Corresponding Parts and Scale Factors

Goals

- Comprehend the phrase "scale factor" and explain (orally) how it relates corresponding lengths of a figure and its scaled copy.
- Explain (orally) what it means to say one part in a figure "corresponds" to a part in another figure.
- Identify and describe (orally and in writing) corresponding points, corresponding segments, or corresponding angles in a pair of figures.

Learning Targets

- I can describe what the scale factor has to do with a figure and its scaled copy.
- In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles.

Lesson Narrative

This lesson develops the vocabulary for talking about scaling and scaled copies more precisely and for identifying the structures in common between two figures. Specifically, students learn to use the term **corresponding** to refer to a pair of points, segments, or angles in two figures that are scaled copies. Students also begin to describe the numerical relationship between the corresponding lengths in two figures using a **scale factor**. They see that when two figures are scaled copies of one another, the same scale factor relates their corresponding lengths. As students identify corresponding parts, they are making use of structure. As students identify the scale factor that relates corresponding sides, they are making use of repeated reasoning.

A look at the angles of scaled copies also begins in this lesson. Students use tracing paper to trace and compare angles in an original figure and its copies. They observe that in scaled copies the measures of corresponding angles are equal.

Student Learning Goal

Let's describe features of scaled copies.

Lesson Timeline

5_{min}

Warm-up

15 min

Activity 1

15 min

Activity 2

10 min

Lesson Synthesis

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Math Talk
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials to Gather

• Geometry toolkits: Activity 1

Required Preparation

Activity 1:

Prepare to display the images of the railroad crossing sign.

For the digital version of the activity, acquire devices that can run the applet.

Assessment

5 min

Cool-down

Warm-up

Math Talk: Multiplying by a Unit Fraction



Activity Narrative

This is the first *Math Talk* activity in the course. See the launch for extended instructions for facilitating this activity successfully.

This *Math Talk* focuses on multiplying a whole number by a unit fraction. It encourages students to think about the relationship between multiplication and division and to rely on properties of operations to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students are identifying scale factors.

Launch

This is the first time students do the *Math Talk* instructional routine in this course, so it is important to explain how it works before starting.

Explain that a *Math Talk* has four problems, revealed one at a time. For each problem, students have a minute to quietly think and are to give a signal when they have an answer and a strategy. The teacher then selects students to share different strategies (likely 2–3, given limited time), and might ask questions such as

O "Who thought about it in a different way?"

The teacher then records the responses for all to see, and might ask clarifying questions about the strategies before revealing the next problem.

Consider establishing a small, discreet hand signal that students can display when they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Instructional Routines

Math Talk

ilclass.com/r/10694967

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

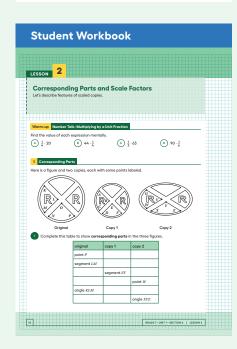
Supports accessibility for: Memory, Organization

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing



Student Task Statement

Find the value of each expression mentally.

```
A.\frac{1}{4} \cdot 20
5
Sample reasoning: 20 \div 4 = 5
B. 44 \cdot \frac{1}{4}
II
Sample reasoning: 44 \div 4 = 11
C.\frac{1}{3} \cdot 63
2I
Sample reasoning: 60 \div 3 = 20, 3 \div 3 = 1, and 20 + 1 = 21.
D.90 \cdot \frac{1}{6}
```

Sample reasoning: $90 \div 3 = 30$ and $30 \div 2 = 15$.

Activity Synthesis

Make sure the connection to division is brought up in the discussion, before moving on to the second expression.

To involve more students in the conversation, consider asking:

☐ "Who can restate ______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway is that these problems all involve multiplying by a unit fraction. One strategy that works in such cases is dividing the other factor by the denominator of the fraction.

Activity 1

Corresponding Parts



Activity Narrative

There is a digital version of this activity.

This activity introduces important language students will apply to describe scaled copies. In particular, it introduces the important idea of corresponding parts. Students have previously analyzed corresponding sides in figures. Here they will begin to examine angles explicitly as well, understanding that corresponding angles in a figure and its scaled copy have the same measure. As students identify corresponding parts, they are making use of structure.

This activity works best when each student has access to tracing paper. If tracing paper is unavailable, consider using the digital version of the activity. In the digital version, students use an applet to drag and rotate a given fixed angle over the images of the railroad sign. Similar to using tracing paper, the applet allows students to compare the size of corresponding angles without needing to find specific numeric values for the measurements.

Launch 22

Tell students that in this lesson, they will look more closely at copies of figures and describe specific parts in them. Display the three images in the activity statement for all to see. Explain that the original design and its two copies have **corresponding** parts, which are parts that are in the same position in relation to the rest of each figure. Point out some of their corresponding parts, such as:

- The X-pattern going across each figure.
- The curved outline of each figure.
- The points *K* in the original figure, *A* in Copy 1, and *U* in Copy 2.

Arrange students in groups of 2 and provide access to their geometry toolkits (especially tracing paper). Give students 2–3 minutes to complete the first two questions and another 2 minutes to discuss their responses with their partner. Ask students to pause their work for a quick class discussion afterward.

Ask a few students to name a set of corresponding points, segments, or angles.

Then, ask students to indicate whether they think either copy is a scaled copy. Invite a couple of students to share their reasoning. When the class reaches an agreement that Copy 1 is a scaled copy and Copy 2 is not, ask students to complete the remaining questions individually and to use tracing paper as a tool.

Consider demonstrating to the class how to use tracing paper to compare angles. Tell or show students that the line segments forming an angle could be extended for easier tracing and comparison.

Instructional Routines

MLR8: Discussion Supports

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Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports.

Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "Line segment KP goes with line segment KP as "Line segment KP corresponds to line segment KP."

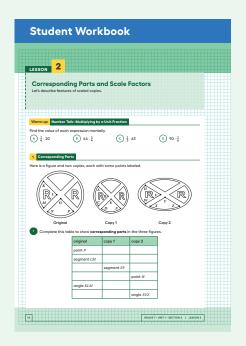
Advances: Speaking, Representing

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.

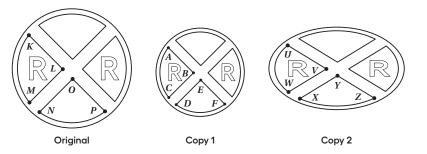
Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of corresponding parts of scaled copies. Terms may include: corresponding points, corresponding line segments, and corresponding angles.

Supports accessibility for: Conceptual Processing, Language



Student Task Statement

Here is a figure and two copies, each with some points labeled.



1. Complete this table to show corresponding parts in the three figures.

| original | сору 1 | сору 2 |
|-------------------|-------------------|------------|
| point P | point F | point Z |
| segment <i>LM</i> | segment BC | segment VW |
| segment OP | segment <i>EF</i> | segment YZ |
| point M | point C | point W |
| angle <i>KLM</i> | angle ABC | angle UVW |
| angle NOP | angle DEF | angle XYZ |

2. Is either copy a scaled copy of the original figure? Explain your reasoning.

Copy I is a scaled copy, but Copy 2 is not.

Sample reasoning: The original figure is a circle. Copy I is also a circle, only smaller. Copy 2 has been stretched sideways and shrunken vertically; its shape has changed into an oval, so it is not a scaled copy.

3. Use tracing paper to compare angle *KLM* with its corresponding angles in Copy 1 and Copy 2. What do you notice?

Angle ABC in Copy I corresponds to and has the same size as angle KLM. Angle UVW in Copy 2 also corresponds to angle KLM but is smaller in size than the original angle.

4. Use tracing paper to compare angle *NOP* with its corresponding angles in Copy 1 and Copy 2. What do you notice?

Angle *DEF* in *Copy* I corresponds to and has the same size as angle *NOP*. Angle *XYZ* in *Copy* 2 also corresponds to angle *NOP* but is larger in size than the original angle.

Activity Synthesis

Select a few students to share their observations about angles. Discuss the size of corresponding angles in figures that are scaled copies and those that are not. Ask questions such as:

"In the scaled copy, Copy 1, did the size of any angle change compared to its corresponding angle in the original sign?"

No

"In Copy 2, did the size of any angle change compared to its corresponding angle in the original sign?"

Yes, for example, angle UVW has a different measure than angle KLM.

"What can you say about corresponding angles in two figures that are scaled copies of one another?"

They have the same measure.

"What can you say about corresponding angles in two figures that are not scaled copies?"

They might not have the same measure.

Activity 2

Scaled Triangles

15 min

Activity Narrative

In this activity, students continue to practice identifying corresponding parts of scaled copies. By organizing corresponding lengths in a table, students see that there is a single factor that relates each length in the original triangle to its corresponding length in a copy. They learn that this number is called a **scale factor**.

As students work on the first question, listen to how they reason about which triangles are scaled copies. Identify groups who use side lengths and angles as the basis for deciding. (Students are not expected to reason formally yet, but should begin to look to lengths and angles for clues.)

As students identify corresponding sides and their measures in the second and third questions, look out for confusion about corresponding parts. Notice how students decide which sides of the right triangles correspond.

If students still have access to tracing paper, monitor for students who use this tool strategically.

This is the first time Math Language Routine 2: *Collect and Display* is suggested in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students that use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. The purpose of this routine is to capture a variety of students' words and phrases—including, especially, everyday or social language and non-English—in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.

Instructional Routines

MLR2: Collect and Display

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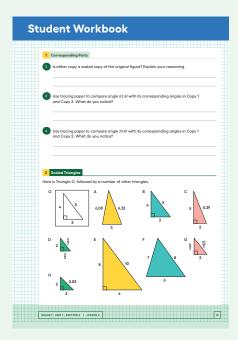
Access for Multilingual Learners (Activity 2, Narrative)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Building on Student Thinking

Students may think that Triangle F is a scaled copy because just like the 3-4-5 triangle, the sides are also three consecutive whole numbers. Point out that corresponding angles are not equal.



Launch

Arrange students into groups of 4. Assign each student one of the following pairs of triangles to examine for the first question.

• A and E

• B and F

• C and G

• D and H

Give students 2 minutes of quiet think time to determine if their assigned triangles are scaled copies of the original triangle. Give students another 2–3 minutes to discuss their responses and complete the first question in groups.

Use Collect and Display to create a shared reference that captures students' developing mathematical language. Collect the language students use to explain how they know whether each triangle is a scaled copy of the original. Display words and phrases such as: "corresponding," "straight," "slanted," "vertical," "right angle," "stretched," "doubled," "halved," "multiplied," "divided," "same amount," "same factor," etc.

Discuss briefly as a class which triangles are scaled copies and select a couple of groups who reasoned in terms of lengths and angles to explain their reasoning. Some guiding questions:

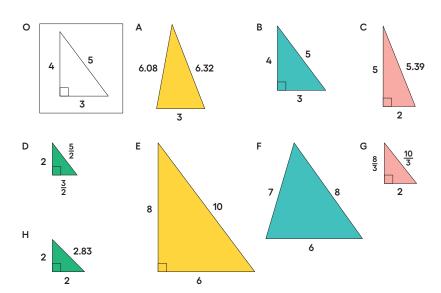
"What information did you use to tell scaled copies from those that are not?"

"How were you able to tell right away that some figures are not scaled copies?"

After the class recognizes that A, C, F, and H are not scaled copies, give students quiet work time to complete the rest of the task.

Student Task Statement

Here is Triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.

Response depends on the pair of triangles students have. Triangles B, D, E, and G are scaled copies.

2. As a group, identify *all* the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.

Triangles B, D, E, and G are scaled copies.

Sample reasoning: B, D, E, and G have not changed in shape (they are still right triangles). Each of their sides is the same number of times as long as the corresponding side in the original triangle. Triangles A and F do not have the same shape as Triangle O (their angles are different), so they are not scaled copies. Triangles C and H are right triangles but their sides are not the same number of times as long as the corresponding sides in the original triangle.

3. List all the triangles that are scaled copies in the table.
Record the side lengths that correspond to the side lengths of Triangle
O listed in each column.

| Triangle O | 3 | 4 | 5 |
|------------|-----|-----|----------|
| Triangle B | 3 | 4 | 5 |
| Triangle D | 3 2 | 2 | <u>5</u> |
| Triangle E | 6 | 8 | 10 |
| Triangle G | 2 | 8/3 | 10 |

4. Explain or show how each copy has been scaled from the original (Triangle O).

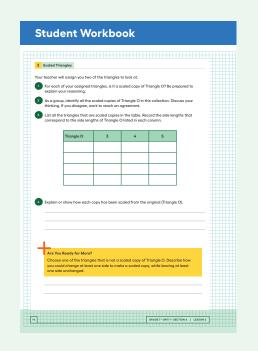
Sample responses:

- Triangle B is a same-size copy of the original. All the lengths stay the same.
- In Triangle D, all the lengths are half of the original ones.
- In Triangle E, all sides double in length.
- In Triangle G, the lengths are $\frac{2}{3}$ times the corresponding lengths in the original triangle.

Are You Ready for More?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

Sample response: On Triangle F, the side of length 7 could be extended to have length 10.



Activity Synthesis

Display the image of all triangles and invite a couple of students to share how they knew which sides of the triangles correspond. Then, display a completed table in the third question for all to see.

Direct students' attention to the reference created using *Collect and Display*. Ask each group to present its observations about one triangle and how the triangle has been scaled from the original. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond. As students present, record or illustrate their reasoning on the table, such as by drawing arrows between rows and annotating with the operation that students are describing, as shown here.

| Triangle O | 3 | 4 | 5 |
|------------|-------|-----|----------------|
| Triangle D | 3 3 2 | 2 | <u>5</u> 2 |
| Triangle E | 6 | 8 | 10 |
| Triangle B | 3 | 4 | 5 |
| Triangle G | 2 | 8 3 | <u>10</u> 3 |

Use the language that students use to describe the side lengths and the numerical relationships in the table to guide students toward **scale factor**. For example:

"You explained that the lengths in Triangle F are all twice those in the original triangle, so we can write those as '2 times' the original numbers. Lengths in Triangle A are half of those in the original; we can write $\frac{1}{2}$ times' the original numbers. We call those multipliers—the 2 and the $\frac{1}{2}$ —scale factors. We say that scaling Triangle O by a scale factor of 2 produces Triangle F, and that scaling Triangle O by a scale factor of $\frac{1}{2}$ produces Triangle A."

Lesson Synthesis

Share with students

C "Today we examined characteristics of scaled copies more closely."

To review the new vocabulary, consider asking students:

"What do we mean by corresponding parts?"

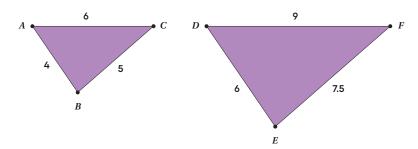
Parts that have the same relative position within each figure

"What is a scale factor? How does it work?"

The segments in a scaled copy are always a certain number of times as long as the corresponding segments in the original figure. We call that number the scale factor.

Students can use informal language to describe corresponding parts and recognize a scale factor as a common ratio between the lengths of corresponding side lengths.

If desired, use this example to review the meanings of these terms. Triangle DEF is a scaled copy of triangle ABC.

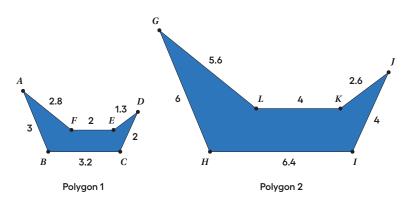


Display this image and invite students to identify examples of corresponding parts as well as the scale factor. For example:

- Vertex E in triangle DEF and vertex B in triangle ABC are corresponding points.
- Segment BC and segment EF are corresponding segments.
- Angle C (or angle BCA) and angle F (or angle EFD) are corresponding angles.
- The scale factor between ABC and its copy triangle DEF is $\frac{3}{2}$ or 1.5 because all lengths in triangle DEF are 1.5 times as long as the corresponding lengths in triangle ABC.

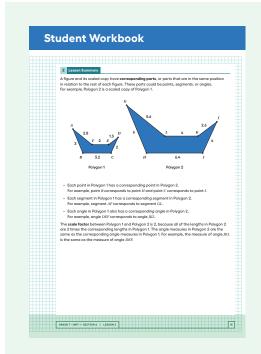
Lesson Summary

A figure and its scaled copy have **corresponding parts**, or parts that are in the same position in relation to the rest of each figure. These parts could be points, segments, or angles. For example, Polygon 2 is a scaled copy of Polygon 1.



- Each point in Polygon 1 has a corresponding point in Polygon 2.
 For example, point B corresponds to point H and point C corresponds to point I.
- Each segment in Polygon 1 has a *corresponding segment* in Polygon 2. For example, segment *AF* corresponds to segment *GL*.
- Each angle in Polygon 1 also has a *corresponding angle* in Polygon 2. For example, angle *DEF* corresponds to angle *JKL*.

The **scale factor** between Polygon 1 and Polygon 2 is 2, because all of the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1. For example, the measure of angle JKL is the same as the measure of angle DEF.



Responding To Student Thinking

Points to Emphasize

If students struggle with corresponding angles or sides, revisit the vocabulary when opportunities arise over the next several lessons.

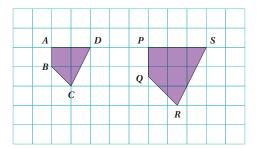
Cool-down

Comparing Polygons ABCD and PQRS



Student Task Statement

Polygon PQRS is a scaled copy of polygon ABCD.



- Name the angle in the scaled copy that corresponds to angle ABC.
 Angle PQR corresponds to angle ABC.
- 2. Name the segment in the scaled copy that corresponds to segment AD.

 Segment PS corresponds to segment AD.
- **3.** What is the scale factor from polygon *ABCD* to polygon *PQRS*? The scale factor is $\frac{3}{2}$ because *PS* = 3 and *AD* = 2.

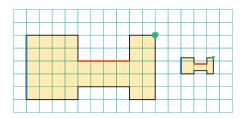
Practice Problems

5 Problems

Problem 1

The second H-shaped polygon is a scaled copy of the first.

Sample responses:



- **a.** Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using colored pencils to highlight corresponding parts or labeling some of the vertices.
- **b.** What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

 $\frac{1}{4}$ or 0.25

Sample reasoning: The sides that are 4 units long in the original polygon are I unit long in the copy, which is one fourth of the original length.

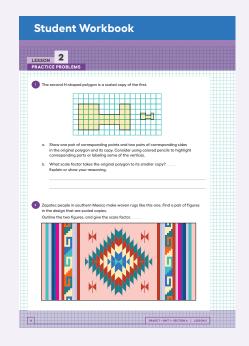
Problem 2

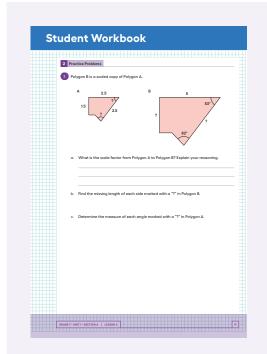
Zapotec people in southern Mexico make woven rugs like this one. Find a pair of figures in the design that are scaled copies. Outline the two figures, and give the scale factor.

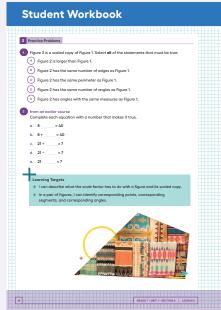


Sample responses:

For the parallelograms, the scale factor is 2. For the triangles, the scale factor is 1.5.

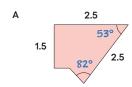


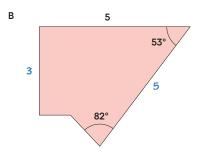




Problem 3

Polygon B is a scaled copy of Polygon A.





- **a.** What is the scale factor from Polygon A to Polygon B? Explain your reasoning.
 - 2 because the top horizontal side has length 2.5 units in Polygon A and 5 units in Polygon B
- **b.** Find the missing length of each side marked with a "?" in Polygon B.

 All sides scale by the same factor of 2, so the side that is 2.5 units in
 - All sides scale by the same factor of 2, so the side that is 2.5 units in Polygon A is 5 units in the copy, and the I.5-unit-long one is 3 units in the copy.
- c. Determine the measure of each angle marked with a "?" in Polygon A. 53° and 82° because scaled copies have the same corresponding angles

Problem 4

Figure 2 is a scaled copy of Figure 1. Select **all** of the statements that must be true:

- **A.** Figure 2 is larger than Figure 1.
- **B.** Figure 2 has the same number of edges as Figure 1.
- C. Figure 2 has the same perimeter as Figure 1.
- **D.** Figure 2 has the same number of angles as Figure 1.
- **E.** Figure 2 has angles with the same measures as Figure 1.

Problem 5

from an earlier course

Complete each equation with a number that makes it true.

- **a.** $8 \cdot 5 = 40$
- **b.** $8 + \frac{32}{2} = 40$
- **c.** $21 \div 3 = 7$
- **d.** $21 \frac{14}{1} = 7$
- **e.** $21 \cdot \frac{1}{3} = 7$