Lesson 13

Measurement Error

Goals

- Compare and contrast (orally) multiple measurements of the same length that result from using rulers with different levels of precision.
- Describe (orally) possible sources of "measurement error" when measuring lengths.
- Generalize a process for calculating measurement error and expressing it as a percentage of the actual length.

Learning Targets

- I can represent measurement error as a percentage of the correct measurement.
- I understand that all measurements include some error.

Lesson Narrative

In this lesson, students express measurement error as a percentage of the correct amount. **Measurement error** is the positive difference between a measured amount and the actual amount. Students see how measurement error can arise in two different ways: from the level of precision in the measurement device and from human error.

The *Warm-up* has students measure line segments with two different rulers, one marked in centimeters and one marked in millimeters, to introduce the concept of measurement error. The next activity introduces the convention of expressing measurement error as a percentage of the correct value. The last activity brings these two ideas together as students measure items around the classroom with the two rulers from the *Warm-up* and calculate the percent error.
Students engage in an important aspect of mathematical modeling as they make simplifying assumptions and approximations of a real situation.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect

Required Materials

Materials to Copy

- Measurement to the Nearest Cutouts (1 copy for every 2 students): Warm-up

Required Preparation

Activity 2:
Measure the height or length of several objects in your classroom to the nearest tenth of a centimeter. If possible, have at least one object per student in the class so that students don't have to wait too long to measure things. Most of the items should be greater than 20 centimeters in length, but some can be less than or equal to 20 centimeters in length. Examples of such objects might be the width of the door, the length of the stick that holds a flag, the length of an eraser, or a side of a table or desk top.

Lesson:
The "Measuring to the Nearest" blackline master contains two versions of a centimeter ruler that students will use to measure things, so card stock would be preferable if available. In the instructions, students are told to cut out the rulers they will use from the blackline master, but to save class time, this may be done for them ahead of time. These same rulers are also used in the "Measuring Your Classroom" activity in this lesson, so they should be used carefully during the *Warm-up*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	10 min	20 min	10 min	5 min

Lesson 13

Measurement Error

Student Learning Goal

Let's use percentages to describe how accurately we can measure.

Warm-up

Measuring to the Nearest

10 min

Activity Narrative

In this *Warm-up*, students use two rulers that have different levels of precision to measure the same lines. Students notice how differences in recorded measurements can result from the level of precision of the measuring device. This prepares students for expressing measurement error as a percentage of the correct amount.

Students will need to use the rulers again later in this lesson, so make sure they keep track of them.

Launch

Arrange students in groups of 2. Distribute one set of 2 pre-cut rulers to each group.

Give students 3–5 minutes of partner work time, then follow with a whole-class discussion.

Student Task Statement

Your teacher will give you two rulers and three line segments labeled A, B, and C.

1. Use the centimeter ruler to measure each line segment to the nearest centimeter. Record these lengths in the first column of the table.
2. Use the millimeter ruler to measure each line segment to the nearest tenth of a centimeter. Record these lengths in the second column of the table.

Sample response:

line segment	length (cm) as measured with the first ruler	length (cm) as measured with the second ruler
A	7	6.7
B	7	6.9
C	7	7.3

Activity Synthesis

Ask students to describe what they notice about the lengths they got when they used the two different measuring devices. Explain to students that one source of measurement error can be the precision level of their measuring device. Ask students,

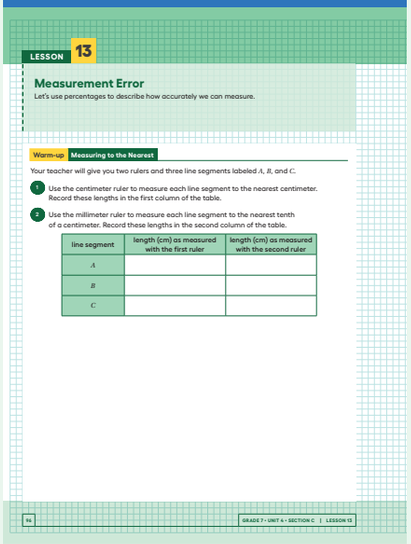
> *"Assuming the measurements to the nearest tenth of a centimeter are exact, how much error was in each measurement when you used the centimeter ruler?"*

7 cm was 0.3 cm too long, 0.1 cm too long, and 0.3 cm too short, respectively.

Building on Student Thinking

Students might not line up the edge of the ruler with the end of the line. Remind students that we need to line up the 0 mark on the ruler (in this case, the edge of the ruler) with the beginning edge of the line being measured.

Student Workbook


LESSON 13

Measurement Error

Let's use percentages to describe how accurately we can measure.

Warm-up Measuring to the Nearest

Your teacher will give you two rulers and three line segments labeled A, B, and C.

1. Use the centimeter ruler to measure each line segment to the nearest centimeter. Record these lengths in the first column of the table.
2. Use the millimeter ruler to measure each line segment to the nearest tenth of a centimeter. Record these lengths in the second column of the table.

line segment	length (cm) as measured with the first ruler	length (cm) as measured with the second ruler
A		
B		
C		

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Activity 1

Measuring a Soccer Field

10 min

Activity Narrative

In this activity, students determine the amount of error in a measurement and express the error as a percentage of the correct value. They brainstorm possible sources of error and discuss how real-world limitations on humans using measuring devices can introduce measurement errors. As students interpret these results, they consider sources for error and improvement in a mathematical model.

The actual length is given in yards, while the measured length is given in feet and inches. Students must choose which unit to use and convert the other measurement before they can calculate the measurement error. Regardless of which unit they use, the percentage that the error is of the correct amount should be the same. Monitor for students who choose to use different units for their calculations: yards, feet, or inches.

Also monitor for mistakes students may make in their solution process, such as:

- Failing to convert the lengths to the same unit.
- Dividing by the measured length instead of the actual length, resulting in the percentage 4.38%.

The key focus is distinguishing between different solution processes that give the same answer versus solution processes that give an incorrect answer.

Launch

Keep students in the same groups. Tell students that a soccer field is 120 yards long, and ask them how they can use a 30-foot-long tape measure to measure that length. (Note the use of two different units of measure, yards and feet.) If not mentioned by students, suggest measuring off 30 feet, making a mark, measuring off another 30 feet, and so on. Ask the class if they would all get exactly the same answer by this method.

Give the students 3–4 minutes of quiet work time to calculate the amount of the error and the percent error, followed by time for partner discussion.

Then hold a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later. Look for a mixture of correct and incorrect solutions.

Student Task Statement

A soccer field is 120 yards long. Han uses a 30-foot-long tape measure to measure the length of the field and gets a measurement of 376 feet 6 inches.

1. What is the amount of the error?

   5.5 yards, 16.5 feet, or 198 inches

2. Express the error as a percentage of the actual length of the field. Explain or show your reasoning.

   About 4.6%

   Sample reasoning:

   - $5.5 \div 120 = 0.04583\overline{3}$
   - $16.5 \div 360 = 0.04583\overline{3}$
   - $198 \div 4{,}320 = 0.04583\overline{3}$

3. What are some possible causes for this error?

   Sample responses:

   - Han did not use the tape measure correctly.
   - He did not position the tape measure precisely every time he measured another 30 feet.
   - He didn't go in a completely straight line.

Activity Synthesis

The goal of this discussion is to contrast different methods that get the correct answer with methods that get an incorrect answer for the percentage that the error is of the correct amount. Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"Why do these different approaches lead to the same outcome?*
- *"Why does this approach lead to a different outcome?"*

The key takeaway is that it doesn't matter which unit students chose. The percentage of the error should be the same as long as their units match on both the measured length and the actual length (and as long as students divided by the actual length, not the measured length).

Explain to students that **measurement error** is the positive difference between the measurement and the actual value. It is often expressed as a percentage of the actual value. We can use words to describe whether the measurement is greater than or less than the actual value. In this case, we might say that the measured length is more than the actual length with an error of about 4.6%.

Building on Student Thinking

If students fail to see the need for converting units of measure, ask,

"How many feet are in 120 yards? How many inches?"

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Develop Expression and Communication.

Invite students to demonstrate their ideas for the last question using a physical ruler as a substitute for Han's tape measure.

Supports accessibility for: Language, Visual-Spatial Processing

Student Workbook


1 Measuring a Soccer Field

A soccer field is 120 yards long. Han uses a 30-foot-long tape measure to measure the length of the field and gets a measurement of 376 feet 6 inches.

1. What is the amount of the error?

2. Express the error as a percentage of the actual length of the field. Explain or show your reasoning.

3. What are some possible causes for this error?

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Student with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.

Begin by enacting a physical demonstration of measuring an item with both the centimeter ruler and millimeter ruler to support understanding of the context.

Supports accessibility for: Conceptual Processing, Language

Activity 2
Measuring Your Classroom

20 min

Activity Narrative

In this activity, students work with their partner to measure three different things around the classroom using the two different rulers from the *Warm-up*. After both students in a group have measured their three objects, the teacher provides them with the actual measurements of those items. The students then calculate the measurement errors as percentages of the actual measurements.

As students measure multiple items and generalize that using the less precise ruler results in greater percent error, they are making use of repeated reasoning.

Monitor for students whose tables clearly show that the measurements taken with the centimeter ruler have greater percent error than the measurements taken with the millimeter ruler.

Launch

Keep students in the same groups. Make sure that students still have access to the two rulers from the *Warm-up* of this lesson.

Assign each group three objects to measure. Explain that after they have measured their objects, you will tell them the actual lengths of the objects so that they can complete the last two columns of the table.

Give students 5–6 minutes of partner work time, then follow with a whole-class discussion.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language students use to describe the measurement error, and express it as a percentage of the actual length. Display words and phrases such as “measurement error,” “measured length,” “actual length,” “more precise,” “closer to the actual value,” and “smaller markings on the ruler.”

Student Task Statement

Your teacher will tell you which three items to measure. Keep using the paper rulers from the earlier activity.

1. Between you and your partner, decide who will use which ruler.

   No written response required.

2. Measure the three items assigned by your teacher, and record your measurements in the second column of the appropriate table.

   See the second column in the completed tables.

   Using the cm ruler:

   Sample response:

item	measured length (cm)	actual length (cm)	difference	percentage
eraser length	19	19.1	0.1	0.5%
table width	106	110.4	4.4	4%
book height	28	27.5	0.5	1.8%

   Using the mm ruler:

   Sample response:

item	measured length (cm)	actual length (cm)	difference	percentage
eraser length	19.1	19.1	0	0%
table width	106	110.4	4.4	4%
book height	27.7	27.5	0.2	0.7%

3. After you finish measuring the items, share your data with your partner. Next, ask your teacher for the actual lengths.

   See the third column in the completed tables.

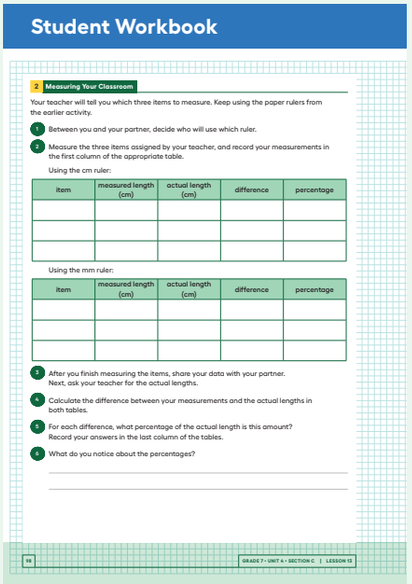
4. Calculate the difference between your measurements and the actual lengths in both tables.

   See the fourth column in the completed tables.

5. For each difference, what percentage of the actual length is this amount? Record your answers in the last column of the tables.

   See the fifth column in the completed tables.

Student Workbook



2 Measuring Your Classroom

Your teacher will tell you which three items to measure. Keep using the paper rulers from the earlier activity.

1. Between you and your partner, decide who will use which ruler.
2. Measure the three items assigned by your teacher, and record your measurements in the first column of the appropriate table.

Using the cm ruler:

item	measured length (cm)	actual length (cm)	difference	percentage

Using the mm ruler:

item	measured length (cm)	actual length (cm)	difference	percentage

3. After you finish measuring the items, share your data with your partner. Next, ask your teacher for the actual lengths.
4. Calculate the difference between your measurements and the actual lengths in both tables.
5. For each difference, what percentage of the actual length is this amount? Record your answers in the last column of the tables.
6. What do you notice about the percentages?

Student Workbook


2 Measuring Your Classroom

Are You Ready for More?

Before there were standard units of measurement, people often measured things using their hands or feet.

1. Measure the length of your foot to the nearest centimeter with your shoe on.

2. How many foot-lengths long is your classroom? Try to determine this as precisely as possible by carefully placing your heel next to your toe as you pace off the room.

3. Use this information to estimate the length of your classroom in centimeters.

4. Use a tape measure to measure the length of your classroom. What is the difference between the two measurements? Which one do you think is more accurate?

GRADE 7 • UNIT 4 • SECTION C | LESSON 13 99

6. What do you notice about the percentages?

Sample responses:

- The percentages are all less than 10%.
- The percentages are generally larger for the cm ruler than for the mm ruler.

Are You Ready for More?

Before there were standard units of measurement, people often measured things using their hands or feet.

1. Measure the length of your foot to the nearest centimeter with your shoe on.

Answers vary.

2. How many foot-lengths long is your classroom? Try to determine this as precisely as possible by carefully placing your heel next to your toe as you pace off the room.

Answers vary.

3. Use this information to estimate the length of your classroom in centimeters.

Answers vary.

4. Use a tape measure to measure the length of your classroom. What is the difference between the two measurements? Which one do you think is more accurate?

The tape measure should be more accurate.

Activity Synthesis

There are two desired outcomes of this activity:

1. To develop a procedure that makes sense to students for computing percent error.
2. To reinforce that less precise measuring devices result in greater percent error.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share what they noticed about their percentages. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. Ask students to suggest ways to update the display:

- *"Are there any new words or phrases that you would like to add? Is there any language you would like to revise or remove?"*

To highlight the effects of using a less precise measuring device, select a pair of students whose tables clearly show that the measurements taken with the centimeter ruler had greater percent error than the measurements taken with the millimeter ruler. Display their work, and ask students to share what they notice about these percentages. Invite students to explain why the measurements taken with the centimeter ruler had greater measurement error.

Lesson Synthesis

Share with students,

- *"Today we saw how percentages can be used to describe measurement error."*

To review these concepts, consider asking:

- *"What is measurement error?"*

  It is the difference between a measurement of an object and its actual quantity.

- *"What causes measurement error?"*

  It may exist due to human error in using a measurement tool or because the tool itself is not precise.

- *"How can we minimize the amount of error?"*

  Use precision tools, and use them carefully.

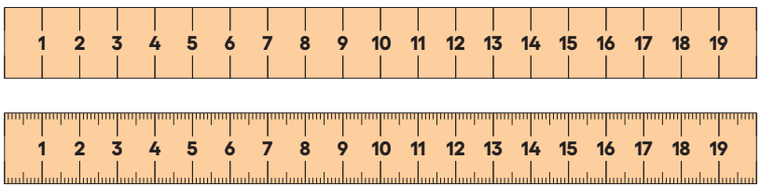
- *"How can measurement error be expressed as a percentage?"*

  Divide the measurement error by the actual quantity.

Lesson Summary

When we use a ruler or measuring tape to measure a length, we can get a measurement that is different from the actual length. This could be because we positioned the ruler incorrectly, or it could be because the ruler is not very precise. There is always at least a small difference between the actual length and a measured length, even if it is a microscopic difference!

Here are two rulers with different markings.

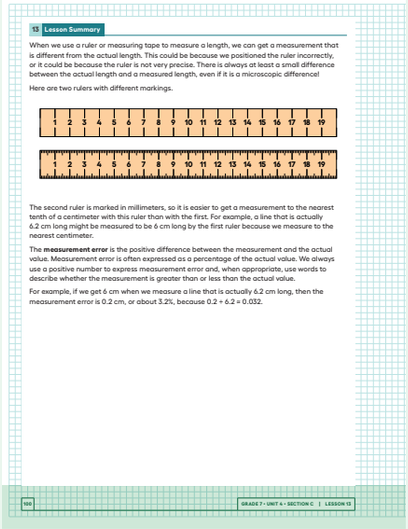


The second ruler is marked in millimeters, so it is easier to get a measurement to the nearest tenth of a centimeter with this ruler than with the first. For example, a line that is actually 6.2 cm long might be measured to be 6 cm long by the first ruler because we measure to the nearest centimeter.

The **measurement error** is the positive difference between the measurement and the actual value. Measurement error is often expressed as a percentage of the actual value. We always use a positive number to express measurement error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value.

For example, if we get 6 cm when we measure a line that is actually 6.2 cm long, then the measurement error is 0.2 cm, or about 3.2%, because $0.2 \div 6.2 \approx 0.032$.

Student Workbook


13 Lesson Summary

When we use a ruler or measuring tape to measure a length, we can get a measurement that is different from the actual length. This could be because we positioned the ruler incorrectly, or it could be because the ruler is not very precise. There is always at least a small difference between the actual length and a measured length, even if it is a microscopic difference!

Here are two rulers with different markings.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

The second ruler is marked in millimeters, so it is easier to get a measurement to the nearest tenth of a centimeter with this ruler than with the first. For example, a line that is actually 6.2 cm long might be measured to be 6 cm long by the first ruler because we measure to the nearest centimeter.

The **measurement error** is the positive difference between the measurement and the actual value. Measurement error is often expressed as a percentage of the actual value. We always use a positive number to express measurement error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value.

For example, if we get 6 cm when we measure a line that is actually 6.2 cm long, then the measurement error is 0.2 cm, or about 3.2%, because 0.2 ÷ 6.2 ≈ 0.032.

160 GRADE 7 • UNIT 4 • SECTION C | LESSON 13

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Off by a Little Bit?

5 min

In this *Cool-down*, students are assessed on their ability to compute measurement error and percent error from two measurements: one estimate and another actual measurement.

Student Task Statement

Clare estimates that her brother is 4 feet tall. When they get measured at the doctor's office, her brother's height is 4 feet, 2 inches.

1. Should Clare's or the doctor's measurement be considered the actual height? Explain your reasoning.

   The doctor's measurement is more precise, while Clare's is only an estimate.

2. What is the error expressed in inches?

   2 inches

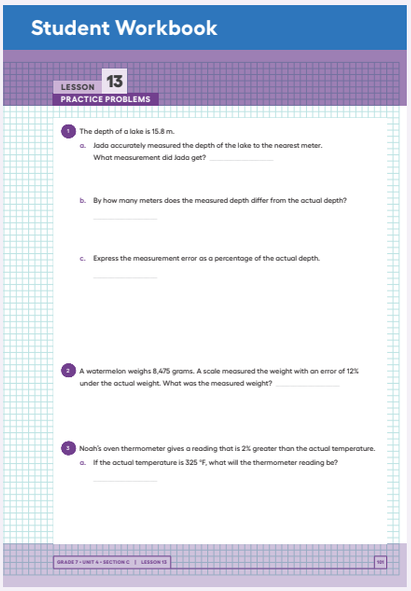
3. What was the error, expressed as a percentage of the actual height?

   4% (4 feet 2 inches is equivalent to 50 inches, and 2 ÷ 50 = 0.04.)

Lesson 13

Practice Problems

5 Problems



Problem 1

The depth of a lake is 15.8 m.

a. Jada accurately measured the depth of the lake to the nearest meter. What measurement did Jada get?

16 m

b. By how many meters does the measured depth differ from the actual depth?

0.2 m

c. Express the measurement error as a percentage of the actual depth.

1.27% (0.2 ÷ 15.8 ≈ 0.01265)

Problem 2

A watermelon weighs 8,475 grams. A scale measured the weight with an error of 12% under the actual weight. What was the measured weight?

7,458 grams (8,475 × 0.88 = 7,458)

Problem 3

Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.

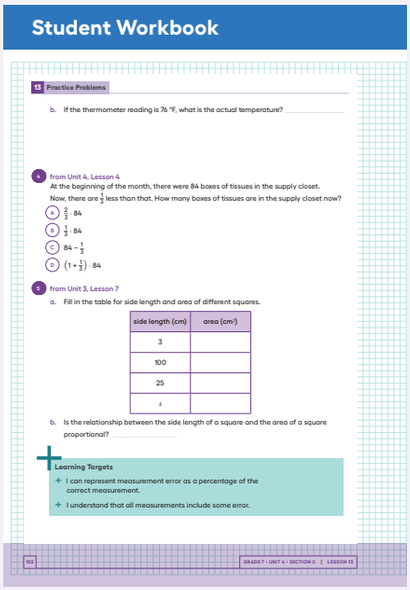
a. If the actual temperature is 325 °F, what will the thermometer reading be?

331.5 °F (325 × 1.02 = 331.5)

b. If the thermometer reading is 76 °F, what is the actual temperature?

74.5 °F (76 ÷ 1.02 ≈ 74.5)

Student Workbook



13 Practice Problems

b. If the thermometer reading is 76 °F, what is the actual temperature? _____

4 from Unit 4, Lesson 4
At the beginning of the month, there were 84 boxes of tissues in the supply closet. Now, there are $\frac{1}{3}$ less than that. How many boxes of tissues are in the supply closet now?
A $\frac{2}{3} \cdot 84$
B $\frac{1}{3} \cdot 84$
C $84 - \frac{1}{3}$
D $\left(1 + \frac{1}{3}\right) \cdot 84$

5 from Unit 3, Lesson 7
a. Fill in the table for side length and area of different squares.

side length (cm)	area (cm²)
3	
100	
25	
s	

b. Is the relationship between the side length of a square and the area of a square proportional? _____

Learning Targets
+ I can represent measurement error as a percentage of the correct measurement.
+ I understand that all measurements include some error.

163 GRADE 7 • UNIT 4 • SECTION C | LESSON 13

Problem 4

from Unit 4, Lesson 4

At the beginning of the month, there were 84 boxes of tissues in the supply closet. Now, there are $\frac{1}{3}$ less than that. How many boxes of tissues are in the supply closet now?

A. $\frac{2}{3} \cdot 84$

B. $\frac{1}{3} \cdot 84$

C. $84 - \frac{1}{3}$

D. $\left(1 + \frac{1}{3}\right) \cdot 84$

Problem 5

from Unit 3, Lesson 7

a. Fill in the table for side length and area of different squares.

side length (cm)	area (cm²)
3	9
100	10,000
25	625
s	s^2

b. Is the relationship between the side length of a square and the area of a square proportional?

No, it is not proportional.

There is no number that the numbers in the first column of the table can be multiplied by to get the numbers in the second column.