Lesson 8

How Much in Each Group? (Part 1)

Goals

- Compare and contrast (orally) strategies for solving problems about "How many groups?" and "How much in 1 group?"
- Create a tape diagram to represent and solve a problem asking "How much in 1 group?" where the size of a group, number of groups, and total amount may be fractions, and explain (orally) the solution method.
- Write multiplication and division equations to represent a problem asking "How much in 1 group?"

Learning Targets

- I can tell when a question is asking for the amount in one group.
- I can use diagrams and multiplication and division equations to represent and answer "How much in each group?" questions.

Lesson Narrative

In this lesson, students encounter situations in which the number of groups is known but the size of each group is not. They interpret division expressions as a way to answer "How much in a group?" questions.

Students begin by inventing a situation for a tape diagram in which the number of groups and the total amount are shown but the amount in each group is represented with a "?" symbol.

Next, students solve problems about finding the amount in 1 batch of a recipe given an amount for a different number of batches. They answer questions such as, "If 2 tablespoons are enough for $\frac{2}{3}$ of a batch, how many tablespoons are in 1 batch?" To support their reasoning, students use familiar representations—multiplication and division equations and tape diagrams—and make use of the same structure of equal-size groups.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 3)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Lesson 8

How Much in Each Group? (Part 1)

Lesson Narrative (continued)

Each of the next two activities prompts students to match three tape diagrams and three descriptions representing an unknown group size. Because all three situations involve the same context and the same total number, represented by a segment of the same length on a tape diagram, students need to make sense of the problems carefully and persevere in interpreting them.

The first of the two activities, "How Much in One Container?," is optional. There, the known total amount is a whole number. Consider doing this activity to scaffold students' thinking, if needed, before they do "How Much in One Section?," where the total amount is a fraction.

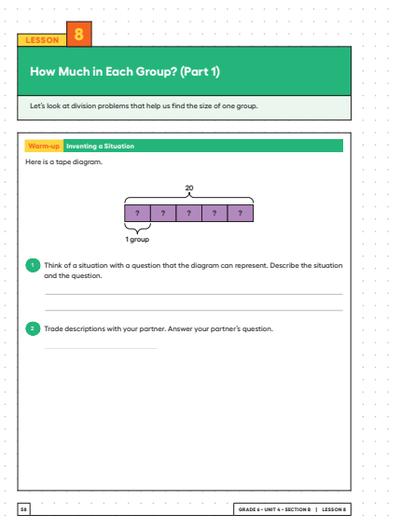
Student Learning Goal

Let's look at division problems that help us find the size of one group.

Building on Student Thinking

Students may struggle to answer their partner's question because the descriptions are unclear or do not match the given expression. Encourage the listening partners to ask clarifying questions about the story or about its connections to the diagram. Urge the story-creating partners to revise their description in response to the questions.

Student Workbook


LESSON 8

How Much in Each Group? (Part 1)

Let's look at division problems that help us find the size of one group.

Warm-up Inventing a Situation

Here is a tape diagram.

1. Think of a situation with a question that the diagram can represent. Describe the situation and the question.

2. Trade descriptions with your partner. Answer your partner's question.

Warm-up

Inventing a Situation

10 min

Activity Narrative

By now, students have created several diagrams based on verbal descriptions of situations. This *Warm-up* invites them to reason in the other direction: to interpret a representation of an equal-group situation and write a story that it could represent. In the given diagram, the number of groups and the total amount are known but the size of one group is not known. The reasoning here prepares students to think about "How much in one group?" questions and create their own representations later in the lesson.

In writing their stories, students have opportunities to communicate with precision, for instance, by including units of measurement or adjusting how "1 group" is referred to based on their chosen quantities. Ask questions to help students clarify their descriptions. Select 2–3 students who write descriptions that match the diagram but about different contexts. Invite them to share during a class discussion.

Launch

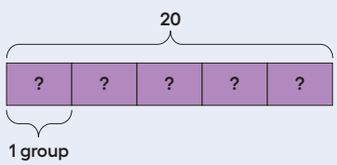
Arrange students in groups of 2. Tell students that there are two parts to this activity: writing a story with a question that can be represented by the tape diagram, and trading stories with their partner and answering each other's question.

Give students **2–3 minutes** to write their story and another minute to read and answer their partner's question.

Follow with a whole-class discussion.

Student Task Statement

Here is a tape diagram.



1. Think of a situation with a question that the diagram can represent. Describe the situation and the question.

   Sample responses:

   - There are 20 envelopes in 5 packs of greeting cards. How many envelopes are in 1 pack?
   - A 20-yard long ribbon is cut into 5 equal pieces. How long is each piece?

2. Trade descriptions with your partner. Answer your partner's question.

   4

   Units vary.

Activity Synthesis

Ask selected students to share their stories with the class. Discuss what the stories have in common. If not mentioned by students, highlight that each story involves finding the amount in one group of something, and the answer is 4 units.

Next, display the following equations for all to see. Give students a minute to think about which equations can represent the diagram (and their stories):

A. $20 \cdot 5 = ?$

B. $20 \div 5 = ?$

C. $20 \cdot ? = 5$

D. $20 \div ? = 5$

E. $5 \cdot ? = 20$

F. $5 \div 20 = ?$

Discuss why equations B, D, and E represent the situation. If time permits, also discuss why options A, C, and F do not represent the situation.

Emphasize that from both the diagram and the equations, we can tell that the value of 1 group (represented by the "?") is 4 units.

- Diagram: 5 groups of 4 makes 20.
- Equations: Using 4 for the "?" in B, D, and E makes each equation true.

Activity 1

How Much in One Batch?

15 min

Activity Narrative

In this activity, students explore division situations in the context of recipes. In each situation, the number of groups and a total amount are given, but the size of 1 group is unknown.

Students write multiplication equations in which the missing factor answers the question "How much in each group?" instead of "How many groups?" They also write division equations and create tape diagrams that represent the given situations. No grid is provided here, but students can use graph paper or refer to the fraction strips from an earlier lesson to support themselves as they create their own diagrams. In doing so, they practice choosing tools strategically.

As students work, monitor how they start their diagrams and which quantity they represent first. If they are not quite sure how to show a particular quantity, ask them to refer to earlier diagrams and notice how the number of groups, the size of each group, and the total amount were represented on a single diagram. Select students whose diagrams would be instructive to others, and ask them to share later.

Building on Student Thinking

When writing multiplication equations, students might simply use the smaller number as a factor and the larger number as the product without attending to what the numbers mean. Support their reasoning with an example that uses only whole numbers. For instance:

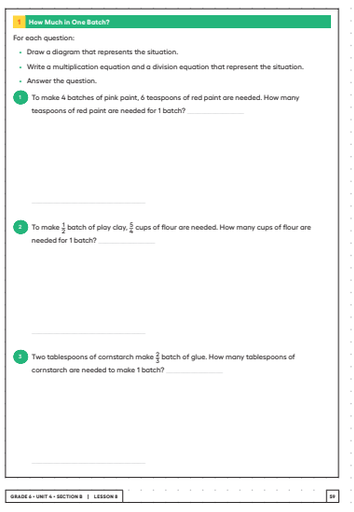
"To make 2 batches of a drink, we need 6 cups of water. How many cups are needed for 1 batch?"

This can be interpreted as "2 times what number equals 6?" and expressed as $2 \cdot ? = 6$. Follow with an example using a non-whole-number total amount. Ask students what multiplication equation can represent this question:

"To make 2 batches of a drink, $\frac{1}{4}$ cup of lemon juice is needed. How many cups are needed for 1 batch?"

$(2 \cdot ? = \frac{1}{4})$

Student Workbook



1 How Much in One Batch?

For each question:

- Draw a diagram that represents the situation.
- Write a multiplication equation and a division equation that represent the situation.
- Answer the question.

1. To make 4 batches of pink paint, 6 teaspoons of red paint are needed. How many teaspoons of red paint are needed for 1 batch?

2. To make $\frac{1}{2}$ batch of play clay, $\frac{5}{4}$ cups of flour are needed. How many cups of flour are needed for 1 batch?

3. Two tablespoons of cornstarch make $\frac{2}{3}$ batch of glue. How many tablespoons of cornstarch are needed to make 1 batch?

GRADE 6 • UNIT 4 • SECTION B | LESSON 8 39

Launch

Tell students that in this lesson they will explore situations in which the size of 1 group is not known but the number of groups is. They will continue to use familiar representations to make sense of and solve problems.

Arrange students in groups of 3–4.

Give groups 5–6 minutes of quiet work time and 2 minutes to share their responses in their group.

Provide access to geometry toolkits (especially graph paper and colored pencils).

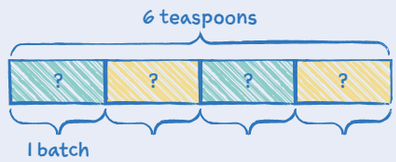
Student Task Statement

For each question:

- Draw a diagram that represents the situation.
- Write a multiplication equation and a division equation that represent the situation.
- Answer the question.

1. To make 4 batches of pink paint, 6 teaspoons of red paint are needed. How many teaspoons of red paint are needed for 1 batch?

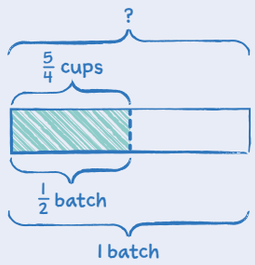
- Sample diagram:



- Multiplication equation: $4 \cdot ? = 6$ (or $? \cdot 4 = 6$)
  Division equation: $6 \div 4 = ?$ (or $6 \div ? = 4$)
- Answer: $1\frac{1}{2}$ teaspoons (or equivalent)

2. To make $\frac{1}{2}$ batch of play clay, $\frac{5}{4}$ cups of flour are needed. How many cups of flour are needed for 1 batch?

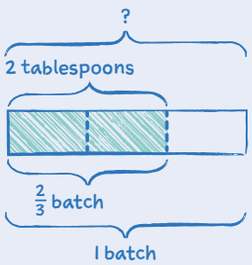
- Sample diagram:



- Multiplication equation: $\frac{1}{2} \cdot ? = \frac{5}{4}$ (or $? \cdot \frac{1}{2} = \frac{5}{4}$)
  Division equation: $\frac{5}{4} \div \frac{1}{2} = ?$ (or $\frac{5}{4} \div ? = \frac{1}{2}$)
- Answer: $\frac{10}{4}$ or $2\frac{1}{4}$ cups (or equivalent)

3. Two tablespoons of cornstarch make $\frac{2}{3}$ batch of glue. How many tablespoons of cornstarch are needed to make 1 batch?

- Sample diagram:



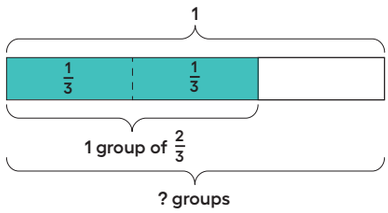
- Multiplication equation: $\frac{2}{3} \cdot ? = 2$ (or $? \cdot \frac{2}{3} = 2$)
  Division equation: $2 \div \frac{2}{3} = ?$ (or $2 \div ? = \frac{2}{3}$)
- Answer: 3 tablespoons

Activity Synthesis

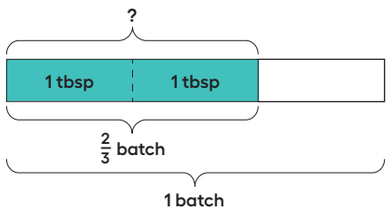
Select previously identified students to share their diagrams and reasoning, or display the ones in the *Student Response* for all to see.

Next, invite students to compare an example from an earlier lesson and one from this activity. For instance, display the following:

- How many groups of $\frac{2}{3}$ are in 1?



- Two tablespoons of cornstarch make $\frac{2}{3}$ batch of glue. How many tablespoons of cornstarch are needed to make 1 batch?



Discuss questions such as:

- *"How are the situations alike?"*

  They both involve $\frac{2}{3}$ and 1. They are both about how many of something are in 1.

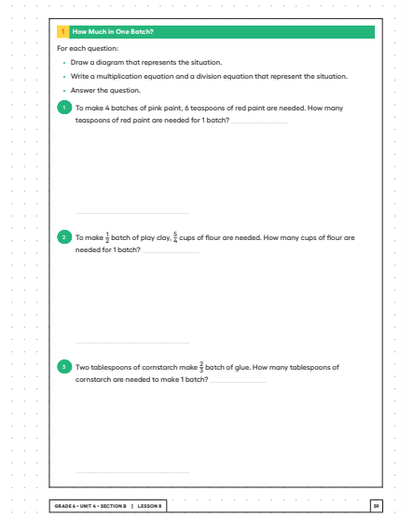
Access for Students with Diverse Abilities (Activity 1, Synthesis)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames, such as the following, to support conversation:

"Both ___ and ___ are alike because ..." "How else could we show this?" "It looks like ___ represents ..."

Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


1 How Much in One Batch?

For each question:

- Draw a diagram that represents the situation.
- Write a multiplication equation and a division equation that represent the situation.
- Answer the question.

1. To make 4 batches of pink paint, 6 teaspoons of red paint are needed. How many teaspoons of red paint are needed for 1 batch? _____

2. To make $\frac{1}{2}$ batch of play clay, $\frac{5}{4}$ cups of flour are needed. How many cups of flour are needed for 1 batch? _____

3. Two tablespoons of cornstarch make $\frac{2}{3}$ batch of glue. How many tablespoons of cornstarch are needed to make 1 batch? _____

Instructional Routines

Notice and Wonder

ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

"How are the situations different?"

In the first situation, the $\frac{2}{3}$ is the amount in 1 group. We are looking for the number of groups in a total amount of 1. In the second, the $\frac{2}{3}$ is the fraction of a group with 2 tablespoons in it. We are looking for how much is in 1 group.

"How are the diagrams alike?"

The same fraction of the diagram is shaded. Both diagrams show $\frac{2}{3}$ of 1.

"How are the diagrams different?"

The shaded portion represents different quantities. In the first diagram, it represents 1 group. In the second, it represents $\frac{2}{3}$ of a group. What is unknown is different.

Emphasize that attending to the meaning of given numbers and their relationships can help us see what we are looking for and how to find an unknown value.

Activity 2

How Much in One Container?

15 min

Activity Narrative

This activity allows students to find the size of 1 group in a concrete and familiar context and in which the total amount is a whole number. Students are prompted to match each of three descriptions with a diagram that represents the same situation. In all three situations, the context is about filling water in containers, and the given total amount is 15 cups. This amount, however, represents different numbers of groups, so students need to make sense of each problem situation carefully.

After matching the given representations, students write equations and answer the question for one situation of their choice.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Display diagrams A, B, and C for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses from 3–5 students without editing or commentary. If possible, record the relevant reasoning on or near the diagrams.

If the amount in 1 container does not come up during the conversation, ask students to discuss this idea.

Tell students to open their student workbooks or devices. Explain that they are to match each of the three diagrams to the description of the situation that it represents.

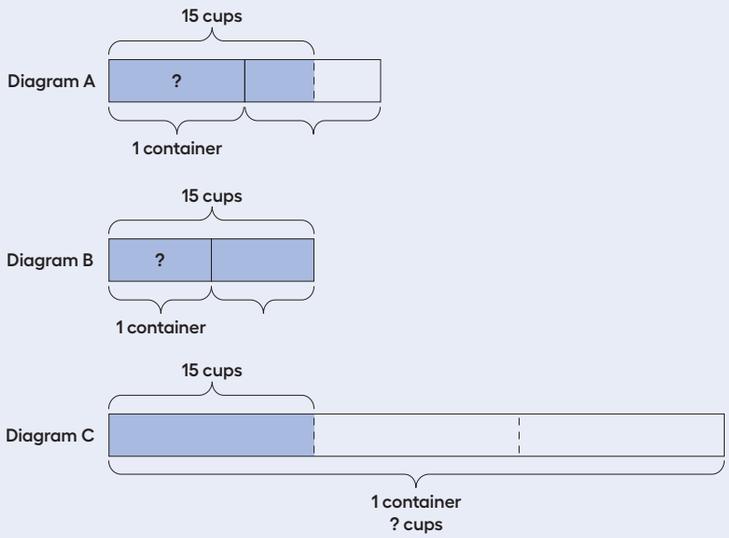
Keep students in groups of 3–4.

Give students **2–3 minutes** of quiet time to do the matching and another **1–2 minutes** to compare their responses with their group and to discuss any questions or discrepancies.

When they reach an agreement, ask students to choose one situation, write multiplication and division equations to represent it, and answer the question.

Student Task Statement

Here are three diagrams and three descriptions that represent situations about filling containers of water.



1. Match each situation to a diagram. Be prepared to explain how you know.

- Tyler filled 2 equal-sized bottles with 15 cups of water. How much water was in each bottle?

  Diagram B, 7.5 cups of water

- Kiran filled $1\frac{1}{2}$ pitchers with 15 liters of water. How much water was in the full pitcher?

  Diagram A, 10 cups of water

- Priya needed 15 cups of water to fill $\frac{1}{3}$ pail. How much water is needed to fill 1 pail?

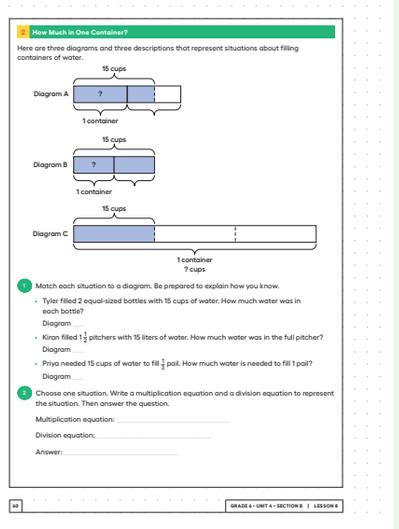
  Diagram C, 45 cups of water

2. Choose one situation. Write a multiplication equation and a division equation to represent the situation. Then answer the question.

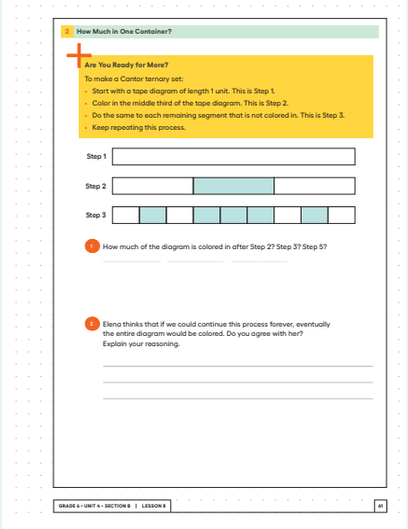
Answers vary based on choice.

- Tyler:
  - Multiplication equation: $2 \cdot ? = 15$
  - Division equation: $15 \div 2 = ?$
  - Answer: 7.5 cups of water
- Kiran:
  - Multiplication equation: $1\frac{1}{2} \cdot ? = 15$
  - Division equation: $15 \div 1\frac{1}{2} = ?$
  - Answer: 10 cups of water
- Priya:
  - Multiplication equation: $\frac{1}{3} \cdot ? = 15$
  - Division equation: $15 \div \frac{1}{3} = ?$
  - Answer: 45 cups of water

Student Workbook



Student Workbook



2 How Much in One Container?

Are You Ready for More?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is Step 1.
- Color in the middle third of the tape diagram. This is Step 2.
- Do the same to each remaining segment that is not colored in. This is Step 3.
- Keep repeating this process.

Step 1

Step 2

Step 3

1 How much of the diagram is colored in after Step 2? Step 3? Step 5?

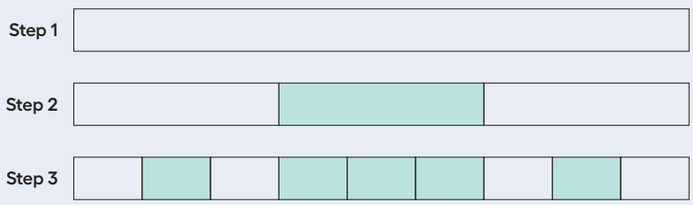
2 Elena thinks that if we could continue this process forever, eventually the entire diagram would be colored. Do you agree with her? Explain your reasoning.

GRADE 6 • UNIT 4 • SECTION B | LESSON 8 41

Are You Ready for More?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is Step 1.
- Color in the middle third of the tape diagram. This is Step 2.
- Do the same to each remaining segment that is not colored in. This is Step 3.
- Keep repeating this process.



1. How much of the diagram is colored in after Step 2? Step 3? Step 5?

   Step 2: $\frac{1}{3}$, Step 3: $\frac{5}{9}$, Step 4: $\frac{19}{27}$, Step 5: $\frac{65}{81}$

2. Elena thinks that if we could continue this process forever, eventually the entire diagram would be colored. Do you agree with her? Explain your reasoning.

   Sample responses:

   - Agree. At each step, the segments that are not colored get smaller and smaller. Eventually there would be no uncolored segments left (or the uncolored segments would be so tiny that they could no longer be seen by our eyes or be partitioned by hand into thirds).
   - Disagree. At any step, two-thirds of any uncolored segment will stay uncolored. Those pieces will get smaller and smaller, but there will always be some left over that are not colored.

Activity Synthesis

Display the three diagrams for all to see. For each diagram, invite a student to share the description that matches the diagram and their reasoning. Ask another student who chose that situation to share the corresponding equations and the answer to the question. Record students' responses near the diagrams.

Discuss any unresolved disagreements about the matches, equations, or answers. To highlight the common structure across the situations and their representations, ask students:

- *"In all three situations, what information was unknown?"*

  The size of one container

- *"What do the multiplication equations have in common?"*

  They all have 15 as the product and one unknown factor.

- *"What do the division equations have in common?"*

  They all have 15 as the dividend or the number being divided.

- *"How can we tell if our answer to each question is correct?"*

  We can record the value in the diagram and see if the relationship of the numbers makes sense. We can substitute the value for the "?" in the multiplication equation and see if it gives a true equation.

Activity 3

How Long is a Section?

10 min

Activity Narrative

This activity prompts students to find the size of 1 group in the context of length and in which the total amount is a fraction less than 1. Students are asked to match each of three descriptions with a diagram that represents the same situation. In all three situations, the context is about cleaning sections of a highway, and the given total amount is $\frac{3}{4}$ mile. This amount, however, represents different numbers of groups, so students need to make sense of each problem situation carefully.

After matching the given representations, students write equations and answer the question for one situation of their choice.

Launch

Ask students if they have seen Adopt-a-Highway or Adopt-a-Road signs. Invite those who are familiar with the program to briefly share what they know. Explain that the program encourages organizations and their volunteers to keep a section of a highway free of litter. In exchange for their cleaning effort, a participating organization can have its name posted on a sign in the section of the highway that it maintains.

Tell students that they are to match three diagrams to three descriptions that represent the same Adopt-a-Highway situation.

Keep students in groups of 3–4.

- Give students **2–3 minutes** of quiet time to do the matching and another **1 minute** to compare their responses with their group and to discuss any questions or discrepancies.

When they reach an agreement, ask students to choose one situation, write multiplication and division equations to represent it, and answer the question.

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 3)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Access for Multilingual Learners (Activity 3, Launch)

MLR8: Discussion Supports.
Display sentence frames, such as the following, that will help students produce verbal justifications:

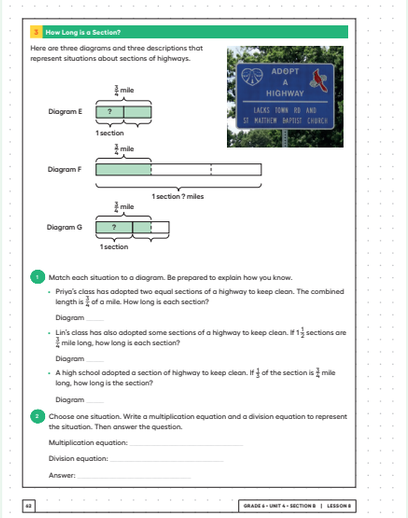
"Diagram ___ represents ___'s situation ___ because ..."

"A multiplication equation that represents this situation is ___ because ..."

"A division equation that represents this situation is ___ because ..."

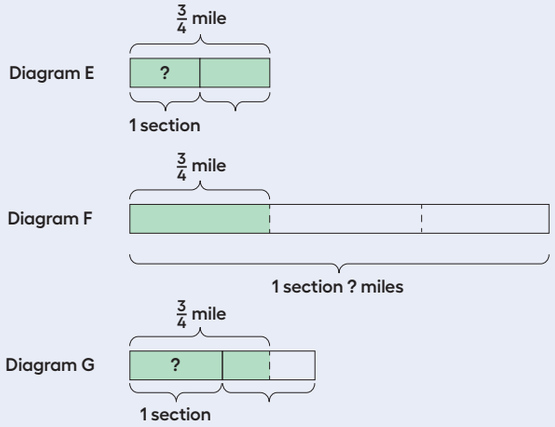
Advances: Speaking, Representing

Student Workbook



Student Task Statement

Here are three diagrams and three descriptions that represent situations about sections of highways.



1. Match each situation to a diagram. Be prepared to explain how you know.

- Priya's class has adopted two equal sections of a highway to keep clean. The combined length is $\frac{3}{4}$ of a mile. How long is each section?

  Diagram **E**

- Lin's class has also adopted some sections of a highway to keep clean. If $1\frac{1}{2}$ sections are $\frac{3}{4}$ mile long, how long is each section?

  Diagram **G**

- A high school adopted a section of highway to keep clean. If $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long, how long is the section?

  Diagram **F**

2. Choose one situation. Write a multiplication equation and a division equation to represent the situation. Then answer the question.

Answers vary based on choice.

- Priya's class:
  - Multiplication equation: $2 \cdot ? = \frac{3}{4}$
  - Division equation: $\frac{3}{4} \div 2 = ?$
  - Answer: $\frac{3}{8}$ mile
- Lin's class:
  - Multiplication equation: $1\frac{1}{2} \cdot ? = \frac{3}{4}$
  - Division equation: $\frac{3}{4} \div 1\frac{1}{2} = ?$
  - Answer: $\frac{1}{2}$ mile
- High school:
  - Multiplication equation: $\frac{1}{3} \cdot ? = \frac{3}{4}$
  - Division equation: $\frac{3}{4} \div \frac{1}{3} = ?$
  - Answer: $\frac{9}{4}$ mile

Activity Synthesis

Display and read the question: "In general, how would you write a multiplication equation and a division equation to represent any situation in this activity?"

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to this question. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- *"___ makes sense, but what do you mean when you say … ?"*
- *"Can you describe that another way?"*
- *"How do you know … ? What else do you know is true?"*

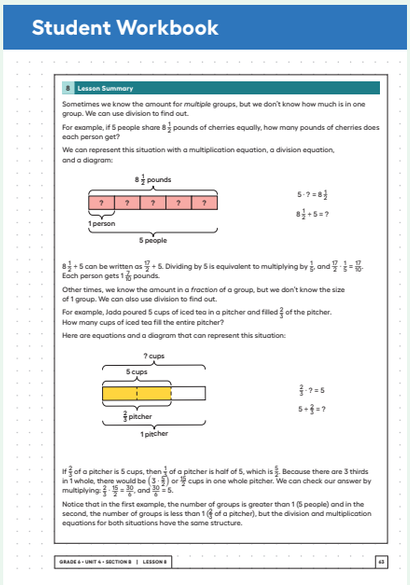
Close the partner conversations, and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Here is an example for a second draft: "For multiplication, I would write: number of sections × length of a section = total length. The length of a section is not known, so I would put a ? there. For division, I would divide the total length by the number of sections to get the length of one section."

After *Stronger and Clearer Each Time*, highlight that we can check if an equation accurately describes a given situation, such as by seeing whether:

- The relationship in the equation matches the relationship in a diagram that represents the same situation.
- The solution to the equation makes sense in the diagram and in terms of the situation.

Student Workbook



Lesson Summary

Sometimes we know the amount for multiple groups, but we don't know how much is in one group. We can use division to find out.

For example, if 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?

We can represent this situation with a multiplication equation, a division equation, and a diagram:

8 $\frac{1}{2}$ pounds

? ? ? ? ?

1 person

5 people

$5 \cdot ? = 8\frac{1}{2}$

$8\frac{1}{2} \div 5 = ?$

$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. Each person gets $1\frac{7}{10}$ pounds.

Other times, we know the amount in *a fraction* of a group, but we don't know the size of 1 group. We can also use division to find out.

For example, Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?

Here are equations and a diagram that can represent this situation:

? cups

5 cups

$\frac{2}{3}$ pitcher

1 pitcher

$\frac{2}{3} \cdot ? = 5$

$5 \div \frac{2}{3} = ?$

If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $(3 \cdot \frac{5}{2})$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both situations have the same structure.

GRADE 6 • UNIT 4 • SECTION B | LESSON 8 43

Lesson Synthesis

Focus the discussion on connecting division and different questions about equal-size groups. Display two questions for all to see:

"How many groups of $\frac{1}{2}$ are in 7?"

"If $\frac{1}{2}$ of a ride is 7 minutes, how long is a ride?"

Give students a minute to think about why both questions can be answered by dividing 7 by $\frac{1}{2}$. Invite students to share their responses. Discuss questions such as:

"What multiplication equation can we write for each question?"

$? \cdot \frac{1}{2} = 7$ and $\frac{1}{2} \cdot ? = 7$

"How can we find the unknown factor in each equation?"

We can divide 7 by $\frac{1}{2}$. We can draw a tape diagram.

"What is the quotient?"

14

"What does the quotient mean for each question?"

In the first case, it is the number of groups. There are 14 groups of $\frac{1}{2}$ in 14. In the second case, it is the size of 1 group. If $\frac{1}{2}$ of a ride is 7 minutes, a whole ride is 14 minutes.

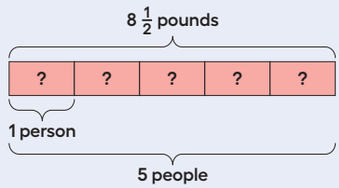
To help students synthesize these meanings of division and the distinction between the two, consider asking students to draw a tape diagram for each interpretation of $7 \div \frac{1}{2}$ and to point out where 14 can be seen in each diagram.

Lesson Summary

Sometimes we know the amount for *multiple* groups, but we don't know how much is in one group. We can use division to find out.

For example, if 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?

We can represent this situation with a multiplication equation, a division equation, and a diagram:



$5 \cdot ? = 8\frac{1}{2}$

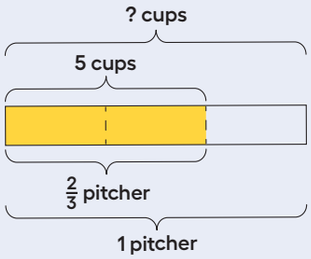
$8\frac{1}{2} \div 5 = ?$

$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. Each person gets $1\frac{7}{10}$ pounds.

Other times, we know the amount in *a fraction* of a group, but we don't know the size of 1 group. We can also use division to find out.

For example, Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?

Here are equations and a diagram that can represent this situation:



$\frac{2}{3} \cdot ? = 5$

$5 \div \frac{2}{3} = ?$

If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $\left(3 \cdot \frac{5}{2}\right)$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both situations have the same structure.

Cool-down

Ice Cubes and Bus Seats

5 min

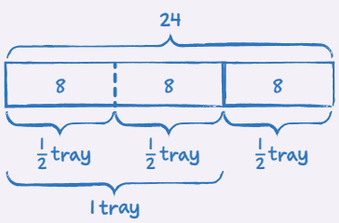
Student Task Statement

Answer each question, and show your reasoning.

1. Kiran filled $1\frac{1}{2}$ ice trays with water and made 24 ice cubes. How many ice cubes are in 1 ice tray?

   16

   Sample reasoning:



   many seats are on the bus?

   60

   Sample reasoning: If $\frac{2}{5}$ of the number of seats is 24, then $\frac{1}{5}$ of it is 12, and all of it is $5 \cdot 12$, which is 60.

Responding To Student Thinking

Points to Emphasize

If students struggle to interpret the quantities in the *Cool-down* in terms of equal groups or to relate the quantities using multiplication or division, focus on making sense of and articulating the relationship between quantities. For example, when discussing the practice problems referred to here, ask students whether each given quantity represents the number of groups (or fraction of a group), the amount in 1 group, or the total amount.

Grade 6, Unit 4, Lesson 8, Practice Problem 2

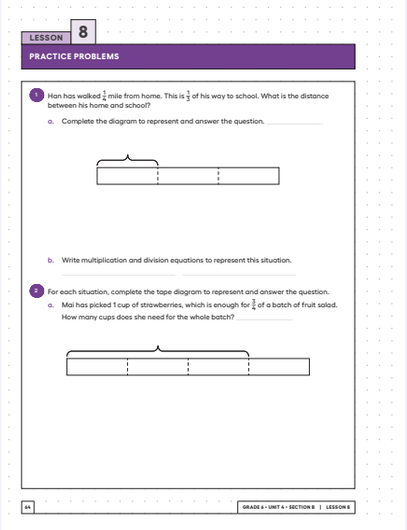
Grade 6, Unit 4, Lesson 8, Practice Problem 3

Lesson 8

Practice Problems

6 Problems

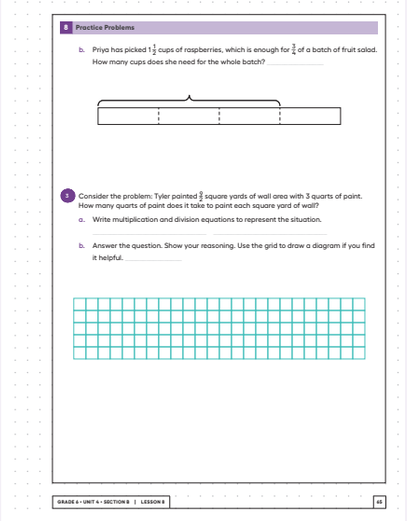
Student Workbook


LESSON 8

PRACTICE PROBLEMS

1. Han has walked $\frac{1}{4}$ mile from home. This is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?
   a. Complete the diagram to represent and answer the question.
   b. Write multiplication and division equations to represent this situation.
2. For each situation, complete the tape diagram to represent and answer the question.
   a. Mai has picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a batch of fruit salad. How many cups does she need for the whole batch?

Student Workbook


8 Practice Problems

b. Priya has picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a batch of fruit salad. How many cups does she need for the whole batch?

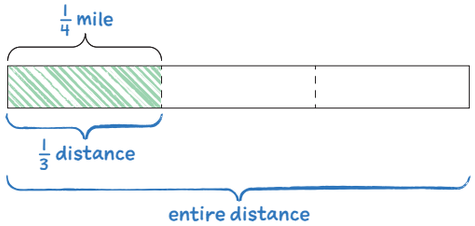
3. Consider the problem: Tyler painted $\frac{2}{3}$ square yards of wall area with 3 quarts of paint. How many quarts of paint does it take to paint each square yard of wall?
   a. Write multiplication and division equations to represent the situation.
   b. Answer the question. Show your reasoning. Use the grid to draw a diagram if you find it helpful.

Problem 1

Han has walked $\frac{1}{4}$ mile from home. This is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?

a. Complete the diagram to represent and answer the question.

Sample response:



The entire distance is $3 \cdot \frac{1}{4}$ or $\frac{3}{4}$ mile.

b. Write multiplication and division equations to represent this situation.

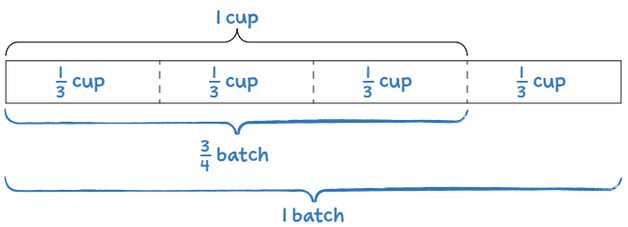
- Multiplication: $\frac{1}{3} \cdot ? = \frac{1}{4}$
- Division: $\frac{1}{4} \div \frac{1}{3} = ?$

Problem 2

For each situation, complete the tape diagram to represent and answer the question.

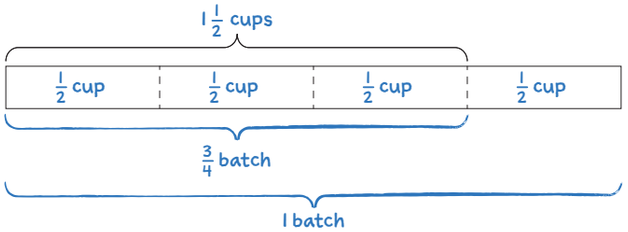
a. Mai has picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a batch of fruit salad. How many cups does she need for the whole batch?

$1\frac{1}{3}$ cups of strawberries



b. Priya has picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a batch of fruit salad. How many cups does she need for the whole batch?

2 cups of raspberries



Problem 3

Consider the problem: Tyler painted $\frac{9}{2}$ square yards of wall area with 3 quarts of paint. How many quarts of paint does it take to paint each square yard of wall?

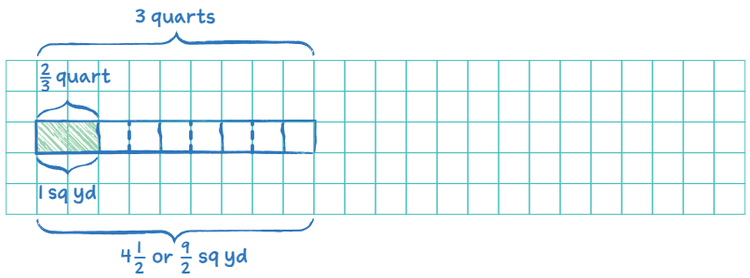
a. Write multiplication and division equations to represent the situation.

- Multiplication: $\frac{9}{2} \cdot ? = 3$
- Division: $3 \div \frac{9}{2} = ?$

b. Answer the question. Show your reasoning. Use the grid to draw a diagram if you find it helpful.

It takes $\frac{2}{3}$ quarts of paint for each square yard of wall.

Sample reasoning:



Problem 4

Lin is working on a project. After working for $2\frac{1}{4}$ hours, she has completed $\frac{3}{5}$ of the project. How long will it take her to complete the entire project?

a. Write multiplication and division equations to represent the situation.

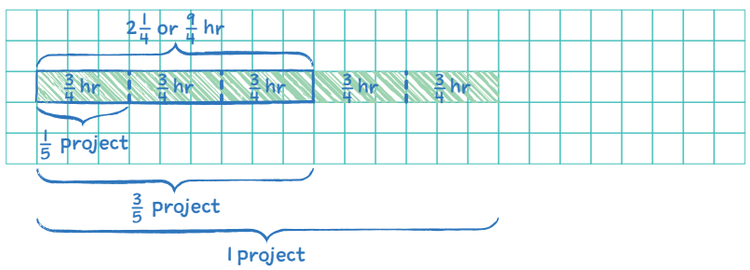
- $? \cdot \frac{3}{5} = 2\frac{1}{4}$ (or equivalent)
- $2\frac{1}{4} \div \frac{3}{5} = ?$

b. Answer the question. Show your reasoning. Use the grid to draw a diagram if you find it helpful.

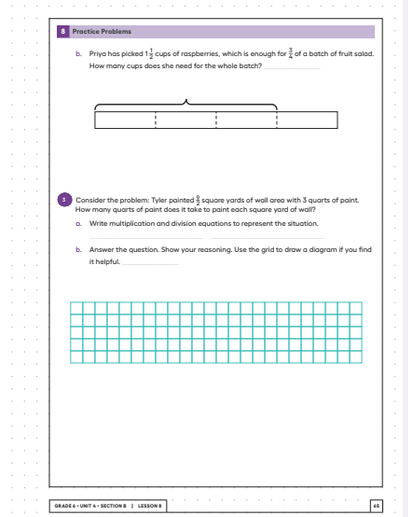
$3\frac{3}{4}$ hours.

Sample reasoning:

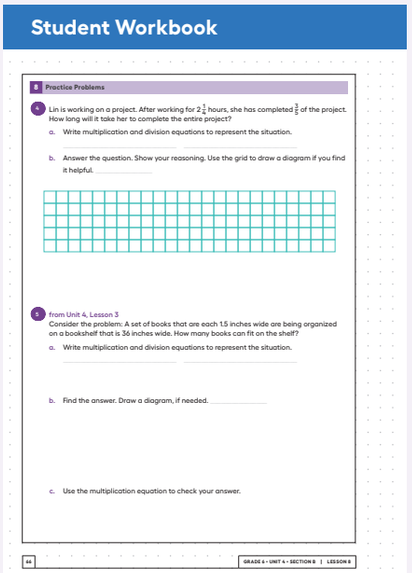
- If $\frac{3}{5}$ of the project took $2\frac{1}{4}$ or $\frac{9}{4}$ hours, each $\frac{1}{5}$ of the project took $\frac{3}{4}$ hour, and the entire project would take $5 \cdot \frac{3}{4}$ or $\frac{15}{4}$ hours, which is $3\frac{3}{4}$ hours.



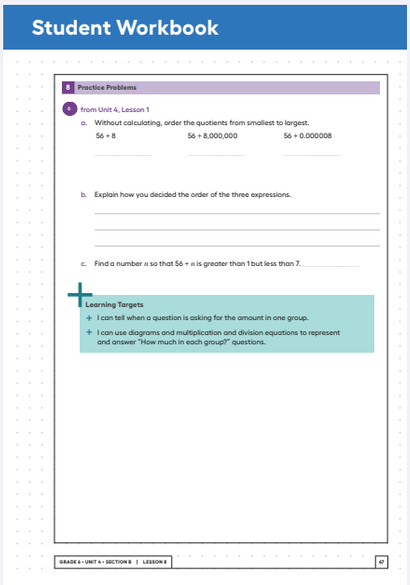
Student Workbook



Student Workbook



Student Workbook



8 Practice Problems

A from Unit 4, Lesson 1

a. Without calculating, order the quotients from smallest to largest.

56 ÷ 8 | 56 ÷ 8,000,000 | 56 ÷ 0.000008

b. Explain how you decided the order of the three expressions.

c. Find a number n so that 56 ÷ n is greater than 1 but less than 7.

Learning Targets

+ I can tell when a question is asking for the amount in one group.
+ I can use diagrams and multiplication and division equations to represent and answer "How much in each group?" questions.

GRADE 4 • UNIT 4 • SECTION B | LESSON 8

47

Problem 5

from Unit 4, Lesson 3

Consider the problem: A set of books that are each 1.5 inches wide are being organized on a bookshelf that is 36 inches wide. How many books can fit on the shelf?

a. Write multiplication and division equations to represent the situation.

- ? · (1.5) = 36 (or equivalent)
- 36 ÷ 1.5 = ? (or equivalent)

b. Find the answer. Draw a diagram, if needed.

24 books can fit on the shelf.

c. Use the multiplication equation to check your answer.

24 · (1.5) = 36

Problem 6

from Unit 4, Lesson 1

a. Without calculating, order the quotients from smallest to largest.

56 ÷ 8 | 56 ÷ 8,000,000 | 56 ÷ 0.000008

56 ÷ 8,000,000, 56 ÷ 8, 56 ÷ 0.000008

b. Explain how you decided the order of the three expressions.

Since the dividend is the same for all three expressions, the larger the divisor, the smaller the quotient.

c. Find a number n so that 56 ÷ n is greater than 1 but less than 7.

Sample response: n = 10 would work since 56 ÷ 8 = 7 and 56 ÷ 56 = 1. Any number between n = 8 and n = 56 would work.