

Benchmark Percentages

Goals

- Explain (orally and in writing) how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
- Generalize (orally) processes for calculating 10%, 25%, 50%, and 75% of a quantity.

Learning Target

When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to.

Lesson Narrative

The goal of this lesson is to help students understand the connection between benchmark percentages and common fractions. In these materials, 10%, 25%, 50%, and 75% are identified as primary benchmark percentages and multiples of 10% as secondary benchmark percentages.

In the first main activity, students find 10%, 50%, and 75% of several given numbers. The repeated reasoning allows students to notice regularity and encourages them to use fractions in computing the answers (for instance, to multiply 2,000 by $\frac{1}{10}$ when finding 10% of 2,000).

Next, students use the connections between percentages and fractions to determine the values for 100% when percentages are known, such as “9 is 50% of what number?” or “9 is 150% of what number?” Finally, students apply their insights to solve problems about discounts.

A note about percentages and fractions:

Percentages are rates, not numbers. In these materials, statements such as “75% equals $\frac{3}{4}$ ” are avoided as they equate rates and numbers. Instead, connections between percentages and fractions are made by saying, for instance, that “75% of a number” is equal to $\frac{3}{4}$ of that number.”

Student Learning Goal

Let's contrast percentages and fractions.

Lesson Timeline

5
min

Warm-up

10
min

Activity 1

10
min

Activity 2

10
min

Activity 3

10
min

Lesson Synthesis

5
min

Cool-down

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 3)

Instructional Routines

- Which Three Go Together?

Warm-up

Which Three Go Together: Shaded Diagram

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four diagrams with a shaded portion in each. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear how students talk about fractions and percentages, including the number of equal parts, the size of each part, and the size of the whole. The reasoning here can remind students that a fraction is determined in relation to one whole, and similarly, that a percentage is determined in relation to 100%.

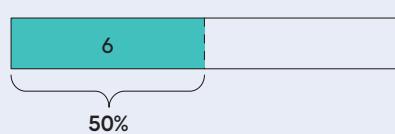
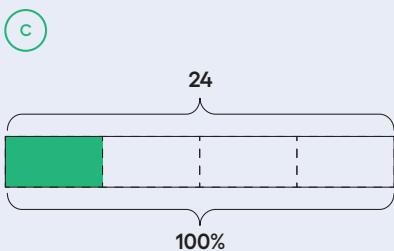
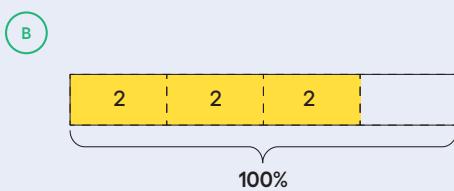
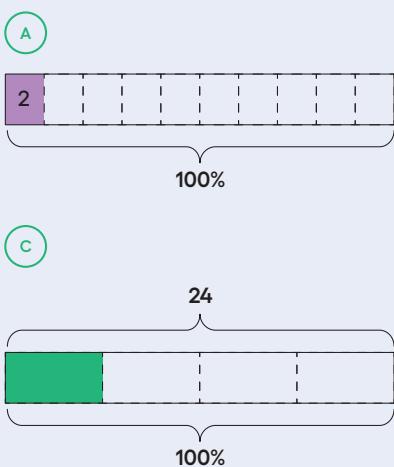
Launch



Arrange students in groups of 2–4. Display the four diagrams for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three diagrams that go together and can explain why. Next, tell each student to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, C go together because:

- The length that represents 100% is marked and labeled.

A, B, D go together because:

- The value of each shaded part is shown.
- The value for 100% is not shown.

A, C, D go together because:

- Only one part in each diagram is shaded.
- The shaded part is a unit fraction ($\frac{1}{10}$, $\frac{1}{4}$, and $\frac{1}{2}$).

Instructional Routines

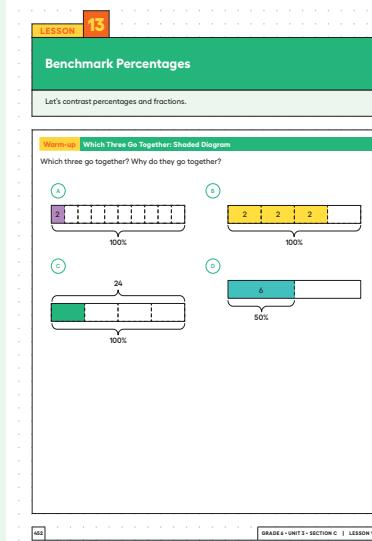
Which Three Go Together?

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Student Workbook



Access for Students with Diverse Abilities (Activity 1, Launch)**Engagement: Develop Effort and Persistence.**

Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as “I noticed _____ so I ...” “How did you get ...?” “It looks like _____ represents ...”

Supports accessibility for: Language, Social-Emotional Functioning

B, C, D go together because:

- The shaded regions all have a value of 6.
- The percentage of shaded regions in each diagram is a multiple of 25 (75%, 25%, 50%).

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students’ explanations, and ensure that the reasons given are correct.

During the discussion, prompt students to clarify their reasoning as needed, especially with regard to the percentage of the shaded region and the value of 100% in each diagram. Consider asking:

“*How do you know ...?*”

“*What do you mean by ...?*”

“*Can you say that in another way?*”

If no students referred to the shaded parts in terms of fractions, ask,

“*Are there other ways to describe the size of each shaded region in each diagram?*”

$\frac{1}{10}$ of A, $\frac{3}{4}$ of B, $\frac{1}{4}$ of diagram C, and $\frac{1}{2}$ of D

Activity 1**Liters, Meters, and Hours**

10
min

Activity Narrative

In this activity, students calculate three different benchmark percentages—50%, 10%, and 75%—given three different values that correspond to 100%. Calculating the same benchmark percentage for different quantities encourages students to notice regularity through repeated reasoning. The goal is for students to generalize the patterns in their calculations and determine how to find those percentages when the 100% value is x .

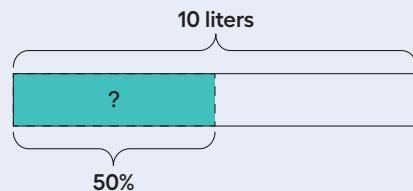
Tape diagrams that represent 50% of different quantities are given for the first set of questions. The diagrams illustrate the same structure across the three situations, encouraging students to look for and make use of structure in answering all questions.

Launch

Ask students to complete the first three sub-questions of each problem mentally. If necessary, clarify that “using mental math” means working out an answer without writing down the calculations and just recording the answer. For the last sub-question, ask them to write a sentence or two to explain their approach. Give students quiet think time to complete the activity and then time to share their explanation with a partner.

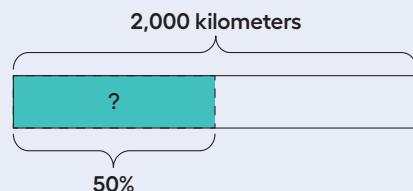
Student Task Statement

- 1. a.** How much is 50% of 10 liters of milk?



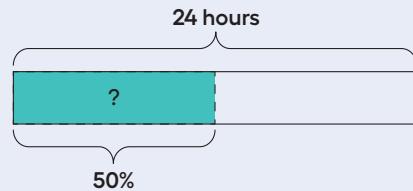
5 liters of milk

- b.** How far is 50% of a 2,000-kilometer trip?



1,000 kilometers

- c.** How long is 50% of a 24-hour day?



12 hours

- d.** How can you find 50% of any number?

Divide by 2, or multiply by $\frac{1}{2}$.

- 2. a.** How far is 10% of a 2,000-kilometer trip?

200 kilometers

- b.** How much is 10% of 10 liters of milk?

1 liter of milk

- c.** How long is 10% of a 24-hour day?

2.4 hours

- d.** How can you find 10% of any number?

Divide by 10, or multiply by $\frac{1}{10}$.

- 3. a.** How long is 75% of a 24-hour day?

18 hours

- b.** How far is 75% of a 2,000-kilometer trip?

1,500 kilometers

- c.** How much is 75% of 10 liters of milk?

7.5 liters of milk

- d.** How can you find 75% of any number?

Divide by 4, and multiply by 3 (or multiply by $\frac{3}{4}$).

Building on Student Thinking

If students are unsure how to begin reasoning mentally about 10%, consider asking questions such as:

"How does 10% relate to 100%?"

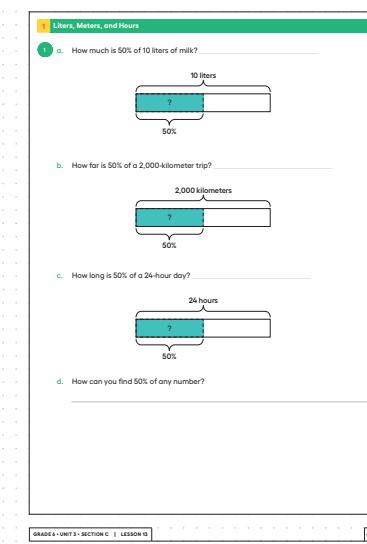
"There are 10 groups of 10% in 100%."

"How would you show 10% on a tape diagram if the length of the entire diagram represents 100%?"

Partition it into 10 equal parts and shading one part.

"How might knowing this help you think about 10% of 2,000?"

I can divide 2,000 into 10 groups for 10% and find the value of 1 group.

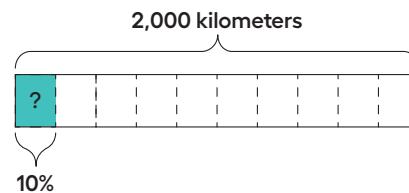
Student Workbook

Activity Synthesis

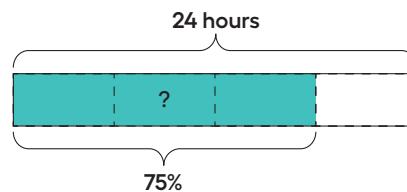
The purpose of the discussion is to highlight the benchmark fractions that correspond to 50%, 10%, and 75%.

Invite students to share their generalizations for finding 50%, 10%, and 75% of any number. Highlight the following points:

- 50% of a quantity is $\frac{1}{2}$ of that quantity. We can calculate it by dividing the quantity by 2 or multiplying the quantity by $\frac{1}{2}$. If no students bring up the latter, ask what number they could multiply the quantity by to get the same answer. (Either $\frac{1}{2}$ or 0.5 is fine.)
- 10% of a quantity is $\frac{1}{10}$ of that quantity. We can calculate it by dividing the quantity by 10 or multiplying the quantity by $\frac{1}{10}$.



- 75% of a quantity is $\frac{3}{4}$ of that quantity. We can calculate it by dividing the quantity by 4 to find 25% of the quantity and then multiplying the result by 3. Another way is to multiply the quantity by $\frac{3}{4}$.



If 25% is not mentioned by students when discussing 75% of a quantity, ask how they might find 25% of any quantity. Highlight that it can be calculated by dividing the quantity by 4 or multiplying it by $\frac{1}{4}$.

Activity 2

Nine is ...

10
min**Activity Narrative**

In this activity, students find the values that correspond to 100% when different benchmark percentages are known. They answer questions such as: “9 is 50% of what number?” Students are asked to calculate these values mentally and to explain their reasoning. To encourage students to continue making connections with fractions, a tape diagram is given to represent the quantities in the first question.

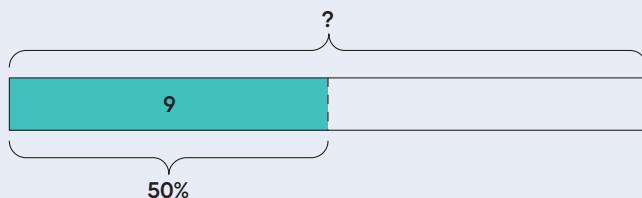
Reasoning repeatedly about the same number (9) as different percentages encourages students to look for and make use of structure. In explaining their reasoning, students practice communicating with precision.

Launch

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

Student Task Statement

Calculate each value mentally. Be prepared to explain your reasoning.



1. 9 is 50% of what number?

18. Sample reasoning: 50% of a quantity is $\frac{1}{2}$ of that quantity. Doubling the value for 50% gives 100% of the quantity. $2 \cdot 9 = 18$

2. 9 is 25% of what number?

36. Sample reasoning: 25% of a number is $\frac{1}{4}$ of that number, so multiplying 9 by 4 gives that number.

3. 9 is 10% of what number?

10. Sample reasoning: 10% is $\frac{1}{10}$ of 100%, so 9 is $\frac{1}{10}$ of that number.

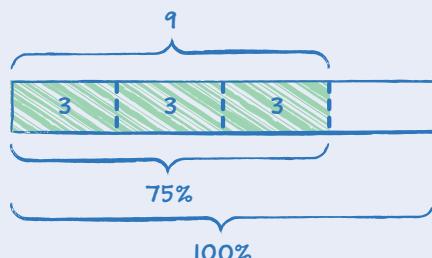
Multiplying 9 by 10 gives that number.

4. 9 is 75% of what number?

12. Sample reasoning:

- If 9 is 75% of a number, then dividing 9 by 3 gives 25% of the number and multiplying the result by 4 gives 100% of the number. $9 \div 3 = 3$ and $4 \cdot 3 = 12$

- 75% is $\frac{3}{4}$ of 100%, so 9 is $\frac{3}{4}$ of a number and 3 is $\frac{1}{4}$ that number. This means that number is 4 times 3, which is 12.

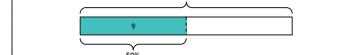
**Student Workbook**

- 1. Leters, Meters, and Hours**
- How far is 10% of a 2,000-kilometer trip?
 - How much is 10% of 10 liters of milk?
 - How long is 10% of a 24-hour day?
 - How can you find 10% of any number?

- 2. Nine is ...**
- How long is 75% of a 24-hour day?
 - How far is 75% of a 2,000-kilometer trip?
 - How much is 75% of 10 liters of milk?
 - How can you find 75% of any number?

3. Nine is ...

Calculate each value mentally. Be prepared to explain your reasoning.



- 9 is 50% of what number?
- 9 is 25% of what number?
- 9 is 10% of what number?
- 9 is 75% of what number?
- 9 is 150% of what number?

456 GRADE 6 • UNIT 3 • SECTION C | LESSON 13

Building on Student Thinking

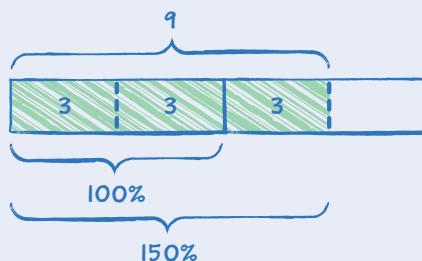
While some students may mentally calculate the answers quickly and be able to explain their reasoning abstractly, others may find it helpful to use diagrams to make sense of the questions or to explain their thinking. Consider asking:

"How might you use a tape diagram to show 9 is 25% of something or 9 is 10% of something?"

Encourage them to look for a pattern in how they partition their diagrams and in the computations they perform to answer the series of questions.

5. 9 is 150% of what number?**6. Sample reasoning:**

- If 9 is 150% of a number, then 3 is 50% of that number, and twice 3 (which is 6) is that number.
- 150% is $\frac{3}{2}$ of 100%, so 9 is $\frac{3}{2}$ of a number and 3 is $\frac{1}{2}$ of that number. This means that number is 6, or 2 times 3.



- 100% is $\frac{2}{3}$ of 150%. $\frac{2}{3}$ of 9 is 6.

Activity Synthesis

Before inviting students to share their responses, use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the first question by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

“9 is 50% of 4.5 because $9 \cdot \frac{1}{2} = 4.5$ ”

Ask,

“What parts of this response are unclear, incorrect, or incomplete?”

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Briefly discuss the responses to the questions about 9 being 25% and 10% of another number. Consider drawing a tape diagram that can illustrate why the responses are $4 \cdot 9$ and $10 \cdot 9$, respectively.

Then, focus the discussion about how students reasoned about the questions about 75% and 150%. If no students use tape diagrams in their reasoning, consider drawing one to illustrate the relationship between the numbers in each question. (See examples in Student Response.)

Emphasize that benchmark fractions such as $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ can be used to make sense of and solve problems about finding the value for 100%.

Activity 3**Shopping with Coupons**10
min**Activity Narrative**

In this activity, students solve percentage problems in the context of shopping. They consider how discounts described as percentages translate into reduced prices and the other way around. (In other words, they find A and C where $A\%$ of B is C). In each problem, students need to first determine what value is associated with 100% and reason accordingly.

Students may choose to reason using double number line diagrams, tape diagrams, tables, or no particular representation. For instance, to find 10% of \$15, they may first find 50% by dividing \$15 by 2, and then divide the resulting \$7.50 by 5 to obtain 10% of \$15. Those who recognize 10% as a benchmark percentage and know that it is one-tenth of 100% may find $15 \cdot \frac{1}{10}$ or $15 \div 10$.

As students work, monitor for students who use different strategies so that they can share later.

Launch 

Invite students to share what they know about coupons. Highlight responses that point out that some coupons specify amounts to be taken off in dollars (such as \$5 off) and some specify percentages (such as 10% off). Consider displaying images of coupons as shown here. Tell students that they will solve a couple of shopping problems that involve discounts.



Keep students in groups of 2. Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Instructional Routines

MLR5: Co-Craft Questions

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**Access for Multilingual Learners
(Activity 3, Student Task)**
MLR5: Co-Craft Questions.

Keep books or devices closed. Display the first statement and the image of two coupons, without revealing the question, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask,

"What do these questions have in common? How are they different?"

Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Student Workbook**Student Task Statement**

Han and Clare go shopping, and they each have a coupon.

1. Han buys an item with a normal price of \$15, and uses a 10% off coupon. How much does he save by using the coupon? Show your reasoning.

\$1.50

Sample reasoning:

- 10% of 15 is $\frac{1}{10}$ of 15, which is 1.5.

value (dollars)	percentage
15	100
1.5	10

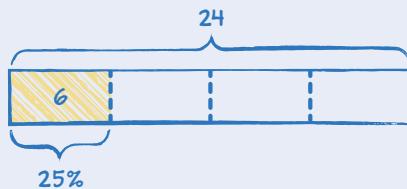
2. Clare buys an item with a normal price of \$24, but saves \$6 by using a coupon. For what percentage off is this coupon? Show your reasoning.

25%

Sample reasoning:

- There are 4 groups of 6 in 24, so 6 is $\frac{1}{4}$ of 24, which is 25% of 24.

- o



value (dollars)	percentage
24	100
6	25

Are You Ready for More?

Clare paid full price for an item. Han bought the same item for 80% of the full price. Clare said, "I can't believe I paid 125% of what you paid, Han!" Is what she said true? Explain your reasoning.

Yes

Sample reasoning: Han paid 80% or $\frac{4}{5}$ what Clare paid. So Clare paid $\frac{5}{4}$ or 125% of what Han paid.

Activity Synthesis

Select 2–3 students who used different representations and strategies to share their reasoning. As students explain, illustrate and display those representations for all to see.

If no students used fractions in their reasoning, ask them to discuss this idea.

Highlight that we can think of 10% of 15 in terms of $\frac{1}{10}$ of 15 and that we can think of 6 as $\frac{1}{4}$ of 24.

If no students created double number line diagrams or tables, demonstrate how one of these representations could be used. Ask students if the same table or double number line diagram could be used to solve both problems and discuss why or why not. Emphasize that two separate double number line diagrams or tables are necessary because the value for 100% is different in each case.

Lesson Synthesis

Certain percentages are easy to think about in terms of fractions.

To highlight this idea, ask students how we can use a fraction to find each benchmark percentage.

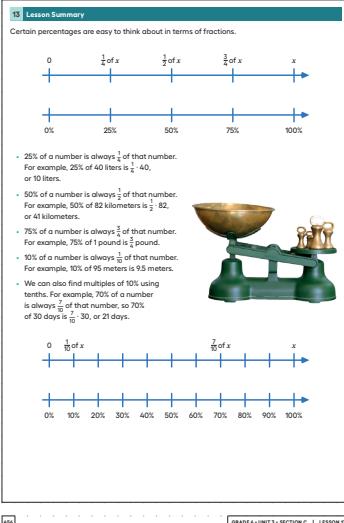
Reiterate that:

- 50% of a number is always $\frac{1}{2}$ of that number.
- 25% of a number is always $\frac{1}{4}$ of that number.
- 75% of a number is always $\frac{3}{4}$ of that number.
- 10% of a number is always $\frac{1}{10}$ of that number.

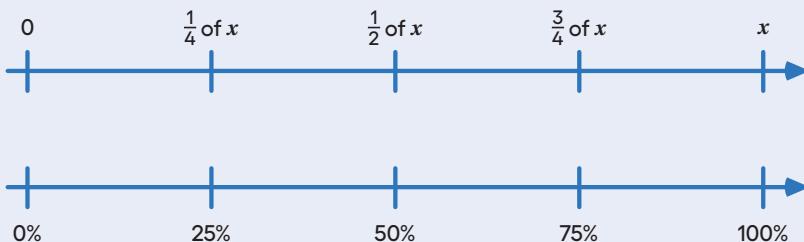
Consider demonstrating the correspondences using a couple of tables as shown—first to find benchmark percentages of a numerical value, such as 40, and then to find benchmark percentages of any value, x .

value	percentage
40	
20	$\frac{1}{2}$
10	$\frac{1}{10}$
30	
4	
	100
	50
	25
	75
	10

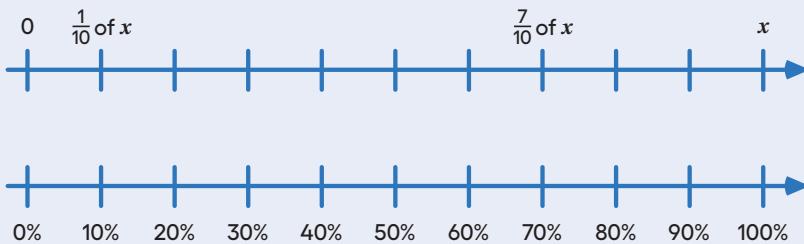
value	percentage
x	
$\frac{1}{4}x$	$\frac{1}{2}$
$\frac{1}{4}x$	$\frac{1}{10}$
$\frac{3}{4}x$	$\frac{3}{4}$
$\frac{1}{10}x$	
	100
	50
	25
	75
	10

Student Workbook**Lesson Summary**

Certain percentages are easy to think about in terms of fractions.



- 25% of a number is always $\frac{1}{4}$ of that number. For example, 25% of 40 liters is $\frac{1}{4} \cdot 40$, or 10 liters.
- 50% of a number is always $\frac{1}{2}$ of that number. For example, 50% of 82 kilometers is $\frac{1}{2} \cdot 82$, or 41 kilometers.
- 75% of a number is always $\frac{3}{4}$ of that number. For example, 75% of 1 pound is $\frac{3}{4}$ pound.
- 10% of a number is always $\frac{1}{10}$ of that number. For example, 10% of 95 meters is 9.5 meters.
- We can also find multiples of 10% using tenths. For example, 70% of a number is always $\frac{7}{10}$ of that number, so 70% of 30 days is $\frac{7}{10} \cdot 30$, or 21 days.



Cool-down**Around the Clock**5
min**Student Task Statement**

Answer each question and explain or show your reasoning.

1. How long is 10% of 60 minutes?

6 minutes

Sample reasoning: It is $\frac{1}{10}$ of an hour.

2. How long is 75% of 60 minutes?

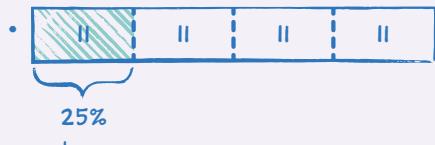
45 minutes

Sample reasoning: It is $\frac{3}{4}$ of an hour.

3. If 25% of a show is 11 minutes, how many minutes is the show?

44 minutes

Sample reasoning:



- If $\frac{1}{4}$ of the show is 11 minutes, the show must be $4 \cdot 11$ or 44 minutes.

Responding To Student Thinking**Points to Emphasize**

If students struggle with the general concept of finding percentages, focus on the way that percentages, such as 10% and 75%, compare to 100%. Do so whenever opportunities arise over the next several lessons. For example, make sure to emphasize the meaning of 100%, 40%, and 60% with multiple representations in this activity:

Unit 3, Lesson 14, Activity 1 Info Gap:
Music Devices

Student Workbook

LESSON 13
PRACTICE PROBLEMS

1. a. How can you find 50% of a number quickly in your head?

b. Andre lives 1.6 km from school. What is 50% of 1.6 km?

c. Diego lives $\frac{1}{2}$ mile from school. What is 50% of $\frac{1}{2}$ mile?

2. There is a 10% off sale on laptop computers. If someone saves \$35 on a laptop, what was its original cost?
 If you get stuck, consider using the table.

savings (dollars)	percentage
35	10
_____	_____
_____	_____

GRADE 4 • UNIT 3 • SECTION C | LESSON 13

Student Workbook

13 Practice Problems

1. Explain how to calculate these mentally.
 a. 15 is what percentage of 30?

b. 3 is what percentage of 12?

c. 6 is what percentage of 10?

2. Noah says that to find 20% of a number he divides the number by 5. For example, 20% of 60 is 12, because $60 \div 5 = 12$. Does Noah's method always work? Explain why or why not.

3. From Unit 3, Lesson 10
 Diego has 75% of \$10. Noah has 25% of \$30. Who has more money?
 Explain your reasoning.

GRADE 4 • UNIT 3 • SECTION C | LESSON 13

Practice Problems

6 Problems

Problem 1

- a. How can you find 50% of a number quickly in your head?

Sample response: Divide the number by 2 (or multiply it by $\frac{1}{2}$).

- b. Andre lives 1.6 km from school. What is 50% of 1.6 km?

0.8 km (or equivalent)

- c. Diego lives $\frac{1}{2}$ mile from school. What is 50% of $\frac{1}{2}$ mile?

$\frac{1}{4}$ mile (or equivalent)

Problem 2

There is a 10% off sale on laptop computers. If someone saves \$35 on a laptop, what was its original cost?

\$350

If you get stuck, consider using the table.

savings (dollars)	percentage
35	10
_____	_____
_____	_____

Problem 3

Explain how to calculate these mentally.

- a. 15 is what percentage of 30?

50%

Fifteen is $\frac{1}{2}$ of 30, so that is 50%.

- b. 3 is what percentage of 12?

25%

Three is $\frac{1}{4}$ of 12, so that is 25%.

- c. 6 is what percentage of 10?

60%

Six is $\frac{6}{10}$ of 10, and each $\frac{1}{10}$ is 10%.

Lesson 13 Practice Problems

Problem 4

Noah says that to find 20% of a number he divides the number by 5. For example, 20% of 60 is 12, because $60 \div 5 = 12$. Does Noah's method always work? Explain why or why not.

Yes

Sample reasoning: Twenty percent of a number is $\frac{20}{100}$ times the number, and $\frac{20}{100} = \frac{1}{5}$. Multiplying by $\frac{1}{5}$ gives the same result as dividing by 5.

Problem 5

from Unit 3, Lesson 10

Diego has 75% of \$10. Noah has 25% of \$30. Who has more money? Explain your reasoning.

They each have \$7.50

Sample reasoning: $\frac{3}{4} \cdot 10 = 7.50$ and $\frac{1}{4} \cdot 30 = 7.50$.

Problem 6

from Unit 3, Lesson 9

Two ants are at the opposite end of a 22-meter long cable. They start walking toward each other at the same time, each at a constant speed. The smaller ant walks at a speed of 2.5 meters per minute. The larger ant walks at a speed of 3 meters per minute.

Here is a table showing the distances the ants traveled and how far apart they are over time.

Use the table to find how much time passes before they meet.

4 minutes

Sample reasoning:

elapsed time (minutes)	distance traveled by smaller ant (meters)	distance traveled by larger ant (meters)	distance apart (meters)
0	0	0	22
1	2.5	3	16.5
2	5	6	11
3	7.5	9	5.5
4	10	12	0

Student Workbook

13 Practice Problems

From Unit 3, Lesson 9
Two ants are at the opposite end of a 22-meter long cable. They start walking toward each other at the same time, each at a constant speed. The smaller ant walks at a speed of 2.5 meters per minute. The larger ant walks at a speed of 3 meters per minute.

Here is a table showing the distances the ants traveled and how far apart they are over time.

Use the table to find how much time passes before they meet.

elapsed time (minutes)	distance traveled by smaller ant (meters)	distance traveled by larger ant (meters)	distance apart (meters)
0	0	0	22
1	2.5	3	16.5
2	5	6	11
3	7.5	9	5.5
4	10	12	0

Learning Targets

- When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to.



GRADE 6 • UNIT 3 • SECTION C | LESSON 13