

Rational and Irrational Numbers

Goals

- Comprehend the term “irrational number” (in spoken language) to mean a number that is not rational and that $\sqrt{2}$ is an example of an irrational number.
- Comprehend the term “rational number” (in written and spoken language) to mean a number that can be expressed as a positive or negative fraction.
- Determine whether or not a given rational number is a solution to the equation $x^2 = 2$ and explain (orally) the reasoning.

Learning Targets

- I know what an irrational number is and can give an example.
- I know what a rational number is and can give an example.

Lesson Narrative

In this lesson, students build on previous work with square roots to learn about irrational numbers. Students recall the definition of **rational numbers**, numbers that can be expressed as positive or negative fractions, as they search for a number x such that $x^2 = 2$.

Students begin by finding positive solutions to equations of the form $x^2 = n$ by finding values of x that will make each given equation true. They build on this idea as they try to find a solution to the specific equation $x^2 = 2$ in three different ways.

Then students consider a square with area 2 square units and are asked to find its side length. Some students may choose a measuring strategy, while others may choose to use square root notation. By comparing these two strategies, students begin to see that the value of $\sqrt{2}$ is close to 1.5.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR7: Compare and Connect (Activity 1)
- MLR2: Collect and Display (Activity 3)

Instructional Routines

- Math Talk
- MLR7: Compare and Connect
- MLR2: Collect and Display

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2
- Tracing paper: Activity 2
- Four-function calculators: Activity 4

Required Preparation

Lesson:

It would be useful throughout this unit to have a list of perfect squares for easy reference. Consider hanging up a poster that shows the 20 perfect squares from 1 to 400. It is particularly handy in this lesson.

Lesson Timeline

5
min

Warm-up

10
min

Activity 1

10
min

Activity 2

10
min

Activity 3

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Rational and Irrational Numbers

Lesson Narrative (continued)

Next, students are given a series of values and must test if the square of any of them equals exactly 2. While the values are specifically chosen to get closer and closer to the value of $\sqrt{2}$, none of them will equal $\sqrt{2}$ exactly. Students continue searching, aided by the use of a calculator (without a square root button). They will not be able to find a rational number whose square equals exactly 2, allowing for the introduction of the term **irrational number**: a number that cannot be expressed as a positive or negative fraction.

Students should not be left with the impression that looking for and failing to find a rational number whose square is 2 is proof that $\sqrt{2}$ is irrational. Proving this is out of the scope of grade 8, and so ultimately students must just accept it as fact for now.

Student Learning Goal

Let's learn about irrational numbers.

Instructional Routines

Math Talk

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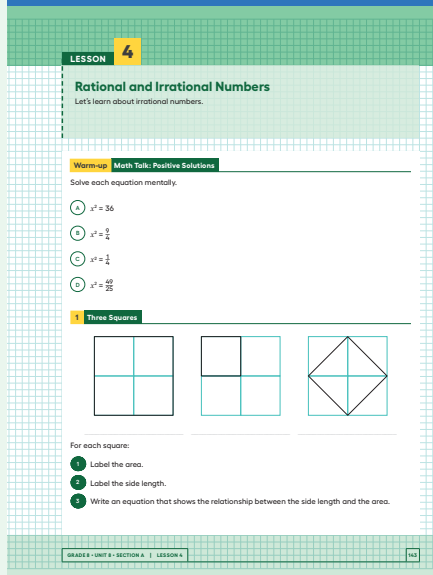
Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



Warm-up

Math Talk: Positive Solutions

5 min

Activity Narrative

This *Math Talk* focuses on reviewing multiplication of fractions. It encourages students to think about squaring integers and to rely on the structure of fractions to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students estimate solutions to the equation $x^2 = 2$.

For this activity it is best if students work with fractions and do not convert numbers to their decimal forms. Answers expressed in decimal form aren't wrong, but working with decimal forms will miss out on the purpose of this *Warm-up*.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time and ask them to give a signal when they have an answer and a strategy.

Invite students to share their strategies and record and display their responses for all to see.

Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A. $x^2 = 36$

6

Sample reasoning: $6 \cdot 6 = 36$

B. $x^2 = \frac{9}{4}$

$\frac{3}{2}$

Sample reasoning: $\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$

C. $x^2 = \frac{1}{4}$

$\frac{1}{2}$

Sample reasoning: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

D. $x^2 = \frac{49}{25}$

$\frac{7}{5}$

Sample reasoning: since $7^2 = 49$ and $5^2 = 25$, we know $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$

Activity Synthesis

To involve more students in the conversation, consider asking:

“Who can restate ____’s reasoning in a different way?”

“Did anyone use the same strategy but would explain it differently?”

“Did anyone solve the problem in a different way?”

“Does anyone want to add on to ____’s strategy?”

“Do you agree or disagree? Why?”

“What connections to previous problems do you see?”

Activity 1

Three Squares

10
min

Activity Narrative

This is the first of three activities where students investigate the value of $\sqrt{2}$. By finding the areas and side lengths of three specific squares, students see that the value of $\sqrt{2}$ is a bit less than 1.5. In following activities, students will look for a more precise value of $\sqrt{2}$.

Monitor for students who:

- Compare lengths directly either by creating a grid ruler or by tracing a segment with tracing paper and bringing it side by side with another segment. Most likely these students will say that the side length of the tilted square is around 1.5 units or a little less.
- Find the area of the tilted square first and recall the square root notation from the previous lesson. Since the area of the tilted square is 2 square units, its side length can be expressed as $\sqrt{2}$ units.

Launch

Provide access to geometry toolkits, including tracing paper.

Give students 3–4 minutes of quiet work time followed by a whole class discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially of students who haven’t shared recently.

Access for Multilingual Learners
(Warm-up, Synthesis)**MLR8: Discussion Supports.**

Display sentence frames to support students when they explain their strategy. For example, “First, I ____ because ...” or “I noticed ____, so I ...” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners
(Activity 1)**MLR7: Compare and Connect**

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Building on Student Thinking

If students say that the side length of the tilted square is 1 unit, consider asking:

“How did you measure the length of the side of the tilted square?”
“How does the area of the tilted square compare to the area of a grid square?”

Student Workbook

LESSON 4

Rational and Irrational Numbers
 Let's learn about irrational numbers.

Warm-up Math Talk: Positive Solutions
 Solve each equation mentally.

- $x^2 = 36$
- $x^2 = \frac{9}{4}$
- $x^2 = \frac{1}{4}$
- $x^2 = \frac{25}{16}$

1 Three Squares

For each square:

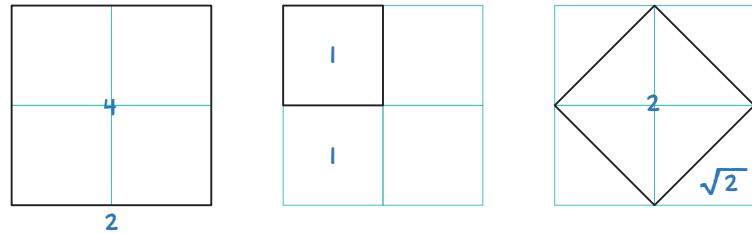
- Label the area.
- Label the side length.
- Write an equation that shows the relationship between the side length and the area.

GRADE 8 • UNIT 8 • SECTION A | LESSON 4

Student Task Statement

For each square:

1. Label the area.



2. Label the side length.

See image.

3. Write an equation that shows the relationship between the side length and the area.

$$2^2 = 4; 1^2 = 1; (\sqrt{2})^2 = 2$$

Activity Synthesis

The goal of this discussion is to connect the idea that $\sqrt{2}$ can be approximated by measuring the side length of a square with an area of 2 square units.

Display strategies from previously selected students for finding the side length of the tilted square. Invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

“How are the two approaches different?”

One is measuring to find an approximate value, and the other one gives us an exact value.

“Are there any benefits or drawbacks to one approach compared to the other?”

The measuring strategy is easier and could be faster, but finding the area of the square gives us an exact answer.

“How can you use what you know about the largest square and the smallest square to determine the side length of the remaining square?”

The value of $\sqrt{2}$ is somewhere between 1 and 2 because an area of 2 square units is between 1 and 4 square units.

Activity 2

Looking for a Solution

10
min

Activity Narrative

This is the second of three activities in which students investigate the value of $\sqrt{2}$. The goal is to achieve a more precise value of $\sqrt{2}$ by checking for possible solutions to $x^2 = 2$, and to revisit the definition of a rational number in preparation for defining an irrational number.

Since $\sqrt{2}$ is a number that can be multiplied by itself to get 2, it is true that $\sqrt{2}^2 = 2$.

Launch

Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

If needed, remind students that 1.5 is equivalent to $\frac{3}{2}$.

Student Task Statement

Are any of these numbers a solution to the equation $x^2 = 2$?
Explain your reasoning.

1. 1

No, because $1^2 = 1$, not 2.

2. $\frac{1}{2}$

No, because $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

3. $\frac{3}{2}$

No, because $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. ($\frac{9}{4} > 2$, since $2 = \frac{8}{4}$.)

4. $\frac{7}{5}$

No, because $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$. ($\frac{49}{25} < 2$, since $2 = \frac{50}{25}$.)

Activity Synthesis

The purpose of this discussion is to revisit the definition of a **rational number** as a number that can be expressed as a positive or negative fraction. Students also add to their understanding that some square roots are rational numbers.

Select students to share their reasoning for why each number is not a solution to $x^2 = 2$. If not brought up in students' explanations, make sure that they notice that $\frac{3}{2}$ is not a terrible approximation for $\sqrt{2}$, since $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, and $\frac{9}{4}$ is only a bit larger than 2. An even better approximation for $\sqrt{2}$ is $\frac{7}{5}$, since $\left(\frac{7}{5}\right)^2 = \frac{49}{25}$, which is just a little bit smaller than $\frac{50}{25}$ (which equals 2). Refer to the display of perfect squares as necessary.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication.

To help get students started, display sentence frames such as "If _____, then _____ because ..." and "That could/couldn't be true because ..."
Supports accessibility for: Language, Organization

Student Workbook

2 Looking for a Solution
Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

1. 1

2. $\frac{1}{2}$

3. $\frac{3}{2}$

4. $\frac{7}{5}$

3 Looking for $\sqrt{2}$
A rational number is a number that can be expressed as a positive or negative fraction.

1. Find some more rational numbers that are close to $\sqrt{2}$.

2. Can you find a rational number that is exactly $\sqrt{2}$?

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Remind students that a fraction is a number on the number line that is the result of dividing the unit interval into equal parts and finding the point that is a of them from 0. We can always write a fraction in the form $\frac{a}{b}$, where a and b are whole numbers (and b is not 0), but there are other ways to write them. For example, 0.4 is a fraction because it is the point on the number line that is the result of dividing the unit interval into 10 equal parts and finding the point that is 4 away from 0. (The same point is a result of dividing the unit interval into 5 equal parts and finding the point that is 2 away from 0.) When we first learned about fractions, we didn't know about negative numbers. Rational numbers are fractions, but they can be positive or negative. So, $-\frac{2}{5}$ is also a rational number.



Here are some examples of rational numbers: $\frac{7}{4}$, 0, $\frac{6}{3}$, 0.2, $-\frac{1}{3}$, -5, $\sqrt{9}$, $-\frac{\sqrt{16}}{\sqrt{100}}$

Invite students to share how each of these examples is a number that can be expressed as a positive or negative fraction, particularly the two values represented using square roots.

Activity 3

Looking for $\sqrt{2}$

10
min

Activity Narrative

This is the third of three activities in which students investigate the value of $\sqrt{2}$. Students look for a fraction that can be multiplied by itself where the product is exactly 2. Since there is no such number, this allows for the term “irrational number” to be defined.

Simply looking for rational solutions to $x^2 = 2$ and not finding any is not a proof that $\sqrt{2}$ is irrational. For the purposes of this course, students are asked to take it as a fact that $\sqrt{2}$ is irrational. They may have opportunities to understand or write a proof that $\sqrt{2}$ is irrational in future studies.

Launch

Provide access to calculators *without* a square root button.

Before students get started, remind them that a rational number is a number that can be expressed as a positive or negative fraction, for example, $\frac{9}{8}$.

Explain that terminating decimals are also rational, for example, $0.7 = \frac{7}{10}$. Some students may focus on numbers in decimal form when searching for a rational number close to $\sqrt{2}$. Encourage these students to recall work in a previous activity where the square of $\frac{7}{5}$ was $\frac{49}{25}$, a number very close to 2.

Monitor for students who find a particularly close value to share later. Since students could search indefinitely for a solution to the last problem with no success, ask students to stop their work in order to leave 3–4 minutes for a whole-class discussion.

Student Task Statement

A **rational number** is a number that can be expressed as a positive or negative fraction.

1. Find some more rational numbers that are close to $\sqrt{2}$.

Answers vary. Sample responses:

- $\frac{10}{7}$, because $\left(\frac{10}{7}\right)^2 = \frac{100}{49}$
- $\frac{17}{12}$, because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$
- $\frac{13}{9}$, because $\left(\frac{13}{9}\right)^2 = \frac{169}{81}$
- $\frac{141}{100}$, because $\left(\frac{141}{100}\right)^2 = \frac{19,881}{10,000}$

2. Can you find a rational number that is exactly $\sqrt{2}$?

No

Are You Ready for More?

If you have an older calculator and evaluate the expression $\left(\frac{577}{408}\right)^2$, it will tell you that the answer is 2, which might lead you to think that $\sqrt{2} = \frac{577}{408}$.

1. Explain why you might be suspicious of the calculator’s result.

Sample response:

One reason to be suspicious is that the calculator might have rounded the answer, so it may just be that $\left(\frac{577}{408}\right)^2$ is close to 2. In fact, we can argue for certain that it is not.

2. Find an explanation for why $408^2 \cdot 2$ could not possibly equal 577^2 . How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?

Sample response:

The equation $577^2 = 2 \cdot 408^2$ could not possibly be true since the left-hand side of that equation is odd, but the right-hand side is even. We know $\left(\frac{577}{408}\right)^2 \neq 2$, since after multiplying both sides by 408^2 we have $577^2 = 2 \cdot 408^2$.

3. Repeat these questions for $\left(\frac{1414213562375}{1000000000000}\right)^2 \neq 2$, an equation that even many modern calculators and computers will get wrong.

Sample response:

This exact argument also works for to explain that $1414213562375^2 \neq 2 \cdot 1000000000000^2$: the left-hand side of the equation is odd and the right-hand side is even.

(In fact, by breaking it into cases depending on which of the numerator and denominator is even or odd, this line of thinking leads to a complete proof that no fraction at all can be squared to get the number 2, proving that $\sqrt{2}$ is irrational.)

Activity Synthesis

The purpose of this discussion is to define an *irrational number*. Invite previously selected students who found particularly close values to share their number with the class and what strategy they used to find it. Applaud students for their perseverance, but then confess that no such rational number exists because $\sqrt{2}$ is an irrational number.

Access for Multilingual Learners
(Activity 3, Student Task)

MLR2: Collect and Display.
Circulate, listen for and collect the language students use as they discuss strategies for finding rational numbers that are close to $\sqrt{2}$. On a visible display, record words and phrases, such as “There is no number that is equal to $\sqrt{2}$,” which can be restated as “We could not find a rational number that is equal to $\sqrt{2}$.” Invite students to borrow language from the display as needed, and update it throughout the activity.
Advances: Conversing, Reading

Student Workbook

2 Looking for a Solution

Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

1

2

$\frac{1}{2}$

3

$\frac{3}{2}$

4

$\frac{2}{3}$

5

$\frac{5}{2}$

3 Looking for $\sqrt{2}$

A rational number is a number that can be expressed as a positive or negative fraction.

1

Find some more rational numbers that are close to $\sqrt{2}$.

2

Can you find a rational number that is exactly $\sqrt{2}$?

GRADE 8 • UNIT 8 • SECTION A | LESSON 4

Student Workbook

3 Are You Ready for More?

If you have an older calculator and evaluate the expression $\left(\frac{577}{408}\right)^2$, it will tell you that the answer is 2, which might lead you to think that $\sqrt{2} = \frac{577}{408}$.

1

Explain why you might be suspicious of the calculator’s result.

2

Find an explanation for why $408^2 \cdot 2$ could not possibly equal 577^2 . How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?

3

Repeat these questions for $\left(\frac{1414213562375}{1000000000000}\right)^2 \neq 2$, an equation that even many modern calculators and computers will get wrong.

GRADE 8 • UNIT 8 • SECTION A | LESSON 4

Student Workbook

Lesson Summary

A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that $\sqrt{25} \cdot \sqrt{25} = 25$.

Since $5 \cdot 5 = 25$, we know that

$$\sqrt{25} = 5.$$

$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. In an earlier grade we learned that $\frac{a}{b}$ is a point on the number line found by dividing the interval from 0 to 1 into b equal parts and finding the point that is a of them to the right of 0. We can always write a fraction in the form $\frac{a}{b}$, where a and b are integers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1} = 5$ or $-\frac{5}{1} = -5$. Because fractions and ratios are closely related ideas, fractions and their opposites are called rational numbers.

Here are some examples of rational numbers:

$$\frac{7}{4}, 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9}, -\frac{\sqrt{16}}{\sqrt{100}}$$

Now consider a square whose area is 2 square units with a side length of $\sqrt{2}$ units. This means that

$$\sqrt{2} \cdot \sqrt{2} = 2.$$



An **irrational number** is a number that is not rational, meaning it cannot be expressed as a positive or negative fraction. For example, $\sqrt{2}$ has a location on the number line (it's a tiny bit to the right of $\frac{7}{5}$), but its location can not be found by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0.

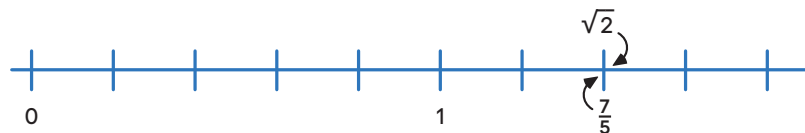


$\frac{7}{5}$ is close to $\sqrt{2}$ because $(\frac{7}{5})^2 = \frac{49}{25} = 1.96$, which is very close to 2 since $\frac{49}{25} \approx 2$. We could keep looking forever for rational numbers that are solutions to $x^2 = 2$, and we would not find any since $\sqrt{2}$ is an irrational number.

The square root of any whole number is either a whole number, like $\sqrt{36} = 6$ or $\sqrt{64} = 8$, or an irrational number. Here are some examples of irrational numbers:

$$\sqrt{10}, -\sqrt{3}, \frac{\sqrt{2}}{2}, \pi$$

Display a number line such as the one shown here for all to see. Tell students that an **irrational number** is a number that is not rational. That is, it is a number that cannot be expressed as a positive or negative fraction. $\sqrt{2}$ is one example of an irrational number. It has a location on the number line, and its location can be made precise (it's a tiny bit to the right of $\frac{7}{5}$), but $\sqrt{2}$ cannot be found on a number line by subdividing the interval from 0 to 1 into b parts and taking a of them. We have to define $\sqrt{2}$ in a different way, such as the side length of a square with area 2 square units.



Lesson Synthesis

The purpose of this discussion is to explicitly point out that while we have collected some evidence that supports the claim that $\sqrt{2}$ is irrational, we have not actually proven this claim. Here are some possible questions for discussion:

“If I told you that there are no purple zebras, and you spent your whole life traveling the world and never saw a purple zebra, does it mean I was right?”

No, it is possible you just failed to find a purple zebra.

“So if we spent our whole lives testing different fractions and never quite got one whose square is 2, does that mean there are no such fractions?”

No, maybe we just haven't found it yet.

Tell students that we haven't learned enough to prove for sure that $\sqrt{2}$ is not equivalent to a fraction. For now, we just have to trust that there are numbers on the number line that are not equivalent to a fraction and that $\sqrt{2}$ is one of them. However, it is possible to get very close estimates with fractions.

Lesson Summary

A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that

$$\sqrt{25} \cdot \sqrt{25} = 25$$

Since $5 \cdot 5 = 25$, we know that

$$\sqrt{25} = 5$$

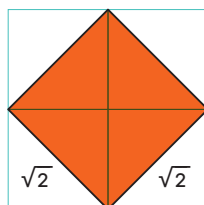
$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. In an earlier grade we learned that $\frac{a}{b}$ is a point on the number line found by dividing the interval from 0 to 1 into b equal parts and finding the point that is a of them to the right of 0. We can always write a fraction in the form $\frac{a}{b}$, where a and b are integers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1} = 5$ or $-\frac{1}{\sqrt{4}} = -\frac{1}{2}$. Because fractions and ratios are closely related ideas, fractions and their opposites are called *rational* numbers.

Here are some examples of rational numbers:

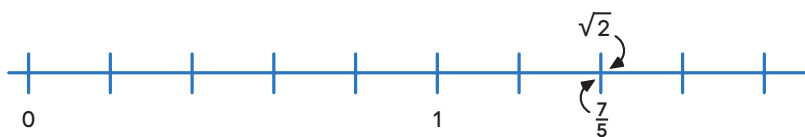
$$\frac{7}{4}, 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9}, -\frac{\sqrt{16}}{\sqrt{100}}$$

Now consider a square whose area is 2 square units with a side length of $\sqrt{2}$ units. This means that

$$\sqrt{2} \cdot \sqrt{2} = 2$$



An **irrational number** is a number that is not rational, meaning it cannot be expressed as a positive or negative fraction. For example, $\sqrt{2}$ has a location on the number line (it's a tiny bit to the right of $\frac{7}{5}$), but its location can not be found by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0.



$\frac{17}{12}$ is close to $\sqrt{2}$ because $(\frac{17}{12})^2 = \frac{289}{144}$, which is very close to 2 since $\frac{288}{144} = 2$. We could keep looking forever for rational numbers that are solutions to $x^2 = 2$, and we would not find any since $\sqrt{2}$ is an irrational number.

The square root of any whole number is either a whole number, like $\sqrt{36} = 6$ or $\sqrt{64} = 8$, or an irrational number. Here are some examples of irrational numbers:

$$\sqrt{10}, -\sqrt{3}, \frac{\sqrt{5}}{2}, \pi$$

Responding To Student Thinking

Points to Emphasize

If most students struggle with defining rational and irrational numbers, revisit irrational numbers as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about irrational numbers on a number line to deepen understanding of its approximation.

Unit 8, Lesson 6, Activity 3 Solutions on a Number Line

Cool-down

Types of Solutions

5
min

Student Task Statement

1. In your own words, say what a rational number is. Give at least three different examples of rational numbers.

Sample response: A rational number is a fraction, like $\frac{1}{2}$, or its opposite, like $-\frac{1}{2}$. Something like 3.98 is rational too because it is equal to $\frac{398}{100}$.

2. In your own words, say what an irrational number is. Give at least two examples.

Answers vary. Sample response: An irrational number is one that is not rational. It is a number that cannot be expressed as a fraction. $\sqrt{2}$ and π are two examples.

Practice Problems

5 Problems

Student Workbook

LESSON 4
PRACTICE PROBLEMS

1 Decide whether each number in this list is rational or irrational.

 $\frac{13}{3}$ 0.1234 $\sqrt{37}$
 -77 $-\sqrt{100}$ $-\sqrt{12}$
2 Which value is an exact solution of the equation $m^2 = 14$?
☒ 7 ☐ $\sqrt{14}$ ☐ 3.74 ☐ $\sqrt{374}$

3 from Unit 7, Lesson 8

Rewrite each expression in an equivalent form that uses a single exponent.

 a. $(10^2)^{-3}$ b. $(3^{-3})^2$
 c. $3^{-5} \cdot 4^{-5}$ d. $2^5 \cdot 3^{-5}$

Problem 1

Decide whether each number in this list is *rational* or *irrational*. $\frac{-13}{3}$, 0.1234, $\sqrt{37}$, -77, $-\sqrt{100}$, $-\sqrt{12}$

- Rational: $\frac{-13}{3}$, 0.1234, -77, $-\sqrt{100}$
- Irrational: $\sqrt{37}$, $-\sqrt{12}$

Problem 2

Which value is an exact solution of the equation $m^2 = 14$?

A. 7

B. $\sqrt{14}$

C. 3.74

D. $\sqrt{3.74}$

Problem 3

from Unit 7, Lesson 8

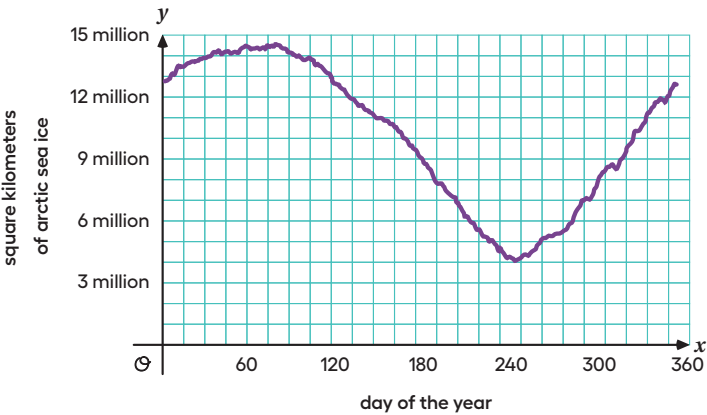
Rewrite each expression in an equivalent form that uses a single exponent.

a. $(10^2)^{-3}$ 10^{-6} (or equivalent)b. $(3^{-3})^2$ 3^{-6} (or equivalent)c. $3^{-5} \cdot 4^{-5}$ 12^{-5} (or equivalent)d. $2^5 \cdot 3^{-5}$ $\left(\frac{2}{3}\right)^5$ (or equivalent)

Problem 4

from Unit 5, Lesson 5

The graph represents the area of arctic sea ice in square kilometers as a function of the day of the year in 2016.



- a. Give an approximate interval of days when the area of arctic sea ice was decreasing.

Sample response: Day 75 to Day 250

- b. On which days was the area of arctic sea ice 12 million square kilometers?

Sample response: Days 135, 350, and 360

Problem 5

from Unit 4, Lesson 14

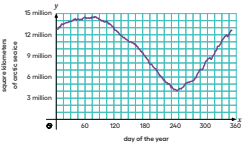
A high school is hosting an event for seniors but will also allow some juniors to attend. The principal approved the event for 200 students and decided the number of juniors should be 25% of the number of seniors. How many juniors will be allowed to attend? If you get stuck, try writing two equations that each represent the number of juniors and seniors at the event.

40 juniors

Sample reasoning: Solve the system $s + j = 200$, $j = 0.25s$ (or equivalent), where s represents the number of seniors and j represents the number of juniors, and get $j = 40$ juniors and $s = 160$ seniors.

Student Workbook

4 Practice Problems
From Unit 5, Lesson 5
The graph represents the area of arctic sea ice in square kilometers as a function of the day of the year in 2016.



- a. Give an approximate interval of days when the area of arctic sea ice was decreasing.
- b. On which days was the area of arctic sea ice 12 million square kilometers?

Student Workbook

4 Practice Problems
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Learning Targets
+ I know what an irrational number is and can give an example.
+ I know what a rational number is and can give an example.