Lesson 10

Calculating Slope

Goal

Generate a method to find slope values given two points on the line.

Learning Target

I can calculate positive and negative slopes given two points on the line.

Lesson Narrative

In this lesson, students extend their work with slope triangles to develop a method for finding the slope of any line given the coordinates of two points on the line. Students begin by finding values that make given equations true, specifically reviewing integer operations that involve negative numbers.

Next, students find the slope of a line given a sketch and the coordinates of two points on that line. They generalize how to find slope for any two points and observe that this generalization holds true for both positive and negative slopes.

Finally, students consider a context that can be represented by a line with a negative slope and they calculate the slope from a graph and interpret the slope in the context.

Student Learning Goal

Let's calculate slope from two points.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR1: Stronger and Clearer Each Time

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Warm-up

Math Talk: Integer Operations

5 min

Activity Narrative

This *Math Talk* focuses on operations with integers. It encourages students to think about positive and negative numbers to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students use two points to calculate the slope of a line with a negative slope.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Mentally find values for a and b that make each equation true.

A. $a + b = -2$

Sample responses: $a = 3, b = -5$; $a = -10, b = 8$

B. $a - b = -2$

Sample responses: $a = 6, b = 8$; $a = -11, b = -9$

C. $\frac{a}{b} = 2$

Sample responses: $a = 4, b = 2$; $a = -20, b = -10$

D. $\frac{a}{b} = -2$

Sample responses: $a = -8, b = 4$; $a = 100, b = -50$

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Instructional Routines

Math Talk

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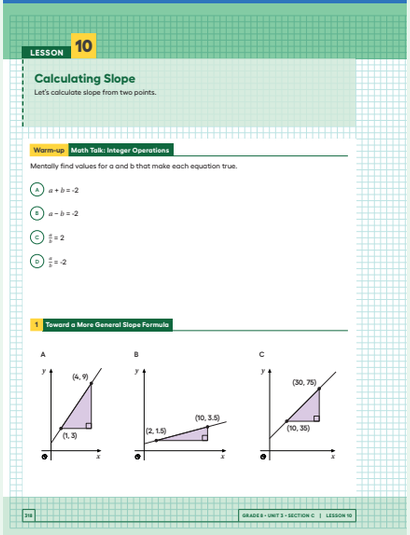
Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 10

Calculating Slope

Let's calculate slope from two points.

Warm-up Math Talk: Integer Operations

Mentally find values for a and b that make each equation true.

A. $a + b = -2$

B. $a - b = -2$

C. $\frac{a}{b} = 2$

D. $\frac{a}{b} = -2$

1 Toward a More General Slope Formula

A B C

(4, 9) (1, 3) (10, 3.5) (2, 1.5) (30, 75) (10, 35)

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 1)

MLR1: Stronger and Clearer Each Time.

This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of slope. Terms may include "pair of points," and "in the same order."
Supports accessibility for: Conceptual Processing, Language

Building on Student Thinking

If students calculate a slope and leave it in a form such as $\frac{-3}{-5}$, consider asking:

- What does the slope $\frac{-3}{-5}$ mean in context?
- How is the slope $\frac{-3}{-5}$ different from $\frac{3}{5}$?

Activity 1

Toward a More General Slope Formula

20 min

Activity Narrative

The purpose of this activity is for students to develop a quick method or formula to calculate the slope of a line based on the coordinates of two points. After students compute the slopes of three specific lines, they generalize the procedure by observing that they can calculate the vertical and horizontal side lengths of the slope triangle without a grid (eventually without even drawing the slope triangle) and that the slope is the quotient of these side lengths. It is not critical or even recommended to use the traditional formula with subscripts. It is more important that students know a technique or way of thinking about slope that works for them than it is that they memorize a particular way to express a formula.

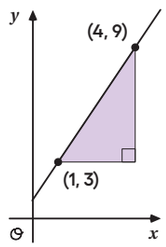
In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

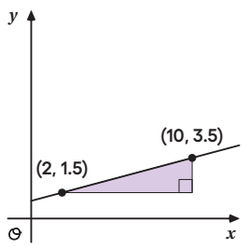
Arrange students in groups of 2. Give them 5–6 minutes to calculate the slopes of the three lines in the first problem and write a draft for the second problem, followed by a whole-class discussion.

Student Task Statement

A



B



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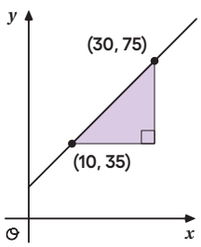


1. For each graph, record:

	vertical change	horizontal change	slope
A	6	3	2
B	2	8	$\frac{1}{4}$
C	40	20	2

2. Describe a procedure for finding the slope between any two points on a line.

Sample response: Subtract the x-coordinates then subtract the y-coordinates in the same order. Divide the difference of the y's by the difference of the x's.

Are You Ready for More?

Find the value of k so that the line passing through each pair of points has the given slope.

1. $(k, 2)$ and $(11, 14)$, slope = 2

   5

2. $(1, k)$ and $(4, 1)$, slope = -2

   7

3. $(3, 5)$ and $(k, 9)$, slope = $\frac{1}{2}$

   11

4. $(-1, 4)$ and $(-3, k)$, slope = $\frac{-1}{2}$

   5

5. $\left(\frac{-15}{2}, \frac{3}{16}\right)$ and $\left(\frac{-13}{22}, k\right)$, slope = 0

   $\frac{3}{16}$

Activity Synthesis

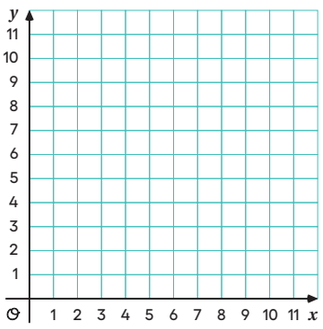
Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their procedure for finding the slope between any two points on a line from the second question. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. Have students take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

Then close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

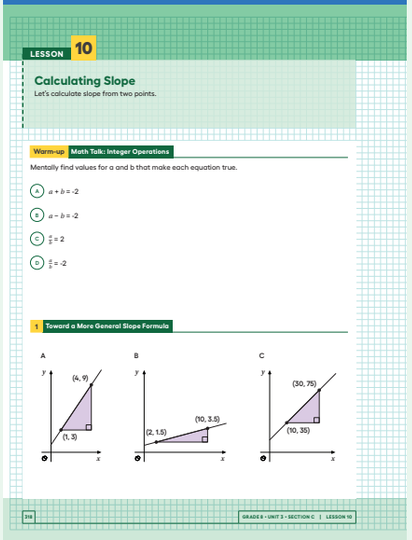
If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After *Stronger and Clearer Each Time,* display the coordinates (1, 11) and (8, 2) for all to see and ask students to predict without calculating, whether the slope of the line that goes through these 2 points will be positive or negative.

Then display the graph below and plot both points. Draw the line that passes through the points and ask students again to predict whether the slope of the line is positive or negative. Invite students who predicted correctly before seeing the graph to share their strategies and record them for all to see.



Student Workbook


LESSON 10

Calculating Slope

Let's calculate slope from two points.

Warm-up Math Talk: Integer Operations

Mentally find values for a and b that make each equation true.

A. $a + b = -2$

B. $a - b = -2$

C. $\frac{a}{b} = 2$

D. $\frac{a}{b} = -2$

1 Toward a More General Slope Formula

A B C

(4, 9) (1, 3) (10, 3.5) (2, 1.5) (30, 75) (10, 35)

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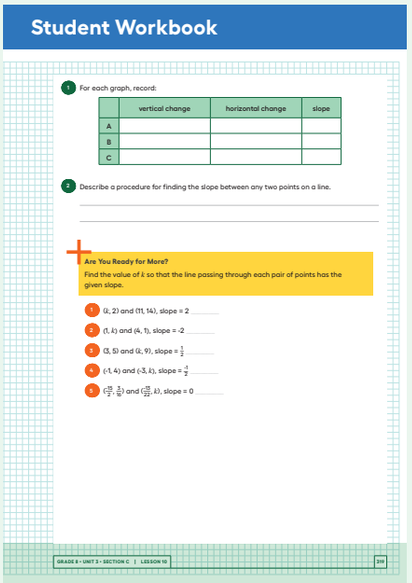
Student Workbook


1 For each graph, record:

	vertical change	horizontal change	slope
A			
B			
C			

2 Describe a procedure for finding the slope between any two points on a line.

Are You Ready for More?

Find the value of k so that the line passing through each pair of points has the given slope.

1. $(k, 2)$ and $(11, 14)$, slope = 2
2. $(1, k)$ and $(4, 1)$, slope = -2
3. $(3, 5)$ and $(k, 9)$, slope = $\frac{1}{2}$
4. $(-1, 4)$ and $(-3, k)$, slope = $\frac{-1}{2}$
5. $\left(\frac{-15}{2}, \frac{3}{16}\right)$ and $\left(\frac{-13}{22}, k\right)$, slope = 0

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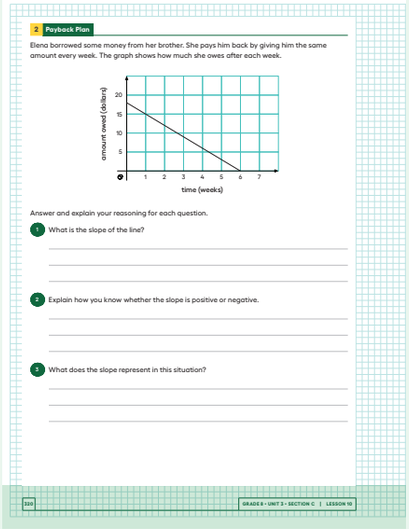
Student Workbook


2 Payback Plan

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.

amount owed (dollars)

time (weeks)

Answer and explain your reasoning for each question.

1. What is the slope of the line?
2. Explain how you know whether the slope is positive or negative.
3. What does the slope represent in this situation?

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Draw a slope triangle connecting the points (1, 11) and (8, 2) and use it to calculate the slope of the line ($\frac{-9}{7}$). Emphasize that since the line is going downhill from left to right, the slope must be negative. Discuss whether or not the procedure they described for calculating slope will work when the slope of the line is negative. (Yes.) Demonstrate how the slope can be calculated using just the coordinates. For example, the vertical change is 11 − 2 = 9 and the horizontal change is 1 − 8 = -7, making the slope $-\frac{9}{7}$.

Activity 2

Payback Plan

10 min

Activity Narrative

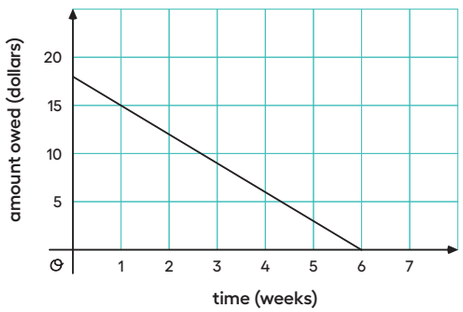
This activity gives students an opportunity to interpret the graph of a line in context, including the meaning of a negative slope and the meaning of the horizontal and vertical intercepts. Students also calculate the slope of a line with a negative slope from a graph.

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time, followed by a partner then whole-class discussion.

Student Task Statement

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.



Answer and explain your reasoning for each question.

1. What is the slope of the line?

The slope is -3.

2. Explain how you know whether the slope is positive or negative.

Sample response: I know the slope is negative because the line is going down from left to right. The amount owed is decreasing as time increases.

3. What does the slope represent in this situation?

Sample responses: The slope represents the amount that Elena pays back her brother each week. The slope represents how much less she owes every time a week goes by.

4. How much did Elena borrow?

   Elena borrowed $18.

5. How much time will it take for Elena to pay back all the money she borrowed?

   It will take Elena 6 weeks to pay back the $18.

Activity Synthesis

The goal of this discussion is to reinforce the ideas of what the slope and intercepts mean in context, with an emphasis on a context with a negative slope. Display the graph from the task for all to see.

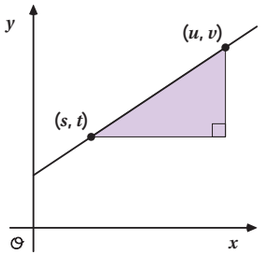
Invite students to share their answers and reasoning for each question, indicating how the graph can be used to find the answers. For example, draw in a slope triangle for the first question, and emphasize how the slope is -3 because the number of dollars she owes decreases over time. Label the vertical intercept for the amount Elena borrowed and the horizontal intercept for the time it takes her to pay back the loan.

Lesson Synthesis

The goal of this discussion is for students to see how the procedure they wrote for finding the slope between any two points on a line can be written using a general formula.

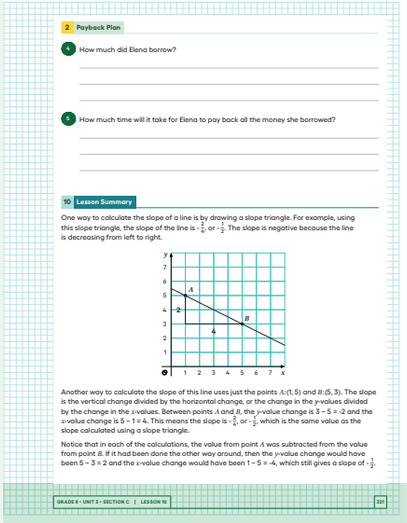
Begin by inviting 1–2 students to share the second draft of their procedure. If necessary, emphasize the importance of subtracting the x-coordinates for the two points in the same order as the y-coordinates.

Then display this image for all to see:



Ask students to use the procedures they wrote for finding slope to find the slope of this line, using the variables s, t, u, and v. Have students share their thinking with a partner before continuing with the whole-class discussion. Record all correct responses next to the graph for all to see, including any equivalent expressions such as $\frac{v-t}{u-s}$ and $(t-v) \div (s-u)$.

Student Workbook


2 Payback Plan

4. How much did Elena borrow?

5. How much time will it take for Elena to pay back all the money she borrowed?

10 Lesson Summary

One way to calculate the slope of a line is by drawing a slope triangle. For example, using this slope triangle, the slope of the line is $-\frac{3}{2}$, or $-1\frac{1}{2}$. The slope is negative because the line is decreasing from left to right.

Another way to calculate the slope of this line uses just the points A (1, 5) and B (5, 3). The slope is the vertical change divided by the horizontal change, or the change in the y-values divided by the change in the x-values. Between points A and B, the y-value change is $3 - 5 = -2$ and the x-value change is $5 - 1 = 4$. This means the slope is $-\frac{2}{4}$, or $-\frac{1}{2}$, which is the same value as the slope calculated using a slope triangle.

Notice that in each of the calculations, the value from point A was subtracted from the value from point B. If it had been done the other way around, then the y-value change would have been $5 - 3 = 2$ and the x-value change would have been $1 - 5 = -4$, which still gives a slope of $-\frac{1}{2}$.

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Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Provide Access by Recruiting Interest.

Invite students to generate a list of additional examples with a negative rate of change that connect to their personal backgrounds and interests.

Supports accessibility for: Conceptual Processing, Attention

Responding To Student Thinking

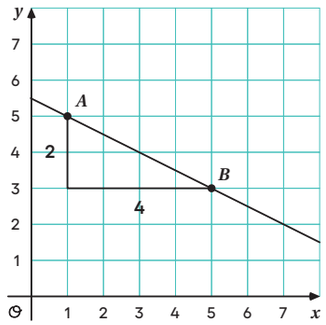
Press Pause

By this point in the unit, there should be some student mastery working with slopes of lines between two points. If most students struggle calculating the slope from two points, make time to revisit related work. For example, using the practice problem referred to here, invite students to explain how they calculated the slope of the graph. The Course Guide provides additional ideas for revisiting earlier work.

Unit 3, Lesson 13, Practice Problem 2

Lesson Summary

One way to calculate the slope of a line is by drawing a slope triangle. For example, using this slope triangle, the slope of the line is $-\frac{2}{4}$, or $-\frac{1}{2}$. The slope is negative because the line is decreasing from left to right.



Another way to calculate the slope of this line uses just the points A: (1, 5) and B: (5, 3). The slope is the vertical change divided by the horizontal change, or the change in the y-values divided by the change in the x-values. Between points A and B, the y-value change is $3 - 5 = -2$ and the x-value change is $5 - 1 = 4$. This means the slope is $-\frac{2}{4}$, or $-\frac{1}{2}$, which is the same value as the slope calculated using a slope triangle.

Notice that in each of the calculations, the value from point A was subtracted from the value from point B. If it had been done the other way around, then the y-value change would have been $5 - 3 = 2$ and the x-value change would have been $1 - 5 = -4$, which still gives a slope of $-\frac{1}{2}$.

Cool-down

Different Slopes

5 min

Student Task Statement

Find the slope of the line that passes through each pair of points.

1. (0, 5) and (8, 2)

$-\frac{3}{8}$ (or equivalent)

2. (2, -1) and (6, 1)

$\frac{1}{2}$ (or equivalent)

3. (-3, -2) and (-1, -5)

$-\frac{3}{2}$ (or equivalent)

Lesson 10

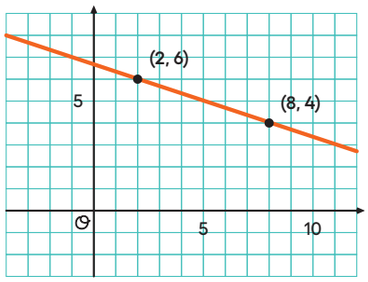
Practice Problems

3 Problems

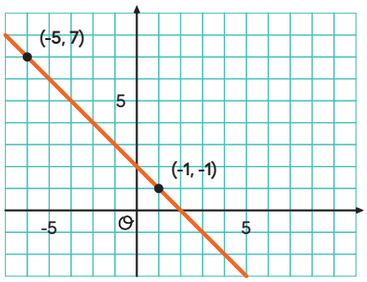
Problem 1

For each graph, calculate the slope of the line.

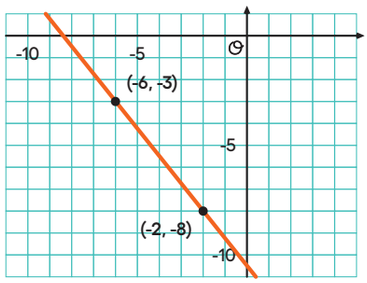
A



B



C



- A: $-\frac{1}{3}$ (or equivalent)
- B: -1 (or equivalent)
- C: $-\frac{5}{4}$ (or equivalent)

Problem 2

Match each pair of points with the slope of the line that passes through each pair.

A. (9, 10) and (7, 2)

B. (-8, -11) and (-1, -5)

C. (5, -6) and (2, 3)

D. (6, 3) and (5, -1)

E. (4, 7) and (6, 2)

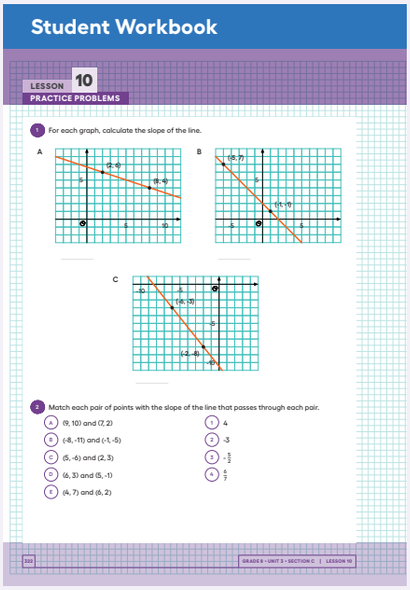
1. 4

2. -3

3. $-\frac{5}{2}$

4. $\frac{6}{7}$

Student Workbook


LESSON 10
PRACTICE PROBLEMS

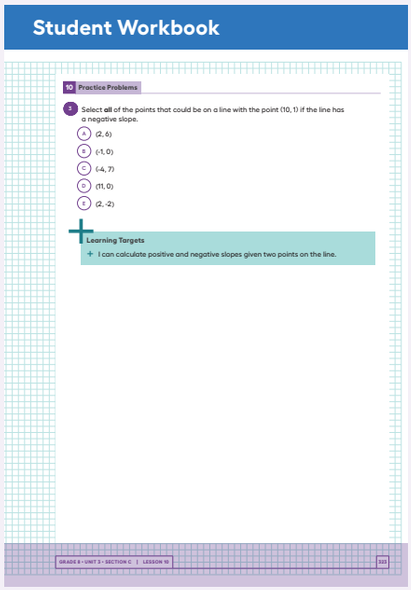
1 For each graph, calculate the slope of the line.

A B C

2 Match each pair of points with the slope of the line that passes through each pair.

A (9, 10) and (7, 2)
B (-8, -11) and (-1, -5)
C (5, -6) and (2, 3)
D (6, 3) and (5, -1)
E (4, 7) and (6, 2)

1 4
2 -3
3 $-\frac{5}{2}$
4 $\frac{6}{7}$


Student Workbook

10 Practice Problems

3 Select **all** of the points that could be on a line with the point (10, 1) if the line has a negative slope.

A (2, 6)
B (-1, 0)
C (-4, 7)
D (11, 0)
E (2, -2)

Learning Targets
+ I can calculate positive and negative slopes given two points on the line.

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Problem 3

Select **all** of the points that could be on a line with the point (10, 1) if the line has a negative slope.

A. (2, 6)

B. (-1, 0)

C. (-4, 7)

D. (11, 0)

E. (2, -2)