Naming the Moves

Goal

Describe (orally and in writing) the movement of shapes informally and formally using the terms "clockwise," "counterclockwise," "translations," "rotations," and "reflections" of figures.

Learning Targets

- I can identify corresponding points before and after a transformation.
- I know the difference between translations, rotations, and reflections.

Student Learning Goal

Let's be more precise about describing moves of figures in the plane.

Lesson Narrative

In this lesson, students are introduced to the terms **translation**, **rotation**, and **reflection** and begin to describe these transformations with greater precision. The collective terms "transformation" and "rigid transformation" are not used until later lessons. Students are also introduced to the terms **clockwise** and **counterclockwise**. Students then use this language to identify the individual moves on various figures.

Students experiment with ways to describe moves precisely enough for another to understand their meaning.

Access for Students with Diverse Abilities

• Engagement (Activity 2)

Access for Multilingual Learners

 MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- · Notice and Wonder
- Take Turns

Required Materials

Materials to Gather

 Geometry toolkits: Warm-up, Activity 1

Materials to Copy

 Move Cards (1 copy for every 3 students): Activity 2

Required Preparation

Lesson:

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles.

Access to tracing paper is particularly important. Each student will need about 10 small sheets of tracing paper (commercially available "patty paper" is ideal). If using large sheets of tracing paper, such as 8.5 inches by 11 inches, cut each sheet into fourths.

Lesson Timeline







Activity 1



Activity 2



Lesson Synthesis

Assessment



Cool-down

Warm-up

Notice and Wonder: A Pair of Quadrilaterals



Activity Narrative

This is the first *Notice and Wonder* activity in the course. Students are shown an image and asked: "What do you notice? What do you wonder?"

Students are given time to write down what they notice and wonder about the image and then time to share their thoughts. Their responses are recorded for all to see. Often, the goal is to elicit observations and curiosities about a mathematical idea students are about to explore. Pondering the two open questions allows students to build interest about and gain entry into an upcoming task.

The purpose of this *Warm-up* is to describe a transformation, which will be useful when students describe transformations with more precise language in a later activity. While students may notice and wonder many things about this image, the angle of rotation and vocabulary used are the important discussion points.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

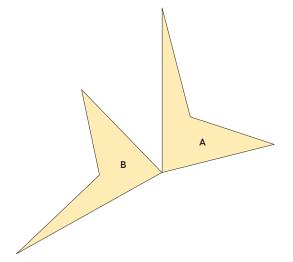
Launch 🙎

Arrange students in groups of 2. Display the two quadrilateral figures for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

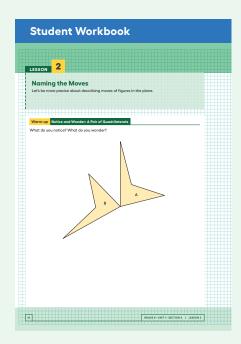
- You can turn Figure A to get Figure B.
- Figure A and Figure B are both quadrilaterals.

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.





Building on Student Thinking

Students may not be sure which angle to measure. They may measure the acute angle between Shape A and Shape B. Ask these students to trace Shape A on tracing paper and rotate it by that angle to see that this does not give Shape B.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.



- Figure A and B share a vertex.
- You cannot slide Figure A to get Figure B.
- The rotation is more than 90°.

Students may wonder:

- Can you get from Figure A to Figure B with only I move?
- Does this shape have a special name?
- Are the shapes the same?
- How many degrees is the rotation from Figure A to Figure B?
- · Which figure is the original?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If determining the angle of rotation does not come up during the conversation, ask students to discuss this idea.

Introduce or reiterate the language of **clockwise** (for rotating in the direction the hands on a clock move) and **counterclockwise** (for rotating in the opposite direction). In this case, the direction of rotation is not specified but it is natural to view Figure A being rotated counterclockwise onto Figure B. Make sure to introduce the language of the *center* of rotation (the vertex shared by A and B is the center of rotation).

Activity 1

How Did You Make That Move?

10 min

Activity Narrative

This activity informally introduces reflections, which appear in addition to some translations and rotations. Students are given a 6-frame cartoon showing the change in position of a polygon. As in the previous lesson, they describe the moves, but this time there are reflections. Students identify the new moves and try to describe them. Since the focus of this activity is informal descriptions, it is not necessary to introduce the formal terms at this time.

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Launch

Keep students in the same groups, and maintain access to geometry toolkits.

Give students 3 minutes of quiet work time, and then invite them to share their responses with their group.

Follow with a whole-class discussion. Tell students that this time there is a new type of move to look out for.

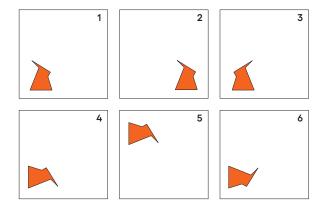
Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Ask students to suggest ways to update the display:

"Are there any new words or phrases that you would like to add?"
"Is there any language you would like to revise or remove?"

Encourage students to use the display as a reference.

Student Task Statement

Here is a set of dance moves.



1. Describe each move and say if it is a new type of move.

Sample responses:

a. Frame 1 to Frame 2

shift to the right

b. Frame 2 to Frame 3

new move, flip the picture from right to left

c. Frame 3 to Frame 4

turn 90° clockwise

d. Frame 4 to Frame 5

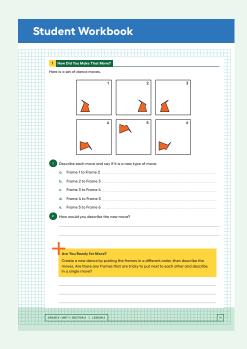
shift up

e. Frame 5 to Frame 6

new move, mirror the picture from top to bottom

2. How would you describe the new move?

The new move is like flipping or mirroring the picture. For the second move, the mirror is vertical and for the last move it is horizontal.



Building on Student Thinking

Students may see a reflection as a translation especially since the figures are not on the same frame. Ask these students to trace Frame 2 on tracing paper. Is there any way to turn it into Frame 3 by sliding it? What do they have to do to turn it into Frame 3? (They have to flip the tracing paper over, so this is a new kind of move.)

In describing reflections, students may confuse the terms horizontal and vertical. Consider posting the terms horizontal and vertical with examples in the room.

Are You Ready for More?

Create a new dance by putting the frames in a different order, then describe the moves. Are there any frames that are tricky to put next to each other and describe in a single move?

Sample response:

New order: 5, 6, 2, 1, 3, 4

Description: Flip down, turn the whole card a quarter turn counterclockwise, slide left, flip left to right in place, turn a quarter turn clockwise in place.

Frames I and 4 were difficult to put next to each other because they needed to turn and flip.

Activity Synthesis

Before students share their thinking:

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their descriptions of the new move. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

The purpose of this discussion is an initial understanding that there is a third type of move that is fundamentally different from the moves previously encountered, because it flips over. Some possible discussion questions are:

"How is the motion from Frame 2 to Frame 3 different from sliding or turning the shape?"

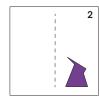
When the shape is flipped, you can't put it back without flipping it again.

- "Is there anywhere else that happens in this set of dance moves?"
 It happens twice, from 2 to 3 and from 5 to 6.
- "What features of the image help us to see that this move is happening?"

 If you put them across from each other, they look like mirror images.

 If you look at the sharp point on the shape, they are pointing in opposite directions.

Display this image with the vertical dotted line.



Ask students,

"What do you think this line represents for the move from 2 to 3?"
It is where the mirror is; it is the line that the shape flips over; it is a line of symmetry.

Use a transparency or tracing paper to demonstrate flipping or mirroring the figure, then ask students how this is different from rotating the figure 180°. Demonstrate the rotation so students can visualize the difference. If time allows, show Frame 5 and ask students where the mirror line or line of symmetry is to go to Frame 6.

It is a horizontal line below the figure.

Activity 2

Card Sort: Move



Activity Narrative

In this partner activity, students take turns identifying translations, rotations, and reflections. There are 3 translations, 3 rotations, and 3 reflections. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

As students work, monitor for groups who have sorted the cards into translations, rotations, and reflections (though not necessarily using those words). Also monitor for descriptions of corresponding points, such as "these points go together" or "here are before and after points."

Launch 🙎

Tell students that their job is to sort the cards into categories by the type of move they show. Explain how to set up and do the activity. If time allows, demonstrate these steps with a student as a partner:

- Mix up the cards and place them face-up.
- One person selects one card and explains to their partner how they know which move it is.
- The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement.
- When both partners agree on the description, they switch roles.

Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of pre-cut cards.

Give students about 8–10 minutes to sort the cards.

Do *not* explicitly instruct students at the beginning to use the words translations, rotations, and reflections. Follow with a whole-class discussion.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to sort the cards into categories according to the type of move they show. Be prepared to describe each category and why it is different from the others.

- 1. For each card, explain to your partner how you know which move it shows.
- For each card that your partner describes, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Translations: 1, 7, 8, 10 Rotations: 2, 6, 9, 12 Reflections: 3, 4, 5, 11

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.



Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.



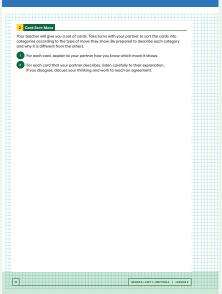
Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students the first 9 cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Student Workbook



Building on Student Thinking

Students may struggle to differentiate between the three moves, confusing reflections with either translations or rotations.

After they make their best decision, encourage these students to use tracing paper to justify their response. In Card 10, students may be confused when the translated figure overlaps the original. For Card 4, students may first think that this is a rotation (much like Cards 6 and 9). Encourage these students here to use tracing paper to check their answers.

Access for Multilingual Learners (Activity 2, Synthesis)

Speaking: MLR8 Discussion Supports.

Use this routine to support the introduction of new terms. As groups share how they categorized and sorted the shapes, revoice their ideas using the terms "translation," "rotation," and "reflection." Some students may benefit from practicing words or phrases or words in context through choral repetition.

Design Principle(s): Optimize output (for explanation)

Activity Synthesis

Once all groups have completed the Card Sort, discuss the following:

"Which cards were tricky to categorize? Explain why."

"Did you need to make adjustments to which category a card was sorted into? What might have caused an error? What adjustments were made?"

The purpose of this discussion is to develop shared vocabulary for the three moves.

Select one or more groups to share the names of their categories. Select one or more groups to share how they sorted the cards into the categories. Ask the class if they disagree with any of the choices, and give students opportunities to justify their reasoning.

Introduce the terms **translation**, **rotation**, and **reflection**. It may be helpful to display an example of each to facilitate discussion:







Alternatively, these applets can be displayed to facilitate discussion:

- Translation: The Geogebra applet 'Move Card Sort' is available here: https://www.geogebra.org/m/wYYvZH7A
- Rotation: The Geogebra applet 'Move Card Sort' is available here: https://www.geogebra.org/m/RUtdpQmN
- Reflection: The Geogebra applet 'Move Card Sort' is available here: https://www.geogebra.org/m/nKQmSnDW

Point out ways to identify which type of move is shown. Translations are a slide with no turning. Rotations are a turn. Reflections are a mirror image.

Lesson Synthesis

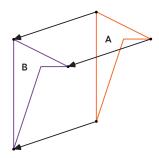
The purpose of this discussion is for students to clearly define "translation," "rotation," and "reflection." Ask students:

"We gave mathematical names to the three types of moves we have seen. What are they called?"

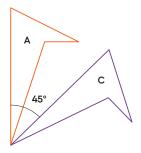
The "slide" is called a "translation," the "turn" is called a "rotation," and the "mirror image" is called a "reflection."

Create a display that shows these three terms and their definitions for reference throughout the unit.

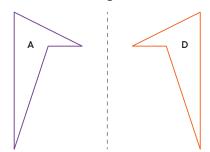
A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.

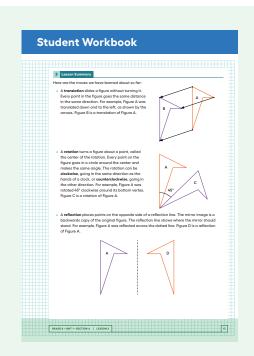


A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.

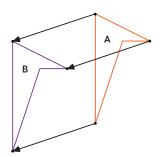




Lesson Summary

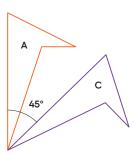
Here are the moves we have learned about so far:

A translation slides a figure without turning it. Every point in the figure
goes the same distance in the same direction. For example, Figure A was
translated down and to the left, as shown by the arrows. Figure B is a
translation of Figure A.

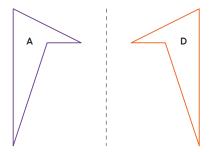


A rotation turns a figure about a point, called the center of the rotation.
 Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be clockwise, going in the same direction as the hands of a clock, or counterclockwise, going in the other direction.

 For example, Figure A was rotated 45° clockwise around its bottom vertex.
 Figure C is a rotation of Figure A.



• A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



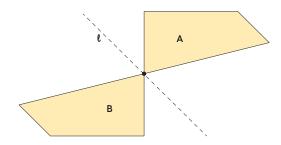
Cool-down

Is It a Reflection?



Student Task Statement

What type of move takes Figure A to Figure B?



Explain your reasoning.

Sample responses:

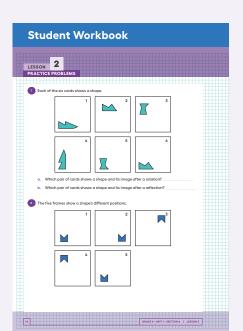
- The move is I rotation. If Figure A is turned around the point shared by Figures A and B, it can land on Figure B.
- The move is 2 reflections. If Figure A is flipped over line l and then flipped over again so that the shared points and angle line up, then it can land on Figure B.

Responding To Student Thinking

Points to Emphasize

If students struggle with identifying the type of transformation, revisit types of transformations as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about how they interpreted the data needed to make a transformation.

Unit 1, Lesson 3, Activity 2 Image Information



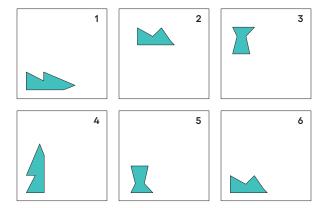
Practice Problems

2

3 Problems

Problem 1

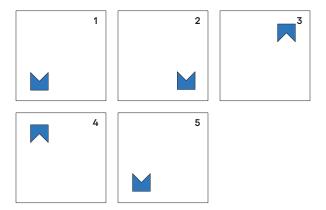
Each of the six cards shows a shape.



- **a.** Which pair of cards shows a shape and its image after a rotation? cards I and 4
- **b.** Which pair of cards shows a shape and its image after a reflection? cards 3 and 5

Problem 2

The five frames show a shape's different positions.



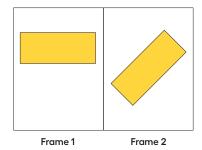
Describe how the shape could move to get from its position in each frame to

Sample response: To get from Position I to Position 2, the shape moves to the right. To get from Position 2 to Position 3, the shape flips over a horizontal line. To get from Position 3 to Position 4, the shape flips over a vertical line. To get from Position 4 to Position 5, the shape may be rotated 180 degrees about a point not on the polygon.

Problem 3

from Unit 1, Lesson 1

The rectangle seen in Frame 1 is rotated to a new position, seen in Frame 2.



Select **all** the ways the rectangle could have been rotated to get from Frame 1 to Frame 2.

- A. 40° clockwise
- **B.** 40° counterclockwise
- C. 90° clockwise
- D. 90° counterclockwise
- **E.** 140° clockwise
- F. 140° counterclockwise

