Lesson 7

Practice with Rational Bases

Goals

- Identify (orally and in writing) misapplications of exponent rules to expressions with multiple bases.
- Use exponent rules to rewrite exponential equations involving negative exponents to have a single positive exponent, and explain (orally) the strategy.

Learning Targets

- I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
- I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

Lesson Narrative

In this lesson, students practice all of the exponent rules they have learned so far and begin to look at expressions with multiple bases. Students begin by comparing four expressions using various bases, exponents, and properties of exponents. Then students investigate different ways of writing expressions that involve exponents. They also analyze the structure of exponents to make sense of expressions with different bases.

Student Learning Goal

Let's practice with exponents.

Instructional Routines

- 5 Practices
- Which Three Go Together?

Access for Students with Diverse Abilities:

- Engagement (Activity 2)

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	20 min	10 min	10 min	5 min

Warm-up

Which Three Go Together: Exponents

5 min

Activity Narrative

This *Warm-up* prompts students to compare four expressions. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the expression for all to see. Ask students to indicate when they have noticed one expression that doesn't belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression doesn't belong and together find at least one reason each expression doesn't belong.

Student Task Statement

Which three go together? Why do they go together?

A. $\frac{2^8}{2^5}$

B. $(4^{-5})^8$

C. $\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$

D. $\frac{10^{-8}}{5^5}$

Sample responses:

A, B, and C go together because:

- They do not have 2 different bases.

A, B, and D go together because:

- They all have bases with whole numbers.
- They are not multiplying two factors together.

A, C, and D go together because:

- They all have 2 bases.
- They are not raising a power to another power.

B, C, and D go together because:

- They all have an exponent that is negative.

Instructional Routines

Which Three Go Together?

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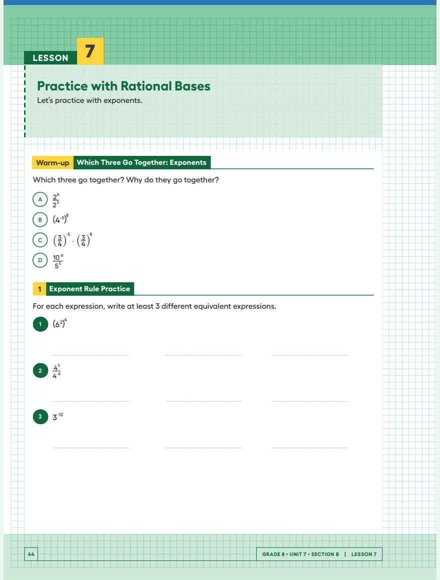
Student Workbook


LESSON 7

Practice with Rational Bases

Let's practice with exponents.

Warm-up Which Three Go Together: Exponents

Which three go together? Why do they go together?

A $\frac{2^8}{2^5}$

B $(4^{-5})^8$

C $\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$

D $\frac{10^{-8}}{5^5}$

1 Exponent Rule Practice

For each expression, write at least 3 different equivalent expressions.

1 $(6^2)^4$

2 $\frac{4^5}{4^{-8}}$

3 3^{-12}

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Instructional Routines

5 Practices

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Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "base," "factor," "power," and "rule," and to clarify their reasoning as needed. Consider asking the following questions:

"How do you know … ?"

"What do you mean by … ?"

"Can you say that in another way?"

Activity 1

Exponent Rule Practice

20 min

Activity Narrative

This activity develops fluency with exponent rules and allows students to think flexibly about equivalent expressions.

Monitor for groups who use these different representations when finding equivalent expressions for 3^{-12}. Here are types of expressions student may find, ordered from more common to less common:

- An expression that uses repeated multiplication, such as $3^{-4} \cdot 3^{-4} \cdot 3^{-4}$.
- An expression that raises a power to another power, such as $(3^{-4})^3$ or $(3^{-2})^6$.
- An expression that shows an understanding of negative exponents, such as 3^{-12}, which is equivalent to $\frac{1}{3^{12}}$ or $\left(\frac{1}{3}\right)^{12}$.
- An expression that uses multiple exponent rules at once, such as $\frac{(3^5)^{-2}}{3^2}$.

Launch

Arrange students in groups of 2. Display this set of expressions for all to see:

$7^5 \cdot 7^4$	$\frac{7^3 \cdot 7^8}{7^4}$	7^{20}
$\frac{7^{10}}{7}$	$(7^3)^3$	$7^0 \cdot 7 \cdot 7^2 \cdot 7^6$

Give students 1–2 minutes to study the expressions and decide which ones are equivalent. ($7^5 \cdot 7^4$, $\frac{7^{10}}{7}$, $(7^3)^3$, and $7^0 \cdot 7 \cdot 7^2 \cdot 7^6$ are equivalent.) Ask students to share their reasoning with a partner before inviting students to share their reasoning with the class.

Explain to students that they will be working to create as many different equivalent expressions to a given expression as they can. Give students 6 minutes to work with a partner before a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

For each expression, write at least 3 different equivalent expressions.

1. $(6^2)^4$
2. $\frac{4^5}{4^{-8}}$
3. 3^{-12}

Answers vary.

Activity Synthesis

The purpose of this discussion is for students to see multiple ways to represent the same expression.

Invite previously selected groups to share their equivalent expression for 3^{-12}. Sequence the discussion of the expressions in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as these:

"Is one type of expression more useful than another type of expression?"

Sometimes a shorter expression may be easier to write, but sometimes a longer expression might model a particular situation better.

"Do you notice any patterns in the different expressions?"

Answers vary.

"How did seeing multiple ways to use the exponent rules enhance your understanding of exponents?"

Answers vary.

Activity 2
Inconsistent Bases

10 min

Activity Narrative

In this activity, students analyze the structure of expressions involving different bases and observe that exponents can be added (or subtracted) only when the powers being multiplied (or divided) have the same base.

It is expected that students will compute the value of the expressions on the left and right sides of the equation to show that they are not actually equal. The last problem alludes to the rule, which will be explored further in a following lesson.

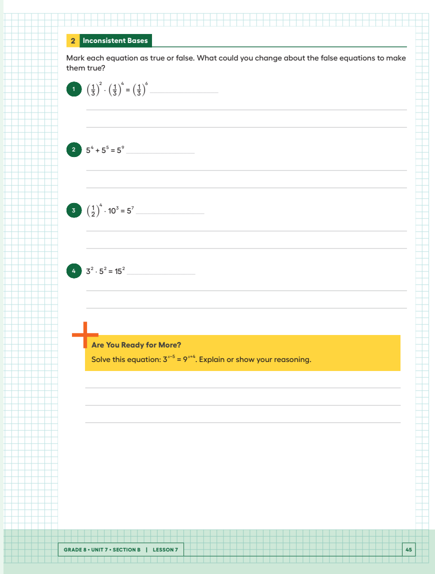
Launch

Display the false equation $2^3 \cdot 5^2 = 10^{3+2} = 10^5$ for all to see. Ask students whether they think the equation is true or false, and invite a few students to explain their reasoning. If not mentioned by students, expand $2^3 \cdot 5^2$ and 10^5 to show their repeated factors. Give students 4 minutes of quiet work time followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Student Workbook


2 Inconsistent Bases

Mark each equation as true or false. What could you change about the false equations to make them true?

Are You Ready for More?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

Student Task Statement

Mark each equation as true or false. What could you change about the false equations to make them true?

1. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^6$

   True because there are 6 factors that are $\frac{1}{3}$ on each side of the equation.

2. $5^4 + 5^5 = 5^9$

   False because $5^4 + 5^5 < 5^9$. Sample change: $5^4 \cdot 5^5 = 5^9$.

3. $\left(\frac{1}{2}\right)^4 \cdot 10^3 = 5^7$

   False because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 10 \cdot 10 \cdot 10 < 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$.

   Sample change: $\left(\frac{1}{2}\right)^3 \cdot 10^3 = 5^3$.

4. $3^2 \cdot 5^2 = 15^2$

   True because $3^2 \cdot 5^2 = 3 \cdot 3 \cdot 5 \cdot 5 = (3 \cdot 5)(3 \cdot 5) = 15 \cdot 15 = 15^2$.

Are You Ready for More?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

$x = -13$. Sample reasoning: Since $9 = 3^2$, the right side of the equation becomes $(3^2)^{x+4}$, or 3^{2x+8}. This means that $x - 5 = 2x + 8$, so $x = -13$.

Activity Synthesis

The goal of this discussion is for students to recognize that the exponent rules work because they capture patterns of repeated multiplication of a single base. Invite students to share their responses and display them for all to see. Consider involving more students in the whole-class discussion with the following questions:

"Who can restate ____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?

For the final question, ask students whether they think it is a coincidence that the equation is true, or if there is another, more general explanation. It is not necessary for students to generalize the relationship here since it will be addressed more fully in a later lesson.

Lesson Synthesis

The goal of this lesson is to reinforce student understanding of the exponent rules and for students to explain when and why the exponent rules don't work. Here are some questions for discussion:

- *"Why is the equation $2^5 \cdot 2^3 = 2^{15}$ false?"*

  Multiplying 5 factors that are 2 by 3 factors that are 2 results in a total of 8 factors that are 2. Multiplying the exponents doesn't make sense in this case.

- *"Why is the equation $\frac{3^5}{3^2} = 3^3$ true?"*

  Expanding the left side, we get $\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$, which is equal to $1 \cdot 3 \cdot 3 \cdot 3$ or just 3^3.

- *"Why is the equation $\frac{6^5}{3^2} = 2^3$ false? Why might someone make this mistake?"*

  Expanding the left side, we get $\frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}$, which is equal to $2 \cdot 2 \cdot 6 \cdot 6 \cdot 6$. Someone might make this mistake because they divide 6 by 3 and use the exponent rule for division to subtract the exponent in the denominator from the exponent in the numerator.

Responding To Student Thinking

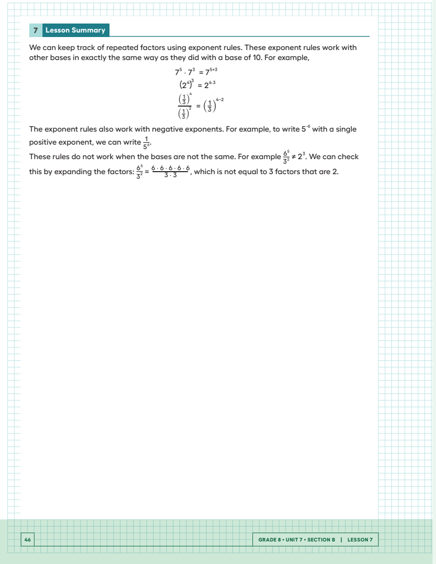
Press Pause

By this point in the unit, there should be some student mastery working with exponent rules. If most students struggle, make time to revisit related work in the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 7, Lesson 8, Practice Problem 1

Unit 7, Lesson 8, Practice Problem 2

Student Workbook



Lesson Summary

We can keep track of repeated factors using exponent rules. These exponent rules work with other bases in exactly the same way as they did with a base of 10. For example,

$$7^5 \cdot 7^3 = 7^{5+3}$$

$$(2^4)^3 = 2^{4 \cdot 3}$$

$$\frac{\left(\frac{1}{3}\right)^4}{\left(\frac{1}{3}\right)^2} = \left(\frac{1}{3}\right)^{4-2}$$

The exponent rules also work with negative exponents. For example, to write 5^{-6} with a single positive exponent, we can write $\frac{1}{5^6}$.

These rules do not work when the bases are not the same. For example $\frac{6^5}{3^2} \neq 2^3$. We can check this by expanding the factors: $\frac{6^5}{3^2} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}$, which is not equal to 3 factors that are 2.

Cool-down

Working with Exponents

5 min

Student Task Statement

1. Rewrite each expression using a single, positive exponent:

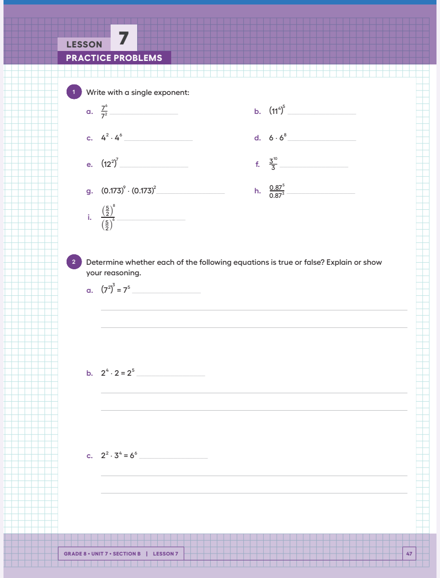
   a. $\frac{9^3}{9^9}$ $\frac{1}{9^6}$

   b. $14^{-3} \cdot 14^{12}$ 14^9

2. Diego wrote $6^4 \cdot 8^3 = 48^7$. Explain what Diego's mistake was and how you know the equation is not true.

   Sample response: Diego multiplied the bases and added their exponents. The equation is not true because 4 repeated factors that are 6 multiplied by 3 repeated factors that are 8 is much smaller than 7 repeated factors that are 48.

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1. Write with a single exponent:

a. $\frac{7^6}{7^2}$ ______ b. $(11^4)^5$ ______

c. $4^2 \cdot 4^6$ ______ d. $6 \cdot 6^8$ ______

e. $(12^2)^7$ ______ f. $\frac{3^{10}}{3}$ ______

g. $(0.173)^9 \cdot (0.173)^2$ ______ h. $\frac{0.87^5}{0.87^3}$ ______

i. $\frac{\left(\frac{5}{2}\right)^8}{\left(\frac{5}{2}\right)^6}$ ______

2. Determine whether each of the following equations is true or false? Explain or show your reasoning.

a. $(7^2)^3 = 7^5$ ______

b. $2^4 \cdot 2 = 2^5$ ______

c. $2^2 \cdot 3^4 = 6^6$ ______

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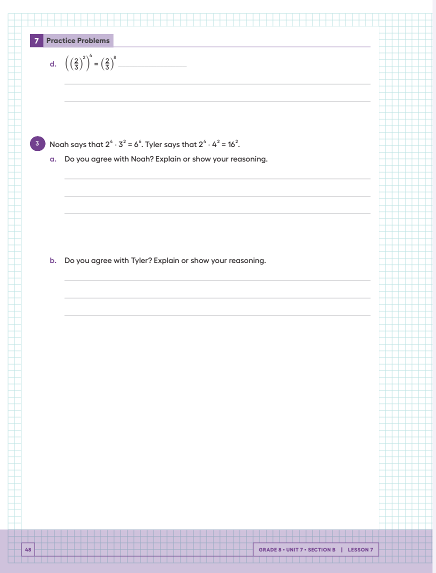
Student Workbook


7 Practice Problems

d. $\left(\left(\frac{2}{3}\right)^2\right)^4 = \left(\frac{2}{3}\right)^8$ ______

3. Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

a. Do you agree with Noah? Explain or show your reasoning.

b. Do you agree with Tyler? Explain or show your reasoning.

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Lesson 7

Practice Problems

4 Problems

Problem 1

Write with a single exponent:

a. $\frac{7^6}{7^2}$ 7^4

b. $(11^4)^5$ 11^{20}

c. $4^2 \cdot 4^6$ 4^8

d. $6 \cdot 6^8$ 6^9

e. $(12^2)^7$ 12^{14}

f. $\frac{3^{10}}{3}$ 3^9

g. $(0.173)^9 \cdot (0.173)^2$ 0.173^{11}

h. $\frac{0.87^5}{0.87^3}$ 0.87^2

i. $\frac{\left(\frac{5}{2}\right)^8}{\left(\frac{5}{2}\right)^6}$ $\left(\frac{5}{2}\right)^2$

Problem 2

Determine whether each of the following equations is true or false? Explain or show your reasoning.

a. $(7^2)^3 = 7^5$

False

Sample reasoning: $(7^2) \cdot (7^2) \cdot (7^2) = 7^6$

b. $2^4 \cdot 2 = 2^5$

True

Sample reasoning: $(2 \cdot 2 \cdot 2 \cdot 2) \cdot 2 = 2^5$

c. $2^2 \cdot 3^4 = 6^6$

False

Sample reasoning: $(2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$ is not the same as 6^6

d. $\left(\left(\frac{2}{3}\right)^2\right)^4 = \left(\frac{2}{3}\right)^8$

True

Sample reasoning: $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^8$

Problem 3

Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

a. Do you agree with Noah? Explain or show your reasoning.

Disagree

Sample reasoning: $2^4 \cdot 3^2 = 16 \cdot 9 = 144$, but 6^6 is much bigger than 144.

b. Do you agree with Tyler? Explain or show your reasoning.

Agree

Sample reasoning: $2^4 = 16$ and $4^2 = 16$, so $2^4 \cdot 4^2$ should equal $16 \cdot 16$ or 16^2.

Problem 4

from Unit 4, Lesson 13

Lin says that the system of equations $2x + 6y = 10$ and $3x + 9y = 15$ has an infinite number of solutions.

a. Do you agree with Lin? Explain your reasoning.

Yes I agree with Lin

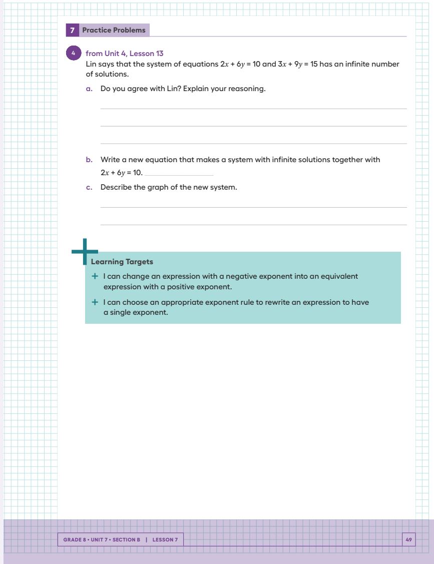
Sample reasoning: The second equation is 1.5 times the first equation, so they will have the same solutions.

b. Write a new equation that makes a system with infinite solutions together with $2x + 6y = 10$. Sample responses: $x + 3y = 5$, $4x + 12y = 20$

c. Describe the graph of the new system.

One line that is the same for both equations

Student Workbook



7 Practice Problems

4 from Unit 4, Lesson 13

Lin says that the system of equations $2x + 6y = 10$ and $3x + 9y = 15$ has an infinite number of solutions.

a. Do you agree with Lin? Explain your reasoning.

b. Write a new equation that makes a system with infinite solutions together with $2x + 6y = 10$. ______

c. Describe the graph of the new system.

Learning Targets

+ I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
+ I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

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