Lesson 16

Applying Volume and Surface Area

Goal

Apply reasoning about surface area and volume of prisms as well as proportional relationships to calculate how much the material to build something will cost, and explain (orally and in writing) the solution method.

Learning Target

I can solve problems involving the volume and surface area of children's play structures.

Lesson Narrative

In this second lesson on applying surface area and volume to solve problems, students solve real-word problems of increased complexity that require them to choose which of the two quantities is appropriate for solving the problem, or whether both are appropriate for different aspects of the problem. They use previous work on ratios and proportional relationships, thus consolidating their knowledge and skill in that area. Students engage in aspects of mathematical modeling as they determine whether to use surface area or volume and consider how the constraints of the situation affect the solution.

Student Learning Goal

Let's explore things that are proportional to volume or surface area.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR3: Critique, Correct, Clarify

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	10 min	5 min

Warm-up

You Decide

5 min

Activity Narrative

In this activity, students are given two contextual situations and determine if the situation requires surface area or volume to be calculated. This prepares students to solve problems involving surface area and volume in situations that will arise in upcoming activities.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time, followed by time to discuss their reasoning with a partner. Follow with a whole-class discussion.

Student Task Statement

For each situation, decide if it requires Noah to calculate surface area or volume. Explain your reasoning.

1. Noah is planning to paint the bird house he built. He is unsure if he has enough paint.

   Surface area

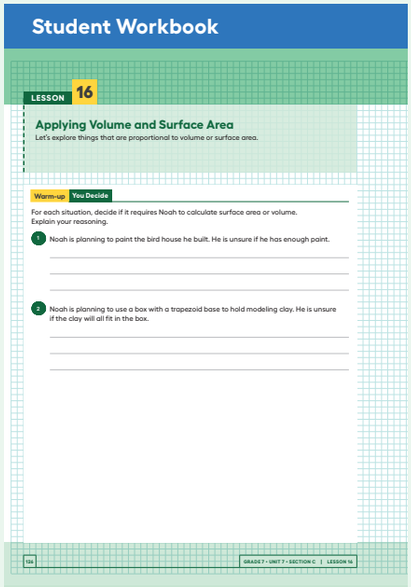
   The surface area is what Noah will calculate because he would need to calculate how much area he needs to cover on the surface of the bird house.

2. Noah is planning to use a box with a trapezoid base to hold modeling clay. He is unsure if the clay will all fit in the box.

   Volume

   The volume is what Noah will calculate because he needs to calculate how much space the box has inside of it to determine if it will hold all of his clay.

Student Workbook



LESSON 16

Applying Volume and Surface Area

Let's explore things that are proportional to volume or surface area.

Warm-up You Decide

For each situation, decide if it requires Noah to calculate surface area or volume. Explain your reasoning.

1. Noah is planning to paint the bird house he built. He is unsure if he has enough paint.

2. Noah is planning to use a box with a trapezoid base to hold modeling clay. He is unsure if the clay will all fit in the box.

Activity Synthesis

Select students to share their responses. Ask students to describe why the bird house situation calls for surface area and why the clay context calls for volume. To highlight the differences between the two uses of the box, ask:

- *"What are the differences in how Noah is using the boxes in these situations?"*
- *"How can you determine if a situation is asking you to calculate surface area or volume?"*

The goal is to ensure that students understand the differences between situations that require them to calculate surface area and those that require them to calculate volume.

Activity 1

Foam Play Structure

15 min

Activity Narrative

In this activity, students apply what they have learned previously about surface area and volume to different situations. Students have to consider whether they are finding the surface area or volume before answering each question. In addition, students apply proportional reasoning to find the cost of the material that is needed. This is an opportunity for students to revisit this prior understanding in a geometry context.

As students work on the task, monitor for students who are using different methods to decompose or compose the base of the object to calculate the area.

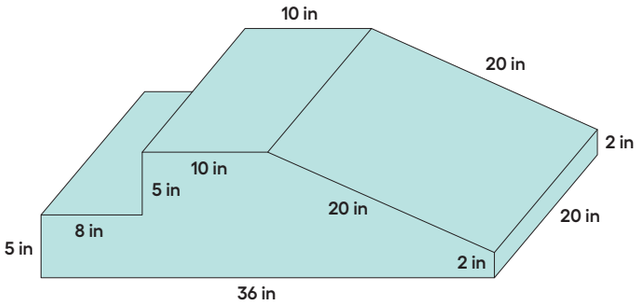
For surface area, monitor for students who calculate the area of each face of the object, as well as those who use the perimeter of the base.

Launch

Arrange students in groups of 2. Make sure that students are familiar with the terms "foam" and "vinyl." For example, it may help to explain that many binders are made out of cardboard covered with vinyl. In the diagram, all measurements have been rounded to the nearest inch. Give students 3–5 minutes of quiet work time, followed by time to share their answers with a partner. Follow with a whole-class discussion.

Student Task Statement

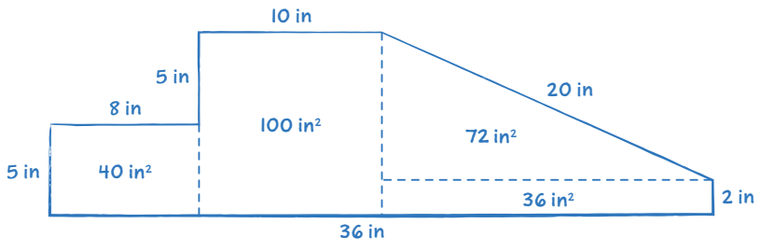
At a daycare, Kiran sees children climbing on this foam play structure.



Kiran is thinking about building a structure like this for his younger cousins to play on.

1. The entire structure is made out of soft foam so the children don't hurt themselves. How much foam would Kiran need to build this play structure?

   The volume of the play structure is 4,960 in^3, because the area of the base is 248 in^2 and 248 · 20 = 4,960. Possible strategy:



2. The entire structure is covered with vinyl so it is easy to wipe clean. How much vinyl would Kiran need to build this play structure?

   The surface area of the play structure is 2,216 in^2, because the perimeter of the base is 86 in and 86 · 20 + 248 · 2 = 2,216.

3. The foam costs 0.8¢ per in^3. Here is a table that lists the costs for different amounts of vinyl. What is the total cost for all the foam and vinyl needed to build this play structure?

vinyl (in^2)	cost ($)
75	0.45
125	0.75

   The total cost is $52.98.

   The foam will cost $39.68 because 4,960 · 0.008 = 39.68. The vinyl will cost $13.30, because 2,216 · 0.006 = 13.296. The total cost is $52.98, because 39.68 + 13.30 = 52.98.

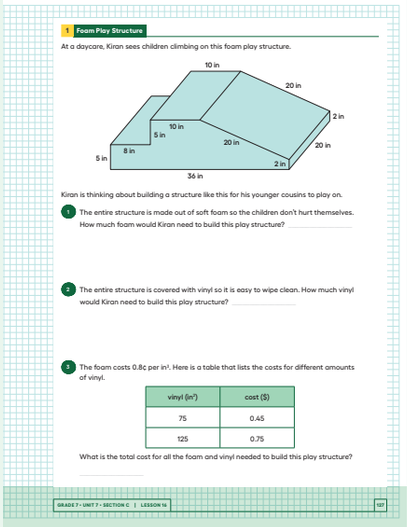
Building on Student Thinking

If students confuse 0.8¢ with $0.80, consider asking:

"Tell me more about the unit price of the foam."

"How can we write the price of the foam to make it more clear?"

Student Workbook


1 Foam Play Structure

At a daycare, Kiran sees children climbing on this foam play structure.

Kiran is thinking about building a structure like this for his younger cousins to play on.

1. The entire structure is made out of soft foam so the children don't hurt themselves. How much foam would Kiran need to build this play structure? _____

2. The entire structure is covered with vinyl so it is easy to wipe clean. How much vinyl would Kiran need to build this play structure? _____

3. The foam costs 0.8¢ per in^3. Here is a table that lists the costs for different amounts of vinyl.

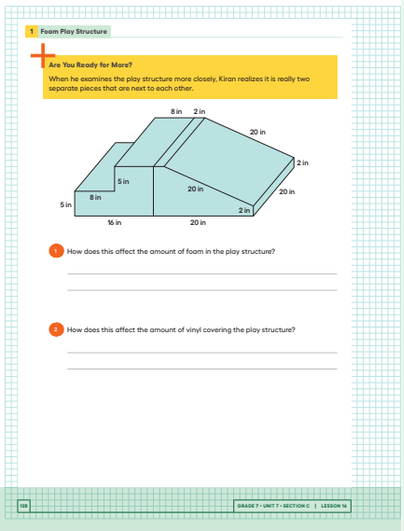
vinyl (in^2)	cost ($)
75	0.45
125	0.75

What is the total cost for all the foam and vinyl needed to build this play structure?

_____

GRADE 7 • UNIT 7 • SECTION C | LESSON 16 127

Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

"How does the triangular portion show up in each representation?"

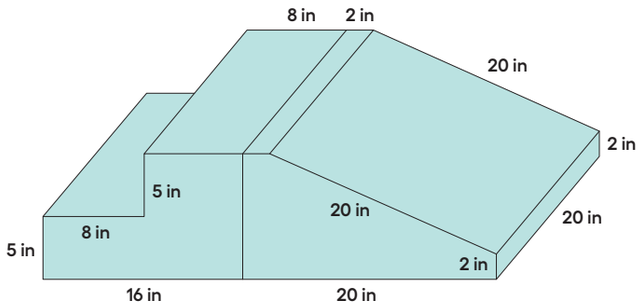
and

"Are there any benefits or drawbacks to one representation compared to another?"

Advances: Representing, Conversing

Are You Ready for More?

When he examines the play structure more closely, Kiran realizes it is really two separate pieces that are next to each other.



1. How does this affect the amount of foam in the play structure?

   The volume of foam stays the same.

2. How does this affect the amount of vinyl covering the play structure?

   The surface area increases by 400 in^2, because there are two new faces, each with an area of $20 \cdot 10$, or 200 in^2.

Activity Synthesis

The purpose of this discussion is to highlight different approaches to this problem. Select previously identified students to share how they calculated the area of the base.

Here are some questions for discussion:

- *"Are there other ways to calculate the area of the base?"*
- *"How did you know when to calculate surface area or volume for this problem?"*
- *"If Kiran buys a big block of foam that is 36 inches wide, 20 inches deep, and 10 inches tall and cuts it into this shape, what shapes would he be cutting off?"*

  A rectangular prism for the step on the left side and a triangular prism for the slide part on the right.

- *"How much more would the big block of foam cost than your calculations?"*

  This method wastes a volume of $36 \cdot 20 \cdot 10 - 4{,}960 = 2{,}240$ cubic inches of foam. This would be an extra \$17.92 since $2{,}240 \cdot 0.008 = 17.92$.

- *"If Kiran decides not to cover the bottom of the structure with vinyl, how much would he save?"*

  This reduces the area of vinyl needed by another $36 \cdot 20 = 720$ square inches. This would save \$4.32 since $720 \cdot 0.006 = 4.32$.

Activity 2
Filling the Sandbox

10 min

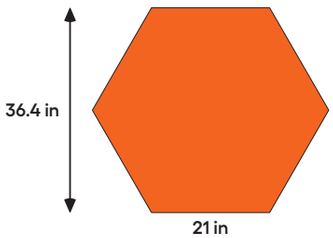
Activity Narrative

This activity provides another opportunity for students to apply what they have previously learned about surface area and volume to different situations. Students will practice using proportions as they apply to volumes of prisms in a real-world application.

As students work on the task, monitor for students who use different strategies to answer the questions.

Launch

Arrange students in groups of 2. If desired, have students close their books or devices, and display, for all to see, this regular hexagon with the dimensions of the sandbox in the problem. Ask students to calculate the base area of the sandbox.



Give students 2–3 minutes of quiet work time followed by time to discuss their work with a partner. Follow with a whole-class discussion.

Instructional Routines

MLR3: Critique, Correct, Clarify

ilclass.com/r/10695504

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify. This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time.
Supports accessibility for: Organization, Attention

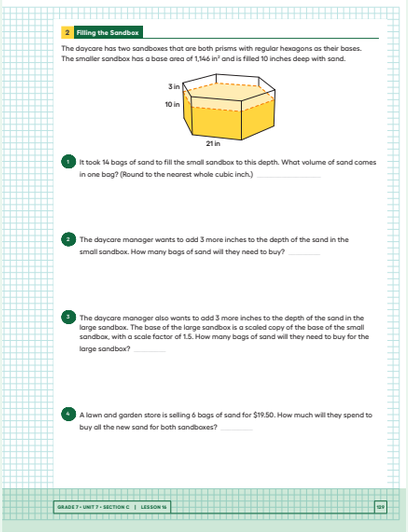
Building on Student Thinking

If students add the number of bags from each sandbox together to get 15 bags of sand, consider asking:

"Exactly how many bags of sand were needed for each sandbox?"

"How could the extra sand be used in this situation?"

Student Workbook


2 Filling the Sandbox

The daycare has two sandboxes that are both prisms with regular hexagons as their bases. The smaller sandbox has a base area of 1,146 in² and is filled 10 inches deep with sand.

3 in
10 in
21 in

1. It took 14 bags of sand to fill the small sandbox to this depth. What volume of sand comes in one bag? (Round to the nearest whole cubic inch.)

2. The daycare manager wants to add 3 more inches to the depth of the sand in the small sandbox. How many bags of sand will they need to buy?

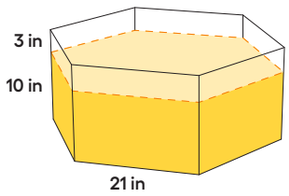
3. The daycare manager also wants to add 3 more inches to the depth of the sand in the large sandbox. The base of the large sandbox is a scaled copy of the base of the small sandbox, with a scale factor of 1.5. How many bags of sand will they need to buy for the large sandbox?

4. A lawn and garden store is selling 6 bags of sand for $19.50. How much will they spend to buy all the new sand for both sandboxes?

GRADE 7 • UNIT 7 • SECTION C | LESSON 16 121

Student Task Statement

The daycare has two sandboxes that are both prisms with regular hexagons as their bases. The smaller sandbox has a base area of 1,146 in^2 and is filled 10 inches deep with sand.



1. It took 14 bags of sand to fill the small sandbox to this depth. What volume of sand comes in one bag? (Round to the nearest whole cubic inch.)

   819 cubic inches of sand

   Sample reasoning: The volume of the sand that is already there is 11,460 in^3 since 1,146 · 10 = 11,460. Since 14 bags were used, each bag must have 819 cubic inches inside, because 11,460 ÷ 14 ≈ 819.

2. The daycare manager wants to add 3 more inches to the depth of the sand in the small sandbox. How many bags of sand will they need to buy?

   5 bags of sand

   Sample reasoning: One way to think about it is that it took 14 bags to fill it to a depth of 10 inches. To fill it another 3 inches, it will take $\frac{3}{10}$ as many bags as the previous time. Since we can't buy part of a bag, we round up to 5 bags, because $\frac{3}{10} \cdot 14 = 4.2$.

3. The daycare manager also wants to add 3 more inches to the depth of the sand in the large sandbox. The base of the large sandbox is a scaled copy of the base of the small sandbox, with a scale factor of 1.5. How many bags of sand will they need to buy for the large sandbox?

   10 bags of sand

   Sample reasoning: One way to think about it is that the larger sandbox's dimensions are all multiplied by 1.5, so the area is multiplied by 2.25, because 1.5 · 1.5 = 2.25. Since the needed depth of sand is the same, the volume is also multiplied by 2.25, so 2.25 · 4.2 = 9.45.

4. A lawn and garden store is selling 6 bags of sand for $19.50. How much will they spend to buy all the new sand for both sandboxes?

   $45.50

   Sample reasoning: Andre's father needs to purchase a total of 14 bags of sand, because 4.2 + 9.45 = 13.65. At $3.25 each, that would cost 14 · 3.25, or $45.50.

Activity Synthesis

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to the last question by correcting errors, clarifying meaning, and adding details.

Display this first draft:

- "It will cost $195.00. They need 10 bags of sand and 10 · 19.50 = 195."

Ask,

💬 *"What parts of this response are unclear, incorrect, or incomplete?"*

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement. Give students 2–4 minutes to work with a partner to revise the first draft.

- Select 1–2 groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, and then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Next, select previously identified students to share their methods for solving the problem. Consider asking the following questions:

💬 *"Are there any other ways to solve this problem?"*

"Did you use any answers from one question (or multiple questions) to help you answer another question? If so, why?"

"Did you use volume or surface area to help you answer any questions?"

yes, volume

💬 *"How did you calculate how much the daycare would spend on sand?"*

Lesson Synthesis

💬 *"How do we use volume and surface area to solve more-complex real-world problems?"*

You may need to calculate volume or surface area to answer a bigger question like how much it would cost to build a toy.

💬 *"What other skills did you have to use to solve the problems in this lesson?"*

ratios and proportional relationships

Explain to students that many times in real-world problems, calculating the volume or surface area is just a small piece of what is needed to be done. There are many other skills involved in solving more-complex problems.

Student Workbook

16 Lesson Summary

Suppose we wanted to make a concrete bench like the one shown in this picture. If we know that the finished bench has a volume of 10 ft³ and a surface area of 44 ft², we can use this information to solve problems about the bench.

For example,

- How much does the bench weigh?
- How long does it take to wipe the whole bench clean?
- How much will the materials cost to build the bench and to paint it?

To figure out how much the bench weighs, we can use its volume, 10 ft³. Concrete weighs about 150 pounds per cubic foot, so this bench weighs about 1,500 pounds, because 10 · 150 = 1,500.

To figure out how long it takes to wipe the bench clean, we can use its surface area, 44 ft². If it takes a person about 2 seconds per square foot to wipe a surface clean, then it would take about 88 seconds to clean this bench, because 44 · 2 = 88. It may take a little less than 88 seconds, since the surfaces where the bench is touching the ground do not need to be wiped.

Would you use the volume or the surface area of the bench to calculate the cost of the concrete needed to build this bench? And for the cost of the paint?

Responding To Student Thinking

Press Pause

If most students struggle with calculating an incomplete surface area, make time to do this optional activity:

Grade 7, Unit 7, Lesson 15, Activity 3
Building a Bat House

Lesson Summary

Suppose we wanted to make a concrete bench like the one shown in this picture. If we know that the finished bench has a volume of 10 ft^3 and a surface area of 44 ft^2, we can use this information to solve problems about the bench.

For example,

- How much does the bench weigh?
- How long does it take to wipe the whole bench clean?
- How much will the materials cost to build the bench and to paint it?

To figure out how much the bench weighs, we can use its volume, 10 ft^3. Concrete weighs about 150 pounds per cubic foot, so this bench weighs about 1,500 pounds, because $10 \cdot 150 = 1{,}500$.

To figure out how long it takes to wipe the bench clean, we can use its surface area, 44 ft^2. If it takes a person about 2 seconds per square foot to wipe a surface clean, then it would take about 88 seconds to clean this bench, because $44 \cdot 2 = 88$. It may take a little less than 88 seconds, since the surfaces where the bench is touching the ground do not need to be wiped. Would you use the volume or the surface area of the bench to calculate the cost of the concrete needed to build this bench? And for the cost of the paint?

Cool-down

Preparing for the Play

5 min

Student Task Statement

Andre is preparing for the school play. He needs to paint a cardboard box to look like a dresser. The box is a rectangular prism that measures 5 feet tall, 4 feet long, and $2\frac{1}{2}$ feet wide. Andre does not need to paint the bottom of the box.

1. How much cardboard does Andre need to paint?

   75 square feet. Sample reasoning: $(2.5 \cdot 4) + 2(5 \cdot 4) + 2(2.5 \cdot 5) = 75$

2. If one bottle of paint covers an area of 40 square feet, how many bottles of paint does Andre need to buy for this project?

   2 bottles of paint. $\frac{75}{40} = 1.875$

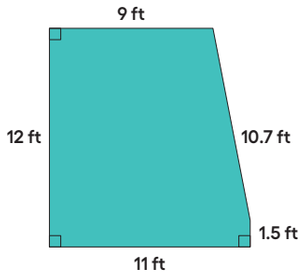
Lesson 16

Practice Problems

4 Problems

Problem 1

A landscape architect is designing a pool that has this top view:



a. How much water will be needed to fill this pool 4 feet deep?

486 ft³

b. Before filling up the pool, it gets lined with a plastic liner. How much liner is needed for this pool?

298.3 ft²

c. Here are the prices for different amounts of plastic liner.

plastic liner (ft²)	cost ($)
25	3.75
50	7.50
75	11.25

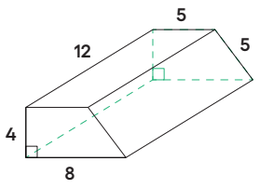
How much will all the plastic liner for the pool cost?

$44.75

Problem 2

from Unit 7, Lesson 13

Shade in a base of the trapezoidal prism. (The base is not the same as the bottom.)



The bases of the prism are the two trapezoids. Students may shade either the trapezoid at the front or the trapezoid at the back.

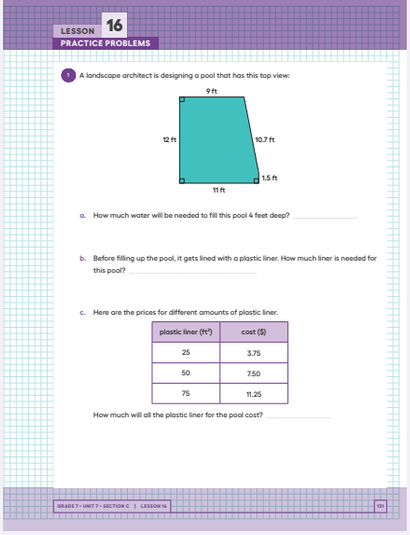
a. Find the area of the base you shaded.

26

b. Find the volume of this trapezoidal prism.

312

Student Workbook


LESSON 16
PRACTICE PROBLEMS

1 A landscape architect is designing a pool that has this top view:

9 ft, 12 ft, 10.7 ft, 1.5 ft, 11 ft

a. How much water will be needed to fill this pool 4 feet deep? _____

b. Before filling up the pool, it gets lined with a plastic liner. How much liner is needed for this pool? _____

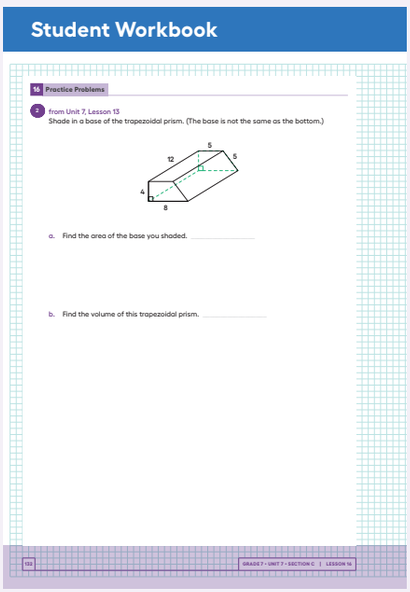
c. Here are the prices for different amounts of plastic liner.

plastic liner (ft²)	cost ($)
25	3.75
50	7.50
75	11.25

How much will all the plastic liner for the pool cost? _____

GRADE 7 • UNIT 7 • SECTION C | LESSON 16 131

Student Workbook


16 Practice Problems

2 from Unit 7, Lesson 13
Shade in a base of the trapezoidal prism. (The base is not the same as the bottom.)

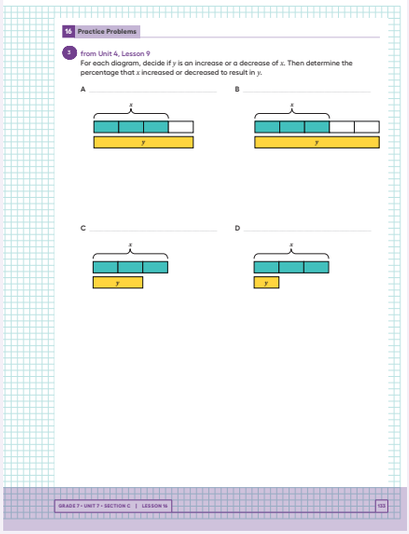
5, 12, 5, 4, 8

a. Find the area of the base you shaded. _____

b. Find the volume of this trapezoidal prism. _____

132 GRADE 7 • UNIT 7 • SECTION C | LESSON 16

Student Workbook


16 Practice Problems

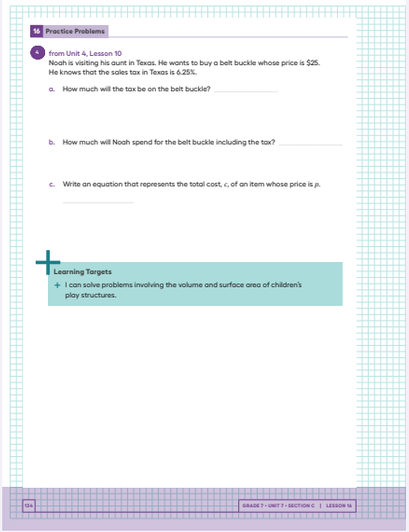
3 from Unit 4, Lesson 9
For each diagram, decide if y is an increase or a decrease of x. Then determine the percentage that x increased or decreased to result in y.

A _____ B _____

C _____ D _____

GRADE 7 • UNIT 7 • SECTION C | LESSON 16 123

Student Workbook


16 Practice Problems

4 from Unit 4, Lesson 10
Noah is visiting his aunt in Texas. He wants to buy a belt buckle whose price is $25. He knows that the sales tax in Texas is 6.25%.

a. How much will the tax be on the belt buckle? _____

b. How much will Noah spend for the belt buckle including the tax? _____

c. Write an equation that represents the total cost, c, of an item whose price is p.

Learning Targets
+ I can solve problems involving the volume and surface area of children's play structures.

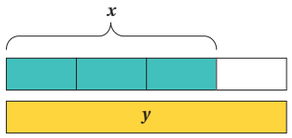
124 GRADE 7 • UNIT 7 • SECTION C | LESSON 16

Problem 3

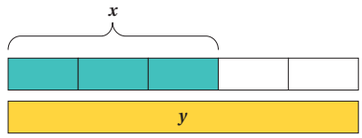
from Unit 4, Lesson 9

For each diagram, decide if y is an increase or a decrease of x. Then determine the percentage that x increased or decreased to result in y.

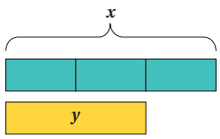
A Increase, $33\frac{1}{3}\%$



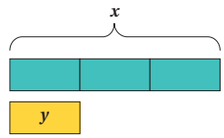
B Increase, $66\frac{2}{3}\%$



C Decrease, $33\frac{1}{3}\%$



D Decrease, $66\frac{2}{3}\%$



Problem 4

from Unit 4, Lesson 10

Noah is visiting his aunt in Texas. He wants to buy a belt buckle whose price is $25. He knows that the sales tax in Texas is 6.25%.

a. How much will the tax be on the belt buckle?

$1.56 (requires rounding)

b. How much will Noah spend for the belt buckle including the tax?

$26.56

c. Write an equation that represents the total cost, c, of an item whose price is p.

$c = 1.0625p$ (or equivalent)