Lesson 3

What a Point in a Scatter Plot Means

Goals

- Coordinate (orally and in writing) data in a table and points on a scatter plot.
- Describe (orally) the trend of the data, and use the trend to predict unknown values.
- Interpret (orally and in writing) a point on a scatter plot in context.

Learning Target

I can describe the meaning of a point in a scatter plot in context.

Lesson Narrative

In this lesson, students interpret points in a scatter plot in terms of a context, and add points to a scatter plot given information about an individual in the population. They compare individuals represented by different points and informally discuss trends in the data.

There are two levels of analysis needed to successfully make sense of scatter plots: what is happening for a particular individual, and what is happening at a global level for the entire population. The ability to move between these two zoom levels develops over time. In this lesson, students spend a lot of time looking at the details of a scatter plot, naming the quantities represented in a scatter plot, and focusing on the meaning of individual points.

Student Learning Goal

Let's investigate points in scatter plots.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Warm-up)

Instructional Routines

- MLR5: Co-Craft Questions

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

What Are These Points?

5 min

Activity Narrative

In this activity, students examine a scatter plot showing weight and fuel efficiency of cars. Students can show their understanding of what data graphed as points means and become familiar with the context of fuel efficiency.

Launch

Arrange students in groups of 2. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

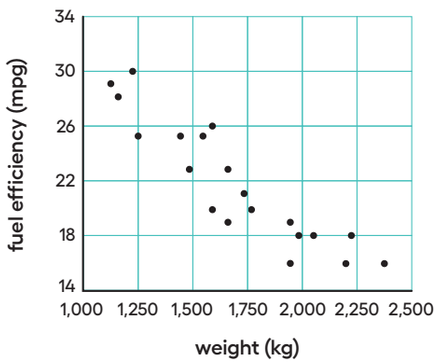
Display the problem stem, table, and scatter plot. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Student Task Statement

What questions do you have about these data?

car	weight (kg)	fuel efficiency (mpg)
A	1,549	25
B	1,610	20
C	1,737	21
D	1,777	20
E	1,486	23
F	1,962	16
G	2,384	16
H	1,957	19
I	2,212	16

car	weight (kg)	fuel efficiency (mpg)
J	1,115	29
K	2,068	18
L	1,663	19
M	2,216	18
N	1,432	25
O	1,987	18
P	1,580	26
Q	1,234	30
R	1,656	23



Sample response:

- Are the weight and fuel efficiency of cars related?
- What is fuel efficiency?
- Which point represents car A?

Access for Multilingual Learners (Warm-up)

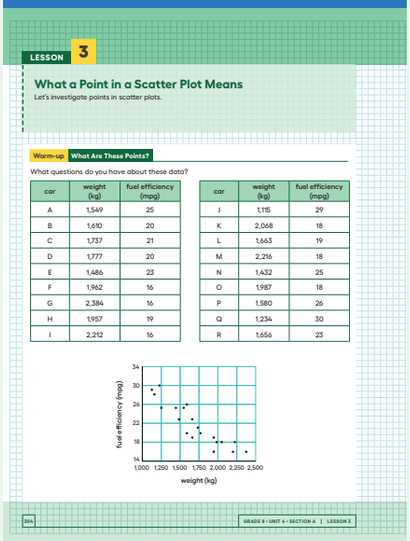
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 3

What a Point in a Scatter Plot Means

Let's investigate points in scatter plots.

Warm-up What Are These Points?

What questions do you have about these data?

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Activity Synthesis

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

💬 *"What do these questions have in common? How are they different?"*

Listen for and amplify language related to the learning goal, such as finding individual cars in the scatter plot or noticing a pattern in the data.

If necessary, explain a bit about fuel efficiency so that students understand this measurement. Fuel efficiency is a measure of the average distance a car will travel using a certain amount of gas. Commonly this is measured in miles per gallon. For example, a car that has a fuel efficiency of 25 miles per gallon (mpg) should be able to drive approximately 25 miles while using up a gallon of gas. Many factors can influence fuel efficiency, including the way an engine is engineered (to produce more power, an engine may use more gasoline), driving conditions (more frequent stopping and starting or driving up and down hills will use fuel less efficiently), and what accessories are being used (air conditioning requires energy from burning fuel that is not used for actually driving the car).

Activity 1

Weight and Fuel Efficiency

15 min

Activity Narrative

There is a digital version of this activity.

The analysis of scatter plots continues with data about the weight of automobiles and their fuel efficiency. Again, students connect points in the scatter plot with rows of data in a table, but now there is a third column that gives a name to each pair.

After picking out information about points in the scatter plot and table, students begin to think about how scatter plots can help them make predictions. Specifically, when looking at two particular points in the scatter plot, students are asked if the results are surprising given the overall trend of the data. In later lessons, students develop more sophisticated tools for answering questions like this (specifically, by fitting lines to the data), so a heuristic discussion is all that is expected at this point.

In the digital version of the activity, students use an applet to create the scatter plot and drag a point to answer a question. The applet allows students to enter data in a table and let the computer create the visualization. Use the digital version if available to allow students to focus on analyzing the scatter plot without worrying about construction.

Launch

Arrange students in groups of 2. If needed, remind students of the context of fuel efficiency for cars.

Student Task Statement

The table and scatter plot show weights and fuel efficiencies of 18 cars.

car	weight (kg)	fuel efficiency (mpg)
A	1,549	25
B	1,610	20
C	1,737	21
D	1,777	20
E	1,486	23
F	1,962	16
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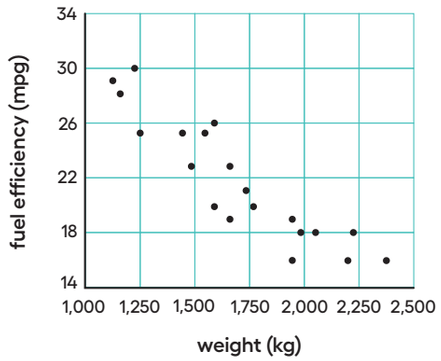


1. Which point in the scatter plot represents Car L's measurements?

   The point with coordinates (1,663, 19)

2. What is the fuel efficiency of the car with the greatest weight?

   16 mpg

3. What is the weight of the car with the greatest fuel efficiency?

   1,234 kg

4. Car S weighs 1,912 kilograms and gets 16 miles per gallon. On the scatter plot, plot a point that represents Car S's measurements.

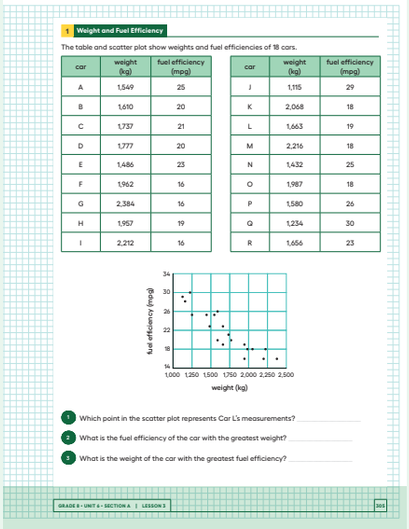
   The point with coordinates (1,912, 16)

5. Cars N and O, shown in the scatter plot, are made by the same company. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?

   Car N has a lower weight and a higher fuel efficiency than car O.

   Sample response: Nothing is surprising about these cars because they fit in with the trend of the data.

Student Workbook


1 Weight and Fuel Efficiency

The table and scatter plot show weights and fuel efficiencies of 18 cars.

car	weight (kg)	fuel efficiency (mpg)	car	weight (kg)	fuel efficiency (mpg)
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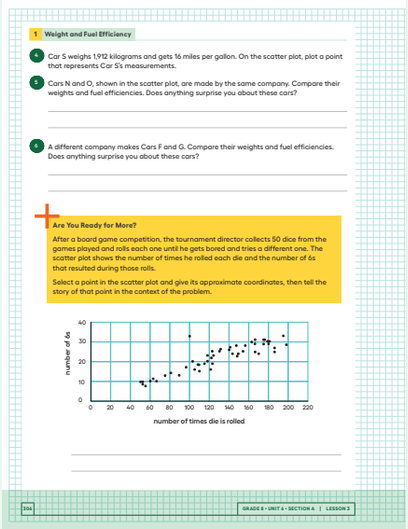
I	2,212	16	R	1,656	23

1. Which point in the scatter plot represents Car L's measurements?
2. What is the fuel efficiency of the car with the greatest weight?
3. What is the weight of the car with the greatest fuel efficiency?

Student Workbook


1 Weight and Fuel Efficiency

4. Car S weighs 1,912 kilograms and gets 16 miles per gallon. On the scatter plot, plot a point that represents Car S's measurements.
5. Cars N and O, shown in the scatter plot, are made by the same company. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?
6. A different company makes Cars F and G. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?

Are You Ready for More?

After a board game competition, the tournament director collects 50 dice from the games played and rolls each one until he gets bored and tries a different one. The scatter plot shows the number of times he rolled each die and the number of 6s that resulted during those rolls.

Select a point in the scatter plot and give its approximate coordinates, then tell the story of that point in the context of the problem.

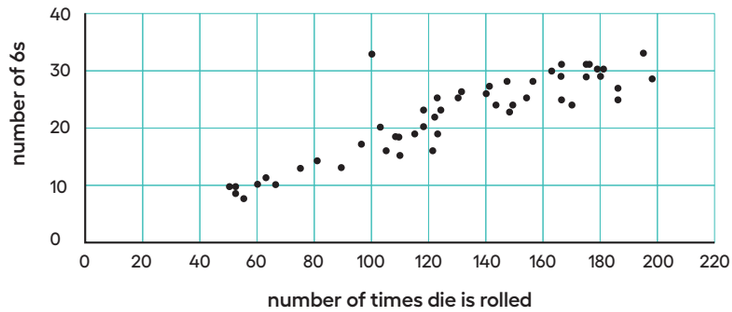
6. A different company makes Cars F and G. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?

Their weights are quite different, but they still have the same fuel efficiency.

Sample response: It is surprising that cars with such different weights made by the same company have the same fuel efficiency since, generally, the heavier the car, the lower the fuel efficiency.

Are You Ready for More?

After a board game competition, the tournament director collects 50 dice from the games played and rolls each one until he gets bored and tries a different one. The scatter plot shows the number of times he rolled each die and the number of 6s that resulted during those rolls.



Select a point in the scatter plot and give its approximate coordinates, then tell the story of that point in the context of the problem.

Sample response: The point at (100, 33) lies apart from the rest of the data. Someone may have brought a die from home that is weighted to roll more 6s, expecting to cheat and win the tournament.

Activity Synthesis

The purpose of the discussion is for students to compare pairs of data in both representations and to begin thinking about trends in the data. Ask students to share their thinking about the last 2 questions. Draw out the fact that Cars N and O seem to fit the overall trend that an increase in weight typically corresponds to a decrease in fuel efficiency but Cars F and G don't fit that trend when compared with each other. Mention that we will see many examples of trends that don't necessarily apply to individuals in a population.

Consider asking some of the following questions to conclude the discussion:

- *"How did you find the automobile with the greatest weight?"*

  I looked for the point farthest to the right.

- *"How did you find the automobile with the greatest fuel efficiency?"*

  I looked for the highest point.

- *"The data in this table is from automobiles that run solely on gasoline. Where do you think a hybrid car would appear on the graph?"*

  A hybrid car would use less gas for the same weight, so its point would be above the other points with a similar weight.

- *"Did the scatter plot match your prediction?"*

  For the most part, yes. I thought that a heavier car would have a lower fuel efficiency and match the general trend of the data.

Access for Students with Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code temperature and coat sales in the graph and table.

Supports accessibility for: Visual-Spatial Processing

Activity 2

Coat Sales

15 min

Activity Narrative

In this activity, students continue to identify points on a scatter plot as representatives of a single month when two variables are measured. Students are also asked to explain the meaning of some abstract points in the context of the problem and identify when it does not make sense to extrapolate information from the graph based on the context.

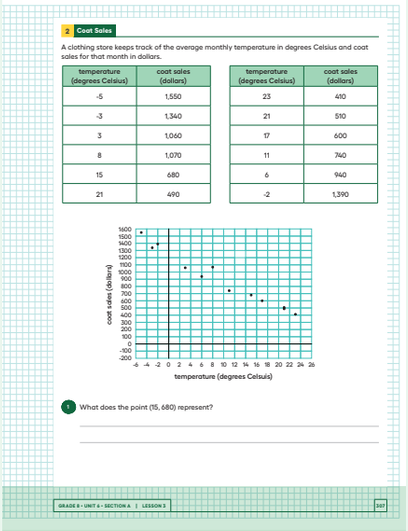
Monitor for students who understand that 60 degrees Celsius is unreasonable for an average monthly temperature in most parts of the world, as well as those who identify the trend to predict negative sales, which does not make sense in this context either.

Launch

Arrange students in groups of 2. Allow students 5 minutes quiet work time followed by 5 minutes partner discussion and whole-class discussion.

Ask students to predict what a scatter plot might look like when the temperature is along the x-axis and the sales of coats at a store is along the y-axis.

Student Workbook



2 Coat Sales

A clothing store keeps track of the average monthly temperature in degrees Celsius and coat sales for that month in dollars.

temperature (degrees Celsius)	coat sales (dollars)
-5	1,550
-3	1,340
3	1,060
8	1,070
15	680
21	490

temperature (degrees Celsius)	coat sales (dollars)
23	410
21	510
17	600
11	740
6	940
-2	1,390

1 What does the point (15, 680) represent?

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Student Workbook

2 Coat Sales

2 Use the table to find the coat sales for the month with the lowest average temperature.

3 Use the graph to estimate the average monthly temperature for the month with the smallest coat sales. Explain how you used the scatter plot to estimate the answer.

4 If there were a point at $(0, A)$ what would it represent? Assume this point follows the trend of the other data and use the scatter plot to estimate a value for A.

5 What would a point at $(B, 0)$ represent? Assume this point follows the trend of the other data and use the scatter plot to estimate a value for B.

6 Would it make sense to use this trend to estimate the value of sales when the average monthly temperature is 60 degrees Celsius? Explain your reasoning.

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Student Task Statement

A clothing store keeps track of the average monthly temperature in degrees Celsius and coat sales for that month in dollars.

temperature (degrees Celsius)	coat sales (dollars)
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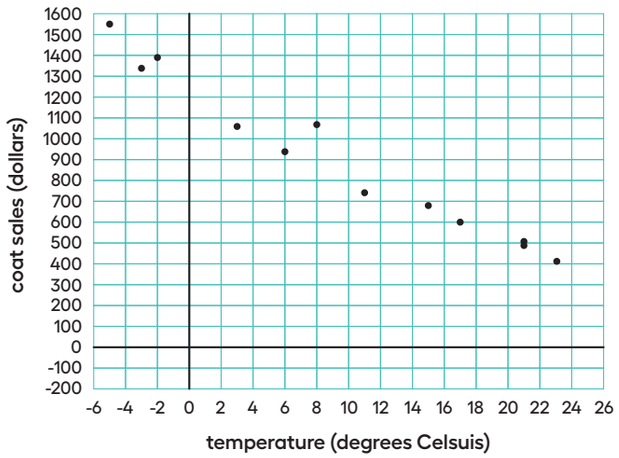

1. What does the point (15, 680) represent?

When the average monthly temperature is 15 degrees Celsius, the store sells $680 of coats.

2. Use the table to find the coat sales for the month with the lowest average temperature.

$1,550

3. Use the graph to estimate the average monthly temperature for the month with the smallest coat sales. Explain how you used the scatter plot to estimate the answer.

23 degrees Celsius

In the graph, I found the lowest point and estimated the x-coordinate.

4. If there were a point at $(0, A)$ what would it represent? Assume this point follows the trend of the other data and use the scatter plot to estimate a value for A.

   When the temperature averaged 0 degrees Celsius for the month, the store sold A dollars of coats.

   Sample response: $1,250

5. What would a point at $(B, 0)$ represent? Assume this point follows the trend of the other data and use the scatter plot to estimate a value for B.

   When the store sold no coats for the month, the average monthly temperature is B degrees Celsius.

   Sample response: 33 degrees Celsius.

6. Would it make sense to use this trend to estimate the value of sales when the average monthly temperature is 60 degrees Celsius? Explain your reasoning.

   It does not make sense. 60 degrees Celsius (140 Fahrenheit) is hotter than has been recorded on Earth. Also, based on the trend, there would be negative dollars sold.

Activity Synthesis

The purpose of the discussion is for students to understand that each point in the graph represents a single month in which two measurements are made. Select students to share their responses, including those identified for the last problem.

Consider asking some of the following questions:

- *"When discussing functions, what is a point of the form $(0, A)$ called? What about a point of the form $(B, 0)$?"*

  The y-intercept; the x-intercept

- *"In a scatter plot, is there a y-intercept?"*

  It depends on the context. For some situations, it may not make sense for the variable represented along the x-axis to be 0. For example, if the variable is the height of a building. On other situations, there may be more than 1 point along the y-axis. For example, in a situation like the one in the task, there may be multiple months where the average temperature is 0 degree Celsius.

- *"When the average monthly temperature goes up, what seems to be happening to sales?"*

  They are decreasing.

Student Workbook


2 Coat Sales

2. Use the table to find the coat sales for the month with the lowest average temperature.

3. Use the graph to estimate the average monthly temperature for the month with the smallest coat sales. Explain how you used the scatter plot to estimate the answer.

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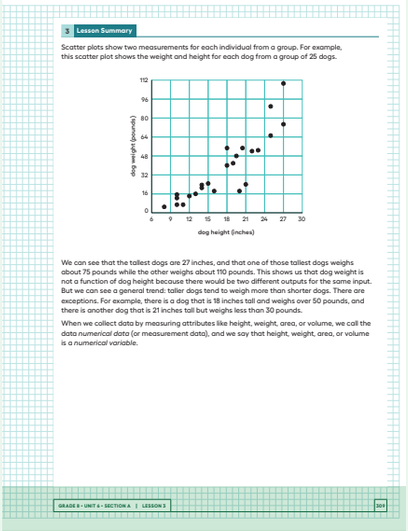
5. What would a point at $(B, 0)$ represent? Assume this point follows the trend of the other data and use the scatter plot to estimate a value for B.

6. Would it make sense to use this trend to estimate the value of sales when the average monthly temperature is 60 degrees Celsius? Explain your reasoning.

Student Workbook


3 Lesson Summary

Scatter plots show two measurements for each individual from a group. For example, this scatter plot shows the weight and height for each dog from a group of 25 dogs.

We can see that the tallest dogs are 27 inches, and that one of those tallest dogs weighs about 75 pounds while the other weighs about 110 pounds. This shows us that dog weight is not a function of dog height because there would be two different outputs for the same input. But we can see a general trend: taller dogs tend to weigh more than shorter dogs. There are exceptions. For example, there is a dog that is 18 inches tall and weighs over 50 pounds, and there is another dog that is 21 inches tall but weighs less than 30 pounds.

When we collect data by measuring attributes like height, weight, area, or volume, we call the data *numerical data* (or measurement data), and we say that height, weight, area, or volume is a *numerical variable*.

Lesson Synthesis

Revisit the idea that a point in a scatter plot represents an individual in a population. Ask students:

- *"What kind of information does a point in a scatter plot represent?"*

  Two measurements for an individual in a population.

- *"How do we know what information is captured by a point in a scatter plot?"*

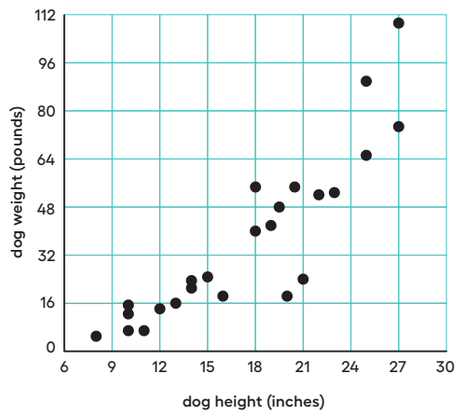
  The axis labels tell us how to interpret the coordinates of the points.

- *"Do trends in a scatter plot necessarily apply to individuals in the population?"*

  No

Lesson Summary

Scatter plots show two measurements for each individual from a group. For example, this scatter plot shows the weight and height for each dog from a group of 25 dogs.



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Cool-down

Quarterbacks

5 min

Students compare data from a table and scatter plot by identifying points in the scatter plot and their corresponding entries in a table. Additionally, students show their understanding of the connection by plotting an additional point to add to the data shown.

Launch

Explain that quarterback rating is a way of rating a quarterback in football on a scale of 0 to 158.3 based on many variables (such as pass attempts, interceptions, and touchdown passes). A higher rating generally represents a quarterback with better passing statistics. When the quarterback starts a game, he is playing from the beginning of the game and is usually the best option available for the team for that position. The data from this activity refers to quarterbacks who were able to start in at least 16 games for their team in a season.

Student Task Statement

In football, a quarterback can be rated by a formula that assigns a number to how well they play. A higher number generally means they played better.

Here are a table and scatter plot that show ratings and wins for quarterbacks who started every game in a season.

player	quarterback rating	number of wins
A	93.8	4
B	102.2	12
C	93.6	6
D	89	8
E	88.2	5
F	97	7
G	88.7	6
H	91.1	7
I	92.7	10

player	quarterback rating	number of wins
J	88	10
K	101.6	9
L	104.6	13
M	84.2	6
N	99.4	15
O	110.1	10
P	95.4	11
Q	88.7	11

Responding To Student Thinking

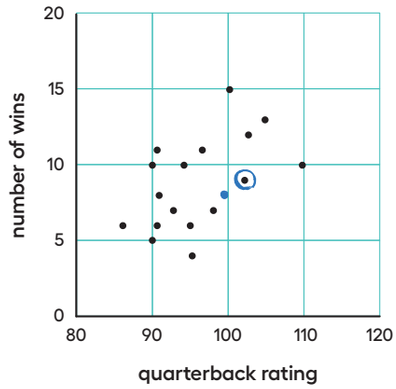
Points to Emphasize

If students struggle with reading scatter plots and identifying points from scatter plots, revisit this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about interpreting data points on the scatterplot and extracting information from the table. Unit 6, Lesson 3, Activity 3 Coat Sales

1. Circle the point in the scatter plot that represents Player K's data.
2. Which quarterback's data are represented by the point farthest to the left?

   Player M

3. Player R is not included in the table. He has a quarterback rating of 99.4 and his team won 8 games. On the scatter plot, plot a point that represents Player R's data.



Lesson 3

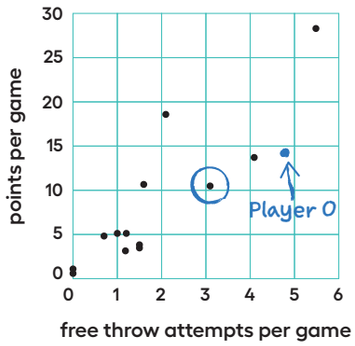
Practice Problems

3 Problems

Problem 1

Here is a table and a scatter plot that compares points per game to free throw attempts for a basketball team during a tournament.

player	free throw attempts	points
player A	5.5	28.3
player B	2.1	18.6
player C	4.1	13.7
player D	1.6	10.6
player E	3.1	10.4
player F	1	5
player G	1.2	5
player H	0.7	4.7
player I	1.5	3.7
player J	1.5	3.5
player K	1.2	3.1
player L	0	1
player M	0	0.8
player N	0	0.6



a. Circle the point that represents the data for Player E.

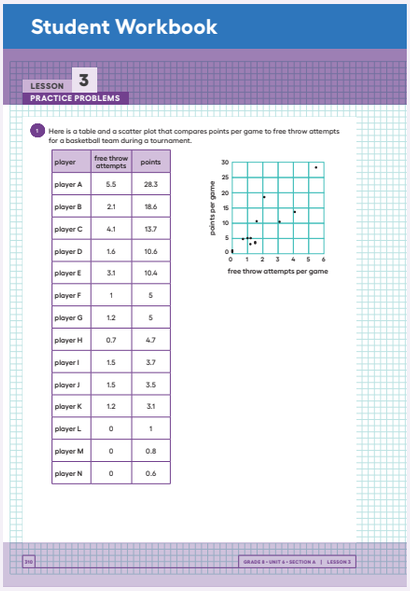
b. What does the point (2.1, 18.6) represent?

The point (2.1, 18.6) represents Player B's data.

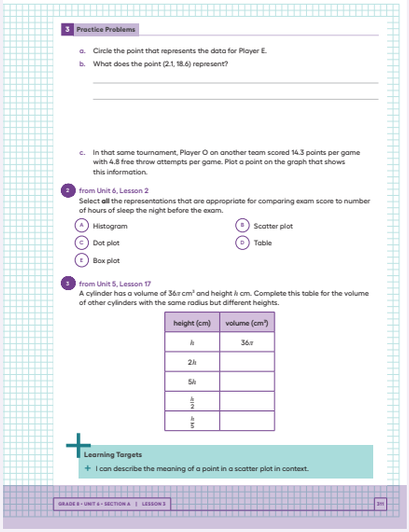
Player B makes 2.1 free throw attempts per game and 18.6 points per game.

c. In that same tournament, Player O on another team scored 14.3 points per game with 4.8 free throw attempts per game. Plot a point on the graph that shows this information.

Student Workbook



Student Workbook



Problem 2

from Unit 6, Lesson 2

Select **all** the representations that are appropriate for comparing exam score to number of hours of sleep the night before the exam.

A. Histogram

B. Scatter plot

C. Dot plot

D. Table

E. Box plot

Problem 3

from Unit 5, Lesson 17

A cylinder has a volume of 36π cm^3 and height h cm. Complete this table for the volume of other cylinders with the same radius but different heights.

height (cm)	volume (cm^3)
h	36π
$2h$	72π
$5h$	180π
$\frac{h}{2}$	18π
$\frac{h}{5}$	$\frac{36}{5}\pi$