Lesson 4

Proportional Relationships and Equations

Goals

- Generalize a process for finding unknown values in a proportional relationship, and justify (orally) why this can be abstracted as $y = kx$, where k is the constant of proportionality.
- Generate an equation of the form $y = kx$ to represent a proportional relationship in a familiar context.
- Write the constant of proportionality to complete a row in a table representing a proportional relationship where the value for the first quantity is 1.

Learning Targets

- I can write an equation of the form $y = kx$ to represent a proportional relationship shown in a table or described in a story.
- I can write the constant of proportionality as an entry in a table.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 3)
- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- MLR3: Critique, Correct, Clarify
- MLR7: Compare and Connect
- Which Three Go Together?

Lesson Narrative

In this lesson, students represent proportional relationships using equations of the form $y = kx$. The activities revisit various contexts from earlier in the unit and continue presenting values in tables. Students see that the relationship in each table can be represented by an equation of the form $y = kx$, where k is the constant of proportionality that relates the two quantities. As students calculate values in the tables and write equations relating the quantities, they practice looking for and expressing regularity in repeated reasoning.

The last activity is optional because it provides an opportunity for additional practice by revisiting another familiar context.

Student Learning Goal

Let's write equations describing proportional relationships.

Lesson Timeline

Warm Up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool Down (Assessment)
5 mins	10 mins	10 mins	10 mins	10 mins	5 mins

Warm-up

Which Three Go Together: Expressions

5 min

Activity Narrative

This *Warm-up* prompts students to compare four expressions. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terminologies students know and how they talk about characteristics of algebraic expressions.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $5 \cdot 2$

B. $4 + ? = 20$

C. $x + 5$

D. $5x$

Sample responses:

A, B, and C go together because:

- They have a symbol that represents the operation.

A, B, and D go together because:

- They all represent multiplication.
- They all involve multiplying by 5.

A, C, and D go together because:

- They are expressions, but not equations.
- They include the numeral 5.

B, C, and D go together because:

- They have a symbol representing an unknown number.

Instructional Routines

Which Three Go Together?

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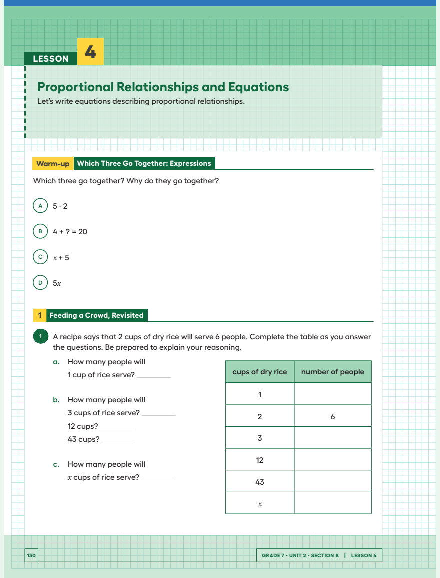
Student Workbook


LESSON 4

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

Warm-up Which Three Go Together: Expressions

Which three go together? Why do they go together?

A $5 \cdot 2$

B $4 + ? = 20$

C $x + 5$

D $5x$

1 Feeding a Crowd, Revisited

A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 cup of rice serve?

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 1)

MLR7: Collect and Display

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "sum," "product," "factor," "term," "expression," "equation," "variable," or "unknown," and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Activity 1

Feeding a Crowd, Revisited

10 min

Activity Narrative

In this activity, students revisit two contexts seen previously and ultimately find equations for the proportional relationships. As students find missing values in the table, they should see that they can always multiply the number of food items by the constant of proportionality to find the number of people served. When students see this pattern and represent the number of people served by x cups of rice as $3x$ (or by n spring rolls as $\frac{1}{2}n$), they are expressing regularity in repeated reasoning.

Monitor for students who use these strategies to complete the tables:

- Find the unit rate, interpret it, and use it to calculate other values.
- Use a scale factor to find the values for the first row and then scale those values to find other rows.
- Find the constant of proportionality that relates the left column to the right column.

Regardless of whether students reason based on the meaning of the unit rate in context or based on the structure of the table, the key takeaway is the constant multiplicative relationship.

Launch

Arrange students in groups of 2–3. Tell students that they will revisit the situation about rice and spring rolls from an earlier activity in this unit.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

cups of dry rice	number of people
1	3
2	6
3	9
12	36
43	129
x	$3x$

a. How many people will 1 cup of rice serve?

3

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

9; 36; 129

c. How many people will x cups of rice serve?

$3x$

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

number of spring rolls	number of people
1	$\frac{1}{2}$
6	3
10	5
16	8
25	12.5
n	$\frac{1}{2}n$

a. How many people will 1 spring roll serve?

$\frac{1}{2}$

b. How many people will 10 spring rolls serve? 5

16 spring rolls? 8

25 spring rolls? 12.5

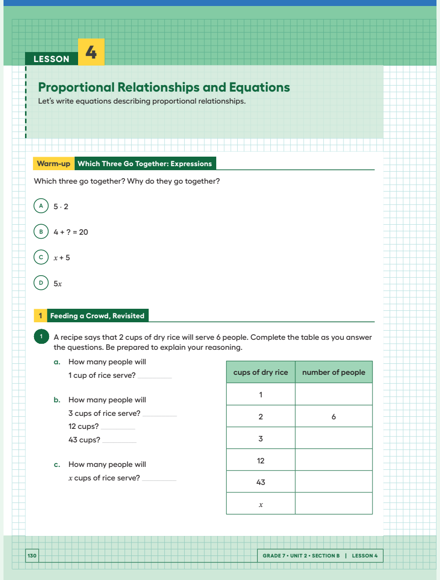
c. How many people will n spring rolls serve?

$\frac{1}{2}n$

Building on Student Thinking

If students have trouble representing each relationship with an expression, encourage them to draw diagrams or to describe the relationship in words.

Student Workbook


LESSON 4

Proportional Relationships and Equations

Let's write equations describing proportional relationships.

Warm-up Which Three Go Together: Expressions

Which three go together? Why do they go together?

A $5 \cdot 2$

B $4 + ? = 20$

C $x + 5$

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1 Feeding a Crowd, Revisited

1 A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 cup of rice serve?

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

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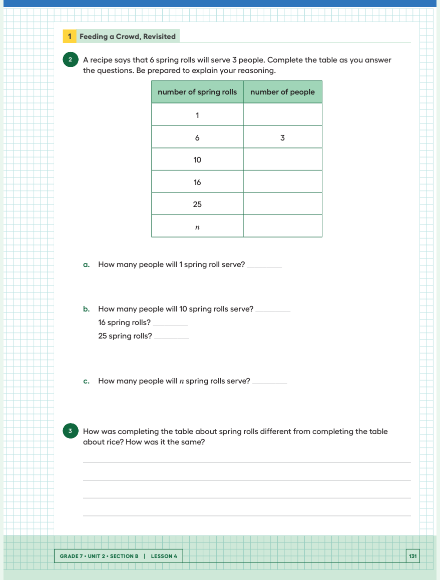
Student Workbook


1 Feeding a Crowd, Revisited

2 A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

a. How many people will 1 spring roll serve?

b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

c. How many people will n spring rolls serve?

3 How was completing the table about spring rolls different from completing the table about rice? How was it the same?

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Instructional Routines

5 Practices

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3. How was completing the table about spring rolls different from completing the table about rice? How was it the same?

Sample response: Both problems involved multiplying (or dividing). The constant of proportionality for the rice was greater than 1, so the number of people is larger than the number of cups. The constant of proportionality for the spring rolls was less than 1, so the number of people is less than the number of spring rolls.

Activity Synthesis

The goal of this discussion is to show how an equation can be used to represent the proportional relationship shown in each table. Display 2–3 approaches to the rice problem from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different representations. Here are some questions for discussion:

"What do the approaches have in common? How are they different?"

"How does the constant of proportionality, 3, show up in each method?"

"Are there any benefits or drawbacks to one method compared to another?"

The key takeaway is that any value in the right column can be found by multiplying the corresponding value in the left column by 3. For the last row, we can represent x times 3 as $3x$.

Next, suggest to students that we let y represent the number of people who can be served by x cups of rice. Ask students to write an equation that gives the relationship between x and y. Display the equation $y = 3x$ and help students interpret its meaning in the context:

"To find y, the number of people served, we can multiply the number of cups of rice, x, by 3."

Lastly, ask students to write an equation that represents the relationship for the spring rolls. Record and display students' equations. Ask them to interpret what the equations tell us about the situation. (To find the number of people served, y, we can divide the number of spring rolls, n, by 2, or multiply it by $\frac{1}{2}$ or 0.5.)

Activity 2

Denver to Chicago

10 min

Activity Narrative

In this activity, students write an equation to represent a proportional relationship between distance and time. The context of airplane flight is similar to that of a previous activity, but not exactly the same. This activity prompts students to move back and forth between the abstract representation and the context as they create an equation and then use it to find other values that aren't in the table. This task increases the level of difficulty by having so much missing information and by using decimals in the table.

There are various ways students may approach the last question. Monitor for students who:

Multiply the elapsed times by the rate 610 miles per hour, without referencing the table or equation.

- Use the structure of the table, adding rows for 3 and 3.5 hours.
- Use the equation to find the distances for 3 and 3.5 hours.

Plan to have students present in this order to support moving them from arithmetic methods towards algebraic methods.

Launch

Tell students that this activity revisits the context of flying in an airplane but it is not exactly the same situation as the airplane activity in the earlier lesson.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

1. Complete the table.

time (hours)	distance (miles)
1	610
1.5	915
2	1,220
2.5	1,525
t	$610t$

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.

Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Building on Student Thinking

Students who are having trouble understanding the task can draw a segment between Denver and Chicago on the map and label it with the distance and the time. Consider prompting them to find the speed in miles per hour (the distance the plane travels in 1 hour at this speed). Then ask students how this number can help them complete the table and answer the questions.


Student Workbook

2 Denver to Chicago

4 If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.

5 How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1 How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

2 Imagine there are two twins. One twin leaves Earth on a spaceship and travels to a planet near Proxima Centauri. The spaceship travels at 90% of the speed of light. The other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri?

3 (Do you think the answer would be the same for the twin on the spaceship? Consider researching "The Twin Paradox" to learn more.)

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2. How far does the plane fly in 1 hour?

   610 miles

   $915 \div 1.5 = 610$ for the speed, and at a speed of 610 miles per hour, it would travel 610 miles after 1 hour.

3. How far would the plane fly in t hours at this speed?

   $610t$ miles

4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.

   $d = 610t$ (or equivalent)

5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

   1,830 miles; 2,135 miles
   Sample reasoning: I multiplied each number of hours by 610.

Are You Ready for More?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

   $1.3 \cdot 3.26 \approx 4.24$ or about 4.24 years

2. Imagine there are two twins. One twin leaves Earth on a spaceship and travels to a planet near Proxima Centauri. The spaceship travels at 90% of the speed of light. The other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri?

   $4.24 \div 0.9 \approx 4.7$ or about 4.7 years

   (Do you think the answer would be the same for the twin on the spaceship? Consider researching "The Twin Paradox" to learn more.)

Activity Synthesis

The purpose of this discussion is to show the value of finding an equation that represents a proportional relationship.

Ask previously selected students to share their solutions to the last question (the distances the plane would travel in 3 and 3.5 hours). Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

"How are these methods the same? How are they different?"

"How does the constant of proportionality show up in each representation?"

"Are there any benefits or drawbacks to one strategy compared to another?"

If not mentioned by students, display the equation $d = 610t$ for all to see, and ask students to interpret its meaning in the context of the situation. (To find d, the distance traveled by the plane in miles, multiply the hours of travel, t, by the plane's speed in miles per hour, 610.)

The key takeaways are:

- An equation is an efficient way to represent a proportional relationship.
- The equation shows the constant of proportionality, and the variables indicate how the quantities are related.
- You can substitute a value in for one of the variables and then multiply (or divide) to find the value for the other variable.

Activity 3: Optional
Revisiting Coco Bread

10 min

Activity Narrative

This activity gives students more practice writing an equation that represents a proportional relationship. It revisits a context examined in a previous lesson—the amounts of coconut milk and flour in a bread recipe. Students can then use their equation to answer additional questions about the situation.

Launch

Tell students that this activity revisits the context of making coco bread from an earlier lesson. Give students quiet work time followed by partner discussion.

Student Task Statement

To bake coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

1. Complete the table.

coconut milk (milliliters)	flour (grams)
100	180
200	360
450	810
c	$1.8c$

2. Use f to represent the grams of flour needed for c milliliters of coconut milk. Write an equation that relates f and c.

   $f = 1.8c$ or equivalent

3. How much flour is needed for 680 milliliters of coconut milk? 945 milliliters? Explain or show your reasoning.

   1,224 grams; 1,701 grams

   Sample reasoning: I multiplied each number by 1.8.

Access for Multilingual Learners (Activity 3)

MLR3: Critique, Collect, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Instructional Routines

MLR3: Critique, Correct, Clarify

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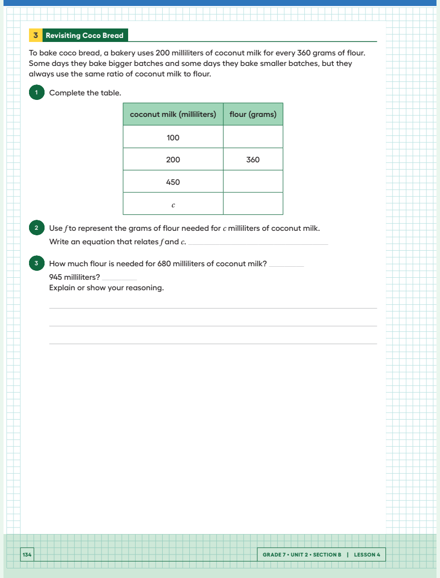
Student Workbook



3 Revisiting Coco Bread

To bake coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

1 Complete the table.

coconut milk (milliliters)	flour (grams)
100	
200	360
450	
c	

2 Use f to represent the grams of flour needed for c milliliters of coconut milk. Write an equation that relates f and c. ______

3 How much flour is needed for 680 milliliters of coconut milk? ______ 945 milliliters? ______
Explain or show your reasoning.

Activity Synthesis

Ask students to compare answers with their partner and discuss their reasoning until they reach an agreement. Then invite students to share with the class how they used their equation from the second question to answer the third question.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response about how to get the equation for this relationship by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"First I wrote the equation $200c = 360f$. Then I wanted to solve it for one of the variables, so I divided each side by 200 to get the equivalent equation $c = 1.8f$."

Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

- Here is an example of a second draft:

"For each row of the table, the grams of flour is 1.8 times the milliliters of coconut milk. For example $200 \cdot 1.8 = 360$. An equation that represents this relationship is $f = 1.8c$."

- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Lesson Synthesis

Share with students,

"Today we used the constant of proportionality to write an equation to represent each proportional relationship."

To review the structure of an equation that represents a proportional relationship, consider asking students:

"What was the constant of proportionality for the relationship between cups of rice and people served?" 3

"What equation did we write for this situation?" $y = 3x$

"What was the constant of proportionality for the relationship between number of spring rolls and people served?" $\frac{1}{2}$

"What equation did we write for this situation?" $p = \frac{1}{2}n$ (or equivalent)

"What was the constant of proportionality for the relationship between the distance the plane traveled and the elapsed time?" 610

"What equation did we write for this situation?" $d = 610t$

"What do these equations have in common?" They show a multiplicative relationship between two quantities. They have a structure like $y = kx$, where k is the constant of proportionality.

Lesson Summary

In this lesson, we wrote equations to represent proportional relationships described in words and shown in tables.

This table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that "parts" can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

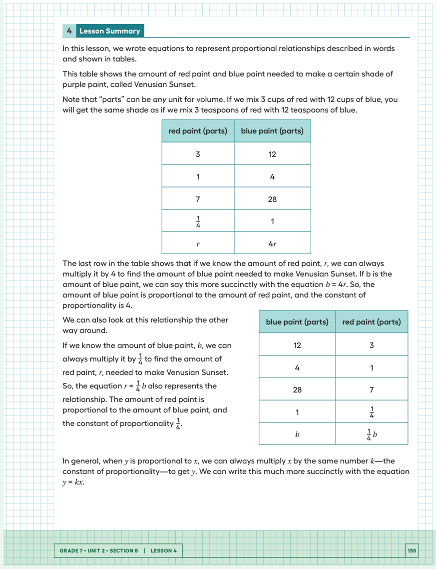
The last row in the table shows that if we know the amount of red paint, r, we can always multiply it by 4 to find the amount of blue paint needed to make Venusian Sunset. If b is the amount of blue paint, we can say this more succinctly with the equation $b = 4r$. So, the amount of blue paint is proportional to the amount of red paint, and the constant of proportionality is 4.

We can also look at this relationship the other way around.If we know the amount of blue paint, b, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint, r, needed to make Venusian Sunset. So, the equation $r = \frac{1}{4}b$ also represents the relationship. The amount of red paint is proportional to the amount of blue paint, and the constant of proportionality $\frac{1}{4}$.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

In general, when y is proportional to x, we can always multiply x by the same number k—the constant of proportionality—to get y. We can write this much more succinctly with the equation $y = kx$.

Student Workbook



Responding To Student Thinking

Points to Emphasize

If students struggle with writing an equation to represent a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activity refered to here, invite multiple students to share their thinking about how they wrote their equations.

Unit 2, Lesson 5, Activity 2 Meters and Centimeters

Cool-down

It's Snowing in Syracuse

5 min

Student Task Statement

Snow is falling steadily in Syracuse, New York. After 2 hours, 4 inches of snow has fallen.

1. If it continues to snow at the same rate, how many inches of snow would you expect after 6.5 hours? If you get stuck, you can use the table to help.

   13 inches

   Two inches fell in 1 hour, 6.5 is $1 \cdot (6.5)$, and $2 \cdot (6.5) = 13$.

2. Write an equation that gives the amount of snow that has fallen after x hours at this rate.

   Sample response: $y = 2x$, where x is the number of hours that have passed and y is the inches of snow that has fallen.

3. How many inches of snow will fall in 24 hours if it continues to snow at this rate?

   48 inches
   Sample reasoning: $24 \cdot 2 = 48$

time (hours)	snow (inches)
0.5	1
1	2
2	4
6.5	13
x	$2x$

Lesson 4

Practice Problems

5 Problems

Problem 1

A ceiling is made up of tiles. Every square meter of the ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	10.75
10	107.5
9.3	100
a	$10.75 \cdot a$

Problem 2

On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d, to the number of hours flying, t, is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

1.6 hours, since $\frac{1}{500} \cdot 800 = 1.6$

Problem 3

Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality: 4

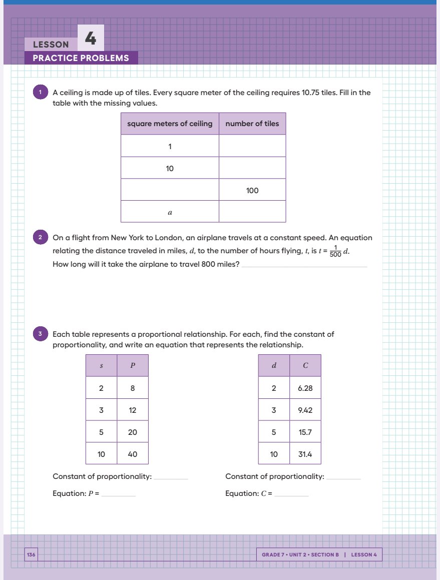
Equation: $P = 4s$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality: 3.14

Equation: $C = 3.14d$

Student Workbook


LESSON 4
PRACTICE PROBLEMS

1 A ceiling is made up of tiles. Every square meter of the ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	
10	
	100
a	

2 On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d, to the number of hours flying, t, is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

3 Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality:

Equation: $P =$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality:

Equation: $C =$

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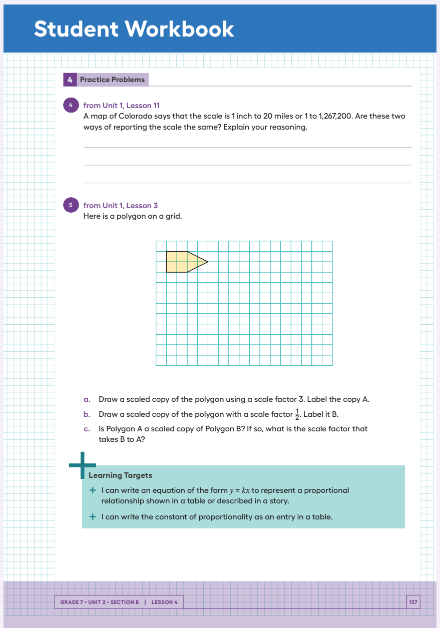


Problem 4

from Unit 1, Lesson 1

A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

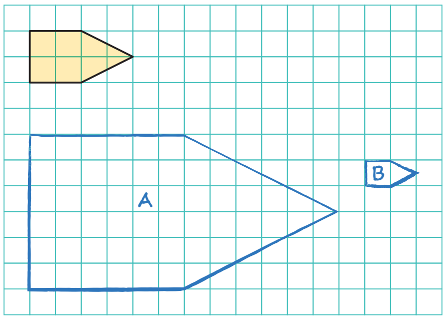
Yes

Sample reasoning: There are 12 inches in a foot and 5280 feet in 1 mile, so that's 63,360 inches in a mile and 1,267,200 inches in 20 miles.

Problem 5

from Unit 1, Lesson 3

Here is a polygon on a grid.



a. Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.

b. Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.

c. Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Yes, B is a scaled copy of A with a scale factor of $\frac{1}{6}$.