Relationships between Quantities

Access for Students with Diverse Abilities
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners
- MLR6 (Activity 1)
- MLR7 (Activity 2)

Instructional Routines
- MLR6: Three Reads
- MLR7: Compare and Connect

Goals
- Determine unknown values in a relationship that is not proportional, and explain (orally and in writing) the solution method.
- Interpret and describe (orally and in writing) relationships that are predictable, but not proportional.
- Justify (orally) that a given relationship is not proportional.

Learning Target
I can think of ways to solve some more complicated word problems.

Lesson Narrative
In this introductory lesson, students encounter some contexts involving relationships that are not proportional. Students are not expected to write expressions or equations, or use any specific representation for these problems. However, they may choose to make diagrams or tables or reason in some other way. The goal here is to help students see that we need some new strategies to solve these new kinds of problems. It is the work of this unit to develop those strategies.

Student Learning Goal
Let's try to solve some new kinds of problems.

Lesson Timeline

10 min	10 min	10 min	10 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis

Assessment

10 min
Cool-down

Warm-up

Pricing Theater Popcorn

10 min

Activity Narrative

Students generate two sets of values: one set could be values in a proportional relationship, and the other set could not. The purpose of this *Warm-up* is to remind students of some characteristics that make a relationship proportional or not proportional, so that later in the lesson, they are better equipped to recognize that a relationship is not proportional and explain why.

Look for students who have a reasonable way to explain why their set of numbers is not proportional, such as "The unit price is different for each size," or "Each size costs a different amount per ounce."

Launch 👥

Invite students to share experiences going to the movies. What do they know about the popcorn for sale? What sizes does it come in? About how much does it cost?

Tell students that in this activity, they will come up with prices for different sizes of popcorn—one set of prices in which the price is in proportion to the size, and another set of prices in which the price is not in proportion to the size, but is still reasonable. Ask students to be ready to explain the reasons they chose the numbers they did. If needed, review what it means for a relationship to be proportional: the values for one quantity are each multiplied by the same number to get the values for the other quantity.

Arrange students in groups of 2.

Give 2 minutes of quiet work time and then invite students to share their response with their partner, followed by whole-class discussion.
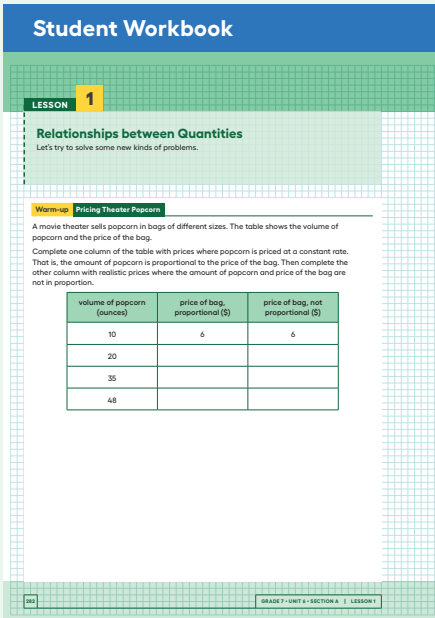
Student Task Statement

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

Complete one column of the table with prices where popcorn is priced at a constant rate. That is, the amount of popcorn is proportional to the price of the bag. Then complete the other column with realistic prices where the amount of popcorn and price of the bag are not in proportion.

Sample response:

volume of popcorn (ounces)	price of bag, proportional ($)	price of bag, not proportional ($)
10	6	6
20	12	11
35	21	20
48	28.8	25

Answers vary for the rightmost column.

Student Workbook

Activity Synthesis

The purpose of this discussion is to elicit different ways of viewing a proportional relationship. For example, for 20 ounces and 35 ounces, students might move from row to row and think in terms of scale factors. This approach is less straightforward for 48 ounces, and some students may shift to thinking in terms of unit rates.

Invite a student to share their prices for the proportional relationship and how they decided on those numbers. Ask if any students thought of it in a different way.

Then invite a student to share their prices for the relationship that is not proportional and record these for all to see. Ask students to explain ways you can tell that the relationship is not proportional.

Activity 1

Entrance Fees

10 min

Activity Narrative

In this activity, students use any strategy to extend a non-proportional pattern. This context was used in an earlier unit as an example of a relationship that is not proportional. However, a different rule for determining the entrance fee is used here.

Watch for students who organize the given information in a table or another visual representation, and for unique, correct approaches to the first two questions.

As students analyze several pairs of values in the relationship and then encapsulate the relationship with a rule, they look for and express regularity in repeated reasoning.

Launch

Tell students that unlike the previous activity where they could choose any numbers, this activity has a relationship where there is a pattern, and part of the work is to figure out what that pattern is. This activity has to do with an entrance fee to a park, where the fee is based on the number of people in the vehicle.

Student Task Statement

A state park charges an entrance fee based on the number of people in a vehicle. A car containing 2 people is charged $14, a car containing 4 people is charged $20, and a van containing 8 people is charged $32.

1. How much do you think a bus containing 30 people would be charged?

$98

Sample reasoning: From 2 people to 4 people, there are 2 additional people that cost 6 additional dollars. From 4 people to 8 people, there are 4 additional people that cost 12 additional dollars. It seems like each additional person costs 3 additional dollars. From 8 people to 30 people is 22 additional people, so they should cost 66 additional dollars, and 32 + 66 = 98.

2. If a bus is charged $122, how many people do you think it contains?

   38 people

   Sample reasoning: $122 is $24 more than $98. An additional $24 is 8 additional people, and 30 + 8 = 38.

3. What rule do you think the state park uses to decide the entrance fee for a vehicle?

   $8 for the vehicle plus $3 for each passenger

   Sample reasoning: 2 people cost $14, so if each person is charged $3, that leaves $8 for the vehicle.

Activity Synthesis

The purpose of this discussion is to elicit different ways of describing the rule for determining the entrance fee based on the number of people, and to notice that the relationship is not proportional.

Invite a student who organized the given information in a table to share. If no students did this, display this table for all to see:

number of people	entrance fee in dollars
2	14
4	20
8	32
30	
	122

Ask:

💬 *"What are some ways that you can tell that this relationship is not proportional?"*

Sample responses:

• 2 people to 4 people is double, but 14 to 20 is not double.

• 14 ÷ 2 = 7, but 20 ÷ 4 = 5. If the entrance fee was in proportion to the number of people, each quotient would be equal.

• You can't describe the situation with an equation like $px = q$.

Access for Multilingual Learners (Activity 1, Task Statement)

MLR6: Three Reads.

Keep books or devices closed. Display only the problem stem, without revealing the questions. Say,

   "We are going to read this problem 3 times."
After the 1st read:

   "Tell your partner what this situation is about."
After the 2nd read:

   "List the quantities. What can be counted or measured?"
For the 3rd read: Reveal and read the first question. Ask,

   "What are some ways we might get started on this?"
Advances: Reading, Representing

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Access for Perception.

Provide visual representations of anything that is read aloud.

Supports accessibility for: Language, Memory

Building on Student Thinking

Students may misunderstand that the first two questions require noticing and extending a pattern, and (because of the *Warm-up*) think that any reasonable number is acceptable. Encourage them to organize the given information and think about what rule the park might use to determine the entrance fee based on the number of people in the vehicle.

Students may come up with "rules" that aren't supported by the context or the given information. For example, they may notice that each additional person costs $3, but then reason that 30 people must cost $90. Whatever their rule, ask students to check whether it works for all of the information given. For example, since 2 people cost $14, we can tell that "$3 per person" is not the rule.

Invite students who had different strategies for answering the first two questions to share their responses. Include as many unique strategies as time allows. For each strategy, ask students to state their rule that the park uses to decide the entrance fee. Record all unique, correct rules, so all students can see different ways of expressing the same idea. For example, the rule might be expressed in the following ways:

- 8 dollars for the vehicle plus 3 dollars per person
- 3 dollars for every person and an additional $8
- 3 times the number of people plus 8

$8 + 3 \cdot$ people

Note: We have the entire rest of the unit to systematically develop relationships like these. There is no need to formalize or generalize anything yet!

Activity 2: Optional
Making Toast

10 min

Activity Narrative

In this activity, students are presented with a different relationship that is not proportional and also doesn't fit a pattern that can be characterized by an equation in the form $y = px + q$ (like the previous activity could be). This optional activity is a good opportunity for students to interpret another context and describe a relationship, but it can be safely skipped if the previous activity takes too much time.

Students must make sense of the problem and persevere in problem solving in this activity because there are many viable ways to represent the relationship and solve the problems, none of which are demonstrated first.

Launch 👥

Keep students in the same groups.

Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

Student Task Statement

A toaster has 4 slots for bread. Once the toaster is warmed up, it takes 35 seconds to make 4 slices of toast, 70 seconds to make 8 slices, and 105 seconds to make 12 slices.

1. How long do you think it will take to make 20 slices? 175 seconds

2. If someone makes as many slices of toast as possible in 4 minutes and 40 seconds, how many slices do you think they can make?

32 slices

Sample reasoning: 4 minutes 40 seconds = 280 seconds

number of slices	seconds it would take to make that number of slices
1	35
2	35
3	35
4	35
5	70
6	70
7	70
8	70
9	105
10	105
11	105
12	105
13–16	140
17–20	175
21–24	210
25–28	245
29–32	280

Are You Ready for More?

What is the smallest number that has a remainder of 1, 2, and 3 when divided by 2, 3, and 4, respectively? Are there more numbers that have this property?

11

Yes, this works for every number that is 1 less than a multiple of 12, for example, 23 or 35.

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Representation: Internalize Comprehension.

Provide a blank two-column table for students to process and organize information. Invite students to share their column labels (for example, number of slices and number of seconds) and how they organized the given information.

Supports accessibility for: Organization, Attention

Access for Multilingual Learners (Activity 2, Synthesis)

Reading, Speaking: MLR7 Compare and Connect.

Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

  "How are the tables or diagrams the same? How are they different?"
  "How does the quantity 20 slices show up in each representation?"
  "Are there any benefits or drawbacks to one representation compared to another?"
Advances: Representing, Conversing

Student Workbook

Student Workbook

Activity Synthesis

Invite students to share their responses and their reasoning. Select as many unique approaches as time allows.

Lesson Synthesis

The goal of this lesson is to recognize that there are situations in the world that are more complicated than what we have studied until this point, and to let students know this unit is about developing tools to solve some more sophisticated problems. Questions for discussion:

💬 *"Describe some rules we encountered in this lesson for how one quantity was related to another quantity."*

"What made these situations more complicated than relationships we have seen in the past?"

"What were some tools or strategies we used that were particularly helpful?"

Lesson Summary

In much of our previous work that involved relationships between two quantities, we were often able to describe amounts as being so much more than another, or so many times as much as another. We wrote equations like $x + 3 = 8$ and $4x = 20$ and solved for unknown amounts.

In this unit, we will see situations where relationships between amounts involve more operations. For example, a pizza store might charge the amounts shown in the table for delivering pizzas.

number of pizzas	total cost in dollars
1	13
2	23
3	33
5	53

We can see that each additional pizza adds $10 to the total cost, and that each total includes a $3 additional cost, maybe representing a delivery fee. In this situation, 8 pizzas will cost $8 \cdot 10 + 3$ and a total cost of $63 means 6 pies were ordered.

In this unit, we will see many situations like this one, and will learn how to use diagrams and equations to answer questions about unknown amounts in the situation.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

💬 *"What norm(s) should stay the way they are?"*

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

Movie Theater Popcorn, Revisited

10 min

Student Task Statement

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

volume of popcorn (ounces)	price of bag ($)
10	6
20	8
35	11
48	13.6

If the theater wanted to offer a 60-ounce bag of popcorn, what would be a good price? Explain your reasoning.

Sample responses:

- $16, because there is a pattern of $4 plus $0.20 per ounce
- $15, because there should be a discount for buying a larger bag of popcorn

Practice Problems

6 Problems

Student Workbook

Problem 1   from Unit 3, Lesson 7

Lin and Tyler are drawing circles. Tyler's circle has twice the diameter of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler? Explain your reasoning.

no

Sample reasoning: The radius and area of a circle are not proportional. The area of Tyler's circle will be 4 times as large as the area of Lin's circle.

Problem 2   from Unit 5, Lesson 15

Jada and Priya are trying to solve the equation $\frac{2}{3} + x = 4$.

- Jada says, "I think we should multiply each side by $\frac{3}{2}$ because that is the reciprocal of $\frac{2}{3}$."

- Priya says, "I think we should add $-\frac{2}{3}$ to each side because that is the opposite of $\frac{2}{3}$."

a. Which person's strategy is correct? Why?

Priya is correct

Sample reasoning: The operation in the expression $\frac{2}{3} + x$ is addition. Adding the additive inverse of $\frac{2}{3}$ to both sides of the equation will change the equation to the form "$x = \ldots$"

b. Write an equation that can be solved using the other person's strategy.

Sample response: $\frac{2}{3}x = 4$.

Problem 3   from Unit 5, Lesson 13

In each equation, the ? represents an operation. Which operation makes each equation true?

a. 48 ? (-8) = (-6)

divide

b. (-40) ? 8 = (-5)

divide

c. 12 ? (-2) = 14

subtract

d. 18 ? (-12) = 6

add

e. 18 ? (-20) = -2

add

f. 22 ? (-0.5) = -11

multiply

Problem 4

from Unit 5, Lesson 14

In football, the team that has the ball has four chances to gain at least 10 yards. If they don't gain at least 10 yards, the other team gets the ball. Positive numbers represent a gain of yards, and negative numbers represent a loss of yards. The Bulldogs just got the ball. Select **all** the sequences of four plays that result in the Bulldogs getting to keep the ball.

A. 8, -3, 4, 21

B. 30, -7, -8, -12

C. 2, 16, -5, -3

D. 5, -2, 20, -1

E. 20, -3, -13, 2

Problem 5

A sandwich store charges $20 to have 3 turkey subs delivered and $26 to have 4 delivered.

a. Is the relationship between the number of turkey subs delivered and the amount charged proportional? Explain how you know.

no

Sample reasoning: If they deliver 3 turkey subs, they charge $6.67 per sub, but for 4 subs, they charge $6.50 per sub.

b. How much does the store charge to deliver 1 additional turkey sub?

$6

c. Describe a rule for determining how much the store charges based on the number of turkey subs delivered.

Sample response: The rule could be $6 per sub plus a $2 delivery fee. 6 times 3 is 18, but they charged $2 more than that for 3 subs. 6 times 4 is 24, but they charged $2 more than that for 4 subs.

Problem 6

Which question **cannot** be answered by the solution to the equation $3x = 27$?

A. Elena read 3 times as many pages as Noah. She read 27 pages. How many pages did Noah read?

B. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?

C. Diego paid $27 to have 3 pizzas delivered and $35 to have 4 pizzas delivered. What is the price of one pizza?

D. A team has 27 players, and 10 of them are in sixth grade. The coach splits all the players into 3 groups to practice skills. How many students are in each group?