Lesson 5

Fermi Problems

Goals

- Calculate a rough estimate for quantities that are difficult or impossible to measure directly and explain (orally) the reasoning.
- Choose an appropriate level of accuracy when reporting estimates of quantities.
- Make simplifying assumptions to solve problems about estimating quantities.

Lesson Narrative

The activities in this optional lesson challenge each student to determine their exact age, how many times their heart has beaten, and how many hairs are on their head. In each, students must decide how precise to be in their response. Determining their age allows more certainty than the other calculations.

This lesson is made of activities sometimes called “Fermi problems” after the famous physicist Enrico Fermi. A Fermi problem requires students to make a rough estimate for quantities that are difficult or impossible to measure directly. Often, they use rates and require several calculations with fractions and decimals, making them well-aligned to grade 7 work. Fermi problems are examples of mathematical modeling because one must make simplifying assumptions, estimates, research, and decisions about which quantities are important and what mathematics to use. They also encourage students to attend to precision because one must think carefully about how to appropriately report estimates and choose words carefully to describe the quantities. This lesson relies on skills developed in Unit 2 and Unit 5.

Access for Students with Diverse Abilities

- Action and Expression (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- Poll the Class

Required Materials

Materials to Gather

- Four-function calculators: Activity 1, Activity 2, Activity 3
- Stopwatches: Activity 2
- Measuring tapes: Activity 3
- String: Activity 3

Activity 2:
Provide a stopwatch or other timing device for each group if students are measuring their own heart rate.

Activity 3:
If students are measuring their heads, provide access to string and measuring tape.

Lesson Timeline

20 min	15 min	20 min
Activity 1	Activity 2	Activity 3

Lesson 5

Fermi Problems

Lesson Narrative (continued)

Any of these tasks can stand on its own. Teachers should select those that they have time for. Doing all three activities would likely take more than a single class period. Teachers should leave plenty of time for discussion, including why the quantities in question are difficult to measure and how precise the estimates should be.

Student Learning Goal

Let's estimate some quantities.

Activity 1: Optional
How Old Are You?

20 min

Activity Narrative

Students attempt to calculate their exact age. Because this is a constantly changing quantity, they attend to precision when they think carefully about how accurately to report the answer. The mathematics involved is multiple unit conversions in the context of time. In addition to the fact that age is always growing, precise times of birth may not be known.

Launch

Arrange students into groups of 4.

Ask them to order themselves according to their age: who is the youngest? Who is the oldest? What information do you need to know to decide?

Tell students that they will be finding their exact age.

Provide access to calculators.

Give students 10 minutes of group work time, followed by whole-class discussion.

Student Task Statement

What is your *exact* age at this moment?

Sample response: I was born on April 4, 2011 at 12:09 p.m. Today is December 17, 2024, and right now it is 10:55 a.m. So I am 13 years, 256 days, 22 hours, and 46 minutes old right now. I can't get any more exact because I don't know the second I was born. Plus, I'm not sure if I should answer the question for when I begin to write my solution, or for when I will be done writing my solution, because I keep getting older.

Activity Synthesis

Invite students to share answers and discuss difficulties in answering the question. The discussion should include the following points:

- We can give an estimate, but the question cannot be answered because we probably do not know the exact time when we were born and "at this moment" keeps moving forward.
- How we express our answer depends on what we know about the time we were born. We probably know the day, but do we know the time? Should we answer to the nearest hour? Minute? Second?
- Did you take into account leap years?

Inspire Math

Cable Cars video

Go Online
Before the lesson, show this video to **introduce** the real-world connection.

ilclass.com/l/614192

Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students struggle with what is meant by "at this moment" in the prompt, consider asking:

"Tell me about your age."
"What does 'at this moment' mean to you?"

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each response or observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 2, Narrative)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

Poll the Class

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Ask students how they usually answer the question,

"How old are you?" This will probably be in whole number of years lived.

Why? Because this whole number communicates enough information for most purposes.

After more than six months have passed since someone's last birthday, the person still doesn't normally round up when they report their age, even though this is customary for reporting many other types of measurements.

Activity 2: Optional

A Heart Stoppingly Large Number

15 min

Activity Narrative

This activity focuses on another complex calculation: the number of times a person's heart has beaten so far in their lifetime. Unlike calculating an exact age, hearts do not beat at a constant rate. Because of this, the answer will be a reasonable estimate and knowing the precise start and end times is unimportant because a few minutes or hours will not significantly impact the answer. Students model with mathematics when they decide what quantities are important and make simplifying assumptions about how to determine the number of heart beats.

Launch

Arrange students in groups of 2.

Tell students they will be investigating how many times their heart has beaten in their lifetime. Ask students to make an estimate and then poll the class. One way to accomplish this is to display a table like this and complete it with the number of students whose estimate is in each range.

estimate for number of heartbeats	number of students
around 10,000	
around 100,000	
around 1,000,000	
around 10,000,000	
around 100,000,000	
around 1,000,000,000	
around 10,000,000,000	
around 100,000,000,000	

Give groups 5 minutes to brainstorm what they need to know or consider and make a plan to answer this question.

Then, make a list of 3–5 things that groups think they will need to figure out. Provide access to stopwatches (if students will estimate their own heartbeats) and calculators. Alternatively, share with them that a normal heartbeat range is 60 to 100 beats per minute, with higher rates during exercise and sometimes lower rates at rest.

Give students 10 minutes of partner work time followed by a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use related to their estimation process. Display words and phrases such as "rate," "per," "constant," and "beats."

Student Task Statement

How many times has your heart beaten in your lifetime?

Sample response: I am about 13 years old. There are 365 days per year, 24 hours per day, and 60 minutes per hour. So that is $13 \cdot 365 \cdot 24 \cdot 60$, which is about 6.8 million minutes. I just took my pulse, and my heart rate is about 75 beats per minute. Even though that changes over time, I'll use it to estimate the total number of times my heart has beaten in my lifetime so far by multiplying the number of minutes I've been alive by 75. That gives about 500 million heartbeats.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their process and solution. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Continue to reference the display while discussing students' strategies and difficulties with questions such as:

- *"How did you estimate how often your heart beats?"*

  by checking my pulse

- *"Is your heart rate always the same?"*

  No, it is faster when I exercise and slower when I sleep.

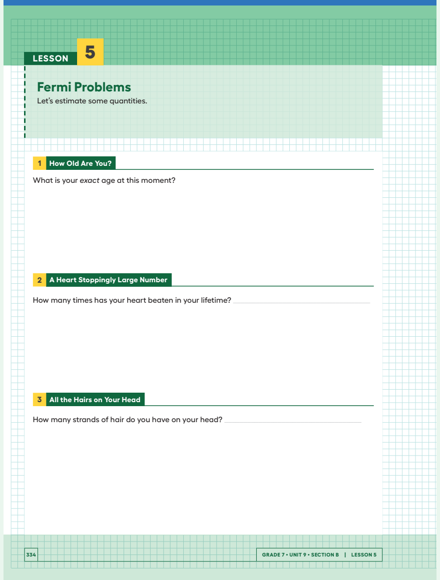
- *"Did you use the information you found for your age?"*

  No because I only have an estimate for my pulse or yes because I already had the information available.

Time permitting, students can be encouraged to check their pulse while resting and then again after a short amount of exercise (jumping jacks, running in place, push ups). This gives them an idea of the variability involved in how frequently a person's heart beats.

The calculation of the number of heartbeats is only an estimate. One way this can be communicated is with the way the final answer is reported. An answer of 524,344,566 would not be appropriate because that makes it look like it is an exact answer. An answer of 500,000,000 makes it clear that only an estimate is being made. Alternatively, students might say that the number of heartbeats is between 400,000,000 and 700,000,000, where the estimate of 400,000,000 comes from the low value of 60 beats per minute, and the estimate of 700,000,000 comes from the high value of 100 beats per minute.

Student Workbook


LESSON 5

Fermi Problems

Let's estimate some quantities.

1 How Old Are You?

What is your exact age at this moment?

2 A Heart Stoppingly Large Number

How many times has your heart beaten in your lifetime?

3 All the Hairs on Your Head

How many strands of hair do you have on your head?

Instructional Routines

Poll the Class

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Access for Students with Diverse Abilities (Activity 3, Student Task)

Action and Expression: Internalize Executive Functions.
To support organization, provide students with a graphic organizer for recording measurements and calculations of areas of circles to represent hair on head.
Supports accessibility for: Language, Organization

Activity 3: Optional
All the Hairs on Your Head

20 min

Activity Narrative

Students estimate how many hairs they have on their head. Although there is a definite number of hairs on each student's head, finding this number exactly is not feasible. While it is a much smaller number than the number of heartbeats in a student's lifetime, it is still too large to count and, even if it could be counted, all of those hairs would need to be cut off to do so! An additional geometric layer of estimation comes into play in this task as students need to estimate the surface area of part of their heads. As students plan their estimation strategy, they make sense of the problem and persevere in solving it. If one or more students in the class are bald or are otherwise sensitive about their hair, consider asking them to estimate the number of hairs on another student's head.

Launch

Arrange students in groups of 2.

Provide access to calculators. Before starting, poll the class to guess how many hairs they have on their head. One way to accomplish this is to display a table like this, and record the number of students with each guess:

number of hairs	number of guesses
about 100	
about 1,000	
about 10,000	
about 100,000	
about 1,000,000	

Next, ask students to brainstorm the information they need to answer the question. Provide the following information below when students ask for it:

- The number of hairs per square cm varies from person to person, but for this analysis, it can be assumed that there are approximately 150 hairs per square cm.
- Students have to be creative in their estimates for the area of their scalp. One possibility is to approximate it with a circle. 600 square centimeters is a good estimate, but students may come up with slightly different estimates.

Provide access to string, tape measures, and calculators. After taking 5 minutes to introduce the activity, give students 10 minutes of partner work time, followed by whole-class discussion.

Student Task Statement

How many strands of hair do you have on your head?

Sample response: A piece of string around the head, from the front of the forehead to the back of the neck, measures about 28 cm. A piece of string going from ear to ear also measures about 28 cm. So it is reasonable to model the scalp with a circle whose diameter is 28 cm. The radius is 14 cm, and the area of the circle is $\pi \cdot 14^2 \approx 600$ square cm. If there are 150 strands of hair per square cm, that means that there are about 90,000 strands of hair on the head.

Activity Synthesis

Invite students to share their answers and methods. Consider asking the following questions:

- *"Why is it challenging to find the exact number of hairs on your head?"*

  There are too many to count, and they are too small to count accurately. The number changes as time goes by.

- *"How can you estimate the number of hairs on your head?"*

  Estimate the area of the part of the head covered by hair. Try to find how many hairs there are on a small part of the head.

- *"How did you estimate the area of your head?"*

  Approximated area with a circle. Covered the head with little pieces of paper [sticky notes] and added up their areas.

This is a situation where there is a definite number of hairs on a head, but because they are so small (and because they are on a head), this number can not be calculated exactly. Fortunately, for almost any purpose, an estimate will do.

Building on Student Thinking

If students struggle with approximating the area of the scalp, consider asking:

"What two-dimensional shape can you use to model the part of your scalp covered with hair?"
"How could you use a string to measure the size of the circle and find its area?"