Lesson 4

How Crowded Is This Neighborhood?

Goals

- Compare and contrast the density of uniformly distributed dots in squares.
- Create an equation and a graph that represent the proportional relationship between the area of a square and the number of dots enclosed by the square.
- Interpret the constant of proportionality in models of housing per square kilometer or population of people per square kilometer.

Lesson Narrative

This optional lesson involves a sequence of four activities that prepare and introduce students to the concept of population density. The first three activities introduce population density as an application of proportional reasoning. The final activity is optional because it compares the population densities of large cities without a visual representation of the populations.

The way the lesson develops helps students make sense of the somewhat abstract idea of density in concrete terms: They start by comparing the number of dots distributed in squares and move on to houses in different neighborhoods. Finally they compare the number of people living in different cities. Unlike speed or unit pricing, density is not likely to be familiar to students, so it provides an opportunity to make sense of an unfamiliar situation by thinking about familiar quantities in a new way. As students move through different representations of density they expand their quantitative reasoning. This lesson relies on skills developed in Unit 2.

Student Learning Goal

Let's see how proportional relationships apply to where people live.

Lesson Timeline

Activity 1	Activity 2	Activity 3	Activity 4
10 min	15 min	15 min	15 min

Access for Students with Diverse Abilities

- Engagement (Activity 3)

Access for Multilingual Learners

- MLR5: Co-Craft Questions (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder

Required Materials

Materials to Gather

- Four-function calculators: Activity 3, Activity 4

Required Preparation

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Activity 3:
If desired, have satellite images showing housing density in your area to share with students.

Activity 4:
If desired, have satellite images showing housing density in your Los Angeles and New York City to share with students.

Instructional Routines

Notice and Wonder

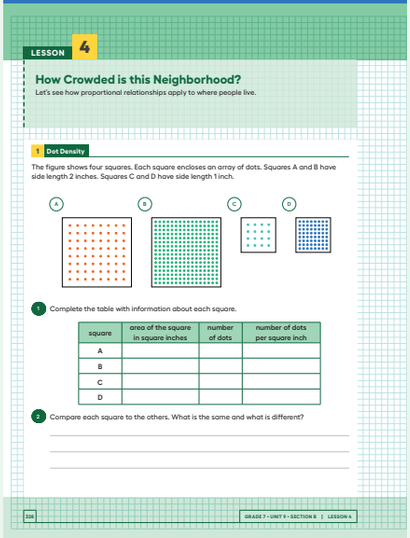
ilclass.com/r/10694948
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students do not understand the purpose of the last column in the table, consider asking:

"Can you explain how you figured out the other parts of the table?"
"The word 'per' has appeared in many places including 'dots per square inch,' 'miles per hour,' and 'price per gallon.' What does 'per' mean?"
"How can you use the parts of the table you already filled in to figure out how many dots per square inch?"

Student Workbook


LESSON 4

How Crowded is this Neighborhood?

Let's see how proportional relationships apply to where people live.

1 Dot Density

The figure shows four squares. Each square encloses an array of dots. Squares A and B have side length 2 inches. Squares C and D have side length 1 inch.

1. Complete the table with information about each square.

square	area of the square in square inches	number of dots	number of dots per square inch
A			
B			
C			
D			

2. Compare each square to the others. What is the same and what is different?

Activity 1
Dot Density

10 min

Activity Narrative

In this activity, students compare dot densities when dots are uniformly distributed. The squares are sized so that students can compare dot density in large and small squares by drawing a partition of the larger square into four smaller squares and comparing the number of dots in squares of the same size. Students reason abstractly and quantitatively as they compare the density of dots and make decisions about how to partition the squares.

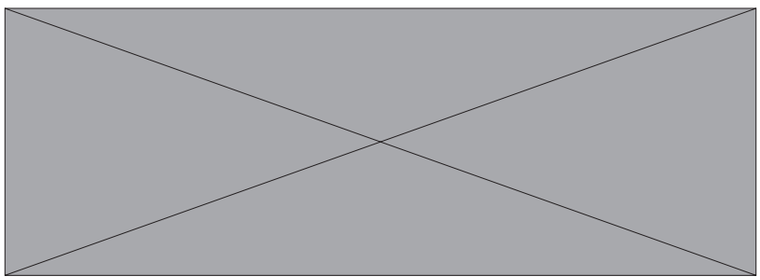
Launch

Display the image of the four squares with dots. Invite students to share what they notice and what they wonder.

Give students 5 minutes of quiet work time followed by whole-class discussion.

Student Task Statement

The figure shows four squares. Each square encloses an array of dots. Squares A and B have side length 2 inches. Squares C and D have side length 1 inch.



1. Complete the table with information about each square.

square	area of the square in square inches	number of dots	number of dots per square inch
A	4	64	16
B	4	256	64
C	1	16	16
D	1	64	64

2. Compare each square to the others. What is the same and what is different?

- Squares A and B have the same area. Squares C and D have the same area.
- The number of dots in Square A is the same as the number of dots in Square D. The two other squares have different numbers of dots.
- The number of dots per square inch is the same for Squares A and C. The number of dots per square inch is the same for Squares B and D.

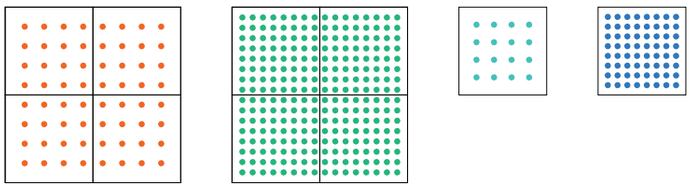
Activity Synthesis

Invite students to share what is similar and what is different about the arrays.

Define *density* as "things per square inch," in this case dots per square inch. Demonstrate the correct use of "dense" and "density" by saying things like:

- *"The green dots in Square B are more densely packed than the red dots in Square A and the blue dots in Square C."*
- *"The density of the red dots in Square A and the blue dots in Square C is the same."*

If students haven't noted it already, point out that Square A can be partitioned into four smaller squares. Each has an array of red dots that is spaced the same as the array of blue dots in Square C. The same is true for Squares B and D.



Activity 2

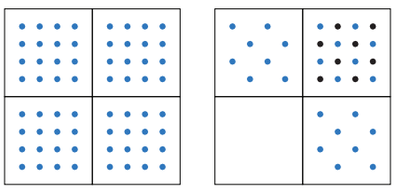
Dot Density with a Twist

15 min

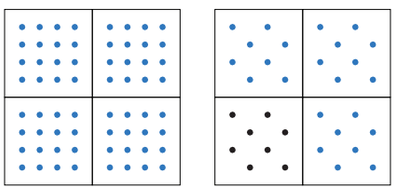
Activity Narrative

There is a digital version of this activity.

In this activity, the dots are distributed uniformly in the first square but not in the second square:



However, these dots are drawn so it is not too hard to see that if they were redistributed, each square inch would have 8 dots:



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Invite students to repeat their reasoning using mathematical language: "Can you say that again, using the word 'density'?"
Advances: Speaking, Conversing

Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 2, Narrative)

MLR5: Co-Craft Questions
This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

The fact that there are 8 dots per square inch means that if the dots were distributed uniformly throughout the square in the right way, there would be 8 dots in each square inch. This prepares students to be able to interpret the constant of proportionality in the next two activities when working with actual houses per square mile or people per square kilometer. Students reason abstractly and quantitatively as they interpret the distributions of dots. The phrase "500 houses per square kilometer" can be thought of as "If all of the houses in the region were spread out uniformly, then there would be 500 houses in every square kilometer."

In the digital version of the activity, students use an applet to graph lines representing the relationships between the squares' areas and the number of dots. The applet allows students to adjust the scale on each axis. The digital version may be helpful for thinking through how to scale each axis to best represent the data.

Launch

Arrange students in groups of 2. Introduce the context of dot density. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions. Display only the problem stem and related image, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

- Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,

💬 ***"What do these questions have in common? How are they different?"***

Listen for and amplify language related to the learning goal, such as comparing dot density.

- Reveal the axes and the question "On this coordinate plane, a represents the area of the square and d represents the number of dots enclosed by the square. For each square, plot a point that represents its values of a and d" and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:

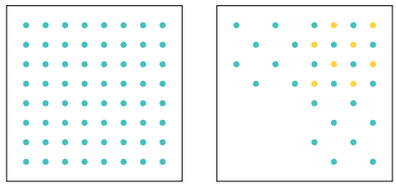
💬 ***"Which of your questions is most similar to or different from the ones given? Why?"***

"Is there a main mathematical concept that is present in both your questions and those given? If so, describe it."

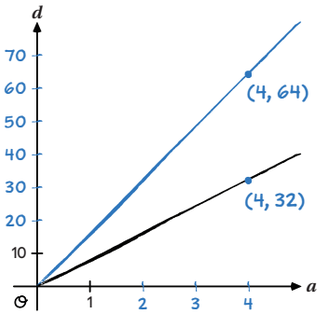
- Have students answer the questions in the statement. If time allows, they can also answer mathematically interesting questions created by their classmates.

Student Task Statement

The figure shows two arrays, each enclosed by a square that is 2 inches wide.



1. On this coordinate plane, a represents the area of the square and d represents the number of dots enclosed by the square. For each square, plot a point that represents its values of a and d.



See figure.

2. Draw lines from (0, 0) to each point. For each line, write an equation that represents the proportional relationship.

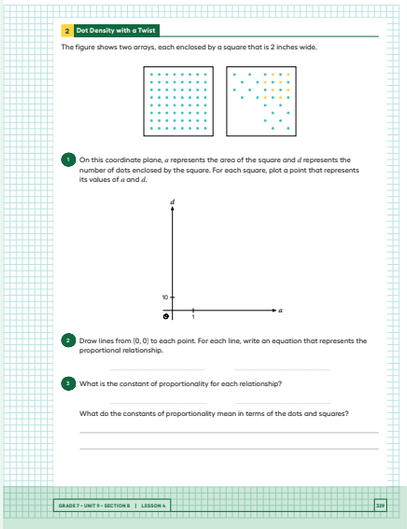
See figure.

The equations are $d = 16a$ and $d = 8a$ respectively.

3. What is the constant of proportionality for each relationship? What do the constants of proportionality mean in terms of the dots and squares?

16 and 8; The constants of proportionality indicate the number of dots per square inch, 16 and 8, respectively. In the first case, it means that if the square were partitioned into square inches, there would be 16 dots in each. In the second case, it means that if the dots were redistributed uniformly, there would be 8 dots per square inch.

Student Workbook


2 Dot Density with a Twist

The figure shows two arrays, each enclosed by a square that is 2 inches wide.

1 On this coordinate plane, a represents the area of the square and d represents the number of dots enclosed by the square. For each square, plot a point that represents its values of a and d.

2 Draw lines from (0, 0) to each point. For each line, write an equation that represents the proportional relationship.

3 What is the constant of proportionality for each relationship?

What do the constants of proportionality mean in terms of the dots and squares?

Access for Students with Diverse Abilities (Activity 3, Synthesis)

Engagement: Provide Access by Recruiting Interest.
Invite students to share how their neighborhood looks compared to the two in the activity.
Supports accessibility for: Conceptual Processing, Memory

Activity Synthesis

The goal of this discussion is for students to make sense of the constant of proportionality in the case where the dots are not uniformly distributed. Invite students to share their interpretations of the constants of proportionality.

Consider asking questions like:

- *"Why is one of the constants of proportionality larger than the other? How can this be seen in the picture of the squares?"*

  There are more dots in the same area.

- *"What are the units of the constants of proportionality?"*

  The number of dots per square inch.

Students will have more opportunities to think about this when working on the activities that follow.

Activity 3
Housing Density

15 min

Activity Narrative

This activity starts to transition students from arrays of dots to real-world objects distributed over the surface of Earth. This task concerns housing density, which is very similar to dot density. The two images in the activity have the following properties:

- It is fairly easy to distinguish the houses and not too tedious to count them all.
- The scale of the images is similar, but the size of each image and the number of houses in each image is not the same.
- In the first image, the houses look fairly uniformly distributed, and in the other, they look less uniformly distributed but not to the point that it is hard to interpret the image.

This task can be customized to any location, for example, different neighborhoods in the school's city. Care should be taken in selecting the images to include noticeably different housing densities and easy-to-count houses.

Launch

Give students 5 minutes of quiet work time followed by partner and whole-class discussion. Provide access to four-function calculators.

Student Task Statement

Here are pictures of two different neighborhoods.

For each neighborhood, find the number of houses per square kilometer.

This image depicts an area that is 0.3 kilometers long and 0.2 kilometers wide.

The first image shows 48 houses. It depicts an area that is 0.06 square kilometers, so the housing density is 800 houses per square kilometer.

This image depicts an area that is 0.4 kilometers long and 0.2 kilometers wide.

The second image shows 9 (or 10) houses. It depicts an area that is 0.08 square kilometers, so the housing density is 112.5 (or 125) houses per square kilometer.

Activity Synthesis

Invite students to share their answers for the two densities. Invite students to share their reasoning. Consider asking the following questions:

- *"One number is a lot bigger than the other. How can this be seen in the images?"*

  There are a lot more houses per the same area.

- *"You only counted 48 houses in the first image but you say that there are 800 houses per square kilometer. Why is that happening?"*

  The area in the image is just a fraction of a square kilometer. If there were a square kilometer with the same density, there would be 800 houses in it.

- *"Some of you said that there are 112.5 houses per square kilometer in the second neighborhood. How can there be half houses?"*

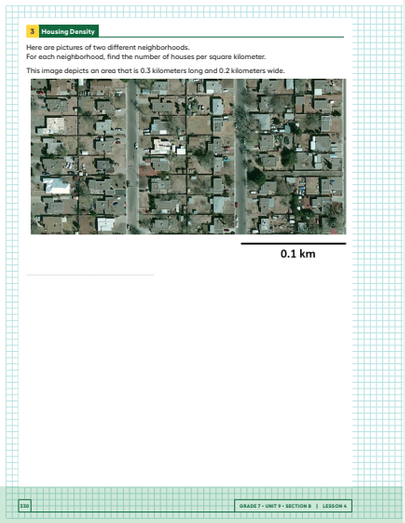
  If there were an image with an area of 2 square kilometers, there would be 225 houses in the image, or if they are uniformly distributed, the image might contain a part of a house.

Building on Student Thinking

If students are accustomed to the area (of a rectangle) being numerically bigger than its length or width, and the areas in the activity seem incorrect to them, consider asking:

"Can you explain how to calculate the area of a rectangle?"
"What happens when 2 fractions that are less than 1 are multiplied? For example, $\frac{1}{2} \cdot \frac{1}{2}$?"

Student Workbook


3 Housing Density

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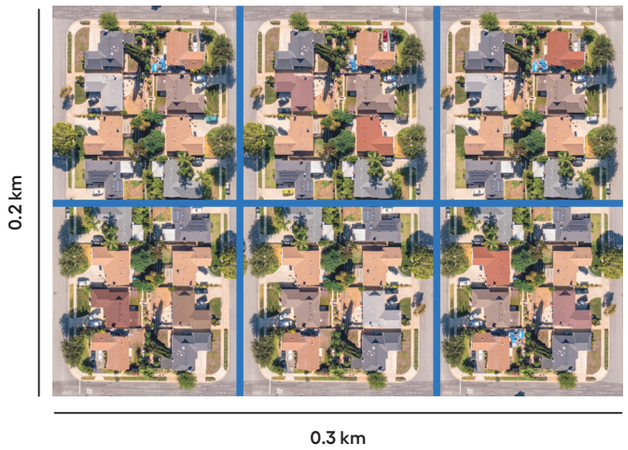
Student Workbook


3 Housing Density

This image depicts an area that is 0.4 kilometers long and 0.2 kilometers wide.

If desired, display this image to help students make sense of their answers.



The map shows a rectangle 0.3 km by 0.2 km. This means that any one of the six squares is 0.1 kilometer by 0.1 kilometer, which has an area of 0.01 square kilometer.

If 10 of these were lined up, there would be a strip 1 kilometer long. 10 of these strips would make a square of 1 kilometer by 1 kilometer; that is, 1 square kilometer. Now it's clear that it takes 100 of these small squares to make a square kilometer, so the small square indeed has an area $\frac{1}{100}$ of a square kilometer. In fact, there are 8 houses in this square, so if the entire square kilometer were filled the same way, there would be 800 houses.

Activity 4: Optional

Population Density

15 min

Activity Narrative

The purpose of this activity is to introduce the concept of *population density*. One added step going from houses in a neighborhood to people in a location is the fact that people do not stay at a fixed spot but rather move around. In this activity, students make sense of what it means to say there are 42.3 people per square kilometer in some location.

This activity gives some information about New York City and Los Angeles and ultimately asks students to decide which city is more crowded. Students may benefit from a demonstration of situations that feel more crowded vs. less crowded.

In the data for this task, people are given in "blocks" of 1000 people. It's perfectly fine to make up a new unit customized to the situation, even if it doesn't have an official name. This is a more sophisticated use of nonstandard units. In earlier grades, nonstandard units tend to be the length of a paper clip, or the length of your shoe. When dealing with any units, it's important to list the units: in the heading of a table, in labels for the axes of a graph, or in writing numbers.

Launch

Arrange students in groups of 2–4. Provide access to calculators.

Consider demonstrating situations that feel more crowded or less crowded by having a certain number of students stand in an area.

- *"What would you have to do to feel more crowded with the same number of people?"*

  Stand in a smaller space, which would make people stand closer to each other

- *"Less crowded?"*

  Take up more space, so that people are farther apart

Then mark off a region on the classroom floor with tape. Ask some students to stand inside it, and then ask,

- *"What would make the space feel more crowded?"*

  If more people stood in the same space

- *"Less crowded?"*

  Fewer people

If desired, share some background information about New York City and Los Angeles and display satellite images:

- New York City is the U.S. city with the largest population. The city has five parts (called boroughs): Manhattan, Brooklyn, The Bronx, Queens, and Staten Island. In Manhattan, most people live in apartment buildings, many in high-rises. In the other boroughs, many people also live in single-family houses. Staten Island is quite different, almost suburban.
- Los Angeles is the U.S. city with the second largest population. Although there are some high-rise apartment buildings, many people live in single-family houses, and many of these houses are single-story.

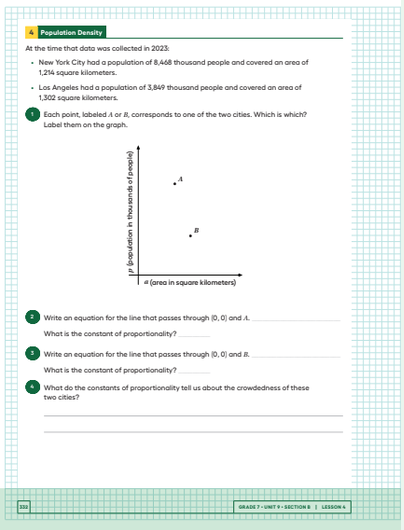
Give students 5 minutes of quiet work time, followed by small-group and whole-class discussion.

Building on Student Thinking

If students have difficulty understanding population being expressed in units of 1,000, consider asking:

"How many people live in New York City?"
"How does expressing large numbers in terms of thousands help one understand them?"

Student Workbook


4 Population Density

At the time that data was collected in 2023:

- New York City had a population of 8,468 thousand people and covered an area of 1,214 square kilometers.
- Los Angeles had a population of 3,849 thousand people and covered an area of 1,302 square kilometers.

1. Each point, labeled A or B, corresponds to one of the two cities. Which is which? Label them on the graph.

p (population in thousands of people)
A
B
a (area in square kilometers)

2. Write an equation for the line that passes through (0, 0) and A.
What is the constant of proportionality?

3. Write an equation for the line that passes through (0, 0) and B.
What is the constant of proportionality?

4. What do the constants of proportionality tell us about the crowdedness of these two cities?

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Student Task Statement

At the time that data was collected in 2023:

- New York City had a population of 8,468 thousand people and covered an area of 1,214 square kilometers.
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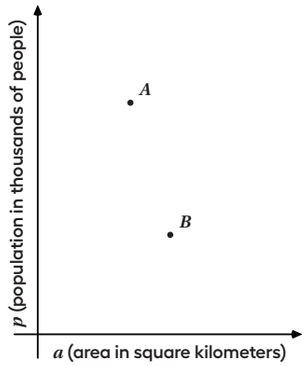


New York City has a greater population and a smaller area, so it must correspond to point A.

2. Write an equation for the line that passes through (0, 0) and A. What is the constant of proportionality?

An equation for the line through point A is $p = 7a$; the constant of proportionality is about 7.

3. Write an equation for the line that passes through (0, 0) and B. What is the constant of proportionality?

An equation for the line through point B is $p = 3a$; the constant of proportionality is about 3.

4. What do the constants of proportionality tell us about the crowdedness of these two cities?

The constants of proportionality tell that in New York City there are about 7 thousand people per square kilometer, or 7,000 people per square kilometer, and in LA there are about 3 thousand people per square kilometer, or 3,000 people per square kilometer.

Are You Ready for More?

1. Predict where these types of regions would be shown on the graph:
   a. A suburban region where houses are far apart, with big yards

      Sample response: Close to (0, 0)

   b. A neighborhood in an urban area with many high-rise apartment buildings

      Sample response: To the left and lower than the point in part a

   c. A rural state with lots of open land and not many people

      Sample response: To the right and lower than the point in part b

2. Next, use this data to check your predictions:

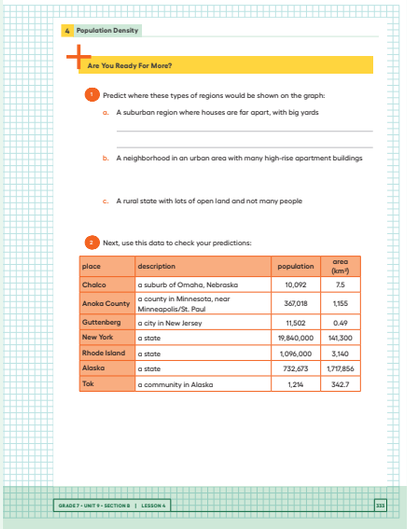
place	description	population	area (km^2)
Chalco	a suburb of Omaha, Nebraska	10,092	7.5
Anoka County	a county in Minnesota, near Minneapolis/St. Paul	367,018	1,155
Guttenberg	a city in New Jersey	11,502	0.49
New York	a state	19,840,000	141,300
Rhode Island	a state	1,096,000	3,140
Alaska	a state	732,673	1,717,856
Tok	a community in Alaska	1,214	342.7

Answers vary based on student predictions.

Note that it's not really possible to see all the points on the same graph: the populations of Los Angeles and New York are so large, and the population of Tok so small, that the point for Tok could be distinguished, LA and NY would be far off the paper or screen. And on the graph above showing LA and NY, Tok's population would be so small that it could not be distinguished from (0, 0).

A computer graphing program can help students understand this as it will take many steps of zooming in or out to switch between very small cities and very large cities.

Student Workbook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Activity Synthesis

Invite some students to display their graphs and equations for all to see. Ask all students if they agree or disagree and why. Once students agree, focus on the meaning of the constants of proportionality and what they reveal about the crowdedness in the two cities.

Consider asking the following questions:

- *"Why did you choose point A to represent New York City?"*

  New York City has more people per square kilometer than Los Angeles, so New York is more crowded than Los Angeles.

- *"What does the constant of proportionality reveal about the crowdedness of the two cities?"*

  Even though people aren't distributed uniformly throughout the cities, if this were the case, there would be about 7,000 people in every square kilometer in NYC and about 3,000 people in every square kilometer in Los Angeles.