Comparing Populations with Friends

Goals

Coordinate (orally) visual displays of data with descriptions of shape, measures of center, and measures of spread.

- Determine what information is needed to solve problems about using samples to compare populations. Ask questions to elicit that information.
- Explain (orally and in writing) whether two populations are likely to be meaningfully different using center and spread.

Learning Target

I can decide what information I need to know to be able to compare two populations based on a sample from each.

Lesson Narrative

Students continue to practice comparing populations by using samples from each population. Outside of the classroom, people who wish to compare groups will not usually have all of the useful statistics presented to them in a nice package, so they will need to determine what information to gather, and then work through the comparison process. In this lesson, students are paired so that one student is presented with a situation and question while the other student has information to help solve the question. They must work together to answer the question by asking their own questions and explaining how each piece of information will be useful. An optional activity is also included in which students are asked to compare data from a sample of one population to statistics from a sample of a second population. By this part of the unit, students should be more precise in their language about statistics involving samples and populations.

Student Learning Goal

Let's ask important questions to compare groups.

Lesson Timeline



Warm-up



Activity 1



Activity 2



Lesson Synthesis

Access for Students with Diverse Abilities

• Action and Expression (Activity 1)

Access for Multilingual Learners

• MLR4: Information Gap Cards (Activity 1)

Instructional Routines

• MLR4: Information Gap Cards

Required Materials

Materials to Gather

· Math Community Chart: Activity 1

Materials to Copy

• Comparing Populations Cards (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

One copy of the blackline master from "Comparing Populations," cut into cards for every 2 students.

Assessment

Cool-down



Warm-up

Features of Graphic Representations



Activity Narrative

In this *Warm-up*, students review the useful information that can be gained from different graphical representations of data in preparation for comparing groups based on samples from each.

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For classes that may need help remembering the different representations, consider displaying an example of each type of graphical representation mentioned.

Student Task Statement

Dot plots, histograms, and box plots are different ways to represent a data set graphically.

Which of those displays would be the easiest to use to find each feature of the data listed here?

A. the mean

dot plot

B. the median

box plot

C.the mean absolute deviation

dot plot

D. the interquartile range

box plot

E. the symmetry

dot plot or histogram

Activity Synthesis

Poll the class for their answers to each of the problems. Select at least one student to share their reasoning for each question.

Activity 1

Info Gap: Comparing Populations



Activity Narrative

In this activity, students compare two groups using data from samples but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.



Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell them that during this activity they are going to choose a norm to focus on and practice. This norm should be one that they think will help themselves and their group during the activity. At the end of the activity, students can share what norm they chose and how the norm did or did not support their group.

Tell students that they will continue to work with comparing measures of center for groups. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

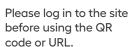
Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Instructional Routines

MLR4: Information Gap Cards

ilclass.com/r/10695522





Access for Multilingual Learners (Activity 1)

MLR4: Information Gap Cards.

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.

Supports accessibility for: Memory, Organization

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

- **1.** Silently read your card and think about what information you need to answer the question.
- 2. Ask your partner for the specific information that you need. "Can you tell me?"
- **3.** Explain to your partner how you are using the information to solve the problem. "I need to know _____ because ..."
 - Continue to ask questions until you have enough information to solve the problem.
- **4.** Once you have enough information, share the problem card with your partner, and solve the problem independently.
- 5. Read the data card, and discuss your reasoning.

Card I: Yes, there is a meaningful difference.

Sample reasoning: Since the data is not symmetric, it makes sense to use the median and IQR to compare the data. The medians are more than 2 IQRs apart (II2.5 - $65 > 2 \cdot 22.5$), so there is a meaningful difference between the flavors.

If your teacher gives you the data card:

- 1. Silently read your card. Wait for your partner to ask for information.
- **2.** Before telling your partner any information, ask, "Why do you need to know _____?"
- **3.** Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

These steps may be repeated.

- **4.** Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
- 5. Share the data card, and discuss your reasoning.

Card 2: No, there is not a meaningful difference.

Sample reasoning: The mean for Teacher A is 82.5, and the MAD is 5.5. The mean for Teacher B is 79, and the MAD is 7. So the difference in means is less than 2 MADs apart $(83 - 79 < 2 \cdot 7)$.

Are You Ready for More?

Is there a meaningful difference between top sports performance in two different decades? Choose a variable from your favorite sport (for example, home runs in baseball, kills in volleyball, aces in tennis, saves in soccer, etc.), and compare the leaders for each year of two different decades. Is the performance in one decade meaningfully different from the other?

Answers vary.

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"What was the most important question you asked for the first problem?"
For the second problem?"

"What are some other ways the information could have been given to solve the problems?"

Instead of the characteristics for the first question, a box plot could have been presented. The second question could have had a dot plot or characteristics like the first problem.

"If the distributions for the first problem had been symmetric, would the answer have been the same?"

Yes. The difference in means is 34.25, which is only I.6 MADs apart, so there would not have been enough information to say that the two population means are meaningfully different.

Highlight for students that it is important to have measures of center as well as measures of variability when comparing groups.

Invite 2–3 students to share the norm they chose and how it supported the work of the group or a realization they had about a norm that would have worked better in this situation. Provide these sentence frames to help students organize their thoughts in a clear, precise way:

\supset "I picked the norm '' It really he	elped me/my group because"
"I picked the norm '' During the	e activity, I realized the norm ''
would be a better focus because	"

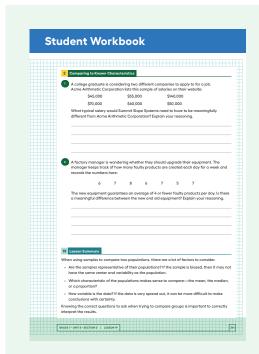
Activity 2: Optional

Comparing to Known Characteristics

15 min

Activity Narrative

In this optional activity, students compare two populations using samples again, but this time only one sample is given. For the other sample, the characteristics have either been computed already or are the focus of the question. This type of analysis is useful when comparing two similar populations as in this activity or when comparing a group against a standard.



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Keep students in groups of 2.

Tell students that sometimes it is useful to compare one group to a standard or another group where the important characteristics have already been computed. In these problems, a random sample from one group is given and characteristics of the second group is either given or sought.

Allow students 10 minutes of partner work time, and follow with a wholeclass discussion.

Student Task Statement

1. A college graduate is considering two different companies to apply to for a job. Acme Arithmetic Corporation lists this sample of salaries on their website:

\$45,000 \$55,000 \$140,000 \$70,000 \$60,000 \$50,000

What typical salary would Summit Slope Systems need to have to be meaningfully different from Acme Arithmetic Corporation? Explain your reasoning.

The median salary would have to be greater than 97,500 or less than 17,500. Sample reasoning: Since Acme Arithmetic Corporation has a large value for a salary far from the other values, the measure of center chosen should be the median. The median salary for Acme Arithmetic Corporation is 57,500, and the IQR is 20,000, so the median salary for Summit Slope Systems must be greater than 2 IQRs above or less than 2 IQRs below Acme's median (57, $500 + 2 \cdot 20,000 = 97,500$, and $57,500 - 2 \cdot 20,000 = 17,500$).

2. A factory manager is wondering whether they should upgrade their equipment. The manager keeps track of how many faulty products are created each day for a week and records the numbers here:

6 7 8 6 7 5

The new equipment guarantees an average of 4 or fewer faulty products per day. Is there a meaningful difference between the new and old equipment? Explain your reasoning.

Yes, there is a meaningful difference.

Sample reasoning: The mean for the sample for the current machine is 6.57 faulty products per day, and the MAD is 0.78 faulty products per day. The difference in means is 2.57 faulty products per day (since 6.57 – 4 = 2.57), which is greater than twice the MAD, so there is a meaningful difference in the mean number of faulty products per day.

Activity Synthesis

The purpose of the discussion is to help students understand how to compare groups when one set of characteristics are known and the other group is represented by sample data.

Select some groups to share their answers and reasoning for the two problems.

Consider asking these discussion questions:

- "How did you determine what to use as a typical value for the first problem?"
 - Since there is one value much greater than the others, the distribution is not symmetric, so median is a more appropriate measure of center.
- "How did you determine what measure of center to use for the second problem?"
 - Since the data are all close, either value could be used, but the new equipment reported the "average," or mean, so the mean should be used for the sample as well.
- "The manufacturer for the new equipment guarantees 4 flaws or fewer per day with the new equipment. If the new equipment produces only 3 flaws per day, does that change the answer for the second problem?"
 - No. There is an even greater difference between the current and new equipment, so it is even more meaningful.
- "What other factors would the college graduate want to consider other than the meaningful difference in median salary between the two companies?"
 - He should consider other factors such as benefits, relationship with coworkers, type of work being done at each company, and so on. The graduate should consider the salary for the type of job he will get at the company. For example, if his degree is in computer science, he may be looking at a job with computers rather than sales or some other department within the company, so he might be able to get a better comparison of salaries that way.
- "What other factors would the factory manager want to consider other than the meaningful difference in flaws for the equipment?"
 - The cost of the frequent flaws as well as the cost of the new equipment will probably factor into her decision to buy new equipment. The age of the current equipment and maintenance for older equipment compared to new equipment may also be important.

Lesson Synthesis

Ask students what information is important to collect when attempting to compare large groups and why each of these pieces of information is useful. Ask students if they can think of other situations in which it might be helpful to compare two large groups by generating a sample and collecting information.

Responding To Student Thinking

Press Pause

By this point in the unit, there should be some student mastery of comparing populations. If students struggle, make time to revisit the work of the activity referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 8, Lesson 18, Activity 2 Do They Carry More?

Lesson Summary

When using samples to compare two populations, there are a lot of factors to consider.

- Are the samples representative of their populations? If the sample is biased, then it may not have the same center and variability as the population.
- Which characteristic of the populations makes sense to compare—the mean, the median, or a proportion?
- How variable is the data? If the data is very spread out, it can be more difficult to make conclusions with certainty.

Knowing the correct questions to ask when trying to compare groups is important to correctly interpret the results.

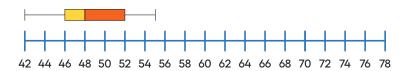
Cool-down

A Different Box Plot

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Student Task Statement

Use the box plot to answer the questions.



- **1.** What measure of center is shown in the box plot? What measure of variability? What are the values for each of these characteristics?
 - Median: 48, IQR: 6
- **2.** Draw another box plot with the same measure of variability that is meaningfully different from the one shown.

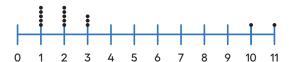
Correct responses show a box plot with an IQR of 6 and median greater than or equal to 60 (or less than or equal to 36).

Practice Problems

4 Problems

Problem 1

An agent at an advertising agency asks a random sample of people how many episodes of a TV show they watch each day. The results are shown in the dot plot.



The agency currently advertises on a different show but wants to change to this one as long as the typical number of episodes is not meaningfully less.

a. What measure of center and measure of variation would the agent need to find for their current show to determine if there is a meaningful difference? Explain your reasoning.

Median and IQR

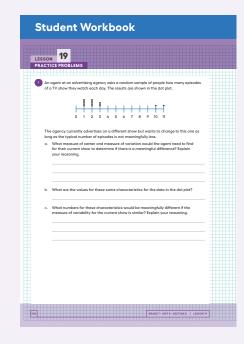
Sample reasoning: The dot plot shows a distribution that is not symmetric, so it would make sense to compare medians.

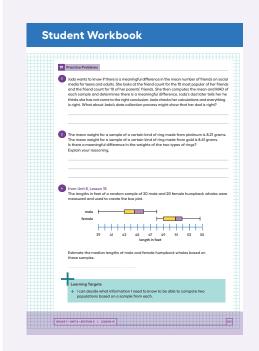
b. What are the values for these same characteristics for the data in the dot plot?

Median: 2 episodes, IQR: 2 episodes

c. What numbers for these characteristics would be meaningfully different if the measure of variability for the current show is similar? Explain your reasoning.

The other show would need to have a median of at least 6 episodes. Sample reasoning: The medians would need to be at least 2 IQRs apart, and $2 + 2 \cdot 2 = 6$.





Problem 2

Jada wants to know if there is a meaningful difference in the mean number of friends on social media for teens and adults. She looks at the friend count for the 10 most popular of her friends and the friend count for 10 of her parents' friends. She then computes the mean and MAD of each sample and determines there is a meaningful difference.

Jada's dad later tells her he thinks she has not come to the right conclusion. Jada checks her calculations and everything is right. What about Jada's data collection process might show that her dad is right?

Sample response: She did not select her samples randomly, so they may not be representative of teens and adults.

Problem 3

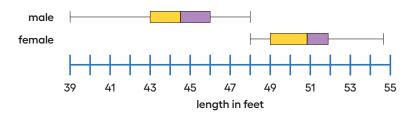
The mean weight for a sample of a certain kind of ring made from platinum is 8.21 grams. The mean weight for a sample of a certain kind of ring made from gold is 8.61 grams. Is there a meaningful difference in the weights of the two types of rings? Explain your reasoning.

The answer is unknown with this information. For example, if the MAD for each is 0.1 grams, then there would be a meaningful difference. If the MAD is greater than 0.2 grams, then there is not a meaningful difference.

Problem 4

from Unit 8, Lesson 15

The lengths in feet of a random sample of 20 male and 20 female humpback whales were measured and used to create the box plot.



Estimate the median lengths of male and female humpback whales based on these samples.

Males: 44.6 feet

• Females: 50.9 feet

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