Lesson 9

Solving Problems About Proportional Relationships

Goals

- Decide whether it makes sense to represent a situation with a proportional relationship, and explain (orally) the reasoning.
- Determine what information is needed to solve a problem involving proportional relationships. Ask questions to elicit that information.
- Write an equation to represent a proportional relationship, and use the equation to solve problems about the situation.

Learning Targets

- I can ask questions about a situation to determine whether two quantities are in a proportional relationship.
- I can solve all kinds of problems involving proportional relationships.

Lesson Narrative

In this lesson, students practice identifying proportional relationships as they complete the first *Info Gap* activity in the grade 7 course. Students are presented with a problem statement without the information needed to solve the problem. They must brainstorm what information is needed and ask questions to obtain that information. In this case, students must determine that the situation involves a constant rate and find the constant of proportionality. Then, they write an equation to represent the proportional relationship and use the equation to solve a problem.

In the *Warm-up* activity, the teacher walks students through the process of asking for information needed to solve a problem. Then, students practice the *Info Gap* routine for themselves in small groups. To obtain all the necessary information, students need to persevere in asking questions and to communicate precisely.

The last activity is optional because it provides an opportunity for additional practice with a new context.

Student Learning Goal

Let's solve problems about proportional relationships.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)
- MLR6: Three Reads (Activity 2)

Instructional Routines

- MLR4: Information Gap Cards

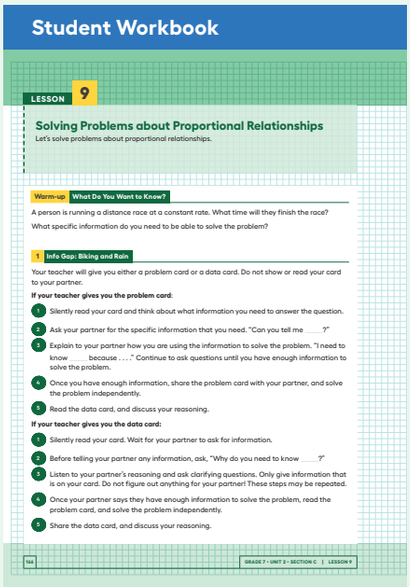
Required Materials

Materials to Copy

- Biking and Rain Cards (1 copy for every 4 students): Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	30 min	10 min	10 min	5 min
				Cool-down


Student Workbook

LESSON 9

Solving Problems about Proportional Relationships

Let's solve problems about proportional relationships.

Warm-up What Do You Want to Know?

A person is running a distance race at a constant rate. What time will they finish the race?

What specific information do you need to be able to solve the problem?

1 Info Gap: Biking and Rain

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me ____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know ____ because . . . ." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know ____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

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Warm-up

What Do You Want to Know?

10 min

Activity Narrative

The purpose of this *Warm-up* is to prepare students for the *Info Gap* activity that follows. First, students are given a problem with incomplete information. They are prompted to brainstorm what they need to know to solve a problem that involves constant speed. Next, they practice asking for information, explaining the rationale for their request, and persevering if their initial questions are unproductive. Once students have enough information, they solve the problem.

Launch

Display the first paragraph of the activity statement for all to see. Ask students to solve the problem. When they recognize that not enough information is given, display the second prompt and ask what they need to know to be able to solve the problem. Display the sentence frame "Can you tell me ..." for all to see and invite students to use it to frame their information request.

Give students 2 minutes of quiet think time.

Student Task Statement

A person is running a distance race at a constant rate. What time will they finish the race?

What specific information do you need to be able to solve the problem?

Sample responses:

- Can you tell me how long the race is?
- Can you tell me how fast the person is running?
- Can you tell me how far the person ran in 1 minute?
- Can you tell me what time they started the race?

Activity Synthesis

Tell students that the problem is a part of an *Info Gap* routine. In the routine, one person has a problem with incomplete information, and another person has data that can help with solving it. Explain that it is the job of the person with the problem to think about what is needed to answer the question, and then request it from the person with information.

Tell students they will try to solve the problem this way as a class to learn the routine. In this round, the students have the problem, and the teacher has the information needed to solve the problem.

- Ask students,

💬 *"What specific information do you need to find out what time they will finish the race?"*

- Select students to ask their questions. Encourage students to use the format of "Can you tell me ... ?" Respond to each question with, "Why do you need to know _____?"

- Once students justify their question, only answer questions if they can be answered using these data:
  - The race is 10,000 meters long.
  - The race started at 9:15 a.m.
  - In 1 minute, the person ran 156 $\frac{1}{4}$ meters.
  - An equation relating distance and time is given by $d = 156\frac{1}{4}t$, where d represents distance in meters and t represents time in minutes.
  - It takes 32 minutes for the person to run 5,000 meters.
  - The person runs at a pace of 6.4 minutes per kilometer (or 1,000 meters).
- If students ask for information that is not on the data card, respond with, "I don't have that information."

When students think they have enough information, give them 2 minutes to solve the problem.

The person should finish the race at 10:19 a.m.

Tell students they will work in small groups and use the routine to solve problems in the next activity.

Activity 1
Info Gap: Biking and Rain

30 min

Activity Narrative

This is the first *Info Gap* activity in the course. In this activity, students write equations for proportional relationships and use the equations to make predictions but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Instructional Routines

MLR4: Information Gap Cards

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Access for Multilingual Learners (Activity 1)

MLR4: Info Gap Cards

This is the first time Math Language Routine 4: *Information Gap* is suggested in this course. This routine facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, thereby creating a need for communication. This routine supports language development by providing students with opportunities to ask for and share information, and to justify their reasoning within conversation.

Access for Students with Diverse Abilities (Activity 1, Launch)

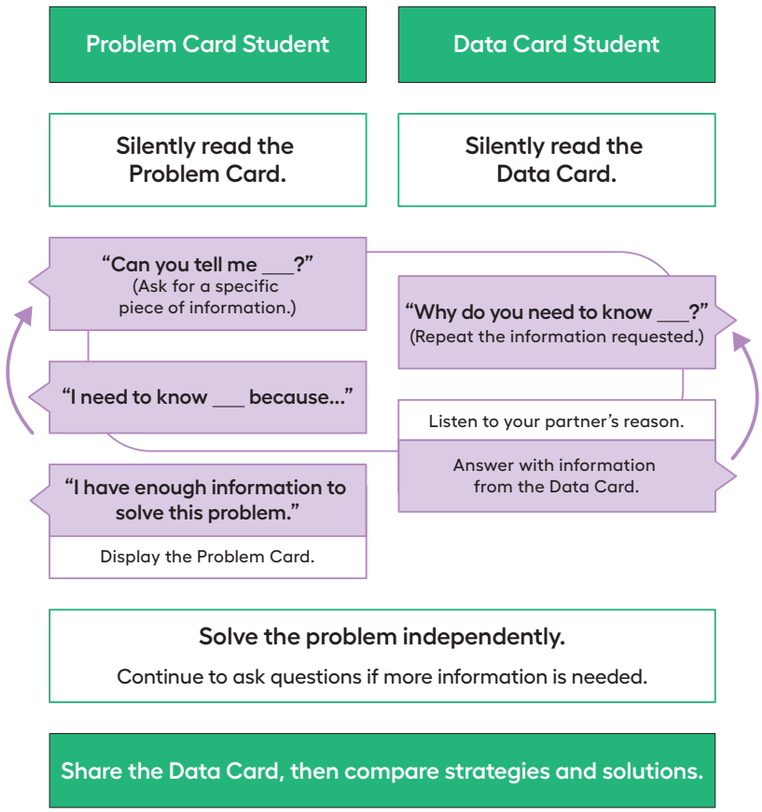
Representation: Access for Perception.

Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.

Supports accessibility for: Language, Memory

Launch

Display for all to see this graphic that illustrates a framework for the *Info Gap* routine:



Explain that in an *Info Gap* routine students work with a partner. One partner gets a problem card with a question that doesn't have enough given information, and the other partner gets a data card with information relevant to the problem card.

The person with the problem card asks questions like "Can you tell me ____?" and is expected to explain what they will do with the information. If that person asks for information that is not on the data card (including the answer!) and gives their reason, then the person with the data card must respond with, "I don't have that information." The person with the data card should just be providing information, not making assumptions. Note that it is okay to help a stuck partner by saying something like "I don't have the time they left. I only have information about distances and speeds."

Once the partner with the problem card has enough information, both partners look at the problem card and solve the problem independently.

Arrange students in groups of 2 or 4. If students are new to the *Info Gap* routine, allowing them to work in groups of 2 for each role supports communication and understanding. In each group, distribute a problem card to one student (or group) and a data card to the other student (or group). After reviewing their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..." Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner! These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

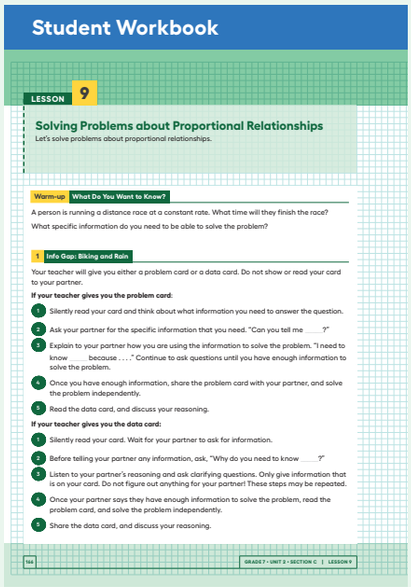
Problem Card 1:

1. For Mai, $d = 250t$, where d represents distance in meters from the park and t represents the amount of time in minutes it will take to get there. For Noah, $d = 300t$.
2. Noah will arrive first, since it will only take him 30 minutes ($9{,}000 \div 300 = 30$) while Mai will take 32 minutes ($8{,}000 \div 250 = 32$).

Problem Card 2:

1. Sample responses:
   - $r = 0.4t$ or $t = 2.5r$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in hours.
   - $r = \frac{1}{150}t$ or $t = 150r$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in minutes.
2. 12.5 hours

Student Workbook



LESSON 9

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Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 2, Launch)

MLR6: Three Reads.

Keep books or devices closed. Display only the problem stem and bullets, without revealing the questions. "We are going to read this problem 3 times."

After the 1st read: "Tell your partner what this situation is about."

After the 2nd read: "List the quantities. What can be counted or measured?"

For the 3rd read: Reveal and read the questions. Ask, "What are some ways we might get started on this?"

Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- *"How did you determine the constant of proportionality for each equation?"*
- *"Why did it make sense to represent each situation with a proportional relationship?"*

Highlight for students that writing an equation to represent the proportional relationship makes it easier to answer questions about the situation.

Activity 2: Optional

Moderating Comments

10 min

Activity Narrative

In this activity students compute rates to decide which job applicant is working the fastest checking online comments. They compare rates and total number of comments checked, then see that using rates is the more useful information in this situation.

Launch

Keep students in the same groups of 2.

Student Task Statement

A company is hiring people to read through all the comments posted on their website to make sure they are appropriate. Four people applied for the job and were given one day to show how quickly they could check comments.

- Person 1 worked for 210 minutes and checked a total of 50,000 comments.
- Person 2 worked for 200 minutes and checked 1,325 comments every 5 minutes.
- Person 3 worked for 120 minutes, at a rate represented by $c = 331t$, where c is the number of comments checked and t is the time in minutes.
- Person 4 worked for 150 minutes, at a rate represented by $t = \left(\frac{3}{800}\right)c$.

1. Order the people from greatest to least in terms of total number of comments checked.
   - Person 2 checked a total of 53,000 comments because $\frac{1,325}{5} \cdot 200 = 53,000$.
   - Person 1 checked a total of 50,000 comments.
   - Person 4 checked a total of 40,000 comments because $\frac{800}{3} \cdot 150 = 40,000$.
   - Person 3 checked a total of 39,720 comments because $331 \cdot 120 = 39,720$.

2. Order the people from greatest to least in terms of how fast they checked the comments.

- Person 3 checked 331 comments per minute.
- Person 4 checked about 267 comments per minute because $800 \div 3 = 266.\overline{6}$.
- Person 2 checked 265 comments per minute because $1{,}325 \div 5 = 265$.
- Person 1 checked about 238 comments per minute because $50{,}000 \div 210 \approx 238$.

Are You Ready for More?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.

- Person 1: $c = \frac{50,000}{210}t \approx 238.1t$
- Person 2: $c = \frac{1,325}{5}t = 265t$
- Person 3: $c = 331t$
- Person 4: $c = \frac{800}{3}t \approx 266.7t$

2. Make a table that allows you to easily compare how many comments the four job applicants can check.

person	time taken in minutes	comments per minute	total comments
1	210	$\frac{50,000}{210} \approx 238.1$	50,000
2	200	$\frac{1,325}{5} = 265$	53,000
3	120	331	39,720
4	150	$\frac{800}{3} \approx 226.7$	40,000

Activity Synthesis

Invite students to share which job applicant should get the job and why.

Possible responses include:

- Person 2 should get the job because they checked the most comments total.
- Person 3 should get the job because they checked the most comments per minute.
- Person 3 should not get the job because they checked comments for the least amount of time.

To involve more students in the conversation, consider asking:

"Do you agree or disagree? Why?"

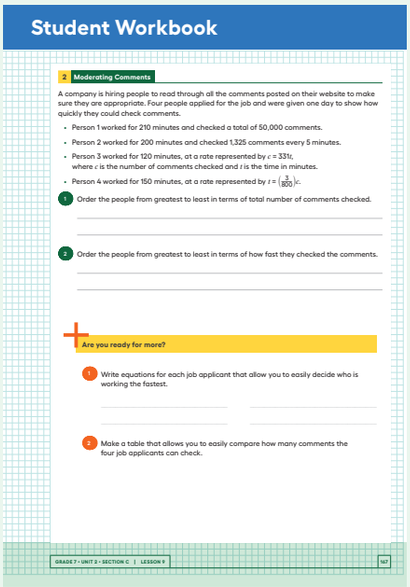
"Who can restate_____'s reasoning in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Did anyone solve the problem in a different way?"

"What connections to previous problems do you see?"

Student Workbook



2 Moderating Comments

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2. Order the people from greatest to least in terms of how fast they checked the comments.

Are you ready for more?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.

2. Make a table that allows you to easily compare how many comments the four job applicants can check.

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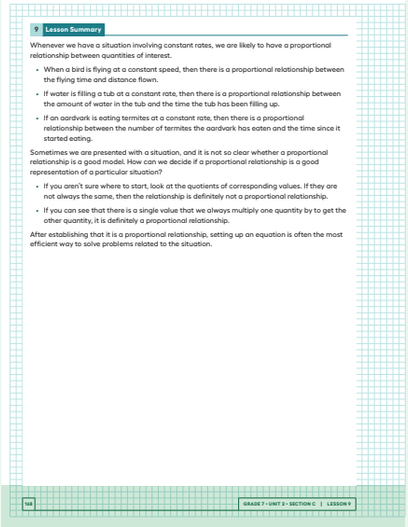
Student Workbook



Lesson Summary

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between quantities of interest.

- When a bird is flying at a constant speed, then there is a proportional relationship between the flying time and distance flown.
- If water is filling a tub at a constant rate, then there is a proportional relationship between the amount of water in the tub and the time the tub has been filling up.
- If an aardvark is eating termites at a constant rate, then there is a proportional relationship between the number of termites the aardvark has eaten and the time since it started eating.

Sometimes we are presented with a situation, and it is not so clear whether a proportional relationship is a good model. How can we decide if a proportional relationship is a good representation of a particular situation?

- If you aren't sure where to start, look at the quotients of corresponding values. If they are not always the same, then the relationship is definitely not a proportional relationship.
- If you can see that there is a single value that we always multiply one quantity by to get the other quantity, it is definitely a proportional relationship.

After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

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Lesson Synthesis

Share with students,

- *"Today we practiced asking questions to get the needed information to solve a problem. In each situation, we had to determine whether there was a constant rate that indicated a proportional relationship."*

To review how to represent a proportional situation with an equation, consider asking students:

- *"What are some types of situations that we have seen where quantities were proportional to each other?"*

  recipes, servings, unit price, constant speed, unit conversions

- *"When we have a situation with a proportional relationship between two quantities, what information do we need to write an equation?"*

  the constant of proportionality between the quantities

- *"Equations are good tools to make predictions or decisions. How did we use an equation to make a prediction or a decision today?"*

  calculating when Mai and Noah will arrive at the park, finding how long it will take for 5 cm of rain to fall, comparing how many comments a person can check per minute

Lesson Summary

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After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

Cool-down

Folding Programs

5 min

The word "program" has several different meanings. If students are unfamiliar with "program" as it is used in this context, clarify that it means a piece of paper that lists the songs that will be performed.

Student Task Statement

Lin is folding programs for the school music concert. She wants to know how long it will take her to finish folding all the programs. What information would you need to know to write an equation that represents this relationship?

Sample responses:

- Is Lin folding the programs at a constant rate?
- How long does it take her to fold 1 program?
- How many programs can she fold in 1 minute?
- How many programs are there total?

Responding To Student Thinking

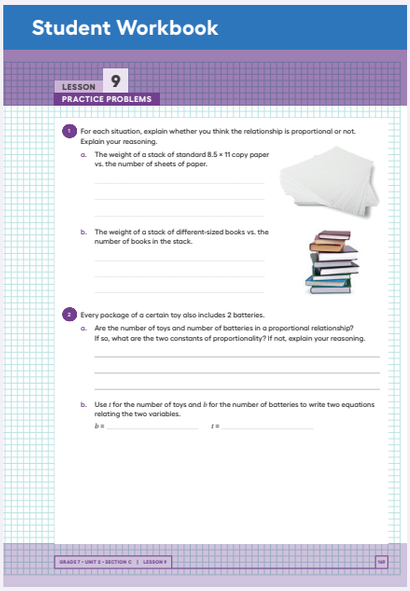
More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 9

Practice Problems

5 Problems


Student Workbook

LESSON 9
PRACTICE PROBLEMS

1 For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

a. The weight of a stack of standard 8.5 × 11 copy paper vs. the number of sheets of paper.

b. The weight of a stack of different-sized books vs. the number of books in the stack.

2 Every package of a certain toy also includes 2 batteries.

a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.

$b =$ _____ $t =$ _____

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Problem 1

For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

a. The weight of a stack of standard 8.5 × 11 copy paper vs. the number of sheets of paper.

There is a proportional relationship between weight and number of sheets of paper. Each piece of paper has the same weight. To find the weight of a stack, multiply the number of sheets of paper by the weight of a single sheet of paper. (We're assuming for this question that each piece of paper is the same weight. Manufacturing being what it is, though, we acknowledge that's not true.)

b. The weight of a stack of different-sized books vs. the number of books in the stack.

The relationship between the number of books and the weight of the stack is not proportional.
Each book has a different weight, so the weight of the stack can't be determined by multiplying the number of books by the weight of one book.

Problem 2

Every package of a certain toy also includes 2 batteries.

a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

Yes; 2 and $\frac{1}{2}$ are the constants of proportionality.

b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.

$b = 2t$ $\qquad$ $t = \frac{1}{2}b$

Problem 3

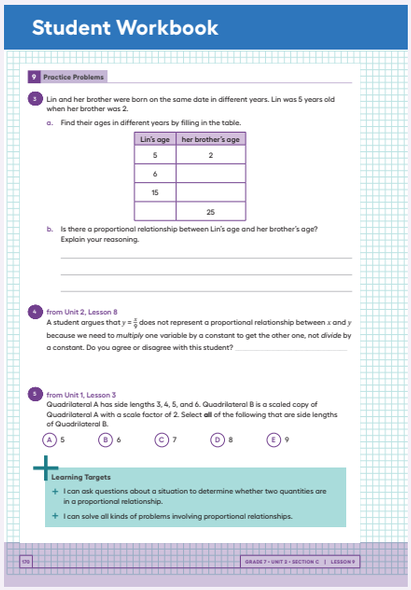
Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

Lin's age	her brother's age
5	2
6	3
15	12
28	25

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

There is no proportional relationship.
Every year, they each get older by one year, so the ratio of their ages changes every year.


Student Workbook

9 Practice Problems

3 Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

Lin's age	her brother's age
5	2
6	
15	
	25

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

4 from Unit 2, Lesson 8
A student argues that $y = \frac{x}{3}$ does not represent a proportional relationship between x and y because we need to multiply one variable by a constant to get the other one, not divide by a constant. Do you agree or disagree with this student?

5 from Unit 1, Lesson 3
Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select **all** of the following that are side lengths of Quadrilateral B.

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Learning Targets
+ I can ask questions about a situation to determine whether two quantities are in a proportional relationship.
+ I can solve all kinds of problems involving proportional relationships.

Problem 4

from Unit 2, Lesson 8

A student argues that $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because we need to *multiply* one variable by a constant to get the other one, not *divide* by a constant. Do you agree or disagree with this student?

Disagree
Dividing by 9 is the same as multiplying by $\frac{1}{9}$. We can look at the equation $y = \frac{x}{9}$ as $y = \frac{1}{9}x$. Also, $\frac{y}{x} = \frac{1}{9}$ for all corresponding values of x and y, so $\frac{y}{x}$ is constant.

Problem 5

from Unit 1, Lesson 3

Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select **all** of the following that are side lengths of Quadrilateral B.

A. 5 **B.** 6 **C.** 7 **D.** 8 **E.** 9