Lesson 8

Percent Increase and Decrease with Equations

Goals

- Explain (orally and in writing) how to calculate the original amount given the new amount and a percentage of increase or decrease.
- Generate algebraic expressions that represent a situation involving percent increase or decrease, and justify (orally) the reasoning.

Learning Target

I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it.

Lesson Narrative

In this lesson, students use equations to represent situations involving percent increase and percent decrease. First, they match given equations to situations and solve the equations to find the unknown value. Next, they invent a situation to match a given equation. Then, students write equations to represent situations. They analyze the structure of the equations with attention to which number in the situation was unknown.

As students relate the situations with the equations, they reason quantitatively and abstractly.

Student Learning Goal

Let's use equations to represent increases and decreases.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1, Activity 3)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Math Talk
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Four-function calculators: Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	15 min	15 min	15 min	10 min	5 min
					Cool-down

Instructional Routines

Math Talk

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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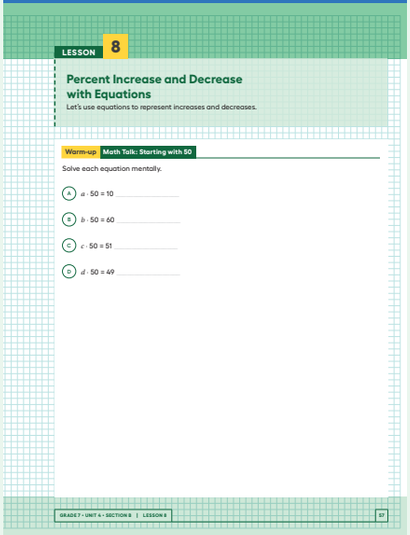
Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 8

Percent Increase and Decrease with Equations

Let's use equations to represent increases and decreases.

Warm-up Math Talk: Starting with 50

Solve each equation mentally.

A. $a \cdot 50 = 10$

B. $b \cdot 50 = 60$

C. $c \cdot 50 = 51$

D. $d \cdot 50 = 49$

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Warm-up

Math Talk: Starting with 50

5 min

Activity Narrative

This *Math Talk* focuses on finding the factor that scales 50 to another number. It encourages students to think about increase and decrease in terms of fractions, decimals, or percentages and to rely on properties of operations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students write equations to represent percent increase or decrease situations.

As students use results from the previous equation to help solve the next equation, they are making use of repeated reasoning.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A. $a \cdot 50 = 10$

$a = 0.2$ (or equivalent)

Sample reasoning: 10 is $\frac{1}{5}$ of 50, so a can be $\frac{1}{5}$, or 0.2, which is the equivalent decimal.

B. $b \cdot 50 = 60$

$b = 1.2$ (or equivalent)

Sample reasoning: 60 is equal to 50 + 10, and by the distributive property, 50 + 10 = 50(1 + 0.2).

C. $c \cdot 50 = 51$

$c = 1.02$ (or equivalent)

Sample reasoning: 1 is $\frac{1}{10}$ of 10, and $\frac{1}{10}$ of 0.2 is 0.02. And 51 is equal to 50 + 1. By the distributive property, 50 + 1 = 50(1 + 0.02).

D. $d \cdot 50 = 49$

$d = 0.98$ (or equivalent)

Sample reasoning: 51 is equal to 50 − 1, and by the distributive property, 50 − 1 = 50(1 − 0.02).

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

If all students give their answers as fractions, encourage them to express the fraction as a decimal. This will help prepare students for working with equations where decimals are used to represent percent increase or decrease.

If any student expresses their reasoning in terms of percentages, for example, by saying that 60 is a 20% increase from 50, highlight this strategy for all to see.

Activity 1
Matching Equations

15 min

Activity Narrative

In this activity, students match equations to verbal descriptions of a percent increase or decrease situation. To help identify the correct equation, students first draw a diagram to represent the situation and make sense of what percentage the new amount is of the original amount. As students relate verbal descriptions of the situations with equations, they reason quantitatively and abstractly.

Launch

Arrange students in groups of 2.

Give students 4–5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole class discussion.

Student Task Statement

For each situation:

- Draw a diagram to represent the situation.
- Match an equation to the situation.
- Solve the equation to find the initial value, x.

$$0.32x = 52$$
$$0.68x = 52$$
$$1.32x = 52$$
$$1.68x = 52$$

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR8: Discussion Supports

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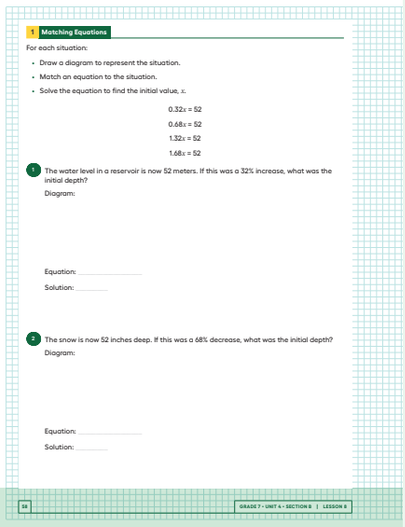
Student Workbook



1 Matching Equations

For each situation:

- Draw a diagram to represent the situation.
- Match an equation to the situation.
- Solve the equation to find the initial value, x.

$0.32x = 52$
$0.68x = 52$
$1.32x = 52$
$1.68x = 52$

1. The water level in a reservoir is now 52 meters. If this was a 32% increase, what was the initial depth?
Diagram:

Equation: _____
Solution: _____

2. The snow is now 52 inches deep. If this was a 68% decrease, what was the initial depth?
Diagram:

Equation: _____
Solution: _____

Student Workbook


1 Matching Equations

2 Write a story for one of the equations that doesn't have a match.

Are You Ready for More?

An astronaut was exploring the moon of a distant planet and found some glowing goo at the bottom of a very deep crater. She brought a 10-gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

3 How much goo will she have after 1 hour? After 2 hours? After 3 hours? After n hours?

4 When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for n hours to return to the original size?

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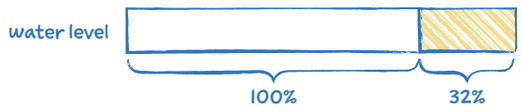
1. The water level in a reservoir is now 52 meters. If this was a 32% increase, what was the initial depth?

   Diagram:

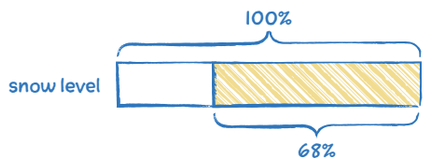


   Equation: $1.32x = 52$

   Solution: $39.\overline{39}$ meters, or 39.4 meters

2. The snow is now 52 inches deep. If this was a 68% decrease, what was the initial depth?

   Diagram:



   Equation: $0.32x = 52$

   Solution: 162.5 inches

3. Write a story for one of the equations that doesn't have a match.

   Sample response: For the equation $0.68x = 52$, there was a 32% decrease, and the new value is 52.

Are You Ready for More?

An astronaut was exploring the moon of a distant planet and found some glowing goo at the bottom of a very deep crater. She brought a 10-gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

1. How much goo will she have after 1 hour? After 2 hours? After 3 hours? After n hours?

   After 1 hour, there will be 20 grams. After 2 hours, there will be 40 grams. After 3 hours, there will be 80 grams. After n hours, there will be $10 \cdot 2^n$.

2. When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for n hours to return to the original size?

   $\frac{n}{2}$ hours

   A 75% decrease is $\frac{1}{4}$ as much, so for every hour, the amount decreases to $\frac{1}{4}$ of what was there at the beginning of the hour. For example, after 2 hours of light exposure, there will be 40 grams of goo, but after only 1 hour in the dark, it will be back to 10 grams.

Activity Synthesis

Ask one or more students to share which equation they matched with each situation, and resolve any discrepancies. Once the matches are agreed upon, ask students how they would solve the equation to find the amount without actually solving the equations.

Activity 2
Decorating Fabric

15 min

Activity Narrative

In this activity, students write equations that represent situations involving percent increase or decrease. They apply the distributive property to find the coefficient that represents the percentage that the new value is of the original value. They solve the equations to find the new value or the original value. As students use the equations to determine whether to multiply or divide to find the unknown value, they are making use of structure.

Launch

Introduce the context of this activity by asking students,

- *"Have you ever taken a plain piece of fabric and decorated it somehow? What are some ways you can think of to decorate fabric?"*

Then explain that *zardozi* (zar-DOO-zee) and *molas* (MOH-lah) are two different ways that some people decorate fabric. Zardozi is a Persian art form that involves sewing metal threads and beads onto silk or other fabrics. Molas are made by the Guna people. They sew multiple layers of fabric together and cut out pieces to make a colorful pattern.

Give students 4–5 minutes of quiet work time followed by time for partner discussion. Then hold a whole-class discussion.

Student Task Statement

Write an equation to represent each situation. Then, solve the equation.

1. A piece of fabric weighed 15 ounces. After it was decorated with zardozi, the weight had increased by 82%. What is the weight of the finished piece?

   $1.82(15) = x$; 27.3 ounces

2. Another finished zardozi piece weighs 22 ounces. This is a 76% increase from the original weight of the fabric. What was the original weight?

   $1.76x = 22$; 12.5 ounces

3. Before making a mola, the layers of fabric weighed 4.7 ounces. When the mola was finished, the weight had decreased by 17%. What is the weight of the finished mola?

   $0.83(4.7) = x$; about 3.9 ounces

4. Another finished mola weighs 4.9 ounces. This is a 21% decrease from the original weight of the fabric. What was the original weight?

   $0.79x = 4.9$; about 6.2 ounces

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Develop Language and Symbols.

Make connections between representations visible. Draw students' attention to correspondences between the diagrams and equations.

Supports accessibility for: Language, Conceptual Processing

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

Use this routine to support whole-class discussion. Provide sentence frames for students to use when they share the equation they matched to each situation: "Situation _____ matches with equation _____ because _____." Call students attention to how the percent increase is represented in the equation and the situation.

Design Principle(s): Support sense-making; Optimize output (for explanation)

Instructional Routines

MLR2: Collect and Display

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Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Show a video for students who are unfamiliar with embroidery or sewing.

Supports accessibility for: Conceptual Processing, Language

Access for Multilingual Learners (Activity 2, Student Task)

MLR2: Collect and Display.

Collect the language students use to describe the differences between each problem. Display words and phrases such as "original amount," "new amount," "percent increase," and "percent decrease." During the *Activity Synthesis*, invite students to suggest ways to update the display:

"What are some other words or phrases we should include?"

Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 3)

MLR1: Stronger and Clearer Each Time

This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 3, Launch)

Action and Expression: Provide Access for Physical Action.

Activate or supply background knowledge. Provide students with access to double number lines to support information processing.

Supports accessibility for: Visual-Spatial Processing, Organization

Are You Ready for More?

A piece of fabric weighed 12 ounces. After it was decorated with zardozi, the finished piece weighed 27 ounces. By what percentage did the weight increase? Explain your reasoning.

125%. Sample reasoning: 27 ÷ 12 = 2.25. Scaling by a factor of 2.25 means an increase of 125%, because 225 – 100 = 125. A 100% increase represents doubling the original amount. The weight of this piece increased by a little more than double.

Activity Synthesis

The purpose of this discussion is to contrast the structure of the equations: those with an unknown value of the original weight and those with an unknown value of the new weight. Invite students to share the equations they wrote for each situation. To help students contrast the equations, consider asking:

- *"What is the same about the equations? What is different?"*

  They all involve multiplication. Two have an unknown factor, while the other two have an unknown product.

- *"How is the percent increase or percent decrease represented in each equation?"*

  For the percent increase situations, the coefficient represents 100% plus the percent increase. For the percent decrease situations, the coefficient represents 100% minus the percent decrease.

- *"Which two equations represent situations where the finished weight was unknown?"*

  the two with the unknown product

- *"Which two equations represent situations where the original weight was unknown?"*

  the two with the unknown factor

- *"How did you determine the unknown value for each equation?"*

  I multiplied to find the unknown product and then divided to find the unknown factor.

Activity 3: Optional

Representing Percent Increase and Decrease: Equations

15 min

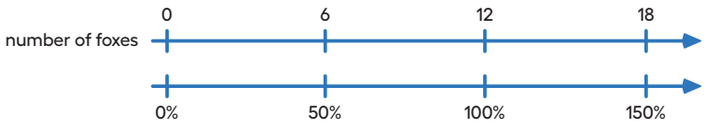
Activity Narrative

In this activity, students practice writing equations to represent situations involving percent increase or decrease. The situations are the same ones that students previously represented with double number line diagrams in a previous lesson.

In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. As students revise their writing, they have an opportunity to attend to precision in the language they use to describe their thinking.

Launch

Show students the double number line from the activity in the previous lesson.



Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the first question, about the equation $(1.5) \cdot 12 = f$. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "_____ makes sense, but what do you mean when you say … ?"
- "Can you describe that another way?"
- "How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

After *Stronger and Clearer Each Time*, give students 3–4 minutes of quiet work time to answer the rest of the questions.

Student Task Statement

1. Last year, scientists counted 12 foxes in a conservation area. This year, they counted 50% more than that. How many foxes did they count this year? Explain why this situation can be represented by the equation $(1.5) \cdot 12 = f$. Make sure that you explain what f represents.

They counted 18 foxes this year. Let f represent the amount of foxes this year. Last year, there were 12 foxes, and this year there are 50% more, or $0.5 \cdot 12$ more, so there is $12 + 0.5 \cdot 12 = 1.5 \cdot 12$. So $1.5 \cdot 12 = f$.

2. Write an equation to represent each of the following situations.

a. After replacing some grass with rocks, a business decreased its water usage by 20%. If their old water usage was 15,000 gallons per week, how much do they use now?

$(0.8) \cdot 15{,}000 = w$, where w is the amount of water now used

b. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?

$(0.75) \cdot t = 12$, where t is the price before the discount

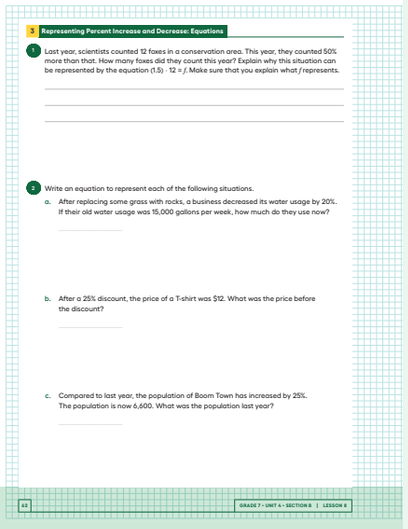
c. Compared to last year, the population of Boom Town has increased by 25%. The population is now 6,600. What was the population last year?

$(1.25) \cdot p = 6{,}600$, where p is the population last year

Building on Student Thinking

Students may continue to struggle to recognize the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

Student Workbook


3 Representing Percent Increase and Decrease: Equations

1. Last year, scientists counted 12 foxes in a conservation area. This year, they counted 50% more than that. How many foxes did they count this year? Explain why this situation can be represented by the equation $(1.5) \cdot 12 = f$. Make sure that you explain what f represents.

2. Write an equation to represent each of the following situations.
   a. After replacing some grass with rocks, a business decreased its water usage by 20%. If their old water usage was 15,000 gallons per week, how much do they use now?
   b. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?
   c. Compared to last year, the population of Boom Town has increased by 25%. The population is now 6,600. What was the population last year?

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Activity Synthesis

Select students to share the different equations they came up with. Points of discussion could include:

- How the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.
- How 100% always corresponds to the original value and when there is an increase in the value. the new value corresponds to a percentage greater than the original 100%.
- How solving problems about percent change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how the unknown number can be found.

Lesson Synthesis

Share with students,

"Today we wrote equations to represent situations involving percent increase or percent decrease."

To review the different types of problems that students solved, for each of these situations, consider asking,

"How could you write an equation to represent the situation? What does the variable represent?":

- "The original amount was 50. There is a 35% increase."

  $1.35 \cdot 50 = x$, where x represents the new amount

- "The new amount is 50. There was a 35% increase."

  $1.35x = 50$, where x represents the original amount

- "The original amount was 50. There is a 35% decrease."

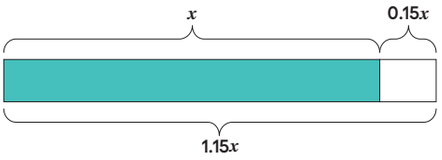
  $0.65 \cdot 50 = x$, where x represents the new amount

- "The new amount is 50. There was a 35% decrease."

  $0.65x = 50$, where x represents the original amount

Lesson Summary

We can use equations to express percent increase and percent decrease. For example, if y is 15% more than x,

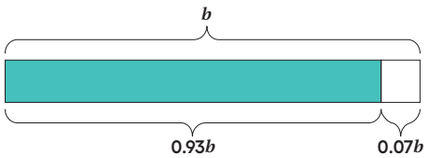


we can represent this by using any of these equations:

$$y = x + 0.15x$$
$$y = (1 + 0.15)x$$
$$y = 1.15x$$

So if someone makes an investment of x dollars, and its value increases by 15% to reach \$1,250, then we can write the equation $1.15x = 1{,}250$ to find the value of the initial investment.

Here is another example: if a is 7% less than b,



we can represent this by using any of these equations:

$$a = b - 0.07b$$
$$a = (1 - 0.07)b$$
$$a = 0.93b$$

So if the amount of water in a tank decreased 7% from its starting value of b to its ending value of 348 gallons, then we can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percent increase or percent decrease.

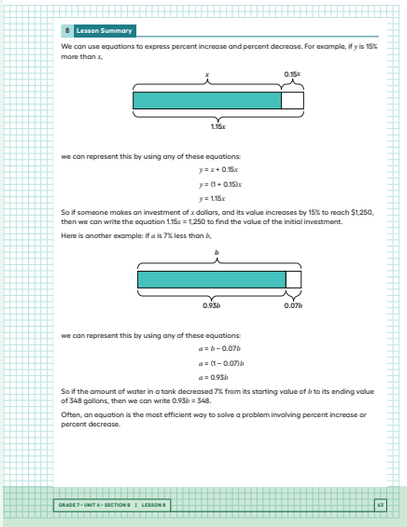
Responding To Student Thinking

Points to Emphasize

If students struggle with writing an equation to represent percent increase, plan to review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about writing equations to represent the situations in this activity:

Grade 7, Unit 4, Lesson 9, Activity 2
Population Growth

Student Workbook



8 Lesson Summary

We can use equations to express percent increase and percent decrease. For example, if y is 15% more than x,

x $0.15x$ $1.15x$

we can represent this by using any of these equations:

$y = x + 0.15x$
$y = (1 + 0.15)x$
$y = 1.15x$

So if someone makes an investment of x dollars, and its value increases by 15% to reach \$1,250, then we can write the equation $1.15x = 1{,}250$ to find the value of the initial investment.

Here is another example: if a is 7% less than b,

b $0.93b$ $0.07b$

we can represent this by using any of these equations:

$a = b - 0.07b$
$a = (1 - 0.07)b$
$a = 0.93b$

So if the amount of water in a tank decreased 7% from its starting value of b to its ending value of 348 gallons, then we can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percent increase or percent decrease.

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Cool-down

Tyler's Savings Bond

5 min

Student Task Statement

Tyler's mom purchased a savings bond for Tyler. The value of the savings bond increases by 4% each year. One year after it was purchased, the value of the savings bond is \$156.

Find the value of the bond when Tyler's mom purchased it. Explain your reasoning.

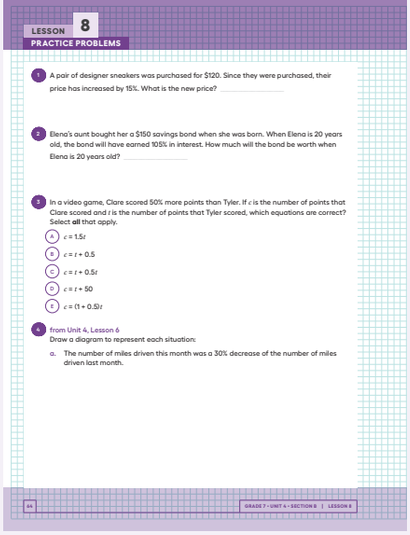
The bond was originally worth \$150.

Sample reasoning: To represent the situation, use the equation $1.04x = 156$, where x represents the value of the savings bond when Tyler's mom purchased it. The solution is $x = 156 \div 1.04 = 150$.

Practice Problems

6 Problems

Student Workbook



LESSON 8
PRACTICE PROBLEMS

1. A pair of designer sneakers was purchased for $120. Since they were purchased, their price has increased by 15%. What is the new price? _____

2. Elena's aunt bought her a $150 savings bond when she was born. When Elena is 20 years old, the bond will have earned 105% in interest. How much will the bond be worth when Elena is 20 years old? _____

3. In a video game, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations are correct? Select **all** that apply.
   - A. $c = 1.5t$
   - B. $c = t + 0.5$
   - C. $c = t + 0.5t$
   - D. $c = t + 50$
   - E. $c = (1 + 0.5)t$

4. from Unit 4, Lesson 6
   Draw a diagram to represent each situation:
   a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

Problem 1

A pair of designer sneakers was purchased for $120. Since they were purchased, their price has increased by 15%. What is the new price?

$138

Problem 2

Elena's aunt bought her a $150 savings bond when she was born. When Elena is 20 years old, the bond will have earned 105% in interest. How much will the bond be worth when Elena is 20 years old?

$307.50

Problem 3

In a video game, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations are correct? Select **all** that apply.

A. $c = 1.5t$

B. $c = t + 0.5$

C. $c = t + 0.5t$

D. $c = t + 50$

E. $c = (1 + 0.5)t$

Problem 4

from Unit 4, Lesson 6

Draw a diagram to represent each situation:

a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

a tape diagram showing 10 equal pieces labeled "number of miles driven last month" on the top with one below it that is just 7 pieces long and is labeled, "number of miles driven this month"

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

a tape diagram showing 4 equal pieces labeled "amount of paper they used last month" on the top with one below it that is 5 pieces long and is labeled, "amount of paper they used this month"

Problem 5

from Unit 4, Lesson 5

Which decimal is the best estimate of the fraction $\frac{29}{40}$?

A. 0.5

B. 0.6

C. 0.7

D. 0.8

Problem 6

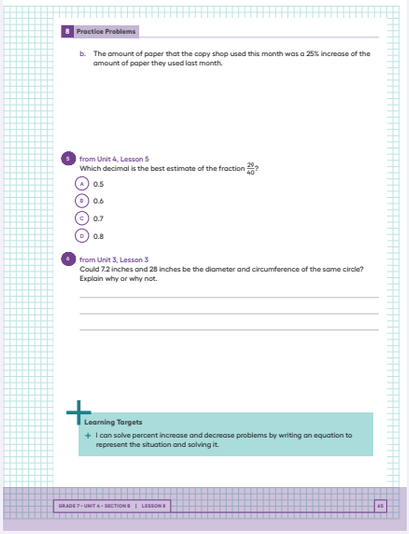
from Unit 3, Lesson 3

Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

No, this cannot be from the same circle.

Sample reasoning: The circumference of a circle with a radius of 7.2 inches is approximately 22.6 inches because $7.2 \cdot \pi \approx 22.6$.

Student Workbook


8 Practice Problems

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

5 from Unit 4, Lesson 5
Which decimal is the best estimate of the fraction $\frac{29}{40}$?
A 0.5
B 0.6
C 0.7
D 0.8

6 from Unit 3, Lesson 3
Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

Learning Targets
+ I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it.

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