Lesson 10

Distinguishing Circumference and Area

Goals

- Critique (orally and in writing) claims about the radius, diameter, circumference, or area of a circle in a real-world situation.
- Decide whether to calculate the circumference or area of a circle to solve a problem in a real-world situation, and justify (orally) the decision.
- Estimate measurements of a circle in a real-world situation, and explain (orally and in writing) the estimation strategy.

Learning Targets

- I can decide whether a situation about a circle has to do with area or circumference.
- I can use formulas for circumference and area of a circle to solve problems.

Lesson Narrative

In this lesson, students apply what they have learned about circumference and area of circles to solve problems in context. Both types of problems are mixed together so students have to determine which measurement is called for in each problem situation. Students continue working with answers expressed in terms of π. Also, in previous lessons students were always given one measurement of each circle, but in this lesson they must rely on their own estimations to solve the problems.

First, students consider eight different situations and determine which situations involve circumference and which involve area. They continue working with these situations throughout the rest of the lesson.

The following three activities are optional, so teachers can choose what best fits the needs of their students. The first optional activity has students create a visual display showing how they solved one of the eight problems. The second optional activity has students critique given solutions to some of the eight problems. The third optional activity uses the *Information Gap* structure to get students thinking about what information is needed to solve these problems.

Student Learning Goal

Let's contrast circumference and area.

Access for Students with Diverse Abilities

- Action and Expression (Activity 3, Activity 4)
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR4: Information Gap Cards (Activity 4)
- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- Card Sort
- MLR4: Information Gap Cards
- Poll the Class
- Take Turns

Required Materials

Materials to Gather

- Math Community Chart: Activity 2

Materials to Copy

- Circle Problems Cards (1 copy for every 2 students): Activity 1
- Visual Display of Circle Problem Handout (1 copy for every 10 students): Activity 2
- Merry-go-round and Unicycle Cards (1 copy for every 4 students): Activity 4

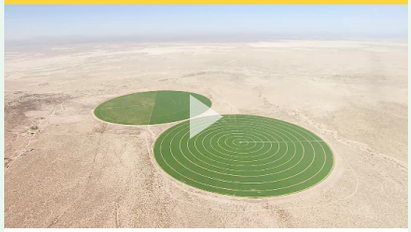
Activity 1:
Be prepared to explain or show images of any of the examples of circles in the sorting activity that may be unfamiliar to your students.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Activity 4	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	20 min	10 min	5 min

Inspire Math

Crop Circles video



Go Online
Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614232

Please log in to the site before using the QR code or URL.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

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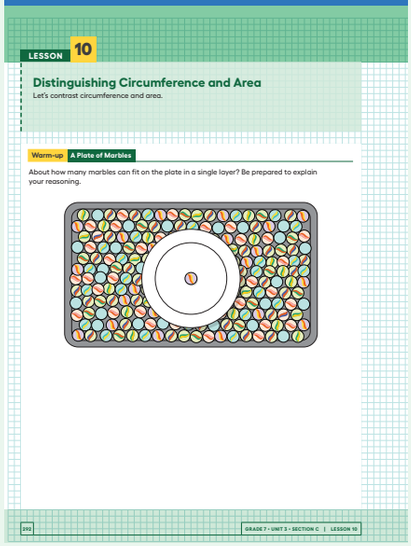
Student Workbook


LESSON 10
Distinguishing Circumference and Area
Let's contrast circumference and area.
Warm-up A Plate of Marbles
About how many marbles can fit on the plate in a single layer? Be prepared to explain your reasoning.

Warm-up

A Plate of Marbles

5 min

Activity Narrative

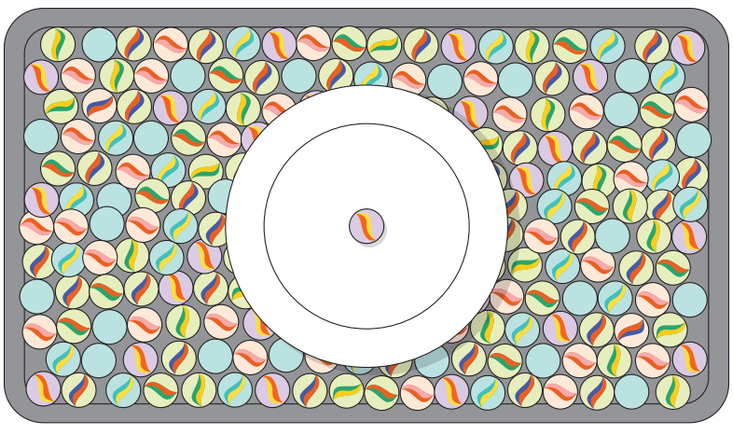
In this *Warm-up*, students apply what they have learned about finding the area of a circle to estimate the area of a circular plate in terms of a smaller circle. Students see a plate with a single marble and, from this information, they are asked to make a reasoned estimate of the number of marbles required to cover the plate.

Launch

Display the image for all to see. Give students 1–2 minutes of quiet think time followed by whole-group discussion.

Student Task Statement

About how many marbles can fit on the plate in a single layer? Be prepared to explain your reasoning.



Sample responses:

- By comparing the plate to the lines of marbles off to the side, the diameter appears to be about 11 marbles. The radius would be 5.5 marbles. $5^2 = 25$ and $6^2 = 36$, so $5.5^2 \approx 30$. The area of the plate is about $3.14 \cdot 30$ times the area of a marble, so about 94 marbles should fit.
- By replicating the marble on the plate, the radius appears to be about 5 marbles. The area of the plate is about $3 \cdot 5^2$ times the area of a marble, so about 75 marbles should fit.

Activity Synthesis

Poll the class on their estimates for the number of marbles that would fit.

Invite students to share their estimation strategies. To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Activity 1

Card Sort: Circle Problems

15 min

Activity Narrative

In this partner activity, students take turns categorizing problems based on whether the question is related to the circumference or the area of a circle. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Next, each group focuses on one of the first five questions. They estimate appropriate measurements for the context and use these measurements to calculate a reasonable answer.

Launch

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice what they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

Tell students that the cards contain questions about circular objects and that they will take turns sorting the cards based on whether the question is related to the circumference or the area of a circle. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of pre-cut cards. If desired, distribute copies of the sorting mat from the blackline master as well. Explain or show images of any of the contexts that may be unfamiliar to your students.

When students have finished sorting the cards, review their work and prompt them to reconsider any cards, if needed. Then, assign each group one problem from Cards 1 through 5 to focus on for the rest of the activity. Consider each groups' familiarity with the contexts and their fluency with circle calculations as you decide which problem to assign them. Problems 2, 4, and 5 involve another step beyond determining the circumference or area of the circle.

Instructional Routines

Card Sort

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.
Direct attention to words collected and displayed from previous lessons. Invite students to borrow language from the display as needed and update it throughout the lesson. Collect the language that students use to decide whether to use the circumference or the area of the circle to answer the question.

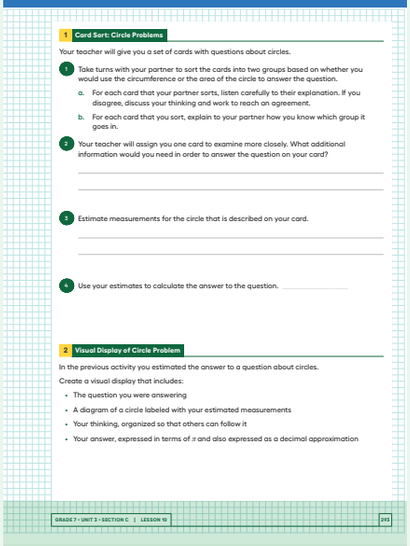
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.
Provide students with a graphic organizer, such as the sorting mat in the blackline master, for students to organize their categories as cards are sorted.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook



1 Card Sort: Circle Problems

Your teacher will give you a set of cards with questions about circles.

1. Take turns with your partner to sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question.
   a. For each card that your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
   b. For each card that you sort, explain to your partner how you know which group it goes in.
2. Your teacher will assign you one card to examine more closely. What additional information would you need in order to answer the question on your card?
3. Estimate measurements for the circle that is described on your card.
4. Use your estimates to calculate the answer to the question.

2 Visual Display of Circle Problem

In the previous activity you estimated the answer to a question about circles.

Create a visual display that includes:

- The question you were answering
- A diagram of a circle labeled with your estimated measurements
- Your thinking, organized so that others can follow it
- Your answer, expressed in terms of π and also expressed as a decimal approximation

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Student Task Statement

Your teacher will give you a set of cards with questions about circles.

1. Take turns with your partner to sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question
   a. For each card that your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
   b. For each card that you sort, explain to your partner how you know which group it goes in.

   Area: 1, 3, 4, 7; Circumference: 2, 5, 6, 8
2. Your teacher will assign you one card to examine more closely. What additional information would you need in order to answer the question on your card?

   For every card, knowing the radius or diameter of the circle would help solve the problem because the circumference and area can both be calculated from the radius or diameter. Additionally, for the Ferris-wheel problem you would need to know the time it takes to go around the Ferris wheel once. For the pizza problem, you would need to know the number of slices the pizza is cut into (and that they are roughly the same size). For the horse-walker problem, you would need to know how to convert 1 mile into a smaller unit of measure.
3. Estimate measurements for the circle that is described on your card.

   Sample responses:
   - Question 1: A radius between 8 and 40 inches
   - Question 2: A diameter between 10 and 160 meters, and about 1 to 30 minutes per rotation
   - Question 3: A radius between 5 and 25 yards
   - Question 4: A radius between 4 and 9 inches, and 8 slices per pizza
   - Question 5: A diameter between 9 and 33 yards, and 1,760 yards per mile
4. Use your estimates to calculate the answer to the question.

   Sample responses:
   - Question 1: Between 64π and 1600π in^2
   - Question 2: From between 10π and 160π meters per minute (1 revolution per minute) to between $\frac{1}{3}\pi$ and $\frac{16}{3}\pi$ meters per minute (1 revolution per 30 minutes)
   - Question 3: Between 25π and 625π yd^2
   - Question 4: Between 2π and $10\frac{1}{8}\pi$ in^2
   - Question 5: Between $\frac{1760}{33\pi}$ and $\frac{1760}{9\pi}$ rotations

Building on Student Thinking

If students are confused about the difference between circumference and area, remind them that circumference measures distance around a circle and uses linear units, and area measures the inside of a circle and uses square units.

Students might think that they need to solve the problems on all 8 cards. Point out that the first question is asking them to think only about how they would solve the problems, not to do any actual calculations.

For the horse-walker problem, students might not realize that they need to convert 1 mile to the same units as their estimated diameter, or that they need to divide by the circumference.

Activity Synthesis

Much of the discussion about sorting the cards will have happened in small groups. The goal of this whole-class discussion is for students to articulate how they decide whether an answer is reasonable.

First, have the students who worked on the same question compare answers and strategies. Display these questions to guide their group discussions:

- Did you use the same units?
- How did you come up with your estimate for the size of the circle (radius or diameter)?
- Are your estimates very close? Are they reasonable?
- How did you calculate your answer to the question?
- Are your answers very close? Are they reasonable?

Next, invite students who have different answers to the same question to share their reasoning with the class. For each group, ask the rest of the class:

💬 *"Which of these answers do you think are reasonable? Why?"*

Make sure students understand that since estimates were called for, there is not one exact correct answer for each of these problems.

Some mistakes that could lead to an unreasonable answer include:

- Making too inaccurate of an initial estimate about the size of the circle.
- Using the diameter as if it is the radius, or vice versa.
- Using the wrong formula for circumference or area.
- Forgetting to address an aspect of the question (such as finding the area of the entire pizza, not one slice).
- Labeling the units incorrectly, like using feet for a measure of area instead of square feet.
- Reporting an answer with more decimals places than is reasonable given the level of precision of the initial estimates.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Provide students with a graphic organizer, such as the ones in the blackline master, to help them ensure that their display includes all the required components.

Supports accessibility for: Visual-Spatial Processing, Organization

Access for Multilingual Learners (Activity 2, Student Task)

MLR7: Compare and Connect.
As students prepare their visual display, encourage them to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students,

"What did the approaches have in common? How were they different?"
"What kinds of additional details or language helped you understand the displays?"
"Were there any additional details or language that you have questions about?"

Advances: Representing, Conversing

Math Community

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

- *"I picked the norm '_____.' It really helped me/my group because _____."*
- *"I picked the norm '_____.' During the activity, I realized the norm '_____' would be a better focus because _____."*

Activity 2: Optional

Visual Display of Circle Problem

15 min

Activity Narrative

In this activity students create a visual display of the circle problem that they solved previously. They can practice explaining their reasoning more clearly on this display than they did in the previous activity. This gives them an opportunity to organize and record their information in a way that can be shared with others who worked on a different problem. The displays can also serve as a record of reasoning about circles, which can be referred back to later in the year.

Launch

Keep students in the same groups. Explain that they are going to create a visual display of the circle problem that they just worked on.

Student Task Statement

In the previous activity you estimated the answer to a question about circles.

Create a visual display that includes:

- The question you were answering
- A diagram of a circle labeled with your estimated measurements
- Your thinking, organized so that others can follow it
- Your answer, expressed in terms of π and also expressed as a decimal approximation

Answers vary.

Activity Synthesis

Arrange for groups that are assigned the same problem to present their visual displays near one another. Give students a few minutes to visit the displays and to see the estimates that others used to answer the question.

Before students begin a gallery walk, ask them to be prepared to share a couple of observations about how their estimates and strategies are the same as or different from others'. After the gallery walk, invite a couple of students to share their observations.

Activity 3: Optional
Analyzing Circle Claims

10 min

Activity Narrative

In this activity students look more closely at the last three situations from the card sort activity (Questions 6 through 8). They analyze and critique two claims about each situation, choosing or supplying the best response and explaining why.

Students must recognize that in the first situation, one of the claims inaccurately estimates the size of the circle. In the second situation, one of the claims calculates the circumference instead of the area. In the third situation, both claims are inaccurate. One of the claims has the right number but uses square units, and the other has the right units but the wrong number.

A note about interpreting the work of others:

It is not possible to know for certain what Clare or Andre were thinking when they made their calculations. For example, it is likely in the second problem that Clare found the circumference of the cookie instead of its area, but it is not possible to know. A wide range of interpretations need to be considered, always keeping an open mind.

Launch

Keep students in the same groups. Tell students they are going to look at how some other students solved the questions on Cards 6, 7, and 8.

For the first situation, make sure that students realize it is referring to the type of merry-go-round at a playground (as pictured in their books or devices), not the larger type of carousel they might see at a fair.

Student Task Statement

Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

1. How many feet are traveled by a person riding once around the merry-go-round?

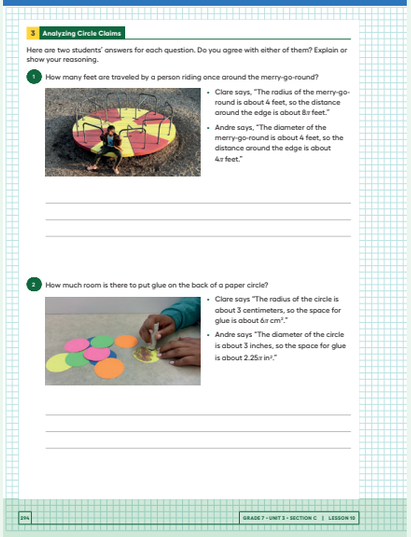
- Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."

Access for Students with Diverse Abilities (Activity 3, Student Task)

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as "I agree/disagree because …", "That could/couldn't be true because …", and "_____'s idea reminds me of …"

Supports accessibility for: Language, Organization

Student Workbook


3 Analyzing Circle Claims

Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

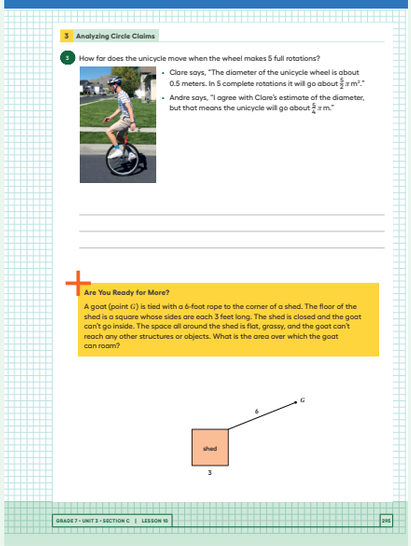
1. How many feet are traveled by a person riding once around the merry-go-round?

- Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."
- Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

2. How much room is there to put glue on the back of a paper circle?

- Clare says "The radius of the circle is about 3 centimeters, so the space for glue is about 6π cm²."
- Andre says "The diameter of the circle is about 3 inches, so the space for glue is about 2.25π in²."

Student Workbook



3 Analyzing Circle Claims

3. How far does the unicycle move when the wheel makes 5 full rotations?

- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."
- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

Are You Ready for More?

A goat (point G) is tied with a 6-foot rope to the corner of a shed. The floor of the shed is a square whose sides are each 3 feet long. The shed is closed and the goat can't go inside. The space all around the shed is flat, grassy, and the goat can't reach any other structures or objects. What is the area over which the goat can roam?

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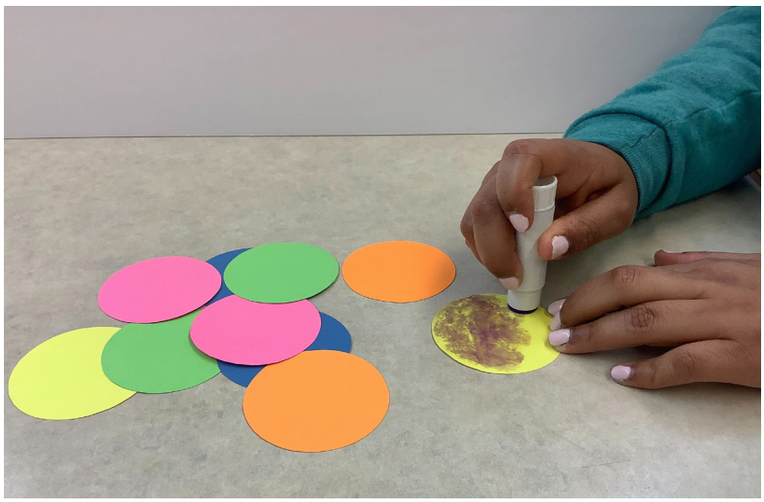
- Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

  Clare's claim is more reasonable. Both people correctly calculated the circumference, given their estimated dimension for the circle. However, Andre's estimated diameter of 4 feet is too small, given the relative size of the child.

2. How much room is there to put glue on the back of a paper circle?



- Clare says "The radius of the circle is about 3 centimeters, so the space for glue is about 6π cm2."
- Andre says "The diameter of the circle is about 3 inches, so the space for glue is about 2.25π in2."

  Andre's claim is more reasonable. Both estimated measurements for the circle are reasonable. However, Clare applied the circumference formula when the problem called for the area.

3. How far does the unicycle move when the wheel makes 5 full rotations?

- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."

- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

  Neither claim is accurate. Clare has the correct number but answered with square units when the problem called for linear units. Andre has the correct units, but he squared the diameter when calculating the numerical value, as if he were using the radius to find the area. (It is also possible that Andre mistakenly used the radius instead of the diameter in the circumference formula.)

Building on Student Thinking

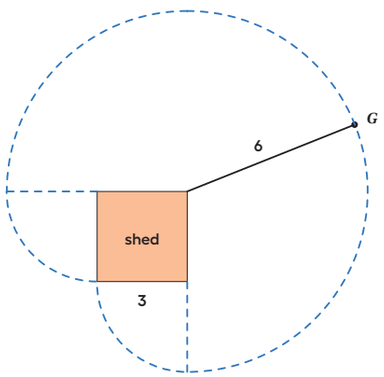
Students might multiply by a decimal approximation, without recognizing that the answers in the claims are all given in terms of π.

Students might not realize that there is an error with both of the claims in the third question.

Finally, students might not realize that they are supposed to analyze the reasonableness of the estimates, not just the mathematical correctness of the calculations.

Are You Ready for More?

A goat (point G) is tied with a 6-foot rope to the corner of a shed. The floor of the shed is a square whose sides are each 3 feet long. The shed is closed and the goat can't go inside. The space all around the shed is flat and grassy, and the goat can't reach any other structures or objects. What is the area over which the goat can roam?



31.5π square feet (or approximately 99 square feet)

The edge of the goat's roaming area is three quarters of a circle with radius 6 feet, until the rope gets caught on the corner of the shed, at which point the goat has two quarter-circles with radius 3 feet. Adding $\frac{3}{4}\pi \cdot 6^2$ and $2 \cdot \frac{1}{4}\pi \cdot 3^2$ gives 31.5π square feet.

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 4)

MLR4: Information Gap Cards. This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Activity Synthesis

For each situation, invite groups to share which claim is more accurate and why. To involve more students in the conversation, consider asking questions like:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Does anyone want to add on to _____s strategy?"*
- *"Do you agree or disagree? Why?"*

If it does not come out during the discussion, point out that the formula for the area of a circle has a squared term and that the units of the answer are square units. On the other hand, the formula for the circumference does not have a squared term, and the units of the answer are linear units.

Activity 4: Optional

Info Gap: Merry-go-round and Unicycle

20 min

Activity Narrative

This activity gives students an opportunity to determine and request the information needed to solve problems that involve the circumference and area of circles. The problems have similar contexts to Cards 6 through 8 of the *Card Sort* activity.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information that they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language that they use and to ask increasingly more precise questions until they get the information that they need.

Launch

Tell students that they will solve problems about circular objects. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it. There is an extra set of cards available for demonstration purposes. The data card in this extra set gives circumference but not diameter or radius. If students ask for the radius or diameter, the teacher can demonstrate saying that they don't have that piece of information, and the students can practice thinking of what other information they could ask for. Going through this process as a class will help prepare students for solving the set of cards about a unicycle.

Arrange students in groups of 2 or 4. If students are new to the *Info Gap* routine, allowing them to work in groups of 2 for each role will support communication and understanding. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

Problem Card 1:

32π feet (approximately 100.5 feet)

Problem Card 2:

80π inches (approximately 21 feet)

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

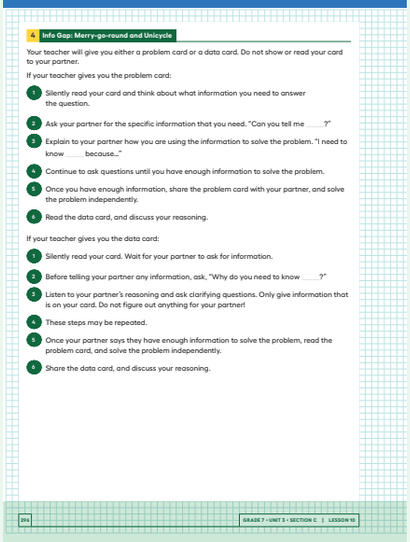
- *"How did you decide whether to calculate the circumference or the area of the circle?"*
- *"Some measurements on the data card were given in terms of pi. How did this affect your solving process?"*

Access for Students with Diverse Abilities (Activity 4, Student Task)

Action and Expression: Internalize Executive Functions.
Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity, or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Workbook


4 Info Gap: Merry-go-round and Unicycle

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

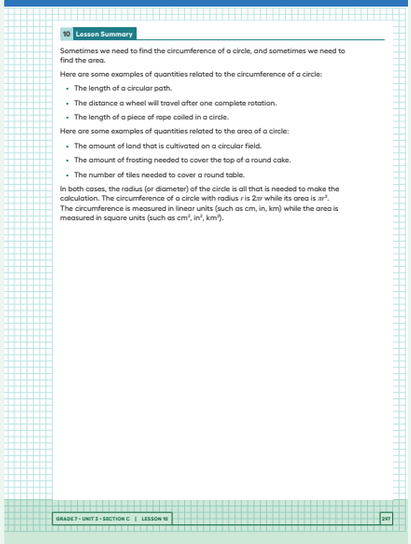
If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because..."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.

Student Workbook



10 Lesson Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area.

Here are some examples of quantities related to the circumference of a circle:

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Here are some examples of quantities related to the area of a circle:

- The amount of land that is cultivated on a circular field.
- The amount of frosting needed to cover the top of a round cake.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to make the calculation. The circumference of a circle with radius r is $2\pi r$ while its area is πr^2. The circumference is measured in linear units (such as cm, in, km) while the area is measured in square units (such as cm^2, in^2, km^2).

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Highlight for students how given only one measurement of a circle (that is, the radius, diameter, circumference, or area), it is possible to calculate all three of the other measurements.

Lesson Synthesis

Share with students:

- *"Today we looked at real-world situations and decided whether they were related to circumference or area of a circle."*

To help students generalize about problems that involve circumference or area of a circle, consider asking:

- *"When would we need to calculate the circumference of a circle?"*

  to answer a question about distance or speed around a circular path

- *"When would we need to calculate the area?"*

  to answer a question about the space in a circular region

- *"What do you need to know to determine the circumference of a circle?"*

  the diameter (or the radius)

- *"What do you need to know in order to determine the area of a circle?"*

  the radius (or the diameter)

Consider posting the students' displays of the problem they solved after the card sorting activity, grouped by circumference or area, so students can refer to them later.

Lesson Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area.

Here are some examples of quantities related to the circumference of a circle:

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- The length of a piece of rope coiled in a circle.

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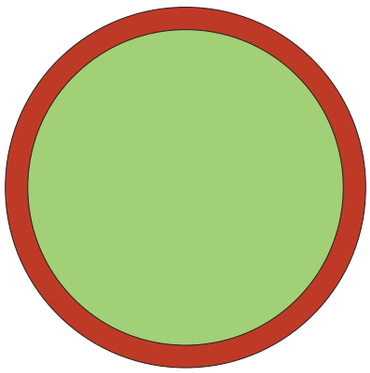
Cool-down

Measuring a Circular Lawn

5 min

Student Task Statement

A circular lawn has a row of bricks around the edge. The diameter of the lawn is about 40 feet.



1. Which is the best estimate for the amount of grass in the lawn?
   - **A.** 125 feet
   - **B.** 125 square feet
   - **C.** 1,250 feet
   - **D.** 1,250 square feet
2. Which is the best estimate for the total length of the bricks?
   - **A.** 125 feet
   - **B.** 125 square feet
   - **C.** 1,250 feet
   - **D.** 1,250 square feet

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of calculating the circumference and area of circles. If students struggle, make time to revisit related work in the sections referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 3, Section A Circumference of a Circle

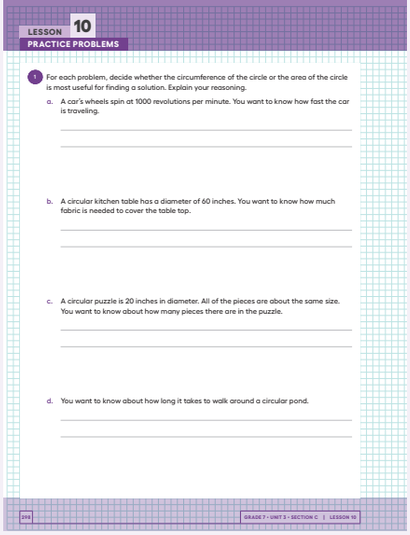
Unit 3, Section B Area of a Circle

Lesson 10

Practice Problems

6 Problems

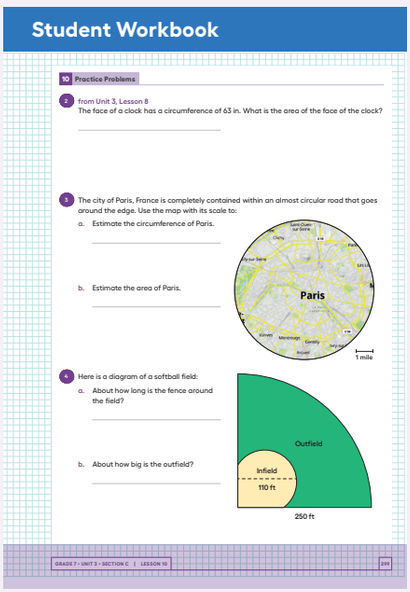
Student Workbook



LESSON 10
PRACTICE PROBLEMS

1. For each problem, decide whether the circumference of the circle or the area of the circle is most useful for finding a solution. Explain your reasoning.
   a. A car's wheels spin at 1000 revolutions per minute. You want to know how fast the car is traveling.
   b. A circular kitchen table has a diameter of 60 inches. You want to know how much fabric is needed to cover the table top.
   c. A circular puzzle is 20 inches in diameter. All of the pieces are about the same size. You want to know about how many pieces there are in the puzzle.
   d. You want to know about how long it takes to walk around a circular pond.

Student Workbook



2. from Unit 3, Lesson 8
   The face of a clock has a circumference of 63 in. What is the area of the face of the clock?
3. The city of Paris, France is completely contained within an almost circular road that goes around the edge. Use the map with its scale to:
   a. Estimate the circumference of Paris.
   b. Estimate the area of Paris.

4. Here is a diagram of a softball field:
   a. About how long is the fence around the field?
   b. About how big is the outfield?

Problem 1

For each problem, decide whether the circumference of the circle or the area of the circle is most useful for finding a solution. Explain your reasoning.

a. A car's wheels spin at 1000 revolutions per minute. You want to know how fast the car is traveling.

circumference, because the circumference of the wheels and the number of revolutions per minute tell you how far the car is traveling and this can be used to calculate the speed

b. A circular kitchen table has a diameter of 60 inches. You want to know how much fabric is needed to cover the table top.

area, because the fabric covers the surface of the table and it is this area that is needed

c. A circular puzzle is 20 inches in diameter. All of the pieces are about the same size. You want to know about how many pieces there are in the puzzle.

area, because the area of the puzzle divided by the area of a puzzle piece will give an estimate of the number of pieces

d. You want to know about how long it takes to walk around a circular pond.

circumference, because you need to know the distance around the pond, which is its circumference

Problem 2

from Unit 3, Lesson 8

The face of a clock has a circumference of 63 in. What is the area of the face of the clock?

about 316 in^2

Sample reasoning: Divide 63 by π and by 2 to determine the radius of the clock. $63 \div 2 \div \pi \approx 10$. To find the area of the face of the clock multiply π by 10^2.

Problem 3

The city of Paris, France, is completely contained within an almost circular road that goes around the edge. Use the map with its scale to:

a. Estimate the circumference of Paris.

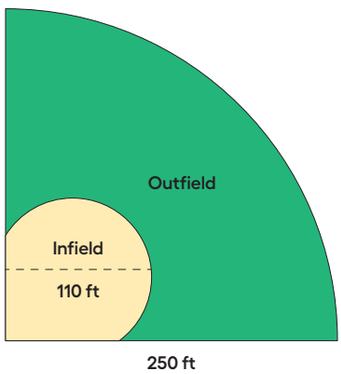
Sample response: about 6π miles (or about 20 miles)

b. Estimate the area of Paris.

Sample response: about $(3)^2\pi$ mi^2 (or about 30 mi^2)

Problem 4

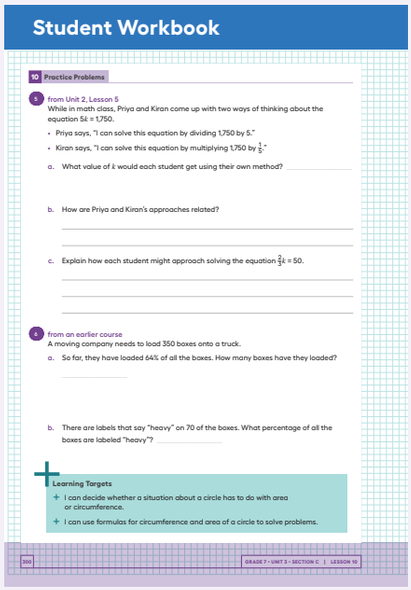
Here is a diagram of a softball field:



a. About how long is the fence around the field?

Sample response: $500 + 125\pi$ (or about 893 ft)

This estimate assumes that the curved boundary of the outfield is modeled by a quarter circle.


Student Workbook

10 Practice Problems

5 from Unit 2, Lesson 5
While in math class, Priya and Kiran come up with two ways of thinking about the equation $5k = 1{,}750$.
- Priya says, "I can solve this equation by dividing 1,750 by 5."
- Kiran says, "I can solve this equation by multiplying 1,750 by $\frac{1}{5}$."

a. What value of k would each student get using their own method?

b. How are Priya and Kiran's approaches related?

c. Explain how each student might approach solving the equation $\frac{2}{3}k = 50$.

6 from an earlier course
A moving company needs to load 350 boxes onto a truck.

a. So far, they have loaded 64% of all the boxes. How many boxes have they loaded?

b. There are labels that say "heavy" on 70 of the boxes. What percentage of all the boxes are labeled "heavy"?

Learning Targets
+ I can decide whether a situation about a circle has to do with area or circumference.
+ I can use formulas for circumference and area of a circle to solve problems.

b. About how big is the outfield?

Sample response: $12{,}600\pi$ (or about 39,600 ft^2)

The area of the full softball field, modeled by a quarter circle, is $\frac{1}{4} \cdot \pi \cdot 250^2$ or $15{,}625\pi$ square feet. The infield, which needs to be subtracted, has about the same area as a circle of radius 55 feet or $3{,}025\pi$ square feet. The difference is $12{,}600\pi$ square feet. Note that if we draw a circle with diameter 110 feet (where the 110 foot measurement is marked), it misses some of the lower left part of the infield but also contains some extra area below the softball field so this is a good estimate.

Problem 5

from Unit 2, Lesson 5

While in math class, Priya and Kiran come up with two ways of thinking about the equation $5k = 1750$.

- Priya says, "I can solve this equation by dividing 1,750 by 5."
- Kiran says, "I can solve this equation by multiplying 1,750 by $\frac{1}{5}$."

a. What value of k would each student get using their own method?

350

b. How are Priya and Kiran's approaches related?

Priya divided each side of the equation by the same number. Seeing that 5 and k were multiplied in the equation, she used division to get k by itself. Meanwhile, Kiran multiplied by the reciprocal of 5.

c. Explain how each student might approach solving the equation $\frac{2}{3}k = 50$.

Priya divides by $\frac{2}{3}$ since k is being multiplied by $\frac{2}{3}$. Her equation is $k = 50 \div \frac{2}{3}$.

Kiran multiplies by the reciprocal of $\frac{2}{3}$. His equation is $k = \frac{3}{2} \cdot 50$.

Problem 6

from an earlier course

A moving company needs to load 350 boxes onto a truck.

a. So far, they have loaded 64% of all the boxes. How many boxes have they loaded?

224 boxes

b. There are labels that say "heavy" on 70 of the boxes. What percentage of all the boxes are labeled "heavy"?

20%