Lesson 8

# Equal and Equivalent

## Goals

- Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values.
- Justify (using words or other representations) whether two expressions are "equivalent," or equal to each other for every value of their variable.

## Learning Targets

- I can explain what it means for two expressions to be equivalent.
- I can use what I know about operations to decide whether two expressions are equivalent.

## Lesson Narrative

In this lesson, students are introduced to **equivalent expressions**. Two expressions are equivalent if they always have the same value. If two expressions with variables are equal whenever the same value is substituted for the variables, the expressions are equivalent.

Students begin by using tape diagrams to justify whether two numerical expressions are equal. They then use tape diagrams to represent two expressions with variables when given a value for the variable. They use the diagrams to distinguish between *equivalent expressions*, which have the same value for all values of the variable, and expressions that may be equal for some, but not all, values of a variable. Students then identify simple equivalent expressions, using the structure of the expressions and what they know about the relationships between operations and their properties.

## Student Learning Goal

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

## Lesson Timeline

| Warm-up | Activity 1 | Activity 2 | Lesson Synthesis | Cool-down (Assessment) |
|---|---|---|---|---|
| 5 min | 20 min | 10 min | 10 min | 5 min |

### Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

### Access for Multilingual Learners

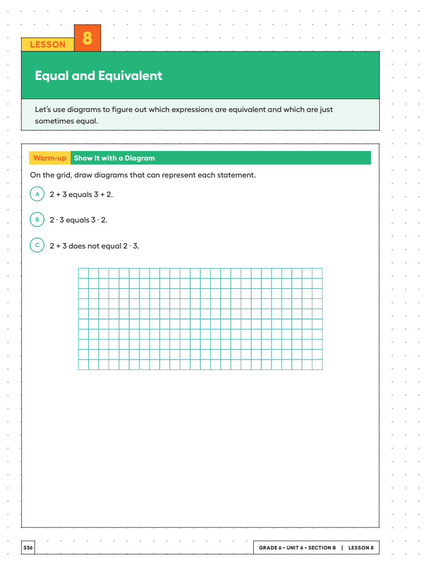
- MLR8: Discussion Supports (Activity 1)

### Required Materials

**Materials to Gather**

- Graph paper: Activity 1

**Student Workbook**


LESSON 8

Equal and Equivalent

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

Warm-up Show It with a Diagram

On the grid, draw diagrams that can represent each statement.

A 2 + 3 equals 3 + 2.

B 2 · 3 equals 3 · 2.

C 2 + 3 does not equal 2 · 3.

## Warm-up
## Show It with a Diagram

5 min

### Activity Narrative

In this *Warm-up*, students use diagrams to explain why two expressions are or are not equal. Students look for and make use of the structure of the diagrams and expressions to explain their reasoning. Later, they will extend the reasoning used here to expressions with variables.

Monitor for students who use the length of the diagram (in grid units) to represent numbers and align their tape diagrams on one side to make it easier to compare them.

When discussing the connections between the diagrams and expressions, students have opportunities to share the language they have for describing the commutative properties of addition and multiplication. This may be some students' introduction to the commutative property by name. Students informally used the commutative property in earlier grades as they made sense of operations and explained computation strategies. It is fine for students to continue to use informal language to describe properties of operations throughout the unit.

### Launch

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

**Student Task Statement**

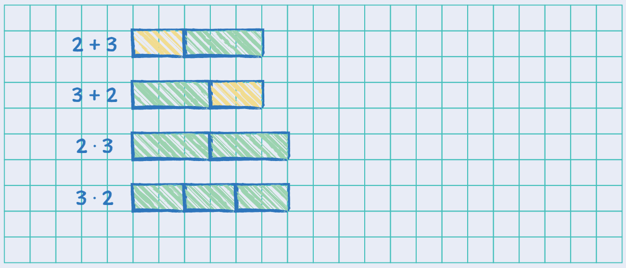
On the grid, draw diagrams that can represent each statement.

**A.** $2 + 3$ equals $3 + 2$.

**B.** $2 \cdot 3$ equals $3 \cdot 2$.

**C.** $2 + 3$ does not equal $2 \cdot 3$.

Sample response:



### Activity Synthesis

Select 1–2 students who used lengths to represent the numbers in the expressions to share their responses. Display their diagrams for all to see. Discuss questions such as:

*"How do the diagrams show whether the expressions are or are not equal?"*

*"How is multiplication shown in the diagrams?"*

$3 \cdot 2$ is shown as 3 rectangles that are each 2 units in length. $2 \cdot 3$ is shown as 2 rectangles that are each 3 units in length.

Highlight that the following key ideas:

- We can tell that $2 + 3$ and $3 + 2$ are equal because the length of the diagrams represent the value of each expression, and the diagrams are the same length. The same can be said about $2 \cdot 3$ and $3 \cdot 2$.
- We can tell that $2 \cdot 3$ and $3 + 2$ are not equal because the lengths of the diagrams that represent them are not the same.
- $2 + 3$ and $3 + 2$ are examples of expressions that are not identical, but are equal in value.

Explain that the statements and diagrams in this activity demonstrate what students already know about addition and multiplication: that numbers can be added or multiplied in any order without affecting the result. Tell students to keep these ideas in mind later, when they look at whether expressions with variables are or are not equal.

If time permits, consider introducing the formal names of these properties and creating a display with the property names and examples of equations that illustrate them.

- The commutative property of addition states that the order of the addends (numbers being added) does not change the value of the sum, $a + b$ is equal to $b + a$.
- The commutative property of multiplication states that the order of the factors does not change the value of the product, $a \cdot b$ is equal to $b \cdot a$.

## Activity 1

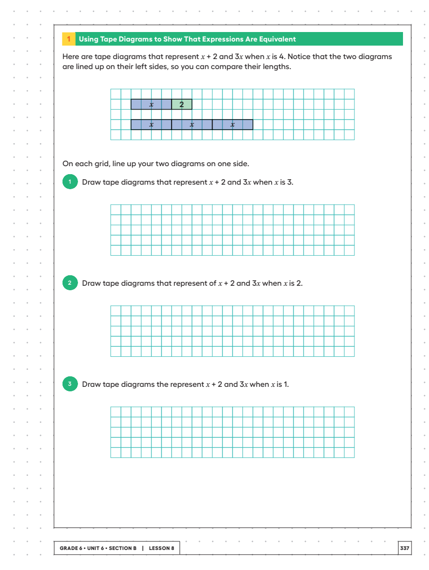
## Using Tape Diagrams to Show That Expressions Are Equivalent

20 min

### Activity Narrative

The purpose of this activity is to develop students' understanding of equivalent expressions. Students use tape diagrams to represent pairs of algebraic expressions when a one-digit whole number is given for the variable. Students see that expressions may be equal at some values for the variables, but not at others. Through repeated reasoning, they also begin to generalize that equivalent expressions are expressions that are equal at all values.

**Student Workbook**



**Access for Students with Diverse Abilities (Activity 1, Launch)**

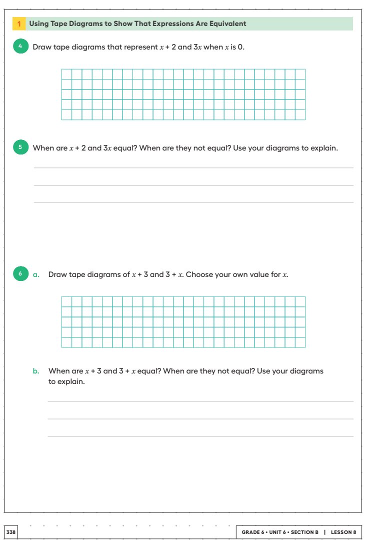
**Action and Expression: Internalize Executive Functions.**
Begin with a small-group or whole-class demonstration and a think-aloud of the first question to remind students how to draw tape diagrams on grids. Keep the worked-out examples on display for students to reference as they work. *Supports accessibility for: Memory, Conceptual Processing*

**Access for Multilingual Learners (Activity 1, Launch)**

**Representing, Conversing, Listening: MLR8: Discussion Supports.**
Display sentence frames to support students when they share their answers and explanations for the questions, "When are $x + 2$ and $3x$ equal? When are they not equal?" Examples: "I know these expressions are equal (or not equal) when ______ because…" This frame can help students use mathematical language as they connect the representations of equal and not equal values of expressions.

*Advances: Speaking, Representing*

**Student Workbook**




## Launch

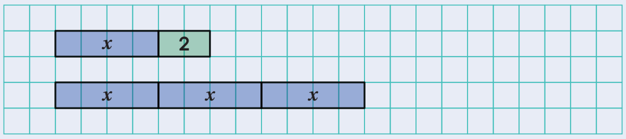
Arrange students in groups of 2. Ask students to work independently on each question and check in with their partner, discussing and resolving any disagreements, before moving on to the next question.

Give students 10–12 minutes of work time, followed by a whole-class discussion. Tell students that they should draw two separate diagrams on each grid—one diagram to represent each expression.

While a grid is provided for each question that requires drawing, some students may wish to have additional drawing space. Provide access to graph paper if requested.

### Student Task Statement

Here are tape diagrams that represent $x + 2$ and $3x$ when $x$ is 4. Notice that the two diagrams are lined up on their left sides, so you can compare their lengths.




On each grid, line up your two diagrams on one side.

1. Draw tape diagrams that represent $x + 2$ and $3x$ when $x$ is 3.

   Diagrams show a length of 5 for $x + 2$ and 9 for $3x$.

2. Draw tape diagrams that represent of $x + 2$ and $3x$ when $x$ is 2.

   Diagrams show a length of 4 for $x + 2$ and 6 for $3x$.

3. Draw tape diagrams the represent $x + 2$ and $3x$ when $x$ is 1.

   Diagrams show a length of 3 for $x + 2$ and 3 for $3x$.

4. Draw tape diagrams that represent $x + 2$ and $3x$ when $x$ is 0.

   Diagrams show a length of 2 for $x + 2$ and 0 for $3x$.

5. When are $x + 2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.

   They are equal when $x = 1$, and not equal for the other values. Sample reasoning: We can check the number of boxes in each diagram or the lengths of the diagrams to see that the expressions have equal value when $x = 1$.

6. a. Draw tape diagrams of $x + 3$ and $3 + x$. Choose your own value for $x$.

      Answers vary. Diagrams should be the same length regardless of choice of $x$.

   b. When are $x + 3$ and $3 + x$ equal? When are they not equal? Use your diagrams to explain.

      They are always equal.

      Sample reasoning: The lengths will always be the same, even though one shows the 3 first and one shows $x$ first, because the order of the parts does not change the total length.

## Activity Synthesis

The goal of this discussion is to make sure students understand what it means for two expressions with a variable to be equivalent.

Invite several students to share the two diagrams they drew for $x + 3$ and $3 + x$. Ask:

*"How are the two diagrams in each pair alike?"*

The two diagrams are always the same length regardless of the value of $x$ chosen.

*"How are the two diagrams in each pair different?"*

The placement of $x$ and 3 are not the same.

Display the equation $x + 3 = 3 + x$. Point out that we can see that this equation is always true regardless of the value of $x$. We call $x + 3$ and $3 + x$ equivalent expressions, because their values are equal no matter what the value of $x$ is.

Display the equation $x + 2 = 3x$, and ask students to consider this equation as they review the diagrams they drew on the grids for the first four problems. Point out that we can see that this equation is true when $x$ is 1, but not for the other values of $x$ that we tried. So, we can say that $x + 2$ is equal to $3x$ when $x$ is 1. Ask:

*"How do you know that $x + 2$ and $3x$ are not equivalent expressions?"*

They are not equal for every value of $x$.

**Access for Students with Diverse Abilities (Activity 2, Synthesis)**

**Engagement: Develop Effort and Persistence.**
Encourage and support opportunities for peer interactions. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class. This will provide students with additional opportunities to compare strategies and hear from others.

*Supports accessibility for: Language, Social-Emotional Functioning*

## Activity 2

## Identifying Equivalent Expressions

**10 min**

## Activity Narrative

In this activity, students apply what they know about the meaning and properties of operations to deepen their understanding of "equivalent expressions." The focus is on looking for and making use of structure, rather than identifying all the types of equivalent expressions appropriate to grade 6.

Monitor for students who apply their knowledge of operations with numbers to reason about operations with variable expressions. For instance:

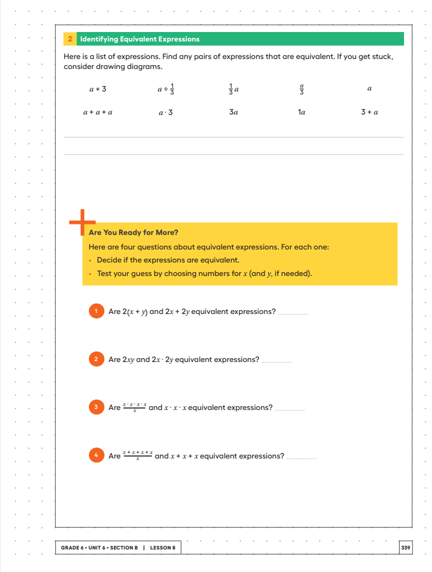
- Having learned that $4 \div \frac{1}{3}$ is equivalent to $4 \cdot 3$, students then reason that $x \div \frac{1}{3} = 3x$.
- Knowing that $4 + 4 + 4 = 3 \cdot 4$, students reason that $a + a + a = 3a$.

Students may also use diagrams to reason or to explain their reasoning.

## Launch

Give students 5 minutes of quiet work time, followed by a whole-class discussion.

**Student Workbook**



## Student Task Statement

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, consider drawing diagrams.

| | | | | |
|---|---|---|---|---|
| $a+3$ | $a \div \frac{1}{3}$ | $\frac{1}{3}a$ | $\frac{a}{3}$ | $a$ |
| $a+a+a$ | $a \cdot 3$ | $3a$ | $1a$ | $3+a$ |

- $a+3$ and $3+a$
- $a \div \frac{1}{3}$ and $a \cdot 3$
- $a+a+a$ and $3a$ (these are also equivalent to $a \div \frac{1}{3}$ and $a \cdot 3$)
- $\frac{1}{3}a$ and $\frac{a}{3}$
- $1a$ and $a$

## Are You Ready for More?

Here are four questions about equivalent expressions. For each one:

- Decide if the expressions are equivalent.
- Test your guess by choosing numbers for $x$ (and $y$, if needed).

1. Are $2(x+y)$ and $2x+2y$ equivalent expressions?

   Yes

2. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

   No

3. Are $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?

   Yes

4. Are $\frac{x+x+x+x}{x}$ and $x+x+x$ equivalent expressions?

   No

## Activity Synthesis

Invite students to share their pairs and reasoning, including those who used diagrams, if any. To highlight the idea that expressions with variables are equivalent only if they have the same value for all values of the variable, ask questions such as:

*"How can you be sure that the expressions on both sides will have the same value no matter what value is used for a?"*

*"Are there two expressions that you knew right away are equivalent, without needing to test values or draw diagrams? If so, which ones, and how did you know?"*

## Lesson Synthesis

The goal of this discussion is to ensure that students understand what is meant by equivalent expressions, and recognize how they are different from expressions that are equal only for a particular value of their variable. Ask questions such as:

*"Why are $x$ and $x \cdot 1$ equivalent expressions?"*

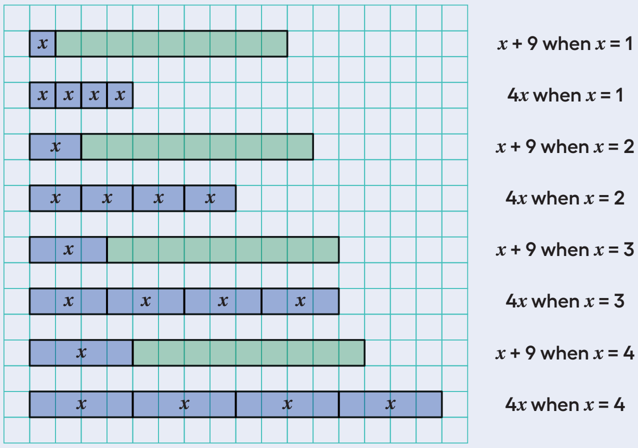
*"The expressions $x \cdot 3$ and $x$ have the same value when $x$ is 0. Why can't we call them equivalent expressions?"*

*"Are $x + 1$ and $1 + x$ equivalent expressions? How do you know?"*

If the commutative properties of addition and multiplication were not introduced to students after the *Warm-up* and if time permits, consider doing so at this time.

### Lesson Summary

We can use tape diagrams to see when expressions are equal. For example, the expressions $x + 9$ and $4x$ are equal when $x$ is 3, but they are not equal for other values of $x$.




Sometimes two expressions are equal for only one particular value of their variable. Other times, they seem to be equal no matter what the value of the variable.

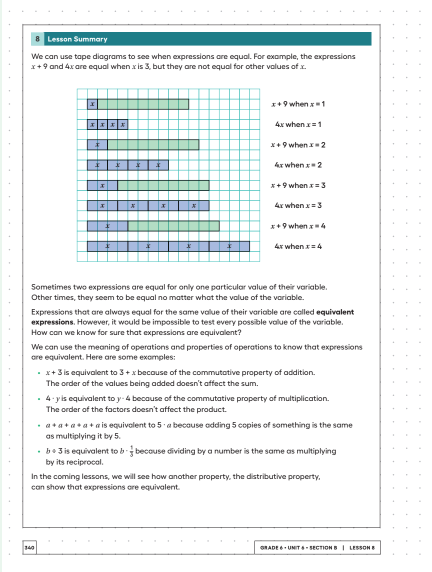
Expressions that are always equal for the same value of their variable are called **equivalent expressions**. However, it would be impossible to test every possible value of the variable. How can we know for sure that expressions are equivalent?

We can use the meaning of operations and properties of operations to know that expressions are equivalent. Here are some examples:

- $x + 3$ is equivalent to $3 + x$ because of the commutative property of addition. The order of the values being added doesn't affect the sum.
- $4 \cdot y$ is equivalent to $y \cdot 4$ because of the commutative property of multiplication. The order of the factors doesn't affect the product.
- $a + a + a + a + a$ is equivalent to $5 \cdot a$ because adding 5 copies of something is the same as multiplying it by 5.
- $b \div 3$ is equivalent to $b \cdot \frac{1}{3}$ because dividing by a number is the same as multiplying by its reciprocal.

In the coming lessons, we will see how another property, the distributive property, can show that expressions are equivalent.

**Student Workbook**



**Responding To Student Thinking**

**Points to Emphasize**
If students struggle with determining if expressions are equivalent, focus on this idea when opportunities arise over the next several lessons. For example, consider inviting students to reflect on the reasoning they go through in:

Grade 6, Unit 6, Lesson 8, Practice Problem 3

Grade 6, Unit 6, Lesson 9, Practice Problem 5

## Cool-down

### Decisions about Equivalence

5 min

Provide access to graph paper, in case requested.

**Student Task Statement**

Decide if the expressions in each pair are equivalent. Explain or show how you know.

1. $x + x + x + x$ and $4x$

   Equivalent

   Sample reasoning: The diagrams representing these expressions would have the same length for any value of $x$.

2. $5x$ and $x + 5$

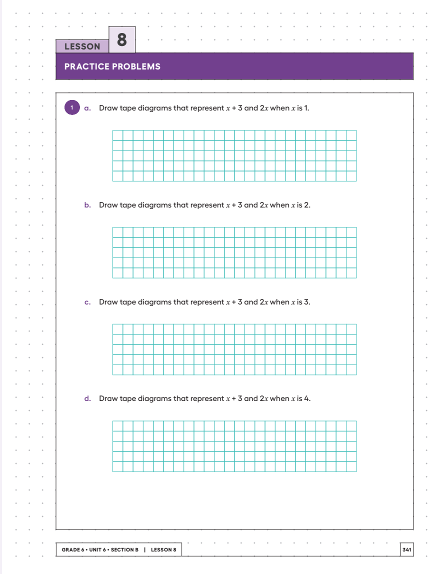
   Not equivalent

   Sample reasoning: if $x = 1$, then $5x = 5$ and $x + 5 = 6$, so they do not have the same value.

## Practice Problems

7 Problems

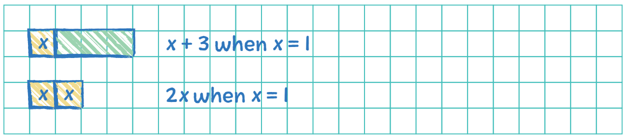
**Student Workbook**




### Problem 1

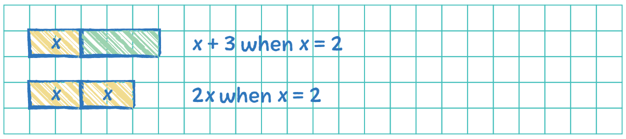
**a.** Draw tape diagrams that represent $x + 3$ and $2x$ when $x$ is 1.

Sample response:



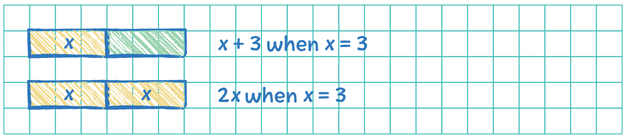

**b.** Draw tape diagrams that represent $x + 3$ and $2x$ when $x$ is 2.

Sample response:



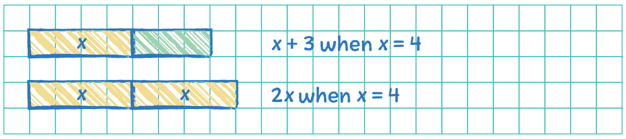

**c.** Draw tape diagrams that represent $x + 3$ and $2x$ when $x$ is 3.

Sample response:




**d.** Draw tape diagrams that represent $x + 3$ and $2x$ when $x$ is 4.

Sample response:

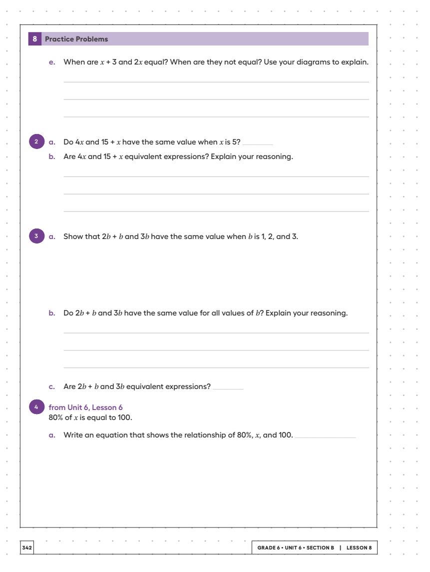



**e.** When are $x + 3$ and $2x$ equal? When are they not equal? Use your diagrams to explain.

When $x = 3$ both expressions are 6.

Sample reasoning: When $x$ is less than 3, the value of $2x$ is less than $x + 3$. When $x$ is greater than 3, the value of $2x$ is greater than $x + 3$.

**Student Workbook**



8 Practice Problems

e. When are $x + 3$ and $2x$ equal? When are they not equal? Use your diagrams to explain.

2 a. Do $4x$ and $15 + x$ have the same value when $x$ is 5?

b. Are $4x$ and $15 + x$ equivalent expressions? Explain your reasoning.

3 a. Show that $2b + b$ and $3b$ have the same value when $b$ is 1, 2, and 3.

b. Do $2b + b$ and $3b$ have the same value for all values of $b$? Explain your reasoning.

c. Are $2b + b$ and $3b$ equivalent expressions?

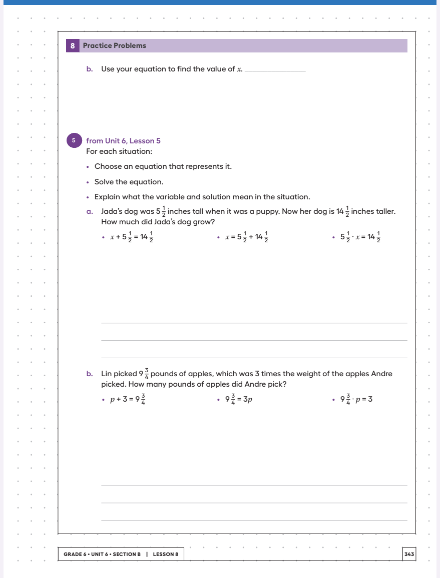
4 from Unit 6, Lesson 6

80% of $x$ is equal to 100.

a. Write an equation that shows the relationship of 80%, $x$, and 100.

342 GRADE 6 • UNIT 6 • SECTION B | LESSON 8

**Student Workbook**



8 Practice Problems

b. Use your equation to find the value of $x$.

5 from Unit 6, Lesson 5

For each situation:

- Choose an equation that represents it.
- Solve the equation.
- Explain what the variable and solution mean in the situation.

a. Jada's dog was $5\frac{1}{2}$ inches tall when it was a puppy. Now her dog is $14\frac{1}{2}$ inches taller. How much did Jada's dog grow?

- $x + 5\frac{1}{2} = 14\frac{1}{2}$
- $x = 5\frac{1}{2} + 14\frac{1}{2}$
- $5\frac{1}{2} \cdot x = 14\frac{1}{2}$

b. Lin picked $9\frac{3}{4}$ pounds of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

- $p + 3 = 9\frac{3}{4}$
- $9\frac{3}{4} = 3p$
- $9\frac{3}{4} \cdot p = 3$

GRADE 6 • UNIT 6 • SECTION B | LESSON 8 343

## Problem 2

**a.** Do $4x$ and $15 + x$ have the same value when $x$ is 5?

Yes, they both have the value 20.

**b.** Are $4x$ and $15 + x$ equivalent expressions? Explain your reasoning.

No. Sample reasoning: Equivalent expressions have the same value no matter what number is used in place of the variable. When $x$ is 1, the expression $4x$ has the value 4, but $15 + x$ has the value 16.

## Problem 3

**a.** Show that $2b + b$ and $3b$ have the same value when $b$ is 1, 2, and 3.

Sample response: When $b = 1$, they both have the value 3. When $b = 2$, they are both 6. When $b = 3$, they both have the value 9.

**b.** Do $2b + b$ and $3b$ have the same value for all values of $b$? Explain your reasoning.

Yes. Sample reasoning: $2b + b$ is the same as $3b$. Both can be written as $b + b + b$.

**c.** Are $2b + b$ and $3b$ equivalent expressions? Yes

Sample reasoning: For any value of $b$, both $2b + b$ and $3b$ give 3 times the value of $b$.

## Problem 4

from Unit 6, Lesson 6

80% of $x$ is equal to 100.

**a.** Write an equation that shows the relationship of 80%, $x$, and 100.

$0.8x = 100$ (or equivalent)

**b.** Use your equation to find the value of $x$.

$x = 125$

## Problem 5

from Unit 6, Lesson 5

For each situation:

- Choose an equation that represents it.
- Solve the equation.
- Explain what the variable and solution mean in the situation.

**a.** Jada's dog was $5\frac{1}{2}$ inches tall when it was a puppy. Now her dog is $14\frac{1}{2}$ inches taller. How much did Jada's dog grow?

- $x + 5\frac{1}{2} = 14\frac{1}{2}$
- $x = 5\frac{1}{2} + 14\frac{1}{2}$
- $5\frac{1}{2} \cdot x = 14\frac{1}{2}$

$x = 9$

Sample reasoning: $x$ represents the number of inches Jada's dog grew. The solution means that Jada's dog grew 9 inches.

**b.** Lin picked $9\frac{3}{4}$ pounds of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

- $p + 3 = 9\frac{3}{4}$
- $9\frac{3}{4} = 3p$
- $9\frac{3}{4} \cdot p = 3$

$p = 3\frac{1}{4}$

Sample reasoning: $p$ represents the weight in pounds of the apples Andre picked. The solution means that Andre picked $3\frac{1}{4}$ pounds of apples.

**Problem 6** from Unit 5, Lesson 8

Find the value of each product.

**a.** $(2.3) \cdot (1.4)$

3.22

**b.** $(1.72) \cdot (2.6)$

4.472

**c.** $(18.2) \cdot (0.2)$

3.64

**d.** $15 \cdot (1.2)$

18

**Problem 7** from Unit 5, Lesson 13

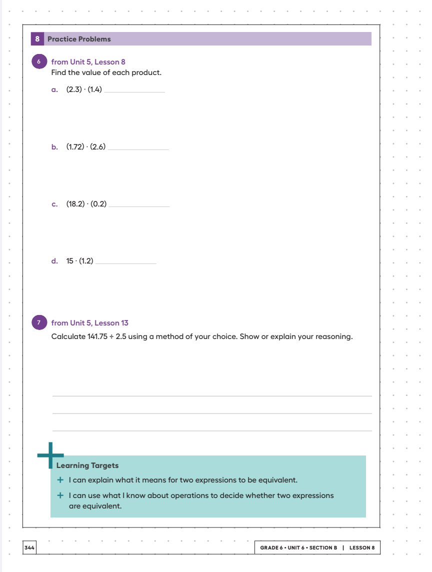
Calculate $141.75 \div 2.5$ using a method of your choice. Show or explain your reasoning.

56.7

Sample reasoning:

- Multiply the dividend and divisor by 10 and calculate $1417.5 \div 25$.
- Multiply the dividend and divisor by 100 and calculate $14175 \div 250$. (Note that the fraction $\frac{14175}{250}$ can be simplified to $\frac{567}{10}$ because both the numerator and denominator are divisible by 25, but simplifying the fraction does not save time because finding $14175 \div 25$ is essentially equivalent to the problem of finding $141.75 \div 2.5$.)
- Write an equivalent expression using fractions $\left(\frac{14175}{100} \div \frac{25}{10}\right)$ and solve by finding $\frac{14175}{100} \cdot \frac{10}{25}$, which equals $\frac{14175}{250}$.

**Student Workbook**



8 Practice Problems

6 from Unit 5, Lesson 8

Find the value of each product.

a. (2.3) · (1.4)

b. (1.72) · (2.6)

c. (18.2) · (0.2)

d. 15 · (1.2)

7 from Unit 5, Lesson 13

Calculate 141.75 ÷ 2.5 using a method of your choice. Show or explain your reasoning.

Learning Targets

+ I can explain what it means for two expressions to be equivalent.

+ I can use what I know about operations to decide whether two expressions are equivalent.