Lesson 2

Adjacent Angles

Goals

- Comprehend the terms "complementary" and "supplementary" (in spoken and written language) as they pertain to pairs of angles.
- Explain (orally and in writing) how to find an unknown angle measure, given adjacent complementary or supplementary angles.
- Generalize (orally) that when a straight angle or a right angle is decomposed, the measures of the resulting angles add up to 180° or 90°, respectively.

Learning Targets

- I can find unknown angle measures by reasoning about complementary or supplementary angles.
- I can recognize when adjacent angles are complementary or supplementary.

Lesson Narrative

In this lesson, students are introduced to the terms **complementary** and **supplementary** for describing specific angle relationships. They practice finding an unknown angle given the measure of another angle that is complementary or supplementary.

Students use repeated reasoning with many pairs of angles to conclude that a pair of complementary angles will have measures that add up to 90 degrees, and a pair of supplementary angles will have measures that add up to 180 degrees.

Student Learning Goal

Let's look at some special pairs of angles.

Access for Students with Diverse Abilities

- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1
- Protractors: Activity 1
- Scissors: Activity 1
- Straightedges: Activity 1

Materials to Copy

- Cutting Rectangles (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

Cut blank paper in half so that each student can have 2 half sheets of paper. It is very important that these cuts are completely straight and exactly perpendicular to the sides being cut for this activity to work.

Prepare to distribute scissors, straightedges, and protractors.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Warm-up

Estimating Angle Measures

5 min

Activity Narrative

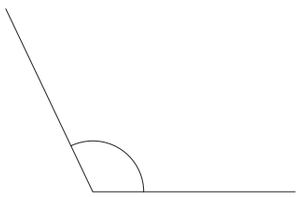
The purpose of this *Warm-up* is for students to estimate degree measures based on angles that are familiar. In the first two rows, an angle that is close to either a right angle or a straight angle is given, and students could use this as a reference angle for the other angles in the row.

Asking students to share estimates that are too low and too high gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it.

As student discuss with their partner, monitor for students who use phrases such as:

- "a little more than 90 degrees"
- "almost a straight line"
- "a little less than 360 degrees"

Launch



Display the image for all to see. Ask students to silently think of a number for the angle measure that they are sure is too low, a number that they are sure is too high, and a number that is about right, and to write these down.

Then, invite students to share their estimates and a short explanation for the reasoning behind their estimate.

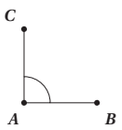
- Too low: 30 degrees, since that is an acute angle and this angle is obtuse.
- Too high: 170 degrees, since that would be almost a straight angle.
- About right: 115 degrees, since it's a little more than a right angle.

Arrange students in groups of 2. Do not supply protractors or pattern blocks; let students know that in this activity they are *estimating* the degree measure of each angle.

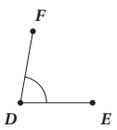
Give students 2 minutes of quiet work time followed by a partner and whole-class discussion.

Student Task Statement

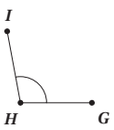
Estimate the degree measure of each indicated angle.



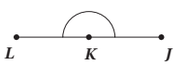
90 degrees



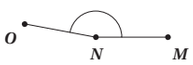
80 degrees



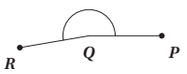
95 degrees



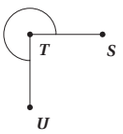
180 degrees



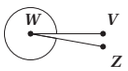
175 degrees



185 degrees



270 degrees



350 degrees

Sample responses:

- Angle *CAB* measures about 90 degrees, since it looks close to a right angle.
- Angle *FDE* measures about 80 degrees, since it's a little less than a right angle.
- Angle *GHI* measures about 95 degrees, since it's a little more than a right angle.
- Angle *JKL* measures about 180 degrees, since it looks like a straight angle.
- Angle *MNO* measures about 175 degrees, since it's a little less than a straight angle.
- Angle *PQR* measures about 185 degrees, since it's a little more than a straight angle.
- Angle *STU* measures about 270 degrees, since it looks like a right angle added to a straight angle.
- Angle *VWZ* measures about 350 degrees, since it's a little less than two straight angles added together.

Activity Synthesis

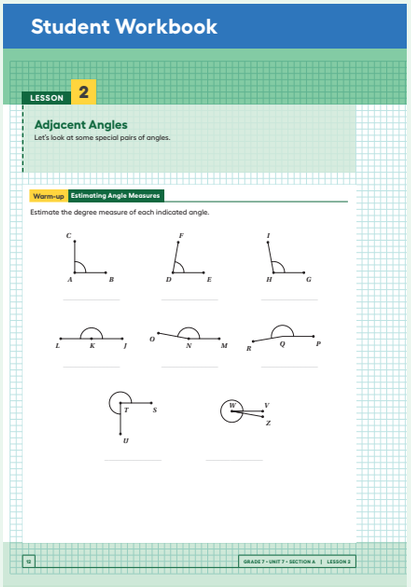
Display the angles for all to see. For each angle, ask a few students to share their estimate and their reasoning, and record the estimates near the displayed angle. If a student is reluctant to commit to an estimate, ask for a range of values. If time allows, ask students,

💬 *"Based on this discussion, does anyone want to revise their estimate?"*

After sharing estimates for the angle measures, ask students what tools they might use to know for sure how many degrees an angle measurement is.

A protractor is a tool to measure angles.

Student Workbook


LESSON 2

Adjacent Angles

Let's look at some special pairs of angles.

Warm-up Estimating Angle Measures

Estimate the degree measure of each indicated angle.

Ask students how they could use a protractor to find the measure of angles like STU or VWZ when the protractors usually only go to 180.

Find the measure of the angle that is less than 180, and subtract it from 360. Draw a straight angle from one side, then measure the remaining portion and add that measure to 180 degrees.

Activity 1
Cutting Rectangles

20 min

Activity Narrative

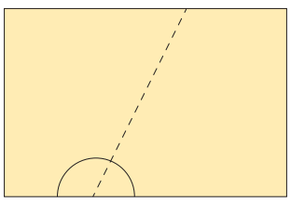
The purpose of this activity is to provide a tangible experience with complementary and supplementary angles, which will be formally defined in a later activity. Students cut sheets of paper in two ways to see the decomposition of straight and right angles. In later activities and lessons, students will continue working with the fact that specific angles can be composed to make straight or right angles as they make sense of the relationship between these pairs of angles. They use informal language to describe the relationships between angles in this activity.

For the whole-group discussion, select students who came up with a variety of angle measures, and ask them to share their angles pairs.

Launch

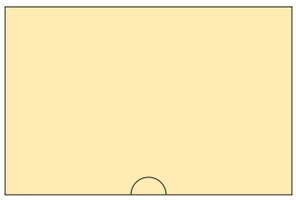
Distribute rectangles cut from copies of the blackline master so that each student receives one rectangle with a half-circle along one side and one rectangle with a quarter-circle in a corner. Provide access to the geometry toolkit. Give students 2–3 minutes of quiet work time, and then 3–5 minutes to work in their groups. Follow that with a whole-class discussion.

Consider demonstrating to students how to cut the paper, starting from the drawn arcs. Share that students can draw and cut any line, provided that the line starts in the center of the arc, is straight, and goes entirely through their paper.



Student Task Statement

Your teacher will give you two small, rectangular papers.



1. On the paper with a half-circle along one side, draw and cut a straight line that starts at the center of the half-circle and goes all the way across the paper, making 2 separate pieces. On each of these two pieces, measure the angle that is marked by the arc created by part of a circle. Label the angle measure on the piece.

2. On the rectangular paper with a quarter-circle in one of the corners, follow the same instructions as you cut, measure, and label the two angles marked by the arc created by part of a circle.
3. Record your group's measurements in the table:

Sample responses:

angles making a side	angles making a corner
40° 140°	40° 50°
27° 153°	36° 54°
70° 110°	53° 37°

4. What do you notice about these angle measures?

For the angles making a side of a rectangle, the two angle measures should have a sum of 180°. For the angles making a corner of a rectangle, the two angle measures should have a sum of 90°.

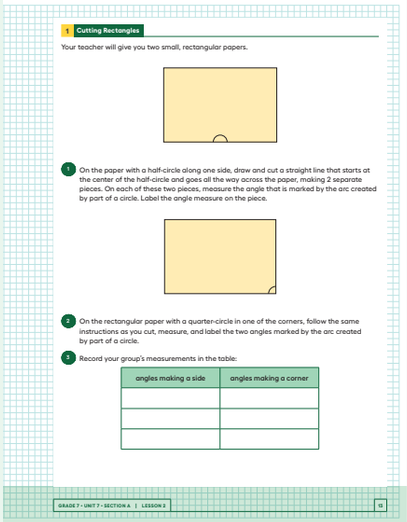
5. For the angles making a side of the rectangle, Clare measured 70 degrees on one of her pieces. Predict the angle measure of her other piece.

110°

6. For the angles making a corner of the rectangle, Priya measured 53 degrees on one of her pieces. Predict the angle measure of her other piece.

37°

Student Workbook



1 Cutting Rectangles

Your teacher will give you two small, rectangular papers.

1. On the paper with a half-circle along one side, draw and cut a straight line that starts at the center of the half-circle and goes all the way across the paper, making 2 separate pieces. On each of these two pieces, measure the angle that is marked by the arc created by part of a circle. Label the angle measure on the piece.
2. On the rectangular paper with a quarter-circle in one of the corners, follow the same instructions as you cut, measure, and label the two angles marked by the arc created by part of a circle.
3. Record your group's measurements in the table:

angles making a side	angles making a corner

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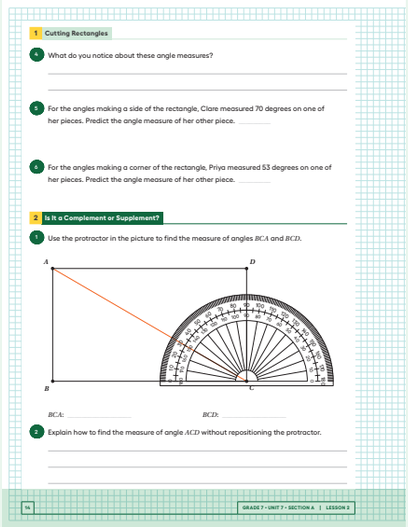
Student Workbook



1 Cutting Rectangles

4. What do you notice about these angle measures?

5. For the angles making a side of the rectangle, Clare measured 70 degrees on one of her pieces. Predict the angle measure of her other piece.

6. For the angles making a corner of the rectangle, Priya measured 53 degrees on one of her pieces. Predict the angle measure of her other piece.

2 Is It a Complement or Supplement?

1. Use the protractor in the picture to find the measure of angles *BCA* and *BCD*.

BCA: _____ *BCD*: _____

2. Explain how to find the measure of angle *ACD* without repositioning the protractor.

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Activity Synthesis

Invite previously selected students to display pairs of angles that make up the side of a rectangle. Ask,

- *"What did your groups notice about the angle pairs that made up the side of the rectangle? What do these angle pairs have in common?"*

  Each angle measure could be different, but the pair added up to 180 degrees. They make the side of a rectangle, which is a straight line.

Invite previously selected students to display pairs of angles that make up the corner of a rectangle. Ask,

- *"What did your groups notice about the angle pairs that made up the corner of the rectangle? What do these angle pairs have in common?"*

  Each angle measure could be different, but the pair added up to 90 degrees. They make the corner of a rectangle, which is a right angle.

Ask students to think about how they solved for the measure of Clare's second angle compared to how they solved for the measure of Priya's second angle.

- *"What was the same, and what was different?"*

  Since Clare's angles made up the side of the rectangle, they had to add up to 180 degrees, but Priya's had to add up to 90 degrees.

Activity 2

Is It a Complement or Supplement?

10 min

Activity Narrative

In this activity, students the generalize process for finding the measures of angles that are complements and supplements of angles with known measures, based on repeated observations of the relationships of these angles. After they have worked on the activity and shared their solutions, they are introduced to the vocabulary terms **complementary** and **supplementary.**

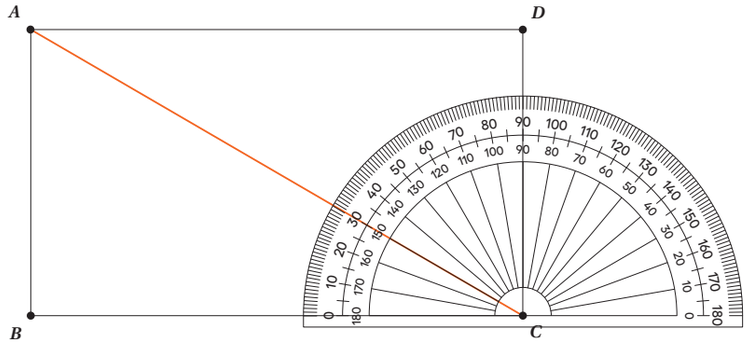
Monitor for students who use or explain different ways to calculate the unknown angle measure. For example, when finding the measure of angle KOM, some might write $38 + x = 180$ and some might write $180 - 38$.

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time, followed by a partner and whole-class discussion.

Student Task Statement

1. Use the protractor in the picture to find the measure of angles BCA and BCD.

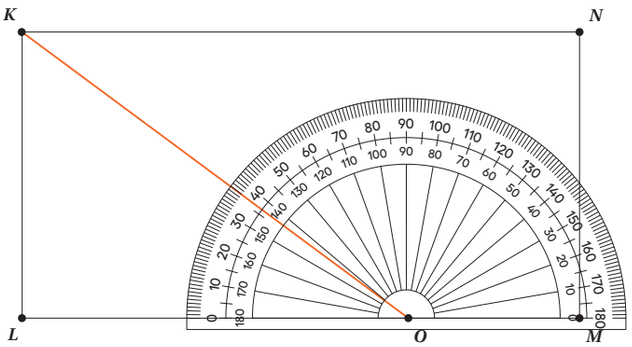


Angle BCA has a measure of 30 degrees. Angle BCD has a measure of 90 degrees.

2. Explain how to find the measure of angle ACD without repositioning the protractor.

Angle ACD has a measure of 60 degrees since it is the angle left when angle BCA is removed from angle BCD, and 90 − 30 = 60.

3. Use the protractor in the picture to find the measure of angles LOK and LOM.

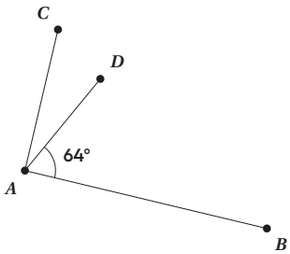


Angle LOK has a measure of 37 degrees. Angle LOM has a measure of 180 degrees.

4. Explain how to find the measure of angle KOM without repositioning the protractor.

Angle KOM is 143 degrees since that is what is left when removing angle LOK from angle LOM, and 180 − 37 = 143.

5. Angle BAC is a right angle. Find the measure of angle CAD.



26° since 90 − 64 = 26

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.
Maintain a visible display to record new vocabulary. Invite students to suggest details (words or pictures) that will help them remember the meaning of complementary angles and supplementary angles.
Supports accessibility for: Language, Memory

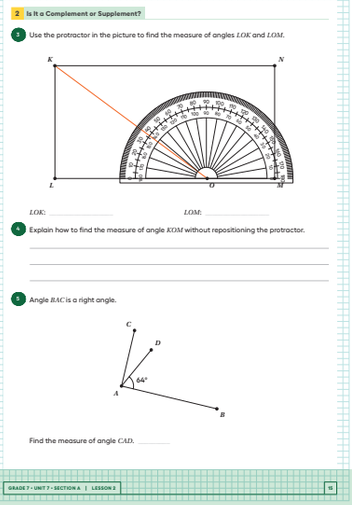
Building on Student Thinking

If students have trouble getting started, ask questions to help students notice angle relationships, such as:

"What do angles BCA and ACD have to add up to?"

"What angle relationships do you already know?"

Student Workbook


2 Is It a Complement or Supplement?

3 Use the protractor in the picture to find the measure of angles LOK and LOM.

LOK: _____ LOM: _____

4 Explain how to find the measure of angle KOM without repositioning the protractor.

5 Angle BAC is a right angle.

Find the measure of angle CAD. _____

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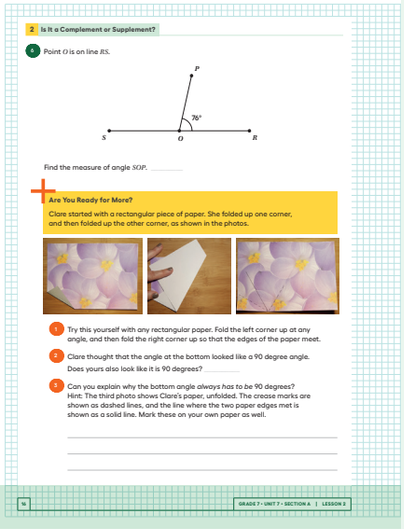
Student Workbook


2 Is It a Complement or Supplement?

6 Point O is on line RS.

Find the measure of angle SOP.

Are You Ready for More?

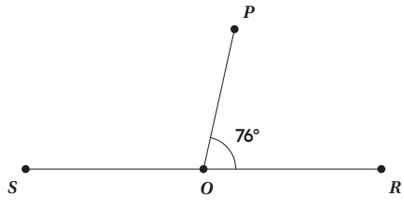
Clare started with a rectangular piece of paper. She folded up one corner, and then folded up the other corner, as shown in the photos.

1 Try this yourself with any rectangular paper. Fold the left corner up at any angle, and then fold the right corner up so that the edges of the paper meet.

2 Clare thought that the angle at the bottom looked like a 90 degree angle. Does yours also look like it is 90 degrees?

3 Can you explain why the bottom angle *always has to be* 90 degrees? Hint: The third photo shows Clare's paper, unfolded. The crease marks are shown as dashed lines, and the line where the two paper edges met is shown as a solid line. Mark these on your own paper as well.

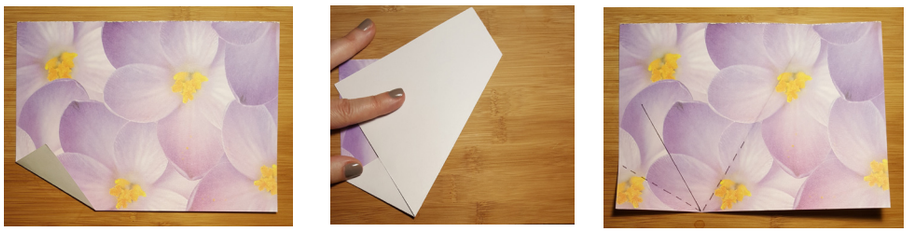
6. Point O is on line RS. Find the measure of angle SOP.



104° since 180 − 76 = 104

Are You Ready for More?

Clare started with a rectangular piece of paper. She folded up one corner, and then folded up the other corner, as shown in the photos.



1. Try this yourself with any rectangular paper. Fold the left corner up at any angle, and then fold the right corner up so that the edges of the paper meet.
2. Clare thought that the angle at the bottom looked like a 90 degree angle. Does yours also look like it is 90 degrees?
3. Can you explain why the bottom angle *always has to be* 90 degrees? Hint: The third photo shows Clare's paper, unfolded. The crease marks are shown as dashed lines, and the line where the two paper edges met is shown as a solid line. Mark these on your own paper as well.

Sample response:

Since they were made by folding, there are two sets of angles with equal measures. Label them $a°$ and $b°$. Since these four angles are adjacent and lie along a line, it must be true that $a + a + b + b = 180$, or $2a + 2b = 180$. Factoring gives $2(a + b) = 2 \cdot 90$. Therefore, it must be true that $a + b = 90$.

Activity Synthesis

The goal of this discussion is to introduce students to the terms complementary and supplementary for describing relationships between pairs of angles.

First, have students compare answers and strategies, with their partners, for the last two questions.

Next, display these images from the activity, and ask students which images are similar and in what way.

Figures 1 and 2 both show a protractor, and Figures 3 and 4 don't. Figures 1 and 3 show angles that add up to 90 degrees, and Figures 2 and 4 show angles that add up to 180 degrees.

Figure 1

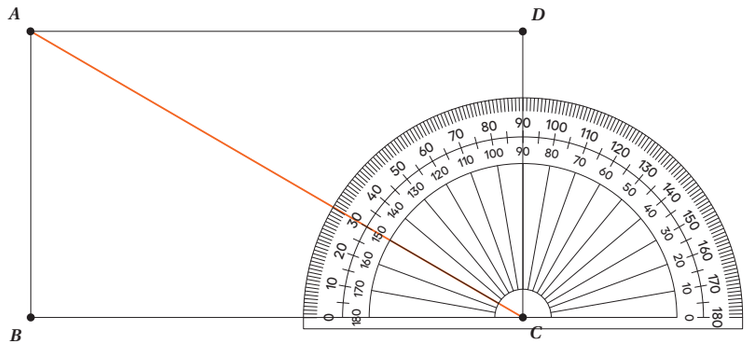


Figure 2

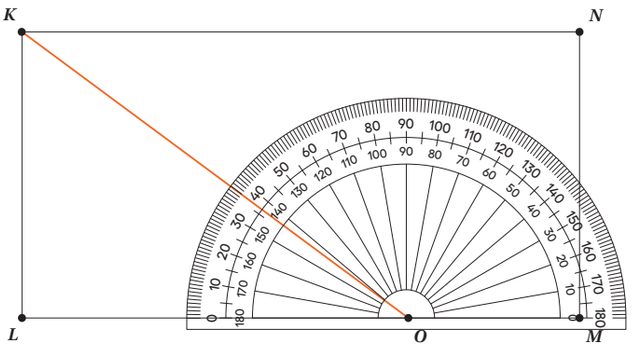


Figure 3

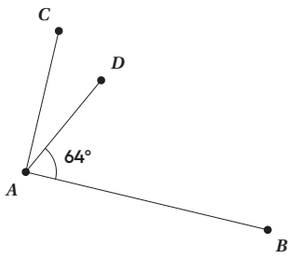
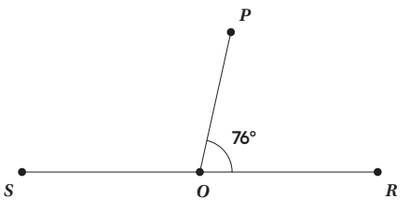


Figure 4



Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support student use of the words *complementary* and *supplementary*. Examples:

- "Angles ___ and ___ are complementary angles because ..." and
- "Angles ___ and ___ are supplementary angles because ..."

Some students may also benefit from choral repetition of these sentences. This will give all students an opportunity to practice these new words in context.
Advances: SELECT 2–3: Speaking, Writing, Conversing, Representing

Tell students that the term **complementary** describes a pair of angles whose measures have a sum of 90 degrees, and the term **supplementary** describes a pair of angles whose measures have a sum of 180 degrees. It is not important at this point to discuss that complementary or supplementary angles do not need to be adjacent, as that will be explored in the next lesson. Ask:

- *"Which angles in this activity were supplementary angles?"*

  angles *SOP* and *POR*, as well as angles *LOK* and *KOM*

- *"Which angles in this activity were complementary angles?"*

  angles *CAD* and *DAB*, as well as angles *ACD* and *BCA* or angles *DAC* and *BAC*, or even angles *OKN* and *OKL*

Invite students to continue practicing using the words complementary and supplementary throughout the rest of this unit, so they can start to feel more comfortable using them in their vocabulary.

Lesson Synthesis

Create a class display with the terms "complementary" and "supplementary." Ask students to describe, in their own words, what it means for two angles to be complementary.

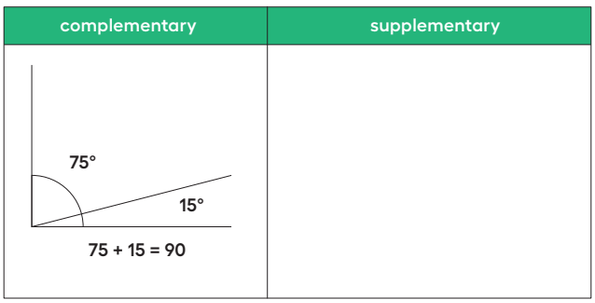
Their measures have a sum of 90°.

Draw a right angle under the term "complementary," and draw a ray that splits the angle into approximately 75 degrees and 15 degrees. Label the 75 degree angle, and ask students,

- *"If these 2 angles are complementary, and this angle is 75 degrees, what does the other angle have to be? How do you know?"*

  It is 15 degrees, since 75 + 15 = 90.

As students explain their thinking, annotate the diagram to look like this:



Next, ask students to describe, in their own words, what it means for two angles to be supplementary.

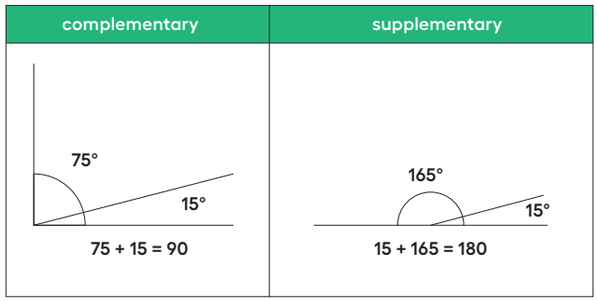
Their measures add up to 180°

Draw a straight angle under the term "supplementary," and draw a ray that splits the angle into approximately 165 degrees and 15 degrees. Label the 15 degree angle, and ask students,

💬 *"If these 2 angles are supplementary, and this angle is 15 degrees, what does the other angle have to be? How do you know?"*

It is 165 degrees, since 15 + 165 = 180.

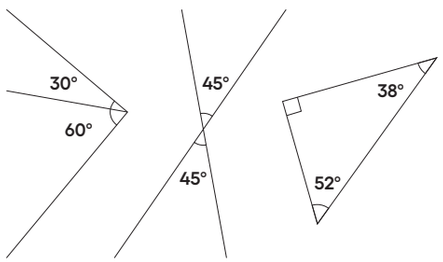
As students explain their thinking, annotate the diagram to look like this:



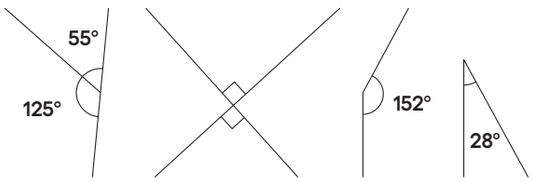
Keep the diagrams and terms displayed throughout the unit, and continue adding to this display as the unit progresses.

Lesson Summary

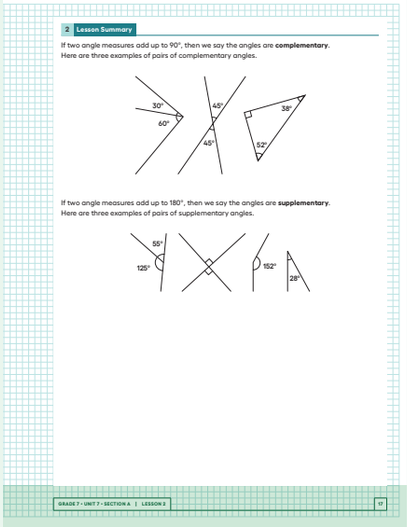
If two angle measures add up to 90°, then we say the angles are **complementary**. Here are three examples of pairs of complementary angles.



If two angle measures add up to 180°, then we say the angles are **supplementary**. Here are three examples of pairs of supplementary angles.



Student Workbook


2 Lesson Summary

If two angle measures add up to 90°, then we say the angles are **complementary**.
Here are three examples of pairs of complementary angles.

30° 60° 45° 45° 38° 52°

If two angle measures add up to 180°, then we say the angles are **supplementary**.
Here are three examples of pairs of supplementary angles.

55° 125° 152° 28°

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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

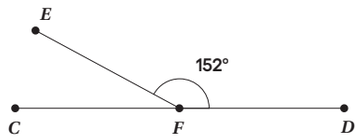
Finding Measurements

5 min

Student Task Statement

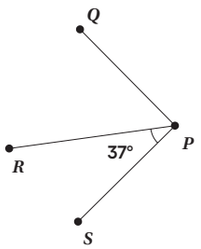
1. Point F is on line CD. Find the measure of angle CFE.

28°



2. Angle SPR and angle RPQ are complementary. Find the measure of angle RPQ.

53°



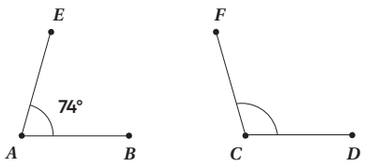
Lesson 2

Practice Problems

5 Problems

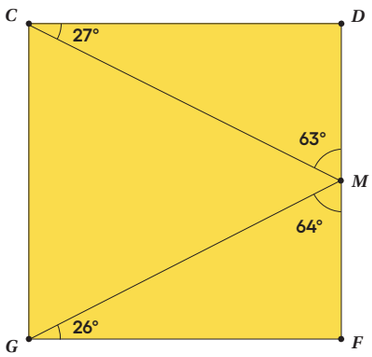
Problem 1

Angles A and C are supplementary. Find the measure of angle C.



106°

Problem 2



a. List two pairs of angles in square $CDFG$ that are complementary.

Any 2 of these pairs: Angles *DCM* and *MCG*, angles *MGF* and *MGC*, angles *MGF* and *GMF*, or angles *DCM* and *DMC*.

b. Name three angles whose measures add up to 180°.

Any 1 of these sets: Angles *DMC*, *CMG*, and *GMF*, angles *FGM*, *GMF*, and *MFG*, angles *CDM*, DMC, and *MCD*, or angles *MCG*, *CGM*, and *GMC*.

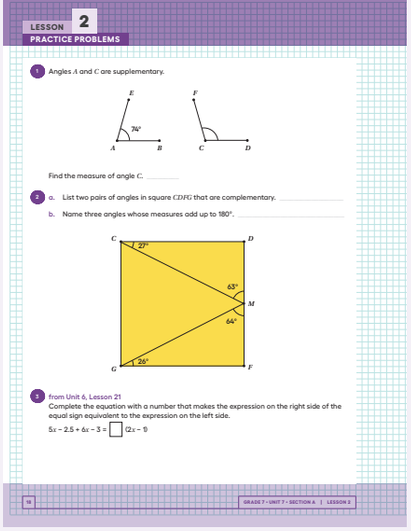
Problem 3

from Unit 6, Lesson 21

Complete the equation with a number that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

$5x - 2.5 + 6x - 3 =$ 5.5 $(2x - 1)$

Student Workbook


LESSON 2
PRACTICE PROBLEMS

1 Angles A and C are supplementary.

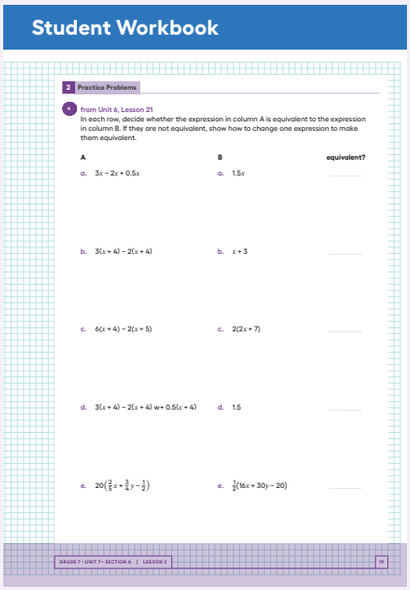
Find the measure of angle C. _____

2 a. List two pairs of angles in square $CDFG$ that are complementary. _____

b. Name three angles whose measures add up to 180°. _____

3 from Unit 6, Lesson 21
Complete the equation with a number that makes the expression on the right side of the equal sign equivalent to the expression on the left side.
$5x - 2.5 + 6x - 3 =$ ☐ $(2x - 1)$

Student Workbook



2 Practice Problems

A from Unit 6, Lesson 21

In each row, decide whether the expression in column A is equivalent to the expression in column B. If they are not equivalent, show how to change one expression to make them equivalent.

	A	B	equivalent?
a.	$3x - 2x + 0.5x$	$1.5x$	
b.	$3(x + 4) - 2(x + 4)$	$x + 3$	
c.	$6(x + 4) - 2(x + 5)$	$2(2x + 7)$	
d.	$3(x + 4) - 2(x + 4) + 0.5(x + 4)$	1.5	
e.	$20\left(\frac{2}{5}x + \frac{3}{4}y - \frac{1}{2}\right)$	$\frac{1}{2}(16x + 30y - 20)$	

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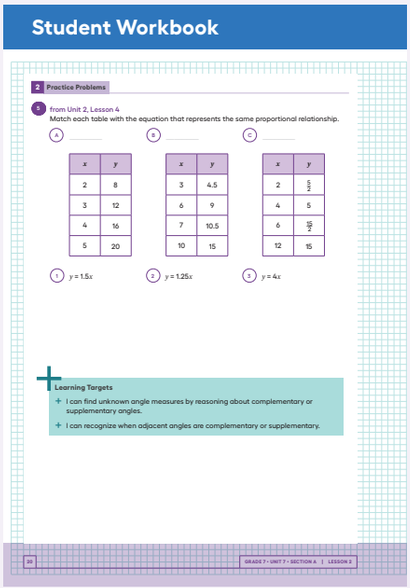
Student Workbook



2 Practice Problems

B from Unit 2, Lesson 4

Match each table with the equation that represents the same proportional relationship.

A

x	y
2	8
3	12
4	16
5	20

B

x	y
3	4.5
6	9
7	10.5
10	15

C

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

1. $y = 1.5x$ 2. $y = 1.25x$ 3. $y = 4x$

Learning Targets

+ I can find unknown angle measures by reasoning about complementary or supplementary angles.
+ I can recognize when adjacent angles are complementary or supplementary.

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Problem 4

from Unit 6, Lesson 21

In each row, decide whether the expression in column A is equivalent to the expression in column B. If they are not equivalent, show how to change one expression to make them equivalent.

	A	B
a.	$3x - 2x + 0.5x$	**a.** $1.5x$
	Equivalent	
b.	$3(x + 4) - 2(x + 4)$	**b.** $x + 3$
	Not equivalent	
	Answers vary. Sample responses: Change the column B entry to $x + 4$, change the column A entry to $3(x + 4) - 2(x + 4) - 1$	
c.	$6(x + 4) - 2(x + 5)$	**c.** $2(2x + 7)$
	Equivalent	
d.	$3(x + 4) - 2(x + 4) + 0.5(x + 4)$	**d.** 1.5
	Not equivalent	
	Answers vary. Sample response: Change the column B entry to $1.5(x + 4)$.	
e.	$20\left(\frac{2}{5}x + \frac{3}{4}y - \frac{1}{2}\right)$	**e.** $\frac{1}{2}(16x + 30y - 20)$
	Equivalent	

Problem 5

from Unit 2, Lesson 4

Match each table with the equation that represents the same proportional relationship.

A.

x	y
2	8
3	12
4	16
5	20

B.

x	y
3	4.5
6	9
7	10.5
10	15

C.

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

1. $y = 1.5x$ **2.** $y = 1.25x$ **3.** $y = 4x$