Lesson 1

What Are Scaled Copies?

Goals

- Describe (orally) characteristics of scaled and non-scaled copies.
- Identify scaled copies of a figure and justify (orally and in writing) that the copy is a scaled copy.

Learning Targets

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

Lesson Narrative

This lesson introduces students to the idea of a **scaled copy**, which is a two-dimensional image that has been recreated at a certain scale factor. Students learn to distinguish scaled copies from those that are not—first informally, and later, with increasing precision. They may start by saying that scaled copies have the same shape as the original figure, or that they do not appear to be distorted in any way, though they may have a different size. Next, students notice that the lengths of segments in a scaled copy vary from the lengths in the original figure in a uniform way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. Students work toward articulating the characteristics of scaled copies quantitatively (for example, "all the segments are twice as long," "all the lengths have shrunk by one third," or "all the segments are one-fourth the size of the segments in the original"), articulating the relationships carefully along the way.

The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to make sense of problems and persevere in solving them from the very beginning of the course.

Math Community
This is the first exercise that focuses on the work of building a mathematical community. Students have the opportunity to think about what a mathematical community is and to share their initial thoughts about what it looks like and sounds like to do math together in a community.

Student Learning Goal

Let's explore scaled copies.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Required Materials

Materials to Gather
- Chart paper: Warm-up
- Sticky notes: Warm-up

Materials to Copy
- 6–12 Blank Math Community Chart (1 copy for every 30 students): Warm-up
- Pairs of Scaled Polygons Cards (1 copy for every 2 students): Activity 2

Required Preparation

Warm-up:
Make a space for students to place their sticky notes at the end of the *Warm-up*. For example, hang a sheet of chart paper on a wall near the door.

For the digital version of the activity, acquire devices that can run the applet.

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the blackline master, if possible, copy each complete set on a different color of paper so that a stray slip can quickly be put back.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	10 min	15 min	10 min	5 min
				Cool-down

Warm-up

Printing Portraits

10 min

Activity Narrative

There is a digital version of this activity.

This opening task introduces the term **scaled copy.** It prompts students to observe several copies of a picture, visually distinguish scaled and unscaled copies, and articulate the differences in their own words. Besides allowing students to have a mathematical conversation about properties of figures, it provides an accessible entry into the concept and gives an opportunity to hear the language and ideas that students associate with scaled figures.

Students are likely to have some intuition about the term "to scale," either from previous work in grade 6 (e.g., scaling a recipe, or scaling a quantity up or down on a double number line) or from outside the classroom. This intuition can help them identify scaled copies. As students apply their previous experience with scaling to analyze the images, they are making sense of problems.

Expect them to use adjectives such as "stretched," "squished," "skewed," "reduced," and so on, in imprecise ways. This is fine because students' intuitive definitions of scaled copies will be refined over the course of the lesson. As students discuss the pictures, note the range of descriptions used. Monitor for students whose descriptions are particularly supportive of the idea that lengths in a scaled copy are found by multiplying the original lengths by the same value. Invite them to share their responses later.

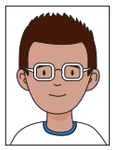
In the digital version of the activity, students use an applet to manipulate copies of the original image. The applet allows students to explore different transformations of the original portrait. This activity works best when each student has access to the applet because students will benefit from seeing the transformations in a dynamic way. If students don't have individual access, displaying the applet for all to see would be helpful during the *Launch* and *Synthesis*.

Launch

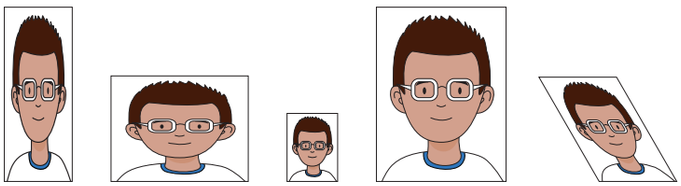
Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and a minute to share their response with their partner.

Student Task Statement

Here is a portrait of a student.



1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?



A B C D E

Sample response:

- Similarities: Pictures A–E are all based on the same original portrait. They all show the same brown-haired boy wearing a blue shirt. They all have the same white background.
- Differences: They each are a different size; some have different shapes. Pictures A, B, and E have been stretched or somehow distorted. Pictures C and D are not stretched or distorted but are each of a different size than the original.

2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

C and D are scaled copies.

Sample reasoning:

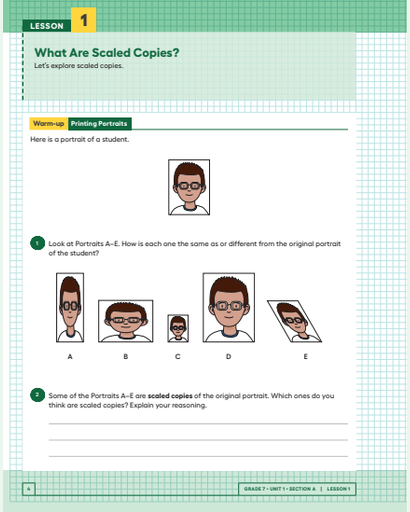
- Pictures A, B, and E are not scaled copies because they have changed in shape compared to the original portrait. Portrait A is stretched vertically, so the vertical side is now much longer than the horizontal side. Picture B is stretched out sideways, so the horizontal sides are now longer than the vertical side. Picture E seems to have its upper-left and lower-right corners stretched out in opposite directions. The portrait is no longer a rectangle.
- Picture C is a smaller copy and Picture D is a larger copy of the original, but their shapes remain the same as the shape of the original.

3. What do you think "scaled copy" means?

Sample responses:

- A scaled copy is a copy of a picture that changes in size but does not change in shape.
- A scaled copy is a duplicate of a picture with no parts of it distorted, though it could be larger, smaller, or the same size.
- A scaled copy is a copy of a picture that has been enlarged or reduced in size but nothing else changes.

Student Workbook


LESSON 1

What Are Scaled Copies?

Let's explore scaled copies.

Warm-up Printing Portraits

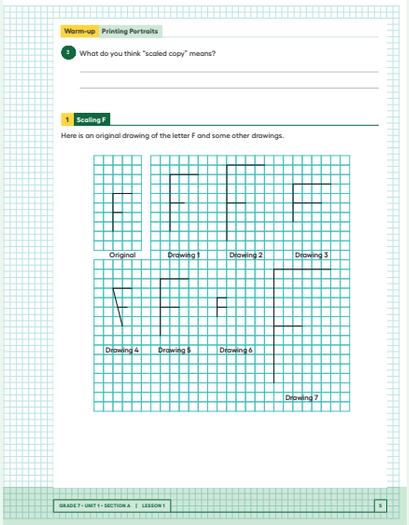
Here is a portrait of a student.

1 Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?

A B C D E

2 Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

Student Workbook


Warm-up Printing Portraits

3 What do you think "scaled copy" means?

1 Scaling F

Here is an original drawing of the letter F and some other drawings.

Original Drawing 1 Drawing 2 Drawing 3

Drawing 4 Drawing 5 Drawing 6 Drawing 7

Activity Synthesis

Select a few students to share their observations. Record and display students' explanations for the second question. Consider organizing the observations in terms of how certain pictures are or are not distorted. For example, students may say that C and D are scaled copies because each is a larger or smaller version of the picture, but the face (or the sleeve, or the outline of the picture) has not changed in shape. They may say that A, B, and E are not scaled copies because something other than size has changed. If not already mentioned in the discussion, guide students in seeing features of C and D that distinguish them from A, B, and E.

Invite a couple of students to share their working definition of scaled copies. Some of the students' descriptions may not be completely accurate. That is appropriate for this lesson because the goal is to build on and refine this language over the course of the next few lessons until students have a more precise notion of what it means for a picture or figure to be a scaled copy.

Math Community

After the *Warm-up*, tell students that today is the start of planning the type of mathematical community they want to be a part of for this school year. The start of this work will take several weeks as the class gets to know one another, reflects on past classroom experiences, and shares their hopes for the year.

Display and read aloud the question "What do you think it should look like and sound like to do math together as a mathematical community?" Give students 2 minutes of quiet think time and then 1–2 minutes to share with a partner. Ask students to record their thoughts on sticky notes and then place the notes on the sheet of chart paper. Thank students for sharing their thoughts and tell them that the sticky notes will be collected into a class chart and used at the start of the next discussion.

After the lesson is complete, review the sticky notes to identify themes. Make a Math Community Chart to display in the classroom. See the blackline master Blank Math Community Chart for one way to set up this chart. Depending on resources and wall space, this may look like a chart paper hung on the wall, a regular sheet of paper to display using a document camera, or a digital version that can be projected. Add the identified themes from the students' sticky notes to the student section of the "Doing Math" column of the chart.

Activity 1
Scaling F

10 min

Activity Narrative

There is a digital version of this activity.

The goal of this activity is for students to describe the characteristics of scaled copies more precisely and to refine the meaning of the term. Students observe copies of a line drawing on a grid and notice how the lengths of line segments and the angles formed by them compare to those in the original drawing.

As students identify distinguishing features of the scaled copies and recognize that corresponding parts increase by the *same* scale factor, they are making use of structure.

For the first question, expect students to explain their choices of scaled copies in intuitive, qualitative terms. For the second question, students should begin to distinguish scaled and unscaled copies in more specific and quantifiable ways. If it does not occur to students to look at lengths of segments, suggest they do so.

As students work, monitor for students who notice the following aspects of the figures. Students are not expected to use these mathematical terms at this point, however.

- The original drawing of the letter F and its scaled copies have equivalent width-to-height ratios.
- We can use a scale factor (or a multiplier) to compare the lengths of different figures and see if they are scaled copies of the original.
- The original figure and scaled copies have corresponding angles that have the same measure.

In the digital version of the activity, students use an applet to create a scaled copy of the original letter F. The applet allows students to draw segments on a grid. The digital version may help students draw quickly and accurately so they can focus more on the mathematical analysis of corresponding lengths.

Launch

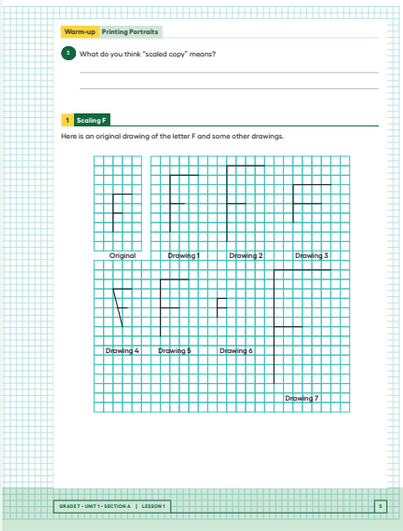
Keep students in the same groups. Give them 3–4 minutes of quiet work time, and then 1–2 minutes to share their responses with their partner. Tell students that how they decide whether each of the drawings is a scaled copy may be very different than how their partner decides. Encourage students to listen carefully to each other's approach and to be prepared to share their strategies.

Building on Student Thinking

Students may make decisions by "eyeballing" rather than observing side lengths and angles. Encourage them to look for quantifiable evidence and notice lengths and angles.

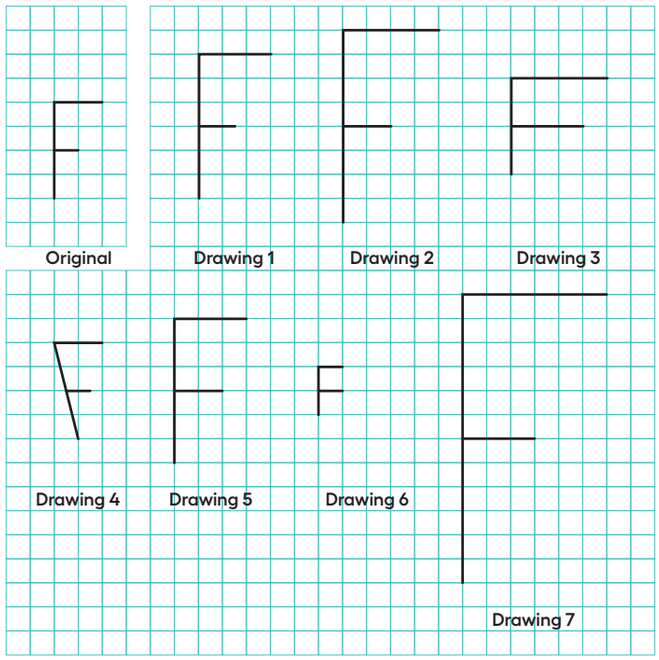
Some may think vertices must land at intersections of grid lines (for example, they may say Drawing 4 is not a scaled copy because the endpoints of the shorter horizontal segment are not on grid crossings). Address this during the whole-class discussion, after students have a chance to share their observations about segment lengths.

Student Workbook



Student Task Statement

Here is an original drawing of the letter F and some other drawings.



1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.

   Drawings 1, 2, and 7 are scaled copies of the original drawing. Sample reasoning: Drawings 1, 2, and 7 are not stretched differently in one direction. They are enlarged evenly in both vertical and horizontal directions.

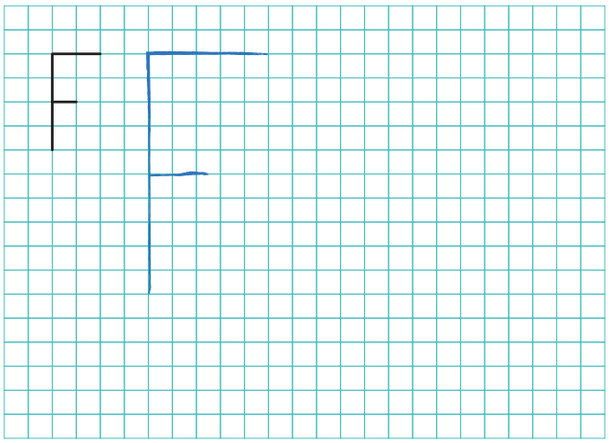
2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?

   Sample responses:

   - In the scaled copies, every segment is the same number of times as long as the matching segment in the original drawing.
   - In the scaled copies, all segments keep the same relationships as in the original. The original drawing of F is 4 units tall. Its top horizontal segment is 2 units wide and the shorter horizontal segment is 1 unit. In Drawing 1, the F is 6 units tall and 3 units wide. In Drawing 2, it is 8 units tall and 4 units wide. In Drawing 7, it is 8 units tall and 4 units wide. In each scaled copy, the width is half of the height, just as in the original drawing of F, and the shorter horizontal segment is half of the longer one.

3. On the grid, draw a different scaled copy of the original letter F.

Sample response:



Activity Synthesis

Display the seven copies of the letter F for all to see. For each copy, ask students to indicate whether they think it is a scaled copy of the original F. Record and display the results for all to see. For any drawing where there is disagreement, ask 1–2 students to share why they think that drawing is not a scaled copy.

To help students generalize about the characteristics of scaled copies, consider asking questions such as:

- *"What features do the scaled copies have in common?"*

  The sides all change by the same amount. The angles stay the same.

- *"How do the other copies fail to show these features?"*

  Some lengths in the copy get bigger or smaller by different amounts. Sometimes the angles in the copy do not match the angles in the original.

If there is a misconception that scaled copies must have vertices on intersections of grid lines, use Drawing 1 (or a relevant drawing by a student) to discuss how that is not the case.

Some students may not be familiar with words such as "twice," "double," or "triple." Clarify the meanings of these words by also saying "two times as long" or "three times as long."

Activity 2

Pairs of Scaled Polygons

15 min

Activity Narrative

In this partner activity, students take turns matching pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as "It looks like ..." "That could/couldn't be true because ..." "One thing that is the same is ..." or "One thing that is different is ..."

Supports accessibility for: Language, Social-Emotional Functioning

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.

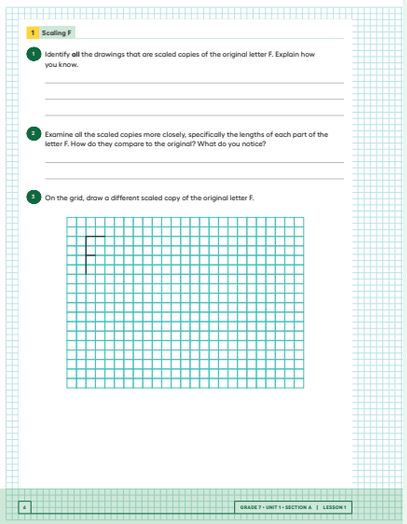
Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.

Student Workbook


1 Scaling F

1 Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.

2 Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?

3 On the grid, draw a different scaled copy of the original letter F.

Access for Students for Diverse Abilities (Activity 2, Launch)

Representation: Internalize Comprehension.
Use multiple examples and non-examples to emphasize relationships between scaled copies.

Supports accessibility for: Conceptual Processing, Attention

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.
Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "_____ matches _____ because ..." and "I noticed _____, so I matched ..."

Advances: Speaking, Conversing

Building on Student Thinking

Some students may think a figure has more than one match. Remind them that there is only one scaled copy for each polygon and ask them to recheck all the side lengths.

Some students may think that vertices must land at intersections of grid lines and conclude that, for example, G cannot be a copy of F because not all vertices on F are on such intersections. Ask them to consider how a 1-unit-long segment would change if scaled to be half its original size. Where must one or both of its vertices land?

As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. Monitor for students who use precise language to articulate their reasoning (for example, "The top side of A is half the length of the top side of G, but the vertical sides of A are a third of the lengths of those in G.").

Launch

Choose a student to be your partner and demonstrate how to set up and do the matching activity:

- Mix up the cards and place them face-up.
- Select two cards and then explain to your partner why you think the cards do or do not match (in other words, whether the polygons are or are not scaled copies).
- The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement. Consider demonstrating productive ways to agree or disagree, such as explaining your mathematical thinking, asking clarifying questions, etc.
- When both partners agree on the match, they switch roles.

Arrange students in groups of 2. Give each group a set of 10 slips cut from the blackline master. Tell students that each polygon has one and only one match. Ask students to take turns finding scaled copies and explaining their thinking.

Student Task Statement

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a polygon with another polygon that is a scaled copy.
   - **a.** For each match you find, explain to your partner how you know it's a match.
   - **b.** For each match your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

   The following polygons are scaled versions of one another:

   - A and C
   - B and D
   - E and I
   - F and G
   - H and J
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

   No written response required.
3. Select one pair of polygons to examine further. Explain or show how you know that one polygon is a scaled copy of the other. You can draw both polygons on the grid if it helps.

   Sample reasoning: For A and C, all the side lengths in C are twice as long as the lengths of the matching sides in A.

Are You Ready for More?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

It's impossible to draw a polygon that is a scaled copy of both Polygon A and Polygon B.

Sample reasonings:

- For a scaled copy of A, all the side lengths would be the same number of times larger or smaller than A, but they won't be the same number of times larger or smaller than B.
- A and B are not scaled copies of each other, so a scaled copy of one polygon will not be a scaled copy of the other.

Activity Synthesis

The purpose of this discussion is to draw out concrete methods for deciding whether or not two polygons are scaled copies of one another, and in particular, to understand that just "eyeballing" to see whether they look roughly the same is not enough to determine that they are scaled copies.

Display the image of all the polygons. Ask students to share their pairings and guide a discussion about how students went about finding the scaled copies. Ask questions such as:

- *"When you looked at another polygon, what exactly did you check or look for?"*

  General shape, side lengths

- *"How many sides did you compare before you decided that the polygon was or was not a scaled copy?"*

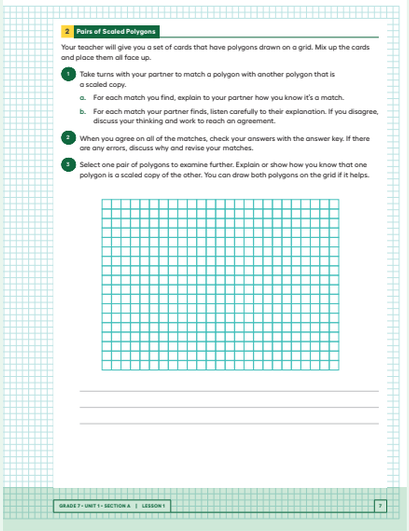
  One pair of sides can be enough to tell that polygons are not scaled copies; all pairs of sides need to be checked to make sure a polygon is a scaled copy.

- *"Did anyone check the angles of the polygons? Why or why not?"*

  No; the sides of the polygons all follow grid lines.

If students do not agree about some pairings after the discussion, ask the groups to explain their case and discuss which of the pairings is correct. Highlight the use of quantitative descriptors such as "half as long" or "three times as long" in the discussion. Ensure that students see that when a figure is a scaled copy of another, all of its segments are the same number of times as long as the corresponding segments in the other.

Student Workbook

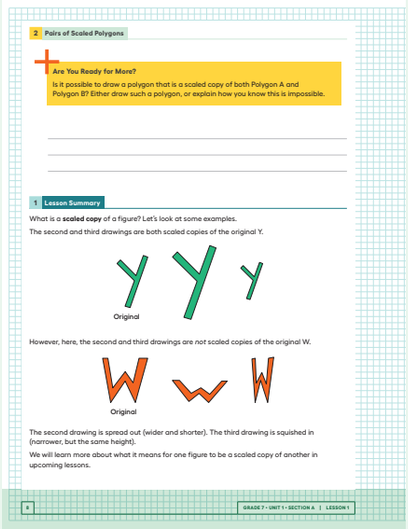

2 Pairs of Scaled Polygons

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a polygon with another polygon that is a scaled copy.
   a. For each match you find, explain to your partner how you know it's a match.
   b. For each match your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. Select one pair of polygons to examine further. Explain or show how you know that one polygon is a scaled copy of the other. You can draw both polygons on the grid if it helps.

GRADE 7 • UNIT 1 • SECTION A | LESSON 1 7

Student Workbook


2 Pairs of Scaled Polygons

Are You Ready for More?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

1 Lesson Summary

What is a **scaled copy** of a figure? Let's look at some examples.

The second and third drawings are both scaled copies of the original Y.

Original

However, here, the second and third drawings are *not* scaled copies of the original W.

Original

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

6 GRADE 7 • UNIT 1 • SECTION A | LESSON 1

Lesson Synthesis

Share with students

"*Today we examined copies of a figure. Some were scaled copies and some were not.*"

To help students refine their definition of what a "scaled copy" is, consider asking students:

"*What is a scaled copy?*"

A scaled copy looks the same but is a different size.

"*What are some characteristics of scaled copies? How are they different from figures that are not scaled copies?*"

A scaled copy is larger or smaller than the original by the same amount in every direction. A copy that is not a scaled copy has been stretched or squished to change the overall shape.

"*What specific information did you look for when determining if something was a scaled copy of an original?*"

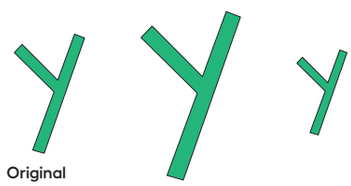
Side lengths and angles

While initial answers need not be particularly precise at this stage of the unit (for example, "scaled copies look the same but are a different size"), guide the discussion toward making careful statements that one could test. The lengths of segments in a scaled copy are related to the lengths in the original figure in a consistent way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. We might say, "All the segments are twice as long," or "All the segments are one-third the size of the segments in the original."

Lesson Summary

What is a **scaled copy** of a figure? Let's look at some examples.

The second and third drawings are both scaled copies of the original Y.



However, here, the second and third drawings are *not* scaled copies of the original W.

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

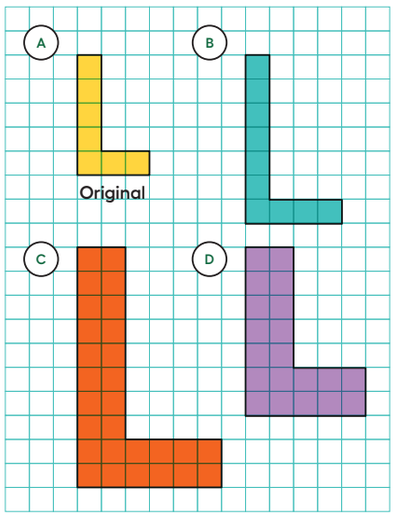
Cool-down

Scaling L

5 min

Student Task Statement

Are any of the figures B, C, or D scaled copies of figure A? Explain how you know.



Only figure C is a scaled copy of figure A.

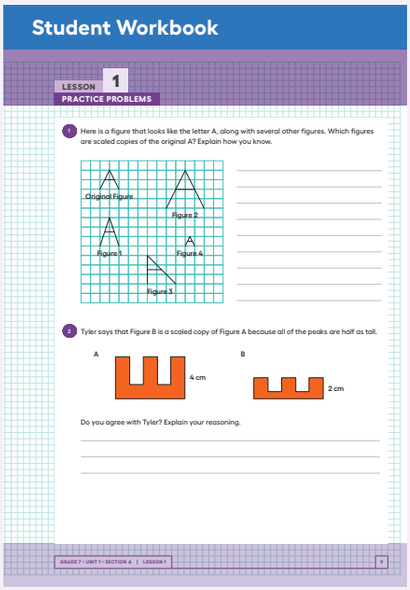
Sample reasoning: In figure C, the length of each segment of the letter L is twice the length of the matching segment in A. Figures B and D are not enlarged evenly. In B, some segments increase and some stay the same. In D, some segments are double in length and some are not.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

4 Problems


Student Workbook

LESSON 1
PRACTICE PROBLEMS

1 Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.

Original Figure
Figure 1
Figure 2
Figure 3
Figure 4

2 Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

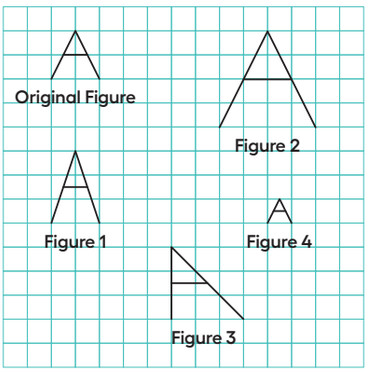
A
4 cm
B
2 cm

Do you agree with Tyler? Explain your reasoning.

GRADE 7 • UNIT 1 • SECTION A | LESSON 1

Problem 1

Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



Figures 2 and 4 are scaled copies.

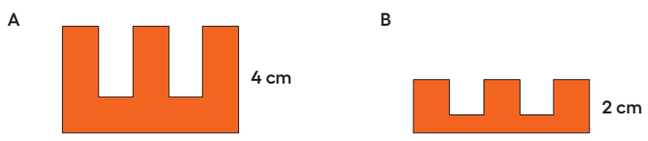
Sample reasonings:

- The original A fits inside a square. The horizontal segment is halfway up the height of the square. The tip of the A is at the midpoint of the horizontal side of the square.
- Figure 1 inside a rectangle, not a square, so it is not a scaled copy. Figure 3 fits inside a square but the shape is different from the original letter A, because one of the legs of the A in Figure 3 is now vertical, so it also is not a scaled copy.
- Figure 2 is twice as high and twice as wide as the original A, and Figure 4 is half as tall and half as wide, but in both figures the locations of the horizontal segment and the tip of the letter A still match the original.

Problem 2

Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

Do you agree with Tyler? Explain your reasoning.

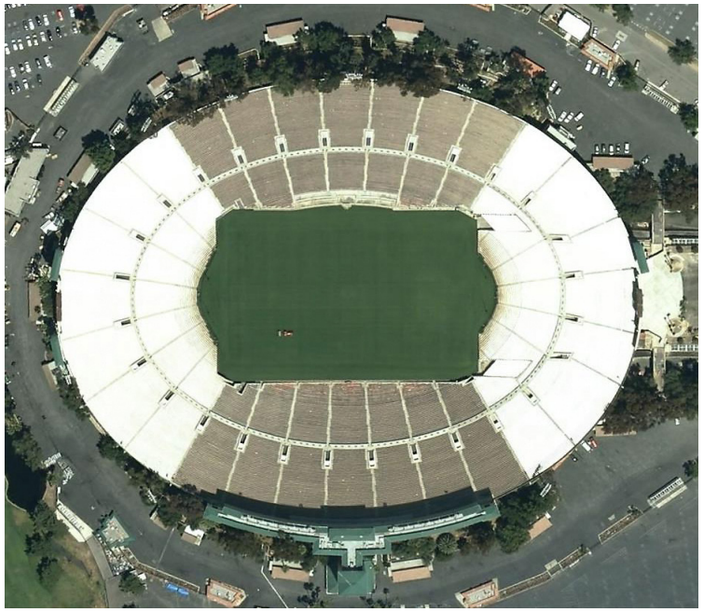


No

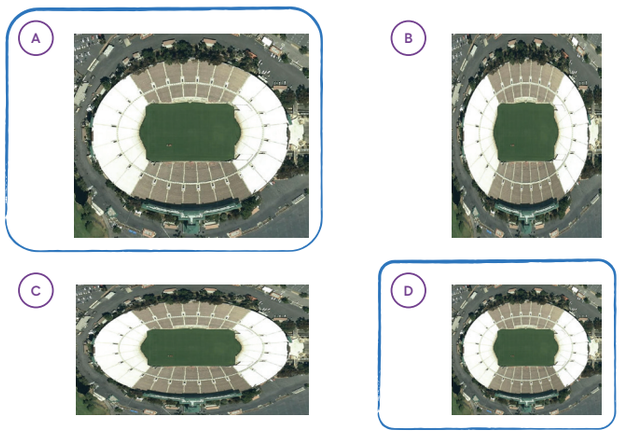
For the smaller figure to be a scaled copy, the figure would have to be half as wide as well.

Problem 3

Here is a picture of the Rose Bowl Stadium in Pasadena, CA.



Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.



Problem 4

from an earlier course

Complete each equation with a number that makes it true.

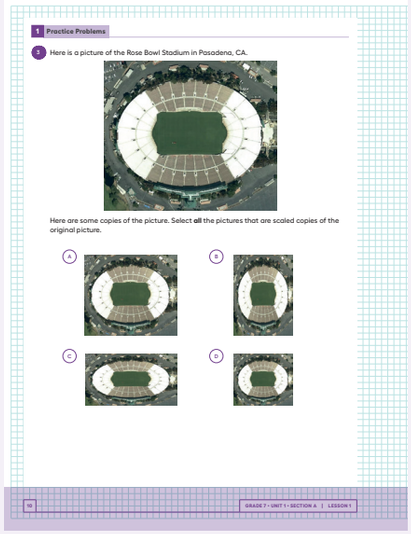
a. $5 \cdot \underline{3} = 15$

b. $4 \cdot \underline{8} = 32$

c. $6 \cdot \underline{1.5}$ (or equivalent) $= 9$

d. $12 \cdot \underline{0.25}$ (or equivalent) $= 3$

Student Workbook



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