Lesson 15

Cube Roots

Goals

- Determine the whole numbers that the cube root of a number lies between, and explain (orally) the reasoning.
- Generalize a process for approximating the value of the cube root of a number, and justify (orally and in writing) that if $x^3 = n$, then $x = \sqrt[3]{n}$.

Learning Target

When I have a cube root, I can reason about which two whole numbers it is between.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Math Talk

Lesson Narrative

The purpose of this lesson is to encourage students to transition from the geometric characterization of cube roots as edge lengths to an algebraic characterization of cube roots as solutions to equations of the form $x^3 = n$ that can be understood as values on a number line.

Students begin the lesson by considering whether statements such as $(\sqrt[3]{5}) = 5$ are true or not. This requires students to make sense of what cube root notation means. Next, students reason about the value of a cube root of a number by finding the two whole numbers it is closest to, creating a stepping stone to help them more accurately place the point on a number line. Finally, students think about $\sqrt[3]{n}$ as the solution to equations of the form $x^3 = n$ and plot these points on a number line.

Student Learning Goal

Let's compare cube roots.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	10 min	10 min	5 min

Warm-up
Math Talk: Cubed

5 min

Activity Narrative

This *Math Talk* focuses on symbolic statements about cube roots. It encourages students to think about the meaning of the cube root symbol and to rely on what they know about cube roots to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students find solutions to equations of the form $x^3 = n$.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

Give students quiet think time and ask them to give a signal when they have an answer and a strategy.

Invite students to share their strategies and record and display their responses for all to see.

Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Decide mentally whether each statement is true.

A. $\left(\sqrt[3]{5}\right)^3 = 5$

True

B. $\left(\sqrt[3]{27}\right)^3 = 3$

Not true

C. $7 = \left(\sqrt[3]{7}\right)^3$

True

D. $\left(\sqrt[3]{64}\right) = 2^3$

Not true

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

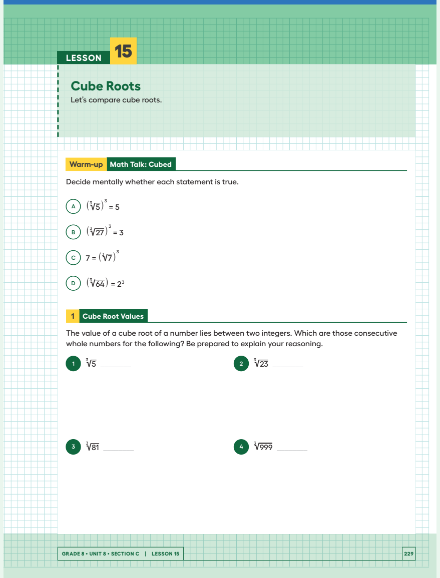
Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Student Workbook



Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Begin by asking,

"Does this problem remind anyone of something we have done before?"

Supports accessibility for: Memory, Attention

Activity 1
Cube Root Values

10 min

Activity Narrative

The purpose of this activity is for students to think about cube roots of numbers in relation to the two integer values they are closest to. Students are encouraged to use the fact that $\sqrt[3]{n}$ is a solution to the equation $x^3 = n$. Students can draw a number line if that helps them reason about the magnitude of the given cube roots, but this is not required.

Monitor for students that multiply non-integers by hand to try and approximate. While this isn't the preferred strategy, their work could be used to think about which integer the cube root is closest to during the *Activity Synthesis*.

Launch

Arrange students in groups of 2. Do not give students access to calculators since the goal of this activity is to reason about the value of cube roots of numbers.

Give 2 minutes of quiet work, and follow with a whole-class discussion.

Student Task Statement

The value of a cube root of a number lies between two integers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

1. $\sqrt[3]{5}$

   1 and 2

   Sample reasoning: $1^3 = 1$ and $2^3 = 8$, so $\sqrt[3]{5}$ is between 1 and 2.

2. $\sqrt[3]{23}$

   2 and 3

   Sample reasoning: $2^3 = 8$ and $3^3 = 27$, so $\sqrt[3]{23}$ is between 2 and 3.

3. $\sqrt[3]{81}$

   4 and 5

   Sample reasoning: $4^3 = 64$ and $5^3 = 125$, so $\sqrt[3]{81}$ is between 4 and 5.

4. $\sqrt[3]{999}$

   9 and 10

   Sample reasoning: $10^3 = 1,000$, so $\sqrt[3]{999}$ is a little bit less than 10.

Activity Synthesis

The purpose of this discussion is for students to share the strategies they used to approximate cube roots by looking at perfect cubes. For each question, ask 1–2 students:

"What strategy did you use to figure out the two whole numbers?"

I made a list of perfect cubes and then found which two the number was between.

During the discussion, make a class display of perfect cubes up to at least $10^3 = 1,000$, or add them to the existing display of perfect squares. This display should be posted in the classroom for the remaining lessons within this unit.

Activity 2

Solutions on a Number Line

10 min

Activity Narrative

In this activity, students use rational approximations of irrational numbers to place both rational and irrational numbers on a number line. This reinforces the definition of a cube root as a solution to the equation of the form $x^3 = n$. This is also the first time that students are asked to think about negative cube roots.

Launch

Arrange students in groups of 2. Do not provide access to calculators since the goal of this activity is to reason about the location of irrational numbers on the number line.

Give students 2 minutes of quiet work time, and follow with a whole-class discussion.

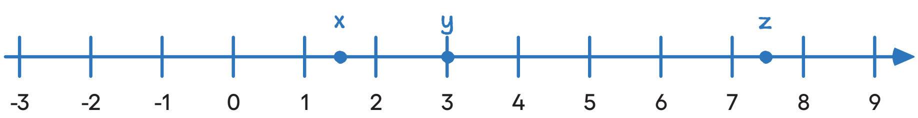
Student Task Statement

The numbers x, y, and z are positive, and:

$x^3 = 5$

$y^3 = 27$

$z^3 = 700$



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.

   x should be between 1 and 2; $y = 3$; z should be between 8 and 9.

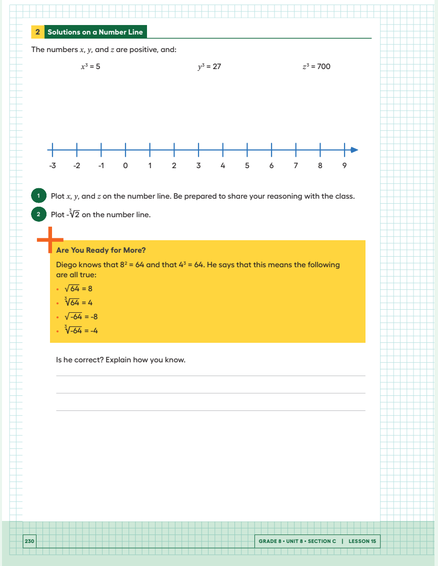
2. Plot $-\sqrt[3]{2}$ on the number line.

   The point for $-\sqrt[3]{2}$ should be between -1 and -2.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Student Workbook


2 Solutions on a Number Line

The numbers x, y, and z are positive, and:

$x^3 = 5$ $y^3 = 27$ $z^3 = 700$

-3 -2 -1 0 1 2 3 4 5 6 7 8 9

1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.
2. Plot $-\sqrt[3]{2}$ on the number line.

Are You Ready for More?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

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Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, use a kinesthetic representation of the number line on a clothesline. Students can place and adjust numbers on folder paper or cardstock on the clothesline in a hands-on manner.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Are You Ready for More?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

Three of Diego's statements are correct, but $\sqrt{-64}$ does not equal -8 because $(-8)^2$ is 64, not -64.

Activity Synthesis

The purpose of this discussion is to make sure students understand how they can use rational numbers to approximate irrational numbers. Display the number line from the *Task Statement* for all to see. Select 1–2 groups to share their reasoning for each value placement and record their values on the displayed number line. After each placement, ask if any students used different reasoning and invite them to share with the class.

Lesson Synthesis

The purpose of this discussion is to generalize a process for approximating the value of a cube root by reinforcing that if $x^3 = n$, then $x = \sqrt[3]{n}$. Here are some questions for discussion:

- *"What is the solution to the equation $a^3 = 47$?"*

  $\sqrt[3]{47}$

- *"Where would $\sqrt[3]{47}$ be located on a number line?*

  $3^3 = 27$ and $4^3 = 64$, so $\sqrt[3]{47}$ has to be between 3 and 4.

- *"What is the solution to the equation $a^3 = 125$?"*

  $\sqrt[3]{125} = 5$

- *"What is the solution to the equation $a^3 = -125$?"*

  $\sqrt[3]{-125} = -5$

- *"How can we plot the cube root of any number on the number line?"*

  Find the two perfect cubes that are closest to the number and use them to determine which integers the cube root of the number will be between and the approximate location between them.

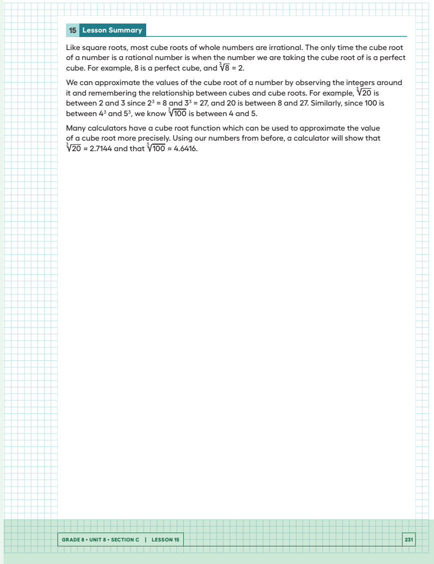
Lesson Summary

Like square roots, most cube roots of whole numbers are irrational. The only time the cube root of a number is a rational number is when the number we are taking the cube root of is a perfect cube. For example, 8 is a perfect cube, and $\sqrt[3]{8} = 2$.

We can approximate the values of the cube root of a number by observing the integers around it and remembering the relationship between cubes and cube roots. For example, $\sqrt[3]{20}$ is between 2 and 3 since $2^3 = 8$ and $3^3 = 27$, and 20 is between 8 and 27. Similarly, since 100 is between 4^3 and 5^3, we know $\sqrt[3]{100}$ is between 4 and 5.

Many calculators have a cube root function which can be used to approximate the value of a cube root more precisely. Using our numbers from before, a calculator will show that $\sqrt[3]{20} \approx 2.7144$ and that $\sqrt[3]{100} \approx 4.6416$.

Student Workbook



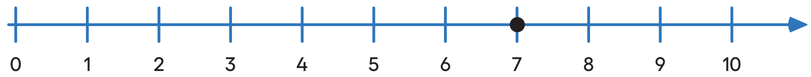
Cool-down

Different Types of Roots

5 min

Student Task Statement

Lin is asked to place a point on a number line to represent the value of $\sqrt[3]{49}$ and she draws:



Where should $\sqrt[3]{49}$ actually be on the number line? How do you think Lin got the answer she did?

Sample response: $\sqrt[3]{49}$ should be between 3 and 4 on the number line. I think Lin placed the point at $\sqrt{49}$ because she thought it was a square root instead of a cube root.

Responding To Student Thinking

Press Pause

By this point in the unit, there should be some student mastery placing numbers on the number line. If most students struggle, make time to revisit related work in the section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

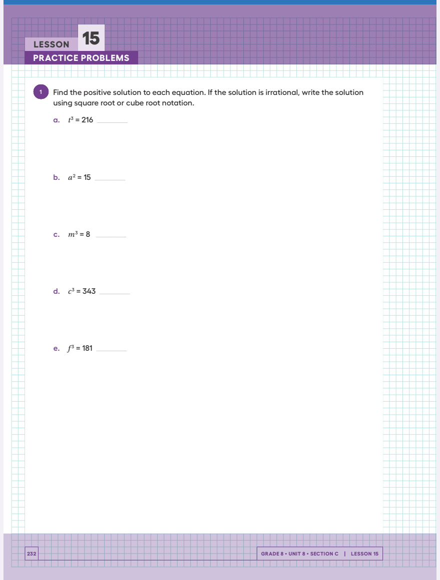
Unit 8, Section C Side Lengths and Volumes of Cubes

Lesson 15

Practice Problems

6 Problems

Student Workbook


LESSON 15
PRACTICE PROBLEMS

1 Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

a. $t^3 = 216$ ______

b. $a^2 = 15$ ______

c. $m^3 = 8$ ______

d. $c^3 = 343$ ______

e. $f^3 = 181$ ______

Student Workbook


15 Practice Problems

2 For each cube root, find the two whole numbers that it lies between.

a. $\sqrt[3]{11}$ ______

b. $\sqrt[3]{80}$ ______

c. $\sqrt[3]{120}$ ______

d. $\sqrt[3]{250}$ ______

3 Order the following values from least to greatest:

$\sqrt[3]{530}$, $\sqrt{48}$, π, $\sqrt{121}$, $\sqrt[3]{27}$, $\frac{19}{2}$

______ ______ ______ ______ ______ ______

4 Select **all** the equations that have a solution of $\frac{2}{7}$:

A. $x^2 = \frac{2}{7}$

B. $x^2 = \frac{4}{14}$

C. $x^2 = \frac{4}{49}$

D. $x^3 = \frac{6}{21}$

E. $x^3 = \frac{8}{343}$

F. $x^3 = \frac{6}{7}$

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Problem 1

Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

a. $t^3 = 216$

$t = 6$

b. $a^2 = 15$

$a = \sqrt{15}$

c. $m^3 = 8$

$m = 2$

d. $c^3 = 343$

$c = 7$

e. $f^3 = 181$

$f = \sqrt[3]{181}$

Problem 2

For each cube root, find the two whole numbers that it lies between.

a. $\sqrt[3]{11}$

2 and 3

b. $\sqrt[3]{80}$

4 and 5

c. $\sqrt[3]{120}$

4 and 5

d. $\sqrt[3]{250}$

6 and 7

Problem 3

Order the following values from least to greatest:

$\sqrt[3]{530}$, $\sqrt{48}$, π, $\sqrt{121}$, $\sqrt[3]{27}$, $\frac{19}{2}$

$\sqrt[3]{27}$ π $\sqrt{48}$ $\sqrt[3]{530}$ $\frac{19}{2}$ $\sqrt{121}$

Problem 4

Select **all** the equations that have a solution of $\frac{2}{7}$:

A. $x^2 = \frac{2}{7}$

B. $x^2 = \frac{4}{14}$

C. $x^2 = \frac{4}{49}$

D. $x^3 = \frac{6}{21}$

E. $x^3 = \frac{8}{343}$

F. $x^3 = \frac{6}{7}$

Problem 5

The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So 5 is a solution, and -5 is also a solution. But! The equation $x^3 = 125$ only has one solution, which is 5. This is because $5 \cdot 5 \cdot 5 = 125$, and there are no other numbers you can cube to make 125. (Think about why -5 is not a solution!)

Find all the solutions to each equation.

a. $x^3 = 8$

2

b. $\sqrt[3]{x} = 3$

27

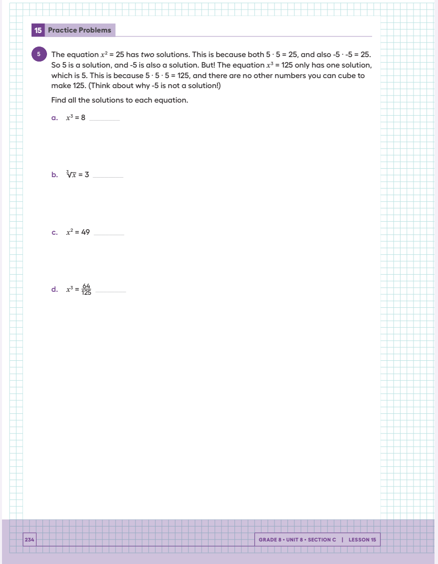
c. $x^2 = 49$

7 and -7

d. $x^3 = \frac{64}{125}$

$\frac{4}{5}$

Student Workbook


15 Practice Problems

The equation $x^2 = 25$ has two solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So 5 is a solution, and -5 is also a solution. But! The equation $x^3 = 125$ only has one solution, which is 5. This is because $5 \cdot 5 \cdot 5 = 125$, and there are no other numbers you can cube to make 125. (Think about why -5 is not a solution!)

Find all the solutions to each equation.

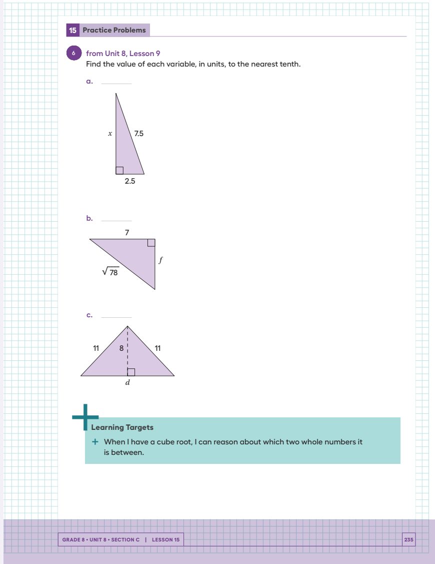
a. $x^3 = 8$

b. $\sqrt[3]{x} = 3$

c. $x^2 = 49$

d. $x^3 = \frac{64}{125}$

Student Workbook


15 Practice Problems

6 from Unit 8, Lesson 9

Find the value of each variable, in units, to the nearest tenth.

Learning Targets

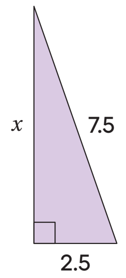
+ When I have a cube root, I can reason about which two whole numbers it is between.

Problem 6

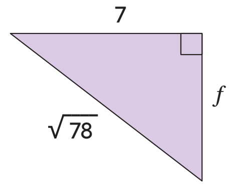
from Unit 8, Lesson 9

Find the value of each variable, in units, to the nearest tenth.

a. $x \approx 7.1$ units



b. $f \approx 5.4$ units



c. $d \approx 15.1$ units