Lesson 6

Estimating Areas

Goals

- Estimate the area of a complex, real-world region, e.g., a state or province, by approximating it with an irregular polygon, and indicate that it is an approximation when expressing the answer (orally and in writing).
- Explain (orally and in writing) how to calculate the area of an irregular polygon by decomposing it.
- Interpret floor plans and maps in order to identify the information needed to calculate area.

Learning Target

I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate.

Lesson Narrative

In this lesson, students practice composing and decomposing irregular regions to calculate their area, in preparation for estimating the area of circles in the next lesson. In the first activity, the region is polygonal, and students can calculate an exact answer for the area of the floorplan. In the second activity, students must approximate the area of the region.

Students use polygons to model regions on a map or floorplan. To complete each task, students need to identify relevant information, choose an appropriate strategy, and make simplifying assumptions. Students also have an opportunity to think about what factors affect the estimates. Students see that the accuracy of an estimate is affected by:

- The nature of the information they have or could obtain.
- The assumptions they make.
- The estimation methods they use.

Student Learning Goal

Let's estimate the areas of weird shapes.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 2)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Instructional Routines

- 5 Practices
- Math Talk
- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits: Lesson
- Geometry toolkits: Activity 1

Instructional Routines

Math Talk

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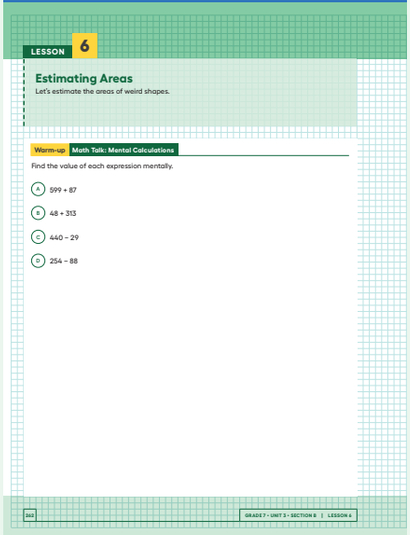
Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 6

Estimating Areas

Let's estimate the areas of weird shapes.

Warm-up Math Talk: Mental Calculations

Find the value of each expression mentally.

A. 599 + 87

B. 48 + 313

C. 440 − 29

D. 254 − 88

Warm-up

Math Talk: Mental Calculations

5 min

Activity Narrative

This *Math Talk* focuses on addition and subtraction problems that can be simplified through compensation. It encourages students to think about adjusting addends by 1 or 2 to mentally solve problems. The strategies elicited here are arithmetic analogues of the composition and decomposition techniques students will use in this lesson to calculate areas of shapes.

As they choose how to rewrite numbers in order to make finding their sum or difference as efficient as possible, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each expression mentally.

A. 599 + 87

686

Sample reasoning: Taking one away from 87 and adding it to 599 turns it into 600 + 86.

B. 48 + 313

361

Sample reasoning: Taking two away from 313 and adding it to 48 turns it into 50 + 311.

C. 440 − 29

411

Sample reasoning: Instead of subtracting 29, it is easier to subtract 30. Since this is subtracting 1 more, 440 needs to be increased by 1, or 441 − 30.

D. 254 − 88

166

Sample reasoning: In order to subtract 88, first subtract 54 and then subtract 34. 254 − 54 = 200 and 200 − 34 = 166.

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

The key takeaway to highlight is the idea of compensation: identifying numbers close to the given ones for which the calculation can be done more efficiently.

- For 599 + 87, because 599 is only one away from 600 (a nice round number), it is natural to change 599 to 600. Adding 1 to 599 means that we need to *subtract* one from 87 to keep the sum the same. So the answer is 600 + 86, or 686.
- For 254 − 88, students may identify 90 or 100 as a nice number near 88 which is simpler to subtract. Subtracting 100 would be subtracting 12 more than 88, so we need to add 12 to 254. So the answer is 266 − 100, or 166. Tell students that in this lesson they are going to use these kinds of strategies with geometric figures to find areas efficiently.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity 1
House Floor Plan

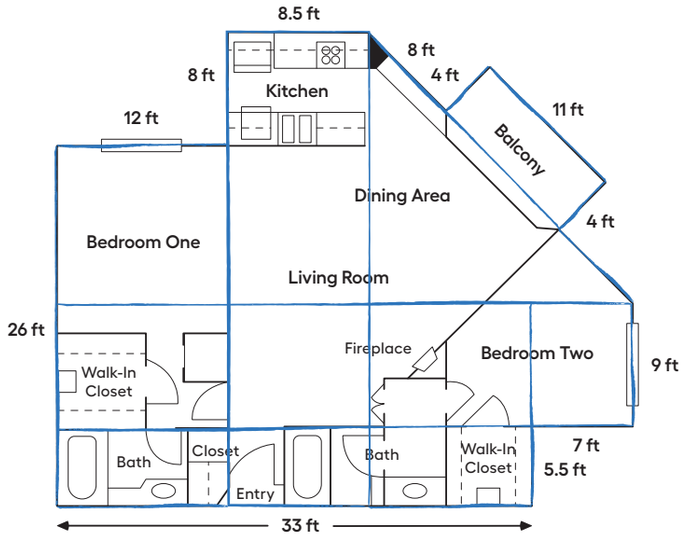
20 min

Activity Narrative

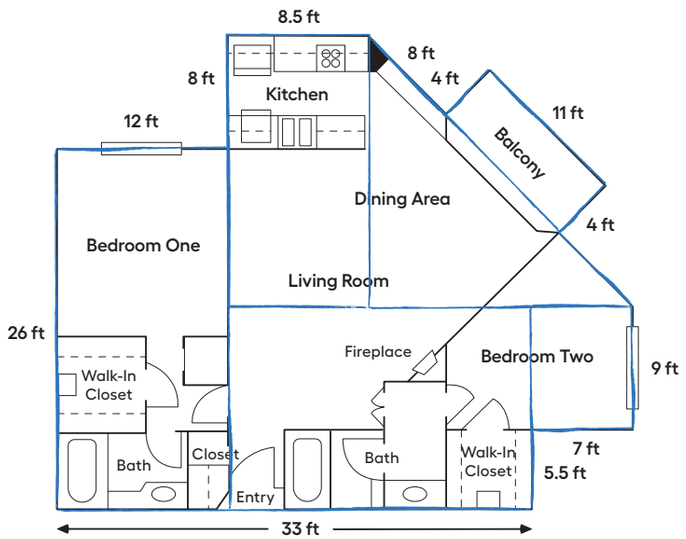
In this activity students calculate the area of an irregular shape presented in a scale drawing. In this case, students can calculate the area exactly by composing and decomposing triangles and parallelograms. As students choose an appropriate way to compose and decompose the floor plan of the house in order to make efficient calculations.

Monitor for students who use these different strategies to determine the area of the floor plan:

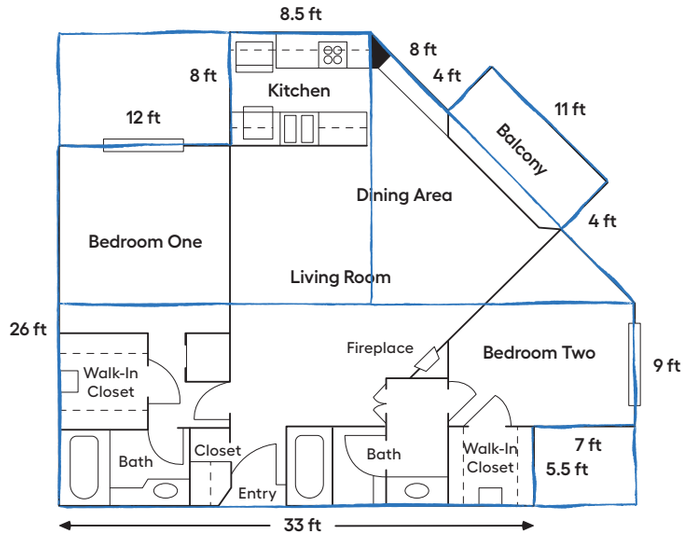
- Decompose the floor plan into lots of small rectangles and a triangle and then add their areas, for example



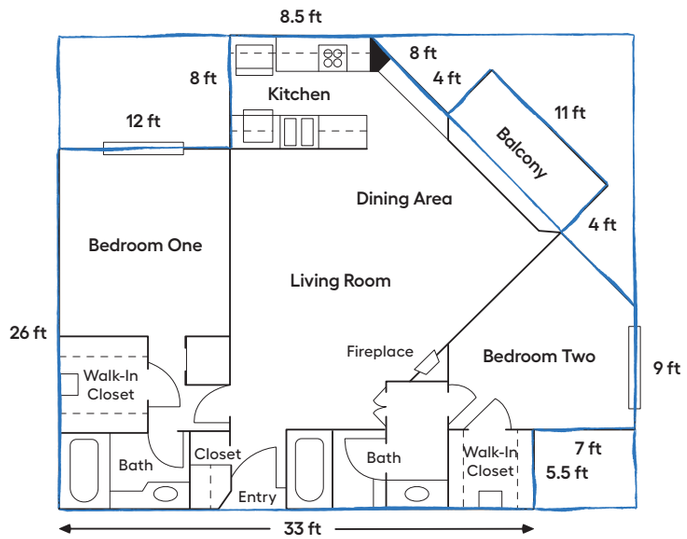
- Decompose the floor plan into a few, larger rectangles and a triangle and then add their areas, for example



- Do a combination of some decomposing and adding with some surrounding and subtracting, for example



- Enclose the floor plan with a large rectangle, subtract the missing pieces, and then add on the balcony



Plan to have students present in this order to support moving them from less efficient to more efficient solution methods.

Launch

Tell students to close their books or devices. Display the image of the floor plan for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image.

Instructional Routines

5 Practices

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Instructional Routines

Notice and Wonder

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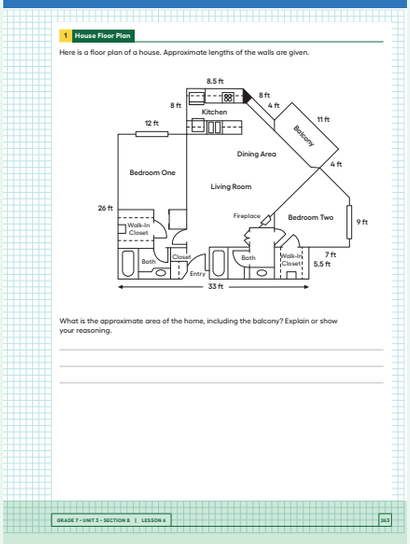
Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.
Activate or supply background knowledge. Provide a reference sheet that includes how to find the area of various shapes such as rectangles, triangles, parallelograms, and trapezoids, for students to use as a reference.

Supports accessibility for: Memory, Organization

Student Workbook


House Floor Plan

Here is a floor plan of a house. Approximate lengths of the walls are given.

8.5 ft, 8 ft, 4 ft, 11 ft, Kitchen, Balcony, 12 ft, Dining Area, Bedroom One, Living Room, 4 ft, 26 ft, Fireplace, Bedroom Two, 9 ft, Walk-in Closet, Bath, Closet, Entry, Bath, Walk-in Closet, 7 ft, 5.5 ft, 33 ft

What is the approximate area of the home, including the balcony? Explain or show your reasoning.

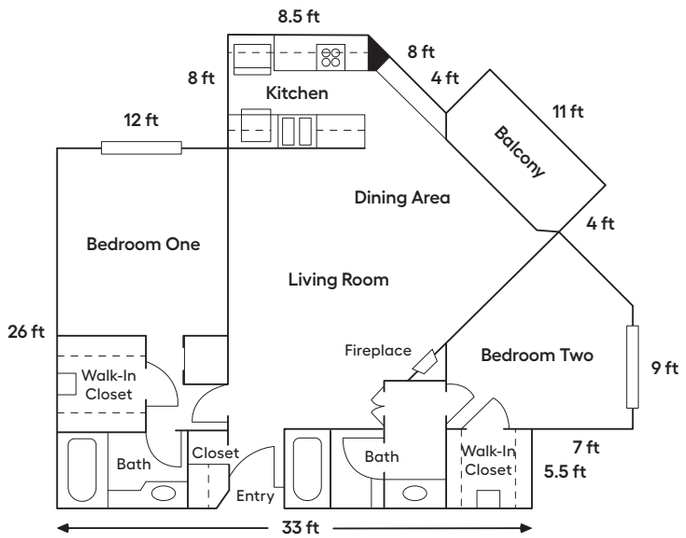
If determining the area of the floor plan does not come up during the conversation, ask students to discuss this idea.

Tell students to open their books or devices and solve the problem. Give students 4–5 minutes of quiet work time followed by whole-class discussion.

Select students who used each strategy described in the activity narrative to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

Here is a floor plan of a house. Approximate lengths of the walls are given. What is the approximate area of the home, including the balcony? Explain or show your reasoning.



about 1,080 ft²

Sample reasoning:

Enclose the house plan with a rectangle. One side of the rectangle is 40 ft because 33 + 7 = 40. The other side is 34 ft because 26 + 8 = 34. So the area of the enclosing rectangle is 1,360 sq ft, because 40 • 34 = 1,360. Next, find the areas of the two smaller rectangles outside the house (at upper left and lower right), then subtract those areas from 1,360 sq ft. The two rectangles are 96 sq ft and 38.5 sq ft because 12 • 8 = 96 and (5.5) • 7 = 38.5 . Next, find the area of the right triangle at upper right, and then subtract it: The triangle includes part of the balcony which will be added back in later. The area of the triangle is about 190 sq ft, because $\frac{1}{2} \cdot (19.5) \cdot (19.5) = 190.125$. The balcony is 44 sq ft, because 4 • 11 = 44. 1360 – 96 – 38.5 – 190 + 44 = 1,079.5. Therefore, the area of the house is about 1,080 sq ft.

Building on Student Thinking

If students struggle getting started finding the area of the floor plan, consider suggesting that they use composition and decomposition to break the floor plan up into familiar shapes whose areas can be calculated.

Activity Synthesis

The purpose of this discussion is to highlight different ways of decomposing the figure and show that they result in the same answer for the area of the region.

Invite previously selected students to share how they calculated the area. Sequence the discussion of the strategies in the order listed in the activity narrative. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"How are the strategies the same? How are they different?"*
- *"Why do the different strategies lead to the same outcome?"*
- *"Are there any benefits or drawbacks to one strategy compared to another?"*

The key takeaway is that any combination of decomposing, adding, enclosing, and subtracting will work to find the area of a complex region, as long as the area of each subpart of the region is accounted for once and only once. The method will be more efficient if students can find ways to combine multiple, smaller sub-regions into fewer, larger sub-regions.

Activity 2

Area of Nevada

10 min

Activity Narrative

In this activity, students use strategies developed in earlier work to estimate the area of the state of Nevada. In this case, the shape is not a polygon, so the area can only be estimated.

First, students identify the information needed to estimate the area of the state from the given map. Next, they choose how to model the region with polygons in order to estimate its area. Monitor for students who:

- Decompose the image of the state into a rectangle and a right triangle, find the area of each, and add the two.
- Enclose the image of the state in a rectangle, find the area of the rectangle, and subtract the area of a right triangle.

These two methods work equally well for this shape. Also, some students may notice and account for the missing area in the southeast corner of the state and others may not.

Instructional Routines

MLR7: Compare and Connect

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Launch

Tell students to close their books or devices (or to keep them closed). Display this map of Nevada that does not have a scale, and invite students to share what they notice and wonder.

Ask students what information they would need to know to calculate the area of Nevada.

the height and width of the state

the height and width of the triangle missing from the southwest corner

the height and width of the approximate rectangle missing from the southeast corner

Instruct students to open their books or devices. Give students 5 minutes of quiet work time followed by partner discussion.

Student Task Statement

Estimate the area of Nevada in square miles.

Explain or show your reasoning.

about 110, 000 square miles

Sample reasoning:

- Enclose Nevada with a 320 mi by 490 mi rectangle, and subtract the area of the right triangle in the lower left corner that has side lengths of 320 mi and 270 mi: $(320 \cdot 490) - \left(\frac{1}{2} \cdot 320 \cdot 270\right) = 113,600$. There is also a small piece missing in the lower right corner, so the area is about 110,000 square miles.
- Decompose Nevada into a rectangle and a right triangle: $(320 \cdot 220) + \left(\frac{1}{2} \cdot 320 \cdot 270\right) = 113,600$. There is also a small piece missing in the lower right corner, so the area is about 110,000 square miles.

Building on Student Thinking

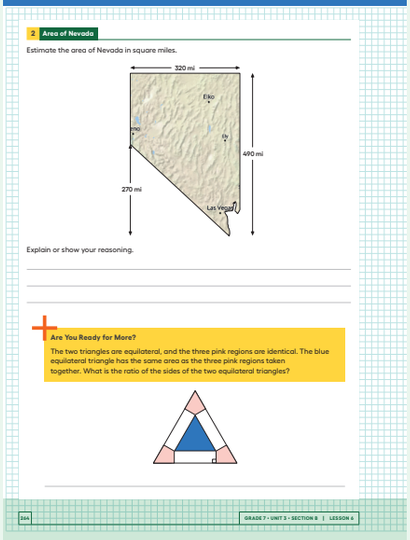
If students decompose the image of the state into a rectangle and triangle, they may use the 270 miles for a side length of the rectangle instead of finding the difference of the 490 miles on the opposite side and 270 miles. Ask them to check their answer with a partner and to reevaluate their calculations.

Access for Students with Diverse Abilities

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "We are trying to …", "How did you get … ?", and "First, I _____ because …"

Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


2 Area of Nevada

Estimate the area of Nevada in square miles.

320 mi
Elko
eno
Ely
490 mi
270 mi
Las Vegas

Explain or show your reasoning.

Are You Ready for More?

The two triangles are equilateral, and the three pink regions are identical. The blue equilateral triangle has the same area as the three pink regions taken together. What is the ratio of the sides of the two equilateral triangles?

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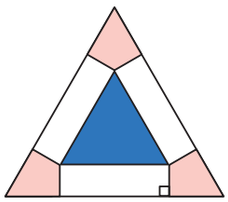
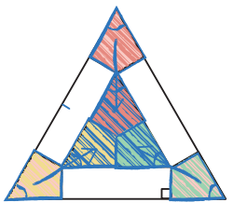


2:1 or 1:2. Since both triangles are equilateral, the three pink regions are identical, and the pink regions have the same area as the blue region, they must fit inside as shown.



Activity Synthesis

The goal of this discussion is for students to understand the distinction between calculating the areas of geometric objects and estimating areas of regions on maps.

First, display these questions for students to discuss with their partners:

- *"How did you make your estimate?"*
- *"If your estimates are not the same, are they close? What accounted for the difference?"*

Ask students how finding the area of Nevada in this activity was the same as finding the area of the floor plan in the previous activity and how it was different.

- One way it was the same was that it was still helpful to decompose the region into rectangles and triangles. Strategies involving addition and strategies involving subtraction were both possible.
- An important difference is that the state is not a polygon. Some of the boundaries are not straight and the overall land is not completely flat. Assuming the state is flat and approximating the boundaries with line segments both lead to some error in the estimate.

Consider telling students that the actual area of Nevada is about 110,560 square miles.

Lesson Synthesis

Share with students:

- *"Today we found the area of complex regions by decomposing them into triangles and rectangles or by approximating them with polygons."*

To review these processes, consider asking students:

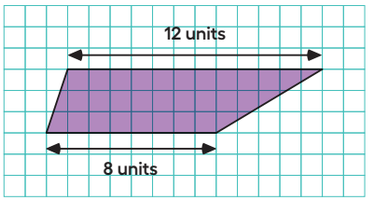
- *"When a shape has straight sides, what can you do to determine its area?"*

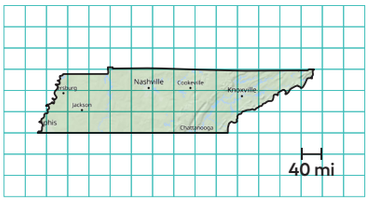
  Decompose it into triangles and rectangles.

- *"When a shape has sides that are not straight, what can you do to determine its area?"*

  Approximate it with polygons.

If desired, use this example to review these concepts.





- *"How could you determine the area of the polygon?"*

  Decompose the region into two right triangles and a rectangle and add their areas.

- *"How could you determine the area of the state of Tennessee?"*

  Approximate the shape with a polygon and calculate its area.

- *"What is different about these two problems?"*

  We can find an exact answer for the area of the polygon, but we can only estimate the area of Tennessee.

- *"What is the same about these two problems?"*

  The overall shape on the grid is roughly the same. We can approximate the state of Tennessee with the given polygon.

Access for Multilingual Learners

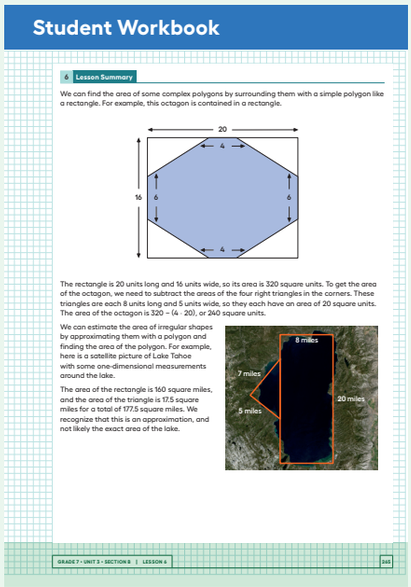
MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Ask:

"Did anyone solve the problem the same way, but would explain it differently?"

"Why do the different approaches lead to the same outcome?"

Advances: Representing, Conversing

Student Workbook



Lesson Summary

We can find the area of some complex polygons by surrounding them with a simple polygon like a rectangle. For example, this octagon is contained in a rectangle.

20
4
16
6
6
4
4

The rectangle is 20 units long and 16 units wide, so its area is 320 square units. To get the area of the octagon, we need to subtract the areas of the four right triangles in the corners. These triangles are each 8 units long and 5 units wide, so they each have an area of 20 square units. The area of the octagon is 320 – (4 · 20), or 240 square units.

We can estimate the area of irregular shapes by approximating them with a polygon and finding the area of the polygon. For example, here is a satellite picture of Lake Tahoe with some one-dimensional measurements around the lake.

8 miles
7 miles
5 miles
20 miles

The area of the rectangle is 160 square miles, and the area of the triangle is 17.5 square miles for a total of 177.5 square miles. We recognize that this is an approximation, and not likely the exact area of the lake.

GRADE 7 • UNIT 3 • SECTION B | LESSON 6 240

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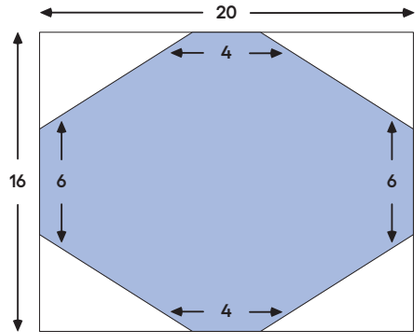

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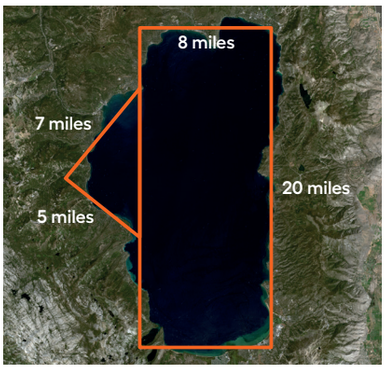


Cool-down

The Area of Alberta

5 min

Launch

Consider telling students that Alberta is a province in Canada.

Student Task Statement

Estimate the area of Alberta in square miles. Show your reasoning.

about 250,000 square miles

Sample response: Alberta can be surrounded with a 410-mile-by-760-mile rectangle with a 290-mile-by-230-mile triangle removed in the lower left corner. The answer has been rounded because the part missing in the lower left is not exactly a triangle.

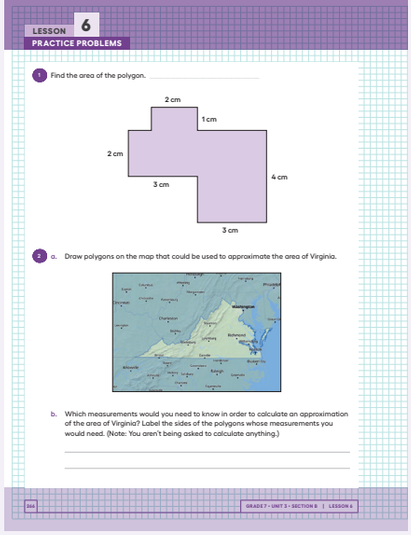
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

5 Problems

Student Workbook


LESSON 6
PRACTICE PROBLEMS

1. Find the area of the polygon.

2 cm, 1 cm, 2 cm, 3 cm, 4 cm, 3 cm

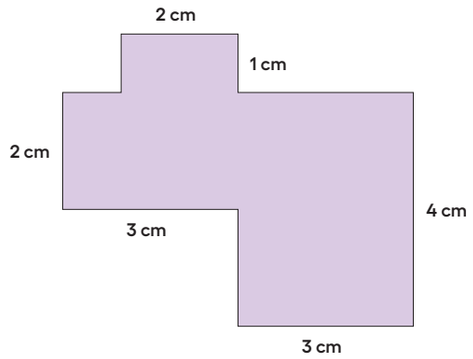
2. a. Draw polygons on the map that could be used to approximate the area of Virginia.

b. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)

GRADE 7 • UNIT 3 • SECTION B | LESSON 6

Problem 1

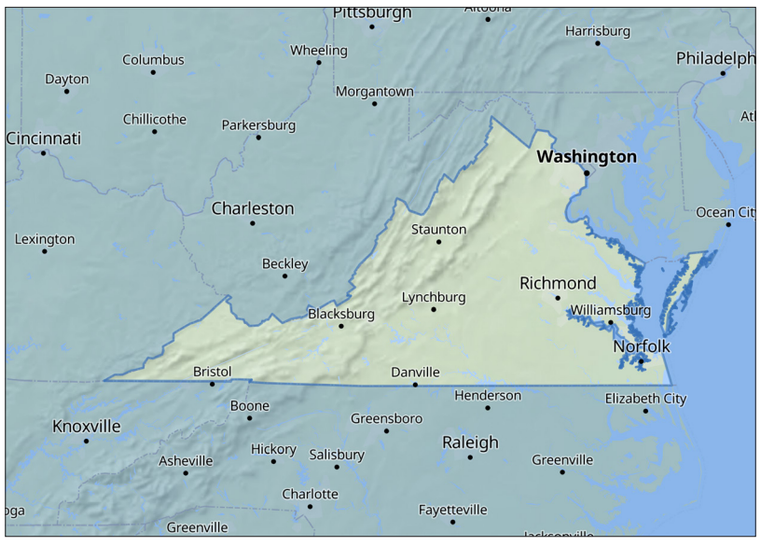
Find the area of the polygon.



20 cm² since the shape can be divided (vertically) into rectangles of area 2, 6, and 12 square centimeters

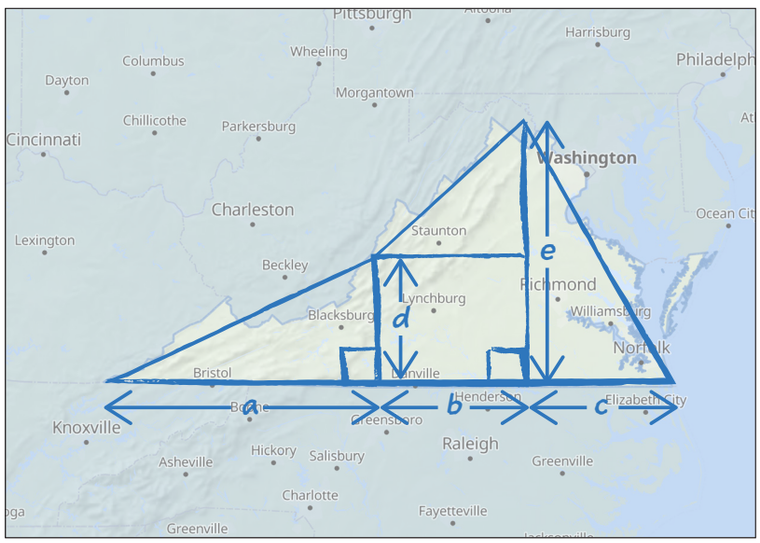
Problem 2

a. Draw polygons on the map that could be used to approximate the area of Virginia.

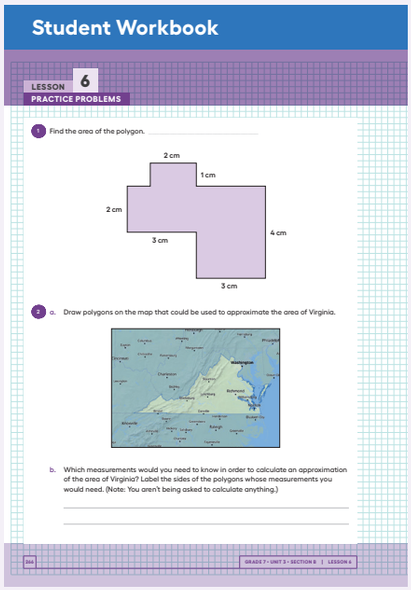


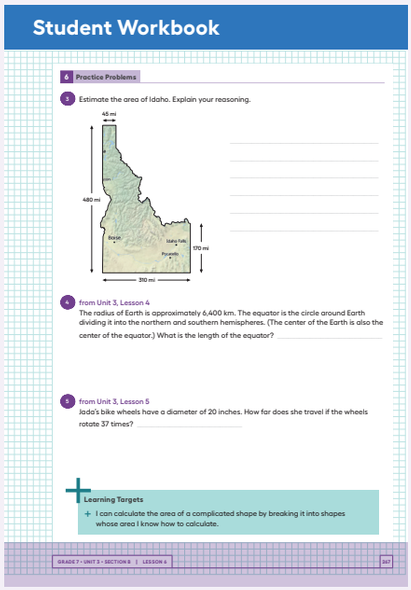
Answers vary. There are many possible ways to draw polygons that would approximate the area of Virginia. One sample response is shown below. Other choices could be made to yield a more or less precise approximation.

b. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)



Answers vary. For rectangles, parallelograms, and triangles, you need both base and height. In the example above, the variables represent measurements needed to find the area of the polygons.




Student Workbook

6 Practice Problems

3 Estimate the area of Idaho. Explain your reasoning.

4 from Unit 3, Lesson 4
The radius of Earth is approximately 6,400 km. The equator is the circle around Earth dividing it into the northern and southern hemispheres. (The center of the Earth is also the center of the equator.) What is the length of the equator?

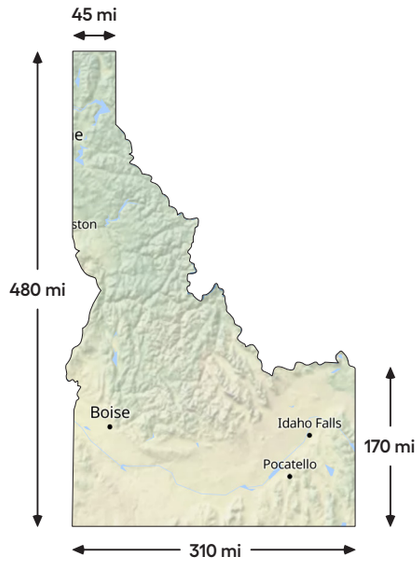
5 from Unit 3, Lesson 5
Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

Learning Targets
+ I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate.

GRADE 7 • UNIT 3 • SECTION B | LESSON 6 247

Problem 3

Estimate the area of Idaho. Explain your reasoning.



Sample response:

about 84,000 square miles

We can approximate the shape of the state with a rectangle on the bottom and a triangle on top.

- The area of the rectangle is $310 \cdot 170$, or 52,700 square miles.
- The base of the triangle is about 200 miles, and the height of the triangle is $480 - 170$, or 310 miles. The area of the triangle is $\frac{1}{2} \cdot 200 \cdot 310$, or 31,000 square miles.

The total area is 52,700 + 31,000, or about 84,000 square miles.

Problem 4

from Unit 3, Lesson 4

The radius of Earth is approximately 6,400 km. The equator is the circle around Earth dividing it into the northern and southern hemispheres. (The center of the Earth is also the center of the equator.) What is the length of the equator?

40,000 km because $6{,}400 \cdot 2 \cdot \pi \approx 40{,}000$

Problem 5

from Unit 3, Lesson 5

Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

about 2,325 in because $37 \cdot 20 \cdot \pi \approx 2324.8$