Lesson 5

Say It with Decimals

Goals

- Comprehend and use the term "repeating" (in spoken language) and the notation $\overline{\ }$ (in written language) to refer to a decimal expansion that keeps having the same number over and over forever.
- Coordinate fraction and decimal representations of situations involving adding or subtracting a fraction of the initial value.
- Use long division to generate a decimal representation of a fraction, and describe (in writing) the decimal that results.

Learning Targets

- I can use the distributive property to rewrite an equation like $x + 0.5x = 1.5x$.
- I can write fractions as decimals.
- I understand that "half as much again" and "multiply by 1.5" mean the same thing.

Student Learning Goal

Let's use decimals to describe increases and decreases.

Lesson Narrative

In this lesson, students use decimal notation to express situations involving fractional increase and decrease. First students see that **long division** can be used to convert a fraction to a decimal. Long division finds the quotient one digit at a time, from left to right. They recognize that the quotient may be a **repeating decimal**, a decimal that has digits that keep going in the same pattern over and over, which they learn to express with bar notation. For example, $1.\overline{3}$ means 1.3333333 … Then students revisit situations where an increase or decrease is given as a fraction of the original amount. They match these situations to equations with decimals. For example, they see that "one quarter less than x" can be expressed as $\frac{3}{4}x$ or as $0.75x$.

As students clarify which values repeat in a decimal expansion, they attend to precision. As students use the distributive property to write equations in a simpler way, they are making use of structure.

The last activity is optional because it provides an opportunity for additional practice matching equations and situations in a familiar context.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)
- Engagement (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- Card Sort
- MLR2: Collect and Display
- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns

Required Materials

Materials to Copy

- More Representations Cards (1 copy for every 2 students): Activity 3

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	10 min	5 min

Warm-up

Notice and Wonder: Fractions to Decimals

5 min

Activity Narrative

The purpose of this *Warm-up* is to get students thinking about decimal expansions of fractions. This will be useful when students use long division to find decimal expansions in a later activity. While students may notice and wonder many things about these equations, the important discussion points are the number of decimal places given for each fraction and any patterns students notice within the decimal expansions.

This prompt gives students opportunities to see and make use of structure. The specific structure they might notice is repeating and terminating decimals.

Launch

Arrange students in groups of 2. Display the equations for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

A calculator gives the following decimal representations for some unit fractions:

$\frac{1}{2} = 0.5$
$\frac{1}{3} = 0.3333333$
$\frac{1}{4} = 0.25$
$\frac{1}{5} = 0.2$
$\frac{1}{6} = 0.1666667$
$\frac{1}{7} = 0.1428571$
$\frac{1}{8} = 0.125$
$\frac{1}{9} = 0.1111111$
$\frac{1}{10} = 0.1$
$\frac{1}{11} = 0.0909091$

What do you notice? What do you wonder?

Students may notice:

- Different fractions have different numbers of digits.
- Some decimals repeat.
- Some decimals don't repeat (not really, but that's what it looks like).
- Some decimals almost repeat, except for the last digit (0.0909091).

Students may wonder:

- Do some decimals really finish after 7 digits, or do they keep going?
- Why are some decimals shorter than others?
- Are there other fractions that only have one digit after the decimal?

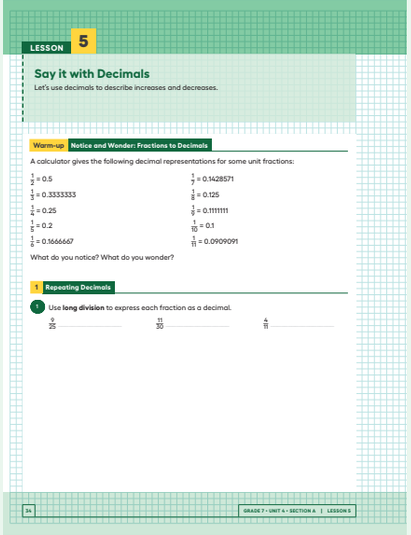
Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Student Workbook



LESSON 5

Say it with Decimals

Let's use decimals to describe increases and decreases.

Warm-up Notice and Wonder: Fractions to Decimals

A calculator gives the following decimal representations for some unit fractions:

$\frac{1}{2} = 0.5$ $\frac{1}{7} = 0.1428571$
$\frac{1}{3} = 0.3333333$ $\frac{1}{8} = 0.125$
$\frac{1}{4} = 0.25$ $\frac{1}{9} = 0.1111111$
$\frac{1}{5} = 0.2$ $\frac{1}{10} = 0.1$
$\frac{1}{6} = 0.1666667$ $\frac{1}{11} = 0.0909091$

What do you notice? What do you wonder?

1 Repeating Decimals

1. Use **long division** to express each fraction as a decimal.

$\frac{9}{25}$ $\frac{11}{20}$ $\frac{4}{11}$

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Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

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Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Provide a document with examples of long division for students to use as a reference.

Supports accessibility for: Memory, Organization

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the equations. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Activity 1

Repeating Decimals

15 min

Activity Narrative

In this activity, students use long division to find the decimal expansion for three different fractions. They see that two of the three fractions result in repeating decimals. As students describe and compare the quotients $0.3\overline{6}$ and $0.\overline{36}$, they attend to precision of language.

Launch

Explain to students that we can use long division to calculate the decimal representation of a fraction. For example, $\frac{7}{8}$ is equal to $7 \div 8$.

$$
\begin{array}{r}
0.875 \\
8\overline{)7} \\
-\ 0 \\
\hline
70 \\
-\ 64 \\
\hline
60 \\
-\ 56 \\
\hline
40 \\
-\ 40 \\
\hline
0
\end{array}
$$

Using long division, we see that $\frac{7}{8}$ is equal to 0.875. This process works for any fraction.

Here is another example: $\frac{7}{12}$ is equal to $7 \div 12$.

$$
\begin{array}{r}
0.58333 \\
12\overline{)7} \\
-\ 0 \\
\hline
70 \\
-\ 60 \\
\hline
100 \\
-\ 96 \\
\hline
40 \\
-\ 36 \\
\hline
40 \\
-\ 36 \\
\hline
40 \\
-\ 36 \\
\hline
4
\end{array}
$$

In this case, the division will never result in a remainder of 0. Because we keep getting 3 over and over again, this is called a **repeating decimal** and can be written as $0.58\overline{3}$.

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time on the first problem and 1–2 minutes to compare their responses and discuss the second question with their partner. Then give students 2–3 minutes of partner work time on the remaining question.

Follow with whole-class discussion.

Student Task Statement

1. Use **long division** to express each fraction as a decimal.

   $\frac{9}{25}$ 0.36 $\frac{11}{30}$ $0.3\overline{6}$ $\frac{4}{11}$ $0.\overline{36}$

2. What is similar about your answers to the previous question? What is different?

   Sample response: All 3 of these decimals have the same two numbers in the tenths and hundredths places, but $\frac{11}{30}$ and $\frac{4}{11}$ are both repeating decimals, while $\frac{9}{25}$ is not.

3. Use the decimal representations to decide which of these fractions has the greatest value. Explain your reasoning.

   $\frac{11}{30}$ is the largest because it has a 6 in the thousandths place, when written as a decimal. $\frac{4}{11}$ has a 3 in the thousandths place and $\frac{9}{25}$ would have a 0 in the thousandths place.

Are You Ready for More?

One common approximation for π is $\frac{22}{7}$. Express this fraction as a decimal. How does this approximation compare to 3.14?

$3.\overline{142857}$ is closer to π than 3.14 is.

Activity Synthesis

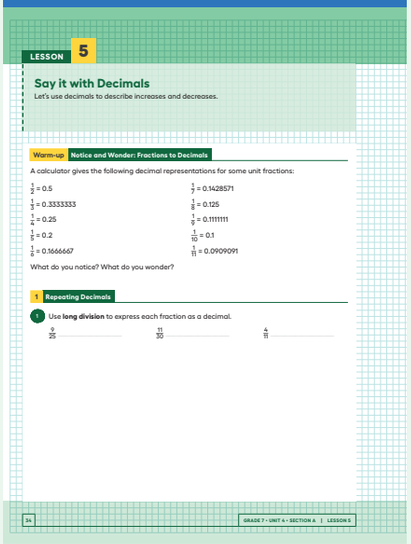
The purpose of this discussion is to help students make sense of the value of repeating decimals. Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they determined which fraction has the greatest value. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases, such as "repeating decimal," "terminating decimal," "place value," "hundredths place," "thousandths place," "bar over _____."

The key takeaway is that 0.36, $0.3\overline{6}$ and $0.\overline{36}$ are different values, even though their decimal representations look quite similar.

Building on Student Thinking

Some students may set up their long division with the divisor and dividend in the wrong places. They will get $2.\overline{7}$, $2.\overline{72}$, and 2.75 as their answers. Prompt them to think about what is being divided and what it is being divided by.

Student Workbook


LESSON 5

Say it with Decimals

Let's use decimals to describe increases and decreases.

Warm-up Notice and Wonder: Fractions to Decimals

A calculator gives the following decimal representations for some unit fractions:

$\frac{1}{2}$ = 0.5 $\frac{1}{7}$ = 0.1428571

$\frac{1}{3}$ = 0.3333333 $\frac{1}{8}$ = 0.125

$\frac{1}{4}$ = 0.25 $\frac{1}{9}$ = 0.1111111

$\frac{1}{5}$ = 0.2 $\frac{1}{10}$ = 0.1

$\frac{1}{6}$ = 0.1666667 $\frac{1}{11}$ = 0.0909091

What do you notice? What do you wonder?

1 Repeating Decimals

1. Use **long division** to express each fraction as a decimal.

$\frac{9}{25}$ $\frac{11}{30}$ $\frac{4}{11}$

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Student Workbook


1 Repeating Decimals

2. What is similar about your answers to the previous question? What is different?

3. Use the decimal representations to decide which of these fractions has the greatest value. Explain your reasoning.

Are You Ready for More?

One common approximation for π is $\frac{22}{7}$. Express this fraction as a decimal. How does this approximation compare to 3.14?

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Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

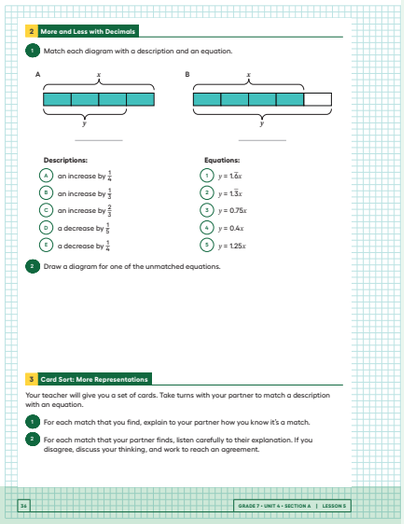
Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Provide a sheet with a list of common fractions and their decimal equivalents for students to use as a reference.

Supports accessibility for: Memory, Organization

Student Workbook


2 More and Less with Decimals

1 Match each diagram with a description and an equation.

A

B

Descriptions:

A an increase by $\frac{1}{4}$

B an increase by $\frac{1}{3}$

C an increase by $\frac{2}{3}$

D a decrease by $\frac{1}{5}$

E a decrease by $\frac{1}{4}$

Equations:

1 $y = 1.\overline{6}x$

2 $y = 1.\overline{3}x$

3 $y = 0.75x$

4 $y = 0.4x$

5 $y = 1.25x$

2 Draw a diagram for one of the unmatched equations.

3 Card Sort: More Representations

Your teacher will give you a set of cards. Take turns with your partner to match a description with an equation.

1 For each match that you find, explain to your partner how you know it's a match.

2 For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

Activity 2

More and Less with Decimals

15 min

Activity Narrative

In this activity students match tape diagrams, verbal descriptions, and equations. The descriptions express the increase or decrease as a fraction of the original amount, while the equations express the scale factor as a decimal. This gives students more practice converting fractions to decimals, including repeating decimals. Then students create their own diagram to represent one of the equations that didn't have a match.

In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

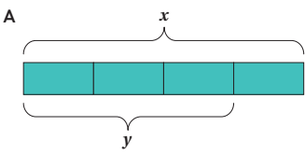
Give students 4–5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Student Task Statement

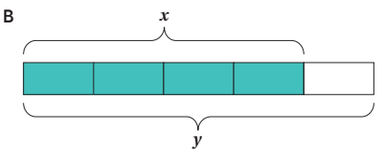
1. Match each diagram with a description and an equation.

A



B, E, 3

B



A, D, 5

Descriptions:

A. an increase by $\frac{1}{4}$

B. an increase by $\frac{1}{3}$

C. an increase by $\frac{2}{3}$

D. a decrease by $\frac{1}{5}$

E. a decrease by $\frac{1}{4}$

Equations:

1. $y = 1.\overline{6}x$

2. $y = 1.\overline{3}x$

3. $y = 0.75x$

4. $y = 0.4x$

5. $y = 1.25x$

2. Draw a diagram for one of the unmatched equations.

Answers vary.

Activity Synthesis

The purpose of this discussion is to emphasize the connection between the numbers in the description and the numbers in the equation. First, ask students to share which description and equation they matched with each diagram. Encourage students to agree or disagree and to restate other students' reasoning in their own words.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to Diagram A by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

"Diagram A shows an increase by $\frac{1}{3}$, so I matched it with $y = 1.\overline{3}x$."

Ask,

- *"What parts of this response are unclear, incorrect, or incomplete?"*

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Here is an example of a second draft:

"In Diagram A, if y is the original, then x is an increase by $\frac{1}{3}$, but the equation $x = 1.\overline{3}y$ wasn't on the list. If we view x as the original, then y is a decrease by $\frac{1}{4}$, which matches with $y = 0.75x$."

- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If time permits, invite students to share the diagram they created. Ask other students to identify the equation that represents the same relationship.

Instructional Routines

Card Sort

ilclass.com/r/10783726

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Activity 3: Optional

Card Sort: More Representations

10 min

Activity Narrative

In this partner activity, students take turns matching situations and equations. The situations are the same as students saw in a previous activity, but this time the equations use decimals instead of fractions. No tape diagrams are given; however, students may choose to draw diagrams if they find this helpful for matching the descriptions with the equations.

As students analyze different representations, they practice reasoning quantitatively and abstractly. In making connections across representations, they practice looking for and making use of structure.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Access for Multilingual Learners (Activity 3, Student Task)

MLR8: Discussion Supports.

Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed ____ , so I matched ..." When students disagree, encourage them to challenge each other using these sentence frames: "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about the different representations.

Advances: Conversing, Representing

Launch

Tell students that the cards contain either situations or equations and that they will take turns matching the cards. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of slips cut from the blackline master.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to match a description with an equation.

1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

The blackline master shows the correct matches.

Activity Synthesis

The purpose of this discussion is to make connections between the numbers in the description and the numbers in the equation. Select 2–3 groups to share one of their sets of cards and how they matched the description with an equation. Discuss as many different sets of cards as the time allows.

Consider asking:

- *"Which matches were tricky? Explain why."*
- *"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"*
- *"How is the number in the equation related to the number in the description?"*

Lesson Synthesis

Share with students,

- *"Today we used long division to convert fractions to decimals. We continued using the distributive property to write expressions that represented situations."*

To review the process for converting a fraction to a decimal, consider asking students:

- *"How could we set up a long division problem to find the decimal equivalent of $\frac{7}{22}$?"*

  Put 7 inside the long division bracket, and put 22 outside on the left.

- *"After doing the long division, we get a quotient of 0.3181818 ... How can we express this value more easily?"*

  Write 0.318, and put a line over the 18.

To review the role of the distributive property in making calculations more efficient, consider prompting students:

- *"Give examples of equivalent expressions that represent an amount, x, plus a decimal part of that amount."*

  $x + 0.5x = 1.5x$; $x + 0.\overline{6}x = 1.\overline{6}x$

- *"Give examples of equivalent expressions that represent an amount, x, minus a decimal part of that amount."*

  $x - 0.25x = 0.75x$; $x - 0.\overline{6}x = 0.\overline{3}x$

Lesson Summary

Long division gives us a way of finding decimal representations for fractions. It finds the quotient one digit at a time, from left to right.

For example, to find a decimal representation for $\frac{9}{8}$, we can divide 9 by 8.

So $\frac{9}{8} = 1.125$.

```
  1.1 2 5
8)9.0 0 0
- 8
  1 0
-  8
    2 0
-   1 6
      4 0
-     4 0
        0
```

To find a decimal representation for $\frac{8}{9}$, we can divide 8 by 9.

So $\frac{8}{9} = 0.\overline{8}$. This is a repeating decimal because the digits keep going in this same pattern over and over.

```
  0.8 8 8
9)8.0 0 0
- 0
  8 0
-  7 2
     8 0
-    7 2
       8 0
-      7 2
         8
```

Sometimes it is easier to work with the decimal representation of a number, and sometimes it is easier to work with its fraction representation. It is important to be able to work with both. For example, consider the following pair of problems:

- Priya earned x dollars doing chores, and Kiran earned $\frac{6}{5}$ as much as Priya. How much did Kiran earn?
- Priya earned x dollars doing chores, and Kiran earned 1.2 times as much as Priya. How much did Kiran earn?

Since $\frac{6}{5} = 1.2$, these are both exactly the same problem, and the answer is $\frac{6}{5}x$ or $1.2x$. When we work with percentages in later lessons, the decimal representation will come in especially handy.

Cool-down

Reading More

5 min

Student Task Statement

Kiran read for x minutes, and Andre read for $\frac{5}{8}$ more than that. Write an equation that relates the number of minutes Kiran read with y, the number of minutes that Andre read. Use decimals in your equation.

$y = 1.625x$ (or equivalent)

Andre read $\frac{5}{8}x = 0.625x$ more minutes than Kiran read. $x + 0.625x = 1.625x$, so $y = 1.625x$.

Responding To Student Thinking

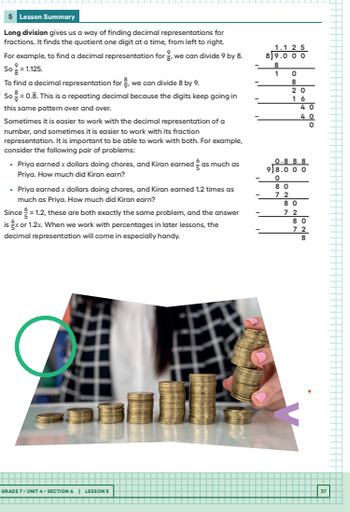
Press Pause

By this point in the unit, there should be some student mastery of writing equations to represent proportional relationships. If students struggle, plan to make time to revisit related work in the lessons referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 4, Lesson 4 More than That, Less than That

Grade 7, Unit 4, Lesson 5 Say It with Decimals

Student Workbook


5 Lesson Summary

Long division gives us a way of finding decimal representations for fractions. It finds the quotient one digit at a time, from left to right.

For example, to find a decimal representation for $\frac{9}{8}$, we can divide 9 by 8.

So $\frac{9}{8} = 1.125$.

To find a decimal representation for $\frac{8}{9}$, we can divide 8 by 9.

So $\frac{8}{9} = 0.\overline{8}$. This is a repeating decimal because the digits keep going in this same pattern over and over.

Sometimes it is easier to work with the decimal representation of a number, and sometimes it is easier to work with its fraction representation. It is important to be able to work with both. For example, consider the following pair of problems:

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- Priya earned x dollars doing chores, and Kiran earned 1.2 times as much as Priya. How much did Kiran earn?

Since $\frac{6}{5} = 1.2$, these are both exactly the same problem, and the answer is $\frac{6}{5}x$ or $1.2x$. When we work with percentages in later lessons, the decimal representation will come in especially handy.

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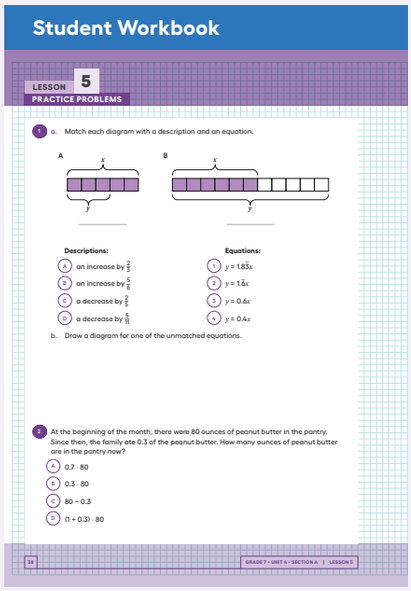
Practice Problems

6 Problems

Student Workbook


LESSON 5
PRACTICE PROBLEMS

1 a. Match each diagram with a description and an equation.

A

B

Descriptions:

A. an increase by $\frac{2}{3}$
B. an increase by $\frac{5}{6}$
C. a decrease by $\frac{2}{5}$
D. a decrease by $\frac{5}{11}$

Equations:

1. $y = 1.8\overline{3}x$
2. $y = 1.\overline{6}x$
3. $y = 0.6x$
4. $y = 0.4x$

b. Draw a diagram for one of the unmatched equations.

2 At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, the family ate 0.3 of the peanut butter. How many ounces of peanut butter are in the pantry now?

A. 0.7 · 80
B. 0.3 · 80
C. 80 − 0.3
D. (1 + 0.3) · 80

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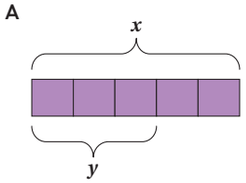
Problem 1

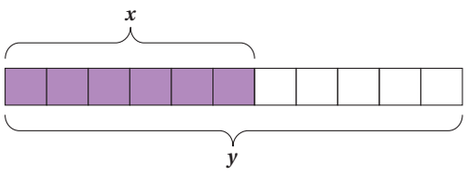
a. Match each diagram with a description and an equation.

A



C, 3

B



B, 1

Descriptions:

A. an increase by $\frac{2}{3}$

B. an increase by $\frac{5}{6}$

C. a decrease by $\frac{2}{5}$

D. a decrease by $\frac{5}{11}$

Equations:

1. $y = 1.8\overline{3}x$

2. $y = 1.\overline{6}x$

3. $y = 0.6x$

4. $y = 0.4x$

b. Draw a diagram for one of the unmatched equations.

Answers vary.

Problem 2

At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, the family ate 0.3 of the peanut butter. How many ounces of peanut butter are in the pantry now?

A. 0.7 · 80

B. 0.3 · 80

C. 80 − 0.3

D. (1 + 0.3) · 80

Problem 3

from Unit 4, Lesson 4

a. On a hot day, a football team drank an entire 50-gallon cooler of water and half as much again. How much water did they drink?

75 gallons

b. Jada has 12 library books checked out, and Han has $\frac{1}{3}$ less than that. How many books does Han have checked out?

8 books

Problem 4

from Unit 4, Lesson 4

If x represents a positive number, select **all** expressions whose value is greater than x.

A. $\left(1-\frac{1}{4}\right)x$

B. $\left(1+\frac{1}{4}\right)x$

C. $\frac{7}{8}x$

D. $\frac{9}{8}x$

Problem 5

from Unit 2, Lesson 6

A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch and counts 8 heartbeats in 10 seconds.

a. Is his heart rate typical? Explain how you know.

No, Noah's heart rate is 48 beats per minute.

Sample reasoning: $10 \cdot 6 = 60$, and $8 \cdot 6 = 48$.

b. Write an equation for h, the number of times Noah's heart beats (at this rate) in m minutes.

$h = 48m$

Problem 6

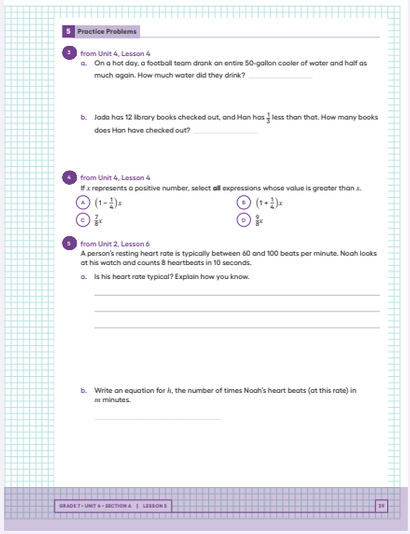
from an earlier course

a. Find 1% of 680. 6.8

b. Find 23% of 680. 156.4

c. Find 23% of 410. 94.3

Student Workbook



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