Lesson 7

All, Some, or No Solutions

Goals

- Compare and contrast (orally and in writing) equations that have no solutions or infinitely many solutions.
- Create linear equations in one variable that have either no solutions or infinitely many solutions, using structure, and explain (orally) the solution method.

Learning Target

I can determine whether an equation has no solutions, one solution, or infinitely many solutions.

Lesson Narrative

In this lesson students encounter equations that have no solutions and equations for which every number is a solution. In the first case, when students try to solve the equation, they end up with a false statement like 0 = 5. In the second case, they end up with a statement that is always true, such as $6x = 6x$. In preparation to predict the number of solutions from the structure of an equation, students add missing terms in three different ways to make them have no solution, one solution, or infinitely many solutions.

Student Learning Goal

Let's think about how many solutions an equation can have.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up

Notice and Wonder: Equation Solutions

5 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that equations can be true for all or no values, which will be useful when students explore what aspects of equations affect the number of solutions in a later activity. While students may notice and wonder many things about these equations, the number of solutions are the important discussion points.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is how more than 1 value can solve the first equation and no values make the second equation true.

Launch

Arrange students in groups of 2. Display the equations for all to see. Ask students to think of at least one thing that they notice and at least one thing they wonder about.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things that they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?

$2t + 5 = 2t + 5$ $\qquad$ $n + 5 = n + 7$

Students may notice:

- The first equation has the same thing on each side.
- Lots of numbers solve the first equations.
- Something seems wrong about the second equation.

Students may wonder:

- Are there any values of n that make the equation true?
- What is the point of the first equation?
- Are the 5s related in the context?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the equations. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the number of solutions does not come up during the conversation, ask students to briefly discuss this idea. Move on to the next activity when students hear that the first equation can be solved by more than 1 value for t and the second equation does not seem to have any solutions.

Instructional Routines

Notice and Wonder

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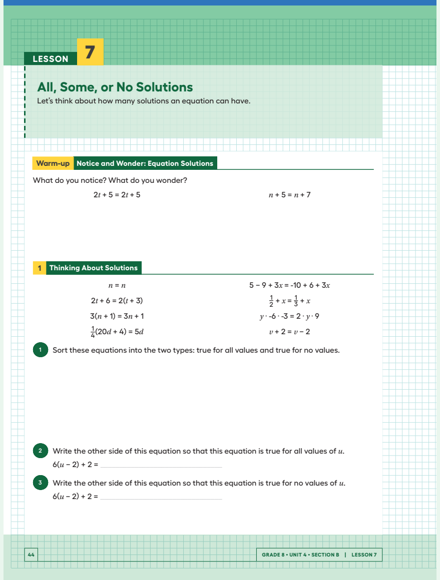
Student Workbook


LESSON 7

All, Some, or No Solutions

Let's think about how many solutions an equation can have.

Warm-up Notice and Wonder: Equation Solutions

What do you notice? What do you wonder?

$2t + 5 = 2t + 5$ $\qquad$ $n + 5 = n + 7$

1 Thinking About Solutions

$n = n$	$5 - 9 + 3x = -10 + 6 + 3x$
$2t + 6 = 2(t + 3)$	$\frac{1}{2} + x = \frac{1}{3} + x$
$3(n + 1) = 3n + 1$	$y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9$
$\frac{1}{4}(20d + 4) = 5d$	$v + 2 = v - 2$

1. Sort these equations into the two types: true for all values and true for no values.
2. Write the other side of this equation so that this equation is true for all values of u.
$6(u - 2) + 2 =$ ____
3. Write the other side of this equation so that this equation is true for no values of u.
$6(u - 2) + 2 =$ ____

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Activity 1

Thinking About Solutions

15 min

Activity Narrative

Students who pause to think about the structure of a complex equation before taking steps to solve it can find the most efficient solution paths and, sometimes, notice that there is no single solution to be found. The goal of this lesson is to encourage students to make this pause part of their routine and to build their skill at understanding and manipulating the structure of equations through the study of two special types of equations: ones that are always true and ones that are never true.

Students begin the activity sorting a variety of equations into categories based on their number of solutions. The activity ends with students filling in the blank side of an equation to make an equation that is always true and then again to make an equation that is never true.

Launch

Keep the equations from the *Warm-up* displayed: $2t + 5 = 2t + 5$ and $n + 5 = n + 7$.

Arrange students in groups of 2. Ask students how they might start to solve the equations. After a brief quiet think time, invite students to share their moves with a partner. Select 1–2 groups per equation to share their thinking while recording student moves for all to see.

Students should notice that the first equation results in $t = t$, $5 = 5$, $0 = 0$, or similar equations that are true for any value used in place of t. This means that all values are solutions to this equation.

The second equation results in $5 = 7$, $n = n + 2$, or similar equations that are false no matter what values are used for n. This means that no values are solutions to this equation.

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time for the first problem, followed by partner discussion, to share how they sorted the equations. Give time for partners to complete the remaining problems, and follow with a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to justify their decisions about the number of solutions for equations while sorting them. Display words and phrases such as "coefficient," "true," "false," and "eliminate the variable."

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Student Task Statement

$n = n$

$2t + 6 = 2(t + 3)$

$3(n + 1) = 3n + 1$

$\frac{1}{4}(20d + 4) = 5d$

$5 - 9 + 3x = -10 + 6 + 3x$

$\frac{1}{2} + x = \frac{1}{3} + x$

$y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9$

$v + 2 = v - 2$

1. Sort these equations into the two types: true for all values and true for no values.

   True for all values: $n = n$, $y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9$, $2t + 6 = 2(t + 3)$, $5 - 9 + 3x = -10 + 6 + 3x$

   True for no values: $\frac{1}{2} + x = \frac{1}{3} + x$, $3(n + 1) = 3n + 1$, $\frac{1}{4}(20d + 4) = 5d$, $v + 2 = v - 2$

2. Write the other side of this equation so that this equation is true for all values of u. $6(u - 2) + 2 =$ $6(u - 2) + 2 = 6u - 10$ (or equivalent)

3. Write the other side of this equation so that this equation is true for no values of u. $6(u - 2) + 2 =$ $6(u - 2) + 2 = 6u$ (or equivalent)

Are You Ready for More?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98. How many sets of two or more consecutive positive integers can be added to obtain a sum of 100?

There are two sets. The first has 8 consecutive integers: 9, 10, 11, 12, 13, 14, 15, 16. The second has five consecutive integers: 18, 19, 20, 21, 22.

Activity Synthesis

Display a list of the equations from the task, leaving enough space to add student ideas next to the equations. The purpose of this discussion is for students to see multiple ways of thinking about and justifying the number of solutions that an equation has.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they categorized each equation. Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond.

Next, ask students for different ways to write the other side of the equation for the second problem, and add these to the display. For example, students may have distributed $6(u - 2) + 2$ to get $6u - 12 + 2$ while others chose $6u - 10$ or something with more terms, such as $6(u - 2 + 1) - 4$.

End the discussion by asking students for different ways to write the other side of the incomplete equation in the last question. It is important to note, if no students point it out, that all solutions should be equivalent to $6u$ + ?, where the question mark represents any number other than -10.

Building on Student Thinking

For the last part of the activity, students may think that any expression that is not equivalent to $6u - 10$ is a good answer.
Ask students:

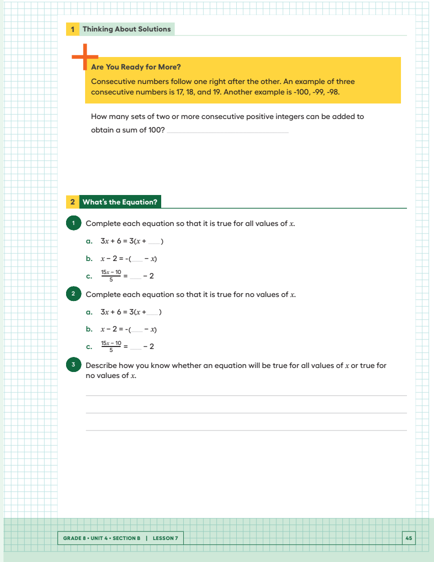
"How many solutions do the equations from the Warm-up have? Why?"
"How many solutions does the equation $2x = 4$ have?

Access for Students with Diverse Abilities (Activity 1, Activity Synthesis)

Action and Expression: Internalize Executive Functions.
Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to determine whether an equation is true for all values or true for no values, and to include examples for each.

Supports accessibility for: Conceptual Processing; Organization

Student Workbook



1 Thinking About Solutions

Are You Ready for More?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

How many sets of two or more consecutive positive integers can be added to obtain a sum of 100? ____

2 What's the Equation?

1. Complete each equation so that it is true for all values of x.
   a. $3x + 6 = 3(x + \_\_)$
   b. $x - 2 = -(\_\_ - x)$
   c. $\frac{15x - 10}{5} = \_\_ - 2$
2. Complete each equation so that it is true for no values of x.
   a. $3x + 6 = 3(x + \_\_)$
   b. $x - 2 = -(\_\_ - x)$
   c. $\frac{15x - 10}{5} = \_\_ - 2$
3. Describe how you know whether an equation will be true for all values of x or true for no values of x.

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Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation such as: "Equations that are always true for x have ____." "Equations that have no solution for any value of x have ____."

Supports accessibility for: Language, Social-Emotional Functioning

Activity 2
What's the Equation?

15 min

Activity Narrative

In this activity, students are presented with three equations, all with a missing term. They are asked to fill in the missing term to create equations with either no solution or infinitely many solutions, building on the work begun in the previous activity. At the end, students summarize what they have learned about how to tell if an equation is true for all values of x or no values of x.

Launch

Give students 3–5 minutes of quiet think time followed by 3–5 minutes of partner discussion. Follow with a whole-class discussion.

Student Task Statement

1. Complete each equation so that it is true for all values of x.
   - **a.** $3x + 6 = 3(x + \underline{\ 2\ })$
   - **b.** $x - 2 = -(\underline{\ 2\ } - x)$
   - **c.** $\frac{15x - 10}{5} = \underline{3x} - 2$
2. Complete each equation so that it is true for no values of x.
   - **a.** $3x + 6 = 3(x + \_\_\_\_)$

     Any number other than 2 will give an equation with no solution.
   - **b.** $x - 2 = -(\_\_\_\_ - x)$

     Any number other than 2 will give an equation with no solution.
   - **c.** $\frac{15x - 10}{5} = \_\_\_\_ - 2$

     Any expression of the form ($3x$ + a number other than 0) will give an equation with no solution. Note: A numerical answer will yield a linear equation of one variable which has one solution.
3. Describe how you know whether an equation will be true for all values of x or true for no values of x.

   Sample response: Equations that are always true for any value of x have equivalent expressions on each side. Equations that have no solution for any value of x simplify to a statement of two unequal numbers being equal, which is always false.

Activity Synthesis

The purpose of this discussion is to record the students' thinking about conditions that must be true for an equation to have no solution or infinite solutions.

Display each equation, leaving a large space for writing. Under each equation, invite students to share what they used to make the equation be true for all values of x, and record these for all to see. Ask:

"What did all these answers have in common?"

There is only one possible answer for each equation that will make it be always true.

"What strategy did you use to figure out what that answer had to be?"

The solution had to be something that would make the right side equivalent to the left.

Next, invite students to share what they used to make the equation true for no values of x, and record these for all to see. Ask:

"Why are there different ways to fill in the blank so that the equation is true for no values of x?"

As long as the answer is different from what we chose in Part 1, then the equation cannot be true for any value of x.

"What was different about Equation C?"

We had to be careful to make sure that the variable coefficient was 3, and we added a constant so that the equation wouldn't have a single solution.

Ask students to share observations that they made for the last question. If no student mentions it, explain that an equation with no solution can always be rearranged or manipulated to say that two unequal values are equal, such as 2 = 3, which means that the equation is never true.

Access for Multilingual Learners (Activity 2, Activity Synthesis)

MLR8: Discussion Supports. For each observation that is shared about the last question, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

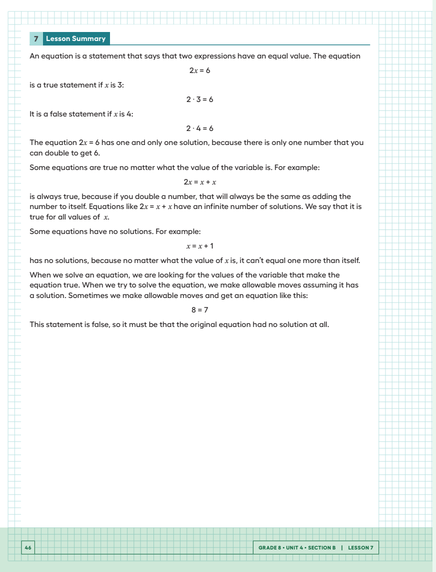
Lesson Synthesis

Ask students to think about some ways in which they are able to determine how many solutions there are to the equations they solved today. Invite students to share some things they did. For example, students may suggest:

- Testing different values for the variable.
- Applying allowable moves to generate equivalent equations.
- Examining the structure of the equation.

Ask students to write a short letter to someone taking the class next year about what they should look for when trying to decide how many solutions an equation has. Tell students to use examples, share any struggles they had when deciding on the number of solutions, and which strategies they prefer for figuring out the number of solutions.

Student Workbook



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Summary

An equation is a statement that says that two expressions have an equal value. The equation

$$2x = 6$$

is a true statement if x is 3:

$$2 \cdot 3 = 6$$

It is a false statement if x is 4:

$$2 \cdot 4 = 6$$

The equation $2x = 6$ has one and only one solution, because there is only one number that you can double to get 6.

Some equations are true no matter what the value of the variable is. For example:

$$2x = x + x$$

is always true, because if you double a number, that will always be the same as adding the number to itself. Equations like $2x = x + x$ have an infinite number of solutions. We say that it is true for all values of x.

Some equations have no solutions. For example:

$$x = x + 1$$

has no solutions, because no matter what the value of x is, it can't equal one more than itself.

When we solve an equation, we are looking for the values of the variable that make the equation true. When we try to solve the equation, we make allowable moves assuming it has a solution. Sometimes we make allowable moves and get an equation like this:

$$8 = 7$$

This statement is false, so it must be that the original equation had no solution at all.

Cool-down

Choose Your Own Solution

5 min

Student Task Statement

$3x + 8 = 3x +$

What value could you write in after "$3x +$" that would make the equation true for:

1. no values of x?

   any value other than 8

2. all values of x?

   8

3. just one value of x?

   any variable term, like x or $2x$, in order to create an equation with one solution

Lesson 7

Practice Problems

7 Problems

Problem 1

For each equation, decide if it is always true or never true.

a. $x - 13 = x + 1$ never true

b. $x + \frac{1}{2} = x - \frac{1}{2}$ never true

c. $2(x + 3) = 5x + 6 - 3x$ always true

d. $x - 3 = 2x - 3 - x$ always true

e. $3(x - 5) = 2(x - 5) + x$ never true

Problem 2

Mai says that the equation $2x + 2 = x + 1$ has no solution because the left hand side is double the right hand side. Do you agree with Mai? Explain your reasoning.

disagree

Sample reasoning: Mai is correct that $2x + 2 = 2(x + 1)$, so the left hand side in this equation is double the right hand side. But $-x$ and -1 can be added to both sides of the equation to get $x + 1 = 0$. So $x = -1$ is a solution.

(This works because 0 is its own double, and it is the only number that is its own double.)

Problem 3

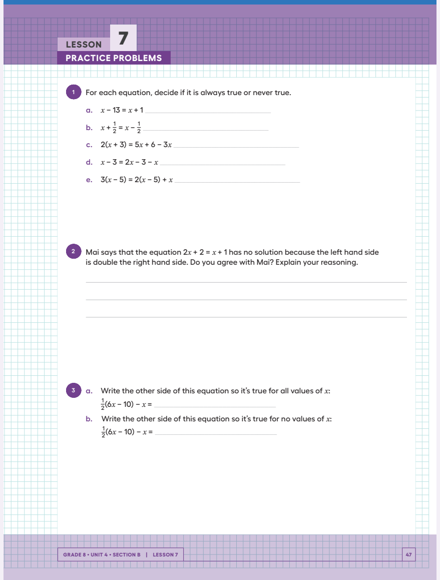
a. Write the other side of this equation so it's true for all values of x:
$\frac{1}{2}(6x - 10) - x =$

$2x - 5$ (or equivalent)

b. Write the other side of this equation so it's true for no values of x:
$\frac{1}{2}(6x - 10) - x =$

Sample response: $2x + 5$

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1 For each equation, decide if it is always true or never true.

a. $x - 13 = x + 1$

b. $x + \frac{1}{2} = x - \frac{1}{2}$

c. $2(x + 3) = 5x + 6 - 3x$

d. $x - 3 = 2x - 3 - x$

e. $3(x - 5) = 2(x - 5) + x$

2 Mai says that the equation $2x + 2 = x + 1$ has no solution because the left hand side is double the right hand side. Do you agree with Mai? Explain your reasoning.

3 a. Write the other side of this equation so it's true for all values of x:
$\frac{1}{2}(6x - 10) - x =$

b. Write the other side of this equation so it's true for no values of x:
$\frac{1}{2}(6x - 10) - x =$

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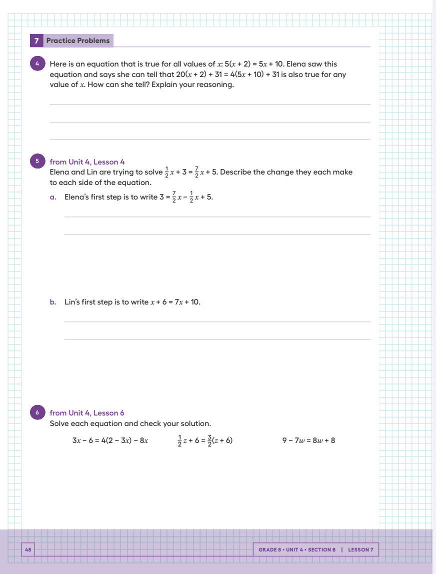
Student Workbook


7 Practice Problems

4 Here is an equation that is true for all values of x: $5(x + 2) = 5x + 10$. Elena saw this equation and says she can tell that $20(x + 2) + 31 = 4(5x + 10) + 31$ is also true for any value of x. How can she tell? Explain your reasoning.

5 from Unit 4, Lesson 4
Elena and Lin are trying to solve $\frac{1}{2}x + 3 = \frac{7}{2}x + 5$. Describe the change they each make to each side of the equation.

a. Elena's first step is to write $3 = \frac{7}{2}x - \frac{1}{2}x + 5$.

b. Lin's first step is to write $x + 6 = 7x + 10$.

6 from Unit 4, Lesson 6
Solve each equation and check your solution.

$3x - 6 = 4(2 - 3x) - 8x$ $\quad \frac{1}{2}z + 6 = \frac{3}{2}(z + 6)$ $\quad 9 - 7w = 8w + 8$

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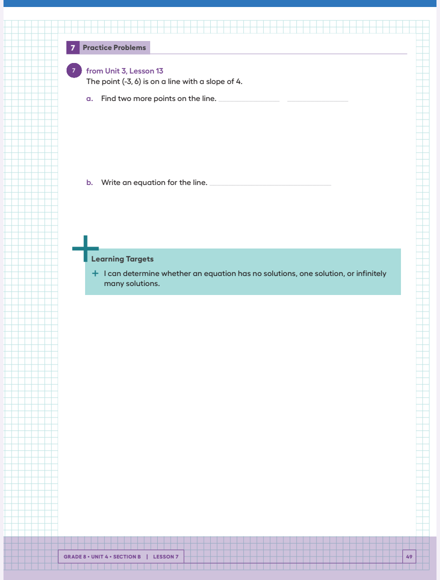
Student Workbook


7 Practice Problems

7 from Unit 3, Lesson 13
The point (-3, 6) is on a line with a slope of 4.

a. Find two more points on the line.

b. Write an equation for the line.

Learning Targets

+ I can determine whether an equation has no solutions, one solution, or infinitely many solutions.

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Problem 4

Here is an equation that is true for all values of x: $5(x + 2) = 5x + 10$. Elena saw this equation and says she can tell that $20(x + 2) + 31 = 4(5x + 10) + 31$ is also true for any value of x. How can she tell? Explain your reasoning.

Elena applied the distributive property.

Sample reasoning: One could distribute the left side of the equation and show that it is equal to the right side, but it is easier to see that each side of the original equation has been multiplied by
4 and added to 31. These moves keep both sides of the equation in balance, and so whatever values of x make the first equation true also make the second equation true.

Problem 5

from Unit 4, Lesson 4

Elena and Lin are trying to solve $\frac{1}{2}x + 3 = \frac{7}{2}x + 5$. Describe the change they each make to each side of the equation.

a. Elena's first step is to write $3 = \frac{7}{2}x - \frac{1}{2}x + 5$.

Elena subtracted $\frac{1}{2}x$ from each side.

b. Lin's first step is to write $x + 6 = 7x + 10$.

Lin multiplied each side by 2.

Problem 6

from Unit 4, Lesson 6

Solve each equation and check your solution.

$3x - 6 = 4(2 - 3x) - 8x$

$x = \frac{14}{23}$

$\frac{1}{2}z + 6 = \frac{3}{2}(z + 6)$

$z = -3$

$9 - 7w = 8w + 8$

$w = \frac{1}{15}$

Problem 7

from Unit 3, Lesson 13

The point (-3, 6) is on a line with a slope of 4.

a. Find two more points on the line.

Sample response: (-2, 10), (-1, 14)

b. Write an equation for the line.

$y = 4x + 18$ (or equivalent)