Lesson 5

Reasoning about Equations and Tape Diagrams (Part 2)

Goals

- Coordinate tape diagrams, equations of the form $p(x + q) = r$, and verbal descriptions of the situations.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form $p(x + q) = r$.
- Interpret (in writing) the solution to an equation in the context of the situation it represents.

Learning Targets

- I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

Lesson Narrative

This lesson parallels the previous one, except the focus is on situations that lead to equations of the form $p(x + q) = r$. Tape diagrams are used to help students understand why these situations can be represented with equations of this form, and to help them reason about solving equations of this form. Students also attend to the meaning of the equation's solution in the context. Note that we are not generalizing solution methods yet; just using diagrams as a tool to reason about solving equations.

Student Learning Goal

Let's use tape diagrams to help answer questions about situations where the equation has parentheses.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8 (Warm-up)

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	5 min

Warm-up

Math Talk: Seeing Structure

10 min

Activity Narrative

This *Math Talk* focuses on analyzing a specific equation structure, $p(x + q) = r$, with an eye toward options for solving such equations. It encourages students to notice when r is divisible by p, and to rely on the structure of expressions in this form to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students solve equations of this form.

To notice that each equation is a template where just two numbers vary, and therefore all the equations have the same solution, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A. $x + 1 = 5$

$x = 4$

Sample reasoning: $5 - 1 = 4$.

B. $2(x + 1) = 10$

$x = 4$

Sample reasoning: When x is 1, $2(x + 1) = 4$. When x is 2, $2(x + 1) = 6$. When x is 3, $2(x + 1) = 8$. When x is 4, $2(x + 1) = 10$.

C. $3(x + 1) = 15$

$x = 4$

Sample reasoning: 15 equals 3 times 5. So, $(x + 1)$ must equal 5.

D. $500 = 100(x + 1)$

$x = 4$

Sample reasoning: 500 equals 100 times 5. So, $(x + 1)$ must equal 5.

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

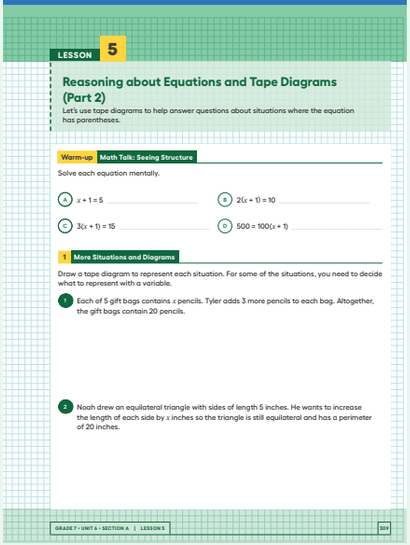
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Task Statement)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook


LESSON 5

Reasoning about Equations and Tape Diagrams (Part 2)

Let's use tape diagrams to help answer questions about situations where the equation has parentheses.

Warm-up Math Talk: Seeing Structure

Solve each equation mentally.

A. $x + 1 = 5$ B. $2(x + 1) = 10$

C. $3(x + 1) = 15$ D. $500 = 100(x + 1)$

1 More Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Each of 5 gift bags contains x pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.

2. Noah drew an equilateral triangle with sides of length 5 inches. He wants to increase the length of each side by x inches so the triangle is still equilateral and has a perimeter of 20 inches.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity Synthesis

Ask students to identify what all the equations have in common. If it doesn't come up, point out that $x + 1$ must equal 5 in every equation. Therefore, the solution to each equation is $x = 4$.

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Activity 1

More Situations and Diagrams

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students draw a tape diagram to represent each situation. The purpose is to show students that some situations can be represented by an equation of the form $p(x + q) = r$ (or equivalent).

For each question, monitor for one student with a correct diagram. Encourage students to explain what any variables used to label the diagram represent in the situation.

Since students have not been shown how to represent situations like these with a tape diagram, they have to do a notable amount of sense-making to complete the task.

In the digital version of the activity, students use an applet to create their tape diagrams. The applet allows students to divide the tape into parts and label the parts and total. The digital version may be helpful for students who struggle with drawing their own diagrams. However, even if the class is using the digital version, some students may prefer to draw the diagrams in their notebooks or on scratch paper.

Launch

Ensure students understand that the work of this task is to draw a tape diagram to represent each situation. There is no requirement to write an equation or solve a problem yet.

Arrange students in groups of 2.

Give 5–10 minutes to work individually or with their partner, followed by a whole-class discussion.

Student Task Statement

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Each of 5 gift bags contains x pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.
2. Noah drew an equilateral triangle with sides of length 5 inches. He wants to increase the length of each side by x inches so the triangle is still equilateral and has a perimeter of 20 inches.
3. An art class charges each student $3 to attend plus a fee for supplies. Today, $20 was collected for the 5 students attending the class.
4. Elena ran 20 miles this week, which was three times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

Sample diagrams:

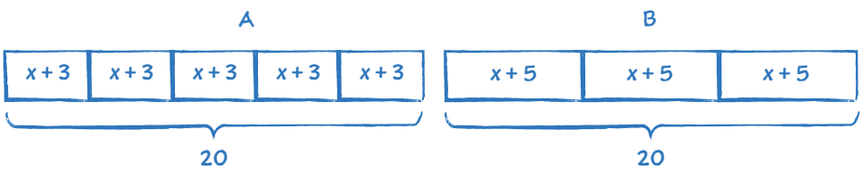


Diagram A corresponds to situations 1 and 3.
Diagram B corresponds to situations 2 and 4.

Activity Synthesis

Select one student for each situation to present their correct diagram. Ensure that students explain the meaning of any variables used to label their diagram.

Possible questions for discussion:

- *"For the situations with no x, how did you decide what quantity to represent with a variable?"*

  Think about which amount is unknown but has a relationship to one or more other amounts in the story.

- *"What does the variable you used to label the diagram represent in the story?"*

  "Did any situations have the same diagrams? How can you tell from the story that the diagrams would be the same?"

  Same number of equal parts, same amount for the total.

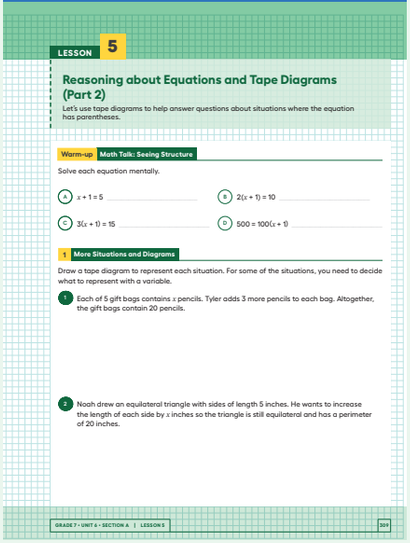
Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Develop Language and Symbols.

Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of modeling with equations. Terms may include: tape diagram, solution, equivalent expressions.

Supports accessibility for: Conceptual Processing, Language

Student Workbook


LESSON 5

Reasoning about Equations and Tape Diagrams (Part 2)

Let's use tape diagrams to help answer questions about situations where the equation has parentheses.

Warm-up Math Talk: Seeing Structure

Solve each equation mentally.

A. $x + 1 = 5$ B. $2(x + 1) = 10$

C. $3(x + 1) = 15$ D. $500 = 100(x + 1)$

1 More Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Each of 5 gift bags contains x pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.

2. Noah drew an equilateral triangle with sides of length 5 inches. He wants to increase the length of each side by x inches so the triangle is still equilateral and has a perimeter of 20 inches.

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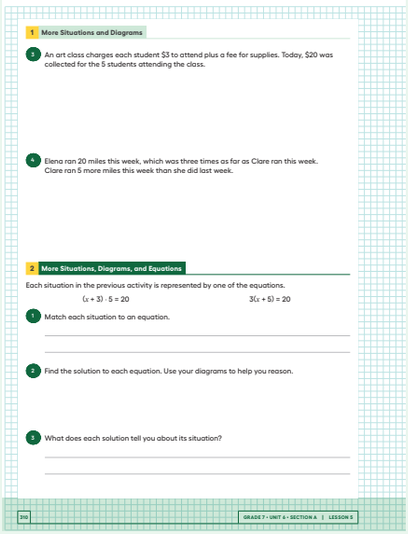
Student Workbook


1 More Situations and Diagrams

3. An art class charges each student $3 to attend plus a fee for supplies. Today, $20 was collected for the 5 students attending the class.

4. Elena ran 20 miles this week, which was three times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

2 More Situations, Diagrams, and Equations

Each situation in the previous activity is represented by one of the equations.

$(x + 3) \cdot 5 = 20$ $3(x + 5) = 20$

1. Match each situation to an equation.

2. Find the solution to each equation. Use your diagrams to help you reason.

3. What does each solution tell you about its situation?

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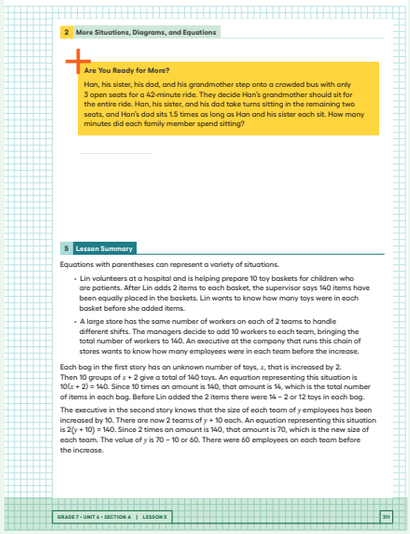
Student Workbook



2 More Situations, Diagrams, and Equations

Are You Ready for More?

Han, his sister, his dad, and his grandmother step onto a crowded bus with only 3 open seats for a 42-minute ride. They decide Han's grandmother should sit for the entire ride. Han, his sister, and his dad take turns sitting in the remaining two seats, and Han's dad sits 1.5 times as long as Han and his sister each sit. How many minutes did each family member spend sitting?

3 Lesson Summary

Equations with parentheses can represent a variety of situations.

- Lin volunteers at a hospital and is helping prepare 10 toy baskets for children who are patients. After Lin adds 2 items to each basket, the supervisor says 140 items have been equally placed in the baskets. Lin wants to know how many toys were in each basket before she added items.
- A large store has the same number of workers on each of 2 teams to handle different shifts. The managers decide to add 10 workers to each team, bringing the total number of workers to 140. An executive at the company that runs this chain of stores wants to know how many employees were in each team before the increase.

Each bag in the first story has an unknown number of toys, x, that is increased by 2. Then 10 groups of $x + 2$ give a total of 140 toys. An equation representing this situation is $10(x + 2) = 140$. Since 10 times an amount is 140, that amount is 14, which is the total number of items in each bag. Before Lin added the 2 items there were $14 - 2$ or 12 toys in each bag.

The executive in the second story knows that the size of each team of y employees has been increased by 10. There are now 2 teams of $y + 10$ each. An equation representing this situation is $2(y + 10) = 140$. Since 2 times an amount is 140, that amount is 70, which is the new size of each team. The value of y is $70 - 10$ or 60. There were 60 employees on each team before the increase.

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Activity 2

More Situations, Diagrams, and Equations

10 min

Activity Narrative

This activity is a continuation of the previous one. Students match each situation from the previous activity with an equation, solve the equation by any method that makes sense to them, and interpret the meaning of the solution. Students are still using any method that makes sense to them to reason about a solution.

For each equation, monitor for a student using their diagram to reason about the solution and a student using the structure of the equation to reason about the solution.

Students reason abstractly and quantitatively when they attend to the meaning of the equation's solution in the context.

Launch

Keep students in the same groups.

Give 5 minutes to work individually or with a partner, followed by a whole-class discussion.

Student Task Statement

Each situation in the previous activity is represented by one of the equations.

$$(x + 3) \cdot 5 = 20 \qquad 3(x + 5) = 2$$

1. Match each situation to an equation.
   - $(x + 3) \cdot 5 = 20$: Situations 1 (gift bags) and 3 (art class)
   - $3(x + 5) = 20$: Situations 2 (triangle perimeter) and 4 (miles run)
2. Find the solution to each equation. Use your diagrams to help you reason.
   - $(x + 3) \cdot 5 = 20$: $x = 1$
   - $3(x + 5) = 20$: $x = 1\frac{2}{3}$ (or equivalent)
3. What does each solution tell you about its situation?
   a. There was originally 1 pencil in each bag.
   b. Noah increased the length of each side by $1\frac{2}{3}$ inches.
   c. The fee for supplies is $1.
   d. Clare ran $1\frac{2}{3}$ miles last week.

Are You Ready for More?

Han, his sister, his dad, and his grandmother step onto a crowded bus with only 3 open seats for a 42-minute ride. They decide Han's grandmother should sit for the entire ride. Han, his sister, and his dad take turns sitting in the remaining two seats, and Han's dad sits 1.5 times as long as Han and his sister each sit. How many minutes did each family member spend sitting?

Han's grandmother: 42, Han's dad: 36, Han: 24, Han's sister: 24

Activity Synthesis

The purpose of this discussion is to connect solving moves on the tape diagram with solving moves on the equation.

Display the first situation next to its matching diagram and equation. Ask one student who reasoned with the diagram and one who reasoned only about the equation to explain their solutions. Point out or demonstrate connections between the two representations and solving moves. For example, "I divided the number of gift bags by 5, leaving me with 4 pencils per gift bag. Since Tyler added 3 pencils to each gift bag, there must have been 1 pencil in each gift bag to start," can be shown on a tape diagram and on a corresponding equation. It is not necessary to invoke the more abstract language of "doing the same thing to each side" of an equation yet.

Lesson Synthesis

Display one of the situations from the lesson and its corresponding equation. Ask students to explain:

- *"What does each number and letter in the equation represent in the situation?"*
- *"What is the reason for each operation (multiplication or addition) used in the equation?"*
- *"What is the solution to the equation? What does it mean to be a solution to an equation? What does the solution represent in the situation?"*

Lesson Summary

Equations with parentheses can represent a variety of situations.

- Lin volunteers at a hospital and is helping prepare 10 toy baskets for children who are patients. After Lin adds 2 items to each basket, the supervisor says 140 items have been equally placed in the baskets. Lin wants to know how many toys were in each basket before she added items.
- A large store has the same number of workers on each of 2 teams to handle different shifts. The managers decide to add 10 workers to each team, bringing the total number of workers to 140. An executive at the company that runs this chain of stores wants to know how many employees were in each team before the increase.

Each bag in the first story has an unknown number of toys, x, that is increased by 2. Then 10 groups of $x + 2$ give a total of 140 toys. An equation representing this situation is $10(x + 2) = 140$. Since 10 times an amount is 140, that amount is 14, which is the total number of items in each bag. Before Lin added the 2 items there were $14 - 2$ or 12 toys in each bag.

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Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: " I noticed _____ so I ...", "_____ corresponds to _____.", "It looks like _____ represents ...", or "How did you get ...?"

Supports accessibility for: Language, Social-Emotional Functioning

Responding To Student Thinking

Points to Emphasize

If most students struggle finding the solution to the equation or show possible misconceptions about tape diagrams, focus on inviting students to explain the connection between tape diagrams and equations in this activity: Grade 7, Unit 6, Lesson 6, Activity 2 Even More Situations, Diagrams, and Equations

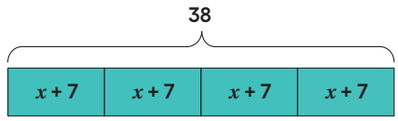
Cool-down

More Finding Solutions

5 min

Student Task Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$4(x + 7) = 38$

$x = 2\frac{1}{2}$

Sample reasoning: The tape diagram has 4 equal pieces, each of which represents $\frac{38}{4}$ (or $9\frac{1}{2}$). $x + 7 = 9\frac{1}{2}$, so x must be $2\frac{1}{2}$.

Lesson 5

Practice Problems

6 Problems

Problem 1

from Unit 4, Lesson 2

Here are some prices customers paid for different items at a farmer's market. Find the cost for 1 pound of each item.

a. \$5 for 4 pounds of apples

\$1.25

b. \$3.50 for $\frac{1}{2}$ pound of cheese

\$7

c. \$8.25 for $1\frac{1}{2}$ pounds of coffee beans

\$5.50

d. \$6.75 for $\frac{3}{4}$ pounds of fudge

\$9

e. \$5.50 for a $6\frac{1}{4}$ pound pumpkin

\$0.88

Problem 2

from Unit 5, Lesson 9

Find the products.

a. $\frac{2}{3} \cdot \left(\frac{-4}{5}\right)$

$\frac{-8}{15}$

b. $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right)$

-1

c. $\left(\frac{-2}{39}\right) \cdot 39$

-2

d. $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right)$

$\frac{-3}{10}$ (or equivalent)

Problem 3

Here are two stories:

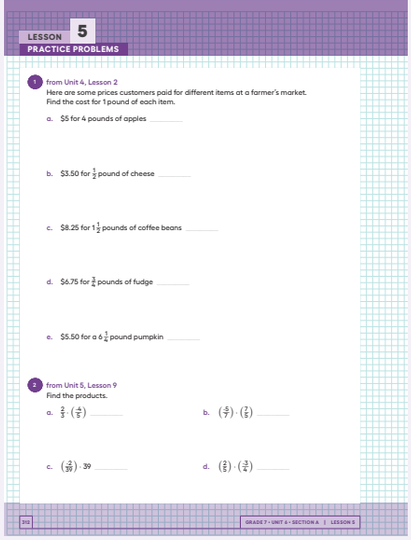
A. A family buys 6 tickets to a show. They also *each* spend \$3 on a snack. They spend a total of \$24 for tickets and snacks.

B. Diego has 24 ounces of juice. He pours equal amounts for each of his 3 friends, and then adds 6 more ounces for each.

Here are two equations:

$3(x + 6) = 24$ $\qquad$ $6(x + 3) = 24$

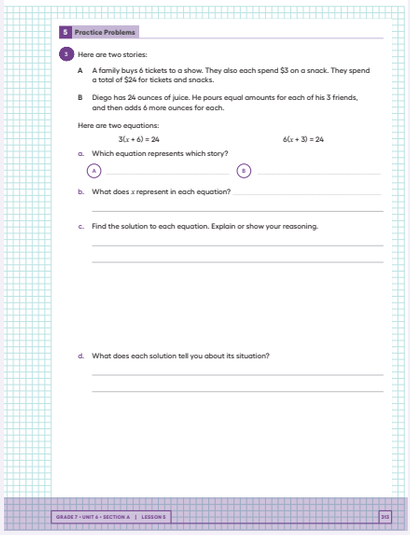
Student Workbook


LESSON 5
PRACTICE PROBLEMS

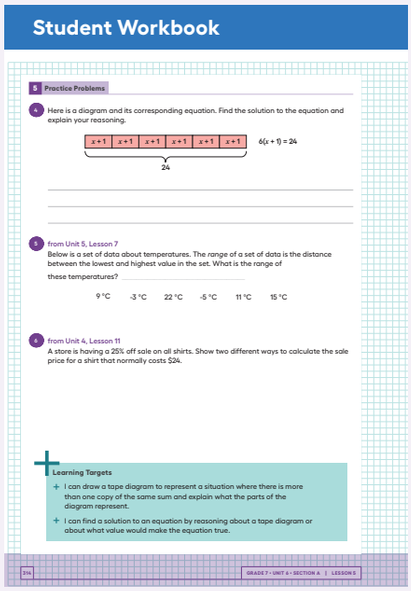
1 from Unit 4, Lesson 2
Here are some prices customers paid for different items at a farmer's market. Find the cost for 1 pound of each item.
a. \$5 for 4 pounds of apples
b. \$3.50 for $\frac{1}{2}$ pound of cheese
c. \$8.25 for $1\frac{1}{2}$ pounds of coffee beans
d. \$6.75 for $\frac{3}{4}$ pounds of fudge
e. \$5.50 for a $6\frac{1}{4}$ pound pumpkin

2 from Unit 5, Lesson 9
Find the products.
a. $\frac{2}{3} \cdot \left(\frac{4}{5}\right)$
b. $\left(\frac{4}{7}\right) \cdot \left(\frac{7}{5}\right)$
c. $\left(\frac{2}{39}\right) \cdot 39$
d. $\left(\frac{2}{5}\right) \cdot \left(\frac{3}{4}\right)$

Student Workbook


5 Practice Problems

3 Here are two stories:
A A family buys 6 tickets to a show. They also each spend \$3 on a snack. They spend a total of \$24 for tickets and snacks.
B Diego has 24 ounces of juice. He pours equal amounts for each of his 3 friends, and then adds 6 more ounces for each.
Here are two equations:
$3(x + 6) = 24$ $\qquad$ $6(x + 3) = 24$
a. Which equation represents which story?
A ____ B ____
b. What does x represent in each equation?
c. Find the solution to each equation. Explain or show your reasoning.
d. What does each solution tell you about its situation?


Student Workbook

5 Practice Problems

4 Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.

x + 1 | x + 1 | x + 1 | x + 1 | x + 1 | x + 1 — 6(x + 1) = 24

24

5 from Unit 5, Lesson 7
Below is a set of data about temperatures. The *range* of a set of data is the distance between the lowest and highest value in the set. What is the range of these temperatures?

9 °C -3 °C 22 °C -5 °C 11 °C 15 °C

6 from Unit 4, Lesson 11
A store is having a 25% off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs $24.

Learning Targets
+ I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent.
+ I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

a. Which equation represents which story?

Story A: Family at the show: $6(x + 3) = 24$

Story B: Diego's juice: $3(x + 6) = 24$

b. What does x represent in each equation?

Family at the show: x represents the cost of a ticket.
Diego's juice: x represents the number of ounces of juice Diego originally poured for each friend.

c. Find the solution to each equation. Explain or show your reasoning.

Story A: $6(x + 3) = 24$: $x = 1$

Story B: $3(x + 6) = 24$: $x = 2$

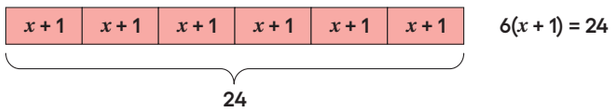
d. What does each solution tell you about its situation?

Story A: Each ticket to the show costs $1.

Story B: Diego originally poured 2 ounces of juice for each friend.

Problem 4

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$x = 3$

Sample reasoning: In the tape diagram, there are 6 units of $x + 1$ that make 24, so $x + 1$ must be $24 \div 6$, which is 4. Since $x + 1 = 4$, $x = 3$.

Problem 5

from Unit 5, Lesson 7

Below is a set of data about temperatures. The *range* of a set of data is the distance between the lowest and highest value in the set. What is the range of these temperatures?

9 °C -3 °C 22 °C -5 °C 11 °C 15 °C

27 degrees Celsius

Problem 6

from Unit 4, Lesson 11

A store is having a 25% off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs $24.

Sample responses:

- $(0.25) \cdot 24 = 6$, and $24 - 6 = 18$. (Find 25% of $24 and subtract this from $24.)
- $1 - 0.25 = 0.75$, and $(0.75) \cdot 24 = 18$. (Find 75% of $24.)
- $24 \div 4 = 6$, and $3 \cdot 6 = 18$. (Find 25% of $24 and multiply this by 3.)