Lesson 6

The Slope of a Fitted Line

Goals

- Describe (orally and in writing) the relationship between two variables using a line fit to data on a scatter plot.
- Interpret (orally and in writing) points on the scatter plot, including points that do and do not lie on a line fit to data.
- Interpret (orally and in writing) the slope of a line fit to data in context.

Learning Target

I can use the slope of a line fit to data in a scatter plot to say how the variables are connected in real-world situations.

Lesson Narrative

In this lesson students focus on the mathematical language of slope using the context of data. Students also identify positive and negative associations of scatter plots without a fitted line shown.

Student Learning Goal

Let's look at how changing one variable changes another.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)
- Representation (Activity 3)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR8: Discussion Supports (Activity 1, Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment
5 min	10 min	10 min	10 min	10 min	5 min
					Cool-down

Warm-up
Estimating Slope

5 min

Activity Narrative

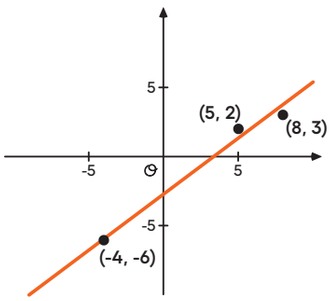
The purpose of this *Warm-up* is for students to estimate the slope of a line given points that are close to the line, but not on the line. This prepares students for thinking about the model's fit to data in the rest of the lesson.

Launch

Arrange students in groups of 2. Give 1 minute of quiet work time followed by 1 minute to discuss their solution with their partner. Follow with a whole-class discussion.

Student Task Statement

Estimate the slope of the line. Be prepared to explain your reasoning.

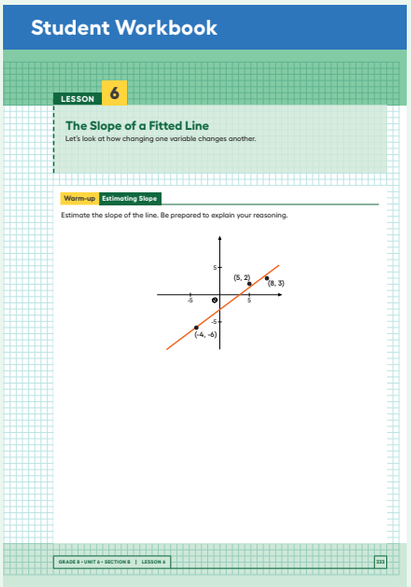


Sample response: About 0.8. It is a little less than $\frac{8}{9}$, which is the slope of a segment connecting $(-4, -6)$ and $(5, 2)$, and a little more than $\frac{3}{4}$, which is the slope of a segment connecting $(-4, -6)$ and $(8, 3)$.

Activity Synthesis

Poll the class and ask students if their estimated slope was close to their partner's estimate. Select 2–3 groups who had close estimates to share their solutions and explain their reasoning. Display the graph with the single line given in the task and record the students' responses next to the graph for all to see.

Student Workbook


LESSON 6

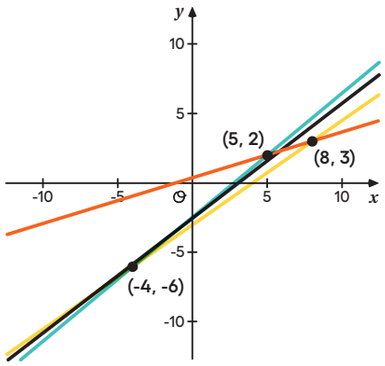
The Slope of a Fitted Line

Let's look at how changing one variable changes another.

Warm-up Estimating Slope

Estimate the slope of the line. Be prepared to explain your reasoning.

If students do not mention that it is better to use points that are far apart rather than close together for estimating the slope, consider displaying this graph for all to see:



To remind students of previous work, draw a slope triangle whose horizontal side has a length of 1, demonstrating that the length of the vertical side is equal to the slope of the line.

Activity 1

Describing Linear Associations

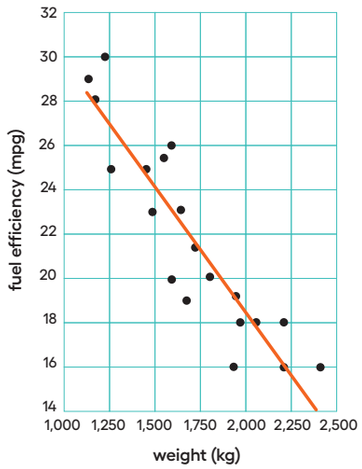
10 min

Activity Narrative

Students practice using precise wording to describe the positive or negative association between two variables given scatter plots of data.

Launch

Display the scatter plot for all to see.



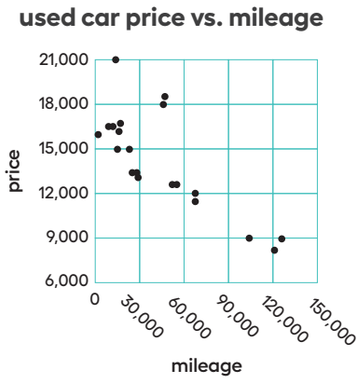
Ask students how they would describe the relationship between weight and fuel efficiency of different cars. After 30 seconds of quiet think time, select 1–2 students to share their responses. (The scatter plot shows that for these cars, as the weight of a car increases, its fuel efficiency decreases.)

Give students 3–5 minutes to construct similar sentences to describe scatter plots of 3 other data sets followed by a whole-class discussion.

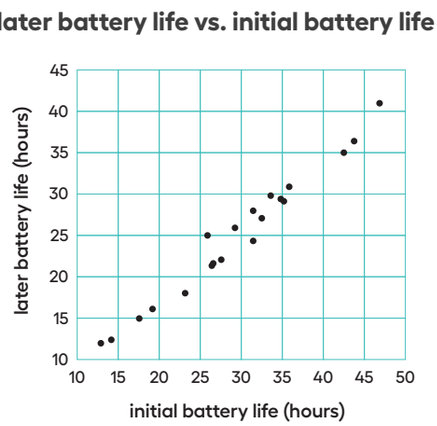
Student Task Statement

For each scatter plot, decide if there is an association between the 2 variables, and describe the situation using one of these sentences:

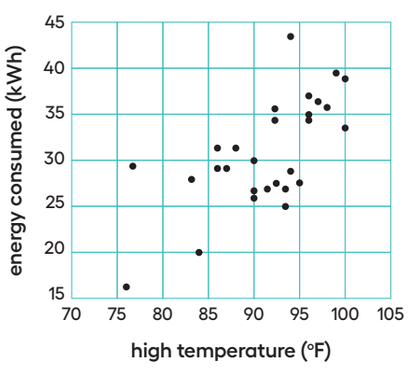
- For these data, as _____ increases, _____ tends to increase.
- For these data, as _____ increases, _____ tends to decrease.
- For these data, _____ and _____ do not appear to be related.



For these data, as car mileage increases, price tends to decrease.



For these data, as initial battery life increases, later battery life tends to increase.

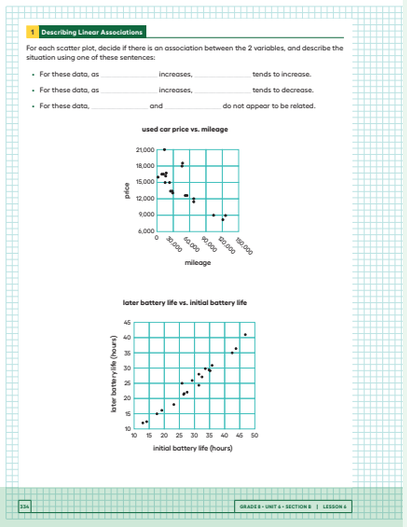


For these data, as daily high temperature increases, energy consumed tends to increase.

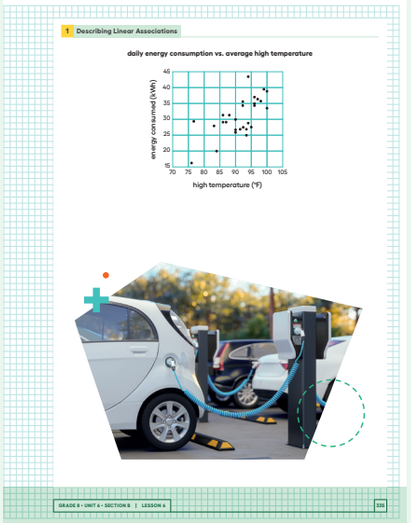
Building on Student Thinking

Students may assume they need to use each sentence exactly 1 time. Let them know it is acceptable to use a sentence more than once and that it is acceptable to not use a sentence.

Student Workbook



Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
Think aloud and use gestures to emphasize the trend in the data. For example, tilt your hand to show the general direction of the data in the scatter plot.
Advances: Listening, Representing

Activity Synthesis

The purpose of the discussion is to talk about trends in data based on the representations in scatter plots.

Consider asking some of the following questions:

- *"Is it surprising that the price decreases as mileage increases? Why doesn't mileage predict price perfectly?"*

  There are other factors, like whether the car has any damage or has any extra features.

- *"Is it surprising that devices with a longer initial battery life have a longer later battery life? Why doesn't the initial battery life predict later battery life perfectly?"*

  It is not surprising. There are other factors, like how much the devices are used or the materials used in the batteries.

- *"Is it surprising that energy consumption goes up as the temperature increases?"*

  No, because of air conditioning.

For the last scatter plot, highlight the outliers by asking:

- *"What might cause more energy consumption on a cool day?"*

  laundry, using power tools, using an electric heater

- *"What might cause less energy consumption on a hot day?"*

  being gone from home, using fans instead of air conditioning, raising the thermostat temperature

Students may notice that the association between high temperature and energy consumed is more variable than the other situations. There is still a positive association or positive trend, but we would describe the association as "weaker."

Activity 2
Interpreting Slopes

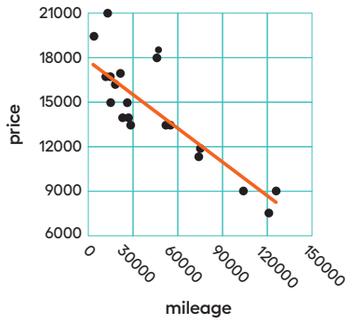
10 min

Activity Narrative

In the previous activity, students noticed trends in the data from the scatter plot. In this activity, the association is made more precise by looking at equations and graphs of linear models for the data to determine the slope. The numerical value of the slope is then interpreted in the context of the problem.

Launch

Remind students that earlier we looked at the price and mileage of some used cars. We saw that for these used cars, the price tends to decrease as the mileage increases. Display the scatter plot and linear model for the data.



Tell students that an equation for the line is $y = -0.073x + 17{,}404.485$. From the equation, we can identify the slope of the line as -0.073. Ask students to think about what that slope tells us and give quiet think time. Select 1–2 students to share their thinking. (It means that for every increase of one mile, the model predicts that the price of the car will decrease by $0.073.) Tell students that in this activity they will determine what the slope of the model means for 3 different sets of data.

Student Task Statement

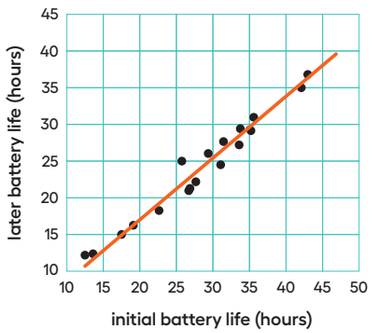
For each of the situations, a linear model for some data is shown.

1. What is the slope of the line in the scatter plot for each situation?

2. What is the meaning of the slope in that situation?

later battery life vs. initial battery life

$y = 0.85x - 0.1$



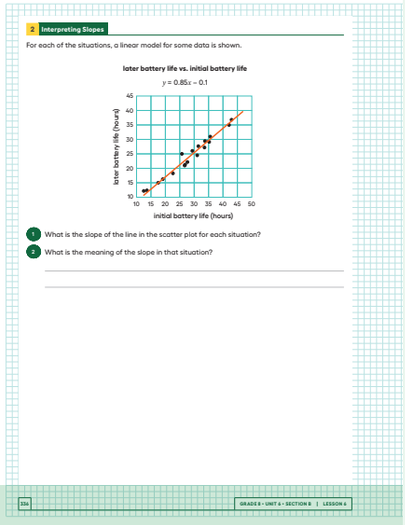
1. The slope is 0.85.

2. If the initial battery life increases by 1 hour, the model predicts that its later battery life increases by 0.85 hours.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. Display sentence frames to describe the meaning of slope in each representation: "If the _____ increases by 1 _____ , the model predicts that _____ (increases/decreases) by _____ ." *Advances: Speaking, Writing*

Student Workbook


2 Interpreting Slopes

For each of the situations, a linear model for some data is shown.

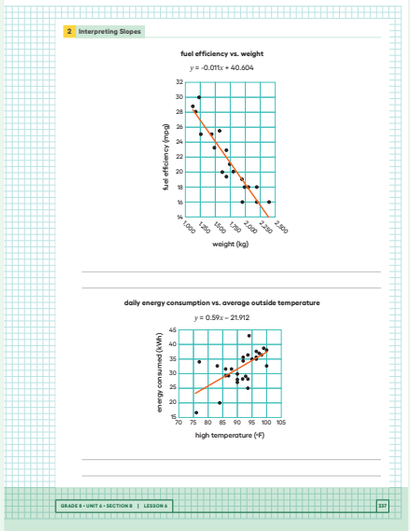
later battery life vs. initial battery life

$y = 0.85x - 0.1$

1 What is the slope of the line in the scatter plot for each situation?

2 What is the meaning of the slope in that situation?

Student Workbook


2 Interpreting Slopes

fuel efficiency vs. weight

$y = -0.011x + 40.604$

fuel efficiency (mpg)

weight (kg)

daily energy consumption vs. average outside temperature

$y = 0.59x - 21.912$

energy consumed (kWh)

high temperature (°F)

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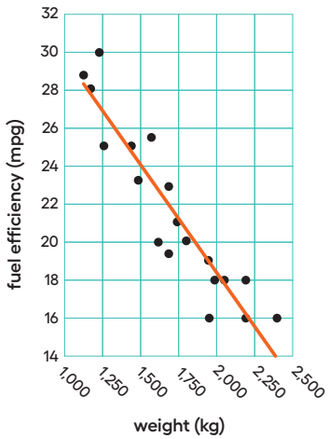
fuel efficiency vs. weight

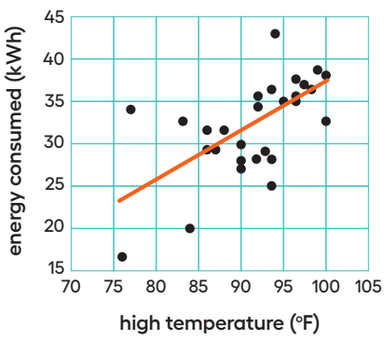
$y = -0.011x + 40.604$



1. The slope is -0.011.
2. If the weight of a car increases by 1 kg, the model predicts that its fuel efficiency decreases by 0.011 m

daily energy consumption vs. average outside temperature

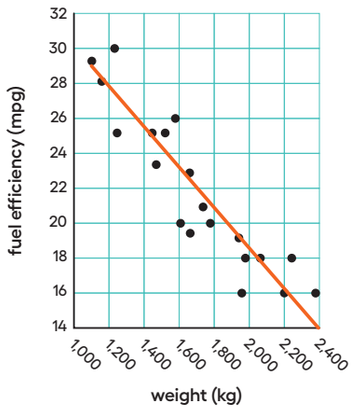
$y = 0.59x - 21.912$



1. The slope is 0.59.
2. If the daily high temperature increases by 1 degree Fahrenheit, the model predicts that the energy consumed increases by 0.59 kilowatt hour.

Are You Ready for More?

The scatter plot shows the weight and fuel efficiency data used in an earlier lesson along with a linear model represented by the equation $y = -0.0114x + 41.3021$.



1. What is the value of the slope and what does it mean in this context?

   -0.0114 is the slope

   It means that for every 1 kg increase in the weight of the car, the fuel efficiency decreases by 0.0114 mile per gallon.

2. What does the other number in the equation represent on the graph? What does it mean in context?

   41.3021 represents the value of the y-coordinate when the x-coordinate is 0. In other words, the y-intercept of the graph. It means that a car that weighs 0 kg would have a fuel efficiency of about 41 miles per gallon.

3. Use the equation to predict the fuel efficiency of a car that weighs 100 kilograms.

   40.16 mpg, since $-0.0114 \cdot 100 + 41.3021 \approx 40.16$

4. Use the equation to predict the weight of a car that has a fuel efficiency of 22 mpg.

   1,693 kg, since $(22 - 41.3021) \div (-0.0114) \approx 1,693$

5. Which of these two predictions probably fits reality better? Explain.

   Sample response: The weight of the car with 22 mpg probably fits reality better. A 100 kilogram car is so small that it likely doesn't fit with the trend of the data we are using to create the linear model.

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present only 1 scatter plot at a time.
Supports accessibility for: Organization, Attention

Access for Multilingual Learners (Activity 3)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479
Please log in to the site before using the QR code or URL.

Activity Synthesis

The purpose of this discussion is to develop a quantitative sense of trends based on linear models of the data.

Consider asking some of the following questions:

- *"What was the easiest way to find the slope for each situation?"*

  The coefficient of the x-coordinate in the equation.

- *"Do the answers for the meaning of the slopes make sense in the contexts of the problem?"*

  Yes, battery life probably decreases over time as a percentage of its initial battery life, so 1 hour increase initially could only give an increase of 0.85 hours later. A 1 kilogram difference in the weight of a car is not very much, so it may lower the gas mileage, but not by much. A 1 degree temperature increase is not significant, so it should not need a lot more energy to cool off a building, but some would be needed.

- *"What is the difference between a positive slope and negative slope in your interpretations?"*

  A positive slope means both variables increase together. A negative slope means that when one variable increases, the other decreases.

- *"The model for energy consumption and temperature predicted a 0.59 kilowatt hour increase in energy consumption for every 1 degree increase in temperature. Estimate how much the temperature would need to increase to raise energy consumption by 6 kilowatt hours."*

  A little more than 10 degrees.

Activity 3

Positive or Negative?

10 min

Activity Narrative

This activity returns to scatter plots without linear models given. Students determine whether the data seems to have a linear association or not. If it does, students are asked to decide whether the variables have a positive or negative association.

Launch

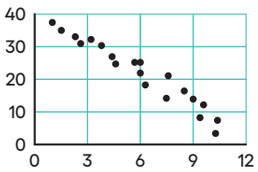
Tell students that while some data sets have a *linear association*, others do not. A linear association is present when the points in a scatter plot suggest that a linear model would fit the data well. Some data sets have a *non-linear association* when the scatter plot suggests that a non-linear curve would fit the data better. Still other data sets have *no association* when the data appears random and no curve would represent the data well.

In this activity, students first need to identify if data has a linear association or not and, if it does, what type of slope a linear model of the data would have.

Student Task Statement

1. For each of the scatter plots, decide whether it makes sense to fit a linear model to the data. If it does, would the graph of the model have a positive slope, a negative slope, or a slope of 0?

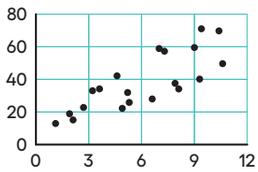
A



yes

negative

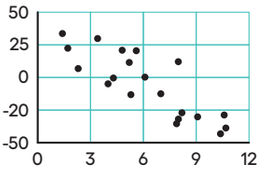
B



yes

positive

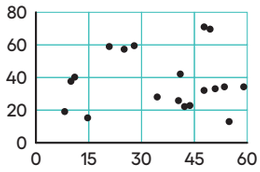
C



yes

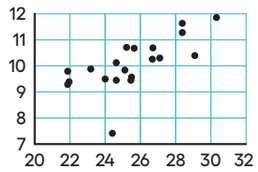
negative

D



no

E



yes

positive

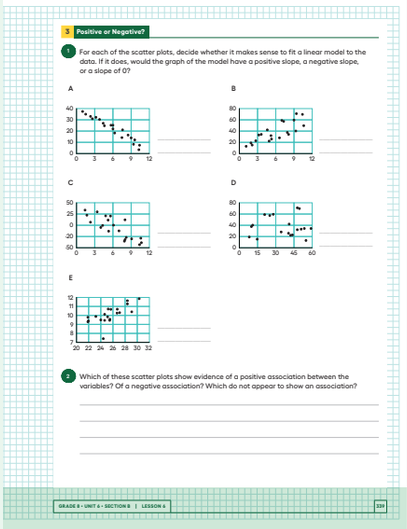
2. Which of these scatter plots show evidence of a positive association between the variables? Of a negative association? Which do not appear to show an association?

- B and E have a positive association.
- A and C have a negative association.
- D has no apparent association.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Representation: Develop Language and Symbols.
Maintain a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of "linear association," "no association," "positive association," and "negative association."
Supports accessibility for: Language, Memory

Student Workbook



Activity Synthesis

The purpose of this discussion is to solidify understanding of trends in scatter plots and look for associations in the data.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the question "How can you tell when a linear model is a good fit for data and what associations the data might have?" In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- *"_____ makes sense, but what do you mean when you say ...?"*
- *"Can you describe that another way?"*
- *"How do you know ...? What else do you know is true?"*

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

After *Stronger and Clearer Each Time*, ask:

- *"How would you begin if you had a table of data?"*

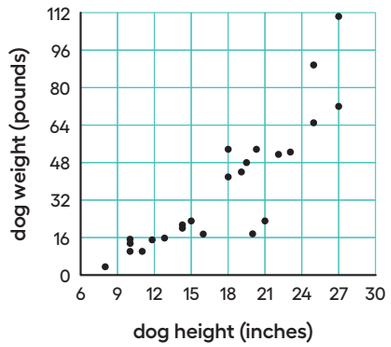
  Rearranging the table so that one of the variables increases as you read down the table might help show how the other variable is trending, but it is difficult to notice whether there is an association in a table, so I would first create a scatter plot.

- *"How does the slope of a good linear model relate to data that has a negative association? A positive association?"*

  If the data has a negative association, the slope of a good linear model should also be negative. Similarly, if the data has a positive association, a good linear model should have a positive slope.

Lesson Synthesis

Display the scatter plot for all to see.



To highlight the main ideas from the lesson about the meaning of the slope of a fitted line, ask:

- *"How would you describe the trend in the scatter plot?"*

  As the height of a dog increases, its weight tends to increase.

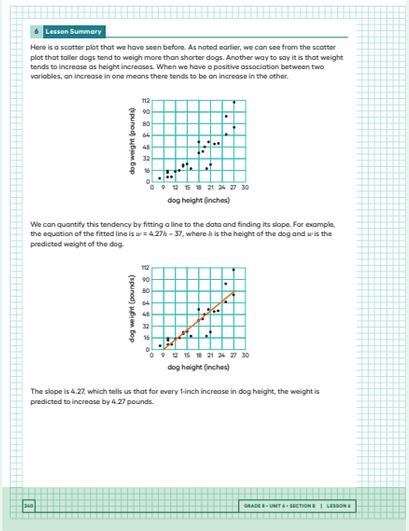
- *"When there is an association between 2 variables, how can we tell if it is a positive association or a negative one?"*

  If the dependent variable tends to increase as the independent variable increases, it is a positive association. A line that is a good fit for the data will have a positive slope. If the dependent variable tends to decrease as the independent variable tends to increase, it is a negative association. A line that is a good fit for the data will have a negative slope.

- *"The slope of a line that models the data is 4.25. What does that tell us about the dogs?"*

  For every 1 inch increase in dog height, the predicted weight increase is 4.25 pounds.

Student Workbook



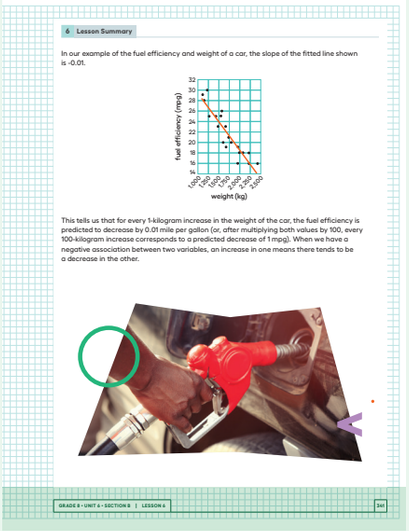
6 Lesson Summary

Here is a scatter plot that we have seen before. As noted earlier, we can see from the scatter plot that taller dogs tend to weigh more than shorter dogs. Another way to say it is that weight tends to increase as height increases. When we have a positive association between two variables, an increase in one means there tends to be an increase in the other.

We can quantify this tendency by fitting a line to the data and finding its slope. For example, the equation of the fitted line is $w = 4.27h - 37$, where h is the height of the dog and w is the predicted weight of the dog.

The slope is 4.27, which tells us that for every 1-inch increase in dog height, the weight is predicted to increase by 4.27 pounds.

Student Workbook



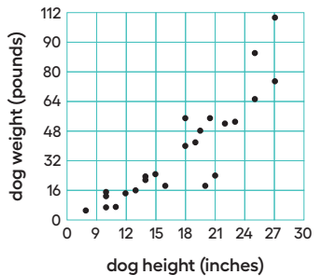
6 Lesson Summary

In our example of the fuel efficiency and weight of a car, the slope of the fitted line shown is -0.01.

This tells us that for every 1-kilogram increase in the weight of the car, the fuel efficiency is predicted to decrease by 0.01 mile per gallon (or, after multiplying both values by 100, every 100-kilogram increase corresponds to a predicted decrease of 1 mpg). When we have a negative association between two variables, an increase in one means there tends to be a decrease in the other.

Lesson Summary

Here is a scatter plot that we have seen before. As noted earlier, we can see from the scatter plot that taller dogs tend to weigh more than shorter dogs. Another way to say it is that weight tends to increase as height increases. When we have a positive association between two variables, an increase in one means there tends to be an increase in the other.



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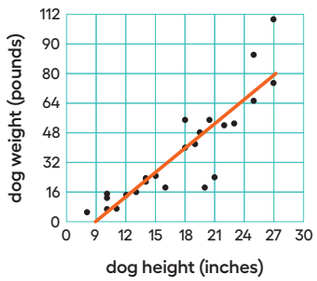
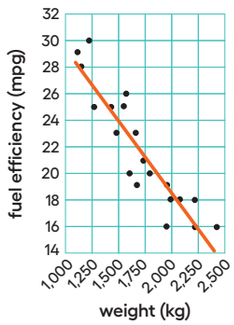


The slope is 4.27, which tells us that for every 1-inch increase in dog height, the weight is predicted to increase by 4.27 pounds.

In our example of the fuel efficiency and weight of a car, the slope of the fitted line shown is -0.01.



This tells us that for every 1-kilogram increase in the weight of the car, the fuel efficiency is predicted to decrease by 0.01 mile per gallon (or, after multiplying both values by 100, every 100-kilogram increase corresponds to a predicted decrease of 1 mpg). When we have a negative association between two variables, an increase in one means there tends to be a decrease in the other.

Cool-down

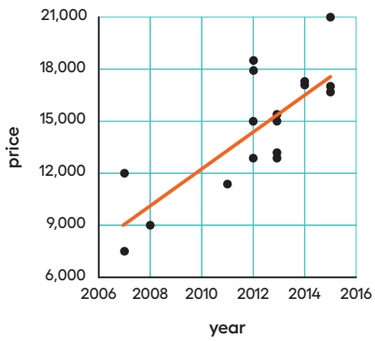
Trends in the Price of Used Cars

5 min

Students use a scatter plot of data representing used car prices and their model year to determine the sign of the slope of a linear model. Then students interpret this slope in the context of the data.

Student Task Statement

Here is a scatter plot that shows the years when some used cars were made and their prices in 2016 together with the graph of a linear model for the relationship between year and price in dollars.



1. Is the slope positive or negative?

   The slope is positive, because as the year of the car increases, the price tends to increase.

2. Which of these values is closest to the slope of the linear model shown in the scatter plot?

   - 1,000
   - 3,000
   - -1,000
   - -3,000

3. Use the value you selected to describe the meaning of the slope in this context.

   The model predicts that when a car is made 1 year later, the price is 1,000 dollars higher.

Responding To Student Thinking

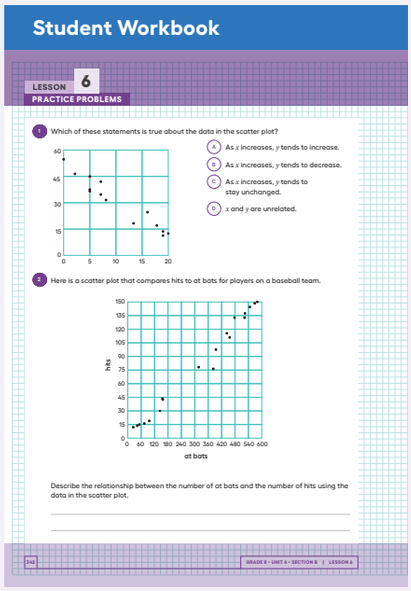
Points to Emphasize

If students struggle with estimating the slope of a line that fits the data, focus on this over the next few lessons. For example, during the Activity Synthesis of the activity referred to here, use Cards C and E to discuss slope estimates.

Unit 6, Lesson 7, Activity 2 Scatter Plot City

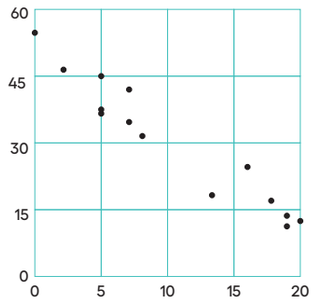
Practice Problems

5 Problems



Problem 1

Which of these statements is true about the data in the scatter plot?



A. As x increases, y tends to increase.

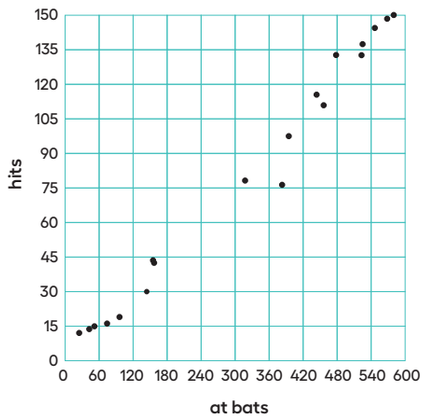
B. As x increases, y tends to decrease.

C. As x increases, y tends to stay unchanged.

D. x and y are unrelated.

Problem 2

Here is a scatter plot that compares hits to at bats for players on a baseball team.

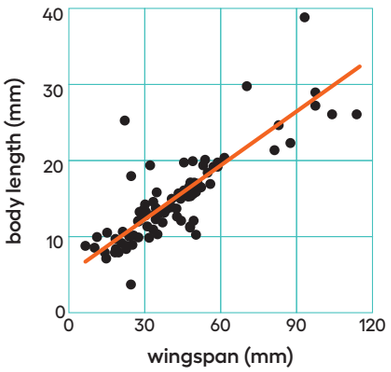


Describe the relationship between the number of at bats and the number of hits using the data in the scatter plot.

Sample response: As the number of at bats increases, the number of hits also increases.

Problem 3

The linear model for some butterfly data is given by the equation $y = 0.238x + 4.642$. Which of the following best describes the slope of the model?



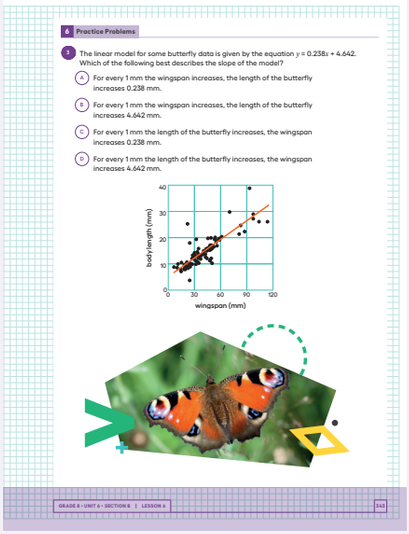
A. For every 1 mm the wingspan increases, the length of the butterfly increases 0.238 mm.

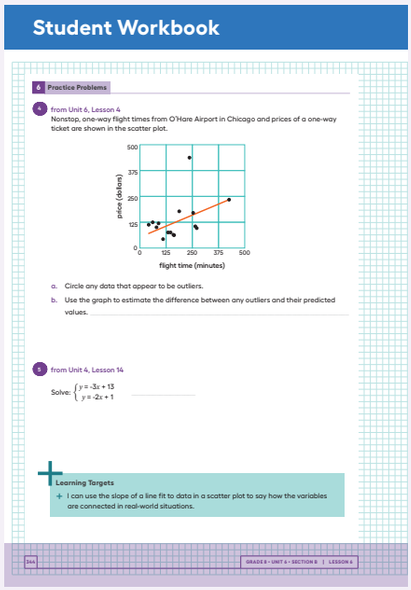
B. For every 1 mm the wingspan increases, the length of the butterfly increases 4.642 mm.

C. For every 1 mm the length of the butterfly increases, the wingspan increases 0.238 mm.

D. For every 1 mm the length of the butterfly increases, the wingspan increases 4.642 mm.

Student Workbook




Student Workbook

6 Practice Problems

4 from Unit 6, Lesson 4
Nonstop, one-way flight times from O'Hare Airport in Chicago and prices of a one-way ticket are shown in the scatter plot.

price (dollars)
flight time (minutes)

a. Circle any data that appear to be outliers.
b. Use the graph to estimate the difference between any outliers and their predicted values.

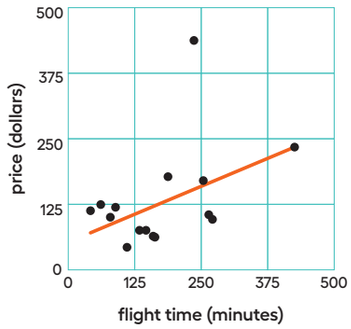
5 from Unit 4, Lesson 14
Solve: $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

Learning Targets
+ I can use the slope of a line fit to data in a scatter plot to say how the variables are connected in real-world situations.

Problem 4

from Unit 6, Lesson 4

Nonstop, one-way flight times from O'Hare Airport in Chicago and prices of a one-way ticket are shown in the scatter plot.



a. Circle any data that appear to be outliers.

The point at (239, 436) appears to be an outlier.

b. Use the graph to estimate the difference between any outliers and their predicted values.

This point represents a flight that costs around 250 dollars more than the model predicts based on its flight time.

Problem 5

from Unit 4, Lesson 14

Solve: $\begin{cases} y = -3x + 13 \\ y = -2x + 1 \end{cases}$

(12, -23)