Lesson 5

More Estimating Probabilities

Goals

- Describe (orally and in writing) reasons why the relative frequency from an experiment may not exactly match the actual probability of the event.
- Recognize that sometimes the outcomes in a sample space are not equally likely.
- Use the results from a repeated experiment to estimate the probability of an event, and justify (orally and in writing) the estimate.

Learning Targets

- I can calculate the probability of an event when the outcomes in the sample space are not equally likely.
- I can explain why results from repeating an experiment may not exactly match the expected probability for an event.

Lesson Narrative

In this lesson students compare the results from running actual trials of an experiment to the calculated expected probabilities. They also use their data to see that additional trials usually produce more accurate results as minor differences even out after many trials.

In the first activity, students spin four different spinners to see that the outcomes in a sample space may not be equally likely, and they examine the spinners to construct arguments about why some outcomes are more likely than others. In the next activity, students draw blocks out of a bag repeatedly and use the relative frequency to estimate the probability of getting a green block.

Student Learning Goal

Let's estimate some probabilities.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)

Required Materials

Materials to Gather

- Paper bags: Activity 1, Activity 2
- Paper cups: Activity 1
- Snap cubes: Activity 2

Materials to Copy

- Making My Head Spin Handout (1 copy for every 4 students): Activity 1

Required Preparation

Activity 1:
Each student will need a pencil and paper clip to use with the spinners.

Activity 2:
Prepare a paper bag containing 5 snap cubes (3 green and 2 of another matching color) for every 3–4 students.

Lesson:
Provide 1 set of 4 spinners cut from the Making My Head Spin blackline master for every 4 students. Each student will need a pencil and paper clip to use with the spinners.

For the How Much Green activity, prepare a paper bag containing 5 snap cubes (3 green and 2 of another matching color) for every 3–4 students.

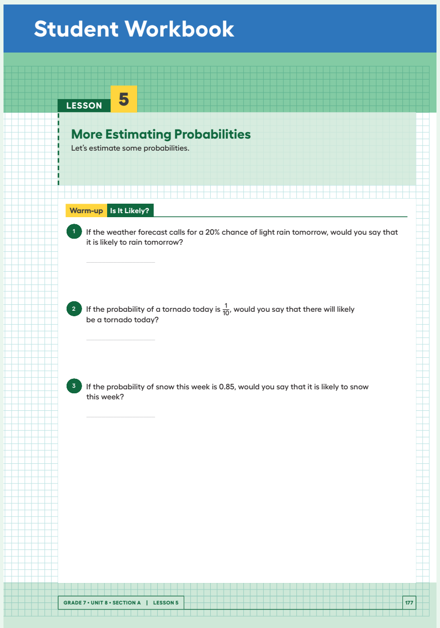
Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	20 min	10 min	10 min	5 min

Assessment

Cool-down (5 min)

Student Workbook


LESSON 5

More Estimating Probabilities

Let's estimate some probabilities.

Warm-up Is It Likely?

1. If the weather forecast calls for a 20% chance of light rain tomorrow, would you say that it is likely to rain tomorrow?

2. If the probability of a tornado today is $\frac{1}{10}$, would you say that there will likely be a tornado today?

3. If the probability of snow this week is 0.85, would you say that it is likely to snow this week?

GRADE 7 • UNIT 8 • SECTION A | LESSON 5 177

Warm-up

Is It Likely?

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to think more deeply about probabilities and what the values actually represent. In this activity, students are asked to compare the likelihood of three events with probabilities given in different formats. In the discussion, students are also asked to think about the context of the situations to see that probabilities are not the only consideration when planning a response.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. If the weather forecast calls for a 20% chance of light rain tomorrow, would you say that it is likely to rain tomorrow?

   It is not likely to rain tomorrow, but it could happen.

2. If the probability of a tornado today is $\frac{1}{10}$, would you say that there will likely be a tornado today?

   The tornado is not likely to happen today, but it could happen.

3. If the probability of snow this week is 0.85, would you say that it is likely to snow this week?

   It is likely that it will snow this week, but it might not happen.

Activity Synthesis

Ask students,

"Which situation would you worry about the most? Is that the same situation that is the most likely?"

Note that our interpretation of the scenarios influences how we feel about how likely an event is to happen. Although the likelihood of rain is higher, the implications of a tornado are much greater, so we may be more likely to worry about the tornado than the rain.

Activity 1
Making My Head Spin

20 min

Activity Narrative

There is a digital version of this activity.

In this activity, students return to calculating probabilities using the sample space, and they compare the calculated probabilities to the outcomes of their actual trials. Students have a chance to construct arguments about why probability estimates based on carrying out the experiment many times might differ from the expected probability. Students use a spinner in this activity, which will be helpful when designing simulations in upcoming lessons.

In the digital version of the activity, students use an applet to spin the spinners virtually. The applet allows students to focus on the mathematical goals rather than creating and using the spinners themselves. The digital version may be preferable if students do not need or want tactile interaction with physical objects.

Launch

Arrange students in groups of 4. Provide 1 set of 4 spinners cut from the blackline master to each group. Each student will need a pencil and paper clip.

Demonstrate how to use a pencil and paper clip to spin the spinner: Unbend one end of the paper clip so that it is straight. Put the paper clip on the end of the pencil and the pencil tip at the center of the spinner. Spin the paper clip so that it rotates around the pencil and the unbent portion points to the result of the spin. If it is difficult to determine which section the end of the paper clip points to, it is okay to disregard that spin and spin again.

Following the teacher demonstration, give students 5 minutes of quiet work time, then 10 minutes of group work.

Follow with a whole-class discussion.

Student Task Statement

Your teacher will give you 4 spinners. Make sure each person in your group uses a different spinner.

1. Spin your spinner 10 times, and record your outcomes.

   Sample response: 3, 2, 5, 1, 2, 4, 6, 1, 3, 4

2. Did you get all of the different possible outcomes in your 10 spins?

   Sample response: yes

3. What fraction of your 10 spins landed on the number 3?

   Sample response: $\frac{2}{10}$

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of directions visible throughout the activity.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Students may think they need to have their probability estimates match the computed probability. Remind them reality may differ from the probability in the short term, but should be fairly close if the chance experiment is repeated many times.

Student Workbook

1 Making My Head Spin

Your teacher will give you 4 spinners. Make sure each person in your group uses a different spinner.

1. Spin your spinner 10 times, and record your outcomes.
2. Did you get all of the different possible outcomes in your 10 spins?
3. What fraction of your 10 spins landed on the number 3?
4. Next, share your outcomes with your group, and record their outcomes.
   a. Outcomes for Spinner A:
   b. Outcomes for Spinner B:
   c. Outcomes for Spinner C:
   d. Outcomes for Spinner D:
5. Do any of the spinners have the same sample space? If so, do they have the same probabilities for each number to result?
6. For each spinner, what is the probability that it lands on the number 3? Explain or show your reasoning.

178 GRADE 7 • UNIT 8 • SECTION A | LESSON 5

Student Workbook


1 Making My Head Spin

7 For each spinner, what is the probability that it lands on something other than the number 3? Explain or show your reasoning.

8 Noah put Spinner D on top of his closed binder and spun it 10 times. It never landed on the number 1. How might you explain why this happened?

9 Han put Spinner C on the floor and spun it 10 times. It never landed on the number 3, so he says that the probability of getting a 3 is 0. How might you explain why this happened?

Are You Ready for More?

Design a spinner that has a $\frac{2}{3}$ probability of landing on the number 3. Explain how you could precisely draw this spinner.

GRADE 7 • UNIT 8 • SECTION A | LESSON 5

4. Next, share your outcomes with your group, and record their outcomes.

Sample response:

a. Outcomes for Spinner A:

3, 2, 5, 1, 2, 4, 6, 1, 3, 4

b. Outcomes for Spinner B:

2, 4, 6, 6, 1, 2, 4, 2, 5, 2

c. Outcomes for Spinner C:

7, 3, 9, 2, 4, 10, 2, 5, 1, 9

d. Outcomes for Spinner D:

1, 4, 1, 1, 3, 3, 2, 1, 2, 1

5. Do any of the spinners have the same sample space? If so, do they have the same probabilities for each number to result?

Sample response: Yes, Spinners A and B have the same sample space. They do not have the same probabilities for the numbers, though. For example, Spinner A has 2 sections with the number 3 out of 12 equal spaces, so the probability of spinning a 3 is $\frac{2}{12}$. Spinner B only has 1 section with the number 3 out of 12 equal spaces, so the probability of spinning a 3 is $\frac{1}{12}$.

6. For each spinner, what is the probability that it lands on the number 3? Explain or show your reasoning.

Spinner A: $\frac{1}{6}$. Spinner B: $\frac{1}{12}$. Spinner C: $\frac{1}{12}$. Spinner D: $\frac{1}{4}$

Sample reasoning: The values for Spinners A, B, and C are computed by counting the number of 3s on the spinner and dividing by the number of equal sections on the spinner. For Spinner D, the section for the number 3 is $\frac{1}{4}$ of the circle.

7. For each spinner, what is the probability that it lands on something other than the number 3? Explain or show your reasoning.

Spinner A: $\frac{5}{6}$. Spinner B: $\frac{11}{12}$. Spinner C: $\frac{11}{12}$. Spinner D: $\frac{3}{4}$

Sample reasoning: The values for Spinners A, B, and C, are computed by counting the number of sections on the spinner that do not have a number 3 and dividing by the total number of sections on the spinner. For Spinner D, the sections that do not have the number 3 make up $\frac{3}{4}$ of the circle.

8. Noah put Spinner D on top of his closed binder and spun it 10 times. It never landed on the number 1. How might you explain why this happened?

Sample response: Since the binder is sloped, gravity may have been pulling the spinner so that it could not land on the number 1.

9. Han put Spinner C on the floor and spun it 10 times. It never landed on the number 3, so he says that the probability of getting a 3 is 0. How might you explain why this happened?

Sample response: Han might have been holding the spinner at an angle like Noah, or maybe he just did not spin enough times. Since it is possible to land on the number 1, the probability should not be 0.

Are You Ready for More?

Design a spinner that has a $\frac{2}{3}$ probability of landing on the number 3. Explain how you could precisely draw this spinner.

Sample response: First, I would draw a circle with a compass. Then I would divide the circle into 3 equal sections by using a protractor and measuring an angle of 120° since $360 \div 3 = 120$. I would write the number 3 in two of the sections and write the number 1 in the other section.

Activity Synthesis

The purpose of this discussion is to think about reasons why the estimate of a probability may be different from the actual probability.

Select some students to share their responses to the last 5 questions.

Ask,

"How does your estimate for the probability of spinning a 3 compare to the probability you expect from just looking at the spinner?"

Explain that although the spinners provided are designed to have equally sized sections (except for Spinner D which has the angles 180 degrees, 45 degrees, and 90 degrees), sometimes it may be difficult to determine when the sections are exactly the same size. For situations where things are not so evenly divided, some experimenting may be needed to determine that the outcomes actually follow the probability we might expect.

There are two main reasons why the fraction of the time an event occurs may differ from the actual probability:

- The simulation was designed or run poorly. For example:
- Maybe the spinner sections were not of equal size even though they should have been, or maybe the spinner was tilted.
- Maybe the items being chosen from the bag were different sizes, so it was more likely to grab one item than another.
- Maybe the coin or number cube were not evenly weighted and were more likely to land with one side up than another.
- Maybe the computer that was programmed to return a random number had a problem with the code, making it return some numbers more often than others.
- Not enough trials were run. For example:
- As in the previous lesson, if a coin were flipped once and it came up heads, that doesn't mean that it always will. Even if it were flipped 100 times, it's not guaranteed to land heads up exactly 50 times. Some slight deviation is to be expected.

Access for Multilingual Learners (Activity 2, Student Task)

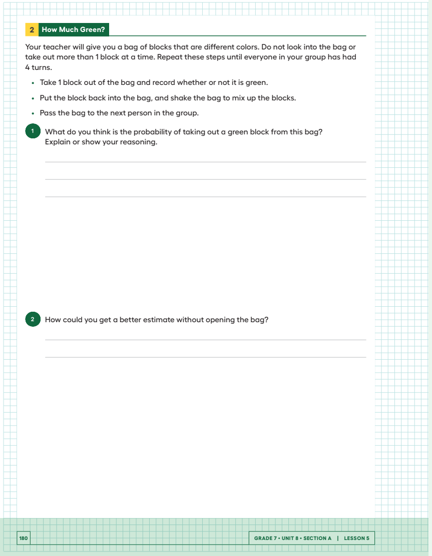
MLR5: Co-Craft Questions. Keep books or devices closed. Provide the bag of blocks and the instructions for the experiment, without revealing the questions. Then ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, “What do these questions have in common? How are they different?” Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Building on Student Thinking

Some students may estimate a probability that is different from the fraction of times they draw a green block. Ask these students for the reason they chose a different value for their estimate.

Student Workbook



2 How Much Green?

Your teacher will give you a bag of blocks that are different colors. Do not look into the bag or take out more than 1 block at a time. Repeat these steps until everyone in your group has had 4 turns.

- Take 1 block out of the bag and record whether or not it is green.
- Put the block back into the bag, and shake the bag to mix up the blocks.
- Pass the bag to the next person in the group.

1. What do you think is the probability of taking out a green block from this bag? Explain or show your reasoning.

2. How could you get a better estimate without opening the bag?

180 GRADE 7 • UNIT 8 • SECTION A | LESSON 5

Activity 2

How Much Green?

10 min

Activity Narrative

In this activity, students see how to approach understanding probability when the entire sample space cannot be known or if the situation is more complex by estimating the probability of an event using the results from repeating trials. In this particular example, the exact probability can be computed when the information is revealed, so students can compare their results to this value. In situations like predicting the weather, estimates may be the best thing available. Students gain exposure to the process of drawing blocks from a bag, which will be useful in designing simulations in future lessons.

Launch

Arrange students in groups of 3–4. Distribute 1 paper bag containing 5 snap cubes (3 green cubes and 2 matching cubes of some other color) to each group.

5 minutes of group work followed by a whole-class discussion.

Student Task Statement

Your teacher will give you a bag of blocks that are different colors. Do not look into the bag or take out more than 1 block at a time. Repeat these steps until everyone in your group has had 4 turns.

- Take 1 block out of the bag and record whether or not it is green.
- Put the block back into the bag, and shake the bag to mix up the blocks.
- Pass the bag to the next person in the group.

1. What do you think is the probability of taking out a green block from this bag? Explain or show your reasoning.

   Sample response: I think the probability is $\frac{7}{12}$ since we got 7 green blocks after 12 trials.

2. How could you get a better estimate without opening the bag?

   Sample response: Continuing to pick out blocks more times might give us a better estimate.

Activity Synthesis

The purpose of the discussion is to show that estimating the probability of an event can be done using repeated trials and is usually improved by including more trials.

Ask each group how many green blocks they got in their trials and display the class results for all to see.

Consider asking these discussion questions:

- *"How can we use the values from the class to estimate the probability of drawing out a green block?"*

  By using the data we have, we can estimate the fraction of blocks that are green.

- *"Based on the class data, what is the estimated probability of choosing a green block from the bag?"*

  "Was the probability estimated from the class data different from the probability estimate based on the data just from your group? Why?"

  Yes, it was different. Not everyone picked out the same thing each time, and in the class data there were more trials.

- *"Some of you may have guessed that there are 5 blocks in the bag. If we use that information, does it change our estimate of the probability?"*

  If there are only 5 blocks, it only makes sense for the probability to be $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$, or $\frac{5}{5}$.

Allow students to open the bags and see what blocks are in there.

- *"What is the probability based on this observation?"*

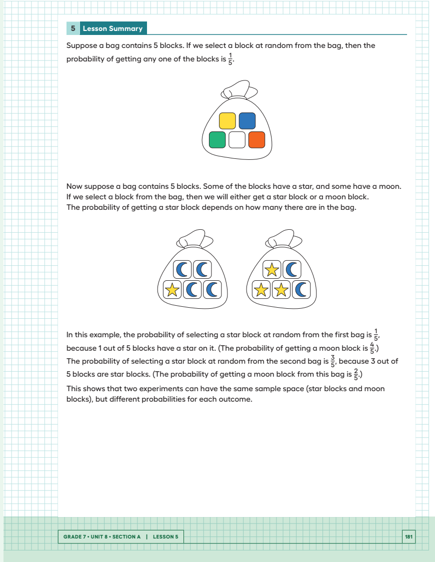
  $\frac{3}{5}$

- *"Does it match your group's estimate? Does it match the class estimate?"*

  "Was the estimate from the class data more accurate than the estimates from the groups?"

  It should be more accurate since more trials are included.

Student Workbook


5 Lesson Summary

Suppose a bag contains 5 blocks. If we select a block at random from the bag, then the probability of getting any one of the blocks is $\frac{1}{5}$.

Now suppose a bag contains 5 blocks. Some of the blocks have a star, and some have a moon. If we select a block from the bag, then we will either get a star block or a moon block. The probability of getting a star block depends on how many there are in the bag.

In this example, the probability of selecting a star block at random from the first bag is $\frac{1}{5}$, because 1 out of 5 blocks have a star on it. (The probability of getting a moon block is $\frac{4}{5}$.) The probability of selecting a star block at random from the second bag is $\frac{3}{5}$, because 3 out of 5 blocks are star blocks. (The probability of getting a moon block from this bag is $\frac{2}{5}$.)

This shows that two experiments can have the same sample space (star blocks and moon blocks), but different probabilities for each outcome.

GRADE 7 • UNIT 8 • SECTION A | LESSON 5

Lesson Synthesis

Consider asking these discussion questions:

- *"A student repeats the process of taking blocks out of a bag and replacing them 100 times. A green block is drawn 67 times. What is a good estimate for the probability of drawing out a green block from the bag?"*

  $\frac{67}{100}$

- *"A chance experiment is done a few times, and the fraction of outcomes in a certain event is used as an estimate for the probability of the event. Even if the experiments were done carefully, how could the estimate be improved?"*

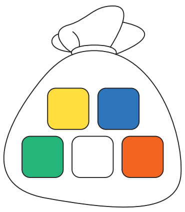
  Usually, the more trials done for an experiment, the closer the estimate will be to a computed probability.

- *"A chance experiment is repeated many times, but the fraction of outcomes for which a certain event occurs does not match the actual probability of the event. What are some reasons this may have happened?"*

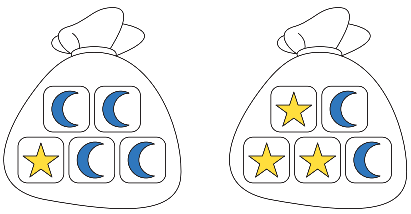
  The experiment may not have been repeated enough times. The experiment was badly done. For example, the experiment may not have been as random as originally thought. We usually expect a little difference between the estimated probability and the actual probability.

Lesson Summary

Suppose a bag contains 5 blocks. If we select a block at random from the bag, then the probability of getting any one of the blocks is $\frac{1}{5}$.



Now suppose a bag contains 5 blocks. Some of the blocks have a star, and some have a moon. If we select a block from the bag, then we will either get a star block or a moon block. The probability of getting a star block depends on how many there are in the bag.



In this example, the probability of selecting a star block at random from the first bag is $\frac{1}{5}$, because 1 out of 5 blocks have a star on it. (The probability of getting a moon block is $\frac{4}{5}$.) The probability of selecting a star block at random from the second bag is $\frac{3}{5}$, because 3 out of 5 blocks are star blocks. (The probability of getting a moon block from this bag is $\frac{2}{5}$.)

This shows that two experiments can have the same sample space (star blocks and moon blocks), but different probabilities for each outcome.

Cool-down

The Probability of Spinning B

5 min

Student Task Statement

Jada, Diego, and Elena each use the same spinner that has four (not necessarily equal sized) sections marked A, B, C, and D.

- Jada says, "The probability of spinning B is 0.3 because I spun 10 times, and it landed on B 3 times."
- Diego says, "The probability of spinning B is 20% because I spun 5 times, and it landed on B once."
- Elena says, "The probability of spinning B is $\frac{2}{7}$ because I spun 7 times, and it landed on B twice."

1. Based on their methods, which probability estimate do you think is the most accurate? Explain your reasoning.

   Sample response: Jada's method is probably the most accurate since she had the most attempts.

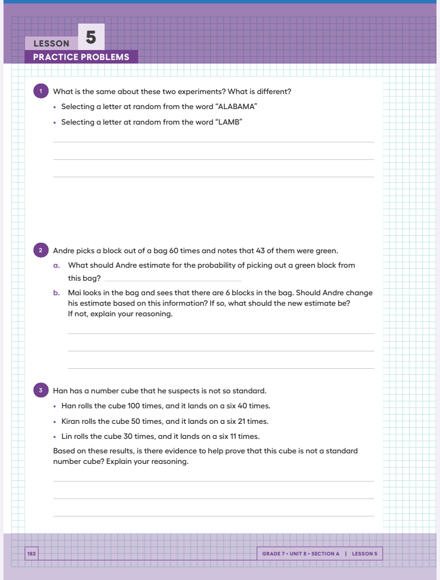
2. Andre measures the spinner and finds that the B section takes up $\frac{1}{4}$ of the circle. Explain why none of the methods match this probability exactly.

   Sample response: Since Jada spun it 10 times, she could only get estimates in increments of 0.1. Since Diego spun it 5 times, he could only get estimates in increments of 20%. Since Elena spun it 7 times, she could only get estimates in increments of $\frac{1}{7}$. If they had spun the spinner more times, their results would probably get closer to $\frac{1}{4}$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1 What is the same about these two experiments? What is different?
• Selecting a letter at random from the word "ALABAMA"
• Selecting a letter at random from the word "LAMB"

2 Andre picks a block out of a bag 60 times and notes that 43 of them were green.
a. What should Andre estimate for the probability of picking out a green block from this bag?
b. Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your reasoning.

3 Han has a number cube that he suspects is not so standard.
• Han rolls the cube 100 times, and it lands on a six 40 times.
• Kiran rolls the cube 50 times, and it lands on a six 21 times.
• Lin rolls the cube 30 times, and it lands on a six 11 times.
Based on these results, is there evidence to help prove that this cube is not a standard number cube? Explain your reasoning.

Lesson 5

Practice Problems

5 Problems

Problem 1

What is the same about these two experiments? What is different?

- Selecting a letter at random from the word "ALABAMA"
- Selecting a letter at random from the word "LAMB"

Sample response: Both these experiments have the same sample space. Also, they are both chance experiments that have to do with selecting letters at random from words. These two experiments are different because in the word "LAMB," each letter is equally likely, but in the word "ALABAMA," the letter "A" is more likely than the other letters.

Problem 2

Andre picks a block out of a bag 60 times and notes that 43 of them were green.

a. What should Andre estimate for the probability of picking out a green block from this bag?

$\frac{43}{60}$

b. Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your reasoning.

Yes. The estimate should be changed to $\frac{4}{6}$.

Sample reasoning: The original estimate is close to $\frac{40}{60}$, which is equal to $\frac{4}{6}$, and which is actually possible with 6 blocks. Since Andre was doing an experiment, it makes sense that he would be close to, but not exactly match, the calculated probability.

Problem 3

Han has a number cube that he suspects is not so standard.

- Han rolls the cube 100 times, and it lands on a six 40 times.
- Kiran rolls the cube 50 times, and it lands on a six 21 times.
- Lin rolls the cube 30 times, and it lands on a six 11 times.

Based on these results, is there evidence to help prove that this cube is not a standard number cube? Explain your reasoning.

Yes

Sample reasoning: A standard number cube should land on a six about $\frac{1}{6}$ of the time. After 100 rolls, it should land on six about 16 or 17 times. For all three people, it lands on six more than twice as often as expected. With this many rolls, there is strong evidence that this cube is not standard.

Problem 4

from Unit 8, Lesson 3

A textbook has 428 pages numbered in order starting with 1. You flip to a random page in the book in a way that it is equally likely to stop at any of the pages.

a. What is the sample space for this experiment?

the numbers 1 through 428

b. What is the probability that you turn to page 45?

$\frac{1}{428}$

c. What is the probability that you turn to an even-numbered page?

$\frac{214}{428}$ or $\frac{1}{2}$ (or equivalent)

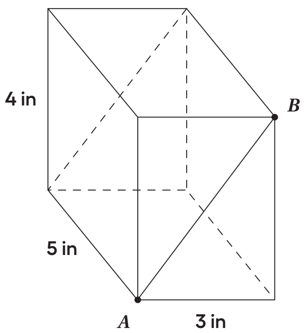
d. If you repeat this experiment 50 times, about how many times do you expect you will turn to an even-numbered page?

about 25 times, because $\frac{1}{2} \cdot 50 = 25$

Problem 5

from Unit 7, Lesson 15

A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between A and B is 5 inches.

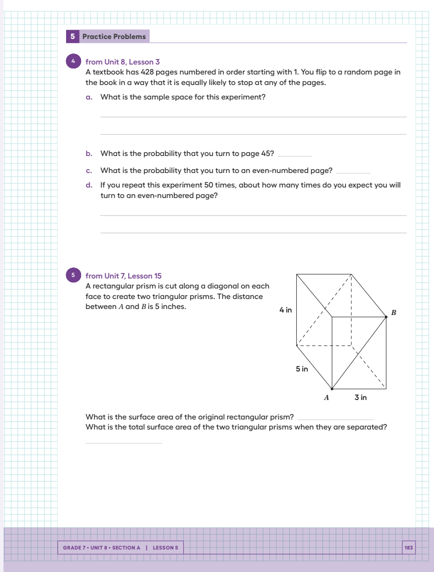


What is the surface area of the original rectangular prism? What is the total surface area of the two triangular prisms when they are separated?

Rectangular prism: 94 square inches.

Two triangular prisms separated: 144 square inches

Student Workbook


5 Practice Problems

4 from Unit 8, Lesson 3
A textbook has 428 pages numbered in order starting with 1. You flip to a random page in the book in a way that it is equally likely to stop at any of the pages.
a. What is the sample space for this experiment?
b. What is the probability that you turn to page 45?
c. What is the probability that you turn to an even-numbered page?
d. If you repeat this experiment 50 times, about how many times do you expect you will turn to an even-numbered page?

5 from Unit 7, Lesson 15
A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between A and B is 5 inches.

What is the surface area of the original rectangular prism?
What is the total surface area of the two triangular prisms when they are separated?