Lesson 3

Exploring Circumference

Goals

- Comprehend the word "pi" and the symbol π to refer to the constant of proportionality between the diameter and circumference of a circle.
- Create and describe (in writing) graphs that show measurements of circles.
- Generalize that the relationship between diameter and circumference is proportional and that the constant of proportionality is a little more than 3.

Learning Targets

- I can describe the relationship between circumference and diameter of any circle.
- I can explain what π means.

Lesson Narrative

In this lesson, students discover that there is a proportional relationship between the diameter and circumference of a circle. They use their knowledge from the previous unit on proportionality to estimate the constant of proportionality. Then they use the constant to compute the circumference given the diameter (and vice versa) for different circles. Students learn that **pi** (π) is the value of the constant and discuss various commonly used approximations. Students will relate the circumference to the radius in a future lesson, so it is not necessary to do so in this lesson.

Determining that the relationship between the circumference and diameter of circles is proportional is an example of looking for and making use of structure and repeated reasoning.

Student Learning Goal

Let's explore the circumference of circles.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Empty toilet paper roll: Warm-up
- Cylindrical household items: Activity 1
- Measuring tapes: Activity 1

Required Preparation

Warm-up:
The empty toilet paper roll is for optional use as a demonstration tool.

Activity 1:
Household items: collect circular or cylindrical objects of different sizes, with diameters from 3 cm to 25 cm. Each group needs 3 items of relatively different sizes. Examples include food cans, hockey pucks, paper towel tubes, paper plates, CD's. Record the diameter and circumference of the objects for your reference during student work time.

Provide one measuring tape per group of 2–4 students. Alternatively, use rulers and string.

To get a good spread of points on the graph, it is important to use circles with a wide variety of diameters, from 3 cm to 25 cm.

For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	25 min	10 min	10 min	5 min

Warm-up

Which Is Greater?

5 min

Activity Narrative

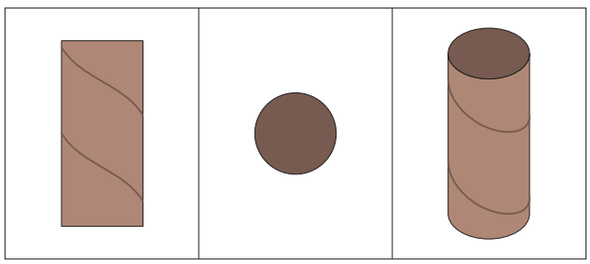
The purpose of this *Warm-up* is to help students visualize circumference as a linear measurement, in preparation for examining the relationship between diameter and circumference in the next activity. Some students may be able to imagine unrolling the tube into a rectangle in order to compare its length and width. Other students may benefit from hands-on experience with a real toilet paper tube.

Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to indicate when they have reasoning to support their response. Give students 1 minute of quiet think time and then time to share their thinking with their group.

Student Task Statement

Clare wonders if the height of the toilet paper tube or the distance around the tube is greater. What information would she need in order to solve the problem? How could she find this out?



Clare needs to measure the length of the tube and the distance around. To find the distance around she could measure the tube with a flexible measuring tape, or cut and flatten the tube.

Building on Student Thinking

Students may not understand what is meant by the height of the tube because it can sit in two different ways. Point these students to the first picture of the tube, and ask them to identify the height as the tube is sitting in that picture.

Activity Synthesis

The goal of this discussion is for students to internalize that the distance around a circle is a length, just like the height of an object is. As a one-dimensional measurement, it can be measured using a ruler.

Survey students on which length they think is greater. Consider displaying the image again for reference while students are explaining their reasoning. To involve more students in the conversation, consider asking questions like:

Inspire Math

Crop Circles video

Go Online

Before the lesson, show this video to introduce the real-world connection.

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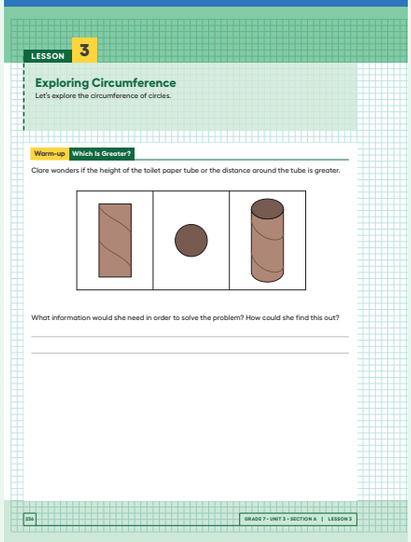
Student Workbook



Instructional Routines

5 Practices

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- *"What was important to you when making your decision?"*
- *"Did anyone think about the measurements in a different way?"*
- *"Do you agree or disagree? Why?"*
- *"What information do you think would help you make a better decision? If an actual toilet paper tube is available, demonstrate measuring the tube by unrolling the tube to measure the circumference and height."*

If an actual toilet paper tube is available, consider demonstrating unrolling the tube to measure the circumference in a straight line and compare it to the height.

Activity 1

Measuring Circumference and Diameter

25 min

Activity Narrative

There is a digital version of this activity.

In this activity, students measure the diameter and circumference of different circular objects and graph their measurements. This echoes the structure of a previous activity in which they measured squares. Students notice that the two quantities appear to be proportional to each other. Based on the graph, they can estimate that the constant of proportionality is close to 3. From the table they may be able to estimate that it is a little bigger than 3.

This activity provides evidence that there is a constant of proportionality between the circumference of a circle and its diameter. The best precision we can expect for the constant of proportionality in this activity is "around 3" or possibly "a little bit bigger than 3." As students measure multiple circles and notice patterns in their measurements, they express regularity in repeated reasoning.

Monitor for students who recognize that the relationship between diameter and circumference appears to be proportional, including:

- Notice that the points on the graph appear to lie on a line through the origin.
- Use the graph to estimate a constant of proportionality.
- Divide the values in the table to estimate a constant of proportionality.

Plan to have students present in this order to support moving from less precise to more precise estimation.

In the digital version of the activity, students use applets to measure the circumference of various circles and plot their measurements on the coordinate plane. The first applet allows students to see an animation of the circumference being unwrapped along a ruler. The second applet allows students to enter values into a table and see the points plotted on a coordinate plane. Use the digital version if physical circular objects are not available.

Launch

Arrange students in groups of 2–3. Distribute two circular objects to each group, along with measuring tapes or string and rulers. Consider demonstrating how to measure the circumference, especially if using string and rulers. Wrap a string around the object, make note of where it completes one circle, unwrap the object, and use a ruler to measure the marked length. Encourage students to be as precise as possible as they measure.

Ask students to complete the first two questions in their group, and then to gather additional information from another group (who measured different objects) for the last two questions.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Access for Students with Diverse Abilities

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Consider giving students one circular object at a time. Check in with students to provide feedback and encouragement after each chunk. Look for students' methods for measuring circumference.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Task Statement

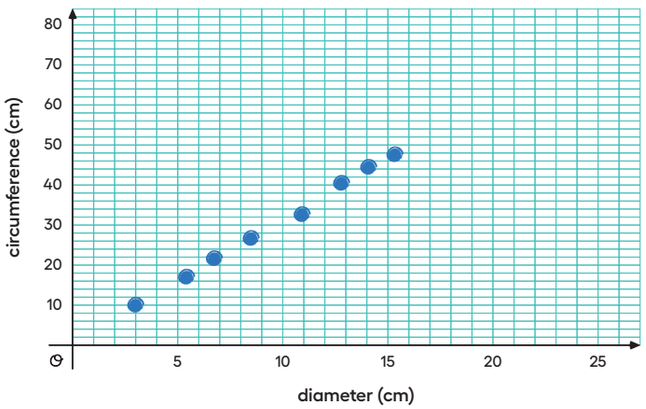
Your teacher will give you two circular objects.

1. Measure the diameter and the circumference of each circle to the nearest tenth of a centimeter. Record your measurements in the first two rows of the table.

   Sample response:

object	diameter (cm)	circumference (cm)
water bottle	7.2	23.3
bowl	22.8	71.5

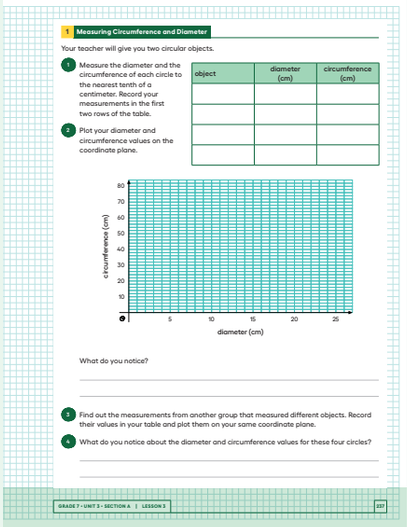
2. Plot your diameter and circumference values on the coordinate plane. What do you notice?



The points look like they are close to lying on a straight line through (0, 0).

3. Find out the measurements from another group that measured different objects. Record their values in your table and plot them on your same coordinate plane.

Student Workbook


1 Measuring Circumference and Diameter

Your teacher will give you two circular objects.

1. Measure the diameter and the circumference of each circle to the nearest tenth of a centimeter. Record your measurements in the first two rows of the table.
2. Plot your diameter and circumference values on the coordinate plane.

object	diameter (cm)	circumference (cm)

circumference (cm)

diameter (cm)

What do you notice?

3. Find out the measurements from another group that measured different objects. Record their values in your table and plot them on your same coordinate plane.
4. What do you notice about the diameter and circumference values for these four circles?

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4. What do you notice about the diameter and circumference values for these four circles?

The points look like they are close to lying on a straight line through (0, 0).

Building on Student Thinking

Students may try to measure the diameter without going across the widest part of the circle, or may struggle with measuring around the circumference. Mentally check that their measurements divide to get approximately 3 or compare with your own prepared table of data and prompt them to re-measure when their measurements are off by too much. If the circular object has a rim or lip, this could help students keep the measuring tape in place while measuring the circumference.

If students are struggling to see the proportional relationship, remind them of recent examples in which they have seen similar graphs of proportional relationships. Ask them to estimate additional diameter-circumference pairs that would fit the pattern shown in the graph. Based on their graphs, do the values of the circumferences seem to relate to those of the diameters in a particular way? What seems to be that relationship?

Activity Synthesis

The purpose of this discussion is for students to see that circumference is proportional to diameter and that the constant of proportionality is close to 3.

Invite previously selected students to share their observations. Sequence the discussion of the observations in the order listed in the activity narrative. If possible, record and display their work for all to see. Connect the different responses to the learning goals by asking questions such as:

- *"What do the representations have in common? How are they different?"*
- *"What do the representations tell us about the relationship between the diameter and circumference of different circles?"*
- *"How does the proportional relationship show up in each representation?"*

The key takeaway is that the relationship between diameter and circumference appears to be proportional.

If no student mentions it, invite students to estimate the constant of proportionality by dividing the values on each row of the table. For example,

object	diameter (cm)	circumference (cm)	circumference ÷ diameter
water bottle	7.2	23.3	3.236
bowl	22.8	71.5	3.136
DVD	12.0	37.0	3.083
medicine bottle	3.1	10.5	3.387

Ask students why these numbers might not be *exactly* the same (measurement error, rounding). Use the average of the quotients, rounded to one or two decimal places, to come up with a "working value" for the constant of proportionality, such as 3.1. This class-generated constant of proportionality will be used in the next activity. There's no need to mention *pi* or its usual approximations yet.

If time permits, consider discussing the accuracy of measurements for circumference and diameter. Measuring the diameter to the nearest tenth of a centimeter can be done pretty reliably with a ruler. Measuring the circumference of a circle to the nearest tenth of a centimeter may or may not be reliable, depending on the method used. Wrapping a flexible measuring tape around the object is likely the most accurate method for measuring the circumference of a circle.

Activity 2

Calculating Circumference and Diameter

10 min

Activity Narrative

In this activity, students apply the proportional relationship between circumference and diameter to calculate unknown measurements of circles. They use the constant of proportionality they estimated in the previous activity. Then, the teacher introduces pi (π) during the whole-class discussion. It is important to save enough time for the discussion about the meaning and value of π at the end of this activity.

When students use the equation $C = kd$ to complete a table showing diameters and circumferences of various circles, they are making use of the structure of proportional relationships.

Launch

If time is limited, consider dividing up the calculations, assigning one or two circles to each group of students. If desired, ask students to label the measurements given in the table on the picture of the circles.

Instruct students to use the constant of proportionality that the class estimated in the previous activity. Give students quiet work time followed by whole-class discussion.

Instructional Routines

MLR8: Discussion Supports

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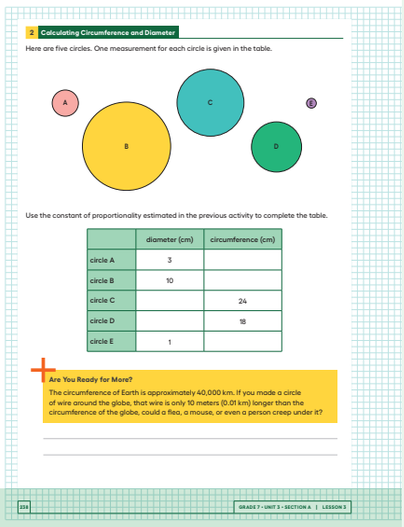
Student Workbook


2 Calculating Circumference and Diameter

Here are five circles. One measurement for each circle is given in the table.

Use the constant of proportionality estimated in the previous activity to complete the table.

	diameter (cm)	circumference (cm)
circle A	3	
circle B	10	
circle C		24
circle D		18
circle E	1	

Are You Ready for More?

The circumference of Earth is approximately 40,000 km. If you made a circle of wire around the globe, that wire is only 10 meters (0.01 km) longer than the circumference of the globe, could a flea, a mouse, or even a person creep under it?

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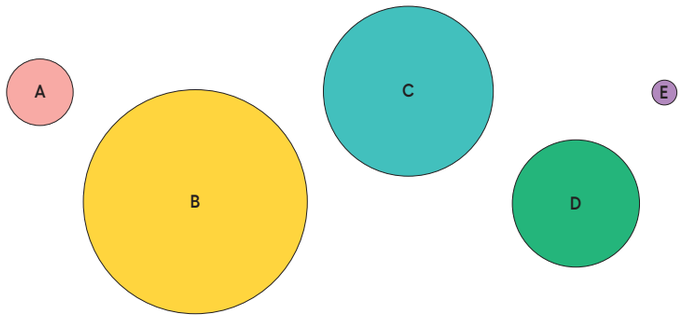
Student Task Statement

Here are five circles. One measurement for each circle is given in the table.



Use the constant of proportionality estimated in the previous activity to complete the table.

Sample response using 3.1 for the constant of proportionality

	diameter (cm)	circumference (cm)
circle A	3	9.3
circle B	10	31
circle C	7.7	24
circle D	5.8	18
circle E	1	3.1

Building on Student Thinking

Some students may multiply the circumference by the constant of proportionality instead of dividing by it. Prompt them to consider whether the diameter can be longer than the circumference of a circle.

Some students may struggle to divide by 3.1 if that is the constant of proportionality decided on in the previous lesson. Ask these students if they could use an easier number as their constant, and allow them to divide by 3 instead. Then ask them how their answer would have changed if they divided by 3.1.

Are You Ready for More?

The circumference of Earth is approximately 40,000 km. If you made a circle of wire around the globe, and that write is only 10 meters (0.01 km) longer than the circumference of the globe, could a flea, a mouse, or even a person creep under it?

Yes, each meter added to the diameter of a circle adds about 3.1 meters to the circumference of the circle. So if the circumference of Earth is increased by 10 meters, this means that a little more than 3 meters have been added to the diameter. So there would be about 1.5 meters of distance between the rope and Earth, making it easy for a flea, mouse, or person to go under the rope!

Activity Synthesis

The goal of this discussion is to introduce the equation $C = \pi d$. First, display the table from the activity for all to see. Invite students to share how they calculated the missing values. Record their reasoning on and around the table. Draw students' attention to their use of multiplication or division by the constant of proportionality.

If no student mentions using an equation of the form $C = kd$, where k is the constant of proportionality agreed upon in the previous activity, ask students how we could summarize this relationship with an equation. Display the equation $C = 3.1d$ (or whatever value of k the class agreed upon) and ask students to describe what this equation means.

Next, ask if students have heard of the number pi or seen the symbol π. Tell students that **pi** is the exact value of the constant of proportionality for this relationship. Explain that the exact value of π is a decimal with infinitely many digits and no repeating pattern, so an approximation is often used. Frequently used approximations for π include $\frac{22}{7}$, 3.14, and 3.14159, but none of these are exactly equal to π. Adjust the displayed equation to say $C = \pi d$.

Lesson Synthesis

Share with students:

- *"Today we measured the diameter and circumference of different circles. We saw that there is a proportional relationship between these quantities. We estimated the constant of proportionality for the relationship."*

To review the relationship between diameter and circumference, consider asking students:

- *"How did the graph help us decide that there is a proportional relationship between diameter and circumference?"*

  When we plotted our measurements for diameter and circumference, the points were close to a straight line through (0, 0).

- *"How did the table help us decide that there is a proportional relationship between diameter and circumference?"*

  When we divided the circumference by the diameter for each row, the quotients were all close to the same value.

- *"If we want to write an equation like $C = kd$ to represent this relationship, what can we use as k, the constant of proportionality?"*

  The exact constant of proportionality is called pi and written π. The value is a little more than 3.

If desired, use this example to review these concepts:

- *"A circle has a diameter of 4 cm. What is its circumference?"*

  approximately 12.4 cm, because $3.1 \cdot 4 = 12.4$

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
To support the transfer of new vocabulary to long-term memory, invite students to chorally repeat these phrases in unison 1–2 times:

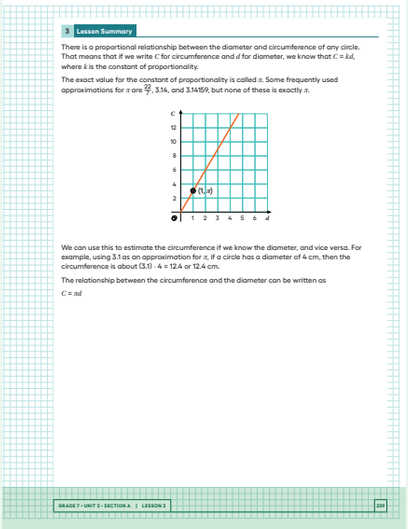
- "Pi is the constant of proportionality between diameter and circumference."
- "Circumference is pi times diameter."
- "Diameter is circumference divided by pi."

Advances: Speaking, Listening

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


3 Lesson Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called π. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.

We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \cdot 4 = 12.4$ or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$C = \pi d$

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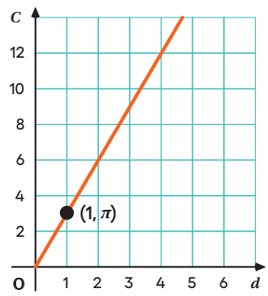
Lesson Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called pi, and its symbol is π. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.



We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \cdot 4 = 12.4$, or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$$C = \pi d$$

Cool-down

Identifying Circumference and Diameter

5 min

Student Task Statement

Select **all** the pairs that could be reasonable approximations for the diameter and circumference of a circle. Explain your reasoning.

5 meters and 22 meters.

does not work, because $22 \div 5 > 4$

19 inches and 60 inches.

does work, because $60 \div 19 \approx 3.158$

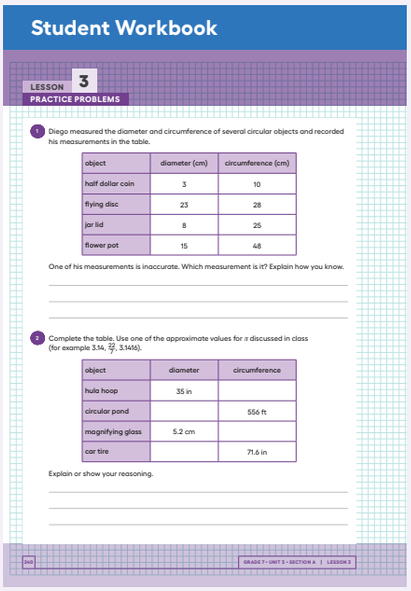
33 centimeters and 80 centimeters.

does not work, because $80 \div 33 < 2.5$

Lesson 3

Practice Problems

4 Problems


Student Workbook

LESSON 3
PRACTICE PROBLEMS

1 Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
half dollar coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

One of his measurements is inaccurate. Which measurement is it? Explain how you know.

2 Complete the table. Use one of the approximate values for π discussed in class (for example 3.14, $\frac{22}{7}$, 3.1416).

object	diameter	circumference
hula hoop	35 in	
circular pond		556 ft
magnifying glass	5.2 cm	
car tire		71.6 in

Explain or show your reasoning.

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Problem 1

Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
half dollar coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

One of his measurements is inaccurate. Which measurement is it? Explain how you know.

The measurement for the flying disc is very inaccurate. It should be about 3 times the diameter (or a little more).

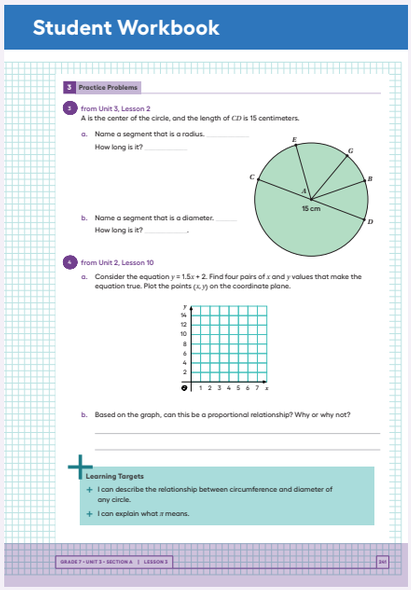
Problem 2

Complete the table. Use one of the approximate values for π discussed in class (for example 3.14, $\frac{22}{7}$, 3.1416). Explain or show your reasoning.

object	diameter (cm)	circumference (cm)
hula hoop	35 in	110 in
circular pond	177 ft	556 ft
magnifying glass	5.2 cm	16 cm
car tire	22.8 in	71.6 in

The constant of proportionality is about 3.14. The given diameters are multiplied by 3.14 to find the missing circumferences. The given circumferences are divided by 3.14 to find the missing diameters. Both the missing circumferences and the missing diameters have been rounded.

Student Workbook


3 Practice Problems

3 from Unit 3, Lesson 2
A is the center of the circle, and the length of CD is 15 centimeters.

a. Name a segment that is a radius. How long is it?

b. Name a segment that is a diameter. How long is it?

4 from Unit 2, Lesson 10

a. Consider the equation $y = 1.5x + 2$. Find four pairs of x and y values that make the equation true. Plot the points (x, y) on the coordinate plane.

b. Based on the graph, can this be a proportional relationship? Why or why not?

Learning Targets
+ I can describe the relationship between circumference and diameter of any circle.
+ I can explain what π means.

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Problem 3

from Unit 3, Lesson 2

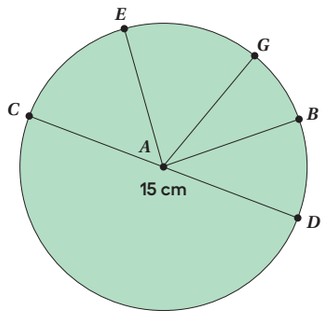
A is the center of the circle, and the length of CD is 15 centimeters.

a. Name a segment that is a radius. How long is it?

Sample responses: *AC, AD, AB, AE, AG, 7.5 cm*

b. Name a segment that is a diameter. How long is it?

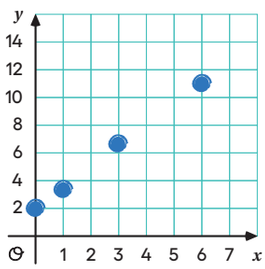
CD, 15 cm



Problem 4

from Unit 2, Lesson 10

a. Consider the equation $y = 1.5x + 2$. Find four pairs of x and y values that make the equation true. Plot the points (x, y) on the coordinate plane.



b. Based on the graph, can this be a proportional relationship? Why or why not?

No, this relationship could not be proportional because the graph does not go through (0, 0).