Lesson 2

Side Lengths and Areas

Goals

- Comprehend the term "square root of n" (in spoken language) and the notation $\sqrt{n}$ (in written language) to mean the side length of a square whose area is n square units.
- Estimate the side length of a square by comparing it to squares with known areas, and explain (orally) the reasoning.

Learning Targets

- I can explain what a square root is.
- If I know the area of a square, I can express its side length using square root notation.

Lesson Narrative

In this lesson students are formally introduced to **square roots** and square root notation where:

$\sqrt{n}$ is the length of the side of a square whose area is n square units.

To get to this definition, students find the area and side length of squares on a grid, starting with squares that have whole number side lengths. They reason that 2 squares with the same area must also have the same side length.

Next, students investigate squares whose side lengths are not whole numbers. By comparing the area of these squares to the areas of squares with known side lengths, students can make reasonable approximations for the unknown side lengths.

Finally, students work with a geometric construction of a square with one corner at the origin of a coordinate plane. Using a rotation about the origin helps students to see the side length as a point on the number line and leads to the introduction of the term "square root" and its notation.

Student Learning Goal

Let's investigate some more squares.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR2: Collect and Display (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2
- Tracing paper: Activity 1, Activity 2

Activity 2:
Make a class display listing perfect squares up to $15^2 = 225$. This display should be posted in the classroom for the remaining lessons within this unit.

Lesson Timeline

5 min	15 min	15 min	10 min
Warm-up	Activity 1	Activity 2	Lesson Synthesis

Assessment

5 min
Cool-down

Warm-up
Estimating Side Lengths from Areas (Part 1)

5 min

Activity Narrative

In this *Warm-up,* students use what they know about finding the area of a square given its side length to think about the converse: finding the side length of a square given its area.

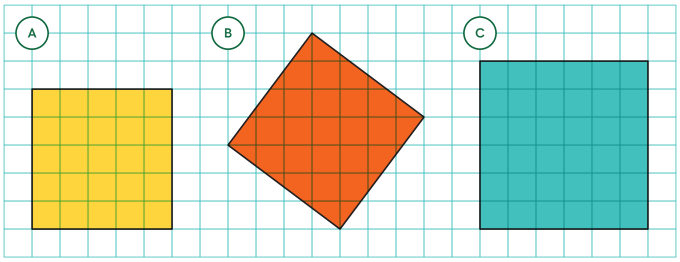
First, students work with squares whose side lengths and areas can be found by counting length or area units. Next, students find the area of a "tilted" square using strategies from a previous lesson. They reason that if two squares have the same area, their side lengths must also be the same, and verify this using tracing paper.

Launch

Arrange students in groups of 2. Provide access to geometry toolkits, including tracing paper.

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement



1. What is the side length of Square A?

   5 units

   What is its area?

   25 square units

2. What is the side length of Square C?

   6 units

   What is its area?

   36 square units

3. What is the area of Square B?

   25 square units

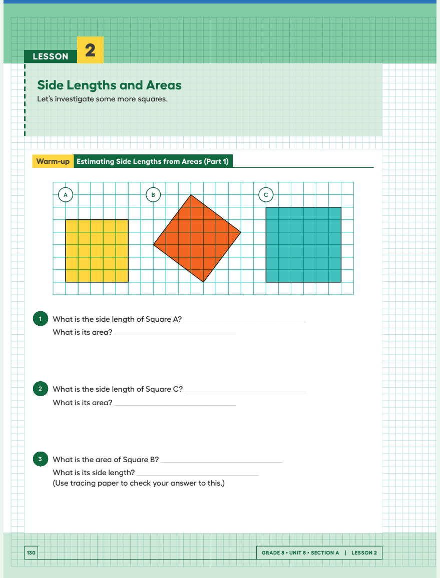
   What is its side length? (Use tracing paper to check your answer to this.)

   5 units

Activity Synthesis

The key takeaway from this activity is that if two squares have the same area then they must also have the same side length. This can be reinforced by measuring with tracing paper. Invite 1–3 students to share how they determined that Square A and Square B have the same area.

Student Workbook



Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

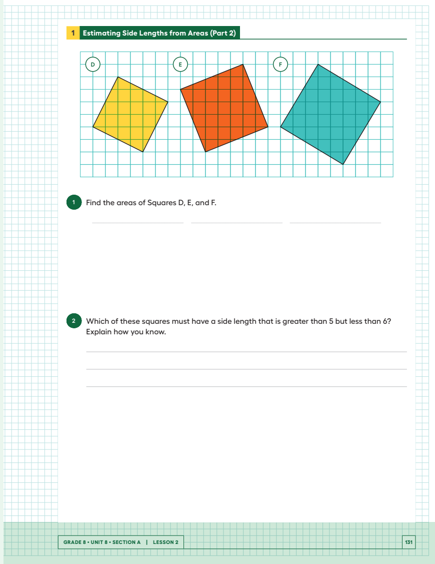
Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their reasoning for the second problem. For example, "I notice that …" and "If _____, then _____ because …"
Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook



Activity 1

Estimating Side Lengths from Areas (Part 2)

15 min

Activity Narrative

In this activity, students find the areas of "tilted" squares and use tracing paper, a ruler, or insights from the *Warm-up* to estimate their side lengths. They will see other techniques for estimating side lengths in later lessons.

Monitor for students who use these strategies for the second question:

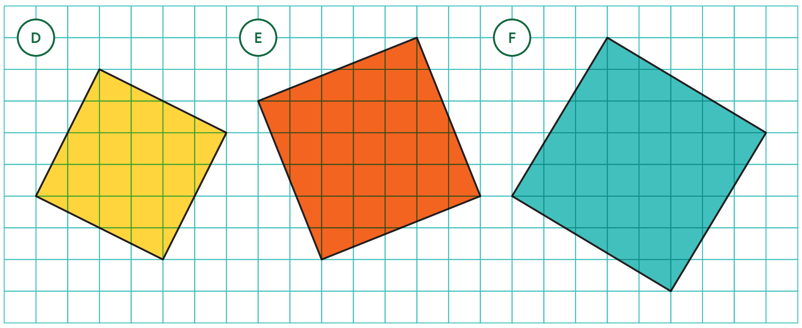
- Reason that the side lengths of Squares E and F are between 5 and 6 units because their areas are between 25 and 36 square units, respectively
- Measure using tracing paper or a ruler

Launch

Arrange students in groups of 2. Provide access to geometry toolkits, including tracing paper.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially of students who haven't shared recently.

Student Task Statement



1. Find the areas of Squares D, E, and F.
   - Square D: 20 square units
   - Square E: 29 square units
   - Square F: 34 square units
2. Which of these squares must have a side length that is greater than 5 but less than 6?

   Squares E and F

   Explain how you know.

   Sample response: They both have side lengths between 5 and 6 because their areas are between 25 square units and 36 square units. I used tracing paper to measure each square and compared the side length to the length of 5 and 6 grid squares.

Activity Synthesis

The goal of this discussion is to establish that if the area of a square is in between the areas of two other squares, then its side length must also be in between the side lengths of the two other squares. This reasoning strategy can be verified with a measuring strategy using tracing paper or a ruler.

Display 2–3 approaches from previously selected students for all to see. Invite students to briefly describe their approach. Introduce the class display listing perfect squares and refer to as needed. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- *"What do the approaches have in common? How are they different?"*

  Both approaches show that Squares E and F have side lengths between 5 and 6. The approaches are different because one uses measurements.

- *"Did anyone solve the problem the same way but would explain it differently?"*

  Answers vary.

- *"Are there any benefits or drawbacks to one approach compared to another?"*

  Answers vary.

It is not necessary at this time to be more precise than knowing that the side lengths of Squares E and F are somewhere between 5 and 6. More precise strategies for estimating the side lengths will be explored in later lessons.

Activity 2
One Square

15 min

Activity Narrative

The purpose of this activity is for students to estimate the side length of a square using a geometric construction that relates the side length of the square to a point on the number line. Students then verify their estimate using techniques from an earlier lesson.

Once students connect the side length to a point on the number line, they learn that this number has a name and a special notation to denote it: square root and the square root symbol. Students will have many opportunities to deepen their understanding of square roots and practice using square root notation in later activities and lessons.

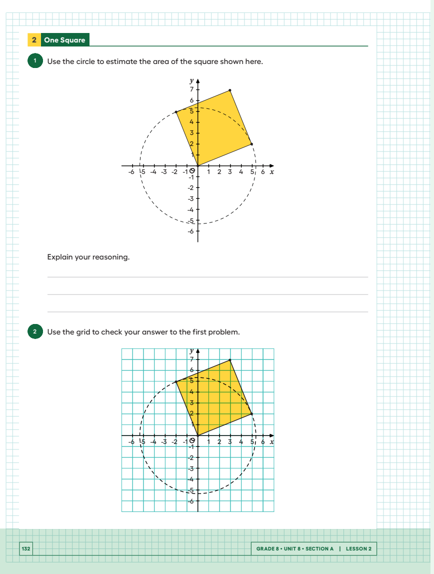
Access for Multilingual Learners (Activity 2, Student Task)

MLR2: Collect and Display. Collect the language students use to estimate the area of the square in the first problem. Display a phrase such as “the side is the same as the radius,” which can be clarified by restating it as “the side length of the square is equal to the radius of the circle.” During the *Synthesis*, invite students to suggest ways to update the display:

> *“What are some other words or phrases we should include?”*

Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Student Workbook

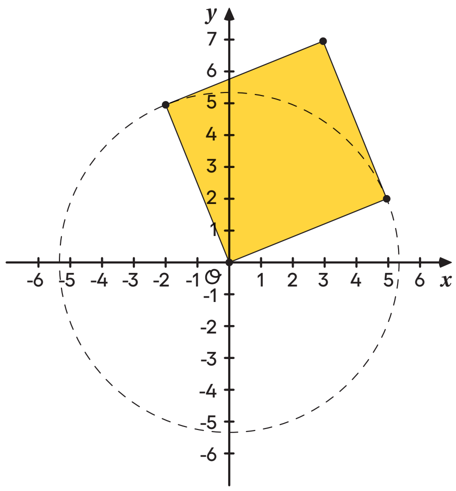


Launch

Arrange students in groups of 2. Give them 1–2 minutes of quiet work time for the first question followed by a brief partner discussion. Have students compare estimates and explain their reasoning to their partner before continuing with the rest of the activity. Follow with a whole-class discussion.

Student Task Statement

1. Use the circle to estimate the area of the square shown here.

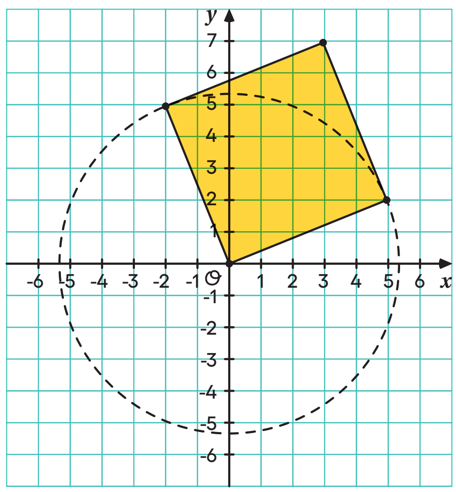


Explain your reasoning.

An estimate between 25 and 36 square units is reasonable.

Sample response: Approximately 28 square units. The radius of the circle is about 5.3 units, which means the area of the square is about 5.3^2 square units.

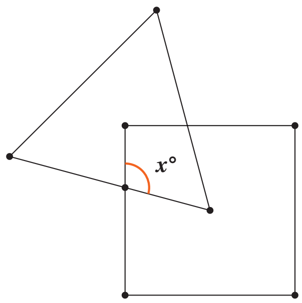
2. Use the grid to check your answer to the first problem.



29 square units

Are You Ready for More?

One vertex of the equilateral triangle is in the center of the square, and one vertex of the square is in the center of the equilateral triangle. What is x?



Draw a segment to connect the center of the square to the center of the equilateral triangle. This segment cuts the 90 degree angle at the center of the equilateral triangle in half because a line from a vertex of an equilateral triangle through its center is a line of symmetry. In the same way, the segment cuts the 60 degree angle at the center of the square in half. So this segment creates a new triangle with angles 45 degrees, x degrees, and 30 degrees. Since the angles in a triangle must sum to 180 degrees, x must be equal to 105 degrees.

Activity Synthesis

The purpose of this discussion is to introduce students to square roots and the square root symbol in the context of area and side length of squares.

Display the image from the second problem and ask students

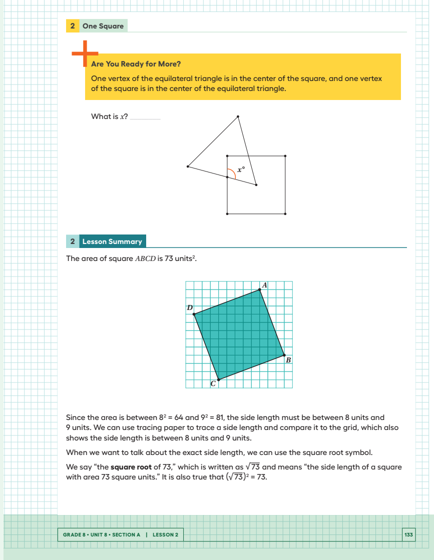
"What do you think the actual side length of the square is? That is, what is the number that when squared is equal to 29?"

As students share, take each guess and square it, noticing out loud how some come very close to 29. For example, point out that 5.3^2 is 28.09, and 5.35^2 is 28.6225, a number closer to 29.

Tell students that some squares like this one have areas that are whole numbers, but their side lengths are not whole numbers. Explain that there is a point on the number line (the x-axis is a number line) that corresponds to these side lengths and can be imagined by rotating the square about the origin so that its sides line up with the x- and y-axes.

Tell students that the exact side length of a square with area 29 square units is called "the **square root** of 29," which can be written as "$\sqrt{29}$," and means that $(\sqrt{29})^2 = 29$.

Student Workbook



2 One Square

Are You Ready for More?

One vertex of the equilateral triangle is in the center of the square, and one vertex of the square is in the center of the equilateral triangle.

What is x? ______

2 Lesson Summary

The area of square $ABCD$ is 73 units^2.

Since the area is between $8^2 = 64$ and $9^2 = 81$, the side length must be between 8 units and 9 units. We can use tracing paper to trace a side length and compare it to the grid, which also shows the side length is between 8 units and 9 units.

When we want to talk about the exact side length, we can use the square root symbol.

We say "the **square root** of 73," which is written as $\sqrt{73}$ and means "the side length of a square with area 73 square units." It is also true that $(\sqrt{73})^2 = 73$.

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Lesson Synthesis

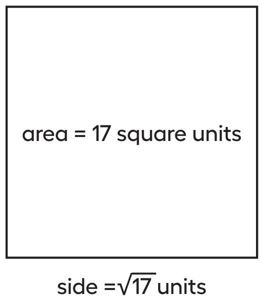
The purpose of this discussion is to check that students understand the definition of square roots as they relate to side lengths of squares. Here are some questions for discussion. After each question, consider displaying a square for all to see and adding labels for the area and side length. An example is provided.

"What does it mean when we write $\sqrt{100} = 10$ in terms of squares and side lengths?"

It means that a square with area 100 has side lengths of 10.

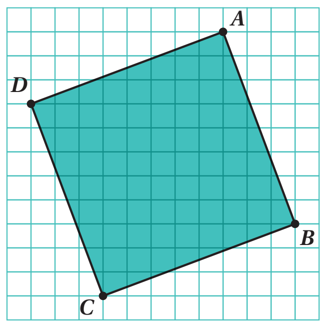
"If $\sqrt{17}$ is a side length of a square, what does that mean about the area?"

The area is 17 square units.



Lesson Summary

The area of square $ABCD$ is 73 units^2.



Since the area is between $8^2 = 64$ and $9^2 = 81$, the side length must be between 8 units and 9 units. We can use tracing paper to trace a side length and compare it to the grid, which also shows the side length is between 8 units and 9 units.

When we want to talk about the exact side length, we can use the square root symbol.

We say "the **square root** of 73," which is written as $\sqrt{73}$ and means "the side length of a square with area 73 square units." It is also true that $(\sqrt{73})^2 = 73$.

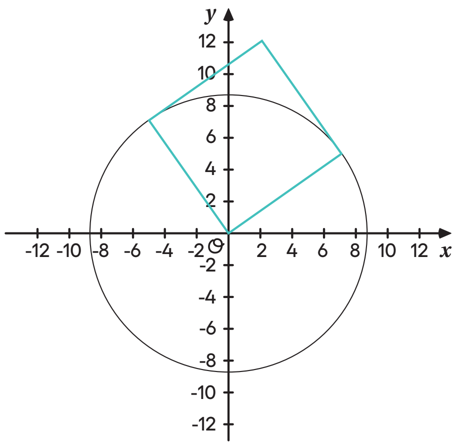
Cool-down

Area Estimate

5 min

Student Task Statement

Mai estimates the area of the square to be somewhere between 70 and 80 square units.



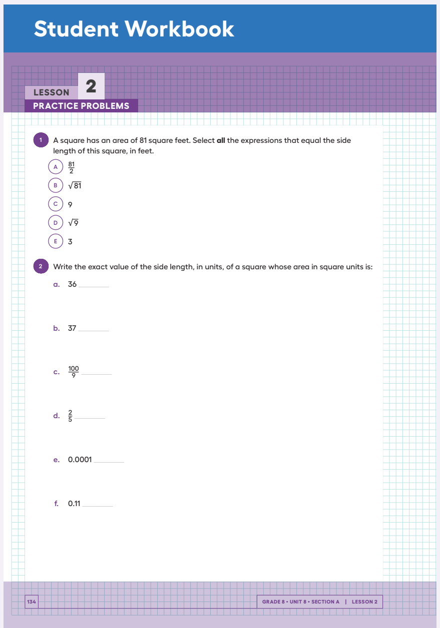
Do you agree with Mai? Explain your reasoning.

I agree with Mai.

Sample reasoning: The side length of the square is the same length as the radius of the circle, which is between 8 and 9 units long. That means the area of the square must be larger than 64 square units but smaller than 81 square units, so Mai's estimate of somewhere between 70 and 80 square units seems reasonable.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.


Student Workbook

LESSON 2
PRACTICE PROBLEMS

1 A square has an area of 81 square feet. Select **all** the expressions that equal the side length of this square, in feet.

A $\frac{81}{2}$
B $\sqrt{81}$
C 9
D $\sqrt{9}$
E 3

2 Write the exact value of the side length, in units, of a square whose area in square units is:

a. 36
b. 37
c. $\frac{100}{9}$
d. $\frac{2}{5}$
e. 0.0001
f. 0.11

Lesson 2

Practice Problems

5 Problems

Problem 1

A square has an area of 81 square feet. Select **all** the expressions that equal the side length of this square, in feet.

A. $\frac{81}{2}$

B. $\sqrt{81}$

C. 9

D. $\sqrt{9}$

E. 3

Problem 2

Write the exact value of the side length, in units, of a square whose area in square units is:

a. 36

6

b. 37

$\sqrt{37}$

c. $\frac{100}{9}$

$\frac{10}{3}$

d. $\frac{2}{5}$

$\sqrt{\frac{2}{5}}$

e. 0.0001

0.01

f. 0.11

$\sqrt{0.11}$

Problem 3

from Unit 8, Lesson 1

Find the area of a square if its side length is:

a. $\frac{1}{5}$ centimeter

$\frac{1}{25}$ square centimeter

b. $\frac{3}{7}$ unit

$\frac{9}{49}$ square unit

c. $\frac{11}{8}$ inches

$\frac{121}{64}$ square inches

d. 0.1 meter

0.01 square meter

e. 3.5 centimeters

12.25 square centimeters

Problem 4

from Unit 7, Lesson 15

Here is a table showing the areas of the seven largest countries.

country	area (km^2)
Russia	1.71×10^7
Canada	9.98×10^6
China	9.60×10^6
United States	9.53×10^6
Brazil	8.52×10^6
Australia	6.79×10^6
India	3.29×10^6

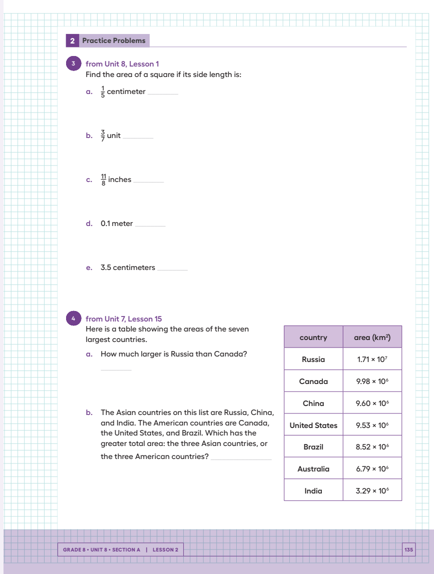
a. How much larger is Russia than Canada?

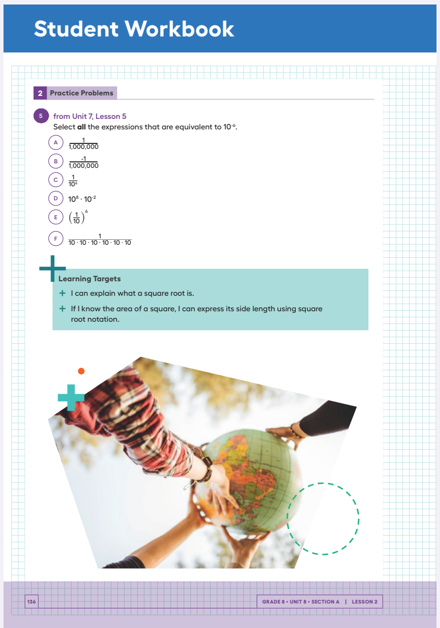
7.12×10^6 km^2

b. The Asian countries on this list are Russia, China, and India. The American countries are Canada, the United States, and Brazil. Which has the greater total area: the three Asian countries, or the three American countries?

The Asian countries, because 2.999×10^7 is larger than 2.803×10^7

Student Workbook





Problem 5

from Unit 7, Lesson 5

Select **all** the expressions that are equivalent to 10^{-6}.

A. $\frac{1}{1{,}000{,}000}$

B. $\frac{-1}{1{,}000{,}000}$

C. $\frac{1}{10^6}$

D. $10^8 \cdot 10^{-2}$

E. $\left(\frac{1}{10}\right)^6$

F. $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$