Lesson 4

Dilations on a Square Grid

Goals

- Create a dilation of a polygon on a square grid given a scale factor and center of dilation.
- Identify the image of a figure on a coordinate grid given a scale factor and center of dilation.

Learning Target

I can apply dilations to figures on a square grid.

Lesson Narrative

In this lesson, students apply dilations to polygons on a grid, both with and without coordinates. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines.

Students begin by estimating scale factors for dilations of points without a grid. Next, they perform dilations on a grid and use the structure of the grid to determine distances. Then students perform a card sort where they match images and descriptions of dilations that are performed in the coordinate plane, allowing students to develop language for precisely communicating figures and their images under dilations.

Student Learning Goal

Let's dilate figures on a square grid.

Access for Students with Diverse Abilities

- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- Card Sort
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1

Materials to Copy

- Matching Dilations on a Coordinate Plane Cards (1 copy for every 2 students): Activity 2

Required Preparation

Warm-up:
Provide access to geometry toolkits.

Activity 1:
Provide access to geometry toolkits.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	10 min	15 min	10 min	5 min
				Cool-down

Inspire Math

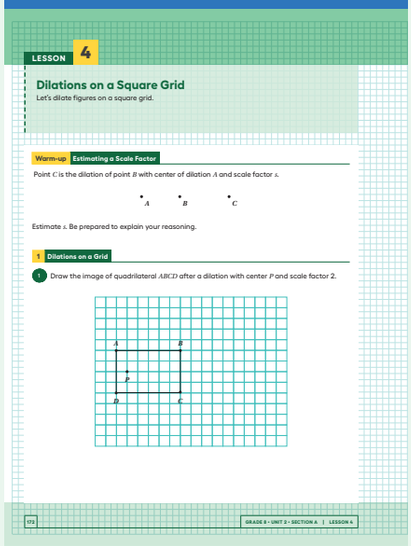
New Perspective video

Go Online
Before the lesson, show this video to introduce the real-world connection.

ilclass.com/l/614134

Please log in to the site before using the QR code or URL.

Student Workbook


LESSON 4

Dilations on a Square Grid

Let's dilate figures on a square grid.

Warm-up Estimating a Scale Factor

Point C is the dilation of point B with center of dilation A and scale factor s.

A B C

Estimate s. Be prepared to explain your reasoning.

1 Dilations on a Grid

1. Draw the image of quadrilateral $ABCD$ after a dilation with center P and scale factor 2.

Warm-up

Estimating a Scale Factor

5 min

Activity Narrative

In this *Warm-up*, students estimate a scale factor based on a picture showing the center of the dilation, a point, and its image under the dilation.

Launch

Provide access to geometry toolkits. Clarify that "estimate" doesn't mean "guess" and encourage students to use any tools available to make a precise estimate. Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Point C is the dilation of point B with center of dilation A and scale factor s.

A B C

Estimate s. Be prepared to explain your reasoning.

Sample response: The scale factor s is about 2.3 because the distance from A to C is a little more than twice the distance from A to B.

Activity Synthesis

The goal of this discussion is to review different ways to determine scale factor. Discuss with students:

- *"How did you measure distances?"*

  I used a ruler. I used an index card and marked off the distances.

- *"Is the scale factor greater than 1?"*

  Yes

- *"How do you know?"*

  The point C is further from A than B.

- *"Is the scale factor greater than 2?"*

  Yes

- *"How do you know?"*

  The distance from C to A is more than twice the distance from B to A.

- *"Is the scale factor greater than 3?"*

  No

- *"How do you know?"*

  The distance from C to A is less than 3 times the distance from B to A.

- *"Is the scale factor greater or less than 2.5?"*

  Less than

- *"How do you know?"*

  The distance from C to A is less than 2.5 times the distance from B to A.

Activity 1
Dilations on a Grid

10 min

Activity Narrative

In this activity, students perform dilations on a square grid, which is particularly helpful when the center of dilation and the points being dilated are grid points. Students will again see that scale factors greater than 1 produce larger copies while scale factors less than 1 produce smaller copies.

Monitor for students who use these strategies to find the dilated points:

- Use a ruler or index card to measure distances along the rays emanating from the center of dilation.
- Count how many squares to the left or right and up or down each point is from the center of dilation.

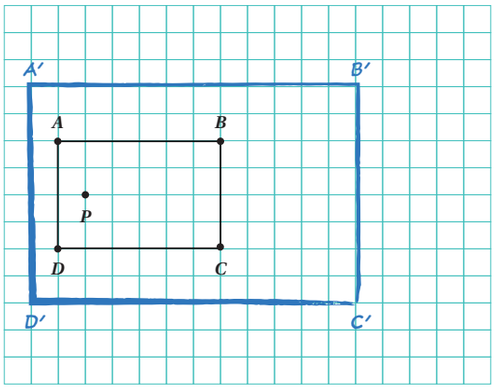
Launch

Arrange students in groups of 2. Provide access to geometry toolkits.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

1. Draw the image of quadrilateral $ABCD$ after a dilation with center P and scale factor 2.



Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1, Launch)

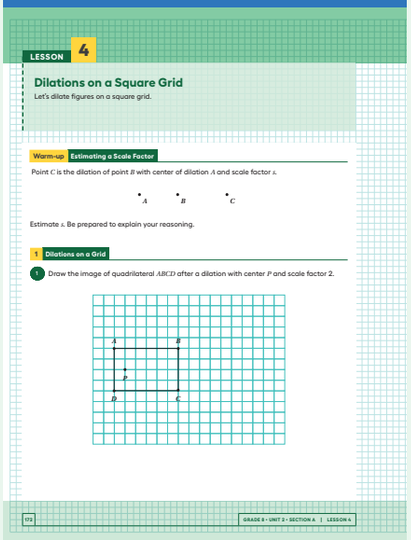
MLR7: Compare and Connect.
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

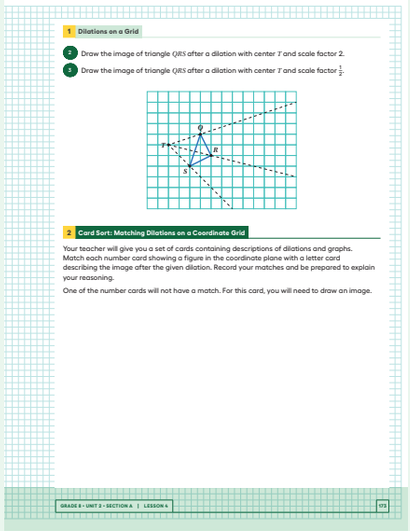
Engagement: Develop Effort and Persistence.
Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lessons on methods for dilating points on a circular grid and on no grid to provide an entry point into this activity.

Supports accessibility for: Social-Emotional Functioning, Conceptual Processing

Student Workbook


LESSON 4
Dilations on a Square Grid
Let's dilate figures on a square grid.
Warm-up Estimating a Scale Factor
Point C is the dilation of point B with center of dilation A and scale factor s.
Estimate s. Be prepared to explain your reasoning.
1 Dilations on a Grid
1 Draw the image of quadrilateral $ABCD$ after a dilation with center P and scale factor 2.

Student Workbook


1 Dilations on a Grid

2 Draw the image of triangle QRS after a dilation with center T and scale factor 2.

3 Draw the image of triangle QRS after a dilation with center T and scale factor $\frac{1}{2}$.

2 Card Sort: Matching Dilations on a Coordinate Grid

Your teacher will give you a set of cards containing descriptions of dilations and graphs. Match each number card showing a figure in the coordinate plane with a letter card describing the image after the given dilation. Record your matches and be prepared to explain your reasoning.

One of the number cards will not have a match. For this card, you will need to draw an image.

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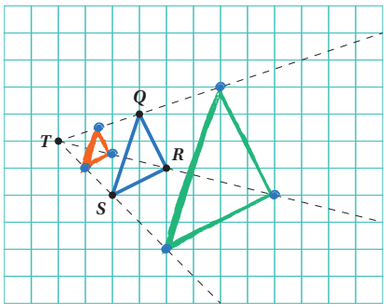
2. Draw the image of triangle QRS after a dilation with center T and scale factor 2.



3. Draw the image of triangle QRS after a dilation with center T and scale factor $\frac{1}{2}$.

See image

Activity Synthesis

The goal of the discussion is to make connections between the 2 strategies described in the *Activity Narrative* for finding a dilation. Measuring with a ruler or index card and reasoning about the grid and counting spaces will both result in the dilated point ending up in the same location.

Display 2–3 approaches or representations from previously selected students for all to see. If time allows, invite students to briefly describe their approach or representation. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches and representations.

Here are some questions for discussion:

- *"What do the 2 strategies have in common? How are they different?"*

  Both strategies are measuring distance, but one method measures distance in a straight line while the other measures distance by counting horizontal and vertical squares.

- *"How does the scale factor show up in each method?"*

  When using a ruler or index card, the scale factor shows up as how many times farther the new point will be. When counting grid squares, the scale factor tells us how many more squares we need to count from the center of dilation.

Activity 2

Card Sort: Matching Dilations on a Coordinate Grid

15 min

Activity Narrative

Students sort different graphs and descriptions into matching pairs during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Students begin by sorting the cards into categories of their choosing. This allows students to familiarize themselves with the content of the cards before finding matching pairs. Monitor for different ways groups choose to match the graphs and the descriptions, but especially for groups who identify similarly for triangles and quadrilaterals and that the dilation of a circle is a circle. Monitor for students who systematically perform the dilations to help identify a match versus those who reason by structure and elimination of possibilities.

As students work, encourage them to refine their explanations of how they made their matches using more precise language and mathematical terms.

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to sort the cards into categories of their choosing.

Pause the class after students have sorted the cards.

Select groups to share their categories and how they sorted their cards/started sorting their cards.

Discuss as many different types of categories as time allows.

Attend to the language that students use to describe their categories, giving them opportunities to describe their category more precisely. Highlight the use of terms like "polygon," "scale factor", "center of dilation," and "coordinates." After a brief discussion, invite students to complete the remaining questions.

Instructional Routines

Card Sort

ilclass.com/r/10783726

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Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports. Display the following sentence frames for all to see: "I noticed _____, so I matched ..." Encourage students to challenge each other when they disagree.

Advances: Speaking, Listening, Conversing

Access for Students with Diverse Abilities (Activity 2, Student Task)

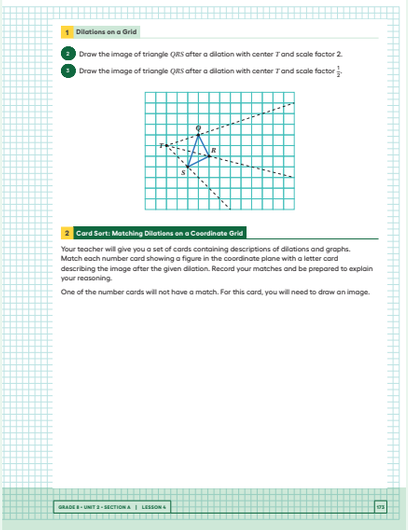
Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Building on Student Thinking

Some students may have trouble finding accurate matches. Prompt them to identify the center of dilation and then consider if the dilation will result in a smaller or larger sized image.

Student Workbook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2 Card Sort: Matching Dilations on a Coordinate Grid

Your teacher will give you a set of cards containing descriptions of dilations and graphs. Match each number card showing a figure in the coordinate plane with a letter card describing the image after the given dilation. Record your matches and be prepared to explain your reasoning.

One of the number cards will not have a match. For this card, you will need to draw an image.

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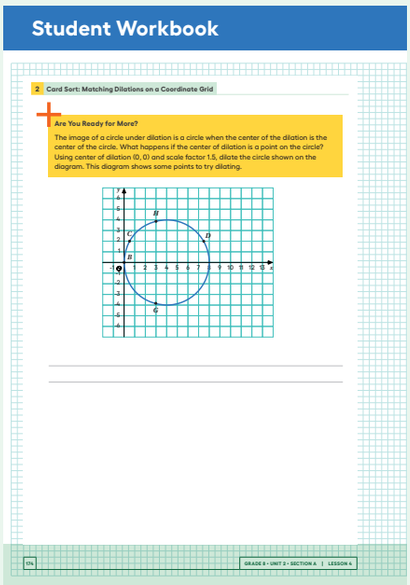
Student Workbook


2 Card Sort: Matching Dilations on a Coordinate Grid

Are You Ready for More?

The image of a circle under dilation is a circle when the center of the dilation is the center of the circle. What happens if the center of dilation is a point on the circle? Using center of dilation (0, 0) and scale factor 1.5, dilate the circle shown on the diagram. This diagram shows some points to try dilating.

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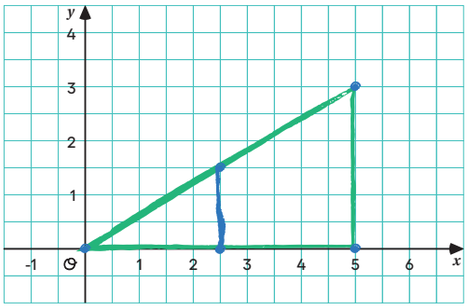
Student Task Statement

Your teacher will give you a set of cards containing descriptions of dilations and graphs. Match each number card showing a figure in the coordinate plane with a letter card describing the image after the given dilation. Record your matches and be prepared to explain your reasoning.

One of the number cards will not have a match. For this card, you will need to draw an image.

The blackline master shows the solution to the matching task. Card #5 does not match any of the lettered cards. See the image on the following grid:



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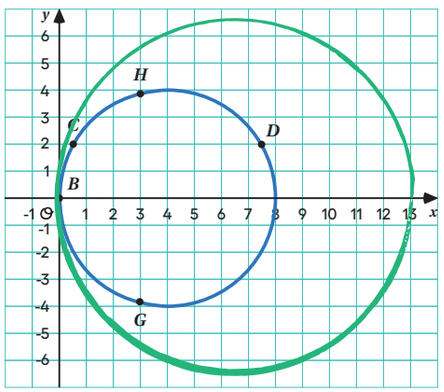
Original has center (4,0) and radius 4. Image has center (6,0) and radius 6.



Activity Synthesis

Once all groups have completed the card sort, discuss the following:

"Which matches were tricky? Explain why."

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

Highlight for students:

A dilation maps a circle to a circle, a quadrilateral to a quadrilateral, and a triangle to a triangle.

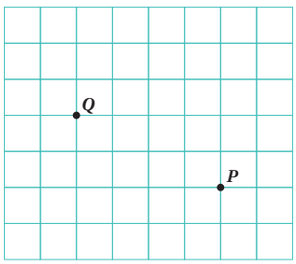
If the center of dilation for a polygon is one of the vertices, then that vertex is on the dilated polygon.

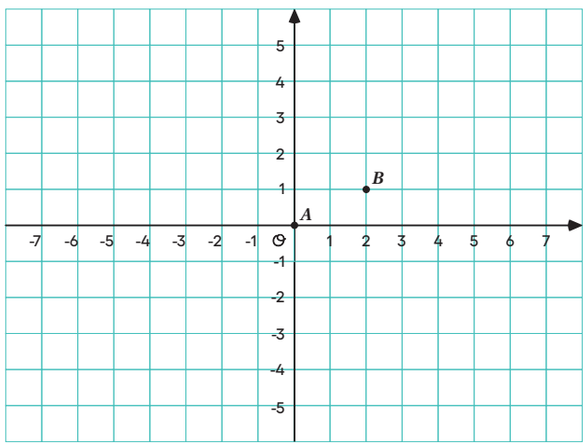
If the scale factor is less than 1 then the dilated image is smaller than the original figure.

If the scale factor is larger than 1 then the dilated image is larger than the original figure.

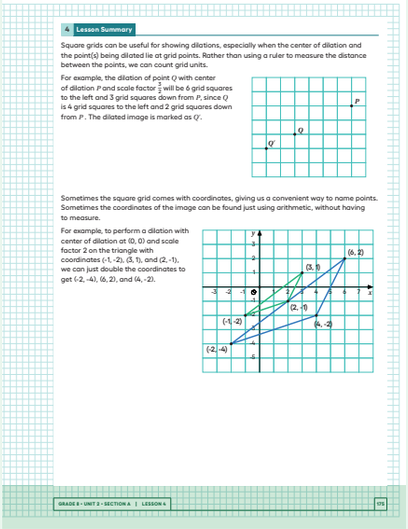
Lesson Synthesis

The purpose of this discussion is to review how to perform a dilation on a square grid. Show the figure for all to see and explain how grid lines can be used as a way to measure distance and direction between points.
Ask students to think about how they would dilate Q with center P and scale factor $\frac{1}{2}$. The image of Q would be half as many grid lines to the left and half has many grid lines up, or 2 grid lines to the left and 1 grid line up from P.





Student Workbook

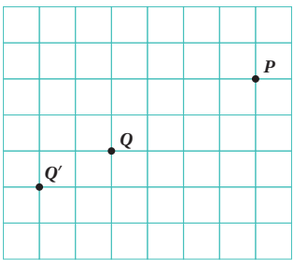


Show the following figure for all to see and ask students how this image is different from the previous figure. (This one has x- and y-axes.) Explain how when the grid has coordinates, it is easier to communicate the location of new points. For example, in the figure, we have A at (0, 0) and B at (2, 1). To communicate the dilation of B with center A and scale factor 3, we can just say (6, 3) because it is 3 times as far to the right and 3 times as far up from A as B.

Lesson Summary

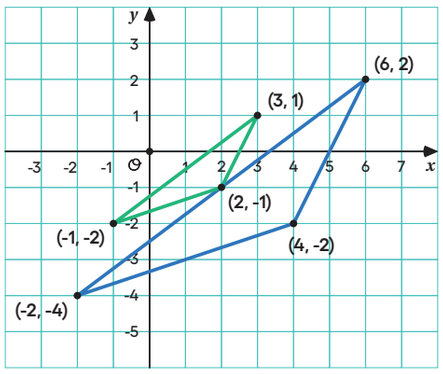
Square grids can be useful for showing dilations, especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, the dilation of point Q with center of dilation P and scale factor $\frac{3}{2}$ will be 6 grid squares to the left and 3 grid squares down from P, since Q is 4 grid squares to the left and 2 grid squares down from P. The dilated image is marked as Q'.



Sometimes the square grid comes with coordinates, giving us a convenient way to name points. Sometimes the coordinates of the image can be found just using arithmetic, without having to measure.

For example, to perform a dilation with center of dilation at (0, 0) and scale factor 2 on the triangle with coordinates (-1, -2), (3, 1), and (2, -1), we can just double the coordinates to get (-2, -4), (6, 2), and (4, -2).



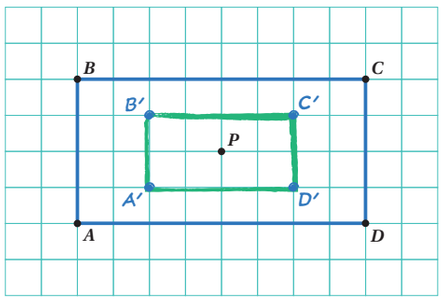
Cool-down

A Dilated Image

5 min

Student Task Statement

Draw the image of rectangle $ABCD$ after a dilation using point P as the center and scale factor $\frac{1}{2}$.



Responding To Student Thinking

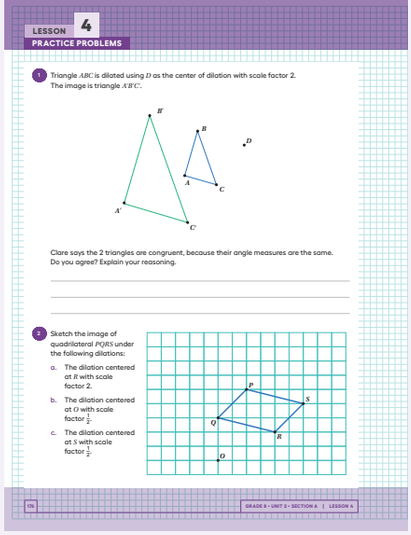
Press Pause
By this point in the unit, there should be some student mastery of working with drawing dilations. If most students struggle, make time to revisit the work of the lesson referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Unit 2, Section A Dilations

Practice Problems

5 Problems

Student Workbook


LESSON 4
PRACTICE PROBLEMS

1. Triangle ABC is dilated using D as the center of dilation with scale factor 2. The image is triangle $A'B'C'$.

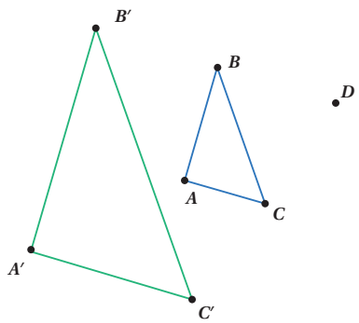
Clare says the 2 triangles are congruent, because their angle measures are the same. Do you agree? Explain your reasoning.

2. Sketch the image of quadrilateral $PQRS$ under the following dilations:
   a. The dilation centered at R with scale factor 2.
   b. The dilation centered at O with scale factor $\frac{1}{2}$.
   c. The dilation centered at S with scale factor $\frac{1}{2}$.

Problem 1

Triangle ABC is dilated using D as the center of dilation with scale factor 2.

The image is triangle $A'B'C'$. Clare says the 2 triangles are congruent, because their angle measures are the same. Do you agree? Explain your reasoning.



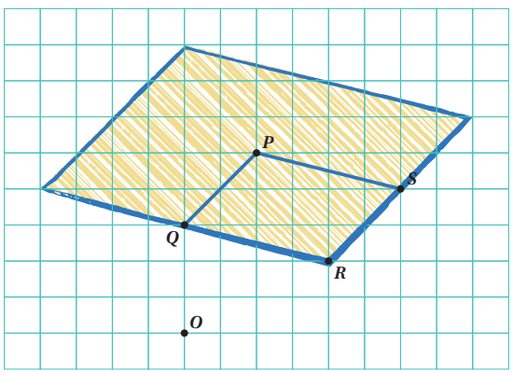
I disagree with Clare.

Sample reasoning: The triangles are not congruent because their side lengths are different.

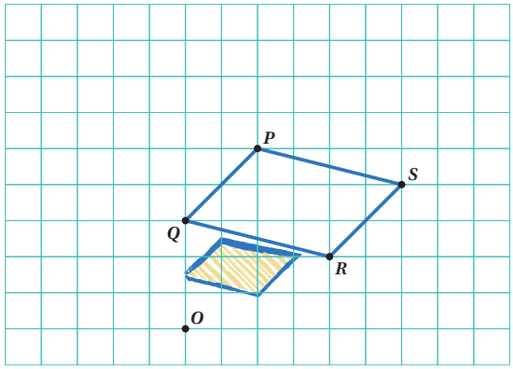
Problem 2

Sketch the image of quadrilateral $PQRS$ under the following dilations:

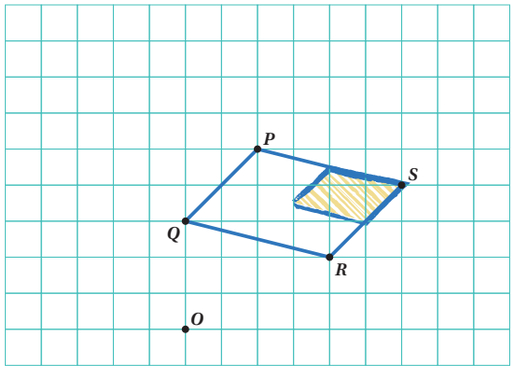
a. The dilation centered at R with scale factor 2.



b. The dilation centered at O with scale factor $\frac{1}{2}$.



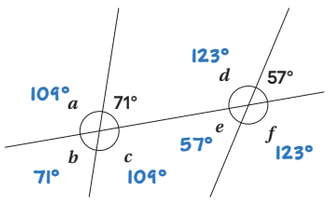
c. The dilation centered at S with scale factor $\frac{1}{2}$.



Problem 3

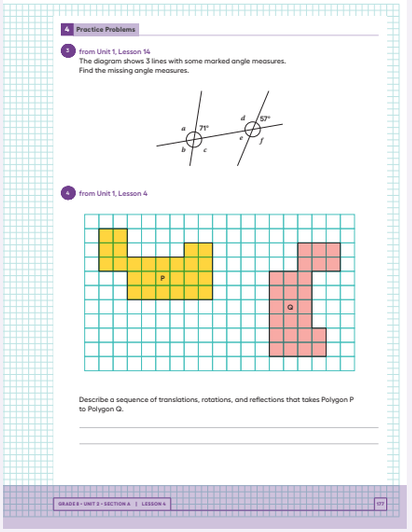
from Unit 1, Lesson 14

The diagram shows 3 lines with some marked angle measures.



Find the missing angle measures.

Student Workbook




Student Workbook

4 Practice Problems

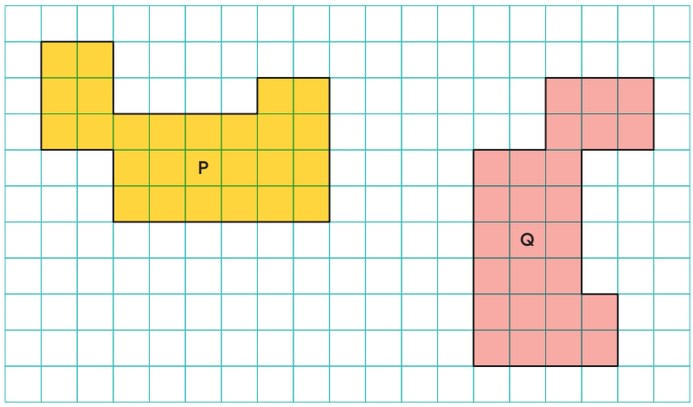
5 from Unit 1, Lesson 6
Point B has coordinates (-2, -5). After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image?

Learning Targets
+ I can apply dilations to figures on a square grid.

Problem 4

from Unit 1, Lesson 4

Describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.



Sample response: Polygon P is rotated 90 degrees clockwise and translated until the corresponding vertices match up.

Problem 5

from Unit 1, Lesson 6

Point B has coordinates (-2, -5). After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image? (2, -3)