Lesson 3

Square Roots

Goals

- Create a table and graph that represents the relationship between side length and area of a square, and use the graph to estimate the side lengths of squares with non-integer side lengths.
- Determine the exact side length of a square and express it (in writing) using square root notation.

Learning Targets

- I can estimate the value of expressions like $\sqrt{29}$ and $\sqrt{3}$.
- I understand the meaning of expressions like $\sqrt{25}$ and $\sqrt{3}$.

Lesson Narrative

In this lesson students use different strategies to approximate the value of square roots when a drawn square is not given. While this lesson offers students some practice with using square root notation and approximating values of square roots, there will be additional opportunities throughout the unit.

Students begin by finding the areas of tilted squares, writing their exact side lengths, and approximating those side lengths using tracing paper. Next, they create a table and graph of side lengths and areas and use them to verify the accuracy of their tracing paper estimates. Lastly, students work in pairs to find the areas of two squares. While the squares have different orientations, both have an area of 50 square units, and so they must also have the same side length of $\sqrt{50}$.

Student Learning Goal

Let's investigate the relationship between side length and area of a square.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Required Materials

Materials to Gather

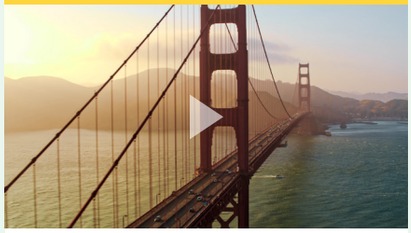
- Geometry toolkits: Activity 1, Activity 3
- Tracing paper: Activity 1
- Four-function calculators: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	15 min	10 min	10 min	10 min
				Cool-down

Inspire Math

Golden Gate Bridge video



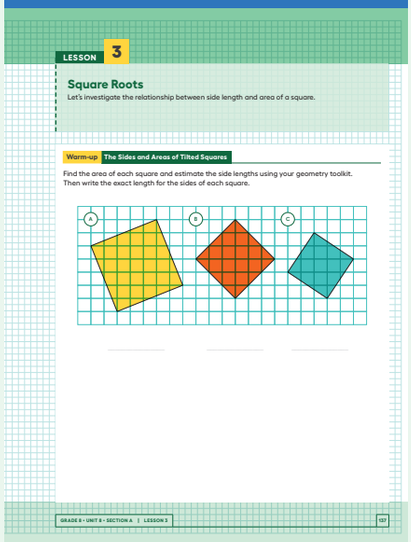
Go Online

Before the lesson, show this video to **introduce** the real-world connection.

ilclass.com/l/614187

Please log in to the site before using the QR code or URL.

Student Workbook



Warm-up

The Sides and Areas of Tilted Squares

10 min

Activity Narrative

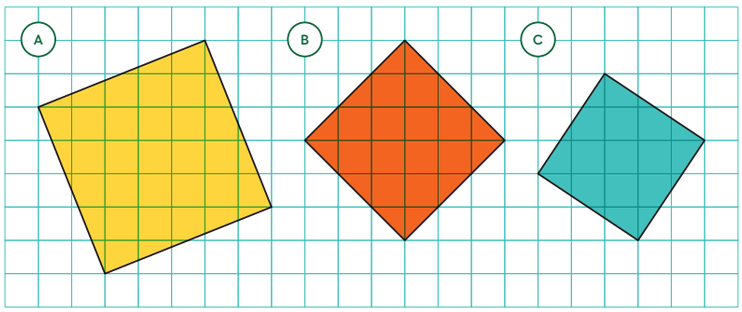
The purpose of this *Warm-up* is for students to continue developing their understanding of square roots. Students find the areas of three squares, estimate their side lengths using tracing paper, and write their exact side lengths using square root notation.

Launch

Arrange students in groups of 2. Provide access to geometry toolkits, including tracing paper. Since the goal of this activity is for students to estimate square roots and not find exact values, do not provide access to calculators. Remind students about the meaning and use of square root notation.

Student Task Statement

Find the area of each square and estimate the side lengths using your geometry toolkit. Then write the exact length for the sides of each square.



Sample response:

A: Area is 29 square units; any side length between 5 and 6 units in reasonable; $s = \sqrt{29}$

B: Area is 18 square units; any side length between 4 and 5 units in reasonable; $s = \sqrt{18}$

C: Area is 13 square units; any side length between 3 and 4 units in reasonable; $s = \sqrt{13}$

Activity Synthesis

The purpose of this discussion is for students to connect the estimate made with tracing paper to the exact length of the side of a square written in square root notation. This helps students to see that a square root is still a number, and it can be approximated by a value that is easier to see.

For each square, invite several students to share their estimate for the side length and display the results for all to see. Here are some questions for discussion:

- *"What do you notice about the estimates?"*

  The estimates are all close to each other. Some have more decimal places than others.

- *"How can we check the accuracy of our estimates?"*

  We can take our estimate and square it to see how close we are to the area of the square.

Activity 1

Side Lengths and Areas of Squares

15 min

Activity Narrative

The purpose of this activity is for students to further their understanding of square roots as numbers by completing a table and creating a graph of side length-area pairs. Students will use the graph to estimate both a square's side length when the value is not an integer and the value of a square root not linked to a drawn square.

Launch

Provide access to calculators. Ask students to complete the table and mark their graphs independently, then check with a partner. Follow with a whole-class discussion.

Student Task Statement

1. Complete the tables with the missing side lengths and areas.

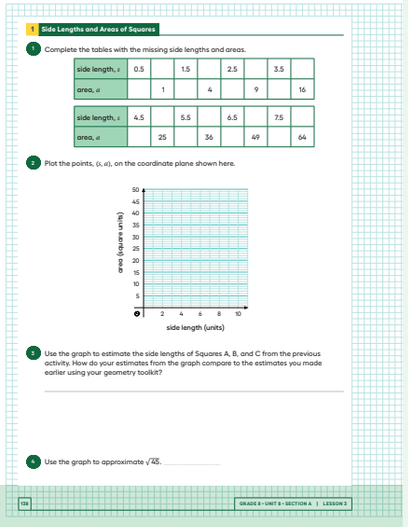
side length, s	0.5	1	1.5	2	2.5	3	3.5	4
area, a	0.25	1	2.25	4	6.25	9	12.25	16

side length, s	4.5	5	5.5	6	6.5	7	7.5	8
area, a	20.25	25	30.25	36	42.25	49	56.25	64

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Ask students to complete the table in the first problem before continuing to create a graph. Check in with students to provide feedback and encouragement after each chunk. *Supports accessibility for: Attention, Social-Emotional Functioning*

Student Workbook


1 Side Lengths and Areas of Squares

1. Complete the tables with the missing side lengths and areas.

side length, s	0.5		1.5		2.5		3.5	
area, a		1		4		9		16

side length, s	4.5		5.5		6.5		7.5	
area, a		25		36		49		64

2. Plot the points, (s, a), on the coordinate plane shown here.

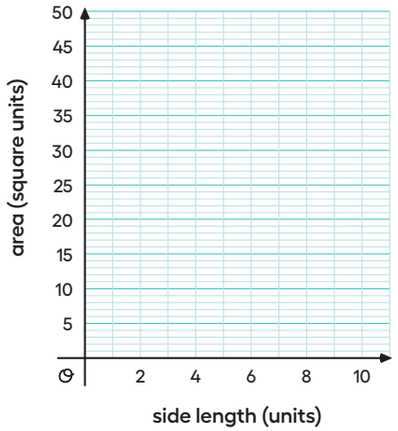
3. Use the graph to estimate the side lengths of Squares A, B, and C from the previous activity. How do your estimates from the graph compare to the estimates you made earlier using your geometry toolkit?

4. Use the graph to approximate $\sqrt{45}$.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each problem, after a student shares their response, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

2. Plot the points, (s, a), on the coordinate plane shown here.



All of the points (s, a) lie on the graph of the equation $y = x^2$.

3. Use the graph to estimate the side lengths of Squares A, B, and C from the previous activity. How do your estimates from the graph compare to the estimates you made earlier using your geometry toolkit?

A: Any estimate between 5 and 5.5 units is reasonable.
B: Any estimate between 4 and 4.5 units is reasonable.
C: Any estimate between 3.5 and 4 units is reasonable.
Sample response: My estimates from the graph are a little more precise than when made with tracing paper.

4. Use the graph to approximate $\sqrt{45}$.

Any estimate between 6.5 and 7 is reasonable. Sample response: 6.7

Activity Synthesis

The purpose of this discussion is to make sure students understand how they can use the table and graph to estimate a square's side length when it is not an integer. This thinking helps students relate numbers expressed using square root notation, such as $\sqrt{45}$, to numbers they are more familiar with, such as 6.5 and 7.

Invite 1–3 students to share their tables and graphs and display them for all to see. Here are some questions for discussion:

- *"What relationship does the graph display?"*

  the relationship between the side length and area of a square

- *"How could we use the table to estimate $\sqrt{45}$?"*

  An area of 45 square units is between the values of 42.25 and 49 in the table. These areas correspond to side lengths of 6.5 and 7 units, so $\sqrt{45}$ has to be between 6.5 and 7.

- *"How did you use the graph to approximate $\sqrt{45}$?"*

  I saw that the point on the graph with a y-value of 45 had an x-value that was a little bigger than 6.5.

Activity 2
Comparing Areas of Squares

10 min

Activity Narrative

The purpose of this activity is to reinforce the difference between the exact and an approximate side length of a square and to practice using different strategies (tracing paper, table, graph) to find them.

In this activity, students are asked to determine which square has the larger area. While the squares appear different, they are actually congruent, both with an area of 50 square units.

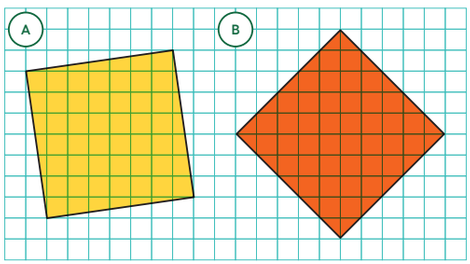
Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Do not provide access to calculators since the goal of the activity is to connect the approximate value with the exact value of a square root.

Display the image of the two squares for all to see and survey the class on which square they think has the larger area. Record the results near the displayed image.

Give students 2–3 minutes of quiet work time, with one student from each pair finding the area of Square A and the other finding the area of Square B. Pause for a partner discussion, followed by a whole-class discussion.

Student Task Statement



1. Find the area of each square and estimate their side lengths.

   Square A and Square B are both 50 square units.
   (Any estimate for the side length between 7 and 7.5 is reasonable).

   Sample response: approximately 7.1 units

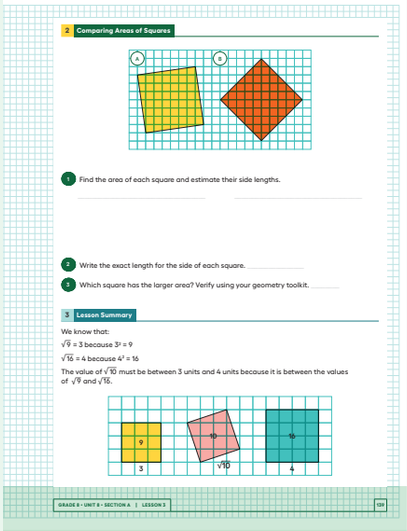
2. Write the exact length for the side of each square.

   Square A: $\sqrt{50}$, Square B: $\sqrt{50}$

3. Which square has the larger area? Verify using your geometry toolkit.

   Both squares have the same area.

Student Workbook


2 Comparing Areas of Squares

1 Find the area of each square and estimate their side lengths.

2 Write the exact length for the side of each square.

3 Which square has the larger area? Verify using your geometry toolkit.

3 Lesson Summary

We know that:
$\sqrt{9} = 3$ because $3^2 = 9$
$\sqrt{16} = 4$ because $4^2 = 16$
The value of $\sqrt{10}$ must be between 3 units and 4 units because it is between the values of $\sqrt{9}$ and $\sqrt{16}$.

Activity Synthesis

The purpose of this discussion is to highlight how $\sqrt{50}$ gives us a way to answer 50 = (side length)^2 exactly, while 7.1 only gets us close.

Here are some questions for discussion:

- *"How did you estimate the side lengths of your square?"*

  I found an area of 50 square units along the y-axis and used the graph from the earlier activity to estimate that the side length was just a little higher than 7.

- *"How can both squares have the same area?"*

  The two squares are the same size but one is just tilted more than the other one.

- *"What rigid transformation takes Square A to Square B?"*

  You can rotate one of the squares to get the other square.

Tell students that this activity shows us that a square with an area of 50 square units has an *exact* side length of $\sqrt{50}$ units, which is *approximately* 7.1 units. This is true for any square with an area of 50 square units no matter the orientation of the square. We can also say that $x = \sqrt{50}$ because $x^2 = 50$ and that $x \approx 7.1$.

Lesson Synthesis

The purpose of this discussion is to check that students understand that it is possible to find the square root of any number and know how to approximate the value of square roots using a table or graph. Here are possible questions for discussion:

- *"What is the exact side length of a square with an area of 35 square units?"*

  $\sqrt{35}$

- *"What is an approximation for the side length of a square with an area of 35 square units?"*

  a little less than 6 or approximately 5.9

- *"Look at the graph of area as a function of side length. Should the points on the graph be connected? Why or why not?"*

  Yes, the points should be connected because it is possible to have any positive value as a side length. Squaring that value gives the area of the square with that specific side length.

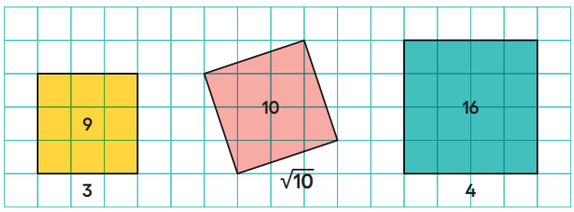
Lesson Summary

We know that:

$\sqrt{9} = 3$ because $3^2 = 9$

$\sqrt{16} = 4$ because $4^2 = 16$

The value of $\sqrt{10}$ must be between 3 units and 4 units because it is between the values of $\sqrt{9}$ and $\sqrt{16}$.



Cool-down

What Is the Side Length?

5 min

Student Task Statement

1. Write the exact value of the side length of a square with each of the following areas.
   - **a.** 100 square units

     10 units
   - **b.** 95 square units

     $\sqrt{95}$ units
   - **c.** 36 square units

     6 units
   - **d.** 30 square units

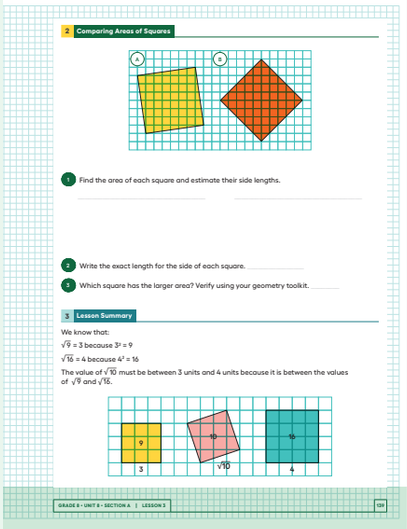
     $\sqrt{30}$ units
2. For each exact value that is not a whole number, estimate the length.

   $\sqrt{95}$ units ≈ 9.7; $\sqrt{30}$ units ≈ 5.5

Responding To Student Thinking

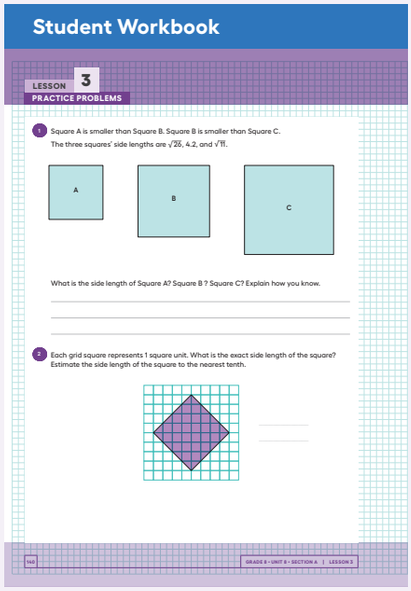
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook



Practice Problems

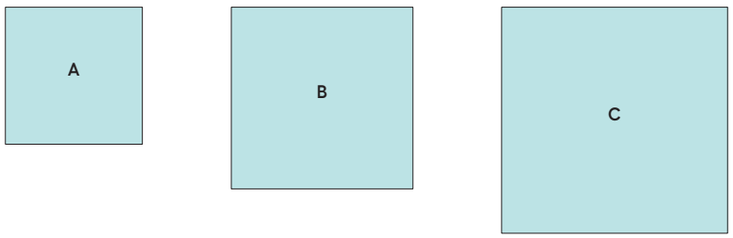
5 Problems



Problem 1

Square A is smaller than Square B. Square B is smaller than Square C.

The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$.



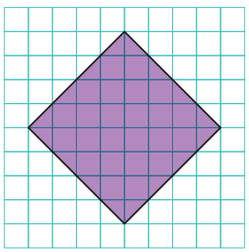
What is the side length of Square A? Square B ? Square C? Explain how you know.

Square A: $\sqrt{11}$ units; Square B: 4.2 units; Square C: $\sqrt{26}$.

Sample reasoning: I know this because $\sqrt{11}$ is between 3 and 4 and $\sqrt{26}$ is between 5 and 6, so $\sqrt{11} < 4.2 < \sqrt{26}$, and the side length of A is less than the side length of B is less than the side length of C.

Problem 2

Each grid square represents 1 square unit. What is the exact side length of the square? Estimate the side length of the square to the nearest tenth.



$\sqrt{32}$ units or about 5.7 units

Problem 3

A square has a side length of $\sqrt{30}$ centimeters. Select **all** the expressions that equal the area of this square in square centimeters.

A. 30

B. $2 \cdot \sqrt{30}$

C. 90

D. $\sqrt{60}$

E. $\sqrt{30} \cdot \sqrt{30}$

F. $\sqrt{30}^2$

Problem 4

from Unit 8, Lesson 2

Find the length of a side of a square if its area is:

a. 81 square inches

9 inches

b. $\frac{4}{25}$ square centimeters

$\frac{2}{5}$ centimeters

c. 0.49 square units

0.7 units

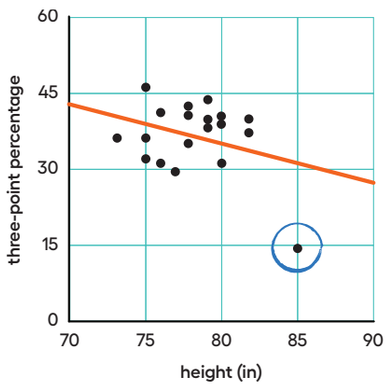
d. m^2 square units

m units

Problem 5

from Unit 6, Lesson 4

The scatter plot shows the heights (in inches) and three-point percentages for different basketball players last season.

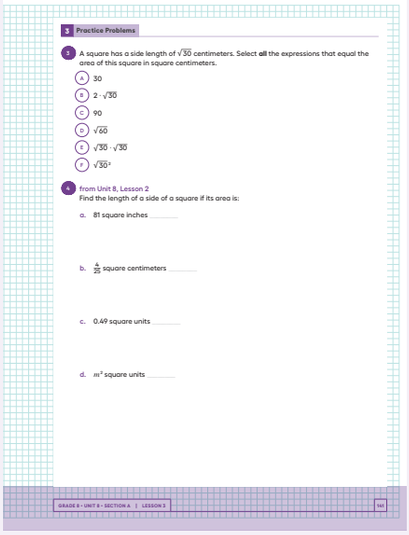


a. Circle any data points that appear to be outliers.

b. Compare any outliers to the values predicted by the model.

Sample response: This point represents a player who had a significantly worse (by about 15% of the attempts) three-point percentage than the model predicts for his height.

Student Workbook


3 Practice Problems

3 A square has a side length of $\sqrt{30}$ centimeters. Select **all** the expressions that equal the area of this square in square centimeters.

A 30
B $2 \cdot \sqrt{30}$
C 90
D $\sqrt{60}$
E $\sqrt{30} \cdot \sqrt{30}$
F $\sqrt{30}^2$

4 from Unit 8, Lesson 2
Find the length of a side of a square if its area is:

a. 81 square inches _____

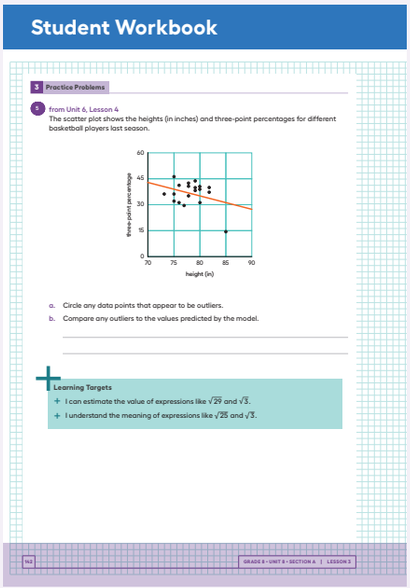
b. $\frac{4}{25}$ square centimeters _____

c. 0.49 square units _____

d. m^2 square units _____

GRADE 8 • UNIT 8 • SECTION A | LESSON 3

Student Workbook


3 Practice Problems

5 from Unit 6, Lesson 4
The scatter plot shows the heights (in inches) and three-point percentages for different basketball players last season.

a. Circle any data points that appear to be outliers.
b. Compare any outliers to the values predicted by the model.

Learning Targets
+ I can estimate the value of expressions like $\sqrt{29}$ and $\sqrt{3}$.
+ I understand the meaning of expressions like $\sqrt{25}$ and $\sqrt{3}$.