# **Estimating Population Proportions**

#### Goals **Learning Target**

- Compare (orally) proportions for the same category from different samples of a population.
- Comprehend that the term "proportion" refers to a number between 0 and 1 that represents the fraction of the data within a certain category.
- Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain (orally and in writing) the reasoning.

I can estimate the proportion of population data that are in a certain category based on a sample.

### **Access for Students with Diverse Abilities**

- Action and Expression (Activity 1)
- Representation (Activity 2)

### **Access for Multilingual Learners**

• MLR1: Stronger and Clearer Each Time (Activity 2)

#### **Instructional Routines**

- MLR1: Stronger and Clearer Each Time (Activity 2)
- · Notice and Wonder

#### **Required Materials**

#### **Materials to Gather**

· Paper bags: Activity 1

### **Materials to Copy**

· Reaction Times Cutouts (1 copy for every 4 students): Activity 1

### **Required Preparation**

#### **Activity 1:**

Prepare one set of slips from the blackline master in a paper bag for every 2 students.

# **Lesson Narrative**

In this lesson, students estimate population proportions based on data from a sample. The term **proportion** is used in statistics to refer to a number from 0 to 1 that represents the fraction of data belonging to a given category.

Students see that if a sample is representative of the population, then we can use proportional reasoning to make predictions about the population. However, students construct an argument that, due to sampling variability, these predictions are estimates, unlike the exact answers they get when working with actual proportional relationships.

The activity about examining a distribution of proportions from many different samples is included as an optional opportunity to deepen students' understanding of sampling variability.

# **Student Learning Goal**

Let's estimate population proportions using samples.

## **Lesson Timeline**

Warm-up

15

**Activity 1** 

15

**Activity 2** 

10

**Activity 3** 

10

**Lesson Synthesis** 

**Assessment** 

Cool-down



### Warm-up

# **Getting to School**

# 5 min

# **Activity Narrative**

The purpose of this *Warm-up* is for students to compute the fraction of individuals whose responses fall in a specified category. This activity gives students time to think about how to compute these fractions from categorical data.

For the last two questions, students may debate whether to include the 10-minute times or not. According to the wording of the question asked, it does ask for more-than-10-minute times, so maybe exactly 10 minutes should not count (because 10 is not greater than 10). On the other hand, all of the values are listed as whole numbers, so a student who takes 10 minutes and 1 second to get to school may have rounded down to 10, but should have been counted. Noticing the large difference in answers for the third question, it may be worth clarifying the data in this instance, even for an estimate.

Monitor for students who include the 10-minute times for the last two questions as well as those who do not.

# Launch

Give students 2 minutes of quiet work time, and follow with a whole-class discussion.

### **Student Task Statement**

A teacher asks all the students in one class how many minutes it takes them to get to school. Here is a list of their responses:

20	10	15	8	5	15	10	5
20	5	15	10	3	10	18	5
25	5	5	12	10	30	5	10

- 1. What fraction of the students in this class say that:
  - a. it takes them 5 minutes to get to school?

b.it takes them more than 10 minutes to get to school?

$$\frac{9}{24}$$
 (or equivalent) or  $\frac{15}{24}$  (or equivalent) if 10 minutes is included

**2.** The whole school has 720 students. Use this data to estimate how many of them would say that it takes them more than 10 minutes to get to school.

Be prepared to explain your reasoning.

About 270 students (or 450 students if IO minutes is included) since  $\frac{9}{24}$  of 720 is 270 (or  $\frac{15}{24}$  of 720 is 450)

# **Activity Synthesis**

Select students to share their methods for computing the solutions. Include previously identified students who did or did not include the 10-minute values in their calculations.

If it does not arise during the discussion, explain that answering the last question with the data at hand is only accurate if the sample data is representative of the school. For example, it is possible that the class happens to contain only students who get a ride to school, but much of the school rides the bus.

# **Activity 1**

### **Reaction Times**



### **Activity Narrative**

In previous lessons, students examined the estimation of the mean and median for populations using data from a sample. In this activity, students apply similar reasoning to estimating the proportion of a population that matches certain characteristics. Students collect a sample of 20 reaction times and compute the fraction of responses in their sample that are in a given range.

Then, in the discussion, students explain why some other values might be reasonable or unreasonable based on the sample of data they collected. They also compare their estimations to the known population proportion and use the class's proportions to gauge the accuracy of their estimate.

# Launch 22

Arrange students in groups of 2. Distribute bags of slips cut from the blackline master.

Tell students that, in statistics, a **proportion** is a number between 0 and 1 that represents the fraction of the data that fits into the desired category. For example, with the data {yes, yes, yes, no, maybe}, the proportion of yes answers is  $\frac{3}{5}$  or 0.6.

Introduce the context: All 120 seniors at a high school are asked to click a button as soon as they notice a box change color, and the response time is recorded in seconds. These 120 response times represent the population for this activity. Their responses are written on the slips of paper in the bag.

When selecting a sample of 20, each value does not need to be replaced before taking the next one.

Allow students 10 minutes of partner work time and follow with a whole-class discussion.

Warm-up

# Access for Students with Diverse Abilities (Activity 1, Student Task)

# Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem solving, chunk this task into more manageable parts. For example, check in with students after 3–5 minutes of work time.

Supports accessibility for: Organization, Attention

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# **Student Task Statement**

The track coach at a high school needs a student whose reaction time is less than 0.4 seconds to help out at track meets. All the twelfth graders in the school measured their reaction times. Your teacher will give you a bag of papers that list their results.

**1.** Work with your partner to select a random sample of 20 reaction times, and record them in the table.

### Sample response:

0.50	0.44	0.51	0.38	0.42	0.79,	0.39	0.46	0.34	0.30
0.36	0.41	0.31	0.82	0.35	0.36	0.48	0.72	0.74	0.45

2. What **proportion** of your sample is less than 0.4 seconds?

$$\frac{8}{20} = 0.4$$

**3.** Estimate the proportion of all twelfth graders at this school who have a reaction time of less than 0.4 seconds. Explain your reasoning.

0.4

Sample reasoning: The sample was chosen at random and is likely representative.

**4.** There are 120 twelfth graders at this school. Estimate how many of them have a reaction time of less than 0.4 seconds.

48 twelfth graders, since 0.4 • 120 = 48

- **5.** Suppose another group in your class comes up with a different estimate than yours for the previous question.
  - a. What is another estimate that would be reasonable?

50 twelfth graders

**b.** What is an estimate you would consider unreasonable?

100 twelfth graders

# **Activity Synthesis**

The purpose of the discussion is for students to see how multiple sample proportions can help revise their estimates and give an idea of how accurate the individual estimates from samples might be.

Ask the groups to share the proportion from their sample that had fast reaction times, and display the results for all to see.

Some questions for discussion:

"Why did you select the values that you did for reasonable and unreasonable estimates?"

It is reasonable for estimates to be close to the value that we got. Because we only had 20 values, I think it is reasonable that another estimate would be within about 10 or so of our result. It is unreasonable that the value would be very different from the value we got, so a very large or small number would not be reasonable.

"Using the class's data, how accurate do you think your group's estimate is? Explain your reasoning."

Answers vary. Students should mention the variability of the proportions from the samples influencing the accuracy of the estimate.

The actual proportion for this population is 0.5. How close was your estimate? Explain why your estimate was not exactly the same."

Answers vary. Each sample might be slightly different since they do not include all of the values, but they should be close.

(i) "If each group had 40 reaction times in their samples instead of 20, do you think the estimate would be more or less accurate?"

The estimate should be more accurate since there is more information available.

### **Activity 2**

# A New Comic Book Hero

**15** min

### **Activity Narrative**

In the previous activity, students collected their own sample and computed an estimate for the population proportion based on the sample. In this activity, students use a different context to practice exploring proportions from samples and their extension to populations. Students must construct an argument for why the character could have the ability to fly using the data to support their argument.

### **Instructional Routines**

# Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.



#### **Instructional Routines**

# MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479

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# Access for Multilingual Learners (Activity 2)

# MLR1: Stronger and Clearer Each Time.

This activity uses the Stronger and Clearer Each Time math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

# Launch

Keep students in groups of 2.

Tell students that three comic books, *The Adventures of Super Sam, Beyond Human, and Mysterious Planets*, are all planning to add a new superhero to their stories. A survey was sent to dedicated readers of each series to ask what type of ability the new hero should have: fly, freeze, strength, or invisibility.

Display the tables from the Task Statement for all to see. Ask students,

○ "What do you notice? What do you wonder?"

### Students may notice:

- There are 4 different responses: fly, freeze, strength, and invisibility.
- The number of responses for each of the 4 different responses.
- There are 20 responses.

### Students may wonder:

- Will the decision for the new hero's power be based only on this survey?
- · What proportion chose each of the different powers?
- Is this sample of 20 responses representative of the population?

Give students 5–7 minutes of partner work time followed by a whole-class discussion.

# **Student Task Statement**

Here are the results of a survey of 20 people who read *The Adventures of Super Sam* regarding what special ability they think the new hero should have.

fly	freeze	freeze	fly	fly
freeze	fly	strength	freeze	fly
freeze	freeze	fly	invisibility	freeze
fly	freeze	fly	strength	freeze

1. What proportion of this sample wants the new hero to have the ability to fly?

$$\frac{8}{20} = 0.4$$

**2.** If there are 2,024 dedicated readers of *The Adventures of Super Sam*, estimate the number of readers who want the new hero to fly.

Two other comic books did a similar survey of their readers.

- In a survey of people who read *Beyond Human*, 42 out of 60 people want a new hero to be able to fly.
- In a survey of people who read *Mysterious Planets*, 14 out of 40 people want a new hero to be able to fly.

About 810 since 0.4 • 2,024 ≈ 810

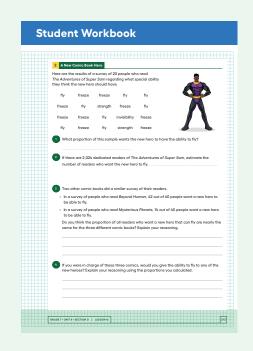
**3.** Do you think the proportion of all readers who want a new hero that can fly are nearly the same for the three different comic books? Explain your reasoning.

The proportion for Mysterious Planets seems close to the proportion for The Adventures of Super Sam but not close to the proportion for Beyond Human. Sample reasoning: The proportion for Mysterious Planets is 0.35, which is close to the 0.4 for The Adventures of Super Sam, while the proportion for Beyond Human is 0.7.

**4.** If you were in charge of these three comics, would you give the ability to fly to any of the new heroes? Explain your reasoning using the proportions you calculated.

## Sample responses:

- I would give the new heroes of all of the comics the ability to fly. Based on these results, about 40% of the readers of Mysterious Planets and The Adventures of Super Sam want the hero to have that new ability, so I think it would satisfy many of their readers. About 70% of the readers of Beyond Human want this new ability, so they will most likely be happy with this choice.
- I would only give the new hero of Beyond Human the ability to fly. In
  the data for The Adventures of Super Sam, more people voted for
  freeze than fly, so I don't think it would make sense to choose flight
  for that comic rather than freeze. It is hard to tell for Mysterious
  Planets without the actual data. The votes might have been split
  among the other choices more evenly and flight might have been the
  most popular choice.



# Access for Students with Diverse Abilities (Activity 2, Synthesis)

# Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, color code each power in the same color to highlight the proportions.

Supports accessibility for: Visual-Spatial Processing

# **Activity Synthesis**

Use Stronger and Clearer Each Time to give students an opportunity to revise and refine their response to "Would you give the ability to fly to any of the new heroes?" In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "\_\_\_\_ makes sense, but what do you mean when you say ...?"
- "What evidence do you have?"
- "How do you know ...? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

After Stronger and Clearer Each Time, ask,

"Explain why you think a sample was used instead of the population for this situation."

The population is too large to ask all of the readers about their preference. Also, the authors may want the power of the new hero to be a surprise to some readers, so they want to get some information without telling everyone about what is coming.

"Although the proportion of responses from the Mysterious Planets sample who chose flight is only 0.35, it is the most popular choice (freeze, strength, and invisibility split the remaining votes). Does this information help to decide whether to give the new hero the power of flight?"

Yes, because it is the most popular, it might be the best choice. Or it may not be the best choice because more than half of the readers prefer something else. It might help to have another question that directly asks whether they would like the new hero to fly.

"Of the three estimates, which do you think is most accurate? Explain your reasoning."

Beyond Human has the largest sample, so it may be the most accurate.

# **Activity 3: Optional**

# Flying to the Shelves



### **Activity Narrative**

This optional activity goes beyond grade-level expectations to deepen students' understanding of sampling variability.

This activity continues the comic book context introduced in the previous activity. There is not a measure of variability such as MAD or IQR for proportions since the data are categorical rather than quantitative, so other methods must be employed to determine the accuracy of an estimate. Students look at dot plots showing the results from multiple samples to gauge the accuracy of the estimates for population proportions. This activity will provide a foundation for work in later grades.

# Launch 22

Arrange students in groups of 2.

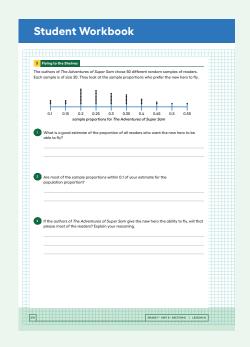
Help students make sense of the dot plot. Each dot in the dot plot represents the proportion from a random sample of 20 readers. 50 random samples were taken, and the 50 proportions are plotted on the dot plot. For example, the proportion of 0.4 from the previous activity would be represented by one of the 4 dots at 0.4 on the first dot plot.

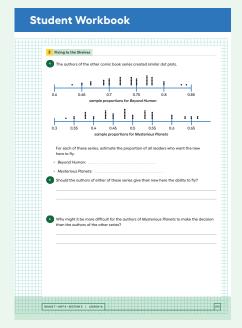
Ask,

"Are any of the sample proportions greater than or equal to 0.5? What does this mean?"

Yes, 2 dots are greater than or equal to 0.5. This means that in those samples, at least half of the people prefer the new hero to have the ability to fly.

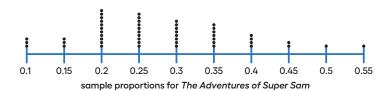
Give students 5–7 minutes of partner work time followed by a whole-class discussion.





# **Student Task Statement**

The authors of *The Adventures of Super Sam* chose 50 different random samples of readers. Each sample is of size 20. They look at the sample proportions who prefer the new hero to fly.



**1.** What is a good estimate of the proportion of all readers who want the new hero to be able to fly?

Sample response: 0.3 since the center of the distribution is near there.

**2.** Are most of the sample proportions within 0.1 of your estimate for the population proportion?

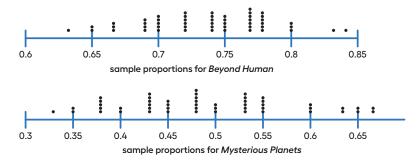
Yes

**3.** If the authors of *The Adventures of Super Sam* give the new hero the ability to fly, will that please most of the readers? Explain your reasoning.

No

Sample reasoning: Only about 30% of readers seem to want the new hero to fly, so there may be a more preferred super power.

4. The authors of the other comic book series created similar dot plots.



For each of these series, estimate the proportion of all readers who want the new hero to fly.

### Sample responses:

• Beyond Human: about 0.73

• Mysterious Planets: about 0.5

**5.** Should the authors of either of these series give their new hero the ability to fly?

Beyond Human should give the power to fly to its new hero since almost  $\frac{3}{4}$  voted for that option.

**6.** Why might it be more difficult for the authors of Mysterious Planets to make the decision than the authors of the other series?

Not only do we estimate the population proportion to be about 0.5, but the values are quite varied among all the sample proportions.

# **Are You Ready for More?**

Draw an example of a dot plot with at least 20 dots that represent the sample proportions for different random samples that would indicate that the population proportion is above 0.6, but there is a lot of uncertainty about that estimate.

Responses should include a majority of the dots greater than or near 0.6, but there should be a lot of variability in the sample proportions.

### **Activity Synthesis**

The purpose of the discussion is for students to talk about how the variability in sample proportions affects their trust in the estimates of the population proportion.

Consider these questions for discussion:

- "When estimating a population mean or median from a random sample, measures of variability from a sample can be used to help gauge the accuracy of the estimate. With proportions there is not a measure of variability in the same way. How did the information in this activity guide your thoughts about the accuracy of the population estimate?"
  - Many samples were taken and their proportions were computed. The variability of these sample proportions showed how much to trust the population estimate.
- "How does the distribution of values in the dot plots of sample proportions affect your trust in an estimate of population proportion?"
  - The more variability, the less certainty in the estimate.
- "How would the distributions change if the number of responses in each sample were increased?"

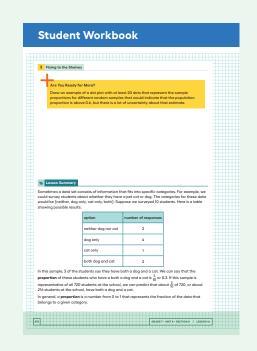
The center should remain about the same and the variability should decrease. In other words, the dots in the dot plot should get closer together towards the center.

### **Lesson Synthesis**

Consider asking these discussion questions to clarify the main ideas of the lesson:

- "When using data, what is a proportion? How is it calculated?"
  - A proportion is the fraction of the data that are in a certain category. It is calculated by counting the number of data values in the category and dividing by the total number of data values in the sample.
- "In order to say that more than half of the people in a sample responded with a certain answer, what would the proportion for that answer be?" any value greater than 0.5
- "A random sample indicates that a 0.45 proportion of people shopping at a certain store prefer wheat bread to white bread. The store has 3,000 customers. Estimate the number of people shopping at the store who prefer wheat bread."

about 1,350 people, since 0.45 • 3,000 = 1,350



### **Responding To Student Thinking**

#### **More Chances**

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

# **Lesson Summary**

Sometimes a data set consists of information that fits into specific categories. For example, we could survey students about whether they have a pet cat or dog. The categories for these data would be {neither, dog only, cat only, both}. Suppose we surveyed 10 students. Here is a table showing possible results:

option	number of responses
neither dog nor cat	2
dog only	4
cat only	1
both dog and cat	3

In this sample, 3 of the students say they have both a dog and a cat. We can say that the **proportion** of these students who have a both a dog and a cat is  $\frac{3}{10}$  or 0.3. If this sample is representative of all 720 students at the school, we can predict that about  $\frac{3}{10}$  of 720, or about 216 students at the school, have both a dog and a cat.

In general, a **proportion** is a number from 0 to 1 that represents the fraction of the data that belongs to a given category.

## Cool-down

# More than 48 Grams

5 mir

# **Student Task Statement**

A chemical engineer is trying to increase the amount of the useful product in a reaction. She performs the reaction with her new equipment 10 times and gets the following amounts of the useful product in grams:

47.1 48.2 48.3 47.5 48.5 48.1 47.2 48.2 48.4 48.3

- What proportion of the reactions are above the 48 grams threshold?
   7, since 7 of the 10 reactions have more than 48 grams of the useful product
- **2.** Other chemists typically get 65% of their reactions to produce more than 48 grams. Should the engineer say that she is able to increase the useful product when compared to the other chemists?

Sample response: She could be optimistic, but her proportion does not seem far from what others have done. She should run more reactions to be more sure of the improvement. With only IO values in her data set, 0.7 (and 0.6) is as close to 0.65 as she could get.

### **Practice Problems**

5 Problems

### **Problem 1**

Tyler wonders what proportion of students at his school would dye their hair blue if they were allowed to. He surveyed a random sample of 10 students at his school, and 2 of them said they would. Kiran didn't think Tyler's estimate was very accurate, so he surveyed a random sample of 100 students, and 17 of them said they would.

**a.** Based on Tyler's sample, estimate what proportion of the students would dye their hair blue.

0.20

**b.** Based on Kiran's sample, estimate what proportion of the students would dye their hair blue.

0.17

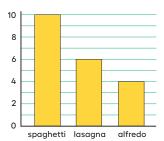
**c.** Whose estimate is more accurate? Explain how you know.

Kiran's estimate is probably more accurate because he used a much larger sample than Tyler.

Sample reasoning: Sample proportions from larger samples tend to be more tightly clustered around that value of the population proportion. It is still possible for Tyler's estimate to be more accurate, coincidentally.

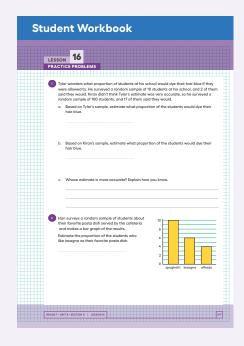
# Problem 2

Han surveys a random sample of students about their favorite pasta dish served by the cafeteria and makes a bar graph of the results.



Estimate the proportion of the students who like lasagna as their favorite pasta dish.

0.3 (or  $\frac{6}{20}$  or equivalent)





## **Problem 3**

Elena wants to know what proportion of people have cats as pets. Describe a process she could use to estimate an answer to her question.

Sample response: Find a random sample of about 50 people, and ask them if they have a pet cat. Once the responses are recorded, count the number of yes answers, and divide that by 50 to get an estimate of the population proportion.

# **Problem 4**

from Unit 8, Lesson 15

The science teacher gives daily homework. For a random sample of days throughout the year, the median number of problems is 5, and the IQR is 2. The Spanish teacher also gives daily homework. For a random sample of days throughout the year, the median number of problems is 10, and the IQR is 1. If you estimate the median number of science homework problems to be 5 and the median number of Spanish problems to be 10, which is more likely to be accurate? Explain your reasoning.

The Spanish estimate is more likely to be accurate.

Sample reasoning: When the measure of variability (the IQR) is larger, it is hard to get a good estimate of the population.

## Problem 5

from Unit 8, Lesson 14

Diego wants to survey a sample of students at his school to learn about the percentage of students who are satisfied with the food in the cafeteria. He decides to go to the cafeteria on a Monday and ask the first 25 students who purchase a lunch at the cafeteria if they are satisfied with the food.

Do you think this is a good way for Diego to select his sample? Explain your reasoning.

No, this is not a good way to select a sample.

Sample reasoning: Students who are buying lunch at the cafeteria may be choosing to buy their lunch because they like the cafeteria food. Students who bring lunch from home won't be included in the sample.

LESSON 16 • PRACTICE PROBLEMS