Reasoning about Contexts with Tape Diagrams

Goals

- Draw and label a tape diagram to represent relationships between quantities in a situation.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in a situation.
- Interpret a tape diagram that represents a relationship of the form px + q = r or p(x + q) = r.

Learning Targets

- I can explain how a tape diagram represents parts of a situation and relationships between them.
- I can use a tape diagram to find an unknown amount in a situation.

Lesson Narrative

In this lesson, students represent and reason about contexts, using tape diagrams. Students may have had experience with tape diagrams in earlier grades, and have seen some examples of their use in prior units. For example, tape diagrams were used to represent percent increase and decrease situations in an earlier unit.

First, students interpret some given tape diagrams. Then they interpret a story and create tape diagrams. While the contexts lead to equations of the forms p(x + q) = r and px + q = r, this lesson is not about writing equations. Likewise, students are asked to find an unknown value in several story problems, but the intention is for them to use any reasoning that makes sense to them. It is not expected that they write and solve equations, or that any particular method is stressed.

Student Learning Goal

Let's use tape diagrams to make sense of different kinds of stories.

Abilities

· Action and Expression (Activity 1, Activity 2)

Access for Students with Diverse

Access for Multilingual Learners

Critique, Correct, Clarify (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- · Notice and Wonder

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline



Warm-up



Activity 1



Activity 2



Lesson Synthesis

Assessment

Cool-down

Warm-up

Remembering Tape Diagrams



Activity Narrative

The purpose of this *Warm-up* is to re-introduce students to tape diagrams as a representation of relationships between quantities. These diagrams will be helpful for reasoning about situations in activities in this lesson. While students may notice and wonder many things about these diagrams, the understanding that the length of a piece of tape is meaningful, and that pieces labeled with the same variable indicate that they have the same length, are the important discussion points.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is multiples of identical sections of the tape diagram, the relative lengths of the sections of the diagram, and the total length.

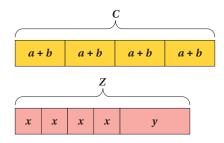
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Arrange students in groups of 2. Display the image for all to see.

Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement

What do you notice? What do you wonder?



Sample responses:

Students may notice:

- There are two diagrams of rectangles with pieces labeled a, b, c, x, y, and z.
- The c and z appear at the top of the diagrams.
- Each diagram consists of a large rectangle partitioned into smaller rectangles.
- In the first diagram, the rectangle contains 4(a + b)'s.
- In the second diagram, the rectangle contains 4 x's and I y.

Students may wonder:

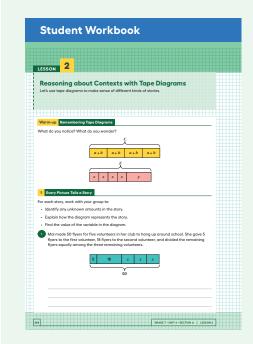
- · What do the diagrams represent?
- · What do the pieces of the diagrams represent?
- Do all of the x's represent the same value? All the y's? All the z's?
- Are longer pieces "worth" more?

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL





Instructional Routines

MLR3: Critique, Correct, Clarify

ilclass.com/r/10695504

Please log in to the site before using the QR code or URL



Access for Multilingual Learners (Activity 1, Launch)

This activity uses the *Critique*, *Correct*, *Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, consider pausing after the first question for a brief class discussion before moving on.

Supports accessibility for: Organization, Attention

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the appropriate parts of the diagrams. Next, ask students,

(a) "Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the idea that pieces labeled with the same variable represent the same value does not come up during the conversation, ask students to discuss this idea.

As an extension, ask students to share possible values for the variables in each diagram. Record and display their responses for all to see. If possible, record the values on the displayed diagram.

In the first diagram, if a = 1 and b = 4, and we assume that c is the total, then c = 1 + 4 \cdot 4 = 20.

In the second diagram, if x = 1 and y = 3, and we assume that z is the total, then $z = 4 \cdot 1 + 3 = 7$.

Activity 1

Every Picture Tells a Story



Activity Narrative

In this activity, students explain how a tape diagram represents a situation. They also use the tape diagram to reason about the value of the unknown quantity. Students are not expected to write and solve equations here; any method they can explain for finding values for x and y is acceptable. While some students might come up with equations to describe the diagram and solve for the unknown, there is no need to focus on developing those ideas at this time.

In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch 222

Arrange students in groups of 2 or 3.

Ask students if they know what a "flyer" is. If any students do not know, explain or ask a student to explain. If possible, reference some examples of flyers hanging in school.

Use *Critique*, *Correct*, *Clarify* to give students an opportunity to improve a sample written response to an explanation of why the first diagram matches the first story by correcting errors, clarifying meaning, and adding details.

Display this first draft:

"The tape diagram has a 5 and an 18, and there's a 5 and an 18 in the story." Ask,

"What parts of this response are unclear, incorrect, or incomplete?"

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

Give students 2-4 minutes to work with a partner to revise the first draft.

Select 1–2 groups to slowly read aloud their draft. Record for all to see as each draft is shared. Then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Student Task Statement

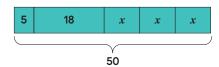
For each story, work with your group to:

Identify any unknown amounts in the story.

Explain how the diagram represents the story.

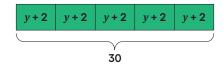
Find the value of the variable in the diagram.

 Mai made 50 flyers for five volunteers in her club to hang up around school. She gave 5 flyers to the first volunteer, 18 flyers to the second volunteer, and divided the remaining flyers equally among the three remaining volunteers.

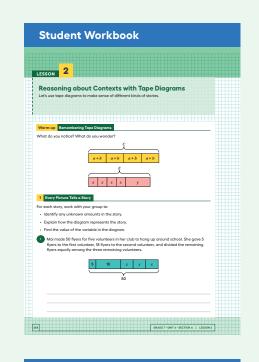


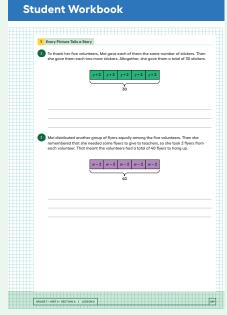
Sample answer: Unknown amounts include the remaining number of flyers (27) and the number of flyers given to each of the 3 remaining volunteers (9). The whole rectangle represents the 50 flyers that Mai made. She split them up into five parts: 5, 18, and 3 equal parts for the rest. The 3 equal parts are shown by 3 same-sized boxes labeled x, and each x represents the number of flyers for each of the 3 remaining volunteers. Together the 3 x boxes represent 27 flyers, since 23 of the total 50 (5 + 18) have already been given out. So each x represents 9 flyers.

2. To thank her five volunteers, Mai gave each of them the same number of stickers. Then she gave them each two more stickers. Altogether, she gave them a total of 30 stickers.



Sample answer: Unknown amounts include the total number of stickers each student receives (6) and the number they received at first (4). The whole rectangle represents the 30 stickers. They are divided into 5 equal parts since the 5 volunteers each got the same number of stickers. They each got some (y) and then each got 2 more, so each one got y+2 stickers. We can find y by thinking that 30 divides into 5 groups of 6. If each volunteer received 6 stickers in total, they got 4 before the extra 2 were added. Another way to think about y is to first take away the 10 extra stickers that were given out. Then 5 groups of 4 would make up the remaining 20 stickers. So y represents 4 stickers.



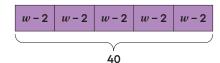


Building on Student Thinking

Students may not realize that when a variable is assigned to represent a quantity in a situation, it has the same value each time it appears. Revisit what x and y represent in these problems and why each occurrence of a variable in a representation must represent the same value.

In the second situation, students might argue that a more accurate representation would be 5 boxes with y to show the first distribution of stickers, and then five boxes with 2 to show the second distribution. Tell students that such a representation would indeed correctly describe the actions in the situation, but that the work of the task is to understand *this* diagram to set us up for success later.

3. Mai distributed another group of flyers equally among the five volunteers. Then she remembered that she needed some flyers to give to teachers, so she took 2 flyers from each volunteer. That meant the volunteers had a total of 40 flyers to hang up.



Sample response: Unknown amounts include the number of flyers each student has in the end (8), the number they received at first (10), and the total number of flyers they started with (50). The whole rectangle represents 40 flyers. They are divided into 5 equal parts since the 5 volunteers each got the same number of flyers. They each got some (w) and then each gave back 2, so each one has w-2 flyers. We can find w by thinking that 40 divides into 5 groups of 8. If each volunteer has 8 flyers, they got 10 before the 2 were taken away. So there were originally 50 flyers, which is the 40 that the volunteers have, plus the 10 that Mai took back.

Activity Synthesis

The goals of this discussion are to make sure students understand how parts of the diagram match the information given in the story, and to encourage students to start reasoning about how the diagrams connect to the operations they can use to find unknown amounts.

Invite one group to provide an explanation for each diagram—both how the diagram represents the story, and how they reasoned about the unknown amounts. After each situation is discussed, ask the class if anyone thought about it a different way. (One additional line of reasoning for each diagram is sufficient.)

Consider asking some of the following questions to encourage students to be more specific:

"What question could you ask about the story?"

"Where in the diagrams do you see equal parts? How do you know they are equal?"

"What quantity does the variable represent in the story? How do you know?"

"In the first story, where in the diagram do we see the 'remaining flyers'?"

"Why don't we see the number 3 in the first diagram to show the three remaining volunteers?"

"In the second diagram, where are the five volunteers represented?"

"How did the diagrams help you find the value of the unknown quantities?"

Activity 2

Every Story Needs a Picture

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students draw tape diagrams to represent three stories. The first story is a bit more scaffolded because it specifies what x represents. For the other two stories, students need to decide which quantity to represent with a variable and choose a letter to use. As with all activities in this lesson, students are not expected to write and solve an equation. This preliminary work supports the understanding needed to be able to represent such situations with equations in later lessons. Since students are defining a quantity of interest and assigning a variable, they are engaging in aspects of mathematical modeling.

Launch

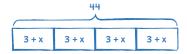
Keep students in the same groups. If time allows, each student should draw all three diagrams and compare them with their groups, working together to resolve any discrepancies. Or if time is short, you might assign each student in the group a different story—ask each student to explain their diagram to their group to see if their group members agree with their interpretation. Another alternative is to only assign the first two stories.

In the digital version of the activity, students use an applet to create their tape diagrams. The applet allows students to divide the tape into equal or unequal parts and label the parts and total. Demonstrate how the controls work in the applet, especially how the tape can be divided into 2 sections and the divider between those sections can be moved to adjust the sizes. Each section can then be divided into equal parts. The digital version may be helpful for students who struggle with drawing their own diagrams. However, even if the class is using the digital version, some students may prefer to draw the diagrams in their notebooks or on scratch paper.

Student Task Statement

Here are three more stories. Draw a tape diagram to represent each story. Then describe how you would find any unknown amounts in the stories.

1. Noah and his sister are making gift bags for a birthday party. Noah puts 3 pencil erasers in each bag. His sister puts *x* stickers in each bag. After filling 4 bags, they have used a total of 44 items.



(or equivalent)

Sample response: Unknown amounts include the number of stickers in each bag (8) and the total number of items in each bag (II). Dividing 44 into 4 equal parts gives II items for each bag, which means 3 erasers and 8 stickers in each. Another way is to subtract the I2 erasers from 44, giving 32 items left. 32 stickers split evenly among 4 bags is 8 in each bag.

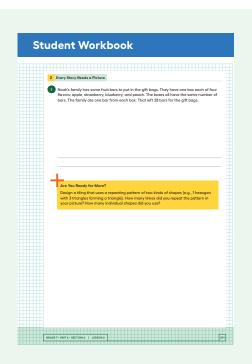
Access for Students with Diverse Abilities (Activity 2, Task Statement)

Action and Expression: Provide Access for Physical Action.

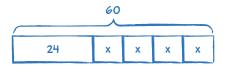
Activate or supply background knowledge. Provide students with access to blank templates of tape diagrams or the digital version of the activity to support information processing.

Supports accessibility for: Visual-Spatial Processing, Organization

Student Workbook 2 Evry 18th Vector Picture Here or the more stories. Town or tope diagram to represent each story. Then describe how you would find any unknown amounts in the tories. 3 In such a before or enging of though for a birthday party, Nosh puts 3 pencil excess in each bag, His sister puts a stickers in each bag. After filling 4 bags, they have used a storial of 44 ferms. 3 Nosh's family also worsts to blow up a total of 60 ballooms for the party. Yesterday they bewup 36 ballooms. Today they want to split the termolning balloons equally between four family members.



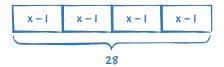
2. Noah's family also wants to blow up a total of 60 balloons for the party. Yesterday they blew up 24 balloons. Today they want to split the remaining balloons equally between four family members.



(or equivalent)

Sample response: Unknown amounts include the number of balloons they need to blow up today (36) and the number that each of the four family members blows up (9). The number of balloons left to blow up is found by 60-24 or 36. Splitting those up equally among four people is $36\div4$ or 9 each.

3. Noah's family has some fruit bars to put in the gift bags. They have one box each of four flavors: apple, strawberry, blueberry, and peach. The boxes all have the same number of bars. The family ate one bar from each box. That left 28 bars for the gift bags.



(or equivalent)

Sample response: Unknown amounts include the number of bars left in each box (7), the number of bars originally in each box (8), and the total number of bars there were in the four boxes (32). Dividing 28 into 4 equal parts gives 7 bars left in each box. Adding I to each gives 8 in each box originally for a total of $8 \cdot 4$ or 32 bars.

Are You Ready for More?

Design a tiling that uses a repeating pattern of two kinds of shapes (e.g., 1 hexagon with 3 triangles forming a triangle). How many times did you repeat the pattern in your picture? How many individual shapes did you use?

Answers vary.

Activity Synthesis

Much of the discussion will take place in groups. Here are some ideas for synthesizing students' learning about creating tape diagrams:

- Ask students if they had any disagreements in their groups and how they resolved them.
- Ask students how they decided which unknown quantity to find in the story. The first story specifies *x* stickers, but the other stories do not define a variable.
- Display one diagram for each story and ask students to explain how the diagrams are alike and how they are different.

Lesson Synthesis

Display one or more of the tape diagrams students encountered or created during the lesson. Ask,

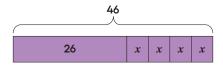
- "What are some ways that tape diagrams give information about a story?"
 Responses to highlight:
- · A total amount is indicated.
- Pieces that represent equal amounts are the same length (or roughly the same length, if sketching by hand).
- Pieces that represent different amounts are not the same length.
- Pieces are labeled with either their amounts, a variable representing an unknown amount, or an expression like x + 1 to mean "1 more than the unknown amount."

Lesson Summary

Tape diagrams are useful for representing how quantities are related and can help us answer questions about a situation.

Example: A school receives 46 copies of a popular book. The library takes 26 copies and the remainder are split evenly among 4 teachers. How many books does each teacher receive?

This situation involves a total formed by 4 equal parts and one other part. We can represent the situation with a diagram labeled 46 for the total number of books. That total length is divided into parts—one long part labeled 26 for the books given to the library and 4 equal-sized parts for the books split among 4 teachers. We label each of those parts with a variable, x, because we don't know how many books each teacher got. Using the same variable, x, in each part means that the same number is represented four times.



Some situations have parts that are all equal, but each part has been increased from an original amount:

Example: A company manufactures a special type of sensor, and packs them in boxes of 4 for shipment. Then a new design increases the weight of each sensor by 9 grams. The new package of 4 sensors weighs 76 grams. How much did each sensor weigh originally?

We can represent this situation with a rectangle representing a total of 76 split into 4 equal parts. Each part shows that the new weight, x + 9, is 9 more than the original weight, x.



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

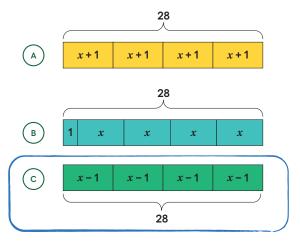
Red and Yellow Apples



Student Task Statement

Here is a story: Lin bought 4 bags of apples. Each bag had the same number of apples. After eating 1 apple from each bag, she had 28 apples left.

1. Which diagram best represents the story? Explain why the diagram represents it.



Sample reasoning: When she ate I apple from each bag, there were x-1 apples left in each bag.

2. Describe how you would find the unknown amount in the story. Each of the 4 pieces of the diagram represents 7 apples, because $28 \div 4 = 7$. If x - 1 = 7, then x is 8.

Practice Problems

5 Problems

Problem 1

from Unit 3, Lesson 1

The table shows the number of apples and the total weight of the apples.

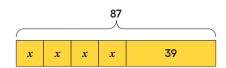
number of apples	weight of apples (grams)
2	511
5	1200
8	2016

Estimate the weight of 6 apples.

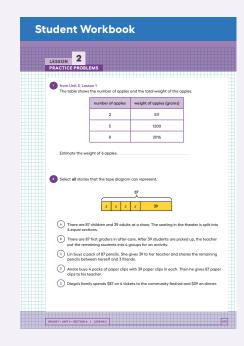
Sample response: about 1500 grams

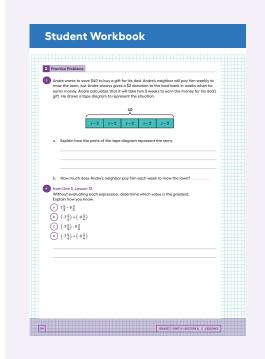
Problem 2

Select all stories that the tape diagram can represent.



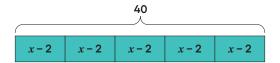
- **A.** There are 87 children and 39 adults at a show. The seating in the theater is split into 4 equal sections.
- **B.** There are 87 first graders in after-care. After 39 students are picked up, the teacher put the remaining students into 4 groups for an activity.
- **C.** Lin buys a pack of 87 pencils. She gives 39 to her teacher and shares the remaining pencils between herself and 3 friends.
- **D.** Andre buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.
- **E.** Diego's family spends \$87 on 4 tickets to the community festival and \$39 on dinner.





Problem 3

Andre wants to save \$40 to buy a gift for his dad. Andre's neighbor will pay him weekly to mow the lawn, but Andre always gives a \$2 donation to the food bank in weeks when he earns money. Andre calculates that it will take him 5 weeks to earn the money for his dad's gift. He draws a tape diagram to represent the situation.



a. Explain how the parts of the tape diagram represent the story.

Sample response: The 5 equal parts represent the 5 weeks. In each week, Andre will earn x dollars for mowing his neighbor's lawn and give \$2 to the food bank, so he will save x-2 dollars. In five weeks, he will save a total of \$40.

b. How much does Andre's neighbor pay him each week to mow the lawn?

Problem 4

from Unit 5, Lesson 13

Without evaluating each expression, determine which value is the greatest. Explain how you know.

A.
$$7\frac{5}{6} - 9\frac{3}{4}$$

B.
$$\left(-7\frac{5}{6}\right) + \left(-9\frac{3}{4}\right)$$

C.
$$\left(-7\frac{5}{6}\right) \cdot 9\frac{3}{4}$$

D.
$$\left(-7\frac{5}{6}\right) \div \left(-9\frac{3}{4}\right)$$

Sample reasoning: It is the only expression with a positive value.

Problem 5

from Unit 5, Lesson 15

Solve each equation.

a.
$$(8.5) \cdot (-3) = a$$

-25.5

b.
$$(-7) + b = (-11)$$

-4

c.
$$c$$
 - (-3) = 15

12

d.
$$d \cdot (-4) = 32$$

-8