Lesson 7

Building Polygons (Part 2)

Goals

- Explain (in writing) how to use circles to locate the point where the sides of a triangle with known side lengths should meet.
- Use manipulatives to justify when it is not possible to make a triangle with three given side lengths.
- Use manipulatives to show that there is a minimum and maximum length that the third side of a triangle could be, given the other two side lengths.

Learning Targets

- I can reason about a figure with an unknown angle.
- I can show whether or not 3 side lengths will make a triangle.

Lesson Narrative

In this lesson, students experiment with constructing triangles given 2 or 3 side lengths. They start by working with cardboard strips and metal fasteners to discover that there are some combinations of lengths that do not make a triangle. Then students move toward using a ruler and compass, seeing that doing so recreates the function of the cardboard strips and metal fasteners more efficiently. The purpose of this transition is to help students move toward a mental understanding that does not depend on physical objects, helping them work toward the understanding that in a triangle the sum of any two sides must be greater than the other side.

Students use repeated reasoning with specific cases to formulate a general rule about which side lengths are possible for triangles.

Student Learning Goal

Let's build more triangles.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2
- Compasses: Activity 1, Activity 2
- Metal paper fasteners: Activity 1, Activity 2
- Copies of blackline masters: Activity 2

Materials to Copy

- Swinging the Sides Around Handout (1 copy for every 1 student): Activity 2

Required Preparation

Activity 1:
Each group needs 2 sets of strips and fasteners from an earlier lesson. If needed, prepare additional sets of strips and fasteners, including punching holes in the strips.

Lesson:
Print the Swinging the Sides Around blackline master. Prepare 1 copy for every 2 students.

Students will also need the cardboard strips and metal paper fasteners from the previous lesson, as well as access to geometry toolkits and compasses.

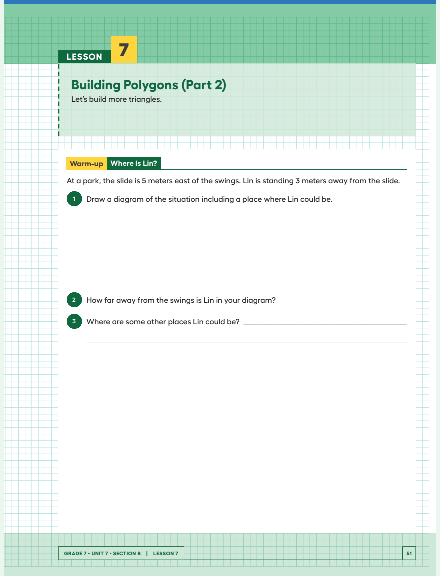
Note: If you are using the digital version of every activity, these supplies will not be needed.

Building on Student Thinking

Some students might assume that the swings, the slide, and Lin are all on a straight line, and that she must be 8 meters away. Ask these students if the problem tells us which direction Lin is from the slide.

Some students may confuse the type of compass discussed in the *Launch* and the type of compass discussed in the *Activity Synthesis*. Consider displaying a sample object or image of each of them and explain that the same name refers to two different tools.

Student Workbook


LESSON 7

Building Polygons (Part 2)

Let's build more triangles.

Warm-up Where Is Lin?

At a park, the slide is 5 meters east of the swings. Lin is standing 3 meters away from the slide.

1 Draw a diagram of the situation including a place where Lin could be.

2 How far away from the swings is Lin in your diagram?

3 Where are some other places Lin could be?

Warm-up
Where Is Lin?

5 min

Activity Narrative

The purpose of this *Warm-up* is to remind students that when you have a fixed starting point, all the possible endpoints for a segment of a given length form a circle (centered around the starting point). The context of finding Lin's position in the playground helps make the geometric relationships more concrete for students. Since there are many possible distances between Lin and the swings (but not infinitely many), this activity serves as an introduction to formalizing rules about what lengths can and cannot be used to form a triangle.

Monitor for students who come up with different locations for Lin, as well as for students who recognize that there are many possible locations, and ask them to share during the whole-class discussion.

Launch

Arrange students in groups of 2. If necessary, remind students of the directions north, south, east, and west and their relative position on a map. Provide access to geometry toolkits. Give students 2 minutes of quiet work time, followed by a partner discussion and a whole-class discussion.

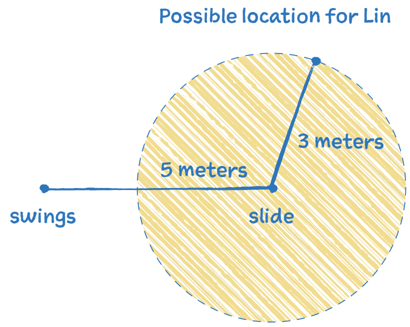
During the partner discussion, have students compare their reasoning with a partner and to discuss until they reach an agreement.

Student Task Statement

At a park, the slide is 5 meters east of the swings. Lin is standing 3 meters away from the slide.

1. Draw a diagram of the situation including a place where Lin could be.

   Answers vary. See diagram.



2. How far away from the swings is Lin in your diagram?

   There is no way to know for sure, because we don't know what direction Lin is from the slide. She could be anywhere between 2 and 8 meters away from the swings.

3. Where are some other places Lin could be?

   Lin could be at any position along a circle that is centered on the slide and that has a radius of 3 meters.

Activity Synthesis

Begin by inviting selected students to share their diagrams of where Lin is located. Discuss the following questions with the whole class:

"Do we know for sure where Lin is?"

No, because we don't know what direction she is from the swings.

"What shape is made by all the possible locations where Lin could be?"

a circle

"What is the closest Lin could be to the swings?"

2 m

"What is the farthest Lin could be away from the swings?"

8 m

Consider displaying the applet to show all the locations where Lin could be.

The Geogebra applet 'Where Is Lin?' is available here: **ilclass.com/l/395042**

Based on their work with drawing circles in a previous unit, some students may suggest that a compass could be used to draw all the possible locations where Lin could be. Consider having a student demonstrate how this could be done. If not mentioned by students, it is *not* necessary for the teacher to bring it up at this point.

Activity 1

How Long Is the Third Side?

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to experience that the sum of the lengths of the two shorter sides of a triangle must be greater than the length of the longest side. Students continue working with the cardboard strips and fasteners from a previous lesson to see how many different triangles they can build given two of the three side lengths. In the Activity Synthesis, the possible triangles are arranged in a way that helps students see the structure of an unknown angle between two known side lengths as a hinge. This prepares students for using compasses to draw triangles with given side lengths. They also continue to work at recognizing when two triangles are identical copies that are oriented differently.

As students work, monitor for those who:

- Find different lengths for the third side of the triangle.
- Use precise language to describe how the two side lengths can move in relation to each other.
- Make a connection to the circle of Lin's possible positions from the previous activity.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and for students who benefit from extra processing time.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Building on Student Thinking

Some students may think that the third side of the triangle cannot be 4 or 5 inches, because then the triangle would have two sides of that length instead of the one asked for in the question. Explain that the triangle is acceptable as long as *at least* one side is 5 inches long and *at least* one side is 4 inches.

If students claim that 8 is the longest the third side can be and that 2 would be the shortest (if they were given a strip of that length), ask them to consider fractional side lengths.

Student Workbook


1 How Long Is the Third Side?

Your teacher will give you some strips of different lengths and fasteners that you can use to attach the corners of the strips.

1. Build as many different triangles as you can that have one side length of 5 inches and one of 4 inches. Record the side lengths of each triangle that you build.

2. Are there any other lengths that could be used for the third side of the triangle but weren't in your set?

3. Are there any lengths that were in your set but could not be used as the third side of the triangle?

Are You Ready for More?

Assuming you had access to strips of any length, and you used the 9-inch and 5-inch strips as the first two sides, complete the sentences:

1. The third side can't be ____ inches or longer.
2. The third side can't be ____ inches or shorter.

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Launch

Arrange students in groups of 4. Distribute *two* sets of strips and fasteners to each group. Give students 7–10 minutes of group work time, followed by a whole-class discussion.

Student Task Statement

Your teacher will give you some strips of different lengths and fasteners that you can use to attach the corners of the strips.

1. Build as many different triangles as you can that have one side length of 5 inches and one of 4 inches. Record the side lengths of each triangle that you build.

   There are 5 possible triangles, with the third side measuring 3 inches, 4 inches, 5 inches, 6 inches, or 8 inches.

2. Are there any other lengths that could be used for the third side of the triangle but weren't in your set?

   Yes

   The third side could be 2 inches, 7 inches, or any fractional length between 1 and 9 inches. Sample responses:

3. Are there any lengths that were in your set but could not be used as the third side of the triangle?

   Yes

   We could not use the 9-inch side to make a triangle, because it is a straight line when we connect it.

Are You Ready for More?

Assuming you had access to strips of any length, and you used the 9-inch and 5-inch strips as the first two sides, complete the sentences:

1. The third side can't be 14 inches or longer.

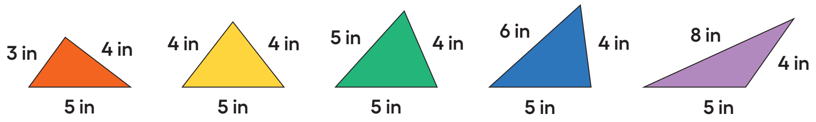
   The longest the third side could be is shorter than 14 inches (such as 13 or 13.9).

2. The third side can't be 4 inches or shorter.

   The shortest the third side could be is longer than 4 inches (such as 5 or 4.1).

Activity Synthesis

Select previously identified groups to share a triangle that they created. Establish whether each new triangle shared is the same as a triangle previously shared or is a different triangle. Collect one example of each possible triangle and display them for all to see, in order of increasing side length for the third side. Continue until students agree that all possible triangles are displayed.



To help students generalize about all the possible triangles that could be built with sides of 4 inches and 5 inches, ask questions like the following:

"What do you notice about the triangles?"

"Why was it impossible to use the 9-inch side to create a triangle?"

"What is the longest the third side of the triangle could be?"

more than 8, but less than 9, e.g., 8.5, 8.75, 8.9

"What is the shortest the third side of the triangle could be?"

less than 2, but more than 1, e.g., 1.5, 1.25, 1.1

"What happens when the third side is 1 inch or 9 inches?"

You get a straight line instead of a triangle.

Display a 5-inch strip fastened to a 4-inch strip for all to see. Demonstrate rotating the 4-inch strip around 180° to line up with each of the displayed triangles, as well as to show the idea that the third side could have a fractional side length. Invite students to share how this relates to the previous activity about Lin's distance from the swings.

If we hold one strip fixed, then all the possible locations where the other strip could end form a circle.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students as they generalize about the length of the third side of the triangle. Examples: "The third side of a triangle will always be ____ because …" "I agree because …" "I wonder if …"
Advances: Speaking, Conversing

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect. This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Activity 2

Swinging the Sides Around

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to relate the process for *building* a triangle given 3 side lengths (using cardboard strips and metal fasteners) to the process for *drawing* a triangle given 3 side lengths (using a compass). Students use the cardboard strips as an informal compass for drawing all the possible locations where the given segments could end. They are reminded of their work with circles in a previous unit: That a circle is the set of all the points that are equally distant from a center point and that a compass is a useful tool, not just for drawing circles, but also for transferring lengths in general. This prepares them for using a compass to draw triangles in future lessons.

In this activity, students also consider what their drawing would look like if the two shorter sides were too short to make a triangle with the third given side length.

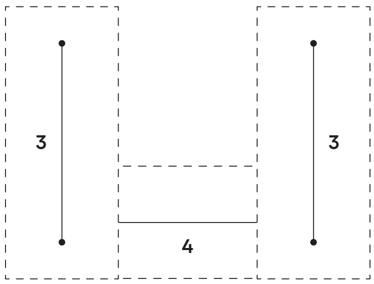
Left-handed students may find it easier to start with drawing the 3-inch circle on the left side of the 4-inch segment.

This activity works best when each student has access to the strips and fasteners. If physical manipulatives are not available, consider using the digital version of the activity. In the digital version, students use an applet to manipulate fixed side lengths of polygons.

Launch

Arrange students in groups of 2. Distribute one copy of the blackline master to each group. Make sure that each group has one complete set of strips and fasteners from the previous activity. Provide access to geometry toolkits and compasses.

Tell students to take one 4-inch piece and two 3-inch pieces and connect them so that the 4-inch piece is in between the 3-inch pieces as seen in the image. If necessary, display the image for all to see. Students should not connect the 3-inch pieces to each other.



Explain to students that the sheet distributed to them is the 4-inch segment that is mentioned in the task statement and that they will be drawing on that sheet.

Select students who used the strips and fasteners as well as a compass to construct circles and triangles, and ask them to share later. Aim to elicit both key mathematical ideas and a variety of student contributions, especially from students who haven't shared recently.

Student Task Statement

We'll explore a method for drawing a triangle that has three specific side lengths. Your teacher will give you a piece of paper showing a 4-inch segment, as well as some instructions for which strips to use and how to connect them.

1. Follow these instructions to mark the possible endpoints of one side:

a. Put your 4-inch strip directly on top of the 4-inch segment on the piece of paper. Hold it in place.

b. For now, ignore the 3-inch strip on the left side. Rotate it so that it is out of the way.

c. In the 3-inch strip on the *right* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where a 3-inch side could end.

d. Remove the connected strips from your paper.

Students draw the circle shown on the right side of the diagram.

2. What shape have you drawn while moving the 3-inch strip around? Why? Which tool in your geometry toolkit can do something similar?

A circle is all the points that are the same distance away from a center point. A compass can be used to draw this.

3. Use your drawing to create two unique triangles, each with a base of length 4 inches and a side of length 3 inches. Use a different color to draw each triangle.

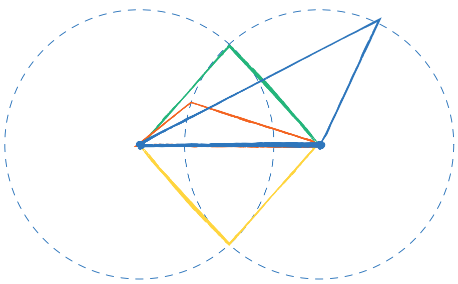
Answers vary. Possible response: Students draw the small red triangle and the large blue triangle shown in the diagram.

4. Reposition the strips on the paper so that the 4-inch strip is on top of the 4-inch segment again. In the 3-inch strip on the *left* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where another 3-inch side could end.

Students draw the circle shown on the left side of the diagram.

5. Using a third color, draw a point where the two circular marks intersect. Using this third color, draw a triangle with side lengths of 4 inches, 3 inches, and 3 inches.

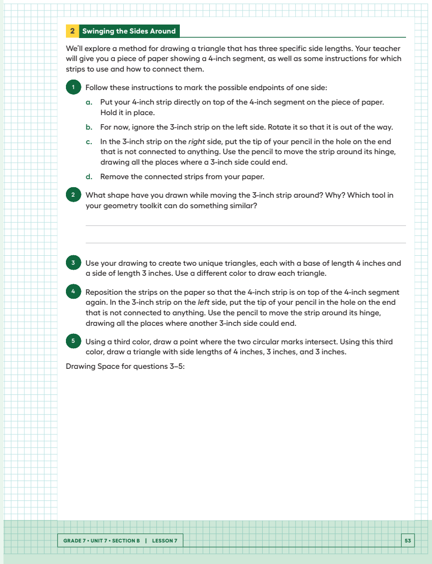
Answers vary. Possible response: Students draw the green or yellow triangle shown in the diagram.



Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Access for Perception.
To support understanding of the context, begin by demonstrating how to use the cardboard strips as an informal compass for drawing all the possible locations where given segments could end.
Supports accessibility for: Conceptual Processing, Language

Student Workbook



2 Swinging the Sides Around

We'll explore a method for drawing a triangle that has three specific side lengths. Your teacher will give you a piece of paper showing a 4-inch segment, as well as some instructions for which strips to use and how to connect them.

1. Follow these instructions to mark the possible endpoints of one side:
   a. Put your 4-inch strip directly on top of the 4-inch segment on the piece of paper. Hold it in place.
   b. For now, ignore the 3-inch strip on the left side. Rotate it so that it is out of the way.
   c. In the 3-inch strip on the *right* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where a 3-inch side could end.
   d. Remove the connected strips from your paper.
2. What shape have you drawn while moving the 3-inch strip around? Why? Which tool in your geometry toolkit can do something similar?
3. Use your drawing to create two unique triangles, each with a base of length 4 inches and a side of length 3 inches. Use a different color to draw each triangle.
4. Reposition the strips on the paper so that the 4-inch strip is on top of the 4-inch segment again. In the 3-inch strip on the *left* side, put the tip of your pencil in the hole on the end that is not connected to anything. Use the pencil to move the strip around its hinge, drawing all the places where another 3-inch side could end.
5. Using a third color, draw a point where the two circular marks intersect. Using this third color, draw a triangle with side lengths of 4 inches, 3 inches, and 3 inches.

Drawing Space for questions 3–5:

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Activity Synthesis

The goal of this discussion is for students to connect building figures with fixed lengths with constructing figures using a compass.

Display 2–3 representations from previously selected students for all to see. If time allows, invite students to briefly describe their approach, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do using the strips and using the compass have in common? How are they different?"

"Did anyone create their triangles in the same way, but would explain it differently?"

"How does the side length of each triangle show up in each method?"

"Are there any benefits or drawbacks to one representation compared to another?"

If time allows, highlight patterns that students noticed in previous activities with questions such as:

"How many different triangles could we draw when we had only traced a circle on one side? Why?"

Lots of different triangles, because we were only using two of the given side lengths.

"What is the longest the third side could have been? And the shortest?"

Less the 7 inches; More than 1 inch

"How many different triangles could we draw once we had traced a circle on each side?"

It looked like there were 2 different triangles, but they are identical copies, so there's really only 1 unique triangle.

Lesson Synthesis

The purpose of this discussion is for students to explain why some sets of side lengths are allowable and some are not for a triangle.

Display these 3 sets of triangle side lengths for all to see. Ask students which sets could create a triangle, and how they know.

- 1, 1, 5
- 4, 4, 4
- 5, 5, 1

Once students have had time to think about the sets of numbers, display this image and ask which set it would correspond to.

1,1,5

Invite students to share their thinking or informal diagrams about the other side lengths.

4,4,4 is all the same, so it will make an equilateral triangle. 5,5,1 could be tall and skinny, but it will work.

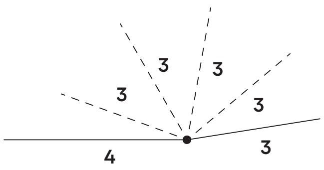
If time allows, ask students,

"If you draw one side of the triangle with circles on each end, and the circles do cross, they will cross twice. Why do we say there's only one possible triangle instead of two?"

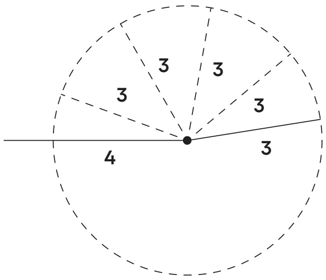
The two triangles are identical copies.

Lesson Summary

If we want to build a polygon with two given side lengths that share a vertex, we can think of them as being connected by a hinge that can be opened or closed:



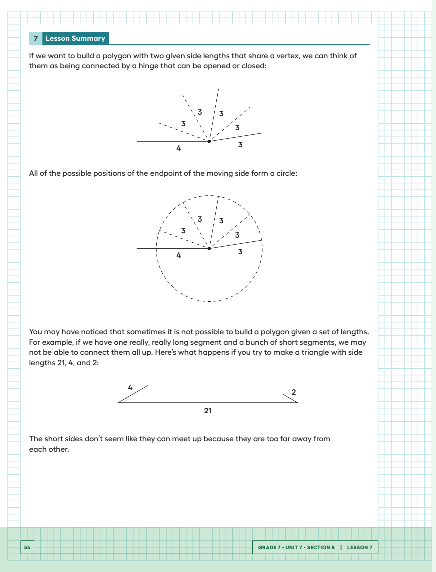
All of the possible positions of the endpoint of the moving side form a circle:



You may have noticed that sometimes it is not possible to build a polygon given a set of lengths. For example, if we have one really, really long segment and a bunch of short segments, we may not be able to connect them all up. Here's what happens if you try to make a triangle with side lengths 21, 4, and 2:

The short sides don't seem like they can meet up because they are too far away from each other.

Student Workbook


7 Lesson Summary

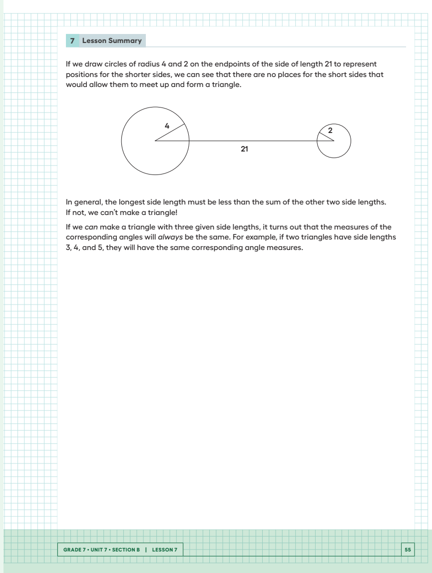
If we want to build a polygon with two given side lengths that share a vertex, we can think of them as being connected by a hinge that can be opened or closed:

All of the possible positions of the endpoint of the moving side form a circle:

You may have noticed that sometimes it is not possible to build a polygon given a set of lengths. For example, if we have one really, really long segment and a bunch of short segments, we may not be able to connect them all up. Here's what happens if you try to make a triangle with side lengths 21, 4, and 2:

The short sides don't seem like they can meet up because they are too far away from each other.

Student Workbook


7 Lesson Summary

If we draw circles of radius 4 and 2 on the endpoints of the side of length 21 to represent positions for the shorter sides, we can see that there are no places for the short sides that would allow them to meet up and form a triangle.

In general, the longest side length must be less than the sum of the other two side lengths. If not, we can't make a triangle!

If we *can* make a triangle with three given side lengths, it turns out that the measures of the corresponding angles will *always* be the same. For example, if two triangles have side lengths 3, 4, and 5, they will have the same corresponding angle measures.

Responding To Student Thinking

Points to Emphasize
If most students struggle with how to finish constructing the triangle, revisit the process of building triangles with side or angle requirements when doing this activity:

Grade 7, Unit 7, Lesson 9, Activity 1
Does Your Triangle Match Theirs?

If we draw circles of radius 4 and 2 on the endpoints of the side of length 21 to represent positions for the shorter sides, we can see that there are no places for the short sides that would allow them to meet up and form a triangle.

In general, the longest side length must be less than the sum of the other two side lengths. If not, we can't make a triangle!

If we *can* make a triangle with three given side lengths, it turns out that the measures of the corresponding angles will *always* be the same. For example, if two triangles have side lengths 3, 4, and 5, they will have the same corresponding angle measures.

Cool-down

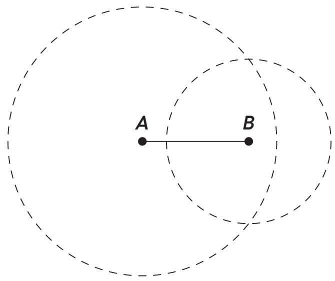
Finishing Elena's Triangles

5 min

Student Task Statement

Elena is trying to draw a triangle with side lengths of 4 inches, 3 inches, and 5 inches.

- She uses her ruler to draw a 4-inch line segment, AB.
- She uses her compass to draw a circle around point B with a radius of 3 inches
- She draws another circle, around point A with a radius of 5 inches.



1. What should Elena do next? Explain and show how she can finish drawing the triangle.

   Elena should put a point where the two circles intersect and draw line segments connecting that point to points A and B to finish her triangle.

Lesson **7**

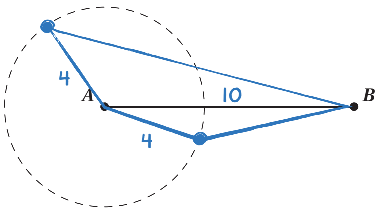
Practice Problems

6 Problems

Problem 1

In the diagram, the length of segment AB is 10 units and the radius of the circle centered at A is 4 units. Use this to create two unique triangles, each with a side of length 10 and a side of length 4. Label the sides that have length 10 and 4.

Answers vary. Possible response:



Problem 2

Select **all** the sets of three side lengths that will make a triangle.

A. 3, 4, 8

B. 7, 6, 12

C. 5, 11, 13

D. 4, 6, 12

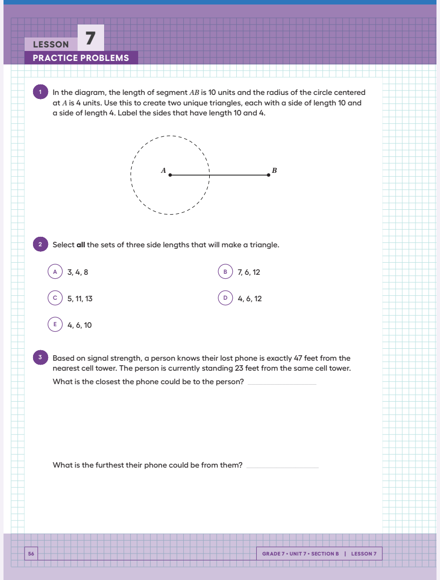
E. 4, 6, 10

Problem 3

Based on signal strength, a person knows their lost phone is exactly 47 feet from the nearest cell tower. The person is currently standing 23 feet from the same cell tower. What is the closest the phone could be to the person? What is the furthest their phone could be from them?

24 feet, 70 feet

Student Workbook



LESSON 7
PRACTICE PROBLEMS

1 In the diagram, the length of segment AB is 10 units and the radius of the circle centered at A is 4 units. Use this to create two unique triangles, each with a side of length 10 and a side of length 4. Label the sides that have length 10 and 4.

2 Select **all** the sets of three side lengths that will make a triangle.

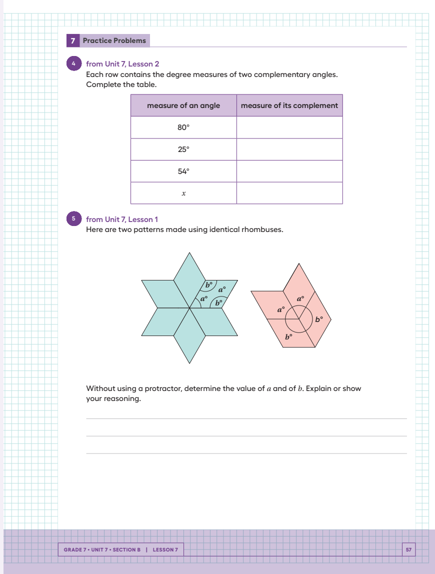
A 3, 4, 8
B 7, 6, 12
C 5, 11, 13
D 4, 6, 12
E 4, 6, 10

3 Based on signal strength, a person knows their lost phone is exactly 47 feet from the nearest cell tower. The person is currently standing 23 feet from the same cell tower.

What is the closest the phone could be to the person?

What is the furthest their phone could be from them?

Student Workbook



7 Practice Problems

4 from Unit 7, Lesson 2
Each row contains the degree measures of two complementary angles. Complete the table.

measure of an angle	measure of its complement
80°	
25°	
54°	
x	

5 from Unit 7, Lesson 1
Here are two patterns made using identical rhombuses.

Without using a protractor, determine the value of a and of b. Explain or show your reasoning.

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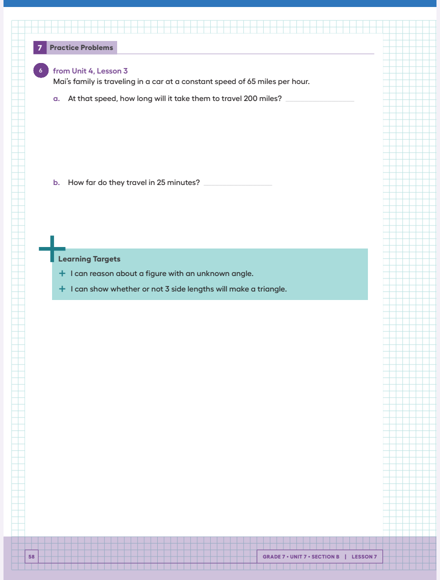
Student Workbook



7 Practice Problems

6 from Unit 4, Lesson 3
Mai's family is traveling in a car at a constant speed of 65 miles per hour.

a. At that speed, how long will it take them to travel 200 miles?

b. How far do they travel in 25 minutes?

Learning Targets
+ I can reason about a figure with an unknown angle.
+ I can show whether or not 3 side lengths will make a triangle.

GRADE 7 • UNIT 7 • SECTION B | LESSON 7

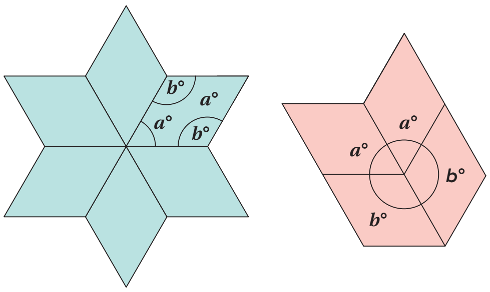
Problem 4 from Unit 7, Lesson 2

Each row contains the degree measures of two complementary angles. Complete the table.

measure of an angle	measure of its complement
80°	10°
25°	65°
54°	36°
x	$90 - x$

Problem 5 from Unit 7, Lesson 1

Here are two patterns made using identical rhombuses. Without using a protractor, determine the value of a and of b. Explain or show your reasoning.



$a = 60$ because 6 a's make 360.

$b = 120$ because 2 a's and 2 b's make 360.

Problem 6 from Unit 4, Lesson 3

Mai's family is traveling in a car at a constant speed of 65 miles per hour.

a. At that speed, how long will it take them to travel 200 miles?

$3\frac{5}{65}$ hours or about 3 hours and 4.6 minutes ($200 \div 65$ hours)

b. How far do they travel in 25 minutes?

$65 \cdot \frac{25}{60}$ miles or about 27.1 miles