Lesson 1

Understanding Proportional Relationships

Goals

- Comprehend that for the equation of a proportional relationship given by $y = kx$, k represents the constant of proportionality.
- Create graphs and equations of proportional relationships in context, including an appropriate scale.
- Interpret diagrams or graphs of proportional relationships in context.

Learning Targets

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

Student Learning Goal

Let's study some graphs.

Lesson Narrative

The purpose of this lesson is to get students thinking about what makes a "good" graph by considering components such as labels and scale. Students add a scale to graphs showing the pace of two bugs and graph an additional line based on a verbal description of a third bug.

This lesson includes graphs with elapsed time on the vertical axis and distance traveled on the horizontal axis. In general, a context that involves a relationship between two quantities does not dictate which quantity is the independent or dependent variable. Consider this situation where a runner is traveling one mile every 10 minutes.

- We can say the number of miles traveled, d, depends on the number of minutes that have passed, t, and write $d = 0.1t$. This way of expressing the relationship might be more useful for questions like "How far does the runner travel in 35 minutes?"
- We can also say that the number of minutes that have passed, t, depends on the number of miles traveled, d, and write $t = 10d$. This way of expressing the relationship might be more useful for questions like "How long does it take the runner to travel 2 miles?"

Both interpretations have meaning, and both could be of interest—it is up to the modeler to decide what questions they want to answer about the context and which way of expressing the relationship will be most useful in answering those questions.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 1)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

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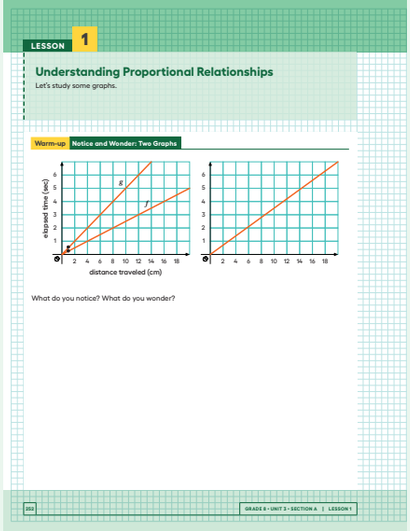
Student Workbook


LESSON 1

Understanding Proportional Relationships

Let's study some graphs.

Warm-up Notice and Wonder: Two Graphs

What do you notice? What do you wonder?

Warm-up

Notice and Wonder: Two Graphs

5 min

Activity Narrative

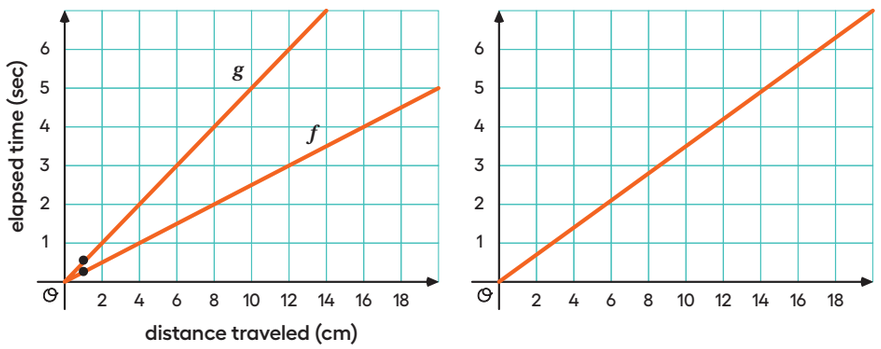
The purpose of this *Warm-up* is to have students discuss which features of a graph are necessary for communicating information. While students may notice and wonder many things about these graphs, the missing labels on the second graph is an important discussion point.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see.

Launch

Arrange students in groups of 2. Display both graphs for all to see.
Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement



What do you notice? What do you wonder?

Students may notice:

- The axes on the second graph are not labeled.
- If the first graph is about speed, then f is twice as fast as g.
- Line g is something going a speed of 2 cm every sec.
- Line f is something going at a pace of about 0.25 sec per 1 cm.

Students may wonder:

- What do the two points mean?
- Why does one image show two lines while the other only has one?
- What do lines g and f represent?
- What does the line in the second graph represent?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the graphs. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the fact that the second graph is missing labels does not come up during the conversation, ask students to discuss this idea.

Activity 1

Moving Through Representations

15 min

Activity Narrative

In this activity, students investigate the paces of two different bugs. Students use quantitative and abstract reasoning as they use the tick-mark diagram at the start of the activity to answer questions about pace, decide on a scale for the axes, and mark and label the time needed to travel 1 centimeter for each bug.

Monitor for students who use different scales on the axes to share later. For example, some students may count by 1 second on the distance axis while others may count by 0.5 second.

Launch

Arrange students in groups of 2. Before students start working, ensure that they understand that each bug's position is measured at the front of its head. For example, after 2 seconds, the ladybug has moved 4 centimeters and the ant has moved 6 centimeters.

Ask students to review the images and the first problem in the activity and give a signal when they have finished. Invite students to share their ideas about which bug is represented by line u and which bug is represented by line v. (The ladybug is u, and the ant is v.) If not mentioned by students, draw attention to how the graph shows the *pace* of the two bugs. The graph shows how much time it takes to go a certain distance, which is different than a graph of speed, which shows how much distance is traveled in a certain amount of time.

Give students work time to complete the remaining problems with their partner followed by a whole-class discussion.

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.

Keep books or devices closed. Display only the *Task Statement* and first image of the bugs, without revealing the line graphs or the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

Building on Student Thinking

If students confuse pace with speed and interpret a steeper line to mean that the ladybug moves faster, consider:

Asking,

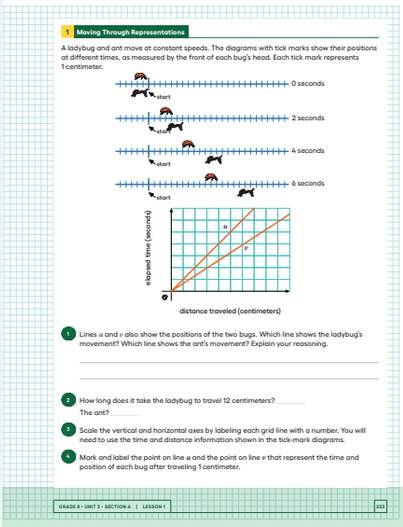
"How do the tick marks on either diagram help show which bug is moving faster?"

Asking,

"With distance on the x-axis and time on the y-axis, where is the ladybug's location after 4 seconds on the graph? Which line does that point correspond with?"

Explaining that moving twice as fast means going at half the pace.

Student Workbook


1 Moving Through Representations

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times, as measured by the front of each bug's head. Each tick mark represents 1 centimeter.

0 seconds
start
2 seconds
start
4 seconds
start
6 seconds
start

elapsed time (seconds)
u
v
O
distance traveled (centimeters)

1. Lines u and v also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning.

2. How long does it take the ladybug to travel 12 centimeters? _____ The ant? _____

3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams.

4. Mark and label the point on line u and the point on line v that represent the time and position of each bug after traveling 1 centimeter.

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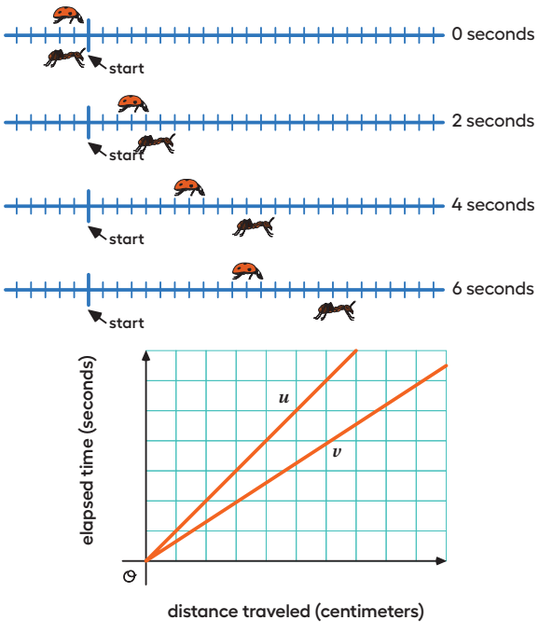
Student Task Statement

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times, as measured by the front of each bug's head. Each tick mark represents 1 centimeter.



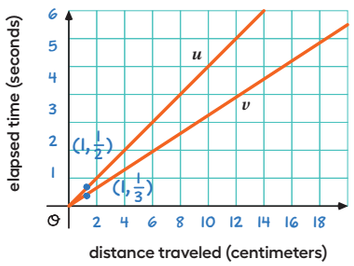
1. Lines u and v also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning.

Line u shows the ladybug and line v shows the ant. Sample reasoning: After 2 seconds, the ant has gone farther than the ladybug, and looking at both lines at 2 seconds, line v shows a greater distance.

2. How long does it take the ladybug to travel 12 centimeters? The ant?

The ladybug travels 12 centimeters in 6 seconds, and it takes the ant 4 seconds.

3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams.



4. Mark and label the point on line u and the point on line v that represent the time and position of each bug after traveling 1 centimeter.

See graph.

Are You Ready for More?

1. How fast is each bug traveling?

   The ladybug is traveling at 2 centimeters/second.

   The ant is traveling at 3 centimeters/second.

2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.

   No, the ant is always one and a half times as far from the start as the ladybug.

Activity Synthesis

Display the images from the *Student Task Statement* for all to see. Invite students to share their solutions for how long it takes each bug to travel 12 centimeters. Encourage students to reference one or both images as they explain their thinking.

Then invite previously selected students to share their graphs and explain how they decided on what scale to use. If possible, display these graphs for all to see. There are many correct ways to choose a scale for this situation, though some scales may have made it easier to answer the last question. Highlight these graphs and encourage students to read all problems when making decisions about how to construct a graph.

Activity 2

Moving Twice as Fast

15 min

Activity Narrative

In this activity, students use the tick-mark diagram and graph representations from the previous activity and add a third bug that is moving twice as fast as the ladybug. Students also write equations for all three bugs. An important aspect of this activity is students making connections between these different representations.

Monitor for students who use these different strategies to write their equations:

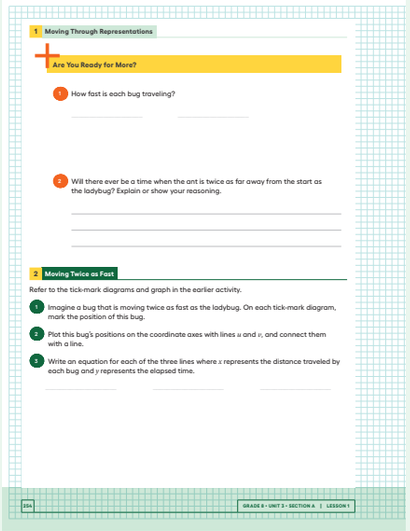
- Reason from the unit rates they can see on their graphs and write equations in the form of $y = kx$, where k is the constant of proportionality
- Use similar triangles to write equations in the form of $\frac{y}{x} = \frac{b}{a}$, where (a, b) is a point on the line

Launch

Arrange students in groups of 2. Give 5–7 minutes work time followed by a whole-class discussion.

Select work from students with different strategies, such as those described in the activity narrative, to share later.

Student Workbook


1 Moving Through Representations

Are You Ready for More?

1 How fast is each bug traveling?

2 Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.

2 Moving Twice as Fast

Refer to the tick-mark diagrams and graph in the earlier activity.

1 Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.

2 Plot this bug's positions on the coordinate axes with lines a and b, and connect them with a line.

3 Write an equation for each of the three lines where x represents the distance traveled by each bug and y represents the elapsed time.

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect. This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

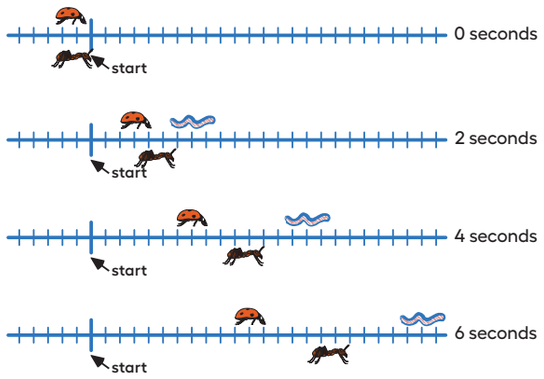
Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts.
For example, show only one question at a time, pausing to check for understanding before moving on.
Supports accessibility for: Organization, Attention

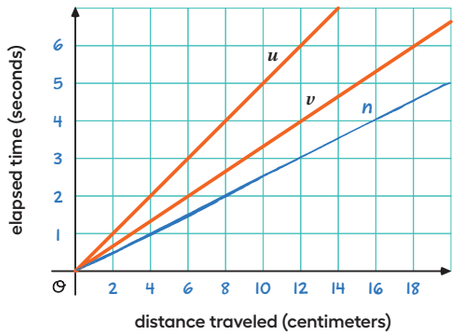
Student Task Statement

Refer to the tick-mark diagrams and graph in the earlier activity.

1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.



2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line.



3. Write an equation for each of the three lines where x represents the distance traveled by each bug and y represents the elapsed time.

Sample responses:

- Ladybug: (line u) $y = \frac{1}{2}x$ or $\frac{y}{x} = \frac{1}{2}$ (or equivalent)
- Ant: (line v) $y = \frac{1}{3}x$ or $\frac{y}{x} = \frac{1}{3}$ (or equivalent)
- New bug: (line n) $y = \frac{1}{4}x$ or $\frac{y}{x} = \frac{1}{4}$ (or equivalent)

Activity Synthesis

The goal of this discussion is to connect the work of using similar triangles to write equations of a line with the work using unit rate to write equations of a line. Display both images from the previous task. Invite previously selected students to share their equations for each bug and record these for all to see.

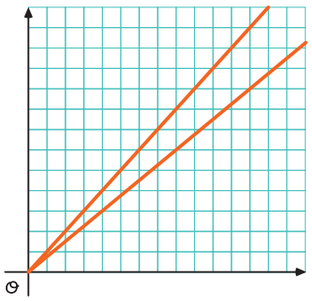
Use *Compare and Connect* to help students compare, contrast, and connect the different approaches and representations. Here are some questions for discussion:

- *"Did anyone write the same equations, but would explain it differently?"*
- *"How does the slope of the line show up in each equation?"*
- *"How do these different representations show the same information?"*

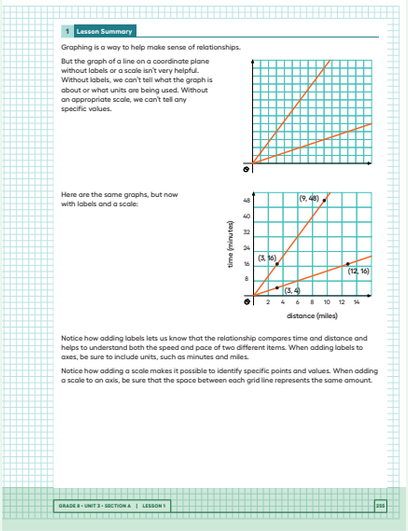
As students share their approaches for writing equations, highlight approaches where students used multiple representations to make sense of their equations. For example, ask students to identify features of the tick-mark diagrams, lines, and equations that show the same information. If time allows, demonstrate how the position of the ladybug in a tick-mark diagram can also be seen in the graph of line u, and how using the distance and elapsed time values in the corresponding equation will make it true.

Lesson Synthesis

The goal of this discussion is for students to see how labels and an appropriate scale on a graph are necessary and can help to make sense of a relationship. For example, display this image for all to see and explain that on longer bike rides, Kiran can ride 4 miles every 16 minutes, and Mai can ride 4 miles every 12 minutes. But without labels or a scale, one can't tell which line represents Kiran and which represents Mai.



Student Workbook


1 Lesson Summary

Graphing is a way to help make sense of relationships.

But the graph of a line on a coordinate plane without labels or a scale isn't very helpful. Without labels, we can't tell what the graph is about or what units are being used. Without an appropriate scale, we can't tell any specific values.

Here are the same graphs, but now with labels and a scale:

time (minutes)
distance (miles)
(9, 48)
(3, 16)
(12, 16)
(3, 4)

Notice how adding labels lets us know that the relationship compares time and distance and helps to understand both the speed and pace of two different items. When adding labels to axes, be sure to include units, such as minutes and miles.

Notice how adding a scale makes it possible to identify specific points and values. When adding a scale to an axis, be sure that the space between each grid line represents the same amount.

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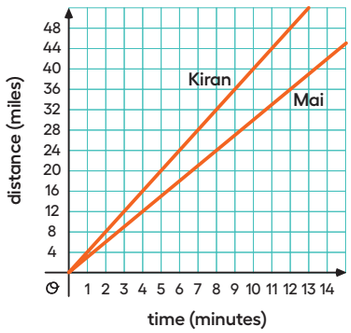
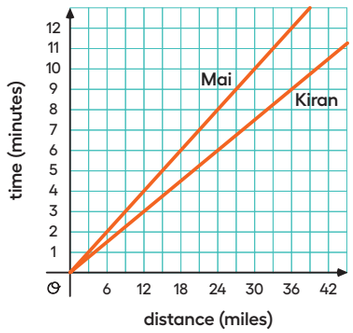
Ask students how to label the axes and add a scale, recording their work for all to see. Then ask students for at least 2 points on each line that will help determine which line is Kiran and which is Mai, and add them to the graphs. Depending on which axis students choose for time and distance, here are two possible labeled and scaled graphs.





If time allows, have students use the completed graph to answer questions such as:

- *"Who rides faster?"*

  Mai

- *"If Kiran and Mai start a bike trip at the same time, how far apart are they after 24 minutes?"*

  2 miles apart

- *"How long will it take each of them to reach the end of a 12 mile bike path?"*

  It will take Kiran 48 minutes and it will take Mai 36 minutes.

Lesson Summary

Graphing is a way to help make sense of relationships.

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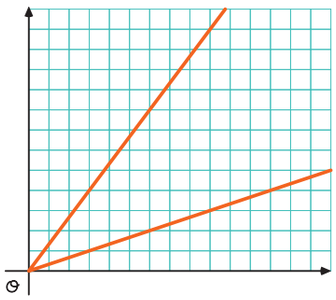
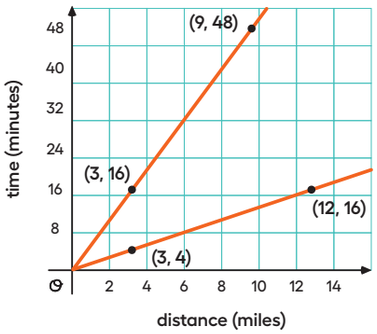




Notice how adding labels lets us know that the relationship compares time and distance and helps to understand both the speed and pace of two different items. When adding labels to axes, be sure to include units, such as minutes and miles.

Notice how adding a scale makes it possible to identify specific points and values. When adding a scale to an axis, be sure that the space between each grid line represents the same amount.

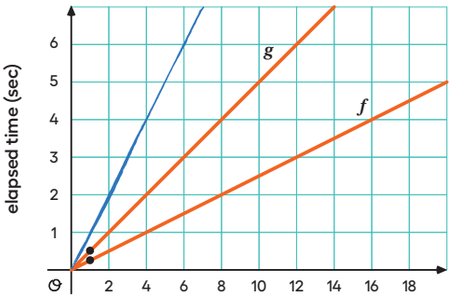
Cool-down

Turtle Race

5 min

Student Task Statement

This graph represents the positions of two turtles in a race.



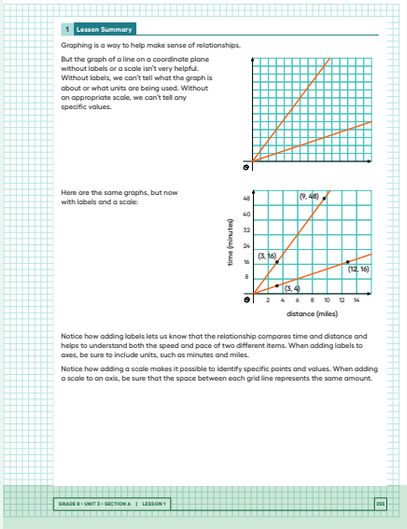
1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line g.

   A line through (0, 0), (1, 1), (2, 2), etc.

2. Explain how your line shows that the turtle is going half as fast.

   Sample reasoning:
   After 2 seconds, the turtle described by line g moved 4 cm, while the third turtle moved only 2 cm. This third turtle covers half the distance in the same amount of time.

Student Workbook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Here are the same graphs, but now with labels and a scale:

(9, 48) (3, 16) (12, 16) (3, 4)

time (minutes)

distance (miles)

Notice how adding labels lets us know that the relationship compares time and distance and helps to understand both the speed and pace of two different items. When adding labels to axes, be sure to include units, such as minutes and miles.

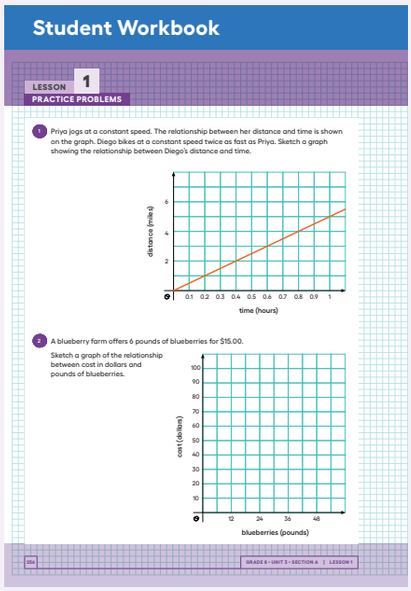
Notice how adding a scale makes it possible to identify specific points and values. When adding a scale to an axis, be sure that the space between each grid line represents the same amount.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

4 Problems


Student Workbook

LESSON 1
PRACTICE PROBLEMS

1. Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.

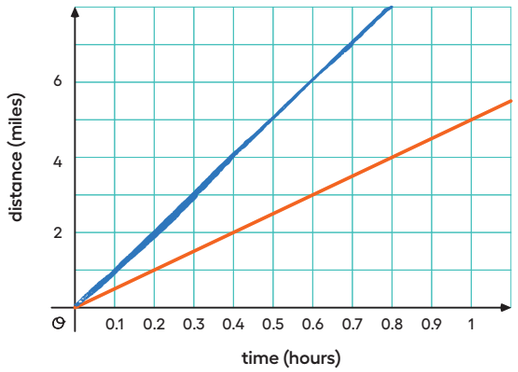
distance (miles)
time (hours)

2. A blueberry farm offers 6 pounds of blueberries for $15.00.
Sketch a graph of the relationship between cost in dollars and pounds of blueberries.

cost (dollars)
blueberries (pounds)

Problem 1

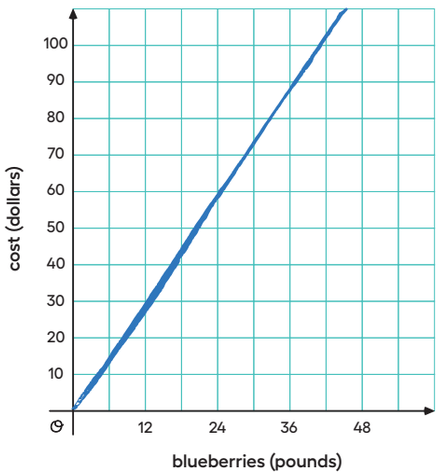
Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.



Problem 2

A blueberry farm offers 6 pounds of blueberries for $15.00.

Sketch a graph of the relationship between cost in dollars and pounds of blueberries.

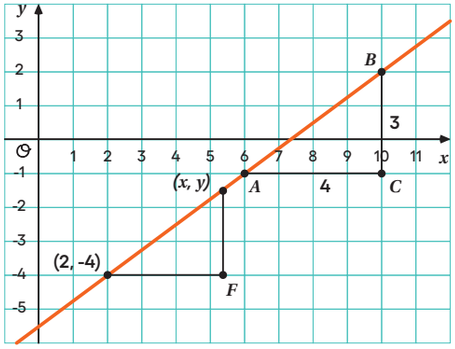


Problem 3

from Unit 2, Lesson 11

The points (2, -4), (x, y), A, and B all lie on the line. Write an equation that describes the line.

$\frac{y+4}{x-2} = \frac{3}{4}$ (or equivalent)



Problem 4

from Unit 2, Lesson 12

The graph shows a line.

Select **all** points that are on this line.

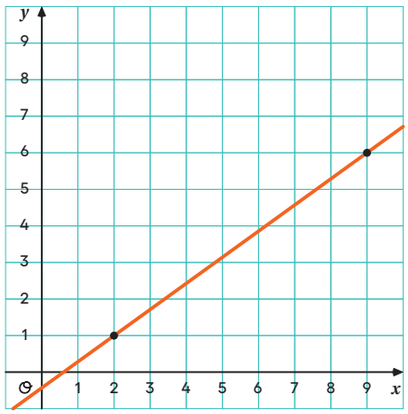
A. (0, 3)

B. (4, 2)

C. (30, 21)

D. (16, 11)

E. (-4, -5)



Student Workbook