Lesson 6

What About Other Bases?

Goals

- Generalize exponent rules for nonzero bases, including bases other than 10.
- Use exponent rules to identify (in writing) equivalent exponential expressions, and explain (orally) the reasoning.

Learning Target

I can use the exponent rules for bases other than 10.

Lesson Narrative

In this lesson, students use their understanding of exponent rules with powers of 10 to make sense of exponent rules for powers of other bases. Students begin by comparing expressions with both positive and negative bases. Then they work through problems analogous to the ones used to develop exponent rules for powers of 10. The same underlying patterns emerge, revealing the fact that the exponent rules are the same for bases other than 10. For simplicity, students do not develop general exponent rules for negative bases. Students continue to practice applying their understanding of exponent rules by analyzing lists of exponential expressions and determining which expressions are not equivalent.

Student Learning Goal

Let's explore exponent patterns with bases other than 10.

Instructional Routines

- Math Talk

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)

Required Preparation

Lesson:
Create a new set of visual displays that generalize the exponent rules. A sample display can be seen in the *Lesson Synthesis*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	15 min	10 min	5 min

Warm-up

Math Talk: Comparing Expressions with Exponents

5 min

Activity Narrative

This *Math Talk* focuses on comparing powers of positive and negative numbers. It encourages students to think about repeated multiplication and to rely on what they know about integers and the meaning of the bases and exponents to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students find equivalent expressions involving exponents.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether each statement is true.

A. $3^5 < 4^6$

True. Sample reasoning: 3^5 has both a smaller base and a smaller exponent than 4^6 so its value will also be smaller.

B. $(-3)^2 < 3^2$

Not true. Sample reasoning: Since a negative number multiplied by itself results in a positive number, both values are equal to 9.

C. $(-3)^3 = 3^3$

Not true. Sample reasoning: A negative number multiplied by itself 3 times will result in a negative number, making $(-3)^3$ less than 3^3.

D. $(-5)^2 > -5^2$

True. Sample reasoning: The expression on the left is equivalent to 25. The expression on the right could be written as $-(5)^2$, meaning that only the 5 is squared and the resulting value is -25.

Instructional Routines

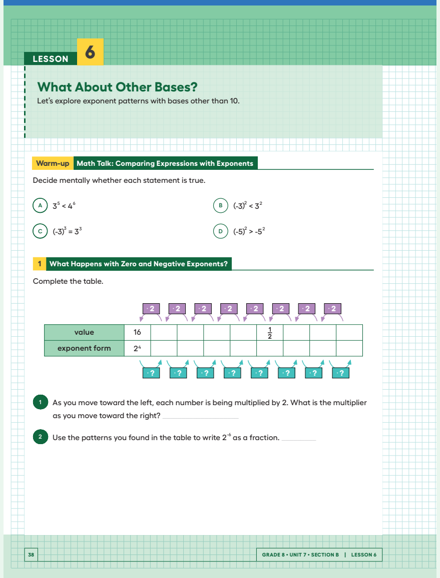
Math Talk

ilclass.com/r/10694967
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook


LESSON 6

What About Other Bases?

Let's explore exponent patterns with bases other than 10.

Warm-up Math Talk: Comparing Expressions with Exponents

Decide mentally whether each statement is true.

A $3^5 < 4^6$

B $(-3)^2 < 3^2$

C $(-3)^3 = 3^3$

D $(-5)^2 > -5^2$

1 What Happens with Zero and Negative Exponents?

Complete the table.

value	16					$\frac{1}{2}$			
exponent form	2^4								

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right? ______
2. Use the patterns you found in the table to write 2^{-6} as a fraction. ______

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Access for Multilingual Learners (Warm-up, Activity Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.
Direct attention to words collected and displayed from a previous lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

What Happens with Zero and Negative Exponents?

10 min

Activity Narrative

The purpose of this activity is to extend the definition of non-positive exponents to other bases. Students notice that patterns that were true for powers of 10 also hold for other bases.

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by 2 minutes to share their reasoning with a partner. Conclude with a whole-class discussion.

Student Task Statement

Complete the table.

$\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$

value	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
exponent form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}

$\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right? $\frac{1}{2}$
2. Use the patterns you found in the table to write 2^{-6} as a fraction. $2^{-6} = \frac{1}{2^6}$ or $\frac{1}{64}$
3. Write $\frac{1}{32}$ as a power of 2 with a single exponent. $\frac{1}{32} = \frac{1}{2^5}$ or 2^{-5}
4. What is the value of 2^0? $2^0 = 1$
5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction? $5^{-3} = \frac{1}{5^3}$ or $\frac{1}{125}$
6. How would you write 3^{-4} as a fraction? $3^{-4} = \frac{1}{3^4}$ or $\frac{1}{81}$

Are You Ready for More?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

   $\left(\frac{3}{2}\right)^{3}$, because the opposite of repeatedly multiplying by $\frac{2}{3}$ is repeatedly multiplying by $\frac{3}{2}$.

2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

   $\left(\frac{5}{4}\right)^{8}$, because the opposite of repeatedly multiplying by $\frac{4}{5}$ is repeatedly multiplying by $\frac{5}{4}$.

3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

   To write $\left(\frac{a}{b}\right)^{-n}$ with a positive exponent, notice that the opposite of repeatedly multiplying by $\frac{a}{b}$ is repeatedly multiplying by $\frac{b}{a}$. The general rule is that a fraction to a negative exponent is equal to the reciprocal, to the positive exponent. So $\left(\frac{b}{a}\right)^{n}$

Activity Synthesis

The goal of this discussion is for students to connect that the exponent rules discovered for powers of 10 work with any base. Discuss the following questions:

- *"How does working with powers of 2 compare to working with powers of 10?"*

  The rules apply in the same way, but computing the value of some powers of 2 can take longer since we can't just look at the number of zeros.

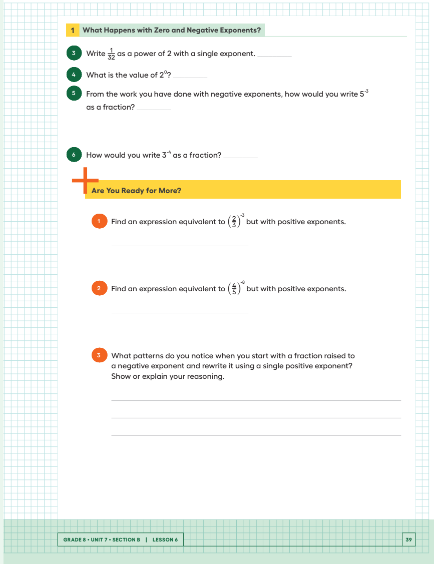
- *"What happens when you use exponent rules when one of the exponents is 0?"*

  The rules apply in the same way. $2^0 = 1$ in the same way that $10^0 = 1$.

- *"How does 10^{-3} compare to 2^{-3}?"*

  Both values are positive and less than 1. When each is written as a fraction, 10^{-3} has greater numbers in the denominator, so it is less than 2^{-3}.

Student Workbook


1 What Happens with Zero and Negative Exponents?

3 Write $\frac{1}{32}$ as a power of 2 with a single exponent.

4 What is the value of 2^0?

5 From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?

6 How would you write 3^{-4} as a fraction?

Are You Ready for More?

1 Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

2 Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

3 What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

Building on Student Thinking

If students think expressions such as 5^{-9} and -5^9 are equivalent, consider:

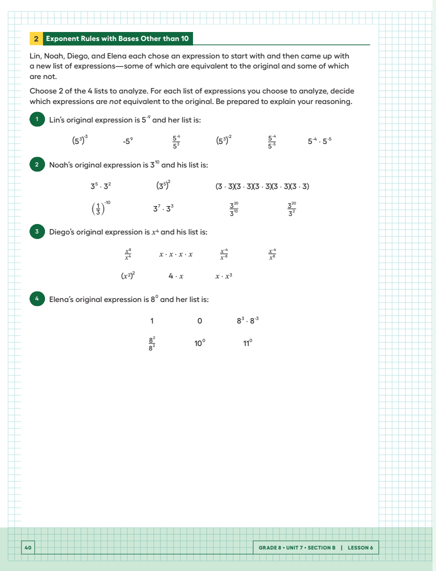
Asking

"What is the difference between 10^3 and 10^{-3}? Between 5^9 and 5^{-9}?"

One is repeated multiplication by 10 (or 5) and the other is repeated multiplication by $\frac{1}{10}$ (or $\frac{1}{5}$).

Explaining that -5^9 is a large negative number while 5^{-9} is a very small positive number.

Student Workbook



Activity 2
Exponent Rules with Bases Other than 10

15 min

Activity Narrative

This activity gives students a chance to practice using exponent rules to analyze expressions and identify equivalent expressions. Students choose 2 out of 4 lists to analyze. As students work, notice the different strategies used to analyze the expressions in each list.

Launch

Arrange students in groups of 2. Tell students to choose 2 out of 4 lists to analyze and give students 6–7 minutes of quiet work time followed by a brief partner discussion. If time allows, have students analyze all 4 lists. Follow with a whole-class discussion.

Student Task Statement

Lin, Noah, Diego, and Elena each chose an expression to start with and then came up with a new list of expressions — some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:

   $(5^3)^{-3}$ -5^9 $\frac{5^{-6}}{5^3}$ $(5^3)^{-2}$ $\frac{5^{-4}}{5^{-5}}$ $5^{-4} \cdot 5^{-5}$

   The following are not equivalent to 5^{-9}:

   - -5^9 because it is negative and 5^{-9} is positive.
   - $(5^3)^{-2}$ because it is equal to 5^{-6}.
   - $\frac{5^{-4}}{5^{-5}}$ because $\frac{5^{-4}}{5^{-5}} = 5^{-4-(-5)} = 5^{-4+5} = 5^1$.

2. Noah's original expression is 3^{10} and his list is:

   $3^5 \cdot 3^2$ $(3^5)^2$ $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$

   $\left(\frac{1}{3}\right)^{-10}$ $3^7 \cdot 3^3$ $\frac{3^{20}}{3^{10}}$ $\frac{3^{20}}{3^2}$

   The following are not equivalent to 3^{10}:

   - $3^5 \cdot 3^2$ because $3^5 \cdot 3^2 = 3^{5+2} = 3^7$.
   - $\frac{3^{20}}{3^2}$ because $\frac{3^{20}}{3^2} = 3^{20-2} = 3^{18}$.

3. Diego's original expression is x^4 and his list is:

   $\frac{x^8}{x^4}$ $x \cdot x \cdot x \cdot x$ $\frac{x^{-4}}{x^{-8}}$ $\frac{x^{-4}}{x^8}$

   $(x^2)^2$ $4 \cdot x$ $x \cdot x^3$

   The following are not equivalent to x^4:

   - $\frac{x^{-4}}{x^8}$ because $\frac{x^{-4}}{x^8} = x^{-4-8} = x^{-12}$.
   - $4 \cdot x$. Sample reasoning: If $x = 10$, then $4 \cdot 10 = 40$ and $10^4 = 10000$.

4. Elena's original expression is 8^0 and her list is:

   1 0 $8^3 \cdot 8^{-3}$ $\frac{8^2}{8^2}$ 10^0 11^0

   The only expression not equivalent to 8^0 in Elena's list is 0, because all of the rest are defined to be equal to 1.

Activity Synthesis

The goal of this discussion is for students to see that the exponent rules are valid for nonzero bases other than 10. For each list, invite students to share which expressions did not match the original expression and to share their reasoning strategies. If necessary for time, consider focusing just on questions 3 and 4 to assess how students have generalized in terms of variables and whether students have internalized the 0 exponent.

As students share their strategies for each list, discuss:

"Do you agree or disagree? Why?"

"Did anyone think about that expression in a different way?"

"Did anyone use the same strategy but would explain it differently?"

Lesson Synthesis

The purpose of the discussion is to check whether students understand that the rules they have developed for doing arithmetic with powers of 10 are valid for any other non-zero rational bases as well. This is also a good opportunity to address possible misconceptions that students have, especially regarding non-positive exponents.

Consider using these discussion questions to emphasize the important concepts in this lesson:

"How do 2^3 and 2^{-3} compare to 10^3 and 10^{-3}?"

When the base is 2 rather than 10, the repeated multiplication works the same way. 2^3 is $2 \cdot 2 \cdot 2$, and 2^{-3} is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$.

"How do 5^3 and 5^{-3} compare to 10^3 and 10^{-3}?"

The exponent rules work in exactly the same way. The base is just 5 instead of 10.

"How is $(-3)^4$ different from -3^4?"

$(-3)^4$ is $(-3)(-3)(-3)(-3)$ whereas -3^4 is $-(3 \cdot 3 \cdot 3 \cdot 3)$.

"Do the other exponent rules work the same way that they did for base 10? Give some examples with different bases."

Yes, the other exponent rules work the same way. With base 7 for example, we have that $\frac{7^5}{7^2} = 7^3$, and $7^3 \cdot 7^6 = 7^9$.

Tell students that the exponent rules with other bases work exactly the same way as they did with a base of 10. They can always check this by expanding the factors of an expression that has exponents. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$a^n \cdot a^m = a^{n+m}$	$a^2 \cdot a^3 = (a \cdot a) \cdot (a \cdot a \cdot a) = a^5$ two factors that are a · three factors that are a = five factors that are a
$(a^n)^m = a^{n \cdot m}$	$(a^2)^3 = (a \cdot a) \cdot (a \cdot a) \cdot (a \cdot a) = a^6$ three groups of two factors that are a = six factors that are a
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a = 1 \cdot a^3 = a^3$ five factors that are a ÷ two factors that are a = three factors that are a
$a^0 = 1$	$\frac{a^6}{a^0} = a^{6-0} = a^6$ this value must be equal to 1
$a^{-n} = \frac{1}{a^n}$	$a^{-3} = \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^3}$ three factors that are one over a

Lesson Summary

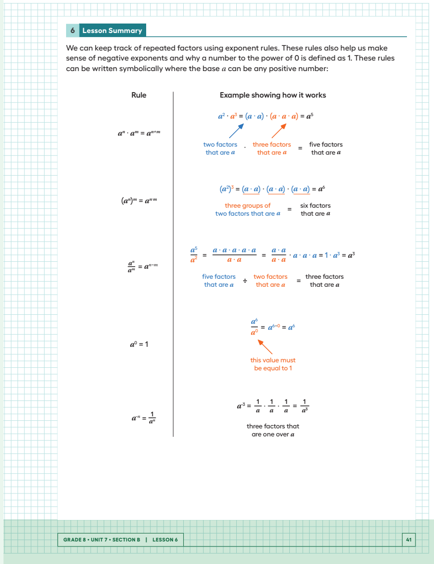
We can keep track of repeated factors using exponent rules. These rules also help us make sense of negative exponents and why a number to the power of 0 is defined as 1. These rules can be written symbolically where the base a can be any positive number:

Rule	Example showing how it works
$a^n \cdot a^m = a^{n+m}$	$a^2 \cdot a^3 = (a \cdot a) \cdot (a \cdot a \cdot a) = a^5$ two factors that are a · three factors that are a = five factors that are a
$(a^n)^m = a^{n \cdot m}$	$(a^2)^3 = (a \cdot a) \cdot (a \cdot a) \cdot (a \cdot a) = a^6$ three groups of two factors that are a = six factors that are a
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a = 1 \cdot a^3 = a^3$ five factors that are a ÷ two factors that are a = three factors that are a
$a^0 = 1$	$\frac{a^6}{a^0} = a^{6-0} = a^6$ this value must be equal to 1
$a^{-n} = \frac{1}{a^n}$	$a^{-3} = \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^3}$ three factors that are one over a

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Cool-down

Spot the Mistake

5 min

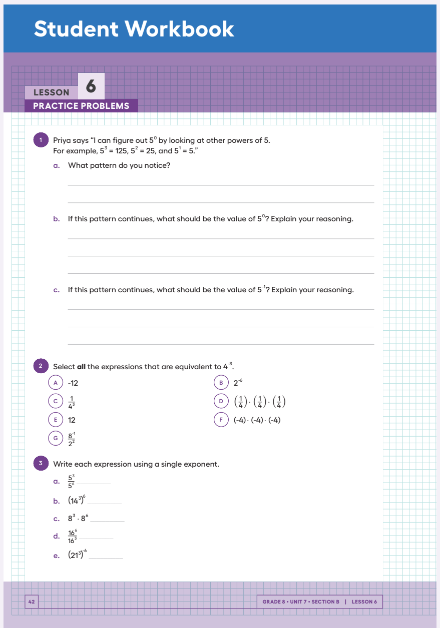
Student Task Statement

1. Diego was trying to write $2^3 \cdot 2^2$ with a single exponent and wrote $2^3 \cdot 2^2 = 2^{3 \cdot 2} = 2^6$. Do you agree with Diego? Explain your reasoning.

   I do not agree with Diego. Sample reasoning: Diego multiplied the exponents when he should have added them. To see this, he could have expanded the expressions: $2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^{3+2} = 2^5$.

2. Andre was trying to write $\frac{7^4}{7^{-3}}$ with a single exponent and wrote $\frac{7^4}{7^{-3}} = 7^{4-3} = 7^1$. Do you agree with Andre? Explain your reasoning.

   I do not agree with Andre. Sample reasoning: Andre did 7^{4-3} when he should have done $7^{4-(-3)}$ to get 7^7.


Student Workbook

LESSON 6
PRACTICE PROBLEMS

1 Priya says "I can figure out 5^0 by looking at other powers of 5. For example, $5^3 = 125$, $5^2 = 25$, and $5^1 = 5$."

a. What pattern do you notice?

b. If this pattern continues, what should be the value of 5^0? Explain your reasoning.

c. If this pattern continues, what should be the value of 5^{-1}? Explain your reasoning.

2 Select **all** the expressions that are equivalent to 4^{-3}.

A -12

B 2^{-6}

C $\frac{1}{4^3}$

D $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

E 12

F (-4) · (-4) · (-4)

G $\frac{8^{-1}}{2^2}$

3 Write each expression using a single exponent.

a. $\frac{5^3}{5^6}$

b. $(14^3)^6$

c. $8^3 \cdot 8^6$

d. $\frac{16^6}{16^3}$

e. $(21^3)^{-6}$

Lesson 6

Practice Problems

4 Problems

Problem 1

Priya says "I can figure out 5^0 by looking at other powers of 5. For example, $5^3 = 125$, $5^2 = 25$, and $5^1 = 5$."

a. What pattern do you notice?

Sample response: When the exponent decreases by 1, the value is divided by 5 or is multiplied by $\frac{1}{5}$.

b. If this pattern continues, what should be the value of 5^0? Explain your reasoning.

Sample reasoning: The value of 5^0 should be the value of 5^1 divided by 5.

c. If this pattern continues, what should be the value of 5^{-1}? Explain your reasoning.

$\frac{1}{5}$

Sample reasoning: The value of 5^{-1} should be the value of 5^0 divided by 5.

Problem 2

Select **all** the expressions that are equivalent to 4^{-3}.

A. -12

B. 2^{-6}

C. $\frac{1}{4^3}$

D. $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

E. 12

F. (-4) · (-4) · (-4)

G. $\frac{8^{-1}}{2^2}$

Problem 3

Write each expression using a single exponent.

a. $\frac{5^3}{5^6}$ 5^{-3}

b. $(14^3)^6$ 14^{18}

c. $8^3 \cdot 8^6$ 8^9

d. $\frac{16^6}{16^3}$ 16^3

e. $(21^3)^{-6}$ 21^{-18}

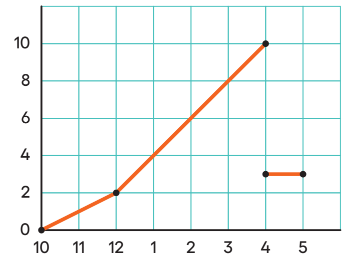
Problem 4

from Unit 5, Lesson 6

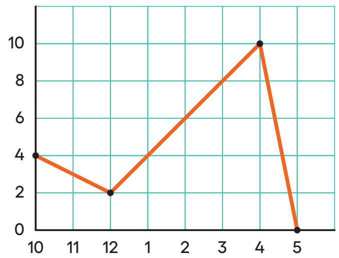
Andre sets up a rain gauge to measure rainfall in his backyard. On Tuesday, it rains off and on all day.

- He starts at 10 a.m. with an empty gauge when it starts to rain.
- Two hours later, he checks, and the gauge has 2 cm of water in it.
- It starts raining even harder, and at 4 p.m., the rain stops, so Andre checks the rain gauge and finds it has 10 cm of water in it.
- While checking it, he accidentally knocks the rain gauge over and spills most of the water, leaving only 3 cm of water in the rain gauge.
- When he checks for the last time at 5 p.m., there is no change.

Graph A



Graph B

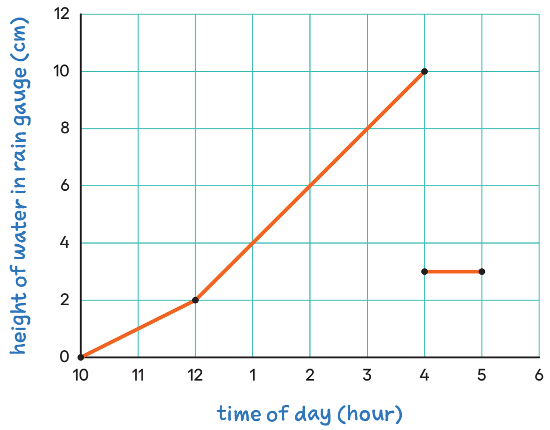


a. Which of the two graphs could represent Andre's story? Explain your reasoning.

Graph A

Sample reasoning: Graph A begins with an empty gauge, while Graph B starts at 4, and between 4 p.m. and 5 p.m. Graph A's function is flat to represent "no change."

b. Label the axes of the correct graph with appropriate units.



c. Use the graph to determine how much total rain fell on Tuesday.

10 cm of rain fell in total on Tuesday, because no rain fell after Andre spilled the rain gauge.

Student Workbook