Lesson 3

Balanced Moves

Goals

- Compare and contrast (orally and in writing) solution paths to solve an equation in one variable by performing the same operation on each side.
- Correlate (orally and in writing) changes on hanger diagrams with moves that create equivalent equations.

Learning Target

I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution.

Lesson Narrative

In this lesson, students continue to transition from hanger diagrams to algebraic equations by identifying a goal for writing equivalent equations. The focus is on using moves to solve for a variable while still describing each move that creates an equivalent equation. Students should begin to recognize which moves are useful for writing equivalent equations that get closer to the goal. Students should also recognize when they have choices for moves such as distributing a value first or dividing by the value instead. Students must pay attention to the structure of the equations to see which moves will effectively work toward a solution.

Student Learning Goal

Let's rewrite equations while keeping the same solutions.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Math Community Chart: Activity 1

Materials to Copy

- Card Sort Matching Equation Moves Cards (1 copy for every 2 students): Activity 1

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	15 min	10 min	10 min	5 min

Warm-up
Matching Hangers

10 min

Activity Narrative

The purpose of this *Warm-up* is for students to connect moves that keep a hanger balanced with moves that create equivalent equations. These moves include:

- Adding or subtracting the same thing on each side.
- Multiplying or dividing each side by the same number.

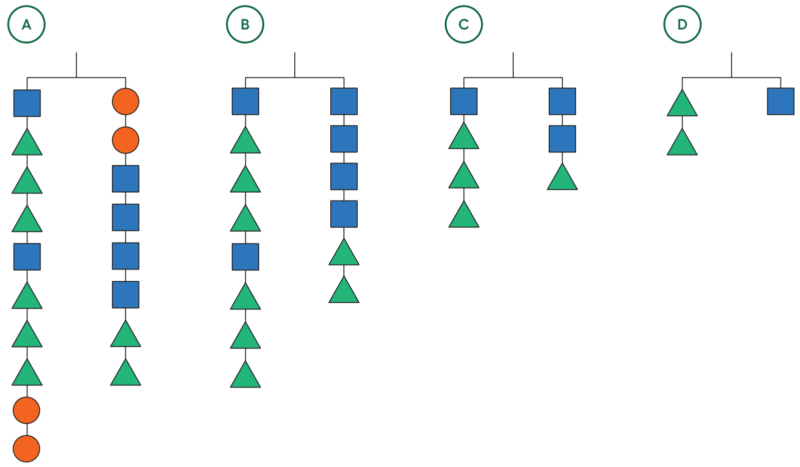
Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Figures A, B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.

Let x be the weight of the blue square, y be the weight of a green triangle, and z be the weight of a red circle.



Here are some equations. Each equation represents one of the hanger diagrams.

1. Draw an arrow from each hanger to the next and describe the move that keeps the hanger in balance near each arrow.
   Match each equation to a hanger.

   Remove 2 circles from each side of the hanger, remove half the shapes on each side, remove 1 triangle and 1 square on each side

Inspire Math

Supply and Demand video

Go Online
Before the lesson, show this video to introduce the real-world connection.

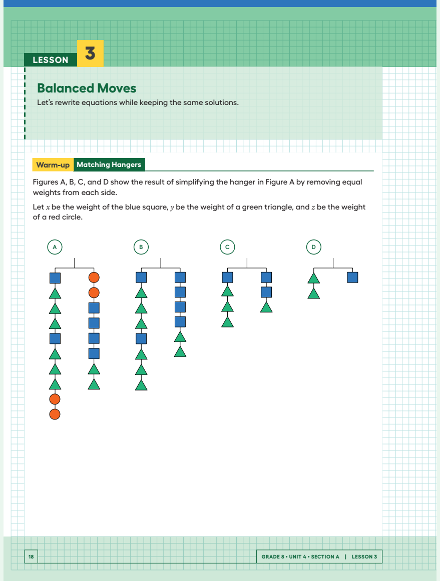
ilclass.com/l/614149

Please log in to the site before using the QR code or URL.

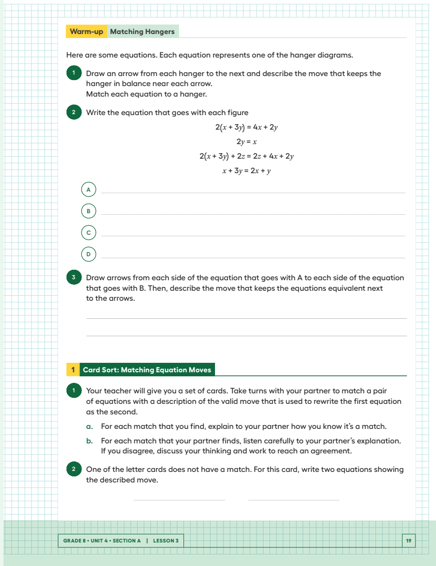
Building on Student Thinking

Some students may be confused about how to match the equations with the parentheses to the hangers. Ask students what they know about the connections between variables and shapes. Then, ask students how many of each shape might be needed for each side of the hanger to match the equation.

Student Workbook



Student Workbook


Warm-up Matching Hangers

Here are some equations. Each equation represents one of the hanger diagrams.

1. Draw an arrow from each hanger to the next and describe the move that keeps the hanger in balance near each arrow.
Match each equation to a hanger.

2. Write the equation that goes with each figure

$$2(x+3y)=4x+2y$$
$$2y=x$$
$$2(x+3y)+2z=2z+4x+2y$$
$$x+3y=2x+y$$

A
B
C
D

3. Draw arrows from each side of the equation that goes with A to each side of the equation that goes with B. Then, describe the move that keeps the equations equivalent next to the arrows.

1 Card Sort: Matching Equation Moves

1. Your teacher will give you a set of cards. Take turns with your partner to match a pair of equations with a description of the valid move that is used to rewrite the first equation as the second.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to your partner's explanation. If you disagree, discuss your thinking and work to reach an agreement.

2. One of the letter cards does not have a match. For this card, write two equations showing the described move.

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2. Write the equation that goes with each figure.

$$2(x + 3y) = 4x + 2y$$
$$2y = x$$
$$2(x + 3y) + 2z = 2z + 4x + 2y$$
$$x + 3y = 2x + y$$

A: $2(x+3y)+2z=2z+4x+2y$

B: $2(x+3y)=4x+2y$

C: $x+3y=2x+y$

D: $2y=x$

3. Draw arrows from each side of the equation that goes with A to each side of the equation that goes with B. Then, describe the move that keeps the equations equivalent next to the arrows.

Subtract $2z$ from each side of the equation.

Activity Synthesis

The purpose of the discussion is to make connections between moves that can be done on a hanger to keep it in balance and moves that can be done to an equation to write an equivalent equation.

Invite students to share the order of equations, then display the image for all to see.

$$\begin{aligned} 2(x+3y)+2z &= 2z+4x+2y \\ 2(x+3y) &= 4x+2y \\ x+3y &= 2x+y \\ 2y &= x \end{aligned}$$

Select students to share their descriptions of the hanger moves and corresponding equation moves, recording for all to see. Focus on the move from Hanger B to Hanger C. If students struggle to describe the move, ask them to look at the equations for an idea of what might be happening. (Half of the weight on each side is removed. One of the square-and-2-triangle patterns is removed on the left and half of the squares and half of the triangles are removed on the right.) Ask students how this is related to the move that is done on the equations. Make sure students understand that this move can be stated as "divide each side by 2" as well as "multiply each side by $\frac{1}{2}$."

Activity 1

Card Sort: Matching Equation Moves

15 min

Activity Narrative

In this partner activity, students take turns matching equations with descriptions of valid moves between them to practice identifying and providing descriptions of moves. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Students also encounter negative values for the first time in the unit, which are not possible when using hangers.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice that they think will help themself and their group during the activity. At the end of the activity, students can share what norms they choose and how the norm did or did not support their group.

Tell students that the cards contain either a pair of equations or a move that can transform an equation into an equivalent equation and that they will take turns matching the cards. Explain how to set up and do the activity. If time allows, demonstrate these steps with a student as a partner:

- Mix up the cards and place them face-up.
- One person selects one card of each type and explains to their partner why the cards match.
- The partner's job is to listen and make sure that they agree. If they don't agree, the partners discuss until they come to an agreement.
- When both partners agree on the match, they switch roles.

Consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of pre-cut cards.

Tell students that hanger diagrams are useful only for reasoning about positive numbers, but all of the usual moves (adding, subtracting, multiplying, or dividing the same value on each side) also work for negative numbers. Ask partners who finish early to write down on a separate sheet of paper what the next move would be for each of the numbered cards if the goal were to solve for x.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, give students a subset of the cards to start with and introduce the remaining cards after students have completed their initial set of matches.

Supports accessibility for: Organization, Attention

Student Task Statement

1. Your teacher will give you a set of cards. Take turns with your partner to match a pair of equations with a description of the valid move that is used to rewrite the first equation as the second.

   1. B

   2. E

   3. D

   4. F

   5. A

   a. For each match that you find, explain to your partner how you know it's a match.

   b. For each match that your partner finds, listen carefully to your partner's explanation. If you disagree, discuss your thinking and work to reach an agreement.

2. One of the letter cards does not have a match. For this card, write two equations showing the described move.

   6. C

   Sample response: $5 - 3x = 2x + 8$, $5 = 5x + 8$.

Activity Synthesis

The purpose of this discussion is to get students using the language of equations and describing changes happening on each side when writing equivalent equations. Select 2–3 groups to share one of their sets of cards and how they matched the pair of equations with a move. Discuss as many different sets of cards as the time allows.

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide a sentence frame to help students organize their thoughts in a clear, precise way:

"I picked the norm '____.' It really helped me/my group because ____."

"I picked the norm '____.' During the activity, I realized the norm '____' would be a better focus because ____."

Activity 2
Keeping Equality

10 min

Activity Narrative

The purpose of this activity is to get students thinking about strategically solving equations by paying attention to their structure.

Monitor for groups who use these different strategies:

- Distribute a value to the terms in parentheses.
- Divide each side by the coefficient of the parenthetical expression. Both methods are equally valid, but there are advantages and disadvantages to each.

Both methods are equally valid, but there are advantages and disadvantages to each.

Launch

Arrange students in groups of 2.

Give partners 3–5 minutes to complete the problems. Follow with a whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative,* to share later.

Student Task Statement

1. Noah and Lin both solve the equation $14a = 2(a - 3)$.

   Do you agree with either of them? Explain your reasoning.

   Noah's solution:

   $14a = 2(a - 3)$

   $14a = 2a - 6$

   $12a = -6$

   $a = -\frac{1}{2}$

   Lin's solution:

   $14a = 2(a - 3)$

   $7a = a - 3$

   $6a = -3$

   $a = -\frac{1}{2}$

   I agree with both Noah and Lin.

   Sample reasoning: Both Noah and Lin followed valid solution paths. Substituting $a = -\frac{1}{2}$ into the original equation yields a true statement, so their solutions are correct.

2. Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?

   Sample response: There are at least two solution paths to this equation: you can divide each side by 5 first, then collect like terms, or you can distribute and collect like terms, then continue to solve.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

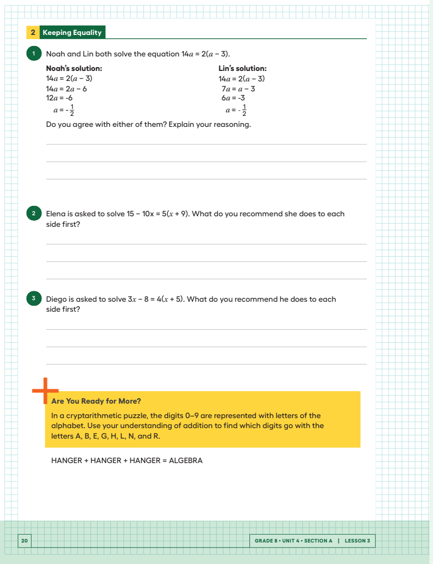
Access for Multilingual Learners (Activity 2)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Building on Student Thinking

Some students may have trouble understanding Noah's or Lin's solutions. Ask them to try drawing arrows to describe each move.

Student Workbook



2 Keeping Equality

1 Noah and Lin both solve the equation $14a = 2(a - 3)$.

Noah's solution: $14a = 2(a - 3)$, $14a = 2a - 6$, $12a = -6$, $a = -\frac{1}{2}$

Lin's solution: $14a = 2(a - 3)$, $7a = a - 3$, $6a = -3$, $a = -\frac{1}{2}$

Do you agree with either of them? Explain your reasoning.

2 Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?

3 Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

Are You Ready for More?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

HANGER + HANGER + HANGER = ALGEBRA

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3. Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

Sample response: There are still two solution paths to this equation, but one is much simpler than the other. Since not all the terms are multiples of 4, dividing first by 4 will give a fractional coefficient of x on one side. Therefore, distributing first and then collecting like terms and solving is the simpler solution path.

Are You Ready for More?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

HANGER + HANGER + HANGER = ALGEBRA

A:2, B:8, E:1, G:6, H:9, L:7, N:0, R:4

Activity Synthesis

The goal of the discussion is to see the advantages and disadvantages of each strategy.

Display 2–3 approaches/representations from previously selected students for all to see. If time allows, invite students to briefly describe their solution paths for the last two questions, then use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"What are the advantages of choosing to distribute first? To divide first?"*

  Distributing first eliminates confusion about which terms can be subtracted from each side. Dividing first makes the numbers smaller and easier to mentally calculate.

- *"What makes it easier to distribute versus divide first on the last question?"*

  In Diego's equation, dividing by 4 before distributing will result in non-integer terms, which can be harder to add and subtract mentally.

- *"Is one path more right than another?"*

  No. As long as we follow valid steps, like adding or multiplying by the same thing on each side of an equation, the steps are right and will give a correct solution.

Lesson Synthesis

Display the equation $6(x + 2) + 12 = 10x - 4$ for all to see. Tell students to think of 3 different things they could do to each side of the question but still maintain equality. Invite students to share their moves. Sample responses include:

- Distribute the 6 on the left side of the equation.
- Divide each side by 6.
- Subtract 12 from each side.

Lesson Summary

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $-2x - 3$ have equal value, we can write the equation

$$4x + 9 = -2x - 3.$$

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add negative numbers! For example, we can add -9 to each side of the equation.

$$\begin{array}{rrcll} & 4x + 9 &=& -2x - 3 & \\ \text{add -9} & 4x + 9 + -9 &=& -2x - 3 + -9 & \text{add -9} \\ \text{combine like terms} & 4x &=& -2x - 12 & \text{combine like terms} \end{array}$$

Because expressions represent numbers, we can also add expressions to each side of an equation. For example, we can add $2x$ to each side and still maintain equality.

$$\begin{array}{rrcll} & 4x &=& -2x - 12 & \\ \text{add } 2x & 4x + 2x &=& -2x - 12 + 2x & \text{add } 2x \\ \text{combine like terms} & 6x &=& -12 & \text{combine like terms} \end{array}$$

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (as long as we do not divide by zero).

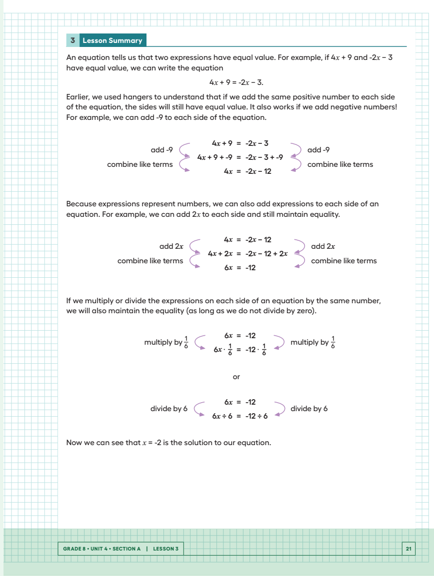
$$\begin{array}{rrcll} & 6x &=& -12 & \\ \text{multiply by } \frac{1}{6} & 6x \cdot \frac{1}{6} &=& -12 \cdot \frac{1}{6} & \text{multiply by } \frac{1}{6} \end{array}$$

or

$$\begin{array}{rrcll} & 6x &=& -12 & \\ \text{divide by 6} & 6x \div 6 &=& -12 \div 6 & \text{divide by 6} \end{array}$$

Now we can see that $x = -2$ is the solution to our equation.

Student Workbook



Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

More Matching Moves

5 min

Student Task Statement

1. Match these pairs of equations with the description of what is done in each step.

Step 1:
$12x - 6 = 10$
$6x - 3 = 5$

Step 2:
$6x - 3 = 5$
$6x = 8$

Step 3:
$6x = 8$
$x = \frac{4}{3}$

A: Add 3 to each side

B: Multiply each side by $\frac{1}{6}$

C: Divide each side by 2

2. You are given the equation $3(x - 2) = 8$. Is your first step to distribute or divide? Explain your reasoning.

Sample responses:

- I would distribute the 3. That way I do not need to deal with fractions like $\frac{8}{3}$ until the end.
- I would divide each side by 3. Then there are fewer terms to manage while solving.

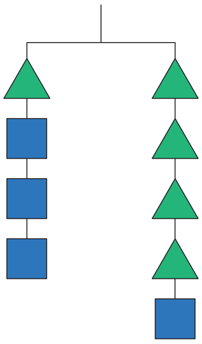
Lesson 3

Practice Problems

5 Problems

Problem 1

In this hanger, the weight of the triangle is x and the weight of the square is y.



a. Write an equation using x and y to represent the hanger.

$x + 3y = 4x + y$ (or equivalent)

b. If x is 6, what is y?

$y = 9$

Problem 2

Andre and Diego are each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each make to the equation.

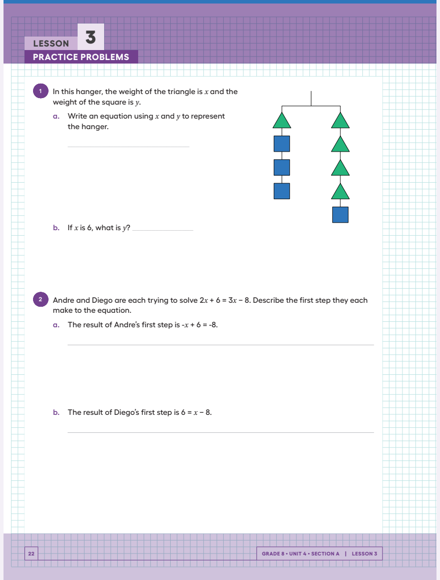
a. The result of Andre's first step is $-x + 6 = -8$.

Andre subtracted $3x$ from each side.

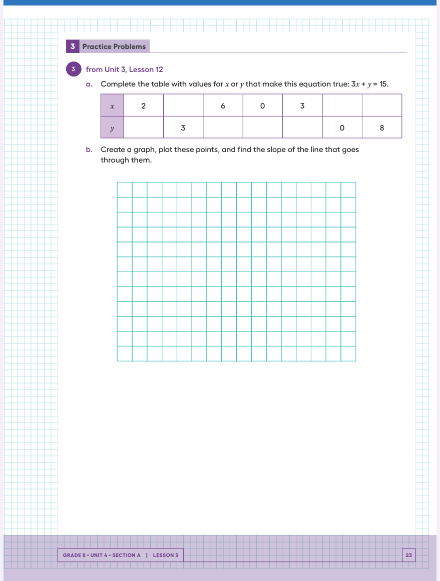
b. The result of Diego's first step is $6 = x - 8$.

Diego subtracted $2x$ from each side.

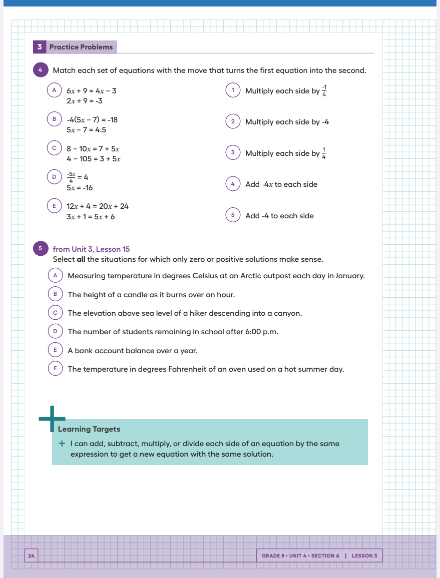
Student Workbook


LESSON 3
PRACTICE PROBLEMS
1 In this hanger, the weight of the triangle is x and the weight of the square is y.
a. Write an equation using x and y to represent the hanger.
b. If x is 6, what is y?
2 Andre and Diego are each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each make to the equation.
a. The result of Andre's first step is $-x + 6 = -8$.
b. The result of Diego's first step is $6 = x - 8$.

Student Workbook



Student Workbook



Problem 3

from Unit 3, Lesson 12

a. Complete the table with values for x or y that make this equation true: $3x + y = 15$.

x	2	4	6	0	3	5	$\frac{7}{3}$
y	9	3	-3	15	6	0	8

b. Create a graph, plot these points, and find the slope of the line that goes through them.

All 7 points plotted correctly with the line drawn. The slope is -3.

Problem 4

Match each set of equations with the move that turns the first equation into the second.

A. $6x + 9 = 4x - 3$
$2x + 9 = -3$

B. $-4(5x - 7) = -18$
$5x - 7 = 4.5$

C. $8 - 10x = 7 + 5x$
$4 - 10x = 3 + 5x$

D. $\frac{-5x}{4} = 4$
$5x = -16$

E. $12x + 4 = 20x + 24$
$3x + 1 = 5x + 6$

1. Multiply each side by $\frac{-1}{4}$

2. Multiply each side by -4

3. Multiply each side by $\frac{1}{4}$

4. Add $-4x$ to each side

5. Add -4 to each side

Problem 5

from Unit 3, Lesson 15

Select **all** the situations for which only zero or positive solutions make sense.

A. Measuring temperature in degrees Celsius at an Arctic outpost each day in January.

B. The height of a candle as it burns over an hour.

C. The elevation above sea level of a hiker descending into a canyon.

D. The number of students remaining in school after 6:00 p.m.

E. A bank account balance over a year.

F. The temperature in degrees Fahrenheit of an oven used on a hot summer day.