Lesson 9

Finding Unknown Side Lengths

Goals

- Calculate unknown side lengths of a right triangle by using the Pythagorean Theorem, and explain (orally) the solution method.
- Label the "legs" and "hypotenuse" on a diagram of a right triangle.

Learning Targets

- If I know the lengths of two sides, I can find the length of the third side in a right triangle.
- When I have a right triangle, I can identify which side is the hypotenuse and which sides are the legs.

Lesson Narrative

In this lesson, students practice applying the Pythagorean Theorem to find unknown side lengths of right triangles. They begin by examining four equations of the type they will encounter later in the lesson, noting any similarities and differences. Students continue by studying a series of triangles and determining which side of each right triangle is the hypotenuse, reinforcing the idea that only right triangles have a hypotenuse and refining students' precision of language. In the last activity, students practice finding the unknown leg or hypotenuse given drawings or a written description.

Student Learning Goal

Let's find missing side lengths of right triangles.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- Which Three Go Together?

Required Materials

Materials to Gather

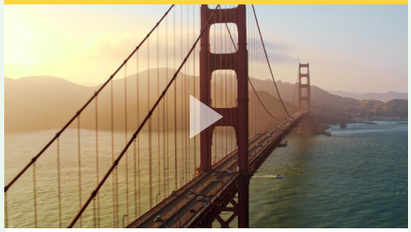
- Blank paper: Lesson

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	5 min	20 min	10 min	5 min

Inspire Math

Golden Gate Bridge video



Go Online
Before the lesson, show this video to **reinforce** the real-world connection.

ilclass.com/l/614221

Please log in to the site before using the QR code or URL.

Instructional Routines

Which Three Go Together?

ilclass.com/r/10690736

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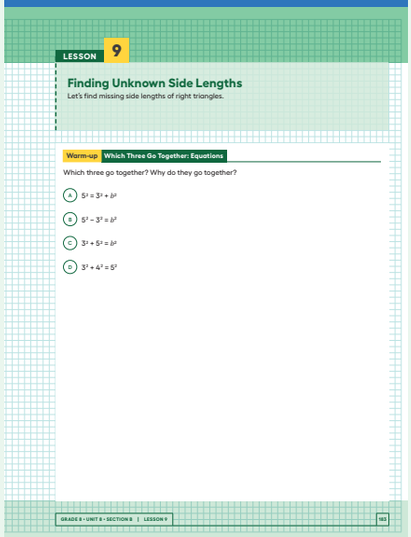
Student Workbook


LESSON 9

Finding Unknown Side Lengths

Let's find missing side lengths of right triangles.

Warm-up Which Three Go Together: Equations

Which three go together? Why do they go together?

A. $5^2 = 3^2 + b^2$

B. $5^2 - 3^2 = b^2$

C. $3^2 + 5^2 = b^2$

D. $3^2 + 4^2 = 5^2$

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Warm-up

Which Three Go Together: Equations

5 min

Activity Narrative

This *Warm-up* prompts students to compare four equations that may arise while using the Pythagorean Theorem. It gives students a reason to use language precisely and it gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the equations in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the equations for all to see.

Give students 1 minute of quiet think time and ask them to indicate when they have noticed three equations that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $5^2 = 3^2 + b^2$

B. $5^2 - 3^2 = b^2$

C. $3^2 + 5^2 = b^2$

D. $3^2 + 4^2 = 5^2$

Sample responses:

A, B, and C go together because:

- They all have a variable (b).
- They all have the terms 3^2, 5^2, and b^2.

A, B, and D go together because:

- They are all based on the 3-4-5 Pythagorean triple.

A, C, and D go together because:

- They don't use subtraction.

B, C, and D go together because:

- They all have two terms on the left side of the equal sign and one term on the right.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as "variable" or "term," and to clarify their reasoning as needed. Consider asking:

- *"How do you know … ?"*
- *"What do you mean by … ?"*
- *"Can you say that in another way?"*

If time allows, invite 2–3 students to share what they notice all of the equations have in common (they all have three terms, all terms are a value squared). The purpose of this concluding share out is to reinforce that all 4 equations show the relationship between three squared values but with different orderings. Paying attention to order while using the Pythagorean Theorem is a focus of later activities.

Activity 1

Which One Is the Hypotenuse?

5 min

Activity Narrative

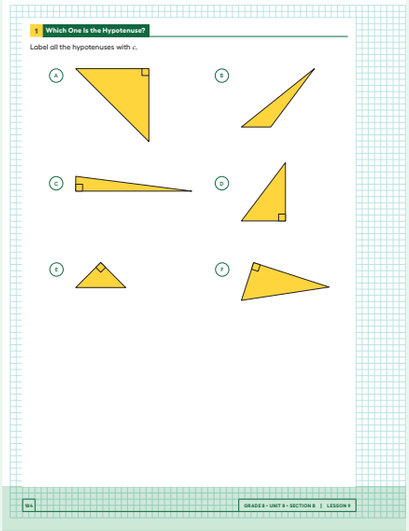
The purpose of this activity is for students to identify the hypotenuse in right triangles in different orientations.

Launch

Arrange students in groups of 2.

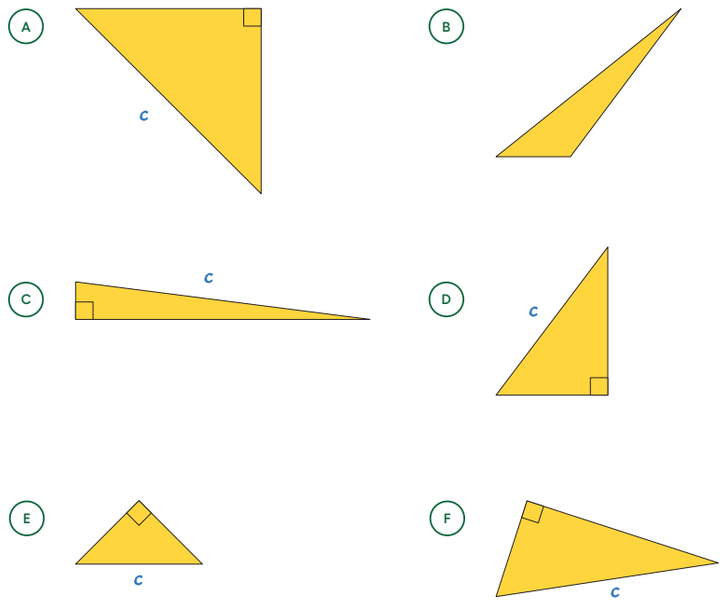
Give students 1 minute of quiet time to work the problem, and then pause for a partner discussion. Follow with a whole-class discussion.

Student Workbook



Student Task Statement

Label all the hypotenuses with c.



Triangle B is not a right triangle, therefore it does not have a hypotenuse.

Activity Synthesis

The goal of this discussion is to make sure students understand that only right triangles have a hypotenuse, and that in a right triangle the hypotenuse is always the side opposite the right angle.

Here are some questions for discussion:

- *"In a right triangle, does it matter which side is a and which is b?"*

  a and b must be the two shorter sides of the right triangle, but it does not matter which side is a or b.

- *"How do you know the side you selected is the hypotenuse?"*

  It is the longest side. It is the side opposite the right angle.

- *"Why does Triangle B not have a hypotenuse?"*

  It is not a right triangle.

Activity 2
Find the Missing Side Lengths

20 min

Activity Narrative

The purpose of this activity is to give students practice finding missing side lengths in a right triangle using the Pythagorean Theorem.

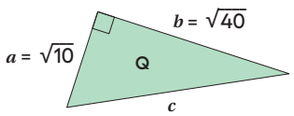
Launch

Arrange students in groups of 2.

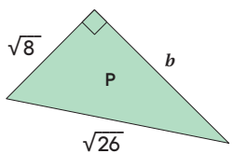
Give students 8–10 minutes of quiet work time followed by a partner discussion. Then follow with a whole-class discussion.

Student Task Statement

1. Find c. $\sqrt{50}$



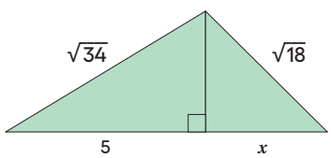
2. Find b. $\sqrt{18}$



3. A right triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse? $\sqrt{48.01}$ cm

4. A right triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side? $\sqrt{\frac{7}{144}}$

5. Find the value of x in the figure. 3



Instructional Routines

MLR3: Critique, Correct, Clarify

ilclass.com/r/10695504

Please log in to the site before using the QR code or URL.

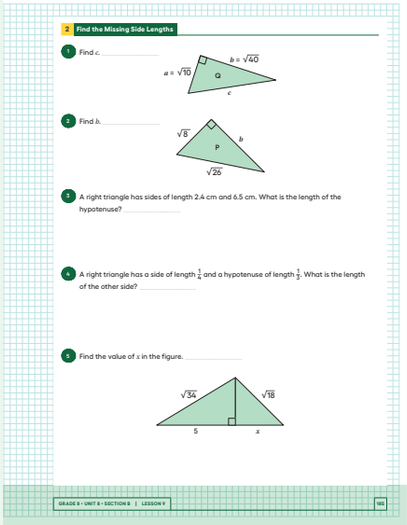
Access for Multilingual Learners (Activity 2)

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

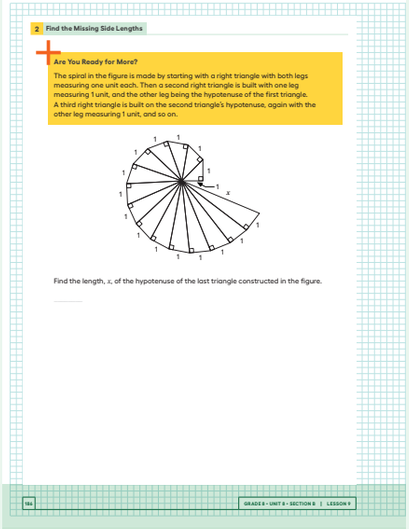
Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Provide Access for Physical Action. Activate or supply background knowledge. Provide or display the Pythagorean Theorem with a labeled diagram for students to use as a reference.
Supports accessibility for: Memory, Organization

Student Workbook



Student Workbook


2 Find the Missing Side Lengths

Are You Ready for More?

The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring 1 unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring 1 unit, and so on.

Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

Are You Ready for More?

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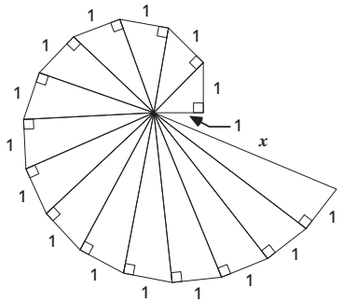


Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

We can repeatedly apply the Pythagorean Theorem. The first hypotenuse equals $\sqrt{2}$, since $(\sqrt{2})^2 = 1^2 + 1^2$. The second right triangle has legs 1 and $\sqrt{2}$, so has a hypotenuse of $\sqrt{3}$, since $(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$. This pattern continues, with the next hypotenuses having length $\sqrt{4}$, $\sqrt{5}$, etc. By counting until the end, we find that the 15th and last hypotenuse has a length x equal to $\sqrt{16}$, so $x = 4$.

Activity Synthesis

The purpose of this discussion is for students to share how they calculated the unknown side lengths. Ask students to share their answers and reasoning for the first two questions. For the third question with sides of length 2.4 cm and 6.5 cm, display any triangles students drew for all to see, noting any differences. For example, students may have drawn triangles with different orientations or labeled different sides as a and b.

Then use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response for finding the missing side length for the triangle with known side length $\frac{1}{4}$ and hypotenuse $\frac{1}{3}$ by correcting errors, clarifying meaning, and adding details.

Draw a right triangle with legs labeled $\frac{1}{4}$ and $\frac{1}{3}$. Then display this first draft next to it: "I know that $a = \frac{1}{4}$ and $b = \frac{1}{3}$, so when I use the Pythagorean Theorem, I get the equation $\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2 = c^2$."

Ask,

💬 *"What parts of this response are unclear, incorrect, or incomplete?"*

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2–4 minutes to work with a partner to revise the first draft.

Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

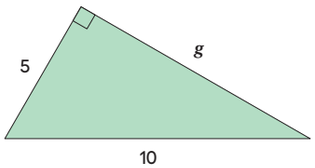
Point out that when two sides of a right triangle are known, the third can always be found by using the Pythagorean Theorem. Remind students it is important to keep track of which side is the hypotenuse.

Lesson Synthesis

The purpose of this discussion is to check that students understand the Pythagorean Theorem and how it can be used to determine information about the side lengths of right triangles. Give each student a blank sheet of paper and ask them to draw a right triangle and label 2 of the 3 sides. Tell students to swap triangles with another student, solve for the missing length, then swap back to check the other person's work. Select a few groups to share their triangles and, if possible, display them for all to see while sharing how they solved for the unknown length.

Lesson Summary

The Pythagorean Theorem can be used to find an unknown side length in a right triangle as long as the length of the other two sides is known. For example, here is a right triangle, where one leg has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other leg is represented by g.



Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse, the side opposite the right angle. For this triangle, the hypotenuse is 10.

$$a^2 + b^2 = c^2$$
$$5^2 + g^2 = 10^2$$
$$g^2 = 10^2 - 5^2$$
$$g^2 = 100 - 25$$
$$g^2 = 75$$
$$g = \sqrt{75}$$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

Cool-down

Could Be the Hypotenuse, Could Be a Leg

5 min

Student Task Statement

A right triangle has sides of length 3, 4, and x.

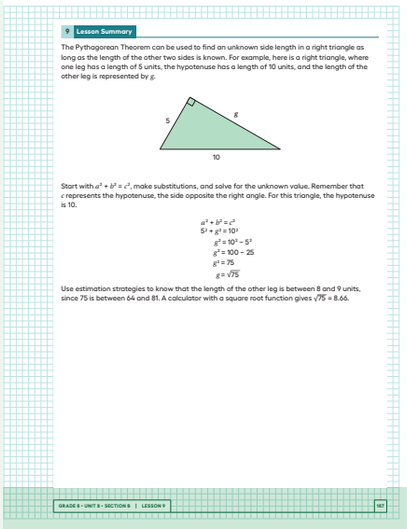
1. Find x if it is the hypotenuse.

$x = \sqrt{25}$ or $x = 5$

2. Find x if it is one of the legs.

$x = \sqrt{7}$

Student Workbook


9 Lesson Summary

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5 g 10

Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse, the side opposite the right angle. For this triangle, the hypotenuse is 10.

$a^2 + b^2 = c^2$
$5^2 + g^2 = 10^2$
$g^2 = 10^2 - 5^2$
$g^2 = 100 - 25$
$g^2 = 75$
$g = \sqrt{75}$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

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Responding To Student Thinking

Points to Emphasize

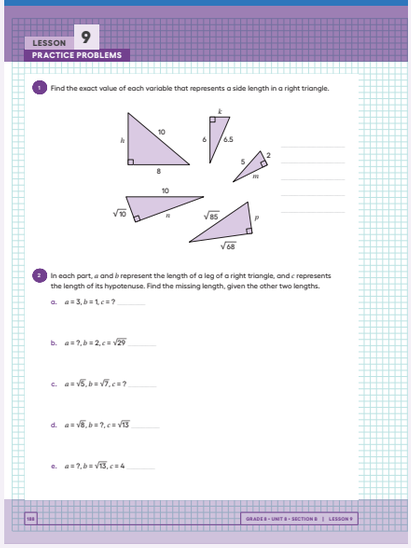
If most students struggle with finding unknown side lengths, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about how right triangles are formed using legs and the hypotenuse.

Unit 8, Lesson 10, Activity 3
Calculating Legs of Right Triangles

Practice Problems

6 Problems

Student Workbook


LESSON 9
PRACTICE PROBLEMS

1 Find the exact value of each variable that represents a side length in a right triangle.

2 In each part, a and b represent the length of a leg of a right triangle, and c represents the length of its hypotenuse. Find the missing length, given the other two lengths.

a. $a = 3, b = 1, c = ?$

b. $a = ?, b = 2, c = \sqrt{29}$

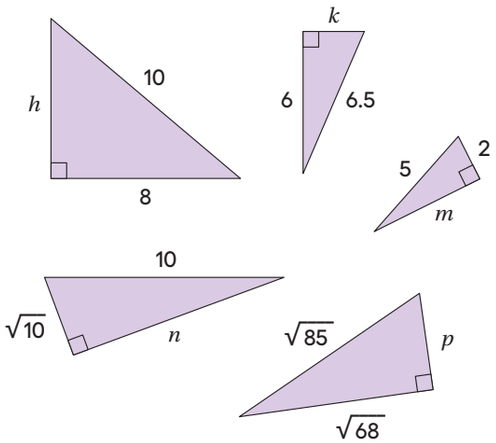
c. $a = \sqrt{5}, b = \sqrt{7}, c = ?$

d. $a = \sqrt{8}, b = ?, c = \sqrt{13}$

e. $a = ?, b = \sqrt{13}, c = 4$

Problem 1

Find the exact value of each variable that represents a side length in a right triangle.



$h = 6$, because $100 - 64 = 36$ and $\sqrt{36} = 6$

$k = 2.5$, because $42.25 - 36 = 6.25$ and $\sqrt{6.25} = 2.5$

$m = \sqrt{21}$, because $25 - 4 = 21$

$n = \sqrt{90}$, because $100 - 10 = 90$

$p = \sqrt{17}$, because $85 - 68 = 17$

Problem 2

In each part, a and b represent the length of a leg of a right triangle, and c represents the length of its hypotenuse. Find the missing length, given the other two lengths.

a. $a = 3, b = 1, c = ?$

$c = \sqrt{10}$

b. $a = ?, b = 2, c = \sqrt{29}$

$a = 5$

c. $a = \sqrt{5}, b = \sqrt{7}, c = ?$

$c = \sqrt{12}$

d. $a = \sqrt{8}, b = ?, c = \sqrt{13}$

$b = \sqrt{5}$

e. $a = ?, b = \sqrt{13}, c = 4$

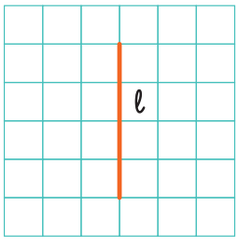
$a = \sqrt{3}$

Problem 3

from Unit 8, Lesson 8

What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)

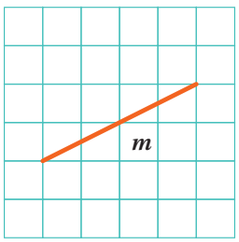
a.



4 units

Sample reasoning: The segment is along the grid lines, so the squares can be counted.

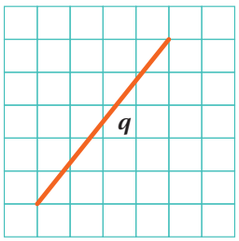
b.



$\sqrt{20}$ units

Sample reasoning: $4^2 + 2^2 = 20$

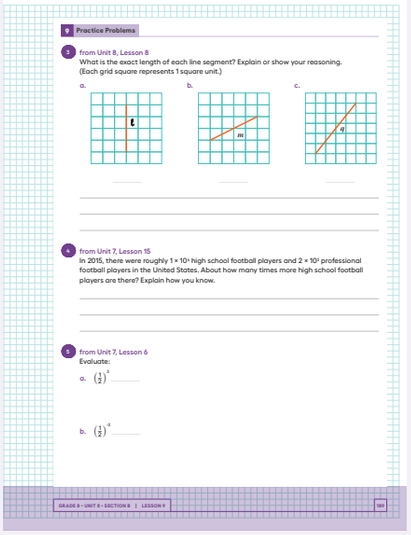
c.



$\sqrt{41}$ units

Sample reasoning: $4^2 + 5^2 = 41$

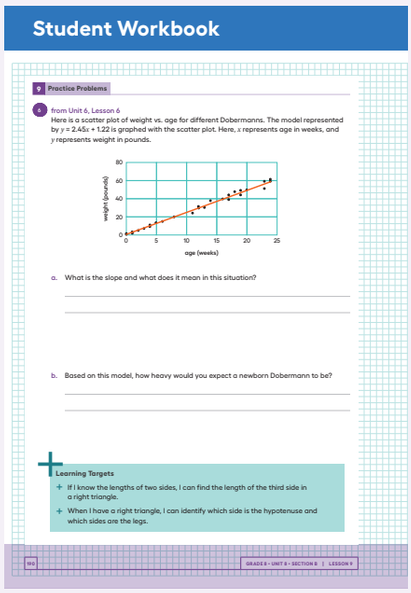
Student Workbook


9 Practice Problems

3 from Unit 8, Lesson 8
What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)
a. b. c.

4 from Unit 7, Lesson 15
In 2015, there were roughly 1×10^6 high school football players and 2×10^5 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

5 from Unit 7, Lesson 6
Evaluate:
a. $\left(\frac{1}{2}\right)^5$
b. $\left(\frac{1}{2}\right)^4$


Student Workbook

Practice Problems

4 from Unit 6, Lesson 6

Here is a scatter plot of weight vs. age for different Dobermanns. The model represented by $y = 2.45x + 1.22$ is graphed with the scatter plot. Here, x represents age in weeks, and y represents weight in pounds.

a. What is the slope and what does it mean in this situation?

b. Based on this model, how heavy would you expect a newborn Dobermann to be?

Learning Targets

+ If I know the lengths of two sides, I can find the length of the third side in a right triangle.
+ When I have a right triangle, I can identify which side is the hypotenuse and which sides are the legs.

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Problem 4

from Unit 7, Lesson 15

In 2015, there were roughly 1×10^6 high school football players and 2×10^3 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

There are approximately 500 times more high school football players.

Sample reasoning: $\frac{1 \times 10^6}{2 \times 10^3} = 0.5 \times 10^3 = 5 \times 10^2$

Problem 5

from Unit 7, Lesson 6

Evaluate:

a. $\left(\frac{1}{2}\right)^3$

$\frac{1}{8}$

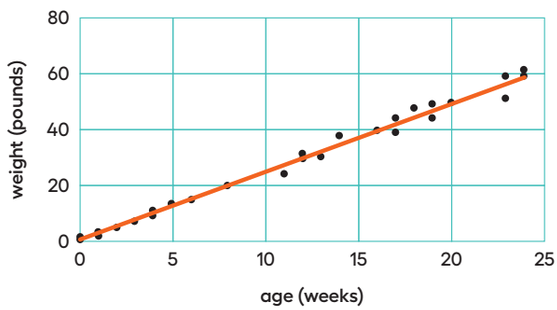
b. $\left(\frac{1}{2}\right)^{-3}$

8

Problem 6

from Unit 6, Lesson 6

Here is a scatter plot of weight vs. age for different Dobermanns.
The model represented by $y = 2.45x + 1.22$ is graphed with the scatter plot.
Here, x represents age in weeks, and y represents weight in pounds.



a. What is the slope and what does it mean in this situation?

Sample response: The slope is 2.45. It means that a Dobermann can be expected to gain 2.45 pounds per week.

b. Based on this model, how heavy would you expect a newborn Dobermann to be?

Sample response: 1.22 pounds (the y-intercept of the function).