Lesson 2

Keeping the Equation Balanced

Goals

- Calculate the weight of an unknown object using a hanger diagram, and explain (orally) the solution method.
- Comprehend that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger by the same amount are moves that keep the hanger balanced.

Learning Targets

- I can add or remove blocks from a hanger and keep the hanger balanced.
- I can represent balanced hangers with equations.

Instructional Routines

- Notice and Wonder

Lesson Narrative

In this lesson, students use hanger diagrams as a way to represent equivalence. Students may have seen this representation in prior grades, but now they explore how to deal with shapes of unknown weight. Students continue to describe moves that maintain equivalence for the representations and should recognize that, as long as the unknown weights are equivalent, they can be removed equally from each side of the diagram. As they move fluently between hanger diagrams and algebraic equations representations, students must reason abstractly and quantitatively.

Student Learning Goal

Let's figure out unknown weights on balanced hangers.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

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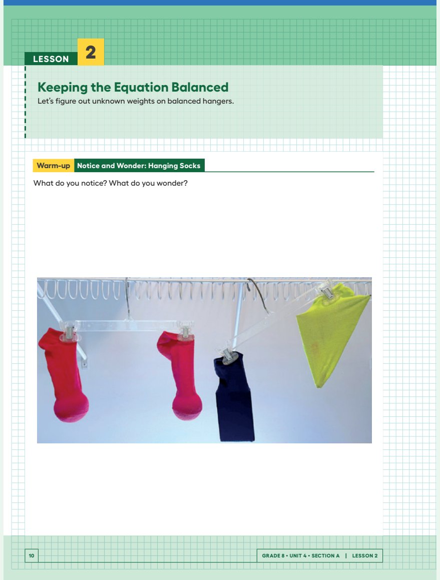
Student Workbook


LESSON 2

Keeping the Equation Balanced

Let's figure out unknown weights on balanced hangers.

Warm-up Notice and Wonder: Hanging Socks

What do you notice? What do you wonder?

Warm-up

Notice and Wonder: Hanging Socks

5 min

Activity Narrative

The purpose of this *Warm-up* is to give students an opportunity to ground their understanding of equality in the context of weight, which will be useful when students examine hanger representations throughout the lesson. While students may notice and wonder many things about this image, the idea that things that weigh the same are balanced and things that weigh different amounts are unbalanced are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder about.

Student Task Statement

What do you notice? What do you wonder?

Students might notice:

- There are four socks on two hangers.
- One hanger is hanging diagonally and one is balanced.
- The navy blue sock looks heavier because it is weighing down that side of its hanger.
- The red socks have balls in them, the navy blue sock has something rectangular in it, the yellow sock has a triangle in it.

Students might wonder:

- Are the hangers a number line and the socks numbers?
- Is this representing an equation?
- How many yellow socks with triangles would it take to balance the hanger on the right?
- What is in each of the socks?

Activity Synthesis

Ask students to share the things they noticed and wondered about. Record and display, for all to see, their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the reasoning behind the left hanger being balanced and the right hanger being unbalanced does not come up during the conversation, ask students to discuss this idea. Students should understand that a hanger will balance only if the weight of the unknown objects in both socks is the same. If they are not the same, then the heavier side is going to be lower than the lighter side.

Activity 1
Hanging Blocks

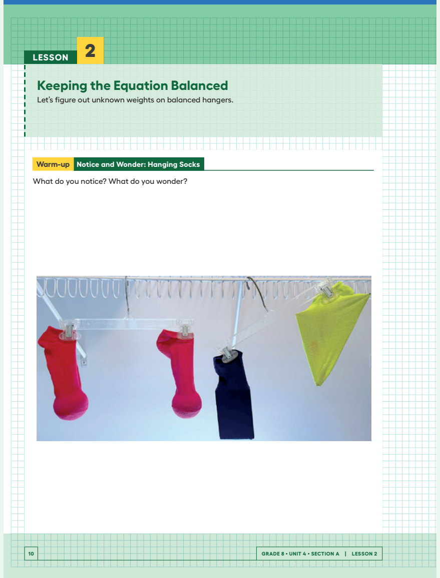
10 min

Activity Narrative

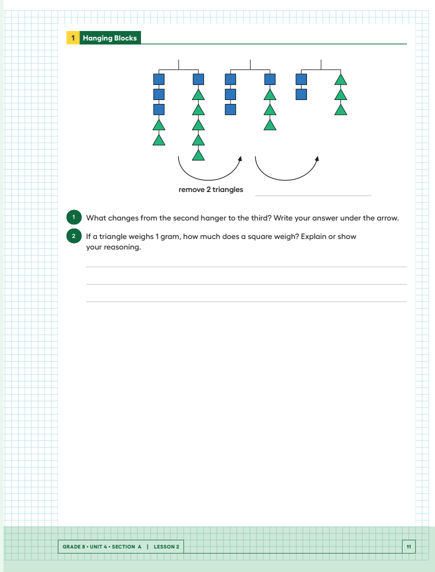
The purpose of this task is for students to understand and explain why they can add or subtract expressions from each side of an equation and still maintain the equality, even if the value of those expressions are not known. Both problems have shapes with unknown weight on each side to promote students' thinking about unknown values in this way before the transition to equations.

While the focus of this activity is on the relationship between both sides of the hanger and not equations, some students may start the second problem by writing and solving an equation to find the weight of a square. While students are working, identify those using equations and those not using equations. Ask them to answer the second problem during the whole-class discussion.

Student Workbook


LESSON 2
Keeping the Equation Balanced
Let's figure out unknown weights on balanced hangers.
Warm-up Notice and Wonder: Hanging Socks
What do you notice? What do you wonder?

Student Workbook


1 Hanging Blocks

remove 2 triangles

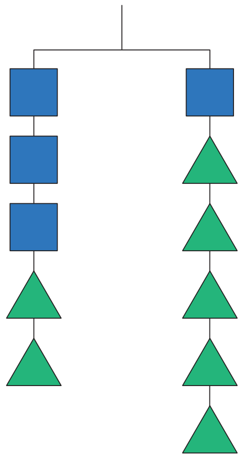
1 What changes from the second hanger to the third? Write your answer under the arrow.

2 If a triangle weighs 1 gram, how much does a square weigh? Explain or show your reasoning.

GRADE 8 • UNIT 4 • SECTION A | LESSON 2

Launch

Display the first hanger image for all to see.



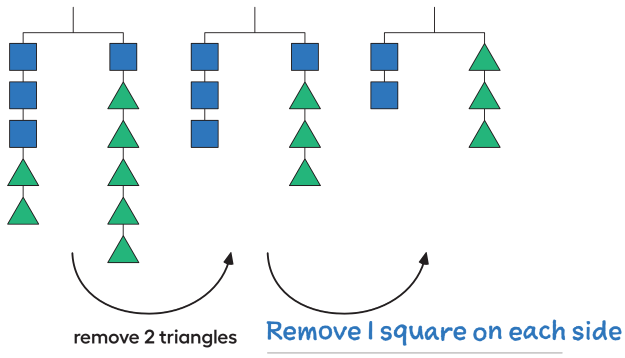
Tell students that this is a hanger problem similar to the one in the *Warm-up*, only instead of the weights hidden inside socks, each block *type* represents a different weight. For example, each green triangle weighs the same amount as any other green triangle, but weighs a different amount than does a blue square.

Ask students,

"What will happen if you take 2 triangles off of the left side and 3 triangles off of the right side?"

The hanger will not balance. The left side will be lower.

Student Task Statement



1. What changes from the second hanger to the third? Write your answer under the arrow.
2. If a triangle weighs 1 gram, how much does a square weigh? Explain or show your reasoning.

   $\frac{3}{2}$ grams or equivalent

   2 squares weigh the same as 3 triangles, which together weigh 3 grams. This means that each square weighs half of 3 grams or $\frac{3}{2}$ grams.

Activity Synthesis

The purpose of this discussion is to examine moves that keep the hanger in balance and how those moves can help in solving for the weight of an unknown object.

Invite students to share their response and reasoning for the weight of a square. Ask if any groups used a different strategy to find the answer. If possible, record and display their work for all to see.

If it does not come up, ask students to consider how the hangers could be turned into equations.

To connect the hangers to equations, ask:

- *"How could the the weight of a square be represented in an equation?"*

  I could use a variable, like x.

- *"How do you account for there being 2 squares on the left side when finding the weight with the hanger diagram? How does it fit into the equation?"*

  I knew I had to divide the weight on the right by 2. In the equation, I used $2x$.

- *"You found the weight of a square using the last hanger. How do you know that is the same weight of a square in the original hanger?"*

  Because the hanger is in balance from each move, the weight of a square for any one of them should be the weight of the square in all of them.

- *"We didn't know the weight of a square. How do we know the hanger will stay balanced when we remove 1 square from each side?"*

  Because all of the squares have the same weight, removing an equal number from each side should keep the hanger balanced.

- *"What other moves could we do to the hanger to keep it in balance?"*

  Add the same number of a shape to each side, double or halve the weight on each side by adding more shapes or cutting them in half.

Activity 2
More Hanging Blocks

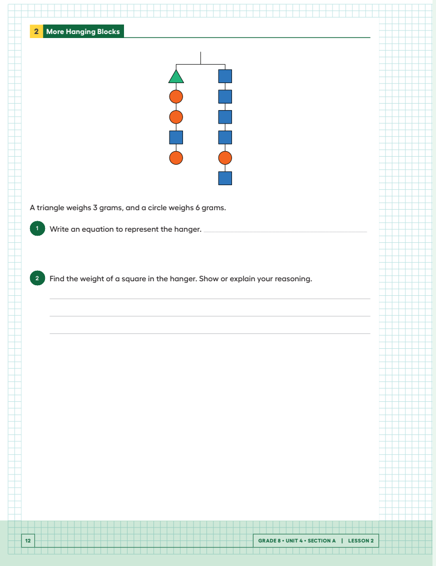
15 min

Activity Narrative

The goal of this activity is for students to transition their reasoning about solving hanger problems by maintaining the equality of each side, to solving equations using the same logic.

Students solve 2 more hanger problems and write equations to represent each hanger. In the first problem, the solution is not an integer, which will challenge any student who has been using guess-and-check in the previous activities and will encourage that student to look for a more efficient method.

Student Workbook

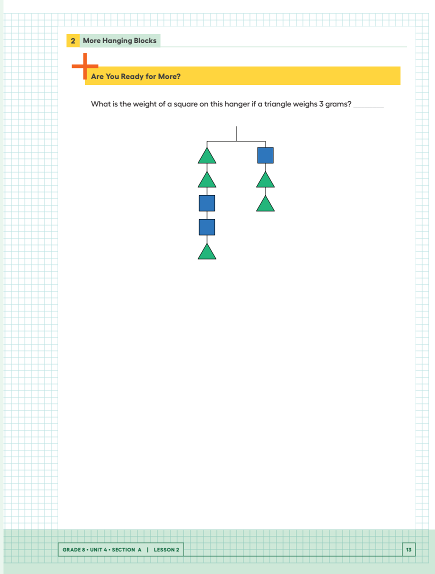

2 More Hanging Blocks

A triangle weighs 3 grams, and a circle weighs 6 grams.

1. Write an equation to represent the hanger.

2. Find the weight of a square in the hanger. Show or explain your reasoning.

Student Workbook


2 More Hanging Blocks

Are You Ready for More?

What is the weight of a square on this hanger if a triangle weighs 3 grams?

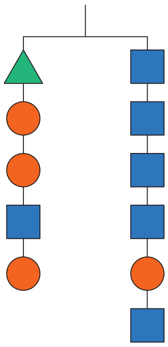
Launch

Arrange students in groups of 2.

Give 5 minutes of quiet work time followed by partner discussion.

Let students know that they should be prepared to share during the whole-class discussion, so they should make sure that their partner understands and agrees with the solution.

Student Task Statement



A triangle weighs 3 grams, and a circle weighs 6 grams.

1. Write an equation to represent the hanger.

   $3 + 18 + x = 5x + 6$ (or equivalent)

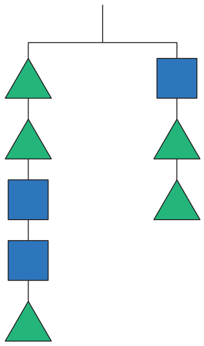
2. Find the weight of a square in the hanger. Show or explain your reasoning.

   $\frac{15}{4}$ grams (or equivalent)

   Sample reasoning: $21 + x = 5x + 6$ is equivalent to $21 = 4x + 6$. Subtract 6 from each side to get $15 = 4x$, then divide each side by 4.

Are You Ready for More?

What is the weight of a square on this hanger if a triangle weighs 3 grams?



This hanger is not possible because removing 1 square and 2 triangles from each side leaves nothing on the right side and 1 square and 1 triangle on the left. This would mean the square would need to have negative weight, which does not make sense.

Activity Synthesis

The purpose of the discussion is to make connections between moves done to hangers that keep them balanced and moves done to equations to write equivalent equations. Prepare for all to see a display with 2 columns with hanger diagrams on the one side and equations on the other. Start the discussion with the original hanger diagram.

Ask students

"What is an equation you wrote for the hanger?"

$3 + 6 + 6 + x + 6 = x + x + x + x + 6 + x$

Record the student response next to the original hanger in the right column.

Next, invite a student to suggest a move they could do to the circles that would keep the hanger balanced.

remove 1 circle from each side

Add a new hanger diagram to the hanger column showing the suggested change. Then ask,

"What is an equation we can write for this new hanger diagram?"

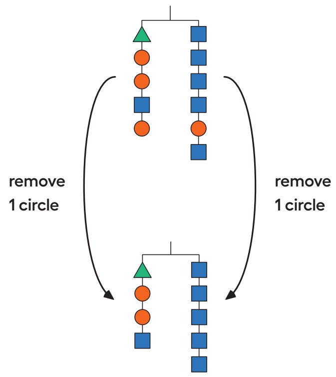
$3 + 6 + 6 + x = x + x + x + x + x$

and record the new equation in the equation column.

Referring back to earlier work using arrows with rows of equivalent equations, in each column draw arrows on each side of the *first* hanger and equation to the *second hanger and equation*. Ask,

"What should we write next to the arrows to describe what happened to the equations?"

remove 1 circle and subtract 6



$$3+6+6+x+6=x+x+x+x+6+x$$

subtract 6 subtract 6

$$3+6+6+x=x+x+x+x+x$$

If time allows, add more rows to each column, each time inviting students to suggest moves that keep the hanger in balance and the equations equivalent.

Here are some questions for discussion:

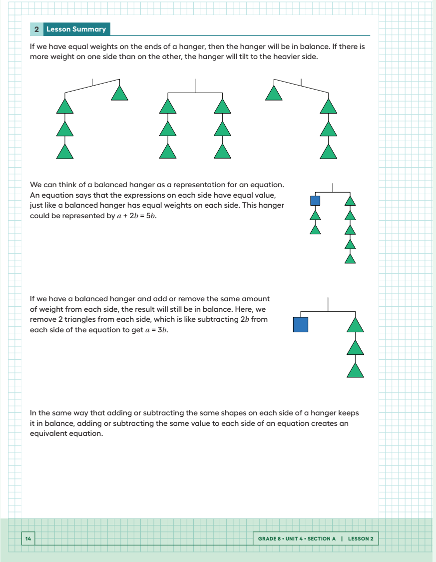
"Could we subtract x from each side of the equation to create an equivalent equation? What does that look like on the hanger diagram?"

Yes, it is the same as removing 1 square from each side.

"How is dealing with removing the triangle different when using the hanger diagrams from using the equation?"

In the equation, we can subtract 3 from each side. For the original hanger diagram, we could remove a triangle from the left side but there is no like item on the right side, so we could substitute two triangles for the circle and then remove one of the triangles.

Student Workbook


2 Lesson Summary

If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than on the other, the hanger will tilt to the heavier side.

We can think of a balanced hanger as a representation for an equation. An equation says that the expressions on each side have equal value, just like a balanced hanger has equal weights on each side. This hanger could be represented by $a + 2b = 5b$.

If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance. Here, we remove 2 triangles from each side, which is like subtracting $2b$ from each side of the equation to get $a = 3b$.

In the same way that adding or subtracting the same shapes on each side of a hanger keeps it in balance, adding or subtracting the same value to each side of an equation creates an equivalent equation.

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Lesson Synthesis

The purpose of this discussion is to have students revisit the *Warm-up* and connect it to the activities, reflecting on why the hanger is an appropriate and helpful analogy for an equation.

Ask these questions:

"What is an equation?"

An equation is a statement that two expressions have the same value.

"What does the equal sign in an equation tell you?"

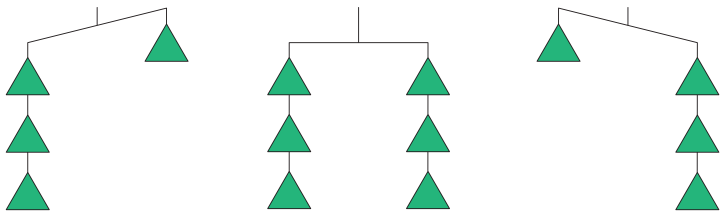
The equal sign tells you that the expressions on either side must have the same value. In these examples, we used weights to represent the values on each side.

"What features do balanced hangers and equations have in common?"

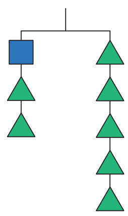
Both representations have sides that are equal in value, even if the actual value of a side is unknown. Each side can contain numbers that we do not know in the form of either shapes or variables. Changing the value of one side of a hanger or equation means changing the value of the other side by the same amount if you want them to stay equal or balanced.

Lesson Summary

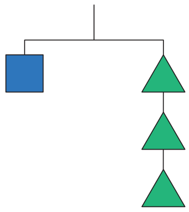
If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than on the other, the hanger will tilt to the heavier side.



We can think of a balanced hanger as a representation for an equation. An equation says that the expressions on each side have equal value, just like a balanced hanger has equal weights on each side. This hanger could be represented by $a + 2b = 5b$.



If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance. Here, we remove 2 triangles from each side, which is like subtracting $2b$ from each side of the equation to get $a = 3b$.



In the same way that adding or subtracting the same shapes on each side of a hanger keeps it in balance, adding or subtracting the same value to each side of an equation creates an equivalent equation.

Responding To Student Thinking

Points to Emphasize

If students struggle with naming moves, have students reflect on and review the types of hanger diagram moves that keep a hanger balanced. For example, after the Warm-up, have students revisit the hanger diagrams to describe why each move keeps the hangers in balance.

Unit 4, Lesson 3, Warm-up Matching Hangers

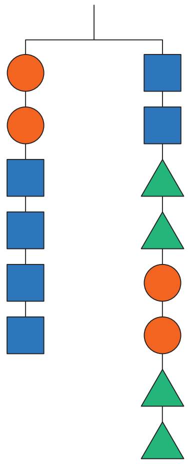
Cool-down

Changing Blocks

5 min

Student Task Statement

Here is a hanger that is in balance. We don't know how much any of its shapes weigh. How could you change the number of shapes on it, but keep it in balance? Describe in words or draw a new diagram.

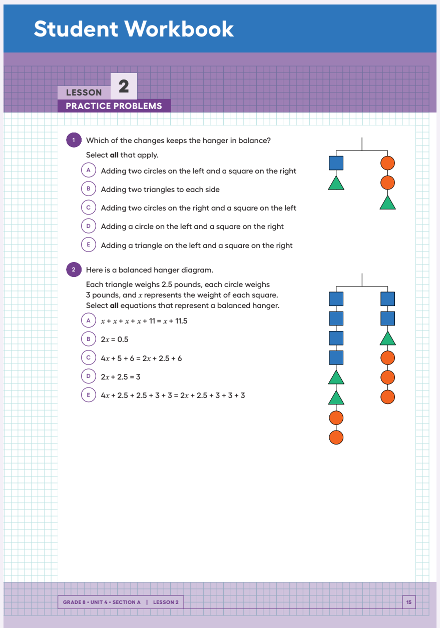


1. How could you remove shapes from the hanger and keep it in balance?

   Sample response: I can remove 2 circles from each side.

2. How could you add shapes to the hanger and keep it in balance?

   Sample response: I can add 1 triangle to each side.


Student Workbook
LESSON 2
PRACTICE PROBLEMS
1 Which of the changes keeps the hanger in balance?
Select **all** that apply.
A Adding two circles on the left and a square on the right
B Adding two triangles to each side
C Adding two circles on the right and a square on the left
D Adding a circle on the left and a square on the right
E Adding a triangle on the left and a square on the right
2 Here is a balanced hanger diagram.
Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square. Select **all** equations that represent a balanced hanger.
A $x + x + x + x + 11 = x + 11.5$
B $2x = 0.5$
C $4x + 5 + 6 = 2x + 2.5 + 6$
D $2x + 2.5 = 3$
E $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

Lesson **2**

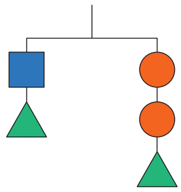
Practice Problems

5 Problems

Problem 1

Which of the changes keeps the hanger in balance?

Select **all** that apply.



A. Adding two circles on the left and a square on the right

B. Adding two triangles to each side

C. Adding two circles on the right and a square on the left

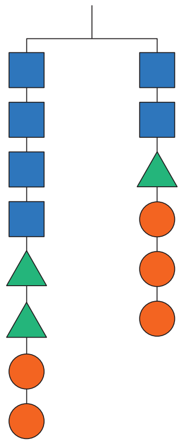
D. Adding a circle on the left and a square on the right

E. Adding a triangle on the left and a square on the right

Problem 2

Here is a balanced hanger diagram.

Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square. Select **all** equations that represent a balanced hanger.



A. $x + x + x + x + 11 = x + 11.5$

B. $2x = 0.5$

C. $4x + 5 + 6 = 2x + 2.5 + 6$

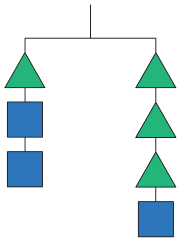
D. $2x + 2.5 = 3$

E. $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

Problem 3

What is the weight of a square if a triangle weighs 4 grams?

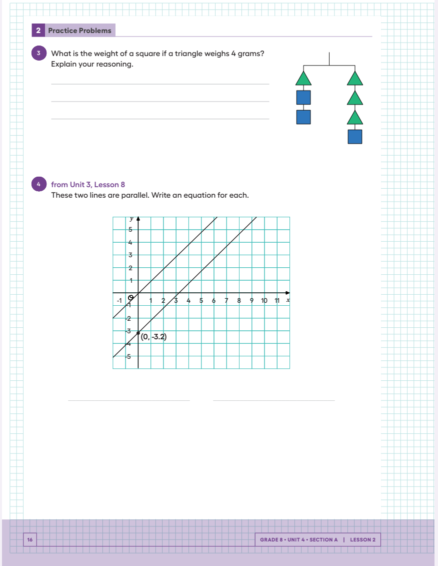
Explain your reasoning.



8 grams

Sample reasoning: There is one more square on the left than on the right and two more triangles on the right than on the left. So the square on the left balances with two triangles on the right.

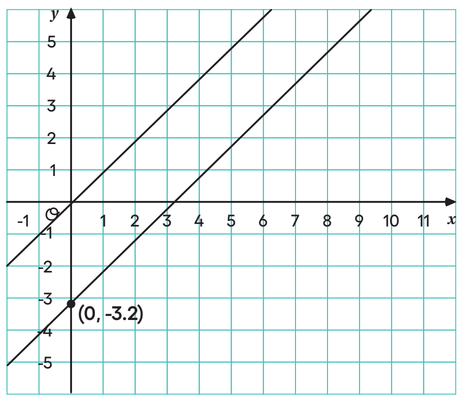
Student Workbook



Problem 4

from Unit 3, Lesson 8

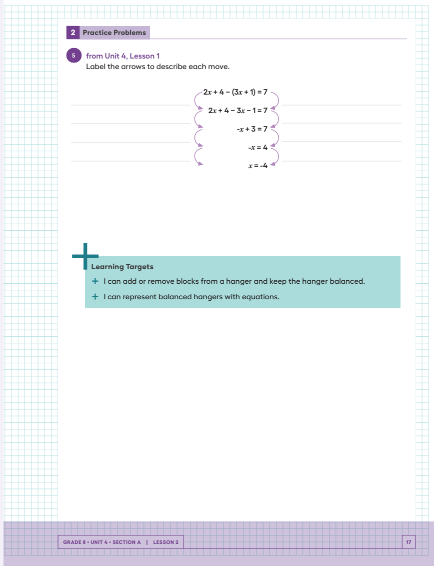
These two lines are parallel. Write an equation for each.



Sample responses:

- $y = \frac{4}{5}x$, $\frac{y}{x} = \frac{4}{5}$, or $\frac{y-4}{x-5} = \frac{4}{5}$ (or equivalent)
- $y = \frac{4}{5}(x-4)$ or $\frac{y}{x-4} = \frac{4}{5}$ (or equivalent)

Student Workbook



Problem 5

from Unit 4, Lesson 1

Label the arrows to describe each move.

	$2x + 4 - (3x + 1) = 7$	
distributive property		distributive property
	$2x + 4 - 3x - 1 = 7$	
combine like terms		combine like terms
	$-x + 3 = 7$	
add -3 on each side (or equivalent)		add -3 on each side (or equivalent)
	$-x = 4$	
multiply by -1 (or equivalent)		multiply by -1 (or equivalent)
	$x = -4$	