Lesson 7

No Bending or Stretching

Goals

- Comprehend the phrase "rigid transformation" to refer to a transformation where all pairs of corresponding distances and corresponding angle measures in the figure and its image are the same.
- Draw and label a diagram of the image of a polygon under a rigid transformation, including calculating side lengths and angle measures.
- Identify (orally and in writing) a rigid transformation using a drawing of a figure and its image.

Learning Target

I can describe the effects of a rigid transformation on the lengths and angles in a polygon.

Access for Students with Diverse Abilities

- Action and Expression (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)
- MLR2: Collect and Display (Activity 2)

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Lesson Narrative

In this lesson, students compare the side lengths and angle measures of a figure and its image under a translation, rotation, or reflection. They observe that the **corresponding** sides are the same length and the corresponding angles have the same measure, and learn that a transformation with this property is called a **rigid transformation**. In order to observe this property, students create a measuring tool using available materials. Then, students identify which figure is the image of an original under a rigid transformation and make arguments to explain their reasoning.

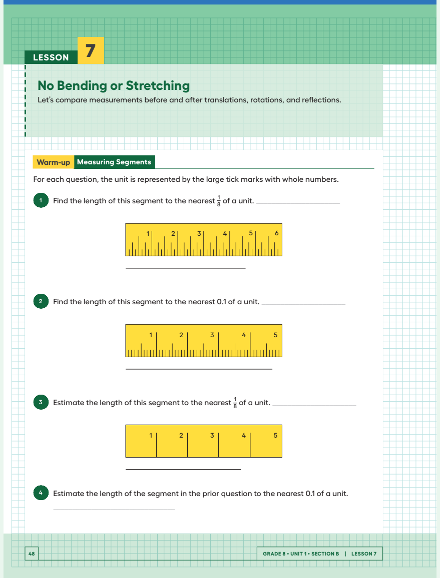
Student Learning Goal

Let's compare measurements before and after translations, rotations, and reflections.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	10 min	5 min

Student Workbook


LESSON 7

No Bending or Stretching

Let's compare measurements before and after translations, rotations, and reflections.

Warm-up Measuring Segments

For each question, the unit is represented by the large tick marks with whole numbers.

1 Find the length of this segment to the nearest $\frac{1}{8}$ of a unit.

2 Find the length of this segment to the nearest 0.1 of a unit.

3 Estimate the length of this segment to the nearest $\frac{1}{8}$ of a unit.

4 Estimate the length of the segment in the prior question to the nearest 0.1 of a unit.

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Building on Student Thinking

Students may struggle with the ruler that is not pre-partitioned into fractional units. Encourage these students to use what they know about eighths and tenths to partition the ruler and estimate their answer.

Warm-up
Measuring Segments

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to consider different aspects of making and recording accurate measurements. It is important to highlight the fractional markings and fraction and decimal equivalents used as students explain how they determined the length of the segment.

Monitor for students who use what they know about eighths and tenths to partition the last ruler and estimate their answer.

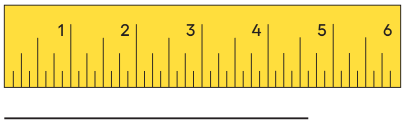
Launch

Give students 2 minutes of quiet work time followed by whole-class discussion.

Student Task Statement

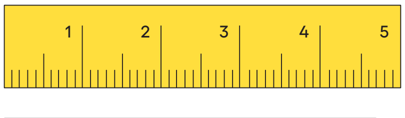
For each question, the unit is represented by the large tick marks with whole numbers.

1. Find the length of this segment to the nearest $\frac{1}{8}$ of a unit.



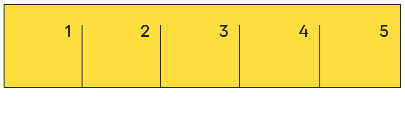
$4\frac{5}{8}$ units

2. Find the length of this segment to the nearest 0.1 of a unit.



4.7 units

3. Estimate the length of this segment to the nearest $\frac{1}{8}$ of a unit.



$3\frac{3}{4}$ (or $3\frac{6}{8}$ units)

4. Estimate the length of the segment in the prior question to the nearest 0.1 of a unit.

3.7 units (or 3.8 units)

Activity Synthesis

Invite students to share their responses and record them for all to see. Ask the class if they agree or disagree with each response. When there is a disagreement, have students discuss possible reasons for the different measurements.

Students are likely to have different answers for their measure of the third segment. The ruler shown is not as accurate as the question requires as it has not been pre-partitioned into fractional units. Ask 2–3 students with different answers to share their strategies for measuring the third segment. There will be opportunities for students to use measuring strategies later in this lesson.

Activity 1

Sides and Angles

15 min

Activity Narrative

The purpose of this activity is for students to see that translations, rotations, and reflections preserve side lengths and angle measures. Students can use tracing paper to help them draw the figures and make observations. While the grid helps measure lengths of horizontal and vertical segments, students may need more guidance when asked to measure diagonal lengths. It is important in the launch to elicit strategies from students to either use tracing paper or an index card to mark off unit lengths using the grid as they use tools strategically.

Since students are creating their own measuring tool, they can only give an estimate, and some flexibility should be allowed in the response. During the discussion, highlight different reasonable answers which are not whole numbers that students find for the lengths.

Monitor for students who use corresponding side lengths and angle measures as well as students who estimate the side lengths using tracing paper or an index card to share.

Launch

Provide students with access to tracing paper or index cards. Display the image of Pentagon C for all to see.

In order for students to measure the side lengths, students will need to create a ruler. Ask students,

"How can you create a ruler to measure the side lengths on this pentagon?"

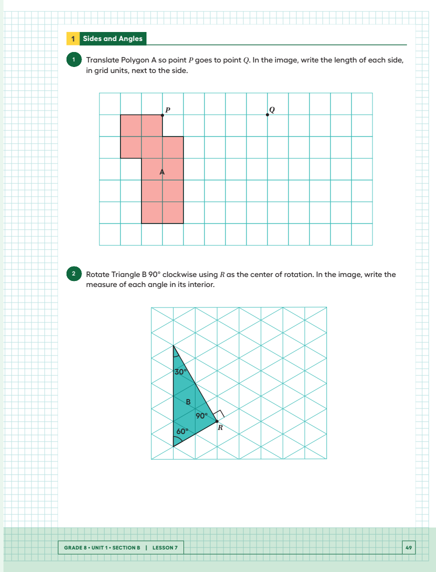
Mark the lengths of the grid lines, then fold the index card to get markings like halves, fourths, and eighths.

If students ask why they need to create their own ruler, point out that they are measuring in grid length units, which may not match up with inches or centimeters.

Give students 5–8 minutes of quiet work time.

Select students with different strategies, such as those described in the *Activity Narrative,* to share later.

Student Workbook


1 Sides and Angles

1 Translate Polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.

2 Rotate Triangle B 90° clockwise using R as the center of rotation. In the image, write the measure of each angle in its interior.

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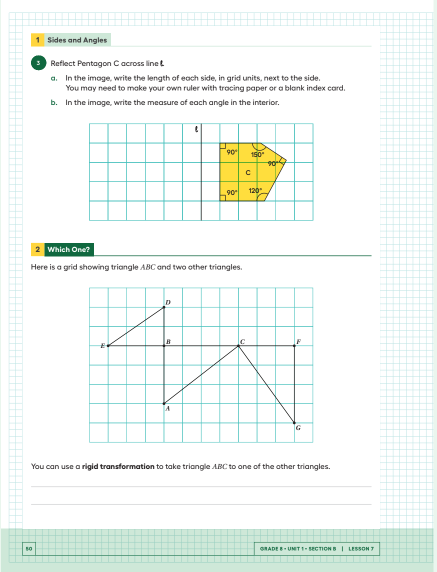
Student Workbook


1 Sides and Angles

3 Reflect Pentagon C across line ℓ.

a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

b. In the image, write the measure of each angle in the interior.

2 Which One?

Here is a grid showing triangle ABC and two other triangles.

You can use a **rigid transformation** to take triangle ABC to one of the other triangles.

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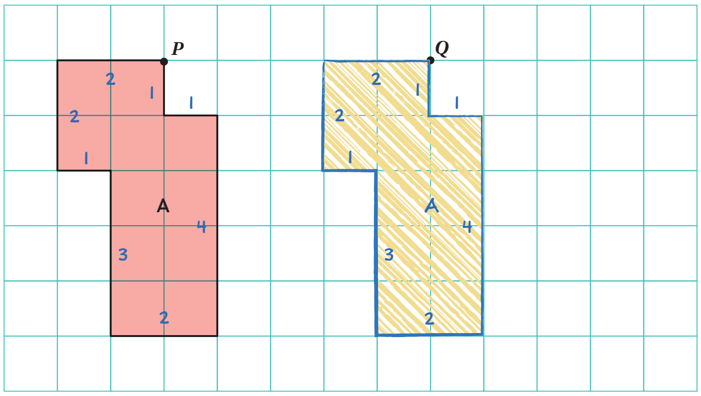
Building on Student Thinking

Students may try to count the grid squares on the diagonal side lengths. Remind students to measure these lengths with their tracing paper or index card. Students may also struggle estimating the diagonal side lengths on their self-marked index card or tracing paper. Remind students of how they estimated the lengths for the questions in the *Warm-up* where the ruler was not marked.

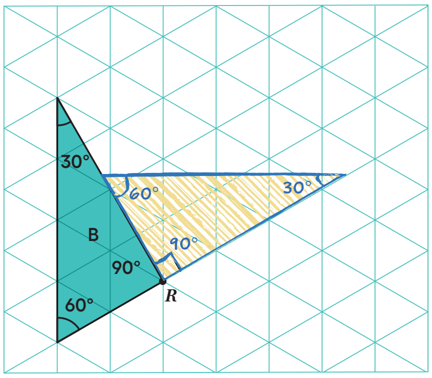
Student Task Statement

1. Translate Polygon A so point P goes to point Q. In the image, write the length of each side, in grid units, next to the side.



The side lengths are measured in units where one unit is the side length of the square in the grid.

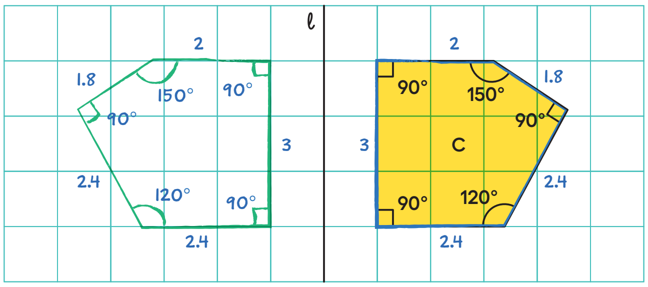
2. Rotate Triangle B 90° clockwise using R as the center of rotation. In the image, write the measure of each angle in its interior.



3. Reflect Pentagon C across line ℓ.

a. In the image, write the length of each side, in grid units, next to the side. You may need to make your own ruler with tracing paper or a blank index card.

b. In the image, write the measure of each angle in the interior.

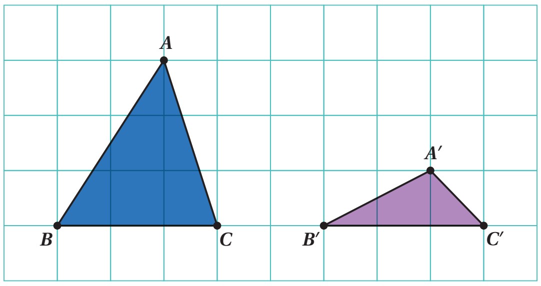


The lengths are measured in grid units. The sides that are not whole numbers have been rounded to the nearest tenth.

Activity Synthesis

Ask selected students to share how they performed the given transformation for each question. After each explanation, ask the class if they agree or disagree. Introduce students to the idea of **corresponding** sides and angles. Ask students to identify the corresponding angles for Polygon A and its image (90 degrees and 270 degrees) and the corresponding side lengths for Triangle B and its image (2 units, 4, units, and about 3.5 units). The point here is not to find the actual values but to note that the corresponding measurements are equal. Since students are creating their own measuring tool, they can only give an estimate, and some flexibility should be allowed in the response. During the discussion, highlight different reasonable answers that students find for the lengths which are not whole numbers.

Point out that for each of the transformations in this activity, the lengths of the sides of the original figure equal the lengths of the corresponding sides in the image, and the measures of the angles in the original figure equal the measures of the corresponding angles in the image. For this reason, we call these transformations **rigid transformations.** They behave as if the shapes move around without stretching, bending, or breaking. An example of a non-rigid transformation is one that compresses a figure vertically, like this:



Tell students that a rigid transformation is a transformation where all pairs of corresponding distances and angle measures in the figure and its image are equal. Translations, reflections, and rotations are the building blocks for *all* rigid transformations, and we will explore that next.

Activity 2
Which One?

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to decide if there is a sequence of translations, rotations, and reflections that take one figure to another and, if so, to produce one such sequence. Deciding whether or not such a sequence is possible uses the knowledge that translations, rotations, and reflections do not change side lengths or angle measures.

Monitor for students who use different transformations to take triangle ABC to triangle CFG and select them to share during the discussion.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Use multimodal examples to show the meaning of corresponding sides and corresponding angles. Use verbal descriptions along with gestures, drawings, or concrete objects to show how the parts of the figures are related through rigid transformations.

Advances: Listening, Representing

Access for Multilingual Learners (Activity 2, Launch)

MLR2: Collect and Display.
Circulate, listen for, and collect the language students use as they work with a partner to transform figures. On a visible display, record words and phrases such as "rotate triangle ABC 90 degrees counterclockwise around point C" and "translate triangle ABC 7 units right." Invite students to borrow language from the display as needed, and update it throughout the lesson. For example, a student may rephrase "the triangles do not go together because the sides do not match" as "the triangles are not connected by rigid transformations because their corresponding sides are not congruent."

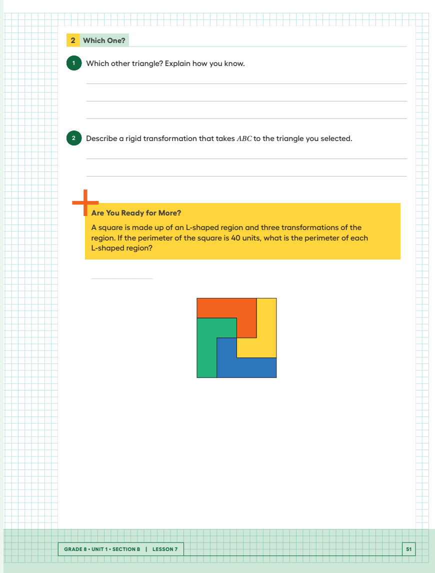
Advances: Conversing, Reading

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Develop Expression and Communication.
Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "Triangle ________ is a rigid transformation of triangle ABC because ... ," "I agree/disagree because ... ," or "Another transformation is ________ because ..."

Supports accessibility for: Language, Organization

Student Workbook



Launch

Provide access to geometry toolkits.

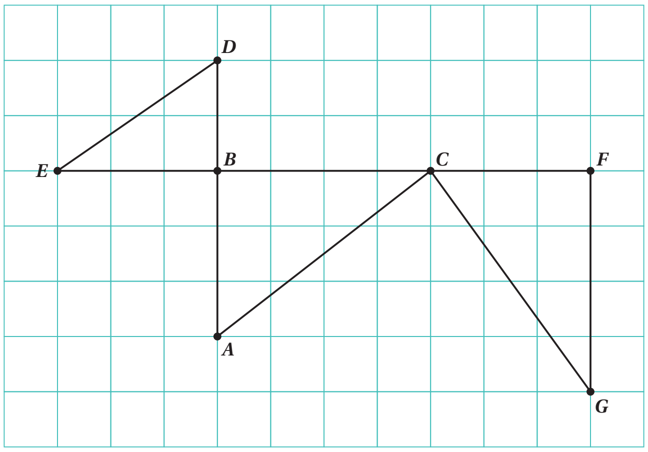
Give students 4 minutes quiet work time, followed by 2 minutes to discuss with a partner.

Select students with different sequences of transformations to share during the whole-class discussion.

In the digital version of the activity, students use an applet to perform transformations. The applet allows students to test multiple transformations. The digital version may be helpful for precisely describing transformations.

Student Task Statement

Here is a grid showing triangle ABC and two other triangles.



You can use a **rigid transformation** to take triangle ABC to one of the other triangles.

1. Which other triangle? Explain how you know.

   Triangle CFG

   Triangle DBE is smaller than triangle ABC, so no rigid transformations can take ABC to triangle DBE.

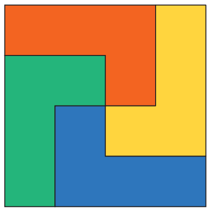
2. Describe a rigid transformation that takes ABC to the triangle you selected.

   Sample responses:

   - Translate triangle ABC 7 units right so that B matches up with F. Then rotate 90 degrees clockwise around F.
   - Rotate triangle ABC 90 degrees counterclockwise around point C, and then rotate 180 degrees around the midpoint of M of segment CG.

Are You Ready for More?

A square is made up of an L-shaped region and three transformations of the region. If the perimeter of the square is 40 units, what is the perimeter of each L-shaped region?



25 units

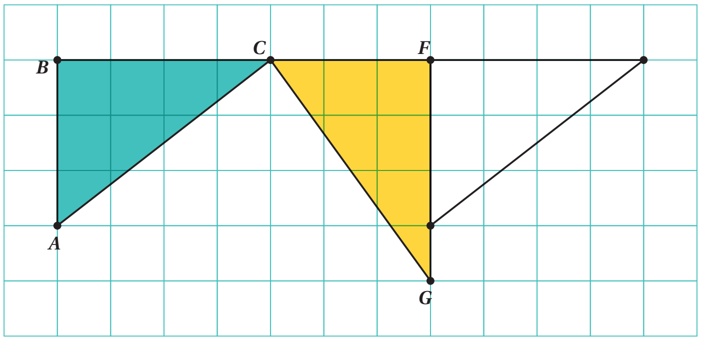
Activity Synthesis

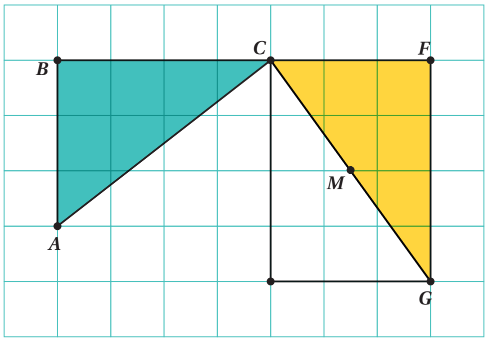
Ask a student to explain why triangle ABC cannot be taken to triangle DBE.

We are only using rigid transformations and therefore the corresponding lengths have to be equal and they are not.

If a student brings up that they think triangle DBE is a scale drawing of triangle ABC, bring the discussion back to translations, rotations, and reflections, rather than talking about how or why triangle DBE isn't actually a scale drawing of triangle ABC.

Offer as many methods for transforming triangle ABC as possible as time permits, inviting previously selected students to share their methods. Include at least two different sequences of transformations. Display student work or these diagrams for all to see as students share their sequences.





Make sure students attend carefully to describing each transformation with the necessary level of precision. For example, for a rotation, specifying the center of rotation, the direction, and the angle of rotation.

If time allows, consider asking the following questions:

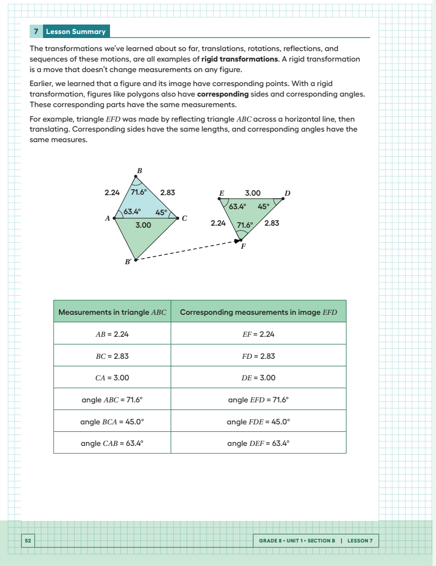
"Can triangle ABC be taken to triangle CFG with only a translation?"

no, since CFG is rotated

"What about with only a reflection?"

no, because they have the same orientation

Student Workbook



7 Lesson Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

For example, triangle EFD was made by reflecting triangle ABC across a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same measures.

Measurements in triangle ABC	Corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
angle $ABC = 71.6°$	angle $EFD = 71.6°$
angle $BCA = 45.0°$	angle $FDE = 45.0°$
angle $CAB = 63.4°$	angle $DEF = 63.4°$

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Lesson Synthesis

Remind students that a **rigid transformation** is a transformation for which all pairs of **corresponding** lengths and angle measures in the original figure and its image are equal. Translations, rotations, and reflections have this property, so they are rigid transformations. Sequences of these are as well—for example, if a figure is translated and then the image is reflected, the side lengths and angle measures stay the same.

Ask students to think of ways they could look at two shapes and tell that one is not the image of the other under a rigid transformation.

Give a moment of quiet think time, and then invite students to share their ideas

> if two shapes have different side lengths or angle measures then there is no rigid transformation taking one shape to the other.

When there is a rigid transformation taking one figure to another, there are many ways to do this. Ask students:

"What are some good ways to tell whether one shape can be taken to another with a sequence of rigid transformations?"

> Measure all of the side lengths and angle measures and ensure that corresponding measurements are equal. Use tracing paper to see if one shape matches up exactly with the other.

"What are the three basic types of rigid transformations?"

> rotations, translations, and reflections

Lesson Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding** sides and corresponding angles. These corresponding parts have the same measurements.

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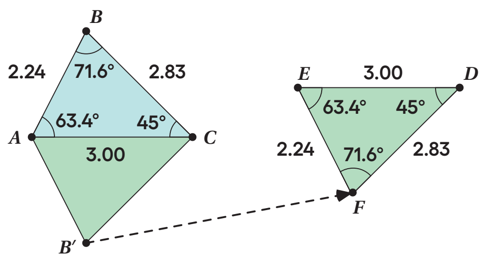

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BC = 2.83	FD = 2.83
CA = 3.00	DE = 3.00
angle ABC = 71.6°	angle EFD = 71.6°
angle BCA = 45.0°	angle FDE = 45.0°
angle CAB = 63.4°	angle DEF = 63.4°

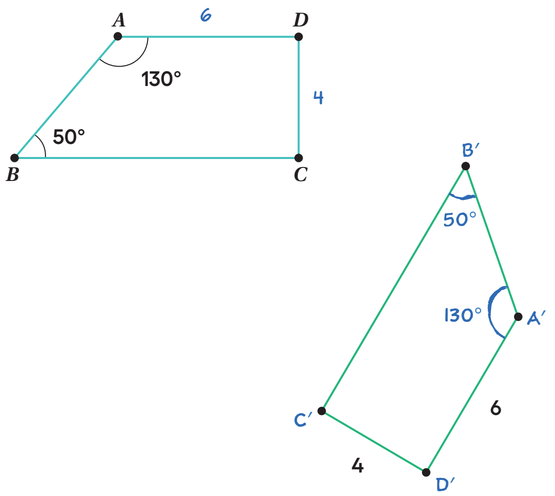
Cool-down

Translated Trapezoid

5 min

Student Task Statement

Trapezoid $A'B'C'D'$ is the image of trapezoid $ABCD$ under a rigid transformation.



1. Label all vertices on trapezoid $A'B'C'D'$.
2. On both figures, label all known side lengths and angle measures.

Responding To Student Thinking

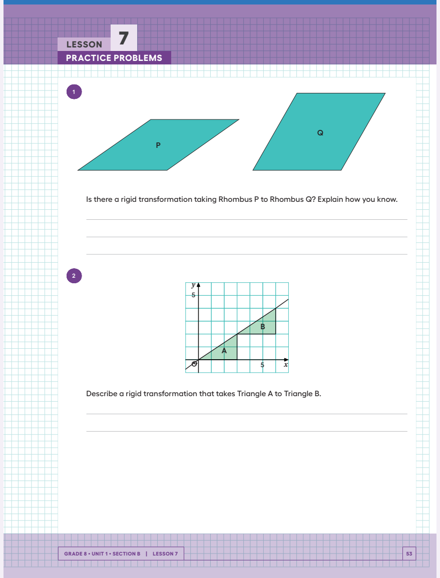
Points to Emphasize
If students struggle with labeling the side lengths and the angle measures, offer an additional opportunity to label corresponding measurements. For example, in the *Activity Synthesis* of the activity referred to here, tell students that the original segment is approximately 2.2 units long, then have students label as many measurements as they know in their final image. Invite multiple students to share their thinking about the side lengths.

Unit 1, Lesson 8, Activity 2 Rotating a Segment

Practice Problems

4 Problems

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1

Is there a rigid transformation taking Rhombus P to Rhombus Q? Explain how you know.

2

Describe a rigid transformation that takes Triangle A to Triangle B.

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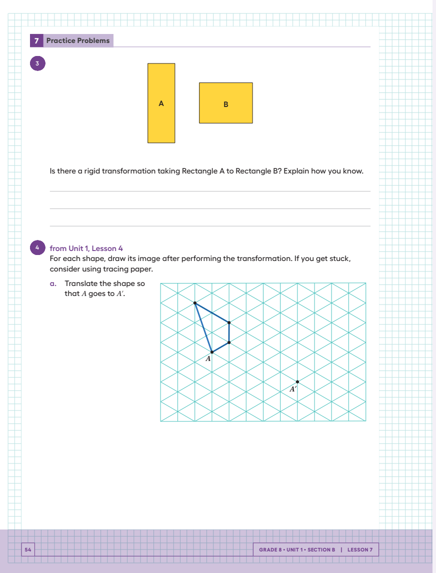
Student Workbook


7 Practice Problems

3

Is there a rigid transformation taking Rectangle A to Rectangle B? Explain how you know.

4 from Unit 1, Lesson 4

For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

a. Translate the shape so that A goes to A'.

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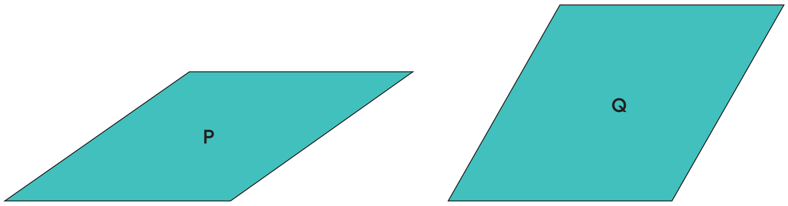
Problem 1

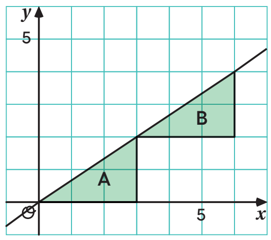


Is there a rigid transformation taking Rhombus P to Rhombus Q? Explain how you know.

no

Sample reasoning: The angle measures of the two polygons are different, and a rigid transformation must preserve all lengths and angle measures.

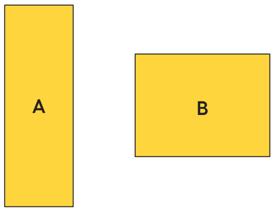
Problem 2



Describe a rigid transformation that takes Triangle A to Triangle B.

Sample response: Translate Triangle A three units right and two units up.

Problem 3



Is there a rigid transformation taking Rectangle A to Rectangle B? Explain how you know.

no

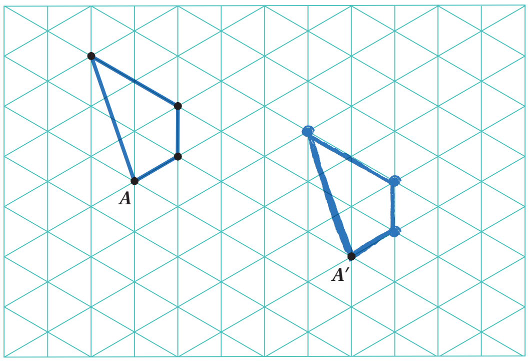
Sample reasoning: The side lengths of the two rectangles are different, and a rigid transformation must preserve all lengths and angle measures.

Problem 4

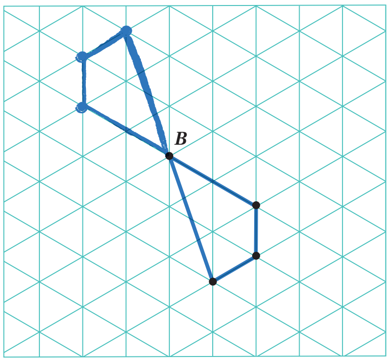
from Unit 1, Lesson 4

For each shape, draw its image after performing the transformation. If you get stuck, consider using tracing paper.

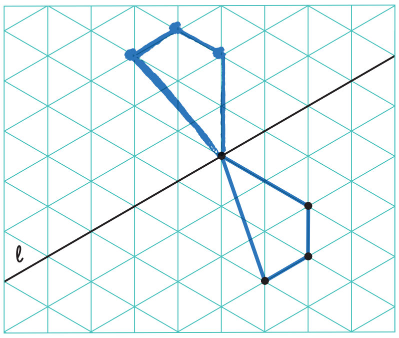
a. Translate the shape so that A goes to A'.



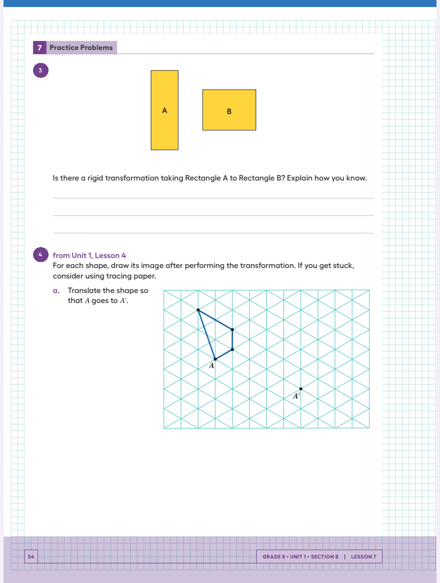
b. Rotate the shape 180° counterclockwise around B.



c. Reflect the shape over the line shown.



Student Workbook



Student Workbook