Lesson 10

Different Options for Solving One Equation

Goals

- Critique (orally and in writing) a given solution method for an equation of the form $p(x + q) = r$.
- Evaluate (orally) the usefulness of different approaches for solving a given equation of the form $p(x + q) = r$.
- Recognize that there are two common approaches for solving an equation of the form $p(x + q) = r$, i.e., expanding using the distributive property or dividing each side by p.

Learning Targets

- For an equation like $3(x + 2) = 15$, I can solve it in two different ways: by first dividing each side by 3, or by first rewriting $3(x + 2)$ using the distributive property.
- For equations with more than one way to solve, I can choose the most efficient way depending on the numbers in the equation.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8 (Warm-up, Activity 1)

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Lesson Narrative

The purpose of this lesson is to practice solving equations of the form $p(x + q) = r$, and to notice that one of the two ways of solving may be more efficient depending on the numbers in the equation.

Student Learning Goal

Let's think about which way is better when we solve equations with parentheses.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Instructional Routines

Math Talk

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Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Warm-up, Task Statement)

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because …" or "I noticed ________ so I …" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Warm-up

Math Talk: Solve Each Equation

5 min

Activity Narrative

This *Math Talk* focuses on seeing structure in equations of the form $p(x+q)=r$. It encourages students to see $(x-3)$ as a chunk in order to mentally solve equations. The understanding elicited here will be helpful later in the lesson when students compare and contrast solution methods.

To work with $(x-3)$ as an object, students need to look for and make use of structure.

Launch

Display one equation at a time.

Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy.

Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Student Task Statement

Solve each equation mentally.

$100(x-3)=1{,}000$

$x=13$ Sample reasoning: 100 times a value is 1,000, so that value must be 10. Since $x-3=10$, the value of x must be 13.

$500(x-3)=5{,}000$

$x=13$ Sample reasoning: 500 times a value is 5,000, so that value must be 10. Since $x-3=10$, the value of x must be 13.

$0.03(x-3)=0.3$

$x=13$ Sample reasoning: 0.03 times a value is 3, so that value must be 10. Since $x-3=10$, the value of x must be 13.

$0.72(x+2)=7.2$

$x=8$ Sample reasoning: 0.72 times a value is 7.2, so that value must be 10. Since $x+2=10$, the value of x must be 8.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ________'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ________'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1
Analyzing Solution Methods

15 min

Activity Narrative

In this activity, students compare three methods for solving the same equation. Monitor for students with different, valid reasons for agreeing or disagreeing with each method.

Students are critiquing the reasoning of others by finding and describing errors in reasoning.

Launch

Arrange students in groups of 2.

Give 5–10 minutes quiet work time and time to share their reasoning with their partner, followed by a whole-class discussion.

Explain to students that their job is to analyze three solution methods for errors. They should share with their partner whether they agree or disagree with each method, and explain why.

Student Task Statement

Three students each attempted to solve the equation $2(x - 9) = 10$, but they got different solutions. Here is their work. Do you agree with any of their methods? Explain or show your reasoning.

Noah's method:

$$
\begin{aligned}
2(x-9) &= 10 \\
2(x-9)+9 &= 10+9 && \text{Add 9 to each side} \\
2x &= 19 \\
2x \div 2 &= 19 \div 2 && \text{Divide each side by 2} \\
x &= \frac{19}{2}
\end{aligned}
$$

Elena's method:

$$
\begin{aligned}
2(x-9) &= 10 \\
2x-18 &= 10 && \text{Apply the distributive property} \\
2x-18-18 &= 10-18 && \text{Subtract 18 from each side} \\
2x &= -8 \\
2x \div 2 &= -8 \div 2 && \text{Divide each side by 2} \\
x &= -4
\end{aligned}
$$

Andre's method:

$$
\begin{aligned}
2(x-9) &= 10 \\
2x-18 &= 10 && \text{Apply the distributive property} \\
2x-18+18 &= 10+18 && \text{Add 18 to each side} \\
2x &= 28 \\
2x \div 2 &= 28 \div 2 && \text{Divide each side by 2} \\
x &= 14
\end{aligned}
$$

Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 1, Task Statement)

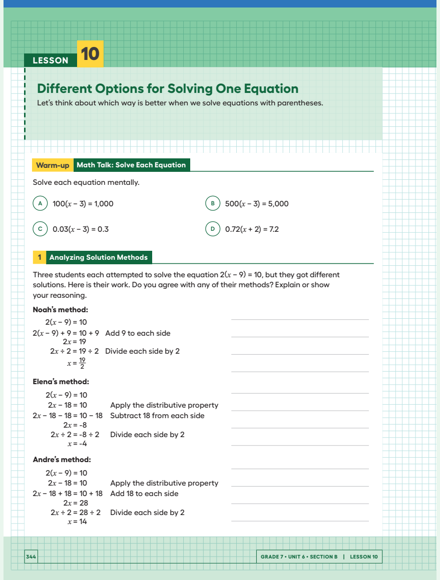
Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. Examples: "It looks like ..." "Is it always true that ... ?" "This method works/doesn't work because ..." "_________ and _________ are different because ..." "Why did they ... ?"

Supports accessibility for: Language, Organization

Building on Student Thinking

If students aren't sure how to begin analyzing Noah's method, ask them to explain what it means for a number to be a solution of an equation. Alternatively, suggest that they draw a tape diagram to represent $2(x - 9) = 10$.

Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support whole-class discussion. Examples: "That could/couldn't be true because ..." and "I agree/disagree because ..."

Advances: Speaking, Conversing

Instructional Routines

MLR8: Discussion Supports

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Sample responses:

1. I disagree with Noah's method, because $2(x - 9) + 9$ is not $2x$. Noah should distribute the 2 before adding a number to each side.
2. I disagree with Elena's method, because $2x - 18 - 18$ is $2x - 36$, not $2x$. Instead of subtracting 18, it would be better to add 18.
3. I agree with Andre's method, because all of his moves are valid, and 14 makes the original equation true when substituted for x.

Activity Synthesis

Invite students to share as many unique reasons they agree or disagree with each method as time allows. The purpose of this discussion is to make explicit a common conceptual error (in Noah's method) and a common conceptual error in Elena's work.

Some possible questions to prompt discussion include:

- *"What are some ways you can tell that a value is* not *a solution to an equation?"*

  Test if it makes the original equation true.

- *"Why isn't $2(x - 9) + 9$ equivalent to $2x$?"*

  The first 9 is being multiplied by 2. So this expression is actually $2x - 18 + 9$, which is $2x - 9$, not $2x$.

- *"Is it possible to solve the original equation without using the distributive property?"*

  Yes, you could start by dividing each side by 2.

Activity 2
Solution Pathways

15 min

Activity Narrative

In this activity, students solve equations of the form $p(x + q) = r$, recognize that there are two valid approaches, and make judgments about which one is more sensible for a given equation.

Deciding on the best approach relies on noticing and using the structure of the equation.

Launch

Display this equation and a hanger diagram to match: $3(x + 2) = 21$.
Tell students,

- *"Any time you want to solve an equation in this form, you have a choice to make about how to proceed. You can either divide each side by 3 or you can distribute the 3."*

If needed, remind students that “distribute the 3” means to multiply each term inside the parentheses by 3. Demonstrate each solution method side by side, using the hanger diagram as a reference point to explain your reasoning.

Keep students in the same groups.

5–10 minutes of quiet or partner work time followed by a whole-class discussion.

Student Task Statement

1. Solve each of these equations twice, one time using each method.

   a. applying the distributive property first:

   $2{,}000(x - 0.03) = 6{,}000$

   dividing each side first:

   $2{,}000(x - 0.03) = 6{,}000$

   $x = 3.03$

   b. applying the distributive property first:

   $2(x + 1.25) = 3.5$

   dividing each side first:

   $2(x + 1.25) = 3.5$

   $x = 0.5$

2. Solve each of these equations once. Choose whichever method you think will be easier for that equation.

   a. $\frac{1}{4}(4 + x) = \frac{4}{3}$

   $x = \frac{4}{3}$ (or equivalent)

   b. $-10(x - 1.7) = -3$

   $x = 2$

   c. $5.4 = 0.3(x + 8)$

   $x = 10$

Activity Synthesis

Reveal the solution to each equation and give students a few minutes to resolve any discrepancies with their partner.

Display the list of equations in the task, and ask students to help you label them with which solution method was the better choice, either “divide first” or “distribute first.”

Some are not so clear. For example, for $-10(x - 1.7) = -3$, one person might distribute the -10 to get rid of the decimal, while someone else might divide each side by -10 to solve in fewer steps.

Some possible questions for discussion:

“For this equation, why is one method preferable to the other?”

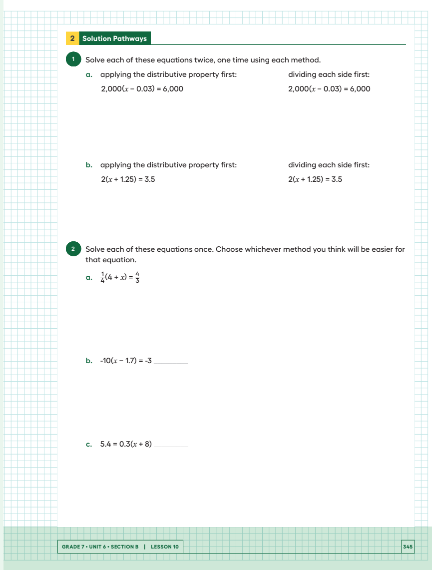
“Which is more important to you—minimizing the number of steps, or getting rid of fractions or decimals?”

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Student Workbook


2 Solution Pathways

1 Solve each of these equations twice, one time using each method.

a. applying the distributive property first: 2,000(x − 0.03) = 6,000 — dividing each side first: 2,000(x − 0.03) = 6,000

b. applying the distributive property first: 2(x + 1.25) = 3.5 — dividing each side first: 2(x + 1.25) = 3.5

2 Solve each of these equations once. Choose whichever method you think will be easier for that equation.

a. ¼(4 + x) = 4/3

b. -10(x − 1.7) = -3

c. 5.4 = 0.3(x + 8)

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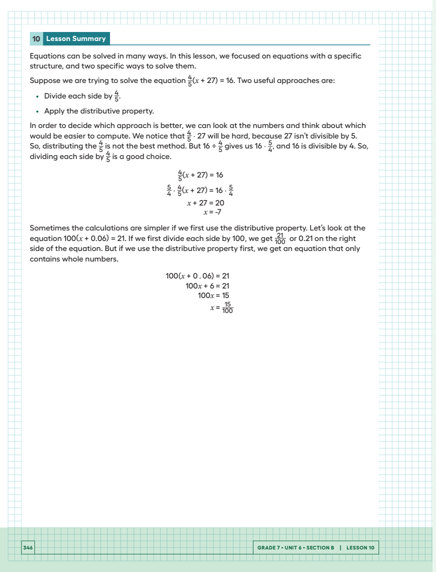
Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
For each observation that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.

Advances: Listening, Speaking

Student Workbook



Lesson Synthesis

Possible questions for discussion:

- *"What are the two main ways we can approach solving equations like the ones we saw today?"*

  Divide first or distribute first.

- *"What kinds of things do we look for to decide which approach is better?"*

  powers of ten, operations that result in whole numbers, moves that will eliminate fractions or decimals

- *"How can we check if our answer is a solution to the original equation?"*

  Substitute our answer for the variable in the original equation and see if it makes the equation true.

Lesson Summary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful approaches are:

- Divide each side by $\frac{4}{5}$.
- Apply the distributive property.

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5}\cdot 27$ will be hard, because 27 isn't divisible by 5. So, distributing the $\frac{4}{5}$ is not the best method. But $16 \div \frac{4}{5}$ gives us $16 \cdot \frac{5}{4}$, and 16 is divisible by 4. So, dividing each side by $\frac{4}{5}$ is a good choice.

$$\begin{aligned}\tfrac{4}{5}(x+27)&=16\\ \tfrac{5}{4}\cdot\tfrac{4}{5}(x+27)&=16\cdot\tfrac{5}{4}\\ x+27&=20\\ x&=-7\end{aligned}$$

Sometimes the calculations are simpler if w first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned}100(x+0.06)&=21\\ 100x+6&=21\\ 100x&=15\\ x&=\tfrac{15}{100}\end{aligned}$$

Cool-down

Solve Two Equations

5 min

Student Task Statement

Solve each equation. Explain or show your reasoning.

1. $8.88 = 4.44(x - 7)$

   $x = 9$

   Sample reasoning: After dividing both sides by 4.44, the equation is $2 = x - 7$. After adding 7 to both sides, the equation is $x = 9$.

2. $5\left(y + \frac{2}{5}\right) = -13$

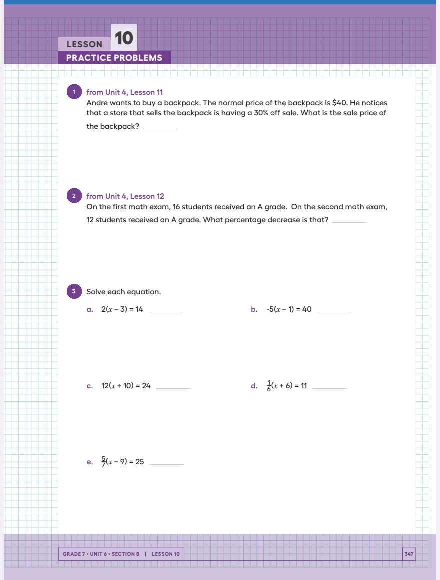
   $y = -3$

   Sample reasoning: After distributing the 5, the equation is $5y + 2 = -13$. After subtracting 2 from each side, it is $5y = -15$. After dividing both sides by 5, it is $y = -3$.

Responding To Student Thinking

Points to Emphasize
If most students struggle with negative numbers when solving equations, plan to focus on strategies when opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about how they solved the word problems in this activity: Grade 7, Unit 6, Lesson 11, Activity 2 Running Around

Student Workbook



LESSON 10
PRACTICE PROBLEMS

1 from Unit 4, Lesson 11
Andre wants to buy a backpack. The normal price of the backpack is $40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

2 from Unit 4, Lesson 12
On the first math exam, 16 students received an A grade. On the second math exam, 12 students received an A grade. What percentage decrease is that?

3 Solve each equation.
a. $2(x-3)=14$
b. $-5(x-1)=40$
c. $12(x+10)=24$
d. $\frac{1}{6}(x+6)=11$
e. $\frac{5}{7}(x-9)=25$

GRADE 7 • UNIT 4 • SECTION B | LESSON 10

Lesson 10

Practice Problems

5 Problems

Problem 1

from Unit 4, Lesson 11

Andre wants to buy a backpack. The normal price of the backpack is $40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

$28

Problem 2

from Unit 4, Lesson 12

On the first math exam, 16 students received an A grade. On the second math exam, 12 students received an A grade. What percentage decrease is that?

25% ($4 \div 16 = 0.25$)

Problem 3

Solve each equation.

a. $2(x-3)=14$

10

b. $-5(x-1)=40$

-7

c. $12(x+10)=24$

-8

d. $\frac{1}{6}(x+6)=11$

60

e. $\frac{5}{7}(x-9)=25$

44

Problem 4

Select **all** expressions that represent a correct solution to the equation $6(x + 4) = 20$.

A. $(20 - 4) \div 6$

B. $\frac{1}{6}(20 - 4)$

C. $20 - 6 - 4$

D. $20 \div 6 - 4$

E. $\frac{1}{6}(20 - 24)$

F. $(20 - 24) \div 6$

Problem 5

Lin and Noah are solving the equation $7(x + 2) = 91$.

Lin starts by using the distributive property. Noah starts by dividing each side by 7.

a. Show what Lin's and Noah's full solution methods might look like.

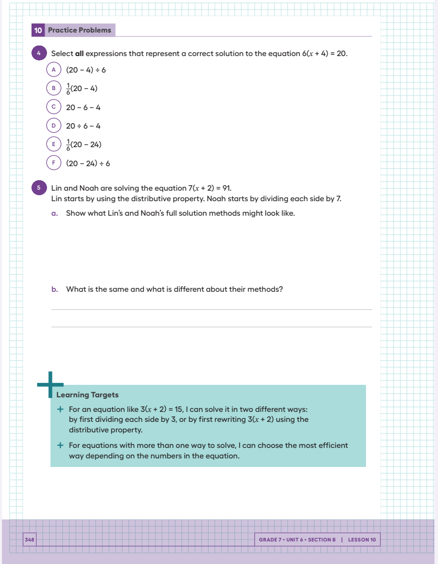
Sample responses:

- Lin's solution method: $7x + 14 = 91$, $7x = 77$, $x = 11$
- Noah's solution method: $x + 2 = 13$, $x = 11$

b. What is the same and what is different about their methods?

Sample response: Both methods involve dividing by 7, but Noah does the division first, while Lin does the division last. Also, Lin's method involves subtracting 14, while Noah's method involves subtracting 2. Both solutions are correct and valid. Noah's solution could be considered more efficient for this example, because it takes fewer steps and has equally complicated arithmetic work.

Student Workbook


10 Practice Problems

4 Select **all** expressions that represent a correct solution to the equation $6(x + 4) = 20$.

A $(20 - 4) \div 6$

B $\frac{1}{6}(20 - 4)$

C $20 - 6 - 4$

D $20 \div 6 - 4$

E $\frac{1}{6}(20 - 24)$

F $(20 - 24) \div 6$

5 Lin and Noah are solving the equation $7(x + 2) = 91$. Lin starts by using the distributive property. Noah starts by dividing each side by 7.

a. Show what Lin's and Noah's full solution methods might look like.

b. What is the same and what is different about their methods?

Learning Targets

+ For an equation like $3(x + 2) = 15$, I can solve it in two different ways: by first dividing each side by 3, or by first rewriting $3(x + 2)$ using the distributive property.
+ For equations with more than one way to solve, I can choose the most efficient way depending on the numbers in the equation.