One of These Things Is Not Like the Others

Goals

- Choose and create representations to compare ratios in the context of recipes or scaled copies.
- Coordinate (orally)
 different representations
 of a situation involving
 equivalent ratios, e.g.,
 discrete diagrams, tables,
 or double number line
 diagrams.
- Determine which recipes or geometric figures involve equivalent ratios, and justify (orally, in writing, and through other representations) that they are equivalent.

Learning Targets

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Access for Students with Diverse Abilities

- Representation (Activity 2)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

• MLR2: Collect and Display

Required Materials

Materials to Gather

- Math Community Chart: Warm-up
- Colored pencils: Activity 1
- Drink mix: Activity 1
- · Measuring cup: Activity 1
- · Measuring spoons: Activity 1
- Mixing containers: Activity 1
- Small disposable cups: Activity 1
- Water: Activity 1
- · Geometry toolkits: Activity 2

Lesson Narrative

In this lesson, students examine situations that involve ratios. They identify features that can be described with **equivalent ratios**. There are opportunities to review work from grade 6 in representing ratios with tables and diagrams. This is intended to support initial, informal conversations about the key ideas in proportional relationships before those ideas are formally introduced.

The tasks are intentionally not well-posed, that is, they do not have exact solutions. They are designed to prompt students to think about how we can use mathematical models to make sense of common perceptual experiences, such as things that taste or look the same or different. The focus is on examination of a feature that can be represented as a unit rate.

Lesson Timeline

5 min

Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

One of These Things is Not Like the Others

Lesson Narrative (continued)

In the first activity, students are given the relevant measurements needed to compare the situations. In the second activity, they are asked to think about how to quantify what they see, in particular, what measurements might help describe the picture. The work of describing observations qualitatively and quantitatively encourages students to communicate with precision.

Math Community

Today's math community building time has two goals. The first is for students to make a personal connection to the math actions chart and to share on their *Cool-down* the math action that is most important to them. The second is to introduce the idea that the math actions that students have identified will be used to create norms for their mathematical community in upcoming lessons.

Student Learning Goal

Let's remember what equivalent ratios are.

Activity 1:

Prepare to show the three drink mixtures, either by making them yourself or by displaying the video.

To make three mixtures:

- 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix
- 2 cups of water with ¹/₂ teaspoon of powdered drink mix
- 1 cup of water with ¹/₄ teaspoon of powdered drink mix

Students will need three small cups each; they just need a few sips of the mixture in each cup.

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Access for Multilingual Learners (Warm-up, Synthesis)

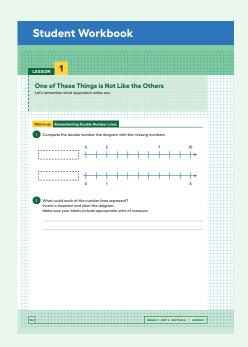
MLR8: Discussion Supports.

Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Building on Student Thinking

Students may struggle thinking of a scenario with a 1:2 ratio. For those students, ask them if they can draw a picture that would represent that ratio and label each line accordingly.



Warm-up

Remembering Double Number Lines



Activity Narrative

This activity prompts students to reason about equivalent ratios on a double number line and think of reasonable scenarios for these ratios. This is a review of their work in grade 6.

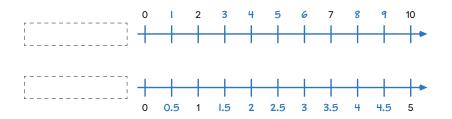
Launch

Arrange students in groups of 2.

Give students 2 minutes of quiet work time followed by partner discussion. As students discuss their answers and reasoning with their partner, select students to share during the whole-class discussion.

Student Task Statement

1. Complete the double number line diagram with the missing numbers.



2. What could each of the number lines represent? Invent a situation and label the diagram.

Make sure your labels include appropriate units of measure.

Sample responses:

- a. number of wheels and number of bicycles
- b. pints of sauce and quarts of sauce
- c. chocolate powder (tablespoons) and milk (cups)

Activity Synthesis

Display the double number line for all to see with correct values filled in. It does not matter whether the bottom line is labeled with fractions, decimals, or mixed numbers.

Invite selected students to share the situations they came up with and the units for each quantity. After each student shares, invite others to agree or disagree with the reasonableness of the diagram representing that situation. For example, is it really reasonable to say that 7 wheels make $3\frac{1}{2}$ bicycles?

Lesson 1 Warm-up **Activity 1** Activity 2 Cool-down Lesson Synthesis

Math Community

After the Warm-up, display the revisions to the class Math Community Chart that were made from student suggestions in an earlier exercise. Tell students that over the next few exercises, this chart will help the class decide on community norms—how they as a class hope to work and interact together over the year. To get ready for making those decisions, students are invited at the end of today's lesson to share which "Doing Math" action on the chart is most important to them personally.

Activity 1

Mystery Mixtures

15

Activity Narrative

The purpose of this activity is to remind students of strategies for representing and comparing ratios. The taste of the mixture depends on the ratio between the amount of water and the amount of drink mix used to make the mixture. As students describe how they know which mixture would taste different, they attend to precision.

Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyze contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can't, make strategic pairings of students for this task.

Monitor for students who:

- Create discrete diagrams, double number line diagrams, or tables to represent the different mixtures.
- Find ratios that are equivalent to the amounts given to help compare the recipes.
- · Calculate unit rates to help compare the recipes.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754





Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display

This activity uses the Collect and Display math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Launch

First, show students the three drink mixtures. Some options for how to accomplish this include:

- Demonstrate making the three drink mixtures yourself.
- Show the video: 'Three Mixtures' available here: ilclass.com/r/18609989
- Display the unlabeled image of the drinks:



Tell students that two of these mixtures taste the same, and one tastes different. Ask students which mixture would taste different and why.

Give students 1 minute of quiet think time and then time to share their thinking with their partner.

If possible, ask one student volunteer to take a small taste of each drink mixture and then describe to the rest of the class how the flavors compare.

Ask students:

- "What does it mean to say that the drink tastes stronger?"
 It has more drink mix for the same amount of water, or it has less water for the same amount of drink mix.
- "The two glasses that taste the same have different amounts of powdered drink mix in them. Why don't the mixtures taste different?"

The amount of water and amount of drink mix were both scaled by the same factor. The ratios of drink mix to water are equivalent.

Give students 4–5 minutes of quiet work time to answer the questions. If students finish quickly, consider asking them to find the amount of drink mix per cup of water in each recipe, thus emphasizing the unit rate.

Use Collect and Display to create a shared reference that captures students' developing mathematical language. Collect the language students use to describe how the amount of water and the amount of drink mix affects the taste of the mixture. Display words and phrases such as: "more drink mix," "more water," "tastes stronger," "tastes weaker," "ratio," "unit rate," "per," etc.

Student Task Statement

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture would taste different? Why?

The first mixture would taste different—stronger than the other two. It looks darker, because it has more drink mix in it.

2. Here are the recipes that were used to make the three mixtures:



1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix



1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix



2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix

Which of these recipes is for the stronger tasting mixture? Explain how you know.

The recipe with $l\frac{1}{2}$ teaspoons of drink mix and I cup of water is the strongest one. There is more drink mix per cup of water.

Are You Ready for More?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar
- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar
- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

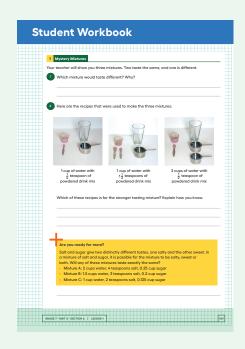
Mixture A and Mixture C will taste exactly the same. Mixture B will taste equally salty, but will be a little bit sweeter.

Activity Synthesis

The key takeaway from this activity is that the flavor depends on the ratio of the amounts of drink mix and water in the mixture. For a given amount of water, the more drink mix you add, the stronger the mixture tastes. Likewise, for a given amount of drink mix, the more water you add, the weaker the mixture tastes.

When comparing mixtures where both the amounts of drink mix and the amounts of water are different, we can find equivalent ratios for one or more of the mixtures. This method enables us to compare the amounts of drink mix for the same amount of water or to compare the amounts of water for the same amount of drink mix. Computing a unit rate for each situation is a particular instance of this strategy.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they knew which recipe was strongest. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.



Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate correspondences between the number line diagrams and ratio tables for each mixture.

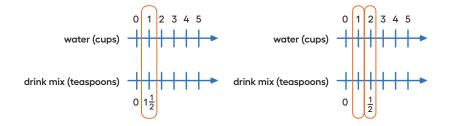
Supports accessibility for: Visual-Spatial Processing

If students do not create diagrams, consider showing how the reasoning they describe could be represented visually. However, it is not necessary to show every type of diagram.

Discrete diagrams:



Double number line diagrams:



Tables:

water (cups)	drink mix (teaspoons)
1	1 1/2
2	3

water (cups)	drink mix (teaspoons)
2	1/2
1	1/4

Identify correspondences between the recipes and various diagrams. For example, ask questions like "On the double number line diagram we see the 1 to $1\frac{1}{2}$ relationship at the first tick mark. Where do we see that relationship in the discrete diagrams? Where do we see it in the tables?"



Crescent Moons



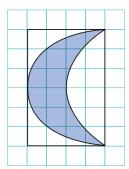
Activity Narrative

There is a digital version of this activity.

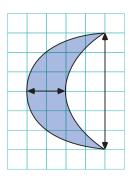
The purpose of this activity is to build on students' recent study of scale drawings to informally introduce the concept of proportional relationships.

Initially, students may describe the difference between Moons A, B, C, and D in qualitative terms, such as "D is more squished than the others." They may also describe Moons A, B, and C as "scaled copies," even though the curved sides make it difficult to compare corresponding side lengths and angle measurements. The activity prompts students to articulate what they mean in quantitative terms. One possibility would be talking about the height relative to the width (after defining "height" and "width" of a moon in some appropriate way).

For example, students can note that the height of the enclosing rectangle is always $1\frac{1}{2}$ times its width for Moons A, B, and C, but not for D.



Alternately, they might note that the distance tip to tip is 3 times the width of the widest part of the moon for Moons A, B, and C, but not for D.



Monitor for different ways students choose to measure the figures. Given the structure of scaled copies, as students work to describe quantitatively what Moons A, B, and C have in common, they are modeling with mathematics.

In the digital version of the activity, students use an applet to compare corresponding parts in multiple images. The applet allows students to add points and measure distances. The digital version may help students measure quickly and accurately so they can focus more on the mathematical analysis.



Arrange students in groups of 2.

Give students 3 minutes of quiet work time followed by 3 minutes of partner discussion.

Based on student conversations, you may want to have a whole-class discussion to ensure that they see a way to measure lengths associated with the moons. Consider asking questions like:

(a) "What does it mean to be 'smashed down'? What measurements might you make to show that this is true?"

"Is there anything else that A, B, and C have in common that you can identify?"

"What things might we measure about these moons to be able to talk about what makes them different in a more precise way?"

Then ask students to complete the last question with their partner.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

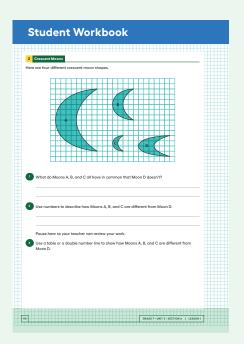
Chunk this task into more manageable parts. Consider having students begin by comparing just two crescent moon shapes, like A and B. Check in with students to provide feedback and encouragement after each chunk. Look for students who create a rectangle around each moon and compare the width-height ratios.

Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

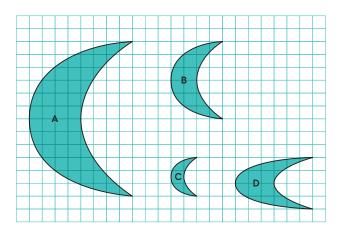
For question 2, students might attempt to find the area of each moon by counting individual square units. Suggest that they create a rectangle around each moon instead and compare the width-height ratios.

For question 3, if students are not sure how to set up these representations, providing a template may be helpful.



Student Task Statement

Here are four different crescent moon shapes.



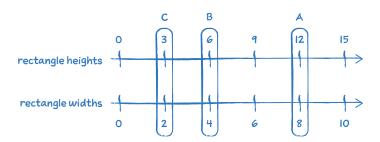
- 1. What do Moons A, B, and C all have in common that Moon D doesn't? Sample response: Moon D is smashed down more than Moons A, B, and C. Moons A, B, and C are all taller than they are wide while Moon D is wider than it is tall.
- **2.** Use numbers to describe how Moons A, B, and C are different from Moon D. Sample responses:
 - We could enclose each moon with a rectangle and compare the ratio of height to width for each moon.
 - We could measure the height of the moon from tip to tip and measure the width of the widest part.

Pause here so your teacher can review your work.

3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Sample responses:

• A double number line for enclosing the moons with a rectangle:



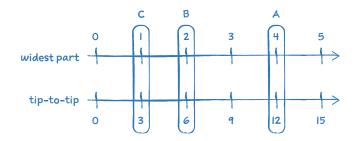
Moon D doesn't fit on this double number line, because 4:6 is not equivalent to these ratios. If we add a tick mark at 4 on the top line, this does not line up with 6 on the bottom line.

• A table for enclosing the moons with a rectangle:

moon	height (units)	width (units)	height ÷ width
A	12	8	12 ÷ 8 = 1.5
В	6	4	6 ÷ 4 = 1.5
С	3	2	3 ÷ 2 = 1.5
D	4	6	4 ÷ 6 = 0.6

For Moon D, the height divided by the width does not equal 1.5, like it does for the other 3 moons.

· A double number line for measuring the widest part and tip-to-tip:



Moon D doesn't fit on this double number line, because 3:4 is not equivalent to these ratios. The tick mark at 3 on the top line does not line up with 4 on the bottom line.

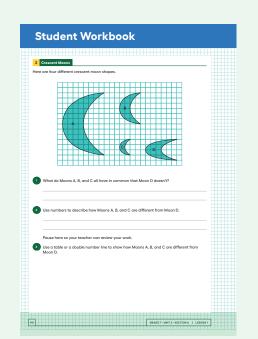
· A table for measuring the widest part and tip-to-tip:

moon	widest part (units)	tip-to-tip (units)	height ÷ width
A	4	12	$4 \div 12 = \frac{1}{3}$
В	2	6	$2 \div 6 = \frac{1}{3}$
С	I	3	$1 \div 3 = \frac{1}{3}$
D	3	4	$3 \div 4 = \frac{3}{4}$

For Moon D, the width divided by the height does not equal $\frac{1}{3}$, like it does for the other 3 moons.

Activity Synthesis

The goal of this discussion is to show various representations of quantities that are in a proportional relationship for Moons A, B, and C but not for Moon D. However, the term "proportional relationship" is not introduced at this time.



Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support whole-class discussion: "All _____ have _____ except _____." and "What makes _____ different from the others is ____."

Advances: Speaking, Representing

Invite groups to share their representations and reasoning. To involve more students in the conversation, consider asking:

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Lesson Synthesis

Share with students,

"Today we examined situations involving ratios. In each case, the ratios of the quantities were equivalent for all but one of the things, and the other thing was different in an important way."

To emphasize ratio and rate language, consider asking students:

"In what important way were the drink mixtures the same or different?"
the amount of drink powder per cup of water

 \bigcirc "How could we tell using ratios that these were the same and different?"

The mixtures that tasted the same were described by equivalent ratios, $1:\frac{1}{4}$ and $2:\frac{1}{2}$. The ratio for the mixture that tasted different was not equivalent.

 \bigcirc "In what important way were the moons the same and different?"

the relationship between the width and the height

"How could we tell using numbers that these were the same and different?"

The moons that looked the same had a height to width ratio of 3:1. The moon that was different had a ratio of 4:3.

Lesson Summary

When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

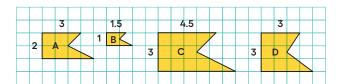
An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were 6:4, then the mixture would taste different.

Notice that the ratios of pairs of corresponding side lengths are equivalent in Figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for Figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other.

This is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like Figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "Which 'Doing Math' action is most important to you, and why?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, review student responses to the community building question. Use the responses to draft a student norm and a teacher norm to use as an example in Exercise 6. For example, if "sharing ideas" is a common choice for students, a possible norm is "We listen as others share their ideas."

For the teacher norms section, if "questioning vs. telling" from the "Doing Math" section is key for your teaching practice, then one way to express that as a norm is "Ask questions first to make sure I understand how someone is thinking."

Cool-down

Orangey-Pineapple Juice

5 min

Student Task Statement

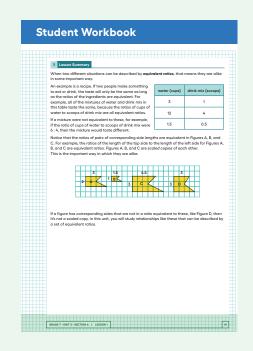
Here are three different recipes for Orangey-Pineapple Juice. Two of these mixtures taste the same and one tastes different.

- Recipe 1: Mix 4 cups of orange juice with 6 cups of pineapple juice.
- Recipe 2: Mix 6 cups of orange juice with 9 cups of pineapple juice.
- Recipe 3: Mix 9 cups of orange juice with 12 cups of pineapple juice.

Which two recipes will taste the same, and which one will taste different? Explain or show your reasoning.

Recipes I and 2 will taste the same.

Sample reasoning: Recipe 3 is different because it requires $l\frac{1}{3}$ cups of pineapple juice for every I cup of orange juice. Recipes I and 2 both require $l\frac{1}{2}$ cups of pineapple juice for every I cup of orange juice.



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

recipe I

orange juice (cups)	pineapple juice (cups)
4	6
2	3
ı	1 1/2

recipe 2

orange juice (cups)	pineapple juice (cups)
6	9
2	3
ı	1 1/2

recipe 3

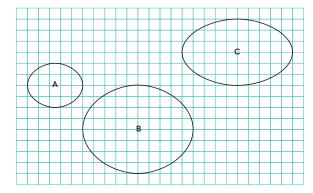
orange juice (cups)	pineapple juice (cups)
9	12
3	4
ı	1 1/3

Double number line diagrams can be used to compare the recipes, for instance, by noting that for Recipes 1 and 2, you use 2 cups of orange juice for every 3 cups of pineapple juice, whereas with Recipe 3, you use $2\frac{1}{4}$ cups of orange juice for 3 cups of pineapple juice.

Practice Problems

5 Problems

Problem 1



Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.

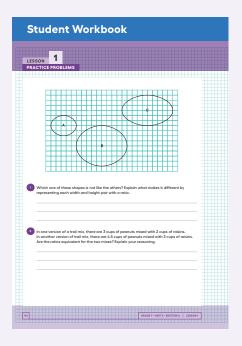
C is different from A and B.

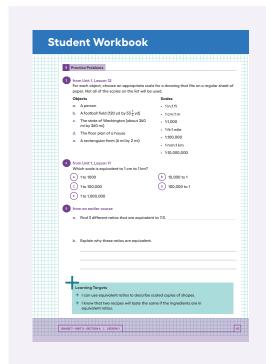
For both A and B, the width:height ratio is 5:4. However, for C, the width is 10 units and the height is 6 units, so the width:height ratio is 5:3.

Problem 2

In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

Yes, since 3 times 1.5 is 4.5 and 2 times 1.5 is 3





Problem 3

from Unit 1, Lesson 12

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects Scales

Sample responses:

- a. A person 1in:1ft
- **b.** A football field (120 yd by $53\frac{1}{3}$ yd) 1 cm:1 m
- c. The state of Washington (about 240 mi by 360 mi)
- d. The floor plan of a house
- e. A rectangular farm (6 mi by 2 mi)

1ft:1mile1:100,000

1 mm:1 km1:10,000,000

Problem 4

from Unit 1, Lesson 11

Which scale is equivalent to 1 cm to 1 km?

A. 1 to 1000

B. 10,000 to 1

C. 1 to 100,000

D. 100,000 to 1

E. 1 to 1,000,000

Problem 5

from an earlier course

a. Find 3 different ratios that are equivalent to 7:3.

Sample response:

- 0 14:6
- 0 21:9
- 0 28:12
- **b.** Explain why these ratios are equivalent.

Sample response: 7 and 3 are each multiplied by 2, 3, and 4, respectively.

LESSON 1 • PRACTICE PROBLEMS