Lesson 1

How Well Can You Measure?

Goals

- Create and describe (in writing) graphs that show measurements of squares.
- Justify (orally and in writing) whether the relationship shown on a graph is close enough to a straight line through the origin that it might be a proportional relationship with some measurement error.
- Recognize that when we measure the quantities in a proportional relationship, measurement error can cause the graph to be not perfectly straight and the quotients to be not exactly constant.

Learning Targets

- I can examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship.
- I understand that it can be difficult to measure the quantities in a proportional relationship accurately.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1, Activity 2)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Activity 1, Activity 2)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Poll the Class

Required Materials

Materials to Gather

- Rulers marked with centimeters: Activity 1
- Four-function calculators: Activity 2

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Narrative

In this lesson, students apply what they have learned about proportional relationships to investigate squares. They measure squares of various sizes and use tables and graphs to analyze the measurements.

First, students see that there is a proportional relationship between the length of a square's diagonal and its perimeter, even though measurement error makes it difficult to identify the exact constant of proportionality. As students make assumptions and approximations to simplify a situation, they are modeling with mathematics. Next, students see that even taking measurement error into account, the relationship between the length of the diagonal and the area of a square is not a proportional relationship.

Student Learning Goal

Let's see how accurately we can measure.

Lesson Timeline

5 min	15 min	15 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Assessment: Cool-down

Warm-up
Perimeter of a Triangle

5 min

Activity Narrative

In this activity, students measure the side lengths of a triangle and calculate its perimeter. This serves as an opportunity to review the meaning of perimeter as well as how to use a ruler to measure to the nearest tenth of a centimeter, in preparation for the next activity.

Launch

Ask students to describe in their own words what is the perimeter of a shape. If needed, display the triangle for all to see, and annotate the image as a student describes the perimeter. Distribute a ruler to each student. Give students 2 minutes of quiet work time.

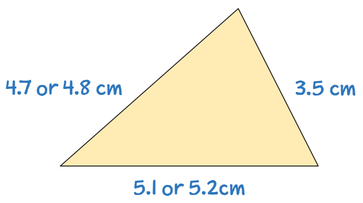
Monitor for how students are measuring and calculating perimeter. If any student has a value for the perimeter that is very far from the correct value, help them identify their error in measurement or calculation.

Student Task Statement

Measure the perimeter of the triangle to the nearest tenth of a centimeter.

Answers vary depending on the printed size of the figure.
Sample response:

About 13.4 cm



Activity Synthesis

The purpose of this discussion is to highlight variability due to measurement error. Poll the class on their answers for the perimeter of the triangle. Point out that many of the answers are pretty close but there is some variation.

Ask students:

- *"Why are these answers not all exactly the same?"*

  When measuring the same thing, people may get slightly different answers due to errors in the measurement.

- *"How do you know if your measurement is actually incorrect?"*

  If your answer is more than just a little bit different from the answer that others have gotten when they measure the same thing, then you may have measured incorrectly.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

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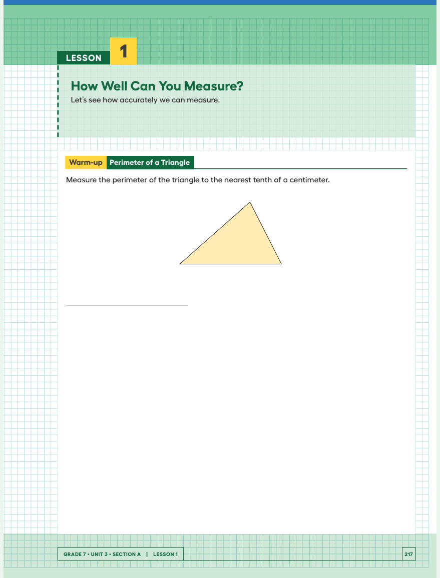
Student Workbook



Activity 1

Perimeter of a Square

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students examine the relationship between the length of the diagonal and the perimeter for squares of different sizes. This prepares students for examining the relationship between the diameter and circumference of circles.

Students graph their measurements and find that the points look like they are close to lying on a line through the origin, suggesting that there could be a proportional relationship between these quantities. This makes sense because the squares can all be viewed as scale copies, so both the side length and the diagonal should change by the same scale factor. As students measure multiple squares and notice patterns in their measurements, they express regularity in repeated reasoning.

Whenever analyzing proportional relationships through experimentation, small errors can be expected. As students make the simplifying assumptions that the variability is likely due to measurement error and that the underlying relationship is proportional, they are modeling with mathematics.

In the digital version of the activity, students use an applet to plot points on the coordinate plane. The applet allows students to add, remove, adjust, and label points. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Launch

Arrange students in groups of 3. Each student book contains a copy of the Perimeter of a Square handout.

Explain that students will work with only 3 of the squares for now and can fill in the other rows of the table at the end of the activity. Assign each group to work with either squares A, B, C, squares D, E, F, or squares G, H, I.

Student Task Statement

Your teacher will give you a picture of 9 different squares and will assign your group 3 of these squares to examine more closely.

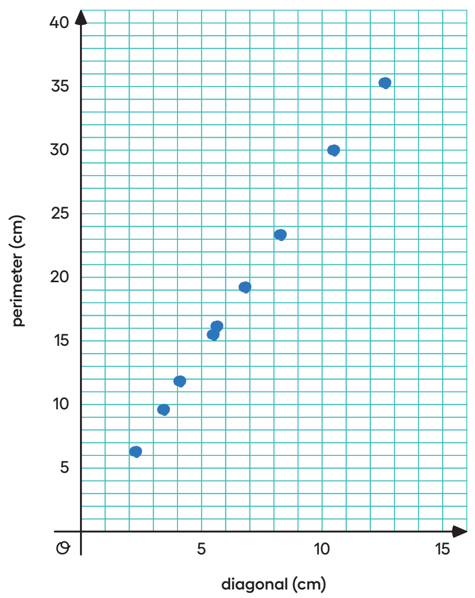
1. For each of your assigned squares, measure the length of the diagonal and the perimeter of the square in centimeters.

   Check your measurements with your group. After you come to an agreement, record your measurements in the table.

	diagonal (cm)	perimeter (cm)
square A	5.7	16.1
square B	8.2	23.3
square C	3.4	9.6
square D	4.2	11.9
square E	6.8	19.2
square F	10.5	30
square G	2.2	6.2
square H	12.6	35.2
square I	5.5	15.6

Answers vary. Each group will have only 3 of these rows filled in, and the numbers may be slightly different. The important thing is that the perimeter of each square should be a little less than 3 times the length of the diagonal.

2. Plot the diagonal and perimeter values from the table on the coordinate plane.

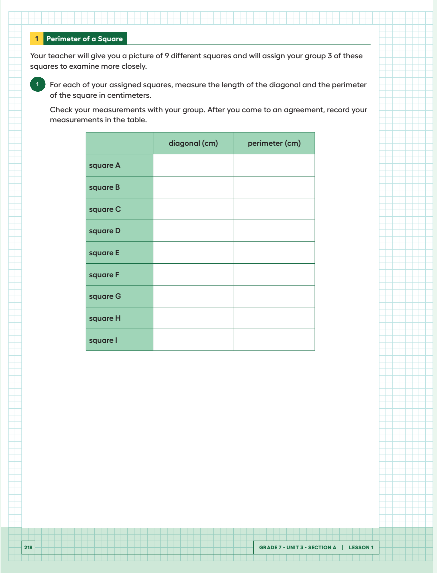


Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "It looks like ...", "I notice that ...", and "What do you notice about ...?"

Supports accessibility for: Language, Organization

Student Workbook



1 Perimeter of a Square

Your teacher will give you a picture of 9 different squares and will assign your group 3 of these squares to examine more closely.

1 For each of your assigned squares, measure the length of the diagonal and the perimeter of the square in centimeters.

Check your measurements with your group. After you come to an agreement, record your measurements in the table.

	diagonal (cm)	perimeter (cm)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

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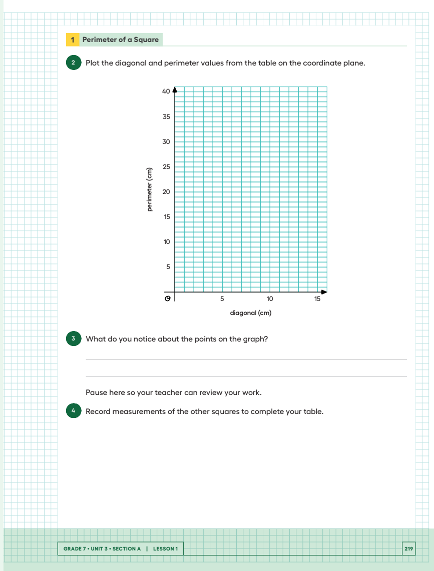
Student Workbook



1 Perimeter of a Square

2 Plot the diagonal and perimeter values from the table on the coordinate plane.

3 What do you notice about the points on the graph?

Pause here so your teacher can review your work.

4 Record measurements of the other squares to complete your table.

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Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language and their own words.

Advances: Listening, Speaking

3. What do you notice about the points on the graph?

   Sample responses:

   - The points almost lie on a straight line through the origin.
   - From the graph it appears that the perimeter is about 3 times as large as the diameter.

   Pause here so your teacher can review your work.

4. Record measurements of the other squares to complete your table.

   See the completed table above.

Activity Synthesis

The goal of this discussion is for students to see that there is a proportional relationship between the length of the diagonal and the perimeter of a square. However, it is difficult to measure accurately enough to get an exact constant of proportionality.

First, ask students to share what they noticed about their graphs. Next, display a table like the one in students' books or devices, ask students to share the measurements they got for each square, and record their answers. If desired, give students 1–2 minutes to plot more points on their graph for the values that they added to their tables.

To lead students to the idea that the length of the diagonal and the perimeter are proportional to each other, ask questions like:

"The data is not perfectly lined up. Do you think it should be? What could be causing the inconsistencies?"

Yes, when we collect data through measurement, we usually will introduce small errors into the data.

"Would it make sense to have (0, 0) as a possible point?"

Yes, since a square with diagonal 0 cm would have a perimeter of 0 cm.

"If we added a point to the graph to represent a square with diagonal 1 cm, what would the perimeter be?"

about 2.8 cm

To help students identify the constant of proportionality, consider adding a column to the table with the quotient of the perimeter divided by the diagonal. For example:

	diagonal (cm)	perimeter (cm)	perimeter ÷ diagonal
square A	5.7	16.1	2.825
square B	8.2	23.3	2.841
square C	3.4	9.6	2.841
square D	4.2	11.9	2.833
square E	6.8	19.2	2.823
square F	10.5	30	2.857
square G	2.2	6.2	2.818
square H	12.6	35.2	2.793
square I	5.5	15.6	2.836

These sample quotients include more decimal places than is justified by the level of precision of the measurements. It is important that students notice the variability in the numbers and come to their own realization that these quotients are all approximately 2.8.

Even though the data does not lie perfectly on a line, the small inconsistencies can be attributed to measurement error. In a situation like this, analyzing the graph is a powerful method to check visually to see if a relationship looks like it may be proportional. Even though the data will look a little bit "bumpy," it will often show the underlying relationships between two quantities.

Activity 2
Area of a Square

15 min

Activity Narrative

In this activity, students examine the relationship between the length of the diagonal and the area for squares of different sizes. Students see that the relationship between diagonal length and area is not close to being proportional. With the table, students identify that there is no constant of proportionality (even after allowing for measurement error). With the graph, students notice that the plotted points are not close to lying on a line.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR8: Discussion Supports
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

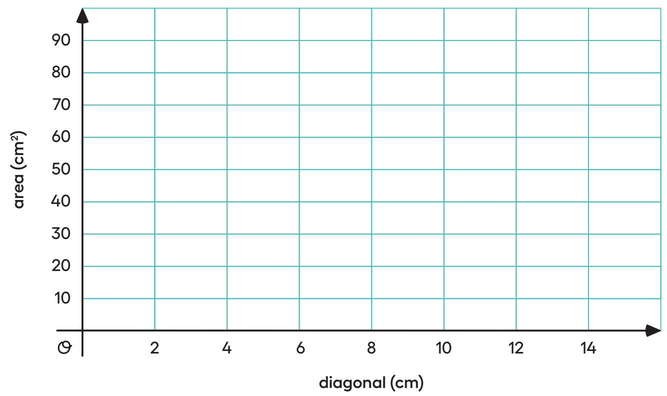
Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Activate or supply background knowledge about calculating the area of a square. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Launch

Keep students in the same groups. Tell them to calculate the *area* of the same 3 squares that they measured in the previous activity. Give students 4 minutes of group work time. Display this blank grid and have students plot points for their measurements.



Give students 2–3 more minutes of group work time followed by whole-class discussion.

Student Task Statement

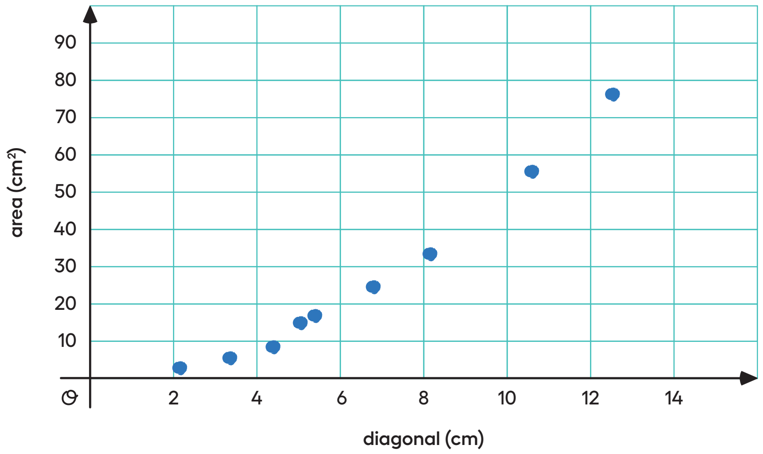
1. In the table, record the length of the diagonal for each of your assigned squares from the previous activity. Next, determine the area of each of your squares.

	diagonal (cm)	area (cm^2)
square A	5.7	16.2
square B	8.2	33.64
square C	3.4	5.76
square D	4.2	8.85
square E	6.8	23.04
square F	10.5	56.25
square G	2.2	2.4
square H	12.6	77.44
square I	5.5	15.21

Pause here so your teacher can review your work. Be prepared to share your values with the class.

Answers vary. Each group will only have 3 of these rows filled in, and the numbers may be slightly different.

2. Examine the class graph of these values. What do you notice?



Sample responses: The graph appears to curve upward. The relationship is not proportional.

3. How is the relationship between the diagonal and area of a square the same as the relationship between the diagonal and perimeter of a square from the previous activity? How is it different?

As the diagonal of a square increases, both perimeter and area increase. However, perimeter is proportional to the length of the diagonal, but area is not.

Building on Student Thinking

Some students may struggle to calculate the area of the squares, or may use the length of the diagonal as if it were the side length. Prompt them with questions like:

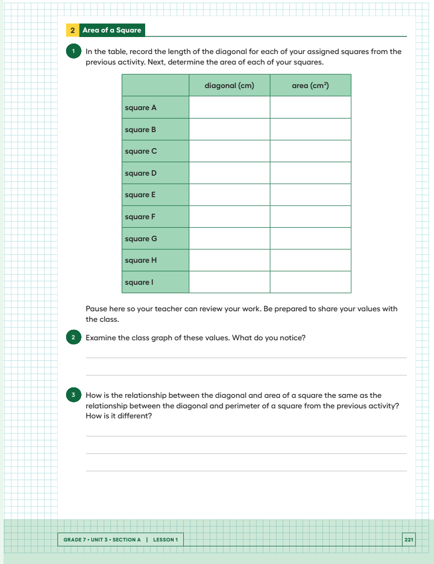
"How can you calculate the area of a rectangle?"

"What is the length and width of your square?"

Some students may measure the side length of each square again, instead of dividing the perimeter from the previous activity by 4. This strategy is allowable, although you can also prompt them to consider if they no longer had access to a ruler, is there a way they could use the information they already recorded to find this measurement.

Some students may struggle to organize the information. Prompt them to add a column to the table in the previous activity to record the side length of the squares.

Student Workbook



2 Area of a Square

1 In the table, record the length of the diagonal for each of your assigned squares from the previous activity. Next, determine the area of each of your squares.

	diagonal (cm)	area (cm²)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

Pause here so your teacher can review your work. Be prepared to share your values with the class.

2 Examine the class graph of these values. What do you notice?

3 How is the relationship between the diagonal and area of a square the same as the relationship between the diagonal and perimeter of a square from the previous activity? How is it different?

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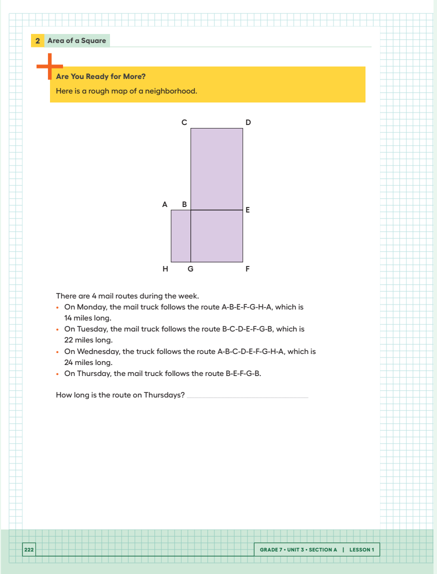
Student Workbook


2 Area of a Square
Are You Ready for More?
Here is a rough map of a neighborhood.
There are 4 mail routes during the week.
On Monday, the mail truck follows the route A-B-E-F-G-H-A, which is 14 miles long.
On Tuesday, the mail truck follows the route B-C-D-E-F-G-B, which is 22 miles long.
On Wednesday, the truck follows the route A-B-C-D-E-F-G-H-A, which is 24 miles long.
On Thursday, the mail truck follows the route B-E-F-G-B.
How long is the route on Thursdays?

Are You Ready for More?

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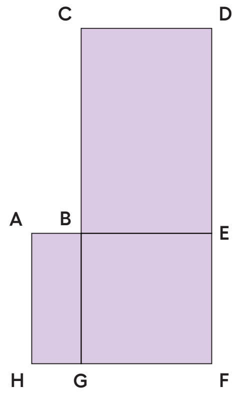

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- On Wednesday, the truck follows the route A-B-C-D-E-F-G-H-A, which is 24 miles long.
- On Thursday, the mail truck follows the route B-E-F-G-B.

How long is the route on Thursdays?

12 miles

Activity Synthesis

The goal of this discussion is for students to recognize that the relationship between the length of the diagonal and the area of squares of different sizes is not a proportional relationship.

First, invite students to share what they noticed about the graph.
Key observations include:

- As the length of the diagonal increases, the area of the square increases.
- The points do not look like they lie on a line and definitely do not lie on a line that goes through (0, 0).
- The area of the square looks to grow at a faster rate for the larger diagonals than for the shorter diagonals.

To reinforce that the relationship is not proportional, ask students to calculate the quotient of the area divided by the length of the diagonal for their squares. Consider displaying a table of their measurements and adding a third column to record their quotients. For example:

	diagonal (cm)	perimeter (cm)	perimeter ÷ diagonal
square A	5.7	16.1	2.825
square B	8.2	23.3	2.841
square C	3.4	9.6	2.824
square D	4.2	11.9	2.833
square E	6.8	19.2	2.823
square F	10.5	30	2.857
square G	2.2	6.2	2.818
square H	12.6	35.2	2.793
square I	5.5	15.6	2.836

Ask questions like:

- *"Can the differences between these quotients be explained by measurement error? Why or why not?"*

  No, the quotients are not close enough to the same value.

- *"Is the relationship between the length of the diagonal and the area of the square a proportional relationship? How do you know?"*

  No, the table does not have a constant of proportionality. The graph is not a straight line through the origin.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the question,

- *"How is this relationship, between the diagonal and the area, different from the relationship we saw earlier, between the diagonal and the perimeter?"*

In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.
If time allows, display these prompts for feedback:

- *"_____ makes sense, but what do you mean when you say … ?"*

  "Can you describe that another way?"

  "How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

Here is an example of a second draft:

- *"The first relationship was proportional, because the quotients of the measurements were approximately the same, and the graph was close to a straight line. The differences could be due to measurement error. This relationship is not proportional, because the quotients are too different to be explained by measurement error, and the graph is not close to a straight line."*

As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Lesson Synthesis

Share with students:

- *"Today we measured squares and decided whether the measurements were in a proportional relationship. We saw that measurement error may make it look like there is not an exact constant of proportionality for a proportional relationship. A graph can be helpful to decide whether the relationship is proportional."*

To review these concepts, consider asking students:

- *"What should the graph of a proportional relationship look like?"*

  a perfectly straight line through the origin

- *"What will the graph of a proportional relationship look like if the values have some measurement error?*

  The points will lie close to a straight line through the origin, but not exactly.

- *"How can you tell from a graph that a relationship is definitely not proportional?"*

  The graph may curve upward or downward. It may meet the y-axis somewhere away from the origin.

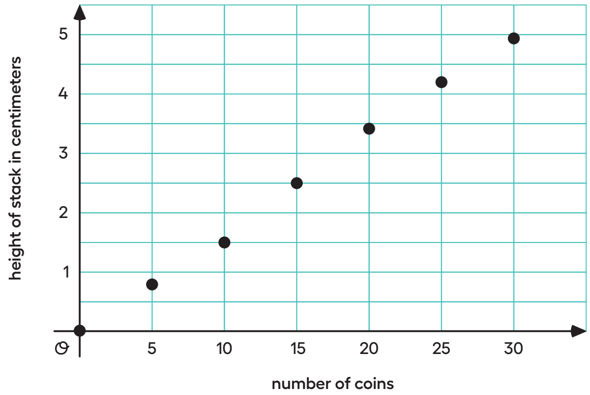
- *"How can you tell from a table that a relationship is definitely not proportional?"*

  Dividing the values on each row gives quotients that are quite different.

Lesson Summary

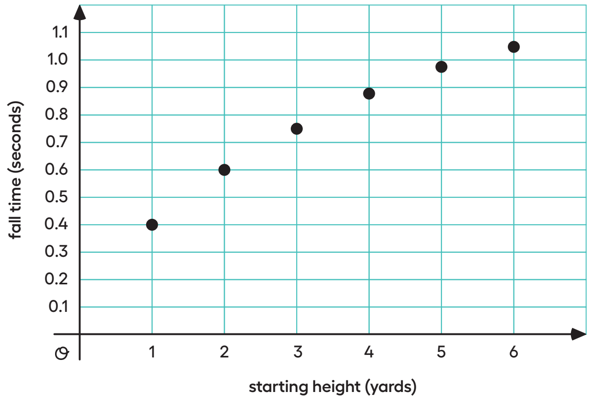
When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through (0, 0), then a proportional relationship is a good model.

This graph shows the height of the stack for different numbers of stacked coins.



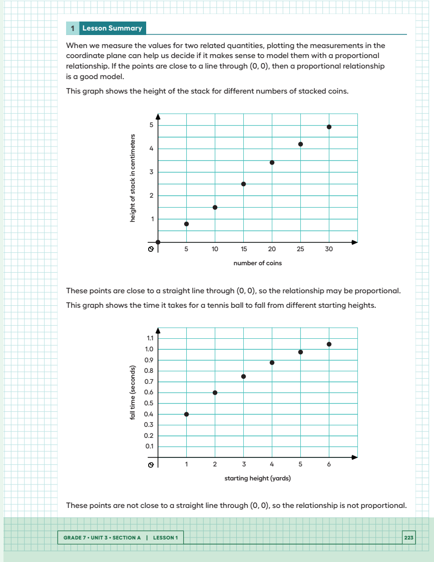
These points are close to a straight line through (0, 0), so the relationship may be proportional.

This graph shows the time it takes for a tennis ball to fall from different starting heights.

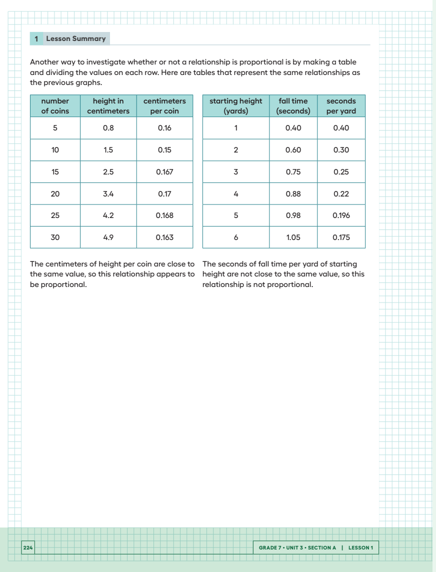


These points are not close to a straight line through (0, 0), so the relationship is not proportional.

Student Workbook


1 Lesson Summary
When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through (0, 0), then a proportional relationship is a good model.
This graph shows the height of the stack for different numbers of stacked coins.
These points are close to a straight line through (0, 0), so the relationship may be proportional.
This graph shows the time it takes for a tennis ball to fall from different starting heights.
These points are not close to a straight line through (0, 0), so the relationship is not proportional.

Student Workbook



1 Lesson Summary

Another way to investigate whether or not a relationship is proportional is by making a table and dividing the values on each row. Here are tables that represent the same relationships as the previous graphs.

The centimeters of height per coin are close to the same value, so this relationship appears to be proportional.

The seconds of fall time per yard of starting height are not close to the same value, so this relationship is not proportional.

Another way to investigate whether or not a relationship is proportional is by making a table and dividing the values on each row. Here are tables that represent the same relationships as the previous graphs.

number of coins	height in centimeters	centimeters per coin
5	0.8	0.16
10	1.5	0.15
15	2.5	0.167
20	3.4	0.17
25	4.2	0.168
30	4.9	0.163

The centimeters of height per coin are close to the same value, so this relationship appears to be proportional.

starting height (yards)	fall time (seconds)	seconds per yard
1	0.40	0.40
2	0.60	0.30
3	0.75	0.25
4	0.88	0.22
5	0.98	0.196
6	1.05	0.175

The seconds of fall time per yard of starting height are not close to the same value, so this relationship is not proportional.

Cool-down

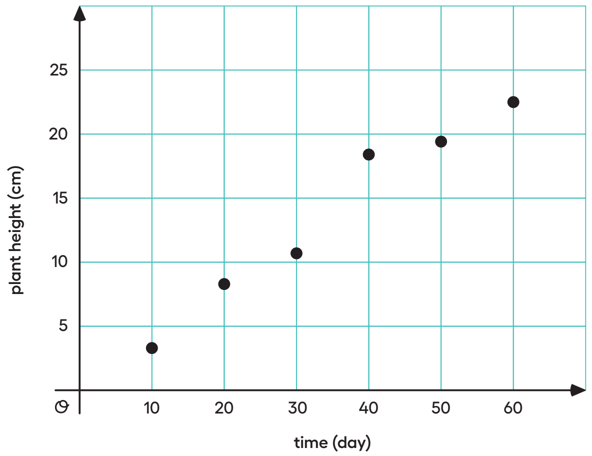
Examining Relationships

5 min

Student Task Statement

For each situation, explain whether the measurements shown on the graph could represent a proportional relationship.

1. The height of a plant was measured every ten days.

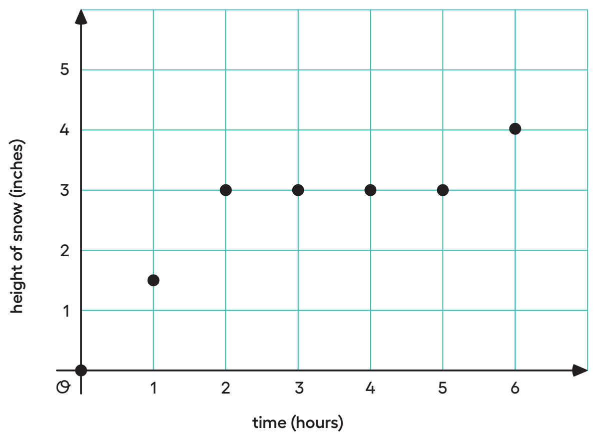


Could the relationship between the number of days and the height of the plant be proportional? Explain your reasoning.

Yes, there may be a proportional relationship.

Sample reasoning: The point $(0, 0)$ is on the graph, the points are close to being on a line, and there could be measurement error. However, it is also possible that the relationship is not proportional. It is not possible to decide for sure from the graph.

2. The height of the snow was measured every hour.



Could the relationship between the number of hours and the height of the snow be proportional? Explain your reasoning.

No, there is not a proportional relationship.

Sample reasoning: For several hours there was no snow falling while some time at the beginning and toward the end there was some snowfall.

Responding To Student Thinking

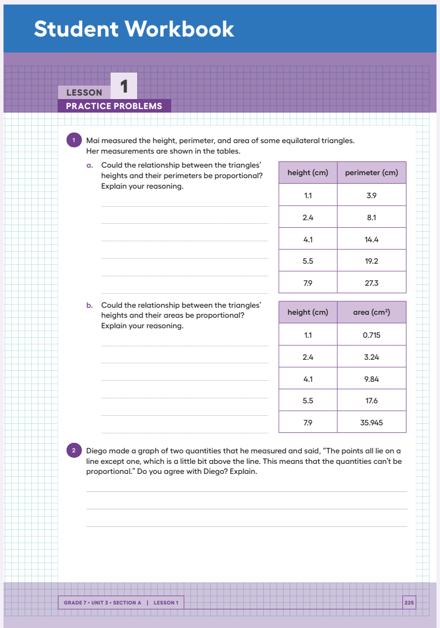
Points to Emphasize
If students struggle with determining whether measurements indicate an underlying proportional relationship, focus on this when opportunities arise over the next several lessons. For example, contrast the relationships in these activities:

Unit 3, Lesson 3, Activity 1 Measuring Circumference and Diameter

Unit 3, Lesson 7, Activity 1 Estimating Areas of Circles.

Practice Problems

4 Problems


Student Workbook

LESSON 1

PRACTICE PROBLEMS

1 Mai measured the height, perimeter, and area of some equilateral triangles. Her measurements are shown in the tables.

a. Could the relationship between the triangles' heights and their perimeters be proportional? Explain your reasoning.

height (cm)	perimeter (cm)
1.1	3.9
2.4	8.1
4.1	14.4
5.5	19.2
7.9	27.3

b. Could the relationship between the triangles' heights and their areas be proportional? Explain your reasoning.

height (cm)	area (cm^2)
1.1	0.715
2.4	3.24
4.1	9.84
5.5	17.6
7.9	35.945

2 Diego made a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities can't be proportional." Do you agree with Diego? Explain.

Problem 1

Mai measured the height, perimeter, and area of some equilateral triangles. Her measurements are shown in the tables.

a. Could the relationship between the triangles' heights and their perimeters be proportional? Explain your reasoning.

Yes, dividing perimeter by height for each row gives values between 3.375 and 3.545. This variability can be explained by measurement error, so there appears to be a constant of proportionality.

height (cm)	perimeter (cm)
1.1	3.9
2.4	8.1
4.1	14.4
5.5	19.2
7.9	27.3

b. Could the relationship between the triangles' heights and their areas be proportional? Explain your reasoning.

No, dividing area by height for each row gives values from 0.65 to 4.55. This is too much variability to be explained by just measurement error, so there does not appear to be a constant of proportionality.

height (cm)	area (cm^2)
1.1	0.715
2.4	3.24
4.1	9.84
5.5	17.6
7.9	35.945

Problem 2

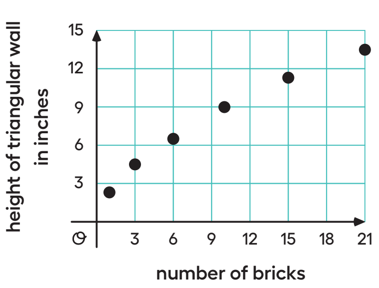
Diego made a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities can't be proportional." Do you agree with Diego? Explain.

Sample response: I don't agree with Diego, since the quantities could be proportional if the line goes through the origin. Measurements are not perfect and the relationship could be proportional.

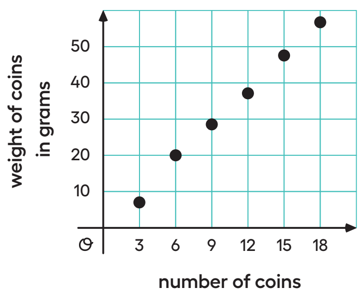
Problem 3

For each graph, explain whether the relationship could be proportional.

A



B

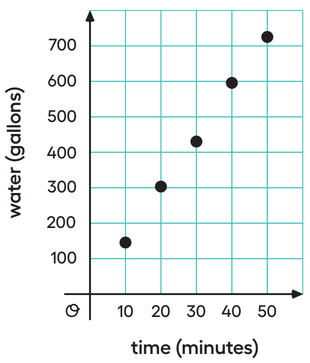


A. No, the points are not close to a line through (0, 0). They appear to be curving to the right.

B. Yes, the points are close to a line through (0, 0). The variation could be due to measurement error.

Problem 4

The graph shows that while it was being filled, the amount of water in gallons in a swimming pool was approximately proportional to the time that has passed in minutes.



a. About how much water was in the pool after 25 minutes?

About 380 gallons

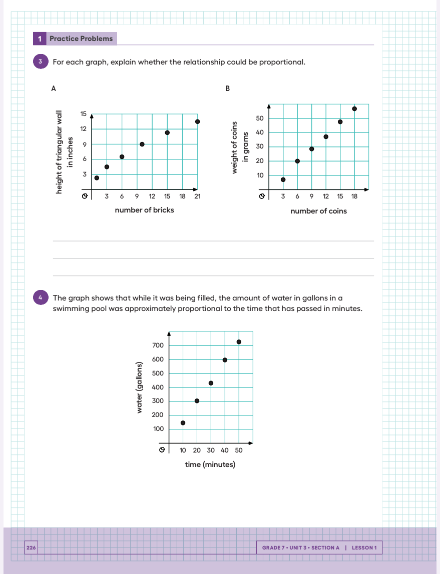
b. Approximately when were there 500 gallons of water in the pool?

After about 35 minutes

c. Estimate the constant of proportionality for the gallons of water per minute going into the pool.

About 15

Student Workbook



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