Lesson 11

Interpreting Graphs of Proportional Relationships

Goals

- Create the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and (0, 0).
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret (orally and in writing) points on the graph of a proportional relationship.

Learning Targets

- I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
- I can find the constant of proportionality from a graph.
- I understand the information given by graphs of proportional relationships that are made up of points or a line.

Lesson Narrative

In this lesson, students interpret points on the graph of a proportional relationship. They make connections between the graph, the equation, and the context modeled by the relationship. Extra attention is given to the meaning of the point (1, k) on the graph, both in terms of the constant of proportionality k in the equation $y = kx$ and in terms of a constant rate in the context. As they relate representations and situations, students practice reasoning quantitatively and abstractly.

In the *Warm-up*, students think about what situation an unlabeled graph might represent and learn the importance of being precise about saying which quantities are represented on each axis.

Student Learning Goal

Let's read stories from the graphs of proportional relationships.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR5: Co-Craft Questions (Activity 1)

Required Materials

Materials to Gather

- Rulers: Activity 2

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	20 min	10 min	10 min	5 min

Warm-up

What Could the Graph Represent?

5 min

Activity Narrative

In this *Warm-up* students are given an unlabeled graph of a proportional relationship and asked to invent a situation that it could represent. This gives students an opportunity to think back to examples of proportional relationships they have encountered. After several possible contexts are shared, students label the axes of the graph, give it a title, and interpret the meaning of a point on the graph. This is an opportunity for students to attend to precision in language. During the discussion, the characteristics of a graph of a proportional relationship are reinforced.

Launch

Tell students that they will look at an unlabeled graph, and their job is to think of a situation that the graph could represent.

Display the problem stem for all to see and give 1 minute of quiet think time.

Ask students to give a signal when they have thought of a situation.

Invite some students to share their ideas and record the responses for all to see. The purpose of this is to provide some inspiration to students who haven't come up with anything.

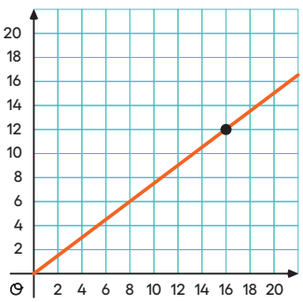
Ask students how they know all of the relationships are proportional.

When one value is 0, the other is 0. The situation involves equivalent ratios. Any pair of values in the relationship has the same unit rate.

Ask students to complete the questions.

Student Task Statement

Here is a graph that represents a proportional relationship.



Invent a situation that could be represented by this graph.

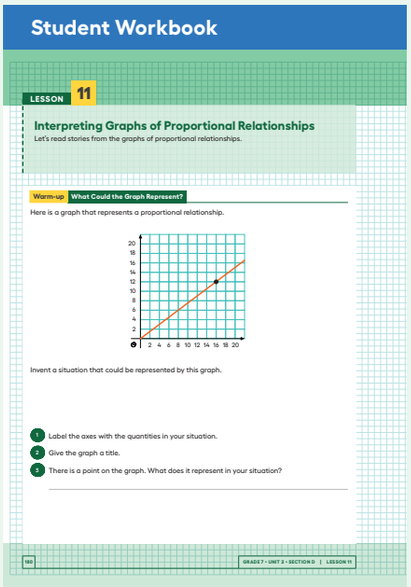
Answers vary

Sample response: A car is moving at a constant speed. Its speed is $\frac{3}{4}$ mile per minute or its pace is $\frac{4}{3}$ minutes per mile.

1. Label the axes with the quantities in your situation.

The horizontal axis is labeled time (minutes) and the vertical axis is labeled distance (miles).

Student Workbook


LESSON 11

Interpreting Graphs of Proportional Relationships

Let's read stories from the graphs of proportional relationships.

Warm-up What Could the Graph Represent?

Here is a graph that represents a proportional relationship.

Invent a situation that could be represented by this graph.

1. Label the axes with the quantities in your situation.
2. Give the graph a title.
3. There is a point on the graph. What does it represent in your situation?

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.

Keep books or devices closed. Display only the problem stem and graph, without revealing the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

2. Give the graph a title.

Distance Traveled by a Car and How Much Time It Takes

3. There is a point on the graph. What does it represent in your situation?

The coordinates of the point are (16, 12). In this situation, it means that the car travels 12 miles in 16 minutes.

Activity Synthesis

Ask a few students to share their situations and other responses. After each, ask the class if they need more information to understand the situation. After a few students have shared, ask the class to think about how all the situations were different and what they had in common. What sorts of things are always true about proportional relationships? Some possible responses might be:

- When one quantity is 0, the other is also 0.
- There is always the same amount of one quantity for every 1 of the other quantity.
- Context-specific considerations like constant speed, the same taste, or the same color.

Remind students that a coordinate point, (x, y) is made up of the "x-coordinate" and the "y-coordinate."

Activity 1

Tyler's Walk

20 min

Activity Narrative

In this activity students interpret points on the graph of a proportional relationship in terms of what they mean about the situation. This activity is intended to further students' understanding of the graphs of proportional relationships in the following respects:

- For any point on the graph except (0, 0), the quotient of the coordinates, $\frac{y}{x}$, is the constant of proportionality.
- When the x-coordinate is 1, the corresponding y-coordinate is k, the constant of proportionality.

Students explain correspondences between parts of the table and parts of the graph. The graph is simple so that students can focus on what a point means in the situation represented. Students need to realize, however, that the axes are marked in 10-unit intervals.

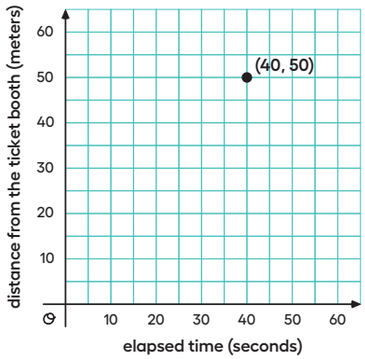
Launch

Arrange students in groups of 2.

Give students 5 minutes of quiet work time, followed by partner and whole-class discussion.

Student Task Statement

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.



time (seconds)	distance (meters)
0	0
20	25
30	37.5
40	50
1	1.25

1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?

   40 seconds after Tyler started walking, he was 50 meters from the ticket booth. The 40 represents the elapsed time in seconds since Tyler started walking away from the ticket booth. The 50 represents Tyler's distance in meters from the ticket booth at that time.

2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.

   Students should write 1.25 in the empty cell of the table and plot (0, 0), (1, 1.25), (20, 25), and (30, 37.5).

3. What does the point (0, 0) mean in this situation?

   Before any time passed, there was no distance between Tyler and the ticket booth.

4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.

   Tyler was 1.25 meters from the ticket booth after 1 second.
   The corresponding point is (1, 1.25).

5. What is the constant of proportionality for the relationship between time and distance?

   The constant of proportionality is 1.25.

   What does it tell you about Tyler's walk?

   It expresses that Tyler is walking at a speed of 1.25 meters per second.

   Where do you see it in the graph?

   It appears as the y-coordinate in (1, 1.25).

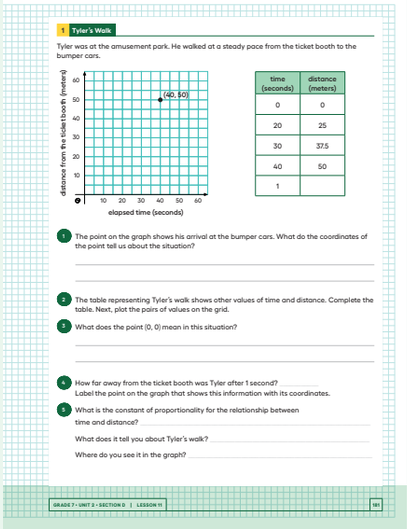
Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

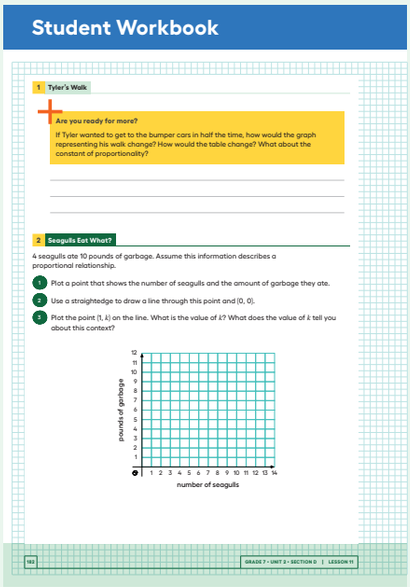
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization, Attention

Student Workbook



Student Workbook


1 Tyler's Walk

Are you ready for more?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

2 Seagulls Eat What?

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straightedge to draw a line through this point and (0, 0).
3. Plot the point (1, k) on the line. What is the value of k? What does the value of k tell you about this context?

pounds of garbage

number of seagulls

Are You Ready for More?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

The graph would be steeper. For the same first coordinate, the second coordinate would be twice as big as in the original situation. The table would include (0, 0), (20, 50), and (1, 2.5). Tyler would already arrive at the bumper cars after 20 seconds and his speed would be 2.5 meters per second.

Activity Synthesis

The goal of the discussion is to make connections between the table and the graph and how they each represent the situation. First, ask students:

- *"How far is the ticket booth from the bumper cars? How do you know?"*

  50 meters, because the point (40, 50) represents his arrival at the bumper cars. 40 is the elapsed time in seconds and 50 is the distance in meters.

Consider clarifying for students that this is assuming that Tyler walked in a straight line. This is an opportunity for attention to precision and making explicit assumptions about a situation.

For each of the following questions, ask students to share how they can tell the answer from the table and how they can tell from the graph.

- *"What quantities are shown?"*

  distance in meters that Tyler is from the ticket booth and elapsed time in seconds since he started walking

- *"How can you tell from the table?"*

  by looking at the column headers

- *"How can you tell from the graph?"*

  by looking at the axes labels

- *"What does each pair of values mean? For example (20, 25) and (0, 0)?"*

  Tyler was 25 meters from the ticket booth after 20 seconds. Tyler was 0 meters away from the ticket booth after 0 seconds.

- *"How can you tell from the table?"*

  The value in the first column gives the elapsed time in seconds since Tyler started walking. The value in the second column tells how many meters away from the ticket booth Tyler was at that time.

- *"How can you tell from the graph?"*

  The x-coordinate gives the elapsed time in seconds since Tyler started walking. The corresponding y-coordinate shows how many meters away from the ticket booth Tyler was at that time.

- *"Is the relationship proportional?"*

  yes

- *"How can you tell from the table?"*

  Dividing the values on any row, except (0, 0), gives the same unit rate.

- *"How can you tell from the graph?"*

  The points lie on a straight line through (0, 0).

- *"What is the constant of proportionality?"*

  1.25

- *"How can you tell from the table?"*

  Find the value for the second column that goes next to the 1 in the first column.

- *"How can you tell from the graph?"*

  Find the point $(1, k)$ that lies on the same line as the other points or find the quotient $\frac{y}{x}$ for any point on the graph besides (0, 0).

After students have seen how the different representations show the same information, consider asking students,

- *"Are there any benefits or drawbacks to one representation compared to the other? Which representation do you prefer?"*

Lastly, ask students to write an equation for this proportional relationship.

Sample responses: $y = 1.25x$ or $d = 1.25t$

Activity 2

Seagulls Eat What?

10 min

Activity Narrative

There is a digital version of this activity.

In this activity students create the graph of a proportional relationship given only one pair of values. Then they use the graph to find the constant of proportionality and interpret it in context.

In previous activities students were given a table with multiple pairs of values to graph. Being given only one pair of values here reinforces the idea that the graph of any proportional relationship makes a straight line through the origin. This activity asks students to connect the points with a solid line. The class discussion revisits the idea that people often connect discrete points with a line to make the relationship more clear, even when the in-between values don't make sense.

In the digital version of the activity, students use an applet to graph a proportional relationship on the coordinate plane. The applet allows students to plot points and draw lines. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Launch

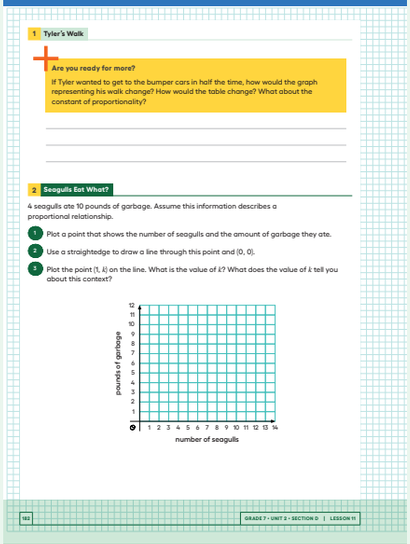
Keep students in the same groups.

Give 5 minutes of quiet work time followed by partner and whole-class discussion.

Building on Student Thinking

If students struggle to find k, encourage them to create a table with a few rows in it and ask them how they can use the table to find k.

Student Workbook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Are you ready for more?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

2 Seagulls Eat What?

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straightedge to draw a line through this point and (0, 0).
3. Plot the point (1, k) on the line. What is the value of k? What does the value of k tell you about this context?

pounds of garbage

number of seagulls

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "What does the value of k tell you about this context?" Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student Task Statement

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.

   Point (4, 10) is plotted.

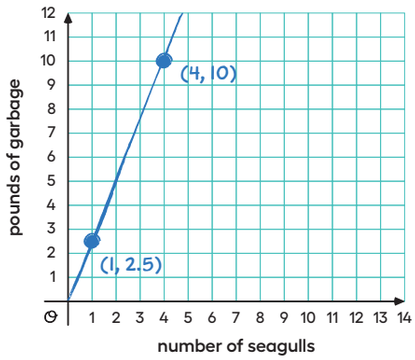
2. Use a straightedge to draw a line through this point and (0, 0).

   See image

3. Plot the point (1, k) on the line. What is the value of k? What does the value of k tell you about this context?

   Point (1, 2.5) is plotted.

   The value of k, 2.5, tells you the number of pounds of garbage consumed per seagull.



Activity Synthesis

Invite students to share their value and interpretation of k. Ask them for different ways to express this information.

Each seagull eats 2.5 pounds of garbage. Or the rate of garbage consumption is 2.5 pounds per seagull.

Ask students:

- *"Is it possible to interpret the meaning of each point on the solid line?"*

  No, only whole numbers of seagulls make sense.

- *"Why is it still useful to draw in the line? How can it help us to learn more about the situation?"*

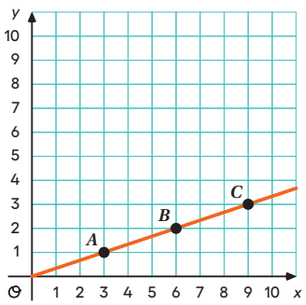
  It helps us to easily find out how much garbage different numbers of seagulls eat. It also helps us to estimate the value of k.

Lesson Synthesis

Share with students,

- *"Today we practiced interpreting the points on a graph in terms of the context it represents."*

If desired, use this example to review these concepts.



- *"What is the constant of proportionality for the relationship shown on this graph?"*

  $\frac{1}{3}$, which is the y-value that goes with the x-value of 1.

- *"Let's say the graph represents the cost of stickers that come in packs of 3."*

  "What does point B represent in this situation?"

  You can buy 6 stickers for $2.

- *"Does it make sense to connect the points with a line for this situation? Why or why not?"*

  No, because it is not possible to buy a fraction of a sticker.

- *"Instead let's say this graph shows the unit conversion between feet and yards."*

  "What does point B represent in this situation?"

  There are 2 yards in 6 feet.

- *"Does it make sense to connect the points with a line for this situation? Why or why not?"*

  Yes, because it is possible for a measurement to be a fraction of a foot.

Lesson Summary

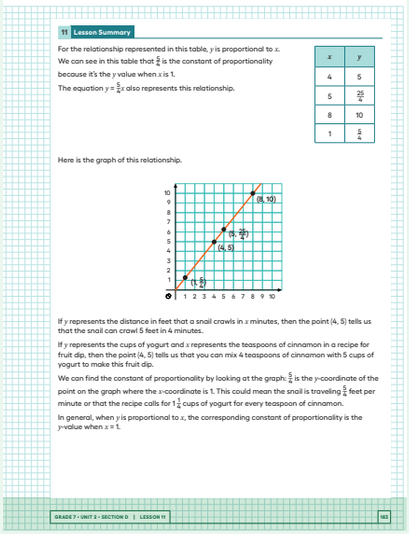
For the relationship represented in this table, y is proportional to x.

We can see in this table that 54 is the constant of proportionality because it's the y value when x is 1.

The equation $y = 54x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Student Workbook


11 Lesson Summary

For the relationship represented in this table, y is proportional to x.
We can see in this table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.
The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.

(8, 10)
(5, 25/4)
(4, 5)
(1, 5/4)

If y represents the distance in feet that a snail crawls in x minutes, then the point (4, 5) tells us that the snail can crawl 5 feet in 4 minutes.

If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point (4, 5) tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

We can find the constant of proportionality by looking at the graph: $\frac{5}{4}$ is the y-coordinate of the point on the graph where the x-coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x, the corresponding constant of proportionality is the y-value when $x = 1$.

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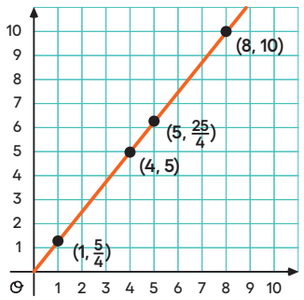
Responding To Student Thinking

Points to Emphasize

If students struggle with interpreting points on the graph of a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about the meaning of various points on the graph.

Unit 2, Lesson 12, Activity 2 Race to the Bumper Cars

Here is the graph of this relationship.



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We can find the constant of proportionality by looking at the graph: $\frac{5}{4}$ is the y-coordinate of the point on the graph where the x-coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x, the corresponding constant of proportionality is the y-value when $x = 1$.

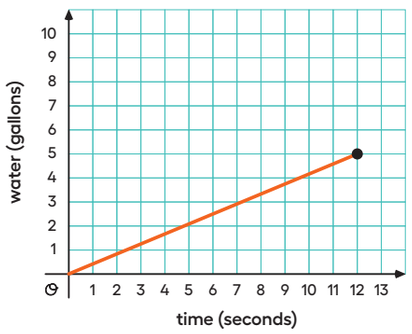
Cool-down

Filling a Bucket

5 min

Student Task Statement

Water runs from a hose into a bucket at a steady rate. The amount of water in the bucket for the time it is being filled is shown in the graph.



1. The point (12, 5) is on the graph. What do the coordinates tell you about the water in the bucket?

   After 12 seconds, there were 5 gallons of water in the bucket.

2. How many gallons of water are in the bucket after 1 second? Label the point on the graph that shows this information.

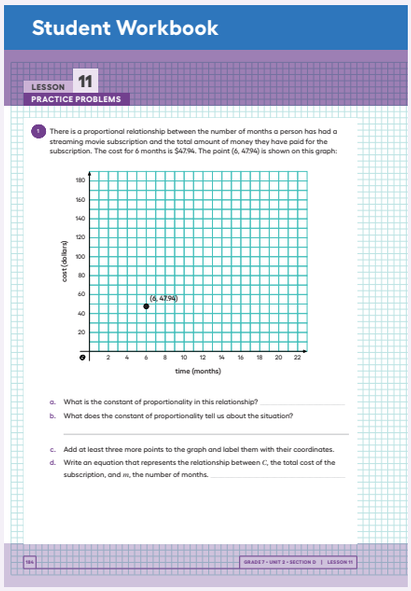
   $\frac{5}{12}$ (or equivalent)

   The point $\left(1, \frac{5}{12}\right)$ should be labeled.

Lesson 11

Practice Problems

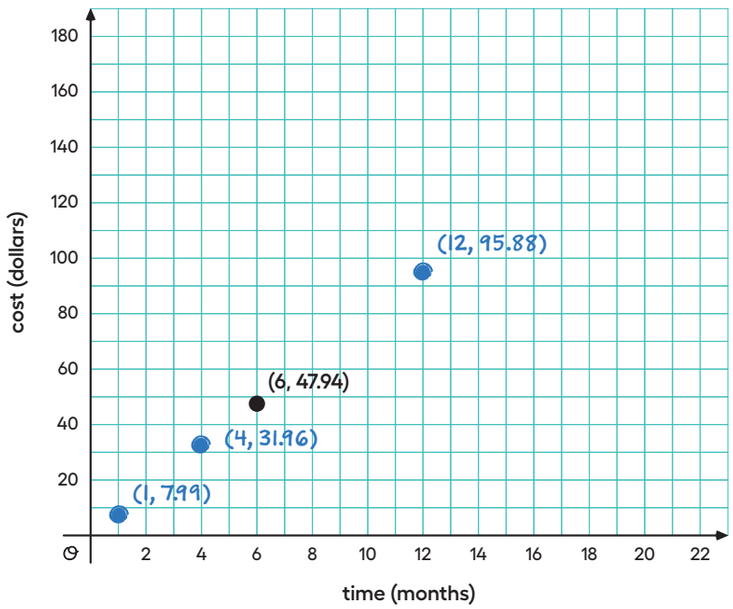
4 Problems



Problem 1

There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is $47.94. The point (6, 47.94) is shown on this graph:

Sample response:



a. What is the constant of proportionality in this relationship?

$7.99

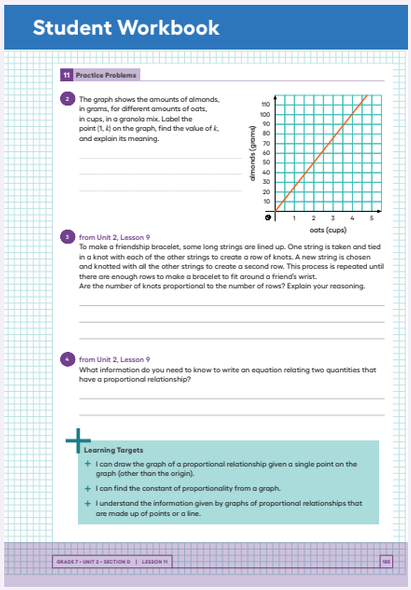
b. What does the constant of proportionality tell us about the situation?

The movie streaming service costs $7.99 for one month of service.

c. Add at least three more points to the graph and label them with their coordinates.

d. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

$C = 7.99m$


Student Workbook

11 Practice Problems

2 The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point (1, k) on the graph, find the value of k, and explain its meaning.

3 from Unit 2, Lesson 9
To make a friendship bracelet, some long strings are lined up. One string is taken and tied in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around a friend's wrist.
Are the number of knots proportional to the number of rows? Explain your reasoning.

4 from Unit 2, Lesson 9
What information do you need to know to write an equation relating two quantities that have a proportional relationship?

Learning Targets
+ I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
+ I can find the constant of proportionality from a graph.
+ I understand the information given by graphs of proportional relationships that are made up of points or a line.

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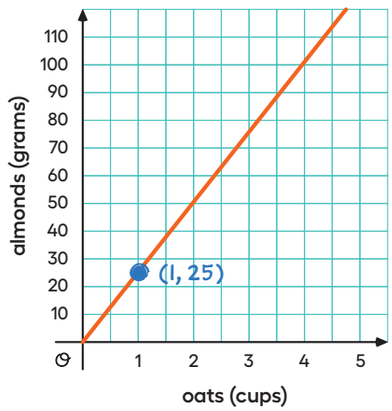
Problem 2

The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point (1, k) on the graph, find the value of k, and explain its meaning.



The point (1, 25) is on the graph. It means that for each cup of oats there are 25 grams of almonds in the granola mix.

Problem 3

from Unit 2, Lesson 9

To make a friendship bracelet, some long strings are lined up. One string is taken and tied in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around a friend's wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

Yes, since each row will have the same number of knots in it, the number of knots will always be a multiple of the number of rows.

Problem 4

from Unit 2, Lesson 9

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

A constant of proportionality and variables for the quantities.