Lesson 11

Writing Equations for Lines

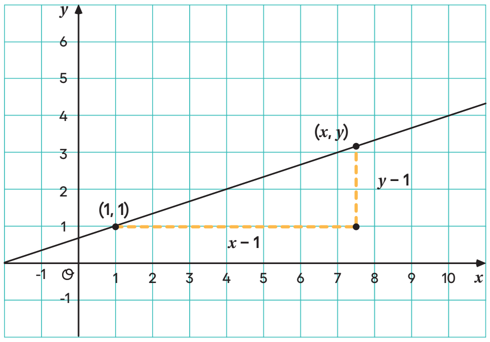
Goals

- Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line.
- Justify (orally) whether a point is on a line by finding quotients of horizontal and vertical distances.

Learning Target

I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.

Lesson Narrative



The purpose of this lesson is for students to use the similarity relationship between slope triangles to write a relationship satisfied by any point on a non-vertical line. Students are shown a line on the coordinate plane that passes through (0, 0) and asked to determine if given points are on that line. Through repeated reasoning, students describe a rule that can test whether or not a point (x, y) is on the line.

When students are shown a line that does not pass through (0, 0), the key idea to introduce is that a point with coordinates (x, y) represents a general point on the line. By noticing that all slope triangles lead to the same value of slope, this general point can be used to write a relationship satisfied by all points on the line.

In this example, the slope of the line is $\frac{1}{3}$ since the points (1, 1) and (4, 2) are on the line. The slope triangle in the picture has vertical length $y - 1$ and horizontal length $x - 1$, giving the equation $\frac{y-1}{x-1} = \frac{1}{3}$, which is satisfied by any point on the line, other than (1, 1).

Student Learning Goal

Let's explore the relationship between points on a line and the slope of the line.

Access for Students with Diverse Abilities

- Engagement (Warm-up)
- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Warm-up)

Instructional Routines

- 5 Practices

Required Materials

Materials to Gather

- Rulers: Warm-up
- Straightedges: Warm-up

Required Preparation

Warm-up:
Provide access to rulers or straightedges.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	15 min	10 min	5 min

Warm-up
Different Slopes of Different Lines

10 min

Activity Narrative

In this activity, students identify lines with different slopes and draw a line with a particular slope. This activity reinforces the idea that different slope triangles whose longest side lies on the same line give the same value for slope.

Monitor for students who use different strategies to construct the line for part F. Here are some strategies students may use:

- Draw slope triangles.
- Count off horizontal and vertical distances without drawing slope triangles.

Select students using each method to share during the discussion.

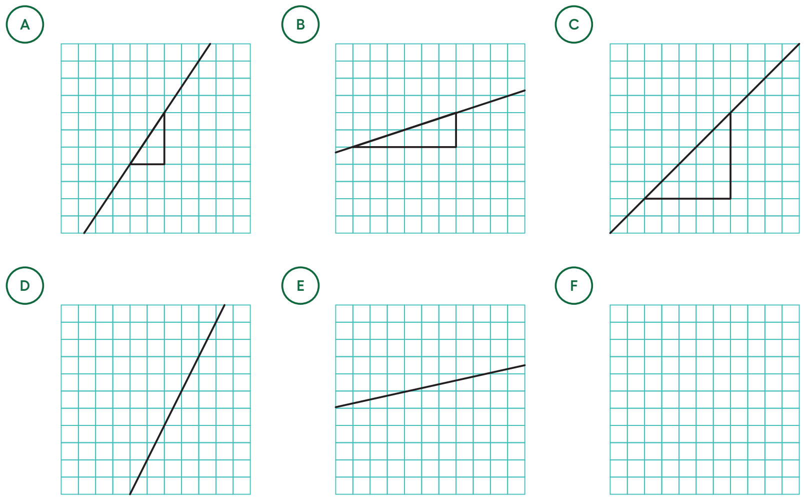
Launch

Provide access to rulers or straightedges. If necessary, refer to the classroom display defining slope.

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are several lines.



1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, $\frac{3}{5}$, 1, 0.25, $\frac{3}{2}$.

$\frac{3}{2}$: A $\quad$ $\frac{1}{3}$: B $\quad$ 1: C

2: D $\quad$ 0.25: E $\quad$ $\frac{3}{5}$: F

Access for Students with Diverse Abilities (Warm-up, Student Task)

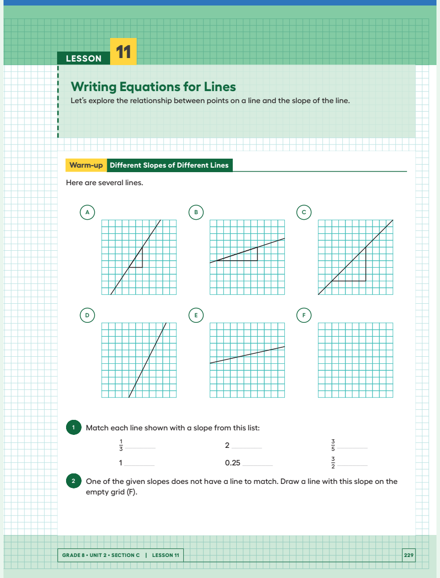
Engagement: Internalize Self-Regulation.
Provide students an opportunity to self-assess and reflect on their own progress. For example, ask students how comfortable they are with identifying the slope of a given line. Ask students what they attribute their level of comfort to and if necessary, brainstorm additional supports that could increase their level of comfort.

Supports accessibility for: Organization, Conceptual Processing

Building on Student Thinking

Some students may find it difficult to draw a slope triangle for a line when one is not given. Prompt them to examine two places where the line crosses an intersection of grid lines.

Student Workbook


LESSON 11

Writing Equations for Lines

Let's explore the relationship between points on a line and the slope of the line.

Warm-up Different Slopes of Different Lines

Here are several lines.

1 Match each line shown with a slope from this list:

2 One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

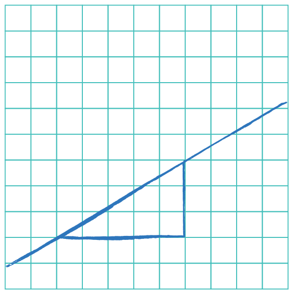
Access for Multilingual Learners (Warm-up, Synthesis)

MLR2: Collect and Display.
Circulate, listen for and collect the language students use to talk about how they matched each line with its slope. On a visible display, record words and phrases such as "I divided the vertical length by the horizontal length" and "I looked for lines that were steeper." Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

Sample response: (A valid response may or may not include a slope triangle similar to the one shown, but all lines should have slope $\frac{3}{5}$.)



Activity Synthesis

The goal is for students to practice finding the slope of a given line on a grid and to understand how different slope triangles can be used to draw or determine the slope of the same line.

Ask previously selected students to share how they drew their lines with a slope of $\frac{3}{5}$. Sequence the discussion so that students who use slope triangles present their work first and students who count horizontal and vertical displacement (without drawing a triangle) present second. Help students see that the second method is the same as the first except that the slope triangle connecting two points on the line is only "imagined" rather than drawn. If time allows, demonstrate that moving up 3 then right 5 results in a line with the same slope as moving right 10 and up 6. Encourage students to draw slope triangles if it helps them to see and understand the underlying structure.

Here are some questions for discussion:

- *"Given a line, how can you determine its slope?"*

  Draw a right triangle where the longest side is on the line and divide the vertical length by the horizontal length.

- *"Given a slope, how can you draw a line with the slope?"*

  Draw a right triangle with vertical and horizontal lengths whose quotient matches the slope, and then extend the longest side of the triangle.

Activity 1
What We Mean by an Equation of a Line

15 min

Activity Narrative

In this activity, students check whether several points lie on a given line and determine a rule to test whether any given point (x, y) is also on the line.

Monitor for students who use these methods for writing a rule in the last question. These approaches are ordered from specific to general, starting with testing the quotient of a single point, to testing a ratio, to testing an equation that can be satisfied by any point (x, y):

- With words and arithmetic: For example, divide the y-coordinate by the x-coordinate and see if it is equivalent to 0.75.
- With a table and proportional relationships: For example, since x and y are in a proportional relationship and $y = 3$ when $x = 4$, see if the constant of proportionality applies to the coordinates of the given point.
- With an equation involving quotients of vertical and horizontal side lengths: For example, see if the point (x, y) satisfies the equation $\frac{y}{x} = \frac{3}{4}$.

If a student writes $y = \frac{3}{4}x$, this can be presented last but it is not essential that students see this now.

Launch

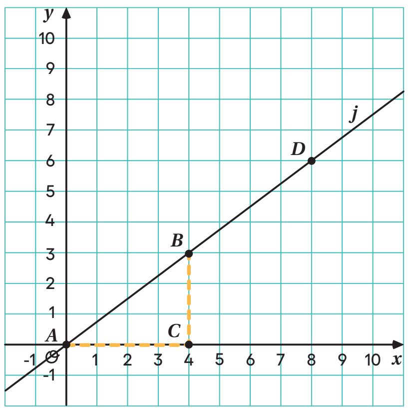
Arrange students in groups of 2.

Give them 3–4 minutes of quiet work time, followed by a partner then whole-class discussion.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

Line j is shown in the coordinate plane.



1. What are the coordinates of B and D?

   $B = (4,3)$ and $D = (8,6)$

Instructional Routines

5 Practices

ilclass.com/r/10690701

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Activity 1, Student Task)

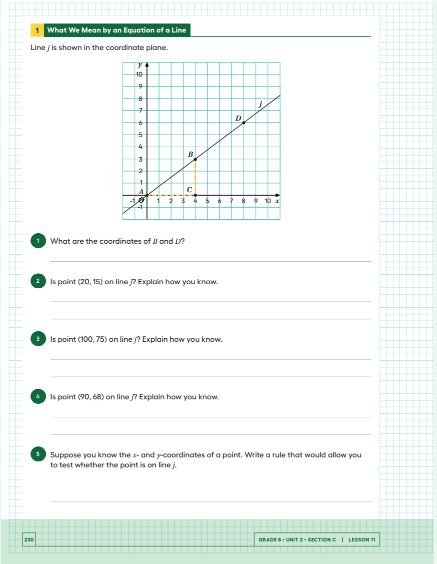
Action and Expression: Develop Expression and Communication
Invite students to talk about their ideas with a partner before recording them. Display sentence frames to support students when they explain their ideas. For example, "This point is/is not on the line because ..." and "First, I _____ because ..."

Supports accessibility for: Language, Organization

Building on Student Thinking

For the last question, a potential issue in saying all points on the line satisfy $\frac{y}{x} = \frac{3}{4}$ is that this equation is not true for the point (0, 0). Unless a student notices this, avoid bringing it up at this time. If necessary, remind students that slope is a ratio of side lengths of a slope triangle, and it doesn't make sense for a triangle to have a side length of 0. Students will learn other ways to think about slope in future lessons.

Student Workbook



1 **What We Mean by an Equation of a Line**

Line j is shown in the coordinate plane.

1. What are the coordinates of B and D?
2. Is point (20, 15) on line j? Explain how you know.
3. Is point (100, 75) on line j? Explain how you know.
4. Is point (90, 68) on line j? Explain how you know.
5. Suppose you know the x- and y-coordinates of a point. Write a rule that would allow you to test whether the point is on line j.

230 GRADE 8 • UNIT 2 • SECTION C | LESSON 11

2. Is point (20, 15) on line j? Explain how you know.

Yes, point (20, 15) is on line j. The y coordinate divided by the x coordinate is 0.75 for points on line j and $\frac{15}{20} = 0.75$.

3. Is point (100, 75) on line j? Explain how you know.

Yes, point (100, 75) is on line j. Sample reasoning: The constant of proportionality is for x- and y-coordinates on the line is $\frac{3}{4}$ and $100 \cdot \frac{3}{4} = 75$.

4. Is point (90, 68) on line j? Explain how you know.

No, point (90, 68) is not on line j. Sample reasoning: The values $x = 90$ and $y = 68$ do not make the equation $\frac{y}{x} = \frac{3}{4}$ true.

5. Suppose you know the x- and y-coordinates of a point. Write a rule that would allow you to test whether the point is on line j.

Sample response: The quotient of the y-coordinate and x-coordinate has to be 0.75.

Activity Synthesis

After reviewing how students answered the first four questions, invite previously selected students to share their response for the last question. Sequence the discussion of the responses in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different methods by asking questions such as:

"How do you see the value $\frac{3}{4}$ in all of the methods?"

In the first method, the quotient had to be equivalent to $\frac{3}{4}$. In the second method, the constant of proportionality was $\frac{3}{4}$. In the third method, one side of the equation was $\frac{3}{4}$.

"What is the slope of this line?"

$\frac{3}{4}$

"Which of these methods would describe the line most clearly without drawing it?"

Answers vary.

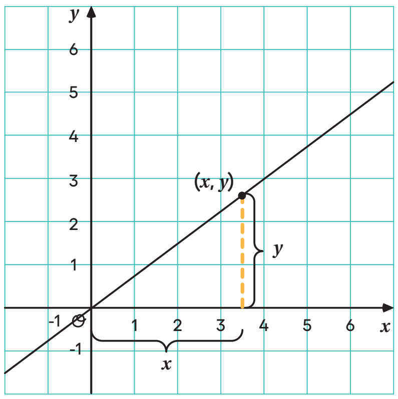
"These methods all look different, but how do they show the same things?"

These methods all describe a proportional relationship with a constant of proportionality of 0.75. They all show how the constant of proportionality can be seen as the relationship between the x- and y-coordinates of a point on the line and also as the slope of the line.

"Would these methods work if the line did not go through (0,0)? Explain your thinking."

No, these methods would not work because the line would not represent a proportional relationship.

Display this image for all to see, or draw a similar diagram.



Ask students how the equation $\frac{y}{x} = \frac{3}{4}$ relates to this situation. (The point (x, y) is on the line when $\frac{y}{x} = \frac{3}{4}$.) The values $\frac{y}{x}$ and $\frac{3}{4}$ are equal because they both represent the quotient of the vertical and horizontal sides of slope triangles for the same line.

The structure of coordinates for points on a line will be examined in greater detail in upcoming lessons. The goal of this activity is to connect student understanding of slope triangles with what it means to be on a line. It is not necessary for students to be able to write equations of a line in the form $y = mx + b$ at this time.

Activity 2

Writing Relationships from Slope Triangles

10 min

Activity Narrative

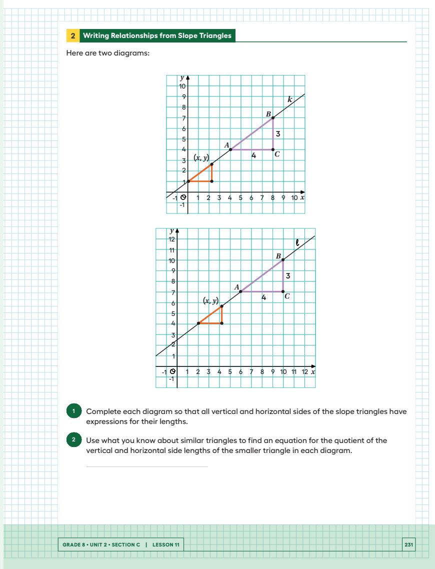
The goal of this activity is for students to write an equation that can determine if a point (x, y) lies on a line that does not contain the point (0, 0) by thinking about similar triangles and their properties. Proportional reasoning based only on the graphed line no longer applies here, but proportional reasoning using similar slope triangles does.

Monitor for different expressions students write in the second question. For example, for line k students may write:

- $\frac{y-1}{x} = \frac{3}{4}$
- $\frac{4-y}{4-x} = \frac{3}{4}$
- $\frac{7-y}{8-x} = \frac{3}{4}$
- $4(y - 1) = 3x$
- $4y = 3x + 4$

Note that the rule or equation for the line is unlikely to come in the form $y = mx + b$. Students do not need to know this for now as it will be addressed in future work.

Student Workbook



Launch

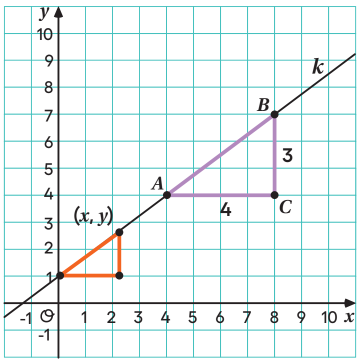
Arrange students in groups of 2.

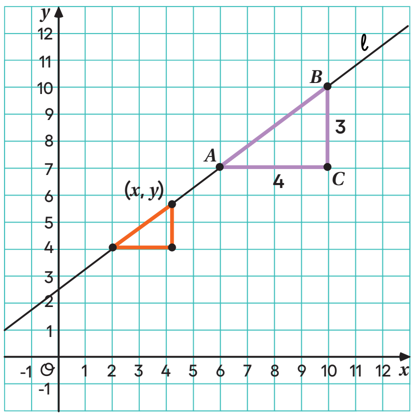
Give students 1–2 minutes of quiet work time to complete the first question for line k.

Pause the class and ensure that all students understand and have a correct expression for each side length before proceeding. Then instruct students to complete the remaining questions.

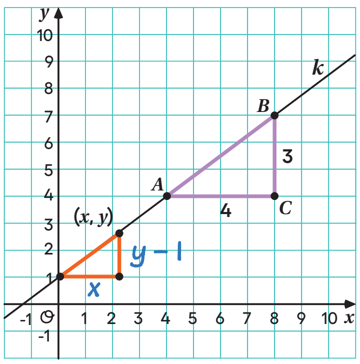
Student Task Statement

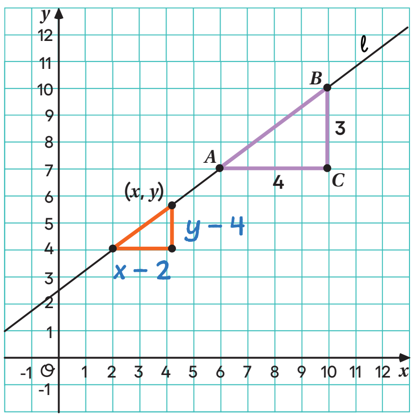
Here are two diagrams:





1. Complete each diagram so that all vertical and horizontal sides of the slope triangles have expressions for their lengths.

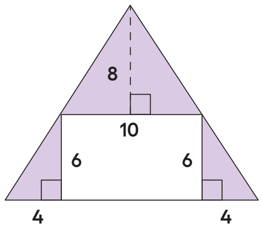




2. Use what you know about similar triangles to find an equation for the quotient of the vertical and horizontal side lengths of the smaller triangle in each diagram.

   Sample responses: line k: $\frac{y-1}{x}=\frac{3}{4}$, line ℓ: $\frac{y-4}{x-2}=\frac{3}{4}$

Are You Ready for More?



1. Find the area of the shaded region by adding the areas of the shaded triangles.

   64 square units

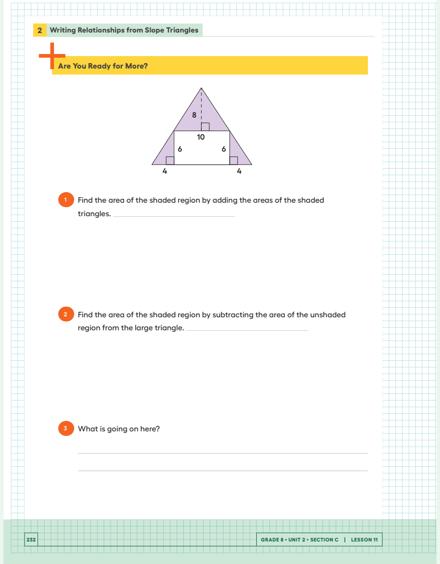
2. Find the area of the shaded region by subtracting the area of the unshaded region from the large triangle.

   66 square units

3. What is going on here?

   Sample response: The areas are different because the so-called "large triangle" is not actually a triangle. Each diagonal side is made up of two segments with slightly different slopes. For example, the bottom left side has a slope of $\frac{3}{2}$ while the top left side is slightly steeper, with a slope of $\frac{8}{5}$. Therefore, the second method of finding the area was not valid for this figure.

Student Workbook



Activity Synthesis

The goal of this discussion is for students to see how different equations relate to different lines.

Invite selected students, as described in the *Activity Narrative*, to share their equations for line k, making sure to note different forms. For example, one equation is $\frac{y-1}{x} = \frac{3}{4}$, though some students might rewrite this as $4y - 4 = 3x$ or even $y = \frac{1}{4}(3x + 4)$.

For line ℓ, one equation is $\frac{y-4}{x-2} = \frac{3}{4}$, and some students may also write $\frac{7-y}{6-x} = \frac{3}{4}$ or $\frac{10-y}{10-x} = \frac{3}{4}$.

There is no reason to manipulate the equations $\frac{y-1}{x} = \frac{3}{4}$ or $\frac{y-4}{x-2} = \frac{3}{4}$, as these two equations contain all of the proportional information from the similar slope triangles. Once the equations are manipulated, this structure is lost, and it is this structure that is of central importance here.

Here are some questions about those two equations for discussion:

"What are the slopes of lines k and ℓ?"

Both lines have a slope of $\frac{3}{4}$.

"How can we see the slope of the lines in their equations?"

Both equations contain the fraction $\frac{3}{4}$.

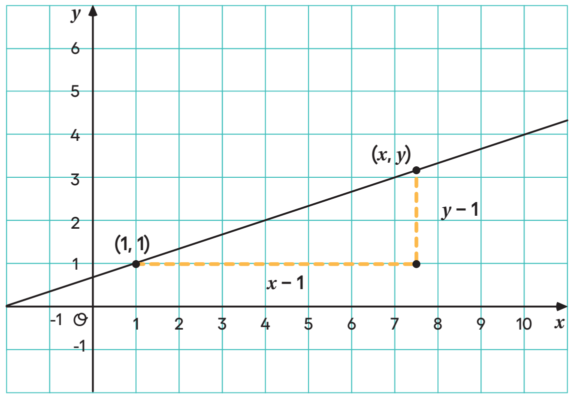
"How are the equations for lines k and ℓ alike and how are they different?"

They can both be written as some quotient involving y and x equal to $\frac{3}{4}$. They have different values subtracted from x and y on the left side of the equations.

If needed, note that the equation does not make sense if the denominator is equal to zero. For example, $\frac{y-1}{x} = \frac{3}{4}$ does not make sense if $x = 0$ and $y = 1$. The reason for this is that the equation is based on using the point (0, 1) as one point. So, a slope triangle cannot be drawn using (0, 1) again as the second point.

Lesson Synthesis

The goal of this discussion is to review how to use slope triangles to find a relationship satisfied by the coordinates of all points on a line. Display the figure for all to see.



Discuss the following questions and record student responses on the display:

"What is the slope of this line?"

It is $\frac{2}{6}$ or $\frac{1}{3}$ (or equivalent)

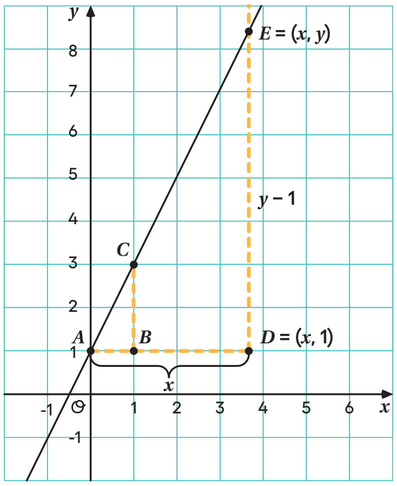
"Using the slope triangle drawn here, what are its vertical and horizontal lengths?"

The vertical side has length $y - 1$ and the horizontal side has length $x - 1$.

"How can you write an equation for this line using this slope triangle?"

Since the quotient of the vertical length and horizontal length is equal to the slope, one equation is $\frac{y-1}{x-1} = \frac{2}{6}$.

Lesson Summary



Here is a line on the coordinate plane. The points A, C, and E are on the same line. Triangles ABC and ADE are slope triangles for the line, so they are similar triangles. We can use their similarity to better understand the relationship between x and y, which are the coordinates of point E.

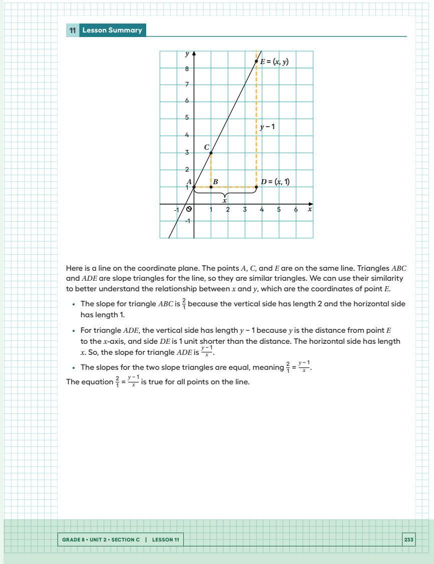
The slope for triangle ABC is $\frac{2}{1}$ because the vertical side has length 2 and the horizontal side has length 1.

For triangle ADE, the vertical side has length $y - 1$ because y is the distance from point E to the x-axis, and side DE is 1 unit shorter than the distance. The horizontal side has length x. So, the slope for triangle ADE is $\frac{y-1}{x}$.

The slopes for the two slope triangles are equal, meaning $\frac{2}{1} = \frac{y-1}{x}$.

The equation $\frac{2}{1} = \frac{y-1}{x}$ is true for all points on the line.

Student Workbook



Responding To Student Thinking

Points to Emphasize
If students struggle to write an equation using slope triangles, as opportunities arise over the next several lessons, focus on identifying similar slope triangles. For example, in the activity referred to here, invite multiple students to share how they arrived at their equations, emphasizing how they created and used slope triangles.

Unit 2, Lesson 12, Activity 2 Writing Relationships from Two Points

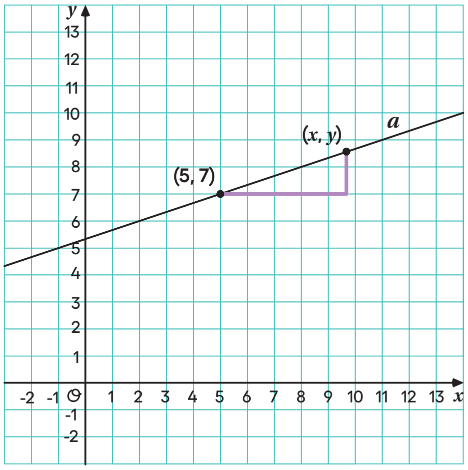
Cool-down

Matching Relationships to Graphs

5 min

Student Task Statement

Line a is shown on the coordinate plane.



1. Explain why the slope of line a is $\frac{2}{6}$.

   Sample reasoning: The points (5, 7) and (11, 9) are on the line. A slope triangle drawn using these points as vertices will have a vertical length of 2 and a horizontal length of 6, giving a slope value of $\frac{2}{6}$.

2. Label the horizontal and vertical sides of the slope triangle with expressions representing their length.

   The vertical side has length $y - 7$, and the horizontal side has length $x - 5$.

3. Use the slope triangle to write an equation for any point (x, y) on line a.

   $\frac{y-7}{x-5} = \frac{2}{6}$ (or equivalent)

   Sample reasoning: Equations such as $\frac{7-y}{5-x} = \frac{1}{3}$ or $\frac{y-6}{x-2} = \frac{2}{6}$ are also correct, being derived from different slope triangles than the one shown.

4. Is the point (95, 37) on line a? Explain or show your reasoning.

   Yes, point (95, 37) is on line a.

   Sample reasoning: Those x- and y-coordinates make the line's equation true: $\frac{37-7}{95-5} = \frac{30}{90} = \frac{2}{6}$.

Lesson 11

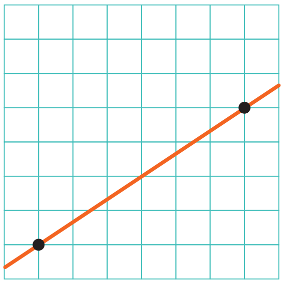
Practice Problems

4 Problems

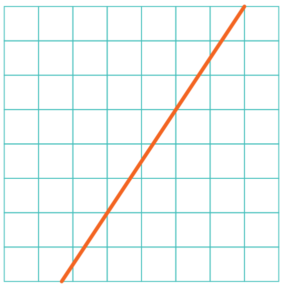
Problem 1

Find the slope of each line.

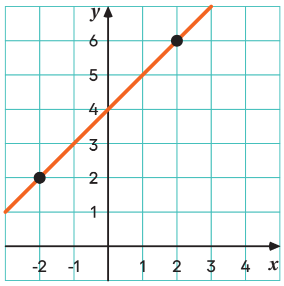
A



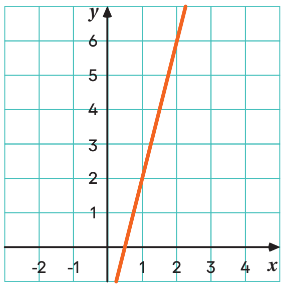
B



C



D



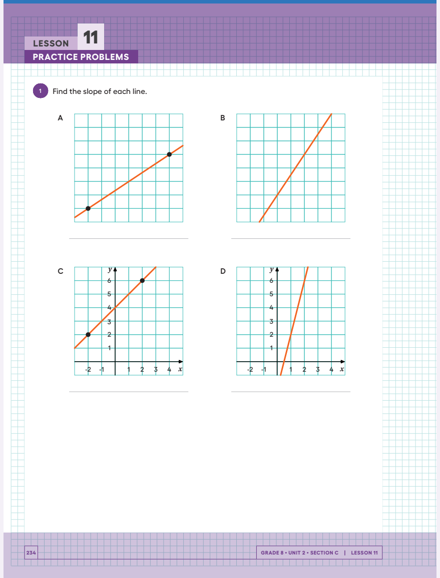
A. $\frac{2}{3}$ (or equivalent)

B. $\frac{3}{2}$ (or equivalent)

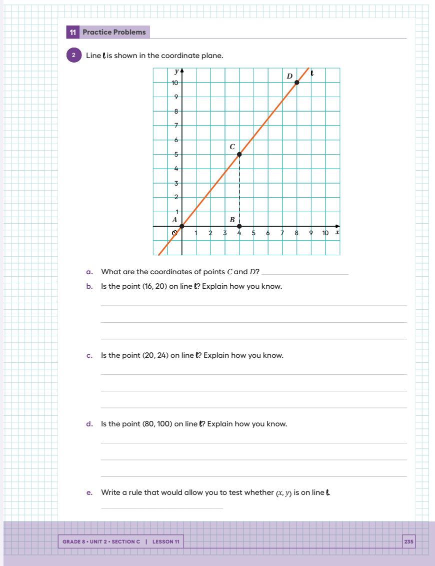
C. 1 (or equivalent)

D. 4 (or equivalent)

Student Workbook



Student Workbook


11 Practice Problems

2 Line ℓ is shown in the coordinate plane.

a. What are the coordinates of points C and D?

b. Is the point (16, 20) on line ℓ? Explain how you know.

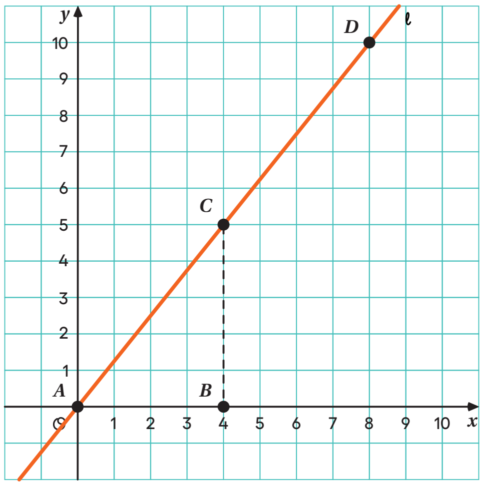
c. Is the point (20, 24) on line ℓ? Explain how you know.

d. Is the point (80, 100) on line ℓ? Explain how you know.

e. Write a rule that would allow you to test whether (x, y) is on line ℓ.

Problem 2

Line ℓ is shown in the coordinate plane.



a. What are the coordinates of points C and D?

$C = (4, 5)$ and $D = (8, 10)$

b. Is the point (16, 20) on line ℓ? Explain how you know.

Yes, point (16, 20) is on line ℓ.

Sample reasoning: Since this is a proportional relationship, the y-coordinate divided by the x-coordinate will be equivalent to $\frac{5}{4}$ for points on line ℓ and $\frac{20}{16} = \frac{5}{4}$.

c. Is the point (20, 24) on line ℓ? Explain how you know.

No, point (20, 24) is not on line ℓ.

Sample reasoning: $\frac{24}{20} \neq \frac{5}{4}$

d. Is the point (80, 100) on line ℓ? Explain how you know.

Yes, point (80, 100) is on line ℓ.

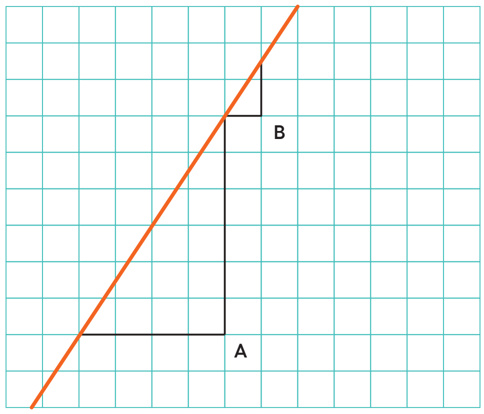
Sample reasoning: $\frac{100}{80} = \frac{5}{4}$

e. Write a rule that would allow you to test whether (x, y) is on line ℓ.

Sample response: $\frac{y}{x} = \frac{5}{4}$

Problem 3

Consider the graphed line.



Mai uses Triangle A and says the slope of this line is $\frac{6}{4}$. Elena uses Triangle B and says the slope of this line is 1.5. Do you agree with either of them? Explain or show your reasoning.

I agree with Mai and Elena.

Sample reasoning: Since both triangles are similar and have their longest side along the same line, using either triangle should give the same slope. They both calculate the slope of the line correctly using their respective triangles, and $\frac{6}{4}$ and 1.5 are equivalent values.

Problem 4

from Unit 2, Lesson 7

A rectangle has dimensions 6 units by 4 units and is similar to quadrilateral $ABCD$. Select **all** statements that are true.

A. The length of side AB is the same as the length of side BC.

B. If the length of side AB is 9 units, then the length of side BC is 7 units.

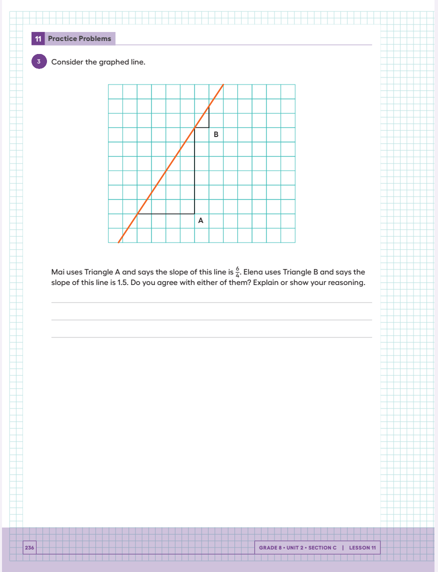
C. The length of the shortest side of quadrilateral $ABCD$ is $\frac{2}{3}$ the length of the longest side.

D. Quadrilateral $ABCD$ is a rectangle.

E. The measure of angle ABC is 90°.

F. The measure of angle BCD is 105°.

Student Workbook



Student Workbook