Lesson 7

Finding Side Lengths of Triangles

Goals

- Comprehend the term "Pythagorean Theorem" (in written and spoken language) as the equation $a^2 + b^2 = c^2$, where a and b are the lengths of the legs, and c is the length of the hypotenuse of a right triangle.
- Describe (orally) patterns in the relationships between the side lengths of triangles.
- Determine the exact side lengths of a triangle in a coordinate grid and express them (in writing) using square root notation.

Learning Target

I can explain what the Pythagorean Theorem says.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- Which Three Go Together?

Lesson Narrative

In this lesson students investigate relationships between the side lengths of right and non-right triangles, leading to the Pythagorean Theorem.

Students begin by comparing aspects of 4 triangles purposefully chosen to prime them to think about patterns. In the next activities, students notice patterns between the side lengths of right and non-right triangles and the areas of squares that share a side length with the right and non-right triangles. Students learn that the **legs** of a right triangle are the sides that make the right angle and the **hypotenuse** is the side of a right triangle that is opposite the right angle. By the end of this lesson, they see that for right triangles only, given legs a and b and hypotenuse c, the side lengths are related by $a^2 + b^2 = c^2$. In the next lesson they will prove this, better known as the **Pythagorean Theorem.**

Student Learning Goal

Let's find triangle side lengths.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	10 min	5 min

Instructional Routines

Which Three Go Together?

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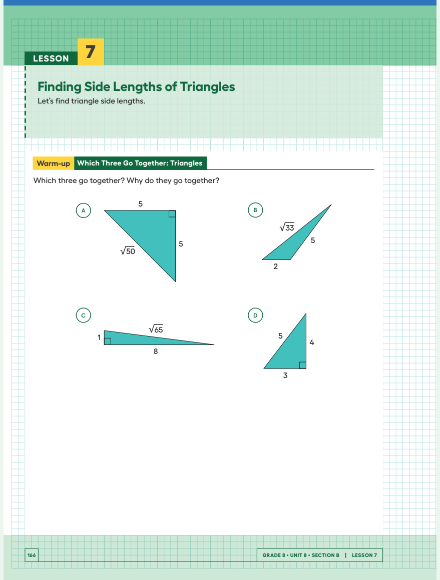
Student Workbook


LESSON 7
Finding Side Lengths of Triangles
Let's find triangle side lengths.
Warm-up Which Three Go Together: Triangles
Which three go together? Why do they go together?

Warm-up

Which Three Go Together: Triangles

5 min

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare features of triangles. In making comparisons, students have a reason to use language precisely. The activity also enables the teacher to hear the terminologies students know and how they talk about characteristics of triangles.

Launch

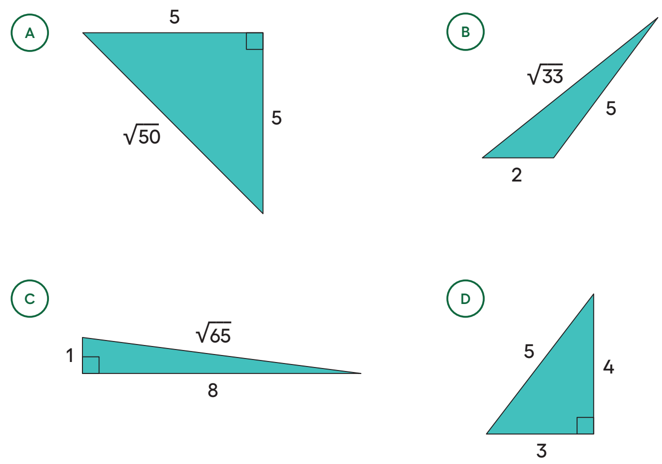
Arrange students in groups of 2–4. Display the figures for all to see.

Give students 1 minute of quiet think time and ask them to indicate when they have noticed three triangles that go together and can explain why.

Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- They all have 1 side that is an irrational number.
- They all have 1 side that is written as a square root.

A, B, and D go together because:

- They all have at least 1 side length that is 5 units.

A, C, and D go together because:

- They are all right triangles.
- They all have 1 right angle.

B, C, and D go together because:

- They are all scalene triangles.
- They all have 3 different side lengths.
- They are not isosceles triangles.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as irrational, scalene, or isosceles, and to clarify their reasoning as needed. Consider asking:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lessons on finding the length of a diagonal of a grid by drawing squares to provide an entry point into this activity.

Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Activity 1
A Table of Triangles

15 min

Activity Narrative

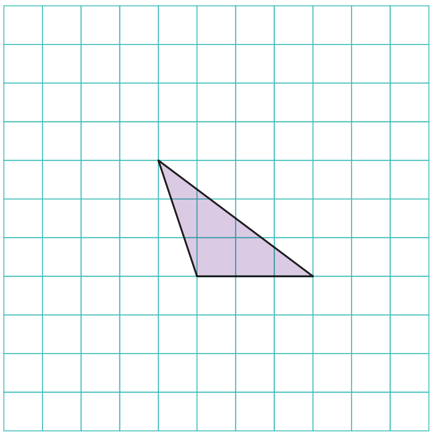
The purpose of this activity is for students to think about the relationships between the squares of the side lengths of triangles as a lead up to the Pythagorean Theorem at the end of this lesson.

Students record both the side lengths and areas of squares and look for patterns. Note that some side lengths are intentionally positioned so that students won't be able to easily draw squares. In these cases the segments are congruent to others whose lengths are already known or could be calculated. Some side lengths lie along gridlines.

Launch

Arrange students in groups of 2–3. Display the image of the triangle on a grid for all to see and ask students to consider how they would find the value of each of the side lengths of the triangle.

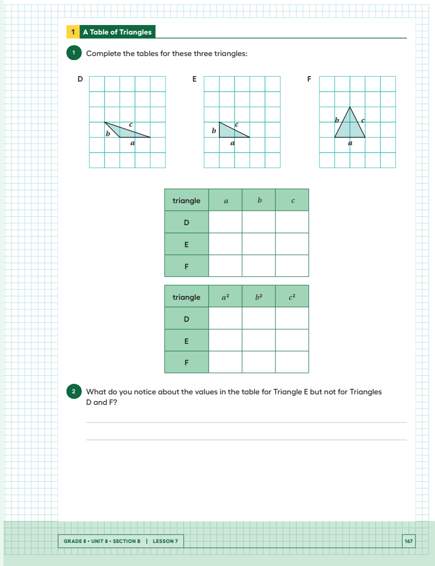
Give students 1–2 minutes of quiet work time and then pause for a brief partner discussion before allowing groups to continue calculating the side lengths.



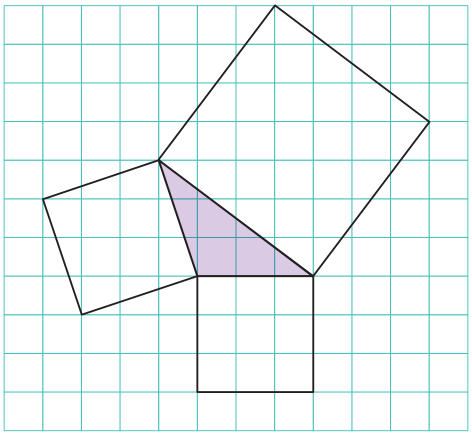
Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.
Collect the language students use to describe what they notice about the values in the table for Triangles E and Q that does not apply to the other triangles. Display words and phrases such as "The values of a^2 and b^2 add up to c^2, which can be restated as "The sum of a^2 and b^2 is c^2." During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Student Workbook



Select 2–3 groups to share their strategies and the values for the side lengths they found ($\sqrt{9} = 3$, $\sqrt{10}$, $\sqrt{25} = 5$). Next, display this image showing the same triangle but with three squares drawn in, each using one of the sides of the triangle as a side length.



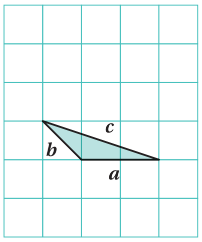
If not mentioned by students, note that for the side that lies along a gridline, the square is not needed to determine the side length. But since $3 = \sqrt{9}$, the strategy of drawing in a square still works.

Encourage groups to divide up the work completing the tables and to discuss strategies for finding the rest of the unknown side lengths.

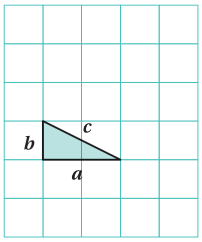
Student Task Statement

1. Complete the tables for these three triangles:

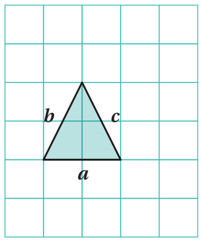
D



E



F



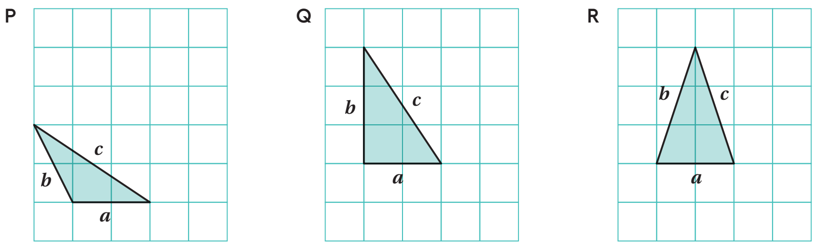
triangle	a	b	c
D	2	$\sqrt{2}$	$\sqrt{10}$
E	2	1	$\sqrt{5}$
F	2	$\sqrt{5}$	$\sqrt{5}$

triangle	a^2	b^2	c^2
D	4	2	10
E	4	1	5
F	4	5	5

2. What do you notice about the values in the table for Triangle E but not for Triangles D and F?

Sample response: The sum of $a^2 = 4$ and $b^2 = 1$ equals $c^2 = 5$.

3. Complete the tables for three more triangles:



triangle	a	b	c
P	2	$\sqrt{5}$	$\sqrt{13}$
Q	2	3	$\sqrt{13}$
R	2	$\sqrt{10}$	$\sqrt{10}$

triangle	a^2	b^2	c^2
P	4	5	13
Q	4	9	13
R	4	10	10

4. What do you notice about the values in the table for Triangle Q but not for Triangles P and R?

Sample response: The sum of $a^2 = 4$ and $b^2 = 9$ equals $c^2 = 13$.

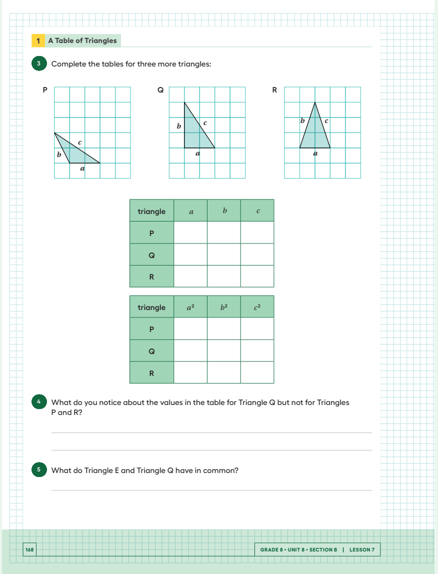
5. What do Triangle E and Triangle Q have in common?

Triangle E and Triangle Q are both right triangles.

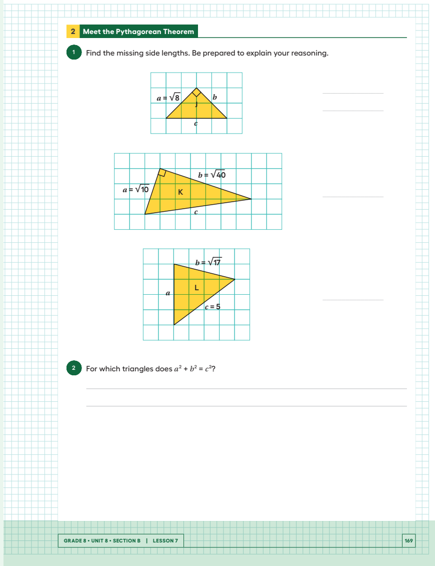
Activity Synthesis

The purpose of this discussion is to allow students to communicate what they noticed using precise mathematical language. Students may notice that $a^2 + b^2 = c^2$ for Triangles E and Q and that they are also right triangles. If so, ask students if any of the other triangles are right triangles. (They are not.) If students do not see these patterns yet, do not give them away. Instead, tell them that they are going to look at more triangles to find a pattern.

Student Workbook



Student Workbook


2 Meet the Pythagorean Theorem

1 Find the missing side lengths. Be prepared to explain your reasoning.

2 For which triangles does $a^2 + b^2 = c^2$?

Activity 2

Meet the Pythagorean Theorem

10 min

Activity Narrative

The goal of this activity is to reach a formal statement of the Pythagorean Theorem. Students will work with a proof of the theorem in a later lesson, so the focus here is on building a basic understanding of what the theorem says and that it is not true for all triangles.

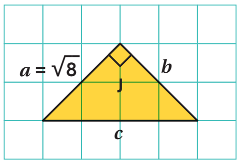
Launch

Arrange students in groups of 2.

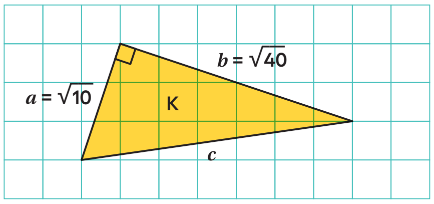
Give students 4 minutes of quiet work time followed by a partner discussion. Then follow with a whole-class discussion.

Student Task Statement

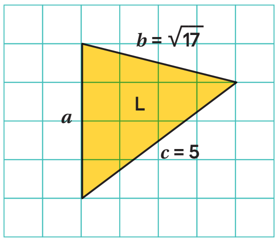
1. Find the missing side lengths. Be prepared to explain your reasoning.



For Triangle J: $c = 4$, which we can see by counting. $b = \sqrt{8}$, which we can see by the fact that it is an isosceles triangle or by drawing a square on the side and finding its area.



For Triangle K: $c = \sqrt{50}$, which we can see by drawing a square on the side and finding its area.



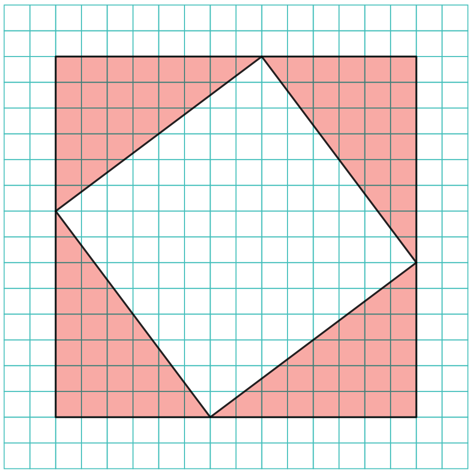
For Triangle L: $a = 4$, which we can see by counting.

2. For which triangles does $a^2 + b^2 = c^2$?

$a^2 + b^2 = c^2$ for Triangles J and K, but not Triangle L.

Are You Ready for More?

If the four shaded triangles in the figure are congruent right triangles, does the inner quadrilateral have to be a square? Explain how you know.



Yes, to prove the inner quadrilateral is a square, we need to show that it has 4 sides of equal length and 4 right angles.

Equal length sides: Since the four triangles are congruent, all four hypotenuses are the same length, so all four sides of the inner quadrilateral have equal length.

Right angles: Each triangle has a 90 degree angle and two other angles. Call the other angles x and y and label them on all of the congruent triangles. Since the angles in a triangle sum to 180 degrees, we know $x + y + 90 = 180$. So $x + y = 90$. There are also straight angles consisting of x, y, and any angle from the inner quadrilateral, which we'll call z. Now $x + y + z = 180$, and $90 + z = 180$, so $z = 90$.

Activity Synthesis

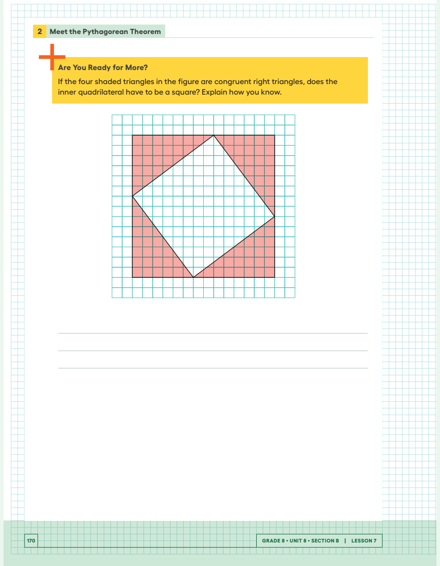
Invite selected students to share their strategies for determining the missing side lengths. Make sure the class comes to an agreement for which triangles $a^2 + b^2 = c^2$. If not brought up in students' explanations, bring to their attention that it works for the two right triangles, Triangles J and K, but not for Triangle L. Then tell students that the **Pythagorean Theorem** says:

If a, b, and c are the sides of a right triangle, where a and b are *legs* and c is the *hypotenuse*, then $a^2 + b^2 = c^2$.

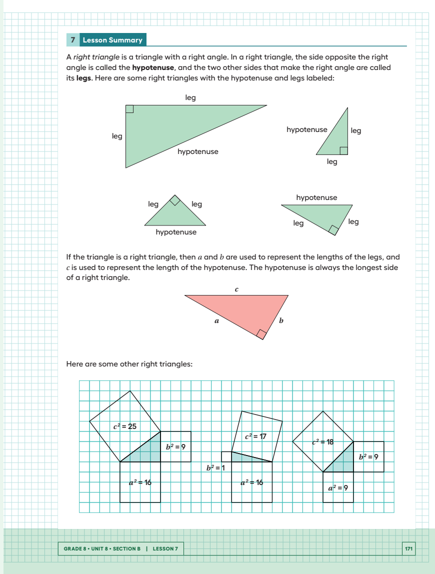
Also give the following definitions to support their understanding of the theorem: The **hypotenuse** is the side opposite the right angle and is the longest side of a right triangle. The **legs** of a right triangle are the sides that make the right angle.

It is important for students to understand that the Pythagorean Theorem only works for right triangles.

Student Workbook



Student Workbook



Lesson Synthesis

The goal of this discussion is for students to describe the relationship they saw and now know as the Pythagorean Theorem. Ask students to describe to a partner the pattern they saw that was true for right triangles. Then invite several students to share their responses and record them for all to see.

Help students use precise mathematical language when describing the relationship. For example, “The sum of the two legs equals the hypotenuse,” could be refined to “The sum of the squares of the legs is equal to the square of the hypotenuse.”

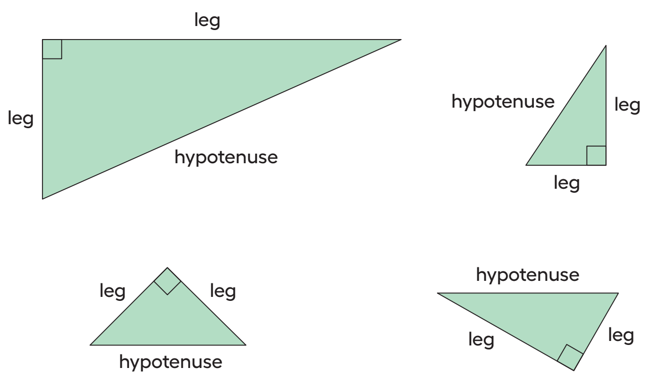
If time allows, display a few right triangles for all to see with labeled side lengths marked a, b, and c. Ask students to check that the Pythagorean Theorem is true for these triangles. As students work, check for common misconceptions:

When calculating $a^2 + b^2$, some students may confuse exponents with multiplying by 2.

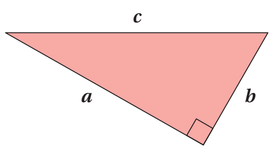
When calculating $a^2 + b^2$, a and b need to be squared first, and then added, rather than adding a and b before squaring.

Lesson Summary

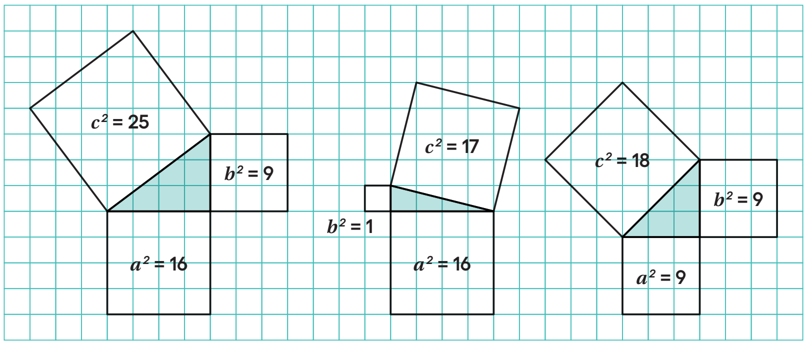
A *right triangle* is a triangle with a right angle. In a right triangle, the side opposite the right angle is called the **hypotenuse**, and the two other sides that make the right angle are called its **legs**. Here are some right triangles with the hypotenuse and legs labeled:



If the triangle is a right triangle, then a and b are used to represent the lengths of the legs, and c is used to represent the length of the hypotenuse. The hypotenuse is always the longest side of a right triangle.

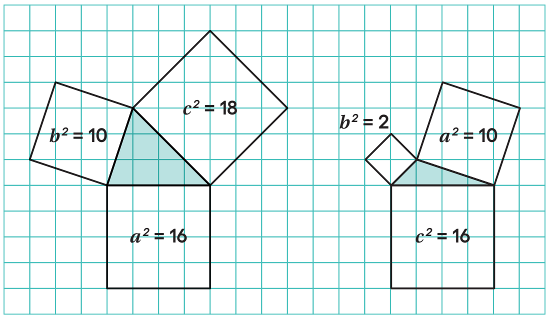


Here are some other right triangles:

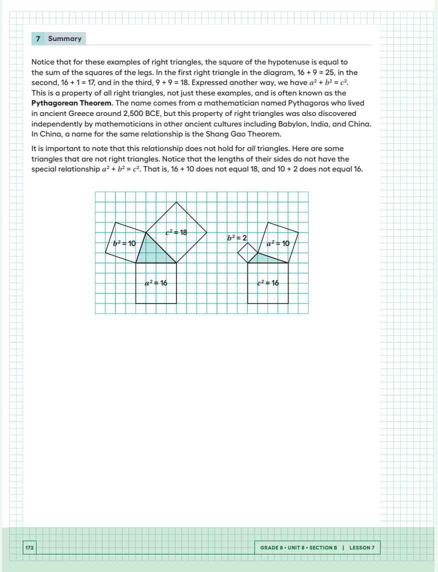


Notice that for these examples of right triangles, the square of the hypotenuse is equal to the sum of the squares of the legs. In the first right triangle in the diagram, 16 + 9 = 25, in the second, 16 + 1 = 17, and in the third, 9 + 9 = 18. Expressed another way, we have $a^2 + b^2 = c^2$ This is a property of all right triangles, not just these examples, and is often known as the **Pythagorean Theorem**. The name comes from a mathematician named Pythagoras who lived in ancient Greece around 2,500 BCE, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures including Babylon, India, and China. In China, a name for the same relationship is the Shang Gao Theorem.

It is important to note that this relationship does not hold for all triangles. Here are some triangles that are not right triangles. Notice that the lengths of their sides do not have the special relationship $a^2 + b^2 = c^2$. That is, 16 + 10 does not equal 18, and 10 + 2 does not equal 16.



Student Workbook



Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

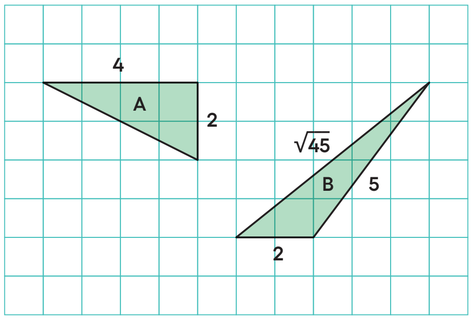
Cool-down

Does a^2 Plus b^2 Equal c^2?

5 min

Student Task Statement

For each of the following triangles, determine if $a^2 + b^2 = c^2$, where a, b, and c are side lengths of the triangle and c is the longest side. Explain how you know.

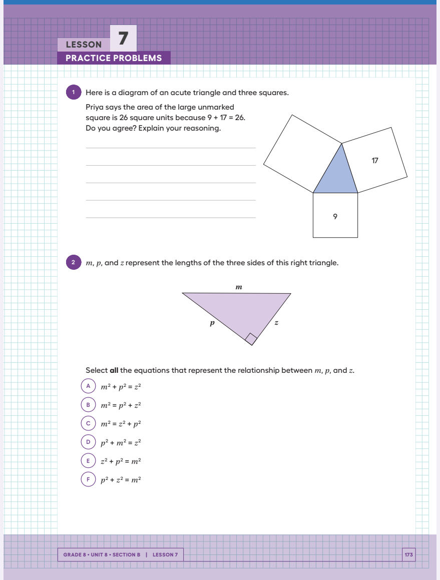


Sample response: It is true for Triangle A because it is a right triangle. You can also find the third side length by constructing a square on it and checking. It is not true for Triangle B. You can see this by squaring the side lengths.

Practice Problems

6 Problems

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1 Here is a diagram of an acute triangle and three squares.

Priya says the area of the large unmarked square is 26 square units because 9 + 17 = 26. Do you agree? Explain your reasoning.

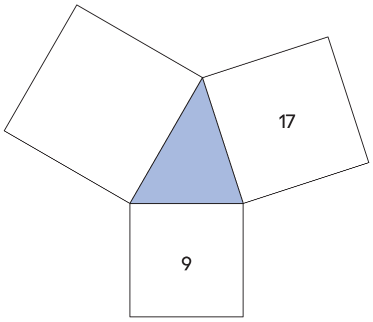
2 m, p, and z represent the lengths of the three sides of this right triangle.

Select **all** the equations that represent the relationship between m, p, and z.

A $m^2 + p^2 = z^2$
B $m^2 = p^2 + z^2$
C $m^2 = z^2 + p^2$
D $p^2 + m^2 = z^2$
E $z^2 + p^2 = m^2$
F $p^2 + z^2 = m^2$

Problem 1

Here is a diagram of an acute triangle and three squares.

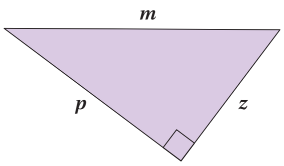


Priya says the area of the large unmarked square is 26 square units because 9 + 17 = 26. Do you agree? Explain your reasoning.

I disagree. Sample reasoning: Priya's pattern only works for right triangles, and this is an acute triangle.

Problem 2

m, p, and z represent the lengths of the three sides of this right triangle.



Select **all** the equations that represent the relationship between m, p, and z.

A. $m^2 + p^2 = z^2$

B. $m^2 = p^2 + z^2$

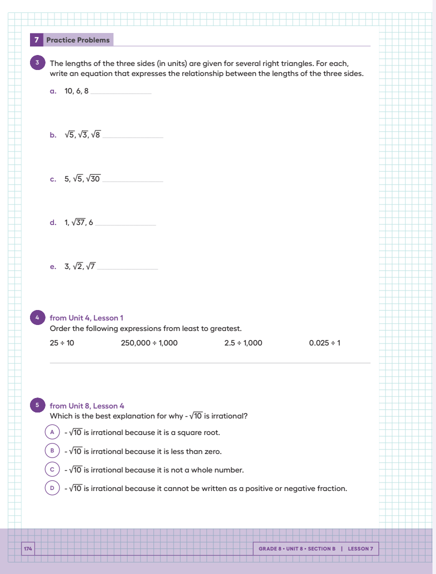
C. $m^2 = z^2 + p^2$

D. $p^2 + m^2 = z^2$

E. $z^2 + p^2 = m^2$

F. $p^2 + z^2 = m^2$

Student Workbook


7 Practice Problems

3 The lengths of the three sides (in units) are given for several right triangles. For each, write an equation that expresses the relationship between the lengths of the three sides.

a. 10, 6, 8

b. $\sqrt{5}, \sqrt{3}, \sqrt{8}$

c. 5, $\sqrt{5}, \sqrt{30}$

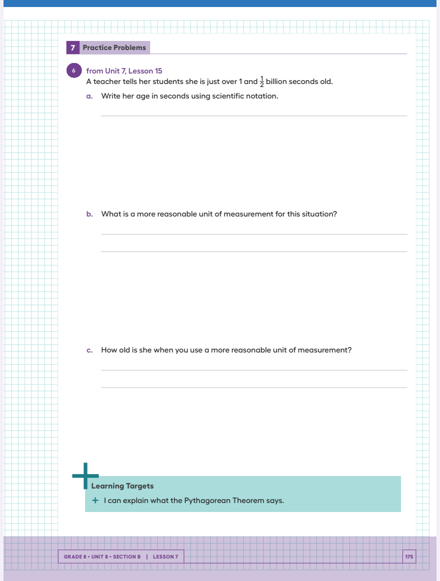
d. 1, $\sqrt{37}$, 6

e. 3, $\sqrt{2}, \sqrt{7}$

4 from Unit 4, Lesson 1
Order the following expressions from least to greatest.
25 ÷ 10 250,000 ÷ 1,000 2.5 ÷ 1,000 0.025 ÷ 1

5 from Unit 8, Lesson 4
Which is the best explanation for why $-\sqrt{10}$ is irrational?
A. $-\sqrt{10}$ is irrational because it is a square root.
B. $-\sqrt{10}$ is irrational because it is less than zero.
C. $-\sqrt{10}$ is irrational because it is not a whole number.
D. $-\sqrt{10}$ is irrational because it cannot be written as a positive or negative fraction.

Student Workbook


7 Practice Problems

6 from Unit 7, Lesson 15
A teacher tells her students she is just over 1 and $\frac{1}{2}$ billion seconds old.
a. Write her age in seconds using scientific notation.
b. What is a more reasonable unit of measurement for this situation?
c. How old is she when you use a more reasonable unit of measurement?

Learning Targets
+ I can explain what the Pythagorean Theorem says.

Problem 3

The lengths of the three sides (in units) are given for several right triangles. For each, write an equation that expresses the relationship between the lengths of the three sides.

a. 10, 6, 8

$6^2 + 8^2 = 10^2$

b. $\sqrt{5}, \sqrt{3}, \sqrt{8}$

$\sqrt{5}^2 + \sqrt{3}^2 = \sqrt{8}^2$

c. 5, $\sqrt{5}, \sqrt{30}$

$5^2 + \sqrt{5}^2 = \sqrt{30}^2$

d. 1, $\sqrt{37}$, 6

$1^2 + 6^2 = \sqrt{37}^2$

e. 3, $\sqrt{2}, \sqrt{7}$

$\sqrt{2}^2 + \sqrt{7}^2 = 3^2$

Problem 4

from Unit 4, Lesson 1

Order the following expressions from least to greatest.

25 ÷ 10	250,000 ÷ 1,000	2.5 ÷ 1,000	0.025 ÷ 1
2.5 ÷ 1,000	0.025 ÷ 1	25 ÷ 10	250,000 ÷ 1,000

Problem 5

from Unit 8, Lesson 4

Which is the best explanation for why $-\sqrt{10}$ is irrational?

A. $-\sqrt{10}$ is irrational because it is a square root.

B. $-\sqrt{10}$ is irrational because it is less than zero.

C. $-\sqrt{10}$ is irrational because it is not a whole number.

D. $-\sqrt{10}$ is irrational because it cannot be written as a positive or negative fraction.

Problem 6

from Unit 7, Lesson 15

A teacher tells her students she is just over 1 and $\frac{1}{2}$ billion seconds old.

Sample responses:

a. Write her age in seconds using scientific notation.

1.5×10^9

b. What is a more reasonable unit of measurement for this situation?

Years

c. How old is she when you use a more reasonable unit of measurement?

She is about 48 years old. There are 31,536,000 seconds in a year.
$1.5 \times 10^9 \div 31{,}536{,}000$ is about 47.6.