Lesson 9

Dealing with Negative Numbers

Goals

- Generalize (orally) that doing the same thing to each side of an equation generates an equivalent equation.
- Solve equations of the form $px + q = r$ or $p(x + q) = r$ that involve negative numbers, and explain (orally and in writing) the solution method.

Learning Target

I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions.

Lesson Narrative

In the previous lessons, we used hanger diagrams to reason about ways to approach equations of the form $px + q = r$ or $p(x + q) = r$. This reasoning can be summed up as "do the same thing to each side until the unknown equals a number." Since the things we do to each side of an equation are just arithmetic operations, and the properties of operations extend to negative numbers, this method of solving equations also works when there are negative numbers. This is true even though it doesn't make physical sense to think about weights on hangers representing negative numbers.

After a warm-up designed to remind students about operating on rational numbers, students are asked to solve some straightforward equations involving negative numbers. "Doing the same thing to each side" is presented as a valid method, even though negative numbers are involved. In the last activity, students do the same thing to each side of an equation and their partner tries to guess what they did. The purpose is to communicate that doing the same thing to each side maintains equality even when the moves aren't intended to lead to the equation's solution.

Student Learning Goal

Let's show that doing the same to each side works for negative numbers too.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR7 (Activity 1)
- MLR8 (Activity 2)

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which Three Go Together?

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
10 min	15 min	15 min	10 min	5 min

Assessment

Cool-down (5 min)

Instructional Routines

Which Three Go Together?

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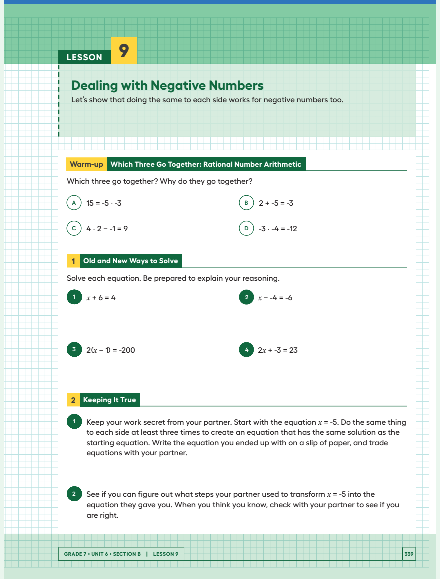
Student Workbook


LESSON 9

Dealing with Negative Numbers

Let's show that doing the same to each side works for negative numbers too.

Warm-up Which Three Go Together: Rational Number Arithmetic

Which three go together? Why do they go together?

A. $15 = -5 \cdot -3$

B. $2 + -5 = -3$

C. $4 \cdot 2 - -1 = 9$

D. $-3 \cdot -4 = -12$

1 Old and New Ways to Solve

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$
2. $x - -4 = -6$
3. $2(x - 1) = -200$
4. $2x + -3 = 23$

2 Keeping It True

1. Keep your work secret from your partner. Start with the equation $x = -5$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.
2. See if you can figure out what steps your partner used to transform $x = -5$ into the equation they gave you. When you think you know, check with your partner to see if you are right.

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Warm-up

Which Three Go Together: Rational Number Arithmetic

10 min

Activity Narrative

This *Warm-up* prompts students to compare four equations. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

During the discussion, listen for strategies for evaluating expressions with rational numbers that will be helpful in the work of this lesson.

Launch

Arrange students in groups of 2–4. Display the equations for all to see.

Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three equations that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $15 = -5 \cdot -3$

B. $2 + -5 = -3$

C. $4 \cdot 2 - -1 = 9$

D. $-3 \cdot -4 = -12$

- A, B, and C go together because they are all true equations.
- A, B, and D go together because they all have the number -3 and they all involve only one operation.
- A, C, and D go together because they all involve multiplication.
- B, C, and D go together because they all have the total on the right side of the equal sign and they all have a negative total.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, especially related to strategies for adding, subtracting, or multiplying signed numbers.

Activity 1
Old and New Ways to Solve

15 min

Activity Narrative

In this activity, students solve four equations. Each solution can be found by asking

"What value would make the equation true?"

However, these equations also present an opportunity to demonstrate that "doing the same thing to each side" still works when there are negative numbers involved. Monitor for students who reason about what value would make the equation true and those who reason by doing the same thing to each side.

Since students have not been shown how to solve equations involving negative numbers, they have to do a notable amount of sense-making to complete the task.

Launch

Give 5–10 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$

   $x = -2$

2. $x - -4 = -6$

   $x = -10$

3. $2(x - 1) = -200$

   $x = -99$

4. $2x + -3 = -23$

   $x = -10$

Activity Synthesis

The goal of this discussion is to connect equation-solving moves with operations on signed numbers. Invite students to share their reasoning for each equation. Consider asking:

"Did anyone solve the equation the same way but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Do you agree or disagree? Why?"

Draw students' attention to the connection between the approaches of "finding the value that makes the equation true" and "doing the same to each side."

Instructional Routines

MLR7: Compare and Connect

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Building on Student Thinking

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations or strategies. Ask,

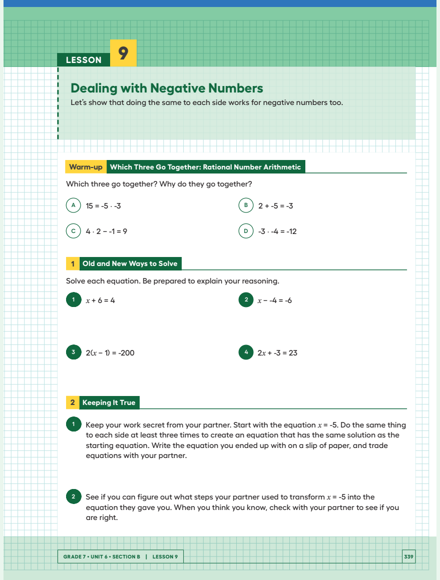
"What do the approaches have in common? How are they different?"

or

"Why do the different approaches lead to the same outcome?"

Advances: Representing, Conversing

Student Workbook


LESSON 9

Dealing with Negative Numbers

Let's show that doing the same to each side works for negative numbers too.

Warm-up Which Three Go Together: Rational Number Arithmetic

Which three go together? Why do they go together?

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C $4 \cdot 2 - -1 = 9$

D $-3 \cdot -4 = -12$

1 Old and New Ways to Solve

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$
2. $x - -4 = -6$
3. $2(x - 1) = -200$
4. $2x + -3 = 23$

2 Keeping It True

1. Keep your work secret from your partner. Start with the equation $x = -5$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.
2. See if you can figure out what steps your partner used to transform $x = -5$ into the equation they gave you. When you think you know, check with your partner to see if you are right.

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Activity 2

Keeping It True

15 min

Activity Narrative

In this activity, students create an equation for their partner by applying a sequence of moves that create an equivalent equation. Their partner tries to guess which moves were made.

To describe moves that could create an equation with the same solutions, students need to attend to precision in the language they use.

Launch

$$x = -6$$
$$x - 3 = -9$$
$$-9 = x - 3$$
$$900 = -100(x - 3)$$
$$900 = (x - 3) \cdot (-100)$$
$$900 = -100x + 300$$

Display this sequence of moves for all to see. Ask students to think about how they know each equation has the same solution as the previous equation. As students share their reasons, write a valid reason next to each new equation. Here's an example of what the reasons might look like:

- Subtract 3 from each side.
- Swap the two sides of the equation.
- Multiply each side by -100.
- Swap the factors -100 and ___. (Apply the commutative property of multiplication.)
- Apply the distributive property.

Arrange students in groups of 2. Explain that they will start with the equation $x = -5$ and use different combinations of things on this list to create new equations with the same solution.

Give them 5–10 minutes to complete the task with their partner.

Student Task Statement

1. Keep your work secret from your partner. Start with the equation $x = -5$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.

   Answers vary.

2. See if you can figure out what steps your partner used to transform $x = -5$ into the equation they gave you. When you think you know, check with your partner to see if you are right.

   Answers vary.

Instructional Routines

MLR8: Discussion Supports

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Access for Multilingual Learners (Activity 2, Task Statement)

MLR8: Discussion Supports. Display sentence frames to support small-group discussion. Examples: "It looks like …" "Another way to look at it is …" "I know ________ because …"

Advances: Speaking, Representing

Access for Students with Diverse Abilities(Activity 2, Task Statement)

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Consider chunking problem 1 by each step. Check in with students to provide feedback and encouragement after each chunk. Be sure that the explanation of each step is mathematically sound.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

Activity Synthesis

Much of the discussion will take place in small groups. Questions for discussion:

- *"Did you have any disagreements, and how did you resolve them?"*
- *"Did anything surprise you? Explain."*
- *"What are some important things to keep in mind when working with negative numbers?"*
- *"Why can you do an operation on both sides of an equation without changing the solution?"*

Lesson Synthesis

Ask students to think of one or two important things they learned in this lesson, and share them with a partner. Points to highlight include:

- Doing the same thing to each side of an equation still keeps the equation balanced, even when there are negative numbers.
- Doing the same thing to each side of an equation still keeps the equation balanced, even when the moves don't get you closer to a solution.

Lesson Summary

To find a solution to some equations, we can just think about what value in place of the variable would make the equation true. Sometimes we also draw diagrams to reason about the solution. Using balanced hanger diagrams helped us understand that doing the same thing to each side of an equation keeps the equation true. So, another way to solve an equation is to perform the same operation on each side in order to get the variable alone on one side.

Doing the same thing to each side of an equation also works when an equation involves negative numbers. Here are some examples of equations that have negative numbers and steps we could take to solve them.

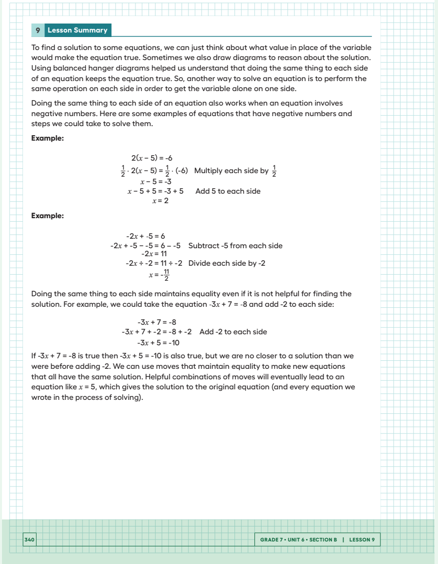
Example:

$$\begin{aligned} 2(x-5) &= -6 \\ \tfrac{1}{2}\cdot 2(x-5) &= \tfrac{1}{2}\cdot(-6) && \text{Multiply each side by } \tfrac{1}{2} \\ x-5 &= -3 \\ x-5+5 &= -3+5 && \text{Add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} -2x + -5 &= 6 \\ -2x + -5 - -5 &= 6 - -5 && \text{Subtract -5 from each side} \\ -2x &= 11 \\ -2x \div -2 &= 11 \div -2 && \text{Divide each side by -2} \\ x &= -\tfrac{11}{2} \end{aligned}$$

Student Workbook


9 Lesson Summary

To find a solution to some equations, we can just think about what value in place of the variable would make the equation true. Sometimes we also draw diagrams to reason about the solution. Using balanced hanger diagrams helped us understand that doing the same thing to each side of an equation keeps the equation true. So, another way to solve an equation is to perform the same operation on each side in order to get the variable alone on one side.

Doing the same thing to each side of an equation also works when an equation involves negative numbers. Here are some examples of equations that have negative numbers and steps we could take to solve them.

Example:

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Example:

$$\begin{aligned} -2x + -5 &= 6 \\ -2x + -5 - -5 &= 6 - -5 && \text{Subtract -5 from each side} \\ -2x &= 11 \\ -2x \div -2 &= 11 \div -2 && \text{Divide each side by -2} \\ x &= -\tfrac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful for finding the solution. For example, we could take the equation $-3x + 7 = -8$ and add -2 to each side:

$$\begin{aligned} -3x + 7 &= -8 \\ -3x + 7 + -2 &= -8 + -2 && \text{Add -2 to each side} \\ -3x + 5 &= -10 \end{aligned}$$

If $-3x + 7 = -8$ is true then $-3x + 5 = -10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x = 5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).

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Responding To Student Thinking

Points to Emphasize
If most students struggle with solving equations, plan to focus on strategies when opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about which method they agree with in this activity: Grade 7, Unit 6, Lesson 10, Activity 1 Analyzing Solution Methods

Doing the same thing to each side maintains equality even if it is not helpful for finding the solution. For example, we could take the equation $-3x + 7 = -8$ and add -2 to each side:

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If $-3x + 7 = -8$ is true then $-3x + 5 = -10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x = 5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).

Cool-down

Solve Two More Equations

5 min

Student Task Statement

Solve each equation. Show your work, or explain your reasoning.

1. $-3x - 5 = 16$

   $x = -7$

   Sample reasoning: After adding 5 to both sides, we get $-3x = 21$. After dividing both sides by -3, we get $x = -7$.

2. $-4(y - 2) = 12$

   $y = -1$

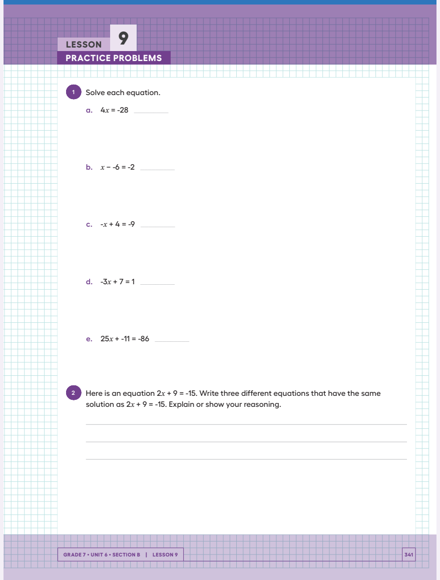
   Sample reasoning: After dividing both sides by -4, we get $y - 2 - 3$. After adding 2 to both sides, we get $y = -1$.

Lesson 9

Practice Problems

5 Problems

Student Workbook


LESSON 9
PRACTICE PROBLEMS

1 Solve each equation.

a. $4x = -28$

b. $x - -6 = -2$

c. $-x + 4 = -9$

d. $-3x + 7 = 1$

e. $25x + -11 = -86$

2 Here is an equation $2x + 9 = -15$. Write three different equations that have the same solution as $2x + 9 = -15$. Explain or show your reasoning.

Problem 1

Solve each equation.

a. $4x = -28$

-7

b. $x - -6 = -2$

-8

c. $-x + 4 = -9$

13

d. $-3x + 7 = 1$

2

e. $25x + -11 = -86$

-3

Problem 2

Here is an equation $2x + 9 = -15$. Write three different equations that have the same solution as $2x + 9 = -15$. Explain or show your reasoning.

Equations vary. Sample equations: $2x + 29 = 5$, $29 + 2x = 5$, $24 + 2x = 0$

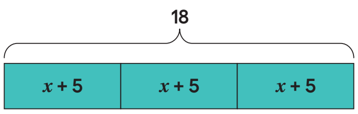
Sample reasoning:

- Start with: $2x + 9 = -15$.
- Add 20 to each side: $2x + 29 = 5$.
- Use the commutative property of addition: $29 + 2x = 5$.
- Subtract 5 from each side: $24 + 2x = 0$.

Problem 3

from Unit 6, Lesson 3

Select **all** the equations that represent the diagram.



A. $x + 5 = 18$

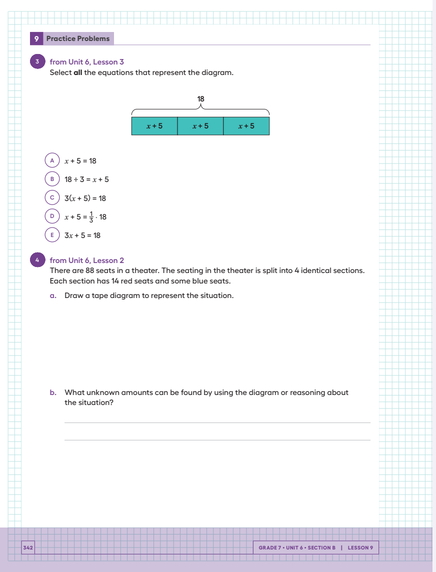
B. $18 \div 3 = x + 5$

C. $3(x + 5) = 18$

D. $x + 5 = \frac{1}{3} \cdot 18$

E. $3x + 5 = 18$

Student Workbook



9 Practice Problems

3 from Unit 6, Lesson 3

Select **all** the equations that represent the diagram.

18

$x+5$ | $x+5$ | $x+5$

A. $x+5=18$

B. $18 \div 3 = x+5$

C. $3(x+5)=18$

D. $x+5=\frac{1}{3} \cdot 18$

E. $3x+5=18$

4 from Unit 6, Lesson 2

There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

a. Draw a tape diagram to represent the situation.

b. What unknown amounts can be found by using the diagram or reasoning about the situation?

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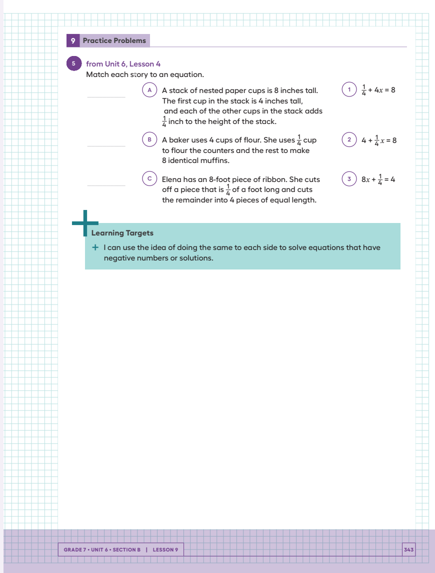
Student Workbook



9 Practice Problems

5 from Unit 6, Lesson 4

Match each story to an equation.

A. A stack of nested paper cups is 8 inches tall. The first cup in the stack is 4 inches tall, and each of the other cups in the stack adds $\frac{1}{4}$ inch to the height of the stack.

B. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 identical muffins.

C. Elena has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into 4 pieces of equal length.

1. $\frac{1}{4}+4x=8$

2. $4+\frac{1}{4}x=8$

3. $8x+\frac{1}{4}=4$

Learning Targets

+ I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions.

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Problem 4

from Unit 6, Lesson 2

There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

a. Draw a tape diagram to represent the situation.

Sample response: A tape diagram with 4 equal parts, each labeled $x + 14$, for a total of 88.

b. What unknown amounts can be found by using the diagram or reasoning about the situation?

Sample response: Each section has 22 seats, of which 8 are blue. There are 32 blue seats and 56 red seats in the theater.

Problem 5

from Unit 6, Lesson 4

Match each story to an equation.

2 **A.** A stack of nested paper cups is 8 inches tall. The first cup in the stack is 4 inches tall, and each of the other cups in the stack adds $\frac{1}{4}$ inch to the height of the stack.

3 **B.** A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 identical muffins.

1 **C.** Elena has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into 4 pieces of equal length.

1. $\frac{1}{4} + 4x = 8$

2. $4 + \frac{1}{4}x = 8$

3. $8x + \frac{1}{4} = 4$