Lesson 10

Composing Figures

Goals

- Draw and label images of triangles under rigid transformations and then describe (orally and in writing) properties of the composite figure created by the images.
- Generalize that lengths and angle measures are preserved under any rigid transformation.
- Identify side lengths and angles that have equivalent measurements in composite shapes and explain (orally and in writing) why they are equivalent.

Learning Target

I can find missing side lengths or angle measures using properties of rigid transformations.

Student Learning Goal

Let's use reasoning about rigid transformations to find measurements without measuring.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 1)

Instructional Routines

- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2, Activity 3

Lesson Narrative

The purpose of this lesson is for students to apply the property that rigid transformations preserve angle measures and side lengths. In this lesson, students create composite shapes using translations, rotations, and reflections of polygons and continue to observe that the side lengths and angle measures do not change. They use this understanding to draw conclusions about the composite shapes. Students apply the property that the image of a line segment under a 180° rotation about a point not on the line is parallel to the original segment in order to explain why a figure must be a parallelogram. Students use the structure of rigid transformations as they apply these properties to various figures.

Students also create a drawing of a composite figure using rigid transformations, then make observations about the corresponding angle measures and side lengths. They identify corresponding parts and explain why they must be the same length or measure.

Consider using the optional activity to reinforce students' belief that rigid transformations preserve distances and angle measures.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
10 min	10 min	15 min	10 min	10 min	5 min

Warm-up

Notice and Wonder: Angles of an Isosceles Triangle

10 min

Activity Narrative

The purpose of this *Warm-up* is to connect prior student knowledge about triangles with rigid transformations, which will be useful when students perform rigid transformations on triangles in a later activity. While students may notice and wonder many things about this image, properties of triangles under rigid transformations are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is that since triangle ABC has two side lengths of the same length, triangle ABD must as well.

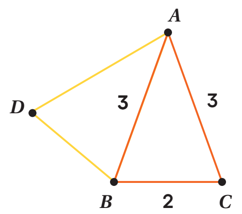
Launch

Arrange students in groups of 2. Display the image for all to see. Tell students that triangle ABD is the image of triangle ABC under a rigid transformation. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- Triangle ABC and triangle ABD share a side.
- Triangle ABC is an isosceles triangle.
- BD must be 2 units, and AD must be 3 units.

Students may wonder:

- Is triangle ABD a reflection or a rotation of triangle ABC?
- Is triangle ABC congruent to triangle ABD?
- How many copies of triangle ABC would it take to go all the way around point A?

Instructional Routines

Notice and Wonder

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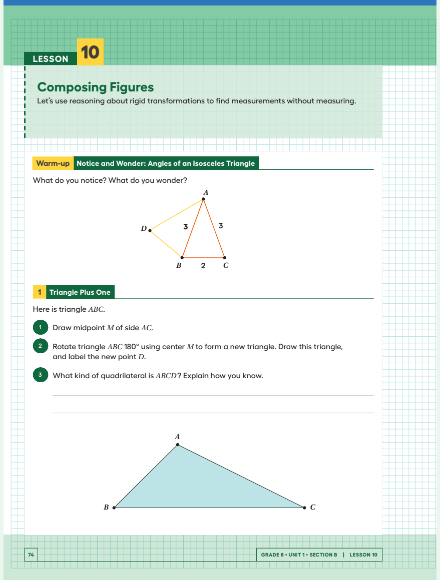
Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 10

Composing Figures

Let's use reasoning about rigid transformations to find measurements without measuring.

Warm-up Notice and Wonder: Angles of an Isosceles Triangle

What do you notice? What do you wonder?

1 Triangle Plus One

Here is triangle ABC.

1 Draw midpoint M of side AC.

2 Rotate triangle ABC 180° using center M to form a new triangle. Draw this triangle, and label the new point D.

3 What kind of quadrilateral is $ABCD$? Explain how you know.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the idea that triangle ABD could be a reflection or a rotation of triangle ABC does not come up during the conversation, ask students to discuss this idea.

Activity 1

Triangle Plus One

10 min

Activity Narrative

The purpose of this task is to use rigid transformations to explore the properties of a parallelogram. In an earlier course, students composed and rearranged two copies of a triangle to form a parallelogram. In this activity, students use the more precise language describing 180-degree rotations to describe this figure.

Students must also use an important property of 180-degree rotations, namely that the image of a line after a 180-degree rotation is parallel to that line. This is what allows them to conclude that the shape they have built is a parallelogram.

Launch

Arrange students in groups of 2. Provide access to geometry toolkits.

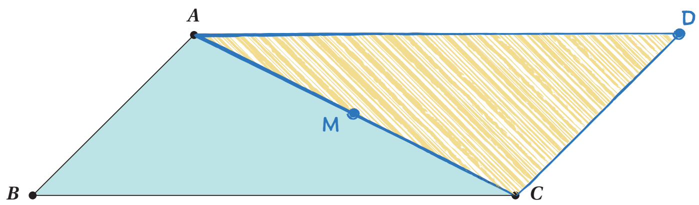
Give 3–4 minutes of quiet work time.

Check in with students within the first 2–3 minutes of work time.

Look for students who struggle with drawing the midpoint of side AC. Consider pausing for a brief discussion and invite 1–2 students to describe or demonstrate a strategy for finding the midpoint of AC. Follow with a partner discussion, then a whole-class discussion.

Student Task Statement

Here is triangle ABC.



1. Draw midpoint M of side AC.
2. Rotate triangle ABC 180° using center M to form a new triangle. Draw this triangle, and label the new point D.
3. What kind of quadrilateral is $ABCD$? Explain how you know.

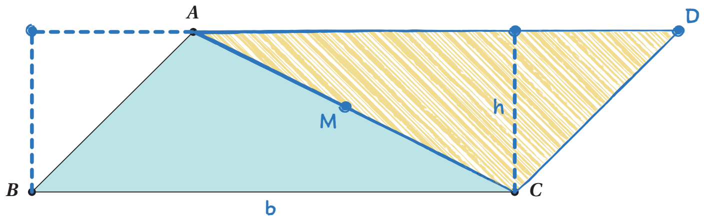
   a parallelogram.

   Sample reasoning: The 180° rotation around M takes line AB to line CD and so these are parallel. It also takes line BC to line AD so these lines are also parallel. That means that $ABCD$ is a parallelogram.

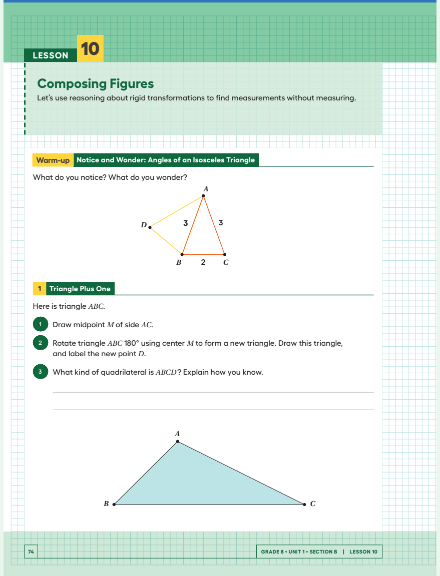
Are You Ready for More?

In the activity, we made a parallelogram by taking a triangle and its image under a 180-degree rotation around the midpoint of a side. This picture helps you justify a well-known formula for the area of a triangle. What is the formula and how does the figure help justify it?

A formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is a base of the triangle and h the corresponding height. In the image we have added b and h to the parallelogram, and marked off two triangles of interest with dashed lines. Since the left and right triangles have the same area, the area of the parallelogram displayed is the same as the area of a rectangle with height h and base b, namely, an area of bh. Since the area of each triangle is half the area of the parallelogram, each triangle has area $A = \frac{1}{2}bh$.



Student Workbook



Building on Student Thinking

Students may struggle to see the 180-degree rotation using center M. This may be because they do not understand that M is the center of rotation or because they struggle with visualizing a 180-degree rotation. Offer these students patty paper, a transparency, or the rotation overlay from earlier in this unit to help them see the rotated triangle.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "What kind of quadrilateral is ABCD? Explain how you know." Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Activity Synthesis

The purpose of this discussion is for students to connect properties of 180-degree rotations with features of a parallelogram. Ask students,

"What happens to points A and C under the rotation?"

They end up at C and A, respectively.

This type of rotation and analysis will happen several times in upcoming lessons.

Next ask,

"How do you know that the lines containing opposite sides of $ABCD$ are parallel?"

They are taken to one another by a 180-degree rotation.

As seen previously, the image of a 180-degree rotation of a line ℓ is parallel to ℓ. Students also saw that when 180-degree rotations were applied to a *pair* of parallel lines it resulted in a (sometimes) new pair of parallel lines which were also parallel to the original lines. The logic here is the same, except that only one line is being rotated 180° rather than a pair of lines. This does not need to be mentioned unless it is brought up by students.

Finally, ask students

"How is the area of parallelogram $ABCD$ related to the area of triangle ABC?"

The area of the parallelogram $ABCD$ is twice the area of triangle ABC because it is made up of ABC and CDA, which has the same area as ABC.

Activity 2

Triangle Plus Two

15 min

Activity Narrative

The purpose of this activity is for students to identify rigid transformations in a figure composed of multiple congruent triangles, then use properties of rigid transformations to identify corresponding line segments and angles. This particular figure will be important later in this unit when students show that the sum of the three angles in a triangle is 180°, so it is particularly important that students recognize corresponding angles with the same measure.

Students draw conclusions about the angle measures and segment lengths by applying the structure of rigid transformations to the composed figure, particularly the property that rigid transformations preserve angles and side lengths.

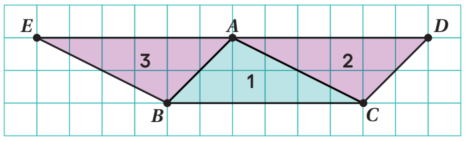
Launch

Keep students in the same groups. Provide access to geometry toolkits.

Give a few minutes of quiet work time, followed by sharing with a partner and a whole-class discussion.

Student Task Statement

The picture shows 3 triangles. Triangle 2 and Triangle 3 are images of Triangle 1 under rigid transformations.



1. Describe a rigid transformation that takes Triangle 1 to Triangle 2. What points in Triangle 2 correspond to points A, B, and C in the original triangle?

   Sample response: A 180-degree rotation using the midpoint of side AC as center. In triangle 2, point C corresponds to A in the original, D corresponds to B in the original, and A corresponds to C in the original.

2. Describe a rigid transformation that takes Triangle 1 to Triangle 3. What points in Triangle 3 correspond to points A, B, and C in the original triangle?

   Sample response: A 180-degree rotation using the midpoint of side AB as center or a 180-degree rotation using the midpoint of segment AC as center followed by a translation taking A to E. In Triangle 3, point A corresponds to B in the original, B corresponds to A in the original, and E corresponds to C in the original.

3. Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

   Sample response: Segment AE and segment BC are the same length and segments AB and CD are also the same length. This is true because a rigid transformation doesn't change a figure's side lengths.

4. Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

   Sample response: $\angle D$ and $\angle ABC$ have the same measure and so do $\angle E$ and $\angle ACB$. This is true because a rigid transformation doesn't change a figure's angle measures.

Activity Synthesis

Ask students to list as many different pairs of matching line segments as they can find. Then, do the same for angles. Record these for all to see. Consider displaying the image from the student task and highlighting or coloring corresponding sides and angle measures. Students may wonder why there are fewer pairs of line segments: this is because of shared sides AB and AC. If they don't ask, there's no reason to bring it up.

Display this statement for all to see: "Under any rigid transformation, lengths and angle measures are preserved." Invite students to restate this statement using their own words and share aloud with the whole class or with a partner.

Access for Students with Diverse Abilities (Activity 2, Student Task)

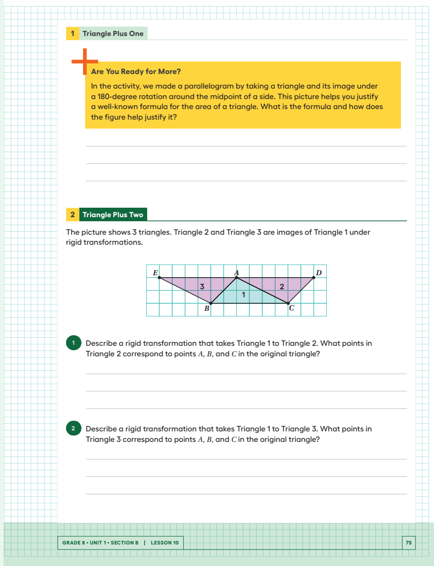
Engagement: Provide Access by Recruiting Interest.
Provide choice and autonomy. Provide access to colored pencils or markers they can use to solve the problem.

Supports accessibility for: Visual-Spatial Processing, Organization

Building on Student Thinking

Students may have trouble understanding which pairs of points correspond in the first two questions, particularly the fact that point A in one triangle may not correspond to point A in another. Use tracing paper to create a transparency of triangle ABC, with its points labeled, and let students perform their rigid transformation. They should see A, B, and C on top of points in the new triangle.

Student Workbook

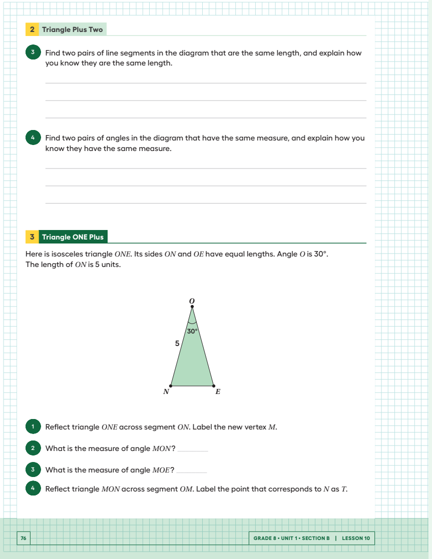


Building on Student Thinking

If students are stuck with the first reflection, suggest that they use tracing paper. If needed, show them the first reflected triangle, then have them continue to answer the problems and do the next reflection on their own.

Some students may have difficulty with the unlabeled length of OT, since it uses the initial information that triangle ONE is isosceles. Ask students what other information is given and if they can use it to figure out the missing length.

Student Workbook


2 Triangle Plus Two

3 Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

4 Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

3 Triangle ONE Plus

Here is isosceles triangle ONE. Its sides ON and OE have equal lengths. Angle O is 30°. The length of ON is 5 units.

1 Reflect triangle ONE across segment ON. Label the new vertex M.

2 What is the measure of angle MON? ______

3 What is the measure of angle MOE? ______

4 Reflect triangle MON across segment OM. Label the point that corresponds to N as T.

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Activity 3: Optional
Triangle ONE Plus

10 min

Activity Narrative

This activity is optional because it provides additional practice for performing rigid transformations on a triangle to compose a new figure. Students may notice different properties of the composed figure, including that two of the sides of the triangles (one side of the original and one side of the 6th) lie on the same line. Students may also notice the right angle made by 3 triangles and reason that they can complete a circle with 4 right angles, or they may notice that the 6 triangle pattern can be reflected over this line to make it "complete" with 12 copies of the original triangle.

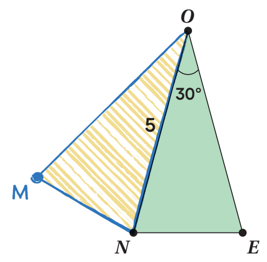
Launch

Provide access to geometry toolkits.

Allow for 8 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Here is isosceles triangle ONE. Its sides ON and OE have equal lengths. Angle O is 30°. The length of ON is 5 units.



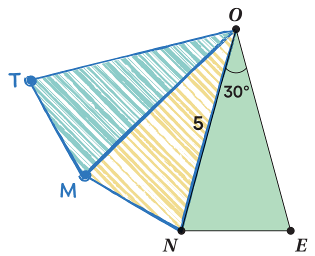
1. Reflect triangle ONE across segment ON. Label the new vertex M.
2. What is the measure of angle MON?

   30° because angle measures are the same after applying a rigid transformation and angle NOE becomes angle NOM after reflection.

3. What is the measure of angle MOE?

   60° because $\angle MOE$ is found by putting together $\angle MON$ and $\angle NOE$ and these are both 30° angles.

4. Reflect triangle MON across segment OM. Label the point that corresponds to N as T.



5. How long is $\overline{OT}$? How do you know?

Segment OT measures 5 units because it is the image of segment ON after a reflection so it has the same length as segment ON.

6. What is the measure of angle TOE?

90°, a right angle because $\angle MOE$ measures 60° and $\angle TOM$ measures 30°. Since $\angle TOE$ is found by putting together $\angle TOM$ and $\angle MOE$ so it is a 90° angle.

7. If you continue to reflect each new triangle this way to make a pattern, what will the pattern look like?

Sample response: Eventually point O will be completely surrounded by triangles, with the 12th triangle touching $\overline{OE}$ again. There are 4 right angles in a full circle and each right angle has 3 copies of the original triangle.

Activity Synthesis

The purpose of this discussion is to apply and reinforce students' belief that rigid transformations preserve distances and angle measures.

Invite students to share how they know the angle measures and side lengths of each part of their completed figure. Highlight student responses that use reasoning, such as the following:

- Angle measures are the same after applying a rigid transformation.
- The image of a segment after a reflection is the same length as the original.
- Since each triangle is made using a sequence of rigid transformations from triangle ONE, the central angle of each triangle in the figure must be the same, which is 30 degrees.

Select a few students, each with a different response, to share their description of the pattern they saw in the last question. The way in which each student visualizes and explains this shape may give insight into the different strategies used to create the final pattern.

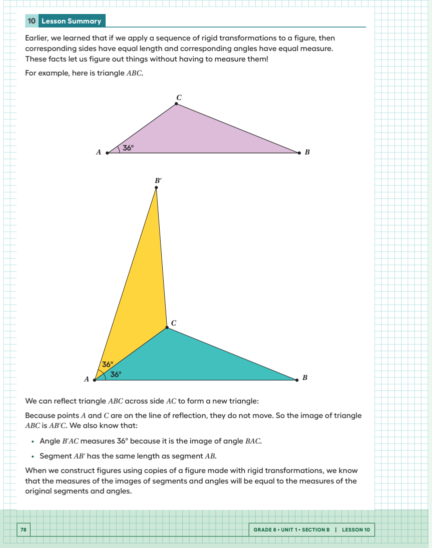
Note: It is *not* important nor required that students know or understand how to find the base angle measures of the isosceles triangles, or even that the base angles have the same measure. Later in this unit, students will show that the sum of the angle measures in a triangle is 180 degrees.

Access for Students with Diverse Abilities (Activity 3, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as: "___ will always ___ because … ," "How do you know … ?" and "Is it always true that … ?"

Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


10 Lesson Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.

We can reflect triangle ABC across side AC to form a new triangle:

Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures 36° because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

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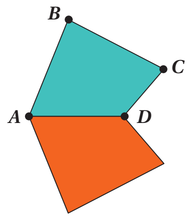
Lesson Synthesis

Display this image for all to see:



This figure was created by reflecting $ABCD$ across line AD. Invite students to identify the corresponding sides and angles in the figure. Consider highlighting or coloring corresponding sides and angle measures as students identify them.

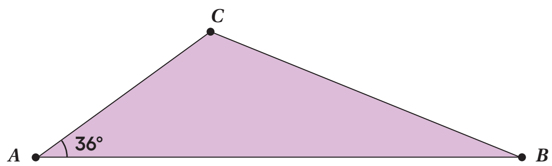
Ask students what knowing that this figure uses a reflection tells them about the corresponding side lengths and angles.

> Reflections are a type of rigid transformation, and rigid transformations preserve lengths and angle measures. This means that the side lengths of corresponding sides are the same, and the corresponding angle measures are the same.

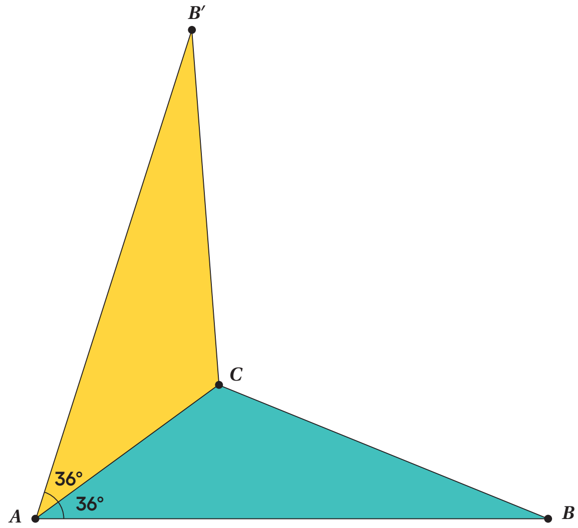
Lesson Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.



We can reflect triangle ABC across side AC to form a new triangle:



Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures 36° because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

Responding To Student Thinking

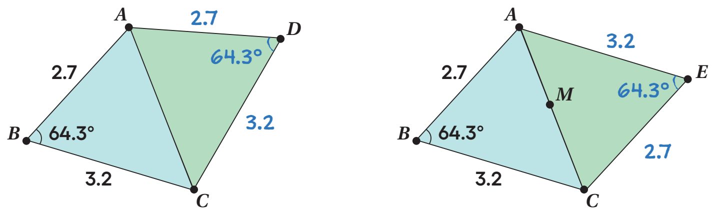
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down
Identifying Side Lengths and Angle Measures

5 min

Student Task Statement

Here is a diagram showing triangle ABC and some transformations of triangle ABC.

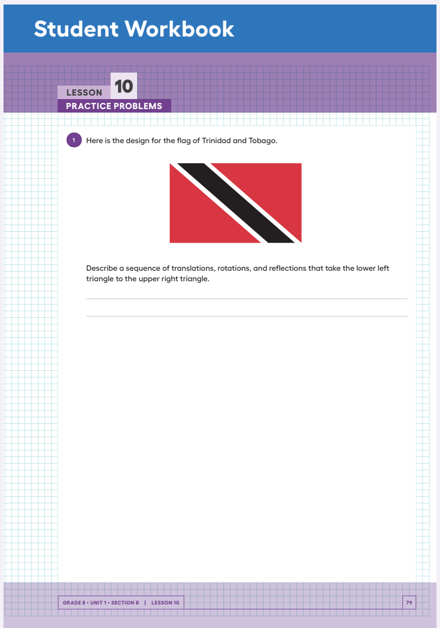


On the left side of the diagram, triangle ABC has been *reflected* across line AC to form quadrilateral $ABCD$. On the right side of the diagram, triangle ABC has been *rotated* 180° using midpoint M as a center to form quadrilateral $ABCE$.

Using what you know about rigid transformations, side lengths and angle measures, label as many side lengths and angle measures as you can in quadrilaterals $ABCD$ and $ABCE$.

Practice Problems

5 Problems


Student Workbook

LESSON 10
PRACTICE PROBLEMS

1 Here is the design for the flag of Trinidad and Tobago.

Describe a sequence of translations, rotations, and reflections that take the lower left triangle to the upper right triangle.

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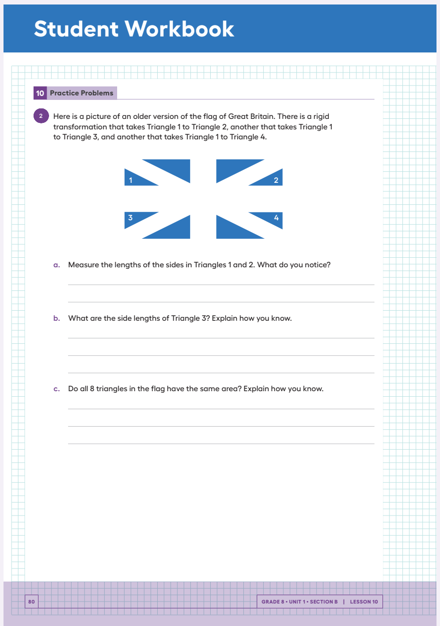
Student Workbook

10 Practice Problems

2 Here is a picture of an older version of the flag of Great Britain. There is a rigid transformation that takes Triangle 1 to Triangle 2, another that takes Triangle 1 to Triangle 3, and another that takes Triangle 1 to Triangle 4.

a. Measure the lengths of the sides in Triangles 1 and 2. What do you notice?

b. What are the side lengths of Triangle 3? Explain how you know.

c. Do all 8 triangles in the flag have the same area? Explain how you know.

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Problem 1

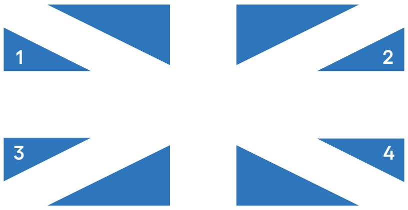
Here is the design for the flag of Trinidad and Tobago.

Describe a sequence of translations, rotations, and reflections that take the lower left triangle to the upper right triangle.

Sample response: The lower left triangle is first translated to the right so that it shares an edge with the upper right triangle. Then it's rotated 180 degrees around the midpoint of the common side.

Problem 2

Here is a picture of an older version of the flag of Great Britain. There is a rigid transformation that takes Triangle 1 to Triangle 2, another that takes Triangle 1 to Triangle 3, and another that takes Triangle 1 to Triangle 4.



a. Measure the lengths of the sides in Triangles 1 and 2. What do you notice?

Sample response: The side lengths of the two triangles are the same.

b. What are the side lengths of Triangle 3? Explain how you know.

The side lengths will be the same as Triangle 1. Sample reasoning: A rigid transformation takes Triangle 1 to Triangle 3.

c. Do all 8 triangles in the flag have the same area? Explain how you know.

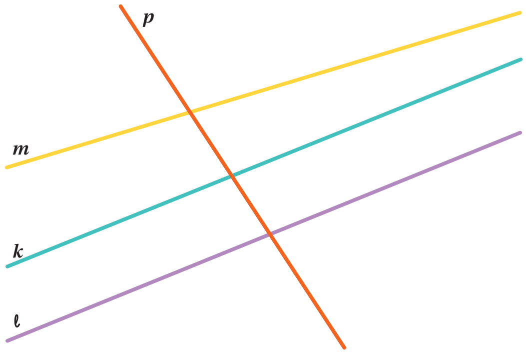
no

Sample reasoning: The four triangles without number labels are larger, so they will not have the same area as the smaller labeled triangles.

Problem 3

from Unit 1, Lesson 9

a. Which of the lines in the picture is parallel to line ℓ? Explain how you know.



k

Sample reasoning: These two lines do not intersect no matter how far out they extend.

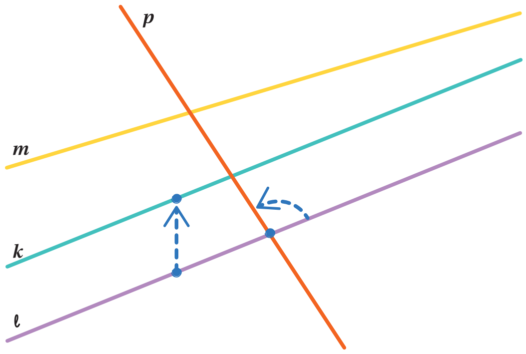
b. Explain how to translate, rotate or reflect line ℓ to obtain line k.

Sample reasoning: Line k can be obtained by translating line ℓ.

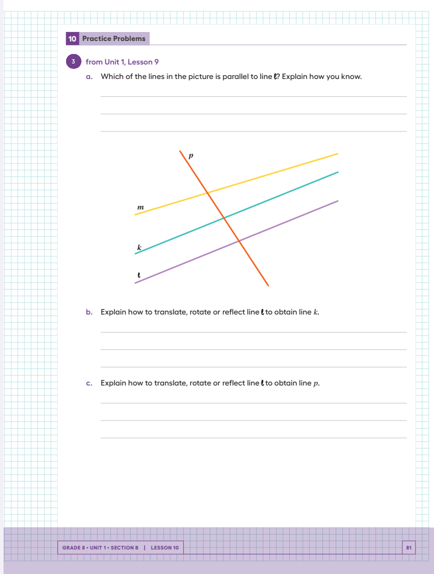
c. Explain how to translate, rotate or reflect line ℓ to obtain line p.

Sample reasoning: Line p can be obtained by rotating lin ℓ.

The picture shows how to translate ℓ to get k and how to rotate ℓ to get p.



Student Workbook



10 Practice Problems

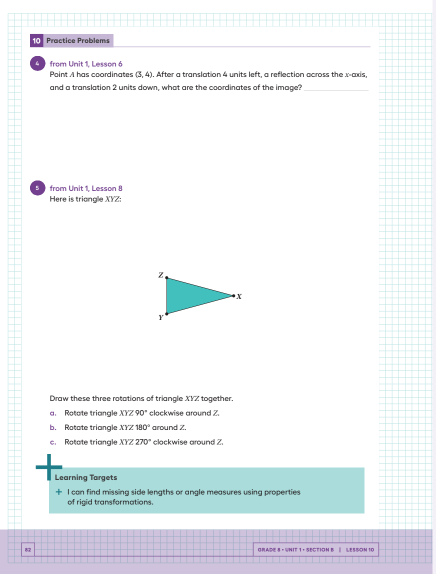
3 from Unit 1, Lesson 9

a. Which of the lines in the picture is parallel to line ℓ? Explain how you know.

b. Explain how to translate, rotate or reflect line ℓ to obtain line k.

c. Explain how to translate, rotate or reflect line ℓ to obtain line p.

Student Workbook


10 Practice Problems

4 from Unit 1, Lesson 6

Point A has coordinates (3, 4). After a translation 4 units left, a reflection across the x-axis, and a translation 2 units down, what are the coordinates of the image?

5 from Unit 1, Lesson 8

Here is triangle XYZ:

Draw these three rotations of triangle XYZ together.

a. Rotate triangle XYZ 90° clockwise around Z.

b. Rotate triangle XYZ 180° around Z.

c. Rotate triangle XYZ 270° clockwise around Z.

Learning Targets

+ I can find missing side lengths or angle measures using properties of rigid transformations.

Problem 4

from Unit 1, Lesson 6

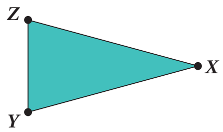
Point A has coordinates (3, 4). After a translation 4 units left, a reflection across the x-axis, and a translation 2 units down, what are the coordinates of the image?

(-1, -6)

Problem 5

from Unit 1, Lesson 8

Here is triangle XYZ:



Draw these three rotations of triangle XYZ together.

a. Rotate triangle XYZ 90° clockwise around Z.

b. Rotate triangle XYZ 180° around Z.

c. Rotate triangle XYZ 270° clockwise around Z.

Each rotation shares vertex Z with triangle XYZ. The four triangles together look like a pinwheel.