Using Graphs to Compare Relationships

Goals

- Create and interpret graphs that show two different proportional relationships on the same axes.
- Generalize (orally and in writing) that when two different proportional relationships are graphed on the same axes, the steeper line has the greater constant of proportionality.

Learning Targets

- · I can compare two, related proportional relationships based on their graphs.
- I know that the steeper graph of two proportional relationships has a larger constant of proportionality.

Lesson Narrative

In this lesson, students examine graphs that have multiple proportional relationships presented on the same set of axes. They compare and interpret the steepness of each graph in terms of the context. First, students use distance-versus-time graphs to decide which person from a group is going the fastest. Then, they work with graphs where the scale is not specified on each axis and realize that they can still use the structure of such graphs to compare rates.

Student Learning Goal

Let's graph more than one relationship on the same grid.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)
- MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

- 5 Practices
- Math Talk
- MLR6: Three Reads
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- · Colored pencils: Activity 1
- · Rulers: Activity 1

Activity 1:

Have available the information from the activity "Tyler's Walk" from the previous lesson.

For the digital version of the activity, acquire devices that can run the applet.

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline



Warm-up

15

Activity 1

15

Activity 2

10

Lesson Synthesis

Assessment

Cool-down

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Building on Student Thinking

Some students may think that the quotient must be expressed as a decimal or mixed number for the problem to be considered finished. Explain that an improper fraction is a valid way of expressing a value and that in some cases this format may be more useful.



Warm-up

Math Talk: More Division



Activity Narrative

This *Math Talk* focuses on various ways to express the result of a division problem. It encourages students to think about the meaning of a remainder and to rely on what they know about equivalent fractions to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students calculate and compare constants of proportionality.

Monitor for different ways students deal with the remainders, such as:

- Representing the quotient as a decimal.
- · Representing the quotient as a fraction or mixed number.

When students use examples to generalize that $a \div b = \frac{a}{b}$, they are using repeated reasoning.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A.3 ÷ 6

 $\frac{1}{2}$ (or equivalent)

Sample reasoning: If I have 3 wholes and I divide them into 6 groups, each group is $\frac{1}{2}$.

B. $4 \div 5$

 $\frac{4}{5}$ (or equivalent)

Sample reasoning:

- 40 ÷ 5 = 8, and 4 is one-tenth of 40, so 4 ÷ 5 = 0.8. This is equivalent to $\frac{8}{10}$ or $\frac{4}{5}$.
- $4 \div 5 = 0R4$. Since the divisor is 5, the remainder equals $\frac{4}{5}$.

C. $5 \div 4$

 $\frac{5}{4}$ (or equivalent)

Sample reasoning:

- 5 ÷ 4 = IRI. Since the divisor is 4, the remainder equals $\frac{1}{4}$. I $\frac{1}{4}$ is equivalent to $\frac{5}{4}$.
- Since the answer to $4 \div 5$ was $\frac{4}{5}$, then the answer to $5 \div 4$ should be $\frac{5}{4}$. The 4 and the 5 swapped places.

D. $10 \div 6$

 $\frac{5}{3}$ (or equivalent)

Sample reasoning: $10 \div 6 = \frac{10}{6}$, which is equivalent to $\frac{5}{3}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate _____'s reasoning in a different way?"
 - "Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway is that the quotient of $a \div b$ can be expressed as $\frac{a}{b}$ or as another fraction that is equivalent to $\frac{a}{b}$. To help highlight this point, ask students if they can think of other ways to express each quotient before moving to the next problem.

Activity 1

Race to the Bumper Cars

15 min

Activity Narrative

There is a digital version of this activity.

This activity builds on "Tyler's Walk" from the previous lesson, as students compare the time-distance relationships for three more people who travel from the ticket booth to the bumper cars. Students create tables and graphs that represent each relationship. They relate the constant of proportionality for each relationship to the person's speed in meters per second.

Students also identify that one of the relationships is not proportional. Because Mai did not start walking at the same time as the others, the line representing her walk does not go through (0, 0).

In the digital version of the activity, students use an applet to graph multiple relationships on the same coordinate plane. The applet allows students to add, remove, adjust, and label points and lines. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis. If students don't have individual access, displaying the applet for all to see would be helpful during the *Synthesis*.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads.

Keep books or devices closed. Display only the problem stem and descriptions of the three journeys, without revealing the questions. "We are going to read this situation 3 times."

- After the 1st read: "Tell your partner what this situation is about."
- After the 2nd read: "List the quantities. What can be counted or measured?"
- For the 3rd read: Reveal and read the question "Which person is moving the most quickly?" Ask, "What are some ways we might get started on this?"

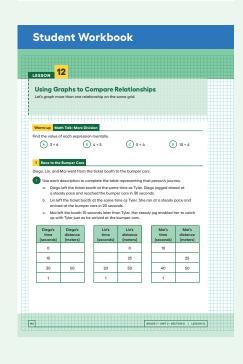
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, use a different color for each person to highlight the connection between the table, graph, and constant of proportionality.

Supports accessibility for: Visual-Spatial Processing



Launch

Arrange students in groups of 2–3. Provide access to colored pencils and rulers.

Tell students that this activity is tied to the activity titled "Tyler's Walk" from the previous lesson. In that activity, we saw that it took Tyler 40 seconds to walk 50 meters from the ticket booth to the bumper cars.

Student Task Statement

Diego, Lin, and Mai went from the ticket booth to the bumper cars.

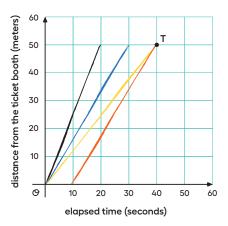
- **1.** Use each description to complete the table representing that person's journey.
 - **a.** Diego left the ticket booth at the same time as Tyler. Diego jogged ahead at a steady pace and reached the bumper cars in 30 seconds.
 - **b.** Lin left the ticket booth at the same time as Tyler. She ran at a steady pace and arrived at the bumper cars in 20 seconds.
 - **c.** Mai left the booth 10 seconds later than Tyler. Her steady jog enabled her to catch up with Tyler just as he arrived at the bumper cars.

Diego's time (seconds)	Diego's distance (meters)
0	0
15	25
30	50
1	<u>5</u> 3

Lin's time (seconds)	Lin's distance (meters)
0	0
10	25
20	50
1	2.5

Mai's time (seconds)	Mai's distance (meters)
10	0
25	25
40	50
1	0

2. Using a different color for each person, draw a graph of all four people's journeys (including Tyler's from the other day).



3. Which person is moving the most quickly?

Lin is moving most quickly.

How is that reflected in the graph?

Sample reasoning:

- · At any given time between 0 and 20 seconds, she has traveled the farthest.
- For any given distance between 0 and 50 meters, it takes her the least amount of time to get there.
- She is traveling at $2\frac{1}{2}$ meters per second, while Diego is traveling at $1\frac{2}{3}$ meters per second and Tyler at $1\frac{1}{4}$ meters per second. You can see this on the graph by looking at the points with x-coordinate I.

Are You Ready for More?

Write equations to represent the relationship between time and distance for each person.

Let t represent elapsed time in seconds and d represent distance from the ticket booth in meters. Lin: d = 2.5t or equivalent. Diego: $d = \frac{5}{3}t$ or equivalent. Tyler: d = 1.25t or equivalent. Mai: $d = \frac{5}{3}(t - 10)$.

Activity Synthesis

The goal of this discussion is to highlight how the constant of proportionality for each relationship is illustrated on the graph and to connect this to each person's speed in meters per second. First, invite students to share their answer to the last question, about who was moving the most quickly. (Lin, because she got to the bumper cars in the least amount of time. She traveled the most meters per second.)

Next, help students make connections between how the constant of proportionality is shown in the different representations. Consider asking:

"For each graph that shows a proportional relationship, what is the constant of proportionality?"

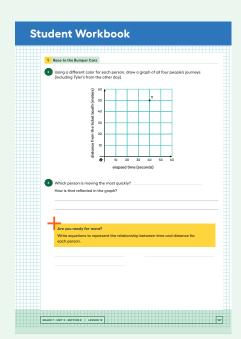
Tyler: 1.25, Diego: $\frac{5}{3}$, Lin: 2.5

"Where can we see the constants of proportionality in the tables?"

the values in the second column that correspond to the value of I in the first column

Building on Student Thinking

Some students may expect the graphs to intersect because everyone arrives at the same location. Point out that they did not all arrive there at the same time. Because all the characters traveled the same distance from the ticket booth and no further, the endpoints of their graphs lie on the same horizontal line, y = 50, that is, they have the same y-coordinate. The points will vary in position from right to left depending on the number of seconds after Tyler left the ticket booth that it took each person to arrive at the bumper cars.



Instructional Routines

5 Practices ilclass.com/r/10690701

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- "Where can we see the constants of proportionality on the graphs?"
 the y-coordinate of points where the x-coordinate is I
- "Which graph does not represent a proportional relationship?"
 Mai's graph does not pass through the origin, so it does not represent a proportional relationship. That is, the distance she traveled is not proportional to the time elapsed since Tyler left the ticket booth.
- "How could we represent the three proportional relationships with equations?"

Tyler: $d = \frac{5}{4}t$, Diego: $d = \frac{5}{3}t$, Lin: $d = \frac{5}{2}t$, or equivalent

 \bigcirc "An equation that represents Mai's journey is $d = \frac{5}{3}$ (t – 10). What do you notice about this equation compared to the other three?"

This equation is not of the form y = kx, while the other three equations are.

Activity 2

Space Rocks and the Price of Rope



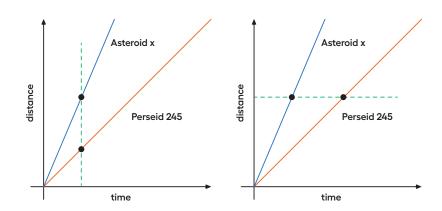
Activity Narrative

There is a digital version of this activity.

In this activity students continue to interpret graphs that show more than one proportional relationship on the same set of axes. To increase the need for abstract reasoning, the graphs in this activity do not include numerical labels. Students can reason abstractly by picking an arbitrary time and comparing the corresponding distances or by picking an arbitrary distance and comparing the corresponding times. The goal is for students to realize that when two proportional relationships are shown on the same set of axes, the steeper graph has the greater constant of proportionality, regardless of how the axes are scaled. As students notice these connections, they are making use of structure.

As students work on the first problem, monitor for groups who use these different approaches:

• Compare the distances each object travels in the same amount of time, such as in the graph with the vertical dashed line



- Compare the times it takes each object to travel the same distance, such as in the graph with the horizontal dashed line
- Recognize that the steeper line has the greater constant of proportionality and interpreting what this means in context of the situation

Plan to have students present in this order to support moving them from more concrete to more abstract reasoning.

In the digital version of the activity, students use an applet to interpret and compare two proportional relationships. The applet allows students to watch the graphs animated over time, as well as add a vertical or horizontal line to facilitate comparing the graphs at a specific time or distance, respectively. This activity works best when each student has access to the applet because students will benefit from seeing the relationship in a dynamic way. If students don't have individual access, displaying the applet for all to see would be helpful during the *Synthesis*.

Launch

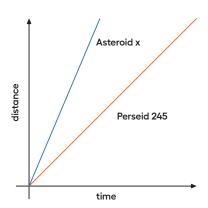
Keep students in the same groups. Explain what an asteroid is and what Perseid 245 is.

Give students 3–4 minutes of quiet work time followed by partner and wholeclass discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Student Task Statement

1. Meteoroid Perseid 245 and Asteroid x travel through the solar system. The graph shows the distance each traveled after a given point in time.



Is Asteroid x traveling faster or slower than Perseid 245? Explain how you know.

Asteroid x is traveling faster than Perseid 245. Sample reasoning: Students might consider the same time on each graph and compare the distance traveled, they might consider the same distance traveled on each graph and compare the time it took, or they might reason about each object's speed in distance units per time unit.

Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.

Use multimodal examples, verbal descriptions along with gestures, drawings, or concrete objects to show two asteroids traveling through the solar system, or invite students to act out the scenario.

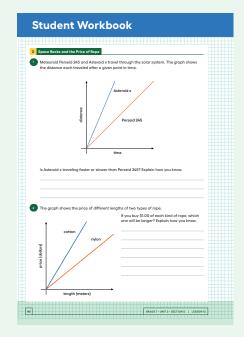
Advances: Listening, Representing

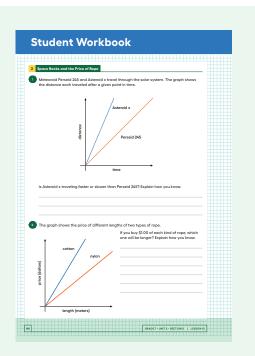
Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

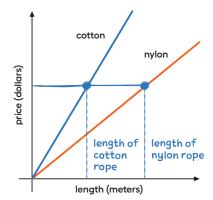
Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as "First, I ______ because ..." "Then, I ..." "I noticed, _____ so I ..." and "I tried _____, and what happened was ..."

Supports accessibility for: Language, Social-Emotional Functioning





2. The graph shows the price of different lengths of two types of rope.



If you buy \$1.00 of each kind of rope, which one will be longer? Explain how you know.

The nylon rope would be longer.

Sample reasoning: The graph shows that a greater length of nylon rope can be purchased for the same price as a shorter length of cotton rope.

Activity Synthesis

The purpose of this discussion is to help students interpret what it means that one graph is steeper than another for cases where there are two different proportional relationships graphed on the same pair of axes. It is important that students do not assume "steeper always means faster," but that they understand why it is in this case by reasoning abstractly and attending to the referents for points on the graphs. If the same relationships were graphed with distance on the horizontal axis and time on the vertical axis, a steeper line would indicate a slower speed. If the same relationships were graphed on separate axes, their scales could be different. Because the graphs share the same axes, it is implicit that comparisons between them occur relative to the same units.

Invite previously selected students to share their reasoning for the first problem. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

"How does each method tell us which object is traveling at a greater speed?"
"How does each method deal with the fact that there are no numbers labeled on the graphs?"

The key takeaways are:

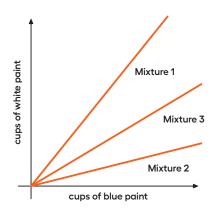
- When comparing proportional relationships, it's important to interpret the constants of proportionality in context of the situation.
- A steeper graph does not always mean that the object is moving faster.
 - If the relationships are on two separate graphs, then the scales on the axes could be different.
- If distance is on the horizontal axis and time is on the vertical axis, a steeper line indicates a slower speed.
- If the quantities are something other than distance and time, then the comparison is of some other aspect besides speed.

Lesson Synthesis

Share with students,

"Today we looked at graphs that showed more than one proportional relationship on the same coordinate grid. We used the graphs to compare the relationships, for example, figuring out who was traveling faster or which item was less expensive."

If desired, use this example to review these concepts.



"A person mixes blue paint and white paint to make different shades of light blue paint, as shown in this graph."

"Which mixture is the darkest? How do you know?"

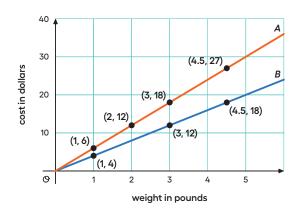
Mixture 2. It uses the least white paint for the same amount of blue paint. Alternately, it uses the most blue paint for the same amount of white paint.

"Which mixture is the lightest? How do you know?"

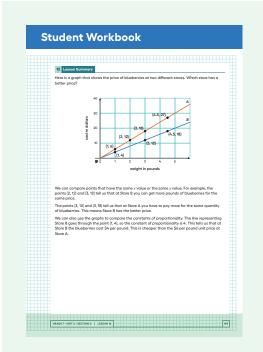
Mixture I. It uses the most white paint for the same amount of blue paint. Alternately, it uses the least blue paint for the same amount of white paint.

Lesson Summary

Here is a graph that shows the price of blueberries at two different stores. Which store has a better price?



We can compare points that have the same x value or the same y value. For example, the points (2, 12) and (3, 12) tell us that at Store B you can get more pounds of blueberries for the same price.



Responding To Student Thinking

Points to Emphasize

If students struggle with comparing two relationships graphed on the same axes, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about Andre and Jada's rates shown on the graph.

Unit 2, Lesson 13, Activity 3 Balloon Animal Contest The points (3, 12) and (3, 18) tell us that at Store A you have to pay more for the same quantity of blueberries. This means Store B has the better price.

We can also use the graphs to compare the constants of proportionality. The line representing Store B goes through the point (1, 4), so the constant of proportionality is 4. This tells us that at Store B the blueberries cost \$4 per pound. This is cheaper than the \$6 per pound unit price at Store A.

Cool-down

Revisiting the Amusement Park



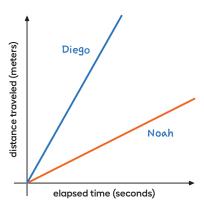
Student Task Statement

Noah and Diego left the amusement park's ticket booth at the same time. Each moved at a constant speed toward his favorite ride. After 8 seconds, Noah was 17 meters from the ticket booth, and Diego was 43 meters away from the ticket booth.

1. Which line represents the distance traveled by Noah, and which line represents the distance traveled by Diego?

The steeper line represents the distance traveled by Diego.

Label each line with one name.



2. Explain how you decided which line represents which person's travel.

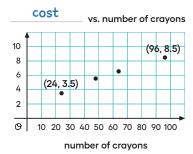
Sample reasoning: Diego had gone farther after 8 seconds. If you pick a time and look at which line represents a person who has gone farther, that is the steeper graph. So that must be Diego's line.

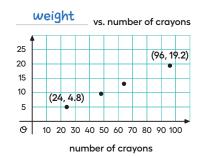
Practice Problems

5 Problems

Problem 1

The following graphs show some information about boxes of crayons at a store. One of the graphs shows cost in dollars vs. number of crayons in the box, and one of the graphs shows weight in ounces vs. number of crayons in the box.





a. Which graph is which? Give them the correct titles.

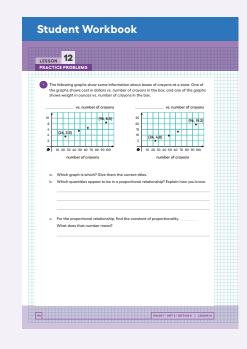
The y-values are appropriate for cost in dollars on the first graph, and the y-values are appropriate for weight in ounces on the second.

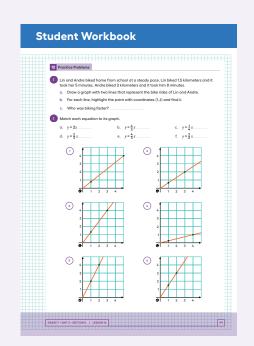
b. Which quantities appear to be in a proportional relationship? Explain how you know.

It appears there is a proportional relationship between weight and number of crayons. The points appear to lie on a line that would pass through the origin. Also, it makes sense that each crayon would weigh the same amount.

c. For the proportional relationship, find the constant of proportionality. What does that number mean?

The constant of proportionality is 0.2, which can be found using $\frac{1.6}{8}$ or $\frac{19.2}{96}$. It means each crayon weighs 0.2 ounces.

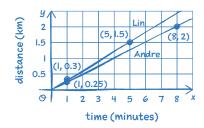




Problem 2

Lin and Andre biked home from school at a steady pace. Lin biked 1.5 kilometers and it took her 5 minutes. Andre biked 2 kilometers and it took him 8 minutes.

a. Draw a graph with two lines that represent the bike rides of Lin and Andre.



- **b.** For each line, highlight the point with coordinates (1, k) and find k. For Lin's graph, k = 0.3. For Andre's graph, $k = \frac{2}{8}$.
- c. Who was biking faster?

Lin is going slightly faster at 0.3 kilometer per minute. Andre is going $\frac{2}{8}$, or 0.25, kilometer per minute.

Problem 3

Match each equation to its graph.

a.
$$y = 2x$$
 5

b.
$$y = \frac{4}{5}x$$

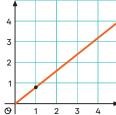
c.
$$y = \frac{1}{4}x$$

d.
$$y = \frac{2}{3}x$$

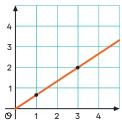
e.
$$y = \frac{4}{3}x$$
 3

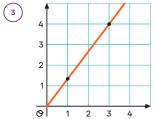
f.
$$y = \frac{3}{2}x$$



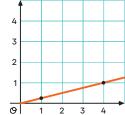


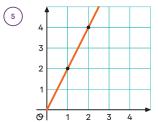


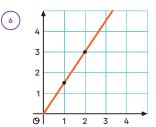










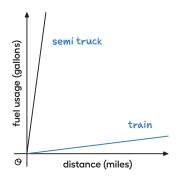


Problem 4

A semi truck can travel 1,300 miles on 200 gallons of fuel. A train can travel 1,000 miles on 2 gallons of fuel. This graph shows the fuel usage in miles per gallon for the two vehicles.

- a. Which line represents the semi truck and which represents the train?
- **b.** Which vehicle uses less fuel to travel the same distance? How can you tell from the graph?

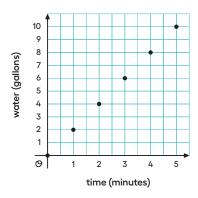
The train uses less fuel to travel the same distance. The line representing the train is less steep than the line representing the semi truck. Since fuel usage is on the vertical axis and distance is on the horizontal axis, this means that the train uses less fuel per mile.

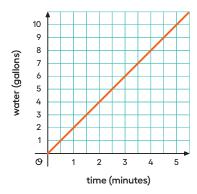


Problem 5

from Unit 2, Lesson 10

Here are two graphs for the relationship between the time that a water faucet has been on and the amount of water in a bucket.





a. What is the same about the two graphs?

The horizontal axis shows time in minutes. The vertical axis shows gallons of water. The graphs show a proportional relationship with a constant of proportionality of 2.

b. What is different about the two graphs?

The graph on the left only has points that represent the amount of water for whole minutes. The graph on the right shows the amount of water for any time between 0 and 5.5 minutes.

c. Which graph makes more sense for representing this situation?

The graph on the right makes more sense for representing the situation, because it is possible to have a fraction of a minute and a fraction of a gallon.

