

## More About Constant of Proportionality

### Goals

- Compare, contrast, and critique (orally and in writing) different ways to express the constant of proportionality for a relationship.
- Explain (orally) how to determine the constant of proportionality for a proportional relationship represented in a table.
- Interpret the constant of proportionality for a relationship in the context of constant speed.

### Learning Targets

- I can find missing information in a proportional relationship using a table.
- I can find the constant of proportionality from information given in a table.

### Lesson Narrative

In this lesson, students continue to work with proportional relationships represented by tables. They identify the constant of proportionality and use it to answer questions about the context. The contexts are familiar from previous grades: unit conversion and constant speed. When students recognize that the conversion factor or the speed are the constants of proportionality for the relationships, they are reasoning abstractly and quantitatively. Although students might continue to reason with equivalent ratios to solve problems, the contexts are designed so that it is more efficient to use the constant of proportionality.

This lesson also introduces students to the idea that there are two ways of viewing any proportional relationship. In other words, if  $y$  is proportional to  $x$ , then  $x$  is also proportional to  $y$ . The two constants of proportionality are **reciprocals**,  $\frac{y}{x}$  and  $\frac{x}{y}$ , respectively. This idea will be developed more in future lessons.

### Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 1)
- Engagement (Activity 2)

### Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Warm-up)

### Instructional Routines

- Math Talk
- MLR2: Collect and Display
- MLR5: Co-Craft Questions

### Required Materials

#### Materials to Gather

- Chart paper: Warm-up
- Math Community Chart: Warm-up

### Lesson Timeline

5  
mins

Warm Up

10  
mins

Activity 1

15  
mins

Activity 2

10  
min

Lesson Synthesis

### Assessment

5  
mins

Cool Down

**More about Constant of Proportionality****Lesson Narrative (continued)****Math Community**

In today's activities, students are introduced to the idea of math norms as expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Students then consider what norms would connect and support the math actions the class recorded so far in the Math Community Chart.

**Student Learning Goal**

Let's solve more problems involving proportional relationships using tables.

## Warm-up

## Math Talk: Division

5  
min

## Activity Narrative

This *Math Talk* focuses on division that results in a decimal. It encourages students to think about how they can use the result of one division problem to find the answer to a similar problem with a different, but related, divisor or dividend. The understanding elicited here will be helpful later in the lesson when students calculate constants of proportionality.

To recognize how a divisor has been scaled and predict how the quotient will be affected, students need to look for and make use of structure.

## Launch

Tell students to close their books or devices (or to keep them closed).  
Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

## Student Task Statement

Find the value of each expression mentally.

A.  $645 \div 10$

64.5

Sample reasoning: 64.5 is one-tenth of 645. This is seen from the location of the decimal point: it is between the 4 and the 5 instead of after the 5.

B.  $645 \div 100$

6.45

Sample reasoning: Since the current divisor, 100, is ten times the previous divisor, 10, the current quotient will be one-tenth the previous quotient. 6.45 is one-tenth of 64.5. This is seen from the location of the decimal point: it is between the 6 and 4 instead of after the 4.

C.  $645 \div 50$

12.9

Sample reasoning: Since the current divisor, 50, is half the previous divisor, 100, the current quotient will be double the previous quotient.

$6.45 \cdot 2 = 12.9$

D.  $64.5 \div 50$

1.29

Sample reasoning: Since the current dividend, 64.5, is one-tenth of the previous dividend, 645, the current quotient will be one-tenth of the previous quotient.  $12.9 \div 10 = 1.29$

## Inspire Math

## Golden Temple video



## Go Online

Before the lesson, show this video to introduce the real-world connection.

[ilclass.com/1/614131](https://ilclass.com/1/614131)

Please log in to the site before using the QR code or URL.



## Instructional Routines

## Math Talk

[ilclass.com/r/10694967](https://ilclass.com/r/10694967)

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**Access for Students with Diverse Abilities (Warm-up, Launch)****Action and Expression: Internalize Executive Functions.**

To support working memory, provide students with access to sticky notes or mini whiteboards.  
*Supports accessibility for: Memory, Organization*

**Access for Multilingual Learners (Warm-up, Synthesis)****MLR8: Discussion Supports.**

Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_\_ because ...” or “I noticed \_\_\_\_\_ so I ...” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Advances: Speaking, Representing*

**Instructional Routines****MLR2: Collect and Display**

[ilclass.com/r/10690754](https://ilclass.com/r/10690754)

Please log in to the site before using the QR code or URL.

**Access for Multilingual Learners (Activity 1)****MLR2: Collect and Display**

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

**Activity Synthesis**

To involve more students in the conversation, consider asking:

☞ “Who can restate \_\_\_\_\_’s reasoning in a different way?”

“Did anyone use the same strategy but would explain it differently?”

“Did anyone solve the problem in a different way?”

“Does anyone want to add on to \_\_\_\_\_’s strategy?”

“Do you agree or disagree? Why?”

“What connections to previous problems do you see?”

The key takeaway to highlight is the effects of multiplying or dividing numbers by powers of 10.

**Math Community**

At the end of the *Warm-up*, display the Math Community Chart. Tell students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Using the Math Community Chart, offer an example of how the “Doing Math” actions can be used to create norms. For example, “In the last exercise, many of you said that our math community sounds like ‘sharing ideas.’ A norm that supports that is ‘We listen as others share their ideas.’ For a teacher norm, ‘questioning vs telling’ is very important to me, so a norm to support that is ‘Ask questions first to make sure I understand how someone is thinking.’”

Invite students to reflect on both individual and group actions. Ask, “As we work together in our mathematical community, what norms, or expectations, should we keep in mind?” Give 1–2 minutes of quiet think time and then invite as many students as time allows to share either their own norm suggestion or to “+1” another student’s suggestion. Record student thinking in the student and teacher “Norms” sections on the Math Community Chart.

Conclude the discussion by telling students that what they made today is only a first draft of math community norms and that they can suggest other additions during the *Cool-down*. Throughout the year, students will revise, add, or remove norms based on those that are and are not supporting the community.

**Activity 1****Centimeters and Millimeters**

10  
min

**Activity Narrative**

This activity has two purposes. First, it involves an important context that can be represented by proportional relationships, namely, measurement conversion. Second, it introduces the idea that there are two constants of proportionality and that they are reciprocals (also known as multiplicative inverses). Students start to use “is proportional to” language to distinguish between the two constants of proportionality:

- The length measured in millimeters is proportional to the length measured in centimeters, and the constant of proportionality is 10.
- The length measured in centimeters is proportional to the length measured in millimeters, and the constant of proportionality is  $\frac{1}{10}$ , or 0.1.

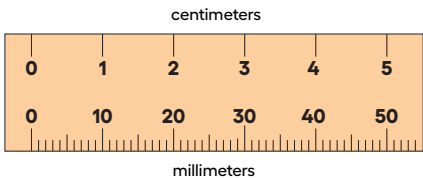
Launch

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language students use to describe the proportional relationships. Display words and phrases such as “is proportional to,” “constant of proportionality,” “reciprocal,” “inverse,” etc.

Give students quiet work time followed by partner and whole-class discussion.

Student Task Statement

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.



There are two ways of thinking about this proportional relationship.

1. If the length of something in centimeters is known, its length in millimeters can be calculated.
- a. Complete the table.

length (centimeters)	length (millimeters)
9	90
12.5	125
50	500
88.49	884.9

- b. What is the constant of proportionality?
- 10

2. If the length of something in millimeters is known, its length in centimeters can be calculated.
- a. Complete the table.

length (millimeters)	length (centimeters)
70	7
245	24.5
4	0.4
699.1	69.91

- b. What is the constant of proportionality?
- $\frac{1}{10}$  or 0.1

Access for Students with Diverse Abilities (Activity 1, Launch)

Action and Expression: Provide Access for Physical Action.

Activate or supply background knowledge. Provide a display showing equivalence between dividing by a number and multiplying by its reciprocal for students to use as a reference.

*Supports accessibility for: Memory, Organization*

Building on Student Thinking

Some students may say that the constants of proportionality are both 10 since you can divide by 10 in the second table. Tell students,

*“The constant of proportionality is what you multiply by. Can you find a way to multiply the numbers in the first column to get the numbers in the second column?”*

Student Workbook

LESSON 3

More about Constant of Proportionality

Let's solve more problems involving proportional relationships using tables.

Warm-up

Math Talk: Division

Find the value of each expression mentally.

☐ A

 $645 \div 10$

☐ B

 $645 \div 100$

☐ C

 $645 \div 50$

☐ D

 $64.5 \div 50$

1

Centimeters and Millimeters

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.

There are two ways of thinking about this proportional relationship.

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LESSON 3

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Student Workbook

1

Centimeters and Millimeters

1

If the length of something in centimeters is known, its length in millimeters can be calculated.

a. Complete the table.

length (centimeters)	length (millimeters)
9	
12.5	
50	
88.49	

b. What is the constant of proportionality?

2

If the length of something in millimeters is known, its length in centimeters can be calculated.

a. Complete the table.

length (millimeters)	length (centimeters)
70	
245	
4	
699.1	

b. What is the constant of proportionality?

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Student Workbook

1

Centimeters and Millimeters

3

How are these two constants of proportionality related to each other?

4

Complete each sentence:

a. To convert from centimeters to millimeters, the value in centimeters is multiplied by \_\_\_\_\_.

b. To convert from millimeters to centimeters, the value in millimeters is divided by \_\_\_\_\_, or multiplied by \_\_\_\_\_.

Are you ready for more?

1

How many square millimeters are there in a square centimeter?

2

How do you convert square centimeters to square millimeters?

How do you convert the other way?

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3. How are these two constants of proportionality related to each other?  
*10 and  $\frac{1}{10}$  are reciprocals.*

4. Complete each sentence:

a. To convert from centimeters to millimeters, the value in centimeters is multiplied by *10*.

b. To convert from millimeters to centimeters, the value in millimeters is divided by *10*, or multiplied by  *$\frac{1}{10}$* .

Are You Ready for More?

1. How many square millimeters are there in a square centimeter?  
*100*

2. How do you convert square centimeters to square millimeters? How do you convert the other way?  
*Multiply by 100; multiply by  $\frac{1}{100}$*

Activity Synthesis

The goal of this discussion is to help students recognize how the structure of proportional relationships applies to this situation involving measurement conversion.

Direct students’ attention to the reference created using *Collect and Display*. Ask students to share what they noticed about the two constants of proportionality, borrowing language from the display as needed. Next, ask students to suggest ways to update the display:

☞ “*Are there any new words or phrases that you would like to add?*”

“*Is there any language you would like to revise or remove?*”

The key takeaway is that the two constants of proportionality are reciprocals. If needed, remind students that dividing by 10 is the same as multiplying by its reciprocal,  $\frac{1}{10}$ . One way to explain why these two constants of proportionality are multiplicative inverses is to imagine starting with a measurement in centimeters, 15 centimeters for example. To convert 15 centimeters to millimeters, we multiply 15 by 10. So 15 centimeters = 150 millimeters. If we convert the measurement in millimeters back to centimeters, it will be 15, so the constant of proportionality we need to multiply by is  $\frac{1}{10}$ .

Activity 2

Pittsburgh to Phoenix

15 min

Activity Narrative

This activity focuses on making connections between constant speed and proportional relationships, with special attention to the constant of proportionality. The numbers are chosen so that students are more likely to use unit rate rather than scale factors. The goal is for students to understand that when speed is constant, time elapsed and distance traveled are proportional. The constant of proportionality indicates the magnitude of the speed. For example, if distance is given in miles and time in hours, then the

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constant of proportionality indicates the speed in miles per hour. As students make these connections, they are reasoning abstractly and quantitatively.

Students might wonder why the route is not a straight line. If they ask about it, teachers can share some reasons why airplane routes are complex, for example the need to avoid congested areas and the fact that the shortest distances on the curved surface of Earth do not always correspond to lines on a map.

This is the first time Math Language Routine 5: *Co-Craft Questions* is suggested in this course. In this routine, students are given a context or situation, often in the form of a problem stem (for example, a story, image, video, or graph) with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: “What mathematical questions could you ask about this situation?” The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students’ awareness of the language used in mathematics problems.

### Launch

Arrange students in groups of 2. Use *Co-Craft Questions* to give students an opportunity to familiarize themselves with the context, and to practice producing the language of mathematical questions.

- Display only the problem stem and related image, without revealing the questions.

Ask students, “What mathematical questions could you ask about this situation?”

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

As partners discuss, support students in using conversation and collaboration skills to generate and refine their questions, for instance, by revoicing a question, seeking clarity, or referring to their written notes.

Listen for how students use language about speed, distance traveled, and elapsed time.

- Invite several students to share one question with the class and record for all to see. Ask the class to make comparisons among the shared questions and their own. Ask, “What do these questions have in common? How are they different?”

Listen for and amplify questions that focus on the relationship between speed, distance traveled, and elapsed time.

Reveal the table and questions and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

💬 “Which of your questions is most similar to or different from the ones provided? Why?”

“Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it.”

Ask students to complete the questions.

### Instructional Routines

#### MLR5: Co-Craft Questions

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### Access for Multilingual Learners (Activity 2)

#### MLR5: Co-Craft Questions

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

### Access for Students with Diverse Abilities (Activity 2, Launch)

#### Engagement: Develop Effort and Persistence.

Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

*Supports accessibility for: Memory, Conceptual Processing*



Building on Student Thinking


Students who need support in understanding the context can trace the segments on the map, labeling the distances they know and putting question marks for unknown distances. An empty double number line could also be a useful tool in helping students reason about the context.

Student Workbook

**Pittsburgh to Phoenix**

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1. What is the distance between Saint Louis and Albuquerque?

2. How many minutes did it take to fly between Albuquerque and Phoenix?

3. What is the proportional relationship represented by this table?

Diego says the constant of proportionality is 550. Andre says the constant of proportionality is  $\frac{9}{10}$ . Do you agree with either of them? Explain your reasoning.

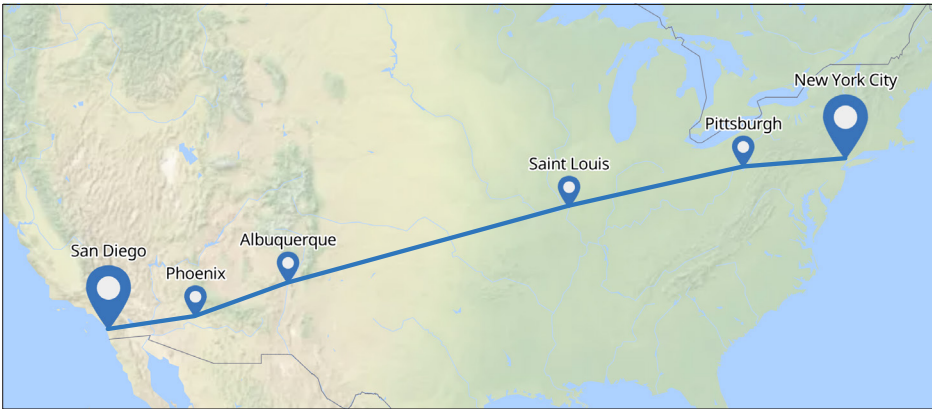
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Student Task Statement

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1 hour 42 minutes	935 miles	550 miles per hour
Albuquerque to Phoenix	36 minutes	330 miles	550 miles per hour

1. What is the distance between Saint Louis and Albuquerque?  
935 miles  
42 minutes is  $\frac{42}{60}$  hours, or  $\frac{7}{10}$  of an hour.  $\frac{7}{10}$  of 550 miles is 385 miles, and  $385 + 550 = 935$ .
2. How many minutes did it take to fly between Albuquerque and Phoenix?  
36 minutes  
330 miles is  $\frac{3}{5}$  of 550 miles, and  $\frac{3}{5}$  of 60 minutes is 36 minutes.
3. What is the proportional relationship represented by this table?  
The distance traveled is proportional to the elapsed time.



4. Diego says the constant of proportionality is 550. Andre says the constant of proportionality is  $9\frac{1}{6}$ . Do you agree with either of them? Explain your reasoning.

Diego uses miles per hour

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1.7 hours	935 miles	550 miles per hour
Albuquerque to Phoenix	0.6 hours	330 miles	550 miles per hour

Andre uses miles per minute.

segment	time	distance	speed
Pittsburgh to Saint Louis	60 minutes	550 miles	$9\frac{1}{6}$ miles per minute
Saint Louis to Albuquerque	102 minutes	935 miles	$9\frac{1}{6}$ miles per minute
Albuquerque to Phoenix	36 minutes	330 miles	$9\frac{1}{6}$ miles per minute

Student Workbook

**2 Pittsburgh to Phoenix**

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1 What is the distance between Saint Louis and Albuquerque?

2 How many minutes did it take to fly between Albuquerque and Phoenix?

3 What is the proportional relationship represented by this table?

4 Diego says the constant of proportionality is 550. Andre says the constant of proportionality is  $9\frac{1}{6}$ . Do you agree with either of them? Explain your reasoning.

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Activity Synthesis

The goal of the discussion is for students to understand that when speed is constant, then distance traveled is proportional to elapsed time. The constant of proportionality is the speed. For example, if distance is given in miles and time in hours, then the constant of proportionality indicates the speed in miles per hour.

Begin by having a student share a table that is all in hours, miles, and miles per hour. Use this example to point out that every number in the first column can be multiplied by the speed to get the number in the second column. Ask:

- ☞ “Which quantities are in a proportional relationship? How do you know?”
- “What is the constant of proportionality in this case?”

If one or more students use minutes for time and miles per minute for speed in the table, the same questions can be asked for these units if time allows, but it is not required. In any case, summarize by making it explicit that when time and distance are in a proportional relationship, the constant of proportionality is the speed (or pace).

Lesson Synthesis

Share with students,

- ☞ “Today we looked at more tables of proportional relationships and found the constant of proportionality for each one.”

Student Workbook

Lesson Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

We can multiply any number in the first column by  $\frac{2}{3}$  to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is  $\frac{2}{3}$ . This means that the bug's pace is  $\frac{2}{3}$  seconds per centimeter. This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

We can multiply any number in the first column by  $\frac{3}{2}$  to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is  $\frac{3}{2}$ . This means that the bug's speed is  $\frac{3}{2}$  centimeters per second.

Notice that  $\frac{3}{2}$  is the reciprocal of  $\frac{2}{3}$ . When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column by to get the values in the second.

Briefly revisit the two contexts, reinforcing the use of new terms. Consider asking students:

“In the first activity, we examined the proportional relationship between millimeters and centimeters from two different perspectives and found two constants of proportionality. What were they?”

10 and  $\frac{1}{10}$

“What is the relationship between the two constants of proportionality?”

They are reciprocals.

“In the second activity, we examined a proportional relationship between the time a plane flies and the distance it travels. What was the constant of proportionality?”

550 or  $9\frac{1}{6}$

“What did the constant of proportionality represent in terms of the situation?”  
the plane’s speed

Lesson Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

We can multiply any number in the first column by  $\frac{2}{3}$  to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is  $\frac{2}{3}$ .

This means that the bug’s pace is  $\frac{2}{3}$  seconds per centimeter.

This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

We can multiply any number in the first column by  $\frac{3}{2}$  to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is  $\frac{3}{2}$ . This means that the bug’s speed is  $\frac{3}{2}$  centimeters per second.

Notice that  $\frac{3}{2}$  is the reciprocal of  $\frac{2}{3}$ . When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column by to get the values in the second.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the norms question “Which norm has not already been listed that you’d like to add to our chart?” Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms question. Use that information to add to the initial draft of the “Norms” sections of the Math Community Chart.

Cool-down

Fish Tank

5 min

Student Task Statement

Mai is filling her fish tank. Water flows into the tank at a constant rate. Complete the table as you answer the questions.

time (minutes)	water (gallons)
0.5	0.8
1	1.6
3	4.8
25	40

1. How many gallons of water will be in the fish tank after 3 minutes?  
Explain your reasoning.  
4.8. If the first row is doubled (scale by 2), there are 1.6 gallons after 1 minute. If the second row is tripled (scale by 3), there are 4.8 gallons after 3 minutes. Or the first row could be scaled by 6 to get 4.8 gallons after 3 minutes.
2. How long will it take to fill the tank with 40 gallons of water?  
Explain your reasoning.  
25 minutes. One way to find a scale factor to use is to divide 40 by 0.8.  
 $\frac{40}{0.8} = 50$  and  $50 \cdot 0.5 = 25$ .
3. What is the constant of proportionality?  
1.6 (or equivalent). You can observe the amount of water that corresponds with 1 minute, or you can divide any value in the right column with its corresponding value in the left column.

Responding To Student Thinking

**Points to Emphasize**

If students struggle with finding the constant of proportionality, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about how they found the constant of proportionality.

Unit 2, Lesson 4, Activity 3 Denver to Chicago

Student Workbook

**Lesson Summary**

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{1}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

We can multiply any number in the first column by  $\frac{2}{3}$  to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is  $\frac{2}{3}$ . This means that the bug's pace is  $\frac{2}{3}$  seconds per centimeter. This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

We can multiply any number in the first column by  $\frac{3}{2}$  to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is  $\frac{3}{2}$ . This means that the bug's speed is  $\frac{3}{2}$  centimeters per second.

Notice that  $\frac{3}{2}$  is the reciprocal of  $\frac{2}{3}$ . When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column by to get the values in the second.

## Practice Problems

6 Problems

## Student Workbook

LESSON 3  
PRACTICE PROBLEMS

- 1 Noah is running a portion of a marathon at a constant speed of 6 miles per hour. Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	
$\frac{1}{2}$	
$1\frac{1}{3}$	
	$1\frac{1}{2}$
	9
	$4\frac{1}{2}$

- 2 One kilometer is 1,000 meters. a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

meters	kilometers
1,000	1
250	
12	
1	

The constant of proportionality tells us that:

kilometers	meters
1	1,000
5	
20	
0.3	

The constant of proportionality tells us that:

- b. What is the relationship between the two constants of proportionality?

## Problem 1

Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8
$\frac{1}{4}$	$1\frac{1}{2}$
$1\frac{1}{2}$	9
$\frac{3}{4}$	$4\frac{1}{2}$

## Problem 2

One kilometer is 1000 meters.

- a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

kilometers	meters
1,000	1,000
250	0.25
12	0.012
1	0.001

The constant of proportionality tells us that:

0.001 kilometers per meter

kilometers	meters
1	1,000
5	5,000
20	20,000
0.3	300

The constant of proportionality tells us that:

1000 meters per kilometer

- b. What is the relationship between the two constants of proportionality?

0.001 and 1000 are reciprocals of each other. This is easier to see if 0.001 is written as  $\frac{1}{1000}$ .

Problem 3

Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is  $\frac{1}{12}$ . Do you agree with either of them? Explain your reasoning.

Both can be correct

Jada is saying that there are 12 inches for every 1 foot. Lin is saying that there is  $\frac{1}{12}$  foot for every 1 inch.

Problem 4

from Unit 1, Lesson 12

The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

1 inch to 50 miles

Problem 5

from Unit 1, Lesson 11

Which of these scales is equivalent to the scale 1 cm to 5 km? Select **all** that apply.

A. 3 cm to 15 km

B. 1 mm to 150 km

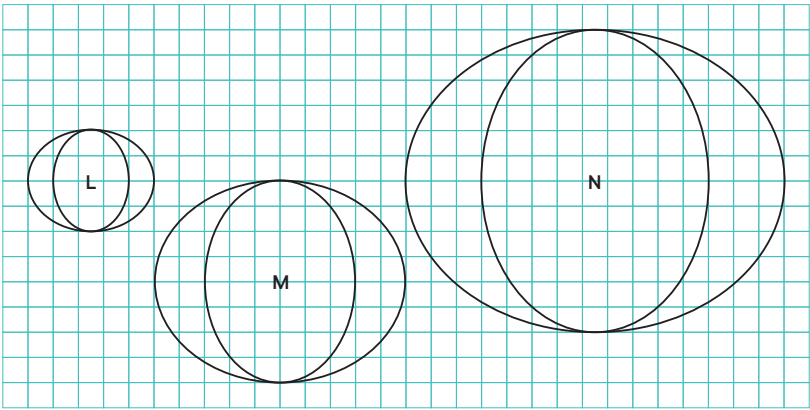
C. 5 cm to 1 km

D. 5 mm to 2.5 km

E. 1 mm to 500 m

Problem 6

from Unit 2, Lesson 1



Which one of these pictures is not like the others? Explain what makes it different using ratios.

M is different from L and N. The width:height ratios for the outsides of the pictures are all equivalent to 5 : 4. However, the width:height ratios of the insides of L and N both have a 3 : 4 ratio of width:height, while the inside of M has a width of 4 units and a height of 8 units, making its ratio 1 : 2. Alternatively, the ratio of height to thickness at the widest part for L and N are both 4 : 1. But M has a height of 8 units and a thickness of 3 units, making that ratio 8 : 3.

Student Workbook

3 Practice Problems

1 Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is  $\frac{1}{12}$ . Do you agree with either of them? Explain your reasoning.

2 from Unit 1, Lesson 12  
The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

3 from Unit 1, Lesson 11  
Which of these scales is equivalent to the scale 1 cm to 5 km? Select all that apply.

A 3 cm to 15 km  
B 1 mm to 150 km  
C 5 cm to 1 km  
D 5 mm to 2.5 km  
E 1 mm to 500 m

GRADE 7 • UNIT 2 • SECTION A • LESSON 3

Student Workbook

3 Practice Problems

1 from Unit 2, Lesson 1

Which one of these pictures is not like the others? Explain what makes it different using ratios.

Learning Targets

- + I can find missing information in a proportional relationship using a table.
- + I can find the constant of proportionality from information given in a table.

GRADE 7 • UNIT 2 • SECTION A • LESSON 3

