Lesson 13

The Volume of a Cylinder

Goals

- Calculate the volume of a cylinder, and compare and contrast (orally) the formula for volume of a cylinder with the formula for volume of a prism.
- Explain (orally) how to find the volume of a cylinder using the area of the base and height of the cylinder.

Learning Targets

- I can find the volume of a cylinder in mathematical and real-world situations.
- I know the formula for volume of a cylinder.

Lesson Narrative

In this lesson students learn that the volume of a cylinder is the area of the base times the height, just like a prism. This is accomplished by considering one-unit-tall layers of a rectangular prism side by side with one-unit-tall layers of a cylinder. After thinking about how to compute the volume of specific cylinders, students learn the general formulas $V = Bh$ and $V = \pi r^2 h$.

Students will continue to work with the formula for the volume of a cylinder in different ways in future lessons.

The third activity is optional. Use this activity if students need additional practice identifying the radius and height in cylinders.

Student Learning Goal

Let's explore cylinders and their volumes.

Access for Students with Diverse Abilities

- Representation (Activity 3)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 3)

Required Materials

Materials to Gather

- Colored pencils: Activity 2

Required Preparation

Activity 1:
If possible, build a rectangular prism from 48 snap cubes to match the first diagram in the *Task Statement*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
10 min	15 min	10 min	10 min	10 min	5 min

Warm-up

A Circle's Dimensions

10 min

Activity Narrative

The purpose of this *Warm-up* is for students to review how to compute the area of a circle, an idea developed in grade 7. This *Warm-up* also gives students an opportunity to revisit language and calculations related to circles in preparation for finding the volume of a cylinder later in the lesson.

Students begin the activity identifying important features of a circle, including its radius and diameter. They use this information and the formula for the area of the circle to choose expressions from a list that are equivalent to the area of the circle. In the final question, students are given the area of the circle and are asked to find the corresponding radius.

Launch

Display the diagram from the *Task Statement* for all to see, and ask students:

- *"Name a segment that is a radius of circle A."*

  AC, AD, and AB, or these segments with the letters reversed, are all radii.

Review the meaning of the radius of a circle.

- *"What do we call a segment like BC, one with endpoints on the circle that contains the center of the circle?"*

  diameter

Review the meaning of the diameter of a circle.

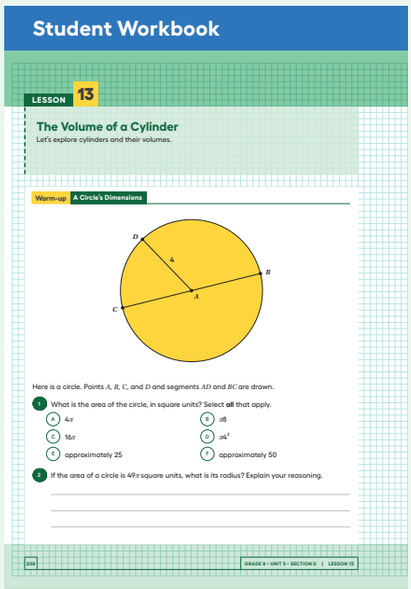
- *"What is the length of segment AB?"*

  4 units

Review the fact that all radii of a circle have the same length.

Give students 3 minutes of quiet work time, and follow with a whole-class discussion.

As students are working, select students who can explain why 16π, $\pi 4^2$, and "approximately 50" square units represent the area of the circle.


Student Workbook

LESSON 13

The Volume of a Cylinder

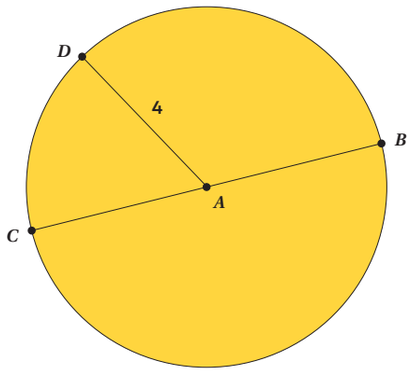
Let's explore cylinders and their volumes.

Warm-up A Circle's Dimensions

Here is a circle. Points A, B, C, and D and segments AD and BC are drawn.

1. What is the area of the circle, in square units? Select **all** that apply.
   - A. 4π
   - B. $\pi 8$
   - C. 16π
   - D. $\pi 4^2$
   - E. approximately 25
   - F. approximately 50
2. If the area of a circle is 49π square units, what is its radius? Explain your reasoning.

Student Task Statement



Here is a circle. Points A, B, C, and D and segments AD and BC are drawn.

1. What is the area of the circle, in square units? Select **all** that apply.

A. 4π **B.** $\pi 8$

C. 16π **D.** $\pi 4^2$

E. approximately 25 **F.** approximately 50

Since the radius is 4, the area of the circle is $\pi \cdot 4^2 = 16\pi$. This is approximately 50.3 square units.

2. If the area of a circle is 49π square units, what is its radius? Explain your reasoning.

7 units

Sample reasoning: The formula for area is times the square of the radius, and the area of this circle is 49 square units. So the square of the radius is 49π, and the radius is 7 units because $49 = 7^2$.

Activity Synthesis

The purpose of this discussion is to make sure students remember that the area of a circle can be found by squaring its radius and multiplying by π.

Ask previously selected students to share answers to the first question and explain why each of the solutions represents the area of the circle. If not brought up during the discussion, tell students that sometimes it is better to express an area measurement in terms of π. Other times it may be better to use an approximation of π, like 3.14, to represent the area measurement in decimal form. In this unit, we will often express our answers in terms of π.

Activity 1
Circular Volumes

15 min

Activity Narrative

The purpose of this activity is for students to connect their previous knowledge of the volume of rectangular prisms to their understanding of the volume of cylinders. From previous work, students should know that the volume of rectangular prisms is found by multiplying the area of the base by the height. Here we expand upon that to compute the volume of a cylinder.

Students start by calculating the volume of a rectangular prism. The goal is for students to then conjecture that the volume of a cylinder is the area of its base multiplied by its height. If some students don't know that they should multiply the area of the base by its height, then they are prompted to connect prisms and cylinders to make a reasonable guess.

Launch

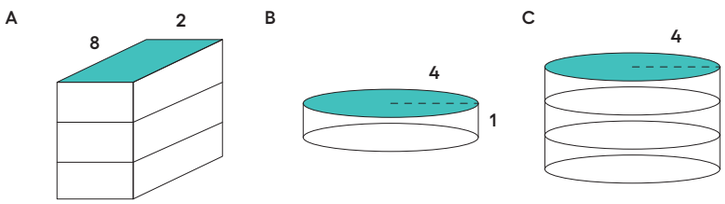
Arrange students in groups of 2. Remind students that a rectangular prism has a base that is a rectangle and that a cylinder has a base that is a circle. It may have been some time since students have thought about the meaning of a result of a volume computation. Consider showing students a rectangular prism built from 48 snap cubes with the same dimensions as Figure A. It may help them to see that one layer is made of 16 cubes.

Give students 3–5 minutes of quiet work time followed by time for partner discussion.

During their discussion, partners compare the volumes they found for the cylinders. If they guessed the volumes, partners explain their reasoning to one another. Follow with a whole-class discussion.

Student Task Statement

What is the volume, in cubic units, of each figure? Even if you aren't sure, make a reasonable guess.



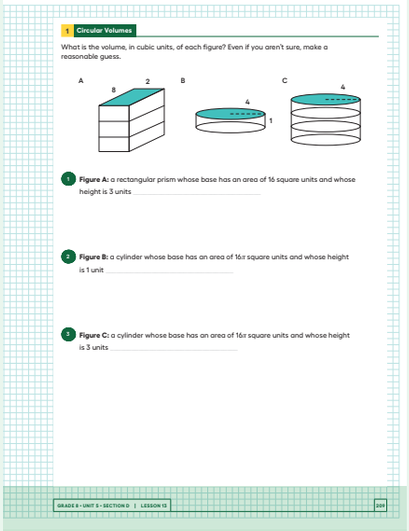
1. **Figure A:** a rectangular prism whose base has an area of 16 square units and whose height is 3 units

   48 cubic units. The area of the rectangular base is 16, which is multiplied by the height of 3 to find the volume.

2. **Figure B:** a cylinder whose base has an area of 16π square units and whose height is 1 unit

   16π cubic units. The area of the circular base is $\pi \cdot 4^2$, which is multiplied by the height of 1 to find the volume.

Student Workbook


1 Circular Volumes

What is the volume, in cubic units, of each figure? Even if you aren't sure, make a reasonable guess.

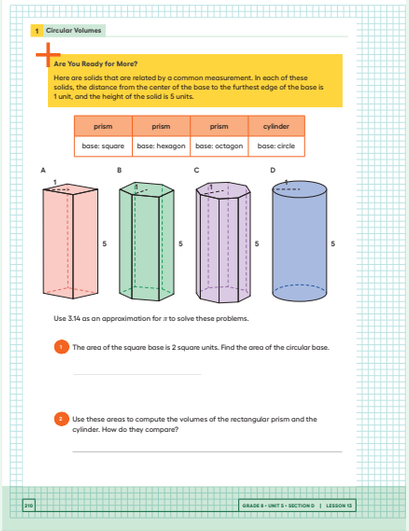
A 8 2 B 4 1 C 4

1 **Figure A:** a rectangular prism whose base has an area of 16 square units and whose height is 3 units

2 **Figure B:** a cylinder whose base has an area of 16π square units and whose height is 1 unit

3 **Figure C:** a cylinder whose base has an area of 16π square units and whose height is 3 units

Student Workbook


1 Circular Volumes

Are You Ready for More?

Here are solids that are related by a common measurement. In each of these solids, the distance from the center of the base to the furthest edge of the base is 1 unit, and the height of the solid is 5 units.

prism	prism	prism	cylinder
base: square	base: hexagon	base: octagon	base: circle

A B C D

Use 3.14 as an approximation for π to solve these problems.

1. The area of the square base is 2 square units. Find the area of the circular base.

2. Use these areas to compute the volumes of the rectangular prism and the cylinder. How do they compare?

3. **Figure C:** a cylinder whose base has an area of 16π square units and whose height is 3 units

   48π cubic units. The area of the circular base is $\pi \cdot 4^2$, which is multiplied by the height of 3 to find the volume.

Are You Ready for More?

Here are solids that are related by a common measurement. In each of these solids, the distance from the center of the base to the furthest edge of the base is 1 unit, and the height of the solid is 5 units.

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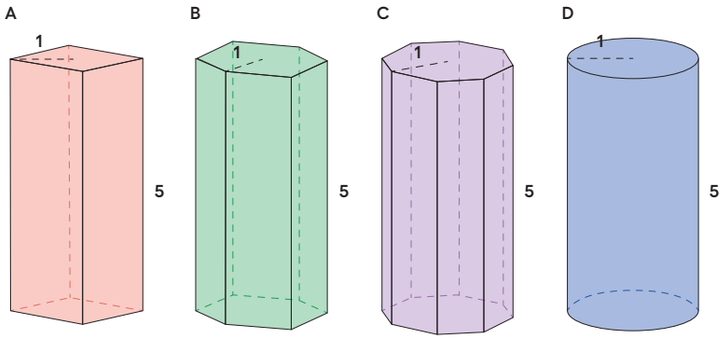


Use 3.14 as an approximation for π to solve these problems.

1. The area of the square base is 2 square units. Find the area of the circular base.

   Area of the circle is approximately 3.14 square units.

2. Use these areas to compute the volumes of the rectangular prism and the cylinder. How do they compare?

   Volume of the rectangular prism is 10 cubic units. Volume of the cylinder is 15.7 cubic units.

3. Without doing any calculations, list the figures from smallest to largest by volume. Use the images and your knowledge of polygons to explain your reasoning.

   Sample response: The areas of the polygons increase as the number of sides increase. This means that the volumes will also increase, since the height stays the same.

4. The area of the hexagon is approximately 2.6 square units, and the area of the octagon is approximately 2.83 square units. Use these areas to compute the volumes of the prisms with the hexagon and octagon bases. How does this match your explanation to the previous question?

   Volume of the hexagonal prism is 13 cubic units. Volume of the octagonal prism is 14.15 cubic units.

Activity Synthesis

Use this discussion to highlight the important features of cylinders and their definitions: The radius of the cylinder is the radius of the circle that forms its base. The height of a cylinder is the length between its circular top and bottom. A cylinder of height 1 can be thought of as a "layer" in a cylinder with height h. To highlight the connection between finding the area of a rectangular prism and finding the area of a cylinder, ask:

- *"How are prisms and cylinders different?"*

  A prism has a base that is a polygon, and a cylinder has a base that is a circle.

- *"How are prisms and cylinders the same?"*

  The volume of cylinders and prisms is found by multiplying the area of the base by the height: $V = Bh$

- *"How do you find the area of the base, B, of a cylinder?"*

  $B = \pi r^2$

Activity 2: Optional
A Cylinder's Dimensions

10 min

Activity Narrative

In this activity, students use colored pencils (or pens or highlighters) to label the radius and height on different pictures of cylinders. Then they sketch their own cylinders and label the radius and heights of those. The purpose of this activity is for students to practice identifying the radius and height of various cylinders, some of which are in context.

This activity can also be abbreviated if students demonstrate prior understanding of how to draw or label cylinders and only need a brief refresh.

Launch

Distribute colored pencils.

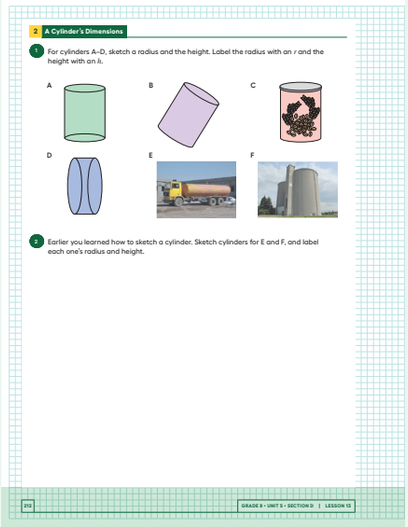
Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Access for Students with Diverse Abilities (Activity 3, Launch)

Representation: Access for Perception.
Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.

Supports accessibility for: Language

Student Workbook


2 A Cylinder's Dimensions

1. For cylinders A–D, sketch a radius and the height. Label the radius with an r and the height with an h.

A B C D E F

2. Earlier you learned how to sketch a cylinder. Sketch cylinders for E and F, and label each one's radius and height.

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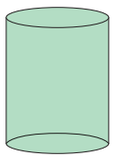
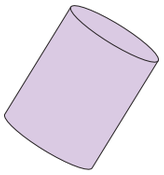
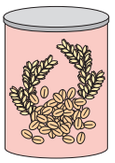
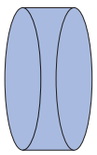
Student Task Statement

1. For cylinders A–D, sketch a radius and the height. Label the radius with an r and the height with an h.

A 

B 

C 

D 

E 

F 

Answers vary.

2. Earlier you learned how to sketch a cylinder. Sketch cylinders for E and F, and label each one's radius and height.

Answers vary.

Activity Synthesis

Select students to share where they marked the radius and height and the cylinders sketched for images E and F. Discuss examples of other cylinders students see in real life.

Activity 3

A Cylinder's Volume

10 min

Activity Narrative

The purpose of this activity is for students to practice calculating volumes for cylinders. By finding the area of the base before finding the volume, students are encouraged to compute the volume by multiplying the area of its base by its height. This way of thinking about volume might be more intuitive for students than the formula $V = \pi r^2 h$.

The second problem of this activity focuses on exploring a cylinder in a context where a picture is not given and students are asked to make an label a sketch. When working with problems in a given context, it is sometimes convenient or practical to use an approximation of π. An example of this is given in the question regarding the volume of a grain silo and interpretations of the answer.

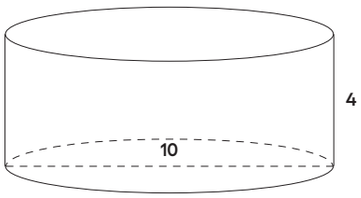
Launch

Provide access to colored pencils to shade the cylinder's base.

Give students 5–6 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

1. Here is a cylinder with height 4 units and diameter 10 units.



a. Shade the cylinder's base.

The image of the cylinder should have either its top or bottom shaded in.

b. What is the area of the cylinder's base? Express your answer in terms of π.

25π. The radius of the base is half of 10 units or 5 units. The area of the base is 25π square units since $\pi 5^2 = 25\pi$.

c. What is the volume of this cylinder? Express your answer in terms of π.

100π. The volume of the cylinder is the area of its base times its height, which is 100π cubic units since $25\pi \cdot 4 = 100\pi$.

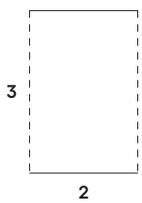
2. A silo is a cylindrical container that is used on farms to hold large amounts of goods, such as grain. On a particular farm, a silo has a height of 30 feet and diameter of 12 feet. Make a sketch of this silo, and label its height and radius. How much volume, in cubic feet, does this silo hold? Use 3.14 as an approximation for π.

The volume of this silo is approximately 3,393 cubic feet since $\pi 6^2 \cdot 30 \approx 3{,}393$.

Are You Ready for More?

One way to construct a cylinder is to take a rectangle (for example, a piece of paper), curl two opposite edges together, and glue them in place.

Which would give the cylinder with the greater volume: gluing the two dashed edges together, or gluing the two solid edges together?



Gluing the two solid edges together will create a cylinder with the greater volume.

Whichever two lines are glued together become the height of the cylinder, and the other lines represent the circumference of the circular base. For the cylinder created by gluing the dashed lines together, the height is 3 units, and the circumference is 2 units. Since circumference of a circle is equal to π multiplied by the diameter, the radius of the circular base must be $\frac{1}{2}\left(\frac{2}{\pi}\right) = \frac{1}{\pi}$. Therefore, the volume can be determined by $V = \pi\left(\frac{1}{\pi}\right)^2(3)$, which is about .95 unit^3.

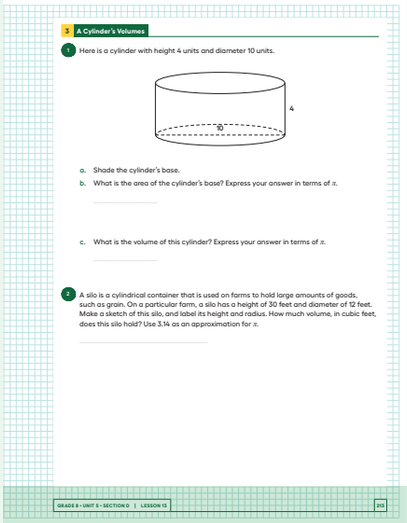
For the cylinder created by gluing the solid lines together, the height is 2 units, and the radius is $\frac{1}{2}\left(\frac{3}{\pi}\right) = \frac{3}{2\pi}$. Therefore, the volume can be determined by $V = \pi\left(\frac{3}{2\pi}\right)^2(2)$, which is about 1.43 units^3.

Building on Student Thinking

If students use the silo's diameter instead of the radius to find the volume, consider saying:

"Tell me more about how you made your sketch."
"Where does the value of 12 feet go on your sketch?"

Student Workbook


3 A Cylinder's Volumes

1 Here is a cylinder with height 4 units and diameter 10 units.

4
10

a. Shade the cylinder's base.
b. What is the area of the cylinder's base? Express your answer in terms of π.

c. What is the volume of this cylinder? Express your answer in terms of π.

2 A silo is a cylindrical container that is used on farms to hold large amounts of goods, such as grain. On a particular farm, a silo has a height of 30 feet and diameter of 12 feet. Make a sketch of this silo, and label its height and radius. How much volume, in cubic feet, does this silo hold? Use 3.14 as an approximation for π.

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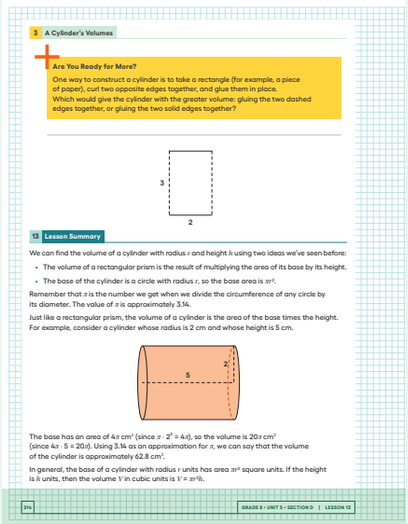
Student Workbook


3 A Cylinder's Volumes

Are You Ready for More?

One way to construct a cylinder is to take a rectangle (for example, a piece of paper), curl two opposite edges together, and glue them in place. Which would give the cylinder with the greater volume: gluing the two dashed edges together, or gluing the two solid edges together?

3
2

13 Lesson Summary

We can find the volume of a cylinder with radius r and height h using two ideas we've seen before:

- The volume of a rectangular prism is the result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is πr^2.

Remember that π is the number we get when we divide the circumference of any circle by its diameter. The value of π is approximately 3.14.

Just like a rectangular prism, the volume of a cylinder is the area of the base times the height. For example, consider a cylinder whose radius is 2 cm and whose height is 5 cm.

5
2

The base has an area of 4π cm^2 (since $\pi \cdot 2^2 = 4\pi$), so the volume is 20π cm^3 (since $4\pi \cdot 5 = 20\pi$). Using 3.14 as an approximation for π, we can say that the volume of the cylinder is approximately 62.8 cm^3.

In general, the base of a cylinder with radius r units has area πr^2 square units. If the height is h units, then the volume V in cubic units is $V = \pi r^2 h$.

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Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports. Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Activity Synthesis

The goal of this discussion is to ensure students understand how to use the area of the cylinder's base to calculate its volume. Consider asking the following questions:

- *"How does knowing the area of a circular base help determine the volume of a cylinder?"*

  The volume is this area multiplied by the height of the cylinder.

- *"If the cylinder were on its side, how do you know which measurements to use for the volume?"*

  It doesn't matter which direction the cylinder is turned. We can always start by finding the radius or diameter of the circle to determine r, and the height is always the distance between the bases.

- *"Do you prefer expressing your answer as a multiple of π or as an approximation?"*

  I prefer answering with an approximation since I can judge the size of the number better when it's written that way. Or, I prefer answering as a multiple of π because I like having the exact value.

Lesson Synthesis

Previously, students computed the volume of prisms by multiplying the area of the base by the prism's height. To help students summarize the ideas in this lesson, ask,

- *"How is finding the volume of a cylinder like finding the volume of a prism?"*

and give students 2–3 minutes to write a response. Encourage students to use diagrams to show their thinking. Invite students to share their ideas, displaying any diagrams for all to see.

Select 1 or 2 student examples to use to make a display that includes the formula for the area of a circle, $A = \pi r^2$, the formula for a cylinder's volume, $V = Bh$, along with labeled diagrams. This display should be kept posted in the classroom for the remaining lessons within this unit.

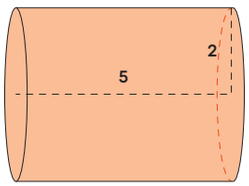
Lesson Summary

We can find the volume of a cylinder with radius r and height h using two ideas we've seen before:

- The volume of a rectangular prism is the result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is πr^2.

Remember that π is the number we get when we divide the circumference of any circle by its diameter. The value of π is approximately 3.14.

Just like a rectangular prism, the volume of a cylinder is the area of the base times the height. For example, consider a cylinder whose radius is 2 cm and whose height is 5 cm.



The base has an area of 4π cm^2 (since $\pi \cdot 2^2 = 4\pi$), so the volume is 20π cm^3 (since $4\pi \cdot 5 = 20\pi$). Using 3.14 as an approximation for π, we can say that the volume of the cylinder is approximately 62.8 cm^3.

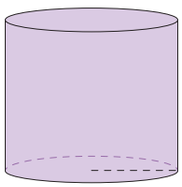
In general, the base of a cylinder with radius r units has area πr^2 square units. If the height is h units, then the volume V in cubic units is $V = \pi r^2 h$.

Cool-down

Liquid Volume

5 min

Student Task Statement



The cylinder shown here has a height of 7 centimeters and a radius of 4 centimeters.

1. What is the area of the base of the cylinder? Express your answer in terms of π.

   16π cm^2, because the square of the radius of the base is $4^2 = 16$, which is multiplied by π, giving $\pi \cdot 4^2 = 16\pi$.

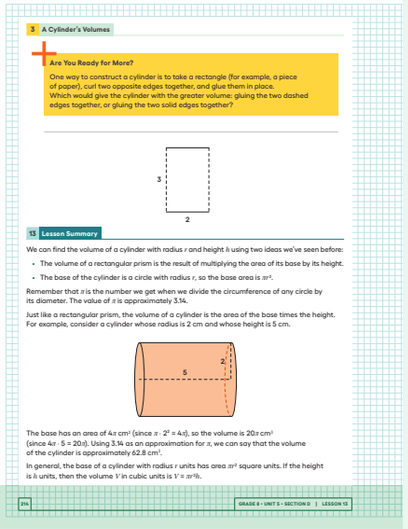
2. How many cubic centimeters of fluid can fill this cylinder? Express your answer in terms of π.

   112π cm^3, because the height of the cylinder is 7, which is multiplied by the area of the base, giving $16\pi \cdot 7 = 112\pi$.

3. Give a decimal approximation of your answer to the previous question using 3.14 to approximate π.

   351.68 cm^3, because $112 \cdot 3.14 \approx 351.68$

Student Workbook


3 A Cylinder's Volumes

Are You Ready for More?

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- The base of the cylinder is a circle with radius r, so the base area is πr^2.

Remember that π is the number we get when we divide the circumference of any circle by its diameter. The value of π is approximately 3.14.

Just like a rectangular prism, the volume of a cylinder is the area of the base times the height. For example, consider a cylinder whose radius is 2 cm and whose height is 5 cm.

5

2

The base has an area of 4π cm^2 (since $\pi \cdot 2^2 = 4\pi$), so the volume is 20π cm^3 (since $4\pi \cdot 5 = 20\pi$). Using 3.14 as an approximation for π, we can say that the volume of the cylinder is approximately 62.8 cm^3.

In general, the base of a cylinder with radius r units has area πr^2 square units. If the height is h units, then the volume V in cubic units is $V = \pi r^2 h$.

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Responding To Student Thinking

Points to Emphasize

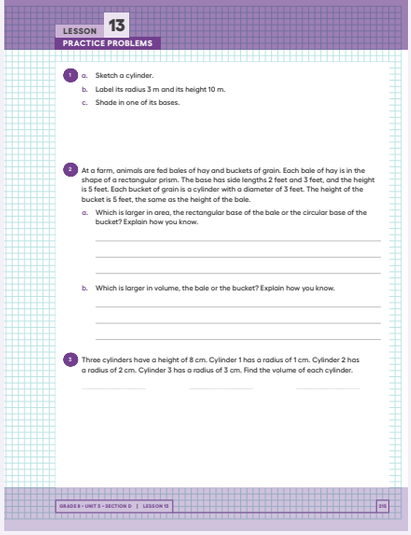
If most students struggle to calculate the volume of the cylinder, focus on these calculations as opportunities arise over the next several lessons. For example, in the activity referred to here, add a row to the table with height and radius dimensions of 7 cm and 4 cm, respectively. Tell students to start with this new row before they begin work on the rest of the table.

Grade 8, Unit 5, Lesson 14, Activity 2
Cylinders with Unknown Dimensions

Practice Problems

6 Problems

Student Workbook



LESSON 13
PRACTICE PROBLEMS

1 a. Sketch a cylinder.
b. Label its radius 3 m and its height 10 m.
c. Shade in one of its bases.

2 At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape of a rectangular prism. The base has side lengths 2 feet and 3 feet, and the height is 5 feet. Each bucket of grain is a cylinder with a diameter of 3 feet. The height of the bucket is 5 feet, the same as the height of the bale.
a. Which is larger in area, the rectangular base of the bale or the circular base of the bucket? Explain how you know.
b. Which is larger in volume, the bale or the bucket? Explain how you know.

3 Three cylinders have a height of 8 cm. Cylinder 1 has a radius of 1 cm. Cylinder 2 has a radius of 2 cm. Cylinder 3 has a radius of 3 cm. Find the volume of each cylinder.

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Problem 1

a. Sketch a cylinder.

b. Label its radius 3 m and its height 10 m.

c. Shade in one of its bases.

Answers vary.

Problem 2

At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape of a rectangular prism. The base has side lengths 2 feet and 3 feet, and the height is 5 feet. Each bucket of grain is a cylinder with a diameter of 3 feet. The height of the bucket is 5 feet, the same as the height of the bale.

a. Which is larger in area, the rectangular base of the bale or the circular base of the bucket? Explain how you know.

The bucket's base

Sample reasoning: The area of the bale's base is 6 square feet. The area of the bucket's base is just over 7 square feet, because $\pi(1.5)^2 \approx 7.07$.

b. Which is larger in volume, the bale or the bucket? Explain how you know.

The bucket

Sample reasoning: The bale and the bucket have the same height, and the bucket's base area is larger.

Problem 3

Three cylinders have a height of 8 cm. Cylinder 1 has a radius of 1 cm. Cylinder 2 has a radius of 2 cm. Cylinder 3 has a radius of 3 cm. Find the volume of each cylinder.

- Cylinder 1 has a volume of $8\pi \approx 25.13$ cm^3.
- Cylinder 2 has a volume of $32\pi \approx 100.53$ cm^3.
- Cylinder 3 has a volume of $72\pi \approx 226.19$ cm^3.

Problem 4

from Unit 5, Lesson 12

A 1-quart container of tomato soup is shaped like a rectangular prism. A soup bowl shaped like a hemisphere can hold 8 oz of liquid. How many bowls will the soup container fill? Recall that 1 quart is equivalent to 32 fluid ounces (oz).

4 bowls

Problem 5

Match each set of information about a circle with the area of that circle.

4 **A.** Circle A has a radius of 4 units.

2 **B.** Circle B has a radius of 10 units.

3 **C.** Circle C has a diameter of 16 units.

1 **D.** Circle D has a circumference of 4π units.

1. 4π square units

2. approximately 314 square units

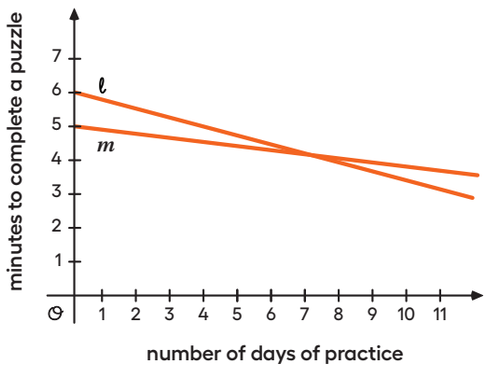
3. 64π square units

4. 16π square units

Problem 6

from Unit 5, Lesson 8

Two students join a puzzle solving club and get faster at finishing the puzzles as they get more practice. Student A improves their times faster than Student B.



a. Match the students to the lines ℓ and m.

Student A is represented by line ℓ. Student B is represented by line m.

b. Which student was faster at puzzle solving before practice?

Student B

Student Workbook