Lesson 11

Slicing Solids

Goals

- Categorize images of planes intersecting pyramids and prisms, and describe (orally) the categories.
- Comprehend that the term "cross section" (in spoken and written language) refers to the two-dimensional face that results from slicing a three-dimensional figure.
- Describe, compare, and contrast (orally and in writing) different cross sections that could result from slicing the same pyramid or prism.

Learning Targets

- I can explain that when a three-dimensional figure is sliced it creates a face that is two dimensional.
- I can picture different cross sections of prisms and pyramids.

Lesson Narrative

This lesson introduces the idea that slicing a three-dimensional figure with a plane results in a two-dimensional cross section. Slicing a fruit or vegetable, dipping the exposed face in paint, and stamping it on a paper helps students make sense of the two-dimensional face that is created by the slice. Given two-dimensional representations of how objects are sliced, students practice visualizing the three-dimensional figures and the resulting cross sections.

Student Learning Goal

Let's see what shapes you get when you slice a three-dimensional object.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- Card Sort
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Take Turns

Required Materials

Materials to Gather

- Fruits or vegetables: Activity 1
- Knife and paint: Activity 1
- Pre-printed cards, cut from copies of the blackline master: Activity 2

Materials to Copy

- Cross Sections Cards (1 copy for every 3 students): Activity 2

Required Preparation

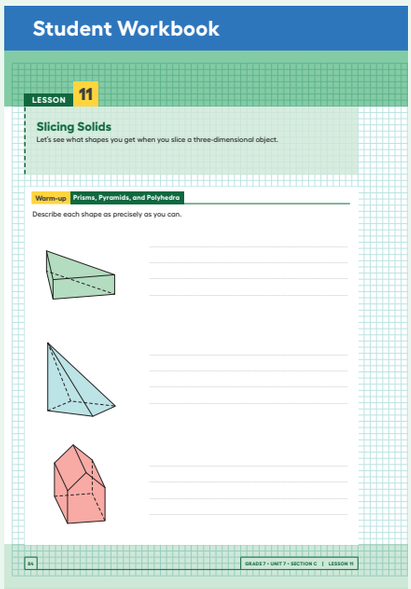
Activity 3:
For the digital version of the activity, acquire devices that can run the applet.

Lesson:
As part of the lesson, the teacher will slice a fruit or vegetable to show a cross section. Recommended are apples, potatoes, or carrots. Tempera or acrylic paint can also be used to stamp the cross section onto paper.

You will need the Cross Section Card Sort blackline master for this lesson. Prepare 1 copy per 3 students, cut the slips, and put each set in an envelope. These slips can be reused from one class to the next. If possible, copy each complete set of cards on a different color of paper, so that a stray card can quickly be put back.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	10 min	20 min	10 min	5 min


Student Workbook

LESSON 11

Slicing Solids

Let's see what shapes you get when you slice a three-dimensional object.

Warm-up Prisms, Pyramids, and Polyhedra

Describe each shape as precisely as you can.

Warm-up

Prisms, Pyramids, and Polyhedra

5 min

Activity Narrative

The purpose of this *Warm-up* is to review important characteristics of prisms, pyramids, and polyhedra. Students should be able to interpret the two-dimensional pictures and three-dimensional objects (understanding that the dotted lines indicate hidden lines) and identify all of the parts of the polyhedra.

Launch

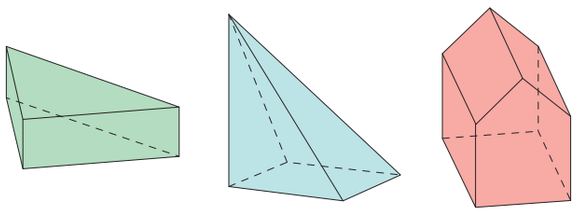
Ask students,

- *"What do you see? Describe the object and its parts as precisely as you can."*

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Describe each shape as precisely as you can.



The first image is a triangular prism with a base that is a right triangle.

The second image is a rectangular pyramid with a vertex that is not centered over the base.

The third image is a prism with a base that is a pentagon.

Activity Synthesis

Ask students to describe each shape. Record and display their responses for all to see. After each student shares, ask the group if they have anything to add before moving on to the next shape.

If not mentioned by students, explain:

- A prism is a polyhedron with two identical polygon bases, connected by rectangles.
- A pyramid is a polyhedron with one polygon base, and all other faces are triangles meeting at a point.
- A pyramid does not have to have the point centered over the base.
- The base of a prism or pyramid does not have to be oriented at the bottom.

Activity 1

What's the Cross Section?

10 min

Activity Narrative

The goal of this activity is to help visualize cross sections of a three-dimensional object. One way to do this is to cut a solid object and use one or both of the pieces to stamp the resulting cross section onto paper. This helps students make sense of the two-dimensional shape that results from cutting a three-dimensional object. During the launch of this activity, students see a demonstration of cutting a fruit or vegetable and are asked to describe the shape of the cross section. Students are then asked to describe the shape of a cross section of a three-dimensional object given to them in the task statement.

As students work on the task, monitor for students who can describe the two-dimensional shape produced from each cross section described.

Launch

Cut the fruit or vegetable so that the cut is in a plane. Some choices: Cut an apple vertically, through the stem. (The cross section will be somewhat heart-shaped, with an indentation.) Cut any fruit or vegetable through the "equator" (The cross section will be a circle.) For a carrot or long potato, cut diagonally (The cross section will be an ellipse, oval, or stretched circle.) Before showing students the cut surface, ask students what shape they think the surface is. Then dip the surface into the paint and stamp on a piece of paper. Then put the cut vegetable back together so that both sides of the cut are painted. Show that the resulting pieces each have a cut surface, and the two surfaces are identical.

Display the paper with the painted cross section for all to see. Invite students to describe the shape of the cross section. Tell students that in this activity they are going to have to describe the shape of something after a cut is made. Give students 2–3 minutes of quiet work time, followed by time to discuss the shapes with their partner. Follow this with a whole-class discussion.

If students *do not* have access to the digital version of the activity, consider projecting the applet and demonstrating for all to see (if possible).

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe cross sections. Display words and phrases, such as "circle," "oval," "both cross sections are rectangles," and "the rectangles are not the same size."

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

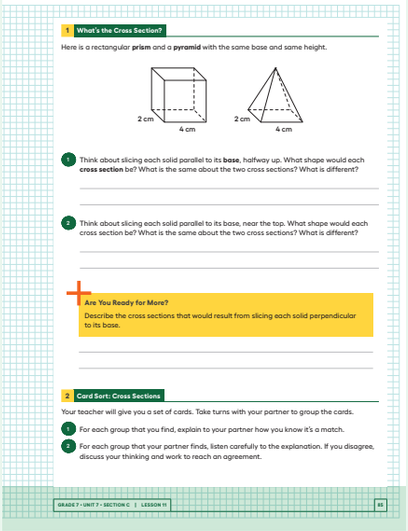
Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display. This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Building on Student Thinking

Some students may struggle to visualize slicing the solids that are shown. It may be helpful to use a three-dimensional model of the rectangular prism and rectangular pyramid to demonstrate where the cut is happening in each question. Building the solids out of salt dough and slicing them with dental floss is another option.

Student Workbook



1 What's the Cross Section?

Here is a rectangular **prism** and a **pyramid** with the same base and same height.

2 cm 4 cm 2 cm 4 cm

1. Think about slicing each solid parallel to its **base**, halfway up. What shape would each **cross section** be? What is the same about the two cross sections? What is different?

2. Think about slicing each solid parallel to its base, near the top. What shape would each cross section be? What is the same about the two cross sections? What is different?

Are You Ready for More?

Describe the cross sections that would result from slicing each solid perpendicular to its base.

2 Card Sort: Cross Sections

Your teacher will give you a set of cards. Take turns with your partner to group the cards.

1. For each group that you find, explain to your partner how you know it's a match.
2. For each group that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

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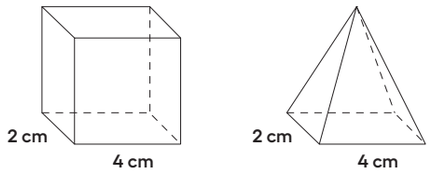
Student Task Statement

Here is a rectangular **prism** and a **pyramid** with the same base and same height.



1. Think about slicing each solid parallel to its **base**, halfway up. What shape would each **cross section** be? What is the same about the two cross sections? What is different?

   Sample response: Both objects have a cross section in the shape of a rectangle. The difference is that the rectangle on the prism is the same size as the base, but the rectangle on the pyramid is smaller.

2. Think about slicing each solid parallel to its base, near the top. What shape would each cross section be? What is the same about the two cross sections? What is different?

   Sample response: Again, both objects have a cross section in the shape of a rectangle. The rectangle on the prism is still the same size as the base, but the rectangle on the pyramid is much smaller.

Are You Ready for More?

Describe the cross sections that would result from slicing each solid perpendicular to its base.

Sample response: Slicing the rectangular prism perpendicular to its base will always result in a rectangular cross section, regardless of the location of the slice.

Sample response: Slicing the rectangular pyramid perpendicular to its base could result in a cross section in the shape of a triangle or a trapezoid, depending on the location of the slice.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their descriptions of the cross sections. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

Select previously identified students to describe the shapes of cross sections of the objects. As students describe the cross sections, consider displaying this applet for all to see:

The Geogebra applet "What's the Cross Section?" is available here: **ilclass.com/l/395072**

If time allows, here are some additional questions for discussion:

- *"How do the cross sections in the different objects compare to one another?"*

  One is a scaled copy of the other.

- *"How do the cross sections in each object compare to its own base?"*

  In the cube, the cross section is the same as the base, in the pyramid, the cross section is a scaled copy of the base.

Instructional Routines

Card Sort

ilclass.com/r/10783726

Please log in to the site before using the QR code or URL.

Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.

Activity 2

Card Sort: Cross Sections

10 min

Activity Narrative

In this partner activity, students take turns visualizing cross sections. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and to critique the reasoning of others.

Launch

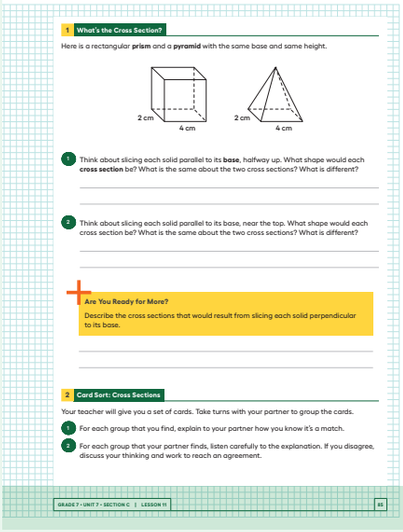
Tell students that the cards contain cross sections of shapes and that they will take turns matching the cards. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Arrange students in groups of 2.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "Both ____ and ____ are alike because ..., ____ and ____ because ..." "____ and ____ are different because ..."
Supports accessibility for: Language; Social-Emotional Skills

Student Workbook



1 What's the Cross Section?

Here is a rectangular **prism** and a **pyramid** with the same base and same height.

2 cm 4 cm 2 cm 4 cm

1. Think about slicing each solid parallel to its **base**, halfway up. What shape would each **cross section** be? What is the same about the two cross sections? What is different?

2. Think about slicing each solid parallel to its base, near the top. What shape would each cross section be? What is the same about the two cross sections? What is different?

Are You Ready for More?

Describe the cross sections that would result from slicing each solid perpendicular to its base.

2 Card Sort: Cross Sections

Your teacher will give you a set of cards. Take turns with your partner to group the cards.

1. For each group that you find, explain to your partner how you know it's a match.
2. For each group that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

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Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to group the cards.

1. For each group that you find, explain to your partner how you know it's a match.
2. For each group that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking and work to reach an agreement.

Sample groupings:

- Cross section is parallel to the base (4 cards), cross section is perpendicular to the base (3 cards), and cross section is oblique to the base (6 cards)
- Cross section is a triangle (5 cards), and cross section is a quadrilateral (8 cards)
- Figure is a rectangular prism (3 cards), figure is a triangle-based pyramid (4 cards), figure is a square-based pyramid (3 cards), and figure is a triangular prism (3 cards)

Activity Synthesis

The purpose of this discussion is to explore the cross sections that are made when shapes are cut in different ways. Select 2–3 groups to share one of their sets of cards and how they grouped the cross sections. Discuss as many different groups of cards as the time allows.

If not mentioned by students, explain that there are a few ways to sort the cards:

- Based on the solid object that is being cut. (rectangular prism, triangular prism, square-based pyramid, triangle-based pyramid)
- Based on the type of cross section made by the cuts. (parallel to the base, perpendicular to the base, oblique to the base)
- Based on the shape of the cross section.

Explain to students that it is possible to create other cross section shapes by cutting these objects in other ways.

Activity 3: Optional
Drawing Cross Sections

20 min

Activity Narrative

There is a digital version of this activity.

In this activity, students are given pictures and descriptions of planes cutting prisms and pyramids. Students are asked to draw cross sections freehand, but this is not a skill that is required in order for students to be able to describe two-dimensional shapes created from cross sections—which is why this is an optional activity. Some pictures are of a moving plane. Students describe how the cross section changes as the plane moves.

In the digital version of the activity, students use an applet to draw cross sections of shapes. The applet allows students to change the plane of the cross section dynamically. This activity works best when students have access to the applet because they will benefit from seeing the cross sections in a dynamic way. If students don't have access, displaying the applet for all to see would be helpful during the synthesis.

Adapted from applets created in GeoGebra by Anthony C.M. OR.

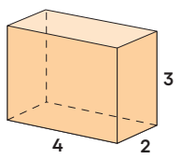
Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time followed by time to discuss shapes of cross sections with a partner. Follow with a whole-class discussion.

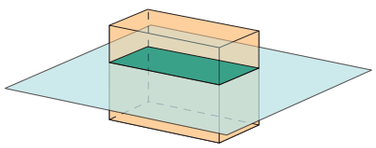
Student Task Statement

Draw and describe each cross section.

1. Here is a picture of a rectangular prism, 4 units by 2 units by 3 units.

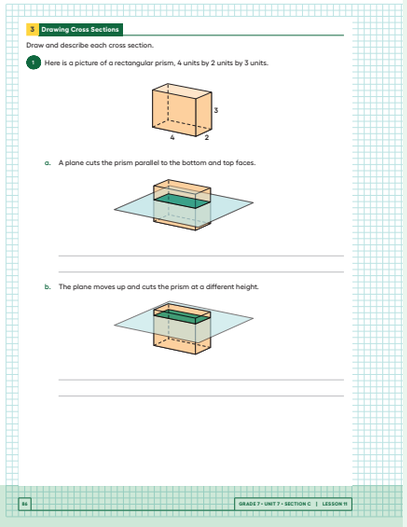


a. A plane cuts the prism parallel to the bottom and top faces.



Drawing of: A rectangle with length 4 and width 2.

Student Workbook


3 Drawing Cross Sections

Draw and describe each cross section.

1 Here is a picture of a rectangular prism, 4 units by 2 units by 3 units.

3
4
2

a. A plane cuts the prism parallel to the bottom and top faces.

b. The plane moves up and cuts the prism at a different height.

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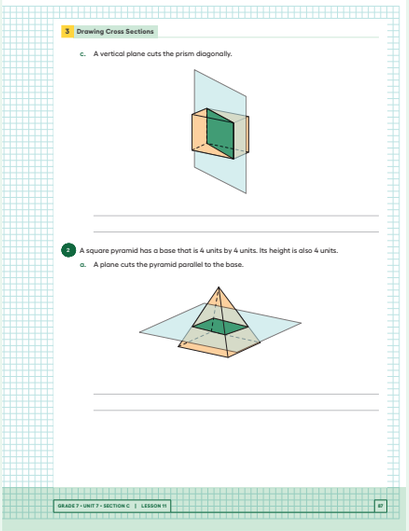
Student Workbook


3 Drawing Cross Sections

c. A vertical plane cuts the prism diagonally.

2 A square pyramid has a base that is 4 units by 4 units. Its height is also 4 units.

a. A plane cuts the pyramid parallel to the base.

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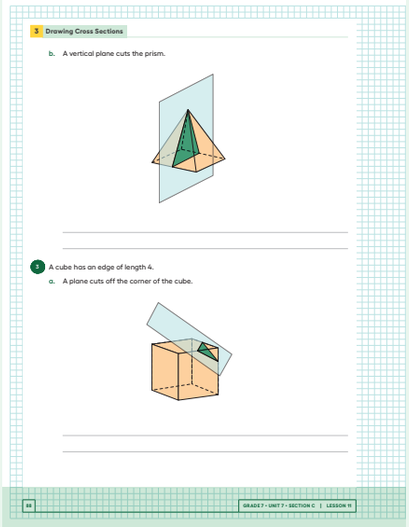
Student Workbook


3 Drawing Cross Sections

b. A vertical plane cuts the prism.

1 A cube has an edge of length 4.

a. A plane cuts off the corner of the cube.

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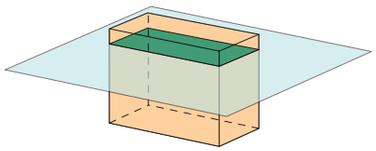
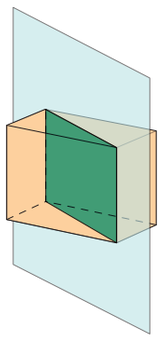
b. The plane moves up and cuts the prism at a different height.



Drawing of: A rectangle with length 4 and width 2.

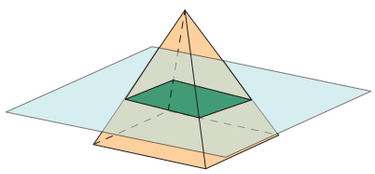
c. A vertical plane cuts the prism diagonally.



Drawing of: A rectangle with height 3 and width the length of the diagonal of the base.

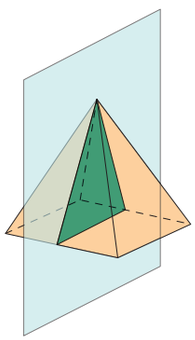
2. A square pyramid has a base that is 4 units by 4 units. Its height is also 4 units.

a. A plane cuts the pyramid parallel to the base.



Drawing of: A square with width and length less than 4.

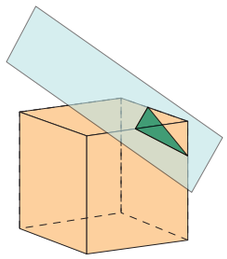
b. A vertical plane cuts the prism.



Drawing of: An isosceles triangle with base 4 and height 4.

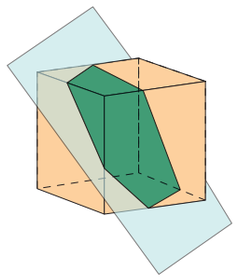
3. A cube has an edge of length 4.

a. A plane cuts off the corner of the cube.



Drawing of: A triangle.

b. The plane moves farther from the corner and makes a cut through the middle of the cube.



Drawing of: A hexagon.

Activity Synthesis

Select students to share their drawings and descriptions. Consider asking some of the following questions:

- *"How did you figure out the shape of the cross section?"*
- *"What helped you visualize the shape?"*
- *"Were any of the shapes you drew here similar to the shapes you described in the previous activity?"*

Lesson Synthesis

Here are some questions for discussion:

- *"What is a cross section?"*

  It is a two-dimensional shape that results from slicing a three-dimensional object.

- *"What are the possible cross sections that can result from a prism that is sliced parallel to its base?"*

  All cross sections will be the same size and shape as the base.

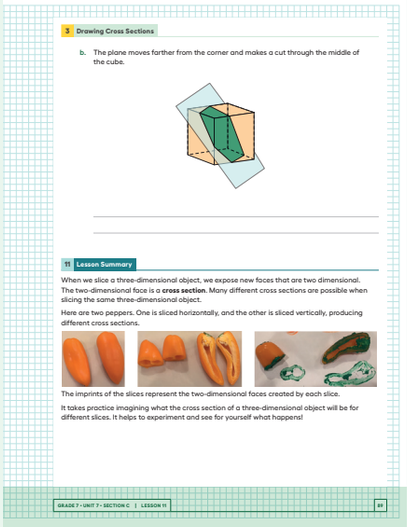
- *"Can cross sections of a prism or pyramid be a different shape than the base? Explain or give an example."*

  Yes, they can be different. For example, slicing off the corner of a cube can result in a triangle.

Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support whole-class discussion. Examples:
"The cross section is ____ because ..."
"A plane cuts ____ as shown by ..."
"The dimensions of a cross section are ____ since ..."
Advances: Speaking, Representing

Student Workbook



3 Drawing Cross Sections

b. The plane moves farther from the corner and makes a cut through the middle of the cube.

11 Lesson Summary

When we slice a three-dimensional object, we expose new faces that are two dimensional. The two-dimensional face is a **cross section**. Many different cross sections are possible when slicing the same three-dimensional object.

Here are two peppers. One is sliced horizontally, and the other is sliced vertically, producing different cross sections.

The imprints of the slices represent the two-dimensional faces created by each slice.

It takes practice imagining what the cross section of a three-dimensional object will be for different slices. It helps to experiment and see for yourself what happens!

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Lesson Summary

When we slice a three-dimensional object, we expose new faces that are two dimensional. The two-dimensional face is a **cross section**. Many different cross sections are possible when slicing the same three-dimensional object.

Here are two peppers. One is sliced horizontally, and the other is sliced vertically, producing different cross sections.



The imprints of the slices represent the two-dimensional faces created by each slice.

It takes practice imagining what the cross section of a three-dimensional object will be for different slices. It helps to experiment and see for yourself what happens!

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- "What norm(s) should stay the way they are?"
- "What norm(s) do you think should be made more clear? How?"
- "What norms are missing that you would add?"
- "What norm(s) should be removed?"

Tell students to first complete the *Cool-down* and then, on the same sheet, to record their answers to one or more of the Math Community questions.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made, and share them in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

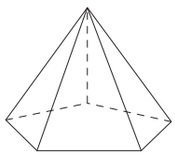
Cool-down

Pentagonal Pyramid

5 min

Student Task Statement

Here is a pyramid with a base that is a pentagon with all sides the same length.



1. Describe the cross section that will result if the pyramid is sliced:
   a. horizontally (parallel to the base).

      Cross section: A pentagon with all sides the same length, but smaller than the base of the pyramid

   b. vertically through the top vertex (perpendicular to the base).

      Cross section: A triangle

2. Describe another way you could slice the pyramid that would result in a different cross section.

   Sample responses:

   - You could slice the pyramid diagonally.
   - You could slice the pyramid vertically but not through the top vertex.

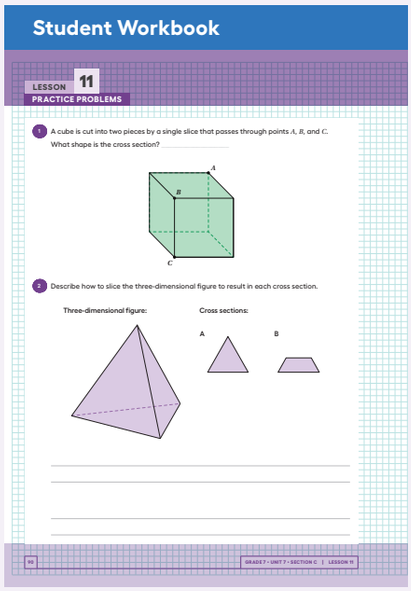
Responding To Student Thinking

Points to Emphasize
If most students struggle with picturing cross sections, focus on what different cross sections would look like when exploring the different shapes in this activity:

Grade 7, Unit 7, Lesson 12, Activity 2
Can You Find the Volume?

Practice Problems

5 Problems


Student Workbook

LESSON 11
PRACTICE PROBLEMS

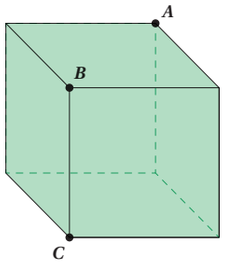
1. A cube is cut into two pieces by a single slice that passes through points A, B, and C. What shape is the cross section?

2. Describe how to slice the three-dimensional figure to result in each cross section.

Three-dimensional figure: Cross sections: A B

Problem 1

A cube is cut into two pieces by a single slice that passes through points A, B, and C. What shape is the cross section?

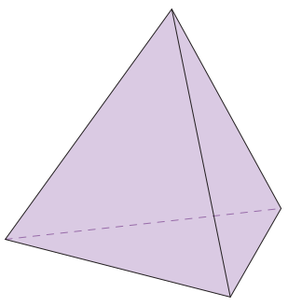


Rectangle

Problem 2

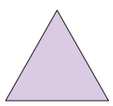
Describe how to slice the three-dimensional figure to result in each cross section.

Three-dimensional figure:



Cross sections:

A

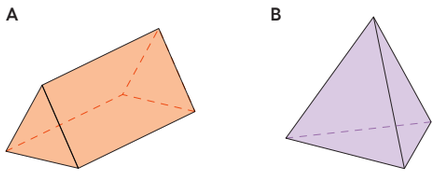


B

To get a cross section that is a triangle, make a slice that is parallel to one of the pyramid's faces. To get a cross section that is a trapezoid, make a slice that is perpendicular to one of the pyramid's faces and that does not pass through the pyramid's opposite vertex.

Problem 3

Here are two three-dimensional figures.



Describe a way to slice one of the figures so that the cross section is a rectangle.

If you slice figure A perpendicular to its triangular bases, the cross section is a rectangle.

Problem 4

from Unit 7, Lesson 2

Each row contains the degree measures of two supplementary angles. Complete the table.

measure of an angle	measure of its supplement
80°	100°
25°	155°
119°	61°
x	180 − x

Problem 5

from Unit 4, Lesson 8

Two months ago, the price, in dollars, of a cell phone was c.

a. Last month, the price of the phone increased by 10%. Write an expression for the price of the phone last month.

1.1c (or equivalent)

Sample reasoning: Because 10% of c is 0.1c, and adding this to c gives 1.1c.

b. This month, the price of the phone decreased by 10%. Write an expression for the price of the phone this month.

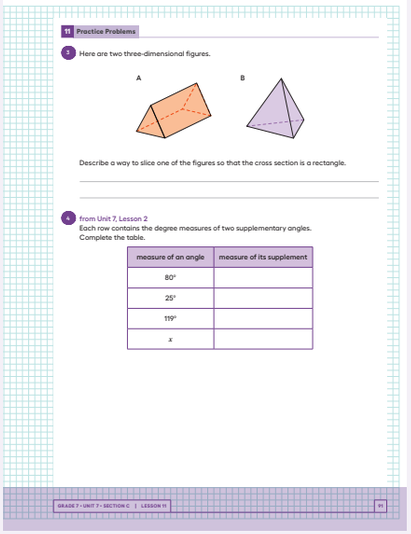
0.99c (or equivalent)

Sample reasoning: Because 10% of 1.1c is 0.11c, and this gives 0.99c when subtracted from 1.1c.

c. Is the price of the phone this month the same as it was two months ago? Explain your reasoning.

No, the phone is a little bit cheaper now than it was a month ago. The 10% discount this month is on the higher price so it is more than the 10% increase a month ago.

Student Workbook


11 Practice Problems

3 Here are two three-dimensional figures.

A B

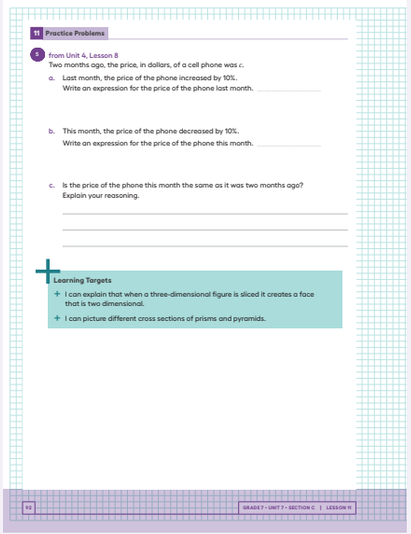
Describe a way to slice one of the figures so that the cross section is a rectangle.

4 from Unit 7, Lesson 2
Each row contains the degree measures of two supplementary angles. Complete the table.

measure of an angle	measure of its supplement
80°	
25°	
119°	
x	

GRADE 7 • UNIT 7 • SECTION C | LESSON 11 P1

Student Workbook


11 Practice Problems

5 from Unit 4, Lesson 8
Two months ago, the price, in dollars, of a cell phone was c.

a. Last month, the price of the phone increased by 10%.
Write an expression for the price of the phone last month.

b. This month, the price of the phone decreased by 10%.
Write an expression for the price of the phone this month.

c. Is the price of the phone this month the same as it was two months ago?
Explain your reasoning.

Learning Targets
+ I can explain that when a three-dimensional figure is sliced it creates a face that is two dimensional.
+ I can picture different cross sections of prisms and pyramids.

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