Lesson 4

How Many Groups? (Part 1) (Optional)

Goals

- Coordinate multiplication equations and pattern block diagrams in which the yellow hexagon represents 1 whole.
- Create a diagram to represent and solve a problem asking "How many groups?" in which the size of a group is a fraction and the answer is a whole number, and explain (orally) the solution method.

Learning Targets

- I can find how many groups there are when the amount in each group is not a whole number.
- I can use diagrams and multiplication and division equations to represent "How many groups?" questions.

Lesson Narrative

In this lesson, students use a concrete reasoning tool—pattern blocks—to explore "How many groups?" questions where the size of a group is a fraction. Here the number of groups—the answer to the question "How many of these are in that?"—is a whole number.

Students begin by identifying the value of different pattern blocks if a hexagon represents 1 whole. They then use the blocks to represent multiplication of a whole number and a fraction (such as $3 \cdot \frac{1}{6} = \frac{1}{2}$) and interpret the situation in terms of equal-size groups (such as 3 groups of $\frac{1}{6}$ make $\frac{1}{2}$).

Then students use the blocks to reason in the other direction and find out how many times a fraction goes into another number (as in, "How many $\frac{2}{3}$s are in 2?"). They relate such questions to multiplication equations (such as $? \cdot \frac{2}{3} = 2$).

Student Learning Goal

Let's play with blocks and diagrams to think about division with fractions.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
5 min	25 min	10 min	5 min

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

Instructional Routines

- MLR1

Required Materials

Materials to Gather

- Pattern blocks: Activity 2

Required Preparation

Activity 2:
Prepare enough pattern blocks so that each group of 3–4 students has at least 2 hexagons and 6 of each of the other shapes (triangle, rhombus, and trapezoid).

For the digital version of the activity, acquire devices that can run the applet.

Lesson:
Prepare enough pattern blocks so that each group of 3–4 students has at least 2 hexagons and 6 of each of the other shapes (triangle, rhombus, and trapezoid).

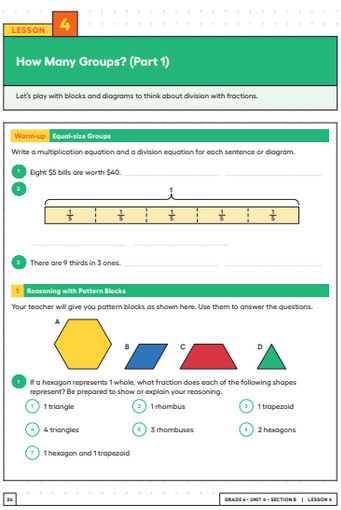
Building on Student Thinking

If students recognize the equal-size groups but represent them with repeated addition (such as $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=1$) instead of multiplication, remind them about the connections between the two. Refer to one of their addition statements and ask questions such as:

"How many same-size groups are being added? What is in each group?"

"How can you express the same information using multiplication?"

Student Workbook


LESSON 4

How Many Groups? (Part 1)

Let's play with blocks and diagrams to think about division with fractions.

Warm-up Equal-size Groups

Write a multiplication equation and a division equation for each sentence or diagram.

1. Eight $5 bills are worth $40.
2. 1 — 1/5 1/5 1/5 1/5 1/5
3. There are 9 thirds in 3 ones.

1 Reasoning with Pattern Blocks

Your teacher will give you pattern blocks as shown here. Use them to answer the questions.

A B C D

1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.
   1. 1 triangle
   2. 1 rhombus
   3. 1 trapezoid
   4. 4 triangles
   5. 3 rhombuses
   6. 2 hexagons
   7. 1 hexagon and 1 trapezoid

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Warm-up

Equal-size Groups

5 min

Activity Narrative

This *Warm-up* prompts students to interpret depictions of equal-size groups and write multiplication and division equations that represent them. In two cases, the size of one group is a fraction. Prior to this point, students have written multiplication equations with one or more fractional factors. Here, students see that division equations can have a fraction as a divisor or a quotient.

The reasoning here further builds students' understanding, from earlier lessons, of the relationship between the number of groups and the size of each group. It also activates what students know, from earlier grades, about multiplying whole numbers and unit fractions.

Launch

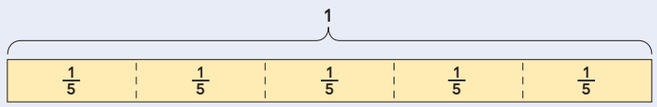
Give students 2 minutes of quiet think time, followed by a whole-class discussion.

Student Task Statement

Write a multiplication equation and a division equation for each sentence or diagram.

1. Eight $5 bills are worth $40.

   $8 \cdot 5 = 40$ (or $5 \cdot 8 = 40$) and $40 \div 5 = 8$ (or $40 \div 8 = 5$)

2. 

   $5 \cdot \frac{1}{5} = 1$ (or $\frac{1}{5} \cdot 5 = 1$) and $1 \div 5 = \frac{1}{5}$ (or $1 \div \frac{1}{5} = 5$)

3. There are 9 thirds in 3 ones.

   $9 \cdot \frac{1}{3} = 3$ (or $\frac{1}{3} \cdot 9 = 3$) and $3 \div 9 = \frac{1}{3}$ (or $3 \div \frac{1}{3} = 9$)

Activity Synthesis

Select 1–2 students to share their responses. Record the responses for all to see.

As students present the equations for each problem, connect the pieces in each equation to the idea of equal-size groups. Ask questions such as:

- *"Which number in the multiplication equation refers to the number of groups?"*
- *"Which number in the multiplication equation refers to how much is in each group?"*
- *"In this case, what does the division equation $1 \div \frac{1}{5} = 5$ tell us?"*
- *"In this case, what does the division equation $3 \div 9 = \frac{1}{3}$ tell us?"*

Activity 1

Reasoning with Pattern Blocks

25 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use the relationships between the areas of geometric shapes to reason about groups of halves, thirds, and sixths, building their intuition for division of fractions. The focus is on the "How many groups?" interpretation of division.

Students start by using pattern blocks to represent multiplication of a whole number and a fraction. For example, if a hexagon represents 1 whole and 6 triangles make a hexagon, then each triangle represents $\frac{1}{6}$. They can then use 6 triangles to represent $6 \cdot \frac{1}{6} = 1$.

Later, students use the blocks to reason in the opposite direction. They consider questions such as, "How many $\frac{1}{2}$s are in 4?" and connect the answer to the missing factor in equations such as $? \cdot \frac{1}{2} = 4$. These kinds of questions serve as a stepping stone to more abstract questions, such as "What is 4 divided by $\frac{1}{2}$?" and equations such as $4 \div \frac{1}{2} = ?$.

As students make sense of the quantities in pattern-block arrangements and the equations that represent them, they practice reasoning quantitatively and abstractly.

This activity works best when each student has access to pattern blocks. If pattern blocks are not available, consider using the digital version of the activity. In the digital version, students use an applet to investigate fractional relationships with pattern blocks. The applet allows students to place and compare pattern blocks with or without a grid.

Launch

Tell students that they will now use pattern blocks to explore equal-size groups involving fractions. Display a hexagon, a rhombus, a trapezoid, and a triangle for all to see. Tell students that these are the shapes they will use.

Draw students' attention to two pattern blocks: a rhombus and a triangle. Ask students:

> *"How might you compare the sizes of the two shapes? How does the size of one shape relate to the size of the other?"*

Highlight responses along the lines of:

- Two triangles have the same area as a rhombus (because two triangles can be composed—without gaps or overlaps—into a shape that matches a rhombus exactly).
- Two triangles make a rhombus.
- A triangle is half of a rhombus.

Remind students that when we refer to fractions such as $\frac{1}{2}$ and $\frac{1}{3}$, it is done in relation to 1 whole. Tell students that in this activity, the hexagon represents 1 whole.

Arrange students in groups of 3–4. Provide access to pattern blocks.

Give students 10–12 minutes to collaborate on the first three questions and 3–4 minutes of quiet think time for the last question.

Access for Students with Diverse Abilities (Activity 1, Launch)

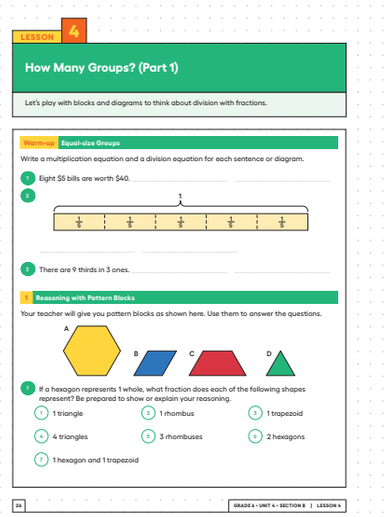
Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Provide 1 or 2 parts of the first question at a time. Check in with students to provide feedback and encouragement after each chunk. Ensure that students are accurately comparing each pattern block or group of pattern blocks to a whole of 1 hexagon.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

Some students may rely on visual comparison and guess incorrectly how the size of each block relates to that of the hexagon. Encourage them to place the blocks on top of the hexagon, to use non-hexagons to compose a hexagon, or to otherwise manipulate the blocks in order to make comparisons.

Student Workbook


LESSON 4

How Many Groups? (Part 1)

Let's play with blocks and diagrams to think about division with fractions.

Warm-up Equal-size Groups

Write a multiplication equation and a division equation for each sentence or diagram.

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2. 1 / 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3
3. There are 9 thirds in 3 ones.

1 Reasoning with Pattern Blocks

Your teacher will give you pattern blocks as shown here. Use them to answer the questions.

A B C D

1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.
   1. 1 triangle
   2. 1 rhombus
   3. 1 trapezoid
   4. 4 triangles
   5. 3 rhombuses
   6. 2 hexagons
   7. 1 hexagon and 1 trapezoid

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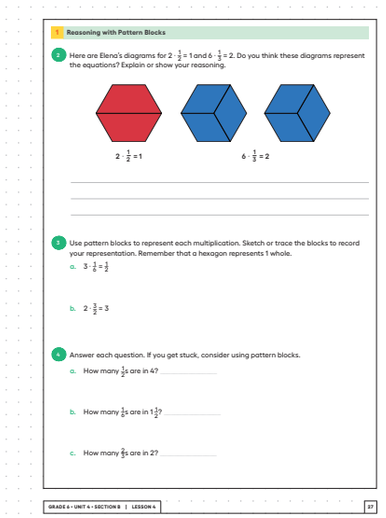
Student Workbook


1 Reasoning with Pattern Blocks

2. Here are Elena's diagrams for $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Do you think these diagrams represent the equations? Explain or show your reasoning.

$2 \cdot \frac{1}{2} = 1$ $6 \cdot \frac{1}{3} = 2$

3. Use pattern blocks to represent each multiplication. Sketch or trace the blocks to record your representation. Remember that a hexagon represents 1 whole.
   a. $3 \cdot \frac{2}{3} = \frac{3}{2}$
   b. $2 \cdot \frac{3}{2} = 3$
4. Answer each question. If you get stuck, consider using pattern blocks.
   a. How many $\frac{1}{2}$s are in 4? _____
   b. How many $\frac{1}{3}$s are in $1\frac{1}{2}$? _____
   c. How many $\frac{2}{3}$s are in 2? _____

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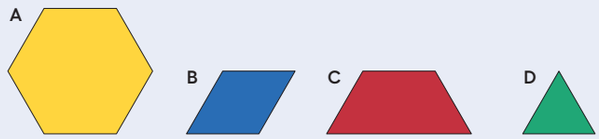
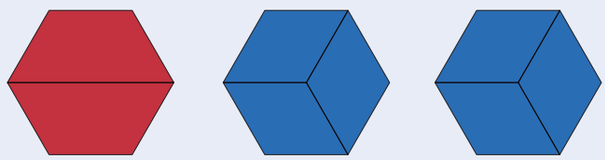
Student Task Statement

Your teacher will give you pattern blocks as shown here. Use them to answer the questions.



1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.
   1. 1 triangle
      $\frac{1}{6}$
   2. 1 rhombus
      $\frac{2}{6}$ (or $\frac{1}{3}$)
   3. 1 trapezoid
      $\frac{1}{2}$
   4. 4 triangles
      $\frac{4}{6}$ (or $\frac{2}{3}$)
   5. 3 rhombuses
      1
   6. 2 hexagons
      2
   7. 1 hexagon and 1 trapezoid
      $1\frac{1}{2}$
2. Here are Elena's diagrams for $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Do you think these diagrams represent the equations? Explain or show your reasoning.



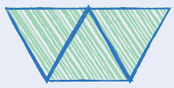
$2 \cdot \frac{1}{2} = 1$ $6 \cdot \frac{1}{3} = 2$

Agree

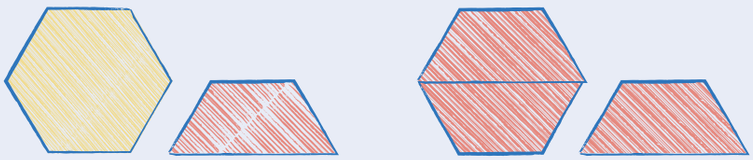
Sample reasoning: In the first representation, each trapezoid is $\frac{1}{2}$ of a hexagon, so 2 of them make 1 whole or 1 hexagon. In the second representation, each rhombus is $\frac{1}{3}$ of a hexagon, so 3 rhombuses make 1 hexagon (or 1 whole), and 6 rhombuses make 2 wholes.

3. Use pattern blocks to represent each multiplication. Sketch or trace the blocks to record your representation. Remember that a hexagon represents 1 whole.

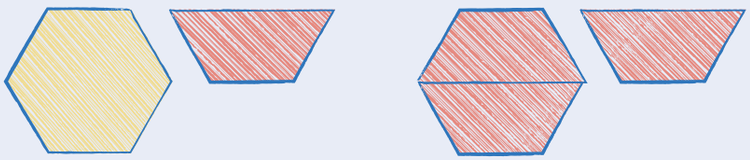
a. $3 \cdot \frac{1}{6} = \frac{1}{2}$



b. $2 \cdot \frac{3}{2} = 3$



or



4. Answer each question. If you get stuck, consider using pattern blocks.

a. How many $\frac{1}{2}$s are in 4?

8

b. How many $\frac{1}{6}$s are in $1\frac{1}{2}$?

9

c. How many $\frac{2}{3}$s are in 2?

3

Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the question about whether Elena's pattern block arrangements represented the equations $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Invite listeners to ask questions and givefeedback that will help their partner clarify and strengthen their ideas and writing. *Give students 3–5 minutes to revise their first draft based on the feedback they receive. Advances: Writing, Speaking, Listening*

Activity Synthesis

Select a few students to show their pattern-block arrangements or drawings for $3 \cdot \frac{1}{6} = \frac{1}{2}$ and $2 \cdot \frac{3}{2} = 3$. After each person shares, ask if others had alternative solutions.

Select other students to share their responses and reasoning for the last set of questions. If no one reasoned about the questions by using pattern blocks, show how the blocks could be used to answer the questions. For instance:

- For "How many $\frac{1}{2}$s are in 4?", we could use 8 trapezoids (each representing $\frac{1}{2}$) to make 4 hexagons.
- For "How many $\frac{1}{6}$s are in $1\frac{1}{2}$?", we could use 9 triangles (each representing $\frac{1}{6}$) to make $1\frac{1}{2}$ hexagons.
- For "How many $\frac{2}{3}$s are in 2?", we could use 2 rhombuses (each representing $\frac{1}{3}$) to represent $\frac{2}{3}$, and use 3 pairs of rhombuses to make 2 hexagons.

Highlight that each of these questions is about finding how many groups (or blocks) are needed to equal a particular value when the size of each group (or each block) is known.

Consider using the applet in the discussion.

Lesson Synthesis

A key point to highlight is that we can reason about equal groups of fractions just as we have done with equal groups of whole numbers. The relationship between the quantities can still be related by multiplication:

(number of groups) · (size of one group) = total amount

Display a question that students answered in this lesson, such as: "How many $\frac{1}{6}$s are in 3?" Ask students:

- *"How can we tell that this question is about equal-size groups?"*

  The question is asking about groups of $\frac{1}{6}$. The groups all have a value of $\frac{1}{6}$.
- *"How can pattern blocks help us find the answer to a question like this?"*

  They allow us to count the number of groups of $\frac{1}{6}$ in a total of 3.
- *"What multiplication equation can we write to represent this question?"*

  $? \cdot \frac{1}{6} = 3$

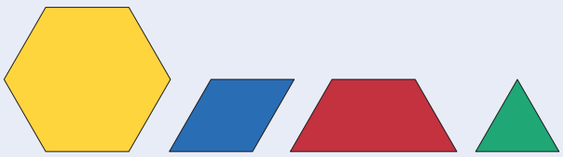
Lesson Summary

Some problems that involve equal-sized groups also involve fractions. Here is an example: "How many $\frac{1}{6}$s are in 2?" We can express this question with multiplication and division equations.

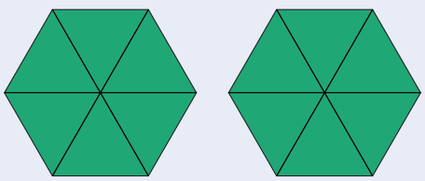
$$? \cdot \frac{1}{6} = 2$$

$$2 \div \frac{1}{6} = ?$$

Pattern-block diagrams can help us make sense of such problems. Here is a set of pattern blocks.



If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$, because 6 triangles make 1 hexagon. We can use the triangle to represent the $\frac{1}{6}$ in the problem.



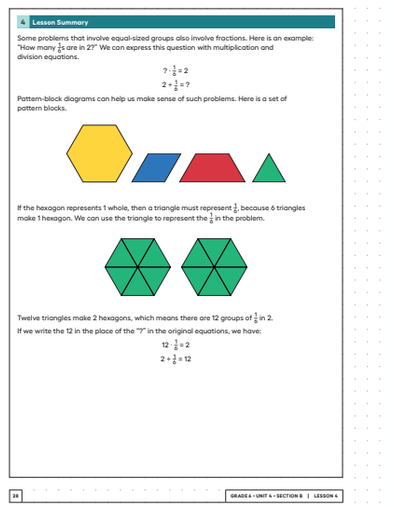
Twelve triangles make 2 hexagons, which means there are 12 groups of $\frac{1}{6}$ in 2.

If we write the 12 in the place of the "?" in the original equations, we have:

$$12 \cdot \frac{1}{6} = 2$$

$$2 \div \frac{1}{6} = 12$$

Student Workbook



4 Lesson Summary

Some problems that involve equal-sized groups also involve fractions. Here is an example: "How many $\frac{1}{6}$s are in 2?" We can express this question with multiplication and division equations.

$? \cdot \frac{1}{6} = 2$

$2 \div \frac{1}{6} = ?$

Pattern-block diagrams can help us make sense of such problems. Here is a set of pattern blocks.

If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$, because 6 triangles make 1 hexagon. We can use the triangle to represent the $\frac{1}{6}$ in the problem.

Twelve triangles make 2 hexagons, which means there are 12 groups of $\frac{1}{6}$ in 2.

If we write the 12 in the place of the "?" in the original equations, we have:

$12 \cdot \frac{1}{6} = 2$

$2 \div \frac{1}{6} = 12$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Halves, Thirds, and Sixths

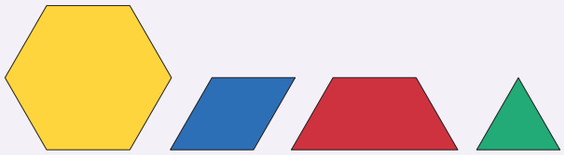
5 min

Launch

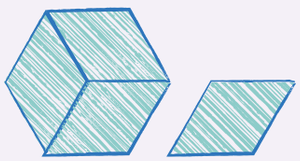
Give students continued access to pattern blocks, if needed.

Student Task Statement

1. The hexagon represents 1 whole.



Draw a pattern-block diagram that represents the equation $4 \cdot \frac{1}{3} = 1\frac{1}{3}$.



2. Answer the following questions. If you get stuck, consider using pattern blocks.

a. How many $\frac{1}{2}$s are in $3\frac{1}{2}$?

There are seven $\frac{1}{2}$s in $3\frac{1}{2}$.

b. How many $\frac{1}{3}$s are in $2\frac{2}{3}$?

There are eight $\frac{1}{3}$s in $2\frac{2}{3}$.

c. How many $\frac{1}{6}$s are in $\frac{2}{3}$?

There are four $\frac{1}{6}$s in $\frac{2}{3}$.

Lesson 4

Practice Problems

7 Problems

Problem 1

We can think of 8 ÷ 2 = ? as asking: "How many 2s are in 8?" or "What is in each group if there are 2 groups in 8?"

Interpret each equation. Write a question to represent your interpretation, and answer the question using a complete sentence.

a. 8 ÷ 3 = ?

Sample response:

Question: "How many 3s are in 8?"

Answer: There are $2\frac{2}{3}$ groups of 3 in 8.

b. $8 \div \frac{1}{3} = ?$

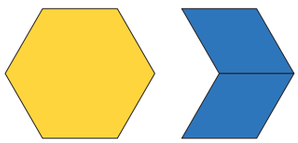
Sample response:

Question: "What is in one group if there are 3 groups in 8?"

Answer: There is $2\frac{2}{3}$ in each group if there are 3 groups in 8.

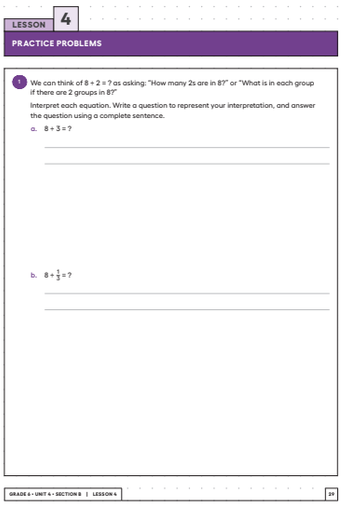
Problem 2

Use the diagram to answer the question: How many $\frac{1}{3}$s are in $1\frac{2}{3}$? The hexagon represents 1 whole. Explain or show your reasoning.



If the hexagon represents 1, then the rhombus represents $\frac{1}{3}$ because the hexagon is composed of three rhombuses. The diagram of one hexagon and two rhombuses matches up exactly with five rhombuses. So there are five $\frac{1}{3}$s in $1\frac{2}{3}$.

Student Workbook



LESSON 4

PRACTICE PROBLEMS

1 We can think of 8 ÷ 2 = ? as asking: "How many 2s are in 8?" or "What is in each group if there are 2 groups in 8?"

Interpret each equation. Write a question to represent your interpretation, and answer the question using a complete sentence.

a. 8 ÷ 3 = ?

b. $8 \div \frac{1}{3} = ?$

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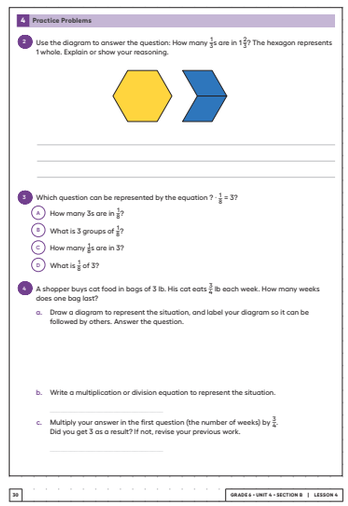
Student Workbook



4 Practice Problems

2 Use the diagram to answer the question: How many $\frac{1}{3}$s are in $1\frac{2}{3}$? The hexagon represents 1 whole. Explain or show your reasoning.

3 Which question can be represented by the equation ? $\cdot \frac{1}{8} = 3$?

A How many 3s are in $\frac{1}{8}$?

B What is 3 groups of $\frac{1}{8}$?

C How many $\frac{1}{8}$s are in 3?

D What is $\frac{1}{8}$ of 3?

4 A shopper buys cat food in bags of 3 lb. His cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

a. Draw a diagram to represent the situation, and label your diagram so it can be followed by others. Answer the question.

b. Write a multiplication or division equation to represent the situation.

c. Multiply your answer in the first question (the number of weeks) by $\frac{3}{4}$. Did you get 3 as a result? If not, revise your previous work.

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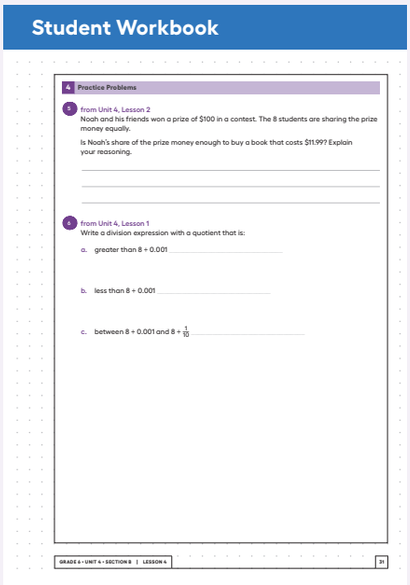
Student Workbook


4 Practice Problems

5 from Unit 4, Lesson 2
Noah and his friends won a prize of $100 in a contest. The 8 students are sharing the prize money equally.
Is Noah's share of the prize money enough to buy a book that costs $11.99? Explain your reasoning.

6 from Unit 4, Lesson 1
Write a division expression with a quotient that is:
a. greater than 8 ÷ 0.001
b. less than 8 ÷ 0.001
c. between 8 ÷ 0.001 and 8 ÷ $\frac{1}{10}$

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Problem 3

Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?

A. How many 3s are in $\frac{1}{8}$?

B. What is 3 groups of $\frac{1}{8}$?

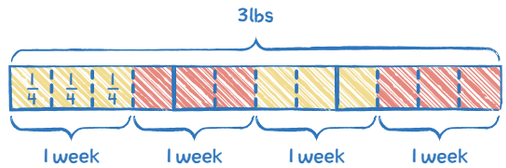
C. How many $\frac{1}{8}$s are in 3?

D. What is $\frac{1}{8}$ of 3?

Problem 4

A shopper buys cat food in bags of 3 lb. His cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

a. Draw a diagram to represent the situation, and label your diagram so it can be followed by others. Answer the question.



There are 4 servings of $\frac{3}{4}$ lb in the 3-lb bag. The bag lasts 4 weeks.

b. Write a multiplication or division equation to represent the situation.

$? \cdot \frac{3}{4} = 3$ or $3 \div \frac{3}{4} = ?$

c. Multiply your answer in the first question (the number of weeks) by $\frac{3}{4}$. Did you get 3 as a result? If not, revise your previous work.

The answer is correct because $4 \cdot \frac{3}{4} = 3$.

Problem 5

from Unit 4, Lesson 2

Noah and his friends won a prize of $100 in a contest. The 8 students are sharing the prize money equally.

Is Noah's share of the prize money enough to buy a book that costs $11.99? Explain your reasoning.

Yes, it is

Sample reasoning: 100 ÷ 8 = 12.5, so each student's share is $12.50, which is more than $11.99.

Problem 6

from Unit 4, Lesson 1

Write a division expression with a quotient that is:

a. greater than 8 ÷ 0 . 001

Sample response: 9 ÷ 0.001 or 8 ÷ 0.0001

b. less than 8 ÷ 0 . 001

Sample response: 7 ÷ 0.01 or 8 ÷ 0.01

c. between 8 ÷ 0 . 001 and 8 ÷ $\frac{1}{10}$

Sample response: 8 ÷ 0.01 or 6 ÷ 0.001

Problem 7

from Unit 3, Lesson 14

Find each unknown number.

a. 12 is 150% of what number?

8

b. 5 is 50% of what number?

10

c. 10% of what number is 300?

3,000

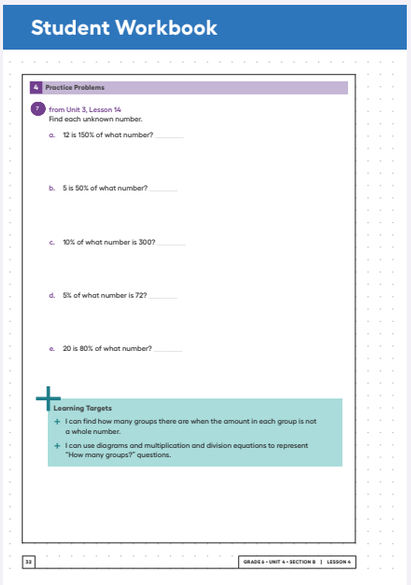
d. 5% of what number is 72?

1,440

e. 20 is 80% of what number?

25

Student Workbook


4 Practice Problems

7 from Unit 3, Lesson 14
Find each unknown number.

a. 12 is 150% of what number? _____

b. 5 is 50% of what number? _____

c. 10% of what number is 300? _____

d. 5% of what number is 72? _____

e. 20 is 80% of what number? _____

Learning Targets
+ I can find how many groups there are when the amount in each group is not a whole number.
+ I can use diagrams and multiplication and division equations to represent "How many groups?" questions.

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