Lesson 3

Reasoning to Find Area

Goals

- Compare and contrast (orally) different strategies for calculating the area of a polygon.
- Find the area of a polygon by decomposing, rearranging, subtracting or enclosing shapes, and explain (orally and in writing) the solution method.
- Include appropriate units (in spoken and written language) when stating the area of a polygon.

Learning Target

I can use different reasoning strategies to find the area of shapes.

Lesson Narrative

This lesson prompts students to find areas of figures—first on a grid and then without a grid. This work reiterates the two key principles about area: that two figures that match exactly when placed one on top of the other have the same area, and that the area of a figure is the sum of the areas of the non-overlapping pieces that compose it.

Students continue to use strategies from earlier explorations to find area, namely:

- Decompose a figure into shapes whose areas they can calculate.
- Decompose and rearrange it into shapes whose areas they can calculate.

The given figures in this lesson allow students to see that they can also:

- consider a figure as a familiar shape with one or more missing pieces, calculate the area of the shape, and then subtract the areas of the missing pieces
- enclose a figure with a shape whose area they can calculate and subtract the area of extra pieces created by the enclosure

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices

Required Materials

Materials to Gather

- Geometry toolkits: Lesson
- Geometry toolkits: Warm-up, Activity 1, Activity 2

Materials to Copy

- Comparing Regions Handout (1 copy for every 1 student: Warm-up

Required Preparation

Warm-up:
Prepare several copies of the pair of figures on the blackline master, in case students propose cutting them out to compare the areas.

Lesson:
Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down

Lesson 3

Reasoning to Find Area

Lesson Narrative (continued)

For now, rectangles are the only shapes whose areas students know how to calculate, but the strategies will become more powerful as students' repertoires grow.

As students consider strategies for finding areas and use them, they practice looking for and making use of structure. In explaining their thinking, students practice constructing logical arguments.

A note about notation:

Starting in this lesson, consider using the "dot" notation instead of the "cross" notation when recording students' solutions. Explain that the · symbol and the × symbol both represent multiplication. Doing so familiarizes students with the use of the notation before they see it in student-facing materials.

Student Learning Goal

Let's decompose and rearrange shapes to find their areas.

Warm-up

Comparing Regions

5 min

Activity Narrative

This activity prompts students to use reasoning strategies from earlier lessons to compare the areas of two figures. It is also an opportunity to use (or introduce) tracing paper as a way to illustrate *decomposing* and *rearranging* a figure.

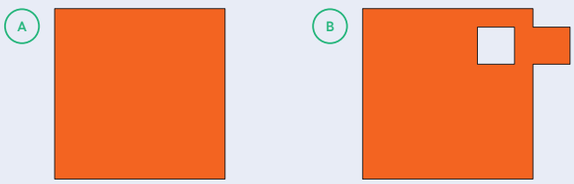
During the activity, look for students who are able to explain or show how they know that the areas are equal. Some students may simply look at the figures and say, with no justification, that they have the same area. Urge them to think of a way to show that their conclusion is true.

Launch

Give students access to their geometry toolkits, and allow for 2 minutes of quiet think time. Ask them to be ready to support their answer, and remind them to use the tools at their disposal. Have copies of the blackline master ready for students who propose cutting the figures out for comparison or as a way to differentiate the activity.

Student Task Statement

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.



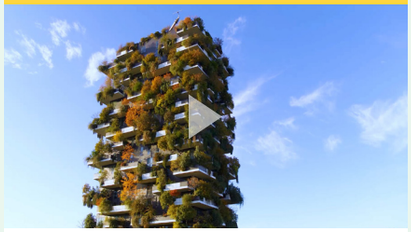
The areas are equal.

Sample reasoning:

- Measuring: Measure the side lengths of the small square-shaped hole and the small shaded square on the side of Figure B. They have the same side lengths, so their areas are equal. This means the square on the side fills the hole on the inside. Measure the side lengths of the large shaded square in Figure A and then in Figure B. They have the same side lengths, so their areas are equal.
- Using scissors: Cut off the little square on the side of Figure B, and use it to fill the hole inside Figure B. The result is a square that matches up exactly with Figure A.
- Using tracing paper: Trace the boundary of the little square on the side of Figure B and move the tracing paper over the unshaded hole. Doing this shows that the little shaded square is exactly the same size as the hole. Moving that little shaded square to fill the unshaded hole creates a big shaded square. If the boundary of that big shaded square is traced and the drawing is placed over Figure A, it would up exactly with Figure A.

Inspire Math

Greening the City video



Go Online

Before the lesson, show this video to reinforce the real-world connection.

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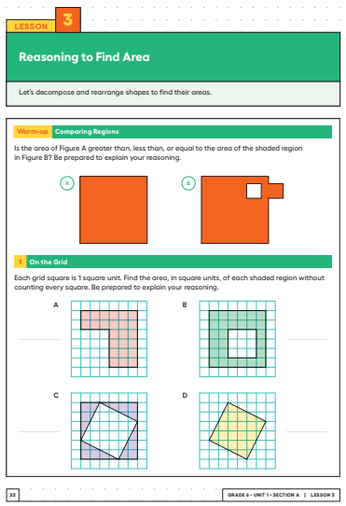
Building on Student Thinking

Students may interpret the area of Figure B as the entire region inside the outer boundary including the unfilled square. Clarify that we want to compare the areas of only the shaded parts of Figure B and Figure A.

Student Workbook



Instructional Routines

5 Practices

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Activity Synthesis

Start the discussion by asking students to indicate which of the three possible responses—area of Figure A is greater, area of Figure B is greater, or the areas are equal—they choose.

Select previously identified students to share their explanations. If no student mentioned using tracing paper, demonstrate the following:

- *Decomposing and rearranging Figure B*: Place a piece of tracing paper over Figure B. Draw the boundary of the small side square, making a dotted auxiliary line to show its separation from the large square. Move the tracing paper so that the boundary of the small square matches up exactly with the boundary of the square-shaped hole in Figure B. Draw the boundary of the large square. Explain that the small square matches up exactly with the hole, so we know the small, shaded square and the hole have equal area.
- *Matching the two figures*: Move the tracing paper over Figure A so that the boundary of the rearranged Figure B matches up exactly with that of Figure A. Say,

💬 *"When two figures that are overlaid one on top of another match up exactly, their areas are equal."*

Highlight the strategies and principles that are central to this unit.
Tell students,

💬 *"We just decomposed and rearranged Figure B so that it matches up exactly with Figure A. When two figures that are overlaid one on top of another match up exactly, we can say that their areas are equal."*

Activity 1

On the Grid

20 min

Activity Narrative

This activity elicits different strategies for reasoning about finding the area of regions:

- decomposing
- decomposing and rearranging
- subtracting
- enclosing and subtracting

The figures are on a grid, which reinforces the meaning of area and supports students in quantifying the square units. Students may start by counting squares, as they had done in earlier grades, but the figures have been chosen to encourage other approaches.

Monitor for students who use at least two different strategies for finding the area of each figure (one strategy as shown in the student responses and at least one other). Here are some approaches students might take for each figure:

- Figure A can be easily decomposed into rectangles.
- Figure B can be decomposed into rectangles. Or, more efficiently, it can be seen as a square with a missing piece. The area of the unshaded inner square can be subtracted from the area of the larger square.

- Figure C can also be seen as having a missing piece, but subtracting the area of the unshaded shape does not work because the side lengths of the inner square are unknown. Instead, the shaded triangles can be decomposed and rearranged into rectangles.
- Figure D can be decomposed and rearranged into rectangles. It can also be viewed as the inner square of Figure C.

Launch

Tell students that they will find the areas of various figures on a grid. To encourage students to use a more grade-appropriate strategy for finding areas, first show them a strategy from earlier grades. As a class, find the area of Figure A by counting the squares aloud, one by one. Confirm that there are 24 square units. Then ask students to think about other ways to find the area of Figure A (or other figures) besides counting each square.

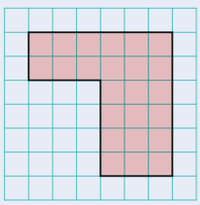
Arrange students in groups of 2. Ask one partner to start with Figures A and C, and the other with B and D. Give students 4–5 minutes of quiet think time and access to their geometry toolkits. Then give them a few minutes to share their responses with their partner. Emphasize that as one partner explains, the other should listen carefully and see if they agree or disagree with the answer and explanations.

For each figure, select at least 2 students with different strategies, such as those described in the Activity Narrative, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

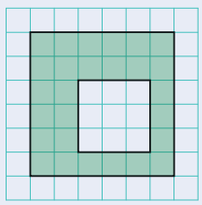
Student Task Statement

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

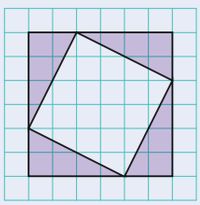
A



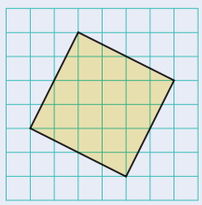
B



C



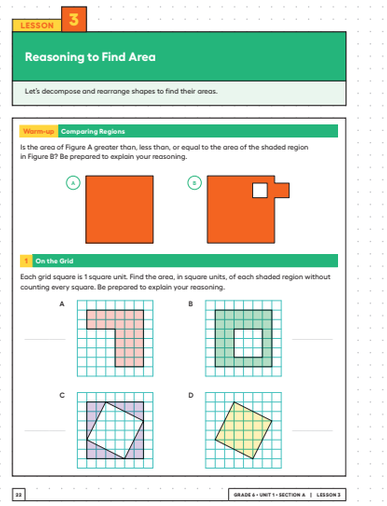
D



Building on Student Thinking

Some students may count both complete and partial grid squares instead of looking for ways to decompose and rearrange larger shapes. Ask them if they can think of a way to find the area by decomposing and rearranging larger pieces. The discussion at the end, during which everyone sees a variety of strategies, is especially important for these students.

Student Workbook



LESSON 3

Reasoning to Find Area

Let's decompose and rearrange shapes to find their areas.

Warm-up Comparing Regions

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.

A B

1 On the Grid

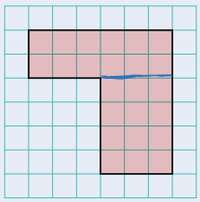
Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

A B C D

A: 24 square units

Sample reasoning:

- Decompose the figure into rectangles. One way is shown here. $(2 \cdot 6) + (4 \cdot 3) = 12$



B: 27 square units

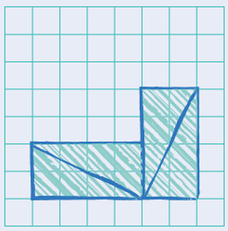
Sample reasoning:

- Decompose the figure into four rectangles.
- Subtract the area of the inner square from the larger square. $(6 \cdot 6) - (3 \cdot 3) = 27$

C: 16 square units

Sample reasoning:

- Decompose into right triangles and rearrange into rectangles. $(2 \cdot 4) + (4 \cdot 2) = 16$



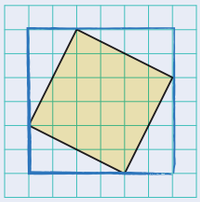
- Find the area of Figure D first, and then subtract it from the 6-by-6 square.

D: 20 square units

Sample reasoning:

- Decompose the shaded square into four right triangles and a 2-by-2 square. (See *Student Response for Are You Ready for More?*) Rearrange the right triangles into two rectangles that are each 2 units by 4 units, with a combined area of 16 square units. Adding the area of the small square (4 square units) gives a total of 20 square units.

- Notice that the shaded square is the inner square of Figure C, enclose it in a square as in Figure C, and subtract the areas of the four right triangles (or the area of Figure C) from the area of the enclosing square.



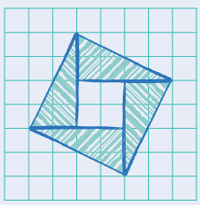
$(6 \cdot 6) - 16 = 20$

Are You Ready for More?

Rearrange the shaded triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

Sample response:

The triangles fit inside the square, with a smaller 2-by-2 square in the center.



Activity Synthesis

The purpose of this discussion is to highlight a few key strategies for finding area. Invite previously selected students to share their work. Before sharing begins, ask students to notice similarities and differences in the strategies and be ready to explain them. Sequence the discussion of the strategies for each figure as shown here, starting with decomposing.

- Figure A:
  - decomposing
  - enclosing and subtracting
- Figure B:
  - decomposing
  - subtracting
- Figure C:
  - decomposing and rearranging
  - subtracting (after finding the area of the inner square, which is Figure D)
- Figure D:
  - decomposing and rearranging
  - enclosing and subtracting

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, create a display that includes multiple copies of each figure. As students describe their strategies, use color and annotation to scribe their thinking so that it is visible for all students. Label each figure with the strategy described (decomposing, rearranging, subtracting, or enclosing).
Supports accessibility for: Visual-Spatial Processing

Instructional Routines

MLR8: Discussion Supports

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As students share their strategies, record and display their moves on each figure for all to see (or ask the presenters to do so).

After each student shares, name the strategy and ask if anyone else reasoned the same way. If one of these strategies does not appear in students' work, illustrate it for the class.

Connect the different responses to the learning goals by asking questions such as:

- *"How are the strategies used to find the areas of figures A and D alike?"*

  We can find the areas by decomposing the figures.

- *"How are the strategies used to find the areas of figures A and D different?"*

  After decomposing A, we can find the area of each piece as they are. After decomposing D, we need to rearrange the pieces before finding their areas.

- *"After decomposing figures C and D, what shapes were the pieces rearranged into?"*

  rectangles

- *"Why might that be?"*

  We know how to find the area of rectangles.

Highlight that there are multiple ways to find the area of each figure and that decomposing, with or without rearranging, is generally a useful strategy.

Activity 2

Off the Grid

10 min

Activity Narrative

In this activity, students apply the strategies they have learned to find the areas of figures, but now the figures are *not* on a grid.

- Figure E can easily be decomposed and rearranged into a rectangle.
- Figure F can be decomposed and rearranged into rectangles (as was done with Figure C in the previous activity). Students cannot use the strategy of subtracting the area of the inner square from that of the outer square because the side lengths of the inner square are unknown.

As students discuss their approaches in groups, support students in naming the strategies and by asking clarifying questions. Monitor for students who observed that the same strategies for reasoning about area can be applied both on and off the grid.

Launch

Tell students that they will now find areas of figures that are not on a grid. Give students access to their geometry toolkits. Remind students that if the side lengths of a rectangle are given in a particular unit (such as meters), then the area is given in square units (such as square meters).

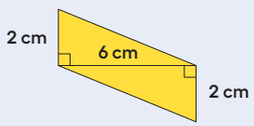
Allow for 3–4 minutes of quiet time to find the areas of the two figures. Then arrange students into groups of 4 and give them 2–3 minutes to compare their responses and strategies. Ask students to discuss the following questions, displayed for all to see:

- *"What units did you use for each area? Why?"*
- *"How are your strategies the same? How are they different?"*
- *"Which strategies are similar to the ones you used when finding the areas of figures on a grid?"*

Student Task Statement

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

E



F

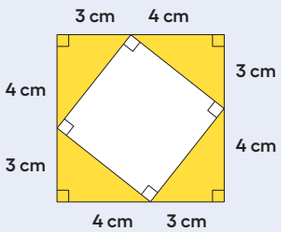


Figure E: 12 square centimeters.

Sample reasoning:

Decompose the two triangles and rearrange them to form a rectangle with side lengths of 6 centimeters and 2 centimeters.

Figure F: 24 square centimeters.

Sample reasoning:

Decompose the triangles and rearrange them to form two rectangles with side lengths of 4 centimeters and 3 centimeters.

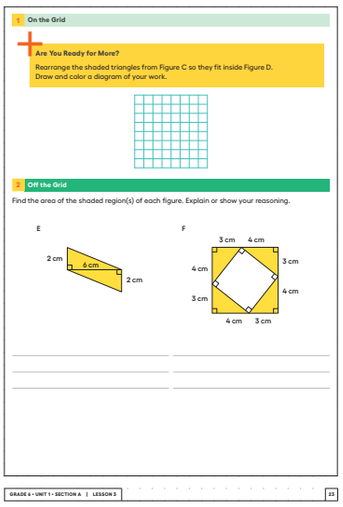
Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. Invite students to repeat their reasoning using mathematical language: "Can you say that again, using the words 'compose,' 'decompose,' or 'rearrange' in your explanation?"
Advances: Speaking, Representing

Building on Student Thinking

For Figure F, students may estimate the side lengths of the inner square so that its area could be subtracted from that of the outer square. They may struggle to see how the triangles could be rearranged. Suggest that they use tracing paper to help them in their thinking.
Students might not be familiar with the symbols that indicate right angles and might think these symbols indicate square units. Remind them that those symbols indicate 90 degree angles.

Student Workbook


1 On the Grid

Are You Ready for More?

Rearrange the shaded triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

2 Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

E F

Activity Synthesis

Briefly discuss the question,

- *"Which strategies are similar to the ones you used in an earlier activity to find the areas of figures on a grid?"*

Select 1–2 previously identified students to share their observations—that in both activities they decomposed and rearranged figures to find their area. (Some students may have also enclosed Figure A and subtracted the areas of unshaded triangles after rearranging them into a rectangle.)

Emphasize that these strategies for finding area can be used whether the measurements are indicated by a grid or given directly (without a grid).

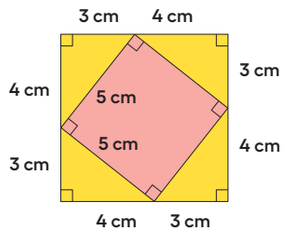
If students did not include appropriate area units for figures E and F, remind them that the side lengths of these figures are given in centimeters, so their areas are in square centimeters.

If time permits, ask students:

- *"For Figure F, can we use the strategy of subtracting the area of the inner square to find the area of the shaded region? Why or why not?"*

  No, because we don't know the side lengths of the inner square or its area.

- Display or sketch Figure F with the sides of the inner square labeled "5 cm."



- *"Can we use subtraction to find the area of the shaded region now? Why or why not?"*

  Yes, because we know the side lengths of the outer and inner squares. We can find the area of the smaller square, 25 square centimeters, and subtract it from the area of the larger square, 49 square centimeters, which gives 24 square centimeters.

Lesson Synthesis

This lesson was all about identifying strategies for finding area and applying them to various figures. Students reasoned about the area of a figure on and off a grid by:

- decomposing it into familiar shapes
- decomposing it and rearranging the pieces into familiar shapes
- considering it as a shape with missing pieces and subtracting the areas of the missing pieces from the area of the shape
- enclosing a figure with a shape whose area can be calculated and subtracting the area of extra pieces created by the enclosure

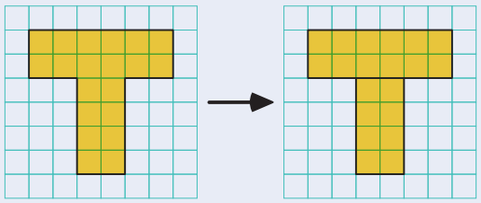
Ask students to go back through this lesson's activities and find problems in which each of these strategies was used.

Tell students we will have many more opportunities to use these strategies in upcoming lessons.

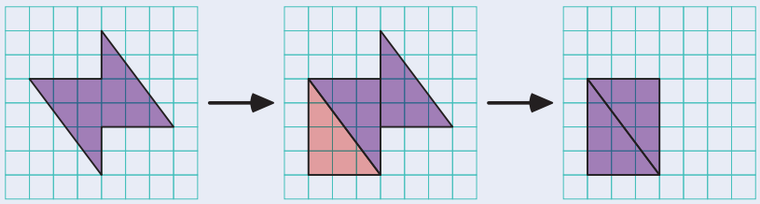
Lesson Summary

There are different strategies we can use to find the area of a region. We can:

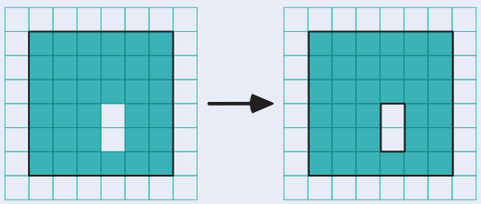
- Decompose it into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.



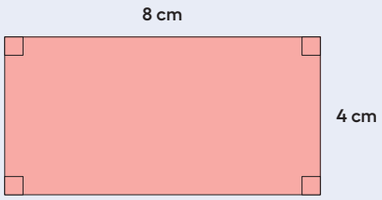
- Decompose it and rearrange the pieces into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.



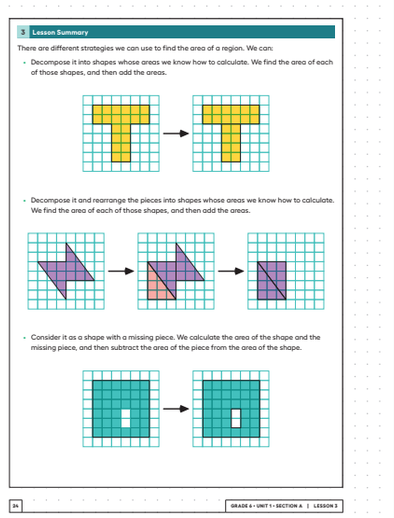
- Consider it as a shape with a missing piece. We calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a figure is always measured in square units. When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters. For example, the area of this rectangle is 32 square centimeters.



Student Workbook


3 Lesson Summary

There are different strategies we can use to find the area of a region. We can:

- Decompose it into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.

- Decompose it and rearrange the pieces into shapes whose areas we know how to calculate. We find the area of each of those shapes, and then add the areas.

- Consider it as a shape with a missing piece. We calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Maritime Flag

5 min

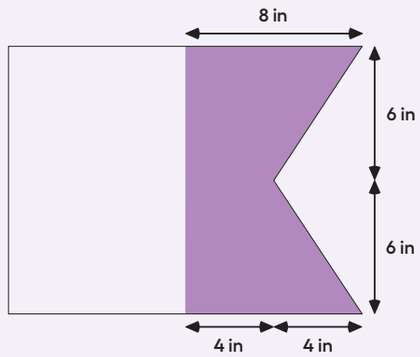
Launch

This task does not explicitly ask students to state area units because one purpose of the task is to assess if students understand what units are appropriate given the information presented.

Give students access to their geometry toolkits.

Student Task Statement

A maritime flag is shown. What is the area of the shaded part of the flag?
72 square inches



Explain or show your reasoning.

Sample reasoning:

If we draw a line down the middle of the shaded area, we would have a 4 inch-by-12 inch rectangle on the left and two right triangles. The 4-by-12 rectangle has an area of 48 square inches. The two triangles on the right can be composed into a 4 inch-by-6 inch rectangle, so their combined area is 24 square inches.

Lesson 3

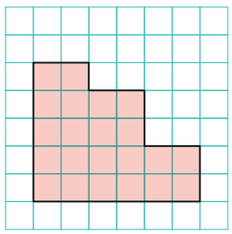
Practice Problems

5 Problems

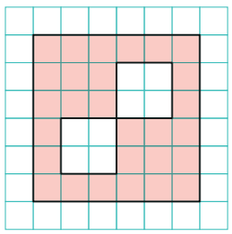
Problem 1

Find the area of each shaded region. Show your reasoning.

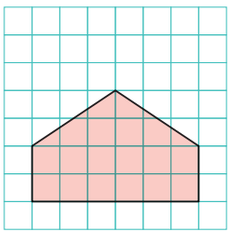
A



B



C



A: 22 square units

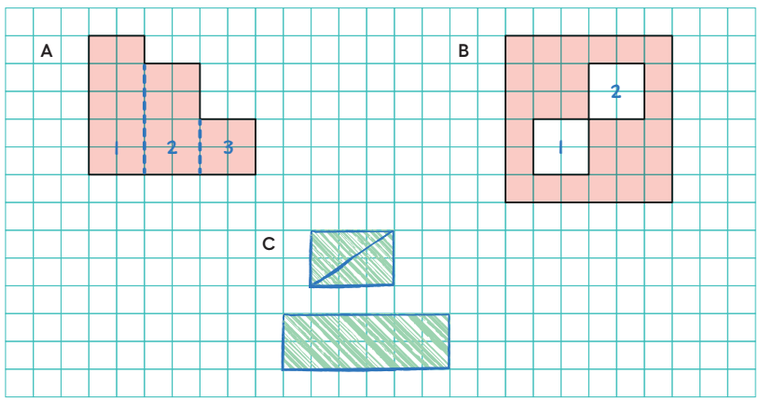
Sample reasoning: The shaded region can be partitioned into rectangles. One way to do this is shown. Rectangle 1 is 2 units by 5 units, so its area is 10 square units. Rectangle 2 is 2 units by 4 units, so its area is 8 square units. The area of Rectangle 3 is 4 square units. The total shaded area is 22 square units, since 10 + 8 + 4 = 22

B: 28 square units

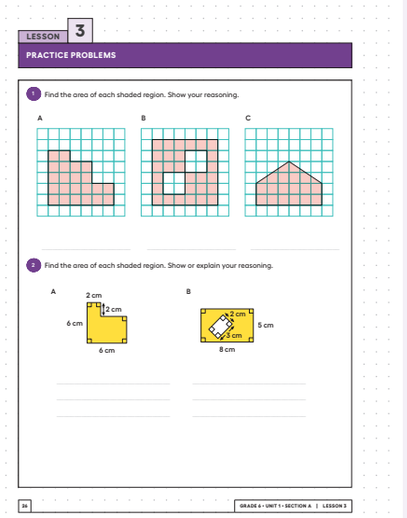
Sample reasoning: The outer square is 6 units by 6 units, so its area is 36 square units. There are two smaller squares inside. Square 1 and Square 2 have been removed. Each small square has an area of 4 square units. To get the shaded area, compute 36 – 4 – 4, which equals 28

C: 18 square units

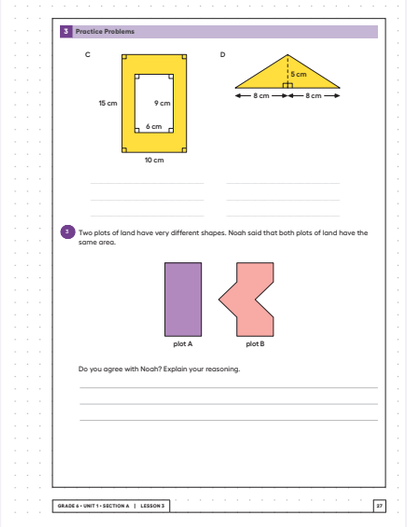
Sample reasoning: The region can be decomposed to form a 2-by-6 rectangle and two right triangles that when rearranged form a 2-by-3 rectangle. $(2 \cdot 6) + (2 \cdot 3) = 18$



Student Workbook



Student Workbook


3 Practice Problems

C

15 cm
9 cm
6 cm
10 cm

D

5 cm
8 cm
8 cm

3 Two plots of land have very different shapes. Noah said that both plots of land have the same area.

plot A
plot B

Do you agree with Noah? Explain your reasoning.

GRADE 6 • UNIT 1 • SECTION A | LESSON 3
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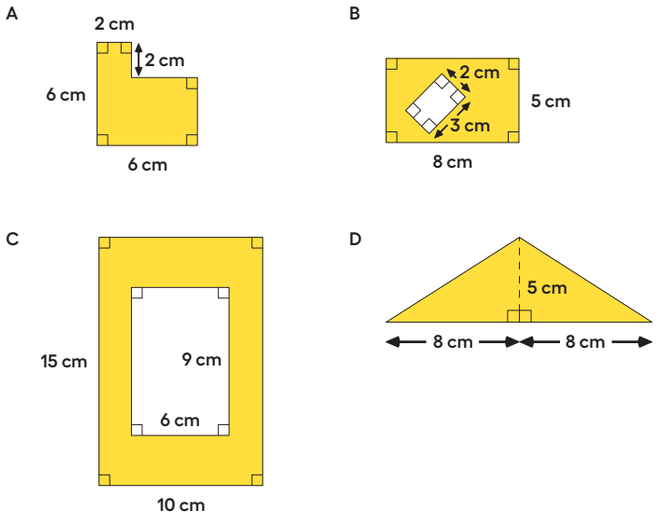
Problem 2

Find the area of each shaded region. Show or explain your reasoning.



A: 28 sq cm

Sample reasoning: A horizontal cut partitions the region into a 2 cm-by-2 cm square (4 sq cm) and a 4 cm-by-6 cm rectangle (24 sq cm).

B: 34 sq cm

Sample reasoning: The outer rectangle has an area of 40 sq cm, while the inner rectangle has an area of 6 sq cm. 40 – 6 = 34

C: 96 sq cm

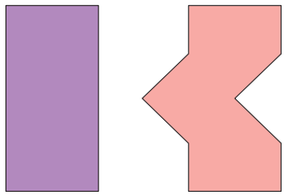
Sample reasoning: The outer rectangle has an area of 150 sq cm, while the inner rectangle has an area of 54 sq cm. 150 – 54 = 96

D: 40 sq cm

Sample reasoning: The two right triangles can be put together to make a 5 cm-by-8 cm rectangle.

Problem 3

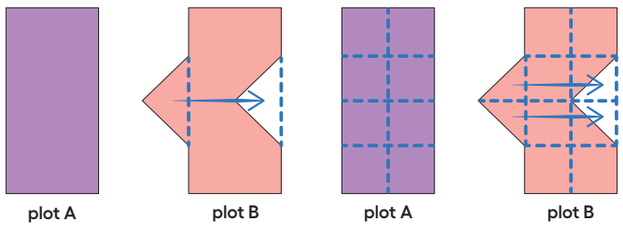
Two plots of land have very different shapes. Noah said that both plots of land have the same area.



Do you agree with Noah? Explain your reasoning.

Agree

Sample reasoning: The triangular shape that juts out from the left side of plot B can be cut off and moved to the right side of plot B. The resulting shape is a rectangle that matches exactly with the shape of plot A. We can use tracing paper to verify. Sample diagrams:



Problem 4

from Unit 1, Lesson 2

A homeowner is deciding on one size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. Tell whether or not you agree with each statement about the tiles. Explain your reasoning.

a. Regardless of the size she chooses, she will need the same number of tiles.

Disagree

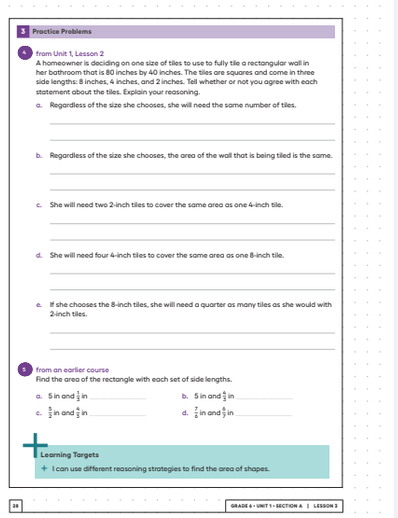
Sample reasoning: She will need fewer of the larger tiles and more of the smaller tiles.

b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.

Agree

Sample reasoning: The region being covered does not change regardless of what tiles she chooses.

Student Workbook


3 Practice Problems

4 from Unit 1, Lesson 2
A homeowner is deciding on one size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. Tell whether or not you agree with each statement about the tiles. Explain your reasoning.
a. Regardless of the size she chooses, she will need the same number of tiles.
b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.
c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.
d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.
e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

5 from an earlier course
Find the area of the rectangle with each set of side lengths.
a. 5 in and $\frac{1}{3}$ in
b. 5 in and $\frac{3}{3}$ in
c. $\frac{5}{3}$ in and $\frac{4}{3}$ in
d. $\frac{7}{6}$ in and $\frac{6}{6}$ in

Learning Targets
+ I can use different reasoning strategies to find the area of shapes.

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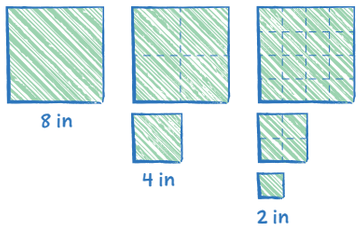
c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.

Disagree

Sample reasoning: She will need four 2-inch tiles to cover the same area as one 4-inch tile.



d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.

Agree

Sample reasoning: Two rows of two 4-inch tiles cover the same area as one 8-inch tile.

e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

Disagree

Sample reasoning: Because one 8-inch tile covers the same area as four 4-inch tiles, she vwill need $\frac{1}{16}$ as many 8-inch tiles as she would with 2-inch tiles.

Problem 5

from an earlier course

Find the area of the rectangle with each set of side lengths.

a. 5 in and $\frac{1}{3}$ in

$\frac{5}{3}$ square inches

b. 5 in and $\frac{4}{3}$ in

$\frac{20}{3}$ square inches

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

$\frac{10}{3}$ square inches

d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

1 square inch