Lesson 12

Applications of Arithmetic with Powers of 10

Goals

- Determine what information is needed to answer a question about large numbers, and explain (orally) how that information would help solve the problem.
- Use exponent rules and powers of 10 to solve problems in context, and explain (orally) the steps used to organize thinking.

Learning Target

I can apply what I learned about powers of 10 to answer questions about real-world situations.

Access for Multilingual Learners:

- MLR8: Discussion Supports (Activity 1)

Access for Students with Diverse Abilities:

- Representation (Activity 1)

Required Materials

Materials to Gather

- Math Community Chart: Activity 2

Lesson Narrative

In this lesson, students apply what they have learned about working with exponents and powers of ten to solve problems in context. Students begin by considering questions about the tallest building in the world and what information would be needed to answer those questions. They have the opportunity to ask for the relevant information and use operations with exponents to compare several very large values. An optional activity allows students an additional opportunity to consider what information is needed to solve a problem about distances to the Moon, ask for that information, and solve the problem. Students must interpret the meaning of their answers in context.

Student Learning Goal

Let's use powers of 10 to help us make calculations with large and small numbers.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	25 min	15 min	10 min	5 min

Warm-up
What Information Do You Need?

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to reason about a real-world situation and consider the essential information required to solve problems.

Launch

Arrange students in groups of 2. Tell students that the Burj Khalifa is the tallest building in the world and is located in the city of Dubai. Then give students 1 minute of quiet think time, followed by 1 minute to share their responses with a partner. Follow with a whole-class discussion.

Student Task Statement

What information would you need to answer these questions?

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?

   Sample responses: the height of the Burj Khalifa, the cost to build the Burj Khalifa, the denomination of money that would be stacked, the thickness of one unit of the specified denomination

2. Which has more mass, the Burj Khalifa or the mass of pennies it cost to build the Burj Khalifa?

   Sample responses: the mass of the Burj Khalifa, the cost to build the Burj Khalifa, the mass of one penny

Activity Synthesis

The goal of this discussion is for students to share their list of information needed to answer the questions. Invite students to share their responses for each question. Record and display the responses for all to see.

Consider asking questions like these to encourage students to reason further about each question:

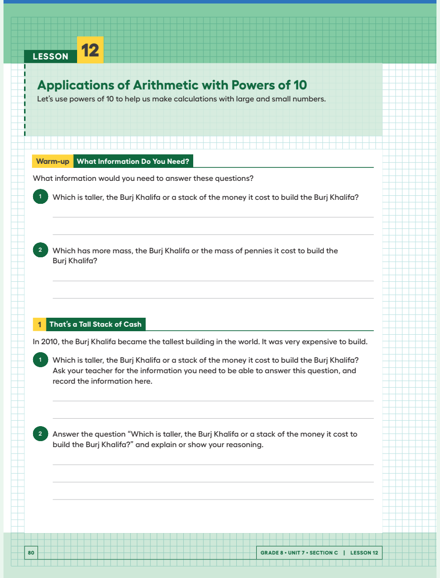
"Why do you need that piece of information?"

"How would you use that piece of information in finding the solution?"

"Where would you look to find that piece of information?"

If time allows, ask students to make predictions for each of the questions in the *Task Statement*. Record and display their responses for all to see.

Student Workbook


LESSON 12

Applications of Arithmetic with Powers of 10

Let's use powers of 10 to help us make calculations with large and small numbers.

Warm-up What Information Do You Need?

What information would you need to answer these questions?

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?

2. Which has more mass, the Burj Khalifa or the mass of pennies it cost to build the Burj Khalifa?

1 That's a Tall Stack of Cash

In 2010, the Burj Khalifa became the tallest building in the world. It was very expensive to build.

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa? Ask your teacher for the information you need to be able to answer this question, and record the information here.

2. Answer the question "Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?" and explain or show your reasoning.

80 GRADE 8 • UNIT 7 • SECTION C | LESSON 12

Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.
Prior to solving the problems, invite students to make sense of the situations and to take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Activate or supply background knowledge. Show a photo or video of some of the tallest buildings and structures in the world for students who are unfamiliar with the Burj Khalifa.
Supports accessibility for: Conceptual Processing, Language

Activity 1
That's a Tall Stack of Cash

25 min

Activity Narrative

The large quantities involved in these questions lend themselves to multiplication and division with powers of 10. Students use numbers and exponents flexibly and interpret their results in context.

Launch

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice. This norm should be one that they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

From the *Warm-up*, students have determined what information they need to solve the problem. Invite students to ask for the information they need. Provide students with only the information they request. Display the information for students to see throughout the activity. If students find they need more information later, provide it to the whole class then.

Here is information students might ask for in order to solve the problems:

- The Burj Khalifa is 830 meters tall.
- The Burj Khalifa cost 1.5 billion dollars to build.
- The stack of money is made out of $100 bills.
- A 1-meter stack of $100 bills is worth about 1,000,000 dollars.
- The Burj Khalifa weighs 500,000,000 kilograms.
- A penny weighs 2.5 grams.
- There are 1,000 grams in 1 kilogram.

Arrange students in groups of 2–4. Give students 6–7 minutes to work on the first 3 problems. Pause the class and allow students to ask for more information. Give students an additional 6–7 minutes to complete the remaining questions.

Student Task Statement

In 2010, the Burj Khalifa became the tallest building in the world. It was very expensive to build.

1. Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa? Ask your teacher for the information you need to be able to answer this question, and record the information here.

   No response necessary.

2. Answer the question "Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?" and explain or show your reasoning.

   The stack of cash is taller. Sample reasoning: The stack of cash is $(1.5)\cdot 10^3$ or 1,500 meters tall, almost twice as tall as the Burj Khalifa. This is because $\frac{(1.5)\cdot 10^9}{10^6} = (1.5)\cdot 10^3$

3. Decide what power of 10 to use to label the rightmost tick mark of the number line so that both the height of the stack of money and the height of the Burj Khalifa can be plotted on the same number line. Label the tick marks, and plot and label both values.

   10^4 for the rightmost tick mark and the first 9 integer multiples of 10^3 for tick marks leading up to it. The height of the stack of cash should be placed between the first and second tick marks. The height of the building should be placed between 0 and the first tick mark, but closer to the first tick mark than to 0.

4. Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa? Ask your teacher for the information you need to be able to answer this question and record the information here.

   No response necessary.

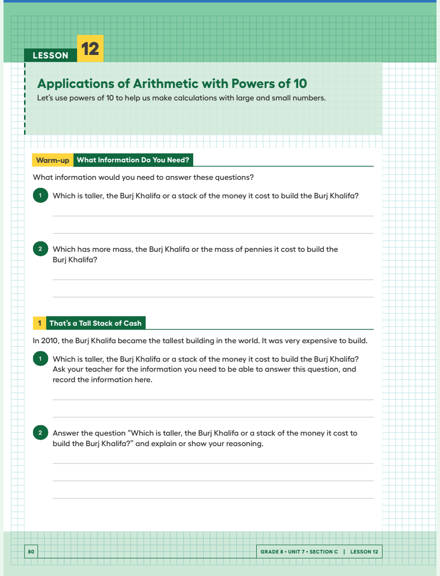
5. Answer the question "Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa?" and explain or show your reasoning.

   The Burj Khalifa has a mass of $5\cdot 10^8$ kg, and a penny has a mass of $(2.5)\cdot 10^{-3}$ kg. Since there are 100 pennies in each dollar, the number of pennies in 1.5 billion dollars is $(1.5)\cdot 10^9\cdot 10^2 = (1.5)\cdot 10^{11}$ pennies. Since each penny has a mass of $(2.5)\cdot 10^{-3}$ kg, then the total mass of the pennies is $(2.5)\cdot 10^{-3}\cdot (1.5)\cdot 10^{11} = (3.75)\cdot 10^8$ kg, which is not quite as massive as the Burj Khalifa.

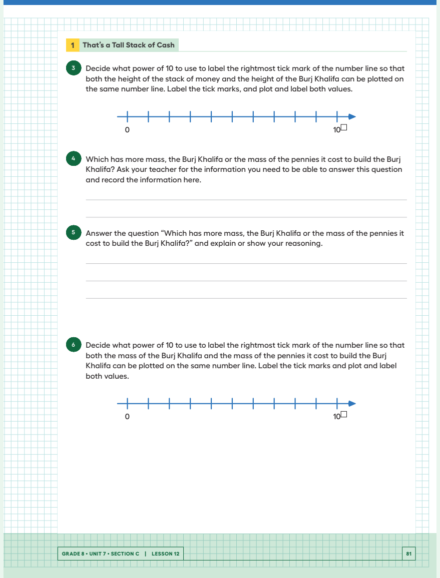
6. Decide what power of 10 to use to label the rightmost tick mark of the number line so that both the mass of the Burj Khalifa and the mass of the pennies it cost to build the Burj Khalifa can be plotted on the same number line. Label the tick marks and plot and label both values.

   10^9 for the rightmost tick mark and the first 9 integer multiples of 10^8 for tick marks leading up to it. The mass of the pennies should be placed $\frac{3}{4}$ of the way between the 3rd and 4th tick marks, and the mass of the Burj Khalifa should be placed between the 4th and 5th tick marks.

Student Workbook



Student Workbook



Activity Synthesis

The goal of this discussion is for students to share their strategies for multiplying and dividing with numbers written as a multiple of a power of 10. Begin by inviting students to share how they used the information to answer the questions. Here are some additional questions for discussion:

"Were the results of your calculations surprising? Why?"

Answers vary.

"How did powers of 10 make the calculations simpler?"

There were fewer zeros and shorter expressions to work with.

As students share their reasoning and strategies, make sure all students understand the following examples:

- To find the product of $6 \cdot 10^3$ and $8 \cdot 10^2$, we can write $(6 \cdot 10^3) \cdot (8 \cdot 10^2) = 48 \cdot 10^5$ because $6 \cdot 8 = 48$ and $10^3 \cdot 10^2 = 10^5$.
- To find the quotient of $6 \cdot 10^3$ and $8 \cdot 10^2$, we can write $\frac{6 \cdot 10^3}{8 \cdot 10^2} = 0.75 \cdot 10^1$ because $6 \div 8 = 0.75$ and $10^3 \div 10^2 = 10^1$.

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

- "I picked the norm '____.' It really helped me/my group because ____."
- "I picked the norm '____.' During the activity, I realized the norm '____' would be a better focus because ____."

Activity 2: Optional
Meter Sticks to the Moon

15 min

Activity Narrative

This optional activity further illustrates the utility of using powers of 10 to work with and interpret very large quantities. Students practice modeling skills, such as identifying essential features of a problem and gathering the required information.

Launch

Ask a student to read the first problem in the *Task Statement* out loud. Invite students to ask for the information they need. Provide students with only the information they request. Display the information for students to see throughout the activity. If students find that they need more information later, provide it to the whole class then.

Here is information students might ask for in order to solve the problems:

- The mass of an average classroom meter stick is roughly 0.2 kg.
- The length of an average classroom meter stick is 1 meter.
- The mass of the Moon is approximately $7 \cdot 10^{22}$ kg.
- The Moon is roughly $(3.8) \cdot 10^{8}$ meters away from Earth.
- The distances to various astronomical bodies the students might recognize, in light years, as points of reference for their last answer. (Consider researching other distances in advance or, if desired, encouraging interested students to do so.)

Arrange students in groups of 2–4, and give 15 minutes of work time followed by a whole-class discussion.

Student Task Statement

1. How many meter sticks does it take to equal the mass of the Moon? Ask your teacher for the information you need to be able to answer this question, and record the information here.

   No response necessary.

2. Answer the question "How many meter sticks does it take to equal the mass of the Moon?" and explain or show your reasoning.

   $(3.5) \cdot 10^{23}$ meter sticks, because the mass of the Moon divided by the mass of a meter stick is $\frac{7 \cdot 10^{22}}{2 \cdot 10^{-1}} = \frac{7}{2} \cdot 10^{22-(-1)} = (3.5) \cdot 10^{23}$

3. Label the number line and plot your answer for the number of meter sticks.

   The right side of the number line should be labeled with 10^{24} with the tick marks labeled as multiples of 10^{23}. The number of meter sticks should be placed between the 3rd and 4th tick marks.

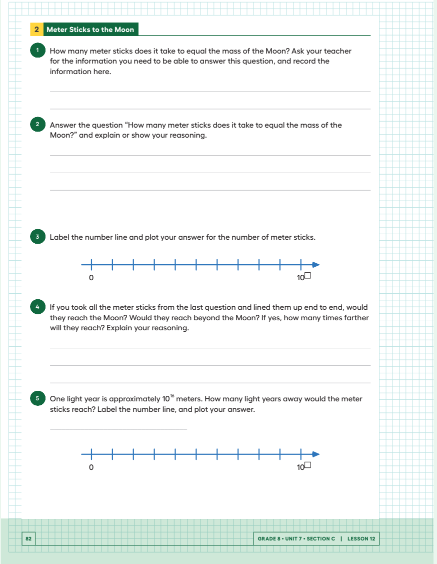
4. If you took all the meter sticks from the last question and lined them up end to end, would they reach the Moon? Would they reach beyond the Moon? If yes, how many times farther will they reach? Explain your reasoning.

   About 10^{15}, or a thousand trillion times as far as the Moon. We can round both 3.5 and 3.8 to 4 (for the sake of comparison) $4 \cdot 10^{23}$ is 10^{15} times as much as $4 \cdot 10^{8}$.

5. One light year is approximately 10^{16} meters. How many light years away would the meter sticks reach? Label the number line, and plot your answer.

   $(3.5) \cdot 10^{7}$, or 35 million light years away. This should be plotted on a number line with 10^{8} as the rightmost tick mark, labeled with multiples of 10^{7}. The number of light years should be placed between the 3rd and 4th tick marks.

Student Workbook


2 Meter Sticks to the Moon

1 How many meter sticks does it take to equal the mass of the Moon? Ask your teacher for the information you need to be able to answer this question, and record the information here.

2 Answer the question "How many meter sticks does it take to equal the mass of the Moon?" and explain or show your reasoning.

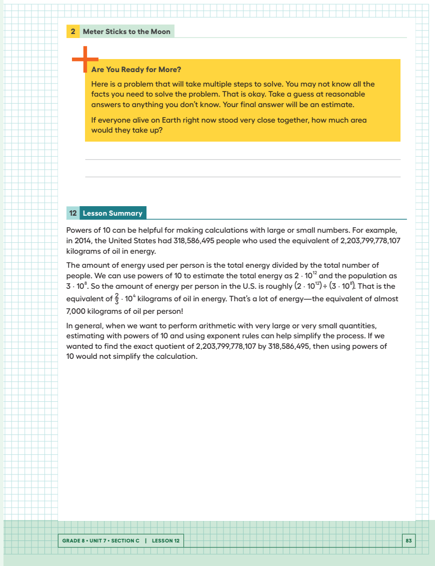
3 Label the number line and plot your answer for the number of meter sticks.

4 If you took all the meter sticks from the last question and lined them up end to end, would they reach the Moon? Would they reach beyond the Moon? If yes, how many times farther will they reach? Explain your reasoning.

5 One light year is approximately 10^{16} meters. How many light years away would the meter sticks reach? Label the number line, and plot your answer.

82 GRADE 8 • UNIT 7 • SECTION C | LESSON 12

Student Workbook


2 Meter Sticks to the Moon

Are You Ready for More?

Here is a problem that will take multiple steps to solve. You may not know all the facts you need to solve the problem. That is okay. Take a guess at reasonable answers to anything you don't know. Your final answer will be an estimate.

If everyone alive on Earth right now stood very close together, how much area would they take up?

12 Lesson Summary

Powers of 10 can be helpful for making calculations with large or small numbers. For example, in 2014, the United States had 318,586,495 people who used the equivalent of 2,203,799,778,107 kilograms of oil in energy.

The amount of energy used per person is the total energy divided by the total number of people. We can use powers of 10 to estimate the total energy as $2 \cdot 10^{12}$ and the population as $3 \cdot 10^8$. So the amount of energy per person in the U.S. is roughly $(2 \cdot 10^{12}) \div (3 \cdot 10^8)$. That is the equivalent of $\frac{2}{3} \cdot 10^4$ kilograms of oil in energy. That's a lot of energy—the equivalent of almost 7,000 kilograms of oil per person!

In general, when we want to perform arithmetic with very large or very small quantities, estimating with powers of 10 and using exponent rules can help simplify the process. If we wanted to find the exact quotient of 2,203,799,778,107 by 318,586,495, then using powers of 10 would not simplify the calculation.

GRADE 8 • UNIT 7 • SECTION C | LESSON 12 83

Are You Ready for More?

Here is a problem that will take multiple steps to solve. You may not know all the facts you need to solve the problem. That is okay. Take a guess at reasonable answers to anything you don't know. Your final answer will be an estimate.

If everyone alive on Earth right now stood very close together, how much area would they take up?

Answers vary (and are likely to vary wildly).

Sample response: There are between 7 billion and 8 billion people on Earth right now, so let's say 8 billion. Most teenagers and adults can fit into a rectangle of about 1 meter by half a meter when standing, so half a square meter. Smaller children would take up less space—maybe about half the space of an adult, so say children take up a quarter of a square meter. Let's guess that about a quarter of the people on Earth are small children. The space the adults will take up should be 6 billion times half a square meter, which is 3 billion square meters. The children will take up 2 billion times a quarter of a square meter, which is half a billion square meters. In total, the people will take up about 3.5 billion square meters.

Students may want to convert this answer to kilometers (or feet to miles, etc). To do this requires the tricky realization that, even though there are 1,000 meters in a kilometer, there are $1{,}000^2$ square meters in a square kilometer. Knowing this, we can divide $(3.5) \cdot 10^9$ by $1 \cdot 10^6$ to get $(3.5) \cdot 10^3$, or 35,000 square kilometers.

Activity Synthesis

The goal of this discussion is for students to see how powers of 10 can be a useful strategy when dealing with problems that involve very large numbers. Begin by inviting students to share how they used the information to answer the question. Here are some questions for discussion:

"Were the results of your calculations surprising? Why?"

Answers vary.

"How did powers of 10 make the calculations simpler?"

There were fewer zeros and shorter expressions to work with.

It might be illuminating to put 35 million light years into some context. It is over a thousand trillion times as far as the distance to the Moon, or about the size of a supercluster of galaxies. The Sun is less than 1.6×10^{-5} light year away from Earth.

Lesson Synthesis

The purpose of this discussion is to prompt students to reflect on the modeling process and on using exponents to solve problems. Consider asking:

"To solve the problems in this lesson you had to determine what information was needed. Did you find that to be fairly straightforward or challenging? What made it straightforward or challenging? Why?"

"Describe your thinking as you planned a solution path for the problems. For example, did you ask for information first and then decide what to do with it, or did you decide what needed to be done first before asking for certain information?"

"Once you had the information you needed, what were some difficulties you encountered? How did you work through them?"

"Would an estimate be an acceptable answer for problems like these? Why or why not? When might we need more precise solutions?"

If we were planning a space exploration, we would likely need a high level of precision to ensure that we hit our targets. But if the answers are for comparison or general information, estimates are likely adequate.

Lesson Summary

Powers of 10 can be helpful for making calculations with large or small numbers. For example, in 2014, the United States had 318,586,495 people who used the equivalent of 2,203,799,778,107 kilograms of oil in energy.

The amount of energy used per person is the total energy divided by the total number of people. We can use powers of 10 to estimate the total energy as $2 \cdot 10^{12}$ and the population as $3 \cdot 10^8$. So the amount of energy per person in the U.S. is roughly $(2 \cdot 10^{12}) \div (3 \cdot 10^8)$. That is the equivalent of $\frac{2}{3} \cdot 10^4$ kilograms of oil in energy. That's a lot of energy—the equivalent of almost 7,000 kilograms of oil per person!

In general, when we want to perform arithmetic with very large or very small quantities, estimating with powers of 10 and using exponent rules can help simplify the process. If we wanted to find the exact quotient of 2,203,799,778,107 by 318,586,495, then using powers of 10 would not simplify the calculation.

Cool-down

That's a Lot of Cells

5 min

Student Task Statement

There are about 260 million adults in the United States and the average adult has 10 pints of blood. If there are 2.4×10^{12} red blood cells in one pint of blood, about how many red blood cells are there in all the adults in the United States?

There are 6.24×10^{21} red blood cells since $(2.6 \times 10^8) \cdot (10) \cdot (2.4 \times 10^{12}) = 6.24 \times 10^{21}$.

Responding To Student Thinking

Points to Emphasize

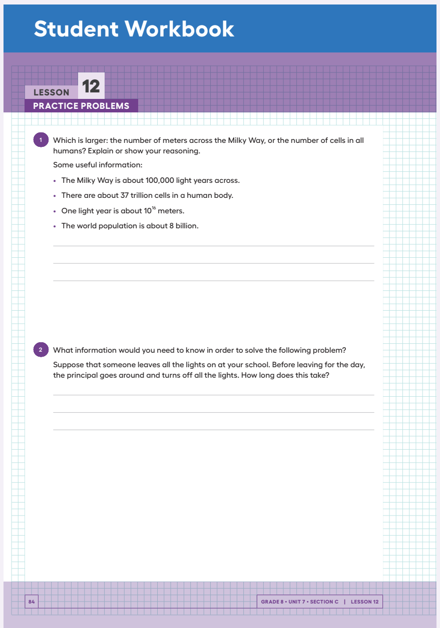
If most students struggle with multiplying and dividing multiples of powers of 10, revisit this concept when multiplying and dividing with numbers in scientific notation. For example, make sure to emphasize the calculation for determining the number of humans per cat in the world in the *Launch* of this activity:

Unit 7, Lesson 14, Activity 3 Info Gap: Distances in the Solar System

Lesson 12

Practice Problems

6 Problems


Student Workbook

LESSON 12

PRACTICE PROBLEMS

1 Which is larger: the number of meters across the Milky Way, or the number of cells in all humans? Explain or show your reasoning.

Some useful information:

- The Milky Way is about 100,000 light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} meters.
- The world population is about 8 billion.

2 What information would you need to know in order to solve the following problem?

Suppose that someone leaves all the lights on at your school. Before leaving for the day, the principal goes around and turns off all the lights. How long does this take?

Problem 1

Which is larger: the number of meters across the Milky Way, or the number of cells in all humans? Explain or show your reasoning.

Some useful information:

- The Milky Way is about 100,000 light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} meters.
- The world population is about 8 billion.

There are more human cells than there are meters across the Milky Way.

Sample reasoning: Since 100,000 is 10^5, it is about $10^5 \cdot 10^{16}$ or 10^{21} meters across the Milky Way. Notice that 37 trillion is $(3.7) \cdot 10^{13}$ and 8 billion is $8 \cdot 10^9$, so the total number of cells of all humans is $(3.7) \cdot 10^{13} \cdot 8 \cdot 10^9$. This gives $(29.6) \cdot 10^{22}$ human cells. This is nearly 300 times larger than 10^{21}, the approximate number of meters across the Milky Way. Using more precise values for population and the number of meters in a light year will yield slightly different results.

Problem 2

What information would you need to know in order to solve the following problem?

Suppose that someone leaves all the lights on at your school. Before leaving for the day, the principal goes around and turns off all the lights. How long does this take?

Answers vary. Sample response: I would need to know how many lights are in the school, how far apart those lights are, how fast the principal walks, and how long it takes the principal to turn off a single light.

Problem 3

Lin's mother has a car that she uses for work, shopping, and trips. About how many times does one tire on her car revolve in a year? Explain or show your reasoning.

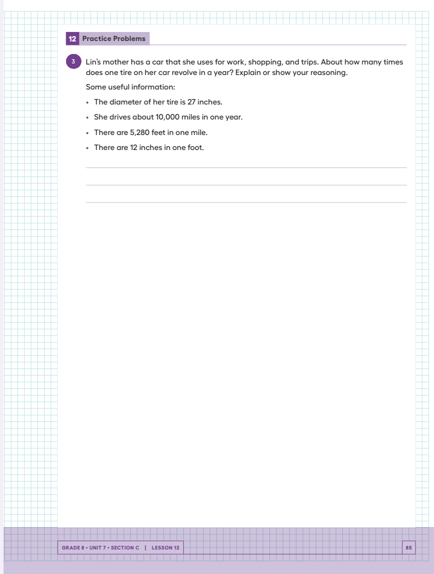
Some useful information:

- The diameter of her tire is 27 inches.
- She drives about 10,000 miles in one year.
- There are 5,280 feet in one mile.
- There are 12 inches in one foot.

About $7.5 \cdot 10^6$ rotations

Sample reasoning: Using the formula $C = 2\pi r$ and a radius of 13.5 inches, the circumference is about 84.823 inches. There are 12 inches in a foot and 5,280 feet in a mile, so that means Lin's mother drives about $6.336 \cdot 10^8$ inches in a year. Dividing the distance Lin's mom drives by the circumference of her tire results in about $7.5 \cdot 10^6$, or 7.5 million, rotations in a year.

Student Workbook


12 Practice Problems

3 Lin's mother has a car that she uses for work, shopping, and trips. About how many times does one tire on her car revolve in a year? Explain or show your reasoning.

Some useful information:

- The diameter of her tire is 27 inches.
- She drives about 10,000 miles in one year.
- There are 5,280 feet in one mile.
- There are 12 inches in one foot.

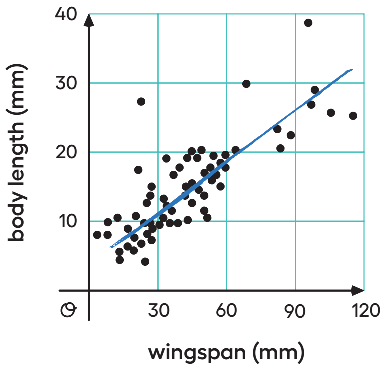
Problem 4

from Unit 6, Lesson 5

Ecologists measure the body length and wingspan of 127 butterfly specimens caught in a single field.

a. Draw a line that you think is a good fit for the data.

Sample response:

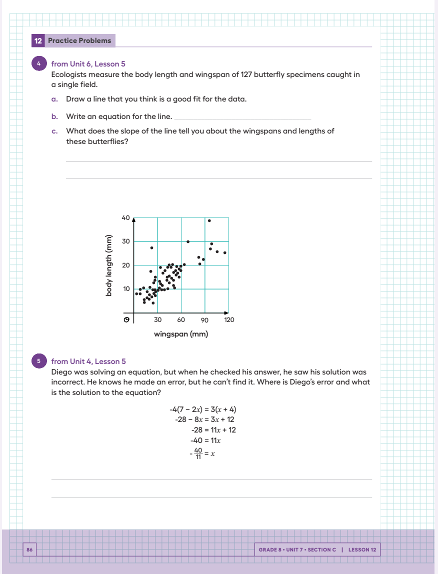


b. Write an equation for the line. $y = \frac{1}{4}x + 5$ (or equivalent)

c. What does the slope of the line tell you about the wingspans and lengths of these butterflies?

Sample response: For every 4 millimeters the length of the wingspan increases, the body length increases 1 millimeter.

Student Workbook


12 Practice Problems

4 from Unit 6, Lesson 5

Ecologists measure the body length and wingspan of 127 butterfly specimens caught in a single field.

a. Draw a line that you think is a good fit for the data.

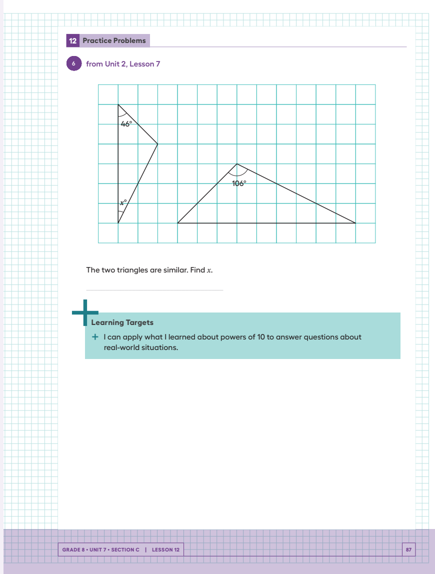
b. Write an equation for the line.

c. What does the slope of the line tell you about the wingspans and lengths of these butterflies?

5 from Unit 4, Lesson 5

Diego was solving an equation, but when he checked his answer, he saw his solution was incorrect. He knows he made an error, but he can't find it. Where is Diego's error and what is the solution to the equation?

Student Workbook



Problem 5

from Unit 4, Lesson 5

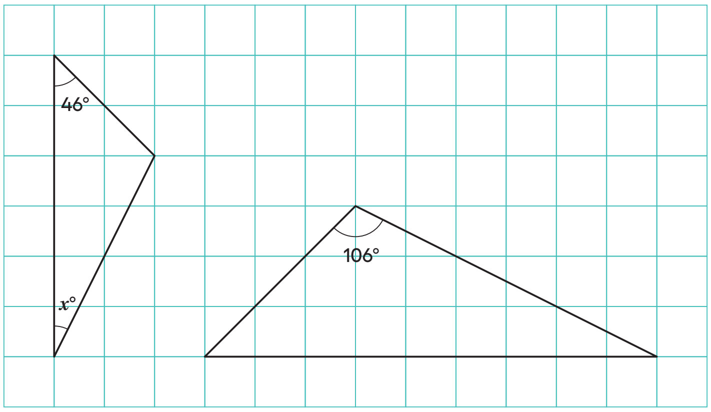
Diego was solving an equation, but when he checked his answer, he saw his solution was incorrect. He knows he made an error, but he can't find it. Where is Diego's error and what is the solution to the equation?

$$\begin{aligned} -4(7-2x) &= 3(x+4) \\ -28-8x &= 3x+12 \\ -28 &= 11x+12 \\ -40 &= 11x \\ -\frac{40}{11} &= x \end{aligned}$$

Sample response: Diego's error occurred in the transition from the first line to the second line. The distributive property with $-4(7-2x)$ should give $-28+8x$. The correct solution is $x=8$.

Problem 6

from Unit 2, Lesson 7



The two triangles are similar. Find x.

$x = 28$, because, given that the triangles are similar and the sum of the three angles is 180°, their obtuse angles both measure 106°