Lesson 11

Dividing Rational Numbers

Goals

- Apply multiplication and division of signed numbers to solve problems involving constant speed with direction, and explain (orally) the reasoning.
- Generalize (orally) a method for determining the quotient of two signed numbers.
- Generate a division equation that represents the same relationship as a given multiplication equation with signed numbers.

Learning Target

I can divide rational numbers.

Student Learning Goal

Let's divide signed numbers

Lesson Narrative

In this lesson, students divide signed numbers. First, students consider a multiplication equation and identify the **solution to the equation**, which is the value of the variable that makes the equation true. Then, they use the relationship between multiplication and division to develop rules for dividing signed numbers. They apply these rules to solve problems about constant velocity and the progress of a drilling rig.

As students describe the rules for dividing signed numbers, they are attending to precision of language.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

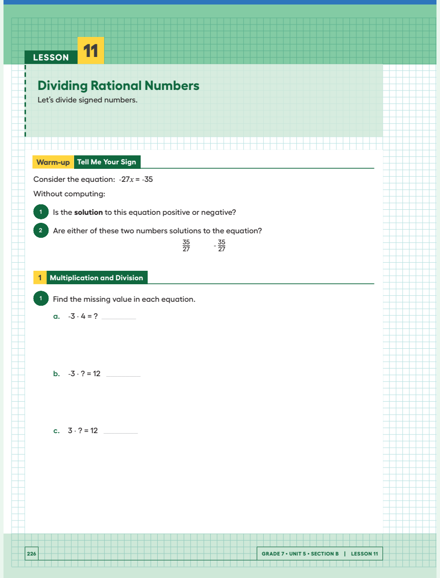
Required Preparation

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	10 min	10 min	5 min

Student Workbook


LESSON 11

Dividing Rational Numbers

Let's divide signed numbers.

Warm-up Tell Me Your Sign

Consider the equation: $-27x = -35$

Without computing:

1. Is the **solution** to this equation positive or negative?
2. Are either of these two numbers solutions to the equation? $\frac{35}{27}$ $-\frac{35}{27}$

1 Multiplication and Division

1. Find the missing value in each equation.

a. $-3 \cdot 4 = ?$

b. $-3 \cdot ? = 12$

c. $3 \cdot ? = 12$

GRADE 7 • UNIT 5 • SECTION B | LESSON 11

Warm-up
Tell Me Your Sign

5 min

Activity Narrative

In this *Warm-up*, students use what they have learned about multiplication and division with rational numbers to answer questions about the solution to an equation. They use the structure of the equation and patterns they have noticed with the signs of products and quotients of positive and negative numbers to determine the sign of the solution.

Launch

Arrange students in groups of 2.

Remind students that the **solution to an equation** is a value that makes the equation true.

Give students 1 minute of quiet think time, and ask them to discuss their reasoning with a partner. Follow with a whole-class discussion.

Student Task Statement

Consider the equation: $-27x = -35$

Without computing:

1. Is the **solution** to this equation positive or negative?

   positive

2. Are either of these two numbers solutions to the equation?

   $\frac{35}{27}$ $\qquad$ $-\frac{35}{27}$

Activity Synthesis

The purpose of this discussion is for students to share their reasoning. Invite students to share their responses, and record them for all to see.

Activity 1
Multiplication and Division

10 min

Activity Narrative

In this activity, students rewrite multiplication equations as division equations and use their understanding of multiplication and division with integers to solve a problem in context. Students use concrete examples and their prior experiences to more precisely articulate a rule for the sign of a quotient given the signs of the dividend and divisor.

Monitor for students who identify and describe the rule clearly.

Launch

Remind students that we can rearrange division equations to be multiplication equations, and vice versa. It may be useful to demonstrate with positive numbers if students struggle to recall this. For example, ask how we could rewrite $10 \div 2 = 5$ as a multiplication equation. Students can also reference the multiplication table from a previous lesson, if needed.

Arrange students in groups of 2. Give students 4 minutes of quiet work time followed by 2 minutes of partner discussion, then follow with a whole-class discussion.

Student Task Statement

1. Find the missing value in each equation.

a. $-3 \cdot 4 = ?$

-12

b. $-3 \cdot ? = 12$

-4

c. $3 \cdot ? = 12$

4

d. $? \cdot -4 = 12$

-3

e. $? \cdot 4 = -12$

-3

2. Rewrite each unknown factor problem (the last four equations of the previous problem) as a division problem.

Sample response:

a. No written response required.

b. $12 \div (-3) = -4$

c. $12 \div 3 = 4$

d. $12 \div (-4) = -3$

e. $-12 \div 4 = -3$

3. Complete the sentences. Be prepared to explain your reasoning.

a. A positive number divided by a positive number equals a positive number.

b. A positive number divided by a negative number equals a negative number.

c. A negative number divided by a positive number equals a negative number.

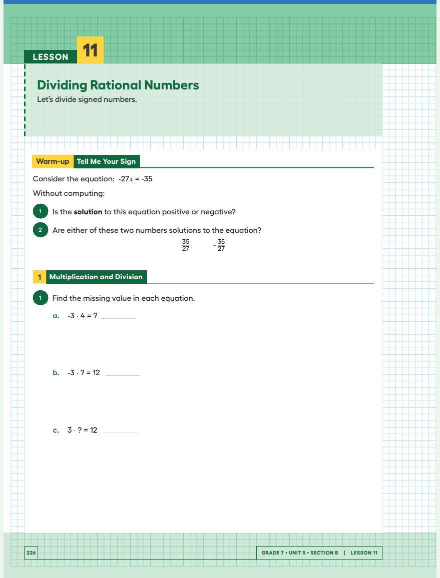
d. A negative number divided by a negative number equals a positive number.

Access for Students with Diverse Abilities (Activity 1, Launch)

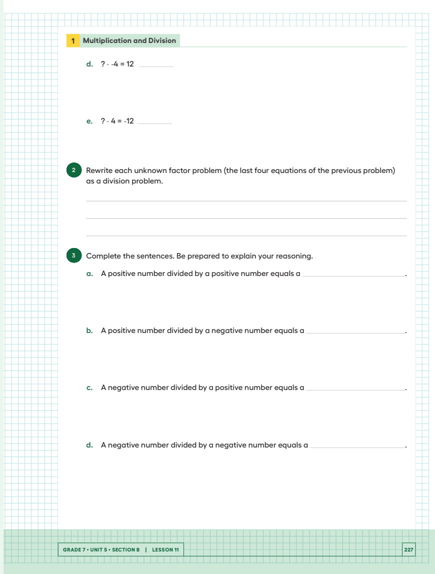
Action and Expression: Internalize Executive Functions.
To support organization, provide students with a graphic organizer that lists the multiplication equations and has space for students to write the associated division equation.

Supports accessibility for: Language, Organization

Student Workbook



Student Workbook


1 Multiplication and Division

d. ? · -4 = 12 ____

e. ? · 4 = -12 ____

2 Rewrite each unknown factor problem (the last four equations of the previous problem) as a division problem.

3 Complete the sentences. Be prepared to explain your reasoning.

a. A positive number divided by a positive number equals a ____.

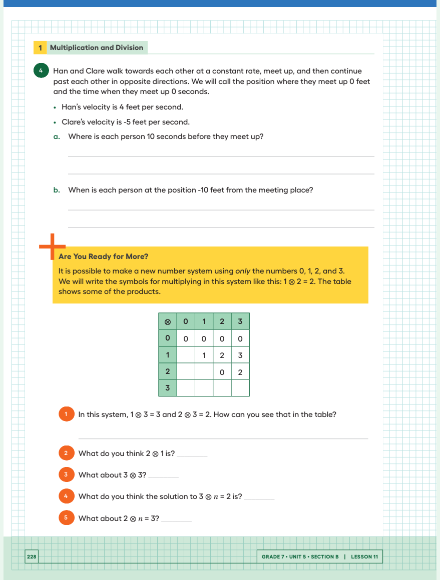
b. A positive number divided by a negative number equals a ____.

c. A negative number divided by a positive number equals a ____.

d. A negative number divided by a negative number equals a ____.

GRADE 7 • UNIT 5 • SECTION B | LESSON 11 227

Student Workbook


1 Multiplication and Division

4 Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.

- Han's velocity is 4 feet per second.
- Clare's velocity is -5 feet per second.

a. Where is each person 10 seconds before they meet up?

b. When is each person at the position -10 feet from the meeting place?

Are You Ready for More?

It is possible to make a new number system using only the numbers 0, 1, 2, and 3. We will write the symbols for multiplying in this system like this: 1 ⊗ 2 = 2. The table shows some of the products.

⊗	0	1	2	3
0	0	0	0	0
1		1	2	3
2			0	2
3				

1 In this system, 1 ⊗ 3 = 3 and 2 ⊗ 3 = 2. How can you see that in the table?

2 What do you think 2 ⊗ 1 is?

3 What about 3 ⊗ 3?

4 What do you think the solution to 3 ⊗ n = 2 is?

5 What about 2 ⊗ n = 3?

228 GRADE 7 • UNIT 5 • SECTION B | LESSON 11

4. Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.

- Han's velocity is 4 feet per second.
- Clare's velocity is -5 feet per second.

a. Where is each person 10 seconds before they meet up?

Han was at -40 feet, and Clare was at 50 feet.

b. When is each person at the position -10 feet from the meeting place?

Han was there at -2.5 seconds, and Clare was there at 2 seconds.

Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for multiplying in this system like this: 1 ⊗ 2 = 2. The table shows some of the products.

⊗	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	

1. In this system, 1 ⊗ 3 = 3 and 2 ⊗ 3 = 2. How can you see that in the table?

The table shows 1 ⊗ 3 = 3 in the cell in the "1" row and the "3" column.
The table shows 2 ⊗ 3 = 2 in the cell in the "2" row and the "3" column.

2. What do you think 2 ⊗ 1 is?

It could be anything, but if multiplication is commutative, then 2 ⊗ 1 = 2.

3. What about 3 ⊗ 3?

It could be anything, but if we fill out the table assuming multiplication is commutative, we can get almost all of the entries.

Looking at the table, setting 3 ⊗ 3 = 1 would make things symmetrical.

A more sophisticated argument requires us to know how to add these numbers, which we saw in an earlier extension. If the distributive property holds and 1 ⊕ 2 = 3, then 3 ⊗ 3 = (1 ⊕ 2) ⊗ 3 = (1 ⊗ 3) ⊕ (2 ⊗ 3) = 3 ⊕ 2. In the earlier extension we suggested that 3 ⊕ 2 = 1.

4. What do you think the solution to 3 ⊗ n = 2 is?

2 ⊗ 3 = 2, so if multiplication is commutative, then 3 ⊗ 2 = 2, so n = 2.

5. What about 2 ⊗ n = 3?

Looking at all of the 2 ⊗, there is no value of n that makes this equation true.

Activity Synthesis

The purpose of this discussion is for students to articulate their understanding about the sign of a quotient given the signs of the dividend and divisor. Note that students do not need to use these terms—they can give examples or use informal language. Begin by inviting students to share a multiplication problem and its corresponding division problem. Then consider discussing the following questions:

- *"What patterns do you notice between the multiplication equation and the division equation you wrote?"*

  The numbers used in both stay the same. The division equations all start with 12 or -12.

- *"What patterns do you notice between the signs of each number in the division equations?"*

  When the two numbers being divided are both positive or both negative, the answer is positive. When the two numbers being divided have different signs, the answer is negative.

- *"How do these patterns compare to what we saw with multiplication?"*

  The patterns for the sign of the answer in both multiplication and division problems are the same.

Activity 2

Drilling Down

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students multiply and divide rational numbers to represent and solve problems in the new context of a drilling rig. They use multiplication and division of negative numbers to work with a relationship that has a negative constant of proportionality. Students use what they know about the structure of proportional relationships to help them represent this situation with a graph.

In the digital version of the activity, students use an applet to plot points on a graph. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Launch

Remind students that we can model positions below the surface with negative values, so drilling 30 feet down is represented with -30 feet.

Ask students what they remember about proportional relationships. Students may say that:

- They are often represented with an equation in the form $y = kx$.
- The constant of proportionality, often called k, is the change in y for a change by 1 in x.
- A graph representing a proportional relationship is a line through (0,0) and (1, k).

Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect.
After all strategies used to determine where Han and Clare are in the last question have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask,

"What did the approaches have in common? How were they different?"
"What kinds of additional details or language helped you understand the displays?"

and

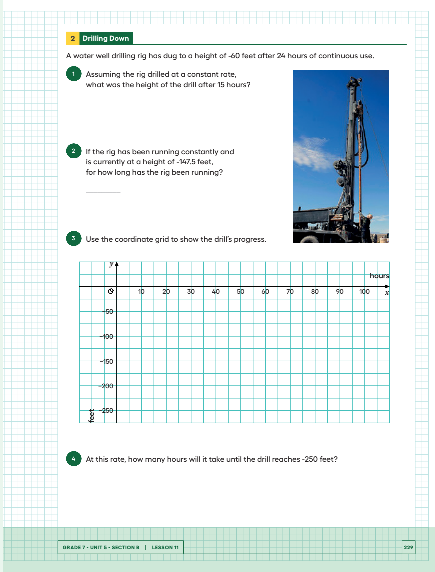
"Were there any additional details or language that you have questions about?"
Advances: Representing, Conversing

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Access for Perception.
Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language

Student Workbook


2 Drilling Down

A water well drilling rig has dug to a height of -60 feet after 24 hours of continuous use.

1 Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

2 If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

3 Use the coordinate grid to show the drill's progress.

4 At this rate, how many hours will it take until the drill reaches -250 feet?

GRADE 7 • UNIT 5 • SECTION B | LESSON 11

If necessary, explain that "one full day of continuous use" means that the drill has been running for 24 hours.

Arrange students in groups of 3 during the discussion.

Student Task Statement

A water well drilling rig has dug to a height of -60 feet after 24 hours of continuous use.

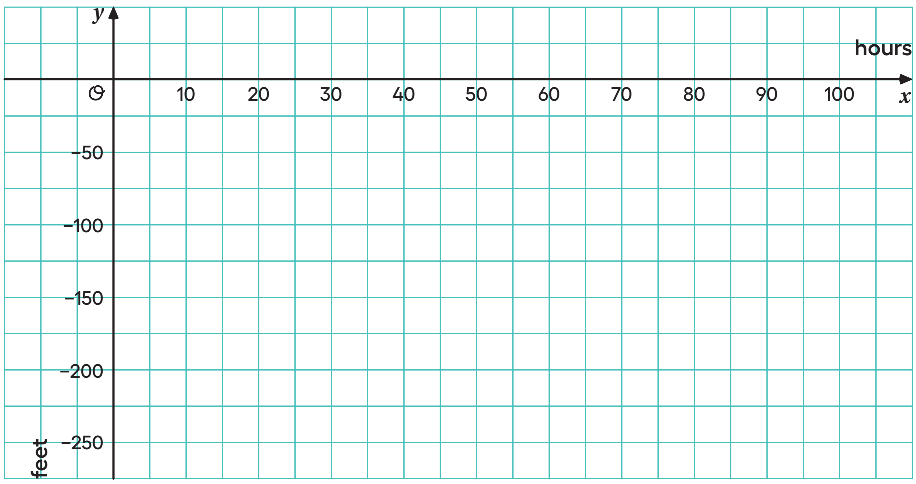
1. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

   -37.5 feet, because the drill drills $-60 \div 24$, or -2.5, feet in one hour, and $-2.5 \cdot 15 = -37.5$ or -37.5, feet after 15 hours.

2. If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

   59 hours, because $-147.5 \div (-2.5)$ is 59

3. Use the coordinate grid to show the drill's progress.



   A ray is drawn starting at (0, 0) and passing through (24, -60) and (59, -147.5).

4. At this rate, how many hours will it take until the drill reaches -250 feet?

   100 hours, because $-250 \div (-2.5) = 100$

Activity Synthesis

The goal of this discussion is for students to share their reasoning. Ask students to share their solutions with each other in groups of 3 and work to come to an agreement.

Lesson Synthesis

Share with students,

“Today we found some patterns that happen when dividing signed numbers.“

To consolidate what students have learned about multiplying and dividing signed numbers, consider asking:

“What kind of number do we get when we divide a positive number by a negative number? Use a multiplication equation to explain why this makes sense.”

We get a negative number. For example, $-3 \cdot -7 = 21$, so $21 \div -3 = -7$.

“What kind of number do we get when we divide a negative number by a negative number? Use a multiplication equation to explain why this makes sense.”

We get a positive number. For example, $-3 \cdot 7 = -21$, so $-21 \div -3 = 7$.

“Give an example of a situation that could be represented by dividing by a negative number.”

finding when a person’s position is -26 if their velocity is -4

Lesson Summary

Any division problem is actually a multiplication problem:

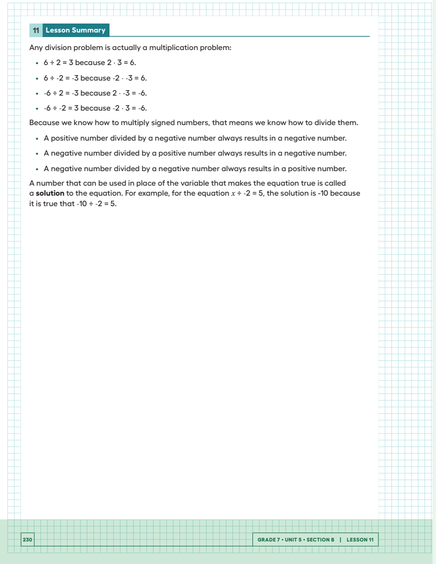
- $6 \div 2 = 3$ because $2 \cdot 3 = 6$.
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$.
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$.
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$.

Because we know how to multiply signed numbers, that means we know how to divide them.

- A positive number divided by a negative number always results in a negative number.
- A negative number divided by a positive number always results in a negative number.
- A negative number divided by a negative number always results in a positive number.

A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. For example, for the equation $x \div -2 = 5$, the solution is -10 because it is true that $-10 \div -2 = 5$.

Student Workbook


11 Lesson Summary

Any division problem is actually a multiplication problem:

- $6 \div 2 = 3$ because $2 \cdot 3 = 6$.
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$.
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$.
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$.

Because we know how to multiply signed numbers, that means we know how to divide them.

- A positive number divided by a negative number always results in a negative number.
- A negative number divided by a positive number always results in a negative number.
- A negative number divided by a negative number always results in a positive number.

A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. For example, for the equation $x \div -2 = 5$, the solution is -10 because it is true that $-10 \div -2 = 5$.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of multiplying and dividing rational numbers. If most students struggle, make time to revisit related work in the lessons referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 5, Lesson 10 Multiply!

Grade 7, Unit 5, Lesson 11 Dividing Rational Numbers

Cool-down

Matching Division Expressions

5 min

Student Task Statement

Match each expression with its value.

1. 15 ÷ 12	**A.** -0.8
2. 12 ÷ (-15)	**B.** 0.8
3. 12 ÷ 15	**C.** -1.25
4. 15 ÷ (-12)	**D.** 1.25

Lesson **11**

Practice Problems 6 Problems

Problem 1

Find the quotients:

a. 24 ÷ -6

-4

b. -15 ÷ 0.3

-50

c. -4 ÷ -20

0.2

Problem 2

Find the quotients.

a. $\frac{2}{5} \div \frac{3}{4}$

$\frac{8}{15}$ (or equivalent)

b. $\frac{9}{4} \div \frac{-3}{4}$

-3

c. $\frac{-5}{7} \div \frac{-1}{3}$

$\frac{15}{7}$ (or equivalent)

d. $\frac{-5}{3} \div \frac{1}{6}$

-10

Problem 3

Is the solution positive or negative?

a. $2 \cdot x = 6$

positive

b. $-2 \cdot x = 6.1$

negative

c. $2.9 \cdot x = -6.04$

negative

d. $-2.473 \cdot x = -6.859$

positive

Problem 4

Find the solution mentally.

a. $3 \cdot -4 = a$

$a = -12$

b. $b \cdot (-3) = -12$

$b = 4$

c. $-12 \cdot c = 12$

$c = -1$

d. $d \cdot 24 = -12$

$d = \frac{-1}{2}$

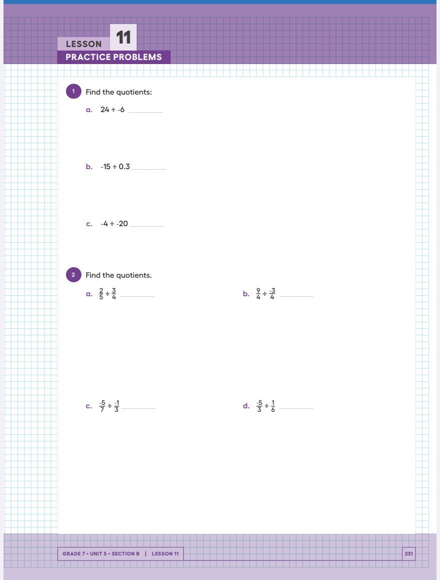
Problem 5

from Unit 4, Lesson 2

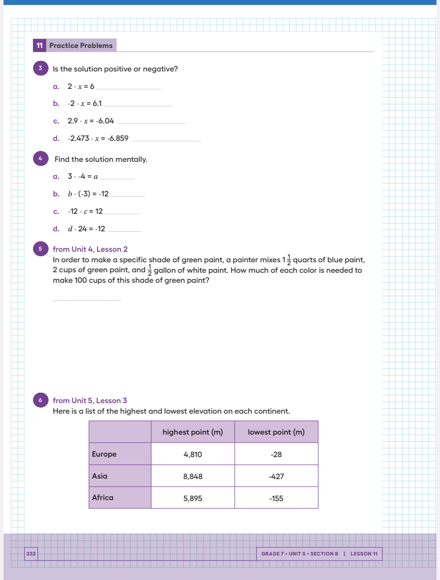
In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

Blue: $37\frac{1}{2}$ cups, green: $12\frac{1}{2}$ cups, white: 50 cups. There are 4 cups in a quart, so $1\frac{1}{2}$ quarts is 6 cups. There are 16 cups in a gallon, so $\frac{1}{2}$ gallon is 8 cups. So the ratio of cups of blue to cups of green to cups of white is 6 to 2 to 8 (or, equivalently, 3 to 1 to 4). 3 + 1 + 4 = 8. And 100 ÷ 8 = 12.5. So, the painter needs 37.5 cups of blue because $3 \cdot (12.5) = 37.5$. The painter needs 12.5 cups of green because $1 \cdot (12.5) = 12.5$. And the painter needs 50 cups of white because $4 \cdot (12.5) = 50$.

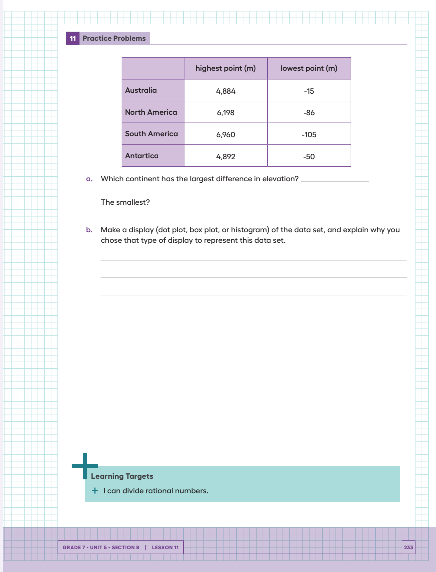
Student Workbook



Student Workbook



Student Workbook


11 Practice Problems

	highest point (m)	lowest point (m)
Australia	4,884	-15
North America	6,198	-86
South America	6,960	-105
Antarctica	4,892	-50

a. Which continent has the largest difference in elevation?

The smallest?

b. Make a display (dot plot, box plot, or histogram) of the data set, and explain why you chose that type of display to represent this data set.

Learning Targets

+ I can divide rational numbers.

Problem 6

from Unit 5, Lesson 3

Here is a list of the highest and lowest elevation on each continent.

	highest point (m)	lowest point (m)
Europe	4,810	-28
Asia	8,848	-427
Africa	5,895	-155
Australia	4,884	-15
North America	6,198	-86
South America	6,960	-105
Antarctica	4,892	-50

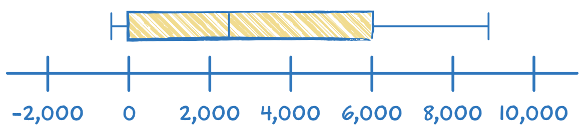
a. Which continent has the largest difference in elevation? The smallest?

Asia has the largest difference in elevation. $8,848 - (-427) = 9,275$.
Europe has the smallest difference in elevation. $4,810 - (-28) = 4,838$

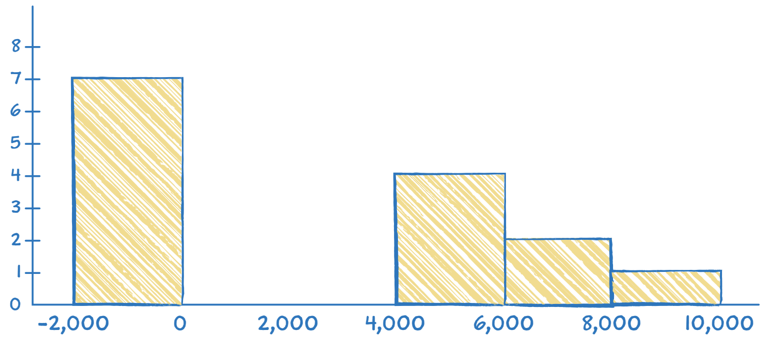
b. Make a display (dot plot, box plot, or histogram) of the data set, and explain why you chose that type of display to represent this data set.

Sample responses:

- Box plot:



- Histogram:



I chose to make a histogram because it can show the gap in the middle of the data better than a box plot can. Also, this data set would be hard to see on a dot plot because some of the elevations are very close to each other, but none of them are exactly the same.