Multiply! (Optional)

Goals

- Identify multiplication expressions that are equal, and justify (orally) that they are equal.
- Multiply rational numbers, including multiplication expressions with three factors, and explain (orally and in writing) the reasoning.

Learning Target

I can solve problems that involve multiplying rational numbers.

Student Learning Goal

Let's get more practice multiplying signed numbers.

Lesson Narrative

This lesson is optional because it provides opportunities for additional practice with multiplying rational numbers. The main activity uses the *Information Gap* routine. To obtain all the necessary information, students need to persevere in asking questions and communicate precisely.

There are two other optional activities in this lesson. In the first, students sort multiplication expressions and match the expressions that have the same value. In the second, they calculate the product of expressions and compare answers with their partner.

Some of the expressions involve fractions or decimals. Multiplying by a unit fraction serves as a bridge to the next lesson in which students will divide rational numbers. Also, some of the expressions have three different factors, so students can apply the associative property. Regardless of which two factors students choose to multiply first, multiplying that product by the remaining factor will result in the same answer.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2, Activity 3)

Access for Multilingual Learners

- MLR4: Information Gap Cards (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- · Card Sort
- MLR4: Information Gap Cards
- · Which Three Go Together?

Required Materials

Materials to Copy

- Temperature and Art Funds Cards (1 copy for every 4 students): Activity 1
- Matching Expressions Cards (1 copy for every 2 students): Activity 2

Required Preparation

Activity 2:

Copy each set of cards on a different color of paper so they can easily be sorted for the next class.

Lesson Timeline

5_{min}

Warm-up

20 min

Activity 1

10 min

Activity 2

10 min

Activity 3

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Warm-up

Which Three Go Together: Expressions



Activity Narrative

This *Warm-up* prompts students to compare four expressions. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three images that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A.7.9x

B. 7.9 + x

C. 7.9 · (-10)

D.-79

Sample responses:

A, B, and C go together because:

- Each expression contains the number 7.9.
- Each expression has more than just a single number.

A, B, and D go together because:

· None of the expressions have parentheses.

A, C, and D go together because:

- · None of the expressions has addition or a sum.
- If x = -10, then the expressions would all be equivalent.

B, C, and D go together because:

- If x = -86.9, then the expressions would all be equivalent.
- None of the expressions are multiplying by a variable.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

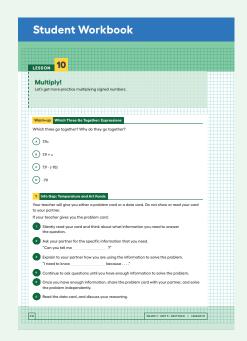
Instructional Routines

Which Three Go Together?

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Instructional Routines

MLR4: Info Gap

ilclass.com/r/10695522

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Access for Multilingual Learners (Activity 1)

MLR4: Information Gap

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.

Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity, or provide students with a physical copy.

Supports accessibility for: Memory, Organization

During the discussion, prompt students to explain the meaning of any terminology they use, such as "positive," "negative," "addition," and "subtraction," and to clarify their reasoning as needed. Consider asking:

"What do you mean by ...?"

"Can you say that in another way?"

Activity 1

Info Gap: Temperature and Art Funds



Activity Narrative

In this activity, students use addition, subtraction, and multiplication with rational numbers to solve problems related to temperature and money, but they do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Launch 🙎

Tell students they will solve problems that involve positive and negative numbers. Display the *Information Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem, and instruct them to switch roles.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

- **1.** Silently read your card and think about what information you need to answer the question.
- **2.** Ask your partner for the specific information that you need. "Can you tell me _____?"
- **3.** Explain to your partner how you are using the information to solve the problem. "I need to know ______ because ..."
- **4.** Continue to ask questions until you have enough information to solve the problem.

- **5.** Once you have enough information, share the problem card with your partner, and solve the problem independently.
- 6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

- 1. Silently read your card. Wait for your partner to ask for information.
- 2. Before telling your partner any information, ask, "Why do you need to know?"
- **3.** Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
- 4. These steps may be repeated.
- **5.** Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
- 6. Share the data card, and discuss your reasoning.

Problem Card I: 1.5 °C

Problem Card 2: -\$14.50

Are You Ready for More?

1. Find the value of the expression without a calculator.

(2)(-30) + (-3)(-20) + (-6)(-10) - (2)(3)(10)

0

2. Write an expression that uses addition, subtraction, and multiplication, only uses negative numbers, and has the same value as the previous expression.

Sample response: -10 + -2 - (-3)(-4)(-1)

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

"What strategies did you use to help you understand or answer the problem?"

I drew a number line diagram. I rewrote a subtraction problem as an addition problem. I wrote an equation to represent the situation.

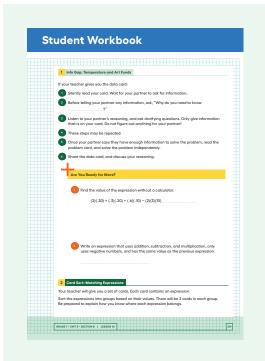
"Which quantities could be represented by a negative number in each situation?"

temperature getting colder and the cost for the museum tickets

"How did you use multiplication in each of these situations?"

The rate the temperature increased or decreased was multiplied by the length of time the temperature was changing. The cost of each ticket was multiplied by the number of students, and the cost of each painting was multiplied by the number of paintings sold.

Highlight for students how operations with positive and negative numbers are used to represent each situation.



Instructional Routines

Card Sort

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Access for Multilingual Learners (Activity 2, Launch)

MLR8: Discussion Supports.

Display sentence frames to support partner discussion. Examples: "I noticed _, so I matched ..." Encourage students to challenge each other when they disagree.

Advances: Listening, Conversing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Activity 2: Optional

Card Sort: Matching Expressions



Activity Narrative

Students sort different expressions during this activity. A sorting task gives students opportunities to analyze representations and structures closely and make connections. Students recall links between positive fractions and multiplication in preparation to think about division as multiplication by the reciprocal.

Monitor for students who determine a way to tell if two expressions are *not* equivalent without computing the product. For example, they may decide whether each product would be positive or negative before doing any arithmetic.

Launch 22

Tell students that the cards contain expressions and that they will take turns sorting the cards into groups with equivalent expressions. Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

If necessary, show an example of finding the product of three integers. Display this expression, or one similar, for all to see, and ask students for strategies on how to determine its value: $(-2) \cdot (-3) \cdot (-4)$.

The key idea is to first consider only one product and replace a pair of numbers with the product. For example, the product (-2) \cdot (-3) can be replaced with 6. The expression turns into (6) \cdot (-4), which students know how to evaluate.

Arrange students in groups of 2. Give each group a set of cards cut from the blackline master. Tell students that each group will have 3 cards.

Student Task Statement

Your teacher will give you a set of cards. Each card contains an expression.

Sort the expressions into groups based on their values. There will be 3 cards in each group. Be prepared to explain how you know where each expression belongs.

-12	-I · I2	-I·(-3)·(-4)	-2 · 6
-15	-I · (-3) · (-5)	- 3 · 5	I · - 15
+15	I · (-3) · (-5)	(-3) · (-5)	I · 15
+8	- <u>1</u> ·(-16)	- -32	2 · 4
-8	- <u>1</u> .16	-64 · 1 8	2 · (-4)
+12	-1 · (-3) · 4	-I · (-2) · <i>6</i>	-I · (-I2)

Activity Synthesis

The purpose of this discussion is for students to share the strategies they used to find equivalent expressions. Once all groups have completed the card sort, discuss the following:

"Which matches were tricky? Explain why."

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

Ask the previously identified students to share their rationale for identifying those that do not match.

If time allows, consider highlighting the link between multiplying by a fraction and dividing by a whole number. For example, ask students to rewrite a multiplication expression, such as -64 $\cdot \frac{1}{8}$, using division (-64 \div 8). It is not necessary for students to know rules for dividing signed numbers at this point, as this will be the focus of future lessons.

Activity 3: Optional

Row Game: Multiplying Rational Numbers

10 min

Activity Narrative

This activity gives students an opportunity to practice multiplying signed numbers. The solutions to the problems in each row are the same, so students can check their work with a partner.

Launch 🙎

Arrange students in groups of 2. Tell students that one person in each group will find the value of the expressions in column A and the other person will find the value of the expressions in column B. Give students 5–6 minutes of partner work time, and follow with a whole-class discussion.

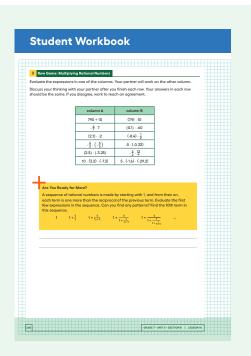
Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation, such as "First, I ________ because ..." "Then, I ..."

"_____ and ____ are the same because ..." or "How did you get ...?"

Supports accessibility for: Language, Social-Emotional Functioning



Student Task Statement

Evaluate the expressions in one of the columns. Your partner will work on the other column.

Discuss your thinking with your partner after you finish each row. Your answers in each row should be the same. If you disagree, work to reach an agreement.

item 2:	item 3:	
790 ÷ 10	(7.9) · 10	Row I: 79
$-\frac{6}{7}\cdot 7$	(0.1) · -60	Row 2: -6
(2.1) · -2	$(-8.4) \cdot \frac{1}{2}$	Row 3: -4.2
$-\frac{4}{3}\cdot\left(-\frac{6}{5}\right)$	-5 · (-0.32)	Row 4: 1.6
(2.5) · (-3.25)	$-\frac{5}{2} \cdot \frac{13}{4}$	Row 5: -8.125
-10 · (3.2) · (-7.3)	5 · (-1.6) · (-29.2)	Row 6: 233.6

Are You Ready for More?

A sequence of rational numbers is made by starting with 1, and from then on, each term is one more than the reciprocal of the previous term. Evaluate the first few expressions in the sequence. Can you find any patterns? Find the 10th term in this sequence.

$$1 + \frac{1}{1}$$
 $1 + \frac{1}{1+1}$ $1 + \frac{1}{1+\frac{1}{1+1}}$ $1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}$

The IOth term is $\frac{84}{55}$. Instead of calculating this from an extremely long fraction representation, we can look for patterns in applying the "one more than the reciprocal" rule. The first few terms are I, 2, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{8}{5}$. To find the next one, applying that rule gives I + $\frac{8}{5}$ = I + $\frac{5}{8}$ = $\frac{8}{8}$ + $\frac{5}{8}$ = $\frac{13}{8}$. This leads to an interesting sequence in the numerators and denominators: Given a term $\frac{a}{b}$ in the sequence, the next term will be $\frac{a+b}{a}$. That is, the old numerator becomes the new denominator, and to get the new numerator, we add together the old numerator and old denominator. This observation makes it very easy to continue the sequence to the IOth term.

 $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}.$

Activity Synthesis

The purpose of this discussion is for students to share strategies they used to multiply signed numbers and to share any patterns they noticed.

Consider asking some of the following questions:

"Were there any rows for which you and your partner did not get the same answer?"

Warm-up

"Did you and your partner use the same strategy for each row?"

"What was the same and different about both of your strategies?"

"Did you learn a new strategy from your partner?"

"Did you try a new strategy while working on these questions?"

Lesson Synthesis

Share with students.

"Today we practiced multiplying signed numbers."

If desired, use this activity to review the different types of multiplication expressions that students encountered in this lesson:

Display a number line with the numbers -1, 0, and 1 labeled. Ask students to give examples of multiplications expressions with a product that is:

- Greater than 1. $(5 \cdot 3 \text{ or } -5 \cdot -3)$
- Less than -1. (5 · -3 or -5 · -3 · -1)
- Between 0 and 1. $(\frac{1}{5} \cdot \frac{1}{3} \text{ or } -\frac{1}{5} \cdot -\frac{1}{3})$
- Between -1 and 0. $(\frac{1}{5} \cdot -\frac{1}{3} \text{ or } -\frac{1}{5} \cdot -\frac{1}{3} \cdot -1)$

Lesson Summary

· A positive number times a positive number always results in a positive number.

For example, $\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$.

• A negative number times a negative number also always results in a positive number.

For example, $-\frac{3}{5} \cdot -\frac{7}{8} = \frac{21}{40}$.

• A negative times a positive number or a positive number times a negative number always results in a negative number.

For example, $\frac{3}{5} \cdot -\frac{7}{8} = -\frac{3}{5} \cdot \frac{7}{8} = -\frac{21}{40}$.

· A negative number times a negative number times a negative number also always results in a negative number.

For example, $-3 \cdot -4 \cdot -5 = -60$.

Student Workbook ple, $\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$. ple, $-\frac{3}{5} \cdot -\frac{7}{8} = \frac{21}{40}$ sle, $\frac{3}{5} \cdot \frac{7}{8} = \frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$.

Warm-up

Responding To Student Thinking

More Chances

Points to Emphasize

If students struggle with multiplying negative fractions or decimals in the cool-down, plan to focus on strategies when opportunities arise over the next several lessons.

Cool-down

Making Mistakes

5 min

Student Task Statement

Noah was doing some homework and answered the following questions. Do you agree with his answers? If you disagree, explain your reasoning.

1.
$$-5 \cdot 8 = 40$$

Disagree

Sample reasoning: A negative number times a positive number is a negative number.

2.
$$(2.7) \cdot (-2.5) = -6.75$$

Agree

3.
$$-\frac{3}{4} \cdot -\frac{5}{7} = -\frac{15}{28}$$

Disagree

Sample reasoning: A negative number times a negative number is a positive number.

Practice Problems

Problem 1

Evaluate each expression:

a. -12
$$\cdot \frac{1}{3}$$

-4

b. -12
$$\cdot -\frac{1}{3}$$

4

c.
$$12 \cdot \left(-\frac{5}{4}\right)$$

-15

d. -12
$$\cdot \left(-\frac{5}{4} \right)$$

15

Problem 2

Evaluate each expression:

-6

6

-6

Problem 3

from Unit 5, Lesson 1

Order each set of numbers from least to greatest.

b. -5, -5.2, 5.5,
$$-5\frac{1}{2}$$
, $\frac{-5}{2}$

$$-5\frac{1}{2}$$
, -5.2 , -5 , $\frac{-5}{2}$, 5.5

Problem 4

from Unit 5, Lesson 3

$$30 + -30 = 0$$
.

a. Write another sum of two numbers that equals 0.

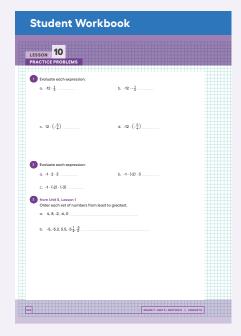
Sample response: -589 + 589

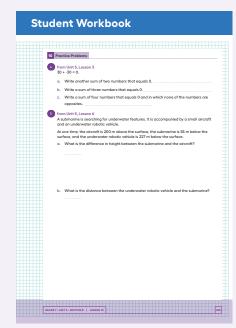
b. Write a sum of three numbers that equals 0.

Sample response: -589 + 500 + 89

c. Write a sum of four numbers that equals 0 and in which none of the numbers are opposites.

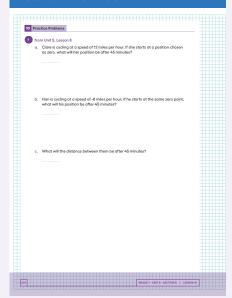
Sample response: -589 + 500 + 90 + (-1)



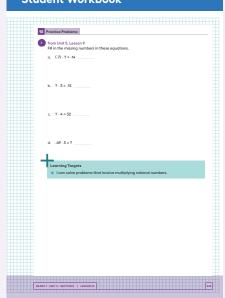


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Student Workbook



Problem 5

from Unit 5, Lesson 6

A submarine is searching for underwater features. It is accompanied by a small aircraft and an underwater robotic vehicle.

At one time, the aircraft is 200 m above the surface, the submarine is 55 m below the surface, and the underwater robotic vehicle is 227 m below the surface.

a. What is the difference in height between the submarine and the aircraft?

We have to assume they are all directly above or below each other to answer the question.

b. What is the distance between the underwater robotic vehicle and the submarine?

172 m

Problem 6

from Unit 5, Lesson 8

a. Clare is cycling at a speed of 12 miles per hour. If she starts at a position chosen as zero, what will her position be after 45 minutes?

9 miles, because 12 · 0.75 = 9

b. Han is cycling at a speed of -8 miles per hour; if he starts at the same zero point, what will his position be after 45 minutes?

-6 miles, because $-8 \cdot 0.75 = -6$

c. What will the distance between them be after 45 minutes?

15 miles

Problem 7

from Unit 5, Lesson 9

Fill in the missing numbers in these equations.

a. $(-7) \cdot ? = -14$

 $(-7) \cdot 2 = -14$

b. $? \cdot 3 = -15$

 $(-5) \cdot 3 = -15$

c. $? \cdot 4 = 32$

 $8 \cdot 4 = 32$

d. $-49 \cdot 3 = ?$

 $(-49) \cdot 3 = -147$