Lesson 6

Strategic Solving

Goals

- Categorize (orally and in writing) linear equations in one variable based on their structure, and solve equations from each category.
- Describe (orally and in writing) features of linear equations that have one solution, no solution, or many solutions.
- Describe (orally) strategies for solving linear equations in one variable with different features or structures.

Learning Target

I can solve linear equations in one variable.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR8: Discussion Supports
- Poll the Class

Lesson Narrative

In this lesson students learn to think strategically before plunging into a solution method. After a *Warm-up* in which they construct their own equation to solve a problem, they look at equations with different structures and decide whether the solution will be positive, negative, or zero, without solving the equation. They judge which equations are likely to be easy for them to solve and which are likely to be difficult for them.

Students must use the structure of equations to look for features of an equation that will tell them something about the solution or help them choose a solution path.

Student Learning Goal

Let's solve linear equations like a boss.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	10 min	20 min	10 min	5 min

Assessment: Cool-down

Warm-up
Equal Perimeters

5 min

Activity Narrative

The purpose of this activity is for students to begin building linear equations and solving them. They use the familiar context of polygon perimeter to find the values of x that give 2 shapes the same perimeter.

Launch

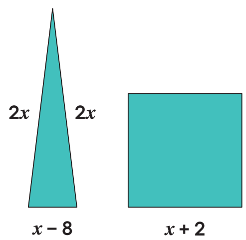
Arrange students in groups of 2.

Give students 2 minutes of quiet think time, and then 2 minutes to discuss their solutions with a partner.

If necessary, remind students how to find the perimeter of a shape. Instruct groups to explain to each other how they came up with expressions and an equation to represent the situation.

Student Task Statement

The triangle and the square have equal perimeters.



1. Find the value of x.

   $x = 16$

   Sample reasoning: Since the perimeters are equal, the perimeter of the triangle must equal the perimeter of the square. The perimeter of the triangle is $2x + 2x + (x - 8)$, which is $5x - 8$, and the perimeter of the square is 4 times the side length, or $4(x + 2)$.

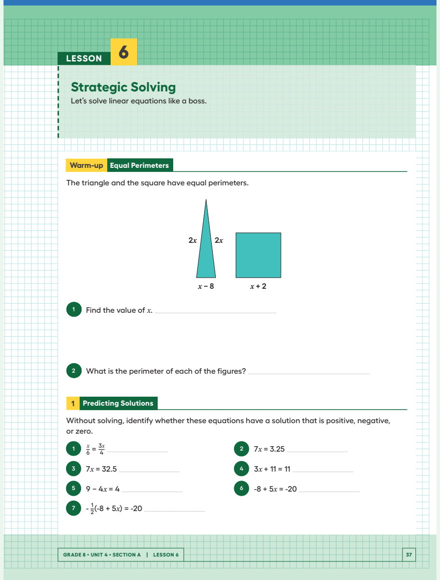
$$5x - 8 = (4x + 2)$$
$$5x - 8 = 4x + 8$$
$$5x = 4x + 16$$
$$x = 16$$

2. What is the perimeter of each of the figures?

   $P = 72$

   Sample reasoning: Since the perimeters are equal, we can use either expression to find the perimeter. From the square: $4(16 + 2) = 72$.

Student Workbook


LESSON 6

Strategic Solving

Let's solve linear equations like a boss.

Warm-up Equal Perimeters

The triangle and the square have equal perimeters.

$2x$ $2x$ $x - 8$ $x + 2$

1 Find the value of x. ____

2 What is the perimeter of each of the figures? ____

1 Predicting Solutions

Without solving, identify whether these equations have a solution that is positive, negative, or zero.

1 $\frac{x}{6} = \frac{3x}{4}$ ____
2 $7x = 3.25$ ____
3 $7x = 32.5$ ____
4 $3x + 11 = 11$ ____
5 $9 - 4x = 4$ ____
6 $-8 + 5x = -20$ ____
7 $-\frac{1}{2}(-8 + 5x) = -20$ ____

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Instructional Routines

MLR8: Discussion Supports

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Activity Synthesis

Ask groups to share their strategies for solving the question. Here are some questions for discussion:

"What expression represents the perimeter of the triangle? The perimeter of the square?"

The expression for perimeter of the triangle is $5x - 8$, and for the perimeter of the square is $4(x + 2)$.

"What was your strategy in making an equation?"

If both perimeters are the same, we can say their expressions are equal.

"What does x mean in the situation?"

It means an unknown value. None of the sides or perimeter is represented by x, so we cannot say it represents a specific thing on the figures.

"Looking at the figures, are there any values that x could not be? Explain your reasoning."

Since the triangles have sides that are $2x$, x cannot be 0 or a negative value. Triangles cannot have sides with 0 or negative side lengths. Since the third side is $x - 8$, we can use this same reasoning to realize that x must actually be greater than 8.

"How does this information help when solving?"

If I make a mistake in my solution and get a value of x that is less than or equal to 8, then I know immediately that my answer is not reasonable, and I can try to find my error.

Activity 1
Predicting Solutions

10 min

Activity Narrative

The purpose of this activity is to shift the focus from solving an equation to thinking about what it means for a number to be a solution of an equation. Students inspect each equation, looking at the structure, the signs, and the operations in it to decide if the solution is positive, negative, or zero. The last two questions have a similar format, so students can take advantage of their thinking from one problem to the next.

Launch

Arrange students in groups of 2.

Display the equation $5x = 6x$ for all to see.

Ask students,

"How might we know whether x is a positive number, negative number, or zero, without solving the equation?"

The variables can be combined into one term, but there are no constant terms. That means eventually the variable term has to equal 0, so x must be 0.

Display the equation $5x$ = -16.5 for all to see and ask the same question.

Without solving, we can see that a positive number of xs has to equal a negative value, so x must be a negative number.

Instruct students to inspect each equation carefully and to use reasoning to answer the questions in the activity rather than trying to solve each equation for a specific value.

Give 5 minutes of quiet think time, and then ask students to compare their work with their partner. For any questions they disagree on, students should work to reach an agreement.

Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.
Display sentence frames to support partner discussions: "I know that equation _____ will have a positive/negative/zero solution because ____." "Some features of equations with a positive/negative/zero solution are ____." "When I look at the structure of this equation I notice that ____."

Advances: Speaking, Conversing

Student Task Statement

Without solving, identify whether these equations have a solution that is positive, negative, or zero.

1. $\frac{x}{6} = \frac{3x}{4}$

   zero

   Sample reasoning: There are only x terms, so there is no constant term for the variable to equal, or the constant term is 0.

2. $7x = 3.25$

   positive

   Sample reasoning: A positive amount of xs equals a positive value.

3. $7x = 32.5$

   positive

   Sample reasoning: A positive amount of xs equals a positive value.

4. $3x + 11 = 11$

   zero

   Sample reasoning: Since the constant terms on each side are equal, the final constant term is 0.

5. $9 - 4x = 4$

   positive

   Sample reasoning: To get 4, a positive number must be subtracted from 9. That means $4x$ is positive, so x must be positive, because the product of 2 positive numbers is positive.

6. $-8 + 5x = -20$

   negative

   Sample reasoning: To get to -20, a negative number must be added to -8. That means $5x$ is negative, so x must be negative because the product of a positive and negative number is positive.

7. $-\frac{1}{2}(-8 + 5x) = -20$

   positive

   Sample reasoning: To get -20, the parentheses must be a positive number because $-\frac{1}{2}$ multiplied by a positive number is negative. To get a positive number, a positive number must be added to -8. That means $5x$ is positive, so x is positive.

Instructional Routines

Poll the Class

ilclass.com/r/10694985

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Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Activity Synthesis

The purpose of this discussion is for students to practice talking about equations and the operations within them and to use logical thinking. There is no need to try to formally generalize student thinking for all cases at this time. In later grades, students will continue the work started here looking for structure in equations.

For each equation, invite groups to share how they decided if the solution was positive, negative, or zero. After each group shares, ask if any other group reasoned about the problem in a different way, and invite them to share their reasoning.

Invite students to suggest ways that knowing the sign of a solution to an equation is helpful. Ensure that students understand that it can help in checking their solution and can often be done quickly mentally when working problems on their own.

Activity 2

Which Would You Rather Solve?

20 min

Activity Narrative

The purpose of this activity is for students to think about what they see as “least difficult” and “most difficult” when looking at equations and to practice solving equations. Students also discuss strategies for dealing with “difficult” parts of equations.

Launch

Keep students in the same groups of 2.

Give students 3–5 minutes of quiet think time to get started and then 5–8 minutes to discuss and work with their partner. Leave ample time for a whole-class discussion.

Student Task Statement

Here are a lot of equations:

A. $-\frac{5}{6}(8 + 5b) = 75 + \frac{5}{3}b$

B. $-\frac{1}{2}(t + 3) - 10 = -6.5$

C. $\frac{10 - v}{4} = 2(v + 17)$

D. $2(4k + 3) - 13 = 2(18 - k) - 13$

E. $\frac{n}{7} - 12 = 5n + 5$

F. $3(c - 1) + 2(3c + 1) = -(3c + 1)$

G. $\frac{4m - 3}{4} = -\frac{9 + 4m}{8}$

H. $p - 5(p + 4) = p - (8 - p)$

I. $2(2q + 1.5) = 18 - q$

J. $2r + 49 = -8(-r - 5)$

1. Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.

Sample response: Equation A would be difficult to solve because there are some fractions on each side of the equation. Equation C would be easy to solve because there is only one fraction on one side of the equation.

2. Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

A. $b = -14$

B. $t = -10$

C. $v = -14$

D. $k = 3$

E. $n = -3.5$

F. $c = 0$

G. $m = -\frac{1}{4}$

H. $p = -2$

I. $q = 3$

J. $r = \frac{3}{2}$

Are You Ready for More?

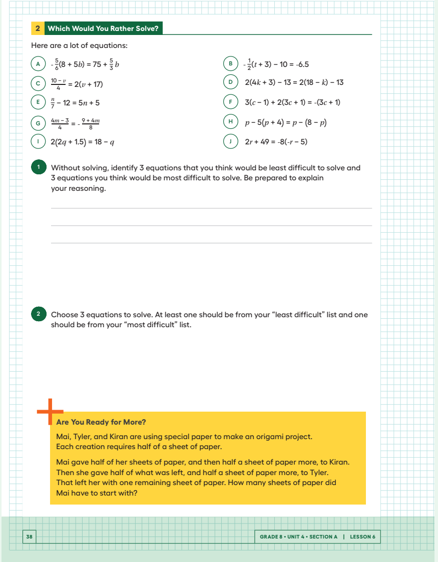
Mai, Tyler, and Kiran are using special paper to make an origami project. Each creation requires half of a sheet of paper.

Mai gave half of her sheets of paper, and then half a sheet of paper more, to Kiran. Then she gave half of what was left, and half a sheet of paper more, to Tyler. That left her with one remaining sheet of paper. How many sheets of paper did Mai have to start with?

7 sheets of paper

Sample reasoning: An equation that represents this scenario is $\frac{x+1}{2} + \frac{x - \frac{x+1}{2} + 1}{2} = x - 1$, where x is the number of papers Mai started with.

Student Workbook



2 Which Would You Rather Solve?

Here are a lot of equations:

1 Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.

2 Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Are You Ready for More?

Mai, Tyler, and Kiran are using special paper to make an origami project. Each creation requires half of a sheet of paper.

Mai gave half of her sheets of paper, and then half a sheet of paper more, to Kiran. Then she gave half of what was left, and half a sheet of paper more, to Tyler. That left her with one remaining sheet of paper. How many sheets of paper did Mai have to start with?

Activity Synthesis

The purpose of this discussion is for students to discuss strategies for solving different types of equations. Some students may have thought that an equation was in the “least difficult” category, while others thought that the same equation was in the “most difficult” category. Remind students that when you feel confident about the strategies for solving an equation, it may move into the “least difficult” category, and recognizing good strategies takes practice and time.

Poll the class for each question as to whether they placed it in the “most difficult” category, “least difficult” category, or if it was somewhere in the middle. Record and display the results of the poll for all to see.

For questions with a split vote, have a group share something that was difficult about it and something that made it seem easy. If there are any questions that everyone thought would be more difficult or everyone thought would be less difficult, ask students why it seemed that way. Ask students,

- *“Were there any equations that were more difficult to solve than you expected? Were there any that were less difficult to solve than you expected?”*

Consider asking some of these questions to further the discussion:

- *“For equation A, what could we do to eliminate the fraction?”*

  Multiply each side by the common denominator of 6. Then the terms will all have integer coefficients.

- *“Which other equations could we use this strategy for?”*

  Any equations that had fractions, such as B, C, E, and G.

- *“What steps do you need to do to solve equation D? Which other equations are like this one?”*

  There is a lot of distributing and collecting like terms. F and H also have to distribute several times.

- *“What other strategies or steps did you use in solving the equations?”*

Lesson Synthesis

Instruct students to write an equation with a variable and a constant term on each side that they would look at and consider difficult to solve.

Select several students’ equations to display for all to see.

Discuss with students:

- *“What are some things that these equations have in common that might be considered difficult to solve?”*

  Fractions, decimals, distribution, and negatives are common ways to increase the level of difficulty.

- *“What strategies do we know for solving equations that have each of these things?”*

  For fractions, we can find a common denominator and multiply each side of the equation by the denominator to eliminate the fractions. For distribution, we can make sure to collect like terms before we do other steps. For use of decimals and negatives, we can make sure that we perform calculations carefully.

Choose one of the displayed equations for students to solve. Have them compare solutions with a partner. Ask students,

"Did any of you use different strategies for solving this equation than your partner did? How many of you followed the same solution path?"

Share a correct solution with the class so they can compare their solutions.

Lesson Summary

Sometimes we are asked to solve equations with a lot of things happening on each side. For example,

$$x - 2(x + 5) = \frac{3(2x - 20)}{6}$$

This equation has variables on each side, parentheses, and even a fraction to think about. Before we start distributing, let's take a closer look at the fraction on the right side. The expression $2x - 20$ is being multiplied by 3 and divided by 6, which is the same as just dividing by 2, so we can re-write the equation as

$$x - 2(x + 5) = \frac{2x - 20}{2}$$

But now it's easier to see that all the terms on the numerator of right side are divisible by 2, which means we can re-write the right side again as

$$x - 2(x + 5) = x - 10$$

At this point, we could do some distribution and then collect like terms on each side of the equation. Another choice would be to use the structure of the equation. Both the left and the right side have something being subtracted from x. But, if the two sides are equal, that means the things being subtracted on each side must also be equal. Thinking this way, the equation can now be rewritten with fewer terms as

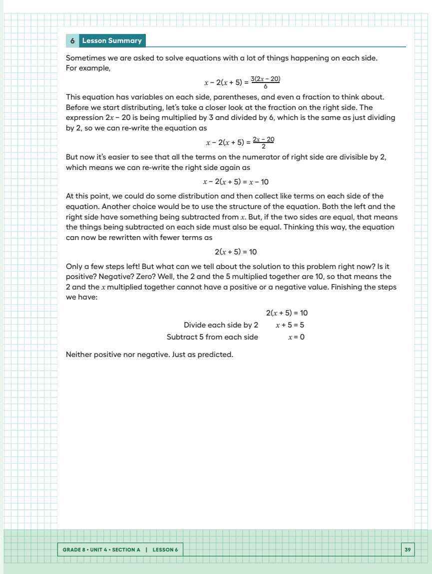
$$2(x + 5) = 10$$

Only a few steps left! But what can we tell about the solution to this problem right now? Is it positive? Negative? Zero? Well, the 2 and the 5 multiplied together are 10, so that means the 2 and the x multiplied together cannot have a positive or a negative value. Finishing the steps we have:

	$2(x + 5) = 10$
Divide each side by 2	$x + 5 = 5$
Subtract 5 from each side	$x = 0$

Neither positive nor negative. Just as predicted.

Student Workbook



Responding To Student Thinking

Points to Emphasize
If students struggle with identifying the sign of the solution, discuss the answer to the first problem in the *Cool-down* so that students can hear how other students recognized that the solution must be positive.
If students struggle to operate with negative numbers in the second part of the *Cool-down*, leverage the practice problems to provide extra attempts with discussion.
Unit 4, Lesson 6, Practice Problem 2

Cool-down

Think Before You Step

5 min

Student Task Statement

1. Without solving, identify whether this equation has a solution that is positive, negative, or zero. Explain your reasoning.

   $3x - 5 = -3$

   positive

   Sample reasoning: If $3x - 5 = -3$, then the x must be positive. If x is negative, then subtracting 5 from $3x$ would result in a number less than -3. For similar reasons, x cannot be zero.

2. Solve the equation.
   $x - 5(x - 1) = x - (2x - 3)$

   $x = \frac{2}{3}$ (or equivalent)

Lesson 6

Practice Problems

4 Problems

Problem 1

Solve each of these equations. Explain or show your reasoning.

$2b + 8 - 5b + 3 = -13 + 8b - 5$

$b = \frac{29}{11}$

Sample reasoning: Collect like terms on each side, add 18 to each side, add $3b$ to each side, then divide each side by 11.

$2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

$x = 0$

Sample reasoning: Collect like terms on the left side, distribute and collect like terms on the right side, add $3x$ to each side, subtract 15 from each side, then divide each side by 9.

$2c - 3 = 2(6 - c) + 7c$

$c = -5$

Sample reasoning: Distribute and collect like terms on each side, subtract 12 from each side, subtract $2c$ from each side, then divide each side by 3.

Problem 2

Solve each equation and check your solution.

$-3w - 4 = w + 3$

$w = \frac{-7}{4}$

$3(3 - 3x) = 2(x + 3) - 30$

$x = 3$

$\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

$z = 8$

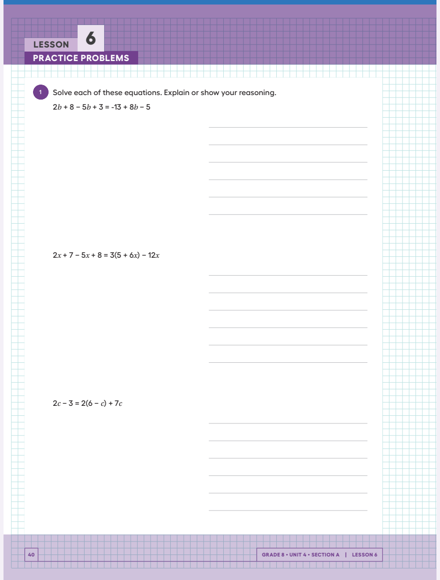
Problem 3

Elena said the equation $9x + 15 = 3x + 15$ has no solutions because $9x$ is greater than $3x$. Do you agree with Elena? Explain your reasoning.

disagree

Sample reasoning: $9x > 3x$ when $x > 0$, but $9x < 3x$ when $x < 0$ and $9x = 3x$ when $x = 0$. The solution to the equation is $x = 0$.

Student Workbook



LESSON 6 PRACTICE PROBLEMS

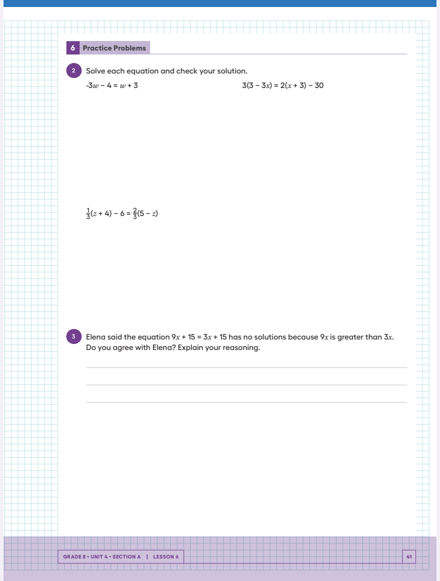
1 Solve each of these equations. Explain or show your reasoning.

$2b + 8 - 5b + 3 = -13 + 8b - 5$

$2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

$2c - 3 = 2(6 - c) + 7c$

Student Workbook



6 Practice Problems

2 Solve each equation and check your solution.

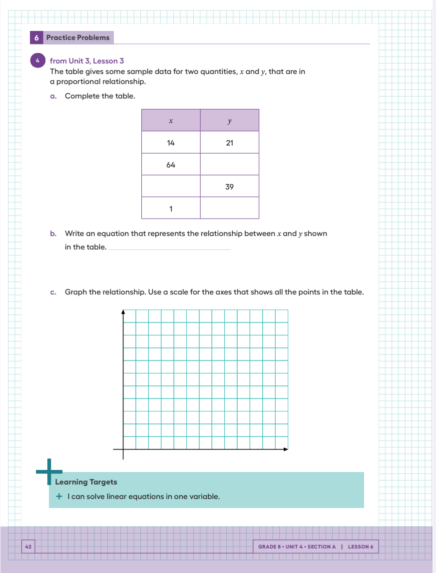
$-3w - 4 = w + 3$

$3(3 - 3x) = 2(x + 3) - 30$

$\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

3 Elena said the equation $9x + 15 = 3x + 15$ has no solutions because $9x$ is greater than $3x$. Do you agree with Elena? Explain your reasoning.

Student Workbook


6 Practice Problems

4 from Unit 3, Lesson 3

The table gives some sample data for two quantities, x and y, that are in a proportional relationship.

a. Complete the table.

x	y
14	21
64	
	39
1	

b. Write an equation that represents the relationship between x and y shown in the table.

c. Graph the relationship. Use a scale for the axes that shows all the points in the table.

Learning Targets

+ I can solve linear equations in one variable.

Problem 4

from Unit 3, Lesson 3

The table gives some sample data for two quantities, x and y, that are in a proportional relationship.

a. Complete the table.

x	y
14	21
64	96
26	39
1	$\frac{3}{2}$

b. Write an equation that represents the relationship between x and y shown in the table.

$y = \frac{3}{2}x$ (or equivalent)

c. Graph the relationship. Use a scale for the axes that shows all the points in the table.