Lesson 16

Solving More Ratio Problems

Goals

- Choose and create diagrams to help solve problems that involve ratios and a total amount.
- Compare and contrast (orally) different representations of and solution methods for the same problem.

Learning Targets

- I can choose and create diagrams to help think through my solution.
- I can solve all kinds of problems about equivalent ratios.
- I can use diagrams to help someone else understand why my solution makes sense.

Lesson Narrative

In this lesson, students use all representations they have learned in this unit—double number lines, tables, and tape diagrams—to solve ratio problems that involve the sum of the quantities in the ratio. They consider when each tool might be useful and preferable in a given situation and why. In so doing, they make sense of situations and representations, and are strategic in their choice of solution method.

Student Learning Goal

Let's compare all our strategies for solving ratio problems.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 1)

Instructional Routines

- MLR6: Three Reads

Required Materials

Materials to Gather

- Graph paper: Activity 2
- Rulers: Activity 2
- Tools for creating a visual display: Activity 2

Materials to Copy

- Cleaning Fluid and Moving Boxes Cards (1 copy for every 4 students): Activity 3

Required Preparation

Activity 3:

Make one copy of the blackline master for every 6 students and cut them up ahead of time. Consider using a different color paper for each of the four pages to make them easier to organize.

Lesson Timeline

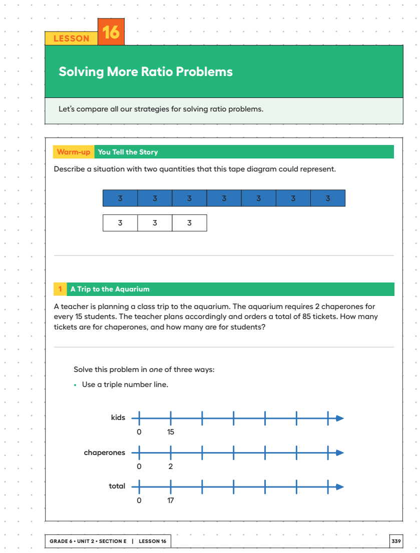
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
10 min	20 min	20 min	10 min	5 min

Building on Student Thinking

Students may misunderstand the meaning of the phrase “with two quantities” and simply come up with a situation involving ten identical groups of three. Point out that the phrase means that the row of seven groups of three should represent something different than the row of three groups of three.

Students may also come up with a situation involving different units, for example, quantity purchased and cost, or distance traveled and time elapsed. Remind them that the parts of tape are meant to represent the same value, so we need a situation that uses the same units for both parts of the ratio.

Student Workbook



Warm-up

You Tell the Story

10 min

Activity Narrative

This *Warm-up* reminds students of previous work with tape diagrams and encourages a different way to reason with them. Students are given only a tape diagram and are asked to generate a concrete context to go with the representation.

Students’ stories should have the following components:

- the same unit for both quantities in the ratio
- a ratio of 7:3
- scaling by 3, or 3 units per part

The stories may also have a quantity of 30 that represents the sum of the two quantities in the ratio.

As students work, identify a few different students whose stories are clearly described and are consistent with the diagram so that they can share later.

Launch

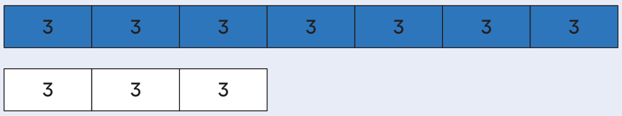
Ask students to share a few things they remember about tape diagrams from the previous lesson. Students may recall that:

- We draw one tape for every quantity in the ratio. Each tape has parts that are the same size.
- We draw as many parts as the numbers in the ratio show. (For instance, for a 2:3 ratio, we draw 2 parts in a tape and 3 parts in another tape).
- Each part represents the same value.
- Tape diagrams can be used to think about a ratio of parts and the total amount.

Tell students their job is to come up with a valid situation to match a given tape diagram. Give students some quiet think time and then time to share their response with a partner.

Student Task Statement

Describe a situation with two quantities that this tape diagram could represent.



Sample responses:

- *One batch of purple paint is made by mixing 7 cups of blue with 3 cups of red. In three batches, there are 30 cups of purple paint, which is made of 21 cups of blue paint and 9 cups of red paint.*
- *There are 30 fish in an aquarium. The ratio of blue fish to red fish is 7:3. There are 21 blue fish and 9 red fish.*

Activity Synthesis

Invite a few students to share their stories with the class. As they share, consider recording key details about each story for all students to see. Then, ask students to notice similarities in the different scenarios.

Guide students to see that they all involve the same units for both quantities of the ratio, a ratio of 7 to 3, and either 3 units per part or scaling by 3. They may also involve an amount of 30 units, representing the sum of the two quantities.

Activity 1

A Trip to the Aquarium

20 min

Activity Narrative

This activity prompts students to solve a single problem using a triple number line, a table, or a tape diagram. Either assign each student a representation or allow students to choose the representation they prefer. During the following discussion, they will compare and contrast the three representations and identify the relative merits of the different representations for the different problems.

Students will have varying opinions about which representation they prefer and why. Their views may stem from observations such as:

- Number lines and tables involve scaling up individual quantities to find the total. Tape diagrams involve starting with the total and breaking it down into equal groups.
- It is hard to take shortcuts with number line diagrams.
- We can use the number line diagram or the table efficiently by thinking, “17 times what is 85?” This value tells us how many “batches” of tickets the teacher had to order.
- When using a tape diagram, it was easier to count 17 groups and compute 85 ÷ 17 to find how many tickets each group needs.

Students may struggle to represent the problem with a tape diagram. As they work, notice any trends that may need to be addressed with the class.

Instructional Routines

MLR6: Three Reads

ilclass.com/r/10695568

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Launch)

MLR6: Three Reads

This is the first time Math Language Routine 6: Three Reads is suggested in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation. During the second read, students identify quantities. During the third read, the final prompt is revealed and students brainstorm possible starting points for answering the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing down a solution path. The purpose of this routine is to support students’ reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Launch

Arrange students in groups of 3–4. Use *Three Reads* to support reading comprehension and sense-making about this problem.

Display only the problem stem (the first two sentences), without revealing the question.

- In the first read, students read the problem with the goal of comprehending the situation.

For the first read, read the problem aloud while everyone else reads along, and then ask,

"What is this situation about? What is going on here?"

Allow 1 minute to discuss with a partner and then share with the whole class.

A typical response may be,

"Students are going on a field trip with chaperones. Everyone needs a ticket."

Listen for and clarify any questions about the context.

- In the second read, students analyze the mathematical structure of the story by naming quantities.

Select a different student to read to the class, and then ask,

"What can be counted or measured in this situation?"

Give students 30 seconds of quiet think time, followed by another 30 seconds to share with their partner.

A typical response may be: number of students and teachers, number of all participants, number of tickets, and cost of tickets.

- In the third read, students brainstorm possible starting points for answering the questions.

Invite students to read the displayed information aloud with their partner, or select a different student to read to the class. After the third read, reveal the question. Ask,

"What are some ways we might get started on this?"

Instruct students to think of ways to approach the questions without actually solving.

Give students 1 minute of quiet think time, followed by another minute to discuss with their group.

Invite students to name some possible strategies referencing quantities from the second read.

Provide these sentence frames as students discuss:

"To find the number of tickets for chaperones (or students), I can start by …"

"I know that … , so I can …"

As students discuss possible solution strategies, select 1–2 students to share their ideas with the whole class. As the selected students present their strategies, create a display that summarizes possible starting points. (Stop students as needed before they share complete solutions or answers.) If no students mention using diagrams or tables, ask them if those might be useful.

Tell students that they will now solve a ratio problem in one of three different ways. Either assign each student one representation or allow them to choose one.

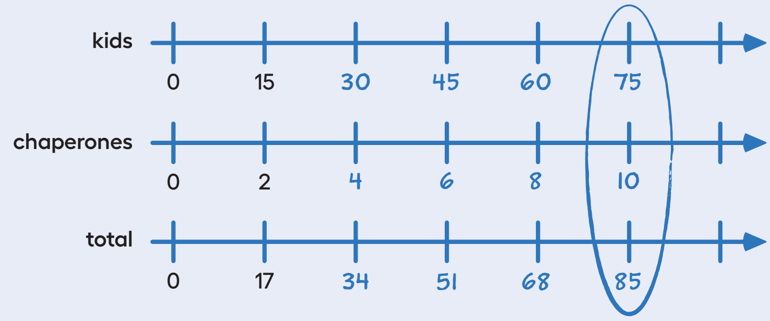
Give students quiet think time to complete the activity and then, optionally, time to share their responses with a small group.

Student Task Statement

A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. The teacher plans accordingly and orders a total of 85 tickets. How many tickets are for chaperones, and how many are for students?

1. Solve this problem in *one* of three ways:

- Use a triple number line.

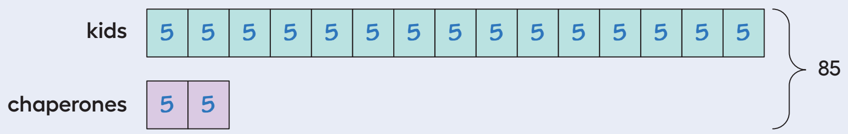


Use a table. (Fill rows as needed.)

kids	chaperones	total
15	2	17
30	4	34
45	6	51
60	8	68
75	10	85

not enough

Use a tape diagram.



Each part of tape is worth 5 tickets, so we can see that the teacher ordered 75 tickets for kids and 10 tickets for chaperones.

2. After discussing all three representations with your class, which representation do you prefer for this problem and why?

Answers vary.

Are You Ready for More?

Use the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

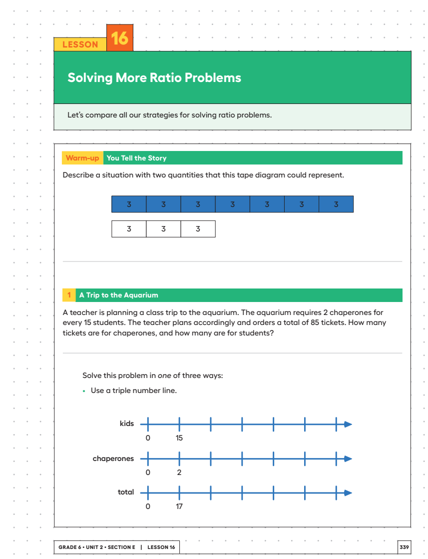
$\boxed{6} : \boxed{3}$ is equivalent to $\boxed{1}\boxed{8} : \boxed{9}$ and $\boxed{5}\boxed{4} : \boxed{2}\boxed{7}$

Note: This problem was inspired by the problem Finding Equivalent Ratios by Graham Fletcher published at openmiddle.com and used here with permission.

Building on Student Thinking

The number line and table representations are organized similarly. For example, one could make progress with both of them simply by skip counting and keeping an eye out for a total of 85 people. The tape diagram, though, is organized in a much different way. Equivalent ratios are not listed out, but rather equivalent ratios arise from thinking about how the diagram could represent any number of batches. Students may thus mistakenly treat the tape diagram like a double number line diagram—they may start writing 15, 30, 45, etc. in the "kids" tape, for example. Once this plays out, students *may* self-regulate once they notice there are only 2 boxes in the chaperones' row. But they may decide to just draw more boxes! Reorient these students by asking how many parts of tape there are (17), and reminding them each part of tape represents equal numbers of people, and that there are 85 total people. The presentation of correct work during the discussion could be used as an opportunity to remediate, as well. For example, consider asking a student to explain what they understand about another student's correct work.

Student Workbook



Building on Student Thinking

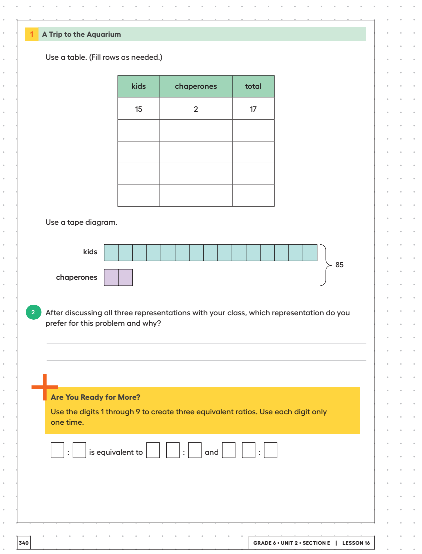
While thinking through problems, it is common for students to hold the meanings of their representations (numbers, quantities, markings, etc.) in their heads without writing them down. When students are getting their solutions ready for others to look at, though, remind them of the importance of labeling quantities and units of measure and making the steps in their thinking clear.

Some students may choose the same strategy or representation each time. If their answers are accurate, this is fine. However, if time allows, ask them to check if they can verify their answer using an alternative strategy.

If using the Info Gap routine: Students holding a Problem Card may have trouble thinking of appropriate questions to ask their partner. Encourage them to revisit the problem at hand and think about the kinds of information that might be helpful or relevant. (For example, if the problem is about how long it would take to perform something, ask students how they usually gauge the amount of time needed for something. Ask,

> *"What would the amount of time depend on?"*

Student Workbook


1 A Trip to the Aquarium

Use a table. (Fill rows as needed.)

kids	chaperones	total
15	2	17

Use a tape diagram.

2 After discussing all three representations with your class, which representation do you prefer for this problem and why?

Are You Ready for More?

Use the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

☐ : ☐ is equivalent to ☐ ☐ : ☐ and ☐ ☐ : ☐

340 GRADE 6 • UNIT 2 • SECTION E | LESSON 16

Activity Synthesis

Before debriefing as a class, consider arranging students in groups of 3, where a group includes one student who used each representation. Give them time to see if they got the same answer and compare and contrast the representations.

Solicit students' reactions to the three strategies, encouraging them to identify what is similar and different about the approaches. Consider polling the class for their preferred strategy. Ask 1–2 students favoring each method to explain why. This can also be a discussion about what worked well in this or that approach, and what might make this or that approach more complete or easy to understand. While there is no right or wrong answer with regards to their preferred strategy, look out for unsupported reasoning or misunderstandings.

Some students may prefer the tape diagram because the solution path seems more direct, but caution them against trying to use a tape diagram any time they see a ratio problem. Because tape diagrams involve equal-sized parts, they can only be used to represent quantities with the same unit. If different units are involved, the parts of one tape and those of the other will not represent an identical quantity.

Activity 2: Optional

Cleaning Fluid and Moving Boxes

20 min

Activity Narrative

This activity gives students a chance to apply the different strategies they have learned to solve ratio problems and to decide on methods that would make the most sense in given situations. If desired, this activity can be completed as an *Info Gap*. Alternatively, students can simply complete the two problems that are printed in the task statement.

Each problem may lend itself better to a particular representation than to others. Here are sample arguments in support of a representation for each problem, though options may vary:

- The cleaning fluid problem can be well represented by all three representations because it involves small numbers and simple multiplication.
- The box-moving problem would be inefficient to represent with a number line diagram, since it would require making 8 half-hour jumps to find the total of 72. Either a tape diagram or a table showing more straightforward multiplication would be more efficient.

The time needed to solve the problems may depend on the representation students choose to use.

Launch

Tell students they will continue to practice using the tools at their disposal—number lines, tables, and tape diagrams—to solve ratio problems.

Provide access to graph paper and rulers. Arrange students in groups of 2.

If using the *Info Gap* routine: Explain the *Info Gap* structure and consider demonstrating the protocol. Instruct students to close their books or devices. In each group distribute a problem card to one student and a data card to the other student. Give students who finish early a different pair of cards and ask them to switch roles.

If not using the *Info Gap* routine: Ask students to complete both questions and then compare their strategies with their partner. Partners should work together to resolve any discrepancies.

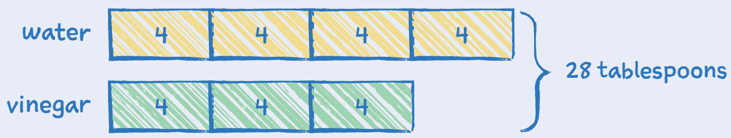
Student Task Statement

Solve each problem and show your thinking. Organize it so it can be followed by others. If you get stuck, consider drawing a double number line diagram, a table, or a tape diagram.

1. A recipe for a cleaning liquid says to mix 4 parts water with 3 parts vinegar. How much water is needed to make a total of 28 tablespoons of the solution?

   16 tablespoons of water

   Sample reasoning: There are 7 parts of water and vinegar combined. If there are 28 tablespoons in the 7 parts, then there are 4 tablespoons in 1 part ($28 \div 7 = 4$) and 16 tablespoons in 4 parts of water ($4 \cdot 4 = 16$).



2. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move all 72 boxes?

boxes moved by Andre	boxes moved by Noah	total boxes moved	elapsed time (hours)
4	5	9	$\frac{1}{2}$
32	40	72	4

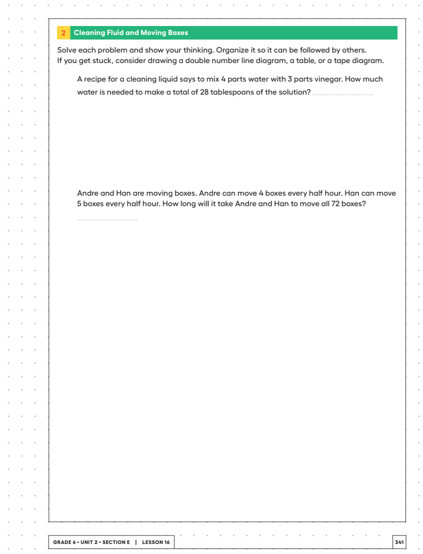
   4 hours

   Sample reasoning: The total number of boxes carried each half hour is 9. $72 = 9 \cdot 8$. Therefore, it takes 8 $\frac{1}{2}$-hour increments, or 4 hours, to move 72 boxes.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example: "We already know …" "We are trying to find …" "We can use _____ to represent …"
Supports accessibility for: Language, Organization

Student Workbook



Activity Synthesis

Share the solutions to the problems or invite students to share theirs. Ask students about their choice(s) of representations. Some guiding questions:

“Is there a particular representation you tend to try first?”

“Does one seem more efficient than the others?”

“Did the situation in a problem affect your choice? If so, what features of a problem might steer you toward or away from a particular strategy?”

Lesson Synthesis

To reinforce the idea that different representations can be used to solve problems involving equivalent ratios, highlight when each representation might be preferred:

- Tape diagrams are most likely to be helpful when the parts of the ratio have the same kind of units (such as cups to cups, miles to miles, or boxes moved to boxes moved) and the sum of the quantities is meaningful in the context.
- Number lines are a good choice when it helps to visualize how far apart numbers are from each other. They are harder to use with very big or very small numbers.
- Tables work well in almost all situations.

To emphasize good problem-solving habits, ask students:

“What should we remember to do when using different representations to solve equivalent ratio problems?”

We should remember to:

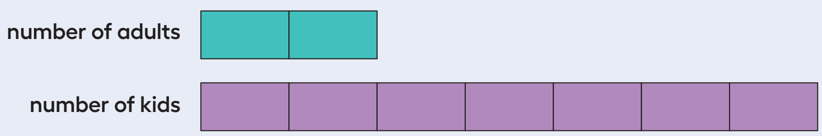
- label each part of the diagram with what it represents
- include units in answer, for example, writing “4 cups” instead of just “4”
- use brackets to indicate total amounts
- read what the question is asking and answering it

Lesson Summary

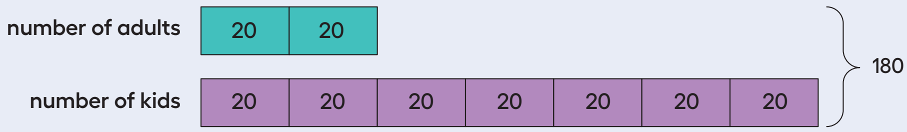
When solving a problem involving equivalent ratios, it is often helpful to use a diagram. Any diagram is fine as long as it correctly shows the mathematics and you can explain it.

Let's compare three different ways to solve the same problem: The ratio of adults to kids in a school is 2:7. If there is a total of 180 people, how many of them are adults?

- *Tape diagrams* are especially useful for this type of problem because both parts of the ratio have the same units ("number of people") and we can see the total number of parts.

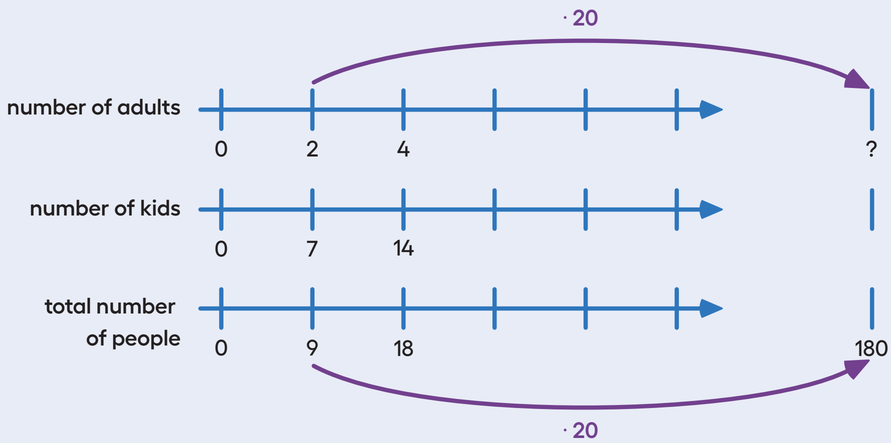


This tape diagram has 9 equal parts, and they need to represent 180 people total. That means each part represents 180 ÷ 9, or 20 people.



Two parts of the tape diagram represent adults. There are 40 adults in the school because $2 \cdot 20 = 40$.

- *Double or triple number lines* are useful when we want to see how far apart the numbers are from one another. They are harder to use with very big or very small numbers, but they could support our reasoning.



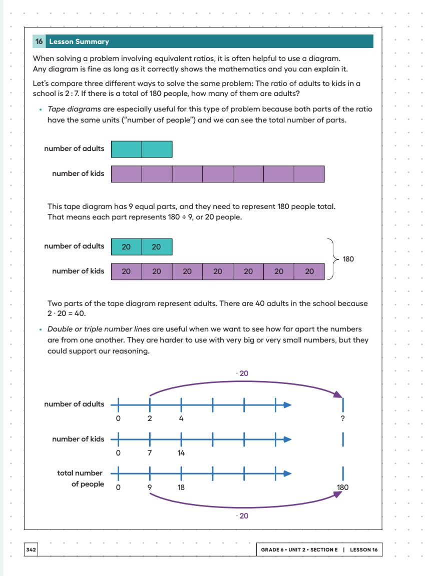
- *Tables* are especially useful when the problem has very large or very small numbers.

adults	kids	total
2	7	9
?		180

$\cdot 20$ (left) $\qquad$ $\cdot 20$ (right)

We ask ourselves, "9 times what is 180?" The answer is 20. Next, we multiply 2 by 20 to get the total number of adults in the school.

Student Workbook



Responding To Student Thinking

Points to Emphasize
If students struggle with solving ratio problems that involve the sum of the quantities in the ratio, as opportunities arise discuss the use of tape diagrams or tables for reasoning. For example, when reviewing these practice problems, encourage students to represent the quantities in each situation with a tape diagram or a table and to use it to make sense of the situation:

Unit 2, Lesson 16, Practice Problem 2
Unit 2, Lesson 16, Practice Problem 3

Another reason to make diagrams is to communicate our thinking to others. Here are some good habits when making diagrams:

- Label each part of the diagram with what it represents.
- Label important amounts.
- Make sure you read what the question is asking and answer it.
- Make sure you make the answer easy to find.
- Include units in your answer. For example, write “4 cups” instead of just “4.”
- Double check that your ratio language is correct and matches your diagram.

Cool-down

Pizza-making Party

5 min

Student Task Statement

You had a pizza-making party for a number of people. Each person needed 6 ounces of dough and 4 ounces of sauce for their pizza. A total of 130 ounces of dough and sauce were used at the party.

1. How many ounces of dough were used at the party?

   78 ounces of dough

2. How many ounces of sauce were used at the party?

   52 ounces of sauce

   Sample reasoning:

dough (ounces)	sauce (ounces)	total (ounces)
6	4	10
78	52	130

3. How many people were at the party?

   13 people

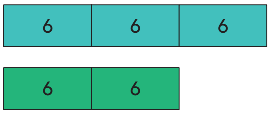
   Sample reasoning: $6 \cdot 13 = 78$ and $4 \cdot 13 = 52$.

Practice Problems

6 problems

Problem 1

Describe a situation that could be represented with this tape diagram.



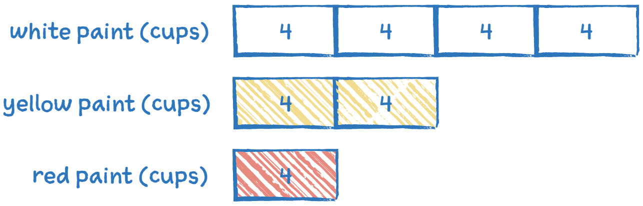
Sample response: There are 30 people at a movie. The ratio of teenagers to adults is 3 to 2. There are 18 teenagers and 12 adults.

Problem 2

One batch of beige paint requires 4 cups of white paint, 2 cups of yellow paint, and 1 cup of red paint. Han made a large amount of this shade of beige paint. If he used 8 cups of yellow paint, how many total cups of beige paint did Han make? Explain or show your reasoning.

28 cups of paint

Sample reasoning:



If there are 8 cups of yellow paint, each piece of the tape diagram represents 4 cups of paint. That means that there are 16 cups of white paint, 8 cups of yellow paint, and 4 cups of red paint. 16 + 8 + 4 = 28.

Problem 3

from Unit 2, Lesson 16

For an activity, a teacher gave each student in the class 4 purple cards and 3 white cards. In total, the teacher gave out 91 cards.

a. How many students are in the class?

13 students

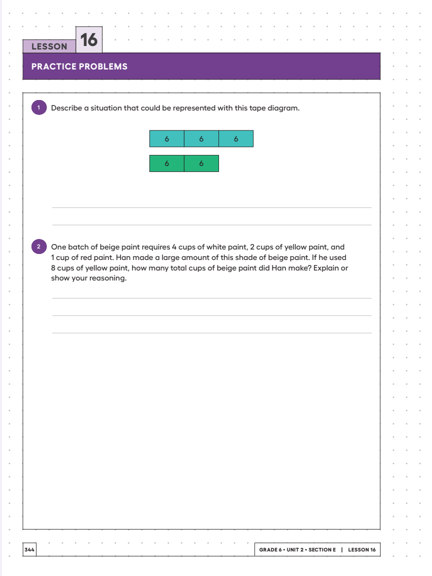
b. How many purple cards were given out?

52 purple cards

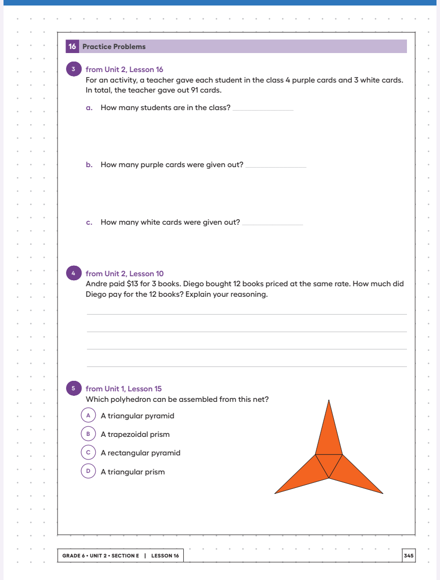
c. How many white cards were given out?

39 white cards

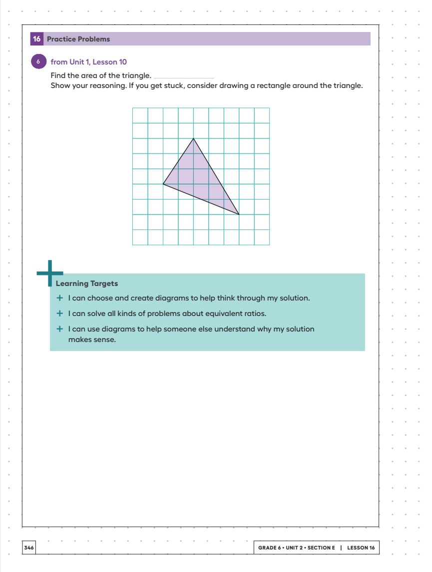
Student Workbook



Student Workbook



Student Workbook



Problem 4

from Unit 2, Lesson 10

Andre paid $13 for 3 books. Diego bought 12 books priced at the same rate. How much did Diego pay for the 12 books? Explain your reasoning.

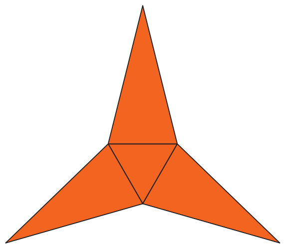
$52

Sample reasoning: Twelve is 4 groups of 3 so Diego's books will cost 4 times as much as Andre's and $4 \cdot 13 = 52$.

Problem 5

from Unit 1, Lesson 15

Which polyhedron can be assembled from this net?



A. A triangular pyramid

B. A trapezoidal prism

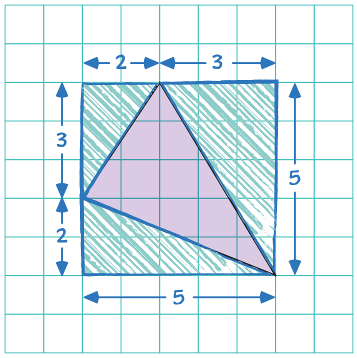
C. A rectangular pyramid

D. A triangular prism

Problem 6

from Unit 1, Lesson 10

Find the area of the triangle. Show your reasoning. If you get stuck, consider drawing a rectangle around the triangle.



9.5 square units

Sample reasoning: Surround the triangle with a 5 by 5 unit square, which has an area of 25 square units. From the area of the square, subtract the areas of the three right triangles. The total area of the right triangles is 15.5 square units, because $3 + 5 + 7.5 = 15.5$. The area of the given triangle is 9.5 square units, since $25 - 15.5 = 9.5$.