Lesson 2

Using Diagrams to Represent Addition and Subtraction (Optional)

Goals

- Compare and contrast (orally and in writing) vertical calculations and base-ten diagrams that represent adding and subtracting decimals.
- Explain (in words and through other representations) that adding and subtracting decimals requires combining digits that represent like base-ten units.
- Interpret and create diagrams that represent 10 like base-ten units being composed into 1 unit of higher place value, such as 10 tenths being composed as 1 one.

Learning Targets

- I can use diagrams to represent and reason about addition and subtraction of decimals.
- I can use place value to explain addition and subtraction of decimals.
- I can use vertical calculations to represent and reason about addition and subtraction of decimals.

Lesson Narrative

This lesson is optional because it reviews place-value concepts from earlier grades. The reasoning here reinforces students' understanding of the relationship between base-ten units, preparing students to compute sums and differences of any decimals algorithmically.

Students use two methods—base-ten diagrams and vertical calculations—to add and subtract decimals. Central to both methods is an understanding about the meaning of each digit in the numbers and how the different digits are related. Students recall that we add the values of two digits only if they represent the same base-ten units. They also recall that when the value of a base-ten unit is 10 or more, we can express it with a larger unit that is 10 times higher in value. For example, 10 tens can be expressed as 1 hundred, and 12 hundredths can be expressed as 1 tenth and 2 hundredths. This idea becomes more explicit to students as they make use of the structure in the diagrams and in vertical calculations.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1, Activity 2)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Base-ten blocks: Activity 1, Activity 2, Activity 3
- Graph paper: Activity 2, Activity 3

Materials to Copy

- Squares and Rectangles Cutouts (1 copy for every 1 student): Activity 1, Activity 2, Activity 3

Required Preparation

Activity 1:
Prepare either physical base-ten blocks or paper cutouts of base-ten representations from the blackline master.

For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
Prepare either physical base-ten blocks or paper cutouts of base-ten representations from the blackline master.

For the digital version of the activity, acquire devices that can run the applet.

Activity 3:
Prepare either physical base-ten blocks or paper cutouts of base-ten representations from the blackline master.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	15 min	10 min	5 min

Lesson 2

Using Diagrams to Represent Addition and Subtraction (Optional)

Lesson Narrative (continued)

In all activities in the lesson, students are prompted to create visual representations of base-ten units. This can be done by drawing base-ten diagrams, using physical base-ten blocks (if available), or using paper cutouts of base-ten representations from the blackline master. Consider providing access to these tools as alternatives to drawing.

Some students might find graph paper helpful for aligning the digits for vertical calculations. Consider making graph paper accessible for activities that involve addition and subtraction of decimals.

Student Learning Goal

Let's represent addition and subtraction of decimals.

Warm-up

Changing Values

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to review place value when working with decimals. There are many ways students might find the numbers represented by the large rectangle and large square. However, the goal is to recognize that each digit in a base-ten number represents a unit that is 10 times larger than the digit immediately to its right. This observation can be made by looking for structure in diagrams and in the numbers represented by each small shape and the larger shape it composes.

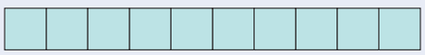
Launch

Give students 1–2 minutes of quiet think time. Encourage students to look for patterns as they work. Select students with correct responses, and ask them to share during the whole-class discussion.

Students may benefit from reviewing place-value names for decimals. Consider displaying a place-value chart for reference, or inviting students to name each number in the *Student Task Statement* before they answer the questions.

Student Task Statement

1. Here is a rectangle.



What number does the rectangle represent if each small square represents:

a. 1

10

b. 0.1

1

c. 0.01

0.1

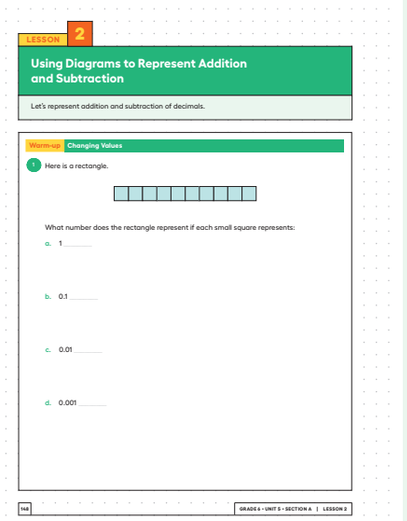
d. 0.001

0.01

Building on Student Thinking

Some students may continually use skip-counting (by 10, by 0.1, and so on) to find the value of the rectangle and the square, rather than making connections to place value. Ask these students if they see a pattern in their skip-counting (for example, in the number of times they skip-counted to answer each question), or if they see a relationship between the value of each of the smaller units to that of the larger unit they compose.

Student Workbook


LESSON 2

Using Diagrams to Represent Addition and Subtraction

Let's represent addition and subtraction of decimals.

Warm-up Changing Values

1. Here is a rectangle.

What number does the rectangle represent if each small square represents:

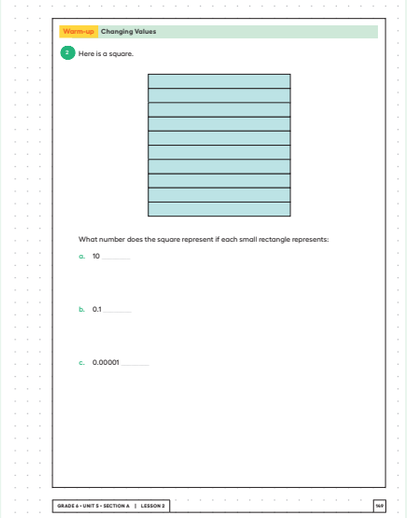
a. 1

b. 0.1

c. 0.01

d. 0.001

Student Workbook


Warm-up Changing Values

2 Here is a square.

What number does the square represent if each small rectangle represents:

a. 10 _____

b. 0.1 _____

c. 0.00001 _____

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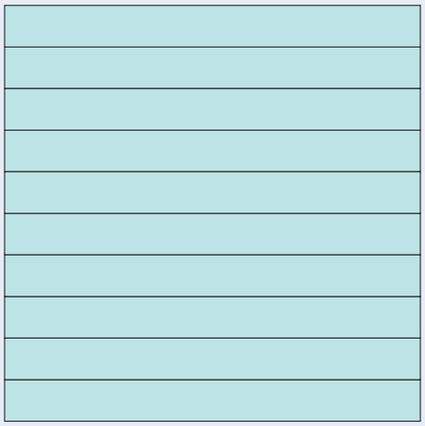
2. Here is a square.



What number does the square represent if each small rectangle represents:

a. 10

100

b. 0.1

1

c. 0.00001

0.0001

Activity Synthesis

Ask previously selected students to share their responses. Record each set of answers in a table, aligning the decimal points vertically, as shown:

value of a small square	value of the rectangle
1	10
0.1	1
0.01	0.1
0.001	0.01

value of a small rectangle	value of the large square
10	100
0.1	1
0.00001	0.0001

Ask students:

- *"What relationship do you see between the values in the two columns of each table?"*

  The values in the right column are 10 times the values in the left column.

- *"What do you notice about the position of the 1 in the two numbers in each row?"*

  It moves one place to the left, for example, from being in the tens place to being in the hundreds place.

- *"How does the value of the 1 change as it shifts one place to the left?"*

  Its value is 10 times as much.

- *"Why might these representations be called 'base-ten diagrams'?"*

  Each larger shape is made of 10 smaller shapes and has a value that is 10 times that of the smaller shape.

Activity 1

Squares and Rectangles

15 min

Activity Narrative

There is a digital version of this activity.

This activity continues to reinforce students' understanding of place value in base-ten numbers. Namely, that the value of a digit is 10 times the value of the same digit immediately to its right (and $\frac{1}{10}$ the value of the same digit immediately to its left).

Students see—in both the diagrams and the numbers being represented—that 10 hundredths has the same value as 1 tenth, that 10 thousandths has the same value as 1 hundredth, and so on. They use this understanding to create equivalent diagrams that represent decimals to thousandths.

In the digital version of the activity, students use an applet to create base-ten diagrams that represent numbers to two decimal places. The applet allows students to generate blocks that represent ones, tenths, and hundredths, but doesn't include blocks for thousandths, which would not be large enough to manipulate. It also allows students to compose 10 hundredths into 1 tenth and 10 tenths into 1. The digital version may reduce barriers for students who need support with fine-motor skills.

Launch

Give students 1 minute of quiet time to study the diagrams and how the squares and rectangles represent base-ten units. Then, ask students to identify the relationship between a few shapes. For example, point to the medium square, and ask what it represents and how it relates to the large rectangle.

Next, give students 7–8 minutes of quiet work time. Provide access to physical base-ten blocks or paper cutouts of the base-ten diagrams, if available.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe 10 of a base-ten unit composing 1 larger unit. Display words and phrases, such as "10 of hundredths make 1 tenth," "there are 10 of tenths in 1," and "a tenth is equivalent to 10 tenths."

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

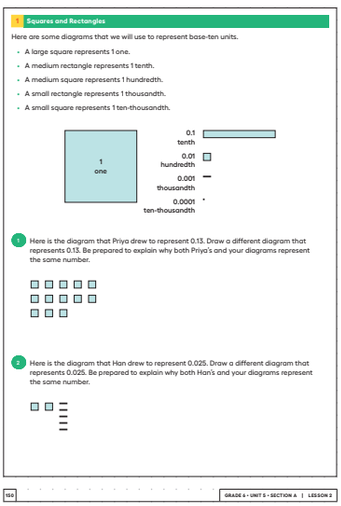
MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Building on Student Thinking

Students may recognize the relationships between numbers such as 0.8 and 0.08 but not connect the numbers to terms such as "tenths" and "hundredths." (They may use terms such as "zero point eight" and "zero point zero eight" instead.) Consider revisiting place-value names or referring to a place-value chart. Prompt students to practice using the terms by listing some numbers that increase by a tenth or a hundredth and asking students to read them aloud. For example, display "6.8, 6.9, 7, 7.1, ..." and invite students to count by tenths: "6 and 8 tenths, 6 and 9 tenths, 7, 7 and 1 tenth ...

Student Workbook


1 Squares and Rectangles

Here are some diagrams that we will use to represent base-ten units.

- A large square represents 1 one.
- A medium rectangle represents 1 tenth.
- A medium square represents 1 hundredth.
- A small rectangle represents 1 thousandth.
- A small square represents 1 ten-thousandth.

1 one

0.1 tenth

0.01 hundredth

0.001 thousandth

0.0001 ten-thousandth

1. Here is the diagram that Priya drew to represent 0.13. Draw a different diagram that represents 0.13. Be prepared to explain why both Priya's and your diagrams represent the same number.

2. Here is the diagram that Han drew to represent 0.025. Draw a different diagram that represents 0.025. Be prepared to explain why both Han's and your diagrams represent the same number.

Student Workbook


1 Squares and Rectangles

3. For each number, draw or describe two different diagrams that represent it.

a. 0.1

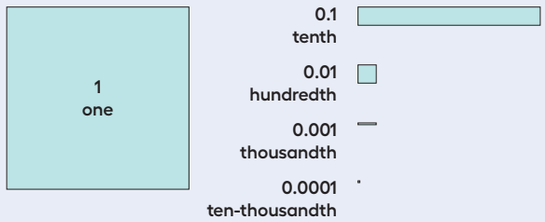
b. 0.02

c. 0.004

Student Task Statement

Here are some diagrams that we will use to represent base-ten units.

- A large square represents 1 one.
- A medium rectangle represents 1 tenth.
- A medium square represents 1 hundredth.
- A small rectangle represents 1 thousandth.
- A small square represents 1 ten-thousandth.



1. Here is the diagram that Priya drew to represent 0.13. Draw a different diagram that represents 0.13. Be prepared to explain why both Priya's and your diagrams represent the same number.

Diagram shows 1 tenth (a medium rectangle) and 3 hundredths (3 medium squares). The tenth piece is equivalent to 10 of the hundredths piece that Priya drew.

2. Here is the diagram that Han drew to represent 0.025. Draw a different diagram that represents 0.025. Be prepared to explain why both Han's and your diagrams represent the same number.

Diagram shows 25 thousandths (25 small rectangles). Twenty of the thousandth pieces are equivalent to the 2 hundredths pieces that Han drew.

3. For each number, draw or describe two different diagrams that represent it.

a. 0.1

1 tenth or 10 hundredths

b. 0.02

2 hundredths or 20 thousandths

c. 0.004

4 thousandths or 40 ten-thousandths

4. Use diagrams of base-ten units to represent each sum. Try to use as few units as possible to represent each number.

a. 0.03 + 0.05

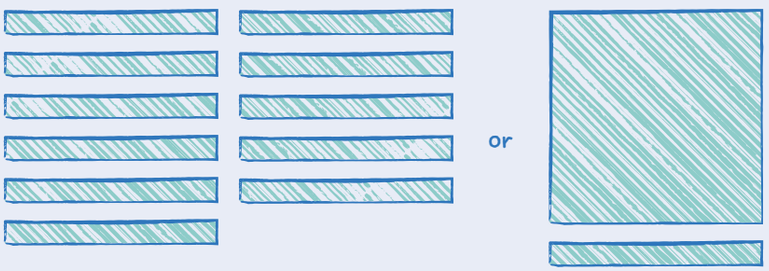
Diagram shows 8 hundredths, 0.08

b. 0.006 + 0.007

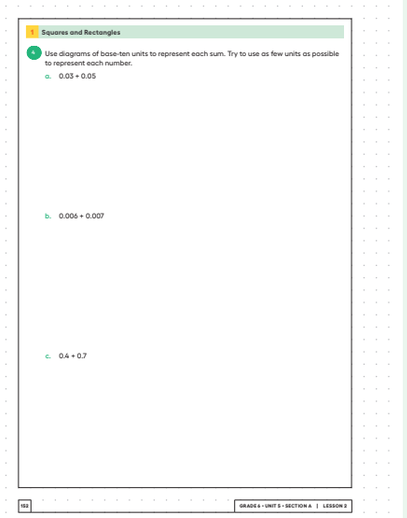
Diagram shows 13 thousandths, or 1 hundredth and 3 thousandths, 0.013

c. 0.4 + 0.7

Diagram shows 13 thousandths, or 1 hundredth and 3 thousandths, 0.013



Student Workbook


1 Squares and Rectangles

4. Use diagrams of base-ten units to represent each sum. Try to use as few units as possible to represent each number.

a. 0.03 + 0.05

b. 0.006 + 0.007

c. 0.4 + 0.7

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Activity Synthesis

The goal of this discussion is to highlight that:

- A base-ten unit can be decomposed into 10 units in the place immediately to its right.
- A group of 10 base-ten units can be composed into 1 larger unit in the place immediately to its left.

Invite students to share their diagrams or display the solutions for all to see. Discuss questions such as:

- *"Why might it be helpful to use as few base-ten blocks as possible when representing a number?"*

  Fewer pieces mean less counting and a smaller likelihood of making a counting mistake.

- *"When is it possible to use fewer shapes or blocks to represent a number?"*

  When there are at least 10 of a unit, the 10 units can be composed into 1 larger unit.

- *"What would a diagram that represents 10 look like?"*

  An extra-large rectangle, composed of 10 big squares.

Instructional Routines

MLR2: Collect and Display

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Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. For students who may be overwhelmed by the entire task statement, provide them with one question at a time. Check in with students to provide feedback and encouragement after each question.

Supports accessibility for: Attention, Social-Emotional Functioning

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

"What about 100?"

A giant square made from 10 of the extra-large rectangles representing 10.

"Why might it be inconvenient to use base-ten diagrams to represent larger multi-digit numbers or numbers with more decimal places?"

It'd mean drawing a lot of squares and rectangles. When there are more units, the diagrams become more complex.

Activity 2

Finding Sums in Different Ways

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use diagrams and the standard algorithm to find a sum that requires regrouping of base-ten units. Students see in both representations how 10 smaller base-ten units can be composed into a larger one. Along the way, students practice looking for and making use of structure.

To illustrate the idea of composing and decomposing more concretely, use of physical base-ten blocks or paper cutouts of base-ten representations (from the blackline master) is recommended.

In the digital version of the activity, students use an applet to create base-ten diagrams that represent sums of numbers to two decimal places. The applet allows students to generate blocks that represent ones, tenths, and hundredths, but doesn't include blocks for thousandths, which would not be large enough to manipulate. It also allows students to compose 10 hundredths into 1 tenth, and 10 tenths into 1. The digital version may reduce barriers for students who need support with fine-motor skills.

Launch

Arrange students in groups of 2.

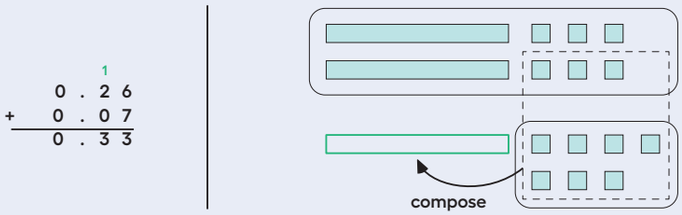
Give groups 3–4 minutes to read and discuss the answers to the first set of questions. Encourage students to use what they know about base-ten units and addition in their explanations.

Pause for a brief class discussion about students' responses before giving students 6–7 minutes of quiet time to complete the remaining questions. Provide access to base-ten representations.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier activity. Collect the language that students use to describe 10 base-ten units being composed into 1 larger unit as they add decimals. Display words and phrases, such as "bundle," "group," "put together," and "compose."

Student Task Statement

1. Here are two ways to calculate the value of 0.26 + 0.07. In the diagram, each rectangle represents 0.1 and each square represents 0.01.



Discuss with your partner:

a. Why can 10 ten squares be composed into a rectangle?

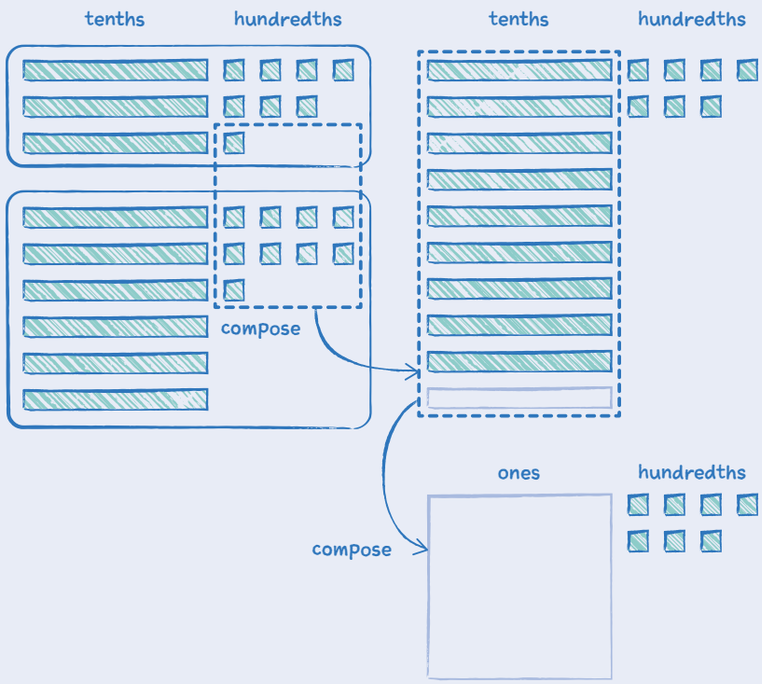
Ten squares can be composed into a rectangle because the squares each represent 1 hundredth (0.01) and the rectangles represent 1 tenth (0.1) There are 10 hundredths in a tenth.

b. How is this composition represented in the vertical calculation?

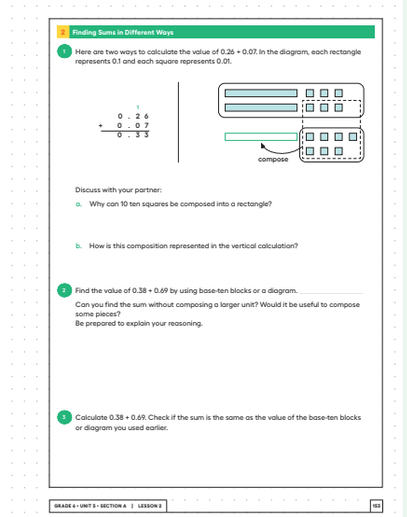
In the computation, the 7 hundredths from 0.07 are combined with 3 of the hundredths from 0.26 to make a tenth.

2. Find the value of 0.38 + 0.69 by using base-ten blocks or a diagram. Can you find the sum without composing a larger unit? Would it be useful to compose some pieces? Be prepared to explain your reasoning.

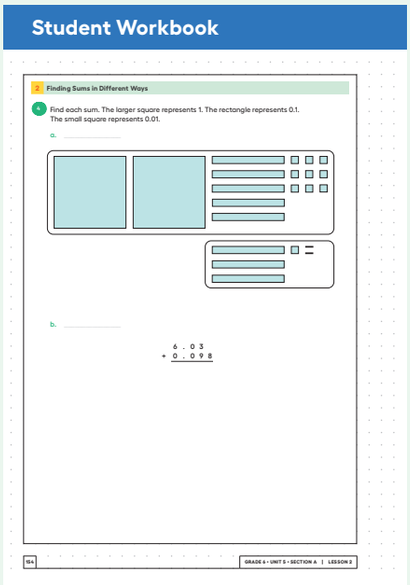
0.38 + 0.69 = 1.07



Student Workbook



Student Workbook


2 Finding Sums in Different Ways

A Find each sum. The larger square represents 1. The rectangle represents 0.1. The small square represents 0.01.

a.

b.

6 . 0 3
+ 0 . 0 9 8

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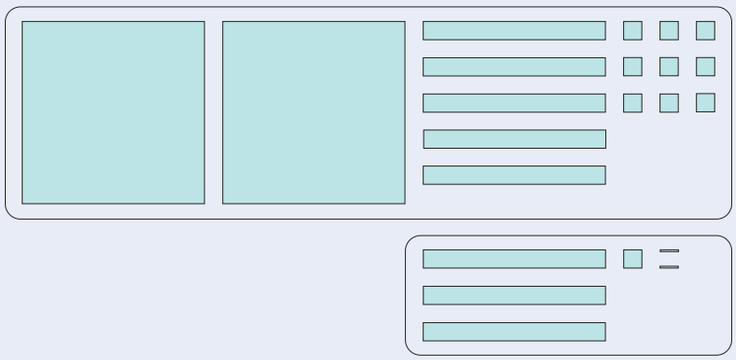
3. Calculate 0.38 + 0.69. Check if the sum is the same as the value of the base-ten blocks or diagram you used earlier.

```
  1   1
  0 . 3 8
+ 0 . 6 9
  1 . 0 7
```

4. Find each sum. The larger square represents 1. The rectangle represents 0.1. The small square represents 0.01.

a. 2.102



b.

```
  6 . 0 3
+ 0 . 0 9 8
  6 . 1 2 8
```

Are You Ready for More?

In a game, special stones are used for bartering. The values of the stones are based on their color and are ranked as shown, with red having the highest value.

red
orange
yellow
green
blue
indigo
violet

Each color is valued at 3 times the color below it in the ranking. So the value of a red stone is 3 times that of an orange stone, and the value of a green stone is 3 times that of a blue stone.

If you had 500 violet stones and wanted to trade so that you would carry as few stones as possible, which stones would you have? Explain or show your reasoning.

2 orange, 1 blue, 1 indigo, 2 violet

Sample reasoning: I would need 3 violet stones for 1 indigo stone, 9 for 1 blue stone, 27 for 1 green stone, 81 for 1 yellow stone, 243 for 1 orange stone, and 729 for 1 red stone. Of the 500 violet stones, 486 (or $2 \cdot 243$) could be traded for 2 orange stones. Of the 14 violet stones left, 9 could be traded for 1 blue stone and 3 for 1 indigo stone, leaving 2 violet stones.

Activity Synthesis

Focus the discussion on the connections across the two representations of addition of decimals. Discuss questions such as:

- *"When adding 0.38 and 0.69, in which place(s) was it possible to compose a unit from 10 of a smaller unit?"*

  In the hundredth and tenth places

- *"Why?"*

  There were a total of 17 hundredths, and 10 hundredths can be composed to make 1 tenth. This 1 tenth is added to the 3 tenths and 6 tenths, which makes 10 tenths. Ten tenths can be composed into 1 one.

- *"Where can we see the process of composing a larger unit in the vertical calculations?"*

  The 1 in the tenths place, above the 3 in 0.38, was composed of 10 of the 17 hundredths from adding 8 hundredths and 9 hundredths. The 1 in the ones place was composed of 10 of the 10 tenths from adding 6 tenths and 3 tenths in the numbers being added, plus the 1 tenth composed from 10 hundredths.

- *"Which method of calculating is more efficient?"*

  It depends on the complexity and size of the numbers. The drawings become hard when there are lots of digits or when the digits are large. The algorithm works well in all cases, but it is more abstract and requires that all bundling be recorded in the right places.

Highlight the idea of choosing appropriate tools to solve a problem, which is an important part of doing mathematics. Point out that physical blocks or base-ten diagrams can effectively help us understand what is happening when we add base-ten numbers, before moving on to a more generalized method.

Activity 3

Representing Subtraction

15 min

Activity Narrative

In this activity, students use base-ten diagrams and vertical calculations to perform subtraction. As with addition of decimals, students need to pay close attention to place value when calculating differences. They identify the need to pair the digits of like base-ten units when subtracting decimals and why it is helpful to line up the decimal points. The subtractions in this activity do not require regrouping or decomposing a larger unit into 10 of a smaller unit.

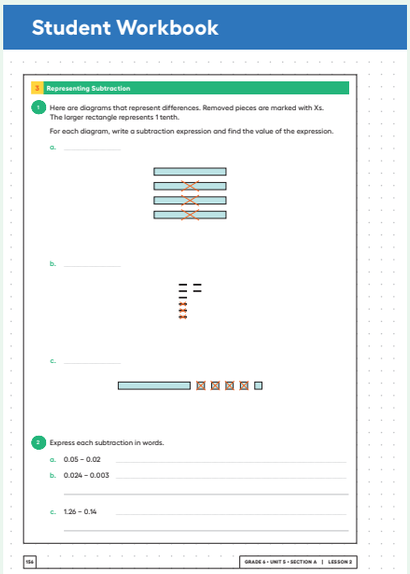
Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect.

Lead a discussion comparing, contrasting, and connecting the different representations. Display three different representations for calculating the sum of 0.38 and 0.69—using base-ten blocks, a diagram, and vertical calculation. Ask students to identify how the action of composing a larger unit from 10 smaller units is represented. For instance, in the base-ten blocks, the pieces are physically joined or traded, in the diagram the hundredths are circled, and in the vertical calculation the "1" is notated.

Advances: Representing, Conversing

Student Workbook


3 Representing Subtraction

1 Here are diagrams that represent differences. Removed pieces are marked with Xs. The larger rectangle represents 1 tenth.
For each diagram, write a subtraction expression and find the value of the expression.

a.

b.

c.

2 Express each subtraction in words.

a. 0.05 – 0.02

b. 0.024 – 0.003

c. 1.26 – 0.14

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Launch

Arrange students in groups of 2. Review that the term "difference" means the result of a subtraction. Tell students that in this activity they will use base-ten diagrams to find differences. Ask them to use X's in the diagrams to indicate what is being taken away.

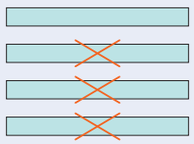
Give students 7–8 minutes of quiet work time, followed by time to discuss their responses with their partner. Provide access to graph paper in case students wish to use it for aligning the digits when using vertical calculations.

Student Task Statement

1. Here are diagrams that represent differences. Removed pieces are marked with Xs. The larger rectangle represents 1 tenth.

For each diagram, write a subtraction expression and find the value of the expression.

a. 0.4 – 0.3 = 0.1



b. 0.008 – 0.003 = 0.005

c. 0.15 – 0.04 = 0.11

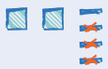
2. Express each subtraction in words.

a. 0.05 – 0.02

Sample response: five hundredths minus two hundredths

b. 0.024 – 0.003

Sample response: the difference between twenty four thousandths and three thousandths



c. 1.26 – 0.14

Sample response: subtract fourteen hundredths from one and twenty six hundredths

3. Find each difference by drawing a diagram and by calculating with numbers. Make sure the answers from both methods match. If not, check your diagram and your numerical calculation.

a. 0.05 – 0.02

$$\begin{array}{r} 0.05 \\ -\ 0.02 \\ \hline 0.03 \end{array}$$

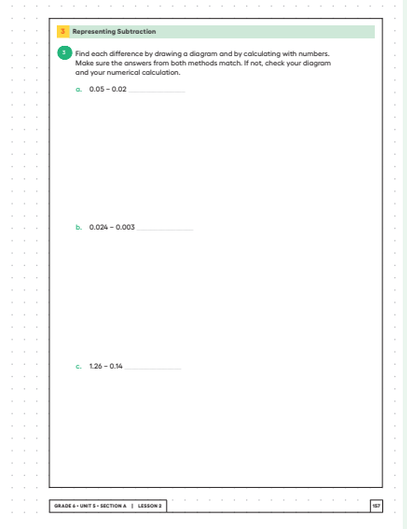
b. 0.024 – 0.003

$$\begin{array}{r} 0.024 \\ -\ 0.003 \\ \hline 0.021 \end{array}$$

c. 1.26 – 0.14

$$\begin{array}{r} 1.26 \\ -\ 0.14 \\ \hline 1.12 \end{array}$$

Student Workbook


3 Representing Subtraction

3 Find each difference by drawing a diagram and by calculating with numbers. Make sure the answers from both methods match. If not, check your diagram and your numerical calculation.

a. 0.05 – 0.02

b. 0.024 – 0.003

c. 1.26 – 0.14

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Activity Synthesis

The goal of the discussion is to make sure students understand that when we perform subtraction without diagrams, it is essential to pay close attention to place value in the numbers.

Select a few students to share their responses and reasoning for the last two questions. Display their diagrams and calculations for all to see. Discuss questions such as:

- *"How are addition and subtraction of decimal numbers alike?"*

  It is important to attend to place value and to add or subtract numbers that represent the same base-ten units.

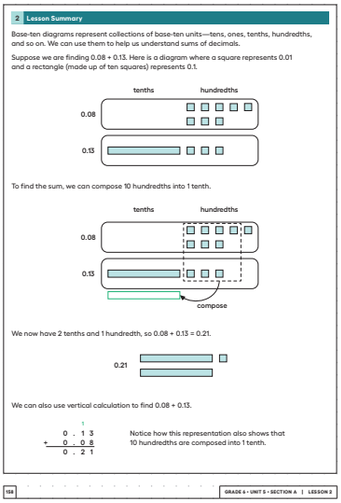
- *"Why is it helpful to line up the decimal points when calculating differences of decimals?"*

  Aligning the points helps us align digits with the same place value.

- *"Which is more efficient for subtracting decimals, using base-ten blocks or using vertical calculations?"*

  For some numbers, such as 1.26 – 0.14, both methods are efficient. If the numbers contain more decimal places or larger digits, the diagrams would take a lot of time to draw.

Student Workbook



2 Lesson Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, and so on. We can use them to help us understand sums of decimals.

Suppose we are finding 0.08 + 0.13. Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.

tenths hundredths

0.08

0.13

To find the sum, we can compose 10 hundredths into 1 tenth.

tenths hundredths

0.08

0.13

compose

We now have 2 tenths and 1 hundredth, so 0.08 + 0.13 = 0.21.

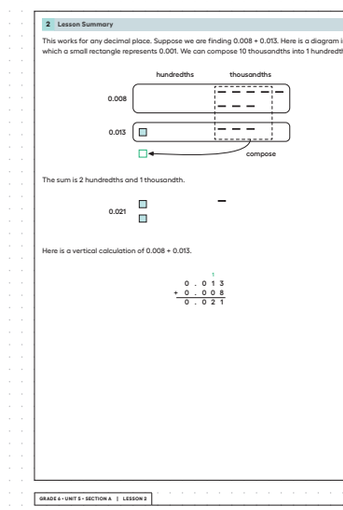
0.21

We can also use vertical calculation to find 0.08 + 0.13.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.13 \\ +\ 0.08 \\ \hline 0.21 \end{array}$$

Notice how this representation also shows that 10 hundredths are composed into 1 tenth.

Student Workbook



2 Lesson Summary

This works for any decimal place. Suppose we are finding 0.008 + 0.013. Here is a diagram in which a small rectangle represents 0.001. We can compose 10 thousandths into 1 hundredth.

hundredths thousandths

0.008

0.013

compose

The sum is 2 hundredths and 1 thousandth.

0.021

Here is a vertical calculation of 0.008 + 0.013.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.013 \\ +\ 0.008 \\ \hline 0.021 \end{array}$$

Lesson Synthesis

One main idea in this lesson is that addition of decimals beyond hundredths works the same way as addition of whole numbers and decimals to hundredths: All of them rely on combining the values of like base-ten units. Another main idea is that of composition: We can group 10 of any base-ten unit into 1 of a base-ten unit that is 10 times as large.

To highlight these ideas, ask questions such as:

- *"In a base-ten diagram, how do the pieces representing ones, tenths, hundredths, and so on help us add two decimals?"*

  When we combine the pieces that represent the same unit, we can see the total value for each decimal place.

- *"When might we want to compose some of the base-ten pieces?"*

  When we have at least 10 of the same unit.

- *"Why?"*

  It would make it simpler to see the sum.

- *"How is adding with vertical calculations similar to and different from using base-ten diagrams?"*

  We still combine numbers based on their place values but without drawing figures to represent each number.

- *"When using vertical calculations, how do we make sure that we add like base-ten units?"*

  We line up digits that represent the same place value or line up the decimal points.

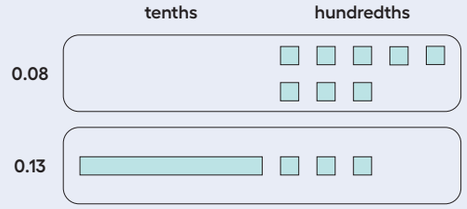
- *Which method of calculating is more efficient?*

  It depends on the size of the numbers, but vertical calculations tend to be quicker. Drawing becomes hard when the numbers have lots of digits, such as 2.315641, or when the digits are large, such as 9.999.

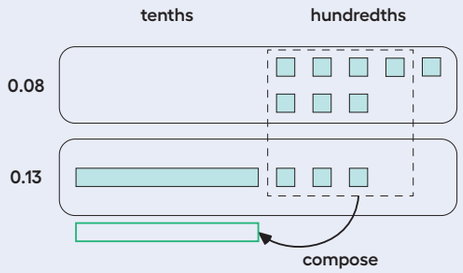
Lesson Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, and so on. We can use them to help us understand sums of decimals.

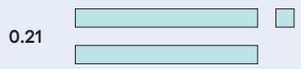
Suppose we are finding 0.08 + 0.13. Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.



To find the sum, we can compose 10 hundredths into 1 tenth.



We now have 2 tenths and 1 hundredth, so 0.08 + 0.13 = 0.21.

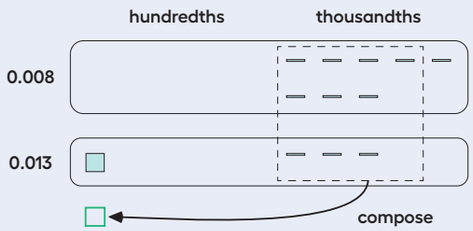


We can also use vertical calculation to find 0.08 + 0.13.

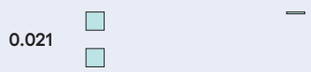
$$\begin{array}{r} {\scriptstyle 1} \\ 0.13 \\ +\ 0.08 \\ \hline 0.21 \end{array}$$

Notice how this representation also shows that 10 hundredths are composed into 1 tenth.

This works for any decimal place. Suppose we are finding 0.008+0.013. Here is a diagram in which a small rectangle represents 0.001. We can compose 10 thousandths into 1 hundredth.



The sum is 2 hundredths and 1 thousandth.



Here is a vertical calculation of 0.008 + 0.013.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.013 \\ +\ 0.008 \\ \hline 0.021 \end{array}$$

Responding To Student Thinking

Points to Emphasize

If students struggle to recognize the place value of the digits in the decimals or to recognize that only digits in the same place can be combined, reinforce these concepts throughout upcoming lessons. For example, before students begin working on the first question in the *Warm-up* referred to here, ask students to say or write the name of each decimal and to identify the place value of each digit.
Grade 6, Unit 5, Lesson 3, Warm-up
Do the Zeros Matter?

Cool-down

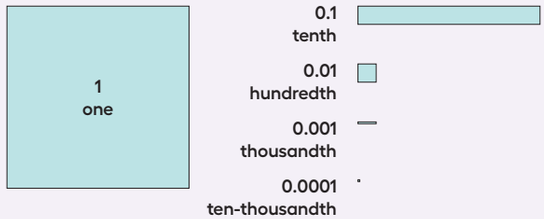
Why or Why Not?

5 min

Student Task Statement

Does adding 0.025 and 0.17 give a sum of 0.042? Explain or show your reasoning.

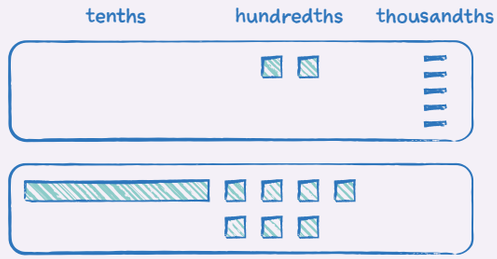
If you choose to use a diagram, you can use the following representations of base-ten units.



No.

Sample reasoning:

- The number 0.17 is greater than 0.042, so 0.042 cannot be the sum of 0.17 and another decimal.
- A diagram showing 1 medium rectangle (1 tenth), 9 medium squares (9 hundredths), and 5 small rectangles (5 thousandths).



- 0.025 + 0.17 = 0.02 + 0.005 + 0.1 + 0.07 = 0.125 + 0.07 = 0.195.
- Calculation with numbers should show the decimal points lining up and a sum of 0.195.

$$\begin{array}{r} 0.025 \\ +\ 0.170 \\ \hline 0.195 \end{array}$$

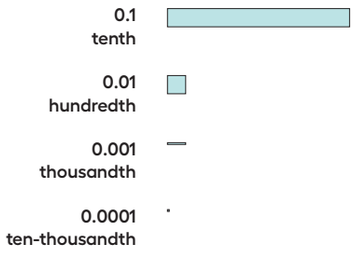
Lesson 2

Practice Problems

4 Problems

Problem 1

Use the given key to answer the questions.

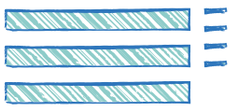


a. What number does this diagram represent?

0.025

b. Draw a diagram that represents 0.216.

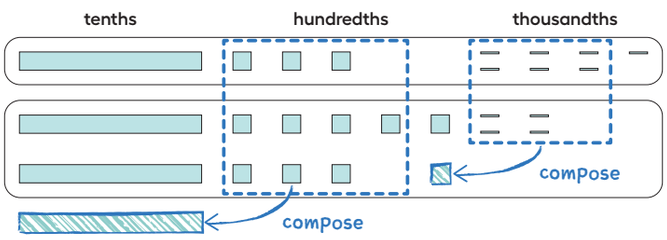
c. Draw a diagram that represents 0.304.



Problem 2

Here are diagrams that represent 0.137 and 0.284.

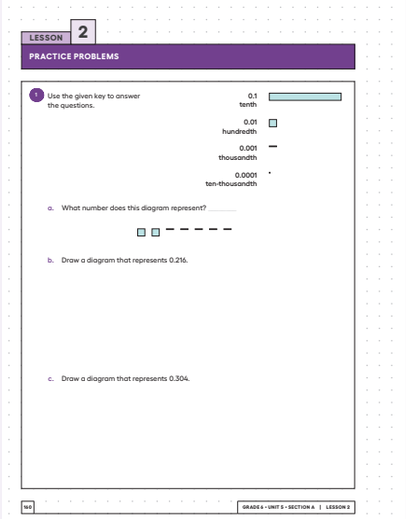
a. Use the diagram to find the value of 0.137 + 0.284. Explain your reasoning.



b. Calculate the sum vertically.

$$\begin{array}{r} 0.137 \\ +\ 0.284 \\ \hline 0.421 \end{array}$$

Student Workbook


LESSON 2

PRACTICE PROBLEMS

1 Use the given key to answer the questions.

0.1 tenth

0.01 hundredth

0.001 thousandth

0.0001 ten-thousandth

a. What number does this diagram represent? _____

b. Draw a diagram that represents 0.216.

c. Draw a diagram that represents 0.304.

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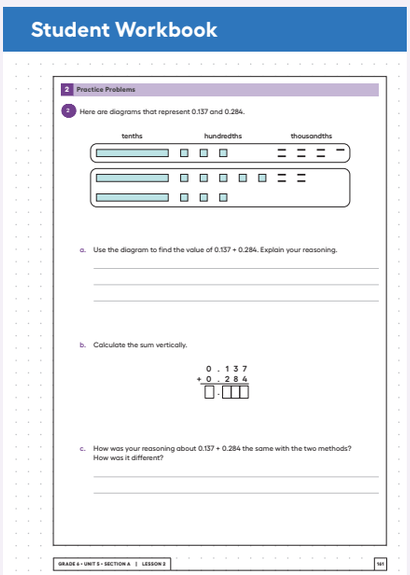
GRADE 6 • UNIT 5 • SECTION A | LESSON 2

Student Workbook


2 Practice Problems

2 Here are diagrams that represent 0.137 and 0.284.

tenths hundredths thousandths

a. Use the diagram to find the value of 0.137 + 0.284. Explain your reasoning.

b. Calculate the sum vertically.

0.137
+ 0.284
□.□□□

c. How was your reasoning about 0.137 + 0.284 the same with the two methods? How was it different?

GRADE 6 • UNIT 5 • SECTION A | LESSON 2

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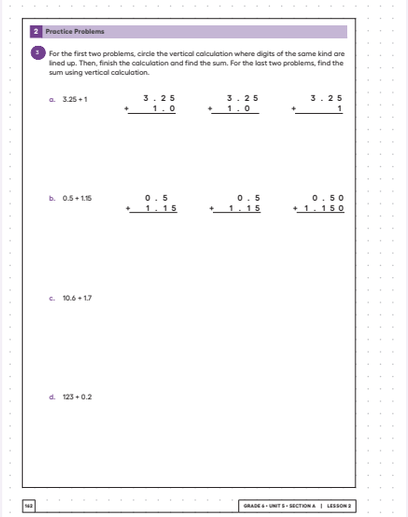
Student Workbook


2 Practice Problems

3 For the first two problems, circle the vertical calculation where digits of the same kind are lined up. Then, finish the calculation and find the sum. For the last two problems, find the sum using vertical calculation.

a. 3.25 + 1

3 . 2 5 + 1 . 0 | 3 . 2 5 + 1 . 0 | 3 . 2 5 + 1

b. 0.5 + 1.15

0 . 5 + 1 . 1 5 | 0 . 5 + 1 . 1 5 | 0 . 5 0 + 1 . 1 5 0

c. 10.6 + 1.7

d. 123 + 0.2

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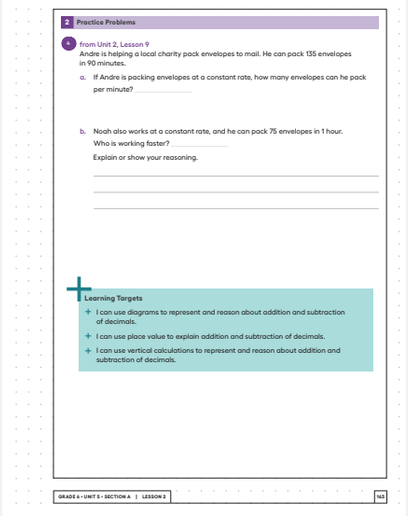
Student Workbook


2 Practice Problems

4 from Unit 2, Lesson 9

Andre is helping a local charity pack envelopes to mail. He can pack 135 envelopes in 90 minutes.

a. If Andre is packing envelopes at a constant rate, how many envelopes can he pack per minute? _____

b. Noah also works at a constant rate, and he can pack 75 envelopes in 1 hour. Who is working faster? _____

Explain or show your reasoning.

Learning Targets

+ I can use diagrams to represent and reason about addition and subtraction of decimals.
+ I can use place value to explain addition and subtraction of decimals.
+ I can use vertical calculations to represent and reason about addition and subtraction of decimals.

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c. How was your reasoning about 0.137 + 0.284 the same with the two methods? How was it different?

Sample response: Using the diagrams, 10 thousandths can be composed into 1 hundredth and 10 hundredths can be composed into 1 tenth. These values can then be combined. Without diagrams, 10 of the thousandths can be converted into 1 hundredth and 10 of the hundredths to 1 tenth. The methods are similar. The diagrams show 10 of the smaller units being composed into 1 larger unit, but the method without a diagram is faster.

Problem 3

For the first two problems, circle the vertical calculation where digits of the same kind are lined up. Then, finish the calculation and find the sum. For the last two problems, find the sum using vertical calculation.

a. 3.25 + 1

$$\begin{array}{r} 3.25 \\ +\ 1.0 \end{array} \qquad \begin{array}{r} 3.25 \\ +\ 1.0 \\ \hline 4.25 \end{array} \qquad \begin{array}{r} 3.25 \\ +\ 1 \end{array}$$

b. 0.5 + 1.15

$$\begin{array}{r} 0.5 \\ +\ 1.15 \\ \hline 1.65 \end{array} \qquad \begin{array}{r} 0.5 \\ +\ 1.15 \end{array} \qquad \begin{array}{r} 0.50 \\ +\ 1.150 \end{array}$$

c. 10.6 + 1.7

12.3

d. 123 + 0.2

123.2

Problem 4

from Unit 2, Lesson 9

Andre is helping a local charity pack envelopes to mail. He can pack 135 envelopes in 90 minutes.

a. If Andre is packing envelopes at a constant rate, how many envelopes can he pack per minute?

1.5 envelopes per minute

Sample reasoning: 135 ÷ 90 = 1.5

b. Noah also works at a constant rate, and he can pack 75 envelopes in 1 hour. Who is working faster? Explain or show your reasoning.

Andre is working faster.

Sample reasoning: There are 60 minutes in an hour. Noah packs envelopes at a rate of 1.25 per minute, because 75 ÷ 60 = 1.25.