Lesson 20

Combining Like Terms (Part 1)

Goals

- Apply properties of operations to justify (orally and in writing) that expressions are equivalent.
- Generate an expression with fewer terms that is equivalent to a given expression.
- Interpret different methods for determining whether expressions are equivalent, and evaluate (orally) their usefulness.

Learning Targets

- I can figure out whether two expressions are equivalent to each other.
- When possible, I can write an equivalent expression that has fewer terms.

Lesson Narrative

In this lesson, students use properties of addition to justify that expressions are equivalent. As students explain how they know two expressions are equivalent for all values of the variables, they attend to precision. Students practice writing equivalent expressions with fewer terms. The given expressions do not include any parentheses.

The last activity is optional because it provides an opportunity for additional practice adding and subtracting signed numbers in the context of writing equivalent expressions.

Student Learning Goal

Let's see how we can tell that expressions are equivalent.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

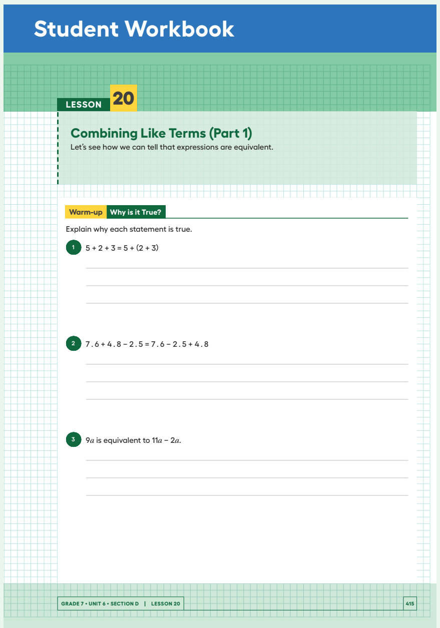
- Collect and Display (Activity 1)
- MLR8 (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	20 min	15 min	10 min	5 min


Student Workbook

LESSON 20

Combining Like Terms (Part 1)

Let's see how we can tell that expressions are equivalent.

Warm-up Why is it True?

Explain why each statement is true.

1. $5 + 2 + 3 = 5 + (2 + 3)$
2. $7.6 + 4.8 - 2.5 = 7.6 - 2.5 + 4.8$
3. $9a$ is equivalent to $11a - 2a$.

Warm-up

Why is it True?

10 min

Activity Narrative

In this *Warm-up,* students encounter some algebraic moves they have studied in the past several lessons and explain why these moves are allowed. The moves discussed here are important to understand as students work toward fluency in writing expressions with fewer terms.

Launch

Display one statement at a time.

Give students 30 seconds of quiet think time for each statement and ask them to give a signal when they have an explanation.

Ask them to share their explanation with a partner and then follow with whole-class discussion.

Student Task Statement

Explain why each statement is true.

1. $5 + 2 + 3 = 5 + (2 + 3)$

   Sample response: Associative property: The convention is to add left to right so $5 + 2$ is added first, but the associative property says grouping differently with addition gives the same result.

2. $7.6 + 4.8 - 2.5 = 7.6 - 2.5 + 4.8$

   Sample response: Subtraction can be written as adding the opposite, and then the order can be switched with the commutative property of addition: $7.6 + 4.8 + -2.5 = 7.6 + -2.5 + 4.8$.

3. $9a$ is equivalent to $11a - 2a$.

   Sample response: Distributive property: $11a - 2a = (11 - 2)a = 9a$

Activity Synthesis

The purpose of this discussion is to refresh students' memories of algebra moves that will be useful in this lesson. Ask students to share their reasons why each statement is true. Record and display their responses for all to see. Highlight correct use of precise, mathematical language and give students opportunities to revise their responses to be more precise.

Some questions to spark discussion include:

"How does the placement of the parentheses change how the first expression is evaluated?"

"Why is it okay to change the order of the terms, even though there's subtraction in the expression?"

"Can you draw a diagram to show why $9a$ is equivalent to $11a - 2a$?"

Activity 1

D's and J's

20 min

Activity Narrative

In this activity, students explain why algebra moves to simplify an expression are valid. In the process, they see two ways to reason about properties to combine like terms. As students explain how they know two expressions are equivalent, they attend to precision of language.

Students begin by substituting a value for the variable into expressions believed to be equivalent, and discover that the expressions are equal for that value. They then substitute other values and find that one of the expressions has a different value than the others. This primes students to see why applying properties is the only reliable way to decide whether two expressions are equivalent.

Students follow up by expanding the terms of the expression to consider each instance of the variables individually, and uncover the properties applied in each step of writing the expression with fewer terms.

In order to see the moves and see the properties, students need to see a group of terms, for example, $(j+j+j+j)$ as an object that can be rearranged. This is an example of noticing and making use of structure.

Launch

Display the first part of the *Task Statement* for all to see:

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to $6j + 4d - 2j + 3d$

- Jada thinks $8j + 1d$ is equivalent to the original expression.
- Diego thinks $4j + 7d$ is equivalent to the original expression.

Remind students that we can tell whether the expressions are equivalent by substituting some different values for j and d and evaluating the expressions.

First, ask students to substitute the values $j = 3$ and $d = 2$ and evaluate the original expression, Jada's expression, and Diego's expression. All the expressions come out to 26. Perhaps Jada's and Diego's are both equivalent to the original expression?

Then ask students to choose some different values for j and d and evaluate: the original expression, Jada's expression, and Diego's expression. For any other values of j and d, Jada and Diego's expressions do not evaluate to the same thing. For example, for $j = 1$ and $d = 1$, the original is 11, Jada's expression is 9, and Diego's is 11.

Emphasize the outcome that the value of Diego's expression matches the original expression's, but Jada's does not. This proves that Jada's expression is not equivalent to the original, but it does *not* prove that Diego's expression is equivalent to the original.

Tell students that experimenting with numbers can tell us that two expressions are not equivalent, but it can't prove that two expressions are equivalent. To prove two expressions are equivalent, we need to show they are equal for every value. But it's impossible to try every value. So, to prove that two expressions are equivalent, we need to reason about the expressions using the properties that we know.

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 1, Launch)

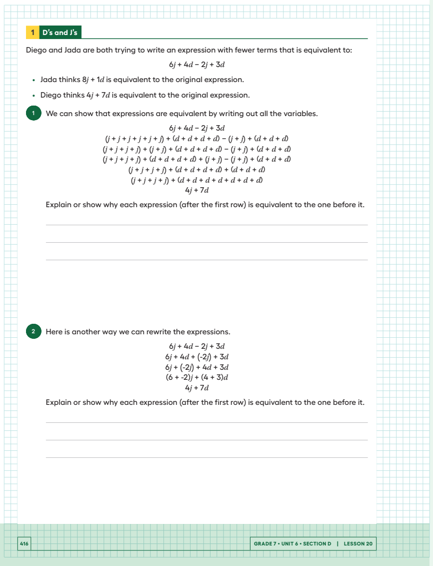
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: students can share their learning orally and kinesthetically, using drawings or physical manipulatives such as connecting cubes or sticky notes.

Supports accessibility for: Language, Visual-Spatial Processing

Student Workbook


1 D's and J's

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to:

$$6j + 4d - 2j + 3d$$

- Jada thinks $8j + 1d$ is equivalent to the original expression.
- Diego thinks $4j + 7d$ is equivalent to the original expression.

1. We can show that expressions are equivalent by writing out all the variables.

$$\begin{gathered} 6j + 4d - 2j + 3d \\ (j+j+j+j+j+j) + (d+d+d+d) - (j+j) + (d+d+d) \\ (j+j+j+j) + (j+j) + (d+d+d+d) - (j+j) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d) + (j+j) - (j+j) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d+d+d+d) \\ 4j + 7d \end{gathered}$$

Explain or show why each expression (after the first row) is equivalent to the one before it.

2. Here is another way we can rewrite the expressions.

$$\begin{gathered} 6j + 4d - 2j + 3d \\ 6j + 4d + (\text{-}2j) + 3d \\ 6j + (\text{-}2j) + 4d + 3d \\ (6 + \text{-}2)j + (4 + 3)d \\ 4j + 7d \end{gathered}$$

Explain or show why each expression (after the first row) is equivalent to the one before it.

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Building on Student Thinking

Students may think the expressions are equivalent after finding them equal for $j = 3$ and $d = 2$. Remind students that equivalent expressions must be equal for every possible value of the variable.

Students might have trouble describing the moves in the last two questions and justifying why the expressions are equivalent. Encourage students to closely examine the changes from row to row and consider why the moves shown do not change the value of the expression.

Arrange students in groups of 2.

Allow 6–7 minutes quiet work time and partner discussions followed by a whole class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to justify why expressions are equivalent. Display words and phrases, such as "rearrange," "expand," "combine," "rewrite," and "distribute."

Student Task Statement

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to:

$$6j + 4d - 2j + 3d$$

Jada thinks $8j + 1d$ is equivalent to the original expression.

Diego thinks $4j + 7d$ is equivalent to the original expression.

1. We can show that expressions are equivalent by writing out all the variables. Explain or show why each expression (after the first row) is equivalent to the one before it.

$$\begin{gathered} 6j + 4d - 2j + 3d \\ (j+j+j+j+j+j) + (d+d+d+d) - (j+j) + (d+d+d) \\ (j+j+j+j) + (j+j) + (d+d+d+d) - (j+j) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d) + (j+j) - (j+j) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d) + (d+d+d) \\ (j+j+j+j) + (d+d+d+d+d+d+d)4j + 7d \end{gathered}$$

   Sample responses:

   - First row: Write products as sums (distributive property).
   - Second row: Group first set of j's differently (associative property).
   - Third row: Switch second and third groups of addends (commutative property).
   - Fourth row: Subtract an addend from itself to get 0: $(j + j) - (j + j)$.
   - Fifth row: Group all the d's together (associative property).
   - Sixth row: Write sums as products (distributive property).

2. Here is another way we can rewrite the expressions. Explain or show why each expression (after the first row) is equivalent to the one before it.

$$\begin{gathered} 6j + 4d - 2j + 3d \\ 6j + 4d + (\text{-}2j) + 3d \\ 6j + (\text{-}2j) + 4d + 3d \\ (6 + \text{-}2)j + (4 + 3)d \\ 4j + 7d \end{gathered}$$

   Sample responses:

   - First row: Write subtraction as addition.
   - Second row: Switch 2nd and 3rd terms (commutative property).
   - Third Row: Write sums as products (distributive property).
   - Fourth row: Evaluate numerical expressions.

Are You Ready for More?

Follow the instructions for a number puzzle:

- Take the number formed by the first 3 digits of your phone number and multiply it by 40.
- Add 1 to the result.
- Multiply by 500.
- Add the number formed by the last 4 digits of your phone number, and then add it again.
- Subtract 500.
- Multiply by $\frac{1}{2}$.

1. What is the final number?
2. How does this number puzzle work?
3. Can you invent a new number puzzle that gives a surprising result?

Sample response:

- Let x represent the 3-digit number, so $40x$
- $40x + 1$
- $500(40x + 1)$
- Let y represent the 4-digit number, so $500(40x + 1) + y + y$ or $500(40x + 1) + 2y$
- $500(40x + 1) + 2y - 500 = 20{,}000x + 500 + 2y - 500 = 20{,}000x + 2y$
- $\frac{20{,}000x + 2y}{2} = 10{,}000x + y$ $10{,}000x + y$ means the 3-digit number, x was moved 4 place values to the left followed 4 zeros and then the 4-digit number was added to the zeros, forming the phone number.

$10{,}000x + y$ means the 3-digit number, x, was moved 4 place values to the left followed by 4 zeros, and then the 4-digit number was added to the zeros, forming the phone number.

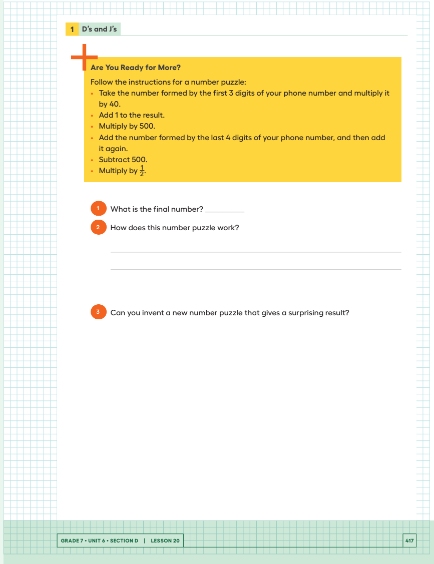
Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their reason each step is equivalent to the last. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. (For example, the display may have "You can rewrite subtracting as adding so you can rearrange the terms" already on it, which can be updated with the more precise statement "You can rewrite subtraction as adding the opposite so you can use the commutative property.")

Invite students to justify that the steps taken by Diego do not change the value of the expressions. Emphasize places where he used the distributive property and the commutative property.

Present a simpler expression, like $3c + 5c$, and ask students how you could rewrite it with fewer terms. Ensure they see it's equivalent to $8c$, whether justified by $c(3 + 5)$ or by writing $(c + c + c) + (c + c + c + c + c)$.

Student Workbook


1 D's and J's

Are You Ready for More?

Follow the instructions for a number puzzle:

- Take the number formed by the first 3 digits of your phone number and multiply it by 40.
- Add 1 to the result.
- Multiply by 500.
- Add the number formed by the last 4 digits of your phone number, and then add it again.
- Subtract 500.
- Multiply by $\frac{1}{2}$.

1. What is the final number? ______
2. How does this number puzzle work?
3. Can you invent a new number puzzle that gives a surprising result?

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Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as "It looks like …" "I noticed _________ so I …" "How do you know … ?" or "That could/couldn't be true because …"

Supports accessibility for: Language, Social-Emotional Functioning

Consider using this opportunity to explain the meaning of this lesson's title. Explain that two terms are called "like terms" when they are both numbers or when they both have the same variable. When we add like terms, we call it "combining like terms." Use these examples to clarify:

- $3c + 5c$ are like terms because they both have the variable c. We can combine them as $8c$.
- $3 + 5$ are like terms because they are both numbers. We can combine them as 8.
- $3c + 5$ are not like terms because one has a variable and one is a number. We cannot combine them.

Activity 2

Making Sides Equal Optional

Activity Narrative

In this activity, students use what they have learned so far to find an unknown term or expression that makes two expressions equivalent. Students have many strategies at their disposal to reason about the unknown term or expression in each equation. For example, for the first equation $6x + ? = 10x$, students might reason as in the previous activity and write out the sum of 6 x's and ? on one side, and a string of 10's on the other side, and reason that 4 x's are needed to make the sides equivalent. Alternatively, they might reason with the distributive property, and rewrite the left side as $x(6 + ?) = 10x$.

When students compare their results with a partner and resolve discrepancies, they have an opportunity to justify their thinking and critique the reasoning of others.

Launch

Arrange students in groups of 2. Explain that they will use what they have learned so far to find an unknown term or expression that will make two expressions equivalent. Draw students' attention to the instructions, which tell them to complete the first set of problems, check in with their partner, and then proceed. If desired, consider asking students to pause after the first set for whole-class discussion.

Student Task Statement

Replace each ? with a term or expression in parentheses that will make the expression on the left side of the equation equivalent to the expression on the right side. Check your results for Set A with your partner and work to reach an agreement before moving on to Set B.

Set A

1. $6x + ? = 10x$ $4x$
2. $6x + ? = 2x$ $-4x$
3. $6x + ? = -10x$ $-16x$
4. $6x + ? = 0$ $-6x$
5. $6x + ? = 10$ $10 - 6x$ (or equivalent)

Set B

1. $6x - ? = 2x$ $4x$
2. $6x - ? = 10x$ $-4x$
3. $6x - ? = x$ $5x$
4. $6x - ? = 6$ $6x - 6$ (or equivalent)
5. $6x - ? = 4x - 10$ $2x + 10$ (or equivalent)

Activity Synthesis

The purpose of the discussion is to highlight alternative ways of reasoning about equivalent expressions.

Ask students to share their expressions for each problem. Record and display their responses for all to see. After each student shares, ask the class if they agree or disagree.

The following questions, when applicable, can be used as students share:

- *"Why didn't you combine x terms and numbers?"*

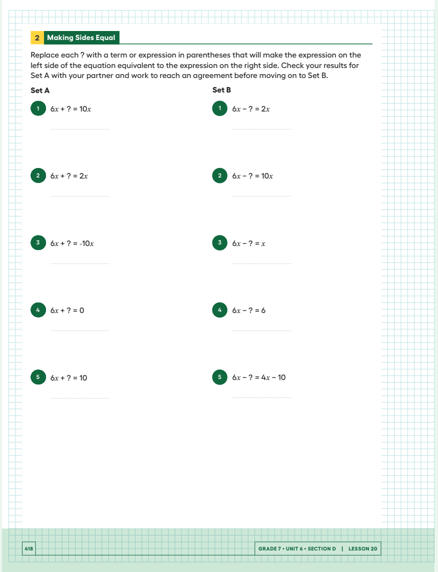
  Rewriting expressions using the properties of multiplication or the distributive property shows why this doesn't result in an equivalent expression.

- *"How did you decide on the components of the unknown term or expression?"*

  "Did you use the commutative property?"

  "Did you use the distributive property?"

Student Workbook



2 Making Sides Equal

Replace each ? with a term or expression in parentheses that will make the expression on the left side of the equation equivalent to the expression on the right side. Check your results for Set A with your partner and work to reach an agreement before moving on to Set B.

Set A

1. $6x + ? = 10x$
2. $6x + ? = 2x$
3. $6x + ? = -10x$
4. $6x + ? = 0$
5. $6x + ? = 10$

Set B

1. $6x - ? = 2x$
2. $6x - ? = 10x$
3. $6x - ? = x$
4. $6x - ? = 6$
5. $6x - ? = 4x - 10$

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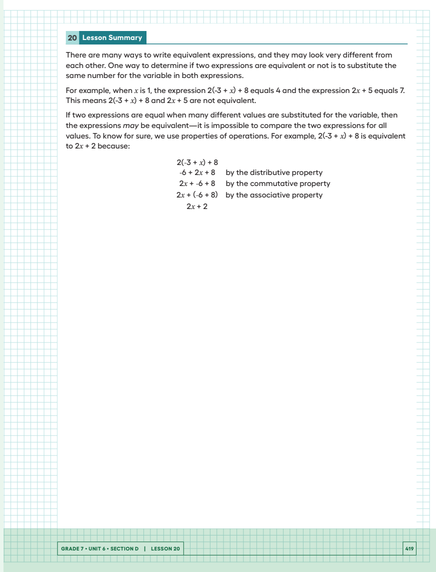
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support whole-class discussion. "I agree/disagree because ...".

Advances: Speaking, Conversing

Student Workbook



Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of combining like terms. If students struggle, plan to make time to revisit related work in the lessons referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 6, Lesson 19 Expanding and Factoring

Grade 7, Unit 6, Lesson 20 Combining Like Terms (Part 1)

Lesson Synthesis

Share with students,

"Today we saw how properties of operations can help us write an equivalent expression with fewer terms."

To review these concepts, consider asking students:

"What are some ways we can tell that $7x + 2$ is not equivalent to $9x$?"

We can substitute any value other than 1 for x. For example, if $x = 2$ then $7x + 2 = 16$ and $9x = 18$.

"How do we know that $7x + 2x$ is equivalent to $9x$?

We can use the distributive property, $7x + 2x = (7 + 2) \cdot x$.

"What are some ways we could rearrange the terms in the expression $-2x + 6y - 6x + 15y$ and create an equivalent expression?"

$-2x + -6x + 6y + 15y$

Lesson Summary

There are many ways to write equivalent expressions, and they may look very different from each other. One way to determine if two expressions are equivalent or not is to substitute the same number for the variable in both expressions.

For example, when x is 1, the expression $2(-3 + x) + 8$ equals 4 and the expression $2x + 5$ equals 7. This means $2(-3 + x) + 8$ and $2x + 5$ are not equivalent.

If two expressions are equal when many different values are substituted for the variable, then the expressions *may* be equivalent—it is impossible to compare the two expressions for all values. To know for sure, we use properties of operations. For example, $2(-3 + x) + 8$ is equivalent to $2x + 2$ because:

$$\begin{array}{rl} 2(-3 + x) + 8 & \\ -6 + 2x + 8 & \text{by the distributive property} \\ 2x + -6 + 8 & \text{by the commutative property} \\ 2x + (-6 + 8) & \text{by the associative property} \\ 2x + 2 & \end{array}$$

Cool-down

Fewer Terms

5 min

Student Task Statement

Write each expression with fewer terms. Show your work or explain your reasoning.

1. $10x - 2x$

   $8x$

2. $10x - 3y + 2x$

   $12x - 3y$

Lesson **20**

Practice Problems

5 Problems

Problem 1

Andre says that $10x + 6$ and $5x + 11$ are equivalent because they both equal 16 when x is 1. Do you agree with Andre? Explain your reasoning.

no

Sample reasoning: Equivalent expressions are equal for any value of their variable. When x is 0, these two expressions are not equal.

Problem 2

Select **all** expressions that can be subtracted from $9x$ to result in the expression $3x + 5$.

A. $-5 + 6x$

B. $5 - 6x$

C. $6x + 5$

D. $6x - 5$

E. $-6x + 5$

Problem 3

Select **all** the statements that are true for any value of x.

A. $7x + (2x + 7) = 9x + 7$

B. $7x + (2x - 1) = 9x + 1$

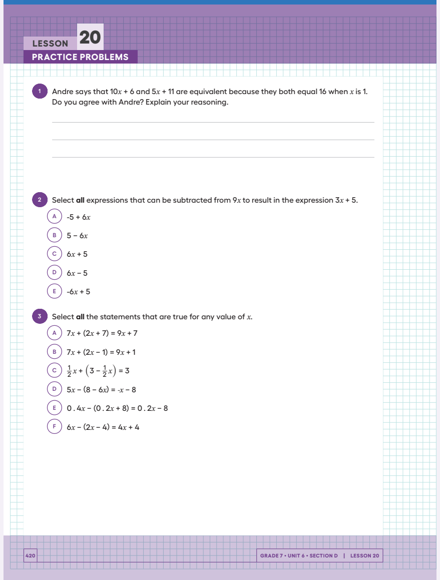
C. $\frac{1}{2}x + \left(3 - \frac{1}{2}x\right) = 3$

D. $5x - (8 - 6x) = -x - 8$

E. $0.4x - (0.2x + 8) = 0.2x - 8$

F. $6x - (2x - 4) = 4x + 4$

Student Workbook


LESSON 20
PRACTICE PROBLEMS

1 Andre says that $10x + 6$ and $5x + 11$ are equivalent because they both equal 16 when x is 1. Do you agree with Andre? Explain your reasoning.

2 Select **all** expressions that can be subtracted from $9x$ to result in the expression $3x + 5$.

A $-5 + 6x$

B $5 - 6x$

C $6x + 5$

D $6x - 5$

E $-6x + 5$

3 Select **all** the statements that are true for any value of x.

A $7x + (2x + 7) = 9x + 7$

B $7x + (2x - 1) = 9x + 1$

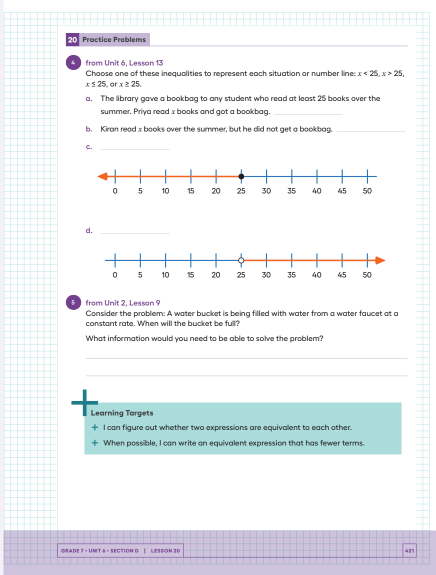
C $\frac{1}{2}x + \left(3 - \frac{1}{2}x\right) = 3$

D $5x - (8 - 6x) = -x - 8$

E $0.4x - (0.2x + 8) = 0.2x - 8$

F $6x - (2x - 4) = 4x + 4$

Student Workbook


20 Practice Problems

4 from Unit 6, Lesson 13

Choose one of these inequalities to represent each situation or number line: $x < 25$, $x > 25$, $x \le 25$, or $x \ge 25$.

a. The library gave a bookbag to any student who read at least 25 books over the summer. Priya read x books and got a bookbag.

b. Kiran read x books over the summer, but he did not get a bookbag.

c.

d.

5 from Unit 2, Lesson 9

Consider the problem: A water bucket is being filled with water from a water faucet at a constant rate. When will the bucket be full?

What information would you need to be able to solve the problem?

Learning Targets

+ I can figure out whether two expressions are equivalent to each other.
+ When possible, I can write an equivalent expression that has fewer terms.

Problem 4

from Unit 6, Lesson 13

Choose one of these inequalities to represent each situation or number line:
$x < 25$, $x > 25$, $x \le 25$, or $x \ge 25$.

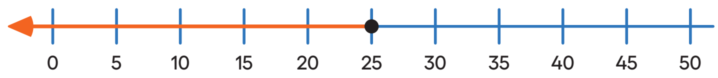
a. The library gave a bookbag to any student who read at least 25 books over the summer. Priya read x books and got a bookbag.

$x \ge 25$

b. Kiran read x books over the summer, but he did not get a bookbag.

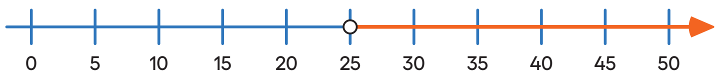
$x < 25$

c.



$x \le 25$

d.



$x > 25$

Problem 5

from Unit 2, Lesson 9

Consider the problem: A water bucket is being filled with water from a water faucet at a constant rate. When will the bucket be full?

What information would you need to be able to solve the problem?

Sample responses:

- How big is the bucket?
- What is the rate of water flow?
- How high is the bucket?
- How high is the water in the bucket after 1 minute?