On or Off the Line?

Goals

- Determine (in writing)

 a point that satisfies
 two relationships
 simultaneously, using
 tables or graphs.
- Interpret (orally and in writing) points that lie on one, both, or neither line on a graph of two simultaneous equations in context.

Learning Targets

- I can identify ordered pairs that are solutions to an equation.
- I can interpret ordered pairs that are solutions to an equation.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Which Three Go Together?

Required Materials

Materials to Gather

• Math Community Chart: Activity 2

Lesson Narrative

In this lesson, students consider pairs of linear equations in familiar contexts such as distance and time or budgeting and interpret the meaning of points on and off the graphs of the equations. After revisiting the idea that coordinates for points on the graph make the associated equation true, students interpret the intersection point of two lines as a special case in which both equations are true. By examining several points on and off the lines, students make use of repeated reasoning to understand their connection to equations.

Student Learning Goal

Let's interpret the meaning of points in a coordinate plane.

Lesson Timeline



Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Lesson Synthesis

Assessment



Cool-down

Warm-up

Which Three Go Together: Lines in the Plane



Activity Narrative

This Warm-up prompts students to compare four graphs. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

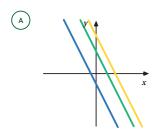
Launch

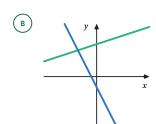
Arrange students in groups of 2–4. Display the graphs for all to see.

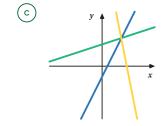
Give students 1 minute of quiet think time and ask them to indicate when they have noticed three graphs that go together and can explain why. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

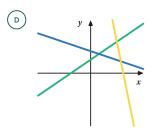
Student Task Statement

Which three go together? Why do they go together?









Sample responses:

A, B, and C go together because:

- · They have 0 or I intersection points.
- · At least one of the lines has a negative vertical intercept.

A, B, and D go together because:

- · The blue line has a negative slope.
- A, C, and D go together because:
- · They have 3 lines.

B, C, and D go together because:

· There are no parallel lines.

Inspire Math

Supply and Demand video



Go Online

Before the lesson, show this video to reinforce the real-world connection.

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Instructional Routines

Which Three Go Together?

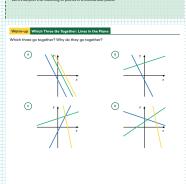
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LESSON 10 On or Off the Line?



Instructional Routines

MLR7: Compare and Connect

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Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.

Represent the same information through different modalities by using coins to represent different combinations that add up to \$2.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "intersection" or "parallel," and to clarify their reasoning as needed. Consider asking:

"What do you mean by ...?"

"Can you say that in another way?"

Activity 1

Pocket Full of Change

15 min

Activity Narrative

In earlier lessons, students have set two expressions equal to one another to find a common value where both expressions are true (if it exists). A system of two equations asks a similar question: At what common pair of values are both equations true? In this activity, students focus on a context involving coins and use multiple representations to think about the context in different ways. The goal of this activity is not for students to write equations or learn the language "system of equations," but rather to investigate the mathematical structure with two stated facts using familiar representations and context while reasoning about what must be true.

Monitor for students who use these representations to solve the last problem:

- Graphs
- Tables
- Words

Launch

Before looking at the task, tell students,

○ "I have money worth \$2 in my pocket. What might be in my pocket?"

Students will likely guess that you have two \$1 bills, but ask what else it might be. Some answers could be 8 quarters, 200 pennies, a \$2 bill, or 20 nickels, 2 quarters, and 5 dimes.

Read the problem context together. Ensure that students understand that we know that Jada has exactly \$2 in her pocket, that she has only quarters and dimes, and that she has exactly 17 coins. Give 1–2 minutes for students to read and complete the first problem. Display the table for all to see, and ask students for values to fill in the table.

Give students 5–7 minutes of quiet work time to finish the remaining problems, and follow that with a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

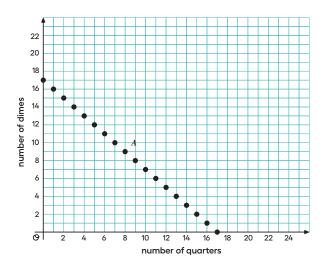
Student Task Statement

Jada told Noah that she has \$2 worth of quarters and dimes in her pocket and 17 coins all together. She asked him to guess how many of each type of coin she has.

1. Here is a table that shows some combinations of quarters and dimes that are worth \$2. Complete the table.

number of quarters	number of dimes
0	20
4	10
8	0
6	5
2	15

2. Here is a graph of the relationship between the number of quarters and the number of dimes when there are a total of 17 coins.



a. What does Point A represent?

8 quarters and 9 dimes

b. How much money, in dollars, is the combination represented by Point ${\cal A}$ worth?

\$2.90, because $8 \cdot 0.25 + 9 \cdot 0.10 = 2.90$

3. Is it possible for Jada to have 4 quarters and 13 dimes in her pocket? Explain how you know.

no

Sample reasoning: Even though 4 quarters and 13 dimes is 17 coins, they are not worth \$2. (They are worth \$2.30.)

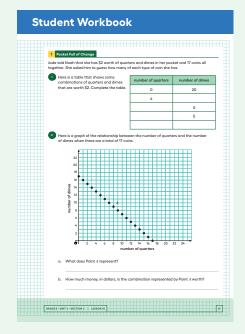
4. How many quarters and dimes must Jada have? Explain your reasoning.

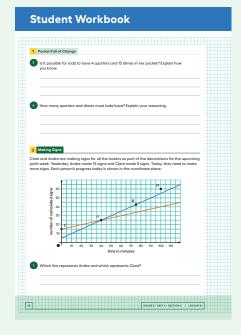
2 quarters and 15 dimes

Sample reasoning: This is 17 coins that are worth \$2 because $2 \cdot 0.25 + 15 \cdot 0.10 = 2.00$. It is the only combination of quarters and dimes that appears both in the table and as a point on the graph.

Building on Student Thinking

Students may wonder why the last row of the table is blank. Tell them they can enter any values that make sense in the context and that are not already in the table.





Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions

This activity uses the Co-Craft Questions math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Activity Synthesis

The goal of this discussion is to focus students on the last problem and what must be true based on the two facts that they know: The coins total \$2 and there are exactly 17 coins. Begin the discussion by asking students about some things that they know cannot be true about the coins in Jada's pocket and how they know. Students may respond that Jada cannot have 20 dimes and 0 quarters because that is not 17 coins or other variations where either the coins do not total \$2, there are not exactly 17 coins, or neither are true.

Display 2–3 approaches/representations from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different representations. Here are some questions for discussion:

"What do the representations have in common? How are they different?"
"How does the solution show up in each representation?"

Ensure that students understand that their solution is one where both facts are true.

Activity 2

Making Signs

15 min

Activity Narrative

In the previous activity, the system of equations was represented in words, a table, and a graph. In this activity, the system of equations is partially given in words, but key elements are provided only in the graph. Students have worked before with lines that represent a context. Now they must work with two lines at the same time to determine whether a point lies on one line, both lines, or neither line.

Launch 🙎

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Arrange students in groups of 2. Introduce the context of making signs for spirit week. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

Display only the problem stem and related image, without revealing the questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

 \bigcirc "What do these questions have in common? How are they different?"

Listen for and amplify language related to the learning goal, such as the meaning of the points and lines in context.

Reveal the question,

© "Based on the lines, mark the statements as true or false for each person."

Give students 1–2 minutes to compare the revealed question to their own question and those of their classmates. Invite students to identify similarities and differences by asking,

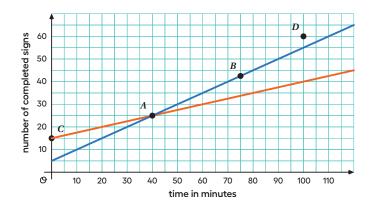
"Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it."

Tell students to complete the table one row at a time, with one person responding for Clare and the other responding for Andre.

Give students 2–3 minutes to finish the table, and follow that with a whole-class discussion.

Student Task Statement

Clare and Andre are making signs for all the lockers as part of the decorations for the upcoming spirit week. Yesterday, Andre made 15 signs and Clare made 5 signs. Today, they need to make more signs. Each person's progress today is shown in the coordinate plane.



1. Which line represents Andre and which represents Clare?

Andre's line has a vertical intercept at (0,15) and a smaller slope. Clare's line has a vertical intercept at (0,5) and a greater slope.

2. Based on the lines, mark the statements as true or false for each person.

point	what it says	Clare	Andre
A	At 40 minutes, I have 25 signs completed.	true	true
В	At 75 minutes, I have 42 and a half signs completed.	true	false
С	At 0 minutes, I have 15 signs completed.	false	true
D	At 100 minutes, I have 60 signs completed.	false	false

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Suggest that students answer all situations for Clare first and then consider Andre's situations. Check in with students to provide feedback and encouragement after each chunk. Ensure that students understand the instructions and how to fill in the table.

Supports accessibility for: Attention, Social-Emotional Functioning

Are You Ready for More?

- 4 toothpicks make 1 square
- 7 toothpicks make 2 squares
- 10 toothpicks make 3 squares



Do you see a pattern? If so, how many toothpicks would you need to make 10 squares according to your pattern? Can you represent your pattern with an expression?

Sample response: Yes, for each new square you will need 3 toothpicks. If you want to make x squares you will need 3x + 1 toothpicks. So 10 squares needs $(3 \cdot 10) + 1 = 31$ toothpicks.

Activity Synthesis

Display the graphs from the *Task Statement*. The goal of this discussion is for students to realize that points that lie on a line make that situation true. If a point is on more than 1 line, such as at an intersection, then it makes all of those situations true. Ask students:

- "Is the point (20,15) on either line? What does that mean in this situation?"
 It is on Clare's line. It means that, 20 minutes after they worked today,
 Clare has made a total of 20 signs.
- "Imagine a point E that is not on either line. What does that information tell us about this point?"

It means that it represents a situation that is not true for either Andre or Clare.

"If a point is on a line, what does it mean? If the point is the intersection of 2 lines, what does that mean?"

A point on a line means that it represents a situation that is true for the situation represented by the line. The intersection point represents a situation that is true for both situations represented by the lines.

Invite groups to share their reasoning about points $A\!-\!D$. Conclude by pointing out to students that, in this context, there are many points true for Clare and many points true for Andre but only one point true for both of them. Future lessons will be about how to figure out that point.

Math Community

Conclude the discussion by inviting 2–3 students to share a norm that they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

\bigcirc "I noticed our norm '.	' in action today, and it really h	elped me/my
group because	<i>"</i> ••	

Lesson Synthesis

Tell students to think about how they found the ordered pair that makes two relationships true using tables and graphs today. Ask:

"What are some advantages of tables? What are disadvantages of using tables?"

Tables are good for knowing the exact values for individual points. If the point you are interested in is not listed, then there's not a good way of figuring out that information.

"What are some advantages of graphs? What are disadvantages of using graphs?"

Graphs give a better overall picture of the relationships and usually make estimating (if not finding exactly) the common point easier. If the point you are interested in is not on a grid, then you can only estimate the coordinates.

"When using graphs, where are the points whose coordinates do not make a given relationship true? Do the coordinates of those points show up in a table of values?"

Points that are off of the line do not make the given relationship true. They can be above or below the line. The coordinates of these points do not show up in a table representing the given relationship.

If time allows, invite students to make up their own stories with two quantities and two relationships to swap with a partner. Have each partner create either two tables of values, two graphs, or one of each to describe the situation and answer a question about the values of the two quantities that make both relationships true.

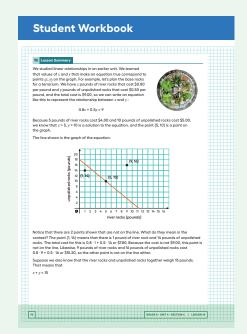
Lesson Summary

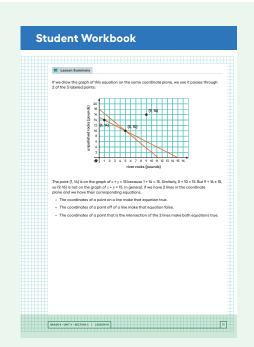
We studied linear relationships in an earlier unit. We learned that values of x and y that make an equation true correspond to points (x, y) on the graph. For example, let's plan the base rocks for a terrarium. We have x pounds of river rocks that cost \$0.80 per pound and y pounds of unpolished rocks that cost \$0.50 per pound, and the total cost is \$9.00, so we can write an equation like this to represent the relationship between x and y:



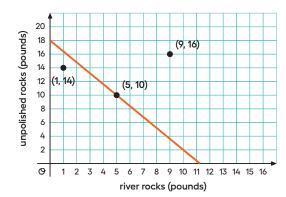
$$0.8x + 0.5y = 9$$

Because 5 pounds of river rocks cost \$4.00 and 10 pounds of unpolished rocks cost \$5.00, we know that x = 5, y = 10 is a solution to the equation, and the point (5, 10) is a point on the graph.





The line shown is the graph of the equation:

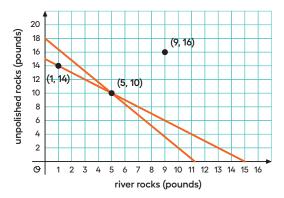


Notice that there are 2 points shown that are not on the line. What do they mean in the context? The point (1, 14) means that there is 1 pound of river rock and 14 pounds of unpolished rocks. The total cost for this is $0.8 \cdot 1 + 0.5 \cdot 14$ or \$7.80. Because the cost is not \$9.00, this point is not on the line. Likewise, 9 pounds of river rocks and 16 pounds of unpolished rocks cost $0.8 \cdot 9 + 0.5 \cdot 16$ or \$15.20, so the other point is not on the line either.

Suppose we also know that the river rocks and unpolished rocks together weigh 15 pounds. That means that

$$x + y = 15$$

If we draw the graph of this equation on the same coordinate plane, we see it passes through 2 of the 3 labeled points:



The point (1, 14) is on the graph of x + y = 15 because 1 + 14 = 15. Similarly, 5 + 10 = 15. But $9 + 16 \neq 15$, so (9, 16) is not on the graph of x + y = 15. In general, if we have 2 lines in the coordinate plane and we have their corresponding equations,

- The coordinates of a point on a line make that equation true.
- The coordinates of a point off of a line make that equation false.
- The coordinates of a point that is the intersection of the 2 lines make both equations true.

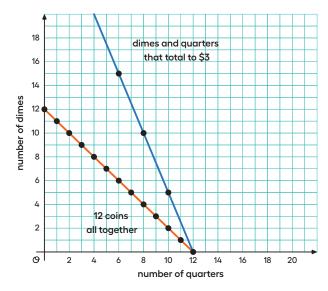
Cool-down

Another Pocket Full of Change



Student Task Statement

On the coordinate plane shown, one line shows combinations of dimes and quarters that are worth \$3. The other line shows combinations of dimes and quarters that total to 12 coins.



1. Name one combination of 12 coins shown on the graph. How does the graph show that the combination is true?

Sample responses: 6 quarters and 6 dimes or II quarters and I dime The point (6,6) (or (II,I)) is on the graph of the line representing I2 coins all together.

- ${\bf 2.}$ Name one combination of coins shown on the graph that total to \$3.
 - Sample responses: 6 quarters and 15 dimes, or 10 quarters and 5 dimes
- 3. How many quarters and dimes would you need to have both 12 coins and \$3 at the same time? How does the graph show that this is true?12 quarters and 0 dimes because the point (12,0) is on both lines.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

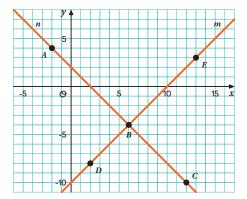
Practice Problems

10

5 Problems

Problem 1

a. Match the lines m and n to the statements they represent:



- i. A set of points where the coordinates of each point have a sum of 2 \underline{n}
- ii. A set of points where the y-coordinate of each point is 10 less than its *x*-coordinate *m*
- **b.** Match the labeled points on the graph to statements about their coordinates:
 - i. Two numbers with a sum of 2 A, B, C
 - ii. Two numbers where the y-coordinate is 10 less than the x-coordinate D, B, E
 - iii. Two numbers with a sum of 2 and where the y-coordinate is 10 less than the x-coordinate $\underline{\mathsf{B}}$

Problem 2

from Unit 4, Lesson 7

Here is an equation: 4x - 4 = 4x +____. What could you write in the blank so the equation would be true for:

a. No values of x

Sample response: 19

4x - 4 = 4x + 19 has no solutions.

b. All values of x

Sample response: -4

4x - 4 = 4x + -4 is true for all values of x.

c. One value of x

Sample response: 4x

4x - 4 = 4x + 4x has one solution (x = -1).

Student Workbook

10

Problem 3

Mai earns \$7 per hour mowing her neighbors' lawns. She also earned \$14 for hauling away bags of recyclables for some neighbors.

Priya babysits her neighbor's children. The table shows the amount of money, m, that Priya earns in h hours. Priya and Mai have agreed to go to the movies the weekend after they have earned the same amount of money for the same number of work hours.

number of hours (h)	amount of money (m)
1	\$8.40
2	\$16.80
4	\$33.60

- a. How many hours do they each have to work before they go to the movies?
 10 hours
- **b.** How much will each of them have earned?
- **c.** Explain where the solution can be seen in tables of values, graphs, and equations that represent Priya's and Mai's hourly earnings.

Sample reasoning: In a table of values for each person, we would see the same entry for h and m in both tables. In the graph, the solution is found in the coordinates of the point (h, m) where the graphs of the two relationships intersect. In the equations, it is the value of h when we set the two expressions for m equal to each other: 8.4h = 7h + 14.

Problem 4

from Unit 4, Lesson 6

For each equation, explain what you could do first to each side of the equation so that there would be no fractions. You do not have to solve the equations (unless you want more practice).

a.
$$\frac{3x-4}{8} = \frac{x+2}{3}$$

Sample reasoning: If you multiply each side by 24 (the least common multiple of 8 and 3), then the equation becomes 3(3x - 4) = 8(x + 2). (The solution is x = 28.)

b.
$$\frac{3(2-r)}{4} = \frac{3+r}{6}$$

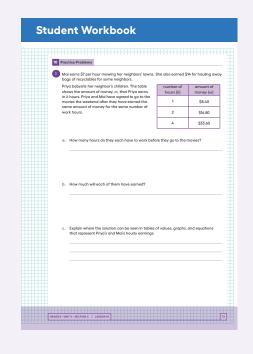
Sample reasoning: If you multiply each side by I2 (the least common multiple of 6 and 4), then the equation becomes 9(2-r)=2(3+r). (The solution is $r=\frac{12}{11}$.)

c.
$$\frac{4p+3}{8} = \frac{p+2}{4}$$

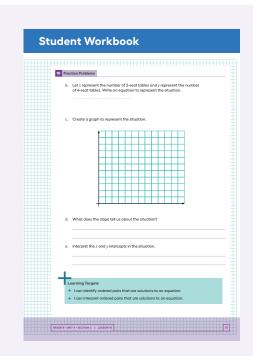
Sample reasoning: If you multiply each side by 8 (the least common multiple of 8 and 4), then the equation becomes 4p + 3 = 2(p + 2). (The solution is $p = \frac{1}{2}$.)

d.
$$\frac{2(a-7)}{15} = \frac{a+4}{6}$$

Sample reasoning: If you multiply each side by 30 (the least common multiple of 6 and 15), then the equation becomes 4(a-7) = 5(a+4). (The solution is a = -48.)



шп		
10 Pro	actice Problems	
Fo		do first to each side of the equation so that we to solve the equations (unless you want
а.	$\frac{3x-6}{8} \equiv \frac{x+2}{3}$	
ь.	$\frac{3(2-r)}{4} \equiv \frac{3+r}{6}$	
c.	$\frac{4p+3}{8}=\frac{p+2}{4}$	
d.	$\frac{2(a-7)}{75} = \frac{a+4}{6}$	
Ti fo		tables and chairs. He wants to have only tables people that can be seated in the restaurant
а.	Describe some possible combinations of 120 customers. Explain how you found	of 2-seat tables and 4-seat tables that will seat them.



Problem 5

from Unit 3, Lesson 15

The owner of a new restaurant is ordering tables and chairs. He wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

a. Describe some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers. Explain how you found them.

No 2-seat and 30 4-seat, 10 2-seat and 25 4-seat, 40 2-seat and 10 4-seat. Sample reasoning: I decided on a number for the 2-seat tables, then figured out how many people that would be (multiply the number of tables by 2) and subtracted that from 120. Then I divided by 4 to get the number of 4-seat tables needed for the remaining people.

b. Let *x* represent the number of 2-seat tables and *y* represent the number of 4-seat tables. Write an equation to represent the situation.

Sample response: 2x + 4y = 120

- **c.** Create a graph to represent the situation.
 - Graph is the line connecting (0, 30) and (60, 0) (or equivalent)
- **d.** What does the slope tell us about the situation? Sample response: The slope is $\frac{-1}{2}$. The $\frac{-1}{2}$ tells us that for every one fewer 4-seat table we can use 2 2-seat tables.
- **e.** Interpret the x and y intercepts in the situation.

Sample response: The intercepts tell us how many tables there will be if only 4-seat tables are used, (0,30), or only 2-seat tables are used, (60,0).

LESSON 10 • PRACTICE PROBLEMS