Lesson 6

More Expressions and Equations

Goals

- Create expressions and equations to represent a linear relationship with two or more related quantities in context.
- Determine the solution to an equation of the form $px + q = r$ and explain (orally) the solution method.
- Interpret expressions, equations, and solutions that represent a linear relationship with two or more related quantities.

Lesson Narrative

In this optional lesson students write algebraic expressions to represent situations and solve equations. They are presented with problems that can be solved using grade 7 mathematics, but are often left for future grades when students have access to a wider variety of algebraic tools. Each of the situations described in the activities involve two or more unknown quantities and multiple relationships or actions. Initially students are walked through the steps of writing an expression to describe a situation, using properties to rewrite the expression with fewer terms, writing an equation to represent the situation, solving the equation, and considering the reasonableness of solutions. As they proceed through the lesson, the supports are slowly removed and students work on their own to reason through the problem. This lesson relies on skills developed in Unit 6.

Student Learning Goal

Let's solve harder problems by writing equivalent expressions.

Lesson Timeline

Activity 1	Activity 2	Activity 3
15 min	15 min	15 min

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 3)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR6: Three Reads

Activity 1
Tickets for the School Play

15 min

Activity Narrative

This activity walks students through the process of defining a variable, writing an expression, writing the expression with fewer terms, estimating a reasonable solution, computing a solution, and finally checking that the solution makes sense and is correct. Through this process, students reason abstractly and quantitatively. Note that there are two unknown quantities (prices for student and adult tickets) and students are guided to express one in terms of the other.

Monitor for students who write the expressions in different ways in the first three questions and invite them to share during the discussion. There are many possible correct answers, but some forms will lead to equations that are easier to solve (such as $px + q$).

Launch

If your school or a nearby school has recently performed a play, consider asking if any students went to see it and have them briefly describe the experience. Alternatively, display photos from any school play, including images of the tickets or ticket booth. Invite students to share what they notice and what they wonder.

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Student tickets for the school play cost $2 less than adult tickets.

1. If a represents the price of 1 adult ticket, write an expression for the price of 1 student ticket. $a - 2$
2. Write an expression that represents the amount of money they collected each night:
   a. The first night, the school sold 60 adult tickets and 94 student tickets. $60a + 94(a - 2)$ (or $154a - 188$)
   b. The second night, the school sold 83 adult tickets and 127 student tickets. $83a + 127(a - 2)$ (or $210a - 254$)
3. Write an expression that represents the total amount of money collected from ticket sales on both nights. $364a - 442$ (or equivalent)
4. Over these two nights, they collected a total of $1,651 in ticket sales.
   a. Write an equation that represents this situation. $364a - 442 = 1651$
   b. What was the cost of each type of ticket? $5.75 adult, $3.75 student
5. Is your solution reasonable? Explain how you know.

   Sample response: Yes, it's reasonable that tickets to the school play would cost somewhere between $0 and $10 and also that the adult tickets would cost more than the students tickets. I can show it is correct using: $5.75(60 + 83) + 3.75(94 + 127) = 822.25 + 828.75 = 1,651$

Student Workbook


LESSON 6

More Expressions and Equations

Let's solve harder problems by writing equivalent expressions.

1 Tickets for the School Play

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4. Over these two nights, they collected a total of $1,651 in ticket sales.
   a. Write an equation that represents this situation.
   b. What was the cost of each type of ticket?
5. Is your solution reasonable? Explain how you know.

Building on Student Thinking

If students are struggling with writing and solving an equation, consider asking:

"Can you explain how you wrote your expressions?"
"How can your expressions be combined to represent both nights?"
"How can you use what you know about the adult ticket price to find out the student ticket price?"

If students represent the price of a student ticket as "2 − a" instead of "a − 2," consider asking:

"Tell me more about this expression."
"How could you use test values to make sure your expression is correct?"

Activity Synthesis

The purpose of the discussion is for students to reflect on the problem solving process. Consider asking these questions:

💬 *"Why was the price of an adult ticket chosen as the variable?"*

This was an arbitrary choice.

💬 *"Could the problem be worked by choosing the price of a student ticket as the variable? How would the expressions be different? The equation? The solution?"*

Yes, the expressions would be $60(s + 2) + 94s$ and $83(s + 2) + 127s$, where s represents the price of 1 adult ticket. The equation would become $364s + 286 = 1{,}651$. The solution to this equation is 5.75. The price of the tickets do not change.

💬 *"What does each term in the expressions you wrote represent?"*

The numbers represent the numbers of each type of ticket sold. The variable a represents the price of one adult ticket.

💬 *"How did you find the price of a student ticket?"*

First, I solved the equation for the price of an adult ticket. Then I used the first expression $a - 2$, to solve for the price of a student ticket because student tickets cost $2 less than adult tickets.

💬 *"How did you check that your solution is correct?"*

I checked using the number of tickets sold and the amount of money earned.

Activity 2
A Souvenir Stand

15 min

Activity Narrative

This activity guides students through a process of writing an equation to solve a problem with three unknown quantities. While this activity has students write their equation in terms of the number of hats, there are other ways to approach this situation. In this activity students reason abstractly and quantitatively when writing expressions and finding values.

Monitor for students who compute the profit in different ways, such as:

- Finding the total income and total cost, and then subtracting them.
- Finding the profit for each item, and then adding them together.

Launch

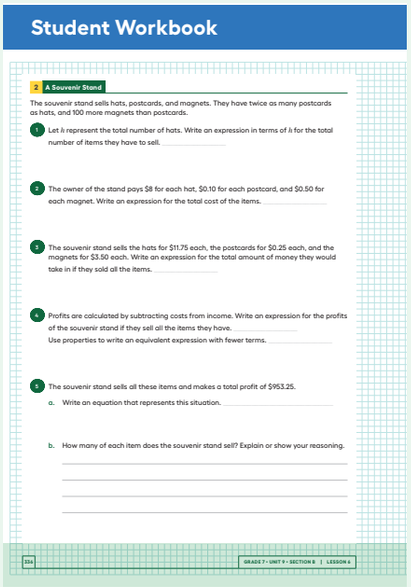
Give students 6–7 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The souvenir stand sells hats, postcards, and magnets. They have twice as many postcards as hats, and 100 more magnets than postcards.

1. Let h represent the total number of hats. Write an expression in terms of h for the total number of items they have to sell. $h + 2h + (2h + 100)$
2. The owner of the stand pays $8 for each hat, $0.10 for each postcard, and $0.50 for each magnet. Write an expression for the total cost of the items.
   $8h + 0.10(2h) + 0.50(2h + 100)$ (or $9.2h + 50$)
3. The souvenir stand sells the hats for $11.75 each, the postcards for $0.25 each, and the magnets for $3.50 each. Write an expression for the total amount of money they would take in if they sold all the items.
   $11.75h + 0.25(2h) + 3.50(2h + 100)$ (or $19.25h + 350$)
4. Profits are calculated by subtracting costs from income. Write an expression for the profits of the souvenir stand if they sell all the items they have. Use properties to write an equivalent expression with fewer terms. $10.05h + 300$
5. The souvenir stand sells all these items and makes a total profit of $953.25.
   a. Write an equation that represents this situation. $10.05h + 300 = 953.25$
   b. How many of each item does the souvenir stand sell? Explain or show your reasoning.
      65. 65 hats, 130 postcards, 230 magnets
      $10.05h + 300 = 953.25$, $10.05h = 653.25$, $h = 65$; $2h = 2 \cdot 65$, $2h = 130$; $2h + 100 = 2 \cdot 65 + 100$, $2h + 100 = 230$

Student Workbook


2 A Souvenir Stand

The souvenir stand sells hats, postcards, and magnets. They have twice as many postcards as hats, and 100 more magnets than postcards.

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2. The owner of the stand pays $8 for each hat, $0.10 for each postcard, and $0.50 for each magnet. Write an expression for the total cost of the items.
3. The souvenir stand sells the hats for $11.75 each, the postcards for $0.25 each, and the magnets for $3.50 each. Write an expression for the total amount of money they would take in if they sold all the items.
4. Profits are calculated by subtracting costs from income. Write an expression for the profits of the souvenir stand if they sell all the items they have. Use properties to write an equivalent expression with fewer terms.
5. The souvenir stand sells all these items and makes a total profit of $953.25.
   a. Write an equation that represents this situation.
   b. How many of each item does the souvenir stand sell? Explain or show your reasoning.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students in explaining how they wrote their expressions: "First, I ___ because ..." and "I noticed ____ so I ..."
Advances: Conversing, Representing

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem.
For example, color code how each unknown is represented in different students' approaches.
Supports accessibility for: Visual-Spatial Processing

Instructional Routines

MLR6: Three Reads

ilclass.com/r/10695568
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 3, Narrative)

MLR6: Three Reads
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Activity Synthesis

Invite students to share their answers to the last two questions (and others, if they wish). Consider asking questions like the following:

- *"Is it a good idea to check your solution by substituting values into the expressions and equations you wrote?"*

  No, you might have written or simplified them incorrectly. The best option is to go back to the original problem.

- *"Why was the number of hats chosen as the unknown quantity to represent with a variable? Would you have chosen differently? How would that change the solution process?"*

  This is just one choice. A different one could be made.

- *"How did you compute the profit?"*

  Compute the total income minus total cost for all items, or add item by item.

- *"I saw that many of you wrote this expression $8h + 0.10(2h) + 0.50(2h + 100)$. What is the meaning of the different parts of the expressions in this situation?"*

  $8h$ is the cost for the hats, $0.10(2h)$ is the cost for the postcards, and $0.50(2h + 100)$ is the cost for the magnets.

Activity 3

Jada Crochets a Scarf

15 min

Activity Narrative

In this activity, students write an equation to represent a situation and solve a problem. Most of the supports for the solution process are removed. Students need to think about how to choose which quantity to represent with a variable, how to represent the other two quantities with expressions in terms of the variable, and how to write an expression for a total. The only guidance offered is the reminder to write the expression with as few terms as possible. Students build on the structure they used in the earlier activities as they write and solve their expressions.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem, without revealing the questions.

- For the first read, read the problem aloud then ask,

"*What is this situation about?*"

the amount of yarn used in different crochet stitches

Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured

length of a single crochet stitch, $\frac{1}{2}$ inch less than twice as much yarn as a single crochet stitch, 1 inch less than three times as much yarn as a single crochet stitch.

After the third read, reveal the question:

"*Write an expression that represents the amount of yarn Jada needs to crochet a scarf that includes 800 single crochet stitches, 400 double crochet stitches, and 200 triple crochet stitches*"

and ask,

"*What are some ways we might get started on this?*"

Invite students to name some possible starting points, referencing quantities from the second read (decide on a variable to represent the amount of yarn used in a single crochet stitch and use that variable and the numbers from the second read to write an expression).

Arrange students in groups of 2.

Give students 6–7 minutes of quiet work time followed by partner and whole-class discussion.

Student Task Statement

Basic crochet stitches are called single, double, and triple. Jada measures her average stitch size and sees that a "double crochet" stitch is not really twice as long as a single crochet stitch. It uses $\frac{1}{2}$ inch less than twice as much yarn as a single crochet stitch. Jada's "triple crochet" stitch uses 1 inch less than three times as much yarn as a single crochet stitch.

1. Write an expression that represents the amount of yarn Jada needs to crochet a scarf that includes 800 single crochet stitches, 400 double crochet stitches, and 200 triple crochet stitches.

   Sample response: Let x represent the length of yarn needed for a single crochet stitch. $800x + 400\left(2x - \frac{1}{2}\right) + 200(3x - 1)$

2. Write an equivalent expression with as few terms as possible.

   $2{,}200x - 400$

3. If Jada uses 5540 inches of yarn for the entire scarf, what length of yarn does she use for a single crochet stitch?

   2.7 inches of yarn for a single crochet stitch
   Sample reasoning: If $2{,}200x - 400 = 5{,}540$, then $2{,}200x = 5{,}940$, and $x = 2.7$

Student Workbook

3 Jada Crochets a Scarf

Basic crochet stitches are called single, double, and triple. Jada measures her average stitch size and sees that a "double crochet" stitch is not really twice as long as a single crochet stitch. It uses $\frac{1}{2}$ inch less than twice as much yarn as a single crochet stitch. Jada's "triple crochet" stitch uses 1 inch less than three times as much yarn as a single crochet stitch.

1. Write an expression that represents the amount of yarn Jada needs to crochet a scarf that includes 800 single crochet stitches, 400 double crochet stitches, and 200 triple crochet stitches.
2. Write an equivalent expression with as few terms as possible.
3. If Jada uses 5540 inches of yarn for the entire scarf, what length of yarn does she use for a single crochet stitch?

Activity Synthesis

The goal of this discussion is to show that the same situation can be represented mathematically in different ways while arriving at the same answer. Invite several students to share their expressions that represent the amount of yarn Jada needs to crochet a scarf. Here are some questions for discussion:

- *"What do the representations have in common? How are they different?"*
- *"Are there any benefits or drawbacks to one representation compared to another?"*
- *"How does the $\frac{1}{2}$ inch less yarn show up in each representation?"*
- *"How could you check your answer?"*

  Use the information from the problem stem to find the length of the other two crochet stitches and use the number of each type of stitch to see if the total length matches the 5540 inches of yarn Jada uses.

If it does not come up during the discussion, ask students about the purpose of the different forms of the expression. For example, the longest expression is helpful for seeing how each term connects to each part of the situation, while the equivalent expression with less terms is helpful for solving for the value of the variable.