

Looking for Associations

Goals

- Calculate relative frequencies, and describe (orally and in writing) associations between variables using a relative frequency table.
- Coordinate (orally and in writing) two-way tables, bar graphs, and segmented bar graphs representing the same data.

Learning Targets

- I can identify the same data represented in a bar graph, a segmented bar graph, and a two-way table.
- I can use a two-way frequency table or relative frequency table to find associations among variables.

Lesson Narrative

In this lesson, students study categorical data displayed in two-way tables, bar graphs, and segmented bar graphs. A **two-way table** is a way to organize categorical data in a table based on 2 variables. A **segmented bar graph** is a way to visualize the percentage of data within categories. The different graphical representations help students visualize the frequencies and relative frequencies. **Relative frequencies** are percentages or fractions of the whole represented by a category. While the concepts and structures in this lesson are not very complex, there are many new terms and representations, and students are given the opportunity to study them carefully so that they can make sense of them.

Student Learning Goal

Let's look for associations in data.

Lesson Timeline

5
min

Warm-up

15
min

Activity 1

15
min

Activity 2

10
min

Lesson Synthesis

Assessment

5
min

Cool-down

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1, Activity 2)

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Materials to Gather

- Pre-printed cards, cut from copies of the blackline master: Activity 1

Materials to Copy

- Matching Representations Cards (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

Print and cut up cards from the Matching Representations blackline master. Prepare 1 set of cards for every 2 students.

Warm-up

Notice and Wonder: Bar Association

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to become familiar with a bar graph by noticing and wondering things about it. While reading a bar graph is a review of a previous grade's work, it is an important for students to look for patterns of association in categorical data.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see.

They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch 

Arrange students in groups of 2. Display the table and images for all to see. Ask students to think of at least 1 thing they notice and at least 1 thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

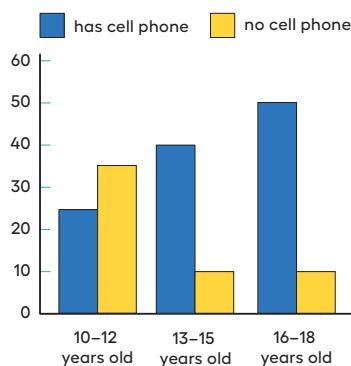
Student Task Statement

What do you notice? What do you wonder?

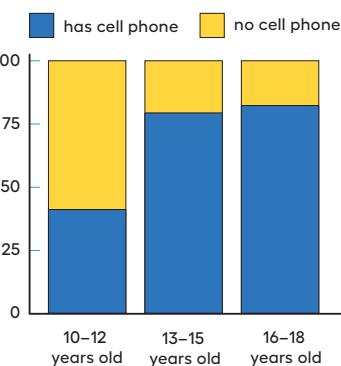
Two-way table

	has cell phone	does not have cell phone	total
10 to 12 years old	25	35	60
13 to 15 years old	40	10	50
16 to 18 years old	50	10	60
total	115	55	170

Bar graph



Segmented bar graph



Students may notice:

- 170 people are represented in the information from the table.
- A much greater percentage of 13–18 year olds have cell phones than 10–12 year olds.
- The graphs have a legend (or key) to show what the different bars (or parts of the bars) mean.

Instructional Routines

Notice and Wonder

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Student Workbook

LESSON 9

Looking for Associations

Let's look for associations in data.

Warm-up Notice and Wonder: Bar Association
What do you notice? What do you wonder?
Two-way table

	has cell phone	does not have cell phone	total
10 to 12 years old	25	35	60
13 to 15 years old	40	10	50
16 to 18 years old	50	10	60
total	115	55	170

Bar graph Segmented bar graph

Bar graph: Y-axis ranges from 0 to 60. Segmented bar graph: Y-axis ranges from 0 to 100.

GRADE 8 • UNIT 6 • SECTION C | LESSON 9

Instructional Routines**MLR2: Collect and Display**

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Access for Students with Diverse Abilities (Activity 1, Student Task)**Engagement: Develop Effort and Persistence.**

Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for:
Conceptual Processing,
Organization, Memory

Access for Multilingual Learners (Activity 1)**MLR2: Collect and Display**

This activity uses the *Collect and Display* math language routine to advance students in developing their mathematical language.

Students may wonder:

- Do all 3 representations show the same data?
- Why aren't the vertical axes labeled?
- Why does the segmented bar graph go up to 100 for all of them when the table shows totals of 60 or 50?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the table and images. Next, ask students,

“Is there anything on this list that you are wondering about now?”

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If a relationship between the 2 variables does not come up during the conversation, ask students to discuss this idea.

Tell students that, in this course, bar graphs are assumed to have a vertical axis representing the frequency of the categories and segmented bar graphs have a vertical axis representing percentage of the category.

Activity 1**Card Sort: Matching Representations**15
min**Activity Narrative**

In this activity students become familiar with two-way tables, clustered bar graphs, and segmented bar graphs by matching different situations.

Students sort different representations of data during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Monitor for different ways groups choose to categorize the representations, but especially for categories that distinguish between two-way tables, bar graphs, and segmented bar graphs.

As students work, encourage them to refine their descriptions of each kind of representation using more precise language and mathematical terms.

Launch

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to describe **two-way tables**, **bar graphs**, and **segmented bar graphs**.

Display words and phrases such as “total,” “stacked,” “legend” or “key,” “percentage,” and “categories.”

Arrange students in groups of 2 and distribute pre-cut cards. Tell them that in this activity, they will sort some cards into categories of their choosing. When they sort the representations, they should work with their partner to come up with categories.

Distribute 1 set of the pre-cut cards from the blackline master to each group.

Student Task Statement

Your teacher will give you a set of cards. Each card contains a representation of some data.

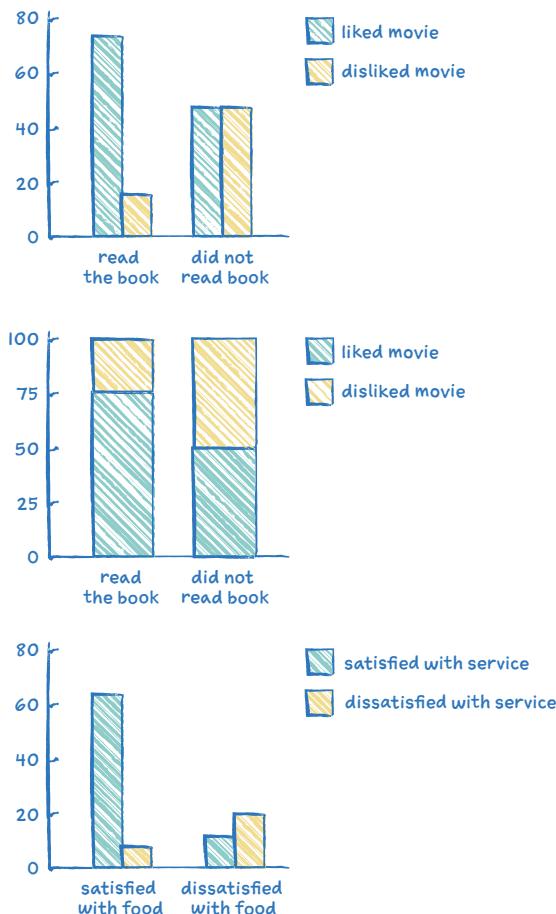
- Sort the systems into groups that represent the same situation.
Be prepared to explain how you know where each representation belongs.
Set 1: A, E, F. Set 2: B, D. Set 3: C, G.
- One of the groups does not have a **two-way table**. Make a two-way table for the situation described by the graphs in the group.

Sample response:

	ate breakfast at home	ate breakfast out	total
office worker	48	12	60
work at home	48	16	64
total	96	28	124

- Label the bar graphs and **segmented bar graphs** so that the categories represented by each bar are indicated.

Sample response:

**Student Workbook**

1 Card Sort: Matching Representations
Your teacher will give you a set of cards. Each card contains a representation of some data.

- Sort the systems into groups that represent the same situation. Be prepared to explain how you know where each representation belongs.
- One of the groups does not have a **two-way table**. Make a two-way table for the situation described by the graphs in the group.

- Label the bar graphs and **segmented bar graphs** so that the categories represented by each bar are indicated.
- Describe in your own words the kind of information shown by a segmented bar graph.

Are You Ready for More?
One of the segmented bar graphs is missing. Construct a segmented bar graph that matches the other representations.

GRADE 8 • UNIT 6 • SECTION C | LESSON 9

Access for Multilingual Learners (Activity 2)

This activity uses the *Collect and Display* math language routine to advance students in developing their mathematical language.

Instructional Routines

MLR2: Collect and Display

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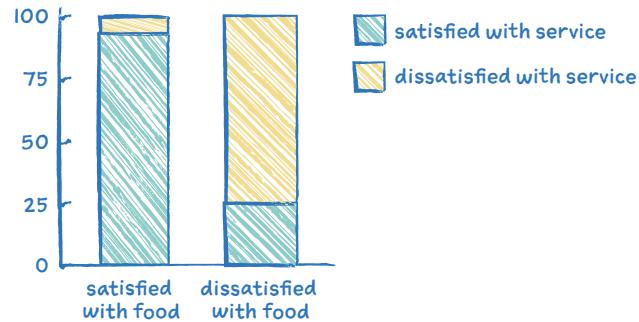


4. Describe in your own words the kind of information shown by a segmented bar graph.

In a segmented bar graph, each bar represents 100% of a category. The bar is split into different subcategories that are stacked on top of each other so their percentages can be compared.

Are You Ready for More?

One of the segmented bar graphs is missing. Construct a segmented bar graph that matches the other representations.



Activity Synthesis

The purpose of this activity is to help students understand the connections among the 3 representations of data. It also helps students see the importance of labeled visual representations.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their matches from the card sort. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. Keep the display for the next activity.

Activity 2

Building Another Type of Two-Way Table

15
min

Activity Narrative

In this activity, students create two-way tables displaying relative frequency. The relative frequency table converts the actual frequency data to percentages which can be useful when comparing groups that include different totals. Finally, students use the relative frequencies to look for a pattern in the data. In the following lesson, students will work with associations in categorical data more explicitly. In this activity, students should use an informal understanding of association to think about whether one is present in the data based on the relative frequency table.

Launch 

Keep students in groups of 2–3. After an introduction to **relative frequency** tables, allow students 3 minutes quiet work time followed by partner discussion and whole-class discussion.

Display the table for all to see.

	watch the news daily	does not watch the news daily	total
younger than 18	30	80	110
18 or older	10	5	15
total	40	85	125

Ask students,

- “Based on this data, who is more likely to watch the news daily:
someone who is younger than 18 or someone who is 18 or older?”

Tell students that, based on the numbers in the table, there are more younger people who watch the news (30) than older (10). On the other hand, the survey reached out to 110 young people and only 15 older people. Without looking at the whole table, that information may have been missed.

In cases like this, finding a relative frequency including percentages can be more helpful than looking at the actual frequency, which is what they are going to do now.

Display the relative frequency table:

	watch the news daily	does not watch the news daily	total
younger than 18	27%, because $30 \div 110 \approx 0.27$	73%	100%
18 or older	67%	33%	100%

Tell students that we would say there is an association between these variables because the relative frequencies of the columns are very different. This means that knowing information about one of the variables, like the age range, would be helpful in making a prediction about the other variable, like whether that person watches the news. If the relative frequencies had been close, then knowing information about one of the variables would not be helpful in making a prediction about the other variable and there would not be an association between the variables.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier activity. Ask students to suggest ways to update the display:

- “Are there any new words or phrases that you would like to add?”

“Is there any language you would like to revise or remove?”

Display additional words or phrases such as “actual frequency” or “relative frequency.” Encourage students to use the display as a reference.

Building on Student Thinking

Students may not calculate relative frequencies correctly. Look to see if they are dividing into the total for each row, instead of some other number in the row, or the total for the entire table.

Students may struggle to decide if there is an association or not. Tell students that an association means that knowing information about one variable makes it easier to predict the other variable. For example, if they wanted to predict whether a person has a cell phone, would their prediction change if they knew which age category the person fit into?

Student Workbook

2 Building Another Type of Two-Way Table

Here is a two-way table that shows data about cell phone usage among children aged 10 to 18.

	has cell phone	does not have cell phone	total
10 to 12 years old	25	35	60
13 to 15 years old	40	10	50
16 to 18 years old	50	10	60
total	115	55	170

- 1 Complete the table. In each row, the entries for “has cell phone” and “does not have cell phone” should have the total 100%. Round entries to the nearest percentage point.
- | | has cell phone | does not have cell phone | total |
|--------------------|----------------|--------------------------|-------|
| 10 to 12 years old | 42% | | |
| 13 to 15 years old | | | 100% |
| 16 to 18 years old | | 17% | |

This is still a two-way table. Instead of showing frequency, this table shows relative frequency.

- 2 Two-way tables that show relative frequencies often don’t include a “total” row at the bottom. Why?
- _____

- 3 Is there an association between age and cell phone use? How does the two-way table of relative frequencies help to illustrate this?
- _____

Student Task Statement

Here is a two-way table that shows data about cell phone usage among children aged 10 to 18.

	has cell phone	does not have cell phone	total
10 to 12 years old	25	35	60
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total	115	55	170

1. Complete the table. In each row, the entries for “has cell phone” and “does not have cell phone” should have the total 100%. Round entries to the nearest percentage point.

	has cell phone	does not have cell phone	total
10 to 12 years old	42%	58%, since $35 \div 60 \approx 0.58$	100%
13 to 15 years old	80%, since $40 \div 50 = 0.8$	20%, since $10 \div 50 = 0.2$	100%
16 to 18 years old	83%, since $50 \div 60 \approx 0.83$	17%	100%

This is still a two-way table. Instead of showing frequency, this table shows **relative frequency**.

2. Two-way tables that show relative frequencies often don’t include a “total” row at the bottom. Why?

The columns are not percentages of the same number, so they can’t be added. The percentages in any given column are not likely to add up to 100%.

3. Is there an association between age and cell phone use? How does the two-way table of relative frequencies help to illustrate this?

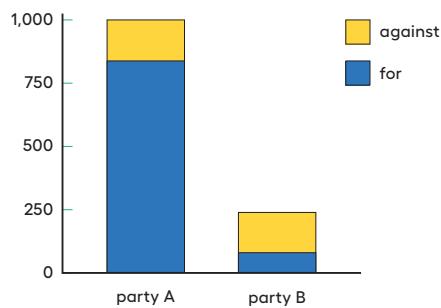
Yes, there is an association between age and cell phone use. A much higher percentage of children from 13 to 18 years old have cell phones than children from 10 to 12 years old do.

Are You Ready for More?

A pollster attends a rally and surveys many of the participants about whether they associate with political Party A or political Party B and whether they are for or against Proposition 3.14 going up for vote soon. The results are sorted into the table shown.

	for	against
party A	832	165
party B	80	160

- A news station reports these results by saying, “A poll shows that about the same number of people from both parties are voting against Proposition 3.14.”
- A second news station shows this graphic.

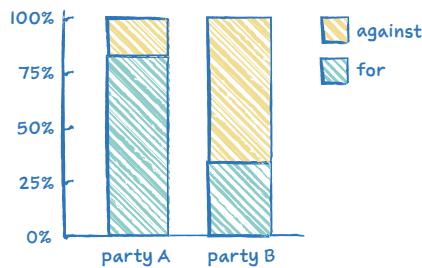


- Are any of the news reports misleading? Explain your reasoning.

Sample response: Yes, the first news station did not take into account that there were not as many people from party B that were polled. The graphic from the second news station is hard to understand because the bars from Party B are so much smaller than from Party A, so it is hard to tell what it is trying to say.

- Create a headline, graphic, and short description that more accurately represents the data in the table.

Sample response: Headline: Party A is For Proposition 3.14 While Party B is Against



Description: Based on this poll, more than 80% of people in Party A are for Proposition 3.14, while only 33% of people in Party B share the same opinion. There seems to be a big divide along party lines for the proposition!

Activity Synthesis

The purpose of this discussion is to help students see the usefulness of two-way tables that display relative frequency.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their solutions to the questions. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Some questions for discussion:

“What does the 42% in the table mean?”

42% of people 10-to-12 years old have a cell phone.

“What is the total of the percentages in the first column? What's wrong with that answer?”

205%. It doesn't make sense since it is greater than 100%. It is combining percentages of different wholes, so it doesn't make sense to add them.

“Did you say there was or was not an association between age and cell-phone use? Explain your reasoning.”

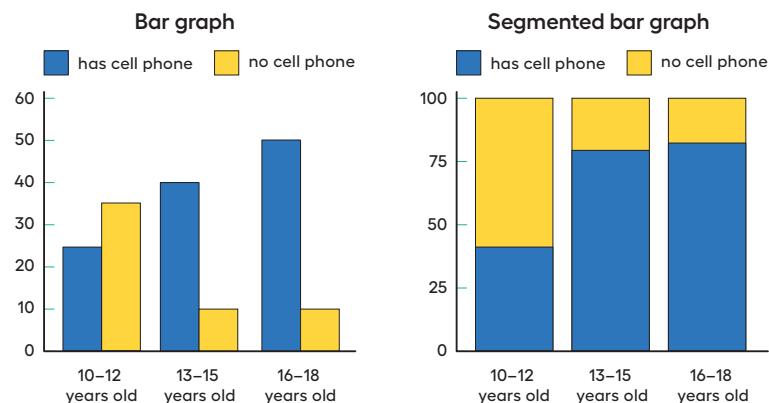
There is an association since the percentages for the 10 to 12 year olds is very different from the other 2 age groups.

“What did the relative frequency table show you that was harder to see in the original two-way table?”

It was much easier to see the association with the percentages than the numbers. For example, the number of people without a cell phone for the older 2 age groups was the same, but the percentages were a little different.

Lesson Synthesis

Display the graphs and tell students to refer to the data from the tables in the “Building Another Type of Two-Way Table” activity.



Consider asking these discussion questions to emphasize the main ideas from the lesson:

❑ “Which graphical representation do you prefer for the data?”

It is easier to see the association with the segmented bar graph, but the actual values are lost.

❑ “In the original table, what did the number 40 represent? How is that group of people represented in the other table and the two graphs?”

There are 40 people who are 13 to 15 years old that have a cell phone. In the relative frequency table, this is represented by the 80%. In the bar graph, this is represented by the taller, blue bar for the set of bars labeled 13–15 years old. In the segmented bar graph, this is represented by the top, blue segment of the bar labeled 13–15 years old.

❑ “What values in the tables represent the same information as the tallest, yellow-striped bar in the bar graph?”

The tallest yellow-striped bar represents the number of people who are 10–12 years old who do not have cell phones. In the original table, this is the value 35, and in the relative frequency table this is about 58%.

Survey students about whether they play a musical instrument or not and whether they play a sport or not. These data will be needed for the next lesson.

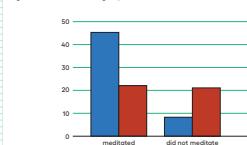
Student Workbook

9 Lesson Summary
When we collect data by counting things in various categories, like red, blue, or yellow, we call the data “categorical data,” and we say that color is a “categorical variable.”

We can use **two-way tables** to investigate possible connections between two categorical variables. For example, this two-way table of frequencies shows the results of a study of meditation and state of mind of athletes before a track meet.

	meditated	did not meditate	total
calm	45	8	53
agitated	23	21	44
total	68	29	97

If we are interested in the question of whether there is an association between meditating and being calm, we might present the frequencies in a bar graph, grouping data about those who meditated and those who did not meditate, so we can compare the numbers of calm and agitated athletes in each group.



Notice that the number of athletes who did not meditate is small compared to the number who meditated (29 as compared to 68, as shown in the table).

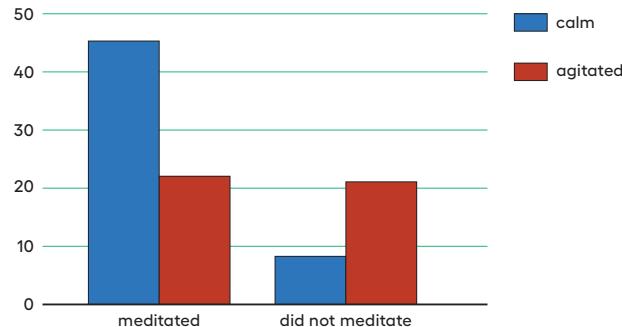
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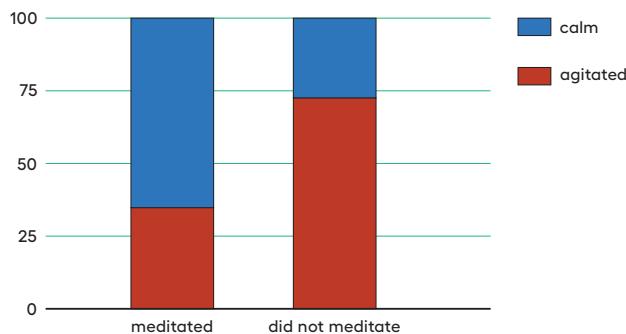
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Notice that the number of athletes who did not meditate is small compared to the number who meditated (29 as compared to 68, as shown in the table).

If we want to know the proportions of calm meditators and calm non-meditators, we can make a two-way table of **relative frequencies** and present the relative frequencies in a **segmented bar graph**.

	meditated	did not meditate
calm	66%	28%
agitated	34%	72%
total	100%	100%



Student Workbook

9 Lesson Summary

If we want to know the proportions of calm meditators and calm non-meditators, we can make a two-way table of relative frequencies and present the relative frequencies in a segmented bar graph.

	meditated	did not meditate
calm	66%	28%
agitated	34%	72%
total	100%	100%

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- What norm(s) should stay the way they are?
- What norm(s) do you think should be made more clear? How?
- What norms are missing that you would add?
- What norm(s) should be removed?

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

Cool-down

Guitar and Golf

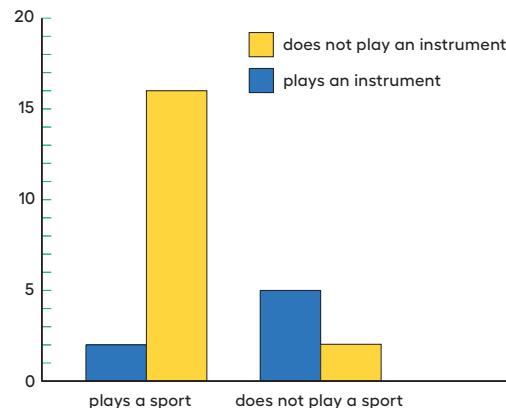
5
min

Students use a bar graph to complete a two-way table. This data is then converted into a relative frequency table to practice the skill needed to create segmented bar graphs.

After collecting the Cool-downs, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Tell students that they should use the bar graph to estimate the values in the two-way table, then find relative frequencies by rows.

Student Task Statement



- 1.** In a class of 25 students, some students play a sport, some play a musical instrument, some do both, and some do neither. Complete the two-way table to show the data from the bar graph.

Sample response:

	plays an instrument	does not play an instrument	total
plays a sport	2	16	18
does not play a sport	5	2	7
total	7	18	25

- 2.** Using the entries from the actual frequency table, complete this table so that it shows relative frequencies based on the rows. Round entries to the nearest percentage point.

	plays an instrument	does not play an instrument	total
plays a sport	11%, since $2 \div 18 \approx 0.11$	89%, since $16 \div 18 \approx 0.89$	100%
does not play a sport	71%, since $5 \div 7 \approx 0.71$	29%, since $2 \div 7 \approx 0.29$	100%

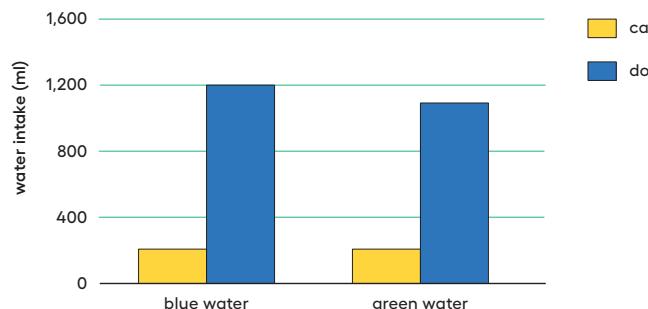
Practice Problems

4 Problems

Problem 1

A scientist wants to know if the color of the water affects how much animals drink. The average amount of water each animal drinks was recorded in milliliters for a week and then graphed.

	cat intake (ml)	dog intake (ml)	total (ml)
blue water	210	1200	1410
green water	200	1100	1300
total	410	2300	2710



Is there evidence to suggest an association between water color and animal?

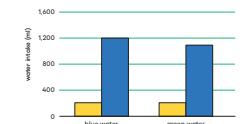
Sample response: No, the relative frequencies of the animals drinking each color of water are about the same, so there is no evidence of association.

Student Workbook

LESSON
PRACTICE PROBLEMS

- 1 A scientist wants to know if the color of the water affects how much animals drink. The average amount of water each animal drinks was recorded in milliliters for a week and then graphed.

	cat intake (ml)	dog intake (ml)	total (ml)
blue water	210	1,200	1,410
green water	200	1,100	1,300
total	410	2,300	2,710



Is there evidence to suggest an association between water color and animal?

GRADE 8 • UNIT 4 • SECTION C | LESSON 9

Lesson 9 Practice Problems

Student Workbook

9 Practice Problems

1 A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else.

	bought apples	did not buy apples
bought lettuce	14	58
did not buy lettuce	8	29

Make a table that shows the relative frequencies for each row. Use this table to decide if there is an association between buying lettuce and buying apples.

2 Researchers at a media company want to study news-reading habits among different age groups. They tracked print and online subscription data and made a 2-way table.

	internet media	print media
18–25 year olds	151	28
26–45 year olds	132	72
46–65 year olds	48	165

a. Create a segmented bar graph using one bar for each row of the table.

Problem 2

A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else.

	bought apples	did not buy apples
bought lettuce	14	58
did not buy lettuce	8	29

Make a table that shows the relative frequencies for each row. Use this table to decide if there is an association between buying lettuce and buying apples.

	bought apples	did not buy apples	total
bought lettuce	19%, since $14 + 58 = 72$ and $14 \div 72 = 0.194$	81%, since $58 \div 72 = 0.805$	100%
did not buy lettuce	22%, since $8 + 29 = 37$ and $8 \div 37 = 0.216$	78%, since $29 \div 37 = 0.783$	100%

Sample response: Since the percentages in each column are close, there is not enough evidence to suggest an association between buying apples and lettuce.

Lesson 9 Practice Problems

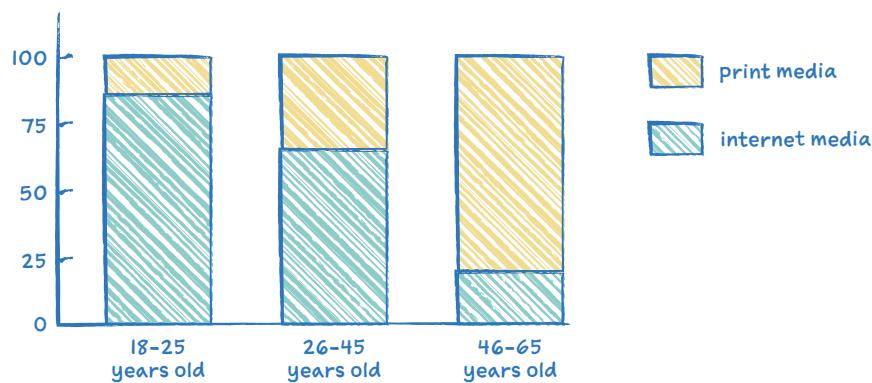
Problem 3

Researchers at a media company want to study news-reading habits among different age groups. They tracked print and online subscription data and made a 2-way table.

	internet media	print media
18–25 year olds	151	28
26–45 year olds	132	72
46–65 year olds	48	165

- a. Create a segmented bar graph using one bar for each row of the table.

Sample response:



- b. Is there an association between age groups and the method they use to read articles? Explain your reasoning.

Yes

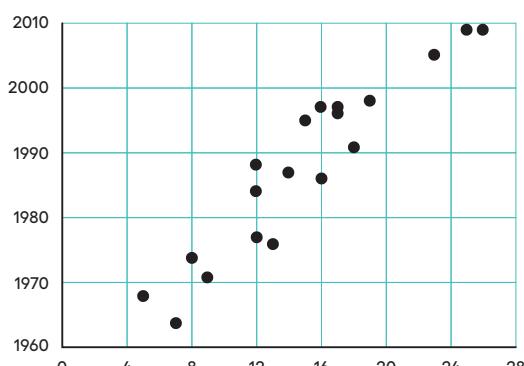
Sample reasoning: The segments of the bars are not very close to being the same size. Younger age groups use the internet articles much more than print articles, while the oldest age group reverses that pattern.

Problem 4

from Unit 6, Lesson 6

Using the data in the scatter plot, what is a reasonable slope of a model that fits this data?

- A. -2.5
- B. -1
- C. 1
- D. 2.5



Student Workbook

9 Practice Problems

1 A farmer brings his produce to the farmer's market and records whether people buy lettuce, apples, both, or something else.

	bought apples	did not buy apples
bought lettuce	14	58
did not buy lettuce	8	29

Make a table that shows the relative frequencies for each row. Use this table to decide if there is an association between buying lettuce and buying apples.

2 Researchers at a media company want to study news-reading habits among different age groups. They tracked print and online subscription data and made a 2-way table.

	internet media	print media
18–25 year olds	151	28
26–45 year olds	132	72
46–65 year olds	48	165

a. Create a segmented bar graph using one bar for each row of the table.

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Student Workbook

9 Practice Problems

b. Is there an association between age groups and the method they use to read articles? Explain your reasoning.

From Unit 6, Lesson 6
Using the data in the scatter plot, what is a reasonable slope of a model that fits this data?

(A) -2.5
(B) -1
(C) 1
(D) 2.5

Learning Targets

- + I can identify the same data represented in a bar graph, a segmented bar graph, and a two-way table.
- + I can use a two-way frequency table or relative frequency table to find associations among variables.

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