Scaling One Dimension (Optional)

Goals

- Create a graph and an equation to represent the function relationship between volume and height for cylinders with the same radius and justify (orally) that the relationship is linear.

- Interpret (in writing) a point on a graph representing the volume of a cone as a function of its height, and explain (orally) how changing one dimension affects the other.

Learning Targets

- I can create a graph of the relationship between volume and height for all cylinders and cones with a fixed radius.

- I can explain in my own words why changing the height by a scale factor changes the volume by the same scale factor.

Lesson Narrative

The purpose of this lesson is for students to see some examples of linear functions, in this case proportional ones, that arise out of geometry. In an earlier lesson, students considered the relationship between the volume of water inside a graduated cylinder and the height of the water. In that example, the height was a function of the volume, and the relationship was proportional.

In the *Warm-up*, students study the graph of a proportional relationship and recall that in a proportional relationship, the two quantities change by the same scale factor: when one of them is multiplied by a scale factor, the other one gets multiplied by the same scale factor.

Next, students consider rectangular prisms that share two edge lengths but have a third length that varies. They graph prism volume as a function of the third edge length and see that the volume is proportional to the length. They conclude that when the length doubles, the volume doubles.

Lesson Timeline

5 min	10 min	15 min	10 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Assessment

Scaling One Dimension (Optional)

Lesson Narrative (continued)

Then they investigate volume as a function of the height for cylinders with a fixed radius. Again they see that the volume is proportional to the height, and that when the height is halved, the volume is halved. During this activity, students have opportunities to explain their reasoning and critique the reasoning of others.

The final activity is optional as it contains content beyond grade level. Consider using this activity to give students additional practice working with the formula for the volume of a cone.
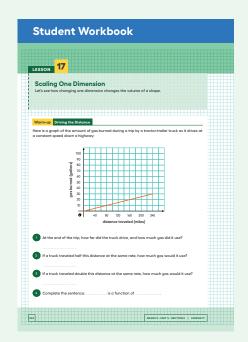
Student Learning Goal

Let's see how changing one dimension changes the volume of a shape.

Building on Student Thinking

If students are unsure how to determine the values for half the distance or double the distance, consider asking:

"How did you identify the values for the first question?"
"How could you use what you know about proportional relationships to help you solve these questions?"

Student Workbook

Warm-up
Driving the Distance

5 min

Activity Narrative
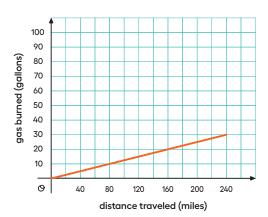
The purpose of this *Warm-up* is for students to jump back into recognizing functions and determining if two quantities are functions of each other. The discussion is meant to get students using the language of functions to describe linear relationships, which continues throughout the rest of the activities.

Launch

Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Here is a graph of the amount of gas burned during a trip by a tractor-trailer truck as it drives at a constant speed down a highway:

distance traveled (miles)

1. At the end of the trip, how far did the truck drive, and how much gas did it use?

   The truck drove 240 miles and used 30 gallons of gas.

2. If a truck traveled half this distance at the same rate, how much gas would it use?

   15 gallons

   Since it is a proportional relationship, if the miles are halved, then the gallons are also halved.

3. If a truck traveled double this distance at the same rate, how much gas would it use?

   60 gallons

   Since it is a proportional relationship, if the miles are doubled, then the gallons are also doubled.

4. Complete the sentence: _Gallons of gas burned_ is a function of _miles traveled._ The number of miles traveled is also a function of the gallons of gas burned.

Activity Synthesis

Invite students to share their answers and their reasoning for why gas burned is a function of distance traveled. Questions to further the discussion about functions:

💬 *"Looking at the graph, what information do you need in order to determine how much gas was used?"*

We need to know the number of miles traveled.

💬 *"What is the independent variable? Dependent variable? How can you tell from the graph?"*

The independent value is the distance traveled. The dependent value is the gas burned. By convention, the independent is on the x-axis, and the dependent is on the y-axis.

💬 *"What are some ways that we can tell from the graph that the relationship between gas burned and distance traveled is proportional?"*

The graph is a line that goes through the origin, and we can see a constant ratio between y and x in some points, like (80, 10), (160, 20), and (240, 30).

💬 *"Is this an example of a linear function? Why or why not?"*

This is a linear function because it is a proportional relationship, which can be written as $y = kx$. For each input value of x, there is one and only one output value for y.

Activity 1
Double the Edge   **10 min**

Activity Narrative

The purpose of this activity is for students to apply what they know about functions and their representations in order to investigate the effect of a change in one dimension on the volume of a rectangular prism. Students use graphs and equations to represent the volume of a rectangular prism with one unknown edge length. They then use these representations to express what happens to the volume when one of the edge lengths is doubled. Groups make connections between the different representations by pointing out how the graph and the equation reflect an edge length of a prism that is doubled and by explaining the relationships between that length and the volume of the prism.

Launch 👥

Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time followed by time to discuss the last question with their partner.

Select groups who make connections between the graph and the question to share during the discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.
Support effective and efficient use of tools and assistive technologies. To use the digital applet, some students may benefit from a checklist.

Supports accessibility for: Organization, Memory, Attention

Building on Student Thinking

If students are not sure how to start labeling their graph, consider asking:

"Can you explain how you wrote your equation between V and s?
"How could making a table of values help you make a graph?"
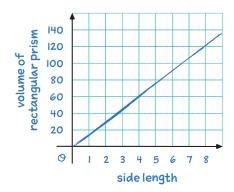
Student Workbook

Student Task Statement

There are many right rectangular prisms with one edge of length 5 units and another edge of length 3 units. Let s represent the length of the third edge and V represent the volume of these prisms.

1. Write an equation that represents the relationship between V and s.

   $V = 15s$

2. Graph this equation, and label the axes.

3. What happens to the volume if you double the edge length s? Where do you see this in the graph? Where do you see it algebraically?

   When the edge length s is doubled, the volume is also doubled. Sample response: In the graph, it can be seen that when the edge length is 4 cm, the volume is 60 cm³. When the edge length doubles to 8 cm, the volume doubles to be 120 cm³, as the graph shows a proportional relationship. Algebraically, if the edge length is doubled from s to $2s$, then the volume goes from $15s$ to $15 \cdot 2s$, or $30s$, which is double the original volume.

Activity Synthesis

Use this discussion to help students make connections between the context, equation, and graph to support the idea that the volume doubles when s is doubled.

Invite previously selected groups to share what happens to the volume when s is doubled. Display their graph and equation for all to see, and ask students to point out where they see the effect of doubling s in the graph (when looking at any two edge lengths that are double each other, their volume will be double also). Ask students:

💬 *"Which of your variables is the independent? The dependent?"*

> The side length, s, is the independent variable, and volume, V, is the dependent variable.

💬 *"Which variable is a function of which?"*

> Volume is a function of side length.

If it has not been brought up in students' explanations, ask what the volume equation looks like when we double the edge length s. Display volume equation $V = 15(2s)$ for all to see. Ask,

💬 *"How can we write this equation to show that the volume doubled when s doubled?"*

> Using algebra, we can rewrite $V = 15(2s)$ as $V = 2(15s)$. Since the volume for s was $15s$, this shows that the volume for $2s$ is twice the volume for s.

Activity 2
Halve the Height

15 min

Activity Narrative

Building on their thinking about an edge length and volume of a prism, in this activity students consider a similar problem with a cylinder. Using different representations of the function relating the volume of a cylinder to its height given a fixed radius, students continue to identify the effect of the changing dimension on the graph and the equation of this function.

In this partner activity, students take turns sharing their initial ideas and first drafts. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch

Keep students in the same groups. Tell students that this activity is similar to the previous one, but they will work with a cylinder instead of a rectangular prism.

Give students 2–3 minutes of quiet work time followed by time to discuss the last question with their partner.

As groups work on the task, identify those who make the connection between the graph and equation representations, and encourage groups to look for similarities and differences between what they see in this activity and what they saw in the previous activity. Select 2–3 groups making these connections to share during the discussion.

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479
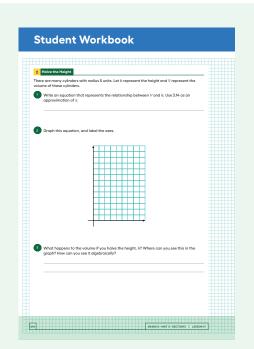Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.
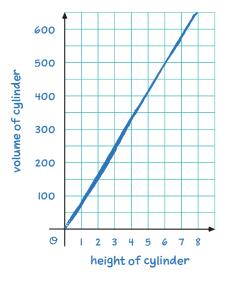

Student Workbook

Student Task Statement

There are many cylinders with radius 5 units. Let h represent the height and V represent the volume of these cylinders.

1. Write an equation that represents the relationship between V and h. Use 3.14 as an approximation of π.

   $V = 78.5h$

2. Graph this equation, and label the axes.

3. What happens to the volume if you halve the height, h? Where can you see this in the graph? How can you see it algebraically?

   When the height h is halved, the volume is also halved. Sample response: In the graph, it can be seen that the graph shows a proportional relationship, and when the height is $\frac{1}{2}$, the corresponding point on the line is half as high as when the height is 1. Algebraically, if the height is halved from h to $\frac{1}{2}h$, then the volume goes from $78.5 \cdot h$ to $78.5 \cdot \frac{1}{2}h$, or $39.25h$, which is half the original volume.

Are You Ready for More?

Suppose we have a rectangular prism with dimensions 2 units by 3 units by 6 units, and we would like to make a rectangular prism of volume 216 cubic units by stretching *one* of the three dimensions.

1. What are the three ways of doing this? Of these, which gives the prism with the smallest surface area?

   The volume of the starting box is 36 cubic units, so to get to 216 cubic units, we need to increase any one of the dimensions by a factor of 6. This gives the following three possibilities:

   ○ 12 units by 3 units by 6 units, with a surface area of 252 square units
   ○ units by 18 units by 6 units, with a surface area of 312 square units
   ○ units by 3 units by 36 units, with a surface area of 372 square units

2. Repeat this process for a starting rectangular prism with dimensions 2 units by 6 units by 6 units.

   The volume of the starting box is 72 cubic units, so to get to 216 cubic units, we need to increase any one of the dimensions by a factor of 3. This gives the following two possibilities (since scaling either side of length 6 would give a box of the same dimensions):

   ○ 6 units by 6 units by 6 units, with a surface area of 216 square units.
   ○ 2 units by 6 units by 18 units, with a surface area of 504 square units.

3. Can you give some general tips to someone who wants to make a box with a certain volume but wants to save cost on material by having as small a surface area as possible?

   All five boxes we have built have a volume of 216 cubic units, but they have surface areas ranging from 216 to 504 square units. This could make quite a difference for the cost of enclosing 216 cubic units! In general, it seems that there is a lot more surface area when there is a lot of imbalance between the side lengths—when one side is significantly shorter or longer than the other two. The best result possible is to make all three sides the same length.

Activity Synthesis

The goal of this discussion is for students to connect that the relationship discussed in this activity about cylinders and in a previous activity about prisms are both examples of linear relationships.

Begin by asking previously selected groups to share their graphs and equations alongside some of the graphs from the earlier activity with the prism. Ask:
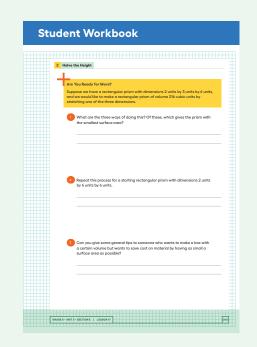
💬 *"Compare the graph in this activity to the graph in the last activity. How are they alike? How are they different?"*

They both look like a line going through the origin. The lines do not have the same slope.

💬 *"Compare the equation in this activity to the equation in the last activity. How are they alike? How are they different?"*

Both equations have the same form of volume equaling a number times a letter. The numbers, 15 and 78.5, are not the same.

💬 *"How can you tell that this is a linear relationship?"*

I can tell it is a linear relationship because it is written as $y = mx + b$, only for both, $b = 0$ since the volume of a shape with a side or height of 0 is 0.

Conclude the discussion by using *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to

💬 *"What happens to the volume if you halve the height, h?"*

In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

💬 *"_____ makes sense, but what do you mean when you say...?"*

"Can you describe that another way?"

"Can you say more about...?"

Close the partner conversations, and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Activity 3: Optional

Figuring Out Cone Dimensions

10 min

Activity Narrative

The purpose of this activity is for students to use representations of functions to explore more about the volume of a cone. This activity differs from the previous ones because students are given a graph that shows the relationship between the volume and height of cones with a fixed radius. They use the graph to reason about the coordinates and their meaning and, along with the formula for the volume of a cone, they calculate the unknown radius.

Launch 👥
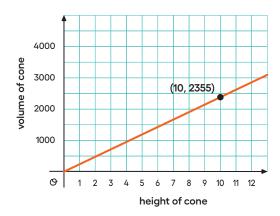
Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time followed by time to discuss the answer and their strategies for the last question with their partner.

Follow with a whole-class discussion.

Student Task Statement

Here is a graph of the relationship between the height and the volume of cones that all have the same radius:

1. What do the coordinates of the labeled point represent?

   A particular cone with a height of 10 units has a volume of 2,355 cubic units.

2. What is the volume of the cone with height 5? With height 30?

   The approximate volume of the cone with a height of 5 units is 1,177.5 cubic units. The approximate volume of the cone with a height of 30 units is 7,065 cubic units.

3. Use the labeled point to find the radius of these cones. Use 3.14 as an approximation for π.

   15 units

   Substituting the values of the labeled point $(10, 2{,}355)$ into the volume equation, we have that $2{,}355 = \frac{1}{3}\pi r^2(10)$, which means $225 \approx r^2$.

4. Write an equation that relates the volume V and height h.

   $V = 75\pi h$

Building on Student Thinking

Students may round π at different points in their work, leading to slightly different answers.

Students can solve the equation $2{,}355 = \frac{1}{3}\pi r^2(10)$ with different series of steps. If they choose to first multiply the numbers on the side with r, they may round the expression $\frac{1}{3}\pi r^2(10)$ to $10.47\, r^2$. This would give $224.9 = r^2$, making r slightly less than 15, though it rounds to 15. Solving by multiplying each side by $3 \cdot \frac{1}{\pi} \cdot \frac{1}{10}$ yields $r^2 = 225$, making r equal 15 exactly (if we accept 3.14 as the value of π). This is an opportunity to discuss how rounding along the way while working toward a solution can introduce imprecision.

Student Workbook

Activity Synthesis

The purpose of this discussion is for students to share the strategies they used to write the equation represented by the graph. Consider asking some of the following questions:

💬 *"How did you write your equation relating V and h?"*

Since the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, I used $\pi = 3.14$, $h = 10$, and $V = 2{,}355$ to get $2{,}355 = \frac{1}{3}\pi r^2(10)$, which means $225 \approx r^2$. So, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 225 \cdot h = 75\pi h$.

💬 *"How do you know that the relationship between volume and height for these cones is a function? How is this shown in the graph? In the equation?"*
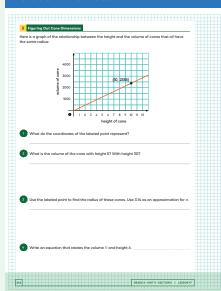
The equation $V = 75\pi h$ is a linear equation, so it must be a function. Each input value h has one and only one output value V.

Student Workbook

💬 *"Identify the independent and dependent variables in this relationship. If they were switched, would we still have a function? Explain how you know."*

The independent variable is the height of the cone h and the dependent variable is the volume of the cone V. If they were switched, it would still be a function since each value of V results in one and only one possible value for h.

Lesson Synthesis

Tell students to imagine a cylindrical water tank with a radius r units and a height h units. Display the following prompts, and ask students to respond to them in writing, encouraging them to include sketches:

💬 *"How can you change a dimension of the water tank so that the volume of the tank increases by a factor of 2?"*

"How can you change a dimension of the water tank so that the volume of the tank increases by a factor of a?"

After quiet work time, invite students to share their responses. If students made a sketch, display them for all to see.
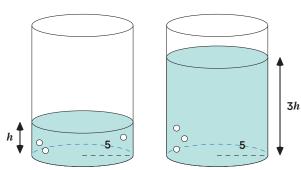
Lesson Summary

Imagine a cylinder with a radius of 5 cm that is being filled with water. As the height of the water increases, the volume of water increases.

We say that the volume of the water in the cylinder, V, depends on the height of the water, h. We can represent this relationship with an equation: $V = \pi \cdot 5^2\, h$, or

$$V = 25\pi h.$$

This equation represents a proportional relationship between the height and the volume. We can use this equation to understand how the volume changes when the height is tripled.

The new volume would be $V = 25\pi(3h) = 75\pi h$, which is precisely 3 times as much as the old volume of $25\pi h$. In general, when one quantity in a proportional relationship changes by a given factor, the other quantity changes by the same factor.

Remember that proportional relationships are examples of linear relationships, which can also be thought of as functions. So in this example, V, the volume of water in the cylinder, is a function of the height, h, of the water.
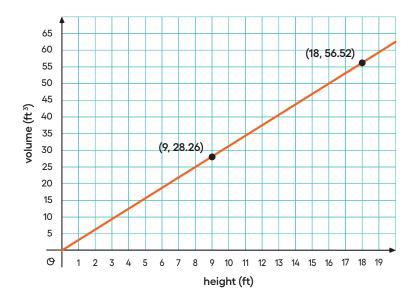
Cool-down

A Missing Radius

5 min

Student Task Statement

Here is a graph of the relationship between the height and volume of some cylinders that all have the same radius, 1 ft. An equation that represents this relationship is $V = \pi h$.

1. Identify and plot another point on the line, and interpret its meaning.

   Sample response: The point (10, 31.4) is on the line. A cylinder with radius 1ft and height 10 ft will have a volume of 31.4 ft³.

2. How can you tell if this relationship is a function?

   Sample response: I know this relationship is a function because the equation is in the form $y = mx + b$ and all linear relationships are functions.

Practice Problems

6 Problems

Problem 1

A cylinder has a volume of 48π cm³ and height h cm. Complete this table for volume of cylinders with the same radius but different heights.

height (cm)	volume (cm³)
h	48π
$2h$	96π
$5h$	240π
$\frac{h}{2}$	24π
$\frac{h}{5}$	$\frac{48}{5}\pi$

Problem 2

A cylinder has a radius of 3 cm and a height of 5 cm.

a. What is the volume of the cylinder?

45π cm³

b. What is the volume of the cylinder when its height is tripled?

135π cm³

c. What is the volume of the cylinder when its height is halved?

$\frac{45}{2}\pi$ cm³

Problem 3

A graduated cylinder that is 24 cm tall can hold 1 L of water. Recall that 1 liter (L) is equal to 1,000 milliliters (ml), and that 1 liter (L) is equal to 1,000 cm³.

a. What is the radius of the cylinder? Explain or show your reasoning.

3.64 cm

Sample reasoning: $\frac{1,000}{24\pi} \approx 13.26$, and $3.64^2 \approx 13.26$.

b. What is the height of the 500 ml mark?

12 cm

c. What is the height of the 250 ml mark?

6 cm

Problem 4

from Unit 5, Lesson 16

An ice cream shop offers two ice cream cones. The waffle cone holds 12 ounces and is 5 inches tall. The sugar cone also holds 12 ounces yet is 8 inches tall. Which cone has a larger radius? Explain your reasoning.

The waffle cone

Sample reasoning: Since its height is smaller, the radius must be larger in order to have the same volume as the sugar cone.

Problem 5

from Unit 5, Lesson 15

A 6 oz paper cup is shaped like a cone with a diameter of 4 inches. How many ounces of water will a plastic cylindrical cup with a diameter of 4 inches hold if it is the same height as the paper cup? Explain your reasoning.

18 oz

Sample reasoning: Since the cups are the same height and have the same radius, the cylindrical cup must have 3 times the volume of the conical cup.

Problem 6

from Unit 5, Lesson 9

Lin's smart phone was fully charged when she started school at 8:00 a.m. At 9:20 a.m., it was 90% charged, and at noon, it was 72% charged.

a. When do you think her battery will be 0% charged? Explain your reasoning.

Approximately 9:20 p.m.

Sample reasoning: Since 10% of battery was lost in 80 minutes, it would take 800 minutes to lose all of the battery. This would give a prediction of 13 hours and 20 minutes after the start time, so approximately 9:20 p.m.

b. Is battery life a function of time? If yes, is it a linear function? Explain your reasoning.

Battery remaining is a function of time, but it is not a linear function. Sample reasoning: It took 80 minutes for her phone to lose the first 10% of the battery and then 160 minutes for her phone to lose another 18%. If the function were linear, it would lose exactly twice as much in 160 minutes as it did in 80 minutes.

Student Workbook

Student Workbook