

## Negative Exponents with Powers of 10

## Goals

- Describe (orally and in writing) how exponent rules extend to expressions involving negative exponents.
- Describe patterns in repeated multiplication and division with 10 and  $\frac{1}{10}$ , and justify (orally and in writing) that  $10^{-n} = \frac{1}{10^n}$ .

## Learning Targets

- I can use the exponent rules with negative exponents.
- I know what it means if 10 is raised to a negative power.

## Lesson Narrative

In this lesson, students make sense of the rule that  $10^{-n} = \frac{1}{10^n}$  and extend the rules they have developed for working with values with exponents to include situations with negative exponents. This type of reasoning appears again in a later course when students extend the rules of exponents to make sense of exponents that are not integers.

Students develop their fluency as they mentally determine missing values from equations involving powers of 10. Next students complete a table comparing expressions written using exponents with those written as a decimal and as a fraction. The table includes both large values and values less than 1. By looking at the multiplier required to increase or decrease the exponent by 1, students correspond a power of 10 with a negative exponent to repeated multiplication by  $\frac{1}{10}$ .

Students reinforce this understanding by matching a series of expressions written with negative exponents to a corresponding expression written using repeated multiplication.

## Student Learning Goal

Let's see what happens when exponents are negative.

## Lesson Timeline

5  
min

Warm-up

15  
min

Activity 1

15  
min

Activity 2

10  
min

Lesson Synthesis

## Assessment

5  
min

Cool-down

## Instructional Routines

- Math Talk

## Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 2)

## Access for Students with Diverse Abilities

- Representation (Warm-up)
- Engagement (Activity 2)

## Required Preparation

## Activity 2:

Create a visual display (or add to an existing display) for  $10^{-n} = \frac{1}{10^n}$  to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

## Warm-up

## Math Talk: What's That Exponent?

5  
min

## Activity Narrative

This *Math Talk* focuses on strategies and fluency regarding exponents and place value. It encourages students to think about the relative values of powers of 10 and to rely on what they know about exponents to mentally solve problems. The ideas elicited here will be helpful later in the lesson when students investigate negative exponents.

## Launch

Tell students to close their student workbooks or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.
- Keep all previous problems and work displayed throughout the talk.

## Student Task Statement

Find the value of  $x$  mentally.

$$\bullet \frac{100}{1} = 10^x$$

$$x = 2$$

Sample reasoning: Dividing 100 by 1 gives 100, and 100 is equal to  $10^2$ .

$$\bullet \frac{100}{x} = 10^1$$

$$x = 10$$

Sample reasoning:  $10^1$  is equal to 10, and 100 divided by 10 is also equal to 10.

$$\bullet \frac{x}{100} = 10^0$$

$$x = 100$$

Sample reasoning:  $10^0$  is equal to 1, so the left side of the equation must be  $\frac{100}{100}$

$$\bullet \frac{100}{1,000} = 10^x$$

$$x = -1$$

Sample reasoning: On the left side of each of the four equations, 100 is being divided by a larger and larger power of 10. On the right side of each equation, the exponent is decreasing by a value of 1 each time. This pattern would suggest that the missing value of  $x$  is  $-1$ .

## Instructional Routines

## Math Talk

[ilclass.com/r/10694967](https://ilclass.com/r/10694967)  
Please log in to the site  
before using the QR  
code or URL.



## Access for Students with Diverse Abilities (Warm-up, Launch)

## Representation: Internalize Comprehension.

To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

## Student Workbook

## LESSON 5

## Negative Exponents with Powers of 10

Let's see what happens when exponents are negative.

## Warm-up Math Talk: What's That Exponent?

Find the value of  $x$  mentally.

- Ⓐ  $\frac{100}{1} = 10^x$   
Ⓑ  $\frac{100}{x} = 10^1$   
Ⓒ  $\frac{x}{100} = 10^0$   
Ⓓ  $\frac{100}{1,000} = 10^x$

## 1 Negative Exponent Table

	100	10	1	0.1	0.01	0.001
using exponents	$10^2$	$10^1$	$10^0$			
as a decimal	1000.0		1.0		0.01	
as a fraction	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$			$\frac{1}{1000}$

- 1 Complete the table to explore what negative exponents mean.  
2 As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?  
3 How does a multiplier of 10 affect the exponent?

How does it affect the value of the decimal and fraction?

Access for Multilingual Learners  
(Warm-up, Activity Synthesis)

**MLR8: Discussion Supports.**  
Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_ because ...” or “I noticed \_\_\_\_ so I ...” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.  
*Advances: Speaking, Representing*

Activity Synthesis

- To involve more students in the conversation, consider asking:
- “Who can restate \_\_\_\_’s reasoning in a different way?”
  - “Did anyone use the same strategy but would explain it differently?”
  - “Did anyone solve the problem in a different way?”
  - “Does anyone want to add on to \_\_\_\_’s strategy?”
  - “Do you agree or disagree? Why?”
  - “What connections to previous problems do you see?”

Activity 1

Negative Exponent Table

15 min

Activity Narrative

In this activity, students extend their understanding of exponents to include negative exponents and explain patterns involving place value when a value is multiplied by 10 or  $\frac{1}{10}$ . Students use repeated reasoning to recognize that negative powers of 10 represent repeated multiplication by  $\frac{1}{10}$  and generalize to the rule  $10^{-n} = \frac{1}{10^n}$ .

A table is used to show different representations of decimals, fractions, and exponents. The table is horizontal to mimic the structure of decimals and to help students connect this work with place value. Monitor for different strategies used to complete the table.

Launch

- Ask a student to read the first question aloud. Explain that the table will help them see patterns if they complete one row at a time.
- Ask a student to read the second question aloud. Select a student to explain the idea of a “multiplier” in this context (a number that each term is multiplied by to get the next term). Give students 4–5 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. Complete the table to explore what negative exponents mean.

· 10

· 10

· 10

· 10

· 10

· 10

using exponents	$10^3$	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$
as a decimal	1000.0	100.0	10.0	1.0	0.1	0.01	0.001
as a fraction	$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

· ?

· ?

· ?

· ?

· ?

· ?

2. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?

- As you move toward the right, the multiplier is  $\frac{1}{10}$  or 0.1.
3. How does a multiplier of 10 affect the exponent?
- A multiplier of 10 increases the exponent by 1.
- How does it affect the value of the decimal and fraction?
- It increases the value of the decimal and the fraction by a factor of 10.

4. How does the other multiplier affect the exponent?

A multiplier of  $\frac{1}{10}$  (or 0.1) decreases the exponent by 1.

How does it affect the value of the decimal and fraction?

It makes each term 10 times smaller, which is the same as dividing each term by 10.

5. Use the patterns you found in the table to write  $10^{-7}$  as a fraction.

$10^{-7} = \frac{1}{10,000,000}$  or  $\frac{1}{10^7}$

6. Use the patterns you found in the table to write  $10^{-5}$  as a decimal.

$10^{-5} = 0.00001$

7. Write  $\frac{1}{100,000,000}$  using a single exponent.  $\frac{1}{100,000,000} = 10^{-8}$

8. Use the patterns in the table to write  $10^{-n}$  as a fraction.  $10^{-n} = \frac{1}{10^n}$

Student Workbook

LESSON 5

Negative Exponents with Powers of 10

Let's see what happens when exponents are negative.

Warm-up Math Talk: What's That Exponent?

Find the value of  $x$  mentally.

$10^3 = 10^x$

$10^2 = 10^x$

$10^1 = 10^x$

$10^0 = 10^x$

1 Negative Exponent Table

using exponents

as a decimal

as a fraction

$10^3$	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$
1000.0	100.0	10.0	1.0	0.1	0.01	0.001
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

1 Complete the table to explore what negative exponents mean.

2 As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?

3 How does a multiplier of 10 affect the exponent?

How does it affect the value of the decimal and fraction?

GRADE 8 • UNIT 7 • SECTION A | LESSON 5

Student Workbook

1 Negative Exponent Table

1 How does the other multiplier affect the exponent?

How does it affect the value of the decimal and fraction?

2 Use the patterns you found in the table to write  $10^{-7}$  as a fraction.

3 Use the patterns you found in the table to write  $10^{-5}$  as a decimal.

4 Write  $\frac{1}{100,000,000}$  using a single exponent.

5 Use the patterns in the table to write  $10^{-n}$  as a fraction.

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### Activity Synthesis

The goal of this discussion is to reinforce the idea that negative exponents can be thought of as repeated multiplication by  $\frac{1}{10}$ , whereas positive exponents can be thought of as repeated multiplication by 10.

Invite students to share their strategies for completing the table. Some students may describe multiplying by  $\frac{1}{10}$  as multiplying by the reciprocal of 10. In grade 8 the focus is on negative exponents with whole number bases, and use of the word reciprocal is not necessary at this time. Record their reasoning for all to see. Here are some strategies students may use:

- Notice that the exponent decreases by 1 as the table moves to the right.
- Notice that for values with a positive exponent — the corresponding decimal and fraction have 1 less zero as the table moves to the right, and the values are greater than or equal to 1.
- Notice that for values with a negative exponent — the corresponding decimal and fraction have 1 more zero as the table moves to the right, and the values are all less than 1.

Students may also mention the following strategies:

- They “moved the decimal to the right or left.”
- They used the value of the exponent to know how many zeros to write.

Validate student thinking as these observations make sense based on the information given in the table. But emphasize the idea that multiplying by 10 or  $\frac{1}{10}$  increases or decreases the value by a factor of 10, and therefore increases or decreases the place value of the digit 1. This will make it appear that the decimal place is moving, or that the exponent is equal to the number of zeros.

Then introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule  
 $10^{-n} = \frac{1}{10^n}$

Example showing how it works

$$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^3}$$

three factors that  
are one tenth

Continue to reinforce student understanding of this idea by writing out an expanded form of each expression when discussing the following questions:

☞ “What is  $10^{-4}$  written with a positive exponent?”

$$10^{-4} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^4}$$

☞ “What is  $10^{-23}$  written with a positive exponent?”

$$10^{-23} = \frac{1}{10^{23}}$$

☞ “What is  $\frac{1}{100,000}$  written with a negative exponent?”

$$10^{-5}$$

## Activity 2

## Follow the Exponent Rules

15  
min

## Activity Narrative

In this activity students make sense of negative powers of 10 as repeated multiplication by  $\frac{1}{10}$  and use this structure in order to distinguish between equivalent exponential expressions.

## Launch

Arrange students in groups of 2. Give students 7–8 minutes of partner work time followed by a whole-class discussion. Tell students to discuss their reasoning with their partner. If there is disagreement, tell students to work to reach an agreement. If students have time, instruct them to write the other expressions in each table as a power of 10 with a single exponent.

## Student Task Statement

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$	$(10 \cdot 10)$	$(10 \cdot 10)$	$(10 \cdot 10)$
$(10^2)^{-3}$	$(\frac{1}{10} \cdot \frac{1}{10})$	$(\frac{1}{10} \cdot \frac{1}{10})$	$(\frac{1}{10} \cdot \frac{1}{10})$
$(10^{-2})^3$	$\frac{1}{10} \cdot \frac{1}{10}$	$\frac{1}{10} \cdot \frac{1}{10}$	$\frac{1}{10} \cdot \frac{1}{10}$
$(10^{-2})^{-3}$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$		

- b. Write  $(10^2)^{-3}$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

$$(10^2)^{-3} = 10^{2 \cdot (-3)} = 10^{-6} \text{ or } (10^2)^{-3} = \frac{1}{(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)} = \frac{1}{10^6} = 10^{-6}$$

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$	$\frac{1}{10} \cdot \frac{1}{10}$
$\frac{10^2}{10^{-5}}$	$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10^{-2}}{10^5}$	$\frac{1}{10} \cdot \frac{1}{10}$
$\frac{10^{-2}}{10^{-5}}$	$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

- b. Write  $\frac{10^{-2}}{10^{-5}}$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

$$\frac{10^{-2}}{10^{-5}} = 10^{-2 - (-5)} = 10^{-2+5} = 10^3 \text{ or } \frac{10^{-2}}{10^{-5}} = \frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}} = \frac{\frac{1}{10^2}}{\frac{1}{10^5}} = \frac{1}{10^2} \cdot \frac{10^5}{1} = \frac{10^5}{10^2} = 10^3$$

Access for Multilingual Learners  
(Activity 2, Student Task)

## MLR8: Discussion Supports.

Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed \_\_\_\_\_, so I matched ..." Encourage students to challenge each other when they disagree by using the sentence frames "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about expressions with negative exponents.

*Advances: Speaking, Listening*

Access for Students with Diverse Abilities  
(Activity 2, Student Task)

## Engagement: Internalize Self Regulation.

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and make 2 matches in each table.

*Supports accessibility for: Organization, Attention*

## Student Workbook

**2 Follow the Exponent Rules**

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$	$(10^3 \cdot 10) \cdot (10^3 \cdot 10) \cdot (10^3 \cdot 10)$
$(10^2)^{-3}$	$(\frac{1}{10} \cdot \frac{1}{10}) \cdot (\frac{1}{10} \cdot \frac{1}{10}) \cdot (\frac{1}{10} \cdot \frac{1}{10})$
$(10^{-2})^3$	$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$
$(10^{-2})^{-3}$	$(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$

b. Write  $(10^2)^{-3}$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$	$\frac{1}{10} \cdot \frac{1}{10}$
$\frac{10^2}{10^{-5}}$	$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10^{-2}}{10^5}$	$\frac{1}{10} \cdot \frac{1}{10}$
$\frac{10^{-2}}{10^{-5}}$	$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

b. Write  $\frac{10^{-2}}{10^{-5}}$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^3 \cdot 10^2$	$(10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$10^3 \cdot 10^{-2}$	$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (10 \cdot 10 \cdot 10)$
$10^3 \cdot 10^3$	$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (10 \cdot 10 \cdot 10)$
$10^3 \cdot 10^{-3}$	$(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write  $10^3 \cdot 10^3$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

GRADE 8 • UNIT 7 • SECTION A | LESSON 5

## Student Workbook

## 2 Follow the Exponent Rules

## Are You Ready for More?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game. Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, "I knew I'd be the only one left, since I went first."

- Record this game on paper a few times with different numbers of players. Does the person who starts always win?
- Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

## 5 Lesson Summary

In this lesson, we observed that when we multiply a positive power of 10 by  $\frac{1}{10}$ , the exponent decreases by 1. For example,  $10^3 \cdot \frac{1}{10} = 10^2$ . This is true for any power of 10. By using the rule  $10^a \cdot 10^b = 10^{a+b}$  with this example, we see that  $10^3 \cdot 10^{-1} = 10^2$ . Notice that for the exponent rules we have developed to work, then  $\frac{1}{10}$  must equal  $10^{-1}$ .

Rule  
 $10^a \cdot \frac{1}{10} = 10^{a-1}$

Example showing how it works  
 $10^3 = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot 10^6$   
three factors that are one tenth

3.a. Match each exponential expression with an equivalent multiplication expression:

$$\begin{array}{ll}
 10^4 \cdot 10^3 & (10 \cdot 10 \cdot 10 \cdot 10) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \\
 10^4 \cdot 10^{-3} & \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \\
 10^{-4} \cdot 10^3 & \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10) \\
 10^{-4} \cdot 10^{-3} & (10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)
 \end{array}$$

b. Write  $10^{-4} \cdot 10^3$  as a power of 10 with a single exponent. Be prepared to explain your reasoning.

$$10^{-4} \cdot 10^3 = 10^{-4+3} = 10^{-1} \text{ or } 10^{-4} \cdot 10^3 = \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10) = \frac{10^3}{10^4} = \frac{1}{10} = 10^{-1}$$

## Are You Ready for More?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, "I knew I'd be the only one left, since I went first."

- Record this game on paper a few times with different numbers of players. Does the person who starts always win?

**No. If you play with five players, for example, the fifth person will win.**

- Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

**The person who starts will win if the number of people is a power of two. Otherwise, someone else will win.**

## Activity Synthesis

The goal of this discussion is for students to understand that the exponent rules work even with negative exponents by making a clear connection between the exponent rules and the process of multiplying repeated factors that are 10 and  $\frac{1}{10}$ .

Display the expressions  $(10^{-2})^3$  and  $(10^2)^{-3}$  from the first problem for all to see. Ask students what is the same and different about these two expressions. (They are both equivalent to  $10^{-6}$ . Both expressions contain one positive exponent and one negative exponent.)

Reinforce students' understanding of the exponent rules by writing out an expanded form of each expression when discussing the following questions:

☞ "What do the 3 and -2 in  $(10^{-2})^3$  mean in terms of repeated multiplication?"

**The 3 means that there are 3 factors that are each  $10^{-2}$ , and the -2 means that there are 2 factors that are  $\frac{1}{10}$ .**

$$\text{So } (10^{-2})^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 10^{-6}.$$

☞ "What do the 2 and -3 in  $(10^2)^{-3}$  mean in terms of repeated multiplication?"

**The 2 means that there are 2 factors that are each 10, and the -3 means that there are 3 factors that are each  $\frac{1}{10 \cdot 10}$ .**

$$\text{So } (10^2)^{-3} = \left(\frac{1}{10 \cdot 10}\right) \left(\frac{1}{10 \cdot 10}\right) \left(\frac{1}{10 \cdot 10}\right) = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 10^{-6}.$$

## Lesson Synthesis

The purpose of this discussion is to show students how negative exponents are a natural part of the decimal place value system.

Remind students how it is possible to write very large numbers in a very small space because of positional notation. For example, in previous courses we sometimes wrote things like  $456 = 4 \cdot 100 + 5 \cdot 10 + 6 \cdot 1$ . This same number can be written with exponents as  $456 = 4 \cdot 10^2 + 5 \cdot 10^1 + 6 \cdot 10^0$ . Ask students:

☞ “How can 2,796 be written as a sum with powers of 10?”

$$2 \cdot 10^3 + 7 \cdot 10^2 + 9 \cdot 10^1 + 6 \cdot 10^0$$

☞ “How can 0.2796 be written as a sum with powers of 10?”

$$2 \cdot 10^{-1} + 7 \cdot 10^{-2} + 9 \cdot 10^{-3} + 6 \cdot 10^{-4}$$

☞ “Think about the meaning of exponents. How is  $10^3$  related to  $10^{-3}$ ?”

Exponents tell us how many factors are being multiplied.  $10^3$  is multiplication by 10 repeated 3 times and  $10^{-3}$  is multiplication by  $\frac{1}{10}$  repeated 3 times.

☞ “Who would need to work with very large numbers? Who would need to work with very small numbers?”

Astronomers might need to work with very large numbers. Biologists, physicists, engineers, and others might need to work with very small numbers.

## Lesson Summary

In this lesson, we observed that when we multiply a positive power of 10 by  $\frac{1}{10}$ , the exponent decreases by 1. For example,  $10^8 \cdot \frac{1}{10} = 10^7$ . This is true for any power of 10.

By using the rule  $10^n \cdot 10^m = 10^{n+m}$  with this example, we see that:  $10^8 \cdot 10^{-1} = 10^7$ .

Notice that for the exponent rules we have developed to work, then  $\frac{1}{10}$  must equal  $10^{-1}$ .

Rule

$$10^{-n} = \frac{1}{10^n}$$

Example showing how it works

$$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^3}$$

three factors that  
are one tenth



## Responding To Student Thinking

## Points to Emphasize

If most students struggle with negative exponents, revisit the connection between positive and negative exponents in:

Unit 7, Lesson 6, Activity 2 What Happens with Zero and Negative Exponents?

## Cool-down

## Negative Exponent True or False

5  
min

## Student Task Statement

Mark each of the following equations as true or false. Explain or show your reasoning.

a.  $10^{-5} = -10^5$

False

Sample reasoning:  $10^{-5} = \frac{1}{100,000}$ , whereas  $-10^5 = -100,000$ .

b.  $(10^2)^{-3} = (10^{-2})^3$

True

Sample reasoning: Both  $(10^2)^{-3}$  and  $(10^{-2})^3$  are equal to  $10^{-6}$ .

c.  $\frac{10^3}{10^{14}} = 10^{-11}$

True

Sample reasoning:  $\frac{10^3}{10^{14}} = 10^{3-14} = 10^{-11}$ .

## Practice Problems

5 Problems

## Problem 1

Write each expression using a single negative exponent.

a.  $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$   $10^{-3}$

b.  $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$   $10^{-7}$

c.  $\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)^2$   $10^{-8}$

d.  $\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)^3$   $10^{-9}$

e.  $(10 \cdot 10 \cdot 10)^{-2}$   $10^{-6}$

## Problem 2

Write each expression as a single power of 10.

a.  $10^{-3} \cdot 10^{-2}$   $10^{-5}$

b.  $10^4 \cdot 10^{-1}$   $10^3$

c.  $\frac{10^5}{10^7}$   $10^{-2}$

d.  $(10^{-4})^5$   $10^{-20}$

e.  $10^{-3} \cdot 10^2$   $10^{-1}$

f.  $\frac{10^{-9}}{10^5}$   $10^{-14}$

## Problem 3

Select **all** of the following that are equivalent to  $\frac{1}{10,000}$ :

A.  $(10,000)^{-1}$

B.  $(-10,000)$

C.  $(100)^{-2}$

D.  $(10)^{-4}$

E.  $(-10)^2$

## Problem 4

from Unit 3, Lesson 2

Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

**Equations:**

$y = 3x$

$\frac{1}{2}x = y$

$y = 3.5x$

$y = \frac{5}{2}x$

**Situations:**

A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.

I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.

A store has a “4 for \$10” sale on hats.

For every 48 apples I pick, my students get 24.

Sample responses:

- $y = 3x$ : For each cup of salt, there are 3 cups of water.
- $\frac{1}{2}x = y$ : For every 2 apples I pick, my students get 1.
- $y = 3.5x$ : The dump truck hauls 3.5 square feet of dirt in each load.
- $y = \frac{5}{2}x$ : Each hat costs \$2.50.

## Student Workbook

LESSON 5

PRACTICE PROBLEMS

1 Write each expression using a single negative exponent.

a.  $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

b.  $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

c.  $\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)^2$

d.  $\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)^3$

e.  $(10 \cdot 10 \cdot 10)^{-2}$

2 Write each expression as a single power of 10.

a.  $10^{-3} \cdot 10^{-2}$

b.  $10^4 \cdot 10^{-1}$

c.  $\frac{10^5}{10^7}$

d.  $(10^{-4})^5$

e.  $10^{-3} \cdot 10^2$

f.  $\frac{10^{-9}}{10^5}$

3 Select all of the following that are equivalent to  $\frac{1}{10,000}$ :

A.  $(10,000)^{-1}$

B.  $(-10,000)$

C.  $(100)^{-2}$

D.  $(10)^{-4}$

E.  $(-10)^2$

4 from Unit 3, Lesson 2

Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

**Equations:**

$y = 3x$

$\frac{1}{2}x = y$

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$y = \frac{5}{2}x$

**Situations:**

A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.

I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.

A store has a “4 for \$10” sale on hats.

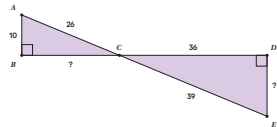
For every 48 apples I pick, my students get 24.

GRADE 5 • UNIT 7 • SECTION A | LESSON 5

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Student Workbook

Practice Problems  
from Unit 2, Lesson 8



a. Explain why triangle  $ABC$  is similar to  $EDC$ .

b. Find the missing side lengths.

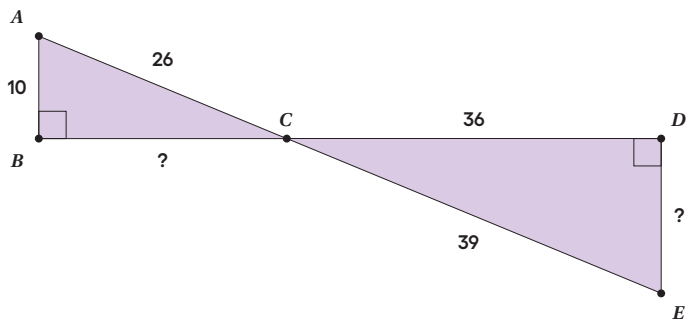


Learning Targets

- + I can use the exponent rules with negative exponents.
- + I know what it means if 10 is raised to a negative power.

Problem 5

from Unit 2, Lesson 8



a. Explain why triangle  $ABC$  is similar to  $EDC$ .

Sample response: Both triangles contain a right angle, and angles  $ACB$  and  $ECD$  are vertical angles. The triangles are similar because two pairs of corresponding angles are congruent.

b. Find the missing side lengths.

Side  $BC$  measures 24, and side  $DE$  measures 15, because the scale factor is  $\frac{39}{26}$  or 1.5