Lesson 14

Four Representations

Goals

- Calculate the constant of proportionality for a proportional relationship in an unfamiliar context, and express it (in writing) using the correct units.
- Critique (orally and in writing) presentations of proportional and nonproportional relationships.
- Invent and describe (in writing and using other representations) a proportional relationship and a nonproportional relationship.

Learning Targets

- I can make connections between the graphs, tables, and equations of a proportional relationship.
- I can use units to help me understand information about proportional relationships.

Student Learning Goal

Let's contrast relationships that are and are not proportional in four different ways.

Lesson Narrative

This lesson is optional because it presents an open-ended problem for students to review everything they have learned in this unit. Students invent their own situation that involves a proportional relationship. They represent it using a sentence, a table, an equation, and a graph. As students identify quantities and relationships to represent a situation, they are modeling with mathematics.

Note that the real-world relationship that students choose to represent may not be exactly proportional. In other words, the rate relating the two quantities may involve some variability and is only roughly constant. It is fine for students to approximate an average constant rate in such cases. Discussions of measurement error and statistical variability will be addressed in later units.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 1

Materials to Copy

- Creating and Representing Situations Handout (1 copy for every 1 student): Activity 1

Lesson Timeline

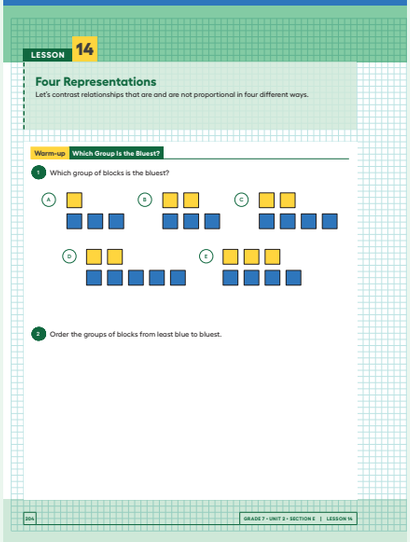
Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
5 min	30 min	10 min	5 min

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to to support whole-class discussion: "I noticed _____, so I ..." "_____'s idea reminds me of ...", and "The next time I put things in order from least to greatest, I will ..."

Advances: Speaking, Conversing, Representing

Student Workbook


LESSON 14

Four Representations

Let's contrast relationships that are and are not proportional in four different ways.

Warm-up Which Group Is the Bluest?

1. Which group of blocks is the bluest?

A B C D E

2. Order the groups of blocks from least blue to bluest.

Warm-up

Which Group Is the Bluest?

5 min

Activity Narrative

In this *Warm-up*, students are asked to reason which group of blocks is the bluest and explain how they arrived at that decision. The goal is to prompt students to visualize and articulate different ways they can use ratios, equivalent ratios and proportions to support their reasoning.

Launch

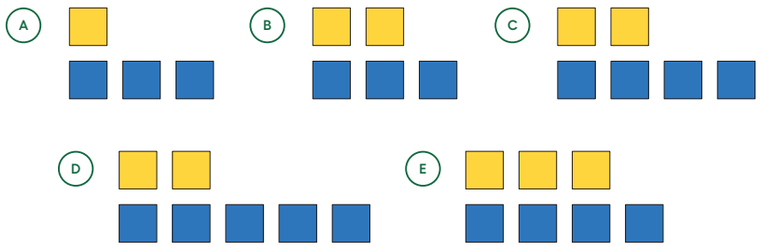
Tell students you will show them five groups of blocks. Their job is to determine which group of blocks is the bluest. Display the image for all to see.

Give students 2 minutes of quiet think time.

Encourage students who have one way of supporting their decision to think about another way while they wait.

Student Task Statement

1. Which group of blocks is the bluest?



A or D

A if looking at the amount of blue per yellow, D if looking at the total amount of blue or the difference between blue and yellow

2. Order the groups of blocks from least blue to bluest.

Sample responses:

- E, B, C, D, A when ordering by the amount of blue blocks per yellow
- A or B, C or E, D when ordering by total amount of blue blocks
- B or E, A or C, D when ordering by difference between blue and yellow blocks

Activity Synthesis

Ask students to share which group of blocks is the bluest and their reasoning. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- Did anyone choose the same group of blocks but would explain it differently?
- Does anyone want to add an observation to the way ____ saw the blocks?
- Do you agree or disagree? Why? Ask students to order the groups of blocks from less blue to bluest after deciding on the bluest group of blocks.

Activity 1
Creating and Representing Situations

30 min

Activity Narrative

In this activity, students create their own proportional and nonproportional relationships. This is a significant change from previous activities where students were always given two quantities and asked to decide if they were proportional or not. This new step gives students the opportunity to think about how quantities are related, which is an important step of modeling with mathematics.

Next, students make a visual display of their scenarios. The posters are displayed around the room so students can view each other's work, give feedback, and critique the reasoning of others.

Launch

Arrange students in groups of 2. Explain that students are going to come up with relationships that represent things in their daily lives.

Give students 1–2 minutes to brainstorm situations that involve proportional relationships.

Invite students to share their ideas. Record the general gist of each idea that is shared and display for all to see. For example, if a student says, "At my favorite arcade, it takes 4 tokens per game of pinball," you could record this idea as "how many tokens for an arcade game."

Give students 8–10 minutes of partner work time to invent a relationship and answer the questions.

If students struggle to think of situations that involve proportional relationships, consider providing them with copies of the first page of the blackline master, which lists some examples of proportional relationships that they have seen during this unit. Make sure students understand that the examples on the blackline master are just samples and they can still choose to write a different type of relationship.

Student Task Statement

1. Make up a situation where there is a proportional relationship between two quantities.

   a. Write one or more sentences describing the relationship.

   Clare and Diego are teammates in a 100-yard three-legged race. Their friend Kiran is timing them. Kiran notices that they pass the 20-yard marker at 0.5 minute, the 40-yard marker at 1 minute, and the 60-yard marker at 1.5 minutes.

   b. Make a table with at least 5 pairs of numbers relating the two quantities.

d	t
20	0.5
40	1
60	1.5
80	2
100	2.5

Access for Multilingual Learners (Activity 1, Launch)

MLR2: Collect and Display.
Direct attention to words collected and displayed from the previous lessons. Invite students to borrow language from the display as needed, and update it throughout the lesson.

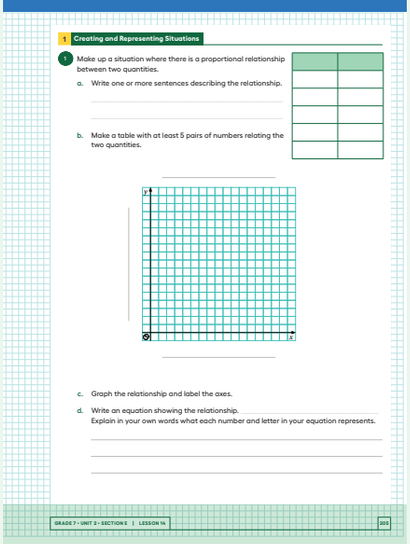
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Provide Access by Recruiting Interest.
Invite students to generate a list of additional examples of proportional and nonproportional relationships that connect to their personal backgrounds and interests. Ask students in advance of the lesson to provide time for students to think about it and make more authentic connections to their everyday life.

Supports accessibility for: Conceptual Processing, Attention

Student Workbook


1 Creating and Representing Situations

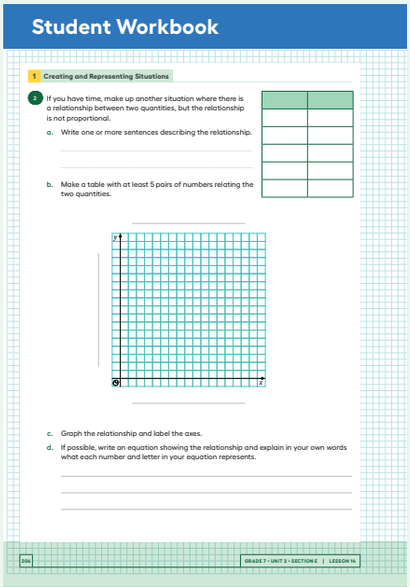
1. Make up a situation where there is a proportional relationship between two quantities.
   a. Write one or more sentences describing the relationship.
   b. Make a table with at least 5 pairs of numbers relating the two quantities.
   c. Graph the relationship and label the axes.
   d. Write an equation showing the relationship. Explain in your own words what each number and letter in your equation represents.

Building on Student Thinking

As students work, pay attention to the numbers they use in their tables. Students can be haphazard when choosing values, and their numbers may end up being unfriendly. Prompt them to reason about what numbers would be more conducive to the situation or easier to calculate.

Some students may struggle to think of a nonproportional situation. Consider providing them with copies of the second page of the blackline master, which lists some examples of nonproportional relationships that they have seen during this unit. However, it is not necessary for every group to come up with a nonproportional relationship, as long as there are a few examples for students to contrast during the discussion.

Student Workbook


1 Creating and Representing Situations

2 If you have time, make up another situation where there is a relationship between two quantities, but the relationship is not proportional.

a. Write one or more sentences describing the relationship.

b. Make a table with at least 5 pairs of numbers relating the two quantities.

c. Graph the relationship and label the axes.

d. If possible, write an equation showing the relationship and explain in your own words what each number and letter in your equation represents.

c. Graph the relationship and label the axes.

d. Write an equation showing the relationship.

$t = \frac{1}{40} d$

Explain in your own words what each number and letter in your equation represents.

2. If you have time, make up another situation where there is a relationship between two quantities, but the relationship is not proportional.

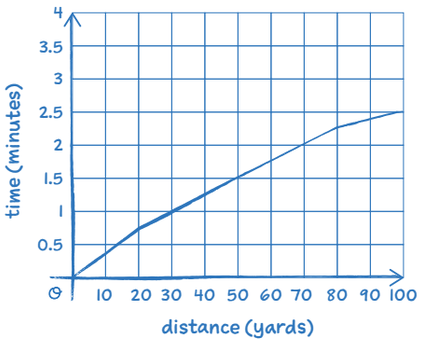
a. Write one or more sentences describing the relationship.

Noah and Elena are teammates in a 100-yard three-legged race. Their friend Lin is timing them. Lin notices that they pass the 20-yard marker at 0.75 minute, the 40-yard marker at 1.25 minutes, and the 60-yard marker at 1.75 minutes.

b. Make a table with at least 5 pairs of numbers relating the two quantities.

d	t
20	0.75
40	1.25
60	1.75
80	2.25
100	2.5

c. Graph the relationship and label the axes.



d. If possible, write an equation showing the relationship and explain in your own words what each number and letter in your equation represents.

No written response required.

3. For each of your situations, explain how you know whether the relationship is proportional or not. Give as many reasons as you can.

The first relationship is proportional because:

- Each value of d in the table can be multiplied by $\frac{1}{40}$ to get the corresponding value of t.
- The graph is part of a line that goes through the origin and Quadrant I.
- The equation can be written in the form $t = kd$.

The second relationship is not proportional because:

- There is not a constant factor that relates the column in the table.
- The graph does not make a straight line through the origin.
- There is not an equation of the form $y = kx$ that relates the quantities.

Activity Synthesis

Give students 5 minutes to trade their work with another group to give feedback about their analysis.

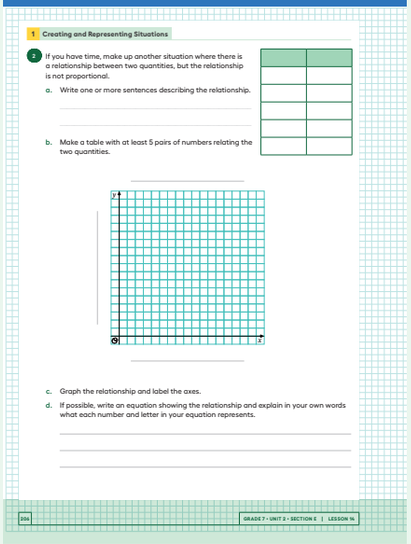
Tell them that they should incorporate the feedback they received when making their posters.

Ask students to create a visual display of the relationship they came up with that includes all four of the different types of representations (sentences, table, graph, equation). When the posters are complete, display them around the room. Provide students with these sentence starters, and give them an opportunity to view their classmates' work and write their responses. This provides a structured way for students to critique the reasoning of others.

- *"The most interesting (or surprising) relationship was ____, because ..."*
- *"The group ____ should check their work where they ..."*
- *"I really liked when the group ____ did this ____ because ..."*

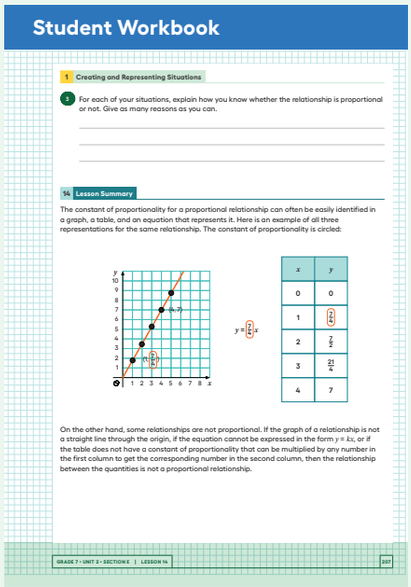
Conclude with a brief whole-class discussion where students can share similarities they noticed about the different relationships and representations.

Student Workbook


1 Creating and Representing Situations

2. If you have time, make up another situation where there is a relationship between two quantities, but the relationship is not proportional.
   a. Write one or more sentences describing the relationship.
   b. Make a table with at least 5 pairs of numbers relating the two quantities.
   c. Graph the relationship and label the axes.
   d. If possible, write an equation showing the relationship and explain in your own words what each number and letter in your equation represents.

Student Workbook



1 Creating and Representing Situations

3 For each of your situations, explain how you know whether the relationship is proportional or not. Give as many reasons as you can.

14 Lesson Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:

On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that can be multiplied by any number in the first column to get the corresponding number in the second column, then the relationship between the quantities is not a proportional relationship.

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Lesson Synthesis

Share with students,

- *"Today we invented our own proportional relationships and represented them with a sentence, a table, an equation, and a graph."*

To help students reflect on their learning, consider asking:

- *"What types of situations have we seen that involve a proportional relationship?"*

  unit price, constant speed, recipes, servings, unit conversion, etc.

- *"What needs to be true about a situation for it to be represented by a proportional relationship?"*

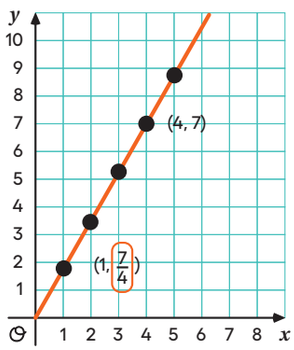
  It needs to have a constant unit rate—a constant of proportionality that you can multiply by one quantity to get the value for the other quantity.

- *"Tell me about something new you learned in this class recently."*

  "Tell me about any questions you still have, or anything that is confusing you."

Lesson Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that can be multiplied by any number in the first column to get the corresponding number in the second column, then the relationship between the quantities is not a proportional relationship.

Cool-down

Explain Their Work

5 min

Student Task Statement

Choose a relationship that another group found and explain why it is a proportional relationship. Make sure to include the quantities they used and any important constants of proportionality.

Sample response: In a 100-yard, three-legged race, distance in yards and time in minutes are proportional since each value of distance could be multiplied by $\frac{1}{40}$ to get the time. The constant of proportionality they used was $\frac{1}{40}$.

Responding To Student Thinking

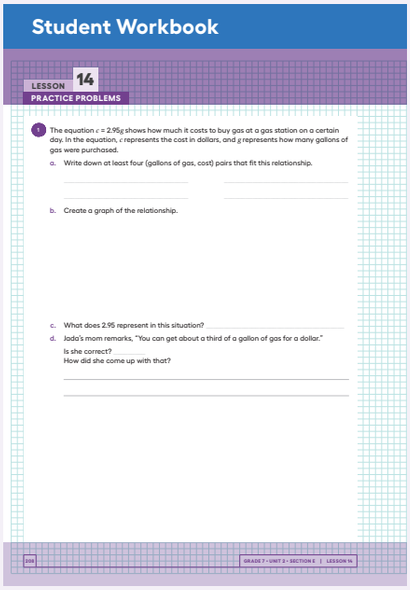
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 14

Practice Problems

2 Problems

Student Workbook


LESSON 14
PRACTICE PROBLEMS

1 The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.

a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.

b. Create a graph of the relationship.

c. What does 2.95 represent in this situation?

d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar."
Is she correct?
How did she come up with that?

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Problem 1

The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.

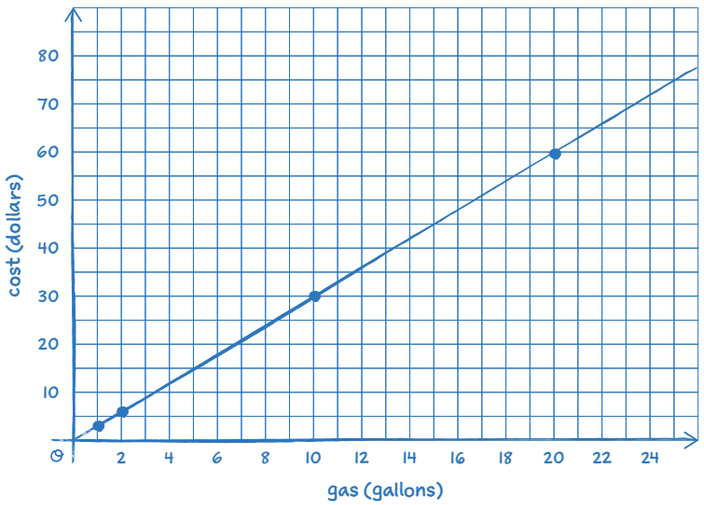
a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.

Sample response:

gallons of gas (g)	cost in dollars (c)
1	2.95
2	5.90
10	29.50
20	59.00

b. Create a graph of the relationship.

Sample response:



c. What does 2.95 represent in this situation?

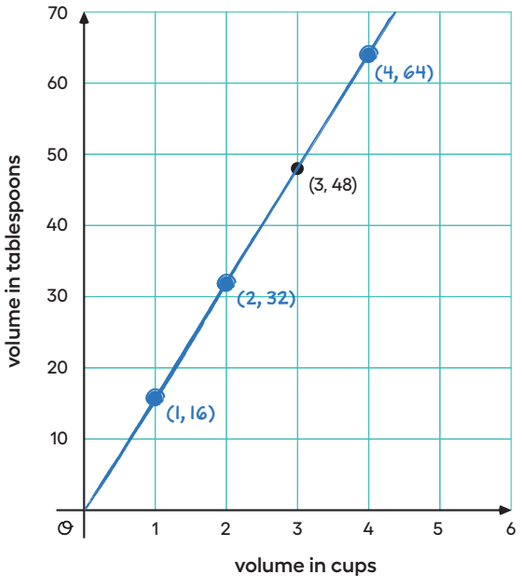
Gas costs $2.95 per gallon, which is the constant of proportionality.

d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? How did she come up with that?

Since 2.95 is close to 3, Jada's mom reasoned that if it cost about 3 dollars per gallon, the reciprocal rate must be $\frac{1}{3}$ gallon per dollar.

Problem 2

There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.



a. Plot and label at least two more points that represent the relationship.

b. Use a straightedge to draw a line that represents this proportional relationship.

c. For which value y is (1, y) on the line you just drew?

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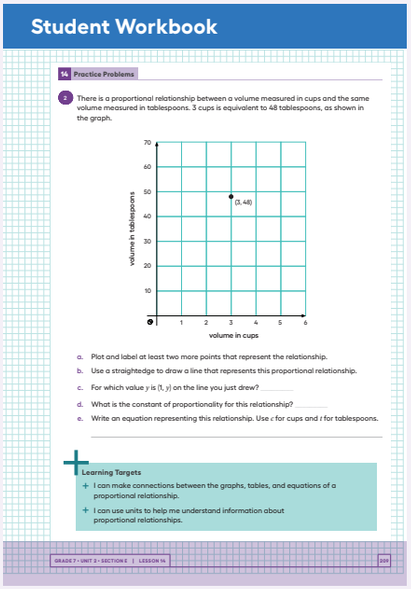
d. What is the constant of proportionality for this relationship?

16 tablespoons per cup

e. Write an equation representing this relationship. Use c for cups and t for tablespoons.

$t = 16c$

Student Workbook


14 Practice Problems

2 There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

volume in tablespoons
volume in cups
(3, 48)

a. Plot and label at least two more points that represent the relationship.
b. Use a straightedge to draw a line that represents this proportional relationship.
c. For which value y is (1, y) on the line you just drew? _____
d. What is the constant of proportionality for this relationship? _____
e. Write an equation representing this relationship. Use c for cups and t for tablespoons.

Learning Targets
+ I can make connections between the graphs, tables, and equations of a proportional relationship.
+ I can use units to help me understand information about proportional relationships.