Expressions with Rational Numbers

Goals

- Evaluate an expression for given values of the variable, including negative values, and compare (orally) the resulting values of the expression.
- Generalize (orally) about the relationship between additive inverses and about the relationship between multiplicative inverses.
- Identify numerical expressions that are equal, and justify (orally) that they are equal.

Learning Targets

- I can add, subtract, multiply, and divide rational numbers.
- I can evaluate expressions that involve rational numbers.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 3)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Card Sort
- Math Talk
- MLR2: Collect and Display
- · Notice and Wonder
- Take Turns

Required Materials

Materials to Copy

• The Same But Different Cards (1 copy for every 2 students): Activity 1

Required Preparations

Activity 1:

Copy each set of cards on a different color of paper so they can easily be sorted for the next class.

Activity 3:

For the digital version of the activity, acquire devices that can run the applet.

Lesson Narrative

The purpose of this lesson is to help students make sense of expressions involving variables and rational numbers and reason about their position on the number line. As students start to gain fluency with rational number arithmetic and they encounter complicated numerical expressions and algebraic expressions with variables, it is important to keep the connection between those expressions and numbers on the number line. This includes whether the numbers are positive or negative, which of two numbers is greater, and whether two expressions represent the same number.

Students work through common misconceptions that can arise about expressions involving variables, such as the misconception that -x must always be a negative number. They also reason about the structure of expressions involving inverse operations as they compare them.

Then students reason about expressions in a and b given the positions of a and b on a number line without a given scale. As they consider the structure of the algebraic expressions on the number line, they connect the letters in an algebraic expression to numbers and compare the expressions' relative values and distance from 0. For example, they see that $\frac{1}{4}a$ is a quarter of the way from 0 to a on the number line, even if the value of a is unknown.

Lesson Timeline

5 min

Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Activity 3

Lesson Synthesis

10

Assessment

5_{min}

Cool-down

Expressions with Rational Numbers

Student Learning Goal

Let's develop our signed number sense.

Inspire Math



Go Online

Before the lesson, show this video to review the real-world connection.

ilclass.com/l/614240

Please log in to the site before using the QR code or URL.



Instructional Routines

Math Talk

ilclass.com/r/10694967

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Warm-up

Math Talk: Rational Numbers



Activity Narrative

This *Math Talk* focuses on reasoning about the values of numeric expressions. It encourages students to think about positive and negative values without necessarily computing anything and to rely on what they know about operations with negative and positive numbers to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students find expressions that have the same value.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether each statement is true.

A. (-38.76)(-15.6) is negative.

False

Sample reasoning: The product of 2 negative numbers is positive.

B.10,000 - 99,999 < 0

True

Sample reasoning: Subtracting a greater number from a lesser number will result in a negative number, which means the difference will be less than 0.

$$\mathbf{C.} \quad \left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 0$$

False

Sample reasoning: Since neither factor is 0, the product cannot be 0.

D.(30)(-80) - 50 = 50 - (30)(-80)

False

Sample reasoning: The left side of the equation is -290 while the right side of the equation is 290, so the two expressions are not equal.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Activity 1

Card Sort: The Same but Different

15 min

Activity Narrative

In this partner activity, students take turns matching expressions that have the same value. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others. Students look for and make use of the relationship between inverse operations to make connections.

Launch 22

Arrange students in groups of 2. Give each group a set of slips cut from the blackline master. If time permits, consider giving students 1 minute to start thinking about possible ways to sort the cards into categories (for example, by the operation, by the number of negative signs in the expression, by the sign of the answer). Discuss as many different categories as time allows before instructing students to match the cards based on their value.

Tell students that the cards contain different expressions and that they will take turns matching the cards. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

Card Sort

ilclass.com/r/10783726

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Multilingual Learners (Activity 1, Student Task)

MLR8: Discussion Supports.

Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "_____ matches ____ because ..." When students disagree, encourage them to challenge each other using these sentence frames: "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about operations with rational numbers.

Advances: Speaking, Conversing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Building on Student Thinking

If some students struggle to find matches, consider asking:

"How else can you think of subtraction/addition/ multiplication/division?" "What do you know about the difference/sum/product/quotient of two signed numbers?"

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to match two expressions with the same value.

- **1.** For each match that you find, explain to your partner how you know it's a match.
- **2.** For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

answer	one expression	on other expression		
-17	-10 + (-7) -10 - 7			
-4	-1 - 4	I ÷ (-1/4)		
-3	-10 + 7	-107		
-2.5	15 ÷ (-6)	-15 · [
-2	8÷(-4)	$8 \cdot \left(-\frac{1}{4}\right)$		
-1	1-2	1+-2		
2	8÷4	(8)(4)		
2.5	15 · [-15 ÷ -6		
3	1+2	I - (-2)		
4	(1)(4)	I ÷ 1/4		

Activity Synthesis

The purpose of this discussion is to make connections between inverse operations in matching expressions. Select 2–3 groups to share one of their sets of cards and how they matched the two expressions. Discuss as many different sets of cards as the time allows. Highlight strategies that compared the structure of the expressions instead of just the final answers. Consider discussing the following questions:

- (How are a number and its opposite the same or different?"
 - They have opposite signs but the same magnitude.

by taking I and dividing by the other.

- "What operation is equivalent to subtracting a given number?"
 Subtracting a number is equivalent to adding the opposite.
- "How are a number and its reciprocal the same or different?"
 They have the same sign but different distances from zero. They are found
- "What operation is equivalent to dividing by a given number?"
 Dividing by a number is equivalent to multiplying by the reciprocal.

Activity 1

Activity 2

Near and Far from Zero

15

Activity Narrative

In this activity, students evaluate expressions using rational numbers. They reason about the relative size of rational numbers and their distance from 0. As they compare the structure of various expressions, they observe that when a variable can be any rational number, it is impossible to compare expressions involving those variables or determine which expression is greater or lesser. For example, if a is positive, then 5a must be greater than 4a, but if a can be any rational number, then that is not necessarily true. If a is negative, then 5a is less than 4a, and if a is 0, 5a is equal to 4a. Students may also use that structure to observe that relative distance from 0 is still possible to tell. For example, 5a is $\frac{5}{4}$ as far away from 0 as 4a, whether that is positive or negative.

Warm-up

Launch

Display the list of expressions for all to see:

$$a, b, -a, -4b, -a + b, a \div -b, a^2, b^3$$

Ask students the following questions, and record their initial responses. It is not expected at this time that they be able to make these comparisons.

"Which expression do you think has the largest value?"

"Which has the smallest?"

"Which is closest to zero?"

If necessary, demonstrate a few of the types of computations that will come up as students work. For example, tell students that a represents 10 and brepresents -2. Ask them to determine the value of the following expressions:

- -b (2)
- b^3 (-8)
- $a \cdot \frac{1}{h}$ (-5)
- $\frac{a}{b} \div a$ (- $\frac{1}{2}$ or equivalent)
- $a + \frac{1}{h} (9 \frac{1}{2})$
- $\left(\frac{1}{h}\right)^2 \left(\frac{1}{h}\right)$

Arrange students in groups of 2. Encourage them to check in with their partner as they evaluate each expression, and work to reach an agreement.

Student Task Statement

1. For each set of values for a and b, evaluate the given expressions, and record your answers in the table.

а	b	- a	-4 b	-a + b	a ÷ -b	a ²	b^3
- 1/2	6	1/2	-24	6 1/2	1/12	14	216
1/2	-6	-1/2	24	-6 ¹ / ₂	1/2	14	1/4
-6	- 1/2	6	2	5 <u>1</u>	-12	36	-18

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

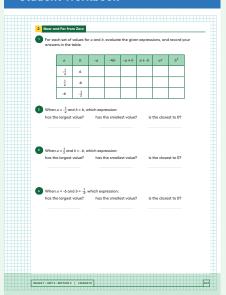
Chunk this task into more manageable parts. For example, present each row of the table separately. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

If students struggle to find the largest value, smallest value, or value closest to zero in the set, encourage them to create a number line to help them reason about the positions of different candidates.

Student Workbook



2. When $a = -\frac{1}{2}$ and b = 6, which expression:

has the largest value? has the smallest value? is the closest to 0?

 $b^3 = 216$ -4b = -24 $a \div -b = \frac{1}{12}$

3. When $a = \frac{1}{2}$ and b = -6, which expression:

has the largest value? has the smallest value? is the closest to 0?

-4b = 24 $b^3 = -216$ $a \div -b = \frac{1}{12}$

4. When a = -6 and $b = -\frac{1}{2}$, which expression:

has the largest value? has the smallest value? is the closest to 0?

 $a^2 = 36$ $a \div -b = -12$ $b^3 = -\frac{1}{8}$

Are You Ready for More?

Are there any values that could be used for a and b that would make all of these expressions have the same value? Explain your reasoning.

No

If a = 0 and b = 0, then most of the expressions will also have a value of 0, but $a \div (-b)$, will be undefined.

Activity Synthesis

Display the completed table for all to see. Invite students to share their values that are largest, smallest, and closest to zero from each set and explain their reasoning.

Ask students if any of these results were surprising and to explain their reasoning. Some possible observations include:

- b^3 was both the largest and smallest value at different times. 6 and -6 are both relatively far from 0, so b^3 is a large number when b is positive and a small number when b is negative.
- a ÷ -b was often the closest to 0, because it had the same absolute value no matter the sign of a and b.

Activity 3: Optional

Seagulls and Sharks Again

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use a familiar context to interpret an expression in terms of the position it represents on a vertical number line. Expressions are equal when they represent the same position on a number line. Students use the structure of the number line to reason about the relative values of expressions.

In a previous course, students have seen fractions with a negative sign in front of the entire fraction, such as $-\frac{a}{2}$. In this activity, students see a fraction with a negative sign in the numerator, such as $\frac{-a}{2}$. Students can apply what they know about dividing signed numbers to make sense of expressions like this.

In the digital version of the activity, students use an applet to place points representing the vertical position of different animals on a vertical number line. The applet allows students to drag points to the number line and quickly check their answers.

Launch

Tell students to close their books or devices (or to keep them closed). Display the image from the *Task Statement* for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image.

Tell students to open their books or devices. If necessary and as time allows, consider demonstrating the placement of one animal before students start working. For example, ask students,

 \bigcirc "If there were a minnow with vertical position m, and $m = \frac{1}{3}b$, where is the minnow?"

Help students interpret the equation and plot the appropriate point on the vertical number line.

Keep students in the same groups as the previous activity. Encourage students to check in with their partner periodically and work together to resolve any disagreements.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

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Instructional Routines

Notice and Wonder ilclass.com/r/10694948

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Access for Multilingual Learners (Activity 3, Student Task)

MLR2: Collect and Display.

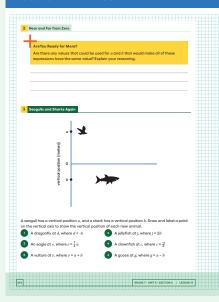
Collect the language that students use as they decide where on the vertical axis to show the position of each new animal. Display words and phrases, such as "twice as far," "opposite direction," and "half the distance." During the Activity Synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

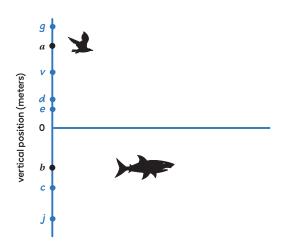
Building on Student Thinking

For students who are struggling to measure out a length of a or b or a sum, difference, or multiple of them, suggest that they measure and cut strips of paper for the lengths of a and b to help guide them. Ask how they could use the strips to find other distances, such as a-b and $\frac{a}{2}$.

Student Workbook



Student Task Statement



A seagull has a vertical position a, and a shark has a vertical position b. Draw and label a point on the vertical axis to show the vertical position of each new animal.

- **1.** A dragonfly at d, where d = -b
- **3.** An eagle at e, where $e = \frac{1}{4}a$
- **5.** A vulture at v, where v = a + b
- **2.** A jellyfish at j, where j = 2b
- **4.** A clownfish at c, where $c = \frac{-a}{2}$
- **6.** A goose at g, where g = a b

Activity Synthesis

The purpose of this discussion is for students to share their reasoning about the vertical location of each animal and to illustrate the meaning of the equal sign. Begin by displaying the diagram from the *Task Statement* for all to see. Invite students to share their responses and reasoning, and record them on the diagram for all to see.

As a point for each animal is placed, illustrate the meaning of the equal sign. For example,

 \bigcirc "We could either label this point with v or with a+b since we know that v=a+b. Since these expressions are equal, they represent the same position on the number line."

If not brought up in students' explanations, consider drawing attention to the equation $c=\frac{-a}{2}$ and discussing how this equation relates to what they have learned about division with signed numbers. A key idea for students to understand is that $-\frac{a}{2}$ is equal to $\frac{-a}{2}$ and $\frac{a}{-2}$, but these are not equal to $\frac{-a}{-2}$.

Activity 2

Lesson Synthesis

Share with students,

"Today we thought about all four operations with rational numbers."

To help students articulate patterns with arithmetic using negative numbers, consider asking:

"What would be another way to write the expression 12–15 without using subtraction?"

12 + -15

"What are some other expressions using positive or negative 12 and 15 that are the same distance from 0 as 12 - 15?"

15 - 12, -15 + 12, -12 + 15

"How are 15 and -15 related?"

They are opposites of each other. They are the same distance from 0, but one is in the negative direction and one is in the positive direction.

"What value of bwould make the expression 7b be equal to 1?"

 \bigcirc "What value of b would make the expression 7b be equal to -1?"

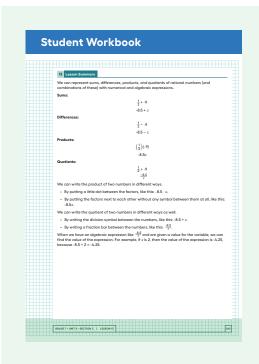
 \bigcirc "If we have the expression $\frac{-9}{x}$, what are some values of x that would make the expression positive?"

any negative number

 \bigcirc "Is there a value of x that will make the expression $\frac{-9}{x}$ equal to 0?"

No, if x is negative, the expression will be greater than 0. If x is positive, the expression will be less than O. Since we cannot divide by O, there are no values of x that will give a number which is exactly 0.

Warm-up



Lesson Summary

We can represent sums, differences, products, and quotients of rational numbers (and combinations of these) with numerical and algebraic expressions.

Sums: $\frac{1}{2}$ + -9

-8.5 + x

Differences: $\frac{1}{2}$ - -9

-8.5 – *x*

Products: $\left(\frac{1}{2}\right)$ (-9)

-8.5*x*

Quotients: $\frac{1}{2} \div -9$

-8.5 x

We can write the product of two numbers in different ways.

- By putting a little dot between the factors, like this: -8.5 \cdot x.
- By putting the factors next to each other without any symbol between them at all, like this: -8.5x.

We can write the quotient of two numbers in different ways as well.

- By writing the division symbol between the numbers, like this: -8.5 \div x.
- By writing a fraction bar between the numbers, like this: $\frac{-8.5}{x}$.

When we have an algebraic expression like $\frac{-8.5}{x}$ and are given a value for the variable, we can find the value of the expression. For example, if x is 2, then the value of the expression is -4.25, because -8.5 ÷ 2 = -4.25.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

Make Them True

5 min

Student Task Statement

Complete each equation with an operation to make it true.

1. 24
$$\frac{3}{4}$$
 = 18

2. 24
$$\frac{\div}{4}$$
 - $\frac{3}{4}$ = -32

5. -18
$$\frac{\bullet}{4}$$
 - $\frac{3}{4}$ = 24

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Student Workbook

13 Practice Problems	
D Procuce Problems	
Find the value of each expression.	
g22+5	
0. 122 + 0	
b22 - (-5)	
c. (-22)(-5)	
d. 22+5	
from Unit 4, Lesson 10 The price for package of highlighters is \$3.25, but it costs \$3.51 with tax.	
What is the soles tax rate?	
Wilde is the states tax rate?	
GRADE 7 - UNIT 5 - SECTION C LESSON S	247

Problem 1

The value of x is $\frac{-1}{4}$. Order these expressions from least to greatest:

$$x$$
 1- x $x-1$ -1 ÷ x $x-1$ x 1- x

The expressions' values are $\frac{-5}{4}$, $\frac{-1}{4}$, $\frac{5}{4}$, and 4.

Problem 2

Here are four expressions that have the value $\frac{-1}{2}$:

$$\frac{-1}{4} + \left(\frac{-1}{4}\right)$$
 $\frac{1}{2} - 1$ $-2 \cdot \frac{1}{4}$ $-1 \div 2$

Write five expressions: a sum, a difference, a product, a quotient, and one that involves at least two operations that have the value $\frac{-3}{4}$.

Sample response:

sum
$$\frac{-1}{4} + \left(\frac{-1}{2}\right)$$

difference $\frac{1}{4} - 1$
product $-3 \cdot \frac{1}{4}$
quotient $-3 \div 4$

two or more operations I ÷ 4 - I

Problem 3

Find the value of each expression.

-4.4

Problem 4

from Unit 4, Lesson 10

The price for package of highlighters is \$3.25, but it costs \$3.51 with tax. What is the sales tax rate?

8%

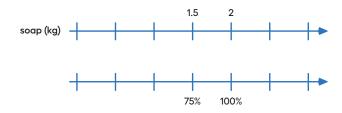
(Any answer between 7.85% and 8.15% is acceptable.)

Problem 5

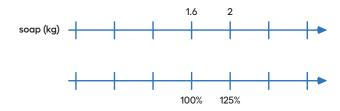
from Unit 4, Lesson 7

Two students are both working on the same problem: A box of laundry soap has 25% more soap in its new box. The new box holds 2 kg. How much soap did the old box hold?

• Here is how Jada set up her double number line.



· Here is how Lin set up her double number line.



Do you agree with either of them? Explain or show your reasoning.

I agree with Lin

Sample reasoning: The soap in the old box represents 100%, and the new box now holds 125%, which is 2 kg.

Problem 6

from Unit 4, Lesson 3

A runner is running a 10 km race. It takes her 17.5 minutes to reach the 2.5 km mark. At this rate, how long will it take her to run the whole race?

70 minutes

It takes her 7 minutes to run I km. Therefore, it would take her 70 minutes to run IO km.

