Lesson 6

Reasoning about Square Roots

Goals

- Comprehend that $-\sqrt{n}$ represents the opposite of $\sqrt{n}$.
- Determine a solution to an equation of the form $x^2 = n$ and represent the solution as a point on the number line.
- Identify the two whole number values that a square root is between and explain (orally) the reasoning.

Learning Target

When I have a square root, I can reason about which two whole numbers it is between.

Lesson Narrative

This lesson continues the transition of student understanding from a geometric characterization of square roots as side lengths, to an algebraic characterization of square roots as specific points on the number line.

Students begin by considering whether statements such as $(\sqrt{5})^2 = 5$ are true, requiring students to make sense of what square root notation means. Next, students reason about the value of a square root by finding the two whole numbers it is closest to in order to more accurately plot the point on a number line and to help justify their placement. Finally, students think about $\sqrt{n}$ as the solution to equations of the form $x^2 = n$ and plot these points on a number line.

Student Learning Goal

Let's approximate square roots.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up)
- MLR1: Stronger and Clearer Each Time (Activity 2)

Instructional Routines

- Math Talk
- MLR1: Stronger and Clearer Each Time

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	10 min	5 min

Warm-up

Math Talk: Squared

5 min

Activity Narrative

This *Math Talk* focuses on analyzing symbolic statements about square roots. It encourages students to think about the meaning of the square root symbol and to rely on what they know about the relationship between squares and square roots to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students plot solutions to equations of the form $x^2 = n$ on the number line.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time and ask them to give a signal when they have an answer and a strategy. Invite students to share their strategies and record and display their responses for all to see.

Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem. Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether or not each statement is true.

A. $(\sqrt{5})^2 = 5$ True

Sample reasoning: $\sqrt{5} \cdot \sqrt{5} = 5$

B. $(\sqrt{9})^2 = 3$ False

Sample reasoning: $\sqrt{9} \cdot \sqrt{9} = 9$, not 3

C. $(\sqrt{10})^2 = 100$ False

Sample reasoning: $\sqrt{10} \cdot \sqrt{10} = 10$, not 100

D. $(\sqrt{16}) = 2^2$ True

Sample reasoning: $\sqrt{16} = 4$ and $2^2 = 4$

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Instructional Routines

Math Talk

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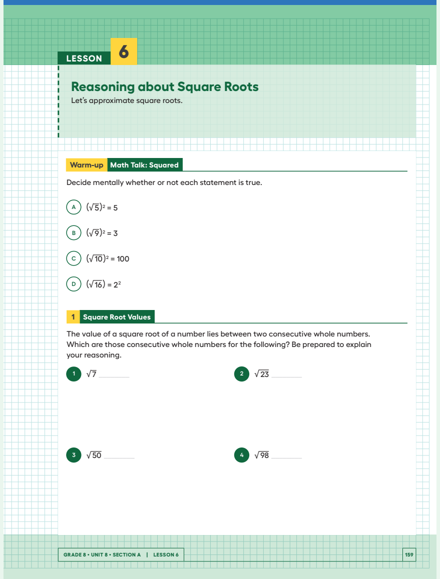
Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Workbook


LESSON 6

Reasoning about Square Roots

Let's approximate square roots.

Warm-up Math Talk: Squared

Decide mentally whether or not each statement is true.

A $(\sqrt{5})^2 = 5$

B $(\sqrt{9})^2 = 3$

C $(\sqrt{10})^2 = 100$

D $(\sqrt{16}) = 2^2$

1 Square Root Values

The value of a square root of a number lies between two consecutive whole numbers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

1 $\sqrt{7}$ ____

2 $\sqrt{23}$ ____

3 $\sqrt{50}$ ____

4 $\sqrt{98}$ ____

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Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Activity 1
Square Root Values

15 min

Activity Narrative

The purpose of this activity is for students to think about square roots in relation to the two whole number values they are closest to. Students are encouraged to use numerical approaches to explain their thinking, especially the fact that $\sqrt{n}$ is a solution to the equation $x^2 = n$.

If needed, students can draw a number line to help them reason about the magnitude of the given square roots, but this is not required.

Launch

Arrange students in groups of 2. Since the purpose of this activity is to reason about the whole numbers that a square root is close to, do not provide access to calculators. Encourage students to reference the class display listing perfect squares.

Give students 2 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

The value of a square root of a number lies between two consecutive whole numbers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

1. $\sqrt{7}$

   2 and 3

   Sample reasoning: 2^2 is 4 and 3^2 is 9, so $\sqrt{7}$ is between 2 and 3.

2. $\sqrt{23}$

   4 and 5

   Sample reasoning: 4^2 is 16 and 5^2 is 24, so $\sqrt{23}$ is between 4 and 5.

3. $\sqrt{50}$

   7 and 8

   Sample reasoning: 7^2 is 49 and 8^2 is 64, so $\sqrt{50}$ is between 7 and 8.

4. $\sqrt{98}$

   9 and 10

   Sample reasoning: 9^2 is 81 and 10^2 is 100, so $\sqrt{98}$ is between 9 and 10.

Are You Ready for More?

Can we do any better than "between 3 and 4" for $\sqrt{12}$? Explain a way to figure out if the value is closer to 3.1 or closer to 3.9.

Answers vary. Sample response: Since $3.5^2 = 12.25$, we know that $\sqrt{12}$ is somewhere between 3 and 3.5. That tells us that it is closer to 3.1 than 3.9.

Access for Students with Diverse Abilities (Activity 1, Launch)

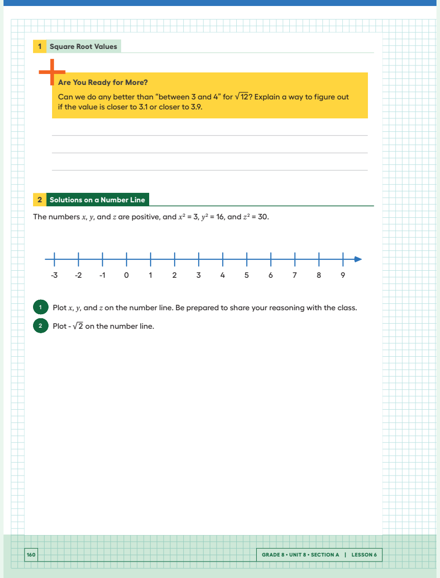
Representation: Internalize Comprehension.
Provide students with a number line that includes rational numbers to record their square root values.
Supports accessibility for: Visual-Spatial Processing, Organization

Building on Student Thinking

If students reason about exponents with statements like "$\sqrt{7}$ is in between 2 and 4 because 2^2 is 4, and 4^2 is 8" or "$\sqrt{23}$ is in between 11 and 12 because 11^2 is 22, and 12^2 is 24," consider:

- Asking students to clarify the meaning of a number raised to the power of 2.
- Prompting students to refer to the class display listing perfect squares.

Student Workbook



Activity Synthesis

The goal of this discussion is to make sure students understand how to reason about the value of a square root by comparing it to square roots that are whole numbers. Ask students:

"What strategy did you use to figure out the two whole numbers?"

I made a list of perfect squares and then found which two the number was between.

"Did anyone use inequality symbols when writing their answers?"

Yes, for the first problem, I wrote $2 < \sqrt{5} < 3$.

When students are in agreement about which two whole numbers each square root lies between, ask students to think more deeply about the relationship by asking them to figure out which of the two whole numbers it is closer to. Give 1–2 minutes of quiet work time, then invite students to share their thinking.

If not brought up by students, highlight that if a number is exactly halfway between two perfect squares, it is not true that the square root of that number is also halfway between the square root of the perfect squares. For example, students may think that $\sqrt{26}$ is halfway between 4 and 6 since 26 is halfway between 16 and 36. It's close since $\sqrt{26} \approx 5.099$, but it's slightly more than "halfway."

If time allows, remind students of the graph they made in an earlier lesson showing the relationship between the side length and area of a square. The graph showed a non-proportional relationship, so making proportional assumptions about relative sizes will not be accurate.

Activity 2

Solutions on a Number Line

10 min

Activity Narrative

The purpose of this activity is for students to use rational approximations of irrational numbers to place both rational and irrational numbers on a number line and to reinforce the definition of a square root as a solution to an equation of the form $x^2 = n$. This is also the first time that students are asked to consider negative square roots.

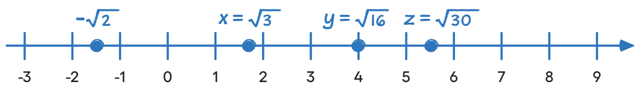
Launch

Arrange students in groups of 2. Since the goal of this activity is for students to approximate the location of a square root on a number line, do not provide access to calculators.

Give students 2 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

The numbers x, y, and z are positive, and $x^2 = 3$, $y^2 = 16$, and $z^2 = 30$.



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.
2. Plot $-\sqrt{2}$ on the number line.

Activity Synthesis

The purpose of this discussion is to reinforce the idea that irrational numbers are still numbers on the number line, though their location cannot be found by subdividing the unit interval into b parts and taking a of them in the way that rational numbers written as $x^2 = n$ can be.

Display the number line from the activity for all to see. Select groups to share how they chose to place values, recording them on the number line as they share. After each placement, survey the class and ask if students used the same or different reasoning. Invite any groups that used different reasoning to share with the class.

Conclude the discussion by asking students to share how they placed $-\sqrt{2}$.

I found the approximate location of $\sqrt{2}$ and then placed $-\sqrt{2}$ the same distance from 0 to the left.

Lesson Synthesis

The goal of this discussion is to make sure that students understand that a square root can be approximated by finding the whole numbers it lies between and then testing values between those two whole numbers to determine a more accurate approximation. Here are some questions for discussion:

- *"How can we find the whole numbers that a square root lies between?"*

  Look at whole numbers whose squares are greater than and less than the number inside the square root symbol.

- *"How can we get a better approximation?"*

  We can test values between those two whole numbers.

- *"What two whole numbers does $\sqrt{68}$ lie between?"*

  8 and 9

- *"Test some numbers between 8 and 9. What is a better approximation?"*

  8.25 is a good approximation because 8.25^2 is only slightly greater than 68.

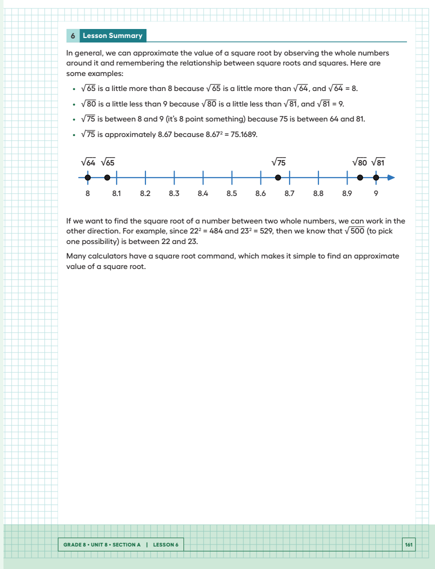
Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, use a kinesthetic representation of the number line on a clothesline. Students can place and adjust numbers on folder paper or cardstock on the clothesline in a hands-on manner.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.
Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to their reasoning for how they placed x, y, and z on the number line in the first problem. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student Workbook



Responding To Student Thinking

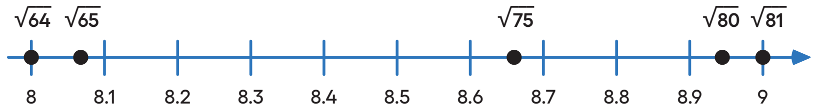
Press Pause
By this point in the unit, there should be some student mastery of approximating the value of an irrational number. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 8, Section A Side Lengths and Areas of Squares

Lesson Summary

In general, we can approximate the value of a square root by observing the whole numbers around it and remembering the relationship between square roots and squares. Here are some examples:

- $\sqrt{65}$ is a little more than 8 because $\sqrt{65}$ is a little more than $\sqrt{64}$, and $\sqrt{64} = 8$.
- $\sqrt{80}$ is a little less than 9 because $\sqrt{80}$ is a little less than $\sqrt{81}$, and $\sqrt{81} = 9$.
- $\sqrt{75}$ is between 8 and 9 (it's 8 point something) because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately 8.67 because $8.67^2 = 75.1689$.



If we want to find the square root of a number between two whole numbers, we can work in the other direction. For example, since $22^2 = 484$ and $23^2 = 529$, then we know that $\sqrt{500}$ (to pick one possibility) is between 22 and 23.

Many calculators have a square root command, which makes it simple to find an approximate value of a square root.

Cool-down

Betweens

5 min

Student Task Statement

Which of the following numbers are greater than 6 and less than 8? Explain how you know.

- $\sqrt{7}$
- $\sqrt{60}$
- $\sqrt{80}$

Only $\sqrt{60}$

Sample reasoning: Since $6^2 = 36$ and $8^2 = 64$, the number inside the square root must be between 36 and 64.

Practice Problems

6 Problems

Problem 1

a. Explain how you know that $\sqrt{37}$ is a little more than 6.

$\sqrt{36}$ is exactly 6, and $\sqrt{37}$ is a little more than that.

b. Explain how you know that $\sqrt{95}$ is a little less than 10.

$\sqrt{100}$ is exactly 10, and $\sqrt{95}$ is a little less than that.

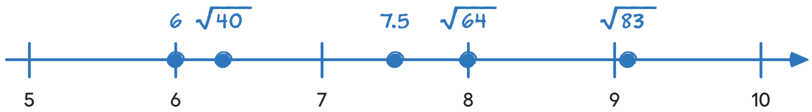
c. Explain how you know that $\sqrt{30}$ is between 5 and 6.

$\sqrt{25} = 5$, $\sqrt{36} = 6$, and $\sqrt{30}$ is in between.

Problem 2

Plot each number on the number line:

$$6, \sqrt{83}, \sqrt{40}, \sqrt{64}, 7.5$$



Problem 3

The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So 5 is a solution, and -5 is also a solution.

Select **all** the equations that have a solution of -4:

A. $10 + x = 6$

B. $10 - x = 6$

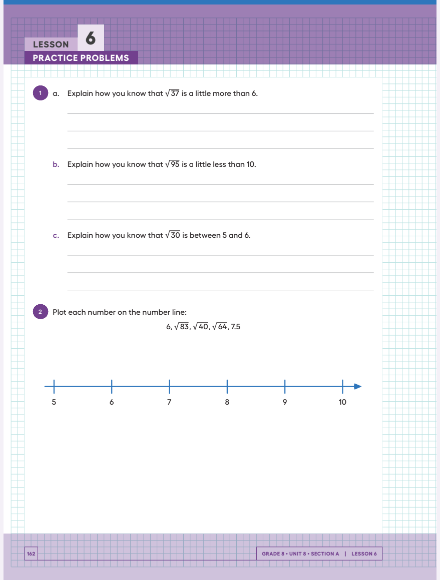
C. $-3x = -12$

D. $-3x = 12$

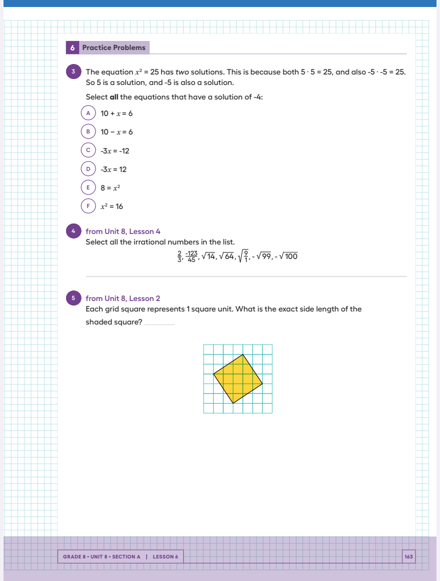
E. $8 = x^2$

F. $x^2 = 16$

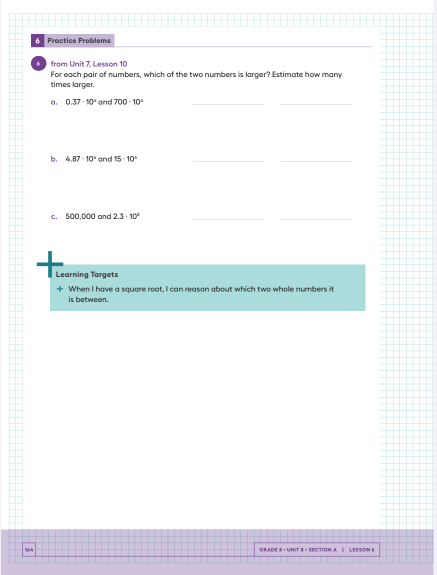
Student Workbook



Student Workbook



Student Workbook



Problem 4

from Unit 8, Lesson 4

Select all the irrational numbers in the list.

$$\frac{2}{3}, \frac{-123}{45}, \sqrt{14}, \sqrt{64}, \sqrt{\frac{9}{1}}, -\sqrt{99}, -\sqrt{100}$$

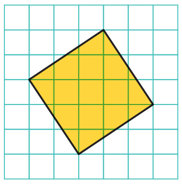
$\sqrt{14}, -\sqrt{99}$

Problem 5

from Unit 8, Lesson 2

Each grid square represents 1 square unit. What is the exact side length of the shaded square?

$\sqrt{13}$ units



Problem 6

from Unit 7, Lesson 10

For each pair of numbers, which of the two numbers is larger? Estimate how many times larger.

a. $0.37 \cdot 10^6$ and $700 \cdot 10^4$

$700 \cdot 10^4$, about 20 times larger

b. $4.87 \cdot 10^4$ and $15 \cdot 10^5$

$15 \cdot 10^5$, about 30 times larger

c. 500, 000 and $2.3 \cdot 10^8$

$2.3 \cdot 10^8$, about 500 times larger