Lesson 10

Piecewise Linear Functions

Goals

- Calculate the different rates of change of a piecewise linear function using a graph, and interpret (orally and in writing) the rates of change in context.
- Create a model of a nonlinear function using a piecewise linear function, and describe (orally) the benefits of having more or less segments in the model.

Learning Target

I can create graphs of nonlinear functions with pieces of linear functions.

Lesson Narrative

This lesson picks up on the idea planted in the previous lesson about creating linear models for data. Specifically, in some situations where a quantity changes at different constant rates over different time intervals, we can model the situation with a piecewise linear function. Students look at temperature data, which change at different rates throughout the day. The rates for these intervals are almost constant. They also use piecewise linear graphs to find information about the real-life situation they represent. The focus of this lesson is not necessarily to find equations for the piecewise linear functions (though students may choose to do so in some instances), but rather to study the graphs qualitatively and to compute and compare the different rates of change.

The fourth activity is optional. Use this activity to give students additional practice describing a piecewise linear function that represents a context.

Student Learning Goal

Let's explore functions built out of linear pieces.

Access for Students with Diverse Abilities

- Engagement (Activity 3)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR5: Co-Craft Questions (Activity 2)

Instructional Routines

- 5 Practices
- MLR5: Co-Craft Questions
- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	15 min	5 min	10 min	5 min

Instructional Routines

Notice and Wonder

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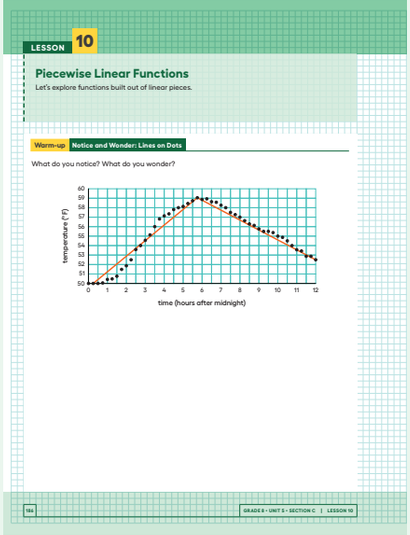
Student Workbook


LESSON 10

Piecewise Linear Functions

Let's explore functions built out of linear pieces.

Warm-up Notice and Wonder: Lines on Dots

What do you notice? What do you wonder?

temperature (°F)

time (hours after midnight)

Warm-up

Notice and Wonder: Lines on Dots

5 min

Activity Narrative

This *Warm-up* prompts students to make sense of a new type of function, a piecewise linear function, before working with it and the mathematics that is involved. Students should notice that the points are not connected and wonder how well the lines model the sections of data they span, which is explored in a later activity. For that reason, the discussion should focus on collecting all the things students notice and wonder but not giving responses that explain the things they wonder about.

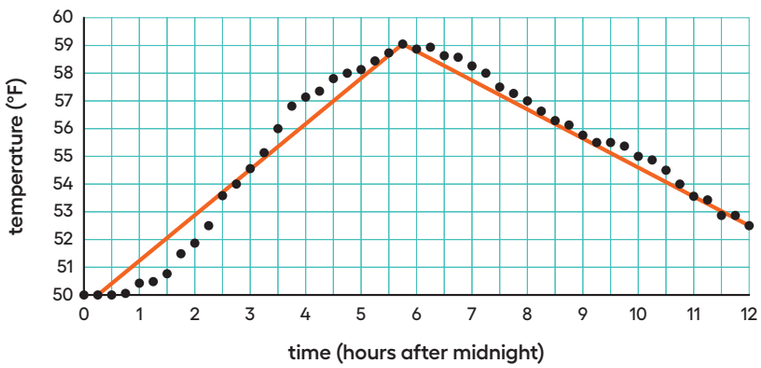
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- Not many of the points are on the blue lines.
- The temperature gets warmer and then cooler as time goes on.
- At about 5:45 a.m., it is the warmest—about 59 degrees.
- The second blue line connects the highest point and the point furthest to the right.

Students may wonder:

- What location does this data represent?
- Why is it warmer at 6:00 a.m. than it is at noon?
- Why aren't the points connected?
- Why is the second line lower than almost all the points?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If no one mentions that the dots are not connected or what they think the blue lines mean, bring these ideas to their attention, and tell them they will be working more with these ideas in the next activity.

Activity 1
Modeling Recycling

10 min

Activity Narrative

In this activity, students work with a graph that clearly cannot be modeled by a single linear function, but pieces of the graph could be reasonably modeled using different linear functions, leading to the introduction of piecewise linear functions. Students find the slopes of their piecewise linear model and interpret them in context.

Monitor for students who use the following numbers of line segments to represent the function, order here from fewest to greatest:

- 3 line segments
- 4 line segments
- 5 line segments

Instructional Routines

5 Practices

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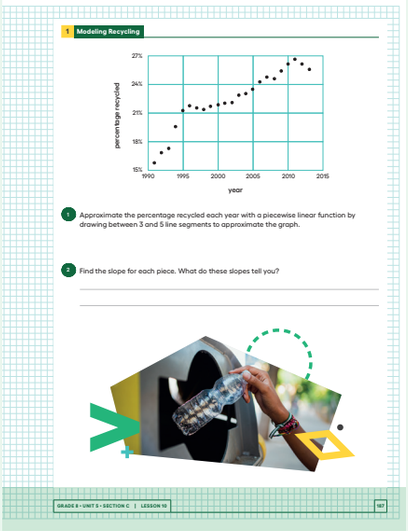
Student Workbook


1 Modeling Recycling

1. Approximate the percentage recycled each year with a piecewise linear function by drawing between 3 and 5 line segments to approximate the graph.

2. Find the slope for each piece. What do these slopes tell you?

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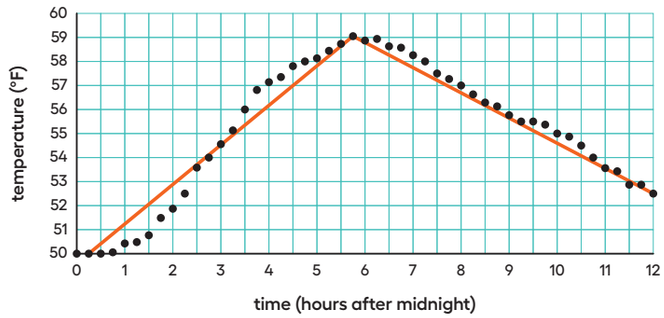
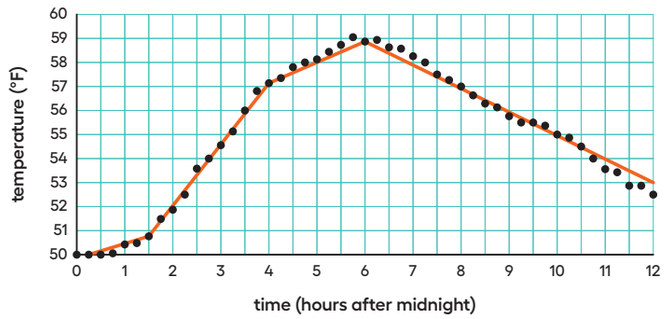
Launch

Arrange students in groups of 2. Display these two images for all to see. Tell students that sometimes we model functions with multiple line segments in different places. These models are called piecewise linear functions. For example, here are two different piecewise linear models of the same temperature data:



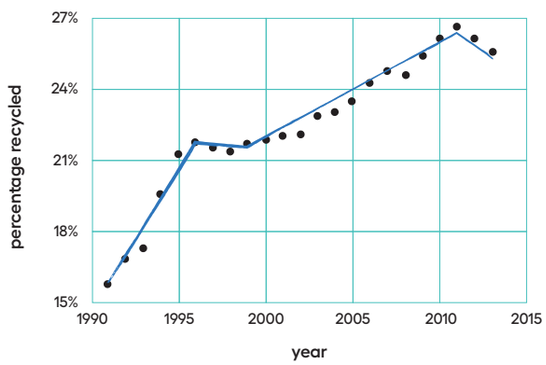


Give students 3–5 minutes of quiet work time and then time to compare their graphs and slope calculations with their partner.

Follow with a whole-class discussion.

Student Task Statement

Sample response:



1. Approximate the percentage recycled each year with a piecewise linear function by drawing between 3 and 5 line segments to approximate the graph.

2. Find the slope for each piece. What do these slopes tell you?

Sample response: The endpoints of the four segments given in the answer to the previous part are (1991, 16), (1996, 22), (1999, 21.5), (2011, 26), and (2013, 25), so we can find that the four line segments have respective slopes of $\frac{6}{5}$, $\frac{-1}{6}$, $\frac{3}{8}$, and $\frac{-1}{2}$. These slopes describe the approximate rate at which the percent recycled increases or decreases over those times. For example, from 2011 to 2013, the percent recycled decreases by approximately 0.5 percent per year.

Activity Synthesis

The purpose of this discussion is for students to see different ways their peers created their models and to consider the benefits and drawbacks of using different numbers of line segments when making a piecewise linear function.

Invite previously selected groups to share their line segments. Sequence the discussion of the approaches in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"What do the slopes of the different lines mean?"*

  The slopes of the lines tell us the rate of change of the different linear pieces for the specific intervals of time.

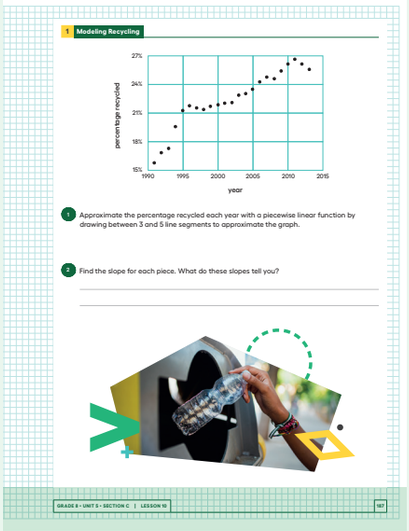
- *"Can we use this information to predict information for recycling in the future?"*

  If the data continues to decrease as it does from 2011 to 2013, yes. If the data starts to increase again, our model may not make very good predictions.

- *"What are the benefits of having fewer segments in the piecewise linear function? What are the benefits of having more segments?"*

  Fewer segments are easier to write equations for and help show long-term trends. More segments give a more accurate model of the data.

Student Workbook


1 Modeling Recycling

1. Approximate the percentage recycled each year with a piecewise linear function by drawing between 3 and 5 line segments to approximate the graph.

2. Find the slope for each piece. What do these slopes tell you?

Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Activity 2
Dog Bath

15 min

Activity Narrative

The purpose of this activity is to give students more practice working with a situation that can be modeled with a piecewise linear function. Here, the situation has already been modeled, and students must calculate the rate of change for the different pieces of the model and interpret it in context. A main discussion point should be what the different rates of change mean in the situation and the connection between features of the graph and the events in the situation.

Launch

Arrange students in groups of 2. Introduce the context of giving a dog a bath. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Display only the problem stem and related image, without revealing the questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

- Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

💬 *"What do these questions have in common? How are they different?" Listen for and amplify language related to the learning goal, such as 'the slope of the line.'*

"Time is the independent variable, so it goes on the horizontal axis."

and

💬 *"The tub was draining from Minute 21 to Minute 30."*

- Reveal the questions, and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

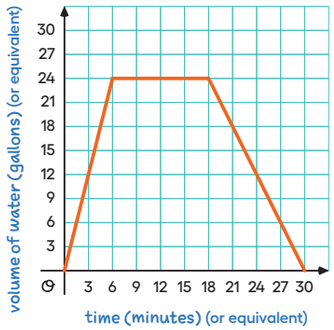
💬 *"How do your questions relate to interpreting rates of change in context?"*

Give students 3–5 minutes to answer the questions with their partner.

Follow with a whole-class discussion.

Student Task Statement

Elena filled up the tub and gave her dog a bath. Then she let the water out of the tub.



1. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes. Add labels to the graph to show this.
2. When did she turn off the water faucet?

   After 6 minutes

   The volume of water is increasing until $t = 6$, so this must be when Elena turned off the faucet.
3. How much water was in the tub when she bathed her dog?

   24 gallons

   She bathed her dog after she turned the water off, so we look to the graph to see the volume of water at $t = 6$.
4. How long did it take for the tub to drain completely?

   12 minutes

   The volume of water starts decreasing at $t = 18$ minutes, and it takes until $t = 30$ to completely drain.
5. At what rate did the faucet fill the tub?

   4 gallons per minute

   It took 6 minutes to fill 24 gallons, giving a rate of 4 gallons per minute $\left(\frac{24}{6}\right)$.
6. At what rate did the water drain from the tub?

   2 gallons per minute

   It took 12 minutes to drain 24 gallons, giving a rate of 2 gallons per minute $\left(\frac{24}{12}\right)$.

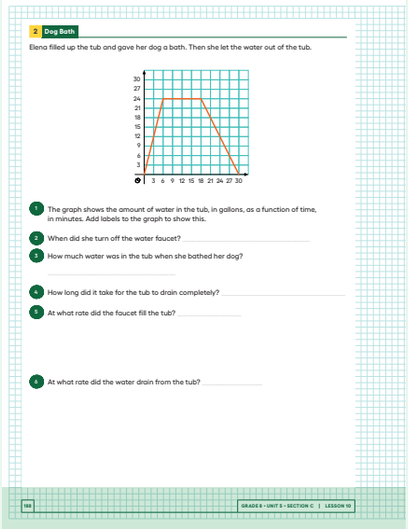
Activity Synthesis

The purpose of this discussion is for students to make sense of what the rate of change and other features of the model mean in the context of this situation. Consider asking the following questions:

- *"Did the tub fill faster or drain faster? How can you tell?"*

  The tub filled faster. The slope of the line representing the interval in which the water was filling the tub is steeper than the slope of the line representing the interval in which the water was draining from the tub.

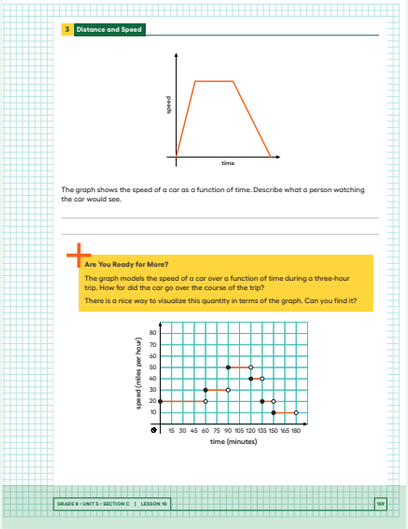
Student Workbook


2 Dog Bath

Elena filled up the tub and gave her dog a bath. Then she let the water out of the tub.

1. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes. Add labels to the graph to show this.
2. When did she turn off the water faucet? _____
3. How much water was in the tub when she bathed her dog? _____
4. How long did it take for the tub to drain completely? _____
5. At what rate did the faucet fill the tub? _____
6. At what rate did the water drain from the tub? _____

Student Workbook


3 Distance and Speed

speed

time

The graph shows the speed of a car as a function of time. Describe what a person watching the car would see.

Are You Ready for More?

The graph models the speed of a car over a function of time during a three-hour trip. How far did the car go over the course of the trip?

There is a nice way to visualize this quantity in terms of the graph. Can you find it?

speed (miles per hour)

time (minutes)

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💬 *"If you were going to write a linear equation for the first piece of the graph, what would you use for m? For b?"*

I would use $m = 4$ and $b = 0$, because the tub filled at a rate of 4 gallons per minute and the initial amount of water was 0.

💬 *"Which part of the graph represents the 2 gallons per minute you calculated?"*

The last part of the piecewise function has a slope of -2, which is when the tub was draining at 2 gallons per minute.

Activity 3: Optional
Distance and Speed

5 min

Activity Narrative

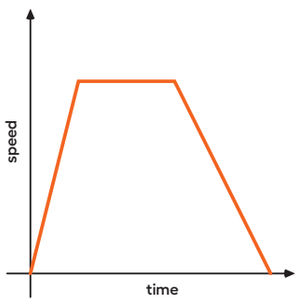
The purpose of this activity is for students to practice their skills interpreting a graph of a piecewise linear function and making sense of the situation the graph represents. Previously, students have had marked values to work from, but those are removed for this activity to encourage students to think more abstractly about what the changes in the graph represent and how they connect to the situation.

Launch

Give students 1–2 minutes of quiet work time.

Select students describing the graph with different levels of detail, particularly students who state that the car got up to speed faster than the car slowed down to 0, to share during the *Activity Synthesis*. Follow with a whole-class discussion.

Student Task Statement



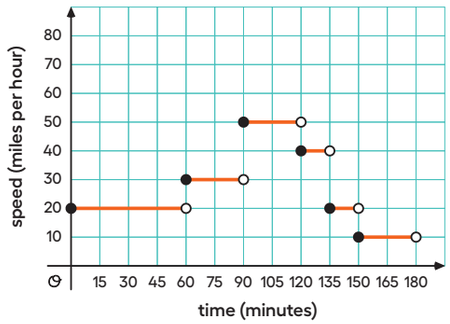
The graph shows the speed of a car as a function of time. Describe what a person watching the car would see.

The car begins at rest and then quickly picks up speed. After some time, it reaches its maximum speed, stays at that speed for a while, and then gradually slows back down until it comes to a stop.

Are You Ready for More?

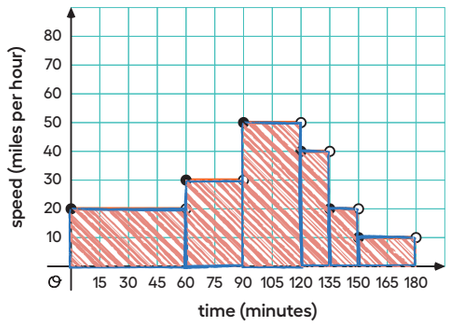
The graph models the speed of a car over a function of time during a three-hour trip. How far did the car go over the course of the trip?

There is a nice way to visualize this quantity in terms of the graph. Can you find it?

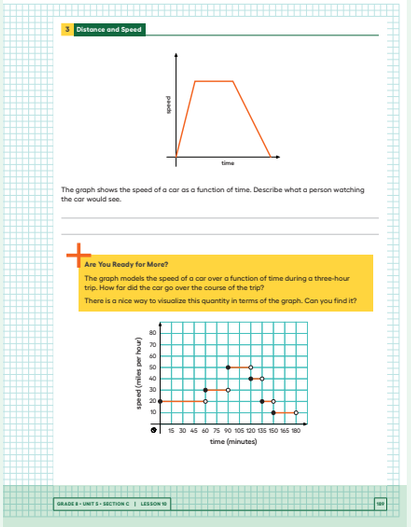


We can divide the trip into segments over which the speed is constant and use "*distance = speed × time*." Over the first hour, the car traveled 20 miles per hour, for a total of 20 miles. Then for half an hour, the car traveled 30 miles per hour, for a total of 15 miles. Similarly, the remaining four segments correspond to distances traveled of 25 miles, 10 miles, 5 miles, and 5 miles. Summing these gives a total of 80 miles.

The quantity "*speed × time*" is represented graphically by a length measured along the *x*-axis times a height measured along the *y*-axis, giving the area of a rectangle. The distance traveled over each segment is the area of the rectangle under that segment, and the total distance is the total area of the shaded region.



Student Workbook


3 Distance and Speed

speed

time

The graph shows the speed of a car as a function of time. Describe what a person watching the car would see.

Are You Ready for More?

The graph models the speed of a car over a function of time during a three-hour trip. How far did the car go over the course of the trip?

There is a nice way to visualize this quantity in terms of the graph. Can you find it?

speed (miles per hour)

time (minutes)

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Access for Students with Diverse Abilities (Activity 3, Synthesis)

Engagement: Provide Access by Recruiting Interest.
Invite students to generate a list of additional examples of graphs that represent speed over time and that connect to their personal backgrounds and interests.
Supports accessibility for: *Conceptual Processing, Attention*

Access for Multilingual Learners (Activity 3, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to

"Describe what a person watching the car would see."

Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Activity Synthesis

The purpose of this activity is for students to connect what is happening in a graph to a situation. Display the graph for all to see. Ask previously selected students to share the situation they came up with. Sequence students from least descriptive to most descriptive. Ask students to point out the parts on the graph as they share their story about the situation.

Consider asking the following questions:

- *"Did the car speed up faster or slow down faster? How do you know?"*

  The car sped up faster because the first part of the model is steeper than the third part of the model.

- *"How did you know that the car stayed that speed for a period of time?"*

  The graph stays at the same height for a while, so the speed was not changing during that time.

Lesson Synthesis

Conclude the lesson by inviting students to reflect on what they have learned about piecewise linear functions. Ask students,

- *"How would you describe a piecewise linear function to someone who has never seen one?"*

  A piecewise linear function is a function whose graph is pieced together out of line segments. For different ranges of input, the output is changing at different approximately constant rates, so a different line is used for each range.

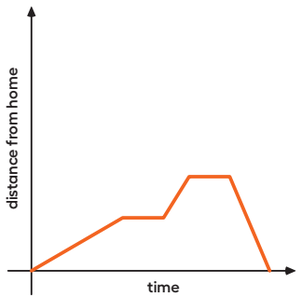
Give 1 minute of quiet think time and then time to share their response with a partner. Invite partners to share their responses with the class, and record them for all to see.

If students don't include that there are different constant rates over different intervals of the independent variable, make sure that is made clear.

If time allows, invite students to think of other situations that change at different constant rates over time, and give partners 1 minute of think time before selecting groups to share their situations.

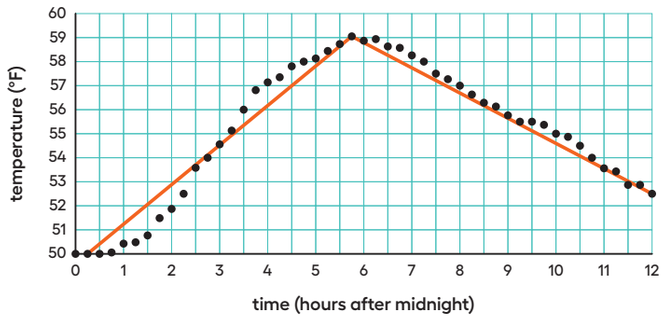
Lesson Summary

This graph shows Andre biking to his friend's house, where he hangs out for a while. Then they bike together to the store to buy some groceries before racing back to Andre's house for a movie night. Each line segment in the graph represents a different part of Andre's travels.

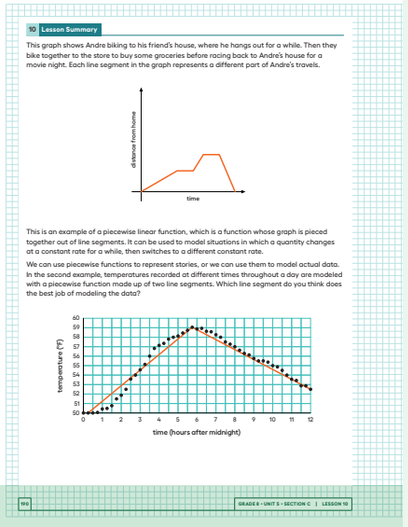


This is an example of a piecewise linear function, which is a function whose graph is pieced together out of line segments. It can be used to model situations in which a quantity changes at a constant rate for a while, then switches to a different constant rate.

We can use piecewise functions to represent stories, or we can use them to model actual data. In the second example, temperatures recorded at different times throughout a day are modeled with a piecewise function made up of two line segments. Which line segment do you think does the best job of modeling the data?



Student Workbook



Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of describing functional relationships between two quantities using a graph. If most students struggle, make time to revisit related work in the Lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work. For example, consider repeating the third activity, but with a different story that uses some specific values.

Grade 8, Unit 5, Lesson 6 Even More Graphs of Function

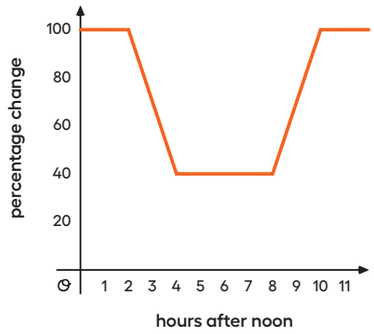
Cool-down

Lin's Phone Charge

5 min

Student Task Statement

Lin uses an app to graph the charge on her phone.



1. When did she start using her phone?

   Lin started using her phone 2 hours after noon, or at 2:00 p.m., since that is where the negative slope begins.

2. When did she start charging her phone?

   Lin started charging her phone 8 hours after noon, or at 8:00 p.m., since that is where the positive slope begins.

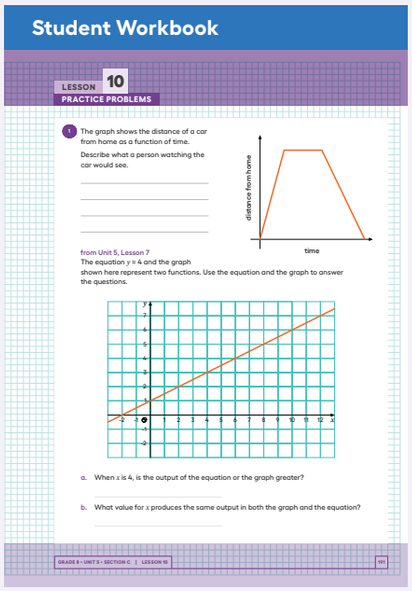
3. While she was using her phone, at what rate was Lin's phone battery dying?

   The battery was dying at 30% per hour since it decreased 60% over 2 hours.

Lesson 10

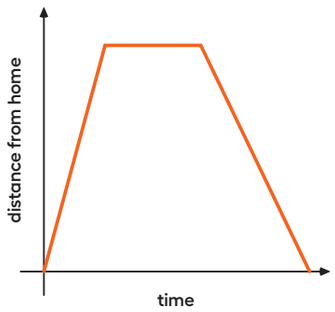
Practice Problems

4 Problems



Problem 1

The graph shows the distance of a car from home as a function of time.



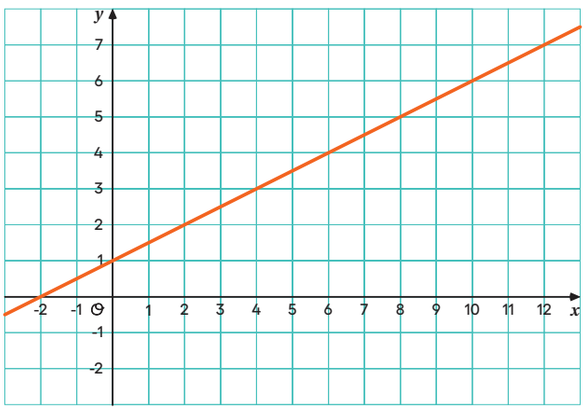
Describe what a person watching the car would see.

Sample response: The car is driven away from home, then waits. The car is then driven back home at a slower speed than it was when driven away from home.

Problem 2

from Unit 5, Lesson 7

The equation $y = 4$ and the graph shown here represent two functions. Use the equation and the graph to answer the questions.

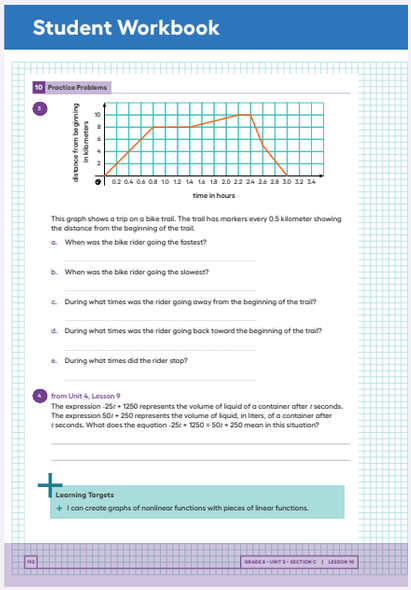


a. When x is 4, is the output of the equation or the graph greater?

equation

b. What value for x produces the same output in both the graph and the equation?

$x = 6$


Student Workbook

10 Practice Problems

3

This graph shows a trip on a bike trail. The trail has markers every 0.5 kilometer showing the distance from the beginning of the trail.

a. When was the bike rider going the fastest?

b. When was the bike rider going the slowest?

c. During what times was the rider going away from the beginning of the trail?

d. During what times was the rider going back toward the beginning of the trail?

e. During what times did the rider stop?

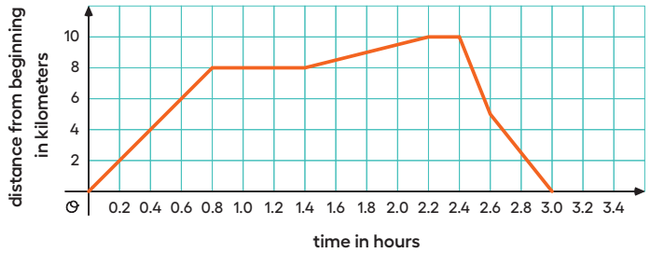
A from Unit 4, Lesson 9

The expression $-25t + 1250$ represents the volume of liquid of a container after t seconds. The expression $50t + 250$ represents the volume of liquid, in liters, of a container after t seconds. What does the equation $-25t + 1250 = 50t + 250$ mean in this situation?

Learning Targets

+ I can create graphs of nonlinear functions with pieces of linear functions.

Problem 3



This graph shows a trip on a bike trail. The trail has markers every 0.5 kilometer showing the distance from the beginning of the trail.

a. When was the bike rider going the fastest?

between 2.4 and 2.6 hours

b. When was the bike rider going the slowest?

between 1.4 and 2.2 hours (not including the time that the rider stopped)

c. During what times was the rider going away from the beginning of the trail?

between 0 and 0.8 hours and between 1.4 and 2.2 hours, because the rider stopped between 0.8 and 1.4 hours

d. During what times was the rider going back toward the beginning of the trail?

between 2.4 and 3 hours

e. During what times did the rider stop?

between 0.8 and 1.4 hours and between 2.2 and 2.4 hours

Problem 4

from Unit 4, Lesson 9

The expression $-25t + 1250$ represents the volume of liquid of a container after t seconds. The expression $50t + 250$ represents the volume of liquid, in liters, of a container after t seconds. What does the equation $-25t + 1250 = 50t + 250$ mean in this situation?

Sample response: The equation says that the volume in one container is equal to the volume in the other container. This equation can be solved for t to find the time at which both containers have the same volume.