Lesson 15

Equivalent Exponential Expressions

Goals

- Describe (orally) the values that result from evaluating expressions in which a fraction is raised to a power.
- Determine whether a given value is a solution to an equation that includes an exponent.
- Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.

Learning Targets

- I can find solutions to equations with exponents in a list of numbers.
- I can replace a variable with a number in an expression with exponents and use the correct order of operations to find the value of the expression.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- Action and Expression (Activity 1)

Instructional Routines

- MLR2: Collect and Display

Lesson Narrative

In this lesson, students encounter expressions and equations that involve variables and exponents. Students first evaluate expressions for given values of their variables. Then students are presented with equations that contain a variable and a list of possible solutions. To identify which value makes each equation true, students may use substitution, consider the structure of the equations, or apply their understanding of exponents and operations.

Student Learning Goal

Let's investigate expressions with variables and exponents.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	10 min	15 min	10 min	5 min

Warm-up

Up or Down?

10 min

Activity Narrative

In this *Warm-up*, students take two numbers to different powers and look for patterns. The first number is a whole number, 3, and the second is its reciprocal, $\frac{1}{3}$. The goal is for students to notice that when a fraction is raised to a positive exponent, its value decreases as the exponent increases. Aside from the presence of exponents, these observations are largely a review of work from grade 5.

As students complete the table, monitor for those who can describe some of the following patterns:

- The values in the 3^x column increase as the exponent increases.
- The values in the $\left(\frac{1}{3}\right)^x$ column decrease as the exponent increases.
- The values in the $\left(\frac{1}{3}\right)^x$ column are reciprocals of the values in the corresponding row of the 3^x column.

Launch

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for each value of x.

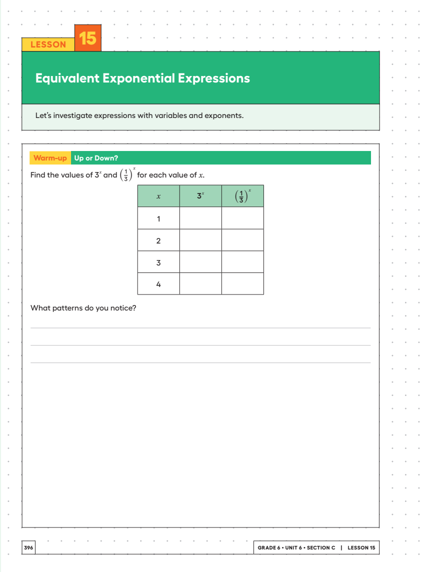
x	3^x	$\left(\frac{1}{3}\right)^x$
1	3	$\frac{1}{3}$
2	9	$\frac{1}{9}$
3	27	$\frac{1}{27}$
4	81	$\frac{1}{81}$

What patterns do you notice?

Sample responses:

- The values in the 3^x column increase as the exponent increases.
- The values in the 3^x column are multiplied by 3 each time you go down a row.
- The values in the $\left(\frac{1}{3}\right)^x$ column decrease as the exponent increases.
- The values in the $\left(\frac{1}{3}\right)^x$ column are multiplied by $\frac{1}{3}$ each time you go down a row.
- The values in the $\left(\frac{1}{3}\right)^x$ column are reciprocals of the values in the corresponding row of the 3 column.

Student Workbook


LESSON 15

Equivalent Exponential Expressions

Let's investigate expressions with variables and exponents.

Warm-up Up or Down?

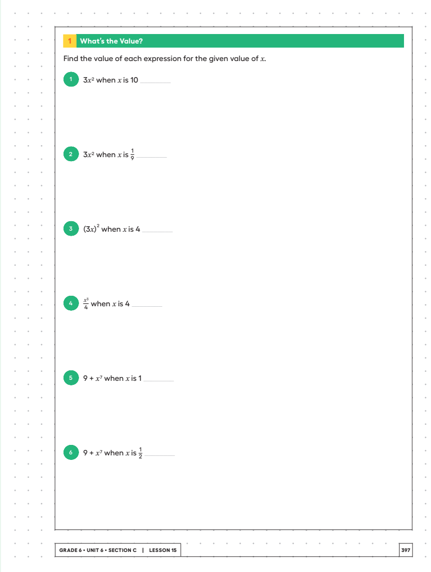
Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for each value of x.

x	3^x	$\left(\frac{1}{3}\right)^x$
1		
2		
3		
4		

What patterns do you notice?

396 GRADE 6 • UNIT 6 • SECTION C | LESSON 15

Student Workbook


1 What's the Value?

Find the value of each expression for the given value of x.

1. $3x^2$ when x is 10 ____
2. $3x^2$ when x is $\frac{1}{9}$ ____
3. $(3x)^2$ when x is 4 ____
4. $\frac{x^3}{4}$ when x is 4 ____
5. $9 + x^7$ when x is 1 ____
6. $9 + x^7$ when x is $\frac{1}{2}$ ____

GRADE 6 • UNIT 6 • SECTION C | LESSON 15 397

Activity Synthesis

Display the table for all to see. Ask students to share their responses and record them in the table. Ask selected students to share the patterns they noticed in the table and ask others to explain why they think these patterns happen. If the ideas described in the *Student Response* do not arise from students during this discussion, bring those ideas to students' attention.

Activity 1

What's the Value?

10 min

Activity Narrative

In this activity, students continue working with exponential expressions with variables. They evaluate expressions when given a value for the variable. As students discuss and compare their thinking with others, they also deepen their understanding of the order of operations, such as when considering the difference between expressions like $3x^2$ and $(3x)^2$. During class discussion, students also have opportunities to practice using mathematical vocabulary related to expressions, such as "coefficient," "variable," and "exponent."

Launch

Reiterate that the expression $6x^2$ means to multiply 6 by the result of x^2. Remind students that the number part of such a product is called the coefficient of the expression, so in this example, 6 is the coefficient of x^2.

Give students 5 minutes of quiet work time to evaluate the expressions. Follow with a whole-class discussion.

Student Task Statement

Find the value of each expression for the given value of x.

1. $3x^2$ when x is 10

   300

2. $3x^2$ when x is $\frac{1}{9}$

   $\frac{1}{27}$

3. $(3x)^2$ when x is 4

   144

4. $\frac{x^3}{4}$ when x is 4

   16

5. $9 + x^7$ when x is 1

   10

6. $9 + x^7$ when x is $\frac{1}{2}$

   $9\frac{1}{128}$

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
Use *Collect and Display* to direct attention to words collected and displayed from earlier lessons. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Access for Multilingual Learners (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
Chunk this task into more manageable parts. After students have evaluated the first 2–3 expressions, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may use the wrong order when evaluating expressions, such as in multiplying the given value of x by 3 first in $3x^2$ and then squaring. Ask students to explain their calculations. Remind them of the conventional order of operations in numeric expressions. Explain that the same order applies to expressions with variables.

Activity Synthesis

The purpose of the discussion is to ensure that students understand how to evaluate exponential expressions with variables for a given value of the variable. It is also an opportunity for students to practice interpreting and using vocabulary like “coefficient,” “variable,” “power,” and “exponent.”

Direct students’ attention to the reference created using *Collect and Display*. Ask students to share the steps they used to find the value of an expression. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional words or phrases.

If more than a few students multiply the coefficient and the variable in $3x^2$ and then square the product, which results in a value of 900 when x is 10 and $\frac{1}{9}$ when x is $\frac{1}{9}$, discuss the difference between $3x^2$ and $(3x)^2$ and how each expression should be evaluated based on the order of operations.

If time permits, consider asking questions such as:

- ***“In each expression, what is the coefficient?”***

  3, 3, 3, $\frac{1}{4}$, 1, 1

- ***“How is evaluating the expressions when x is a fraction similar to when x is a whole number? How is it different?”***

  It’s similar because we’re still just multiplying x by itself a certain number of times. It’s different because multiplying a fraction by a fraction is a bit more complicated than multiplying a whole number by itself.

Activity 2
Exponent Experimentation

15 min

Activity Narrative

In this activity, students recall what is meant by a *solution* to an equation as they look to replace a variable with a number that makes two expressions equal. Here, one or both expressions in each equation contains an exponent.

Students are not expected to use algebraic methods to solve equations such as $64 = x^2$ or $4^3 = 8^x$ in grade 6. Instead, they are to look for and make use of the structure to strategically choose and test values that could be solutions from a list, or by reasoning about the meaning of each expression. For example, to find the solution for $64 = x^2$, they may think: “What number, when squared, will give 64?”

Note that some of the equations also have solutions that are negative. Because operations on negative numbers are not part of grade 6 standards, students are only expected to consider positive values in this activity.

Launch

Ask students to close their books or devices. Display the equation $x^2 = 100$ and the values 2, 10, and 50. Discuss what it would mean for 2, 10, or 50 to be a solution to the equation. As needed, remind students that a *solution* is a value for x that makes the equation true. Discuss why 10 is a solution, and why 2 or 50 are not solutions.

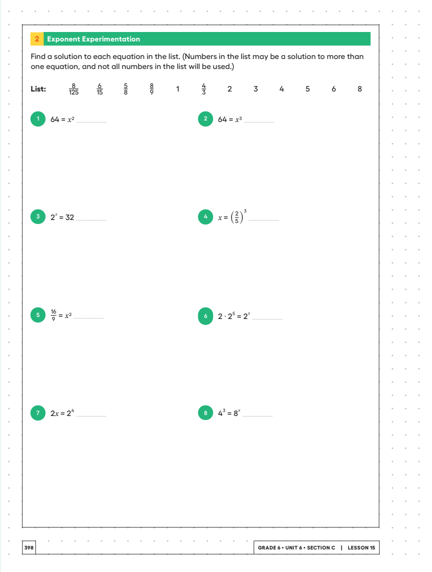
Arrange students in groups of 2. Give students 5 minutes of quiet work time and then time to discuss their responses with a partner, followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Activate or supply background knowledge about exponential expressions and finding a solution to an equation. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


Exponent Experimentation

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

Student Task Statement

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

List: $\frac{8}{125}$ $\frac{6}{15}$ $\frac{5}{8}$ $\frac{8}{9}$ 1 $\frac{4}{3}$ 2 3 4 5 6 8

1. $64 = x^2$

   8

2. $64 = x^3$

   4

3. $2^x = 32$

   5

4. $x = \left(\frac{2}{5}\right)^3$

   $\frac{8}{125}$

5. $\frac{16}{9} = x^2$

   $\frac{4}{3}$

6. $2 \cdot 2^5 = 2^x$

   6

7. $2x = 2^4$

   8

8. $4^3 = 8^x$

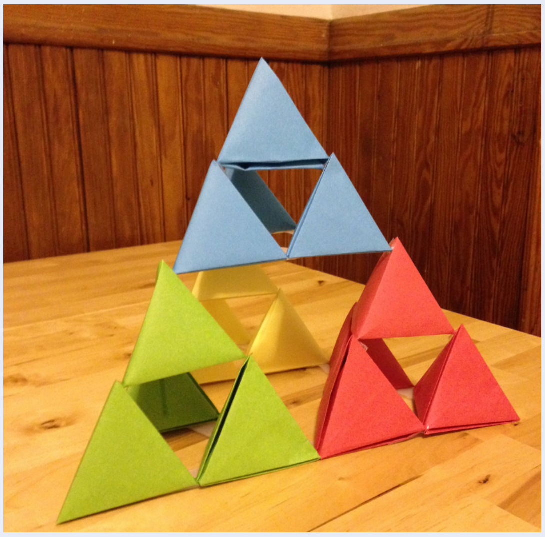
   2

Are You Ready for More?

A fractal is a special kind of geometric pattern. In a fractal, each part makes up a larger part with the same features, or is formed by smaller parts with the same features.

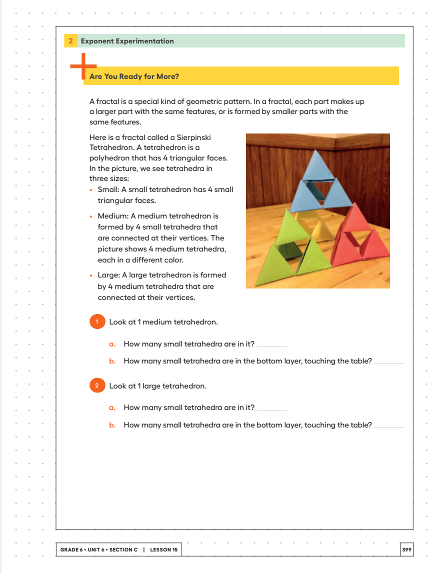
Here is a fractal called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has 4 triangular faces. In the picture, we see tetrahedra in three sizes:

- Small: A small tetrahedron has 4 small triangular faces.
- Medium: A medium tetrahedron is formed by 4 small tetrahedra that are connected at their vertices. The picture shows 4 medium tetrahedra, each in a different color.
- Large: A large tetrahedron is formed by 4 medium tetrahedra that are connected at their vertices.



1. Look at 1 medium tetrahedron.
   - **a.** How many small tetrahedra are in it?
     4
   - **b.** How many small tetrahedra are in the bottom layer, touching the table?
     3
2. Look at 1 large tetrahedron.
   - **a.** How many small tetrahedra are in it?
     16
   - **b.** How many small tetrahedra are in the bottom layer, touching the table?
     9

Student Workbook



2 Exponent Experimentation

Are You Ready for More?

A fractal is a special kind of geometric pattern. In a fractal, each part makes up a larger part with the same features, or is formed by smaller parts with the same features.

Here is a fractal called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has 4 triangular faces. In the picture, we see tetrahedra in three sizes:

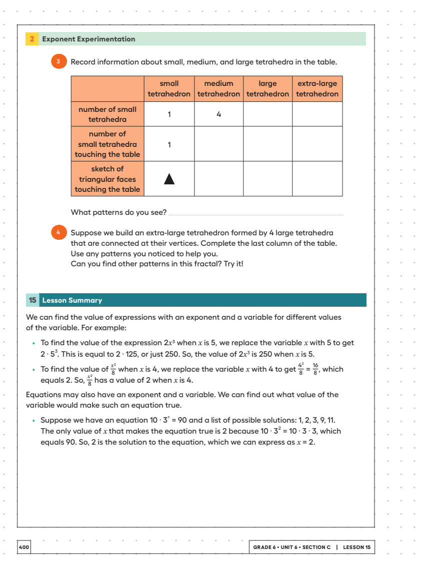
- Small: A small tetrahedron has 4 small triangular faces.
- Medium: A medium tetrahedron is formed by 4 small tetrahedra that are connected at their vertices. The picture shows 4 medium tetrahedra, each in a different color.
- Large: A large tetrahedron is formed by 4 medium tetrahedra that are connected at their vertices.

1 Look at 1 medium tetrahedron.
a. How many small tetrahedra are in it?
b. How many small tetrahedra are in the bottom layer, touching the table?

2 Look at 1 large tetrahedron.
a. How many small tetrahedra are in it?
b. How many small tetrahedra are in the bottom layer, touching the table?

GRADE 6 • UNIT 6 • SECTION C | LESSON 15

Student Workbook



2 Exponent Experimentation

3. Record information about small, medium, and large tetrahedra in the table.

	small tetrahedron	medium tetrahedron	large tetrahedron	extra-large tetrahedron
number of small tetrahedra	1	4		
number of small tetrahedra touching the table	1			
sketch of triangular faces touching the table	▲			

What patterns do you see?

4. Suppose we build an extra-large tetrahedron formed by 4 large tetrahedra that are connected at their vertices. Complete the last column of the table. Use any patterns you noticed to help you.
Can you find other patterns in this fractal? Try it!

15 Lesson Summary

We can find the value of expressions with an exponent and a variable for different values of the variable. For example:

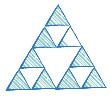
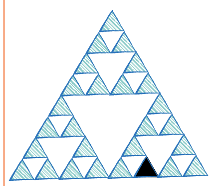
- To find the value of the expression $2x^3$ when x is 5, we replace the variable x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$, or just 250. So, the value of $2x^3$ is 250 when x is 5.
- To find the value of $\frac{x^2}{8}$ when x is 4, we replace the variable x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So, $\frac{x^2}{8}$ has a value of 2 when x is 4.

Equations may also have an exponent and a variable. We can find out what value of the variable would make such an equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: 1, 2, 3, 9, 11. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So, 2 is the solution to the equation, which we can express as $x = 2$.

400 GRADE 6 • UNIT 6 • SECTION C | LESSON 15

3. Record information about small, medium, and large tetrahedra in the table.

	small tetrahedron	medium tetrahedron	large tetrahedron	extra-large tetrahedron
number of small tetrahedra	1	4	16	64
number of small tetrahedra touching the table	1	3	9	27
sketch of triangular faces touching the table				

What patterns do you see?

Sample responses:

- Each time the fractal grows, there are 4 times as many small tetrahedra in the figure and 3 times as many small tetrahedra in the bottom layer.
- Each sketch shows 3 copies of the sketch before it (or the sketch before it plus 2 copies added to the bottom).

4. Suppose we build an extra-large tetrahedron formed by 4 large tetrahedra that are connected at their vertices. Complete the last column of the table. Use any patterns you noticed to help you.

See table.

Can you find other patterns in this fractal? Try it!

Activity Synthesis

The goal of the discussion should focus on how the meaning of the equal sign, exponent, and solution to an equation can help us find a value that makes each equation true.

Invite students to share how they found the solution to each equation. Discuss questions such as:

"What was your strategy when x is by itself on one side of the equation?"

"What was your strategy when x is the number being multiplied or raised to a power?"

"What was your strategy when x was the exponent?"

"Did you reason about the last equation the same way you did with the first equation? What did you do differently, if anything?"

Lesson Synthesis

Summarize the key ideas from the lesson by asking questions such as:

- *"In this lesson, we saw expressions such as 3^x, $\left(\frac{1}{3}\right)^x$, and $3x^4$. How are these different from those in earlier lessons?"*

  They involve both a variable and an exponent.

- *"If the value of x is 2, what is the value of 3^x and $\left(\frac{1}{3}\right)^x$?"*

  9 and $\frac{1}{3}$

- *"If we use a greater value of x, say 4, 5, or 10, how would the value of each expression change?"*

  The value of 3^x would get larger. The value of $\left(\frac{1}{3}\right)^x$ would get smaller.

- *"If x is 2, to find the value of $3x^4$, do we multiply 3 and 2 first and then raise the product to the power of 4, or do we find 2^4 first, and then multiply the result by 3?"*

  The latter.

- *"Why?"*

  Following the order of operations, we compute the expression with an exponent first before multiplying.

- *"Which number is a solution to $4x^2 = 1$? How do you know?"*

  $\frac{1}{2}$, because it is the value that would make the equation true.

Lesson Summary

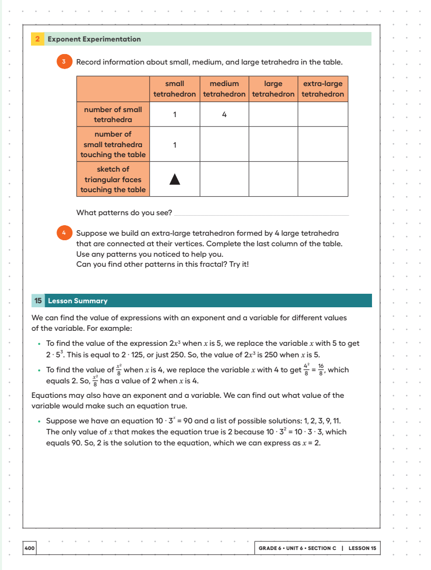
We can find the value of expressions with an exponent and a variable for different values of the variable. For example:

- To find the value of the expression $2x^3$ when x is 5, we replace the variable x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$, or just 250. So, the value of $2x^3$ is 250 when x is 5.
- To find the value of $\frac{x^2}{8}$ when x is 4, we replace the variable x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So, $\frac{x^2}{8}$ has a value of 2 when x is 4.

Equations may also have an exponent and a variable. We can find out what value of the variable would make such an equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: 1, 2, 3, 9, 11. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So, 2 is the solution to the equation, which we can express as $x = 2$.

Student Workbook


2 Exponent Experimentation

3 Record information about small, medium, and large tetrahedra in the table.

	small tetrahedron	medium tetrahedron	large tetrahedron	extra-large tetrahedron
number of small tetrahedra	1	4		
number of small tetrahedra touching the table	1			
sketch of triangular faces touching the table	▲			

What patterns do you see? ____

4 Suppose we build an extra-large tetrahedron formed by 4 large tetrahedra that are connected at their vertices. Complete the last column of the table. Use any patterns you noticed to help you.
Can you find other patterns in this fractal? Try it!

15 Lesson Summary

We can find the value of expressions with an exponent and a variable for different values of the variable. For example:

- To find the value of the expression $2x^3$ when x is 5, we replace the variable x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$, or just 250. So, the value of $2x^3$ is 250 when x is 5.
- To find the value of $\frac{x^2}{8}$ when x is 4, we replace the variable x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So, $\frac{x^2}{8}$ has a value of 2 when x is 4.

Equations may also have an exponent and a variable. We can find out what value of the variable would make such an equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: 1, 2, 3, 9, 11. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So, 2 is the solution to the equation, which we can express as $x = 2$.

400 GRADE 6 • UNIT 6 • SECTION C | LESSON 15

Responding To Student Thinking

Points to Emphasize
If most students struggle with evaluating an expression with an exponent for a given value of the variable, focus on this idea when opportunities arise over the next several lessons. For example, consider inviting students to reflect on the reasoning behind these practice problems:

Grade 6, Unit 6, Lesson 16, Practice Problem 4

Grade 6, Unit 6, Lesson 17, Practice Problem 5

Cool-down

Expressions with Exponents

5 min

Student Task Statement

Find the value of each expression for the given value of x.

1. $\left(\frac{1}{4}\right)^x$ when x is 3

   $\frac{1}{64}$

2. $4 + x^5$ when x is 2

   36

3. $4x^2$ when x is 10

   400

4. $(4x)^2$ when x is 10

   1,600

Lesson 15

Practice Problems

6 Problems

Problem 1

Find the value of each expression when $x = 3$.

a. 2^x

8

b. x^2

9

c. 1^x

1

d. x^1

3

e. $\left(\frac{1}{2}\right)^x$

$\frac{1}{8}$

Problem 2

Find the value of each expression for the given value of x.

a. $2 + x^3$ when x is 3

29

b. x^2 when x is $\frac{1}{2}$

$\frac{1}{4}$

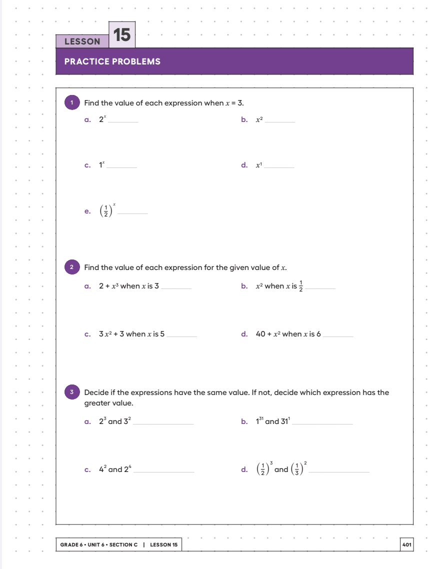
c. $3x^2 + 3$ when x is 5

78

d. $40 + x^2$ when x is 6

76

Student Workbook


LESSON 15

PRACTICE PROBLEMS

1 Find the value of each expression when $x = 3$.

a. 2^x ____ b. x^2 ____

c. 1^x ____ d. x^1 ____

e. $\left(\frac{1}{2}\right)^x$ ____

2 Find the value of each expression for the given value of x.

a. $2 + x^3$ when x is 3 ____ b. x^2 when x is $\frac{1}{2}$ ____

c. $3x^2 + 3$ when x is 5 ____ d. $40 + x^2$ when x is 6 ____

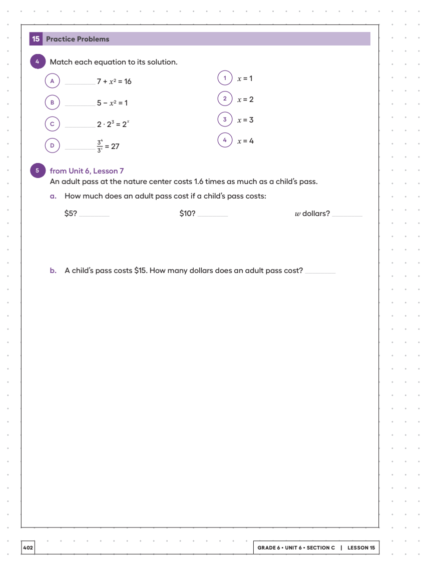
3 Decide if the expressions have the same value. If not, decide which expression has the greater value.

a. 2^3 and 3^2 ____ b. 1^{31} and 31^1 ____

c. 4^2 and 2^4 ____ d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$ ____

GRADE 6 • UNIT 6 • SECTION C | LESSON 15 401

Student Workbook



Problem 3

Decide if the expressions have the same value. If not, decide which expression has the greater value.

a. 2^3 and 3^2

Not equal. 3^2 has the greater value.

Sample reasoning: $2^3 = 8$ and $3^2 = 9$.

b. 1^{31} and 31^1

Not equal. 31^1 has the greater value.

Sample reasoning: $1^{31} = 1$ and $31^1 = 31$.

c. 4^2 and 2^4

Equal

Sample reasoning: They both have 16 as their value.

d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$

Not equal. $\left(\frac{1}{2}\right)^3$ has the greater value.

Sample reasoning: $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and $\frac{1}{8} > \frac{1}{9}$.

Problem 4

Match each equation to its solution.

A. $7 + x^2 = 16$ 3	**1.** $x = 1$
B. $5 - x^2 = 1$ 2	**2.** $x = 2$
C. $2 \cdot 2^3 = 2^x$ 4	**3.** $x = 3$
D. $\frac{3^4}{3^x} = 27$ 1	**4.** $x = 4$

Problem 5

from Unit 6, Lesson 7

An adult pass at the nature center costs 1.6 times as much as a child's pass.

a. How much does an adult pass cost if a child's pass costs:

\$5?

8 dollars

$1.6 \cdot 5 = 8$

\$10?

16 dollars

$1.6 \cdot 10 = 16$

w dollars?

$1.6w$ dollars

b. A child's pass costs \$15. How many dollars does an adult pass cost?

24 dollars

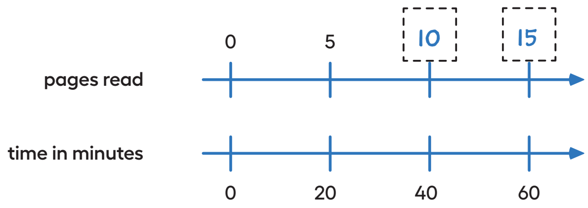
$1.6w = 24$ and $1.6 \cdot 15 = 24$

Problem 6

from Unit 2, Lesson 14

Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour?

- Use a double number line diagram to find the answer.



- Use a table to find the answer.

Sample responses:

pages read	time in minutes
5	20
0.25	1
15	50

pages read	time in minutes
5	20
10	40
15	60

Which strategy do you think is better, and why?

Sample response: The table is more efficient, because I can skip values.

Student Workbook