

Representing Ratios with Diagrams

Goals

- Coordinate discrete diagrams and multiple written sentences describing the same ratios.
- Draw and label discrete diagrams to represent situations involving ratios.
- Practice reading and writing sentences describing ratios, for example, “The ratio of these to those is $a:b$. The ratio of these to those is a to b . For every a of these, there are b of those.”

Learning Targets

- I can draw a diagram that represents a ratio and explain what the diagram means.
- I include labels when I draw a diagram that represents a ratio, so that the meaning of the diagram is clear.

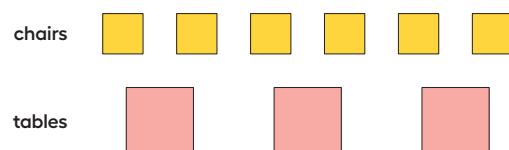
Lesson Narrative

In this lesson, students use diagrams to represent situations involving ratios and continue to develop ratio language.

Previously, students used words to describe ratios in collections of objects. Here, they also draw diagrams to represent ratios and interpret statements about ratios. Both the visual and verbal descriptions of ratios demand careful attention to language.

There is not really a right or wrong way to draw a diagram. What matters is that the diagram accurately represents the quantities, makes sense to the student and others, and can be explained. That said, one goal of this lesson is to help students draw useful diagrams efficiently.

For example, here is a diagram that represents 6 chairs and 3 tables in a room.



Lesson Timeline

5
min

Warm-up

15
min

Activity 1

15
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- Card Sort
- Take Turns
- Which Three Go Together?

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 1

Materials to Copy

- Pencil Case Cards (1 copy for every 1 student): Activity 2

Required Preparation

Activity 2:

Make 1 copy of the blackline master for each group of 2 students, plus a few extras. Place two copies of uncut blackline masters in envelopes to serve as answer keys. Cut the remaining copies blackline master for students to use. Discard the cut slips that say “The above diagram also matches this sentence.”

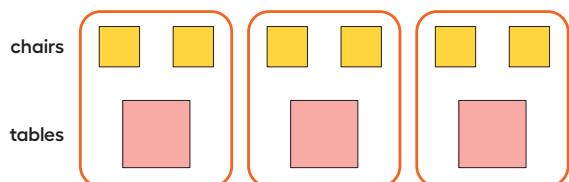
Consider using a different color of paper for each group’s copy of the blackline master so that misplaced slips can quickly be put back.

Representing Ratios with Diagrams

Lesson Narrative (continued)

When asked to draw diagrams, students may include unnecessary details, for instance, by making each chair look like an actual chair. Examples of very simple diagrams, such as the discrete diagrams shown here, can guide students toward more abstract representations while still offering visual or spatial cues to support reasoning. (Students do not need to know the term “discrete diagrams.”)

Diagrams can also help students see associations between quantities in different ways. For example, we can see that there are 2 chairs for each table by grouping the items as shown.



Students may say “for every 2 chairs there is 1 table,” but for now, we do not suggest writing the association as $2:1$. Equivalent ratios will be carefully developed in upcoming lessons.

While the discrete diagrams here reflect the parallel structure of the double number line diagrams that they will see later in the unit, students do not need to draw them this way as long as they can explain their diagrams and interpret those shown in the lesson.

Student Learning Goal

Let's use diagrams to represent ratios.

Instructional Routines

Which Three Go Together?

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Student Workbook

LESSON 2

Representing Ratios with Diagrams

Let's use diagrams to represent ratios.

Warm-up Which Three Go Together: Representations

Which three go together? Why do they go together?

(A)

(B)

(C)

(D)

2 green objects

Blue Paint and Play Clay

Elena mixed 2 cups of white paint with 6 tablespoons of blue paint. Here is a diagram that represents this situation.

white paint (cups)

blue paint (tablespoons)

GRADE 6 • UNIT 2 • SECTION A | LESSON 2

Warm-up

Which Three Go Together: Representations

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four representations of quantities in preparation for describing ratios of quantities. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terms that students know and how they talk about visual representations.

Launch



Arrange students in groups of 2–4. Display the representations for all to see. Give students 1 minute of quiet think time.

Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

(A)



(B)



(C)



(D)

2 green objects

Sample responses:

- A, B, and C go together because they show a picture or an image.
- A, B, and D go together because they all involve green objects.
- A, C, and D go together because they represent 2 objects.
- B, C, and D go together because they show representations of objects and not actual objects.

Activity Synthesis

Invite each group to share one reason why a particular set of three goes together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

If students use different terms (such as “picture,” “drawing,” or “diagram”) to describe the same representation, invite students to notice how they are referring to the representation. Acknowledge that we may use different terms to describe visual representations and may interpret the same term in different ways.

Activity 1**Blue Paint and Play Clay**15
min**Activity Narrative**

In this activity, students continue to draw connections between a diagram and the ratios it represents. Students discuss with a partner different ways to use ratio language to describe discrete diagrams. They first identify statements that would correctly describe a given diagram. Then, they create both a diagram and corresponding statements to represent a new situation involving ratio.

Monitor for the different kinds of diagrams that students create and the different statements that they write to describe the play clay recipe. Here are some ways in which students may create diagrams, from more elaborate to more simple:

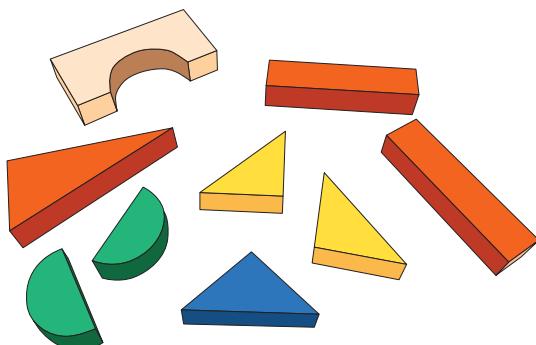
- draw literal drawings of cups and pints
- draw rectangles, circles, or other shapes
- use letters or other symbols

They may write statements that:

- Use the given values, such as: “the ratio of cups of corn flour to pints of glue is 8 to 2.”
- Regroup the quantities, such as: “for every 4 cups of corn flour, Jada used 1 cup of glue.”
- Use equivalent ratios, even though these have not been explicitly taught, such as: “the ratio of cups of corn flour to pints of glue is 4:1.” While this is correct and we have regrouped quantities and used language such as: “for every 4 of something, there is 1 of another thing,” we have not yet asserted that the ratio can be written as 4:1. The idea of equivalent ratios is sophisticated and will be developed over the next several lessons.
- Include fractions, such as: “for every 1 cup of corn flour, Jada used $\frac{1}{4}$ pint of glue.” Although students are not expected to work with fractions in this lesson, responses involving fractions are fine.

Launch

Arrange students in groups of 2. Display the following image of blocks or an actual collection of blocks. To reinforce the ratio language from an earlier lesson, ask students to describe to a partner a ratio that they see, using the ratio language they learned. Invite a few students to share.

**Instructional Routines****5 Practices**

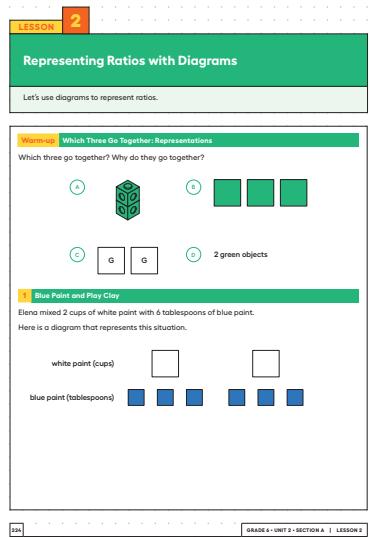
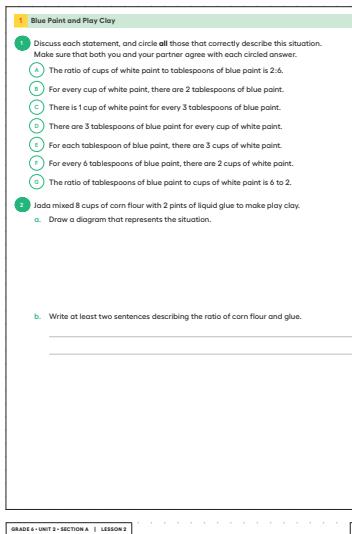
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Building on Student Thinking

Some students may think that all of the statements about the paint mixture are accurate descriptions. If so, suggest that there are two false statements. Have students discuss the statements again as they determine which two are false.

Student Workbook**Student Workbook**

If no one mentions a ratio in which the numbers are the same (such as “2 to 2”), present a statement such as “the ratio of yellow blocks to green blocks is 2 to 2.” Ask if the sentence represents a ratio in the image and why or why not. If students suspect that ratios are only used to associate quantities with different values, clarify that this is not the case.

For the first part of the task, ensure that students understand that they are supposed to select more than one statement. Consider having students take turns reading each statement and deciding whether they think it describes the situation or not.

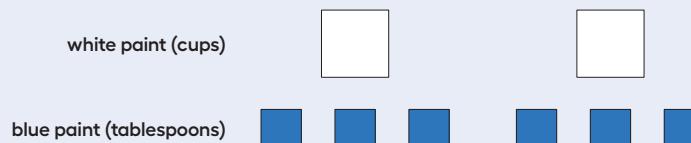
For the second part, clarify what play clay or modeling clay is or show an example, if needed. Explain that homemade clay can be made by mixing ingredients such as flour, water, salt, and oil. Some recipes use only two ingredients, such as corn flour and liquid glue.

Select groups of students who draw different diagrams and use ratio language differently and, later, ask them to share.

Student Task Statement

Elena mixed 2 cups of white paint with 6 tablespoons of blue paint.

Here is a diagram that represents this situation.



1. Discuss each statement, and circle **all** those that correctly describe this situation. Make sure that both you and your partner agree with each circled answer.

- A. The ratio of cups of white paint to tablespoons of blue paint is 2:6.
- B. For every cup of white paint, there are 2 tablespoons of blue paint.
- C. There is 1 cup of white paint for every 3 tablespoons of blue paint.
- D. There are 3 tablespoons of blue paint for every cup of white paint.
- E. For each tablespoon of blue paint, there are 3 cups of white paint.
- F. For every 6 tablespoons of blue paint, there are 2 cups of white paint.
- G. The ratio of tablespoons of blue paint to cups of white paint is 6 to 2.

2. Jada mixed 8 cups of corn flour with 2 pints of liquid glue to make play clay.

- a. Draw a diagram that represents the situation.
- b. Write at least two sentences describing the ratio of corn flour and glue.

Sample responses:

- The ratio of cups of corn flour to pints of glue is 8:2.
- The ratio of pints of glue to cups of corn flour is 2 to 8.
- For each pint of glue, there are 4 cups of corn flour.
- For every 8 cups of corn flour, there are 2 pints of glue.
- For every 4 cups of corn flour, there is 1 pint of glue.
- There are 2 pints of glue for every 8 cups of corn flour.

Activity Synthesis

Invite previously selected students to share. Sequence the diagrams in the order listed in the *Activity Narrative*.

Connect the various ways in which the quantities in the play clay can be represented and described. For example, ask questions such as:

- ❑ “How can all the drawings—of objects, shapes, and letters—represent the same situation?”

They all show the ratio of corn flour to glue but with different levels of detail.

- ❑ “How can we keep track of what each drawing or each mark represents?”

Labels can help us keep track of the quantities.

- ❑ “Does it matter the order in which we draw the quantities or name them in a sentence?”

No, we can draw or name the items in different orders, but the numbers must match the ratio in the situation.

- ❑ “How might we decide whether to create detailed drawings or draw diagrams?”

It might depend on the quantities or situation. In this case, a discrete diagram is a more efficient way of showing the recipe.

Instructional Routines**Card Sort**ilclass.com/r/10783726

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**Instructional Routines****Take Turns**ilclass.com/r/10573524

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**Access for Students with Diverse Abilities (Activity 2, Launch)****Action and Expression: Develop Expression and Communication.**

To help get students started, display sentence frames such as, “_____ and _____ are a match because ...”

Supports accessibility for: Language; Organization

Activity 2**Card Sort: Pencil Cases**15
min**Activity Narrative**

Writing and using ratio language requires attention to detail. This activity further develops students' ability to describe ratio situations precisely by attending carefully to the quantities, their units, and their order in the ratio.

Students are given a set of cards showing discrete diagrams and statements that represent ratios of objects in pencil cases. They work with a partner to find diagrams and statements that represent the same ratio. Students take turns making a match and explaining their reasoning. In doing so, they practice constructing viable arguments and critiquing the reasoning of others.

This is the first *Card Sort* activity of the course. An important aspect of this routine is to allow students time at the start to sort the cards into categories of their choosing. This step gives students the opportunity to familiarize themselves with the content of the cards without the additional pressure of organizing them in a specific fashion. It also provides insight into the aspects of each card students attend to and the language they have to describe their observations.

Launch

Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to sort the cards into categories of their choosing.

- Pause the class after students have sorted the cards.
- Select groups to share their categories and how they sorted their cards.
- Discuss as many different types of categories as time allows.

Next, demonstrate how to set up and play the matching game: Choose a student to be your partner. Discuss what all the symbols mean. Mix up the cards, place them face-up, and reiterate that they contain either diagrams or sentences, which students are likely to have noticed earlier. Select one of each style of card, and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree, for instance, by explaining your mathematical thinking, asking clarifying questions, and so on.

Arrange students in groups of 2, and give each group the pre-cut cards. Tell students to use the answer keys to check their matches after they complete the activity.

Student Task Statement

Your teacher will give you cards showing diagrams or sentences that describe the items in different pencil cases. In the diagrams:

- A circle represents a pencil.
- A square represents an eraser.
- A triangle represents a paper clip.

1. Take turns with your partner to match a sentence with a diagram.
 - a. For each match that you find, explain to your partner how you know it's a match.
 - b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
 - **Diagram A matches with sentence 4.**
 - **Diagram B matches with sentences 2 and 8.**
 - **Diagram C matches with sentence 1.**
 - **Diagram D matches with sentence 5.**
 - **Diagram E matches with sentences 3 and 7.**
 - **Diagram F matches with sentence 6.**

2. After you and your partner have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

No response required.

3. There were two diagrams that each matched with two different sentences. Which were they?

- Diagram B matched with both sentences 2 and 8.
- Diagram E matched with both sentences 3 and 7.

4. Select one of the other diagrams and invent another sentence that could describe the ratio shown in the diagram.

Sample responses:

- For diagram A, the ratio of pencils to erasers is 3:1.
- For diagram D, the ratio of erasers to pencils is 2 to 5.

Building on Student Thinking

If students disagree about a match, encourage them to figure out the correct answer through discussion and use of the answer key. Make sure that when students use the answer key, they discuss any errors rather than just make changes.

Students may think that the shapes in the diagram need to be drawn in the same order in which the ingredients appear in the description. This is not the case. We could turn a diagram card upside down and it would still represent the same situation. The purpose of the diagram is to show the items in the pencil cases. It is the case, however, that within the description, the order of the words in the sentence must correspond with the terms within the ratio.

Student Workbook

Card Sort: Pencil Cases

Your teacher will give you cards showing diagrams or sentences that describe the items in different pencil cases. In the diagrams:

- A circle represents a pencil.
- A square represents an eraser.
- A triangle represents a paper clip.

1. Take turns with your partner to match a sentence with a diagram.
 - a. For each match that you find, explain to your partner how you know it's a match.
 - b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. After you and your partner have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. There were two diagrams that each matched with two different sentences. Which were they?
 - Diagram _____ matched with both sentences _____ and _____.
 - Diagram _____ matched with both sentences _____ and _____.
4. Select one of the other diagrams and invent another sentence that could describe the ratio shown in the diagram.

Are You Ready for More?

Andre has markers, pens, and rulers in his drawer. There are 2 markers for every ruler. There are 3 pens for every marker.

What is the ratio of pens to markers to rulers? _____

Draw a diagram to show your reasoning.

**Access for Multilingual Learners
(Activity 2, Synthesis)**
MLR8: Discussion Supports.

To demonstrate and amplify mathematical language and to help students communicate their reasoning more clearly, revoice students' ideas and press for more details. For example, if a student says that they matched Diagram D with Sentence 5, ask,

"What did you see in Diagram D that matched with the words of Sentence 5?"

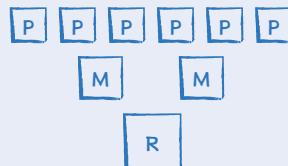
Advances: Speaking

Are You Ready for More?

Andre has markers, pens, and rulers in his drawer. There are 2 markers for every ruler. There are 3 pens for every marker.

1. What is the ratio of pens to markers to rulers? Draw a diagram to show your reasoning.

The ratio of pens to markers to rulers are 6:2:1.



2. Keeping that ratio, could there be 9 pens in Andre's drawer? Explain your reasoning.

No

Sample reasoning: If there were 9 pens, then there would be 3 markers and $1\frac{1}{2}$ rulers. It is not possible to have a fraction of a ruler.

Activity Synthesis

After all groups have completed the matching, discuss the following:

Which matches were tricky? Explain why.

Did any groups need to make adjustments in their matches? What might have caused an error? What adjustments were made?

Lesson Synthesis

This lesson used diagrams to represent ratios. These diagrams omit details that are not necessary for understanding and solving the problem at hand. Discuss:

❑ “What are some good things to remember when we draw a diagram of a ratio?”

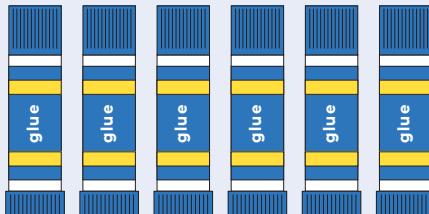
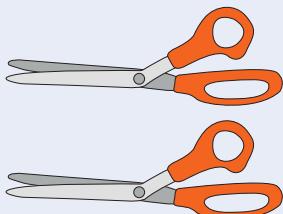
We need only necessary information. We can use shapes, color-coded boxes, or initials to represent the objects. It is helpful to organize the types of items in rows, and to arrange smaller groups so they are easier to see.

❑ “How can a diagram help you make sense of a situation involving a ratio?”

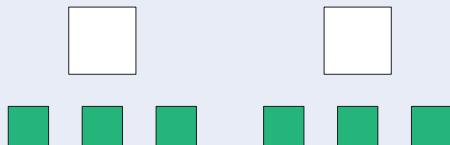
It can help us see how the objects can be grouped. It can help us write correct statements about the objects.

Lesson Summary

Ratios can be represented using diagrams. The diagrams do not need to include realistic details. For example, there are 2 pairs of scissors and 6 glue sticks in a bin. Instead of this:



We can draw something like this:



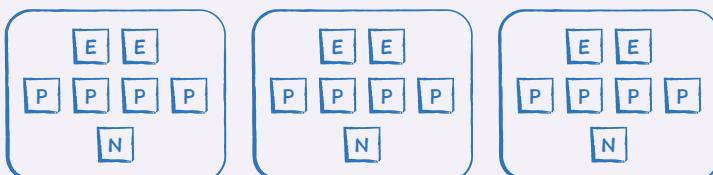
This diagram shows that the ratio of glue sticks to pairs of scissors is 6 to 2. We can also see that for every pair of scissors, there are 3 glue sticks.

Cool-down**Stationery Sets**5
min**Student Task Statement**

Lin has 3 sets of stationery. Each set has 2 erasers, 4 pencils, and 1 notepad.

1. Draw a diagram that shows an association between the numbers of erasers, pencils, and notepads that Lin has.

Sample response:



- a. The ratio of erasers to pencils to notepads is 6 : 12 : 3.
- b. There are 4 pencils for every notepad.
- c. There are 2 pencils for every eraser.

Student Workbook

Card Sort: Pencil Cases

1 Keeping the ratio, could there be 9 pens in Andre's drawer? Explain your reasoning.

2 Lesson Summary

Ratios can be represented using diagrams. The diagrams do not need to include realistic details. For example, there are 2 pairs of scissors and 6 glue sticks in a bin. Instead of this:

We can draw something like this:

This diagram shows that the ratio of glue sticks to pairs of scissors is 6 to 2. We can also see that for every pair of scissors, there are 3 glue sticks.

GRADE 6 • UNIT 2 • SECTION A | LESSON 2

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Responding To Student Thinking**Points to Emphasize**

If students struggle with describing ratios, when opportunities arise over the next several lessons, focus on the verbal description of ratios. Emphasize that the order in which the quantities are described in the sentence must correspond with the order in the ratio. For example, ask students to attend to how the recipe ingredients are described (such as “teaspoons of drink mix to cups of water”) and to how the amounts are listed (such as “4 to 1”) in this activity: Unit 2, Lesson 3, Activity 2 Powdered Drink Mix

Student Workbook

LESSON 2
PRACTICE PROBLEMS

1. Here is a diagram that describes the cups of green and white paint in a mixture.
-

- Select all the statements that correctly describe this diagram.
- The ratio of cups of white paint to cups of green paint is 2 to 4.
 - For every cup of green paint, there are two cups of white paint.
 - The ratio of cups of green paint to cups of white paint is 4 : 2.
 - For every cup of white paint, there are two cups of green paint.
 - The ratio of cups of green paint to cups of white paint is 2 : 4.

2. To make a snack mix, combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

- a. Create a diagram to represent the quantities of each ingredient in this recipe.

- b. Use your diagram to complete each sentence.

- The ratio of _____ to _____ is _____.
- There are _____ cups of pretzels for every cup of raisins.
- There are _____ cups of almonds for every cup of raisins.

ANSWER

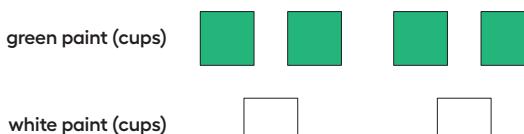
GRADE 4 • UNIT 3 • SECTION A | LESSON 2

Practice Problems

5 Problems

Problem 1

Here is a diagram that describes the cups of green and white paint in a mixture.



Select all the statements that correctly describe this diagram.

A. The ratio of cups of white paint to cups of green paint is 2 to 4.

B. For every cup of green paint, there are two cups of white paint.

C. The ratio of cups of green paint to cups of white paint is 4:2.

D. For every cup of white paint, there are two cups of green paint.

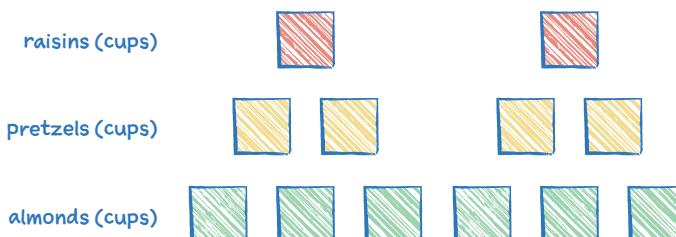
E. The ratio of cups of green paint to cups of white paint is 2:4.

Problem 2

To make a snack mix, combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

a. Create a diagram to represent the quantities of each ingredient in this recipe.

Sample response:



b. Use your diagram to complete each sentence.

Sample response:

- The ratio of cups of raisins to cups of pretzels to cups of almonds is 2 : 4 : 6.
- There are 2 cups of pretzels for every cup of raisins.
- There are 3 cups of almonds for every cup of raisins.

Lesson 2 Practice Problems

Problem 3

from Unit 1, Lesson 17

- a. A square is 3 inches by 3 inches. What is its area?

9 square inches ($3 \cdot 3 = 9$)

- b. A square has a side length of 5 feet. What is its area?

25 square feet ($5 \cdot 5 = 25$)

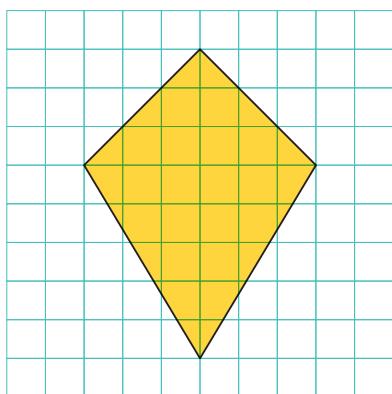
- c. The area of a square is 36 square centimeters. What is the length of each side of the square?

6 centimeters ($6 \cdot 6 = 36$)

Problem 4

from Unit 1, Lesson 11

Find the area of this quadrilateral. Explain or show your strategy.



24 square units

Sample reasoning: Decompose the quadrilateral into two triangles with a horizontal cut. The top triangle has a base of 6 units and a height of 3 units. Its area is 9 square units, as $(6 \cdot 3) \div 2 = 9$. The bottom triangle has a base of 6 units and a height of 5 units. Its area is 15 square units, as $(6 \cdot 5) \div 2 = 15$. $9 + 15 = 24$. Because $9 + 15 = 24$, the area of the quadrilateral is 24 square units.

Problem 5

from Unit 2, Lesson 1

Complete each equation with a number that makes it true.

$$\frac{1}{8} \cdot 8 = \underline{\hspace{2cm}}$$

1 (or equivalent)

$$\frac{1}{8} \cdot 7 = \underline{\hspace{2cm}}$$

$\frac{7}{8}$ (or equivalent)

$$\frac{3}{8} \cdot 8 = \underline{\hspace{2cm}}$$

3 (or equivalent)

$$\frac{3}{8} \cdot 7 = \underline{\hspace{2cm}}$$

$\frac{21}{8}$ (or equivalent, $2\frac{5}{8}$ for example)

Student Workbook

2 Practice Problems

from Unit 1, Lesson 17

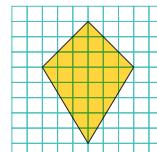
- a. A square is 3 inches by 3 inches. What is its area? _____

- b. A square has a side length of 5 feet. What is its area? _____

- c. The area of a square is 36 square centimeters. What is the length of each side of the square? _____

from Unit 1, Lesson 11

Find the area of this quadrilateral. Explain or show your strategy.



Student Workbook

3 Practice Problems

from Unit 2, Lesson 1

Complete each equation with a number that makes it true.

$$\frac{1}{8} \cdot 8 = \underline{\hspace{2cm}} \quad \frac{1}{8} \cdot 7 = \underline{\hspace{2cm}}$$

$$\frac{3}{8} \cdot 8 = \underline{\hspace{2cm}} \quad \frac{3}{8} \cdot 7 = \underline{\hspace{2cm}}$$

Learning Targets

- + I can draw a diagram that represents a ratio and explain what the diagram means.
- + I include labels when I draw a diagram that represents a ratio, so that the meaning of the diagram is clear.

