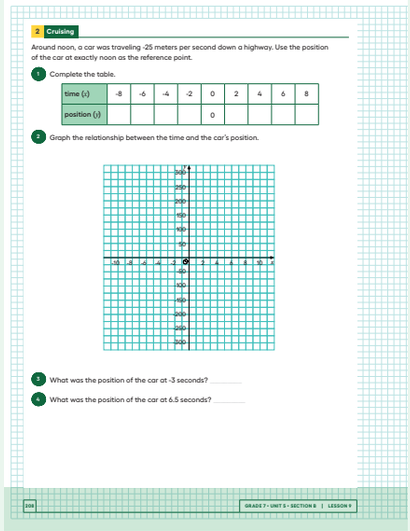
There is a digital version of this activity.

In this activity, students revisit the representation of a multiplication chart that has been extended to include negative numbers. Students complete the chart and use repeated reasoning to identify patterns about the sign of the products of positive and negative numbers.

If necessary, remind students how a multiplication chart works—the factors are listed at the end of the rows and columns, and their products go in the boxes.

In the digital version of the activity, students use an applet to complete the multiplication chart. The applet reveals additional rows and columns when prompted and shows incorrect answers in a different color. This activity works best when students have access to the applet because students can focus on fewer numbers at a time and receive immediate feedback on whether their answers are correct.

Student Workbook


2 Cruising

Around noon, a car was travelling -25 meters per second down a highway. Use the position of the car at exactly noon as the reference point.

1. Complete the table.

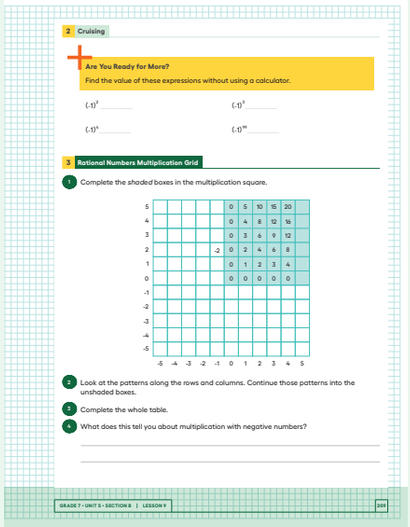
time (x)	-8	-6	-4	-2	0	2	4	6	8
position (y)					0				

2. Graph the relationship between the time and the car's position.

3. What was the position of the car at -3 seconds?

4. What was the position of the car at 6.5 seconds?

Student Workbook


2 Cruising

Are You Ready for More?

Find the value of these expressions without using a calculator.

$(-1)^2$ $(-1)^3$

$(-1)^4$ $(-1)^{99}$

3 Rational Numbers Multiplication Grid

1. Complete the *shaded* boxes in the multiplication square.

					0	5	10	15	20
					0	4	8	12	16
					0	3	6	9	12
				-2	0	2	4	6	8
					0	1	2	3	4
					0	0	0	0	0

2. Look at the patterns along the rows and columns. Continue those patterns into the unshaded boxes.

3. Complete the whole table.

4. What does this tell you about multiplication with negative numbers?

Access for Students with Diverse Abilities (Activity 3, Launch)

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as the digital applet.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

The blackline master has a multiplication chart that also includes the factors 1.5, -1.5, 3.2, and -3.2 so that students can see how the patterns extend to rational numbers that are not integers. Encourage students to complete the rows and columns for the integers first and then come back to 1.5, -1.5, 3.2, and -3.2 later. Directions are included on the blackline master for a way that students can fold their papers to hide the nonintegers while they fill in the integers. If students use this technique, it would be beneficial to demonstrate and walk students through the process of folding their paper.

Launch

Arrange students in groups of 3. If desired, distribute 1 copy of the blackline master to every student, and instruct students to ignore the chart printed in their books or devices. (Also if desired, instruct students to fold their papers according to the directions on the top and right sides of the chart so that the decimal rows and columns are temporarily hidden.) Give students 30 seconds of quiet think time. Have them share what patterns they notice about the numbers that are already filled in. Give the groups 5 minutes of work time, and follow with a whole-class discussion.

Student Task Statement

1. Complete the *shaded* boxes in the multiplication square.

1–3. For the table in student workbooks and on the blackline master:

	-5	-4	-3	-2	-1	0	1	2	3	4	5
5	-25	-20	-15	-10	-5	0	5	10	15	20	-25
4	-20	-16	-12	-8	-4	0	4	8	12	16	-20
3	-15	-12	-9	-6	-3	0	3	6	9	12	-15
2	-10	-8	-6	-4	-2	0	2	4	6	8	-10
1	-5	-4	-3	-2	-1	0	1	2	3	4	-5
0	0	0	0	0	0	0	0	0	0	0	0
-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
-5	25	20	15	10	5	0	-5	-10	-15	-20	-25

2. Look at the patterns along the rows and columns. Continue those patterns into the unshaded boxes.

3. Complete the whole table.

4. What does this tell you about multiplication with negative numbers?

Sample response: A positive number multiplied by a negative number is negative, while a negative number multiplied by a negative number is positive.

Activity Synthesis

The goal of this discussion is to emphasize that the product of two negative numbers is a positive number. This is true whether or not the numbers are integers.

Display a complete chart for all to see, and begin by asking students to share any patterns they see. Record these observations for all to see. Then ask students to explain the ways in which the chart shows that the product of two negative numbers is a positive number. The general argument involves assuming that a pattern observed in a row or column will continue on the other side of 0.

Lesson Synthesis

Share with students,

- *"Today we used time before a certain starting point to make sense of multiplying negative numbers."*

To help students consolidate their learning about multiplying signed numbers, consider asking:

- *"What does it mean if a time is negative?"*

  It means the amount of time before a chosen reference time.

- *"What kind of number do we get when we multiply a positive number by a negative number? Use a situation from the lesson to explain why this makes sense."*

  We get a negative number. For example, if you were walking from left to right, then before you crossed the reference point, you would have been to the left.

- *"What kind of number do we get when we multiply a negative number by a negative number? Use a situation from the lesson to explain why this makes sense."*

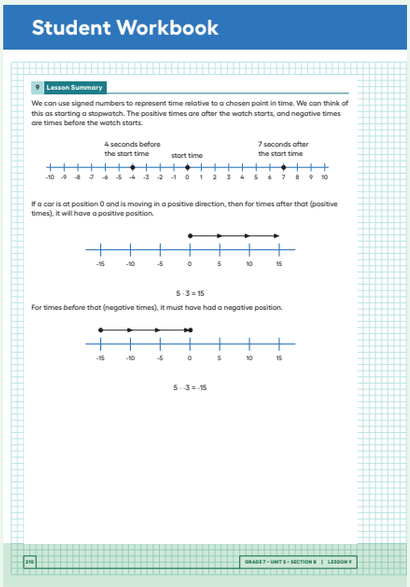
  We get a positive number. For example, if you were walking from right to left, then before you crossed the reference point, you would have been to the right.

Access for Multilingual Learners (Activity 3, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question about what the patterns in the table tell about the multiplication of negative numbers. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Workbook


Lesson Summary

We can use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.

4 seconds before the start time
start time
7 seconds after the start time

If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position.

5 · 3 = 15

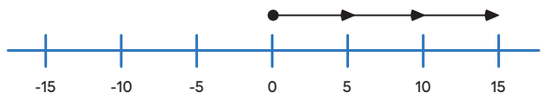
For times *before* that (negative times), it must have had a negative position.

5 · -3 = -15

Lesson Summary

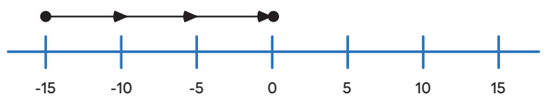
We can use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.

If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position.



$5 \cdot 3 = 15$

For times *before* that (negative times), it must have had a negative position.

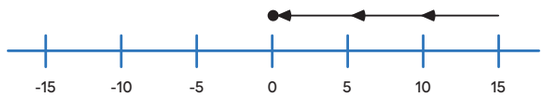


$5 \cdot -3 = -15$

If a car is at position 0 and is moving in a negative direction, then for times after that (positive times), it will have a negative position.

$-5 \cdot 3 = -15$

For times *before* that (negative times), it must have had a positive position.



$-5 \cdot -3 = 15$

Here is another way of seeing this:

- A positive number times a positive number always results in a positive number.
- A negative number times a positive number or a positive number times a negative number always results in a negative number.
- A negative number times a negative number always results in a positive number.

Cool-down

True Statements

5 min

Student Task Statement

Decide if each equation is true or false.

1. $7 \cdot 8 = 56$

true

2. $-7 \cdot 8 = 56$

false

3. $-7 \cdot -8 = -56$

false

4. $-7 \cdot -8 = 56$

true

5. $(3.5) \cdot 12 = 42$

true

6. $(-3.5) \cdot -12 = -42$

false

7. $(-3.5) \cdot -12 = 42$

true

8. $-12 \cdot \frac{7}{2} = 42$

false

Responding To Student Thinking

Points to Emphasize

If most students struggle with multiplying signed numbers, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the multiplication expressions in these activities:

Grade 7, Unit 5, Lesson 10, Activity 3 Card Sort: Matching Expressions

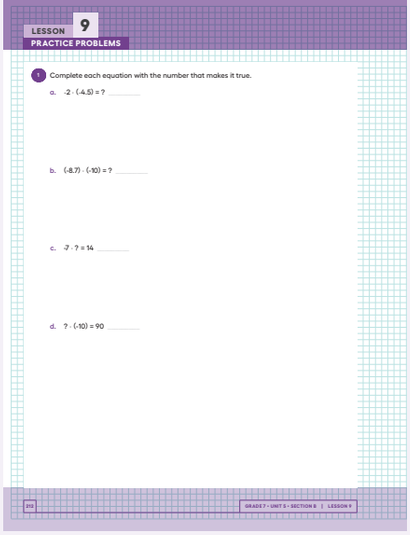
Grade 7, Unit 5, Lesson 11, Activity 1 Multiplication and Division

Lesson 9

Practice Problems

6 Problems

Student Workbook


LESSON 9
PRACTICE PROBLEMS

1 Complete each equation with the number that makes it true.

a. -2 · (-4.5) = ? _____

b. (-8.7) · (-10) = ? _____

c. -7 · ? = 14 _____

d. ? · (-10) = 90 _____

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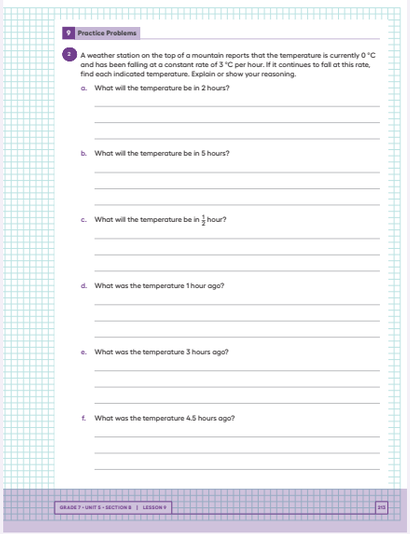
Student Workbook


9 Practice Problems

2 A weather station on the top of a mountain reports that the temperature is currently 0 °C and has been falling at a constant rate of 3 °C per hour. If it continues to fall at this rate, find each indicated temperature. Explain or show your reasoning.

a. What will the temperature be in 2 hours?

b. What will the temperature be in 5 hours?

c. What will the temperature be in $\frac{1}{2}$ hour?

d. What was the temperature 1 hour ago?

e. What was the temperature 3 hours ago?

f. What was the temperature 4.5 hours ago?

GRADE 7 • UNIT 3 • SECTION B | LESSON 9

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Problem 1

Complete each equation with the number that makes it true.

a. $-2 \cdot (-4.5) = ?$

9

b. $(-8.7) \cdot (-10) = ?$

87

c. $-7 \cdot ? = 14$

-2

d. $? \cdot (-10) = 90$

-9

Problem 2

A weather station on the top of a mountain reports that the temperature is currently 0 °C and has been falling at a constant rate of 3 °C per hour. If it continues to fall at this rate, find each indicated temperature. Explain or show your reasoning.

a. What will the temperature be in 2 hours?

-6 °C, because $-3 \cdot 2 = -6$

b. What will the temperature be in 5 hours?

-15 °C, because $-3 \cdot 5 = -15$

c. What will the temperature be in $\frac{1}{2}$ hour?

-1.5 °C, because $-3 \cdot 0.5 = -1.5$

d. What was the temperature 1 hour ago?

3 °C, because $-3 \cdot (-1) = 3$

e. What was the temperature 3 hours ago?

9 °C, because $-3 \cdot (-3) = 9$

f. What was the temperature 4.5 hours ago?

13. °C, because $-3 \cdot (-4.5) = 13.5$

Problem 3

Find the value of each expression.

a. $\frac{1}{4} \cdot (-12)$

-3

b. $-\frac{1}{3} \cdot 39$

-13

c. $\left(-\frac{4}{5}\right) \cdot (-75)$

60

d. $-\frac{2}{5} \cdot \left(-\frac{3}{4}\right)$

$\frac{3}{10}$ (or equivalent)

e. $\frac{8}{3} \cdot -42$

-112

Problem 4

from Unit 5, Lesson 5

George Washington Carver was a scientist who experimented with natural ways that farmers could improve their soil and grow more crops.

a. In 1897 he spent about $16 on his experiment, and his crops earned about $10. How much profit was made?

-$6

b. After improving the soil for 5 years, in 1902 he spent $12 on his experiment, and his crops earned $50. How much profit was made?

$38

c. How much greater was the profit in 1902 than in 1897?

$44

Problem 5

from Unit 5, Lesson 7

a. Here are the vertices of rectangle $FROG$: (-2, 5),(-2, 1),(6, 5),(6, 1). Find the perimeter of this rectangle. If you get stuck, try plotting the points on a coordinate plane.

24 units

The short side of the rectangle has length 4 units, because 5 − 1 = 4. The long side has length 8 units, because 6 − (−2) = 8. The perimeter is 24 units, because 4 + 4 + 8 + 8 = 24.

b. Find the area of the rectangle $FROG$.

32 square units, because $8 \cdot 4 = 32$

c. Here are the coordinates of rectangle $PLAY$: (-11, 20),(-11, -3),(-1, 20),(-1, -3). Find the perimeter and area of this rectangle. See if you can figure out its side lengths without plotting the points.

The perimeter is 66 units, and the area is 230 square units. The short side has length 10 units, because −1 − (−11) = 10. The long side has length 23 units, because 20 − (−3) = 23. The perimeter is 66 units, because 2(10 + 23) = 66.

Problem 6

from Unit 4, Lesson 2

To make a specific hair dye, a hair stylist uses a ratio of $1\frac{1}{8}$ oz of red tone, $\frac{3}{4}$ oz of gray tone, and $\frac{5}{8}$ oz of brown tone.

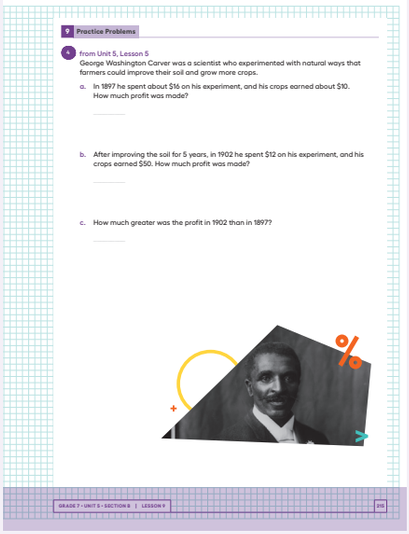
a. If the stylist needs to make 20 oz of dye, how much of each dye color is needed?

9 oz red, 6 oz gray, and 5 oz brown, because the given amounts of dye make $2\frac{1}{2}$ oz, so each quantity needs to be multiplied by 8 to get 20 oz

b. If the stylist needs to make 100 oz of dye, how much of each dye color is needed?

45 oz red, 30 oz gray, and 25 oz brown, because 100 oz is 5 batches of 20 oz

Student Workbook



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