Lesson 8

Combining Bases

Goal

Generalize a process for multiplying expressions with different bases having the same exponent, and justify (orally and in writing) that $(ab)^n = a^n \cdot b^n$.

Learning Target

I can use and explain a rule for multiplying terms that have different bases but the same exponent.

Lesson Narrative

In this lesson, students explore what happens with exponent rules when the bases are different. They make use of structure when decomposing numbers into their constituent factors and regrouping them. This leads to the new rule $a^n \cdot b^n = (a \cdot b)^n$. Students also attend to precision in the language they use to explain this new rule.

Student Learning Goal

Let's multiply expressions with different bases.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Access for Multilingual Learners:

- MLR1: Stronger and Clearer Each Time (Activity 1)

Access for Students with Diverse Abilities:

- Representation (Activity 1)

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 2

Required Preparation

Activity 1:
Create a visual display (or add to an existing display) of the exponent rule $a^n \cdot b^n = (a \cdot b)^n$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

Lesson Timeline

5 min	20 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Same Exponent, Different Base

5 min

Activity Narrative

The purpose of this *Warm-up* is to encourage students to relate expressions of the form $a^n \cdot b^n$ to $(a \cdot b)^n$ by exploring the structure of the factors. Evaluating and expanding expressions will be useful when students explore products of bases with the same exponent in a following activity.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. Evaluate $5^3 \cdot 2^3$. $5^3 \cdot 2^3 = 125 \cdot 8 = 1{,}000$

2. Evaluate 10^3. $10^3 = 1{,}000$

Activity Synthesis

The purpose of this discussion is to help students make connections between the two expressions. Here are some questions for discussion:

"What do you notice about the two expressions?"

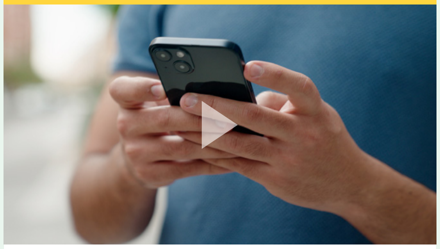
The product of the bases in the first expression is equal to the base in the second expression: $2 \cdot 5 = 10$. The exponents are the same in both expressions.

"How can we show that the two expressions are equivalent without evaluating?"

Since there are 3 factors that are 5 and 3 factors that are 2, group the 2s and 5s together to get 3 factors that are 10.
$5^3 \cdot 2^3 = (5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2) \cdot (5 \cdot 2) \cdot (5 \cdot 2) = 10 \cdot 10 \cdot 10 = 10^3$.

Inspire Math

Going Viral video

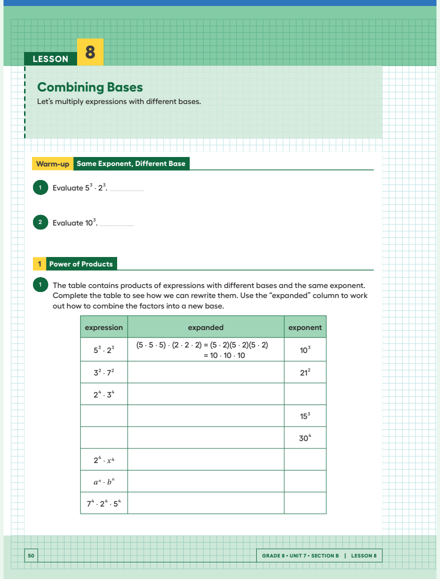


Go Online
Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/614219

Please log in to the site before using the QR code or URL.

Student Workbook



LESSON 8

Combining Bases

Let's multiply expressions with different bases.

Warm-up **Same Exponent, Different Base**

1. Evaluate $5^3 \cdot 2^3$. ______

2. Evaluate 10^3. ______

1 **Power of Products**

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

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Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Building on Student Thinking

If some students write $2x^4$ instead of $(2x)^4$, or $a \cdot b^n$ instead of $(a \cdot b)^n$, consider asking:

> *"How do parentheses affect the value of -3^2 versus $(-3)^2$?"*
> $-3^2 = -(3 \cdot 3) = -9$ and $(-3)^2 = (-3) \cdot (-3) = 9$
> *"What is the difference between $3 \cdot 4^2$ and $(3 \cdot 4)^2$?"*
> $3 \cdot 4^2 = 3 \cdot (4 \cdot 4) = 48$ and $(3 \cdot 4)^2 = (3 \cdot 4) \cdot (3 \cdot 4) = 144$

Activity 1
Power of Products

20 min

Activity Narrative

In this activity, students use repeated reasoning to discover the rule $a^n \cdot b^n = (a \cdot b)^n$. When students articulate their reasoning for what happens when neither the bases nor the exponents are the same, they have an opportunity to attend to precision in the language they use to describe their thinking. They might first propose less formal or imprecise language, and after sharing with a partner, revise their explanation to be clearer and stronger.

Launch

Arrange students in groups of 2. Give students 6–7 minutes to complete the table and answer the questions.

Student Task Statement

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$	$(3 \cdot 3) \cdot (7 \cdot 7) = (3 \cdot 7)(3 \cdot 7) = 21 \cdot 21$	21^2
$2^4 \cdot 3^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$ $= (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$ $= 6 \cdot 6 \cdot 6 \cdot 6$	6^4
$3^3 \cdot 5^3$	$15 \cdot 15 \cdot 15 = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$ $= (3 \cdot 3 \cdot 3)(5 \cdot 5 \cdot 5)$	15^3
Answers vary. Sample: $3^4 \cdot 10^4$	Answers vary. Sample: $30 \cdot 30 \cdot 30 \cdot 30$ $= (3 \cdot 10)(3 \cdot 10)(3 \cdot 10)(3 \cdot 10)$ $= (3 \cdot 3 \cdot 3 \cdot 3)(10 \cdot 10 \cdot 10 \cdot 10)$	30^4
$2^4 \cdot x^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x \cdot x)$ $= (2 \cdot x)(2 \cdot x)(2 \cdot x)(2 \cdot x)$ $= 2x \cdot 2x \cdot 2x \cdot 2x$	$(2x)^4$
$a^n \cdot b^n$	$(a \cdot \ldots \cdot a)(b \cdot \ldots b) = (a \cdot b) \cdot \ldots \cdot (a \cdot b)$	$(a \cdot b)^n$
$7^4 \cdot 2^4 \cdot 5^4$	$(7 \cdot 7 \cdot 7 \cdot 7)(2 \cdot 2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5 \cdot 5)$ $= (7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)$ $= 70 \cdot 70 \cdot 70 \cdot 70$	70^4

2. Can you write $2^3 \cdot 3^4$ with a single exponent? Explain or show your reasoning.

No

Sample reasoning: Regrouping $2^3 \cdot 3^4$ into factors that are 6 will leave an extra factor of 3.

3. What happens when multiplying bases where neither the exponents nor the bases are the same?

Sample response: If the bases and the exponents are not the same when multiplying, the factors cannot be grouped together evenly.

Activity Synthesis

The goal of this discussion is to clarify what happens when multiplying two factors with different bases that cannot be rewritten to have the same base.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to "What happens if neither the exponents nor the bases are the same?" In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

"____ makes sense, but what do you mean when you say … ?"

"Can you describe that another way?"

"How do you know … ? What else do you know is true?"

Close the partner conversations, and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

Here is an example of a second draft: "When multiplying expressions where the bases are the same, the exponents can be added together. If the bases are different but the exponents are the same, the bases can be regrouped and multiplied. If the bases and the exponents are different, and the expressions cannot be rewritten to have the same base, then the bases can not be grouped together evenly, and the expression cannot be expressed with a single exponent."

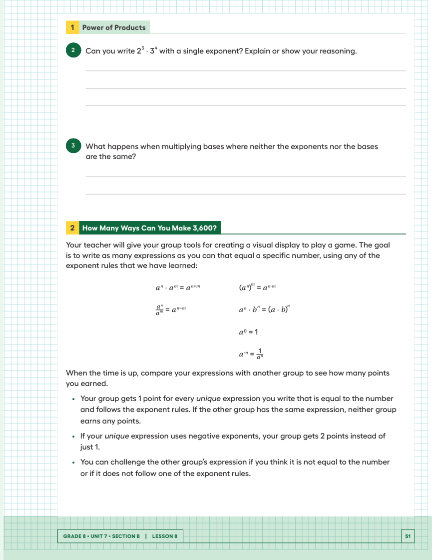
Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$a^n \cdot b^n = (a \cdot b)^n$	$2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$ three factors that are 2 · three factors that are 5 = regrouped factors = three factors that are ten

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use multiple examples and non-examples to re-inforce when an expression can be written with a single exponent.
Supports accessibility for: Conceptual Processing, Attention

Student Workbook


1 Power of Products

2 Can you write $2^3 \cdot 3^4$ with a single exponent? Explain or show your reasoning.

3 What happens when multiplying bases where neither the exponents nor the bases are the same?

2 How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. The goal is to write as many expressions as you can that equal a specific number, using any of the exponent rules that we have learned:

$a^n \cdot a^m = a^{n+m}$ $\quad (a^n)^m = a^{n \cdot m}$

$\frac{a^n}{a^m} = a^{n-m}$ $\quad a^n \cdot b^n = (a \cdot b)^n$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

When the time is up, compare your expressions with another group to see how many points you earned.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rules. If the other group has the same expression, neither group earns any points.
- If your *unique* expression uses negative exponents, your group gets 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the exponent rules.

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Activity 2: Optional

How Many Ways Can You Make 3,600?

15 min

Activity Narrative

This activity gives students an opportunity to deepen their thinking by generating different equivalent expressions using the rules of exponents. The process of generating different expressions requires students to look for and make use of structure when considering the numerous ways numbers can be broken into factors and how to combine those factors and express the result using exponents.

Launch

Arrange students in groups of 2–3. Provide students with tools for creating a visual display.

There will be several rounds in which students generate multiple expressions equivalent to a given number. As an example, invite the class to generate expressions equivalent to 1,000 using any combination of the exponent rules that we have learned so far. Display exponent rules and examples (such as those shown in the table) for all to see.

$a^n \cdot a^m = a^{n+m}$	$(a^n)^m = a^{n \cdot m}$	$\frac{a^n}{a^m} = a^{n-m}$	$a^n \cdot b^n = (a \cdot b)^n$	$a^0 = 1$	$a^{-n} = \frac{1}{a^n}$
$10^1 \cdot 10^2 = 10^3$	$(10^1)^3 = 10^3$	$\frac{10^5}{10^2} = 10^3$	$5^3 \cdot 2^3 = 10^3$	$10^3 \cdot 10^0 = 10^3$	$10^5 \cdot 10^{-2} = 10^3$

Explain that after each round, groups will be paired up to score each other's display. If there are an odd number of groups, have a group of 3 score each other.

- A group gets 1 point for every *unique* expression they find that is equivalent to the target number. If the two groups find the same expression, neither group gets a point for it.
- A group gets 2 points for every unique expression that uses negative exponents.
- Students can challenge another group's expressions if they think it isn't equivalent to the target number, or if a group didn't use any of the exponent rules.

Keep the examples on display for all to see while students are working to generate their own expressions. Set a timer for 2 minutes (or other duration, depending on time available) and let students work with their groups.

Play as many rounds of this game as time allows. In subsequent rounds, pair groups up with different opponents. Consider using the following numbers in different rounds as time permits: 3,600, $\frac{1}{200}$, 810,000, $\frac{1}{64}$, and 3,375.

Student Task Statement

Your teacher will give your group tools for creating a visual display to play a game. The goal is to write as many expressions as you can that equal a specific number, using any of the exponent rules that we have learned:

$$a^n \cdot a^m = a^{n+m} \qquad (a^n)^m = a^{n \cdot m}$$

$$\frac{a^n}{a^m} = a^{n-m} \qquad a^n \cdot b^n = (a \cdot b)^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

When the time is up, compare your expressions with another group to see how many points you earned.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rules. If the other group has the same expression, neither group earns any points.
- If your *unique* expression uses negative exponents, your group gets 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the exponent rules.

Sample responses:

- 3,600: 60^2, $6^2 \cdot 10^2$, $6^{-3} \cdot 6^5 \cdot \frac{10^8}{10^6}$
- $\frac{1}{200}$: $2^{-3} \cdot 5^{-2}$, $\frac{2^6}{2^9} \cdot \frac{5^4}{5^6}$

Are You Ready for More?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.

   Sample response: 10, 13, 16

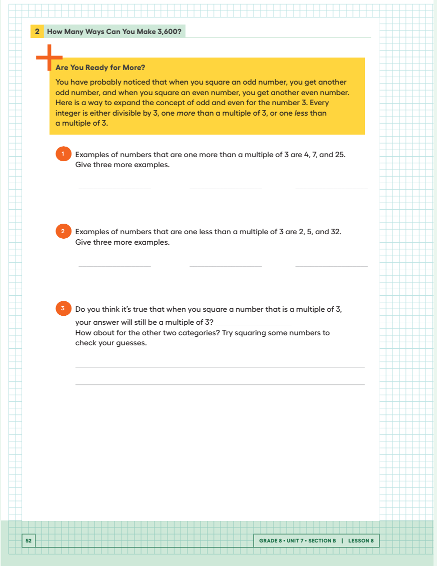
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.

   Sample response: 8, 11, 14

3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

   This is true for multiples of 3 and numbers that are one more than a multiple of 3, but not for numbers that are one less than a multiple of 3.

Student Workbook


2 How Many Ways Can You Make 3,600?

Are You Ready for More?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

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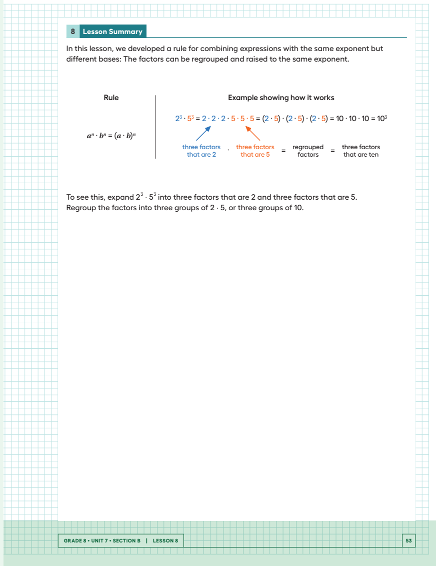
Student Workbook


8 Lesson Summary
In this lesson, we developed a rule for combining expressions with the same exponent but different bases: The factors can be regrouped and raised to the same exponent.
Rule
Example showing how it works
$a^n \cdot b^n = (a \cdot b)^n$
$2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$
three factors that are 2 · three factors that are 5 = regrouped factors = three factors that are ten
To see this, expand $2^3 \cdot 5^3$ into three factors that are 2 and three factors that are 5. Regroup the factors into three groups of $2 \cdot 5$, or three groups of 10.
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Activity Synthesis

The purpose of this discussion is for students to share what they learned about the rules of exponents by working in groups and playing the game. Here are some questions for discussion:

- *"Which exponent rule(s) did your group use the most? Why?"*

  "What is something you learned about exponents from your group?"

  "How did your group change your strategy for finding equivalent expressions from one round to the next?"

  "Did you notice any patterns?"

Lesson Synthesis

The goal of the discussion is to check that students understand why the exponent rule $a^n \cdot b^n = (a \cdot b)^n$ works. Here are questions for discussion:

- *"Is it possible to write $4^5 \cdot 5^5$ using a single exponent?"*

  Yes, $4^5 \cdot 5^5 = 20^5$.

- *"What about $4^3 \cdot 5^5$?"*

  No. You could combine 3 factors that are 4, and 3 factors that are 5 to make 3 factors that are 20, but there are still 2 factors that are 5 left over.

- *"When is it possible to combine different bases together to get an expression with a single exponent?"*

  It is only possible when both bases have the same exponent.

Lesson Summary

In this lesson, we developed a rule for combining expressions with the same exponent but different bases: The factors can be regrouped and raised to the same exponent.

Rule	Example showing how it works
$a^n \cdot b^n = (a \cdot b)^n$	$\underbrace{2^3}_{\text{three factors that are 2}} \cdot \underbrace{5^3}_{\text{three factors that are 5}} = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = \underbrace{(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)}_{\text{regrouped factors}} = \underbrace{10 \cdot 10 \cdot 10}_{\text{three factors that are ten}} = 10^3$

To see this, expand $2^3 \cdot 5^3$ into three factors that are 2 and three factors that are 5. Regroup the factors into three groups of $2 \cdot 5$, or three groups of 10.

Cool-down

Equivalent or Not?

5 min

Student Task Statement

Determine whether each of these expressions is equivalent or not equivalent to $(12 \cdot 4)^7$. Explain or show your reasoning.

1. $12^7 \cdot 4^7$

   Equivalent

   Sample reasoning: Since there are 7 factors that are 12 and 7 factors that are 4, they can be regrouped as 7 factors of $(12 \cdot 4)$.

2. $12 \cdot 4^7$

   Not equivalent

   Sample reasoning: This expression has only 1 factor that is 12 instead of 7 factors that are 12.

3. $(12 \cdot 7) \cdot (4 \cdot 7)$

   Not equivalent

   Sample reasoning: In this expression 12 and 4 are each being multiplied by 7 instead of being raised to the power of 7.

4. 48^7

   Equivalent

   Sample reasoning: Since $12 \cdot 4 = 48$ these expressions are equivalent.

Responding To Student Thinking

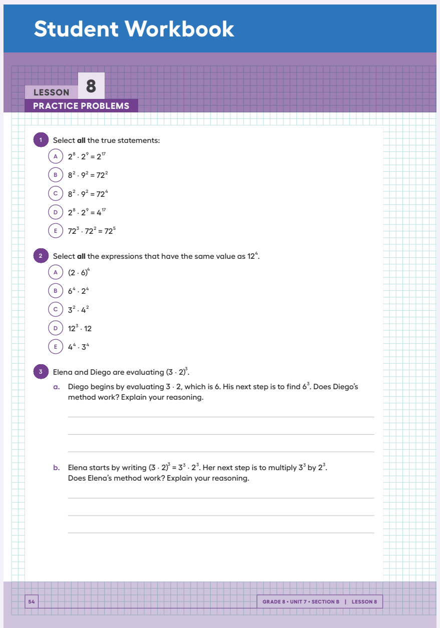
Press Pause
By this point in the unit, there should be some student mastery working with exponent rules. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 7, Section B More Exponent Rules

Lesson 8

Practice Problems — 4 Problems

Student Workbook


LESSON 8
PRACTICE PROBLEMS

1 Select **all** the true statements:

2 Select **all** the expressions that have the same value as 12^4.

3 Elena and Diego are evaluating $(3 \cdot 2)^3$.

a. Diego begins by evaluating $3 \cdot 2$, which is 6. His next step is to find 6^3. Does Diego's method work? Explain your reasoning.

b. Elena starts by writing $(3 \cdot 2)^3 = 3^3 \cdot 2^3$. Her next step is to multiply 3^3 by 2^3. Does Elena's method work? Explain your reasoning.

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Problem 1

Select **all** the true statements:

- **A.** $2^8 \cdot 2^9 = 2^{17}$
- **B.** $8^2 \cdot 9^2 = 72^2$
- **C.** $8^2 \cdot 9^2 = 72^4$
- **D.** $2^8 \cdot 2^9 = 4^{17}$
- **E.** $72^3 \cdot 72^2 = 72^5$

Problem 2

Select **all** the expressions that have the same value as 12^4.

- **A.** $(2 \cdot 6)^4$
- **B.** $6^4 \cdot 2^4$
- **C.** $3^2 \cdot 4^2$
- **D.** $12^3 \cdot 12$
- **E.** $4^4 \cdot 3^4$

Problem 3

Elena and Diego are evaluating $(3 \cdot 2)^3$.

a. Diego begins by evaluating $3 \cdot 2$, which is 6. His next step is to find 6^3. Does Diego's method work? Explain your reasoning.

Answers vary. Sample response: Yes, Diego is correct. He is using order of operations by first doing the operation in parentheses, then the exponent.

b. Elena starts by writing $(3 \cdot 2)^3 = 3^3 \cdot 2^3$. Her next step is to multiply 3^3 by 2^3. Does Elena's method work? Explain your reasoning.

Answers vary. Sample response: Yes, Elena is correct. She starts by using the exponent rule $a^n \cdot b^n = (ab)^n$ and finishes the problem from there.

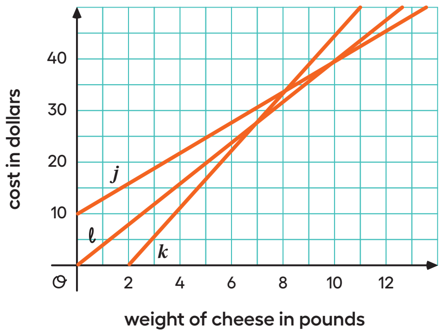
Problem 4

from Unit 5, Lesson 8

The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

- Store A sells the cheese for a dollars per pound.
- Store B sells the same cheese for b dollars per pound, and a customer has a coupon for $5 off the total purchase at that store.
- Store C is an online store, selling the same cheese at c dollars per pound, but with a $10 delivery fee.

This graph shows the total cost functions for stores A, B, and C after discounts are applied.



a. Match Stores A, B, and C with Graphs j, k, and ℓ.

Store A: Graph ℓ

Store B: Graph k

Store C: Graph j

b. What is the price per pound for cheese at each store?

Store A charges $4 per pound.

Store B charges $5 per pound.

Store C charges $3 per pound.

c. How many pounds of cheese does the coupon for Store B pay for?

1 pound of cheese.

d. At which store will the customer pay the lowest amount for a half a pound of cheese?

Store B

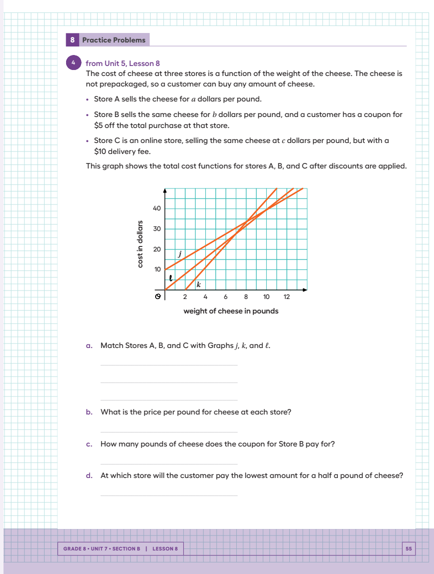
e. A customer wants to buy 5 pounds of cheese for a party. Which store has the lowest total purchase price for 5 pounds of cheese?

Stores A and B charge the same total purchase price for 5 lb of cheese, which is less than the total purchase price at Store C.

f. How many pounds would a customer need to order to make Store C a good option?

If a customer orders more than 10 pounds of cheese, Store C has the lowest price.

Student Workbook



Student Workbook