## **Interpreting Negative Numbers**

## Goals

- Interpret signed numbers in the contexts of temperature and elevation.
- Order rational numbers, and justify (orally) the comparisons.
- Plot points on a vertical or horizontal number line to represent rational numbers.

## **Learning Targets**

- I can compare rational numbers.
- I can use rational numbers to describe temperature and elevation.

## Lesson Narrative

In this lesson, students review what they learned previously about positive and negative numbers. They remember that **negative numbers** are less than 0 degrees, while **positive numbers** are greater than 0 degrees. First, students look at thermometers, reading temperatures above and below zero and comparing them. Next, they examine the vertical position of animals above or below sea level and describe the vertical distances between them. Both activities involve a number line that is oriented vertically to help students make sense of the context. As students relate the relative position of numbers on the number line to the meaning of "above zero" and "below zero" in each context, they are reasoning abstractly and quantitatively.

The last activity is optional because it provides an opportunity for additional practice comparing and ordering a set of rational numbers that includes fractions.

#### Student Learning Goal

Let's review what we know about signed numbers.

# **Access for Students with Diverse Abilities**

- Engagement (Activity 3)
- Representation (Activity 2)

#### **Access for Multilingual Learners**

- MLR2: Collect and Display (Activity 2)
- MLR5: Co-Craft Questions (Warm-up)

#### **Instructional Routines**

- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- Take Turns

#### **Required Materials**

#### **Materials to Copy**

 Rational Numbers Cards (1 copy for every 3 students): Activity 3

#### **Required Preparation**

#### **Activity 3:**

Copy each set of cards on a different color of paper so they can easily be sorted for the next class.

#### **Lesson Timeline**



Warm-up



**Activity 1** 



**Activity 2** 



**Activity 3** 

10 min

**Lesson Synthesis** 



5 min

Cool-down

#### **Instructional Routines**

## MLR5: Co-Craft Questions

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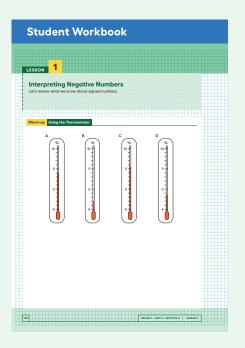
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# Access for Multilingual Learners (Warm-up)

#### **MLR5: Co-Craft Questions**

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.



#### Warm-up

### **Using the Thermometer**

## 5 min

## **Activity Narrative**

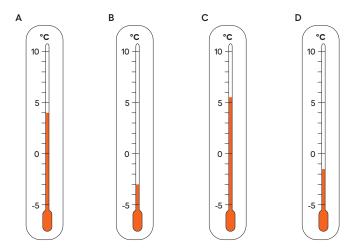
The purpose of this *Warm-up* is to remind students about negative numbers using a familiar context. The context of a thermometer also reminds students of vertical number lines.

## Launch 🙎

Tell students to close their student workbooks or devices (or to keep them closed). Arrange students in groups of 2. Introduce the context image, and explain that degrees Celsius is a way of measuring temperature, like degrees Fahrenheit. If necessary, tell students that 0 degrees Celsius is the freezing point of water and 0 degrees Fahrenheit is the freezing point of brine, a type of saltwater. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

## **Student Task Statement**



#### Sample responses:

- Which thermometer shows the warmest temperature? The coldest temperature?
- What temperature is shown on each thermometer?
- What is the difference between the warmest and coldest temperatures shown?

#### **Activity Synthesis**

Invite several partners to share one question with the class, and record responses. Ask the class to make comparisons among the shared questions and their own. Ask, "What do these questions have in common? How are they different?" Listen for and amplify language related to the learning goal, such as "positive," "negative," and "difference."

## **Activity 1**

### Fractions of a Degree



#### **Activity Narrative**

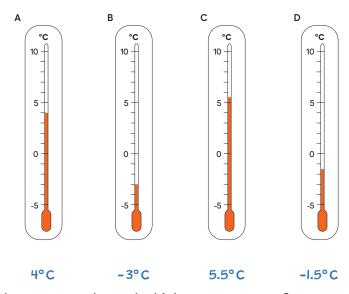
In this activity, students return to the context of a thermometer to examine rational numbers that are not integers. Students compare and interpret the signed numbers, including a temperature that is not pictured, to make sense of them in context.

### Launch

Give students 2–3 minutes of quiet work time, and follow with a whole-class discussion.

### **Student Task Statement**

1. What temperature is shown on each thermometer?



- 2. Which thermometer shows the highest temperature? C
- 3. Which thermometer shows the lowest temperature? B
- **4.** Suppose the temperature outside is 4° C. Is that colder or warmer than the coldest temperature shown? Explain your reasoning.

#### Colder

Sample reasoning: Since -4 is less than -3, which is the coldest temperature shown, -4 would be colder.

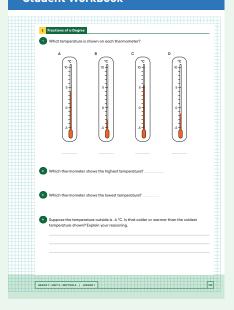
### **Building on Student Thinking**

If some students struggle to estimate temperatures that are between two markings, consider asking:

"What would the temperatures be at the markings directly above and directly below the thermometer's level?"

"What temperature would be halfway in between those two numbers?"

### Student Workbook



# Access for Students with Diverse Abilities (Activity 2, Launch)

# Representation: Internalize Comprehension.

Provide students with an annotated diagram that includes a horizontal line extending from an animal's eye to the vertical axis for one or more of the animals. This can help with determining the height or depth of the animal.

Supports accessibility for: Visual-Spatial Processing, Organization

### **Activity Synthesis**

The purpose of this activity is to remind students of how to compare two values using inequality notation. Begin by inviting students to share their responses for the temperature shown on each thermometer.

Then discuss the last question, asking students to explain their reasoning until they come to an agreement that -4° C is colder than -3° C. If not brought up in students' explanations, introduce the notation -4 < -3, and remind students that this is read, "Negative 4 is less than negative 3." Explain that -4 is farther away from 0 than -3 is, and point to the location of -4 on a thermometer to show that it is below -3. On the negative side of the number line, that means -4 is less than -3. Familiarity with "less than" notation will be useful for describing their reasoning in the next activity.

## **Activity 2**

#### Seagulls Soar, Sharks Swim

10 min

### **Activity Narrative**

In this activity, students continue to interpret signed numbers in context and begin to compare their relative locations on a vertical number line in order to make sense of signed number. The number line is labeled in 5-meter increments, so students have to interpolate the height or depth for some of the animals. Students are also given the height or depth of other animals that are not pictured and asked to compare these to the height or depth of the animals shown.

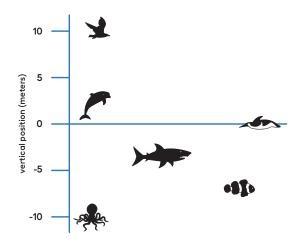
Monitor for students who recognize that there are two possible answers to the last question depending on whether the new dolphin is 3 meters above or below the original dolphin.

## Launch

Display the image for all to see. Tell students to measure the height or depth of each animal's eyes to the nearest meter. Remind students that we choose sea level to be our 0 level, in the same way that we chose a 0 level for temperature.

## **Student Task Statement**

Here is a picture of some sea animals. The number line on the left shows the vertical position, in meters, of each animal above or below sea level.



**1.** How far above or below sea level is each animal? Measure to their eye level.

Seagull is at 10 m. Dolphin is at 3 m. Octopus is at -10 m. Shark is at -3 m. Fish is at -7 m. Penguin is at 0 m.

**2.** A mobula ray is 3 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

The jumping dolphin? The flying seagull? The octopus?

0 m above 7 m below 13 m above

**3.** An albatross is 5 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

The jumping dolphin? The flying seagull? The octopus?

2 m above 5 m below 15 m above

**4.** A clownfish is 2 meters below the surface of the ocean. How does its vertical position compare to the height or depth of:

The jumping dolphin? The flying seagull? The octopus?

5 m below 12 m below 8 m above

**5.** The vertical distance between a new dolphin and the dolphin in the picture is 3 meters. What is the new dolphin's distance from the surface of the ocean?

either 0 m or 6 m, depending on whether the new dolphin is 3 m above or below the dolphin in the picture

#### **Are You Ready for More?**

The north pole is in the middle of the ocean. A person at sea level at the north pole would be about 3,949 miles from the center of Earth. The sea floor below the north pole is at an elevation of approximately -2.7 miles. The elevation of the south pole is about 1.7 miles. How far is a person standing on the south pole from a submarine at the sea floor below the north pole?

About 7,897 miles.

# Access for Multilingual Learners (Activity 2, Student Task)

#### MLR2: Collect and Display.

Circulate to listen for and collect the language that students use as they describe and compare the heights and depths of the different sea animals. On a visible display, record words and phrases, such as "the albatross is 3 meters above the penguin" or "the penguin is 3 meters below the albatross." Invite students to borrow language from the display as needed, and update it throughout the lesson. During the Activity Synthesis, invite students to suggest ways to update the display:

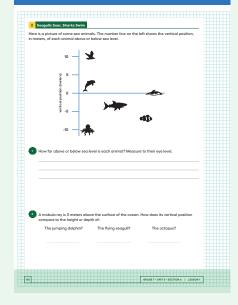
"What are some other words or phrases we should include?" Advances: Conversing, Reading

#### **Building on Student Thinking**

If students measure to the top or bottom of the animal, remind them that we are using the eyes of the animal to measure their height or depth.

Some students may struggle to visualize where the albatross, seagull, and clownfish are on the graph. Consider having them draw or place a marker where the new animal is located while comparing it to the other animals in the picture.

#### Student Workbook



#### **Instructional Routines**

#### **Take Turns**

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## **Activity Synthesis**

The purpose of this discussion is for students to share their reasoning for the height or depth of each new animal. A key idea of this discussion is that distances above and below sea level can be represented using signed numbers. The depths of the shark, fish, and octopus can be expressed as negative numbers because they are below sea level, while the heights of the remaining animals can be expressed as positive numbers because they are above sea level.

Begin by inviting students to share their responses and reasoning to the fourth question about the relative position of the clownfish. Record and display their verbal descriptions using signed numbers. For example, if a student says the clownfish is 5 meters below the dolphin, write "-5".

Finally, ask students to share their responses and reasoning to the last question about the height of the new dolphin. Invite a previously selected student to explain why there are two possible answers to the last question.

The new dolphin could be 3 meters above or below the original dolphin.

## **Activity 3: Optional**

#### **Rational Numbers Cards**

15 min

#### **Activity Narrative**

In this partner activity, students take turns ordering rational numbers from least to greatest. Students start with a set of integers first, then add rational numbers second. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

## Launch 22

Arrange students in groups of 2, and explain that they will be putting a set of cards in order from least to greatest. If time allows, choose a student as a partner, and demonstrate how to set up and do the activity. Otherwise, share these steps:

- Place the card with the value of 0 in front of both partners.
- One partner picks a card and places it so that the cards are in order from least to greatest and explains their placement.
- The other partner listens and makes sure they agree with the placement and the reasoning.
- If they don't agree, the partners discuss until they come to an agreement.
- For the next card placement, the students swap roles.

Give students 5 minutes to order the first set of cards. When a group finishes, check their ordering before giving them the second set of cards. Give students another 5 minutes to order the second set of cards, and follow with a whole-class discussion.

## **Student Task Statement**

Your teacher will give you a set of number cards. Take turns with your partner placing a card from the set in order from least to greatest.

- 1. For each placement that you make, explain your reasoning to your partner.
- **2.** For each placement that your partner makes, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.
- 3. Pause after the first set so your teacher can review your ordering.
- **4.** Your teacher will give you a second set of cards to add in order with the first set.

Set 1: -23, -10, -9, -7, -6, -4, -3, -2, -1, 0, 1, 2, 3, 5, 8, 10, 11, 15, 22, 23 Set 2 added to Set 1: -23, -22 $\frac{3}{8}$ , - $\frac{31}{3}$ , -10, -9, -7.7, -7, -6, -5 $\frac{5}{6}$ , -4, -3, - $\frac{8}{3}$ , -2.5, -2, - $\frac{9}{8}$ , -1, - $\frac{1}{4}$ , 0,  $\frac{1}{4}$ , 1,  $\frac{9}{8}$ , 2, 2.5,  $\frac{8}{3}$ , 3, 5, 5 $\frac{5}{6}$ , 7.7, 8, 10,  $\frac{31}{3}$ , 11, 15, 22, 22 $\frac{3}{8}$ , 23

## **Activity Synthesis**

The purpose of this discussion is for students to share their strategies for comparing and ordering rational numbers. Highlight strategies that use the magnitude of a number and its additive inverse. Here are some questions to consider:

"Which numbers did you place first? Why?"

"Which numbers were hardest to place, and which were the easiest?"

"How does placing negative numbers compare to placing positive numbers?"

"How did you use numbers you had already placed to reason about where to place new numbers?"

"How did you decide where to put the fractions?"

"How is  $-\frac{9}{8}$  related to  $\frac{9}{8}$ ?"

"Describe any difficulties you experienced and how you resolved them."

Introduce the convention that number lines are usually drawn horizontally, with the negative numbers to the left of 0. If any groups put their slips in order vertically, consider asking them to reposition their slips to match the orientation of a horizontal number line.

#### **Lesson Synthesis**

Share with students,

"Today we worked with positive and negative numbers that represented temperatures and elevations."

To review the meaning and relative positive of negative numbers, consider asking:

"When a number line is arranged vertically, where are the negative numbers?"

at the bottom, below O

"When a number line is arranged horizontally, where are the negative numbers?"

to the left of O

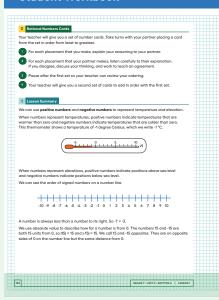
## Access for Students with Diverse Abilities (Activity 3, Student Task)

# Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

#### **Student Workbook**



#### **Responding To Student Thinking**

#### **More Chances**

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

"What does it mean for a temperature to be negative?"

It is colder than the reference point, for example, colder than the freezing point of water.

"What does it mean for an elevation to be negative?"

It is below the reference point, for example, below sea level.

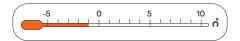
"Which number is greater, -7 or -12? Why?"

-7 is greater than -12 because they are both negative and -7 is closer to 0.

## **Lesson Summary**

We can use **positive numbers** and **negative numbers** to represent temperature and elevation.

When numbers represent temperatures, positive numbers indicate temperatures that are warmer than zero and negative numbers indicate temperatures that are colder than zero. This thermometer shows a temperature of -1 degree Celsius, which we write -1 °C.



When numbers represent elevations, positive numbers indicate positions above sea level and negative numbers indicate positions below sea level.

We can see the order of signed numbers on a number line.



A number is always less than a number to its right. So -7 < -3.

We use absolute value to describe how far a number is from 0. The numbers 15 and -15 are both 15 units from 0, so |15| = 15 and |-15| = 15. We call 15 and -15 opposites. They are on opposite sides of 0 on the number line but the same distance from 0.

#### Cool-down

#### **Signed Numbers**

## 5 mir

#### **Student Task Statement**

Here is a set of signed numbers:

7

-3

 $\frac{1}{2}$ 

-0.8

0.8

-<u>1</u>

-2

1. Order the numbers from least to greatest.

-3

-2

-0.8

\_\_\_\_\_\_

1/2

0.8

7

**2.** If these numbers represent temperatures in degrees Celsius, which is the coldest?

-3

**3.** If these numbers represent elevations in meters, which is the farthest away from sea level?

7

#### **Practice Problems**

## 5 Problems

## Problem 1

It was -5° C in Copenhagen and -12° C in Oslo. Which city was colder?

#### Oslo

-12 is less than -5.

## Problem 2

a. A fish is 12 meters below the surface of the ocean. What is its elevation?

-12 m

**b.** A sea bird is 28 meters above the surface of the ocean. What is its elevation?

28 m

c. If the bird is directly above the fish, how far apart are they?

40 m

### **Problem 3**

Compare the numbers in each statement using >, =, or <.

**g.** -1.5 
$$=\frac{-3}{2}$$

**h.** 
$$\frac{-4}{5} > \frac{-5}{4}$$

i. 
$$\frac{-3}{5} = \frac{-6}{10}$$

**j.** 
$$\frac{-2}{3} < \frac{1}{3}$$

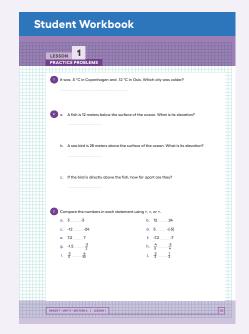
## **Problem 4**

from Unit 4, Lesson 7

Han wants to buy a \$30 ticket to a game, but the pre-order tickets are sold out. He knows there will be more tickets sold on the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game?

## \$90

A 100% increase of a \$30 ticket is an additional \$30. Therefore a 200% increase of a \$30 ticket would be an additional \$60.





## Problem 5

from Unit 2, Lesson 1

27

A type of green paint is made by mixing 2 cups of yellow with 3.5 cups of blue.

- **a.** Find a mixture that will make the same shade of green but a lesser amount. Sample response: I cup of yellow and 1.75 cups of blue
- **b.** Find a mixture that will make the same shade of green but a greater amount.
  - Sample response: 4 cups of yellow and 7 cups of blue
- **c.** Find a mixture that will make a different shade of green that is bluer. Sample response: 2 cups of yellow and 4 cups of blue
- **d.** Find a mixture that will make a different shade of green that is more yellow. Sample response: 2 cups of yellow and 2 cups of blue

LESSON 1 • PRACTICE PROBLEMS