Lesson 3

Powers of Powers of 10

Goal

Generalize a process for finding a power raised to a power, and justify (orally and in writing) that $(10^n)^m = 10^{n \cdot m}$.

Learning Target

I can explain and use a rule for raising a power of 10 to a power.

Lesson Narrative

In this lesson, students make use of repeated reasoning to discover the exponent rule $(10^n)^m = 10^{n \cdot m}$. Students begin by computing the volume of a very large cube. Next, they expand expressions written as a value with an exponent raised to another power and notice patterns when asked to write the expression using a single power of 10. Students will extend this exponent rule to cases where the exponents are zero or negative in following lessons, but the focus here is on cases with positive exponents.

Student Learning Goal

Let's look at powers of powers of 10.

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Required Preparation

Activity 2:
Create a visual display (or add to an existing display) of the exponent rule $(10^n)^m = 10^{n \cdot m}$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis.*

Lesson Timeline

5 min	15 min	10 min	10 min
Warm-up	Activity 1	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Big Cube

5 min

Activity Narrative

The purpose of this *Warm-up* is to introduce the idea of raising a value with an exponent to another power. Computing the volume of a cube whose side lengths are themselves powers of 10 introduces the basic structure of a power to a power, which will lead to a general exponent rule in later activities.

Monitor for students who use different strategies, such as counting zeros to keep track of place value or writing 10,000 as 10^4 and then using exponent rules.

Launch

Give students 1–2 minutes of quiet work time followed by a brief whole-class discussion.

Student Task Statement

What is the volume of a giant cube that measures 10,000 km on each side? Be prepared to explain your reasoning.

1,000,000,000,000 km^3. Sample reasoning: $10^4 \cdot 10^4 \cdot 10^4 = 10^{(4+4+4)} = 10^{12}$

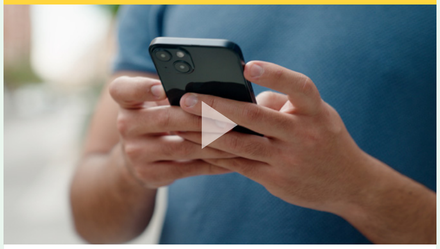
Activity Synthesis

The purpose of this discussion is to introduce the idea that $10^4 \cdot 10^4 \cdot 10^4 = (10^4)^3$, which is equal to 10^{12} before this pattern is generalized in a following activity.

Invite previously identified students to share their strategies for computing the volume. If not brought up in students' explanations, show students the strategy of re-writing 10,000 as a power of 10 and raising 10^4 to the power of 3. Ask students what patterns they notice between $(10^4)^3$ and 10^{12}. If students mention the strategy of counting zeros to multiply powers of 10, emphasize that the exponent describes the number of factors that are multiplied together and not necessarily the number of zeros.

Inspire Math

Going Viral video

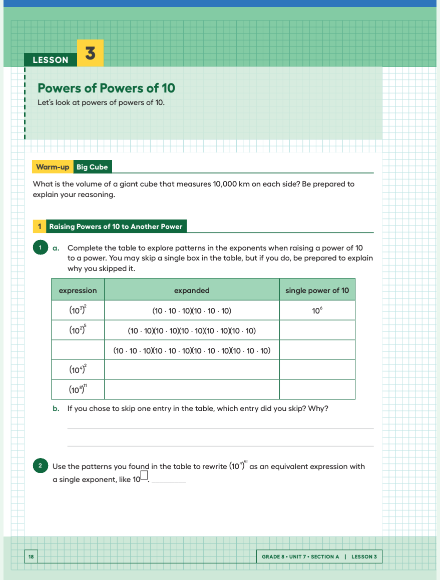


Go Online
Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614181

Please log in to the site before using the QR code or URL.

Student Workbook



LESSON 3

Powers of Powers of 10

Let's look at powers of powers of 10.

Warm-up Big Cube

What is the volume of a giant cube that measures 10,000 km on each side? Be prepared to explain your reasoning.

1 Raising Powers of 10 to Another Power

1 a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

2 Use the patterns you found in the table to rewrite $(10^n)^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

18 GRADE 8 • UNIT 7 • SECTION A | LESSON 3

Access for Multilingual Learners (Activity 1, Launch)

MLR2: Collect and Display.
Collect the language that students use to expand expressions with a power raised to another power. Display words and phrases such as "factors," and "groups of factors," "multiply the exponents." During the synthesis, invite students to suggest ways to update the display:

> *"What are some other words or phrases we should include?"*

Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Have students complete one problem at a time. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Social-Emotional Functioning

Activity 1

Raising Powers of 10 to Another Power

15 min

Activity Narrative

In this activity, students explore patterns to discover the property $(10^n)^m = 10^{n \cdot m}$ for values of n and m that are positive integers.

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before inviting 1–2 students to share and explain their answers. When it is clear that students understand how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 5–6 minutes to complete the remaining questions before a whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	10^{10}
$(10^3)^4$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^{12}
$(10^4)^2$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$(10^8)^{11}$	skip	10^{88}

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

   I chose to skip the expanded column of $(10^8)^6$ because the table cannot fit 88 factors that are 10.

2. Use the patterns you found in the table to rewrite $(10^n)^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   $(10^n)^m = 10^{n \cdot m}$ because there are m groups of n factors that are 10

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side.

   How many cubic meters of oil is this? 10^9 cubic meters of oil because $(10^3)^3 = 10^9$

   Do you think this would be enough to fill a pond, a lake, or an ocean?

   Sample response: This is enough to fill a lake.

Are You Ready for More?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Since $4{,}096 = 2^{12}$, it can be broken down into other representations of the form $(2^m)^n$ so that $m \cdot n = 12$. For example, $(2^2)^6 = 4^6$, $(2^3)^4 = 8^4$, $(2^4)^3 = 16^3$, $(2^6)^2 = 64^2$, and $(2^{12})^1 = 4{,}096^1$.

Activity Synthesis

The goal of this discussion is to reinforce the exponent rule for raising powers of 10 to another power. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$
	three groups of two factors that are ten = six factors that are ten

Continue to reinforce student understanding of this rule by writing out an expanded form of each expression when discussing the following questions:

- *"What is $(10^2)^4$ written as a single power of 10?"*

  $(10^2)^4 = (10^2) \cdot (10^2) \cdot (10^2) \cdot (10^2) = 10^{2 \cdot 4} = 10^8$

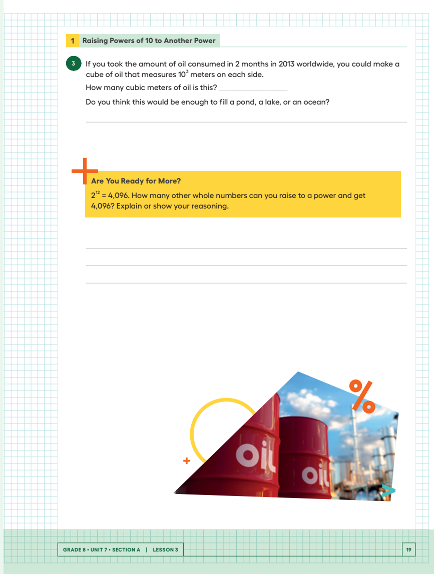
- *"What is $(10^7)^3$ written as a single power of 10?"*

  $(10^7)^3 = (10^7) \cdot (10^7) \cdot (10^7) = 10^{7 \cdot 3} = 10^{21}$

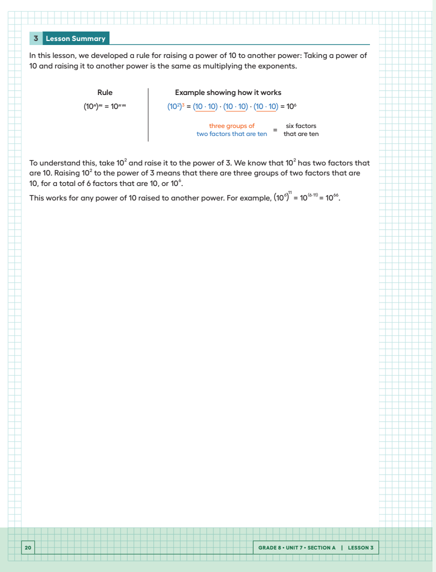
- *"What are some different ways to write 10^{24} as a power of 10 raised to another power?"*

  $(10^4)^6$, $(10^{12})^2$, $(10^3)^8$

Student Workbook



Student Workbook



Lesson Synthesis

The purpose of this discussion is to emphasize the similarities and differences between the rule for multiplying values with the same base and the rule for raising a value with an exponent to another power. Display these two statements for all to see, and explain that Andre and Elena were both trying to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

Andre: $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$

Elena: $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$

Ask students if they agree with either Andre or Elena. Give students time to share their reasoning with a partner before inviting students to share their thinking with the class. Record student responses for all to see. Consider asking the following questions for discussion:

- *"How are these statements the same? How are they different?"*

  Both statements are attempting to write $10^2 \cdot 10^2 \cdot 10^2$ as a number with a single exponent. The first statement uses the rule for multiplying values with the same base while the second statement tries to use the rule for raising a value with an exponent to another power.

- *"How can the second statement be corrected to make it true?"*

  When raising a value with an exponent to another power, the two exponents can be multiplied together, giving $(10^2)^3 = 10^{(2 \cdot 3)} = 10^6$

- *"How can $10^5 \cdot 10^5 \cdot 10^5 \cdot 10^5$ be written with exponents instead of repeated multiplication?"*

  $(10^5)^4 = 10^{20}$

Lesson Summary

In this lesson, we developed a rule for raising a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$
	three groups of two factors that are ten = six factors that are ten

To understand this, take 10^2 and raise it to the power of 3. We know that 10^2 has two factors that are 10. Raising 10^2 to the power of 3 means that there are three groups of two factors that are 10, for a total of 6 factors that are 10, or 10^6.

This works for any power of 10 raised to another power. For example, $(10^6)^{11} = 10^{(6 \cdot 11)} = 10^{66}$.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- "What norm(s) should stay the way they are?"
- "What norm(s) do you think should be made more clear? How?"
- "What norms are missing that you would add?"
- "What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made, to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Discuss the potential revisions over the next few lessons.

Responding To Student Thinking

Points to Emphasize
If most students struggle with multiplying powers of 10, make time to do the optional activity referred to here. Focus on equivalent expressions using multiplication.
Unit 7, Lesson 4, Activity 4 Making Millions

Cool-down

Making a Million

10 min

Student Task Statement

Here are some equivalent ways of writing 10^4:

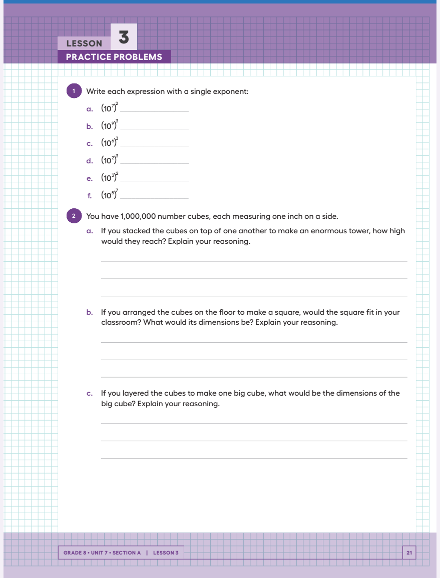
- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6.

Answers vary. Sample responses:

- 1,000,000
- $1{,}000 \cdot 1{,}000$
- $10^2 \cdot 10^4$
- $(10^3)^2$
- $1 \cdot 10^6$
- $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Student Workbook


LESSON 3
PRACTICE PROBLEMS

1 Write each expression with a single exponent:

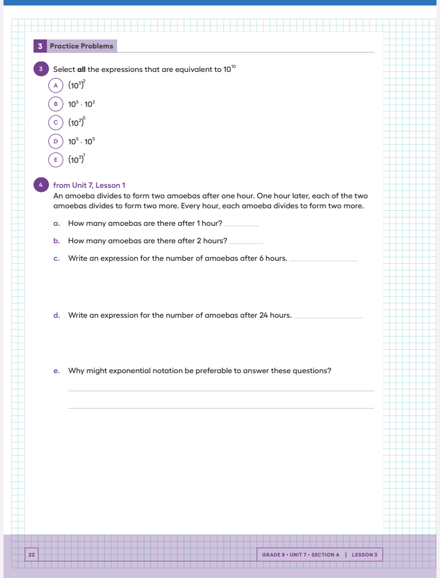
2 You have 1,000,000 number cubes, each measuring one inch on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

Student Workbook


3 Practice Problems

3 Select **all** the expressions that are equivalent to 10^{10}

4 from Unit 7, Lesson 1
An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

a. How many amoebas are there after 1 hour?

b. How many amoebas are there after 2 hours?

c. Write an expression for the number of amoebas after 6 hours.

d. Write an expression for the number of amoebas after 24 hours.

e. Why might exponential notation be preferable to answer these questions?

Lesson 3

Practice Problems

5 Problems

Problem 1

Write each expression with a single exponent:

a. $(10^7)^2$ 10^{14}

b. $(10^9)^3$ 10^{27}

c. $(10^6)^3$ 10^{18}

d. $(10^2)^3$ 10^{6}

e. $(10^3)^2$ 10^{6}

f. $(10^5)^7$ 10^{35}

Problem 2

You have 1,000,000 number cubes, each measuring one inch on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

1,000,000 inches tall, or about 83,000 feet, or about 16 miles

Sample reasoning: $1{,}000{,}000 \div 12 \approx 83{,}333$ feet, which is almost 16 miles ($83{,}333 \div 5{,}280 \approx 15.78$).

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

1,000-inch side length

Sample reasoning: $1{,}000 \cdot 1{,}000 = 1{,}000{,}000$.
1,000 inches is about $83\frac{1}{3}$ feet long and would probably be longer than most classrooms.

c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

100-inch side length

Sample reasoning: $100 \cdot 100 \cdot 100 = 1{,}000{,}000$.
This is a side length of $8\frac{1}{3}$ feet.

Problem 3

Select **all** the expressions that are equivalent to 10^{10}

A. $(10^5)^2$

B. $10^5 \cdot 10^2$

C. $(10^2)^5$

D. $10^5 \cdot 10^5$

E. $(10^3)^7$

Problem 4

from Unit 7, Lesson 1

An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

a. How many amoebas are there after 1 hour? 2

b. How many amoebas are there after 2 hours? 4

c. Write an expression for the number of amoebas after 6 hours.
2^6 (or equivalent)

d. Write an expression for the number of amoebas after 24 hours.
2^{24} (or equivalent)

e. Why might exponential notation be preferable to answer these questions?

Sample response: Exponential notation is simpler to write than very large or small numbers, and the expression 2^{24} visibly includes the information that the amoebas have divided 24 times.

Problem 5

from Unit 3, Lesson 13

You have two numbers, (x, y). If you triple the second number and subtract it from the first, the difference is 12.

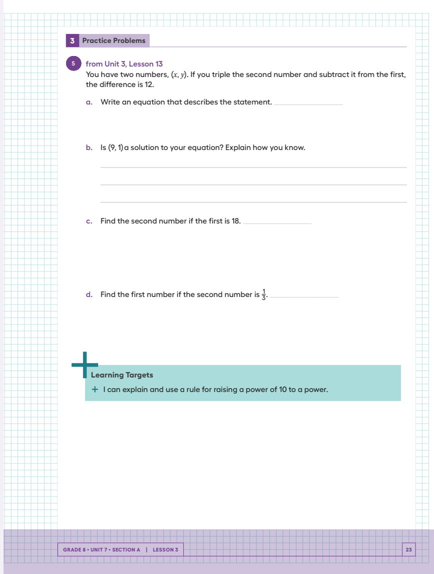
a. Write an equation that describes the statement. $x - 3y = 12$

b. Is (9, 1) a solution to your equation? Explain how you know.
No, because $9 - 3(1)$ is not 12

c. Find the second number if the first is 18. 2

d. Find the first number if the second number is $\frac{1}{3}$. 13

Student Workbook


3 Practice Problems

5 from Unit 3, Lesson 13
You have two numbers, (x, y). If you triple the second number and subtract it from the first, the difference is 12.

a. Write an equation that describes the statement.

b. Is (9, 1) a solution to your equation? Explain how you know.

c. Find the second number if the first is 18.

d. Find the first number if the second number is $\frac{1}{3}$.

Learning Targets
+ I can explain and use a rule for raising a power of 10 to a power.