Lesson 15

Distinguishing Volume and Surface Area

Goals

- Compare and contrast (orally and in writing) problems that involve surface area and volume of prisms.
- Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation, and justify (orally) the decision.
- Estimate measurements of a prism in a real-world situation, and explain (orally) the estimation strategy.

Learning Target

I can decide whether I need to find the surface area or the volume, when solving a problem about a real-world situation.

Lesson Narrative

This is the first of two lessons in which students apply their knowledge of surface area and volume to solve real-world problems. The purpose of this first lesson is to help students distinguish between surface area and volume and to choose which of the two quantities is appropriate for solving a problem. They solve problems that require finding the surface area or volume of a prism, or both, then consider whether the quantities make sense in terms of the context.

Student Learning Goal

Let's work with surface area and volume in context.

Access for Students with Diverse Abilities

- Engagement (Activity 2)
- Representation (Activity 3)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- Card Sort
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Poll the Class
- Take Turns

Required Materials

Materials to Gather

- Pre-printed slips, cut from copies of the blackline master: Activity 2

Materials to Copy

- Surface Area or Volume Cards (1 copy for every 2 students): Activity 2

Required Preparation

Lesson:
Make 1 copy of the Card Sort: Surface Area or Volume blackline master for every 2 students, and cut them up ahead of time.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	5 min	10 min	5 min

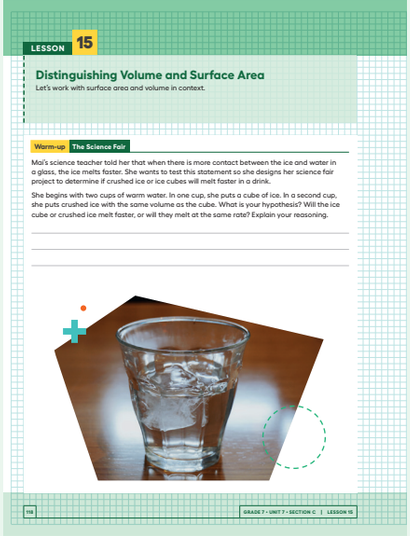
Instructional Routines

Poll the Class

ilclass.com/r/10694985

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 15

Distinguishing Volume and Surface Area

Let's work with surface area and volume in context.

Warm-up The Science Fair

Mai's science teacher told her that when there is more contact between the ice and water in a glass, the ice melts faster. She wants to test this statement so she designs her science fair project to determine if crushed ice or ice cubes will melt faster in a drink.

She begins with two cups of warm water. In one cup, she puts a cube of ice. In a second cup, she puts crushed ice with the same volume as the cube. What is your hypothesis? Will the ice cube or crushed ice melt faster, or will they melt at the same rate? Explain your reasoning.

Warm-up

The Science Fair

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to reason, within a context, about two objects that have the same volume but different surface areas.

Launch

Give students 1 minute of quiet think time, followed by a whole-class discussion.

Student Task Statement

Mai's science teacher told her that when there is more contact between the ice and water in a glass, the ice melts faster. She wants to test this statement so she designs her science fair project to determine if crushed ice or ice cubes will melt faster in a drink.

She begins with two cups of warm water. In one cup, she puts a cube of ice. In a second cup, she puts crushed ice with the same volume as the cube. What is your hypothesis? Will the ice cube or crushed ice melt faster, or will they melt at the same rate? Explain your reasoning.

The crushed ice will melt faster because there are more parts touching the warm water. The increased surface area will make the crushed ice melt faster.

Activity Synthesis

The goal of this discussion is to focus on the role of surface area in the speed of ice melting in water. Poll the students on their hypotheses. Record and display their responses for all to see. If all students agree that the crushed ice will melt faster, ask them to share their reasoning. If there are different hypotheses, ask students to explain their choice and ask questions of one another. Continue the discussion until the students reach an agreement on the crushed ice. Important ideas to highlight during the discussion are:

- Since the crushed ice has more places touching the warm water, it should melt faster.
- The center of the cube is surrounded by cold ice and will not melt until it is touching the water.
- The *surface area* of the crushed ice is greater, so it will melt faster than the ice cube even though they have the same volume.

If any of these ideas are not mentioned by students, bring them to their attention at the end of the discussion.

Activity 1
Revisiting the Box of Chocolates

10 min

Activity Narrative

In this activity students are presented with a prism that was used in a previous lesson to calculate volume. Here, they calculate the surface area of the prism. This provides students with the opportunity to work, within a given context, with complex shapes to find surface area.

Monitor for students who use these different strategies:

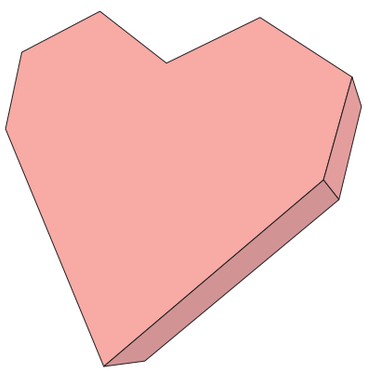
- Decomposing the heart into shapes to calculate the area.
- Calculating the area of a square around the heart and removing the negative space.

In each approach, students will be decomposing the figure into smaller areas to do calculations.

Launch

Display the image for all to see throughout the activity. Tell students that they calculated the volume of this heart-shaped box in a previous lesson and today they are going to calculate a different measurement. Ask students what additional information they need in order to find the total amount of cardboard in the box. When students recognize that they need the lengths of the diagonal sides of the box, give them the measurements for those sides (2.2 inches for the sides around the top and 6.4 inches for the sides around the bottom). Give students 2–3 minutes of quiet work time, followed by a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, and ask them to share later.



Instructional Routines

MLR7: Compare and Connect

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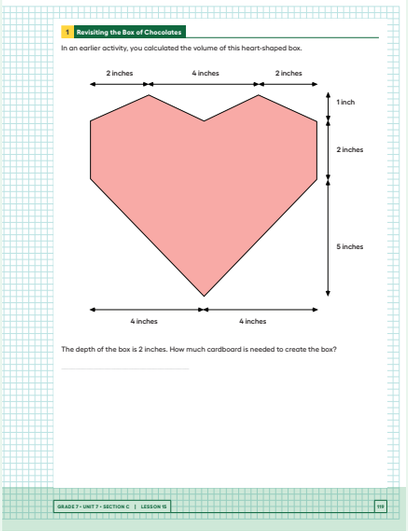
Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect. This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Building on Student Thinking

Students who are familiar with actual heart-shaped boxes of chocolate may want to double the lateral area to represent the way the top and bottom pieces nest together.

Student Workbook


1 Revisiting the Box of Chocolates

In an earlier activity, you calculated the volume of this heart-shaped box.

The depth of the box is 2 inches. How much cardboard is needed to create the box?

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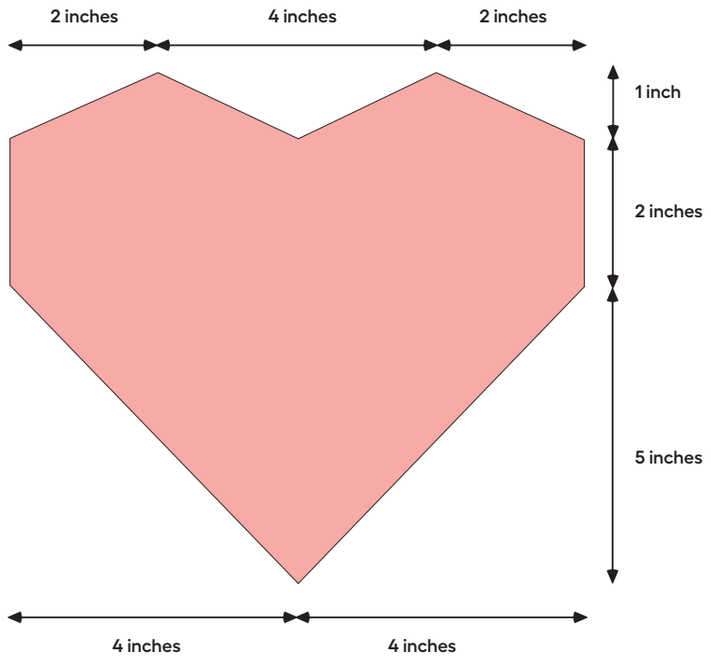
Student Task Statement

In an earlier activity, you calculated the volume of this heart-shaped box.



The depth of the box is 2 inches. How much cardboard is needed to create the box?

131.2 in^2

The area of the heart base is 40 in^2, as we figured out the other day. The perimeter of the base is 25.6 in, so the area of the long rectangle of the prism is 51.2 in^2, because $25.6 \cdot 2 = 51.2$. Therefore, the total surface area is found with $40 + 40 + 51.2 = 131.2$.

Activity Synthesis

The goal of this discussion is to connect different methods of calculating the surface area of the box. Focus on connecting a few different strategies and how they arrive at the same answer.

Display, for all to see, 2–3 approaches/representations from previously selected students. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"What do the approaches have in common? How are they different?"*
- *"Did anyone solve the problem the same way, but would explain it differently?"*
- *"Are there any benefits or drawbacks to one representation compared to another?"*

Activity 2
Card Sort: Surface Area or Volume

15 min

Activity Narrative

In this partner activity, students take turns sorting cards and deciding if the context is better represented with surface area or volume. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Here, students are asked to sort questions with a context to determine if it makes more sense to think about surface area or volume when answering the question. After sorting, students think about what information they need to answer a question and estimate reasonable measurements to calculate the answer to their question.

Launch

Tell students that the cards contain questions that would best be answered with either volume or surface area and that they will take turns sorting the cards. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Distribute pre-printed slips, cut from copies of the blackline master. Once each group has sorted their cards, ask the class where they placed each card. Ask students to come to agreement on any differences.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to sort each card based on whether it would make more sense to think about the surface area or the volume of the figure referred to, when answering the question.

1. For each card that you sort, explain to your partner how you know it's in the right category.
2. For each card that your partner sorts, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

The first, second, and fourth cards in the first column of blackline master are volume, the rest of the cards refer to surface area.

Instructional Routines

Card Sort

ilclass.com/r/10783726

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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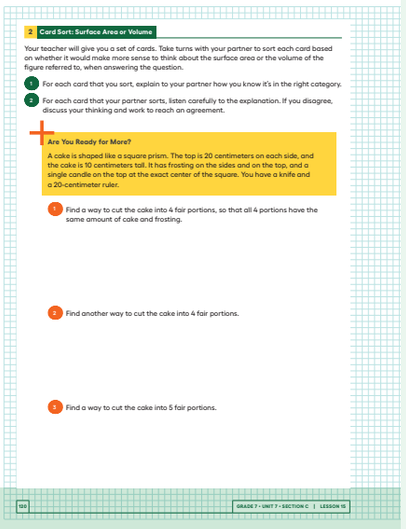
Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "I noticed ____ so I knew it was ..."
Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


2 Card Sort: Surface Area or Volume

Your teacher will give you a set of cards. Take turns with your partner to sort each card based on whether it would make more sense to think about the surface area or the volume of the figure referred to, when answering the question.

1. For each card that you sort, explain to your partner how you know it's in the right category.
2. For each card that your partner sorts, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

Are You Ready for More?

A cake is shaped like a square prism. The top is 20 centimeters on each side, and the cake is 10 centimeters tall. It has frosting on the sides and on the top, and a single candle on the top at the exact center of the square. You have a knife and a 20-centimeter ruler.

1. Find a way to cut the cake into 4 fair portions, so that all 4 portions have the same amount of cake and frosting.
2. Find another way to cut the cake into 4 fair portions.
3. Find a way to cut the cake into 5 fair portions.

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Are You Ready for More?

A cake is shaped like a square prism. The top is 20 centimeters on each side, and the cake is 10 centimeters tall. It has frosting on the sides and on the top, and a single candle on the top at the exact center of the square. You have a knife and a 20-centimeter ruler.

1. Find a way to cut the cake into 4 fair portions, so that all 4 portions have the same amount of cake and frosting.

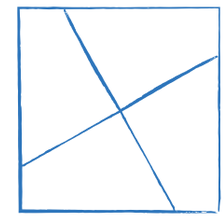
   The simplest way is to cut the cake in half on one diagonal, and then on the other diagonal.

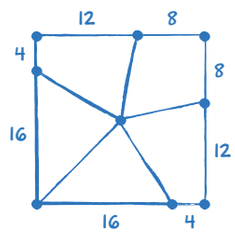
2. Find another way to cut the cake into 4 fair portions.

   The key is to pick points that are equally spaced from the corners, as shown.

3. Find a way to cut the cake into 5 fair portions.

   The perimeter of the cake is 80 cm, and 80 ÷ 5 = 16. Find 5 points around the perimeter of the cake that are each 16 centimeters apart, and connect each of them to the candle in the center with a cut. Each piece gets 16 cm of the edge of the cake, so the same amount of frosting. Through decomposing into triangles, we can show that each piece of the cake has a volume of 800 cubic centimeters. (Note that this solution presumes that the frosting has no thickness, which is not likely to be true, but is a handy simplifying assumption. In reality, people who love frosting should still take a corner piece.)





Activity Synthesis

Once all groups have completed the *Card Sort*, discuss:

- *"Which cards were tricky to sort? Explain why."*
- *"Did you need to make adjustments in your choices? What might have caused an error? What adjustments were made?"*

Select students to share their explanations of how they determined if a question referred to volume or to surface area. The goal is to ensure that students can verbally describe their reasoning because they are asked to write about the similarities and differences in the *Cool-down*.

Activity 3: Optional
Building a Bat House

5 min

Activity Narrative

This activity reinforces work that students have done in previous activities with regard to surface area and volume. Students work with a contextual problem to determine the surface area and volume of an object.

Launch

Arrange students in groups of 2. Invite students to share what they know about the role of bats in the ecosystem and why building a house for a bat colony is helpful.

Here are some points for discussion, if no students bring them up:

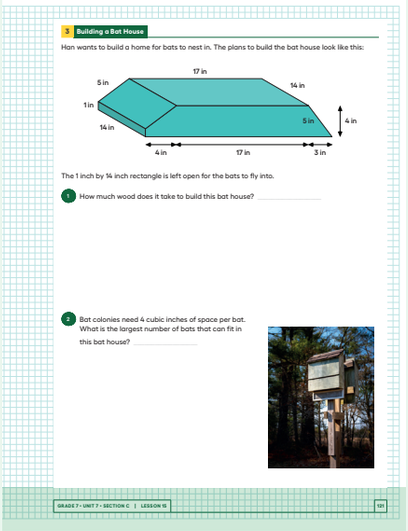
- Bats are an important part of any ecosystem.
- Bats eat insects and small pests.
- The world's largest bat colony is located near San Antonio, Texas, with up to 20 million bats.
- 23 bat species are critically endangered, 85 bat species are endangered, and 113 are considered vulnerable.

Tell students that one way to help bats is to provide roosting places for them, often called bat boxes or bat houses. Give students 3–5 minutes of quiet work time followed by time to share their answers with a partner. Follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 3, Launch)

Representation: Internalize Comprehension.
Activate or supply background knowledge by reviewing an image or video of a bat house. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual Processing

Student Workbook


3 Building a Bat House

Han wants to build a home for bats to nest in. The plans to build the bat house look like this:

The 1 inch by 14 inch rectangle is left open for the bats to fly into.

1. How much wood does it take to build this bat house?

2. Bat colonies need 4 cubic inches of space per bat. What is the largest number of bats that can fit in this bat house?

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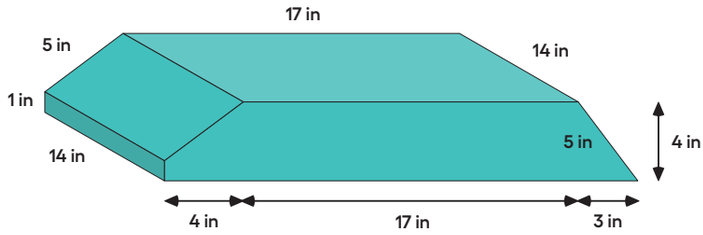
Access for Multilingual Learners (Activity 3, Synthesis)

Speaking: MLR8: Discussion Supports. Display sentence frames to support partner discussion, such as: "First I _____. Then I _____, because ..." "The volume is _____ because ..." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language.
Design Principle(s): Optimize Output (for explanation); Cultivate Conversation

Student Task Statement

Han wants to build a home for bats to nest in. The plans to build the bat house look like this:



The 1 inch by 14 inch rectangle is left open for the bats to fly into.

1. How much wood does it take to build this bat house?

   714 in^2

   Sample reasoning: The perimeter of the base is 52 in, and $52 \cdot 14 = 728$. Since the bottom rectangle is left open, the surface area of the bat house is $728 - 14$, or 714 in^2.

2. Bat colonies need 4 cubic inches of space per bat. What is the largest number of bats that can fit in this bat house?

   294 bats

   Sample reasoning: The area of the base is 84 in^2, so the volume is $84 \cdot 14$, or 1176 in^3. The number of bats that can fit is $1176 \div 4$, or 294 bats.

Activity Synthesis

The purpose of this discussion is to clarify when to calculate volume and when to calculate surface area, as well as acknowledging that there are times that a partial surface area is needed. Here are some questions for discussion:

- *"Are there other ways to calculate the area of the base?"*
- *"How did you know when to calculate surface area or volume for this problem?"*

Lesson Synthesis

Create or display an anchor chart that shows a prism and its surface area and volume.

Refer to the display during the discussion. Here are some questions to elicit student thinking:

- *"When is it better to know surface area than volume?"*

  When you are covering an object, when you want to know how much is exposed to the environment, etc.

- *"When is it better to know volume than surface area?"*

  When you are filling up the object, when you need to know how much is already inside, etc.

- *"If you cut an object in half and still consider both pieces, how does that affect the volume and surface area?"*

  The volume remains unchanged, but the surface area will increase.

Lesson Summary

Sometimes we need to find the volume of a prism, and sometimes we need to find the surface area.

Here are some examples of quantities related to volume:

- How much water a container can hold
- How much material it took to build a solid object

Volume is measured in cubic units, like in^3 or m^3.

Here are some examples of quantities related to surface area:

- How much fabric is needed to cover a surface
- How much of an object needs to be painted

Surface area is measured in square units, like in^2 or m^2.

Cool-down

Surface Area Differences

5 min

Student Task Statement

Describe some similarities and differences between a situation that involves calculating surface area and a situation that involves calculating volume.

Sample response: Volume refers to how much of something fits inside an object. Surface area refers to how much of something is needed to cover the outside of an object.

Student Workbook


15 Lesson Summary

Sometimes we need to find the volume of a prism, and sometimes we need to find the surface area.

Here are some examples of quantities related to volume:

- How much water a container can hold
- How much material it took to build a solid object

Volume is measured in cubic units, like in^3 or m^3.

Here are some examples of quantities related to surface area:

- How much fabric is needed to cover a surface
- How much of an object needs to be painted

Surface area is measured in square units, like in^2 or m^2.

Responding To Student Thinking

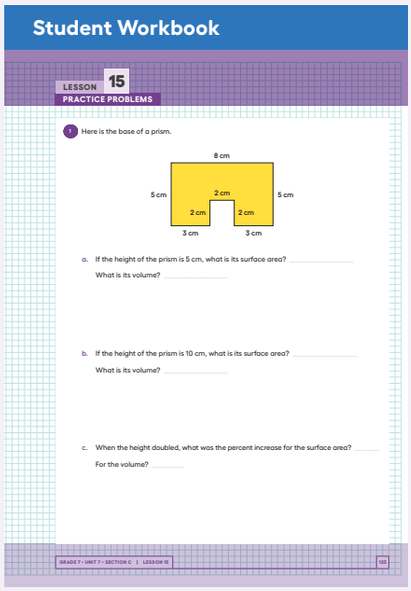
Points to Emphasize

If most students struggle with differentiating between situations that require surface area and volume, focus on this distinction when doing this activity:

Grade 7, Unit 7, Lesson 16, Warm-up You Decide

Practice Problems

5 Problems


Student Workbook

LESSON 15
PRACTICE PROBLEMS

1 Here is the base of a prism.

8 cm
5 cm
2 cm
5 cm
2 cm
2 cm
3 cm
3 cm

a. If the height of the prism is 5 cm, what is its surface area? _____
What is its volume? _____

b. If the height of the prism is 10 cm, what is its surface area? _____
What is its volume? _____

c. When the height doubled, what was the percent increase for the surface area? _____
For the volume? _____

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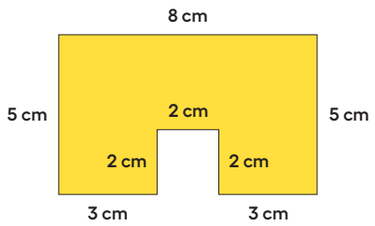
Problem 1

Here is the base of a prism.



a. If the height of the prism is 5 cm, what is its surface area? What is its volume?

$V = 180$ cm^3

b. If the height of the prism is 10 cm, what is its surface area? What is its volume?

$SA = 372$ cm^2, $V = 360$ cm^3

c. When the height doubled, what was the percent increase for the surface area? For the volume?

The surface area increased by about 67.6%.

The volume increased by 100% (doubled).

Problem 2

Select **all** the situations where knowing the volume of an object would be more useful than knowing its surface area.

A. Determining the amount of paint needed to paint a barn.

B. Determining the monetary value of a piece of gold jewelry.

C. Filling an aquarium with buckets of water.

D. Deciding how much wrapping paper a gift will need.

E. Packing a box with watermelons for shipping.

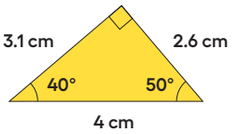
F. Charging a company for ad space on your race car.

G. Measuring the amount of gasoline left in the tank of a tractor.

Problem 3

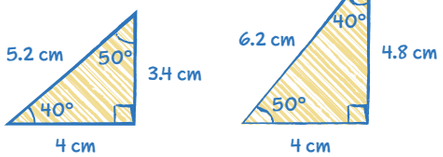
from Unit 7, Lesson 9

Han draws a triangle with a 50° angle, a 40° angle, and a side of length 4 cm as shown. Can you draw a different triangle with the same conditions?



Answers vary.

Sample response: Yes, if we rearrange the angles and side, there are more possibilities.



Problem 4

from Unit 7, Lesson 3

Angle H is half as large as angle J. Angle J is one fourth as large as angle K. Angle K has a measurement of 240 degrees. What is the measurement of angle H?

30°

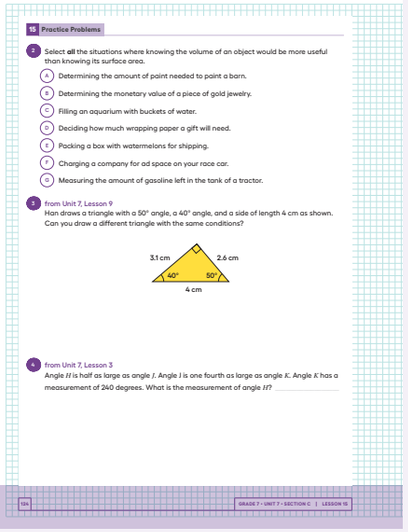
Problem 5

from Unit 4, Lesson 9

The Colorado state flag consists of three horizontal stripes of equal height. The side lengths of the flag are in the ratio 2:3. The diameter of the gold-colored disk is equal to the height of the center stripe. What percentage of the flag is gold?

Approximately 5.82% (the exact proportion is $\frac{\pi}{54}$ or equivalent)

Student Workbook


15 Practice Problems

2 Select **all** the situations where knowing the volume of an object would be more useful than knowing its surface area.

A Determining the amount of paint needed to paint a barn.
B Determining the monetary value of a piece of gold jewelry.
C Filling an aquarium with buckets of water.
D Deciding how much wrapping paper a gift will need.
E Packing a box with watermelons for shipping.
F Charging a company for ad space on your race car.
G Measuring the amount of gasoline left in the tank of a tractor.

3 from Unit 7, Lesson 9
Han draws a triangle with a 50° angle, a 40° angle, and a side of length 4 cm as shown. Can you draw a different triangle with the same conditions?

3.1 cm 2.6 cm 40° 50° 4 cm

4 from Unit 7, Lesson 3
Angle H is half as large as angle J. Angle J is one fourth as large as angle K. Angle K has a measurement of 240 degrees. What is the measurement of angle H? _____

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Student Workbook


15 Practice Problems

5 from Unit 4, Lesson 9
The Colorado state flag consists of three horizontal stripes of equal height. The side lengths of the flag are in the ratio 2 : 3. The diameter of the gold-colored disk is equal to the height of the center stripe. What percentage of the flag is gold? _____

Learning Targets
+ I can decide whether I need to find the surface area or the volume, when solving a problem about a real-world situation.

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