Finding Solutions to Inequalities in Context

Goals

Interpret inequalities that represent situations with a constraint.

- Solve an equation of the form px + q = r to determine the boundary point for an inequality of the form px + q > r or px + q < r.
- Use substitution or reasoning about the context to justify (orally and in writing) whether the values that make an inequality true are greater than or less than the boundary point.

Learning Targets

- I can describe the solutions to an inequality by solving a related equation and then reasoning about values that make the inequality true.
- I can write an inequality to represent a situation.

Lesson Narrative

In this lesson, students compare equations and inequalities that represent the same situation. They interpret what each part of the equation or inequality means in the context. They solve the related equation and use the solution to the equation to reason about what values are **solutions to** the inequality, the values that make the inequality true.

Several of the inequalities involve negative coefficients. Students may notice that the inequality symbol in the solution is reversed from the symbol that was in the original inequality that represented the situation. The goal is for students to use context to make sense of these reversals, for example, by thinking "since x represents the amount of money to spend, Diego would need to spend less than this much if he wants to have more than a certain amount left over."

As students interpret equations, inequalities, and their solutions, they are reasoning abstractly and quantitatively.

Student Learning Goal

Let's solve more complicated inequalities.

Lesson Timeline

10

Warm-up

15

Activity 1

15

Activity 2

10

Lesson Synthesis

Access for Students with Diverse Abilities

• Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- · Co-Craft (Activity 1)
- MLR8 (Activity 2)

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

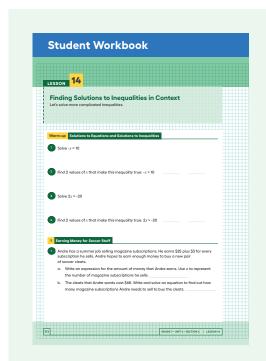
Required Preparation

Lesson:

· Several activities suggest providing students with blank number lines to use for scratch work. One way to accomplish this is to print a line with unlabeled, evenly-spaced tick marks, and place these into sheet protectors. Students can write on these with dryerase markers and wipe them off.

Assessment

Cool-down



Warm-up

Solutions to Equations and Solutions to Inequalities



Activity Narrative

In this activity, students solve some equations and some related inequalities. This *Warm-up* highlights the link between an inequality and its associated equation. Monitor for students who use the value -10 as a boundary as they test values to find solutions to the inequalities.

Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion. Optionally, provide students with blank number lines for scratch work.

Student Task Statement

- **1.** Solve -x = 10
- -10
- **2.** Find 2 values of x that make this inequality true: -x > 10Sample responses: -12, -28.7, -209 (Any value that is less than -10 works.)
- **3.** Solve 2x = -20
- -10
- **4.** Find 2 values of x that make this inequality true: 2x > -20Sample responses: -9, 0, $82\frac{3}{4}$ (Any value that is greater than -10 works.)

Activity Synthesis

The purpose of this discussion is for students to understand the term *solution* to an inequality and for them to recognize that the number that makes the two sides of an inequality equal is a good region of the number line to start looking for solutions.

Display two number lines for all to see that each include -10 and some integral values to its left and right. Ask a few students to share their responses to the first two questions, recording their responses on one number line and gauging the class for agreement. Ask a few students to share their responses to the last two questions, recording their responses on the other number line and gauging the class for agreement.

Highlight the fact that -x = 10 and 2x = -20 have the same solution (-10), but the inequalities -x > 10 and 2x > -20 don't have the same solutions. Ask:

"How do you know when a value is a solution to an equation?"

"How do you know when a value is a solution to an inequality?"

"How are solutions to an inequality different from solutions to an equation?"

Select students to share strategies they had for finding solutions to the inequalities in this activity. If not mentioned by students, discuss the fact that since -10 makes the two sides of the inequality equal, the region of values around -10 is a good place to start looking for solutions.

Activity 1

Earning Money for Soccer Stuff



Activity Narrative

In this activity, students write an equation to represent a situation, and then they write an associated inequality. They notice that they can express not just that an outcome can be equal to a value, but that an outcome can be at least as much as a value by using the new notation \geq .

When students start with an equation representing a context and use its structure to write a related inequality, they notice and make use of structure.

Launch

Arrange students in groups of 2. Introduce the context of earning money to save up for sports supplies. Use Co-Craft Questions to orient students to the context and elicit possible mathematical questions.

Display only the first problem stem, without revealing the questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as "greater than" or "less than."

Reveal the questions and give students 1–2 minutes to compare them to their own questions and those of their classmates.

Invite students to identify similarities and differences before proceeding.

Student Task Statement

- 1. Andre has a summer job selling magazine subscriptions. He earns \$25 plus \$3 for every subscription he sells. Andre hopes to earn enough money to buy a new pair of soccer cleats.
 - **a.** Write an expression for the amount of money that Andre earns. Use nto represent the number of magazine subscriptions he sells.

3n + 25 (or equivalent)

b. The cleats that Andre wants cost \$68. Write and solve an equation to find out how many magazine subscriptions Andre needs to sell to buy the cleats.

3n + 25 = 68 (or equivalent), Sample reasoning: $n = 14\frac{1}{3}$ so Andre would need to sell 15 magazine subscriptions to reach his goal, since he can't sell part of a subscription.

c. If Andre sold 16 magazine subscriptions this week, would he reach his goal? Explain your reasoning.

Yes. Sample reasoning: 16 > 14 $\frac{1}{3}$. He made \$73, which is more than enough to buy the cleats.

Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners

This activity uses the Co-Craft Questions math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Access for Perception.

Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language

Student Workbook



d. What are some numbers of magazine subscriptions Andre could sell and still reach his goal?

Sample responses: 15, 17, 100. (Any whole number greater than 14 would make sense.)

e. Write an inequality to represent that Andre wants to earn at least \$68.

 $3n + 25 \ge 68$ (or equivalent)

f. Write an inequality to represent the number of subscriptions Andre must sell to reach his goal.

 $n \ge 14\frac{1}{3}$ (or equivalent)

- **2.** Diego has budgeted \$35 to buy shorts and socks for soccer. He needs 5 pairs of identical socks and a pair of shorts. The shorts he wants cost \$19.95.
 - **a.** Write an expression for the total cost of the socks and shorts. Use x to represent the price of one pair of socks.

5x + 19.95

b. Write and solve an equation that represents Diego spending exactly \$35 on the socks and shorts.

5x + 19.95 = 35, x = 3.01. In this situation, Diego paid \$3.01 for each pair of socks

c. List some other possible prices for the socks that would still allow Diego to stay within his budget.

Sample responses: Any price under \$3.01.

d. Write an inequality to represent the amount Diego can spend on a single pair of socks.

 $x \le 3.01$.

Activity Synthesis

The purpose of this discussion is for students to understand that to find the solution to an inequality, it helps to find the solution to the related equation. That value is important to know because it separates numbers that are solutions to the inequality from numbers that are not solutions.

Ask students to share the inequality they wrote to represent the number of subscriptions Andre must sell to reach his goal. For example, $3n+25 \ge 68$. Explain that to find whether the solution to this inequality is $n \ge 14\frac{1}{3}$ or $n \le 14\frac{1}{3}$, we can substitute some values of n that are greater than $14\frac{1}{3}$ and some that are less than $14\frac{1}{3}$ to check. Alternatively, we can think about the context: If Andre wants to make more money, he needs to sell more magazines, not fewer.

In the same way, we can think: If Diego wants to spend less than \$35, he needs to spend less for socks, not more. This will help us understand that why $x \le 3$. 01 is the solution, not $x \ge 3$. 01.

To promote thinking about a general solving strategy, ask:

"How does solving the related equation help us solve an inequality? What does the solution to the equation tell us about solutions to the inequality?"

"What are some ways we can determine whether the solution to an inequality should use 'less than' or 'greater than' symbols?"

"How can we check whether a value is a solution to the inequality?"

Lesson 14 Activity 1 **Activity 2** Lesson Synthesis Cool-down Warm-up

Activity 2

Granola Bars and Savings



Activity Narrative

In this activity, students interact with contexts in which the direction of inequality is the opposite of what they might expect if they were to solve it like an equation. For example, in the second problem, the original inequality is 9 $(7 - x) \le 36$, but the solution to the inequality is $x \ge 3$.

Some students might solve the associated equation and then test values of x to determine the direction of inequality. That method will be introduced in more generality in the next lesson. This activity emphasizes thinking about the context in deciding the direction of inequality.

Since the task requires students to interpret the meaning of their answer in a context, they are reasoning abstractly and quantitatively.

Launch

Keep students in the same groups.

Give 5–10 minutes of quiet work time and partner discussion followed by a whole-class discussion.

Student Task Statement

- 1. Kiran has \$100. He wants to know how much he could spend each month to still have at least \$25 left one year from now.
 - **a.** To represent this situation, Kiran writes the inequality $-12x + 100 \ge 25$. What does -12x represent? Why is it negative?
 - It represents the total amount Kiran can spend in a year. It's negative because it is money spent.
 - **b.** Find some values of x that would work for Kiran.
 - Sample responses: I, 2, 6. (Any value less than or equal to 6.25 works.)
 - c. We could express all the values that would work using either $x \le$ or $x \ge$. Which one should we use? Explain your reasoning.

 $X \leq$

Sample reasoning: Kiran must spend less than a certain amount each month in order to end up with at least \$25 in the account at the end of the year.

- 2. A teacher wants to buy 9 boxes of granola bars for a school trip. Each box usually costs \$7, but many grocery stores are having a sale on granola bars this week. Different stores are selling boxes of granola bars at different discounts.
 - a. If x represents the dollar amount of the discount, then the amount the teacher will pay can be expressed as 9(7 - x). What does the quantity 7 - x represent?

the price of I box after the discount

Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Task Statement)

Representation: Internalize Comprehension.

Provide students with a partially completed graphic organizer, such as a two-column table with the columns labeled "amount Kiran withdraws each month" and "account balance one year from now" to help students see the relationship between the variables.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook

Gra	nola Bars and Savings
	an has \$100. He wants to know how much he could spend each month to still have least \$25 left one year from now.
a.	To represent this situation, Kiran writes the inequality -12 x + 100 \ge 25. What does -12 x represent? Why is it negative?
b.	Find some values of x that would work for Kiran.
	We could express all the values that would work using either $x \le - 0$ or $x \ge - 0$.
٠.	Which one should we use? Explain your reasoning.
bu	eacher wants to buy 9 baxes of granola bars for a school trip. Each box usually costs \$1 t many grocery stores are having a sale on granola bars this week. Different stores are ling boxes of granola bars at different discounts.
	If x represents the dollar amount of the discount, then the amount the teacher will
ь.	pay can be expressed as $9(7 - x)$. What does the quantity $7 - x$ represent? The teacher has \$36 to spend on the aranola bars. The equation $9(7 - x) = 36$
	represents her spending all \$36. Solve this equation.
	What does the solution mean in this situation?
c.	The teacher does not have to spend all \$36. Write an inequality that represents her spending at most \$36.



b. The teacher has \$36 to spend on the granola bars. The equation 9 (7 - x) = 36 represents her spending all \$36. Solve this equation. What does the solution mean in this situation?

x = 3

If the discount is \$3, then the cost for the granola bars will be exactly \$36.

c. The teacher does not have to spend all \$36. Write an inequality that represents her spending at most \$36.

$$9(7-x) \le 36$$
 (or equivalent)

d. The solution to this inequality must either look like $x \le$ or $x \ge$. Which one is it? Explain your reasoning.

x ≥ ___

Sample reasoning: The solution is $x \ge 3$. This makes sense in the context because a discount higher than \$3 per box means that the teacher pays a lower price for granola bars (and spends less than \$36). With the opposite solution, $x \le 3$, the lower discount amount causes the teacher to spend more than \$36.

Are You Ready for More?

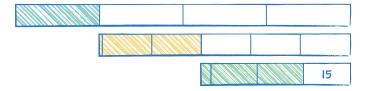
Jada and Diego grew plants from seedlings.

- They selected $\frac{1}{4}$ of the plants to give to their teachers.
- Next, they threw away 1 damaged plant.
- They delivered $\frac{2}{5}$ of the remaining plants to a local nursing home.
- Then they gave 3 plants to some neighbors.
- They boxed up $\frac{2}{3}$ of the remaining plants to save for their friends.

After all this, Jada and Diego had 15 plants left. How many plants did they grow from seedlings?

108 plants

Possible strategy: Draw a diagram to represent the situation:



Next, work backwards:

$$48 \div 3 = 16, 16 \cdot 5 = 80, 80 + 1 = 81$$

$$81 \div 3 = 27, 27 \cdot 4 = 108$$

An equation that represents the number of plants remaining would be $\frac{1}{3}(\frac{3}{5}(\frac{3}{4}x-1)-3)=15$. To solve this equation, we would multiply by 3, add 3, multiply by $\frac{5}{3}$, add 1, and then multiply by $\frac{4}{3}$. Compare these steps to the steps we took to solve with the diagram to see they are the same. (Note that divide by 3, then multiply by 5 is the same as multiply by $\frac{5}{3}$.)

Activity Synthesis

The purpose of the discussion is to let students voice their reasoning about the direction of the inequality symbol using the context. Ask students to share their reasons for choosing the direction of inequality in their solutions. Some students may notice that the algebra in both problems involves multiplying or dividing by a negative number. Honor this observation, but again, the goal is not to turn this observation into a rule for students to memorize and follow. Interpreting the meaning of the solution in the context should be the focus.

As students model real-world situations, questions about the interpretation of the mathematical solution should continue to come up in the conversation. For instance, the amount of the granola bar discount cannot be \$3.5923, even though this is a solution to the inequality $x \ge 3$. The value -10 is a solution to Kiran's inequality $-12x + 100 \ge 25$, even though he can't spend a negative number of dollars. Students can argue that negative values for x simply don't make sense in this context. Some may argue that we should interpret x = -10 to mean that Kiran deposits or earns \$10 every month.

Lesson Synthesis

Share with students,

166

Today we solved problems that can be represented with inequalities."

If desired, use this example to help students differentiate between equations and inequalities. Display 3x + 8 = 20 and $3x + 8 \ge 20$ for all to see. Consider asking:

"What is different about these two statements?"

The first is an equation, and the second is an inequality.

"What is different about the situations that these two statements could represent?"

The equation represents reaching exactly that value, while the inequality represents getting at least that amount. You could also have more.

"What is different about the solutions to these two statements?"

The solution to the equation is just one number, while there are many possible values that are solutions to the inequality.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

For each observation that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.

Advances: Listening, Speaking

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

We can write and solve inequalities to solve problems.

Example: Elena has \$5 and sells pens for \$1.50 each. Her goal is to save \$20. We could solve the equation 1.5x + 5 = 20 to find the number of pens, x, that Elena needs to sell in order to save exactly \$20. Adding -5 to both sides of the equation gives us 1.5x = 15, and then dividing both sides by 1.5 gives the solution x = 10 pens.

What if Elena wants to save more than \$20? The inequality 1.5x + 5 > 20 tells us that the amount of money Elena saves needs to be *greater* than \$20. The solution to the previous equation will help us understand what the solutions to the inequality will be. We know that if Elena sells 10 pens, she will save *exactly* \$20. Since each pen gives her more money, she needs to sell more than 10 pens to save more than \$20. So, we can represent all the solutions to the inequality with another inequality: x > 10. A **solution to an inequality** is a number that can be used in place of the variable to make the inequality true.

Cool-down

Colder and Colder

5 mir

Student Task Statement

It is currently 10 degrees outside. The temperature is dropping 4 degrees every hour.

1. Explain what the equation 10 - 4h = -2 represents.

Sample response:

when the temperature is exactly -2 degrees

2. What value of h makes the equation true?

h=3

3. Explain what the inequality 10 - 4h < -2 represents.

Sample response:

When the temperature is colder than -2 degrees

4. Does the solution to this inequality look like $h < _$ or $h > _$? Explain your reasoning.

h>

Sample reasoning: The solution is h > 3. Since the temperature is dropping, it will be colder than -2 degrees after 3 hours.

Practice Problems

7 Problems

Problem 1

The solution to 5 - 3x > 35 is either x > -10 or -10 > x. Which solution is correct? Explain how you know.

x < -10

Sample reasoning: If we replace x with -IOO in the original inequality 5-3x>35, we get 305>35, which is true. Any value of x that is less than -IO makes the inequality true. -IO > x refers to all values of x that are less than -IO, so it is not the correct solution.

Problem 2

The school band director determined from past experience that if they charge t dollars for a ticket to the concert, they can expect attendance of 1000 - 50t. The director used this model to figure out that the ticket price needs to be \$8 or greater in order for at least 600 to attend. Do you agree with this claim? Explain your reasoning.

No, I do not agree.

Sample reasoning: If ticket prices are higher, fewer people will attend (this can be seen by trying some different values of t in 1000 – 50t). The value t = 8 is the solution to 1000 – 50t = 600, but they need to charge \$8 or less if they want 600 people or more to attend (1000 – 50t ≥ 600).

Problem 3

from Unit 6, Lesson 13

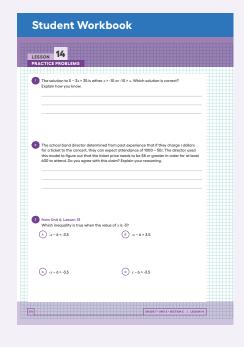
Which inequality is true when the value of x is -3?

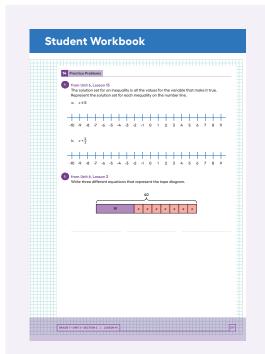
A. -x - 6 < -3.5

B. -x - 6 > 3.5

C. -x - 6 > -3.5

D. x - 6 > -3.5





Problem 4

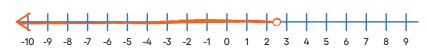
from Unit 6, Lesson 13

The solution set for an inequality is all the values for the variable that make it true. Represent the solution set for each inequality on the number line.

a. $x \le 5$



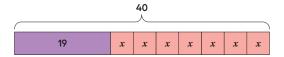
b. $x < \frac{5}{2}$



Problem 5

from Unit 6, Lesson 3

Write three different equations that represent the tape diagram.



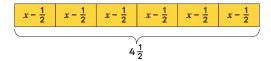
Sample responses:

- a. 7x + 19 = 40
- b. 40 = 7x + 19
- c. 7x = 40 19

Problem 6

from Unit 6, Lesson 2

A baker wants to reduce the amount of sugar in his cake recipes. He decides to reduce the amount used in 1 cake by $\frac{1}{2}$ cup. He then uses $4\frac{1}{2}$ cups of sugar to bake 6 cakes.



a. Describe how the tape diagram represents the story.

Sample response: The 6 equal parts of the diagram represent the 6 cakes the baker bakes. The label $x-\frac{1}{2}$ in each part represents the amount of sugar, measured in numbers of cups, that the baker used in each cake. x represents the original amount of sugar used in each cake and $x-\frac{1}{2}$ represents the original number of cups reduced by $\frac{1}{2}$ cup. $4\frac{1}{2}$ is the total amount of sugar, measured in cups, used for the 6 cakes.

b. How much sugar was originally in each cake recipe?

I 4 cups

Problem 7

from Unit 4, Lesson 12

One year ago, a tree was 4 feet 6 inches tall. Now the tree is 4 feet 10 inches tall. By what percentage did the tree's height increase in the last year?

about 7% (4 feet 6 inches is 54 inches and it grew 4 inches: $\frac{4}{54} \approx 0.07$)

