Lesson 18

When Is the Same Size Not the Same Size?

Goals

- Apply ratios and the Pythagorean Theorem to solve a problem involving the aspect ratio of screens or photos, and explain (orally) the reasoning.
- Describe (in writing and using other representations) characteristics of rectangles with the same aspect ratio or with different aspect ratios.

Learning Targets

- I can apply what I have learned about the Pythagorean Theorem to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem using the Pythagorean Theorem.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 2)

Instructional Routines

- Notice and Wonder

Required Materials

Materials to Gather

- Scientific calculators: Activity 2, Activity 3

Lesson Narrative

In this culminating lesson for the unit, students use the Pythagorean Theorem to compare the diagonal lengths of rectangles with various dimensions. They investigate different aspect ratios of items such as photographs and smartphone screens.

There is an element of mathematical modeling in the last activity, because in order to quantify the screens' sizes to compare them, students need to refine the question that is asked.

Student Learning Goal

Let's figure out how aspect ratio affects screen area.

Lesson Timeline

5 min	20 min	20 min	10 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis

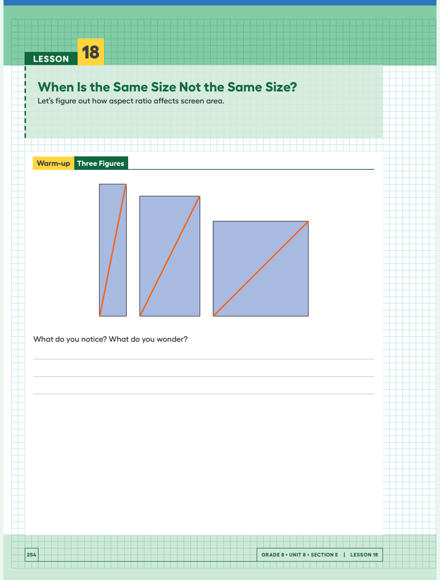
Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Student Workbook



Warm-up
Three Figures

5 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that rectangles can have the same diagonal length but different areas, which will be useful when students work with aspect ratio in a later activity. While students may notice and wonder many things about these figures, the fact that all of the diagonals are the same length and that all of the rectangles have different areas are the important discussion points.

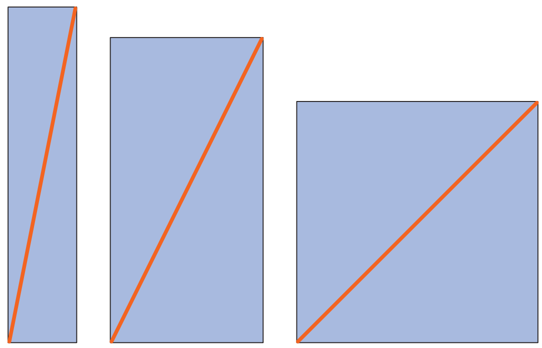
This *Warm-up* prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved.

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement



What do you notice? What do you wonder?

Students may notice:

- They are all rectangles.
- They all have a diagonal drawn in.
- Their diagonals are all the same length.
- One of the rectangles looks like a square.

Students may wonder:

- Do the rectangles all have the same area?
- Do the rectangles all have the same perimeter?
- Are the diagonals all the same length? (if they did not measure)

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the idea that the diagonals are all the same length but the areas of the rectangles are different does not come up during the conversation, encourage students to use a ruler or the edge of a piece of paper to measure and verify these claims.

Activity 1

A 4:3 Rectangle

20 min

Activity Narrative

The purpose of this activity is to understand what an aspect ratio is and how to use this concept along with the Pythagorean Theorem to solve problems.

Launch

Arrange students in groups of 2. Provide access to calculators that can take the square root of a number.

Explain to students that in photography, film, and some consumer electronics with a screen, the ratio of the two sides of a rectangle is often called its "aspect ratio." For example, in the rectangles in *Warm-up*, the aspect ratios are 5:1, 2:1, and 1:1. Demonstrate by showing how the length of one side is a multiple of the other.

Give students 10 minutes to work with their partner to answer the questions, and follow with a whole-class discussion.

Instructional Routines

MLR6: Three Reads

ilclass.com/r/10695568

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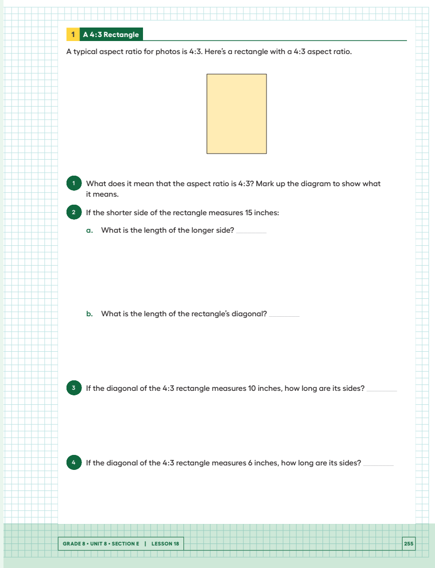
Student Workbook


1 A 4:3 Rectangle

A typical aspect ratio for photos is 4:3. Here's a rectangle with a 4:3 aspect ratio.

1 What does it mean that the aspect ratio is 4:3? Mark up the diagram to show what it means.

2 If the shorter side of the rectangle measures 15 inches:

a. What is the length of the longer side?

b. What is the length of the rectangle's diagonal?

3 If the diagonal of the 4:3 rectangle measures 10 inches, how long are its sides?

4 If the diagonal of the 4:3 rectangle measures 6 inches, how long are its sides?

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Student Task Statement

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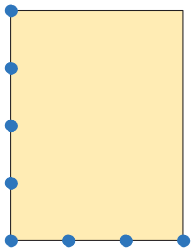
1. What does it mean that the aspect ratio is 4:3? Mark up the diagram to show what it means.



2. If the shorter side of the rectangle measures 15 inches:
   a. What is the length of the longer side? 20 inches
   b. What is the length of the rectangle's diagonal? 25 inches
3. If the diagonal of the 4:3 rectangle measures 10 inches, how long are its sides? 8 inches and 6 inches
4. If the diagonal of the 4:3 rectangle measures 6 inches, how long are its sides? 4.8 inches and 3.6 inches

Activity Synthesis

The purpose of this discussion is for students to see different strategies for solving the fourth question of the problem. Invite students to share their solutions. If not brought up in students' explanations, here are some possible solution strategies:

Solve the equation $(3x)^2 + (4x)^2 = 6^2$, where x represents the length in inches of one of the "pieces" in the diagram. x is $\frac{6}{5}$, which can be multiplied by 3 and 4 to find the side lengths.

Scale the 6 and 8 found previously by a factor of $\frac{6}{10}$ since this triangle would be similar to the triangle formed in the third question.

Activity 2
The Screen Is the Same Size … Or Is It?

20 min

Activity Narrative

In this activity, students solve an application problem that requires an understanding of aspect ratio, the Pythagorean Theorem, and the realization that a good way to compare the sizes of two screens is to compare their areas.

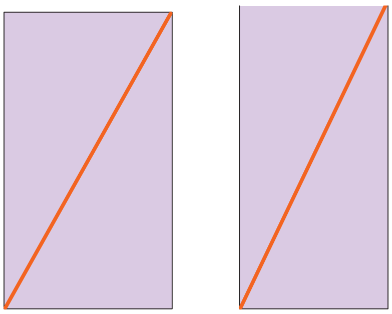
Monitor for students using different approaches and strategies. Students may benefit from more time to think about this problem than is available during a typical class meeting.

Launch

Keep students in groups of 2. Provide access to calculators that can calculate the square root of a number.

Display and read aloud the problem stem.

Scale diagrams are not included in the student materials since it becomes somewhat obvious that the new phone design has a smaller screen area. However, if desired, here is an image to show or provide to students:



Student Task Statement

Before 2017, a smart phone manufacturer's phones had a diagonal length of 5.8 inches and an aspect ratio of 16:9. In 2017, they released a new phone that also had a 5.8-inch diagonal length, but an aspect ratio of 18.5:9. Some customers complained that the new phones had a smaller screen. Were they correct? If so, how much smaller was the new screen compared to the old screen?

Sample response: For the 16:9 screen, it's approximately 5.06 inches by 2.84 inches, for an area of approximately 14.4 square inches. This can be found by solving $(16x)^2 + (9x)^2 = 5.8^2$, which gives $x \approx 0.316$. Multiply this value by 16 and 9 to find the lengths of the two sides. Then multiply the lengths of the sides to find the area in square inches.

For the 18.5:9 screen, it's approximately 5.22 inches by 2.54 inches, for an area of approximately 13.3 square inches. This can be found by solving $(18.5x)^2 + (9x)^2 = 5.8^2$, which gives $x \approx 0.282$. Multiply this value by 18.5 and 9 to find the lengths of the two sides. Then multiply the lengths of the sides to find the area in square inches.

Since 13.3 square inches is less than 14.4 square inches, the newer phones did, in fact, have a smaller screen when measured in terms of area. The difference was about 1.1 square inches.

Another possible solution method is to find that the diagonal of a 16-by-9-inch rectangle is approximately 18.36 inches, and then scale each of the three measures down by a factor of $\frac{5.8}{18.36}$.

Access for Multilingual Learners (Activity 2, Launch)

MLR6: Three Reads.
Keep books or devices closed. Display only the problem, without revealing the questions. Say,

> ***"We are going to read this information 3 times."***

After the 1st Read:

> ***"Tell your partner what this situation is about."***

After the 2nd Read:

> ***"List the quantities. What can be counted or measured?"***

Reveal the question(s). After the 3rd Read:

> ***"What are some ways we might get started on this?"***

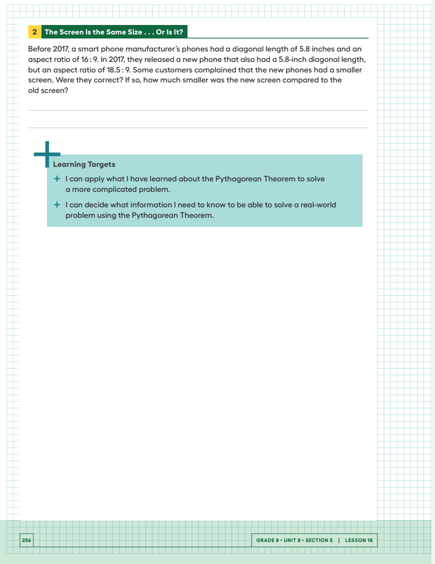
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Provide Access by Recruiting Interest.
Optimize meaning and value. Invite students to share the dimensions of their phone or to measure a screen in the classroom.

Supports accessibility for: Conceptual Processing, Social-Emotional Functioning

Student Workbook


2 The Screen Is the Same Size . . . Or Is It?

Before 2017, a smart phone manufacturer's phones had a diagonal length of 5.8 inches and an aspect ratio of 16 : 9. In 2017, they released a new phone that also had a 5.8-inch diagonal length, but an aspect ratio of 18.5 : 9. Some customers complained that the new phones had a smaller screen. Were they correct? If so, how much smaller was the new screen compared to the old screen?

Learning Targets
+ I can apply what I have learned about the Pythagorean Theorem to solve a more complicated problem.
+ I can decide what information I need to know to be able to solve a real-world problem using the Pythagorean Theorem.

Activity Synthesis

The purpose of this discussion is to give students the opportunity to present their solution. Invite students to share their ideas and progress with the class. If appropriate, students may benefit from an opportunity to clearly present their solution in writing.

Lesson Synthesis

The goal of this discussion is to summarize takeaways from this lesson. Here are key ideas to highlight and discuss:

- *"How can the Pythagorean Theorem help us tackle difficult problems?"*

  If a problem can be broken down into a right triangle, the Pythagorean Theorem can help us find a missing side length.

- *"How did you use mathematical modeling in this lesson?"*

  I had to draw a diagram to show the situation.

- *"What part(s) of the activity about smartphones did you have to try multiple strategies in order to get an answer?"*

  I started out using guess and check but realized that it wouldn't work.