Lesson 4

Dividing Powers of 10

Goals

- Generalize a process for dividing powers of 10, and justify (orally and in writing) that $\frac{10^n}{10^m} = 10^{n-m}$.
- Use exponent rules to multiply and divide with 10^0, and justify (orally) that 10^0 is 1.

Learning Targets

- I can evaluate 10^0 and explain why it makes sense.
- I can explain and use a rule for dividing powers of 10.

Lesson Narrative

In this lesson, students continue to make use of repeated reasoning to discover the exponent rule $\frac{10^n}{10^m} = 10^{n-m}$.

Students begin by examining different ways to write fractions that are equivalent to 1. Next, they expand expressions written as the division of two powers of 10 and notice patterns when asked to write the quotient using a single power of 10.

Students extend this rule to see what happens when dividing by a value with an exponent of 0. They notice that the original value remains unchanged, helping students to make sense of why $10^0$is defined to be equal to 1. Cases in which the exponent is negative will be explored in following lessons.

Student Learning Goal

Let's explore patterns with exponents when we divide powers of 10.

Instructional Routines

- MLR3: Critique, Correct, Clarify
- Notice and Wonder

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR7: Compare and Connect (Warm-up)

Access for Students with Diverse Abilities

- Representation (Activity 1)

Required Preparation

Activity 2:
Create a visual display (or add to an existing display) of the exponent rule $\frac{10^n}{10^m} = 10^{n-m}$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

Activity 3:
Create a visual display (or add to a previous display) for the rule $10^0 = 1$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	10 min	5 min

Warm-up
A Surprising One

5 min

Activity Narrative

In this activity, students investigate fractions that are equal to 1. This concept helps students make sense of the exponent division rule explored in a following activity. It is expected that students will try to compute the numerator and denominator of the fraction. Monitor for students who instead make use of structure to find factors in the numerator and denominator that can be used to show multiplication by 1.

Launch

Give students 5 minutes of quiet work time. Expect students to attempt to work out all of the multiplication without using exponent rules. Follow with a brief whole-class discussion.

Student Task Statement

What is the value of the expression?

The expression is equal to 1.

Be prepared to explain your reasoning.

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

Sample reasoning:

- The numerator and denominator both compute to 23,328 and $\frac{23,328}{23,328} = 1$.
- The numerator and denominator both have 5 factors that are 2 and 6 factors that are 3. Since the numerator and denominator have the same value, the entire fraction is equal to 1.

Activity Synthesis

The goal of this discussion is for students to see that a fraction is often easier to analyze when dividing matching factors from the numerator and denominator to show multiplication by 1. Invite students to share their answer and reasoning. If not brought up in students' explanations, provide the following example and ask students how it could be used in this situation:

$$\frac{2 \cdot 3 \cdot 7 \cdot 11}{5 \cdot 3 \cdot 7 \cdot 11} = \frac{3 \cdot 7 \cdot 11}{3 \cdot 7 \cdot 11} \cdot \frac{2}{5} = 1 \cdot \frac{2}{5} = \frac{2}{5}.$$

If time allows, ask students "What has to be true about a fraction for it to equal 1?" (The numerator and denominator must be the same value and something other than 0.) Then, invite students to create their own fraction that is equivalent to 1 and has several bases and several exponents.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR7: Compare and Connect. After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask,

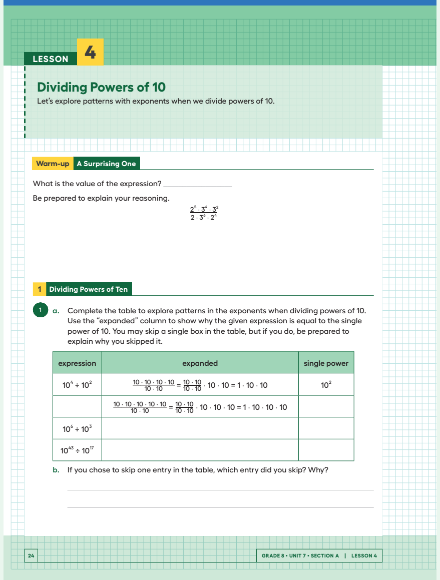
> *"What did the different approaches have in common? How were they different?"*

and

> *"Why did the different approaches lead to the same outcome?"*

Advances: Representing, Conversing

Student Workbook


LESSON 4

Dividing Powers of 10

Let's explore patterns with exponents when we divide powers of 10.

Warm-up A Surprising One

What is the value of the expression? ____________

Be prepared to explain your reasoning.

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

1 Dividing Powers of Ten

1 a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the "expanded" column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

24 GRADE 8 • UNIT 7 • SECTION A | LESSON 4

Instructional Routines

Notice and Wonder

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Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem.
For example, color code the connections between the expression, the "expanded" column, and the single power to support students' use of structure.

Supports accessibility for: Visual-Spatial Processing

Activity 1

Dividing Powers of Ten

15 min

Activity Narrative

In this activity, students explore patterns to discover the rule that $\frac{10^n}{10^m} = 10^{n-m}$. Students will work only with cases where $n > m$ and will extend the rule to include cases where $n = m$ or $n < m$ in following activities and lessons.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Display the table from the *Task Statement* for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses for all to see without editing or commentary.

If not mentioned by students, make sure to emphasize the following ideas:

- An expression using the division symbol can be written as a fraction.
- The first expression in the "expanded" column shows each power of 10 expanded into factors.
- In the "expanded" column, a certain number of factors in the numerator and denominator are grouped together because their quotient is 1.

Tell students to open their student workbookss or devices and explain that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 5–6 minutes to complete the remaining questions before a whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the "expanded" column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
$10^5 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	10^3
$10^6 \div 10^3$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	10^3
$10^{43} \div 10^{17}$	skip	10^{26}

b. If you chose to skip one entry in the table, which entry did you skip? Why?

I chose to skip the expanded column of $10^{43} \div 10^{17}$ because there is not enough space in the table for all of the factors.

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression with a single exponent, like $10^{\square}$.

   $\frac{10^n}{10^m} = 10^{n-m}$ because m factors in the numerator and denominator are divided to make 1, leaving $n - m$ factors remaining.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

   There are roughly 100 trees per person because 10^{12} trees divided equally among 10^{10} people is $10^{12-10} = 10^2$ trees per person.

Are You Ready for More?

expression	expanded	single power
$10^4 \div 10^6$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 1 \cdot \frac{1}{10} \cdot \frac{1}{10}$	$\frac{1}{10^2}$

Activity Synthesis

The goal of this discussion is to reinforce the exponent rule for dividing powers of 10. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$\frac{10^n}{10^m} = 10^{n-m}$	$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10^3 = 10^3$ five factors that are ten $\div$ two factors that are ten $=$ three factors that are ten

Continue to reinforce student understanding of this rule by writing out an expanded form of each expression when discussing the following questions:

- *"What is $10^6 \div 10^4$ written as a single power of 10?"*

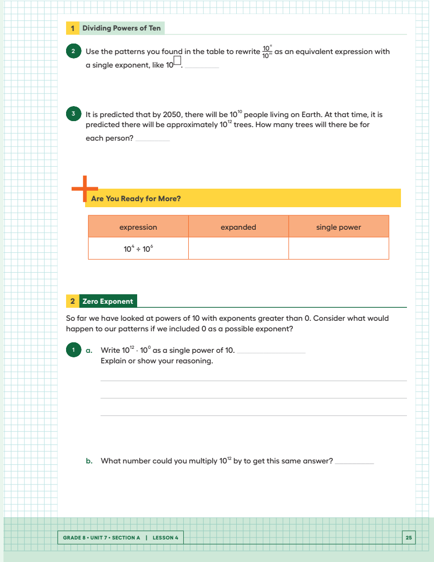
  $10^{20} \cdot 10^{17} = 10^{20+17} = 10^{37}$

- *"When writing $10^{80} \div 10^{20}$ as a single power of 10, how many factors of 10 from the numerator and denominator can be re-written as a fraction equivalent to 1?"*

  20 factors of 10 from the numerator and 20 factors of 10 from the denominator can be written as a fraction equivalent to 1.

- *"What is $10^{80} \div 10^{20}$ as a single power of 10?"*

  $\frac{10^{80}}{10^{20}} = 10^{(80-20)} = 10^{60}$


Student Workbook

1 Dividing Powers of Ten

2 Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression with a single exponent, like $10^{\square}$. ____

3 It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person? ____

Are You Ready for More?

expression	expanded	single power
$10^4 \div 10^6$		

2 Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. Consider what would happen to our patterns if we included 0 as a possible exponent?

1 a. Write $10^{12} \cdot 10^0$ as a single power of 10. ____ Explain or show your reasoning.

b. What number could you multiply 10^{12} by to get this same answer? ____

GRADE 8 • UNIT 7 • SECTION A | LESSON 4 25

Instructional Routines

MLR3: Critique, Correct, Clarify

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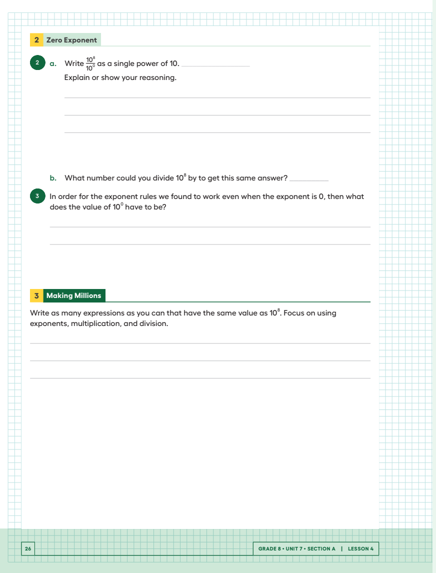
Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Student Workbook


2 Zero Exponent

a. Write $\frac{10^8}{10^0}$ as a single power of 10.
Explain or show your reasoning.

b. What number could you divide 10^8 by to get this same answer?

In order for the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?

3 Making Millions

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents, multiplication, and division.

GRADE 8 • UNIT 7 • SECTION A | LESSON 4

Activity 2
Zero Exponent

15 min

Activity Narrative

In this activity, students extend exponent rules to discover why it makes sense to define 10^0 as 1. Students also critique a statement that is intentionally incorrect and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by a partner discussion and then a whole-class discussion.

Student Task Statement

So far we have looked at powers of 10 with exponents greater than 0. Consider what would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \cdot 10^0$ as a single power of 10. 10^{12}
   Explain or show your reasoning.
   Sample reasoning: $10^{12} \cdot 10^0 = 10^{12+0} = 10^{12}$
   b. What number could you multiply 10^{12} by to get this same answer?
   1 because $10^{12} \cdot 1 = 10^{12}$.
2. a. Write $\frac{10^8}{10^0}$ as a single power of 10. 10^8
   Explain or show your reasoning.
   Sample reasoning: $\frac{10^8}{10^0} = 10^{8-0} = 10^8$
   b. What number could you divide 10^8 by to get this same answer?
   1 because $\frac{10^8}{1} = 10^8$.
3. In order for the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?
   The value of 10^0 has to be 1 in order for the exponent rules to work when the exponent is 0.

Activity Synthesis

The purpose of this discussion is to solidify the concept that $10^0 = 1$. Invite students to share their answers to the questions, and then ask them to share their thinking about what 10^0 means.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response by correcting errors, clarifying meaning, and adding details.

Display this first draft: "10^0 means zero factors of 10 and so 10^0 must equal 0."

Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement. Give students 2–4 minutes to work with a partner to revise the first draft.

Select 1–2 individuals to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Next, introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$10^0 = 1$	$\frac{10^6}{10^0} = 10^{6-0} = 10^6$ this value must be equal to 1

Activity 3: Optional
Making Millions

10 min

Activity Narrative

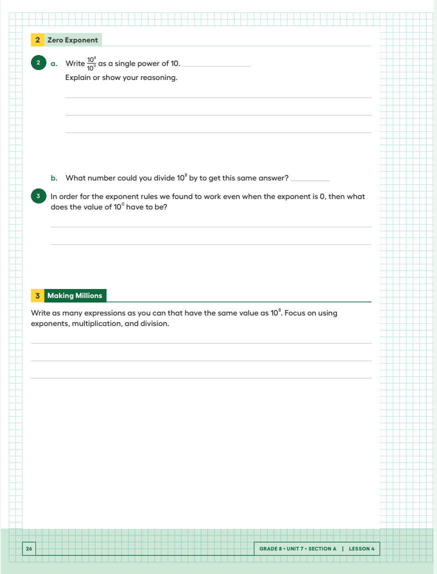
This activity expands on a previous *Cool-down* as students generate different representations of the same number to solidify what they have learned about exponents.

Students are not expected to make an exponent of 8 using negative exponents, but do not discourage it if they do. Explain to these students that, while the rules still work when using negative exponents, it is not yet clear what the value of 10^{-1} is, and this will be explored in following lessons.

Launch

Arrange students in groups of 2. Give them 5 minutes of quiet work time before asking students to share their responses with their partner. Follow with a whole-class discussion.

Student Workbook


2 Zero Exponent

2 a. Write $\frac{10^8}{10^0}$ as a single power of 10. ______

Explain or show your reasoning.

b. What number could you divide 10^8 by to get this same answer? ______

3 In order for the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?

3 Making Millions

Write as many expressions as you can that have the same value as 10^8. Focus on using exponents, multiplication, and division.

26 GRADE 8 • UNIT 7 • SECTION A | LESSON 4

Student Task Statement

Write as many expressions as you can that have the same value as 10^8. Focus on using exponents, multiplication, and division.

Sample responses:

- 100,000,000
- $10^4 \cdot 10^4$
- $(10^2)^4$
- $10^{10} \div 10^2$
- $\frac{10^{15}}{10^7}$
- $10^5 \cdot 10 \cdot 10 \cdot 10$

Activity Synthesis

The goal of this discussion is for students to share different expressions that have the same value as 10^8. Tell students to compare responses with their partner, and invite students to share examples that show creativity or that combine multiple rules together.

To include more students in the discussion, consider asking:

"What is similar or different between yours and your partner's representations?"

"How can you build on ____'s method to come up with another expression?"

Lesson Synthesis

The goal of this discussion is to check that students can explain why the exponents are subtracted when rewriting a quotient of powers of 10 with a single exponent, and why it makes sense to define 10^0 as equal to 1. Consider recording student responses and displaying them for all to see.

Here are some questions for discussion:

"How can you write $\frac{10^{36}}{10^{12}}$ using a single exponent?"

10^{24}

"Why are the values of the exponents subtracted from each other when finding the quotient of 2 powers of 10 instead of dividing them?"

The exponents are subtracted because we are counting the number of factors that are 10 that survive division. In this case, we have $\frac{10^{36}}{10^{12}} = \frac{10^{12} \cdot 10^{24}}{10^{12}} = 10^{24}$.

"Why is 10^0 defined to be equal to 1?"

10^0 is defined to be equal to 1 because that is the value for which the exponent rules we have discovered still work. For example, the rules indicate $10^4 \cdot 10^0$ should be equal to 10^{4+0}, which is just 10^4. So 10^0 is a number that doesn't change the value of other numbers when it is multiplied. The only number with this property is 1, so it only makes sense to define 10^0 as 1.

Lesson Summary

In this lesson, we developed a rule for dividing powers of 10: Dividing powers of 10 is the same as subtracting the exponent of the denominator from the exponent of the numerator. To see this, take 10^5 and divide it by 10^2.

Rule

$$\frac{10^n}{10^m} = 10^{n-m}$$

Example showing how it works

$$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10^3 = 10^3$$

five factors that are ten ÷ two factors that are ten = three factors that are ten

We know that 10^5 has 5 factors that are 10, and 2 of these factors can be divided by the 2 factors of 10 in 10^2 to make 1. That leaves $5 - 2 = 3$ factors of 10, or 10^3.

This will work for other factors of 10, too. For example $\frac{10^{56}}{10^{23}} = 10^{56-23} = 10^{33}$.

This rule also extends to 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0doesn't change its value. That means if we want the rule to work when the exponent is 0, then 10^0 must equal 1.

Rule

$$10^0 = 1$$

Example showing how it works

$$\frac{10^6}{10^0} = 10^{6-0} = 10^6$$

this value must be equal to 1

Cool-down

Why Subtract?

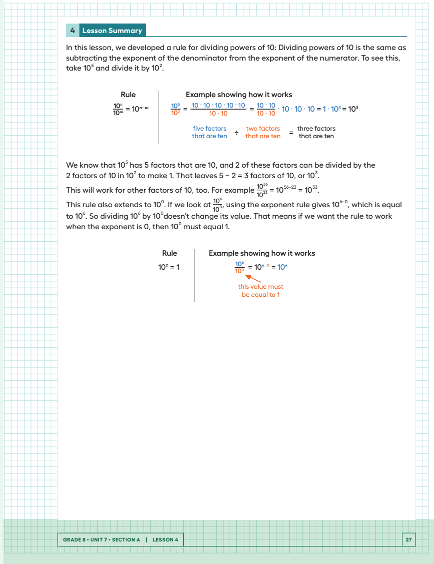
5 min

Student Task Statement

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11}? Explain or show your thinking.

Sample response: $\frac{10^{15}}{10^4} = 10^{11}$ because 4 factors that are 10 in the numerator and denominator are used to make 1, leaving 11 remaining factors that are 10. In other words, $\frac{10^{15}}{10^4} = \frac{10^4 \cdot 10^{11}}{10^4} = 10^{11}$.

Student Workbook

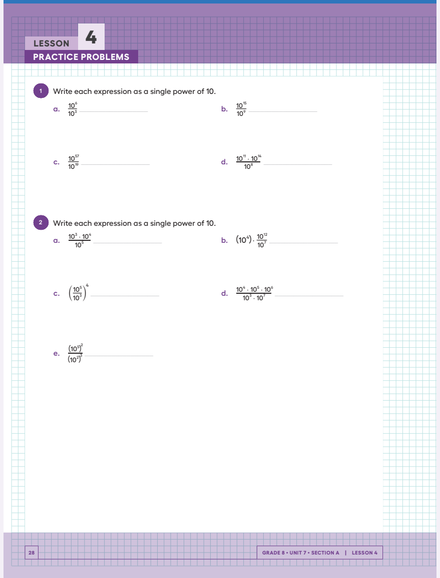


Responding To Student Thinking

Points to Emphasize

If most students struggle with dividing expressions with exponents, revisit the rule for division with exponents. For example, make sure to focus on the division expressions that were and were not equivalent during the discussion in this activity:
Unit 7, Lesson 6, Activity 3 Exponent Rules with Bases Other than 10

Student Workbook



Student Workbook

Lesson **4**

Practice Problems

5 Problems

Problem 1

Write each expression as a single power of 10.

a. $\frac{10^6}{10^3}$ 10^3

b. $\frac{10^{15}}{10^9}$ 10^6

c. $\frac{10^{57}}{10^{19}}$ 10^{38}

d. $\frac{10^{11} \cdot 10^{14}}{10^8}$ 10^{17}

Problem 2

Write each expression as a single power of 10.

a. $\frac{10^3 \cdot 10^4}{10^5}$ 10^2

b. $(10^4) \cdot \frac{10^{12}}{10^7}$ 10^9

c. $\left(\frac{10^5}{10^3}\right)^4$ 10^8

d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$ 10^5

e. $\frac{(10^5)^2}{(10^2)^3}$ 10^4

Problem 3

The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? 10^3

Explain how you know.

Sample reasoning: This can be determined by calculating $\frac{10^5}{10^2}$, since both the Sun and KW Sagittarii's widths can be compared to the width of Earth.

Problem 4

from Unit 5, Lesson 3

Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

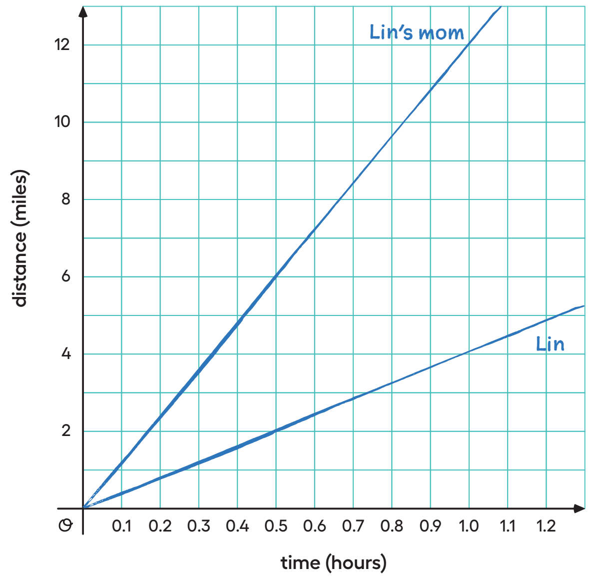
a. Write an equation relating the two variables. $1.5b + 3g = 12$ (or equivalent)

b. How many pounds of bananas can Kiran buy if he buys 2 pounds of guavas? 4 pounds of bananas

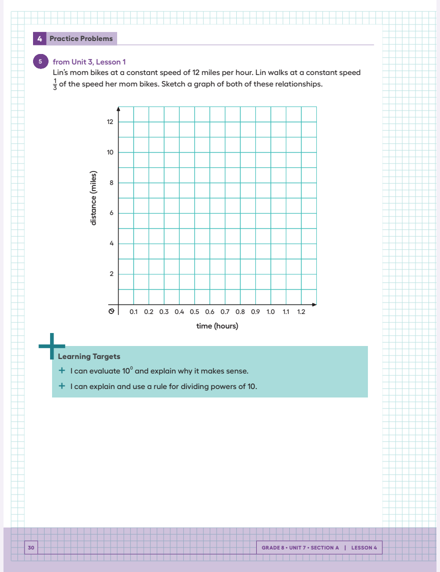
Problem 5

from Unit 3, Lesson 1

Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



Student Workbook


Practice Problems

from Unit 3, Lesson 1

Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.

Learning Targets

+ I can evaluate 10^0 and explain why it makes sense.
+ I can explain and use a rule for dividing powers of 10.