Lesson 12

Volume of Right Prisms

Goals

- Determine the volume of a right prism by counting how many unit cubes it takes to build one layer and then multiplying by the number of layers.
- Generalize (orally) the relationship between the volume of a prism, the area of its base, and its height.
- Identify whether a given figure is a prism, and if so, identify its base and height.

Learning Target

I can explain why the volume of a prism can be found by multiplying the area of the base by the height of the prism.

Lesson Narrative

In this lesson, students learn that they can calculate the volume of any right prism by multiplying the area of the base by the height of the prism. Students make sense of this formula by picturing the prism decomposed into identical layers, each 1 unit tall. These layers are composed of a number of cubic units equal to the number of square units in the area of the base. The height of the prism tells how many of these layers there are. Therefore, multiplying the number of cubic units in one layer by the number of layers gives the total number of cubic units in the prism, regardless of the shape of the base.

Students construct and critique arguments about whether or not some three-dimensional figures are prisms. They then decide whether they can apply the formula $V = Bh$ to calculate the volume. If so, they identify the base and measure the height, and they then calculate the volume. Students also apply the formula $V = Bh$ to find the height of a prism, given its volume and the area of its base.

Access for Students with Diverse Abilities

- Representation (Activity 3)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR3: Critique, Correct, Clarify

Required Materials

Materials to Gather

- Copies of blackline masters: Activity 1, Activity 2
- Snap cubes: Activity 1
- Pre-assembled polyhedra: Activity 2
- Rulers marked with centimeters: Activity 2

Materials to Copy

- Finding Volume with Cubes Handout (1 copy for every 6 students): Activity 1
- Can You Find the Volume Cutouts (1 copy for every 3 students): Activity 2

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson:
You will need the Finding Volume with Cubes blackline master for this lesson. You will use only one of the two pages. If your snap cubes measure $\frac{3}{4}$ inch, print the first page of the blackline master, with the slightly smaller shapes. If your snap cubes measure 2 cm, print the second page of the blackline master, with the slightly larger shapes. Make sure to print the blackline master at 100% scale so that the dimensions are accurate. Prepare 1 copy for every 6 students, and cut the pages in half so that each group of 3 students has one half-page.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	10 min	10 min	5 min

Lesson 12

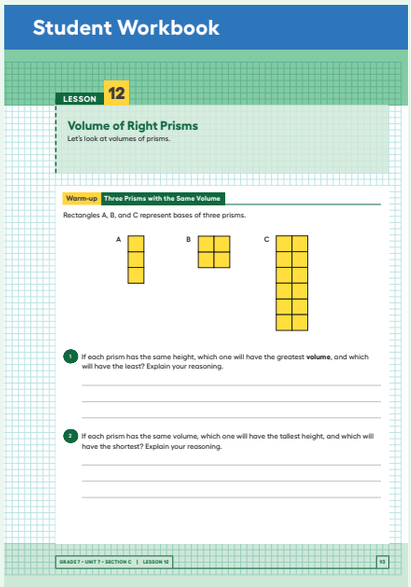
Volume of Right Prisms

Student Learning Goal

Let's look at volumes of prisms.

Print, cut, and assemble the nets from the Can You Find the Volume? blackline master. Card stock paper is recommended. Make sure to print the blackline master at 100% scale so the dimensions are accurate. Prepare 1 polyhedron for every 3 students (1 copy of the entire file for every 18 students).

Make sure that students have access to rulers marked in centimeters and snap cubes.


Student Workbook

LESSON 12

Volume of Right Prisms

Let's look at volumes of prisms.

Warm-up Three Prisms with the Same Volume

Rectangles A, B, and C represent bases of three prisms.

A B C

1. If each prism has the same height, which one will have the greatest **volume**, and which will have the least? Explain your reasoning.

2. If each prism has the same volume, which one will have the tallest height, and which will have the shortest? Explain your reasoning.

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Warm-up

Three Prisms with the Same Volume

5 min

Activity Narrative

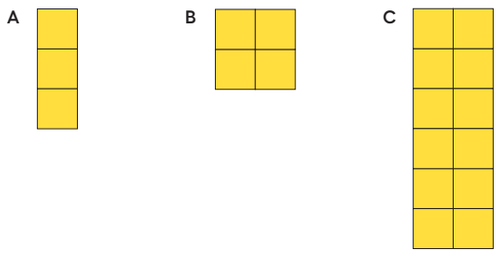
The purpose of this *Warm-up* is to encourage students to think about how the area of the base affects the volume of prisms with the same height and affects the height of prisms with the same volume. This is a review of previous work students have done with volume, in which they found the volume of a rectangular prism by multiplying the area of a base by the corresponding height of the prism. The ideas in this *Warm-up* are revisited later in this lesson, so it is important that students can clearly explain how they ordered their prisms based on them having the same volume and how they found the height of the prism with base C.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by time to discuss their explanations with a partner. Follow with a whole-class discussion.

Student Task Statement

Rectangles A, B, and C represent bases of three prisms.



1. If each prism has the same height, which one will have the greatest **volume**, and which will have the least? Explain your reasoning.

   Prism C will have the greatest volume and prism A will have the least. Since the volume is the area of the base multiplied by the height, the base with the greatest area will have the greatest volume, even if all the heights are the same.

2. If each prism has the same volume, which one will have the tallest height, and which will have the shortest? Explain your reasoning.

   Prism A will have the tallest height and prism C will have the shortest. Since the volume is the area of the base multiplied by the height, the base with the least area will have the tallest height, and the base with the greatest area will have the shortest height.

Activity Synthesis

The purpose of this discussion is to clarify the relationship between a prism's area, height, and volume. Select students to share which prisms they found to have the greatest and least volume and the tallest and shortest height. Record and display their responses for all to see. Ask the class if they agree or disagree. If students all agree, ask a few students to share their reasoning. If they do not agree, ask students to share their reasoning until they reach an agreement.

If there is time, display this question for all to see:

- "If each prism has the same volume and the prism associated with base B has a height of 6 units, what is the height of the prism associated with base C?"

Have students share the volume of the prism with base C and their reasoning. Record and display the responses for all to see.

Activity 1

Finding Volume with Cubes

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students extend their understanding of prisms to see that even when the base is not a rectangle, they can still calculate the volume of a prism by multiplying the area of the base by the height of the prism.

Students use snap cubes or the digital applet to build a prism with a base that matches the shape from the blackline master. Each group needs 30 snap cubes for the first question and a total of 60 snap cubes for the second question. If there are not enough snap cubes, two groups of 3 students may combine together (after answering the first question) to form one group of 6 students.

If using snap cubes that measure $\frac{3}{4}$ inch, make copies of the first page of the blackline master, which has the slightly smaller shapes. If using snap cubes that measure 2 cm, make copies of the second page of the blackline master, which has the slightly larger shapes.

In the digital version of the activity, students use an applet to build a prism. The applet allows students to manipulate digital versions of snap cubes. Use the digital version if you do not have access to enough snap cubes.

Launch

Arrange students in groups of 3. Distribute copies of the blackline master, one half-page to each group, and 30–60 snap cubes to each group. Give students 5–8 minutes of quiet work time followed by a whole-class discussion.

For students using digital materials: depending on the needs of your class, either demonstrate how to build figures using the applet, or instruct students to read and follow the instructions for using the applet.

Student Workbook

1 Finding Volume with Cubes

Your teacher will give you a paper with a shape on it and some snap cubes.

1. Using the face of a snap cube as your area unit, what is the area of the shape? Explain or show your reasoning.

2. Use snap cubes to build the shape from the paper. Add another layer of cubes on top of the shape you have built. Describe this three-dimensional object.

3. What is the volume of your object? Explain your reasoning.

4. Right now, your object has a height of 2. What would the volume be:
   a. if it had a height of 5?
   b. if it had a height of 8.5?

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Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "How do you know if a three-dimensional figure is a prism?" Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive. *Advances: Writing, Speaking, Listening*

Student Task Statement

Your teacher will give you a paper with a shape on it and some snap cubes.

1. Using the face of a snap cube as your area unit, what is the area of the shape? Explain or show your reasoning.

   27 units^2

   Sample reasoning: $3 \cdot 3 + 3 \cdot 6 = 27$.

2. Use snap cubes to build the shape from the paper. Add another layer of cubes on top of the shape you have built. Describe this three-dimensional object.

   Sample response: It is a prism with a base in the shape of an "L."

3. What is the volume of your object? Explain your reasoning.

   54 units^3, because $2 \cdot 27 = 54$.

4. Right now, your object has a height of 2. What would the volume be:

   a. if it had a height of 5?

      135 units^3, because it would take 5 layers and $5 \cdot 27 = 135$.

   b. if it had a height of 8.5?

      229.5 units^3, because $8.5 \cdot 27 = 229.5$.

Activity Synthesis

The goal of this discussion is to clarify that the volume of a prism can be calculated by multiplying the area of the base by the height of the prism, even if the base is not a rectangle.

Select students to share their reasoning.

Here are some questions for discussion:

- *"How do you know this figure is a prism?"*

  Cross sections parallel to the base are identical copies.

- *"What is the area of the base of this figure?"*

  It is the number of cubes in one layer of the prism.

- *"How do you calculate the total number of cubes to make the prism?"*

  Multiply the number of cubes in one layer by the number of layers.

- *"What is the volume of this prism?"*

  The volume is the same as calculating the number of cubes to make the prism.

- *"If you find the area of the base, how do you use that information to calculate the volume of the prism?"*

  Multiply the area of the base by the height of the prism.

- *"How would the volume of the prism change if we changed the shape of the base but still used 27 cubes to build it?"*

  The volume would not change.

If not mentioned by students, explain that calculating the total number of cubes to make the prism is the same as calculating the volume of the prism. We can find the area of the base of the prism and multiply that by the number of layers in the prism, which is the same as the height of the prism. The height of the prism is measured in units, the area of the base is measured in units^2, and the volume of the prism is measured in units^3.

Activity 2
Can You Find the Volume?

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to get hands-on experience with polyhedra, recognizing whether or not a figure is a prism and, if it is, determining which face is the base of the prism. Once students determine which face is the base, they use a ruler marked in inches to measure the height of the prism. The area of each face is labeled on the shape so that students do not get bogged down with calculating the base area and can focus on using the area to find the volume.

In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Instead of creating enough sets of polyhedra for every group to have one of every shape at the same time, consider having the students pass the shapes from one group to the next or rotate around to different stations so that fewer sets of shapes have to be constructed.

In the digital version of the activity, students use an applet to explore the polyhedra. The applet allows students to turn and view the shapes from any direction. Use the digital version if it would be easier to use devices than the shapes made from the nets.

If students don't have individual access to the applet, displaying it for all to see would be helpful during the synthesis.

Launch

Arrange students in groups of 3. Distribute the three-dimensional figures that were already assembled from the nets in the blackline master and rulers marked in centimeters. Give students 1–2 minutes of quiet work time with the polyhedra given to their group, have groups exchange objects so that each group gets to examine each figure. Follow with a whole-class discussion.

Student Task Statement

Your teacher will give you a set of three-dimensional figures.

1. For each figure, determine whether the shape is a prism.

   A. Yes, it is a square prism. Any cross section parallel to any of the faces will be the same size square, so it is a prism.

   B. Yes, it is a pentagonal prism. Any cross section parallel to the pentagon base will be the same size pentagon, so it is a prism.

   C. No, it is a hexagonal pyramid. Cross sections parallel to the hexagon will be smaller hexagons.

   D. Yes, it is a triangular prism. Any cross section parallel to the triangle base will be the same size triangle, so it is a prism.

   E. No. Cross sections are not the same all the way through the object.

   F. Yes, it is a prism. Any cross section parallel to the rhombus base will be the same size rhombus, so it is a prism.

Instructional Routines

MLR3: Critique, Correct, Clarify

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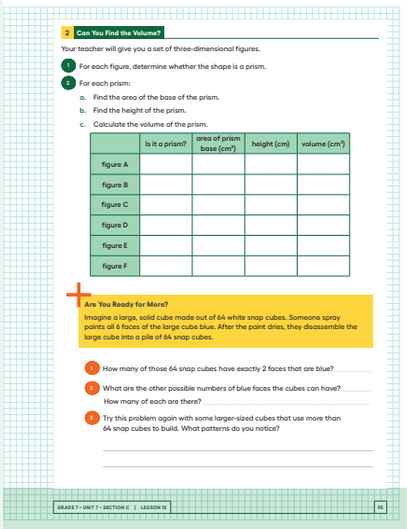
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify. This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Student Workbook



2 Can You Find the Volume?

Your teacher will give you a set of three-dimensional figures.

1. For each figure, determine whether the shape is a prism.
2. For each prism:
   a. Find the area of the base of the prism.
   b. Find the height of the prism.
   c. Calculate the volume of the prism.

	Is it a prism?	area of prism base (cm^2)	height (cm)	volume (cm^3)
figure A				
figure B				
figure C				
figure D				
figure E				
figure F				

Are You Ready for More?

Imagine a large, solid cube made out of 64 white snap cubes. Someone spray paints all 6 faces of the large cube blue. After the paint dries, they disassemble the large cube into a pile of 64 snap cubes.

1. How many of those 64 snap cubes have exactly 2 faces that are blue?
2. What are the other possible numbers of blue faces the cubes can have? How many of each are there?
3. Try this problem again with some larger-sized cubes that use more than 64 snap cubes to build. What patterns do you notice?

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2. For each prism:

a. Find the area of the base of the prism.

b. Find the height of the prism.

c. Calculate the volume of the prism.

	Is it a prism?	area of prism base (cm^2)	height (cm)	volume (cm^3)
figure A	Yes	25	5	125
figure B	Yes	15	15	225
figure C	No			
figure D	Yes	24	6	144
figure E	No			
figure F	Yes	35	3	105

Are You Ready for More?

Imagine a large, solid cube made out of 64 white snap cubes. Someone spray paints all 6 faces of the large cube blue. After the paint dries, they disassemble the large cube into a pile of 64 snap cubes.

1. How many of those 64 snap cubes have exactly 2 faces that are blue?

24 cubes have 2 faces painted blue

2. What are the other possible numbers of blue faces the cubes can have? How many of each are there?

- 3 faces: 8 cubes
- 1 face: 24 cubes
- 0 faces: 8 cubes.

3. Try this problem again with some larger-sized cubes that use more than 64 snap cubes to build. What patterns do you notice?

Sample response: There are always 8 cubes with 3 blue faces, there is a cube on the inside with side length 2 units less than the large cube that doesn't get painted at all.

(Students can make similar observations and perhaps find formulas for the number of cubes with 1 or 2 blue faces.)

Activity Synthesis

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to determine whether each shape is a prism by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"Figure A is not a prism because it is a cube."

Ask,

- *"What parts of this response are unclear, incorrect, or incomplete?"*

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits, to make the final draft even more clear and more convincing.

Activity 3: Optional

What's the Prism's Height?

10 min

Activity Narrative

The purpose of this activity is for students to work backward from the volume to the height of a prism. Students see that for two prisms to have the same volume, the one with the smaller base has the taller height, and the one with the larger base has the shorter height. The grid helps students find the area of the base so they can focus their attention on what it means to have a prism made out of stacks of layers of the same base.

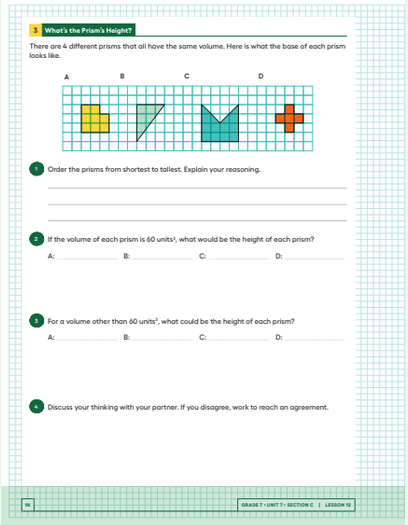
Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by time to discuss their thinking with a partner. Follow this with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 3, Launch)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with geometric solids or snap cubes and a printed copy of the student task statement to draw on or annotate.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Student Workbook


3 What's the Prism's Height?

There are 4 different prisms that all have the same volume. Here is what the base of each prism looks like.

A B C D

1. Order the prisms from shortest to tallest. Explain your reasoning.

2. If the volume of each prism is 60 units³, what would be the height of each prism?

A: B: C: D:

3. For a volume other than 60 units³, what could be the height of each prism?

A: B: C: D:

4. Discuss your thinking with your partner. If you disagree, work to reach an agreement.

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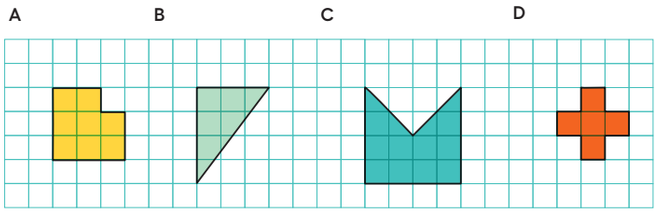
Student Task Statement

There are 4 different prisms that all have the same volume. Here is what the base of each prism looks like.



1. Order the prisms from shortest to tallest. Explain your reasoning.

   C, A, B, D, since a larger base area means the height of the prism must be shorter to maintain the same volume.

2. If the volume of each prism is 60 units³, what would be the height of each prism?

   If the volume of all 4 prisms is 60 units³, prism A is 7.5 units tall, prism B is 10 units tall, prism C is 5 units tall, and prism D is 12 units tall.

3. For a volume other than 60 units³, what could be the height of each prism?

   Sample responses:

   - The volume of all 4 prisms is 48 units³ if prism A is 6 units tall, B is 8 units tall, C is 4 units tall, and D is 9.6 units tall.
   - The volume of all 4 prisms is 120 units³ if prism A is 15 units tall, B is 20 units tall, C is 10 units tall, and D is 24 units tall.

4. Discuss your thinking with your partner. If you disagree, work to reach an agreement.

Activity Synthesis

The goal of this discussion is to solidify the connection between volume, height, and area of the base of a prism. Select students to share their responses and reasoning. If not brought up in a student's explanation, explain that for the last problem, there is more than one possible correct answer. The smallest possible volume that involves all whole number side lengths is 120 units³, but there is nothing in the problem that requires all the heights to be whole numbers.

Here are some questions for discussion:

- *"How do you calculate the volume of a prism?"*

Display the equation for all to see: $V = B \cdot h$.

- *"Since $V = B \cdot h$, how could we find the area of the base if we knew the volume and height of the prism?"*

Display the equation for all to see: $B = V \div h$.

- *"If we keep the volume the same, what happens to the height when we increase the area of the base?"*

  It decreases.

- *"If we keep the height the same, what happens to the volume when we increase the area of the base?"*

  It increases.

Lesson Synthesis

"What information do we need to calculate the volume of a prism?"

Area of the base and the height

"Explain how you could use layers to find the volume of a prism."

If you look at the first layer of a prism, you can find how many cubes are in that layer by finding the area of the base. Once you find the number of cubes on the first layer, you multiply that by the number of layers it takes to stack up to the height of the prism.

"Two prisms have the same base area and height, but they have different base shapes. Which prism has a greater volume? Explain."

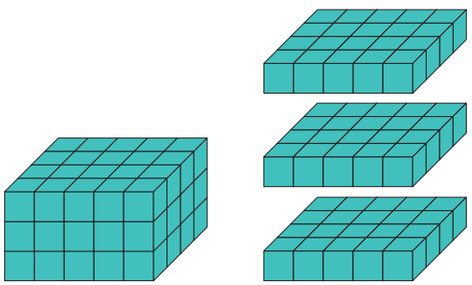
The two prisms have the same volume. The shape of the base does not matter if it is a prism—only the base area matters.

"Two clay prisms use the same amount of clay to make them, but the first has a larger height than the second has. Which prism has a larger base area?"

The second prism will have a larger base area since a shorter height means a larger base area if the volume is held constant. Imagine squashing the first one down in a nice way to make a shorter, fatter version.

Lesson Summary

Any cross section of a prism that is parallel to the base will be identical to the base. This means we can slice prisms up to help find their volume. For example, if we have a rectangular prism that is 3 units tall and has a base that is 4 units by 5 units, we can think of this as 3 layers, where each layer has $4 \cdot 5$ cubic units. The **volume** of the figure is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.

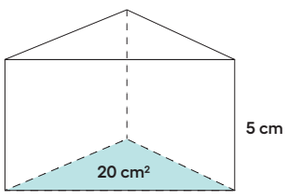


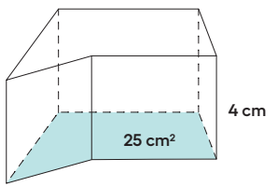
That means the volume of the original rectangular prism is $3(4 \cdot 5)$, or 60, cubic units.

This works with any prism! If we have a prism with a height of 3 cm that has a base with an area of 20 cm^2, then the volume is $3 \cdot 20$ cm^3 regardless of the shape of the base. In general, the volume of a prism with height h and area B is

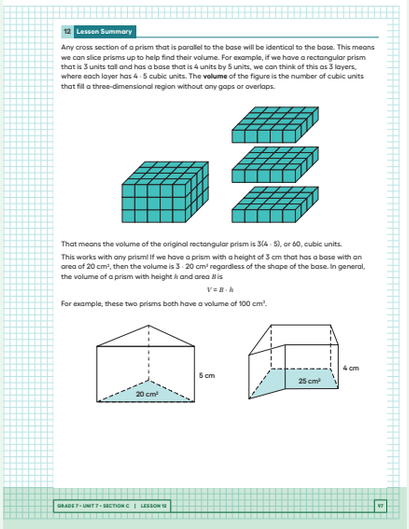
$V = B \cdot h$

For example, these two prisms both have a volume of 100 cm^3.





Student Workbook


12 Lesson Summary

Any cross section of a prism that is parallel to the base will be identical to the base. This means we can slice prisms up to help find their volume. For example, if we have a rectangular prism that is 3 units tall and has a base that is 4 units by 5 units, we can think of this as 3 layers, where each layer has 4 · 5 cubic units. The **volume** of the figure is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.

That means the volume of the original rectangular prism is 3(4 · 5), or 60, cubic units.

This works with any prism! If we have a prism with a height of 3 cm that has a base with an area of 20 cm², then the volume is 3 · 20 cm³ regardless of the shape of the base. In general, the volume of a prism with height h and area B is

$V = B \cdot h$

For example, these two prisms both have a volume of 100 cm³.

5 cm
20 cm²
4 cm
25 cm²

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

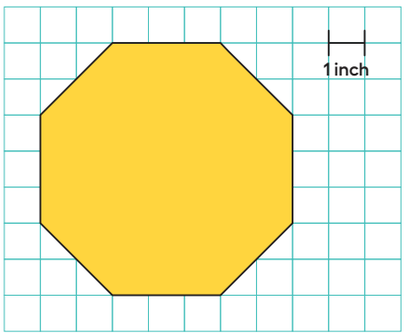
Cool-down

Octagonal Box

5 min

Student Task Statement

A box is shaped like an octagonal prism. Here is what the base of the prism looks like.



For each question, make sure to include the unit with your answer and explain or show your reasoning.

1. If the height of the box is 7 inches, what is the volume of the box?

   287 in^3, because the base has an area of 41 in^2, and $41 \cdot 7 = 287$.

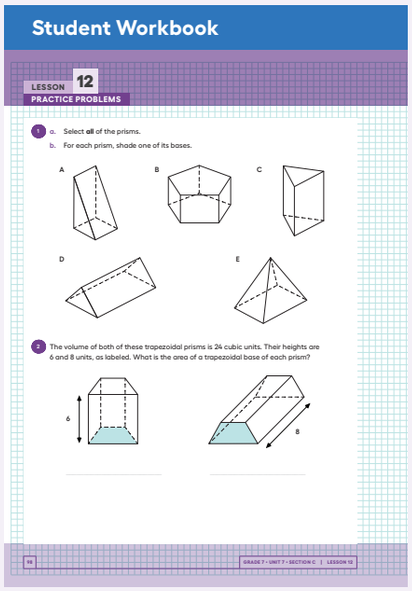
2. If the volume of the box is 123 in^3, what is the height of the box?

   3 in, because $41 \cdot 3 = 123$.

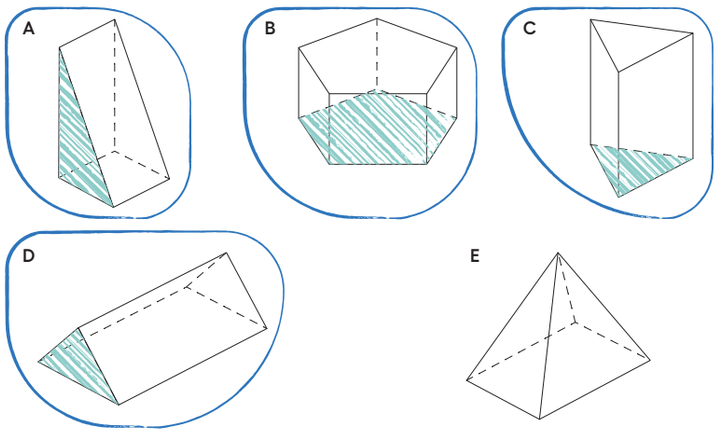
Lesson 12

Practice Problems

6 Problems



Problem 1

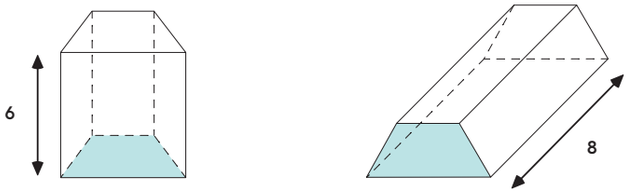


a. Select **all** of the prisms.

b. For each prism, shade one of its bases.

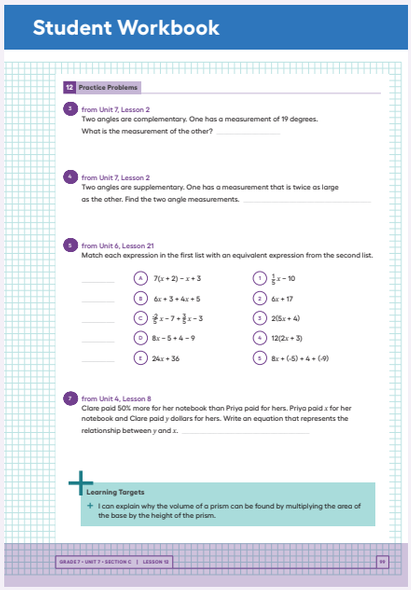
Problem 2

The volume of both of these trapezoidal prisms is 24 cubic units. Their heights are 6 and 8 units, as labeled. What is the area of a trapezoidal base of each prism?



The prism with a height of 6 units has a base with area of 4 square units, because 24 ÷ 6 = 4.

The prism with a height of 8 units has a base with area of 3 square units, because 24 ÷ 8 = 3.


Student Workbook

12 Practice Problems

3 from Unit 7, Lesson 2
Two angles are complementary. One has a measurement of 19 degrees. What is the measurement of the other? _____

4 from Unit 7, Lesson 2
Two angles are supplementary. One has a measurement that is twice as large as the other. Find the two angle measurements. _____

5 from Unit 6, Lesson 21
Match each expression in the first list with an equivalent expression from the second list.

_____	A $7(x+2)-x+3$	1 $\frac{1}{5}x-10$
_____	B $6x+3+4x+5$	2 $6x+17$
_____	C $\frac{-2}{5}x-7+\frac{3}{5}x-3$	3 $2(5x+4)$
_____	D $8x-5+4-9$	4 $12(2x+3)$
_____	E $24x+36$	5 $8x+(-5)+4+(-9)$

7 from Unit 4, Lesson 8
Clare paid 50% more for her notebook than Priya paid for hers. Priya paid x for her notebook and Clare paid y dollars for hers. Write an equation that represents the relationship between y and x. _____

Learning Targets
+ I can explain why the volume of a prism can be found by multiplying the area of the base by the height of the prism.

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Problem 3

from Unit 7, Lesson 2

Two angles are complementary. One has a measurement of 19 degrees. What is the measurement of the other?

71 degrees

Problem 4

from Unit 7, Lesson 2

Two angles are supplementary. One has a measurement that is twice as large as the other. Find the two angle measurements.

60° and 120°

Problem 5

from Unit 6, Lesson 21

Match each expression in the first list with an equivalent expression from the second list.

A. $7(x+2)-x+3$	**1.** $\frac{1}{5}x-10$
B. $6x+3+4x+5$	**2.** $6x+17$
C. $\frac{-2}{5}x-7+\frac{3}{5}x-3$	**3.** $2(5x+4)$
D. $8x-5+4-9$	**4.** $12(2x+3)$
E. $24x+36$	**5.** $8x+(-5)+4+(-9)$

Problem 6

from Unit 4, Lesson 8

Clare paid 50% more for her notebook than Priya paid for hers. Priya paid x for her notebook and Clare paid y dollars for hers. Write an equation that represents the relationship between y and x.

$y = 1.5x$ (or equivalent)