Lesson 6

Writing Equations to Represent Relationships

Goals

- Generate an equation for a proportional relationship, given a description of the situation but no table.
- Interpret (orally) each part of an equation that represents a proportional relationship in an unfamiliar context.
- Use an equation to solve problems involving a proportional relationship, and explain (orally) the reasoning.

Learning Targets

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson Narrative

In this lesson, students continue to write equations of the form $y = kx$ to represent proportional relationships. They begin to recognize situations where using the equation is a more efficient way of solving problems than other methods they have been using, such as using tables or equivalent ratios.

The activities introduce new contexts without providing tables. Students who still need tables should be given a chance to realize that fact and create tables for themselves. The activities are intended to motivate the usefulness of representing proportional relationships abstractly with equations. The repeated calculations called for in the activities serve as scaffolding for finding the equations.

Math Community
Today's activity is for students to individually reflect on the norms generated so far. During the *Cool-down*, students provide feedback on the norms, sharing those they agree with and those they feel need revision or removal. These suggestions will inform the next version of the classroom norms.

Student Learning Goal

Let's use equations to solve problems involving proportional relationships.

Lesson Timeline

5 min	15 min	15 min	10 mins	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 2)
- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Warm-ups)

Instructional Routines

- Math Talk
- MLR6: Three Reads
- MLR7: Compare and Connect

Required Materials

Materials to Gather
- Math Community Chart: Warm-up

Required Preparation

Lesson:
Calculators can optionally be made available to take the focus off computation.

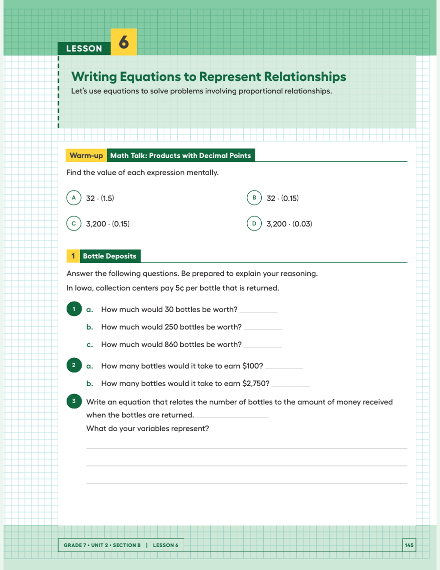
Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 6

Writing Equations to Represent Relationships

Let's use equations to solve problems involving proportional relationships.

Warm-up Math Talk: Products with Decimal Points

Find the value of each expression mentally.

A 32 · (1.5)

B 32 · (0.15)

C 3,200 · (0.15)

D 3,200 · (0.03)

1 Bottle Deposits

Answer the following questions. Be prepared to explain your reasoning.

In Iowa, collection centers pay 5¢ per bottle that is returned.

1 a. How much would 30 bottles be worth? ______

b. How much would 250 bottles be worth? ______

c. How much would 860 bottles be worth? ______

2 a. How many bottles would it take to earn $100? ______

b. How many bottles would it take to earn $2,750? ______

3 Write an equation that relates the number of bottles to the amount of money received when the bottles are returned. ______

What do your variables represent?

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Warm-up

Math Talk: Products with Decimal Points

5 min

Activity Narrative

This *Math Talk* focuses on multiplication by a decimal. It encourages students to think about how they can use the result of one multiplication problem to find the answer to a similar problem with a different, but related, factor. The understanding elicited here will be helpful later in the lesson when students evaluate equations where the constant of proportionality is a decimal.

To recognize how a factor has been scaled and predict how the product will be affected, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A. $32 \cdot (1.5)$

48

Sample reasoning: $32 + \frac{1}{2} \cdot 32 = 32 + 16$.

B. $32 \cdot (0.15)$

4.8

Sample reasoning: 0.15 is $\frac{1}{10}$ of 1.5, and $\frac{1}{10}$ of 48 is 4.8.

C. $3{,}200 \cdot (0.15)$

480

Sample reasoning: 3,200 is 100 times 32, and 100 times 4.8 is 480.

D. $3{,}200 \cdot (0.03)$

96

Sample reasoning:

- 0.03 is $\frac{1}{5}$ of 0.15, and $\frac{1}{5}$ of 480 is 96.
- To get from 0.15 to 0.03 you can multiply by 2 and divide by 10. $480 \cdot 2 \div 10 = 96$.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway to highlight is how we can use the structure of place value and properties of operations to find products involving decimals.

Math Community

After the *Warm-up*, display the Math Community Chart. Remind students that norms are agreements that everyone in the class shares responsibility for, so it is important that everyone understands the intent of each norm and can agree with it. Tell students that today's *Cool-down* includes a question asking for feedback on the drafted norms. This feedback will help identify which norms the class currently agrees with and which norms need revising or removing.

Activity 1

Bottle Deposits

15 min

Activity Narrative

In this activity, students practice writing an equation to represent a proportional relationship in a new context. Students reason about quantities and prices, calculating several values before writing an equation. No table is provided for students to organize their thinking in order to encourage them to look for regularity in repeated reasoning and to notice the efficiency of using an equation to express the relationship.

This activity also emphasizes the interpretation of the constant of proportionality in the context, as students may choose to express the relationship as 5 cents per bottle, 0.05 dollars per bottle, or 20 bottles per dollar. Students compare these different approaches during the whole-class discussion.

Monitor for students who:

- Write many calculations, without any organization
- Create a table to organize their work
- Write an equation to record their repeated reasoning
- Use 5 as the constant of proportionality
- Use 0.05 as the constant of proportionality
- Use 20 as the constant of proportionality

Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because…" or "I noticed _____ so I …" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

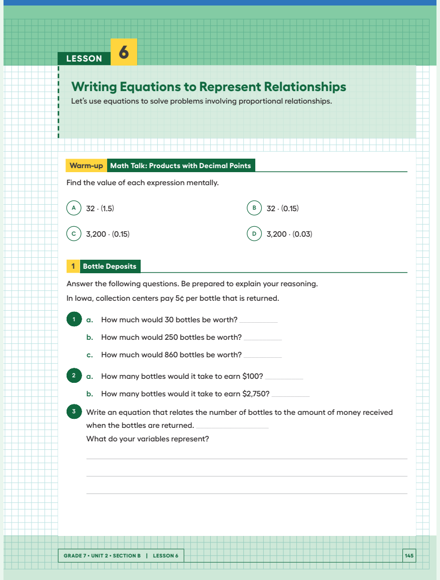
Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


LESSON 6

Writing Equations to Represent Relationships

Let's use equations to solve problems involving proportional relationships.

Warm-up Math Talk: Products with Decimal Points

Find the value of each expression mentally.

A $32 \cdot (1.5)$

B $32 \cdot (0.15)$

C $3{,}200 \cdot (0.15)$

D $3{,}200 \cdot (0.03)$

1 Bottle Deposits

Answer the following questions. Be prepared to explain your reasoning.

In Iowa, collection centers pay 5¢ per bottle that is returned.

1. a. How much would 30 bottles be worth?
   b. How much would 250 bottles be worth?
   c. How much would 860 bottles be worth?
2. a. How many bottles would it take to earn $100?
   b. How many bottles would it take to earn $2,750?
3. Write an equation that relates the number of bottles to the amount of money received when the bottles are returned. What do your variables represent?

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Launch

Provide access to calculators.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Answer the following questions. Be prepared to explain your reasoning.

In Iowa, collection centers pay 5¢ per bottle that is returned.

1. **a.** How much would 30 bottles be worth?

   $1.50. Sample reasoning: $30 \cdot 5 = 150$ and $150 \div 100 = 1.5$

   b. How much would 250 bottles be worth?

   $12.50. Sample reasoning: $250 \cdot 0.05 = 12.5$

   c. How much would 860 bottles be worth?

   $43. Sample reasoning: $860 \div 20 = 43$

2. **a.** How many bottles would it take to earn $100?

   2,000 bottles. Sample reasoning: $100 \cdot 20 = 2{,}000$

   b. How many bottles would it take to earn $2,750?

   55,000 bottles. Sample reasoning: $2{,}750 \div 0.05 = 55{,}000$

3. Write an equation that relates the number of bottles to the amount of money received when the bottles are returned. What do your variables represent?

   Sample responses:

   - $y = 5x$, where x represents the number of bottles and y represents the amount of money, in cents
   - $m = 0.05b$, where b represents the number of bottles and m represents the amount of money, in dollars
   - $b = 20m$, where b represents the number of bottles and m represents the amount of money, in dollars

Activity Synthesis

The two goals of this discussion are:

- To highlight the efficiency of expressing the relationship as an equation
- To contrast different ways the relationship could be expressed as an equation

Invite previously selected students to share how they found the amounts of money and the numbers of bottles. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

“How could we represent this reasoning with an equation? What does the constant of proportionality in this equation represent?”

“Why do the different approaches lead to the same outcome?”

“Are there any benefits or drawbacks to one approach compared to another?”

Here are some different strategies for finding the value of some number of bottles, along with one or more ways to record that reasoning with an equation.

sample repeated reasoning	possible equations	meaning of the constant of proportionality
multiply by 5 (and then divide by 100)	$y = 5x$	5 cents per bottle
multiply by 0.05	$y = 0.05x$	0.05 dollars per bottle
divide by 20*	$m = \frac{1}{20}b$ or $b = 20m$	$\frac{1}{20}$ dollar per bottle or 20 bottles per dollar

*If students suggest an equivalent equation that uses division, such as $m = b \div 20$, confirm that that equation is correct and also ask if they can think of a way to express that equation in the form $y = kx$.

The key takeaway is that defining variables and writing an equations can be an efficient way to describe the proportional relationship between two quantities. However, there is more than one way to represent a given situation with an equation. It is important to specify what each variable represents so others can interpret the equation.

Activity 2

Recycling

15 min

Activity Narrative

This activity is intended to further develop students' ability to write equations to represent proportional relationships. It involves work with decimals and asks for equations that represent proportional relationships of different pairs of quantities, which increases the challenge of the task. As students identify the constants of proportionality between each pair of quantities to represent the relationships with equations, they are reasoning quantitatively and abstractly.

Students may solve the first two problems in different ways. Monitor for different solution approaches, such as using computations, using tables, finding the constant of proportionality, and writing equations.

Launch

Arrange students in groups of 2. Provide access to calculators.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem, without revealing the questions.

- For the first read, read the problem aloud then ask,

"What is this situation about?"

weight of cans and the amount of money made from recycling.

- Listen for and clarify any questions about the context.

Access for Multilingual Learners (Activity 2)

MLR6: Three Reads

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Instructional Routines

MLR6: Three Reads

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.

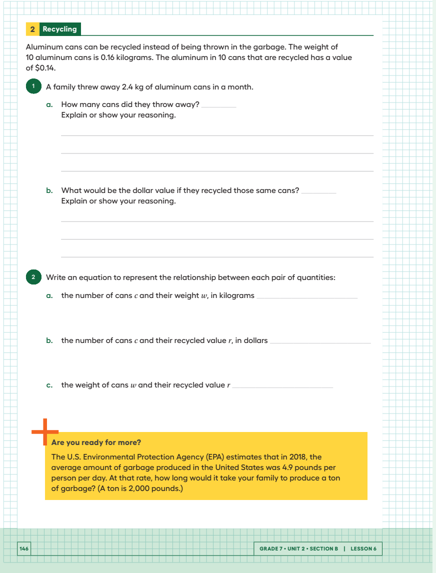
Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organize the information provided.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Building on Student Thinking

If students have trouble getting started, encourage them to create representations of the relationships, like a diagram or a table. If they are still stuck, suggest that they first find the weight and dollar value of 1 can.

Student Workbook


2 Recycling

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of $0.14.

1. A family threw away 2.4 kg of aluminum cans in a month.
   a. How many cans did they throw away? ____
   Explain or show your reasoning.
   b. What would be the dollar value if they recycled those same cans? ____
   Explain or show your reasoning.

2. Write an equation to represent the relationship between each pair of quantities:
   a. the number of cans c and their weight w, in kilograms ____
   b. the number of cans c and their recycled value r, in dollars ____
   c. the weight of cans w and their recycled value r ____

Are you ready for more?

The U.S. Environmental Protection Agency (EPA) estimates that in 2018, the average amount of garbage produced in the United States was 4.9 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

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- After the second read, ask students to list any quantities that can be counted or measured.

  number of aluminum cans; total weight of aluminum cans, in kilograms; money earned, in dollars

- After the third read, reveal the questions:

"A family threw away 2.4 kg of aluminum in a month. How many cans did they throw away? What would be the dollar value if they recycled those same cans?"

and ask,

"What are some ways we might get started on this?"

- Invite students to name some possible starting points, referencing quantities from the second read.

  Calculate the weight of aluminum in 1 can and the amount of money earned from 1 can

Give 5 minutes of quiet work time followed by partner discussion.

Student Task Statement

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of $0.14.

1. A family threw away 2.4 kg of aluminum cans in a month.

   a. How many cans did they throw away? Explain or show your reasoning.

   150 cans. Sample reasoning: $2.4 \div 0.16 = 15$ and $10 \cdot 15 = 150$. There are 15 groups of 10 cans.

   b. What would be the dollar value if they recycled those same cans? Explain or show your reasoning.

   $2.10. Sample reasoning: $(0.14) \cdot 15 = 2.1$

2. Write an equation to represent the relationship between each pair of quantities:

   a. the number of cans c and their weight w, in kilograms

   $c = 62.5w$ (or equivalent)

   b. the number of cans c and their recycled value r, in dollars

   $r = 0.014c$ (or equivalent)

   c. the weight of cans w and their recycled value r

   $r = 0.875w$ (or equivalent)

Here is one way to organize the given information and solutions in a table:

number of cans (c)	weight in kilograms (w)	recycled value in dollars (r)
10	0.16	0.14
150	2.4	2.10
1	0.016	0.014
62.5	1	0.875

Are You Ready for More?

The U.S. Environmental Protection Agency (EPA) estimates that in 2018, the average amount of garbage produced in the United States was 4.9 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

Sample responses: A family of two would take about 29 weeks. A family of three would take about $19\frac{1}{2}$ weeks. A family of four would take about $14\frac{1}{2}$ weeks.

Activity Synthesis

Invite several students to share their methods for solving the first two problems, such as using computations, using tables, finding the constant of proportionality, and writing equations. If students did not use equations to solve the first two problems, ask them how they can use the equations they found later in the activity to answer the first two questions.

If time permits, highlight connections between the equations generated, illustrated by the following sequence of equations.

$r = 0.014c$

$r = 0.014(62.5w)$

$r = 0.875w$

Lesson Synthesis

Share with students,

"Today we wrote equations to represent proportional relationships where no tables were given. We saw that it is important to state what the variables in the equation represent."

Briefly revisit some equations from the activities. For each equation, ask students:

"In this equation, what did each variable represent?"

"What did the number mean?"

To help students generalize about equations of proportional relationships, consider asking students:

"What do all these equations have in common?"

They are of the form $y = kx$, where k is the constant of proportionality. They have two variables and one number. The constant of proportionality is being multiplied by one of the variables.

"How did writing an equation help you solve the problems?"

It made it easier to see what number you should multiply or divide by to answer each question.

Student Workbook

6 Lesson Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,310 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5{,}280m$$

where f represents a distance measured in feet and m represents the same distance measured in miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20{,}310 = 5{,}280m$$

Solving this equation for m gives $m = \frac{20,310}{5,280} \approx 3.85$, so we can say that Denali is approximately 3.85 miles above sea level.

Responding To Student Thinking

Points to Emphasize

If students struggle with writing an equation to represent a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activities referred to here, use the tables to help students see a pattern of constant change.

Unit 2, Lesson 8, Activity 2 More Conversions

Unit 2, Lesson 8, Activity 4 All Kinds of Equations

Lesson Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,310 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

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Solving this equation for m gives $m = \frac{20{,}310}{5{,}280} \approx 3.85$, so we can say that Denali is approximately 3.85 miles above sea level.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet. Make sure students know they can make suggestions for both student and teacher norms.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about.

Cool-down

More Recycling

5 min

Student Task Statement

Glass bottles can be recycled. At one recycling center, 1 ton of clear glass is worth \$25. (1 ton = 2,000 pounds)

1. How many pounds of clear glass is worth \$10?

   800 pounds, because $2{,}000 \div 25 = 80$ and $80 \cdot 10 = 800$

2. How much money is 40 pounds of clear glass worth?

   \$0.50, because $40 = 80 \cdot 0.50$

3. Write an equation to represent the relationship between the weight of clear glass and the value of the glass.

   Sample response: If v represents the value, in dollars, of p pounds of clear glass, then the equation could be either $p = 80v$ or $v = 0.0125p$.

Lesson 6

Practice Problems

6 Problems

Problem 1

A car is traveling on a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

a. What does the 65 tell us in this situation?

The car is traveling 65 miles per hour (which is the constant of proportionality).

b. How many miles does the car travel in 1.5 hours?

97.5 miles

c. How long does it take the car to travel 26 miles at this speed?

$\frac{2}{5}$ of an hour or 24 minutes

Problem 2

Elena has some bottles of water that each holds 17 fluid ounces.

a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.

$w = 17b$ or $b = \frac{1}{17}w$

b. How much water is in 51 bottles?

867 fluid ounces, because $17 \cdot 51 = 867$

c. How many bottles does it take to hold 51 fluid ounces of water?

3 bottles, because $51 \div 17 = 3$

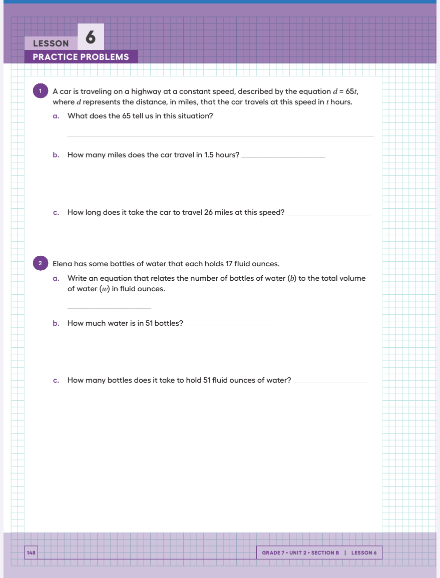
Problem 3

from Unit 2, Lesson 5

There are about 1.61 kilometers in 1 mile. Use x to represent a distance measured in kilometers and y to represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

- $x = 1.61y$
- $y = \frac{1}{1.61}x$
- $y = 0.62x$

Student Workbook



LESSON 6 PRACTICE PROBLEMS

1 A car is traveling on a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

a. What does the 65 tell us in this situation?

b. How many miles does the car travel in 1.5 hours?

c. How long does it take the car to travel 26 miles at this speed?

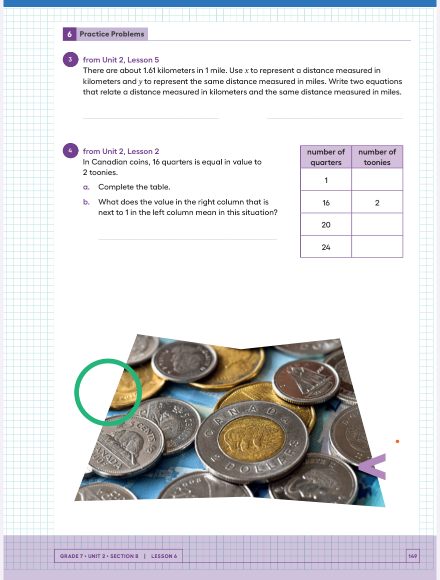
2 Elena has some bottles of water that each holds 17 fluid ounces.

a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.

b. How much water is in 51 bottles?

c. How many bottles does it take to hold 51 fluid ounces of water?

Student Workbook



6 Practice Problems

3 from Unit 2, Lesson 5

There are about 1.61 kilometers in 1 mile. Use x to represent a distance measured in kilometers and y to represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

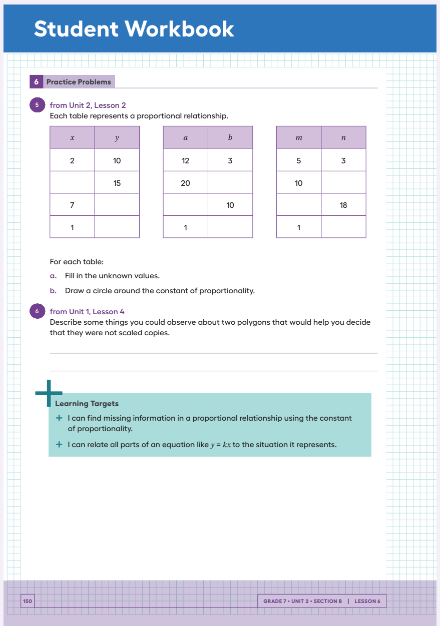
4 from Unit 2, Lesson 2

In Canadian coins, 16 quarters is equal in value to 2 toonies.

a. Complete the table.

b. What does the value in the right column that is next to 1 in the left column mean in this situation?

number of quarters	number of toonies
1	
16	2
20	
24	



Problem 4

from Unit 2, Lesson 2

In Canadian coins, 16 quarters is equal in value to 2 toonies.

a. Complete the table.

number of quarters	number of toonies
1	$\frac{1}{8}$
16	2
20	2.5
24	3

b. What does the value in the right column that is next to 1 in the left column mean in this situation?

$\frac{1}{8}$

Sample response: 1 Canadian quarter has the same value as $\frac{1}{8}$ of a toonie.

Problem 5

from Unit 2, Lesson 2

Each table represents a proportional relationship.

For each table:

$\cdot \frac{3}{2}$

x	y
2	10
3	15
7	35
1	5

$\cdot 2$

a	b
12	3
20	5
40	10
1	$\frac{1}{4}$

$\cdot 3$

m	n
5	3
10	6
30	18
1	$\frac{3}{5}$

a. Fill in the unknown values.

b. Draw a circle around the constant of proportionality.

Problem 6

from Unit 1, Lesson 4

Describe some things you could observe about two polygons that would help you decide that they were not scaled copies.

Sample response: I could find an angle measure in one that is not an angle measure of the other. I could find that a different scale factor would have to be used on one part of the pair than on another.