Lesson 4

Applying Circumference

Goals

- Compare and contrast (orally) values for the same measurements that were calculated using different approximations for π.
- Explain (orally) how to calculate the radius, diameter, or circumference of a circle, given one of these three measurements.

Learning Targets

- I can choose an approximation for π based on the situation or problem.
- If I know the radius, diameter, or circumference of a circle, I can find the other two.

Lesson Narrative

In this lesson, students apply the relationships between diameter, radius, and circumference to solve problems in a variety of contexts. Students develop flexibility with using the relationships rather than memorizing a variety of formulas. Understanding the equation $C = 2\pi r$ will help with the transition to the study of area in future lessons.

Next, students solve problems about distances of paths composed of multiple circles or parts of circles and other shapes. They think strategically about how to decompose and recompose complex shapes and need to choose an appropriate level of precision for π and for their final calculations.

The last activity is optional because it provides an additional opportunity to apply circumference to solve a problem that requires composing and decomposing parts of circles.

Student Learning Goal

Let's use π to solve problems.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Warm-up)
- MLR5: Co-Craft Questions (Activity 3)
- MLR7: Compare and Connect (Activity 2)

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Four-function calculators: Activity 1

Required Preparation

Activity 1:
For the *Are You Ready for More?* activity, acquire devices that can run the applet.

Activity 2:
For the *Are You Ready for More?* activity, acquire devices that can run the applet.

Lesson Timeline

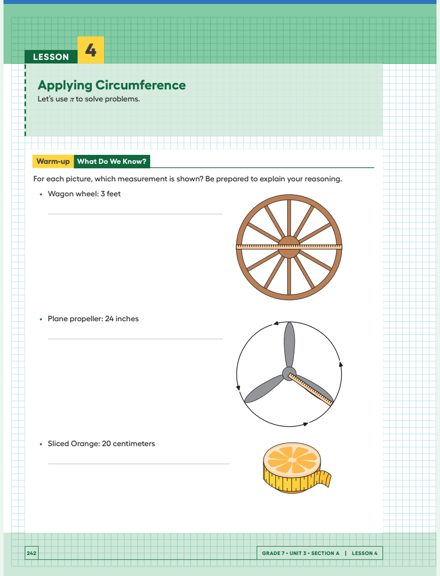
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	15 min	10 min	5 min

Access for Multilingual Learners (Warm-up, Student Task)

MLR2: Collect and Display.
Direct attention to words collected and displayed from a previous lesson. Invite students to borrow language from the display as needed and update it throughout the lesson.

Advances: Conversing, Reading

Student Workbook


LESSON 4

Applying Circumference

Let's use π to solve problems.

Warm-up What Do We Know?

For each picture, which measurement is shown? Be prepared to explain your reasoning.

- Wagon wheel: 3 feet
- Plane propeller: 24 inches
- Sliced Orange: 20 centimeters

GRADE 7 • UNIT 3 • SECTION A | LESSON 4

Warm-up
What Do We Know?

5 min

Activity Narrative

In this *Warm-up*, students examine pictures of circular objects that have one measurement given. They identify which measurement is shown and then use the given measurement to reason about the other measurements of the circle. Students can find the diameter given the radius, or vice versa. However, they do not need to do any calculations that involve pi at this time. They will continue working with these situations in a later activity.

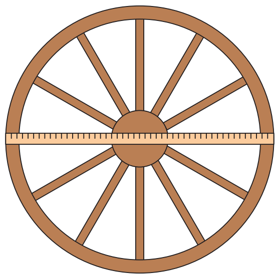
Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by 2 minutes of partner discussion.

Student Task Statement

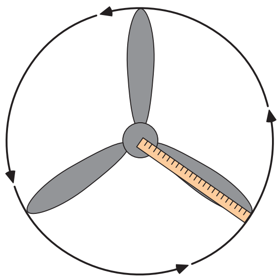
For each picture, which measurement is shown? Be prepared to explain your reasoning.

- Wagon wheel: 3 feet



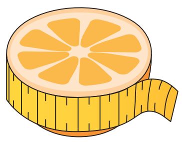
The measurement of the diameter is shown. The measuring tape goes all the way across the wheel.

- Plane propeller: 24 inches



The measurement of the radius is shown. The measuring tape goes from the center to the edge of the propeller.

- Sliced orange: 20 centimeters



The circumference is shown. The measuring tape goes around the outside of the orange.

Activity Synthesis

Display this table for all to see, without any measurements filled in.

object	radius	diameter	circumference
wagon wheel			
airplane propeller			
sliced orange			

For each picture, ask students to share which measurement is shown in the picture. Record the given measurement in the corresponding cell of the table. Then, ask students what else we know about the measurements of this circle.

- For the wagon wheel, we know the radius is 1.5 feet, because $3 \div 2 = 1.5$.
- For the airplane propeller, we know the diameter is 48 inches, because $24 \cdot 2 = 48$.
- We also know that we could find the other measurements by multiplying or dividing by pi. However, it is not necessary to do any calculations involving pi at this time.

Continue recording measurements in the table as students share their reasoning and come to an agreement. By the end of the discussion, the table should have these five measurements in place.

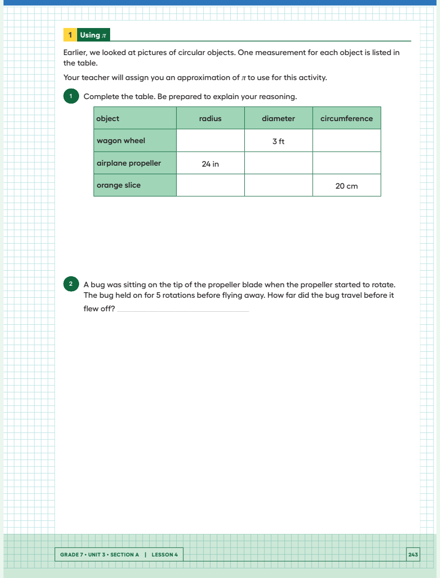
object	radius	diameter	circumference
wagon wheel	1.5 ft	3 ft	
airplane propeller	24 in	48 in	
sliced orange			20 cm

Access for Students with Diverse Abilities

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, consider giving the students the information for one row at a time.

Supports accessibility for: Organization, Attention

Student Workbook


1 Using π

Earlier, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

1 Complete the table. Be prepared to explain your reasoning.

object	radius	diameter	circumference
wagon wheel		3 ft	
airplane propeller	24 in		
orange slice			20 cm

2 A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

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Activity 1

Using π

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students calculate the remaining measurements for the circles in the previous activity. Different students use different approximations of π: $\frac{22}{7}$, 3.14, and 3.1415927. The last approximation for π is the level of precision that many calculators show.

The different approximations for π lead to different estimates for the missing measurements. During the whole-class discussion, students share their different answers and discuss which approximation is the most useful in these cases.

Launch

Divide students into 3 groups. Assign each group a different approximation for π to use in their calculations: 3.1415927, 3.14, and $\frac{22}{7}$. Give students 4–5 minutes of quiet work time followed by whole-class discussion.

Student Task Statement

Earlier, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

1. Complete the table. Be prepared to explain your reasoning.

object	radius	diameter	circumference
wagon wheel	1.5 ft	3 ft	9.4247781 ft 9.42 ft $9\frac{3}{7}$ ft
airplane propeller	24 in	48 in	150.7964496 in 150.72 in $150\frac{6}{7}$ in
sliced orange	3.183098815 cm 3.184713376 cm $3\frac{2}{11}$ cm	6.366197763 cm 6.369426752 cm $6\frac{4}{11}$ cm	20 cm

2. A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

about 754 inches

$(150.7964496) \cdot 5 = 753.982248$

Activity Synthesis

The goal of this discussion is for students to consider the appropriate level of precision for reporting measurements of these objects.

First, invite one student who used each approximation of pi to share their answer for the circumference of the wagon wheel. (9.42 ft, 9.4247781 ft, $\frac{66}{7}$ or 9.428571 ft)

Ask questions like:

- *"Why are these answers different?"*

  They used different values for π.

- *"Which answer is correct?"*

  They are all approximations. None of them is exact.

- *"How different are these answers?"*

  They are very close, within 0.01 of each other. They agree in the first three digits.

- *"Does it make sense to report the circumference of a wagon wheel to 7 decimal places?"*

  Not really, because the given measurement, 3 ft, didn't have any decimal places.

- *"Could the wagon wheel actually be measured that precisely?"*

  No, at least not with normal measurement tools.

If time permits, repeat some of these questions for the airplane propeller and sliced orange.

The key takeaways are:

- People use different approximations for pi depending on the situation and the precision of the measurement.
- For situations like these where the measurements themselves do not have too much accuracy, either 3.14 or $\frac{22}{7}$ is probably the most appropriate value of π to use.
- Using a more precise value for π is always acceptable, but the final answer should not be reported with more precision than the measurements had.

Finally, ask students to share their reasoning for the problem about the bug on the propeller blade. Understanding that the bug travels 5 times the circumference of the circle will help prepare students to solve upcoming problems that involve multiple revolutions around a circle.

Activity 2: Optional

Hopi Basket Weaving

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students calculate the circumference of a circle given its radius, and vice versa. It is not necessary for students to memorize the formula $C = 2\pi r$. However, it is helpful to recognize that the circumference of a circle is proportional to its radius as well as to its diameter.

Instructional Routines

MLR7: Compare and Connect

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Instructional Routines

MLR7: Compare and Connect
This actiivity uses MLR7 which supports students in comparing, contrasting, and connecting different approaches.

The questions in this activity do not use the word "circumference." Students use quantitative and abstract reasoning as they connect the length of the willow frame to the circumference of the basket in order to solve problems.

Monitor for students who use these different strategies:

- Use the radius to find the diameter, and then use the diameter to calculate the circumference
- Multiply the radius by 2π to find the circumference directly

Launch

Introduce the context of this activity by asking students:

"What types of containers does your family use to store things?"

Then, explain that Hopi weavers in northern Arizona make baskets out of local plants, like yucca, willow, rabbitbrush, and grasses. They use the baskets to gather and store food. They also use the baskets in their ceremonies.

Display the image of the sifter basket for all to see. Point out the location of the circular willow frame around the outer edge of the basket to familiarize students with what the question is asking.

Give students 3–4 minutes of quiet work time, followed by partner and whole-class discussion.

Watch for students who calculate the diameter and not the radius. If needed, direct their attention to the word "radius" in the question.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

Hopi weavers make baskets by weaving thin strips of yucca onto a circular willow frame.

Sifter Basket

Tray with Handles

1. To make a basket with a radius of $6\frac{1}{2}$ inches, how long does the piece of willow for the circular frame need to be?

   about 41 inches

   $6\frac{1}{2} \cdot 2 = 13$ and $\pi \cdot 13 \approx 41$

2. If a weaver uses a piece of willow that is 33 inches long, what will the radius of the basket be?

   about $5\frac{1}{4}$ inches

   $33 \div \pi \approx 10\frac{1}{2}$ and $10\frac{1}{2} \div 2 = 5\frac{1}{4}$

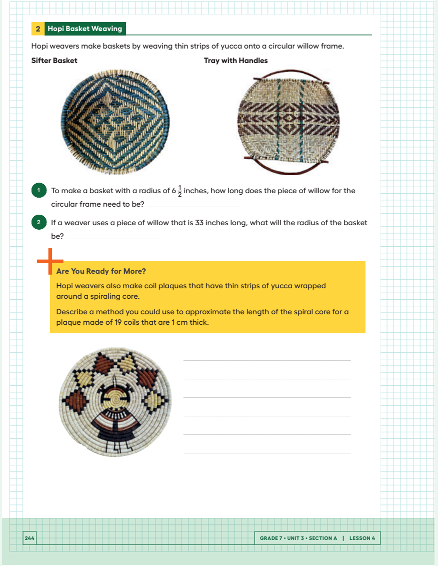
Are You Ready for More?

Hopi weavers also make coil plaques that have thin strips of yucca wrapped around a spiraling core.

Describe a method you could use to approximate the length of the spiral core for a plaque made of 19 coils that are 1 cm thick.

To approximate the total length, the spiral can be modeled as a set of circles nested inside each other. Find the circumferences for a series of circles that have radii of 1, 2, 3, … 18, and 19 cm. Then, add all the circumferences together.

Student Workbook



2 Hopi Basket Weaving

Hopi weavers make baskets by weaving thin strips of yucca onto a circular willow frame.

Sifter Basket **Tray with Handles**

1. To make a basket with a radius of $6\frac{1}{2}$ inches, how long does the piece of willow for the circular frame need to be? ______

2. If a weaver uses a piece of willow that is 33 inches long, what will the radius of the basket be? ______

Are You Ready for More?

Hopi weavers also make coil plaques that have thin strips of yucca wrapped around a spiraling core.

Describe a method you could use to approximate the length of the spiral core for a plaque made of 19 coils that are 1 cm thick.

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Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 3)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Begin by decomposing the track shape and directing students to find the distance of each of the three distinct parts. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Attention, Social-Emotional Functioning

Activity Synthesis

The goal of this discussion is to highlight the relationship between radius and circumference.

Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"Why do the different approaches lead to the same outcome?"

"How does pi show up in each method?"

"Are there any benefits or drawbacks to one method compared to another?"

As students share their thinking, highlight these connections:

- Because multiplication is associative and commutative, changing the order in which you multiply several factors doesn't change the product.
- To find the circumference of a circle given its radius, you need to multiply by 2π. You can do this in two separate steps or in one combined step.

If time permits, consider displaying the equations $d = 2r$ and $C = \pi d$ for all to see. Ask students to describe what each equation means. Then, demonstrate substituting $2r$ in place of d to get the equation $C = \pi \cdot 2r$. Ask students to describe what this equation represents. If desired, explain that the standard way to write expressions like this is to put the number first. Rewrite the third equation as $C = 2\pi r$.

Activity 3: Optional
Around the Running Track

15 min

Activity Narrative

In this activity, students compute the length of a figure that is composed of two semicircles and two line segments. Students need to determine how to decompose the figure to calculate its length. For the second question, students have to compose several line segments to determine the diameter of the semicircle.

When students decide how to decompose the figure to find its perimeter, they are making sense of problems and persevering in solving them.

Launch

Arrange students in groups of 2. Introduce the context of a running track. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

- Display only the problem stem and related image, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.
- Invite several partners to share one question with the class, and record their responses. Ask the class to make comparisons among the shared questions and their own. Ask,

"What do these questions have in common? How are they different?"

- Listen for and amplify language related to the learning goal, such as the distance around the track or the difference in length between the inside and outside of the track.
- Reveal the questions,

"What is the distance around the inside of the track?"

and

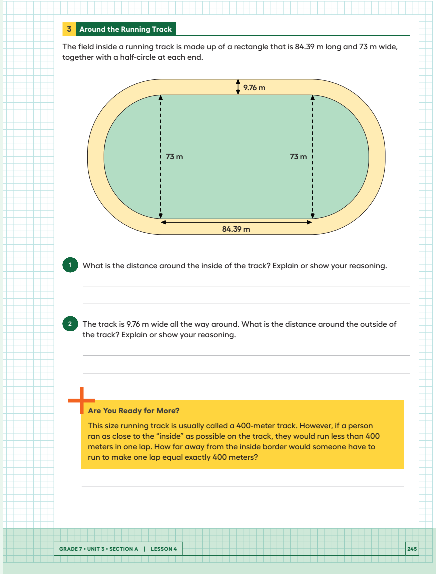
"What is the distance around the outside of the track?"

- Give students 1–2 minutes to compare them to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

"Which of your questions is most similar to or different from the ones provided? Why?"

Ensure that students understand what is meant by "the inside of the track" and "the outside of the track." Point out each black outline and label them if needed. Give students 4–5 minutes of quiet work time followed by partner discussion.

Student Workbook


3 Around the Running Track

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.

1 What is the distance around the inside of the track? Explain or show your reasoning.

2 The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

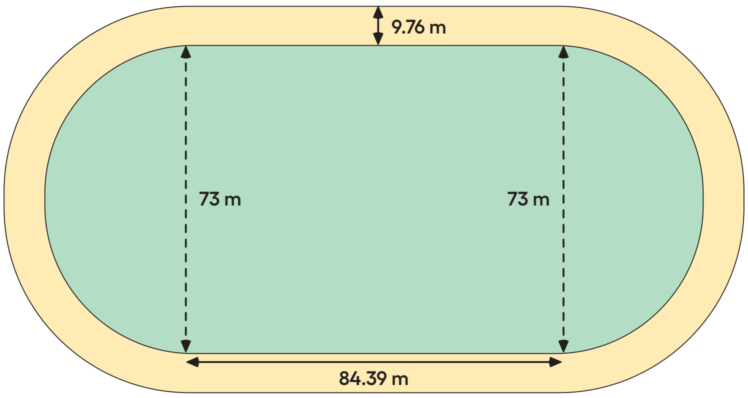
Are You Ready for More?

This size running track is usually called a 400-meter track. However, if a person ran as close to the "inside" as possible on the track, they would run less than 400 meters in one lap. How far away from the inside border would someone have to run to make one lap equal exactly 400 meters?

GRADE 7 • UNIT 3 • SECTION A | LESSON 4

Student Task Statement

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.



1. What is the distance around the inside of the track? Explain or show your reasoning.

   The inside of the track is 398 m long. The distance around each half-circle is 114.61 m, because $73 \cdot 3.14 \div 2 = 114.61$. Add two line segments and two half-circles: $84.39 + 84.39 + 114.61 + 114.61 = 398$.

2. The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

   The outside of the track is 459.3 m long. The diameter of the larger half-circles is 92.52 m, because $73 + 9.76 + 9.76 = 92.52$. The distance around the large half-circle is 145.26 m, because $92.52 \cdot 3.14 \div 2 = 145.26$. Add two line segments and two half-circles: $84.39 + 84.39 + 145.26 + 145.26 = 459.3$.

Building on Student Thinking

If students have trouble seeing the circle and rectangle that compose the figure, suggest that they draw additional lines to decompose the figure.

Students might apply the formula for the circumference of a circle to find the circumference of the oval-shaped field and track. If this happens, remind students that the track is not in the shape of a circle. Ask if they see a way to form a circle and a rectangle within the space.

Students who are familiar with the fact that this size track is referred to as a 400-meter track may be confused when their answer does not equal 400 meters. Explain that a runner does not run right on the edge of the track and possibly direct the student to look at the extension.

Are You Ready for More?

This size running track is usually called a 400-meter track. However, if a person ran as close to the "inside" as possible on the track, they would run less than 400 meters in one lap. How far away from the inside border would someone have to run to make one lap equal exactly 400 meters?

The length of the straight part of the track is not affected by the distance from the border that a person runs. Excluding the straight parts, the rest of the distance is 231.22 meters, because $400 - 2 \cdot 84.39 = 231.22$. The half-circles
must have a diameter of 73.6 meters, because $231.22 \div \pi \approx 73.6$. The runner must run 0.3 meters in from the inside border of the track, because $(73.6 - 73) \div 2 = 0.3$.

Activity Synthesis

Most of the discussion will occur in small groups. However, the whole class can debrief on the following questions:

- *"In what ways did you decompose the figure into different shapes?"*
- *"Why did you choose a particular approximation for π, and what was the resulting answer?"*
- *"How are different students' answers related, and are they all reasonable lengths for this situation?"*

Students who use more digits for their approximation of π may come up with a slightly different answer, such as 398.1. In general, when making calculations, if only an estimate is desired, then using 3.14 for π is usually good enough. In a situation like this, where the given measurements are quite precise, it can be worth trying more digits in the expansion of π, but it turns out not to make much difference in this case.

Lesson Synthesis

Share with students:

- *"Today we applied the relationship between diameter and circumference to solve multi-step problems."*

To help students generalize about applications of circumference, consider asking students:

- *"If I know the radius of a circle, how do I find its diameter and circumference?"*

  Multiply the radius by 2 to find the diameter, and then multiply that by π to find the circumference.

- *"If I know the circumference of a circle, how do I find its diameter and radius?"*

  Divide the circumference by π to find diameter and then divide that by 2 to find radius.

- *"What types of situations involve finding the circumference of a circle?"*

  Determining the perimeter of a circular object, calculating the distance something has moved along a circular path, etc.

- *"What are some approximations of π?"*

  3.1, 3.14, 3.14159, $\frac{22}{7}$

- *"Why isn't there just one value for π?"*

  The exact value of π is a decimal with infinitely many digits and no repeating pattern, so an approximation is often used.

Lesson Summary

The circumference of a circle, C, is π times the diameter, d. The diameter is twice the radius, r. So if we know any one of these measurements for a particular circle, we can find the others. We can write the relationships between these different measures using equations:

$d = 2r$

$C = \pi d$

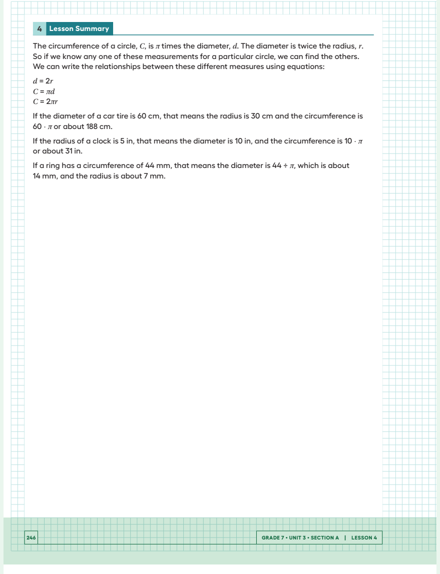
$C = 2\pi r$

If the diameter of a car tire is 60 cm, that means the radius is 30 cm, and the circumference is $60 \cdot \pi$, or about 188 cm.

If the radius of a clock is 5 in, that means the diameter is 10 in, and the circumference is $10 \cdot \pi$, or about 31 in.

If a ring has a circumference of 44 mm, that means the diameter is $44 \div \pi$, which is about 14 mm, and the radius is about 7 mm.

Student Workbook



Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of calculating circumference. If students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 2, Section A Circumference of a Circle

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs,* identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

Circumferences of Two Circles

5 min

Student Task Statement

Circle A has a diameter of 9 cm. Circle B has a radius of 5 cm.

1. Which circle has the larger circumference?

   Circle B has the larger circumference. Circle A has a diameter of 9 cm, and Circle B has a diameter of $5 \cdot 2$, or 10 cm. Since Circle B's diameter is larger than Circle A's diameter, and circumference is proportional to diameter, that means Circle B's circumference is also larger.

2. About how many centimeters larger is it?

   The difference is about 3.14 cm because the circumference of Circle A is 9π, or about 28.26 cm, and the circumference of Circle B is 10π, or about 31.4 cm. The difference is $31.4 - 28.26$, or about 3.14 cm.

Lesson 4

Practice Problems

5 Problems

Problem 1

Here is a picture of a Ferris wheel. It has a diameter of 80 meters.

a. On the picture, draw and label a diameter.

Answers vary. Possible response:

b. How far does a rider travel in one complete rotation around the Ferris wheel?

In one complete rotation, a rider travels the circumference of the Ferris wheel. This distance is $80 \cdot \pi$, or about 251 meters. Because the gondola where the rider is seated is a little bit farther than 40 meters from the center of the Ferris wheel, the distance that the rider travels is actually a little more.

Problem 2

Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

a. The length of the minute hand on a clock is 5 inches.

radius; diameter: 10 in, circumference: about 31 in

b. The fence around a circular pool is 75 feet long.

circumference; diameter: about 24 ft, radius: about 12 ft

c. The tires on a mining truck are 14 feet tall.

diameter; radius: 7 ft, circumference: about 44 ft

d. The distance from the tip of a slice of pizza to the crust is 7 inches.

radius; diameter: 14 in, circumference: about 44 in

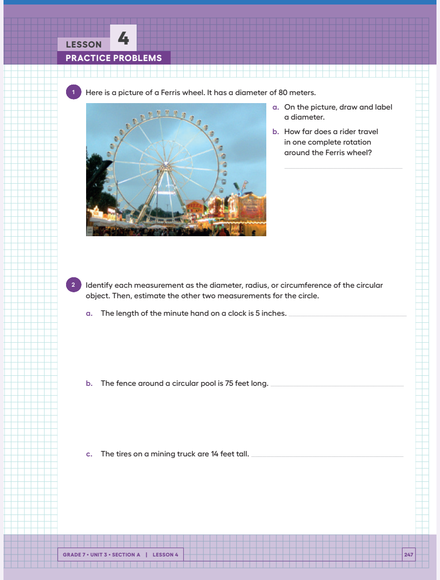
e. Folding a tortilla in half creates a straight side 16 centimeters long.

diameter; radius: 8 cm, circumference: about 50 cm

f. The length of the metal rim around a glass lens is 190 millimeters.

circumference; diameter: about 60 mm, radius: about 30 mm

Student Workbook



LESSON 4
PRACTICE PROBLEMS

1 Here is a picture of a Ferris wheel. It has a diameter of 80 meters.

a. On the picture, draw and label a diameter.

b. How far does a rider travel in one complete rotation around the Ferris wheel?

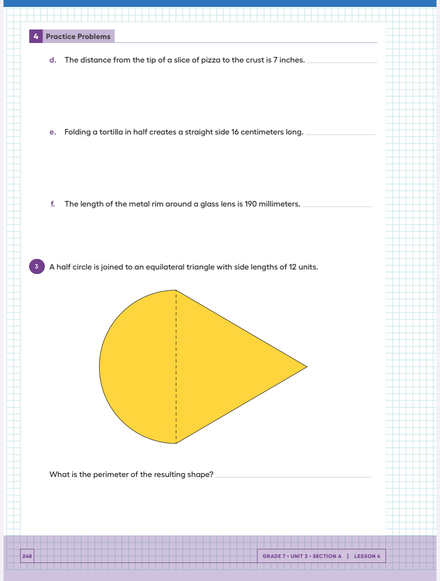
2 Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

a. The length of the minute hand on a clock is 5 inches.

b. The fence around a circular pool is 75 feet long.

c. The tires on a mining truck are 14 feet tall.

Student Workbook



4 Practice Problems

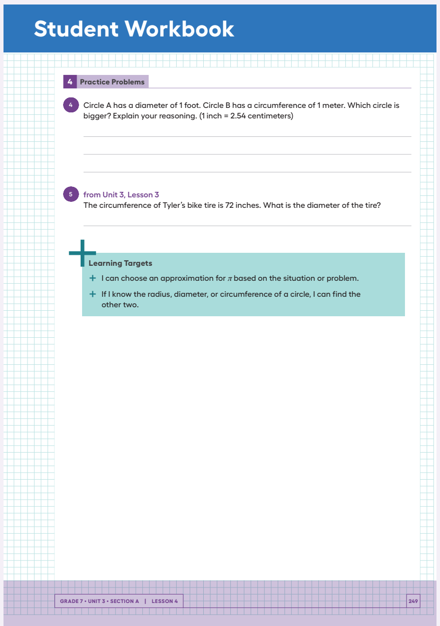
d. The distance from the tip of a slice of pizza to the crust is 7 inches.

e. Folding a tortilla in half creates a straight side 16 centimeters long.

f. The length of the metal rim around a glass lens is 190 millimeters.

3 A half circle is joined to an equilateral triangle with side lengths of 12 units.

What is the perimeter of the resulting shape?



Student Workbook

4 Practice Problems

4 Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

5 from Unit 3, Lesson 3
The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

Learning Targets

+ I can choose an approximation for π based on the situation or problem.
+ If I know the radius, diameter, or circumference of a circle, I can find the other two.

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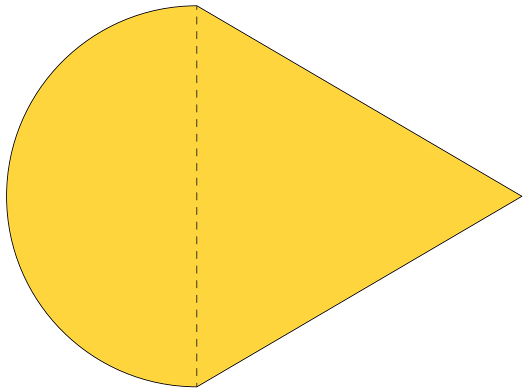
Problem 3

A half circle is joined to an equilateral triangle with side lengths of 12 units.



What is the perimeter of the resulting shape?

about 42.84 units

The two sides of the triangle each contribute 12 units and the semi-circle has a perimeter of $6 \cdot \pi$ or about 18.84 units.

Problem 4

Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

Circle B is bigger.

Answers vary. Possible explanation: There are 12 inches in 1 foot. The circumference of Circle A is about 95.8 cm because $1 \cdot 12 \cdot 2.54 \cdot \pi \approx 95.8$. The circumference of Circle B is 100 cm because there are 100 cm in 1 m.

Problem 5

from Unit 3, Lesson 3

The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

about 23 inches because $72 \div \pi \approx 22.9$