Revisiting Proportional Relationships

Goals

- Calculate and interpret (orally) the constant of proportionality for a proportional relationship involving fractional quantities.
- Explain (orally and in writing) how to use a table with only two rows to solve a problem involving a proportional relationship.
- Write an equation to represent a given proportional relationship with a fractional constant of proportionality.

Learning Targets

- I can use a table with 2 rows and 2 columns to find an unknown value in a proportional relationship.
- When there is a constant rate, I can identify the two quantities that are in a proportional relationship.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)
- Representation (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR1: Stronger and Clearer Each Time (Activity 3)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR3: Critique, Correct, Clarify

Lesson Narrative

In this lesson, students practice solving problems that involve proportional relationships and fractions. They see that setting up a table with two rows is an efficient method for making sense of the problem and for deciding what to multiply or divide.

In the first activity, the table is given and students describe how they could use the structure of the table to help solve the problem. In the next activity, students practice creating such a table for themselves. When students describe their multiplication or division steps, they are encouraged to interpret these operations in terms of finding and using scale factors or the constant of proportionality.

The last activity is optional because it provides an opportunity for additional practice solving problems where no table is given.

Student Learning Goal

Let's use constants of proportionality to solve more problems.

Lesson Timeline

5 min

Warm-up

15 min

Activity 1

10 min

Activity 2

10 min

Activity 3

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

5 min

Warm-up

Agua Fresca

The purpose of this *Warm-up* is to bring up two main methods for figuring outmissing numbers in a table that represents a proportional relationship. The two methods students might use for this activity are:

- Using a scale factor to find equivalent ratios, for example multiplying the first row by $1\frac{1}{2}$ to get the second row.
- Using the constant of proportionality, 2, between the first column and the second column.

This activity is to get students thinking about the second method as a more efficient method, since it works for every row. This lays the groundwork for solving problems using proportional relationships and for the activities in this lesson.

Launch 🙎

Introduce the context of this activity by asking students,

When it is hot outside, what do you like to drink to refresh yourself?"

Then, explain that people in Mexico make a drink called *agua fresca* (AH-gwah FREH-skah) by blending fresh fruit with water and ice.

Arrange students in groups of 2.

Give 1 minute of quiet work time followed by time to compare their table with a partner.

Then hold a whole-class discussion.

Student Task Statement

A recipe for watermelon *agua fresca* calls for $\frac{1}{2}$ cup of cubed, seeded watermelon and 1 cup of ice. Complete the table to show how much watermelon and ice to use in different numbers of batches of the recipe.

watermelon (cups)	ice (cups)
1/2	1
<u>3</u> 4	l <u>1</u>
7/8	1 3/4
1	2
14	2 1 2

Inspire Math

Sport Success video



Go Online

Before the lesson, show this video to introduce the real-world connection.

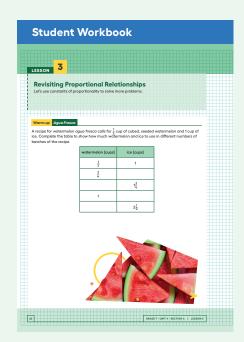
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Building on Student Thinking

Some students may assume the watermelon column will continue to increase by the same amount if they do not pay close attention to the values in the sugar column. Ask these students what they notice about the values in the sugar column and if it makes sense for the watermelon amount to increase by the same amount each time.



Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

MLR2: Collect and Display

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Activity Synthesis

The purpose of this discussion is to contrast two different methods for completing the table: using scale factors between rows and using the constant of proportionality between columns.

Display the table for all to see, and invite students to share their answers and reasoning for each missing entry. Record their ideas directly on the table if possible. Ask students if they agree or disagree with the values in the table.

To help students compare, contrast, and connect the different approaches, consider asking:

"Why do the different approaches lead to the same outcome?"
"Are there any benefits or drawbacks to one approach compared to another?"

If not mentioned by students, highlight that the constant of proportionality is the same for every row in the table, while the scale factor may differ for each pair of rows.

Activity 1

The Price of Rope

15 min

Activity Narrative

In this activity, students see how an abbreviated table can be used to solve a problem about a proportional relationship. In grade 6, students learned to use a table with 3 rows, where the middle row had a "1" in the left column. Now they learn to use a table with only 2 rows, making use of structure.

For teachers accustomed to setting up a proportion and cross-multiplying, this approach is structurally very similar. One benefit to using an abbreviated table is that students have the column headings to help make sure they get the numbers in the right places. In addition, students should have a better idea for why they are multiplying and dividing, which is that they are finding and using either a scale factor or the constant of proportionality. In this case, the numbers are selected such that using the constant of proportionality is easier. It is also a natural way to think about calculating the price of any amount of something.



Students should be comfortable with Kiran's method from their work with tables of equivalent ratios in grade 6. However, if needed, show them this even longer solution method first, and let them examine it. Ask why Lin decided to multiply by $\frac{1}{3}$. Once students are comfortable with the reasoning shown, explain that you will be looking at more efficient ways of solving this problem with a table.

Lin's method:

	length of rope (feet)	price of rope (dollars)	
	6	7.50	
	12	15.00	
. 1/3	24	30.00	. 1/3
	48	60.00	
	2	2.50	
	50	62.50	

Arrange students in groups of 2.

Give students 2–3 minutes of quiet work time followed by time for partner discussion. Then hold a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language students use to describe how the table can be used to solve the problem. Display words and phrases, such as "row," "column," "scale factor," "constant of proportionality," "unit price," and "per."

Student Task Statement

Two students are solving the same problem: At a hardware store, they can cut a length of rope off of a big roll so that the customer can buy any length they like. The cost for 6 feet of rope is \$7.50. How much would the customer pay for 50 feet of rope at this rate?

1. Kiran knows he can solve the problem this way.

length of rope (feet)	price of rope (dollars)
6	7.50
1	1.25
50	2/

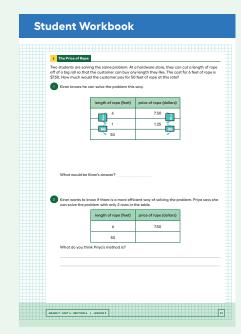
What would be Kiran's answer?

\$62.50, because $(1.25) \cdot 50 = 62.5$

Building on Student Thinking

Some students may struggle to progress with Priya's method because the arrows are not drawn in the image and none of the values given are easily divisible. There are many supporting questions that could be asked.

- What if we knew the price of 1 foot of rope?
- If 6 times something is 7.5, how can we find the something?



2. Kiran wants to know if there is a more efficient way of solving the problem. Priya says she can solve the problem with only 2 rows in the table.

length of rope (feet)	price of rope (dollars)
6	7.50
50	

What do you think Priya's method is?

Sample responses:

- (preferred) I foot of rope costs \$1.25 because $7.5 \div 6 = 1.25$. (Or, $6 \cdot 1.25 = 7.5$.) So multiply 50 by 1.25 to find the cost of 50 feet of rope. $50 \cdot (1.25) = 62.5$.
- Since $50 \div 6 = 8\frac{1}{3}$, that means $6 \cdot 8\frac{1}{3} = 50$. So multiply 7.5 by $8\frac{1}{3}$ to get 62.5.

Activity Synthesis

The purpose of this discussion is to highlight how solving this problem can be accomplished in two steps—dividing and multiplying—and to interpret what those two operations represent in terms of the situation.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how the table can be used to solve the problem. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

If no students come up with one of these methods, display it for all to see. Scale factor method:

length of rop (feet)	price of rope (dollars)
.50 6	7.50
50	4

Constant of proportionality method:



Consider asking:

- "How do we find the scale factor between the two rows?"
 Divide 50 by 6.
- \bigcirc "What does the scale factor, $8\frac{1}{3}$, mean in this situation?"

 The customer is buying $8\frac{1}{3}$ "batches" of rope.

- "How do we find the constant of proportionality?"
 Divide 7.50 by 6.
- "What does the constant of proportionality, 1.25, mean in this situation?"
 The rope costs \$1.25 per foot.
- "Which method do you prefer? Why?"

Although either method will work, there are reasons to prefer using the constant of proportionality to approach problems like these. The constant of proportionality means something important in the situation—it's the price of 1 foot of rope. Because of that, the 1.25 could be used to easily compute the price of any length of rope. If no students bring it up, point out that the equation y = 1.25x could be used to relate any length of rope, x, to its price, y.

Activity 2

Swimming, Manufacturing, and Painting

10 min

Activity Narrative

In this activity, students practice using abbreviated tables to solve problems involving proportional relationships. The scaffolding is slowly removed as each problem has less of the table already completed, leaving more of the table for students to fill in.

In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Give students 4–5 minutes of partner work time, and follow with a whole-class discussion.

Student Task Statement

1. Tyler swims at a constant speed, 5 meters every 4 seconds. How long does it take him to swim 114 meters?

distance (meters)	time (seconds)
5	4
114	91.2

91.2 seconds

Tyler swims I meter in 0.8 seconds because $4 \div 5 = 0.8$. It takes him 91.2 seconds to swim II4 meters, because II4 \cdot 0.8 = 91.2.

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique*, *Correct*, *Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.

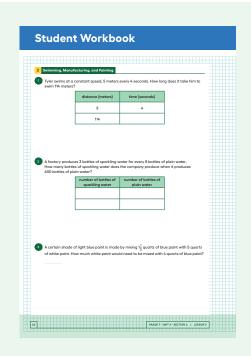
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time, and monitor students to ensure they are using the constant of proportionality to find missing numbers.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Some students may struggle to continue working as the scaffolding is decreased. Consider using these questions to prompt students:

"What are the two associated quantities in this problem?" "How many quarts of blue paint are needed for 1 quart of white paint?"





2. A factory produces 3 bottles of sparkling water for every 8 bottles of plain water. How many bottles of sparkling water does the company produce when it produces 600 bottles of plain water?

Cool-down

number of bottles of sparkling water	number of bottles of plain water
3	8
225	600

225 bottles

The factory produces 0.375 of a bottle of sparkling water per bottle of plain water because $3 \div 8 = 0.375$. The factory produces 225 bottles of sparkling water when it produces 600 bottles of plain water, because $600 \cdot 0.375 = 225$.

3. A certain shade of light blue paint is made by mixing $1\frac{1}{2}$ quarts of blue paint with 5 quarts of white paint. How much white paint would need to be mixed with 4 quarts of blue paint?

There are $3\frac{1}{3}$ quarts of white paint per quart of blue paint, because $5 \div 1\frac{1}{2} = 3\frac{1}{3}$. So $13\frac{1}{3}$ quarts of white paint would need to be mixed with 4 quarts of blue paint, because $4 \cdot 3\frac{1}{3} = 4 \cdot \frac{10}{3} = 13\frac{1}{3}$.

blue paint (quarts)	white paint (quarts)
1 1/2	5
4	13 3

4. For each of the previous three situations, write an equation to represent the proportional relationship.

Sample responses:

- $t = \frac{4}{5}d$
- $p = \frac{8}{3}s$
- $\circ w = \frac{10}{3}b$

Are You Ready for More?

Different nerve signals travel at different speeds.

- Pressure and touch signals travel about 250 feet per second.
- Dull pain signals travel about 2 feet per second.
- 1. How long does it take a person to feel an ant crawling on their foot?

Sample response: The distance between your foot and your brain depends on how tall you are. If you are 5.5 feet tall, then:

It takes about $5.5 \div 250 = 0.022$ seconds for the signal to reach their brain.

2. How much longer does it take to feel a dull ache in their foot?

Sample response: The distance between your foot and your brain depends on how tall you are. If you are 5.5 feet tall, then:

It takes about $5.5 \div 2 = 2.75$ seconds for the pain signal to reach their brain, which is 2.728 seconds longer than it takes to feel the ant.

Activity Synthesis

The purpose of this discussion is to highlight how the structure of a table can help identify the calculations that are needed to solve a problem involving a proportional relationship. Invite students to share how they solved the third problem about the paint mixture. Highlight strategies that involved setting up a table.

To get students analyzing the structure of the table, consider asking:

"Does it matter which heading goes in which column?"

"If you were to switch the columns, would you get a different answer? Why or why not?"

"If you were to switch the columns, would it change what calculations you had to do? Why or why not?"

Use *Critique*, *Correct*, *Clarify* to give students an opportunity to improve a sample written response to the question "How much white paint would need to be mixed with 4 quarts of blue paint?" by correcting errors, clarifying meaning, and adding details.

Display this first draft:

"There are 0.3 quarts of blue paint to white paint, so 1.2 quarts of white paint would be needed."

Ask.

- "What parts of this response are unclear, incorrect, or incomplete?"
 - Give students 2–4 minutes to work with a partner to revise the first draft.
 - Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If time permits, invite students to share the equations they wrote for each proportional relationship. Consider discussing how there are different, equivalent ways to express each relationship and how the equation can be used to help solve the problem.

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Lesson Synthesis

Access for Students with Diverse Abilities (Activity 3, Student Task)

Warm-up

Representation: Internalize Comprehension.

Represent the same information through different modalities by using a table to help organize the information provided.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Access for Multilingual Learners (Activity 3, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2-3 partners to share and get feedback on their first draft response to "Which mixture has a stronger lime flavor?" Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Workbook

3 Finishing the Race an	d More Agua Fresca			
Lin runs $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Tyler runs $8\frac{2}{5}$ miles in $\frac{4}{3}$ hours. How long does it take each of them to run 10 miles at that rate? Explain or show your reasoning.				
3 Lesson Summary				
can calculate the constar situation. For example, Ar does it take him to run 91	If we identify two quantities in a problem and one quantity is proportional to the other, then we can accludate the constant of proportionality and use it to answer other questions about the situation. For example, Andre runs at a constant speed de 5 meters every 2 seconds. How long does it take him to run 91 meters at this rate?			
is running at a constant s	In this problem there are two quantities, time (in seconds) and distance (in meters). Since Andre is running at a constant speed, time is proportional to distance. We can make a table with distance and time as column ha			
	distance (meters)	time (seconds)		
	5	2		
	91			
To find a value in the righ			umn by $\frac{2}{5}$ because	
$\frac{2}{5} \cdot 5 = 2$. This means that it takes Andre $\frac{2}{5}$ of a second to run 1 meter. At this rate, it would take Andre $\frac{2}{5} \cdot 91 = \frac{92}{5} \cdot or 364$, seconds to walk 91 meters. More generally, if t is the time it takes to walk d meters at that pace, then $t = \frac{2}{5}d$.				
24		GRADE	7 - UNIT 4 - SECTION A LESSON:	

Activity 3: Optional

Activity 1

Finishing the Race and More Agua Fresca



Activity Narrative

In this activity, students solve problems involving proportional relationships and fractions without any tables provided. As students choose to create a table or other representation, they make sense of problems and persevere in solving them.

Monitor for students who create a table and use the constant of proportionality to find unknown values.

Launch

Give students 3-5 minutes of guiet work time, and follow with a wholeclass discussion.

If students struggle with getting started, consider asking:

"How many cups of lime juice are used for one cup of watermelon juice?" "How many miles do they each run in one hour?"

Student Task Statement

- 1. To make watermelon agua fresca:
- Diego mixes $\frac{1}{4}$ cup of lime juice into $\frac{3}{4}$ gallon of watermelon juice.
- Elena mixes $\frac{1}{3}$ cup of lime juice into $\frac{7}{8}$ gallon of watermelon juice. Which mixture has a stronger lime flavor? Explain or show your reasoning.

Elena's mixture. Sample reasoning: Diego's mixture uses $\frac{1}{3}$ cup of lime juice per gallon of watermelon juice, because $\frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$. Elena's mixture uses $\frac{8}{21}$ cup of lime juice per gallon of watermelon juice, because $\frac{1}{3} \div \frac{7}{8} = \frac{8}{21}$. Elena's mixture has a stronger lime flavor because $\frac{8}{21}$ is greater than $\frac{1}{3}$.

2. Lin runs $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Tyler runs $8\frac{2}{3}$ miles in $\frac{4}{3}$ hours. How long does it take each of them to run 10 miles at that rate? Explain or show your reasoning.

Warm-up

Lin takes $I\frac{5}{11}$ hours and Tyler takes $I\frac{7}{13}$ hours. Sample reasoning: Lin takes $\frac{8}{55}$ of an hour to run each mile because $\frac{2}{5} \div 2\frac{3}{4} = \frac{8}{55}$. Lin takes $I\frac{5}{11}$ of an hour to run IO miles because $IO \cdot \frac{8}{55} = I\frac{5}{11}$. Tyler takes $\frac{2}{13}$ hours to run each mile because $\frac{4}{3} \div 8\frac{2}{3} = \frac{2}{13}$. Tyler takes $I\frac{7}{13}$ of an hour to run IO miles because $IO \cdot \frac{2}{13} = I\frac{7}{13}$.

Activity Synthesis

The purpose of this discussion is for students to share how they solved the problems involving proportional relationships with fractions. Invite students to share their method for finding a solution. If time is limited, pick only one of the problems to discuss.

If not mentioned by students, ask students to describe why they chose to calculate the constant of proportionality for this problem and how that helped them with finding the solution.

Lesson Synthesis

Share with students,

"Today we used tables to solve problems involving proportional relationships with fractions."

Invite students to share how they use a table with proportional relationships. Here are some questions to elicit student thinking:

"How can creating a table help us solve a problem about a proportional relationship?"

"Why are the column headers important?"

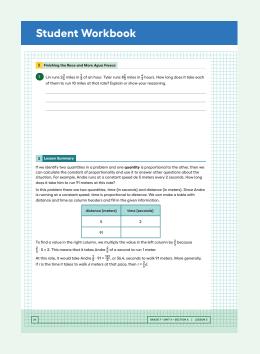
"How do you decide what calculations to do?"

"How can you find and use the constant of proportionality?"

Lesson Summary

If we identify two quantities in a problem and one **quantity** is proportional to the other, then we can calculate the constant of proportionality and use it to answer other questions about the situation. For example, Andre runs at a constant speed of 5 meters every 2 seconds. How long does it take him to run 91 meters at this rate?

In this problem there are two quantities, time (in seconds) and distance (in meters). Since Andre is running at a constant speed, time is proportional to distance. We can make a table with distance and time as column headers and fill in the given information.



Responding To Student Thinking

Points to Emphasize

If students struggle with solving proportional problems involving fractions, plan to review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about completing the table in this activity:

Grade 7, Unit 4, Lesson 4, Activity 1 Walking Half as Much Again

distance (meters)	time (seconds)
5	2
91	

To find a value in the right column, we multiply the value in the left column by $\frac{2}{5}$ because $\frac{2}{5} \cdot 5 = 2$. This means that it takes Andre $\frac{2}{5}$ of a second to run 1 meter.

At this rate, it would take Andre $\frac{2}{5} \cdot 91 = \frac{182}{5}$, or 36.4, seconds to walk 91 meters. More generally, if t is the time it takes to walk d meters at that pace, then $t = \frac{2}{5}d$.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

○ "What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Cool-down

The Price of Wire

5 mir

Student Task Statement

It costs \$3.45 to buy $\frac{3}{4}$ foot of electrical wire. How much would it cost to purchase $7\frac{1}{2}$ feet of wire? Explain or show your reasoning.

\$34.50

Sample reasoning:

- It costs IO times as much to buy 7.5 ft of wire as to buy $\frac{3}{4}$ ft of wire because $\frac{3}{4}$ IO = 7.5 and 3.45 IO = 34.50.
- The wire costs \$4.60 per foot because $3.45 \div 0.75 = 4.60$. At this rate, it will cost \$34.50 for 7.5 ft wire because $4.60 \cdot 7.5 = 34.50$.

Practice Problems

8 Problems

Problem 1

It takes an ant farm 3 days to consume $\frac{1}{2}$ of an apple. At that rate, in how many days will the ant farm consume 3 apples?

18 days

Problem 2

To make a shade of paint called jasper green, mix 4 quarts of green paint with $\frac{2}{3}$ cup of black paint. How much green paint should be mixed with $4\frac{1}{2}$ cups of black paint to make jasper green?

27 quarts

Problem 3

An airplane is flying from New York City to Los Angeles. The distance it travels in miles, d, is related to the time in seconds, t, by the equation $d = \frac{3}{20}t$.

- **a.** How fast is it flying? Be sure to include the units.
 - 0.15 mile per second (or equivalent)
- **b.** How far will it travel in 30 seconds?
 - $4\frac{1}{2}$ miles
- **c.** How long will it take to go $12\frac{3}{4}$ miles?

85 seconds

Problem 4

A grocer can buy strawberries for \$1.38 per pound.

 ${\bf a.}\,$ Write an equation relating c, the cost, and p, the pounds of strawberries.

c = 1.38p

b. A strawberry order costs \$241.50. How many pounds did the grocer order?

175 pounds



Student Workbook 3 Presetics Problems 4 groot can bey strewberries for \$1.58 per pound. a. Write an equation relating c, the cost, and p, the pounds of strewberries. b. A strawberry order costs \$241.50. How many pounds did the grocer order? 4 from Unit A, Lesson 1 Mancack (mohn A/Athoh) is a board game from East Africa. Here is a photo of a game progress. Left store 1 fight store 1 fight store 2 fight store 2 fight store 3 The pieces in the right store are what percentage of all the pieces on the game board? (There are 48 total pieces.)



Problem 5

from Unit 4, Lesson 1

Mancala (mahn-KAH-lah) is a board game from East Africa. Here is a photo of a game in progress.



a. The pieces in the right store are what percentage of all the pieces on the game board? (There are 48 total pieces.)

18.75%

b. The pieces in the right store are what percentage of the pieces in the left store?

225%

Problem 6

from Unit 3, Lesson 10

Crater Lake in Oregon is shaped like a circle with a diameter of about 5.5 miles.

a. How far is it around the perimeter of Crater Lake? about 17 miles (5.5π)

b. What is the area of the surface of Crater Lake?

about 24 square miles $(\pi \cdot 2.75^2)$

Problem 7

from Unit 3, Lesson 8

A 50-centimeter piece of wire is bent into a circle. What is the area of this circle?

 $\frac{625}{\pi}$, or about 199, cm²

Problem 8

from Unit 1, Lesson 2

Suppose Quadrilaterals A and B are both squares. Are A and B necessarily scaled copies of one another? Explain.

Yes, A and B are scaled copies of one another. Since all four side lengths of a square are the same, whatever scale factor works to scale one edge of A to an edge of B takes all edges of A to all edges of B. Since scaling a square gives another square, B is a scaled copy of A.

LESSON 3 • PRACTICE PROBLEMS