Lesson 14

More Solutions to Linear Equations

Goals

- Calculate the solution to a linear equation given one variable, and explain (orally) the solution method.
- Determine whether a point is a solution to an equation of a line using a graph of the line.

Learning Target

I can find solutions (x, y) to linear equations given either the x- or y-value to start from.

Lesson Narrative

In this lesson, students consider graphically and algebraically what it means to be a solution to an equation with two variables. Students begin by choosing a value for one variable in an equation and finding the value for the other variable that makes the equation true.

Next, they analyze statements about potential solutions to the equations defining three graphs. Students observe that when a point lies on two lines, such as at their point of intersection, then the coordinates of that point are solutions to both equations represented by the lines. This will be useful for thinking about what it means to be a solution to a system of linear equations in a following unit.

Then students consider equations given in different forms, ask their partner for either the x- or y-coordinate of a solution to the equation, and then find the missing value for the coordinate. Students are encouraged to use the structure of an equation and decide whether it would be more efficient to solve if given x or given y.

Student Learning Goal

Let's find solutions to more linear equations.

Access for Students with Diverse Abilities

- Engagement (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- Take Turns

Required Materials

Materials to Copy

- I'll Take an X Please Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up
Coordinate Pairs

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to practice solving an equation for an unknown value while thinking about a coordinate pair, (x, y), that makes the equation true. While the steps to solve the equation are the same regardless of which value of x students choose, there are strategic choices that can make solving the resulting equation simpler.

Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion. If necessary, encourage students to not pick 0 for x each time.

Student Task Statement

For each equation choose a value for x and then find the corresponding y-value that makes that equation true.

1. $6x = 7y$

Sample response: $x = 7, y = 6$

2. $5x + 3y = 9$

Sample response: $x = 3, y = -2$

3. $y + 5 - \frac{1}{3}x = 7$

Sample response: $x = 3, y = 3$

Activity Synthesis

The goal of this discussion is to reinforce the idea that the solutions to a given equation will all lie on the same line, and that line represents the set of all possible solutions to the equation. Begin by collecting the pairs of x's and y's students calculated and graphing them on a coordinate plane. It may be useful to graph each set of points in a different color. Here are some questions for discussion:

- *"How did you pick your x-values?"*

  For the first problem, choosing x to be a multiple of 7 makes y an integer. For the last problem, picking x to be a multiple of 3 makes y an integer.

- *"What do you notice about all the points?"*

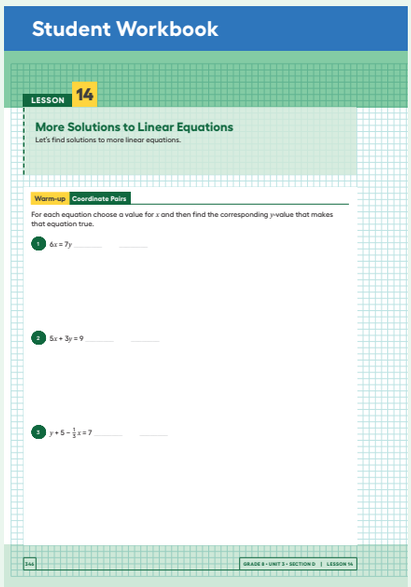
  The points collected for each equation form a different line.

Access for Students with Diverse Abilities (Warm-up, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use a calculator to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


LESSON 14

More Solutions to Linear Equations

Let's find solutions to more linear equations.

Warm-up Coordinate Pairs

For each equation choose a value for x and then find the corresponding y-value that makes that equation true.

1. $6x = 7y$
2. $5x + 3y = 9$
3. $y + 5 - \frac{1}{3}x = 7$

Instructional Routines

MLR3: Critique, Correct, Clarify

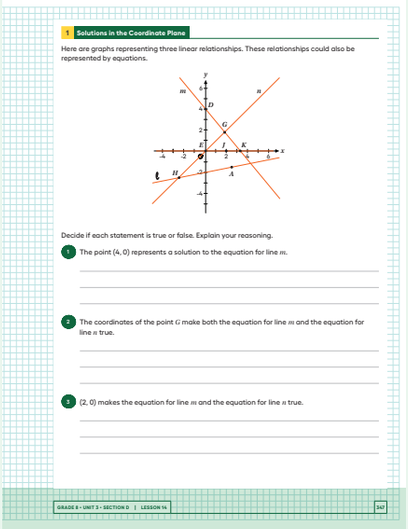
ilclass.com/r/10695504

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR3: Critique, Correct, Clarify. This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Student Workbook


1 Solutions in the Coordinate Plane

Here are graphs representing three linear relationships. These relationships could also be represented by equations.

Decide if each statement is true or false. Explain your reasoning.

1. The point (4, 0) represents a solution to the equation for line m.
2. The coordinates of the point G make both the equation for line m and the equation for line n true.
3. (2, 0) makes the equation for line m and the equation for line n true.

GRADE 8 • UNIT 3 • SECTION D | LESSON 14 247

Activity 1

Solutions in the Coordinate Plane

15 min

Activity Narrative

In this activity, students are given graphs of lines and asked to consider statements about solutions to the equations that define each line. While there is not enough information to accurately find the equations for these lines, some students may try to find them. The important understanding of this activity is that a coordinate pair lies on a line if it is a solution to the equation of that line, and points not on the line are not solutions.

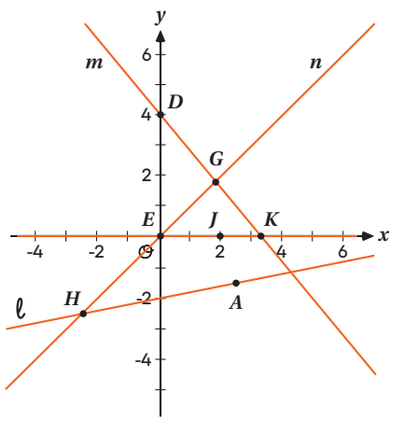
In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Arrange students in groups of 2. Give students 4–5 minutes of quiet work time followed by a partner discussion. If partners disagree on an answer, encourage them to work to reach an agreement before continuing with a whole-class discussion.

Student Task Statement

Here are graphs representing three linear relationships. These relationships could also be represented by equations.



Decide if each statement is true or false. Explain your reasoning.

1. The point (4, 0) represents a solution to the equation for line m.

   False

   Sample reasoning: The point (4, 0) does not lie on line m, so it is not a solution to the equation for line m.

2. The coordinates of the point G make both the equation for line m and the equation for line n true.

   True

   Sample reasoning: Point G lies on both lines, so its coordinates are a solution to both equations.

3. (2, 0) makes the equation for line m and the equation for line n true.

False

Sample reasoning: The point (2, 0) does not lie on either line, so it is not a solution to either equation.

4. There is not a solution to the equation for line ℓ that has $y = 0$.

False

Sample reasoning: Line ℓ will pass through a point where $y = 0$ when it passes through the x-axis. While this point is not visible within the scale of this graph, the point does exist and is still a solution.

5. The coordinates of point H are a solution to the equation for line ℓ.

True

Sample reasoning: Point H lies on line ℓ, so its coordinates are a solution.

6. There are exactly two solutions to the equation for line ℓ.

False

Sample reasoning: There are infinitely many solutions for line ℓ.

7. There is a point whose coordinates make the equations of all three lines true.

False

Sample reasoning: There is no point that lies on all three lines.

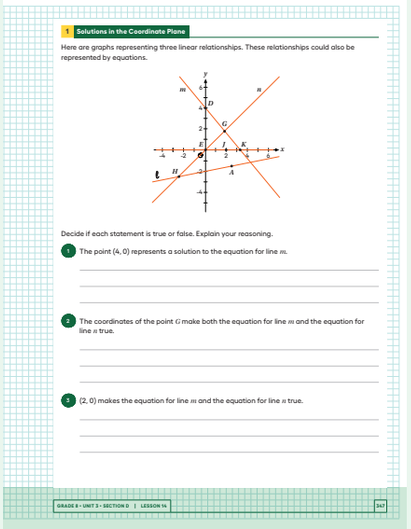
8. $x = 0$ is a solution to the equation for line n.

False

Sample response: Since the equation has two variables, a solution must be a pair of numbers or both coordinates of a point on the line.

Discuss your thinking with your partner. If you disagree, work to reach an agreement.

Student Workbook


1 Solutions in the Coordinate Plane

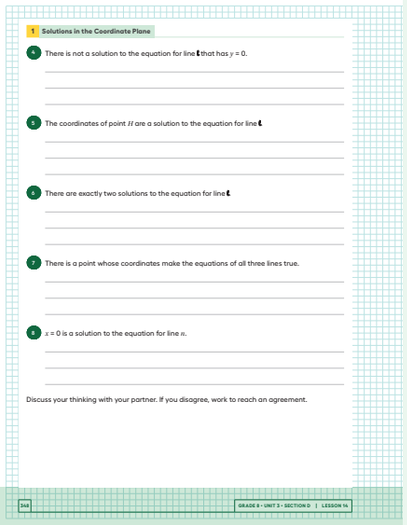
Here are graphs representing three linear relationships. These relationships could also be represented by equations.

Decide if each statement is true or false. Explain your reasoning.

1. The point (4, 0) represents a solution to the equation for line m.
2. The coordinates of the point G make both the equation for line m and the equation for line n true.
3. (2, 0) makes the equation for line m and the equation for line n true.

GRADE 8 • UNIT 3 • SECTION D | LESSON 14 347

Student Workbook


1 Solutions in the Coordinate Plane

4. There is not a solution to the equation for line ℓ that has $y = 0$.
5. The coordinates of point H are a solution to the equation for line ℓ.
6. There are exactly two solutions to the equation for line ℓ.
7. There is a point whose coordinates make the equations of all three lines true.
8. $x = 0$ is a solution to the equation for line n.

Discuss your thinking with your partner. If you disagree, work to reach an agreement.

348 GRADE 8 • UNIT 3 • SECTION D | LESSON 14

Activity Synthesis

The purpose of this discussion is to reinforce the idea that solutions to equations with two variables are a pair of numbers. Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

  Yes, $x = 0$ is a solution of the equation for line n because line n passes through the point (0, 0).

- Ask,

💬 *"What parts of this response are unclear, incorrect, or incomplete?"*

Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Access for Perception.
Students may benefit from watching or hearing the example demonstration from the *Launch* of how to solve for one variable after asking for the other variable in a solution to an equation with two variables more than once.

Supports accessibility for: Language, Attention

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

The answer is incorrect because $x = 0$ is not a solution. $x = 0$ defines a vertical line but the solution to an equation with two variables is a point. A solution to an equation with two variables is a pair of values, and $x = 0$ is only a single value. The values $x = 0$ and $y = 0$ are a solution to the equation for line n.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Activity 2

I'll Take an X, Please

15 min

Activity Narrative

In this partner activity, students are given linear equations in various forms, as well as a solution to their partner's equations in the form of a coordinate pair. Students take turns asking for either the x or y value of the solution, and then solving their equation for the other variable. Students analyze and make use of the structure of an equation when they decide whether it would be easier to solve if given x- or given y-. This activity also reinforces the idea that solutions to equations with two variables are a pair of numbers.

Launch

Arrange students in groups of 2.

Display the equation $y = 5x - 11$ and the coordinate pair (1, -6) for all to see. Explain that each person will receive a set of cards, some of which contain equations in different forms and some of which contain the coordinates of a point. For each equation they have, their partner has the coordinates of a point representing a solution to their equation. Choose a student to be your partner and demonstrate how to do the activity using the displayed equation and coordinate pair. Then share these steps:

- One partner receives Cards A through F from the left side of the blackline master and the other receives Cards a through f from the right side.
- One partner asks for either the x- or y-coordinate value and uses it to solve their equation for the other value.
- The other partner gives the requested information and checks their partner's work.
- If the coordinate pair does not match, both partners should look through the steps to find and correct any errors.
- For the next equation, the students swap roles until they have completed all 6 cards.

Student Task Statement

Your teacher will give you a set of cards. One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. Cards with the same letter, for example Cards A and a, have an equation on one card and a coordinate pair (x, y) that makes the equation true on the other card. Take turns asking your partner for either the x- or y-coordinate value and using it to solve your equation for the other value.

1. The partner with the equation asks the partner with a solution for either the x-value or the y-value.
2. The partner with the equation uses this value to find the other value, explaining each step as they go.
3. The partner with the coordinate pair checks their partner's work. If the coordinate pair does not match, both partners should look through the steps to find and correct any errors. Otherwise, both partners move onto the next set of cards.

   Keep playing until you have finished all the cards.

   The cards with a coordinate pair contain both the x- and y-values for each equation.

Are You Ready for More?

Consider the equation $ax + by = c$, where a, b, and c are positive numbers.

1. Find the coordinates of the x- and y-intercepts of the graph of the equation.

   x-intercept: $\left(\frac{c}{a}, 0\right)$; y-intercept $\left(0, \frac{c}{b}\right)$.

   Putting 0 in for y, the equation becomes $ax = c$, so $x = \frac{c}{a}$, and the x-intercept is $\left(\frac{c}{a}, 0\right)$. Putting 0 in for x, the equation becomes $by = c$, so $y = \frac{c}{b}$, and the y-intercept is $\left(0, \frac{c}{b}\right)$.

2. Find the slope of the graph.

   $-\frac{a}{b}$

   Using the two intercepts to calculate the slope, the equation becomes

   slope $= \frac{\frac{c}{b} - 0}{0 - \frac{c}{a}} = -\frac{\frac{c}{b}}{\frac{c}{a}} = -\frac{a}{b}$

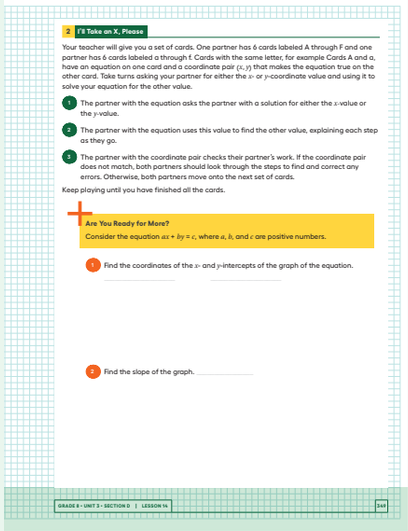
Activity Synthesis

The purpose of this discussion is to share non-graphical strategies for finding a solution to an equation with two variables and to reinforce the idea that solutions to equations with two variables are a pair of numbers.

Once all groups have completed all 6 sets of cards, consider discussing the following:

- *"How did you decide whether you wanted the value of x or the value of y?"*
- *"What strategy did you use to solve your equation once you had the value for one of the variables?"*
- *"Describe any difficulties you experienced and how you resolved them."*

Student Workbook



2 I'll Take an X, Please

Your teacher will give you a set of cards. One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. Cards with the same letter, for example Cards A and a, have an equation on one card and a coordinate pair (x, y) that makes the equation true on the other card. Take turns asking your partner for either the x- or y-coordinate value and using it to solve your equation for the other value.

1. The partner with the equation asks the partner with a solution for either the x-value or the y-value.
2. The partner with the equation uses this value to find the other value, explaining each step as they go.
3. The partner with the coordinate pair checks their partner's work. If the coordinate pair does not match, both partners should look through the steps to find and correct any errors. Otherwise, both partners move onto the next set of cards.

Keep playing until you have finished all the cards.

Are You Ready for More?

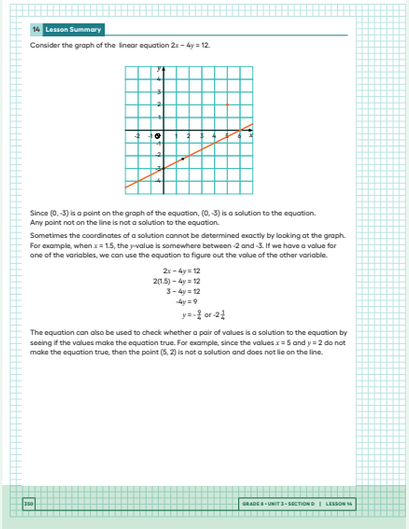
Consider the equation $ax + by = c$, where a, b, and c are positive numbers.

1. Find the coordinates of the x- and y-intercepts of the graph of the equation.

2. Find the slope of the graph.

GRADE 8 • UNIT 3 • SECTION D | LESSON 14

Student Workbook


14 Lesson Summary

Consider the graph of the linear equation $2x - 4y = 12$.

Since (0, -3) is a point on the graph of the equation, (0, -3) is a solution to the equation. Any point not on the line is not a solution to the equation.

Sometimes the coordinates of a solution cannot be determined exactly by looking at the graph. For example, when $x = 1.5$, the y-value is somewhere between -2 and -3. If we have a value for one of the variables, we can use the equation to figure out the value of the other variable.

$2x - 4y = 12$
$2(1.5) - 4y = 12$
$3 - 4y = 12$
$-4y = 9$
$y = -\frac{9}{4}$ or $-2\frac{1}{4}$

The equation can also be used to check whether a pair of values is a solution to the equation by seeing if the values make the equation true. For example, since the values $x = 5$ and $y = 2$ do not make the equation true, then the point (5, 2) is not a solution and does not lie on the line.

220 GRADE 8 • UNIT 3 • SECTION D | LESSON 14

Lesson Synthesis

The goal of this discussion is to highlight student thinking about different strategies for finding a solution to a linear equation. Discuss with students:

💬 *"What are different ways to find a solution to the linear equation $3y + x = 12$?"*

Graph the equation and find points that lie on the line. Substitute in a value for one variable and solve for the other.

💬 *"How can you determine if a given pair of values is a solution to the equation $3y + x = 12$?"*

Substitute the values into the equation for the variable they each represent to see if they make the equation true. Write the pair of values as a coordinate point and see if that point lies on the graph of the equation.

💬 *"What is a solution to the equation $3y + x = 12$ if $x = 0$?"*

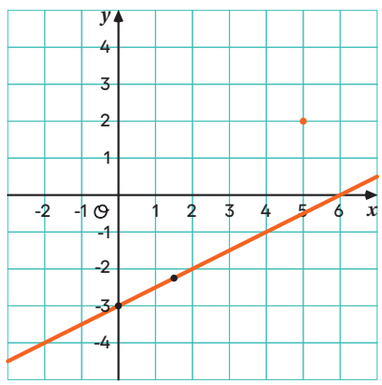
$x = 0$ and $y = 4$ since $3(4) + 0 = 12$

💬 *"What is a solution to the equation $3y + x = 12$ if $y = 0$?"*

$x = 12$ and $y = 0$ since $3(0) + 12 = 12$

Lesson Summary

Consider the graph of the linear equation $2x - 4y = 12$.



Since (0, -3) is a point on the graph of the equation, (0, -3) is a solution to the equation. Any point not on the line is not a solution to the equation.

Sometimes the coordinates of a solution cannot be determined exactly by looking at the graph. For example, when $x = 1.5$, the y-value is somewhere between -2 and -3. If we have a value for one of the variables, we can use the equation to figure out the value of the other variable.

$$\begin{aligned} 2x - 4y &= 12 \\ 2(1.5) - 4y &= 12 \\ 3 - 4y &= 12 \\ -4y &= 9 \\ y &= -\frac{9}{4} \text{ or } -2\frac{1}{4} \end{aligned}$$

The equation can also be used to check whether a pair of values is a solution to the equation by seeing if the values make the equation true. For example, since the values $x = 5$ and $y = 2$ do not make the equation true, then the point (5, 2) is not a solution and does not lie on the line.

Cool-down

Intercepted

5 min

Student Task Statement

Does the graph of the line for $3x - y = -6$ pass through the points (-2, 0) and (0, -6)? Explain your reasoning.

The graph passes through the point (-2, 0) but not through the point (0, -6). Sample reasoning: Since $3(-2) - 0 = -6$, the point (-2, 0) is a solution to the equation and will lie on the line. Since $3(0) - (-6) = 6$, and not -6, the point (0, -6) is not a solution and will not lie on the line.

Responding To Student Thinking

Points to Emphasize

If most students struggle with determining if a point is on a line, revisit how to substitute values into an equation to determine if the values make the equation true. For example, in the activity referred to here, after students have written their equations, give them pairs of values and ask them to determine if the pair of values is a solution to the equation.

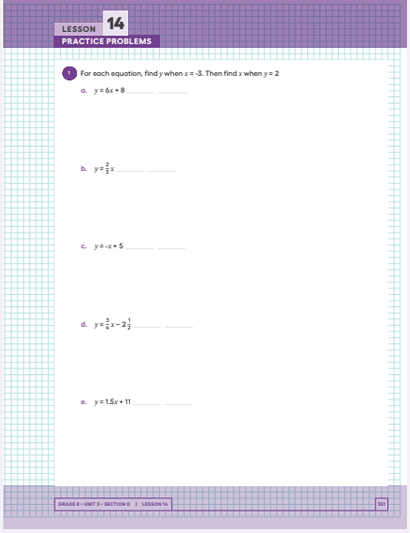
Unit 3, Lesson 15, Warm-up Writing Equations

Lesson 14

Practice Problems

5 Problems

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 For each equation, find y when $x = -3$. Then find x when $y = 2$

a. $y = 6x + 8$ _____ _____

b. $y = \frac{2}{3}x$ _____ _____

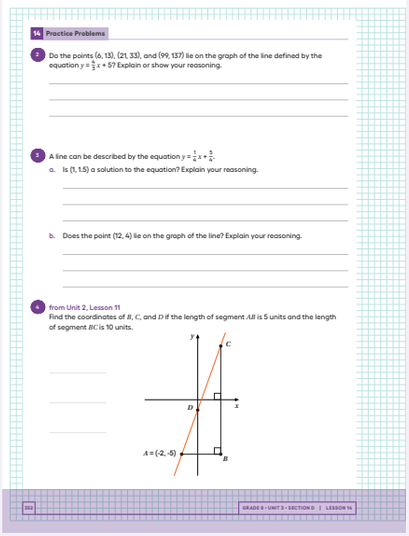
c. $y = -x + 5$ _____ _____

d. $y = \frac{3}{4}x - 2\frac{1}{2}$ _____ _____

e. $y = 1.5x + 11$ _____ _____

GRADE 8 • UNIT 3 • SECTION D | LESSON 14 181

Student Workbook


14 Practice Problems

2 Do the points (6, 13), (21, 33), and (99, 137) lie on the graph of the line defined by the equation $y = \frac{4}{3}x + 5$? Explain or show your reasoning.

3 A line can be described by the equation $y = \frac{1}{4}x + \frac{5}{4}$.

a. Is (1, 1.5) a solution to the equation? Explain your reasoning.

b. Does the point (12, 4) lie on the graph of the line? Explain your reasoning.

4 from Unit 2, Lesson 11
Find the coordinates of B, C, and D if the length of segment AB is 5 units and the length of segment BC is 10 units.

182 GRADE 8 • UNIT 3 • SECTION D | LESSON 14

Problem 1

For each equation, find y when $x = -3$. Then find x when $y = 2$

a. $y = 6x + 8$

$y = -10$, $x = -1$

b. $y = \frac{2}{3}x$

$y = -2$, $x = 3$

c. $y = -x + 5$

$y = 8$, $x = 3$

d. $y = \frac{3}{4}x - 2\frac{1}{2}$

$y = \frac{-19}{4}$, $x = 6$

e. $y = 1.5x + 11$

$y = 6.5$, $x = -6$

Problem 2

Do the points (6, 13), (21, 33), and (99, 137) lie on the graph of the line defined by the equation $y = \frac{4}{3}x + 5$? Explain or show your reasoning.

Yes, all three points lie on the line.

Sample reasoning: $13 = \frac{4}{3}(6) + 5$; $33 = \frac{4}{3}(21) + 5$; $137 = \frac{4}{3}(99) + 5$

Problem 3

A line can be described by the equation $y = \frac{1}{4}x + \frac{5}{4}$.

a. Is (1, 1.5) a solution to the equation? Explain your reasoning.

Yes, (1, 1.5) is a solution to the equation.

Sample reasoning: the ordered pair satisfies the equation since $1.5 = \frac{1}{4}(1) + \frac{5}{4}$.

b. Does the point (12, 4) lie on the graph of the line? Explain your reasoning.

No, the point (12, 4) does not lie on the graph of the line.

Sample reasoning: When $x = 12$, y would be 4.25, not 4. Since the x- and y-values do not make the equation true, the point will not be on the line.

Problem 4

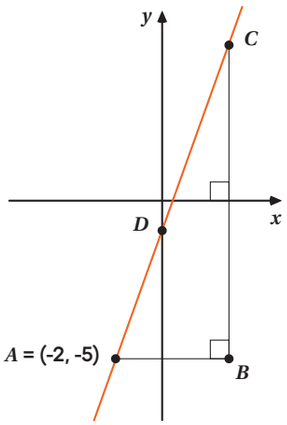
from Unit 2, Lesson 11

Find the coordinates of B, C, and D if the length of segment AB is 5 units and the length of segment BC is 10 units.

B = (3, -5)

C = (3, 5)

D = (0, -1)

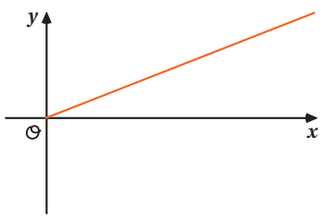


Problem 5

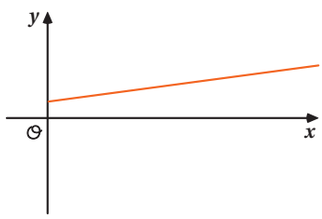
from Unit 3, Lesson 9

Match each graph of a linear relationship to a situation that most reasonably reflects its context.

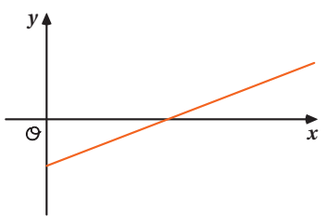
A 4



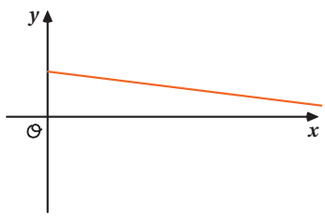
B 1



C 3

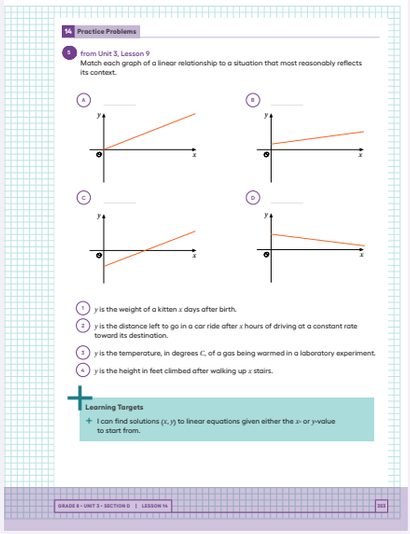


D 2



1. y is the weight of a kitten x days after birth.
2. y is the distance left to go in a car ride after x hours of driving at a constant rate toward its destination.
3. y is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.
4. y is the height in feet climbed after walking up x stairs.

Student Workbook


14 Practice Problems

5 from Unit 3, Lesson 9
Match each graph of a linear relationship to a situation that most reasonably reflects its context.

A B C D

1. y is the weight of a kitten x days after birth.
2. y is the distance left to go in a car ride after x hours of driving at a constant rate toward its destination.
3. y is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.
4. y is the height in feet climbed after walking up x stairs.

Learning Targets
+ I can find solutions (x, y) to linear equations given either the x- or y-value to start from.