Lesson 7

Using Histograms to Answer Statistical Questions

Goals

- Compare and contrast (in writing) histograms that represent two different data sets measuring the same quantity.
- Critique (orally) a description of a distribution, recognizing that there are multiple valid ways to describe its center and spread.
- Describe (orally and in writing) the distribution shown on a histogram, including making claims about the center and spread.

Learning Targets

- I can draw a histogram from a table of data.
- I can use a histogram to describe the distribution of data and determine a typical value for the data.

Lesson Narrative

In this lesson, students create, read, and interpret histograms in situations. They characterize the distribution displayed in a histogram in terms of its shape and spread, and identify a measurement that is typical for the data set by looking for the center in a histogram. Students also use histograms to make comparisons and to better understand what different spreads and values of center mean in a given context.

Student Learning Goal

Let's draw histograms and use them to answer questions.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

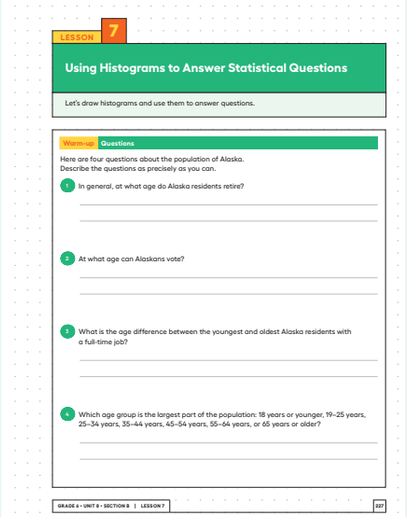
Materials to Gather

- Rulers marked with centimeters: Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down

Student Workbook



LESSON 7

Using Histograms to Answer Statistical Questions

Let's draw histograms and use them to answer questions.

Warm-up Questions

Here are four questions about the population of Alaska.
Describe the questions as precisely as you can.

1. In general, at what age do Alaska residents retire?
2. At what age can Alaskans vote?
3. What is the age difference between the youngest and oldest Alaska residents with a full-time job?
4. Which age group is the largest part of the population: 18 years or younger, 19–25 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, or 65 years or older?

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Warm-up

Questions

5 min

Activity Narrative

In this *Warm-up*, students use the terminology they have learned in the unit so far to describe questions. Students are reminded to talk about statistical questions, numerical and categorical data, and typical values. Students have the chance to practice being precise in their mathematical language.

Launch

Arrange students in groups of 2–4. Display mathematical vocabulary related to questioning and data already discussed in the unit such as:

- Numerical data.
- Categorical data.
- Statistical question.
- Typical value.
- Spread.
- Variability.

Student Task Statement

Here are four questions about the population of Alaska.

Describe the questions as precisely as you can.

1. In general, at what age do Alaska residents retire?

   Sample response: A statistical question that requires numerical data. The answer will be the typical value.

2. At what age can Alaskans vote?

   Sample response: A non-statistical question that will have a numerical answer.

3. What is the age difference between the youngest and oldest Alaska residents with a full-time job?

   Sample response: A statistical question that requires numerical data. The answer is based on how spread out the data are.

4. Which age group is the largest part of the population: 18 years or younger, 19–25 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, or 65 years or older?

   Sample response: A statistical question that requires categorical data. The category with the most data is the answer.

Activity Synthesis

For each question, invite a group to share how they described the question. After the group shares, invite other groups to add any additional information. If necessary, direct students to the displayed list of words to help students use more formal language.

Activity 1
Measuring Earthworms

20 min

Activity Narrative

In this activity, students practice drawing a histogram for a given data set and using it to answer statistical questions. To help students understand the lengths involved in the data set, students are asked to draw various lengths used to group the worms in the first histogram.

Monitor for groups who identify a typical length as:

- An interval.
- A value within a bin.
- A boundary value for a bin.

Any of these choices are valid due to the vagueness of histograms, but should be backed by a reasoning that makes sense with the distribution.

Launch

Arrange students in groups of 2. Provide access to centimeter rulers.

Consider giving students a brief overview of the context for the problems in the activity. Tell students that there are nearly 6,000 species of earthworms in the world. Some earthworms help the environment, while others (generally not native to the region in which they are found) may harm the environment. Earthworms that are native to a particular region of the world are often raised, by farmers, in terrariums (a container or bin similar to an aquarium but it contains soil and leaves). The terrarium-raised earthworms provide bait for people who fish, provide food for various wildlife, and decompose food waste into soil. Food waste and water are added to the terrariums as food for raising and growing worms. Soil produced by the worms as they eat the food waste is often used as fertilizer.

Explain that the lengths of the worms in the terrariums provide information about the ages of the worms, which can be useful for the farmer. In this activity, students will organize the lengths of the earthworms in several terrariums.

Give students 8–10 minutes of quiet work time, and then 3–4 minutes to discuss their work and to complete the activity with a partner.

Select work from students with different strategies, such as those described in the Activity Narrative, to share later.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect. Support effective and efficThis activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Provide Access for Physical Action. Support effective and efficient use of tools and assistive technologies. To use a ruler, some students may benefit from a demonstration.

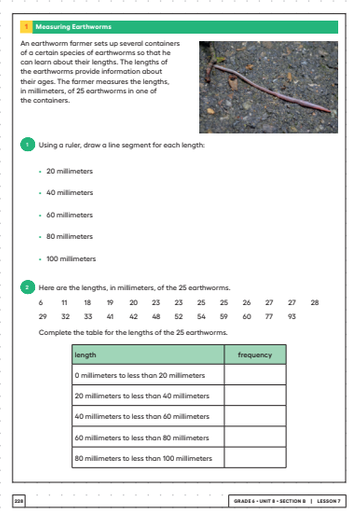
Supports accessibility for: Organization, Memory, Attention

Building on Student Thinking

When determining frequencies of data values, students might lose track of their counting. Suggest that they use tally marks to keep track of the number of occurrences for each bin.

When drawing the histogram, students might mistakenly use bar graphs as a reference and leave spaces between the bars. Ask them to look at the bars in other histograms they have seen so far and to think about what the gaps might mean considering that the bars are built on a number line.

Student Workbook


1 Measuring Earthworms

An earthworm farmer sets up several containers of a certain species of earthworms so that he can learn about their lengths. The lengths of the earthworms provide information about their ages. The farmer measures the lengths, in millimeters, of 25 earthworms in one of the containers.

1. Using a ruler, draw a line segment for each length:
- 20 millimeters
- 40 millimeters
- 60 millimeters
- 80 millimeters
- 100 millimeters

2. Here are the lengths, in millimeters, of the 25 earthworms.

6 11 18 19 20 23 23 25 25 26 27 27 28
29 32 33 41 42 48 52 54 59 60 77 93

Complete the table for the lengths of the 25 earthworms.

length	frequency
0 millimeters to less than 20 millimeters	
20 millimeters to less than 40 millimeters	
40 millimeters to less than 60 millimeters	
60 millimeters to less than 80 millimeters	
80 millimeters to less than 100 millimeters	

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Student Task Statement

An earthworm farmer sets up several containers of a certain species of earthworms so that he can learn about their lengths. The lengths of the earthworms provide information about their ages. The farmer measures the lengths, in millimeters, of 25 earthworms in one of the containers.

1. Using a ruler, draw a line segment for each length:

- 20 millimeters
- 40 millimeters
- 60 millimeters
- 80 millimeters
- 100 millimeters

Drawings should show segments of 20 mm, 40 mm, 60 mm, 80 mm, and 100 mm.

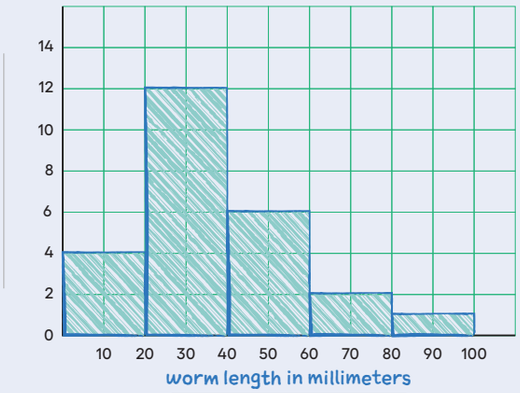
2. Here are the lengths, in millimeters, of the 25 earthworms.

6 11 18 19 20 23 23 25 25 26 27 27 28
29 32 33 41 42 48 52 54 59 60 77 93

Complete the table for the lengths of the 25 earthworms.

length	frequency
0 millimeters to less than 20 millimeters	4
20 millimeters to less than 40 millimeters	12
40 millimeters to less than 60 millimeters	6
60 millimeters to less than 80 millimeters	2
80 millimeters to less than 100 millimeters	1

3. Use the grid and the information in the table to draw a histogram for the worm length data. Be sure to label the axes of your histogram.



4. Based on the histogram, what value could be given as the center as a typical length for these 25 earthworms? Explain how you know.

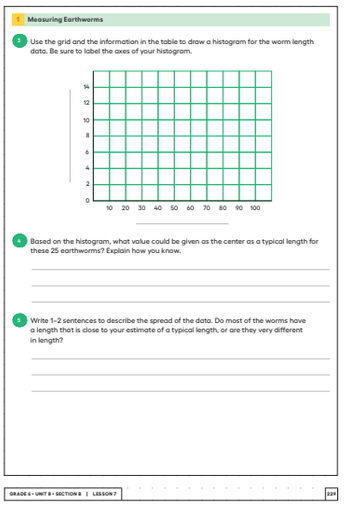
Sample responses:

- Between 20 and 40 mm because this is the interval with the greatest frequency.
- About 30 mm because that is the middle of the interval with the greatest frequency.
- About 35 mm because it is in the interval with the greatest frequency, but there are more longer worms than shorter ones, so I chose a number a little greater than the middle of the interval.

5. Write 1–2 sentences to describe the spread of the data. Do most of the worms have a length that is close to your estimate of a typical length, or are they very different in length?

Sample response: Most of the worms appear to be shorter than my estimate of the typical value, but they are all pretty close to it. There are worms that are longer than my estimate, but not as many.

Student Workbook


1 Measuring Earthworms

3 Use the grid and the information in the table to draw a histogram for the worm length data. Be sure to label the axes of your histogram.

4 Based on the histogram, what value could be given as the center as a typical length for these 25 earthworms? Explain how you know.

5 Write 1–2 sentences to describe the spread of the data. Do most of the worms have a length that is close to your estimate of a typical length, or are they very different in length?

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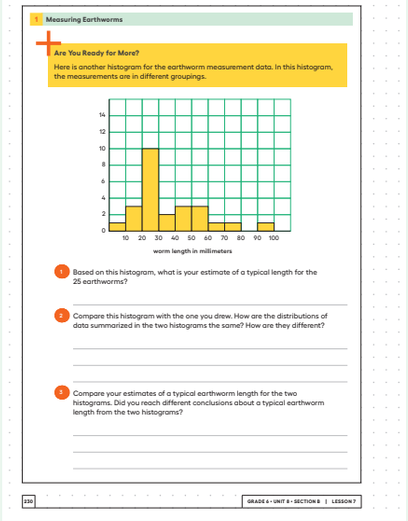
Student Workbook


1 Measuring Earthworms

Are You Ready for More?

Here is another histogram for the earthworm measurement data. In this histogram, the measurements are in different groupings.

worm length in millimeters

1. Based on this histogram, what is your estimate of a typical length for the 25 earthworms?

2. Compare this histogram with the one you drew. How are the distributions of data summarized in the two histograms the same? How are they different?

3. Compare your estimates of a typical earthworm length for the two histograms. Did you reach different conclusions about a typical earthworm length from the two histograms?

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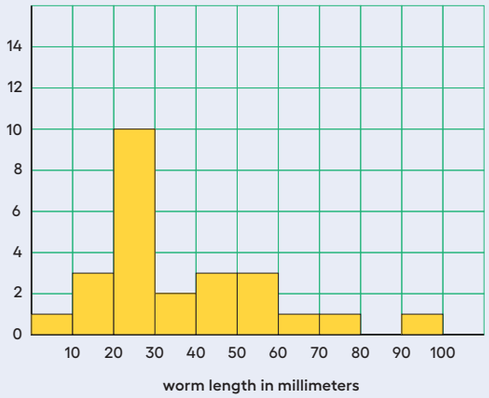
Are You Ready for More?

Here is another histogram for the earthworm measurement data. In this histogram, the measurements are in different groupings.



1. Based on this histogram, what is your estimate of a typical length for the 25 earthworms?

   Sample response: About 30 mm

2. Compare this histogram with the one you drew. How are the distributions of data summarized in the two histograms the same? How are they different?

   Sample response: Both histograms have clusters of data around 20 and 30 mm. The one I drew has more data values between 30 and 60 mm, and this histogram has data that is more spread out.

3. Compare your estimates of a typical earthworm length for the two histograms. Did you reach different conclusions about a typical earthworm length from the two histograms?

   Sample response: The typical earthworm lengths are both around 30 mm, but the typical length from this histogram is a little smaller.

Activity Synthesis

Ask one or two students to display their completed histograms for all to see and briefly describe the overall distribution.

Display 2–3 ways of estimating the center of the distribution from previously selected students for all to see. If time allows, invite students to briefly describe their reasoning for their choice. Then, use Compare and Connect to help students compare, contrast, and connect the different estimates. Here are some questions for discussion:

- *"What do the estimates have in common? How are they different?"*
- *"How does the shape of the distribution show up in each method?"*

Focus the discussion on how identifying the center and spread using a histogram is different from doing so using a dot plot. Discuss:

- *"In a histogram, are we able to see clusters of values in the distribution?"*

  In a way, yes. The values that are close together are probably grouped together into the same bin.

- *"Can we see the largest and smallest values? Can we tell the overall spread?"*

  We cannot get exact values for the largest and smallest values. We can only vaguely describe the spread from the histogram.

- *"How do we identify the center of a distribution?"*

  It is somewhere near the middle of the distribution, but not necessarily halfway between the greatest and least values.

Depending on how the description of the distribution using center and spread are being used, it may be okay that the description is vague. If you don't know about these worms, it might be ok to know "most worms are around 20 to 40 millimeters long, but can get as large as about 100 millimeters." If you are a worm expert or are comparing 2 worm farms, you might want more detail and need to know more precisely how long the worms typically are at this particular farm.

Activity 2

Tall and Taller Players

10 min

Activity Narrative

In this activity, students use histograms to compare two groups by studying the shape, center, and spread of each distribution. Although histograms are not precise, often they can be enough to make a general comparison of groups.

Launch

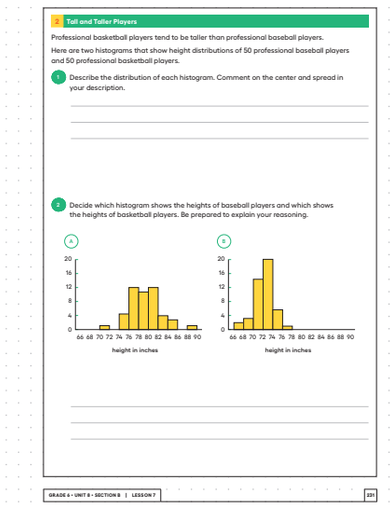
Arrange students in groups of 2. Give students 4–5 minutes of quiet work time and 1–2 minutes to share their responses with a partner.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example: "Histogram _____ represents the heights of _____ players because ..." and "I agree/disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


Tall and Taller Players

Professional basketball players tend to be taller than professional baseball players.

Here are two histograms that show height distributions of 50 professional baseball players and 50 professional basketball players.

1. Describe the distribution of each histogram. Comment on the center and spread in your description.

2. Decide which histogram shows the heights of baseball players and which shows the heights of basketball players. Be prepared to explain your reasoning.

A

B

height in inches

height in inches

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the questions about describing the distribution of heights of basketball and baseball players. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback that they receive.

Advances: Writing, Speaking, Listening

Student Task Statement

Professional basketball players tend to be taller than professional baseball players.

Here are two histograms that show height distributions of 50 professional baseball players and 50 professional basketball players.

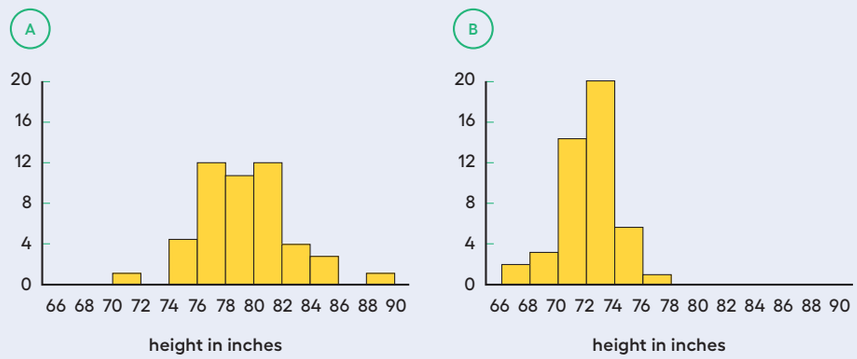
1. Describe the distribution of each histogram. Comment on the center and spread in your description.

   Sample response:

   a. Histogram A has a distribution centered at around 80–81 inches. Except for two players at the shortest and tallest ends, most players are within 6–7 inches of the center.

   b. Histogram B has a distribution centered at around 72 inches. The spread is also similar to the distribution in the other histogram. All players are within 6 inches of the center of the data.

2. Decide which histogram shows the heights of baseball players and which shows the heights of basketball players. Be prepared to explain your reasoning.



Histogram A shows the heights of professional basketball players.

Sample explanation: The center is located higher on the number line compared to those in Histogram B, which means they represent taller players in general.

Activity Synthesis

Select a few students to share their descriptions about basketball players and baseball players. After each student shares, ask others if they agree with the descriptions and, if not, how they might revise or elaborate on them. In general, students should recognize that the distributions of the two groups of athletes are different and be able to describe how they are different.

Highlight the fact that students are using approximations of center and different adjectives to characterize a distribution or a typical height and that, as a result, there are variations in our descriptions. In some situations, these variations might make it challenging to compare groups more precisely.

If time allows, remind students that this type of analysis uses trends to compare groups, not individuals. There are some baseball players that are taller than some basketball players in these groups, so we cannot determine which sport each person plays based on their height.

Lesson Synthesis

In this lesson, we learn how to draw a histogram and how to use it to describe characteristics of a data set.

"What are some decisions we should think about and make before drawing a histogram?"

We should consider how large to make the axes and how large the bins should be.

"Does the width of each bar have to represent a distance of 5 units, or can it represent another number of units?"

They can be any size that balances the precision of smaller bins with the convenience of combining data to reveal the distribution shape. All of the bars must be the same bin size once it is decided, though.

"What does the horizontal axis of a histogram tell us? What about the vertical axis?"

The horizontal axis tells us the range of values from the data that are included in the bar. The vertical axis is the frequency of values in the data that fall in that bin.

"How do we know how tall to make each bar?"

Count the number of values in the interval or bin in the data set to find the frequency. That is the height of the bar.

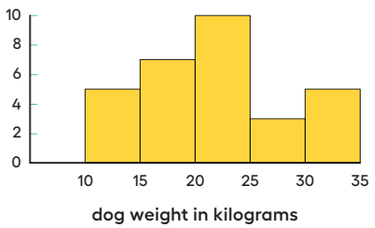
Once we have a histogram drawn, we can use it to answer some questions about a data set.

"How would you describe a typical weight for this group of dogs?"

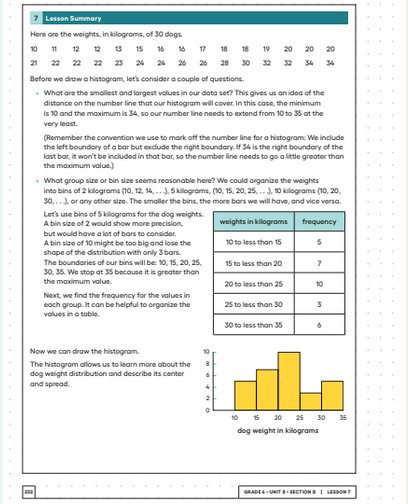
The center of the distribution seems to be around 20–25 kilograms. I might choose 22 because it's in that interval and there seem to be a few more smaller dogs than larger ones.

"What can we say about the spread of the dog weights based on this histogram?"

The dogs range in weight from around 10 kilograms up to almost 35 kilograms. It is hard to be precise with a histogram, but those are the outer edges of the bars shown.



Student Workbook



7 Lesson Summary

Here are the weights, in kilograms, of 30 dogs.

10 11 12 12 13 15 16 16 17 18 18 19 20 20 20
21 22 22 22 23 24 24 26 26 28 30 32 32 34 34

Before we draw a histogram, let's consider a couple of questions.

- What are the smallest and largest values in our data set? This gives us an idea of the distance on the number line that our histogram will cover. In this case, the minimum is 10 and the maximum is 34, so our number line needs to extend from 10 to 35 at the very least.

  (Remember the convention we use to mark off the number line for a histogram: We include the left boundary of a bar but exclude the right boundary. If 34 is the right boundary of the last bar, it won't be included in that bar, so the number line needs to go a little greater than the maximum value.)

- What group size or bin size seems reasonable here? We could organize the weights into bins of 2 kilograms (10, 12, 14, . . .), 5 kilograms, (10, 15, 20, 25, . . .), 10 kilograms (10, 20, 30, . . .), or any other size. The smaller the bins, the more bars we will have, and vice versa.

Let's use bins of 5 kilograms for the dog weights. A bin size of 2 would show more precision, but would have a lot of bars to consider. A bin size of 10 might be too big and lose the shape of the distribution with only 3 bars. The boundaries of our bins will be: 10, 15, 20, 25, 30, 35. We stop at 35 because it is greater than the maximum value.

weights in kilograms	frequency
10 to less than 15	5
15 to less than 20	7
20 to less than 25	10
25 to less than 30	3
30 to less than 35	5

Next, we find the frequency for the values in each group. It can be helpful to organize the values in a table.

Now we can draw the histogram.

The histogram allows us to learn more about the dog weight distribution and describe its center and spread.

Lesson Summary

Here are the weights, in kilograms, of 30 dogs.

10 11 12 12 13 15 16 16 17 18 18 19 20 20 20
21 22 22 22 23 24 24 26 26 28 30 32 32 34 34

Before we draw a histogram, let's consider a couple of questions.

- What are the smallest and largest values in our data set? This gives us an idea of the distance on the number line that our histogram will cover. In this case, the minimum is 10 and the maximum is 34, so our number line needs to extend from 10 to 35 at the very least.

  (Remember the convention we use to mark off the number line for a histogram: We include the left boundary of a bar but exclude the right boundary. If 34 is the right boundary of the last bar, it won't be included in that bar, so the number line needs to go a little greater than the maximum value.)

- What group size or bin size seems reasonable here? We could organize the weights into bins of 2 kilograms (10, 12, 14, ...), 5 kilograms, (10, 15, 20, 25, ...), 10 kilograms (10, 20, 30, ...), or any other size. The smaller the bins, the more bars we will have, and vice versa.

Let's use bins of 5 kilograms for the dog weights. A bin size of 2 would show more precision, but would have a lot of bars to consider. A bin size of 10 might be too big and lose the shape of the distribution with only 3 bars. The boundaries of our bins will be: 10, 15, 20, 25, 30, 35. We stop at 35 because it is greater than the maximum value.

weights in kilograms	frequency
10 to less than 15	5
15 to less than 20	7
20 to less than 25	10
25 to less than 30	3
30 to less than 35	5

Next, we find the frequency for the values in each group. It can be helpful to organize the values in a table.

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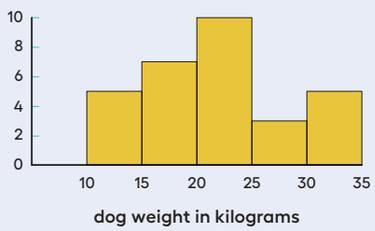
The histogram allows us to learn more about the dog weight distribution and describe its center and spread.



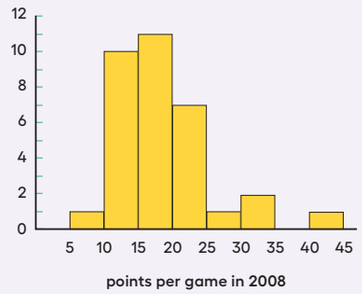
Cool-down

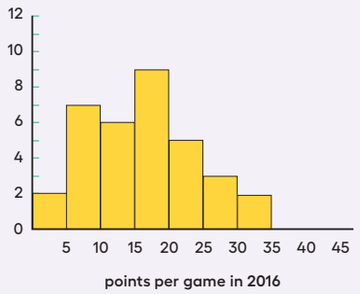
A Tale of Two Seasons

5 min

Student Task Statement

The two histograms show the points scored per game by a basketball player in 2008 and 2016.





1. Describe the center of each distribution represented by the histograms. Explain your reasoning.

   Sample response: In both seasons, the player typically scored around 15 to 20 points in a game. In each histogram, there seems to be a similar frequency of values on each side of this interval.

2. Write 2–3 sentences that describe the spreads of the two distributions, including what spreads might tell us in this context.

   Sample response: The spread of the distribution for 2008 seems less than the spread for the 2016 distribution. There are only 5 games in which the player did not score between 10 and 25 points per game in 2008, but in 2016 the data are more spread out. This means that, from game to game, the player was more consistent in 2008 than in 2016.

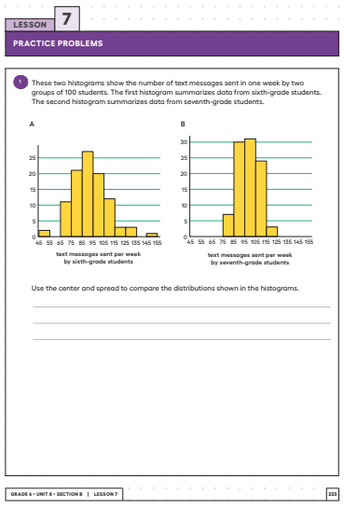
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

4 Problems

Student Workbook


LESSON 7

PRACTICE PROBLEMS

1 These two histograms show the number of text messages sent in one week by two groups of 100 students. The first histogram summarizes data from sixth-grade students. The second histogram summarizes data from seventh-grade students.

A

B

text messages sent per week by sixth-grade students

text messages sent per week by seventh-grade students

Use the center and spread to compare the distributions shown in the histograms.

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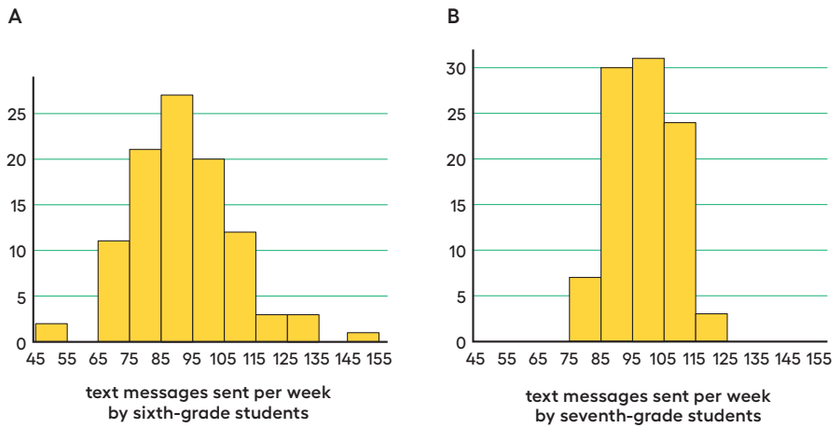
Problem 1

These two histograms show the number of text messages sent in one week by two groups of 100 students. The first histogram summarizes data from sixth-grade students. The second histogram summarizes data from seventh-grade students.



Use the center and spread to compare the distributions shown in the histograms.

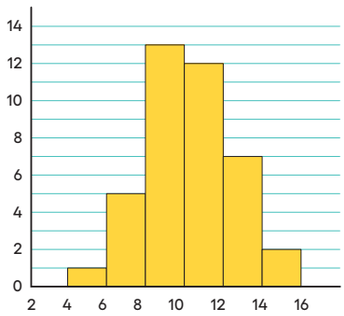
Sample response: Both distributions have a center around 100 text messages. The distribution for the sixth-grade students has a wider spread, so their texting habits are quite different when compared to the more consistent seventh-grade students.

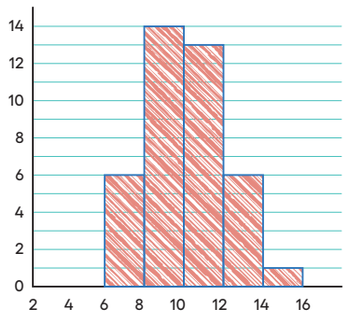
Problem 2

Forty sixth-grade students ran 1 mile. Here is a histogram that summarizes their times, in minutes. The center of the distribution is approximately 10 minutes.

On the blank axes, draw a second histogram that has all three of these characteristics:

- A distribution of times for a different group of 40 sixth-grade students.
- A center at 10 minutes.
- Less variability than the distribution shown in the first histogram.





Problem 3

from Unit 7, Lesson 9

Jada has d dimes. She has more than 30 cents but less than a dollar.

a. Write two inequalities that represent how many dimes Jada has.

$d > 3$ and $d < 10$ (or $d \geq 4$ and $d \leq 9$)

b. Can d be 10?

No, this does not make $d < 10$ true. 10 dimes is a dollar, which is too much.

c. How many possible solutions make both inequalities true? If possible, describe or list the solutions.

6 possible solutions: 4, 5, 6, 7, 8, and 9

Problem 4

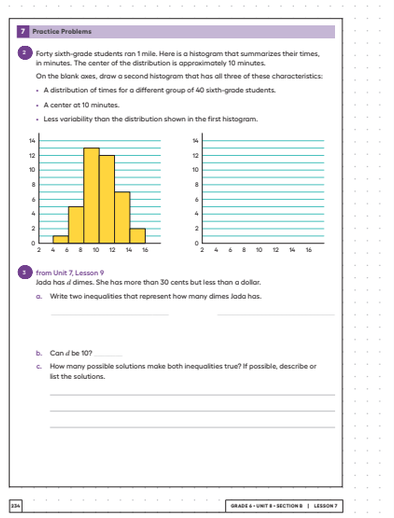
from Unit 7, Lesson 4

Order these numbers from greatest to least:

-4 $\quad \frac{1}{4}$ $\quad 0$ $\quad 4$ $\quad -3\frac{1}{2}$ $\quad \frac{7}{4}$ $\quad -\frac{5}{4}$

4 $\quad \frac{7}{4}$ $\quad \frac{1}{4}$ $\quad 0$ $\quad -\frac{5}{4}$ $\quad -3\frac{1}{2}$ $\quad -4$

Student Workbook


7 Practice Problems

2 Forty sixth-grade students ran 1 mile. Here is a histogram that summarizes their times, in minutes. The center of the distribution is approximately 10 minutes.

On the blank axes, draw a second histogram that has all three of these characteristics:

- A distribution of times for a different group of 40 sixth-grade students.
- A center at 10 minutes.
- Less variability than the distribution shown in the first histogram.

3 from Unit 7, Lesson 9

Jada has d dimes. She has more than 30 cents but less than a dollar.

a. Write two inequalities that represent how many dimes Jada has.

b. Can d be 10?

c. How many possible solutions make both inequalities true? If possible, describe or list the solutions.

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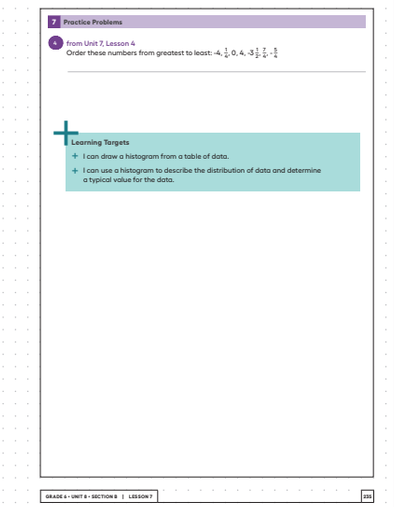
Student Workbook


7 Practice Problems

4 from Unit 7, Lesson 4

Order these numbers from greatest to least: $-4, \frac{1}{4}, 0, 4, -3\frac{1}{2}, \frac{7}{4}, -\frac{5}{4}$

Learning Targets

- I can draw a histogram from a table of data.
- I can use a histogram to describe the distribution of data and determine a typical value for the data.

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