Scale Drawings

Goals

- Describe (orally) what a "scale drawing" is.
- Explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances.
- Interpret the "scale" of a scale drawing.

Learning Targets

- I can explain what a scale drawing is, and I can explain what its scale means.
- I can use actual distances and a scale to find scaled distances.
- I can use a scale drawing and its scale to find actual distances

Lesson Narrative

In this lesson, students begin to look at **scale drawings**, or scaled twodimensional representations of actual objects or places. Students see that although scale drawings capture three-dimensional objects or places, they show scaled measurements in only two of the dimensions, and that all information is projected onto a plane.

They see that the principles and strategies they used when reasoning about scaled copies are applicable to scale drawings. For example, previously they saw scale factor as a number that describes how lengths in a figure correspond to lengths in a copy of the figure (and vice versa). Now they see that **scale** serves a similar purpose: It describes how the lengths in an actual object are related to the lengths on a drawn representation of it. First students work with a drawing on which the scale is given in words ("1 cm represents 2 m"). Then they work with a drawing on which the scale is shown as a line segment on the drawing. As students use scales and scale drawings to find actual and scaled lengths, they reason quantitatively and abstractly.

Math Community

Today's community building centers on the teacher sharing their draft commitments as part of the mathematical community. At the end of the lesson, students are invited to suggest additions to the teacher sections of the chart.

Student Learning Goal

Let's explore scale drawings.

Lesson Timeline

5 min

15 min 15 min

Activity 2

10 min

Lesson Synthesis

Access for Students with Diverse Abilities

• Representation (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Geometry toolkits: Activity 1, Activity 2

Required Preparation

Warm-up

Prepare to display the examples and non-examples of scale drawings for all to see. Consider adding to the collection a local map showing the actual route of a train or bus line (example of scale drawing) and a diagrammatic transit map (non-example).

In the "Doing Math" teacher section of the Math Community Chart, add 2–5 commitments you have for what your teaching practice "looks like" and "sounds like" this year.

Assessment

5 min

Cool-down

Warm-up

What is a Scale Drawing?



Activity Narrative

This activity encourages students to notice characteristics of scale drawings by observing examples and counterexamples and to articulate what a scale drawing is. Though students are not expected to come up with precise definitions, they are likely able to intuit that scale drawings are accurate two-dimensional depictions of what they represent, in the sense that all shapes, arrangements of parts, and relative sizes match those of the actual objects.

Expect student observations about scale drawings to be informal and not mathematical. For example, they might say that a scale drawing looks just like the object it is portraying, with the parts shown having the right size and being in the right places in the drawing. Or that in a scale drawing, a smaller part in the actual object does not end up being larger in the drawing.

Like any mathematical model of a real situation, a scale drawing captures some important aspects of the real object and ignores other aspects. It may not be apparent to students that scale drawings prioritize features of one plane of the object (and sometimes features of other planes parallel to it) and ignore other surfaces and dimensions. Monitor for students who show insights around this idea.

Launch 22

Arrange students in groups of 2. Before students look at the materials, poll the class to find out who has seen scale drawings. Ask a few students who are familiar with them to give a couple of examples of scale drawings they have seen. Then, give students 2 minutes to observe the examples and counterexamples of scale drawings and discuss in groups what they think a scale drawing is.

Student Task Statement

Here are some drawings of a school bus, a quarter, and the subway lines around Boston, Massachusetts.

The first three drawings are **scale drawings** of these objects.





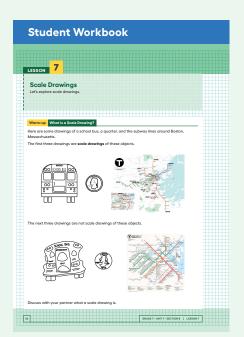


Building on Student Thinking

If students struggle to characterize scale drawings, offer prompts to encourage them to look closer. For example, ask:

"How do the shapes and sizes of the objects in the drawings compare to those of the actual objects?"

Students may say that sizes of the objects in the scale drawings are smaller than those of the actual objects. Ask them if any parts of the scale drawings are distorted, compared to the actual object—ask them to focus on the two images of the quarter, one of which is circular in shape while the other is not.



The next three drawings are not scale drawings of these objects.







Discuss with your partner what a scale drawing is.

Sample responses:

- A scale drawing is a drawing that shows the object accurately and all parts in the drawing match the parts in the actual object.
- · No parts in a scale drawing are distorted.
- A scale drawing is like a scaled copy of a real object, but it is a drawing that shows one flat surface of the object.

Activity Synthesis

Ask a few students to share what they noticed about characteristics of scale drawings and to compare and contrast scaled copies and scale drawings. Discuss questions such as the following. Record common themes and helpful descriptions.

O "What do the examples have or show that the counterexamples do not?"

"How are scale drawings like scaled copies you saw in earlier lessons? How are they different from scaled copies?"

"What aspects of the bus, coin, and the city of Boston do the scale drawings show? What aspects of the actual objects do scale drawings not show?"

Notice misconceptions, but it is not necessary to address them right away, as students' understanding will be shaped in this and upcoming lessons. Tell students that they will continue to analyze scale drawings and revise their definitions in upcoming activities.

Math Community

After the Warm-up, display the Math Community Chart with the "Doing Math" actions added to the teacher section for all to see. Give students 1 minute to review. Then share 2–3 key points from the teacher section and your reasoning for adding them. For example,

- If "questioning vs. telling," a shared reason could focus on your belief that students are capable mathematical thinkers and your desire to understand how students are making meaning of the mathematics.
- If "listening," a shared reason could be that sometimes you want to sit quietly with a group just to listen and hear student thinking and not because you think the group needs help or is off-track.

After sharing, tell students that they will have the opportunity to suggest additions to the teacher section during the *Cool-down*.

Activity 1

Sizing Up a Basketball Court



Activity Narrative

In this introductory activity, students explore the meaning of **scale**. They begin to see that a scale communicates the relationship between lengths on a drawing and corresponding lengths in the objects that they represent, and they learn some ways to express this relationship:

- "a units on the drawing represent b units of actual length"
- "at a scale of a units (on the drawing) to b units (actual)"
- "a units (on the drawing) for every b units (actual)"

Students measure lengths on a scale drawing and use a given scale to find corresponding lengths on a basketball court in their student workbooks. Because students are measuring to the nearest tenth of a centimeter, some of the actual measurements they calculate will not have the precision of the official measurements. For example, the official measurement for d is 0.9 m.

Launch 🙎

Ask students if they have ever played basketball or seen a basketball court. If so, where? If some students have played basketball or seen a basketball court, ask them if they could throw a basketball across the width of a basketball court. What about across the full length of the court?

Arrange students in groups of 2. Distribute a ruler to each student. Give students 6–7 minutes of quiet work time to complete the first three questions. Ask them to share their responses with their partner before completing the remaining questions.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language students use to describe how they are using the scale drawing and scale to find the actual distances. Display words and phrases such as: "scale," "scale drawing," "represents," "scaled distance," "actual distance," "multiply," "for every."

Student Task Statement

Look at the scale drawing of a basketball court on the previous page. The drawing does not have any measurements labeled, but it says that 1 centimeter represents 2 meters.

1. To the nearest tenth of a centimeter, measure the distances on the scale drawing that are labeled a–d. Record your results in the first row of the table.

Measurements may vary based on rounding.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754





Access for Multilingual Learners (Activity 1, Narrative)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

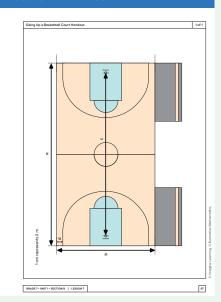
Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of scale drawings. Terms may include: scale, scaled distance, scale drawing.

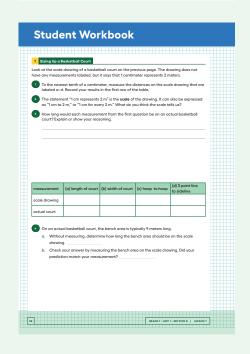
Supports accessibility for: Conceptual Processing, Language

Student Workbook



Building on Student Thinking

Instead of using the scale to find actual measurements, students might try to convert distances in centimeters to meters (14 cm is 0.14 m). Explain that the distances they measured on paper could be converted to meters, but then the results are still lengths on paper, just expressed in meters, rather than the measurements of the actual basketball court. Draw students' attention to the statement "1 cm represents 2 m" on the scale drawing and ask them to think about how to use it to find actual measurements.



2. The statement "1 cm represents 2 m" is the **scale** of the drawing. It can also be expressed as "1 cm to 2 m," or "1 cm for every 2 m." What do you think the scale tells us?

Sample responses:

- The scale tells us how the lengths on the drawing compare to actual lengths.
- The scale tells us how to use the measurements on the drawing to find actual measurements.
- **3.** How long would each measurement from the first question be on an actual basketball court? Explain or show your reasoning.

Sample reasoning: Since every centimeter represents 2 meters, each measurement from the drawing can be multiplied by 2 to find the actual measurement in meters.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing	14 cm	7.5 cm	12.4 cm	0.5 cm
actual court	28 m	15 m	24.8 m	Lm

- **4.** On an actual basketball court, the bench area is typically 9 meters long.
 - a. Without measuring, determine how long the bench area should be on the scale drawing. 4.5 cm
 - **b.** Check your answer by measuring the bench area on the scale drawing. Did your prediction match your measurement? **Answers vary.**

Activity Synthesis

Before debriefing as a class, display the table showing only the scaled distances so students can do a quick check of their measurements (which they may round differently). Explain to students that the distances on a scale drawing are often referred to as "scaled distances." The distances on the basketball court, in this case, are called *actual* distances.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they used the given scale to find actual distances. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

To further students' understanding of scale, discuss:

- "Does a scale of '1 cm for every 2 m' mean that the actual distance is twice the distance on the drawing? Why or why not?"
 - "Which parts of the court can be drawn using the '1 cm for every 2 m' rule?"

Everything that is flat on the court can be drawn using this scale, but the heights of things like the basketball hoop or the bleachers cannot be shown on this drawing.

Can we reverse the order in which we list the scaled and actual distances? For example, could we say '2 m of actual distance to 1 cm on the drawing' or just '2 m to 1 cm' for the scale?"

The scaled distance is conventionally stated first, but the actual distance represented could also come first as long as the meaning is clear from the context.

If needed, a short discussion about accuracy of measurements on the scale drawing might highlight some possible sources of measurement error. For example, the lines on the scale drawing have width, and this could contribute a small error depending on whether the measurement is from the inside or the outside of the lines.

Activity 2

Tall Structures

15 min

Activity Narrative

This activity introduces students to graphic scales. Students interpret a scale drawing that has a graphic scale. They use it to find actual measurements and express it non-graphically. Students choose what strategy and tools they will use to compare the scaled heights of several buildings with the graphic scale.

Monitor for students who use these strategies to determine the actual heights:

- Use their fingers to estimate the length of the graphic scale and compare it to the heights of the buildings.
- Use an index card to copy the length of the graphic scale once and use that benchmark to measure the heights of the buildings.
- Use an index card to copy the length of the graphic scale multiple times, end-to-end, creating an informal ruler that can be used to measure the heights of the buildings.
- Draw multiple copies of the graphic scale end-to-end on the image, creating a vertical axis that can be used to measure the heights of the buildings.
- Use a ruler to measure both the graphic scale and the heights of the buildings and then use ratio reasoning to calculate the actual heights.

Plan to have students present in this order, from less precise to more precise.

Launch

Display the scale drawing of the structures. Before beginning the work of the task, students may be interested in or eager to share the locations of the structures. Consider taking a few minutes to elicit what they know, or display a world map showing the locations of the structures.

Ask students what the segment labeled with "0 m" and "100 m" might mean. Some students are likely to say that it also conveys a scale. Verify that a scale can indeed be communicated graphically; an actual distance is not represented by a numerical measurement, but rather, by the length of the segment.

Instructional Routines

5 Practices

ilclass.com/r/10690701

Please log in to the site before using the QR code or URL.



Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

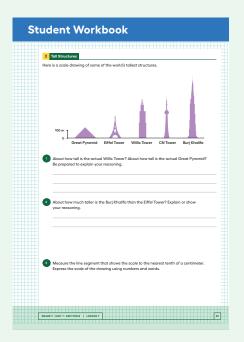
Please log in to the site before using the QR code or URL.



Building on Student Thinking

Students may not measure heights of the buildings at a right angle from the ground line. Remind students that heights are to be measured perpendicular to the ground or base line

If needed, demonstrate how to use the edge of a sheet of paper or an index card to measure lengths on a scale drawing that has a graphic scale.

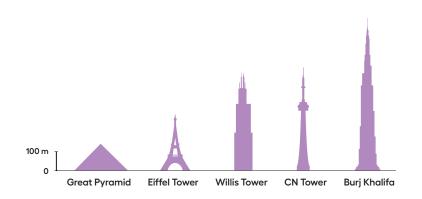


Provide access to geometry toolkits, including index cards or sheets of paper that students can use to measure. Tell students to check their answers to the first question with a partner and discuss to reach an agreement before proceeding to work on the rest of the problems. Give students 4–5 minutes of quiet work time and partner discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later. As students work, encourage them to be as precise as possible in estimating lengths that are less than 1 scale-segment long.

Student Task Statement

Here is a scale drawing of some of the world's tallest structures.



1. About how tall is the actual Willis Tower? About how tall is the actual Great Pyramid? Be prepared to explain your reasoning.

The Willis Tower is a bit more than 500 m tall. It takes about 5 of the segment lengths to measure the height. The Great Pyramid is approximately 150 m. Its height is about $1\frac{1}{2}$ segments long.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Explain or show your reasoning.

The Burj Khalifa is approximately 550 m taller than the Eiffel Tower. Sample reasonings:

- It takes about 3 of the IOO-m segments to measure the Eiffel Tower, so it is about 300 m tall. It takes about $8\frac{1}{2}$ of the IOO-m segments to measure the Burj Khalifa, so it is about 850 meters tall. 850 300 = 550.
- It takes about 3 of the IOO-m segments to measure the Eiffel Tower and about $8\frac{1}{2}$ segments to measure the Burj Khalifa. This is a difference of $5\frac{1}{2}$ segments on the scale drawing, which would be an actual difference of about 550 m.
- The Burj Khalifa looks almost 3 times as tall as the Eiffel Tower, which would be a difference of between 500 and 600 m.
- **3.** Measure the line segment that shows the scale to the nearest tenth of a centimeter. Express the scale of the drawing using numbers and words.

Answers vary depending on the printed size of the scale. Sample responses:

- o 0.7 cm on the drawing represents 100 m
- o 0.7 cm to 100 m
- 0.7 cm for every 100 m in actual height

Are You Ready for More?

The tallest mountain in the United States, Mount Denali in Alaska, is about 6,190 m tall. If this mountain were shown on the scale drawing, how would its height compare to the heights of the structures? Explain or show your reasoning.

Mount Denali will be far taller than the Burj Khalifa.

Sample reasonings:

- The Burj Khalifa is about 830 m. Mount Denali is 6,190 m, and 6,190 divided by 830 is about 7.5, so the mountain will be about 7.5 times taller than the Burj Khalifa.
- It takes about 62 of the IOO-m segments to measure the mountain (6,190 ÷ 100 = 61 · 9). It takes about $8\frac{1}{4}$ of the IOO-m segments to measure the Burj Khalifa. This means the mountain is about 7.5 times taller than the Burj, because 62 divided by $8\frac{1}{4}$ is about 7.5.

Activity Synthesis

The purpose of this discussion is to familiarize students with using a scale drawing with a graphic scale to calculate actual distances. Invite previously selected students to share how they estimated the heights of the buildings. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. (It is not necessary to demonstrate every possible method. The goal is for students to see some methods that are less precise and some methods that are more precise.) If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

"How was the scale used in each method?"

"What worked well in _____'s approach? What did not work well?"

"How did each approach deal with potential measurement error?"

After several methods have been shared, discuss the scale drawing of the towers more broadly. Consider asking:

"Besides height information, what other information about the towers does the drawing show?"

Widths of the buildings and their overall shapes.

"What information does it not show?

The depth of each building, any projections or protrusions, shapes of different parts of the buildings, distances between the different buildings, etc.

"How is this scale drawing the same as that of the basketball court? How are the two scale drawings different?"

They both show information on a single plane and are drawn using a scale. The basketball court is a flat surface, like the drawing of the court. The drawing of the towers is a side view or front view. The actual objects represented are not actually flat objects.

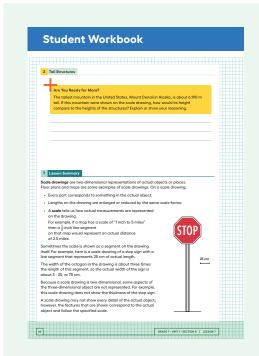
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

The tablest from the survey of the structures? Explain or show your reasoning. The tablest from the survey of the structures? Explain or show your reasoning. The tablest from the structures? Explain or show your reasoning. The stablest from the structures? Explain or show your reasoning. Scale drawings are two-dimensional representations of actual objects or places. There prises and maps, are some examples of scale drawings. On a scale drawing: - Every port corresponds to something in the actual object. - Lengths on the drawing are enlarged or reduced by the same scale factor. - A scale table is to have octual measurements are expresented on the drawing. For example, if a map have a scale of "I hinch to 5 miles" on the map would represent an actual distance of 2.5 miles. Somethines the scale is shown as a segment on the drawing itself for example, here is a scale drawing of a stop sign with a line segment that represents 25 mm of actual lengths. The width of the actual with of the actual object of the sign is clouds. The drawing itself for example, here is a scale drawing is tabout three firms. - Because a scale drawing is two-dimensional, some expects of the three-dimensional object or not represented. For example, the color of follow the specified scale.



Lesson Synthesis

Share with students

"Today we used scale drawings to calculate actual distances."

To help students generalize about scale drawings, consider asking:

"What is a scale drawing?"

A scale drawing is a flat, scaled representation of an actual object.

○ "How can we describe the purpose of the scale for a scale drawing?"

The scale tells us how lengths on the drawing relate to lengths on the actual object. Some distance on the drawing represents a specified amount of actual distance.

"How do we find actual distances using a scale drawing?"

We measure the drawing and then multiply or divide by the numbers in the scale.

If desired, use the basketball court activity as an example to review the meaning of these terms.

"We looked at a scale drawing of a basketball court. What was the scale on that drawing?"

I centimeter on the drawing represented 2 meters on the actual court.

"If a line segment in the scale drawing was 5 centimeters long, how long would the corresponding segment be on the actual court?"

10 meters, because $2 \cdot 5 = 10$

It is important to remember that a scale drawing shows scaled measurements in only two dimensions; i.e., measurements of a particular surface of an object and those that have been projected onto a particular plane. For example, the drawing of the basketball court did not show the height of the basketball hoops.

Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings. On a scale drawing:

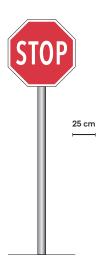
- Every part corresponds to something in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A scale tells us how actual measurements are represented on the drawing. For example, if a map has a scale of "1 inch to 5 miles" then a $\frac{1}{2}$ -inch line segment on that map would represent an actual distance of 2.5 miles.

Sometimes the scale is shown as a segment on the drawing itself. For example, here is a scale drawing of a stop sign with a line segment that represents 25 cm of actual length.

The width of the octagon in the drawing is about three times the length of this segment, so the actual width of the sign is about $3 \cdot 25$, or 75 cm.

Because a scale drawing is two-dimensional, some aspects of the three-dimensional object are not represented. For example, this scale drawing does not show the thickness of the stop sign.

A scale drawing may not show every detail of the actual object; however, the features that are shown correspond to the actual object and follow the specified scale.



Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "What additions would you make to the teacher 'Doing Math' section of the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the teacher "Doing Math" section of the Math Community Chart before Exercise 4.

Cool-down

Length of a Bus and Width of a Lake



Launch

If desired, provide access to four-function calculators.

Student Task Statement

1. A scale drawing of a school bus has a scale of $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is $4\frac{1}{2}$ inches on the scale drawing, what is the actual length of the bus? Explain or show your reasoning.

45 ft

Sample reasoning: There are 9 groups of $\frac{1}{2}$ in $4\frac{1}{2}$. If $\frac{1}{2}$ inch represents 5 feet, then $4\frac{1}{2}$ inches represents 9 · 5 or 45 feet.

2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1,000 m, what is the width of the lake on the scale drawing?

12.5 cm

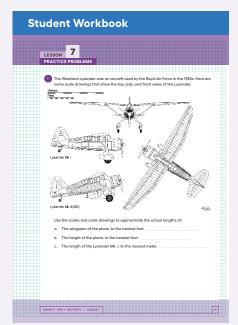
Sample reasoning: Since every 80 m is represented by I cm, I,000 m is represented by I,000 \div 80 or I2.5 cm.

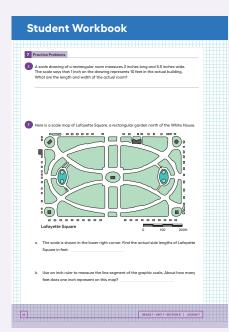
Responding To Student Thinking

Points to Emphasize

If students struggle with finding actual lengths from a scale drawing, revisit this when opportunities arise over the next several lessons. For example, in this activity, invite students to share how they found the actual distance between the two cities:

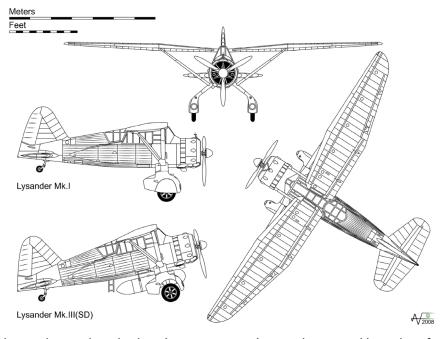
Unit 1, Lesson 8, Activity 1 Biking through Kansas





Problem 1

The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.



Use the scales and scale drawings to approximate the actual lengths of:

- a. The wingspan of the plane, to the nearest foot. 46 feet
- **b.** The height of the plane, to the nearest foot. <u>I2 feet</u>
- **c.** The length of the Lysander Mk. I, to the nearest meter. 9 meters

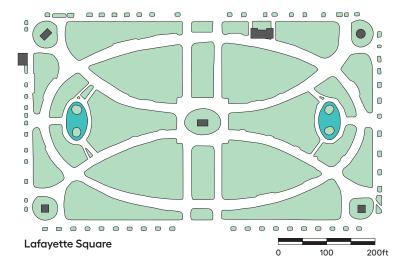
Problem 2

A scale drawing of a rectangular room measures 3 inches long and 5.5 inches wide. The scale says that 1 inch on the drawing represents 10 feet in the actual building. What are the length and width of the actual room?

30 feet long and 55 feet wide

Problem 3

Here is a scale map of Lafayette Square, a rectangular garden north of the White House.



a. The scale is shown in the lower right corner. Find the actual side lengths of Lafayette Square in feet.

About 800 feet by 500 feet

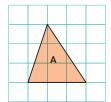
b. Use an inch ruler to measure the line segment of the graphic scale. About how many feet does one inch represent on this map?

Answers vary depending on the size of the printed scale. Sample response: I in represents 300 feet.

Problem 4

from Unit 1, Lesson 6

Here is Triangle A. Lin created a scaled copy of Triangle A with an area of 72 square units.



- a. How many times larger is the area of the scaled copy compared to that of Triangle A? 16 times larger (72 ÷ 4 . 5 = 16)
- **b.** What scale factor did Lin apply to the Triangle A to create the copy? <u>4</u>
- c. What is the length of the bottom side of the scaled copy? 12 units

