Lesson 15

Part-Part-Whole Ratios

Goals

- Comprehend the word “parts” as an unspecified unit in sentences (written and spoken) that describe ratios.
- Draw and label a tape diagram to solve problems that involve both ratios and a total amount. Explain (orally) the solution method.

Learning Targets

- I can create tape diagrams to help me reason about problems that involve both a ratio and a total amount.
- I can solve problems when I know a ratio and a total amount.

Lesson Narrative

In this lesson, students encounter situations in which the sum of two or more quantities is meaningful in context. So far, students have worked with ratios of quantities where the units are the same (such as cups to cups) and ratios of quantities where the units are different (such as miles to hours). In some instances of the former, it makes sense to ask questions about the total amount. For example, mixing 3 cups of yellow paint and 2 cups of blue paint gives 5 cups of green paint. (Note that the sum of quantities in the same unit doesn’t always make sense in context. For instance, 3 cups of water mixed with 2 cups of dry oatmeal will not make 5 cups of soggy oatmeal.)

Students begin by using snap cubes to represent a ratio. For instance, 3 blue cubes and 2 yellow cubes can represent blue paint and yellow paint in a ratio of 3 to 2. Students see that each cube can represent a variety of amounts and use this insight to reason about individual quantities (the parts) in the ratio and the total quantity (the whole). For example, if each cube represents 10 ml, then 30 ml of blue paint and 20 ml of yellow paint would be mixed, producing 50 ml of green paint.

Students then learn to use **tape diagrams** to represent and reason about such ratios—sometimes called “part-part-whole” ratios—and solve problems. They also see that ratios can be expressed in “parts” (such as “2 parts of glue and 1 part of water”) rather than in specific units (such as cups or liters).

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)
- Engagement (Activity 3)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 1)
- MLR2: Collect and Display (Activity 2)

Instructional Routines

- Math Talk

Required Materials

Materials to Gather

- Graph paper: Lesson
- Snap cubes: Lesson
- Tools for creating a visual display: Lesson
- Snap cubes: Activity 1, Activity 2, Activity 3
- Graph paper: Activity 2, Activity 3
- Tools for creating a visual display: Activity 3

Required Preparation

Activity 1:

Prepare a set of 50 red snap cubes and 30 blue snap cubes for each group of students.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down (Assessment)
5 mins	15 mins	15 mins	10 mins	10 min	5 mins

Part-Part-Whole Ratios

Lesson Narrative (continued)

As they interpret statements about ratios in context and consider ways to represent them, students practice making sense of problems, persevering in solving them, and reasoning quantitatively and abstractly.

This lesson includes an optional activity that allows students to invent problems that involve part-part-whole ratios.

Student Learning Goal

Let's look at situations where you can add the quantities in a ratio together.

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Warm-up

Math Talk: A Whole Number and a Unit Fraction

5 min

Activity Narrative

This *Warm-up* focuses on multiplication of a whole number and a unit fraction. It encourages students to use the meaning of fractions and properties of operations to reason about equations. While students may evaluate each side of the equation to determine if it is true or false, elicit the following ideas:

- The first equation: Dividing is the same as multiplying by the reciprocal of the divisor.
- The second equation: Adjusting the factors adjusts the products. If both factors increase, the resulting product will be greater than the original.
- The third equation: Multiplication is commutative. Changing the order of the factors doesn't change the product.

The fourth equation: Decomposing a dividend into two numbers and dividing each by the divisor is a way to find the quotient of the original dividend.

In explaining their reasoning, students need to be precise in their word choice and use of language.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.
- Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether each statement is true.

A. $\frac{1}{5} \cdot 45 = \frac{45}{5}$ True

Sample reasoning:

- One-fifth of 45 is 9, so is $45 \div 5$.
- $\frac{1}{5} \cdot 45$ is the same as $45 \cdot \frac{1}{5}$, which is $\frac{45}{5}$.
- Division is the same as multiplying by the reciprocal.

B. $\frac{1}{5} \cdot 20 = \frac{1}{4} \cdot 24$ False

Sample reasoning:

- The expression on the left has a value of 4. The expression on the right has a value of 6.
- Both factors on the right are greater than those on the left.

C. $42 \cdot \frac{1}{6} = \frac{1}{6} \cdot 42$ True

Sample reasoning: The two factors are the same, just switched in order, which doesn't affect the product (commutative property of multiplication).

D. $486 \cdot \frac{1}{12} = \frac{480}{12} + \frac{6}{12}$ True

Sample reasoning: $486 = 480 + 6$, so $\frac{486}{12}$ is $\frac{480}{12} + \frac{6}{12}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ___'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

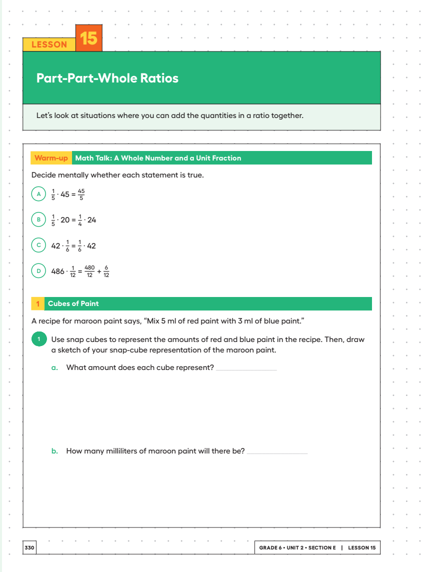
"Does anyone want to add on to ___'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

After each true equation, ask students if they could rely on the reasoning used on the given problem to think about or solve other problems that are similar in type. After each false equation, ask students how we could make the equation true.

Student Workbook


LESSON 15

Part-Part-Whole Ratios

Let's look at situations where you can add the quantities in a ratio together.

Warm-up Math Talk: A Whole Number and a Unit Fraction

Decide mentally whether each statement is true.

A $\frac{1}{5} \cdot 45 = \frac{45}{5}$

B $\frac{1}{5} \cdot 20 = \frac{1}{4} \cdot 24$

C $42 \cdot \frac{1}{6} = \frac{1}{6} \cdot 42$

D $486 \cdot \frac{1}{12} = \frac{480}{12} + \frac{6}{12}$

1 Cubes of Paint

A recipe for maroon paint says, "Mix 5 ml of red paint with 3 ml of blue paint."

1 Use snap cubes to represent the amounts of red and blue paint in the recipe. Then, draw a sketch of your snap-cube representation of the maroon paint.

a. What amount does each cube represent?

b. How many milliliters of maroon paint will there be?

330 GRADE 6 • UNIT 2 • SECTION E | LESSON 15

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes, blocks, or other tactile objects that can represent the ratio of red and blue paint.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Activity 1
Cubes of Paint

15 min

Activity Narrative

Up until now, students have worked with ratios of quantities given in terms of specific units such as milliliters, cups, teaspoons, etc. This task introduces students to the use of the more generic "parts" as a unit in ratios, and the use of **tape diagrams** to represent such ratios. In addition to thinking about the ratio between two quantities, students also begin to think about the ratio between the two quantities and their total.

Two important ideas to make explicit through the task and discussion:

- A ratio can associate quantities given in terms of a specific unit (as in 4 teaspoons of this to 3 teaspoons of that). A ratio can also associate quantities of the same kind without specifying particular units, in terms of "parts" (as in 4 parts of this to 3 parts of that). Any appropriate unit can be used in place of "parts" without changing the 4 to 3 ratio.
- A ratio can tell us about how two or more quantities relate to one another, but it can also tell us about the combined quantity (when that makes sense) and allow us to solve problems.

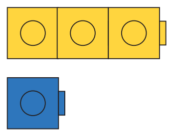
As students work, notice in particular how they approach the last two questions. Identify students who add snap cubes to represent the larger amount of paint, and those who use the original number of snap cubes but adjust their reasoning about what each cube represents. Be sure to leave enough time to debrief as a class and introduce tape diagrams afterwards.

Launch

Explain to students that they will explore paint mixtures and use snap cubes to represent them. Tell students:

"To make a particular green paint, we need to mix 1 ml of blue paint to 3 ml of yellow."

Represent this recipe with 1 blue snap cube and 3 yellow ones and display each set horizontally (to mimic the appearance of a tape diagram).



Ask:

"How much green paint will this recipe yield?"

4 ml of green paint

"If each cube represents 2 ml instead of 1 ml, how much of blue and yellow do the snap cubes represent? How many ml of green paint will we have?"

2 ml of blue, 6 ml of yellow, and 8 ml of green

"Is there another way to represent 2 ml of blue and 6 ml of yellow using snap cubes?"

We could use 2 blue snap cubes and 6 yellow ones

"How do we refer to 2 ml of blue and 6 ml of yellow in terms of 'batches'?"

2 batches

Highlight the fact that they could either represent 2 ml of blue and 6 ml of yellow with 2 blue snap cubes and 6 yellow ones (show this representation, if possible), or with 1 blue snap cube and 3 yellow ones (show representation), with the understanding that each cube stands for 2 ml of paint instead of 1 ml.

Explain to students that, in the past, they had thought about different amounts of ingredients in a recipe in terms of batches, but in this task they will look at another way to mix the right amounts specified by a ratio.

Arrange students in groups of 3–5. Provide 50 red snap cubes and 30 blue snap cubes to each group. Give groups time to complete the activity, and then debrief as a class.

Student Task Statement

A recipe for maroon paint says, "Mix 5 ml of red paint with 3 ml of blue paint."

1. Use snap cubes to represent the amounts of red and blue paint in the recipe. Then, draw a sketch of your snap-cube representation of the maroon paint.

   Representations show 5 red snap cubes and 3 blue ones.

   a. What amount does each cube represent?

      Each snap cube represents 1 ml.

   b. How many milliliters of maroon paint will there be?

      $1 + 1 + 1 + 1 + 1 = 5$, so there is 5 ml of red paint. $1 + 1 + 1 = 3$, so there is 3 ml of blue paint. $5 + 3 = 8$, so there is 8 ml of maroon paint.

2. a. Suppose each cube represents 2 ml. How much of each color paint is there?

      Red: 10 ml   Blue: 6 ml   Maroon: 16 ml

      $2 + 2 + 2 + 2 + 2 = 10$, so there is 10 ml of red paint. $2 + 2 + 2 = 6$, so there is 6 ml of blue paint. $10 + 6 = 16$, so there is 16 ml of maroon paint.

   b. Suppose each cube represents 5 ml. How much of each color paint is there?

      Red: 25 ml   Blue: 40 ml   Maroon: 16 ml

      There is 25 ml of red, since $5 \cdot 5 = 25$, and 15 ml of blue, since $5 \cdot 3 = 15$. $25 + 15 = 40$, so there is 40 ml of maroon paint.

3. a. Suppose you need 80 ml of maroon paint. How much red and blue paint would you mix? Be prepared to explain your reasoning.

      Red: 50 ml   Blue: 30 ml   Maroon: 80 ml

      $80 \div 8 = 10$ and $10 \cdot 5 = 50$, so there is 50 ml red. $10 \cdot 3 = 30$, so there is 30 ml blue. $50 + 30 = 80$, so there is 80 ml maroon.

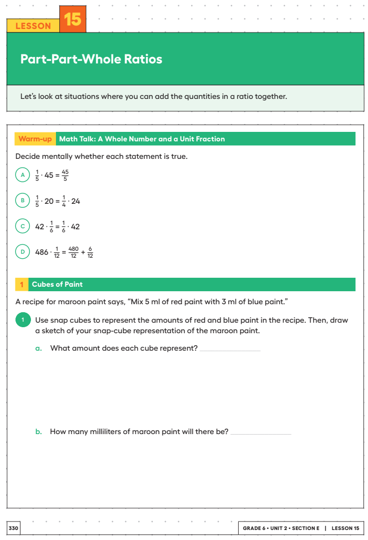
   b. If the original recipe is for one batch of maroon paint, how many batches are in 80 ml of maroon paint?

      There are 10 batches of paint, because each part changed from a value 1 ml to a value of 10 ml.

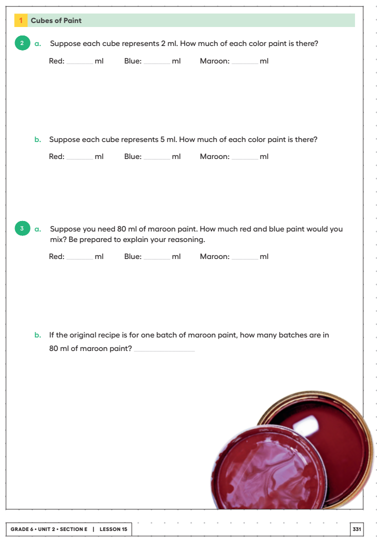
Building on Student Thinking

Students may need help interpreting "Suppose each cube represents 2 ml." If necessary, suggest they keep using one cube to represent 1 ml of paint. So, for example, the second question would be represented by 5 *stacks of 2* red cubes and 3 *stacks of 2* blue cubes. If they use that strategy, each part of the tape diagram would represent one stack.

Student Workbook



Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
To help students connect ratio language and ratio reasoning, invite them to represent their reasoning using snap cubes or a tape diagram. Support students in using precise language by requesting that they challenge an idea, elaborate on an idea, or give an example.
Advances: Reading, Representing

Activity Synthesis

Class discussion should center around how students used snap cubes to answer the questions and their approach to the last two questions. Invite some students to share their group's approach. Ask:

"How did the snap cubes help you solve the first few problems?"

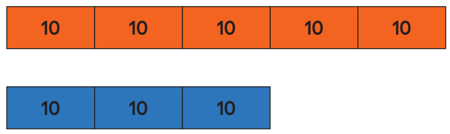
"In one of the problems, you were only given the total amount of maroon paint. How did you find out the amounts of red and blue paint needed to produce 80 ml of maroon?"

"How did you approach the last question?"

Add more cubes, or use the same representation of 5 red cubes and 3 blue ones.

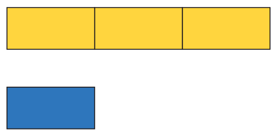
Discuss how the same 5 red cubes and 3 blue ones can be used to represent a total of 80 ml of blue paint. Explain that this situation can be represented with a **tape diagram**. A **tape diagram** is a horizontal strip that is partitioned into parts. Each part (like each snap cube) represents a value. It can be any value, as long as the same value is used throughout.

Show a tape diagram representing a 5:3 ratio of red paint to blue paint yielding 80 ml of maroon paint. Ask students where they see the 5, the 3, and the 80 being represented in the diagram. Discuss how many batches of paint are represented.

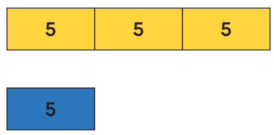


Show the tape diagram for green paint mixture discussed earlier. Students should be able to say that the ratio of blue to yellow paint is 1:3. Ask:

"What value each part of the diagram would have to take to show a 20 ml mixture of green paint? How do you know?"



Guide students to see that, if each of the 4 total parts must be equal in value and amount to 20 ml, we could divide 20 by 5 to find out what each part represents. $20 \div 4 = 5$, so each part represents 5 ml of paint.



Activity 2

Seasoning, Sneakers, and Concrete

15 min

Activity Narrative

This activity allows students to practice reasoning about situations involving ratios of two quantities and their sum. It also introduces students to using "parts" in recipes (for instance, 4 parts chili powder, 1 part garlic powder, and 2 parts ground cumin), instead of more familiar units such as cups, teaspoons, milliliters, and so on. Students may use tape diagrams to support their reasoning, or they may use other representations learned so far—discrete diagrams, number lines, tables, or equations. All approaches are welcome as long as students use them to represent the situations appropriately to support their reasoning.

As students work, monitor for different ways students reason about the problems, with or without using tape diagrams.

When solving the last problem, students may say that more than 21 gallons of gravel, 14 gallons of sand, and 7 gallons of cement will be needed to make 42 gallons of concrete. They may reason that the finer ingredients—sand and cement—will fill the spaces between the coarser pieces of gravel. Acknowledge the validity of their reasoning. Clarify that we can make an assumption that the gravel is also on the smaller side, so there are minimal empty spaces.

Launch

Keep students in the same groups. Provide graph paper and snap cubes (any three colors). Explain that they will now practice solving problems involving ratios and their combined quantities (similar to the green and purple paint in the previous task).

Draw students' attention to a ratio that uses "parts" as its unit. Ask students what they think "one part" means or amounts to, and how situations expressed in terms of "parts" could be diagrammed. Make sure they understand that "parts" do not represent specific amounts, that the value of "one part" can vary but the size of all parts is equal, and that a tape diagram can be used to show these parts.

If students are unfamiliar with cumin, clarify that it is an herb in the parsley family and its seed-like fruit is used as a spice.

Access for Multilingual Learners (Activity 2, Student Task)

MLR2: Collect and Display. Circulate, listen for and collect the language students use as they make sense of and talk about ratios that use "parts" for their units. On a visible display, record words and phrases such as: "one part of _____ represents ...," "for every part of _____, there are _____ parts of ...," "the parts are the same size, so ...," "all the parts make _____, so ...," and so on. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

Building on Student Thinking

Students using a tape diagram may think of each segment of a tape diagram as representing a particular unit of measurement, rather than as a flexible representation of an increment of a quantity. Help them set up the tapes with the correct number of sections and then discuss how many parts there are in all.

Student Workbook


2 Seasoning, Sneakers, and Concrete

Solve each of the following problems and show your thinking. If you get stuck, consider drawing a **tape diagram** to represent the situation.

1. Elena makes taco seasoning using 4 parts chili powder, 1 part garlic powder, and 2 parts ground cumin. If Elena uses 20 teaspoons of ground cumin, how much taco seasoning will she have?

2. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers?

3. A recipe for concrete says, "Mix 3 parts gravel with 2 parts sand and 1 part cement." If a project needs 42 gallons of concrete in all, how much of each ingredient is needed?

Student Task Statement

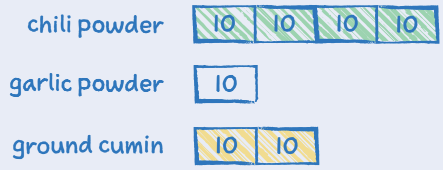
Solve each of the following problems and show your thinking. If you get stuck, consider drawing a **tape diagram** to represent the situation.

1. Elena makes taco seasoning using 4 parts chili powder, 1 part garlic powder, and 2 parts ground cumin. If Elena uses 20 teaspoons of ground cumin, how much taco seasoning will she have?

   70 teaspoons of taco seasoning.

   Sample reasoning:

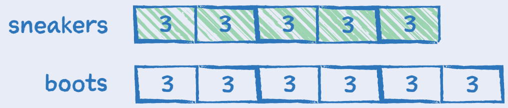
   - If 2 parts represent 20 teaspoons, then each part is 10 teaspoons. One part of garlic powder is 10 teaspoons. Four parts of chili pepper is $4 \cdot 10$ or 40 teaspoons. $40 + 10 + 20 = 70$.

   - There are 7 parts of all three ingredients in total. If 20 teaspoons make 2 parts, then each part is 10 teaspoons, and 7 parts is $7 \cdot 10$ or 70 teaspoons.

2. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers?

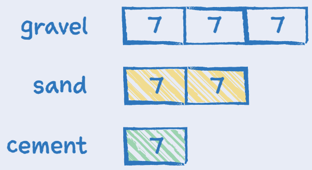
   15 students are wearing sneakers. $33 \div 11 = 3$. The value of each unit is 3. $3 \cdot 5 = 15$.

3. A recipe for concrete says, "Mix 3 parts gravel with 2 parts sand and 1 part cement." If a project needs 42 gallons of concrete in all, how much of each ingredient is needed?

   21 gallons of gravel, 14 gallons of sand, and 7 gallons of cement.

   Sample reasoning: There are 6 parts of ingredients making 42 gallons of concrete. This means 7 gallons for 1 part ($42 \div 6 = 7$), 14 gallons in 2 parts, and 21 gallons in 3 parts.

Are You Ready for More?

Using the recipe from earlier, how much taco seasoning can you make if you have 32 teaspoons of chili powder, 9 teaspoons of garlic powder, and 15 teaspoons of cumin? Explain or show your reasoning.

52.5 teaspoons of taco seasoning.

Sample reasoning: First, we need to figure out for each ingredient what 1 part can be:

- If 32 teaspoons of chili powder are used for the 4 parts, then each part is 8 teaspoons.
- If 9 teaspoons of garlic powder are used for the 1 part, then each part is 9 teaspoons.
- If 15 teaspoons of ground cumin are used for the 2 parts, then each part is 7.5 teaspoons.

The amount of ground cumin limits us to 7.5 teaspoons per part. There are 7 parts of ingredients in the recipe and $7 \cdot (7.5) = 52.5$.

Activity Synthesis

Select students to share their responses and reasoning. Discuss the ways in which tape diagrams are used to represent the quantities in the problems. Ask questions such as:

"For the sneakers and boots problem, where do we see the 6 to 5 ratio in the tape diagram? How do we show the total number of students?"

"For the taco seasoning problem, how do we represent the 20 teaspoons of ground cumin?"

Highlight that the ratio of quantities expressed using "parts" and the ratio expressed using specific units such as "gallons" or "teaspoons" are equivalent. For example, the ratio of parts of gravel to sand to cement in the concrete recipe is 3 to 2 to 1. We can make concrete in a ratio of 3 gallons to 2 gallons to 1 gallon or 21 gallons to 14 gallons to 7 gallons because these ratios are equivalent. Number line diagrams, tables, and tape diagrams can all help us find these equivalent ratios, regardless of the units being used.

Activity 3: Optional

Invent Your Own Ratio Problem

10 min

Activity Narrative

In this activity, students have an opportunity to create their own equivalent ratio problem.

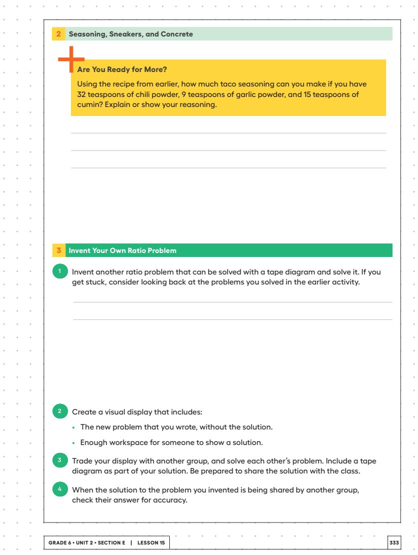
Launch

Keep students in the same groups. Provide graph paper, snap cubes (any three colors), and tools for creating a visual display.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Internalize Self Regulation.
Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to first decide on a context and some quantities in that context. Next, ask them to consider 1–2 questions that could be asked about the quantities and choose a strategy for answering the questions. Then, ask them to create a visual display as described in the activity.
Supports accessibility for: Organization, Attention

Student Workbook


2 Seasoning, Sneakers, and Concrete

Are You Ready for More?

Using the recipe from earlier, how much taco seasoning can you make if you have 32 teaspoons of chili powder, 9 teaspoons of garlic powder, and 15 teaspoons of cumin? Explain or show your reasoning.

3 Invent Your Own Ratio Problem

1. Invent another ratio problem that can be solved with a tape diagram and solve it. If you get stuck, consider looking back at the problems you solved in the earlier activity.
2. Create a visual display that includes:
   - The new problem that you wrote, without the solution.
   - Enough workspace for someone to show a solution.
3. Trade your display with another group, and solve each other's problem. Include a tape diagram as part of your solution. Be prepared to share the solution with the class.
4. When the solution to the problem you invented is being shared by another group, check their answer for accuracy.

GRADE 6 • UNIT 2 • SECTION E | LESSON 15 333

Student Task Statement

1. Invent another ratio problem that can be solved with a tape diagram and solve it. If you get stuck, consider looking back at the problems you solved in the earlier activity.
2. Create a visual display that includes:
   - The new problem that you wrote, without the solution.
   - Enough work space for someone to show a solution.
3. Trade your display with another group, and solve each other's problem. Include a tape diagram as part of your solution. Be prepared to share the solution with the class.
4. When the solution to the problem you invented is being shared by another group, check their answer for accuracy.

   Answers vary.

Activity Synthesis

Have each group share the peer-generated question it was assigned and the solution. Though the group that wrote the question will be responsible for confirming the answer, encourage all to listen to the reasoning each group used.

Lesson Synthesis

The ratio problems in this lesson are different from the ones in previous lessons because they include an additional piece of information. Consider asking students the following questions and using examples from the activities to highlight each point:

- *"What made the problems in this lesson different from the ones in earlier lessons?"*

  They include the combined or total amount of the quantities in the ratio. The quantities always have the same unit. Combining the quantities made sense in the context.

- *"How can a tape diagram represent these types of situations?"*

  The number of parts in the tape for each quantity tells us the ratio in the situation. Each part of the tape represents a particular value, and the sum of those values represents the total amount.

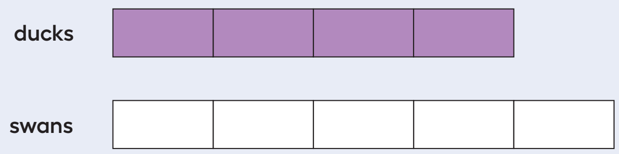
- *"Does changing the value of each part of the tape change the ratio? Why or why not?"*

  No, changing the value of each part changes the amount of each quantity and the total but not the ratio. The ratio is equivalent to the original ratio because we're multiplying each part by the same value.

Lesson Summary

A **tape diagram** is another way to represent a ratio. All the parts of the diagram that are the same size have the same value.

For example, this tape diagram represents the ratio of ducks to swans in a pond, which is 4:5.

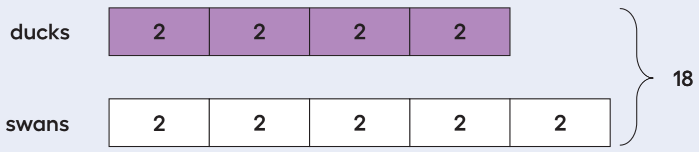


The first tape represents the number of ducks. It has 4 parts.

The second tape represents the number of swans. It has 5 parts.

There are 9 parts in all, because 4 + 5 = 9.

Suppose we know there are 18 of these birds in the pond, and we want to know how many are ducks.



The 9 equal parts on the diagram need to represent 18 birds in all. This means that each part of the tape diagram represents 2 birds, because 18 ÷ 9 = 2.

There are 4 parts of the tape representing ducks, and 4 · 2 = 8, so there are 8 ducks in the pond.

Cool-down

Room Sizes

5 min

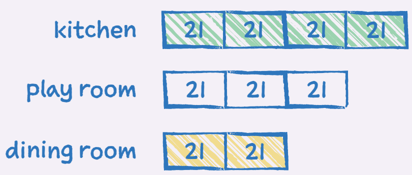
Provide access to graph paper.

Student Task Statement

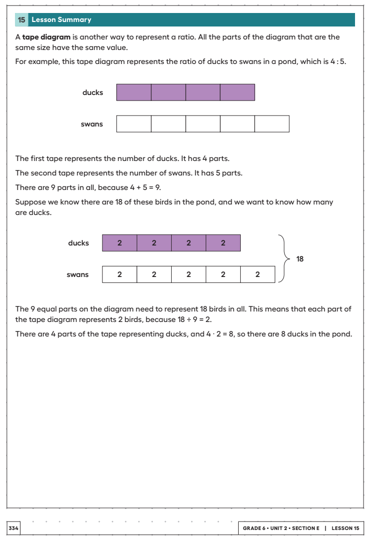
A house has a kitchen, a playroom, and a dining room on the first floor. The areas of the kitchen, playroom, and dining room in square feet are in the ratio 4:3:2. The combined area of these three rooms is 189 square feet. What is the area of each room?

The area of the kitchen is 84 square feet, the area of the playroom is 63 square feet, and the area of the dining room is 42 square feet.

Sample reasoning: All three rooms amount to 9 parts. All three rooms amount to 9 units. All three rooms make 189 square feet. 189 ÷ 9 = 21, so each part of the tape diagram represents 21 square feet. 4 • 21 = 84, 3 • 21 = 63, and 2 • 21 = 42.



Student Workbook



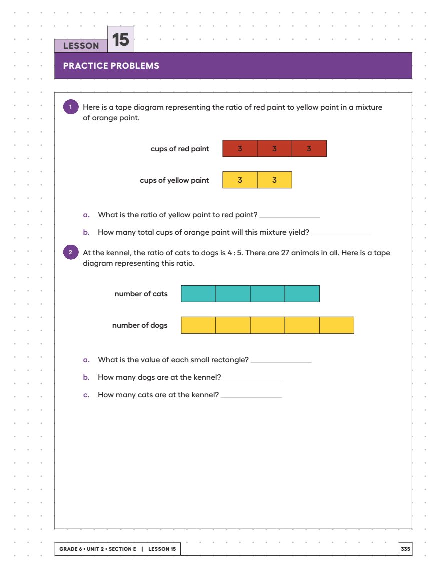
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

6 Problems

Student Workbook


LESSON 15

PRACTICE PROBLEMS

1 Here is a tape diagram representing the ratio of red paint to yellow paint in a mixture of orange paint.

cups of red paint 3 3 3

cups of yellow paint 3 3

a. What is the ratio of yellow paint to red paint?

b. How many total cups of orange paint will this mixture yield?

2 At the kennel, the ratio of cats to dogs is 4 : 5. There are 27 animals in all. Here is a tape diagram representing this ratio.

number of cats

number of dogs

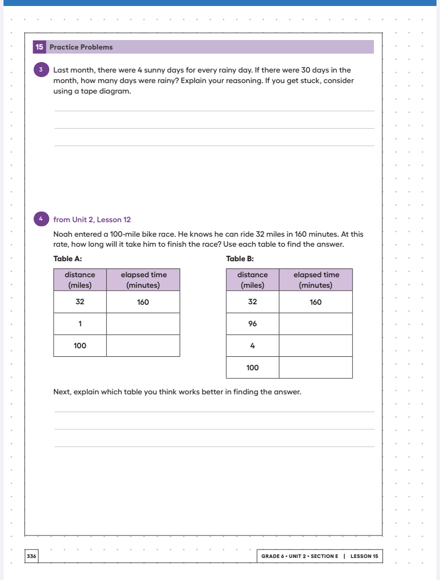
a. What is the value of each small rectangle?

b. How many dogs are at the kennel?

c. How many cats are at the kennel?

GRADE 6 • UNIT 2 • SECTION E | LESSON 15 225

Student Workbook


15 Practice Problems

3 Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Explain your reasoning. If you get stuck, consider using a tape diagram.

4 from Unit 2, Lesson 12

Noah entered a 100-mile bike race. He knows he can ride 32 miles in 160 minutes. At this rate, how long will it take him to finish the race? Use each table to find the answer.

Table A:

distance (miles)	elapsed time (minutes)
32	160
1	
100	

Table B:

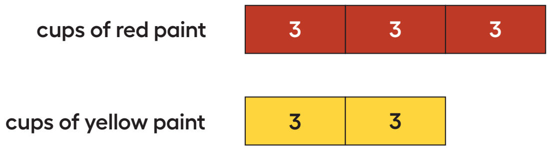
distance (miles)	elapsed time (minutes)
32	160
96	
4	
100	

Next, explain which table you think works better in finding the answer.

226 GRADE 6 • UNIT 2 • SECTION E | LESSON 15

Problem 1

Here is a tape diagram representing the ratio of red paint to yellow paint in a mixture of orange paint.



a. What is the ratio of yellow paint to red paint?

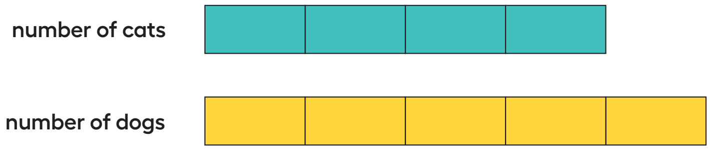
2:3 (or equivalent)

b. How many total cups of orange paint will this mixture yield?

15 cups

Problem 2

At the kennel, the ratio of cats to dogs is 4 : 5. There are 27 animals in all. Here is a tape diagram representing this ratio.



a. What is the value of each small rectangle?

3, because $4 + 5 = 9$ and $27 \div 9 = 3$

b. How many dogs are at the kennel?

15 dogs, because $3 \cdot 5 = 15$

c. How many cats are at the kennel?

12 cats, because $3 \cdot 4 = 12$

Problem 3

Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Explain your reasoning. If you get stuck, consider using a tape diagram.

6 rainy days

Sample reasoning: $4 + 1 = 5$, so there are 5 units total. $30 \div 5 = 6$, so each unit is worth 6.

Problem 4

from Unit 2, Lesson 12

Noah entered a 100-mile bike race. He knows he can ride 32 miles in 160 minutes. At this rate, how long will it take him to finish the race? Use each table to find the answer.

Table A:

distance (miles)	elapsed time (minutes)
32	160
1	5
100	100

Table B:

distance (miles)	elapsed time (minutes)
32	160
96	480
4	20
100	500

Next, explain which table you think works better in finding the answer.

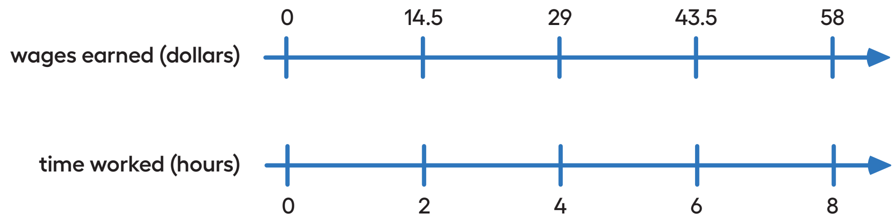
He will finish the race in 500 minutes (or equivalent).

Sample response: The first table is more efficient, but they both work in getting the answer.

Problem 5

from Unit 2, Lesson 13

A cashier worked an 8-hour day, and earned \$58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your reasoning.



a. How much does the cashier earn per hour?

\$7.25 per hour

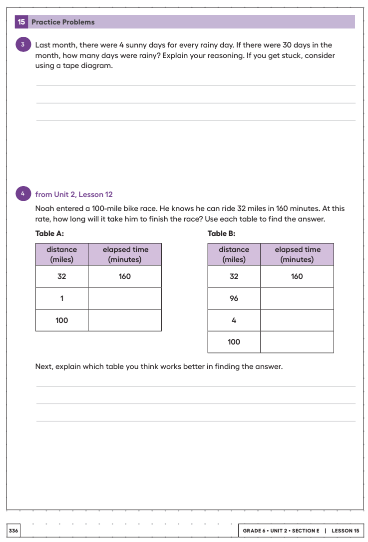
Sample reasoning: $14.5 \div 2 = 7.25$

b. How much does the cashier earn if she works 3 hours?

\$21.75

Sample reasoning: $(7.25) \cdot 3 = 21.75$

Student Workbook



15 Practice Problems

3 Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Explain your reasoning. If you get stuck, consider using a tape diagram.

4 from Unit 2, Lesson 12

Noah entered a 100-mile bike race. He knows he can ride 32 miles in 160 minutes. At this rate, how long will it take him to finish the race? Use each table to find the answer.

Table A:

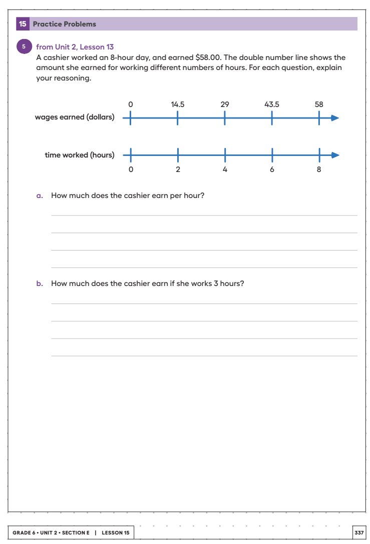
distance (miles)	elapsed time (minutes)
32	160
1	
100	

Table B:

distance (miles)	elapsed time (minutes)
32	160
96	
4	
100	

Next, explain which table you think works better in finding the answer.

Student Workbook



15 Practice Problems

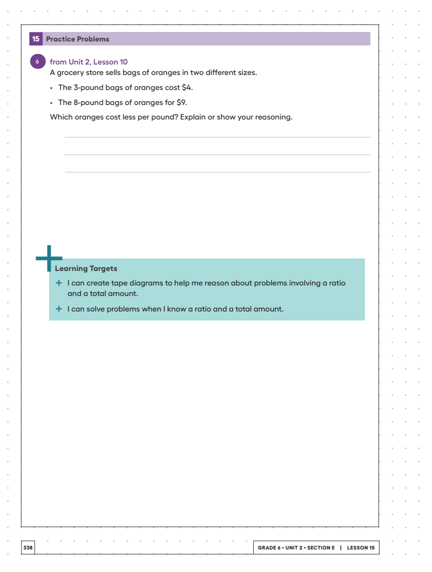
5 from Unit 2, Lesson 13

A cashier worked an 8-hour day, and earned \$58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your reasoning.

a. How much does the cashier earn per hour?

b. How much does the cashier earn if she works 3 hours?

Student Workbook


15 Practice Problems

6 from Unit 2, Lesson 10

A grocery store sells bags of oranges in two different sizes.

- The 3-pound bags of oranges cost $4.
- The 8-pound bags of oranges for $9.

Which oranges cost less per pound? Explain or show your reasoning.

Learning Targets

- I can create tape diagrams to help me reason about problems involving a ratio and a total amount.
- I can solve problems when I know a ratio and a total amount.

338 GRADE 6 • UNIT 2 • SECTION E | LESSON 15

Problem 6

from Unit 2, Lesson 10

A grocery store sells bags of oranges in two different sizes.

- The 3-pound bags of oranges cost $4.
- The 8-pound bags of oranges for $9.

Which oranges cost less per pound? Explain or show your reasoning.

The 8-pound bags cost less per pound.

Sample reasoning:

- Compare the cost for 24 pounds of oranges for both types of bags. When sold in 3-pound bags, 24 pounds cost $32. When sold in 8-pound bags, 24 pounds cost $27.
- Compare how much can be bought for the same amount of money. $36 can buy 27 pounds of oranges in 3-pound bags, or it can buy 32 pounds in 8-pound bags.