Lesson 12

Units in Scale Drawings

Goals

- Comprehend that the phrase "equivalent scales" refers to different scales that relate scaled and actual measurements by the same scale factor.
- Generate a scale without units that is equivalent to a given scale with units, or vice versa.
- Justify (orally and in writing) that scales are equivalent, including scales with and without units.

Learning Targets

- I can tell whether two scales are equivalent.
- I can write scales with units as scales without units.

Lesson Narrative

In this lesson, students analyze various scales and find that sometimes it is helpful to rewrite scales with units as scales without units in order to compare them. They see that equivalent scales relate scaled and actual measurements by the same scale factor, even though the scales may be expressed differently. For example, the scale 1 inch to 2.5 feet is equivalent to the scale 5 m to 150 m, because they are both at a scale of 1 to 30. As students identify equivalent scales, they construct arguments and attend to precision.

This lesson is also the culmination of students' work on scaling and area. Students have seen many examples of the relationship between scaled area and actual area, and now they must use this realization to find the area of an irregularly-shaped pool.

There is a blackline master that gives some information about equal lengths that students may want to refer to during these activities.

Student Learning Goal

Let's use different scales to describe the same drawing.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR3: Critique, Correct, Clarify (Activity 3)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- Card Sort
- MLR1: Stronger and Clearer Each Time
- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns

Required Materials

Materials to Gather

- Metric and customary unit conversion charts: Activity 2
- Geometry toolkits: Activity 3

Materials to Copy

- Scales Cards (1 copy for every 4 students): Activity 1
- Units of Length Reference Sheet (1 copy for every 2 students): Activity 1
- Units of Length Reference Sheet (1 copy for every 2 students): Activity 2

Required Preparation

Activity 1:
For the blackline master, if possible, copy each complete set on a different color of paper so that a stray slip can quickly be put back.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	15 min	10 min	5 min

Warm-up
Equal Measures

5 min

Activity Narrative

This *Warm-up* prompts students to review work from grade 5 about converting standard units within a given measurement system. Later in this lesson students will examine equivalent scales; that is, scales that lead to the same size scale drawing. Checking whether or not two scales are equivalent often involves converting given quantities to other units.

Launch

Arrange students in groups of 2. Ask students to use the following numbers and units to record as many equivalent measurements as they can. Tell them that they are allowed to reuse numbers and units. Give students 2 minutes of quiet think time followed by 1 minute to share their equations with their partner.

Student Task Statement

Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write "30 minutes is $\frac{1}{2}$ hour."

You can use the numbers and units more than once.

1	$\frac{1}{2}$	0.3	centimeter (cm)
12	40	24	meter (m)
0.4	100	$\frac{1}{10}$	kilometer (km)
8	$3\frac{1}{3}$	6	inch (in)
50	30	2	foot (ft)
		$\frac{2}{3}$	yard (yd)

Answers vary.

Sample responses:

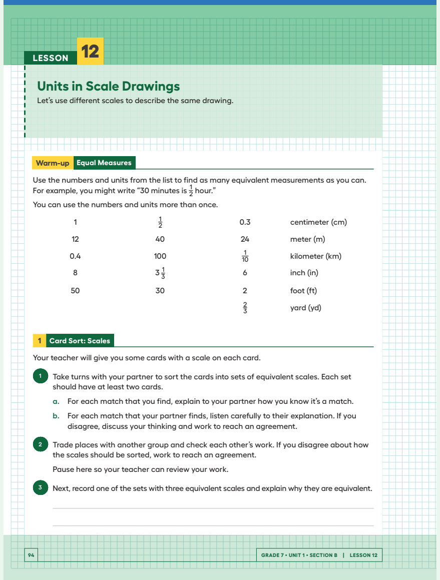
- 30 cm is 0.3 m, 40 cm is 0.4 m, 50 cm is $\frac{1}{2}$ m, 100 cm is 1 m
- 100 m is $\frac{1}{10}$ km
- 6 in is $\frac{1}{2}$ ft, 8 in is $\frac{2}{3}$ ft, 12 in is 1 ft, 24 in is 2 ft, 40 in is $3\frac{1}{3}$ ft
- 0.3 ft is $\frac{1}{10}$ yd, 2 ft is $\frac{2}{3}$ yd, 6 ft is 2 yd, 24 ft is 8 yd
- 24 in is $\frac{2}{3}$ yd

Building on Student Thinking

Some students may write "30 cm is 1 ft" because 30 is often the largest centimeter measurement labeled on a 1-foot ruler. Explain that these values are approximately equal, but not exact. If students want to keep such pairings on their list, prompt them to use a phrase like "is close to." For example:

- 30 cm is close to 1 ft
- 0.3 m is close to 1 ft
- 40 in is close to 1 m
- $3\frac{1}{3}$ ft is close to 1 m

Student Workbook


LESSON 12

Units in Scale Drawings

Let's use different scales to describe the same drawing.

Warm-up Equal Measures

Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write "30 minutes is $\frac{1}{2}$ hour."

You can use the numbers and units more than once.

1	$\frac{1}{2}$	0.3	centimeter (cm)
12	40	24	meter (m)
0.4	100	$\frac{1}{10}$	kilometer (km)
8	$3\frac{1}{3}$	6	inch (in)
50	30	2	foot (ft)
		$\frac{2}{3}$	yard (yd)

1 Card Sort: Scales

Your teacher will give you some cards with a scale on each card.

1. Take turns with your partner to sort the cards into sets of equivalent scales. Each set should have at least two cards.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. Trade places with another group and check each other's work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.
3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

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Instructional Routines

Card Sort

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed _________, so I matched ..." Encourage students to challenge each other when they disagree.

Advances: Speaking, Conversing.

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to begin with a subset of the cards to start with and introduce the remaining cards after students have completed their initial set of matches.

Supports accessibility for: Organization, Social-Emotional Functioning

Activity Synthesis

Invite a few students to share equations that they had in common with their partner and ones that were different. Record these answers for all to see. After each equation is shared, ask the class to give a signal if they had the same one recorded. Display the following questions for all to see and discuss:

"What number(s) did you use the most? Why?"

"If you could include two more cards in this selection, what would they be? Why?"

Activity 1: Optional

Card Sort: Scales

15 min

Activity Narrative

In this partner activity, students take turns identifying equivalent scales, including some expressed without units and some with units. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

A key insight to uncover here is that when comparing scales, it can be helpful to convert them into equivalent scales in a particular format (for example, without units, or using the same units).

Launch

Tell students that the cards contain different scales and that they will take turns matching scales that are equivalent. Explain how to set up and do the activity. If time allows, demonstrate these steps with a student as a partner:

- Mix up the cards and place them face-up.
- One person selects two cards and explains to their partner why the cards match.
- The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement.
- When both partners agree on the match, they switch roles.

Consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Arrange students in groups of 2–3. Give each group a set of pre-cut cards.

Give students 5–6 minutes to sort the slips, and another 2–3 minutes to check another group's work, followed by whole-class discussion.

Student Task Statement

Your teacher will give you some cards with a scale on each card.

1. Take turns with your partner to sort the cards into sets of equivalent scales. Each set should have at least two cards.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
      - 1 cm to 1 m, and 1 to 100
      - 1 cm to 1 km, $\frac{1}{2}$ cm to 500 m, and 1 to 100,000
      - 1 in to 8 ft, $\frac{1}{8}$ in to 1 ft, and 1 to 96
      - 1 cm to 10 m, 1 in to 1,000 in, and 1 mm to 1 m
      - 1 ft to 1 mi and 1 to 5,280
      - 1 in to 1 mi and 1 to 63,360
2. Trade places with another group and check each other's work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.

   No written response required.
3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

   Sample response: The scales 1 cm to 1 km, $\frac{1}{2}$ cm to 500 m, and 1 to 100,000 are equivalent. There are 100,000 cm in one km, so 1 cm to 1 km and 1 to 100,000 are equivalent. Also, 100,000 groups of $\frac{1}{2}$ centimeter is 50,000 cm. This is the same length as 500 m, because $50,000 \div 100 = 500$.

Activity Synthesis

Much of the discussion takes place between partners, so a whole-class debrief may be necessary only to tie up any loose ends. Invite a few students to share how their group reasoned about a couple of the scales (for example, $\frac{1}{2}$ cm to 500 m, 1 mm to 1 m).

"What were some ways you handled …"

"Describe any difficulties you experienced and how you resolved them."

"Which matches were tricky? Explain why."

Address any questions that arose during sorting, common misconceptions, or unsettled disagreements between groups. For example, students may still be unclear about whether scales in customary and metric units can be equivalent. Consider asking

"Can '1 inch to 1,000 inches' and '1 centimeter to 10 meters' both go in the same group? Why or why not?"

Help students see that as long as the two scales represent the same scale factor, they are equivalent and will produce the same scale drawing.

If time permits, consider asking students to order their groups of equivalent scales, starting with the ones that would produce the smallest drawing of the same actual thing to the ones that would produce the largest drawing. Invite students to explain their reasoning.

Building on Student Thinking

If groups have trouble getting started, encourage them to think about different ways to express a scale, both with units and without units.

Students may sort the cards by the types (metric or customary; with units or without units) rather than by common scale factors. Remind students that scales that are equivalent have the same factor relating their scaled lengths to actual lengths.

Students may think that scales in metric units and those in customary units cannot be equivalent. For example, they may think that "1 inch to 1,000 inches" belongs in one group and "1 cm to 10 m" belongs in another. If this misconception arises and is not resolved in group discussions, address it during the *Activity Synthesis*.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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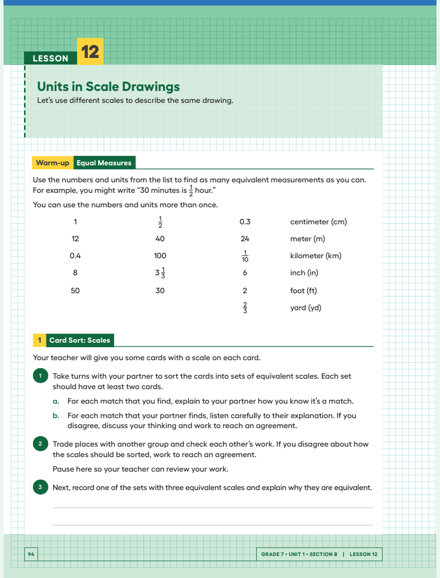
Student Workbook


LESSON 12

Units in Scale Drawings

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Warm-up Equal Measures

Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write "30 minutes is $\frac{1}{2}$ hour."

You can use the numbers and units more than once.

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1 Card Sort: Scales

Your teacher will give you some cards with a scale on each card.

1. Take turns with your partner to sort the cards into sets of equivalent scales. Each set should have at least two cards.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. Trade places with another group and check each other's work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.
3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

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Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Building on Student Thinking

Students may be confused about whether to multiply or divide by 2,000 (or to multiply by 2,000 or by $\frac{1}{2,000}$) when finding the missing lengths. Encourage students to articulate what a scale of 1 to 2,000 means, or remind them that it is a shorthand for saying "1 unit on a scale drawing represents 2,000 of the same units in the object it represents." Ask them to now think about which of the two—actual or scaled lengths—is 2,000 times the other and which is $\frac{1}{2,000}$ of the other.

For the third question relating the area of the real flag to the scale model, if students are stuck, encourage them to work out the dimensions of each explicitly and to use this to calculate the scale factor between the areas.

Activity 2
The World's Largest Flag

15 min

Activity Narrative

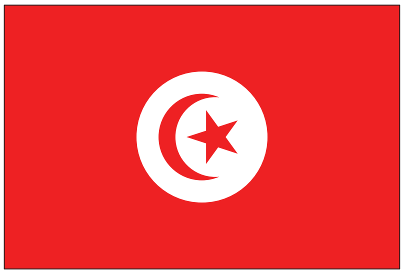
In this activity, students use a scale without units to find actual and scaled distances that involve a wider range of numbers, from 0.02 to 2,000. They also return to thinking about how the area of a scale drawing relates to the area of the actual thing. As students make sure to include the correct units on their answers and while explaining their reasoning, they are attending to precision.

Students are likely to find scaled lengths in one of two ways: 1) by first converting the measurement in meters to centimeters and then dividing by 2,000, or 2) by dividing the measurement by 2,000 and then converting the result to centimeters. To find actual lengths, similar paths are likely, except that students will multiply by 2,000 and reverse the unit conversion.

Launch

Tell students to close their books or devices. Display an image of Tunisia's flag. Explain that Tunisia holds the world record for the largest version of a country flag. The record-breaking flag is nearly four soccer fields in length. Solicit from students a few guesses for a scale that would be appropriate to create a scale drawing of the flag on a sheet of paper. If asked, provide the length of the flag (396 m) and the size of the paper (letter size: $8\frac{1}{2}$ inches by 11 inches, or about 21.5 cm by 28 cm).

After hearing some guesses, explain to students that they will now solve problems about the scale of the giant Tunisian flag.



Arrange students in groups of 3–4. Provide access to a metric unit conversion chart. Give students 4–5 minutes of quiet work time, and then another 5 minutes to collaborate and discuss their work in groups.

Student Task Statement

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

	flag length	flag height	height of crescent moon
actual	396 m	264 m	99 m
at 1 to 2,000 scale	19.8 cm	13.2 cm	4.95 cm

Sample reasoning:

- Length of flag: 396 ÷ 2,000 = 0.198. 0.198 m is 19.8 cm.
- Height of flag: (13.2) · 2,000 = 26,400. 26,400 cm is 264 m.
- Height of crescent moon on flag: 99 ÷ 2,000 = 0.0495. 0.0495 m is 4.95 cm.

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

The following correct answers are followed by sample reasonings:

a. 1 cm to 2,000 cm. 2,000 times 1 cm is 2,000 cm.

b. 1 cm to 20 m. 2,000 cm is 20m because 2,000 ÷ 100 = 20.

c. 1 cm to 0.02 km. 2000 cm to 0.02 km because 2,000 ÷ 100,000 = 0.02.

d. 2 m to 4,000 m. 2,000 times 2 m is 4,000 m.

e. 5 cm to 100 m. 1 cm represents 20 m, so 5 cm represents 5 · 20 or 100m.

f. 50 cm to 1,000 m. Dividing 1,000 m by 2,000 gives 0.5 m, or 50 cm.

g. 10 mm to 20 m. 1 cm represents 20 m and 1 cm is 10 mm, so 10 mm represents 20 m.

3. a. What is the area of the large flag?

The large flag is 396 m by 264 m, so its area is 104,544 m².

b. What is the area of the smaller flag?

The small flag is 19.8 cm by 13.2 cm, so its area is 261.36 cm².

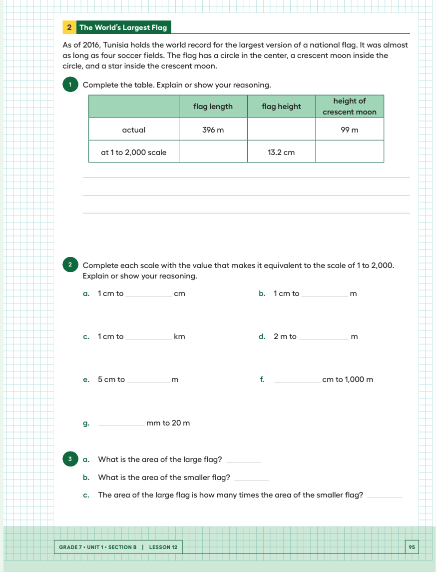
c. The area of the large flag is how many times the area of the smaller flag?

4,000,000 times. Sample reasoning: The scale factor for the height is 2,000 and the scale factor for the length is 2,000, so the area of the actual flag is 2,000 · 2,000, or 4,000,000 times the area of the scale drawing.

Activity Synthesis

Select a few students with differing solution paths to share their responses to the first question. Record and display their reasoning for all to see. Highlight two different ways for dealing with unit conversions. For example, in finding scaled lengths, one can either first convert the actual length in meters to centimeters and then multiply by $\frac{1}{2,000}$, or multiply by $\frac{1}{2,000}$ first, and then convert the quotient to centimeters.

Student Workbook


2 The World's Largest Flag

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

	flag length	flag height	height of crescent moon
actual	396 m		99 m
at 1 to 2,000 scale		13.2 cm	

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

a. 1 cm to ______ cm
b. 1 cm to ______ m
c. 1 cm to ______ km
d. 2 m to ______ m
e. 5 cm to ______ m
f. ______ cm to 1,000 m
g. ______ mm to 20 m

3. a. What is the area of the large flag? ______
b. What is the area of the smaller flag? ______
c. The area of the large flag is how many times the area of the smaller flag? ______

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the first question (explaining how they completed the table). Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Instructional Routines

MLR3: Critique, Correct, Clarify

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Instructional Routines

Notice and Wonder

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Access for Multilingual Learners (Activity 3, Narrative)

MLR3: Critique, Correct, and Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Poll the class on their responses for the second question. If there is disagreement about any of the answers, invite a few students to share their reasoning. Emphasize that all of the scales are equivalent because in each scale, a factor of 2,000 relates scaled distances to actual distances. Reiterate the fact that a scale does not have to be expressed in terms of 1 scaled unit, as is shown in the last three sub-questions, but that 1 is often chosen because it makes the scale factor easier to see and can make calculations more efficient.

If time permits, help students recognize that the areas of the two flags are related by a factor of 4,000,000. Both the length and the height of the large flag are 2,000 times the corresponding side of the small flag, so the area of the large flag is $2{,}000 \cdot 2{,}000$ times the area of the small flag. Another way to look at it is that there are 10,000 square centimeters in a square meter, so in square centimeters, the area of the large flag is 1,045,440,000. Dividing this by the area of the small flag in square centimeters, 261.36, also gives 4,000,000.

Activity 3: Optional

Pondering Pools

15 min

Activity Narrative

In this activity, students use the floor plan of a recreation center in their student workbooks to calculate the area of two swimming pools. This helps reinforce the relationship that the area of an actual region is related to its area on a scale drawing by the $(\text{scale factor})^2$.

Previously, whenever students were asked to use a scale drawing to calculate the area of an actual region, they were able to find the dimensions of the actual region as an intermediate step. Each time, students were prompted to notice the $(\text{scale factor})^2$ pattern. Some students may have already become comfortable using this relationship to calculate the actual area directly from the scaled area, without needing to calculate the actual dimensions as an intermediate step.

The purpose of this activity is to help all students internalize this more efficient method. The question about the rectangular pool can be solved either way, but for the question about the kidney-shaped pool, students must rely on the relationship between scaled area and actual area. As students apply the $(\text{scale factor})^2$ pattern they have seen in other activities to determine the scaled area, they are making use of structure.

As students work, monitor for the different ways that students find the area of the large rectangular pool, such as:

- By first finding the actual side lengths of the pool in meters and then multiplying them.
- By calculating the scaled area in square centimeters and multiplying that area by 25 (or 5^2).

Launch

Ask students to look at the floor plan of a recreation center in their student workbooks. Ask them to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Students may notice:

- There are three different swimming pools on the floor plan.
- This floor plan has more details than others they have worked with previously, such as stairs and doors.
- It looks like it could be a scale drawing, but there is no scale given.

Some may wonder:

- What is the scale of this drawing?
- How deep are these pools?
- Where is this recreation center located?
- How much does it cost to use these pools?

Provide access to centimeter rulers. Give students 4–5 minutes of quiet work time, followed by whole-class discussion.

Student Task Statement

Look at the floor plan of a recreation center on the previous page.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.

   1 to 500 and 1 cm to 5 m (or equivalent)

   Sample reasoning: The side length of the square pool is 15 m. On the drawing, the side length measures 3 cm. The scale is 3 cm to 15 m, or 1 cm to 5 m. There are 500 cm in 5 m.

2. Find the actual area of the large rectangular pool. Show your reasoning.

   About 412.5 m^2

   Sample reasonings:

   - In the drawing, the pool is about 5.5 cm by 3 cm. The pool's actual measurements are 27.5 m by 15 m. Its area is 412.5 m^2, because $(27.5) \cdot 15 = 412.5$.
   - In the drawing, the pool is about 5.5 cm by 3 cm, so its area is about 16.5 cm^2. If 1 cm represents 5 m, then 1 cm^2 is 25 m^2 in actual area, so the area is 412.5 m^2, because $(16.5) \cdot 25 = 412.5$.

3. The kidney-shaped pool has an area of 3.2 cm^2 on the drawing. What is its actual area? Explain or show your reasoning.

   About 80 m^2

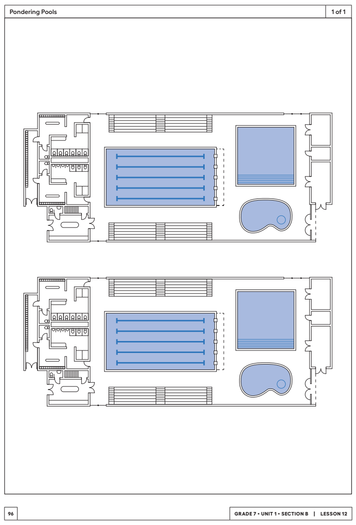
   Sample reasoning: 1 cm^2 represents 25 m^2 so 3.2 cm^2 represents 80 m^2 because $(3.2) \cdot 25 = 80$.

Building on Student Thinking

Students may multiply the scaled area by 5 instead of by 5^2. Remind them to consider what 1 *square* centimeter represents, rather than what 1 centimeter represents.

Students may think that the last question cannot be answered because not enough information is given. Encourage them to revisit their previous work regarding how scaled area relates to actual area.

Student Workbook



Student Workbook

3 Pondering Pools

Look at the floor plan of a recreation center on the previous page.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.
2. Find the actual area of the large rectangular pool. Show your reasoning.
3. The kidney-shaped pool has an area of 3.2 cm^2 on the drawing. What is its actual area? Explain or show your reasoning.

Are You Ready for More?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 $units^2$, what is the area of Square B?
2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 $units^3$, what is the volume of Cube B?
3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the "volume" of Hypercube A is 10 $units^4$, what do you think the "volume" of Hypercube B is?

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Are You Ready for More?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 units^2, what is the area of Square B? $\frac{10}{2^2}$
2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 units^3, what is the volume of Cube B? $\frac{10}{2^3}$
3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the “volume” of Hypercube A is 10 units^4, what do you think the “volume” of Hypercube B is?

   The answer to this depends on what it means to scale a hypercube in 4 dimensions! Assuming the pattern we see in 2 and 3 dimensions holds, we might suspect that the answer is $\frac{10}{2^4}$. That might even help us think about how to define scaling in four dimensions. If we use coordinates and think of scaling by a factor of 2 as multiplying all of the coordinates by a factor of 2, then it does, in fact, work the way we think it should based on the pattern in 2 and 3 dimensions.

Activity Synthesis

The goal of this discussion is to help students appreciate the efficiency of using the (scale factor)^2 relationship to calculate the area of an actual region.

Select a few students with differing solution paths to share their responses to the question about the large rectangular pool. Record and display their reasoning for all to see. Highlight the two different strategies described in the activity narrative. Consider asking questions such as:

“Why do the different approaches lead to the same outcome?”

“How is the scale used in each approach?”

“What might be some benefits of using one method over another for finding the actual area?”

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question (about the kidney-shaped pool) by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

“Since the scale is 1 to 500, I multiplied 3.2 times 500. This equals 1,600 so the kidney-sized pool has an actual area of 1,600.”

- Ask,

“What parts of this response are unclear, incorrect, or incomplete?”

- As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.
- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Lesson Synthesis

Share with students

"Today we saw that scales can be expressed in many different ways, including using different units or not using any units."

To review the relationship between scales with units and scales without units, consider asking:

"How can we express the scale '1 inch to 5 miles' without units?"

Since there are 12 inches in a foot and 5,280 feet in a mile, this is the same as 1 inch to 63,360 inches, or 1 to 63,360.

If time allows, consider reviewing how scaling affects area.

"A scale tells us how a distance on a scale drawing corresponds to an actual distance, and it can also tell us how an area on a scale drawing corresponds to an actual area."

"If a map uses the scale '1 inch to 5 miles,' how can we find the actual area of a region represented on the map?"

Find the area on the map in square inches and multiply by 25, because 1 square inch represents 25 square miles.

Lesson Summary

Sometimes scales come with units, and sometimes they don't. For example, a map of Nebraska may have a scale of 1 mm to 1 km. This means that each millimeter of distance on the map represents 1 kilometer of distance in Nebraska. Notice that there are 1,000 millimeters in 1 meter and 1,000 meters in 1 kilometer. This means there are 1,000 · 1,000 or 1,000,000 millimeters in 1 kilometer. So, the same scale without units is 1 to 1,000,000, which means that each unit of distance on the map represents 1,000,000 units of distance in Nebraska. This is true for *any* choice of unit to express the scale of this map.

Sometimes when a scale comes with units, it is useful to rewrite it without units. For example, let's say we have a different map of Rhode Island, and we want to use the two maps to compare the size of Nebraska and Rhode Island. It is important to know if the maps are at the same scale. The scale of the map of Rhode Island is 1 inch to 10 miles. There are 5,280 feet in 1 mile, and 12 inches in 1 foot, so there are 63,360 inches in 1 mile (because 5,280 · 12 = 63,360). Therefore, there are 633,600 inches in 10 miles. The scale of the map of Rhode Island without units is 1 to 633,600. The two maps are not at the same scale, so we should not use these maps to compare the size of Nebraska to the size of Rhode Island.

Here is some information about equal lengths that you may find useful.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

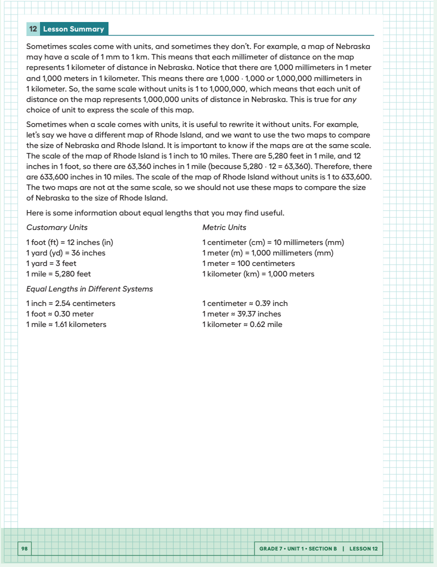
Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers

Metric Units

1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters
1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Student Workbook



Responding To Student Thinking

Points to Emphasize

If students struggle with comparing scales expressed in different units, focus on this when opportunities arise in the next lesson. For example, in this activity, discuss the scales that students chose and how to interchange smaller units of measurement with larger ones:

Unit 1, Lesson 13, Activity 2 Creating a Floor Plan (Part 2)

Cool-down

Drawing the Backyard

5 min

Student Task Statement

Lin and her brother each created a scale drawing of their backyard, but at different scales. Lin used a scale of 1 inch to 1 foot. Her brother used a scale of 1 inch to 1 yard.

1. Express the scales for the drawings without units.

   Lin's scale of 1 inch to 1 foot can be written as 1 to 12. Her brother's scale of 1 inch to 1 yard can be written as 1 to 36.

2. Whose drawing is larger? How many times as large is it? Explain or show your reasoning.

   Lin's drawing is larger.

   Sample reasonings:

   - The lengths on Lin's plan are 3 times the corresponding lengths on her brother's drawing. The area of Lin's drawing is 9 times the area of her brother's drawing.
   - Since 1 yard equals 3 feet, the scale of Lin's brother's drawing is equivalent to 1 inch to 3 feet. Each inch on his drawing represents a longer distance than on Lin's drawing, so his drawing will require less space on paper.
   - At 1 inch to 1 foot, Lin's drawing will show $\frac{1}{12}$ of the actual the distances. At 1 inch to 1 yard, or 1 inch to 3 feet, her brother's drawing will show $\frac{1}{36}$ of the actual distances. Since $\frac{1}{12}$ is larger than $\frac{1}{36}$, Lin's drawing will be larger.

Practice Problems

6 Problems

Problem 1

The Empire State Building in New York City is about 1,450 feet high (including the antenna at the top) and 400 feet wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-inch-by-11-inch piece of paper. Select a scale that you think is the most appropriate for the scale drawing. Explain your reasoning.

A. 1 inch to 1 foot

B. 1 inch to 100 feet

C. 1 inch to 1 mile

D. 1 centimeter to 1 meter

E. 1 centimeter to 50 meters

F. 1 centimeter to 1 kilometer

Sample reasoning: With A, B, and D, the scaled image will not fit on the page. For C and F, the image will be too small. Option E is just right because at 1 cm to 50 m, the height of the building is about 10 cm, and the width is about 3 cm.

Problem 2

Elena finds that the area of a house on a scale drawing is 25 square inches. The actual area of the house is 2,025 square feet. What is the scale of the drawing? 1 inch to 9 feet

Problem 3

Which of these scales are equivalent to 3 cm to 4 km? Select **all** that apply. Recall that 1 inch is 2.54 centimeters.

A. 0.75 cm to 1 km

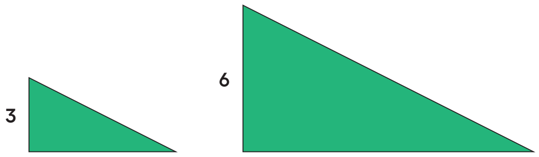
B. 1 cm to 12 km

C. 6 mm to 2 km

D. 0.3 mm to 40 m

E. 1 in to 7.62 km

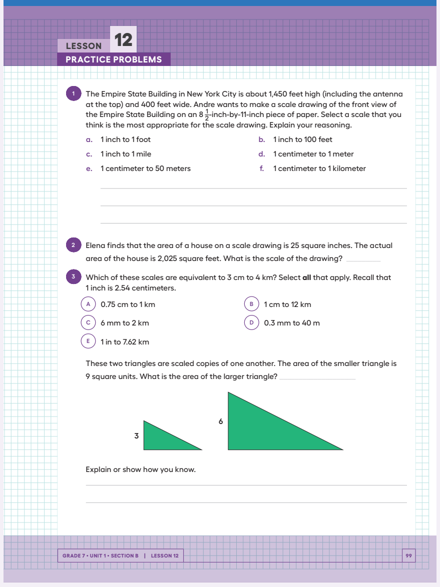
These two triangles are scaled copies of one another. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Explain or show how you know.



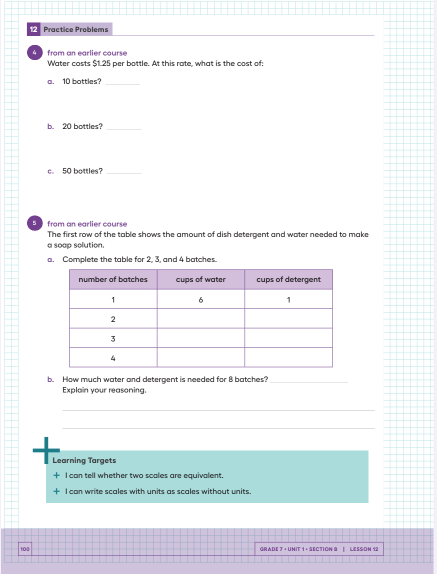
36 square units

When the lengths of a scaled copy are 2 times those of the original figure, the area of the copy is 4 times that of the original area: $4 \cdot 9 = 36$.

Student Workbook



Student Workbook


12 Practice Problems

4 from an earlier course

Water costs $1.25 per bottle. At this rate, what is the cost of:

a. 10 bottles?

b. 20 bottles?

c. 50 bottles?

5 from an earlier course

The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

a. Complete the table for 2, 3, and 4 batches.

number of batches	cups of water	cups of detergent
1	6	1
2		
3		
4		

b. How much water and detergent is needed for 8 batches? Explain your reasoning.

Learning Targets

+ I can tell whether two scales are equivalent.
+ I can write scales with units as scales without units.

Problem 4

from an earlier course

Water costs $1.25 per bottle. At this rate, what is the cost of:

a. 10 bottles?

$12.50, because $10 \cdot 1.25 = 12.5$

b. 20 bottles?

$25, because $20 \cdot 1.25 = 25$

c. 50 bottles?

$62.50, because $50 \cdot 1.25 = 62.5$

Problem 5

from an earlier course

The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

a. Complete the table for 2, 3, and 4 batches.

number of batches	cups of water	cups of detergent
1	6	1
2	12	2
3	18	3
4	24	4

b. How much water and detergent is needed for 8 batches? Explain your reasoning.

48 cups of water and 8 cups of dish detergent.

Sample reasoning: 8 batches is 2 times 4 batches. Doubling 24 gives 48 and doubling 4 gives 8.