Lesson 14

Finding Cylinder Dimensions

Goals

- Calculate the value of one dimension of a cylinder, and explain (orally and in writing) the reasoning.
- Create a table of dimensions of cylinders, and describe (orally) patterns that arise.

Learning Target

I can find missing information about a cylinder if I know its volume and some other information.

Lesson Narrative

In this lesson, students use the formula $V = \pi r^2 h$ for the volume of a cylinder to solve a variety of problems. They compute volumes given radius and height and find radius or height given a cylinder's volume and the other dimension by reasoning about the structure of the volume formula.

Student Learning Goal

Let's figure out the dimensions of cylinders.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
A Cylinder of Unknown Height

5 min

Activity Narrative

The purpose of this *Warm-up* is to get students thinking about the structure of the volume formula for cylinders as preparation for the work in the rest of the lesson. Previously, students were given enough information to determine the radius and the height of a cylinder before calculating its volume. Here, students are given information to find the area of the cylinder's base, but they are not given the height. An important takeaway is that any positive value for the volume is possible given the right height.

Launch

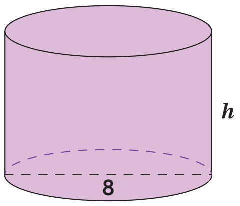
Arrange students in groups of 2. Remind students of the display of the volume formula for a cylinder created in a previous lesson.

Give students 1–2 minutes of quiet work time followed by time to explain their reasoning to their partner.

Follow this with a whole-class discussion.

Student Task Statement

What is a possible volume for this cylinder if the diameter is 8 cm? Explain your reasoning.



Sample response: The radius of the cylinder's base is 4 cm, which means the area of the base is 16π cm^2 since $4^2 \cdot \pi = 16\pi$. If the height is 1 cm, then the volume would be 16π cm^3 since $16\pi \cdot 1 = 16\pi$.

Activity Synthesis

The goal of this discussion is for students to see how the height of a cylinder is related to its volume.

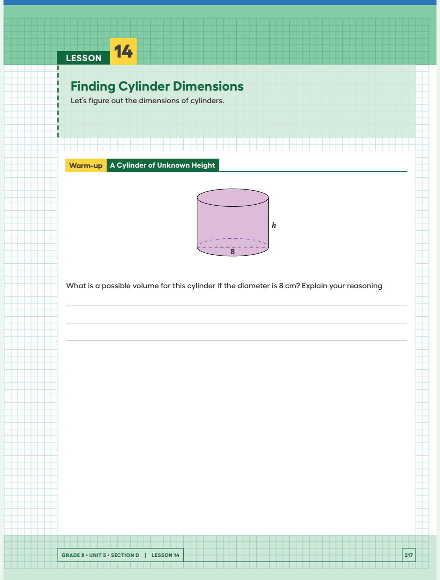
Invite 2–5 groups in which partners had very different values for the volume of the cylinder to share. Record and display the dimensions and volumes of cylinders that correspond to solutions given by students to show the range of possible volumes. For example, if one student picked $h = 0.5$, while the other picked $h = 100$, the volumes of the two resulting cylinders are quite different even though they each have the same area for their bases.

Building on Student Thinking

If students are not sure how to get started, consider asking:

"Tell me more about what you know about this cylinder."
"How could the formula for the volume of a cylinder help you?"

Student Workbook



Activity 1

What's the Dimension?

15 min

Activity Narrative

In this activity, students find the missing dimensions of cylinders when given the volume and the other dimension. A volume equation representing the cylinder is given for each problem to help students focus on solving for the unknown value instead of setting up the equations themselves, which will come later.

In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Give students 2–3 minutes of quiet work time to get started on the first problem.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to

"What does the height of this cylinder have to be?"

In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

"____ makes sense, but what do you mean when you say …?"

"Can you describe that another way?"

"How do you know h is …?"

Close the partner conversations, and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.
If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time*, tell students to complete the remaining problem. Select students who use different strategies to share during the discussion.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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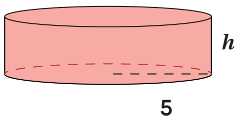
Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Student Task Statement

The volume V of a cylinder with radius r is given by the formula $V = \pi r^2 h$.

1. The volume of this cylinder with radius 5 units is 50π cubic units.
   This statement is true: $50\pi = 5^2 \pi h$.

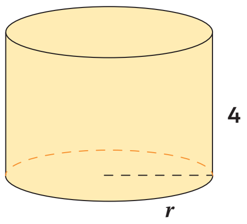


   What does the height of this cylinder have to be? Explain how you know.

   2 units

   Sample reasoning: The statement $50\pi = 5^2 \pi h$ is equivalent to $50 = 25 \cdot h$. Since 50 is 25 times 2, $h = 2$.

2. The volume of this cylinder with height 4 units is 36π cubic units.
   This statement is true: $36\pi = r^2 \pi 4$.



   What does the radius of this cylinder have to be? Explain how you know.

   3 units

   Sample reasoning: The statement $36\pi = r^2 \pi 4$ is equivalent to $36 = r^2 \cdot 4$. Since 36 is 4 times 9, $r^2 = 9$. This implies that $r = 3$.

Are You Ready for More?

Suppose a cylinder has a volume of 36π cubic inches, but it is not the same cylinder as the one you found earlier in this activity.

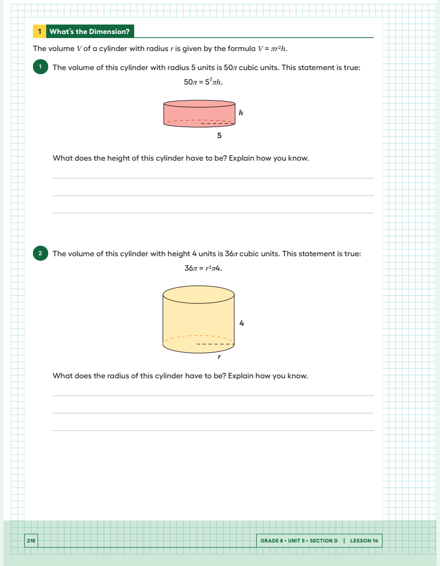
1. What are some possibilities for the dimensions of the cylinder?

   The volume for the cylinder is $36\pi = \pi \cdot r^2 \cdot h$, which implies $36 = h \cdot r^2$.
   Sample response: The cylinder could have $r = 3$ and $h = 4$ or $r = 9$ and $h = \frac{4}{9}$.

2. How many different cylinders can you find that have a volume of 36π cubic inches?

   There are an infinite number of cylinders with a volume of 36π cubic inches. No matter what value for r is chosen, a value for h can be calculated using the formula $36\pi = h \cdot r^2 \cdot \pi$.

Student Workbook


1 What's the Dimension?

The volume V of a cylinder with radius r is given by the formula $V = \pi r^2 h$.

1. The volume of this cylinder with radius 5 units is 50π cubic units. This statement is true: $50\pi = 5^2 \pi h$.

What does the height of this cylinder have to be? Explain how you know.

2. The volume of this cylinder with height 4 units is 36π cubic units. This statement is true: $36\pi = r^2 \pi 4$.

What does the radius of this cylinder have to be? Explain how you know.

Activity Synthesis

The purpose of this discussion is to compare the different strategies used to calculate the unknown values. Invite previously selected students to share their strategies. If one of these strategies is not brought up, share it with the class:

- Guess and check: Plug in numbers for h to find a value that makes the statements true. Since the solutions for these problems are small whole numbers, this strategy works well. In other situations, this strategy may be less efficient.
- Divide each side of the equation by the same value to solve for the missing variable: For example, divide each side of $36\pi = r^2 \pi 4$ by the common factor, 4π. It's important to remember that π is a number that can be multiplied and divided like any other factor.
- Use the structure of the equation to reason about the missing variable: For example, 50π is double 25π, so the missing value must be 2. For the second cylinder, $36 = 9 \cdot 4$, so the radius must be 3 since $9^2 = 3$.

Conclude the discussion by inviting students to share which strategy they liked best. It is important to note that while all three of these strategies work for the cylinders here, the numbers will not always lead to guess and check being efficient, especially if the value of the volume is approximated instead of written in terms of π.

Activity 2

Cylinders with Unknown Dimensions

15 min

Activity Narrative

The purpose of this activity is for students to use the structure of the volume formula for cylinders to find missing dimensions of a cylinder given other dimensions. The students are given the image of a generic cylinder with marked dimensions for the radius, diameter, and height to help their reasoning about the different rows in the table.

While completing the table, students work with exact values of π as well as statements that require reasoning about squared values. The final row of the table asks students to find missing dimensions given an expression representing volume that uses letters to represent the height and the radius. This requires students to manipulate expressions consisting only of variables representing dimensions.

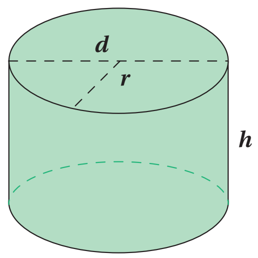
Encourage students to make use of work done in some rows to help find missing information in other rows. By paying attention to what rows have values in common, students can use the structure of the table and their knowledge of the volume formula to calculate related values more efficiently.

Launch

Give students 6–8 minutes of work time, and follow with a whole-class discussion.

If short on time, consider assigning students only some of the rows to complete.

Student Task Statement



Each row of the table has information about a particular cylinder. Complete the table with the missing dimensions.

	diameter (units)	radius (units)	area of the base (square units)	height (units)	volume (cubic units)
a	6	3	9π	4	36π
b	12	6	36π	3	108π
c	6	3	9π	8	72π
d	10	5	25π	1	25π
e	0.8	0.4	0.16π	100	16π
f	20	10	100π	0.2 (or $\frac{1}{5}$)	20π
g	20	10	100π	1	100π
h	$2j$	j	$j^2\pi$	k	$\pi \cdot k \cdot j^2$

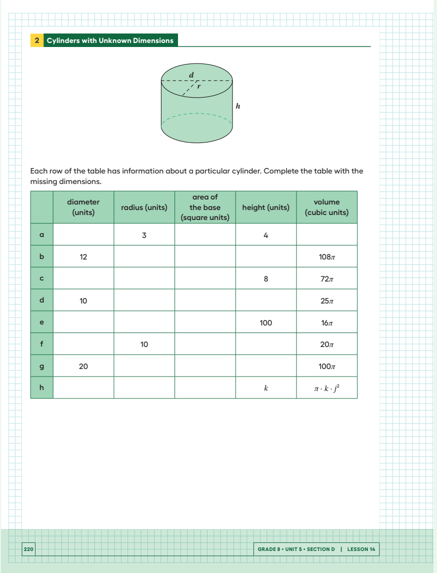
Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Internalize Self Regulation.
Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to 5 out of 8 rows in the table. Encourage students to select rows that contain varied information.
Supports accessibility for: Organization, Attention

Building on Student Thinking

Students might try to quickly fill in the missing dimensions without the proper calculations. Encourage students to use the volume of a cylinder equation and the given dimensions to figure out the unknown dimensions.

Student Workbook


2 Cylinders with Unknown Dimensions

Each row of the table has information about a particular cylinder. Complete the table with the missing dimensions.

	diameter (units)	radius (units)	area of the base (square units)	height (units)	volume (cubic units)
a		3		4	
b	12				108π
c				8	72π
d	10				25π
e				100	16π
f		10			20π
g	20				100π
h				k	$\pi \cdot k \cdot j^2$

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students as they share their strategies for completing the table. Examples: “I noticed ______ in the (rows/columns)” or “I noticed ______, and it tells me that ______ .”
Advances: Speaking, Conversing

Activity Synthesis

The purpose of this discussion is to make visible the different strategies students used to calculate the values in the table and to highlight some key relationships between radius, height, and volume.

Display the table from the *Task Statement* for all to see. Invite students to share their answers for a row along with any patterns they noticed while filling out the table, such as seeing that since rows f and g have the same radius, the area of the base must also be the same.

If not brought up by students, highlight the following rows:

- Rows a and c have the same radius, which means they have the same base area. The height in row c is double the height in row a, and the volume in row c is double the volume in row a. The height and volume increase proportionally.
- Rows d and g have the same height, but the radius for row g is double the radius for row d, and the volume for row g is quadruple the volume for row d. The radius and volume do not increase proportionally.

Give students 30 seconds of quiet think time, then invite 1–2 students to share why they think this is happening. (The volume formula for a cylinder is $V = \pi r^2 h$. Doubling the value of h doubles the volume V, but doubling the value of r quadruples the volume because the radius is squared and $(2r)^2 = 4r$.

Students will have more opportunities to consider this aspect of volume formulas in future lessons.

Lesson Synthesis

Conclude the lesson by giving students a chance to put some of the skills they learned today to practice. Arrange students in groups of 2. Tell students to choose one partner to name a value for the radius and one partner to name a value for the volume of a cylinder. Together, partners make a sketch of their cylinder, including labels for the dimensions of their sketch, and determine the height. Invite as many partners as time allows to share their sketches and their strategies for determining height.

Lesson Summary

In an earlier lesson we learned that the volume V of a cylinder with radius r and height h is

$$V = \pi r^2 h.$$

We say that the volume depends on the radius and height, and if we know the radius and height, we can find the volume. It is also true that if we know the volume and one dimension (either radius or height), we can find the other dimension.

For example, imagine a cylinder that has a volume of 500π cm^3 and a radius of 5 cm, but the height is unknown. From the volume formula we know that

$$500\pi = \pi \cdot 25 \cdot h$$

must be true. Looking at the structure of the equation, we can see that $500 = 25h$. That means that the height has to be 20 cm, since $500 \div 25 = 20$.

Now imagine another cylinder that also has a volume of 500π cm^3 with an unknown radius and a height of 5 cm. Then we know that

$$500\pi = \pi \cdot r^2 \cdot 5$$

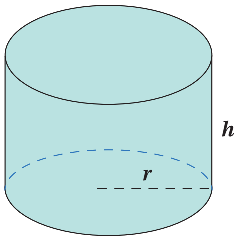
must be true. Looking at the structure of this equation, we can see that $r^2 = 100$. So the radius must be 10 cm.

Cool-down

Find the Height

5 min

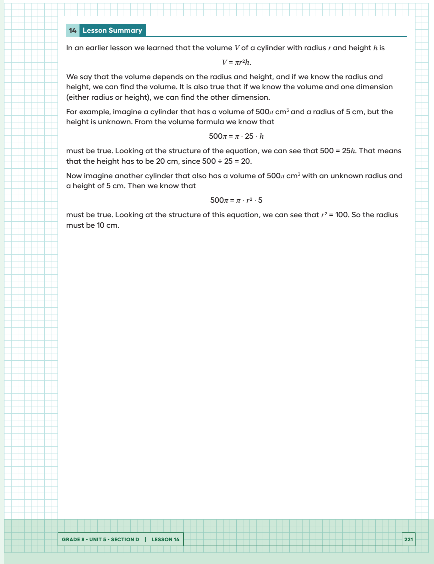
Student Task Statement



This cylinder has a volume of 12π cubic inches and a diameter of 4 inches. Find the cylinder's radius and height.

The radius is 2 inches, and the height is 3 inches. Since the diameter is 4 inches, the radius is half of 4 inches. The volume is $12\pi = 2^2\pi h$, which means $12\pi = 4\pi h$ and $h = 3$.

Student Workbook



Responding To Student Thinking

Points to Emphasize

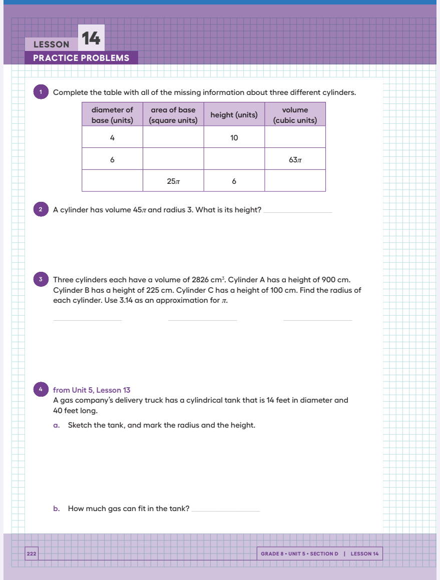
If most students struggle using the formula to solve for an unknown value, focus on this skill in this activity:

Grade 8, Unit 5, Lesson 16, Activity 2
Cones with Unknown Dimensions

Practice Problems

5 Problems

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 Complete the table with all of the missing information about three different cylinders.

diameter of base (units)	area of base (square units)	height (units)	volume (cubic units)
4		10	
6			63π
	25π	6	

2 A cylinder has volume 45π and radius 3. What is its height?

3 Three cylinders each have a volume of 2826 cm^3. Cylinder A has a height of 900 cm. Cylinder B has a height of 225 cm. Cylinder C has a height of 100 cm. Find the radius of each cylinder. Use 3.14 as an approximation for π.

4 from Unit 5, Lesson 13
A gas company's delivery truck has a cylindrical tank that is 14 feet in diameter and 40 feet long.

a. Sketch the tank, and mark the radius and the height.

b. How much gas can fit in the tank?

Problem 1

Complete the table with all of the missing information about three different cylinders.

diameter of base (units)	area of base (square units)	height (units)	volume (cubic units)
4	4π	10	40π
6	9π	7	63π
10	25π	6	150π

Problem 2

A cylinder has volume 45π and radius 3. What is its height?

5 units

solve $45\pi = \pi \cdot 3^2 \cdot h$

Problem 3

Three cylinders each have a volume of 2826 cm^3. Cylinder A has a height of 900 cm. Cylinder B has a height of 225 cm. Cylinder C has a height of 100 cm. Find the radius of each cylinder. Use 3.14 as an approximation for π.

- Cylinder A has a radius of 1 cm.
- Cylinder B has a radius of 2 cm.
- Cylinder C has a radius of 3 cm.

Problem 4

from Unit 5, Lesson 13

A gas company's delivery truck has a cylindrical tank that is 14 feet in diameter and 40 feet long.

a. Sketch the tank, and mark the radius and the height.

A sketch is drawn with the radius labeled as 7 feet and the height labeled as 40 feet.

b. How much gas can fit in the tank?

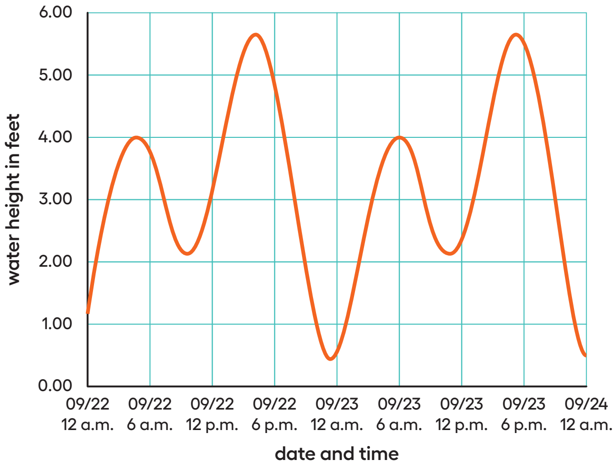
About 6,158 cubic feet

The volume of the cylinder is given by $V = \pi \cdot r^2 \cdot h$, where $r = 7$ and $h = 40$. Using a close approximation of π gives an approximate volume of 6,158 cubic feet, but different answers may be found if a different approximation of π is used.

Problem 5

from Unit 5, Lesson 5

Here is a graph that shows the approximate water height of the ocean between September 22 and September 24, 2016 in Bodega Bay, CA.



a. Estimate the water height at 12 p.m. on September 22.

3 feet

b. How many times was the water height 5 feet? Find two times when this happens.

4 times: at approximately 2 p.m. and 6 p.m. on 9/22 and 3 p.m. and 7 p.m. on 9/23

c. What was the lowest the water got during this time period? When does this occur?

The lowest height was at about 11 p.m. on 9/22. The water height at that time was about 0.5 foot.

d. Does the water ever reach a height of 6 feet?

No, the highest it ever got was about 5.8 feet.

Student Workbook