Lesson 9

Linear Models

Goals

- Compare and contrast (orally and in writing) different linear models of the same data, and determine (in writing) the range of values for which a given model is a good fit for the data.
- Create a model of a nonlinear data using a linear function, and justify (orally and in writing) whether the model is a good fit for the data.

Learning Targets

- I can decide when a linear function is a good model for data and when it is not.
- I can use data points to model a linear function.

Lesson Narrative

In this lesson, students use linear functions to model real-world situations. In the *Warm-up*, they are given data for an almost linear relationship and develop a linear model. They use their model to make predictions and discuss the reasonableness of the model. Then students consider a situation, the amount of the moon that appears illuminated from Earth over time, that they can make a linear model for, but it isn't very accurate. In the garbage recycling activity, different linear models apply to different time periods.

When given data isn't perfectly linear, students have to determine if a linear model is reasonable and what types of restrictions the model might need. The work here leads to future work modeling with piecewise linear functions.

The third activity is optional and offers additional practice with a situation that seems linear at first glance but is not. This activity works best when each student has access to a device that can run the applet, which allows students to plot values accurately.

Student Learning Goal

Let's model situations with linear functions.

Lesson Timeline

10 min	10 min	10 min	10 min	10 min	5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Assessment: Cool-down

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR6: Three Reads (Activity 2)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- Poll the Class

Required Materials

Materials to Gather

- Straightedges: Activity 3

Required Preparation

Warm-up:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Warm-up
Candlelight

10 min

Activity Narrative

There is a digital version of this activity.

In this *Warm-up*, students work with data to determine if the situation represented by the data could be modeled by a linear function. Students are given three different data points and use what they know about linear relationships to estimate when the candle will burn out.

In the digital version of the activity, students use an applet to visualize the height of the candle at different times. The applet allows students to plot points quickly and accurately without having to set up the axes from scratch.

Launch

Arrange students in groups of 2. Display the problem stem for all to see. Give students 30 seconds to make a guess at when the candle will burn out completely, then poll the class, displaying their responses for all to see.

Students should work with their partner on the questions. If they don't agree, partners should work to understand each other's thinking. If any students attempt to guess a linear equation that fits the data, ask them to share during the discussion. Follow with a whole-class discussion.

Student Task Statement

A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.

1. When do you think the candle will burn out completely?

   Sample response: Since it burns about 2 inches every hour, it will burn out between 5 and 6 hours after it was lit.

2. Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

   The height of a candle is a function of time because at any given time, the candle will have one and only one height. Sample reasoning: It is not exactly linear, although it looks close enough to use a linear function since the rate of burning is almost constant (2 inches per hour).

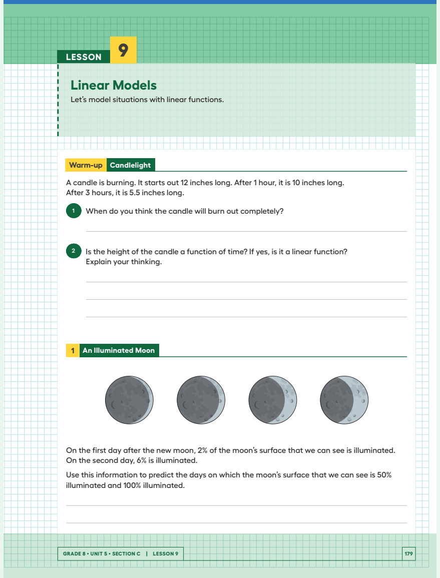
Instructional Routines

Poll the Class

ilclass.com/r/10694985

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 9

Linear Models

Let's model situations with linear functions.

Warm-up Candlelight

A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.

1 When do you think the candle will burn out completely?

2 Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

1 An Illuminated Moon

On the first day after the new moon, 2% of the moon's surface that we can see is illuminated. On the second day, 6% is illuminated.

Use this information to predict the days on which the moon's surface that we can see is 50% illuminated and 100% illuminated.

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Activity Synthesis

The purpose of this discussion is for students to justify how this situation can be modeled by a linear equation. Select students who answered yes to the last question, and ask:

- *"Was the data exactly linear? If not, what made you decide that you could treat it as such?"*

  I couldn't draw a line exactly through all three points, so it isn't linear, but my line is very close to going through all three that it seems like a decent estimate.

- *"What was the slope between the first two data points? What was the slope between the last two data points? What does it mean that their slopes are different?"*

  The slope from 0 to 1 hour is 2 inches per hour, while the slope from 1 hour to 3 hours is 2.25 inches per hour. Different values mean all three points are not on a straight line.

- *"Did your estimate of when the candle would burn out increase, decrease, or stay the same?"*

  I first guessed 6 hours since the candle starts at 12 inches and loses 2 inches of height in the first hour. The height after 3 hours makes me think it will burn out sooner, so my guess decreased.

Tell students that although the data is not precisely linear, it does make sense to model the data with a linear function because the points resemble a line when graphed. We can then use different data points to help predict when the candle would burn out. Answers might vary slightly, but it results in a close approximation.

Conclude the discussion by asking students to reconsider the range of values posted earlier for the first question, and ask if they think that range is acceptable or if it needs to change (for example, students may now think the range should be smaller after considering the different slopes).

Activity 1
An Illuminated Moon

10 min

Activity Narrative

The purpose of this activity is for students to consider a situation that seems like it could be modeled with a linear function from the limited information given, but where a linear model fails with just a bit more data. While study of periodic functions is in high school, students' familiarity with the cyclical nature of the moon should be enough for them to make sense of the non-linear nature of the situation.

Launch

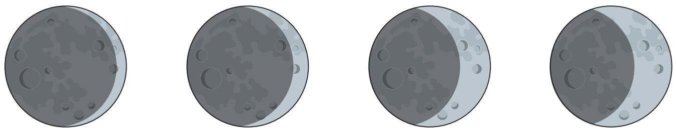
Arrange students in groups of 2. Tell students to close their student workbooks or devices, and display the image for all to see. Ask, “What do you know about the moon?” After a brief quiet think time, invite students to share, recording responses near the image. While students may know many things about the moon, highlight any information shared about moon phases and how the amount of the moon that reflects sunlight changes each day.

If the fact that a full cycle of the moon is about 28 days or that it takes about 14 days for a moon to go from “new” to “full” is shared, students may use this information in the *Task Statement* instead of making their prediction using the data provided. Invite students to use the data to see how close the two numbers are.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Provide students with a graphic organizer, such as a two-column table or a graph, to record relationships between the day and the percent of illumination to help make their estimate and decide if the relationship is linear.
Supports accessibility for: Visual-Spatial Processing, Organization

Student Task Statement

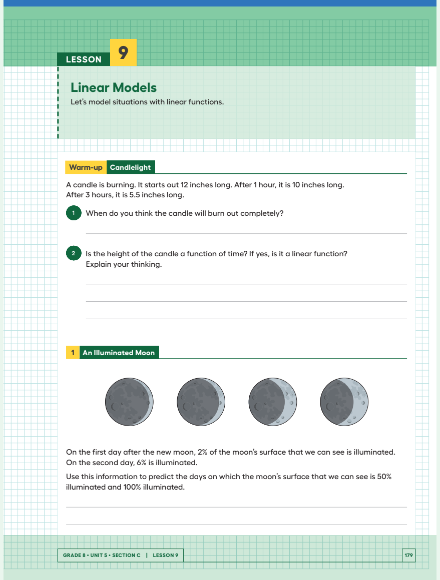


On the first day after the new moon, 2% of the moon’s surface that we can see is illuminated. On the second day, 6% is illuminated.

Use this information to predict the days on which the moon’s surface that we can see is 50% illuminated and 100% illuminated.

Sample response: If the illumination increases by 4% every day, then 11 days after Day 2, it reaches 50%. Then 13 days after that, the illumination reaches 100%. This gives a prediction of Day 13 for 50% and Day 26 for 100%.

Student Workbook



LESSON 9

Linear Models

Let’s model situations with linear functions.

Warm-up Candlelight

A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.

1 When do you think the candle will burn out completely?

2 Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

1 An Illuminated Moon

On the first day after the new moon, 2% of the moon’s surface that we can see is illuminated. On the second day, 6% is illuminated.

Use this information to predict the days on which the moon’s surface that we can see is 50% illuminated and 100% illuminated.

Activity Synthesis

The goal of this discussion is for students to understand that while we can create a linear function out of 2 data points, doing so does not always lead to accurate predictions. Here are some questions for discussion:

- *“The Moon’s surface viewed from Earth is actually 100% illuminated on day 14. Does this agree with the prediction you made?”*

  No, I assumed a 4% increase each day which means the Moon would be 100% illuminated on day 26.

- *“Is the percentage illumination of the Moon’s surface a linear function of the day?”*

  No, the amount of the Moon’s surface that is illuminated increases and decreases repeatedly over a year about every 28 days. That is not linear.

Tell students that if we only use two data points, it is always possible to model a situation with a linear function. We need additional data to help us determine if a linear model is appropriate. In this case, we have observed ourselves that the moon phases go in cycles that repeat over an over again, which isn’t something a linear function is meant to model.

Activity 2: Optional

Shadows

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to determine if a given set of data can be modeled by a linear function. Students first view a set of pictures and data for the length of a shadow at 0, 20, and 60 minutes. Then, they make a prediction about how long a shadow will be after 95 minutes. Students then compare their estimate with the actual length of the shadow and make conclusions about the model they used to make their estimate.

Monitor for students using different strategies to make their prediction. For example, students may use different pairs of points to make their prediction, or they may try to use all three. The discussion for this activity focuses on how if we are given two input-output pairs, we can always find a linear function with these inputs and outputs, but that doesn't mean a linear function is actually appropriate for the situation. In this case, we can see when we get more data that a linear function is not appropriate.

In the digital version of the activity, students use an applet to estimate the length of the shadow. The applet allows students to plot points quickly and accurately. This activity works best when students have access to the applet because the points are close to linear but make a distinct downward curve.

Launch

Arrange students in groups of 2. Tell students to close their student workbooks or devices, and display the image and given data for all to see.

Give students 1–2 minutes of quiet think time to estimate the length of the shadow after 95 minutes and discuss their responses with their partner.

Encourage partners to discuss their estimation strategy and why their estimate makes sense. Invite groups to share their estimate and reasoning with the whole class.

Tell students to open their student workbooks or devices, and give work time for the remaining questions. Follow with a whole-class discussion.

Access for Multilingual Learners (Activity 2, Launch)

MLR6: Three Reads.
Keep student workbooks or devices closed. Display only the problem stem and the three images, without revealing the questions.

"We are going to read these sentences 3 times."

After the 1st read:

"Tell your partner what this situation is about."

After the 2nd read:

"List the quantities. What can be counted or measured?"

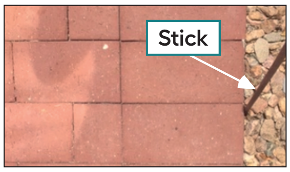
For the 3rd read: Reveal and read the questions. Ask,

"What are some ways we might get started on this?"

Advances: Reading, Representing

Student Task Statement

When the sun was directly overhead, the stick had no shadow.
After 20 minutes, the shadow was 10.5 centimeters long. After 60 minutes, it was 26 centimeters long.



1. Use this information to estimate how long it will be after 95 minutes.

   Sample response: If we model this with the linear function that goes through $(0,0)$ and $(20,10.5)$, we would predict that the length would be growing at a rate of $\frac{10.5}{20}$ centimeters per minute. After 95 minutes, this would give a prediction of about 50 centimeters since $95 \cdot \frac{10.5}{20} \approx 49.88$.

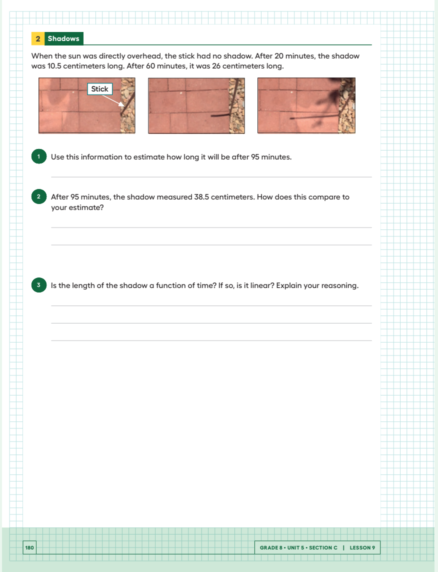
2. After 95 minutes, the shadow measured 38.5 centimeters. How does this compare to your estimate?

   Sample response: The prediction we made overestimated the length by about 11.4 centimeters.

3. Is the length of the shadow a function of time? If so, is it linear? Explain your reasoning.

   The length of the shadow is a function of time since every time determines only one length. It is not a linear function of the time, since the points $(0,0)$, $(20,10.5)$, $(60,26)$ and $(95,38.5)$ do not lie on any one line. Based on the answer to the last part, it is not even very well approximated by a linear function.

Student Workbook


2 Shadows

When the sun was directly overhead, the stick had no shadow. After 20 minutes, the shadow was 10.5 centimeters long. After 60 minutes, it was 26 centimeters long.

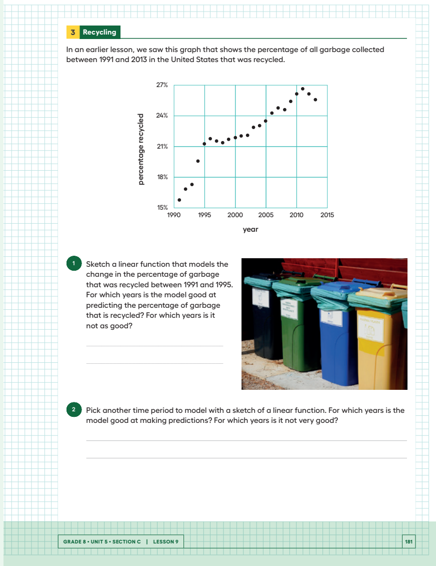
1. Use this information to estimate how long it will be after 95 minutes.
2. After 95 minutes, the shadow measured 38.5 centimeters. How does this compare to your estimate?
3. Is the length of the shadow a function of time? If so, is it linear? Explain your reasoning.

Activity Synthesis

Select groups that had different strategies for making their original prediction to share their reasoning about whether or not a linear model is a good fit for predicting the length of the shadow. In particular, make sure it is pointed out how much using the rate of change determined by the first two points overpredicts the length of the shadow after 95 minutes.

If not brought up by students, ask, “What will the length of the shadow be near sunset? After sunset?” (Near sunset the shadow will be very long, and after sunset there will not be a shadow.) Emphasize that over the short term, a linear model might be useful even for nonlinear data, but if the actual situation isn’t considered, like here where the sun will eventually set, conclusions will be wrong if we try to force a linear model.

Student Workbook


3 Recycling

In an earlier lesson, we saw this graph that shows the percentage of all garbage collected between 1991 and 2013 in the United States that was recycled.

1. Sketch a linear function that models the change in the percentage of garbage that was recycled between 1991 and 1995. For which years is the model good at predicting the percentage of garbage that is recycled? For which years is it not as good?

2. Pick another time period to model with a sketch of a linear function. For which years is the model good at making predictions? For which years is it not very good?

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Activity 3

Recycling

10 min

Activity Narrative

The purpose of this activity is for students to approximate different parts of a graph with an appropriate line segment. While they have seen previously that not everything can be modeled with a linear function, students see in this activity that it is possible to model pieces of a situation but that the model should not be stretched beyond the piece it is meant for. This activity prepares for work with piecewise linear functions in a future lesson.

The graph is one students encountered in a previous activity, but here students interact with it differently by sketching a linear function that models a certain part of the data. They take this model and consider its ability to predict inputs and outputs for other parts of the graph. This helps students think about subsets of data that might have different models from other parts of the data.

Launch

Arrange students in groups of 2. Provide students with access to straightedges.

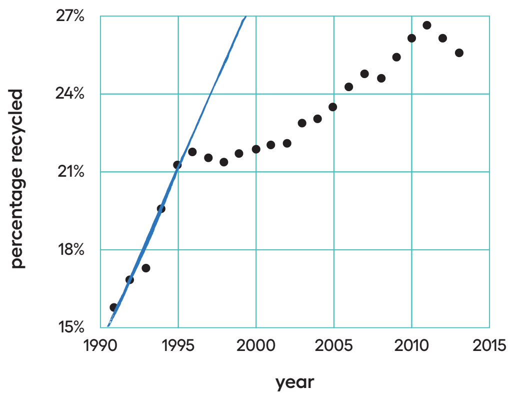
Give students 3–5 minutes of quiet work time and then time to share their responses with their partner.

Follow with a whole-class discussion.

As students work, select students who draw in different lines for the first question to share during the *Activity Synthesis*.

Student Task Statement

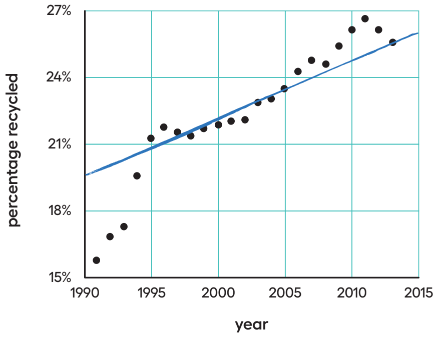
In an earlier lesson, we saw this graph that shows the percentage of all garbage collected between 1991 and 2013 in the United States that was recycled.



1. Sketch a linear function that models the change in the percentage of garbage that was recycled between 1991 and 1995. For which years is the model good at predicting the percentage of garbage that is recycled? For which years is it not as good?

   Sample response: The linear function displayed is a reasonable approximation of the data between 1991 and 1996, and a bad approximation from then on.

2. Pick another time period to model with a sketch of a linear function. For which years is the model good at making predictions? For which years is it not very good?



Sample response: The linear function displayed is a reasonable approximation of the data between 1994 and 2013, but a bad approximation from 1991 to 1993.

Activity Synthesis

The purpose of this discussion is for students to understand that although they might find a good model for one part of a graph, that does not mean that model will work for other parts.

Select previously identified students to share their models. Display these for all to see throughout the entire discussion. Questions for discussion:

"How much does your model for 1991 to 1995 overestimate the percent recycled in 1996? In 1997?"

"If we drew in a single line to model 1997 to 2013, what would that line predict well? What would that line predict poorly?"

A single line modeling those years would reasonably predict the percent recycled from 1997 to 2010, but it wouldn't be able to show how the percent recycled from 2011 to 2013 is decreasing.

Conclude the discussion by telling students that there is a trade-off between the accuracy of the model and the number of years to include in the interval. We could "connect the dots" and be accurate about everything, but then our model has limited use and is complicated with so many parts. (Just imagine writing an equation for each piece!)

Lesson Synthesis

Tell students that a mathematical *model* is a mathematical object, like an equation, a function, or a geometric figure, that we use to represent a real-life situation. Sometimes a situation can be modeled by a linear function. We have to use judgment about whether this is a reasonable thing to do based on the information we are given. We must also be aware that the model makes imprecise predictions, or it may only be appropriate for certain ranges of values.

Give students 1–2 minutes to think of a situation that may seem linear but actually is not.

Invite them to share their situations. For example, the height of humans may look linear for short periods of time, but eventually growth stops, so we wouldn't want to use a linear model for height over a large period of time.

Student Workbook


3 Recycling

In an earlier lesson, we saw this graph that shows the percentage of all garbage collected between 1991 and 2013 in the United States that was recycled.

1 Sketch a linear function that models the change in the percentage of garbage that was recycled between 1991 and 1995. For which years is the model good at predicting the percentage of garbage that is recycled? For which years is it not as good?

2 Pick another time period to model with a sketch of a linear function. For which years is the model good at making predictions? For which years is it not very good?

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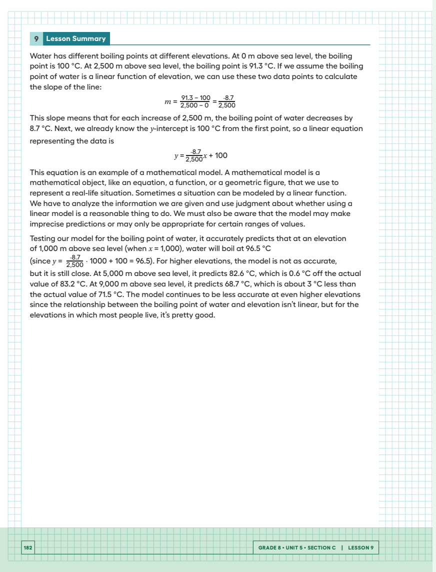
Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports.
For each model that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Student Workbook



Responding To Student Thinking

Points to Emphasize
If most students struggle with identifying a single linear model, revisit linear models as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about the lines they drew and the meaning of the slopes for each piece. Also highlight the question in the *Activity Synthesis* asking about using the information to make predictions.

Grade 8, Unit 5, Lesson 10, Activity 1
Modeling Recycling

Lesson Summary

Water has different boiling points at different elevations. At 0 m above sea level, the boiling point is 100 °C. At 2,500 m above sea level, the boiling point is 91.3 °C. If we assume the boiling point of water is a linear function of elevation, we can use these two data points to calculate the slope of the line:

$$m = \frac{91.3 - 100}{2{,}500 - 0} = \frac{-8.7}{2{,}500}$$

This slope means that for each increase of 2,500 m, the boiling point of water decreases by 8.7 °C. Next, we already know the y-intercept is 100 °C from the first point, so a linear equation representing the data is

$$y = \frac{-8.7}{2{,}500}x + 100$$

This equation is an example of a mathematical model. A mathematical model is a mathematical object, like an equation, a function, or a geometric figure, that we use to represent a real-life situation. Sometimes a situation can be modeled by a linear function. We have to analyze the information we are given and use judgment about whether using a linear model is a reasonable thing to do. We must also be aware that the model may make imprecise predictions or may only be appropriate for certain ranges of values.

Testing our model for the boiling point of water, it accurately predicts that at an elevation of 1,000 m above sea level (when $x = 1{,}000$), water will boil at 96.5 °C (since $y = \frac{-8.7}{2{,}500} \cdot 1000 + 100 = 96.5$). For higher elevations, the model is not as accurate, but it is still close. At 5,000 m above sea level, it predicts 82.6 °C, which is 0.6 °C off the actual value of 83.2 °C. At 9,000 m above sea level, it predicts 68.7 °C, which is about 3 °C less than the actual value of 71.5 °C. The model continues to be less accurate at even higher elevations since the relationship between the boiling point of water and elevation isn't linear, but for the elevations in which most people live, it's pretty good.

Cool-down

Board Game Sales

5 min

Student Task Statement

A small company is selling a new board game, and they need to know how many to produce in the future.

After 12 months, they sold 4 thousand games. After 18 months, they sold 7 thousand games. And after 36 months, they sold 15 thousand games.

Could this information be reasonably estimated using a single linear model? If so, use the model to estimate the number of games sold after 48 months. If not, explain your reasoning.

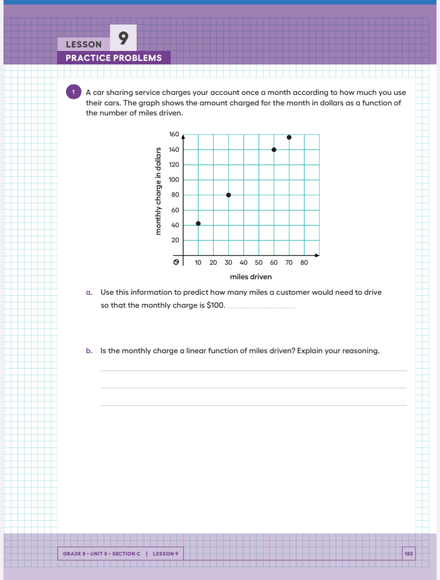
Predictions between 20 and 22 thousand sales, depending on the data points used for the model, are reasonable.

Sample response: Yes. After 48 months, they sold about 20.5 thousand games. From Month 12 to Month 38, the rate of games sold was about $\frac{11}{24}$ thousand games per month. This means the amount sold during the 12 months from Month 36 to Month 48 was 5.5 thousand, since $\frac{11}{24} \cdot 12 = 5.5$, and 5.5 thousand added to 15 thousand is 20.5 thousand.

Practice Problems

5 Problems

Student Workbook

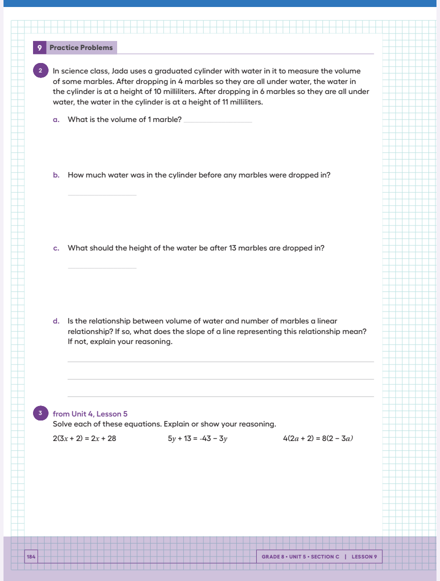

LESSON 9
PRACTICE PROBLEMS

1 A car sharing service charges your account once a month according to how much you use their cars. The graph shows the amount charged for the month in dollars as a function of the number of miles driven.

a. Use this information to predict how many miles a customer would need to drive so that the monthly charge is $100.

b. Is the monthly charge a linear function of miles driven? Explain your reasoning.

Student Workbook


9 Practice Problems

2 In science class, Jada uses a graduated cylinder with water in it to measure the volume of some marbles. After dropping in 4 marbles so they are all under water, the water in the cylinder is at a height of 10 milliliters. After dropping in 6 marbles so they are all under water, the water in the cylinder is at a height of 11 milliliters.

a. What is the volume of 1 marble?

b. How much water was in the cylinder before any marbles were dropped in?

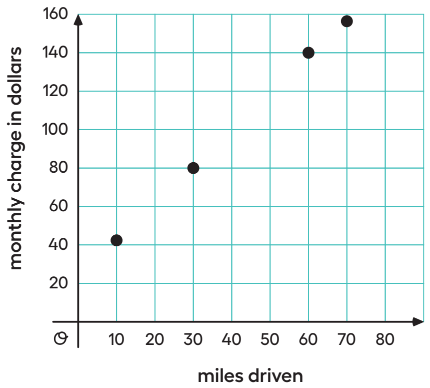
c. What should the height of the water be after 13 marbles are dropped in?

d. Is the relationship between volume of water and number of marbles a linear relationship? If so, what does the slope of a line representing this relationship mean? If not, explain your reasoning.

3 from Unit 4, Lesson 5
Solve each of these equations. Explain or show your reasoning.
$2(3x+2)=2x+28$ $5y+13=-43-3y$ $4(2a+2)=8(2-3a)$

Problem 1

A car sharing service charges your account once a month according to how much you use their cars. The graph shows the amount charged for the month in dollars as a function of the number of miles driven.



a. Use this information to predict how many miles a customer would need to drive so that the monthly charge is $100. about 40 miles

b. Is the monthly charge a linear function of miles driven? Explain your reasoning.

Yes

Although the values do not line up exactly, the function $y = 2x + 20$ (where x represents the miles driven and y represents the monthly charge) is very close to all the values.

Problem 2

In science class, Jada uses a graduated cylinder with water in it to measure the volume of some marbles. After dropping in 4 marbles so they are all under water, the water in the cylinder is at a height of 10 milliliters. After dropping in 6 marbles so they are all under water, the water in the cylinder is at a height of 11 milliliters.

a. What is the volume of 1 marble?

0.5 milliliter

b. How much water was in the cylinder before any marbles were dropped in?

8 milliliters

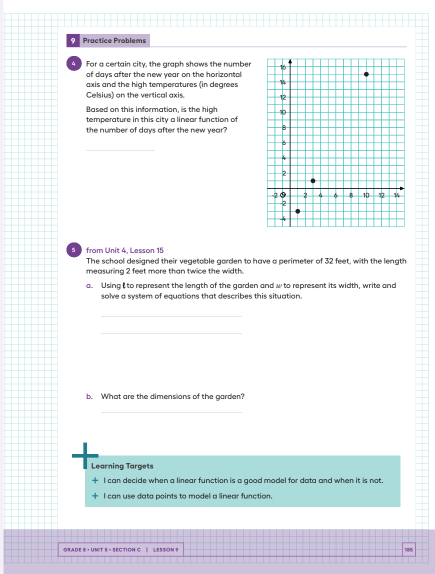
c. What should the height of the water be after 13 marbles are dropped in?

14.5 milliliters

d. Is the relationship between volume of water and number of marbles a linear relationship? If so, what does the slope of a line representing this relationship mean? If not, explain your reasoning.

The relationship is linear. The slope of the line represents the volume of 1 marble.

Student Workbook



Problem 3

from Unit 4, Lesson 5

Solve each of these equations. Explain or show your reasoning.

$2(3x + 2) = 2x + 28$

$x = 6$

Sample reasoning: Distribute 2 on the left side, add -4 to each side, add $-2x$ to each side, then divide each side by 4.

$5y + 13 = -43 - 3y$

$y = -7$

Sample reasoning: Add $3y$ to each side, subtract 13 from each side, then divide each side by 8.

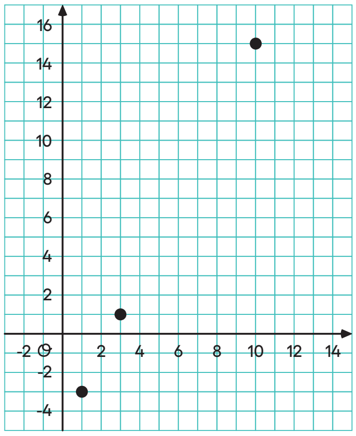
$4(2a + 2) = 8(2 - 3a)$

$a = \frac{1}{4}$

Sample reasoning: Divide each side by 4, distribute 2 on the right side, subtract 2 from each side, add $6a$ to each side, then divide each side by 8.

Problem 4

For a certain city, the graph shows the number of days after the new year on the horizontal axis and the high temperatures (in degrees Celsius) on the vertical axis.



Based on this information, is the high temperature in this city a linear function of the number of days after the new year?

Sample response: Although this data does fit a linear model, it does not make sense to use a linear model for this situation. For example, after only 2 months, the high temperature would be more than the boiling point of water, which is unlikely.

Problem 5

from Unit 4, Lesson 15

The school designed their vegetable garden to have a perimeter of 32 feet, with the length measuring 2 feet more than twice the width.

a. Using ℓ to represent the length of the garden and w to represent its width, write and solve a system of equations that describes this situation.

$2\ell + 2w = 32$, $\ell = 2w + 2$ (or equivalent)

b. What are the dimensions of the garden?

$\ell = 11\frac{1}{3}$ feet, $w = 4\frac{2}{3}$ feet