Solving Equations with Rational Numbers

Goals

- Explain (orally and in writing) how to solve an equation of the form x + p = q or px = q, where p, q, and x are rational numbers.
- Generalize (orally) the usefulness of additive inverses and multiplicative inverses for solving equations of the form x + p = q or px = q.
- Generate an equation of the form x + p = q or px = q to represent a situation involving rational numbers.

Learning Target

I can solve equations that include rational numbers and have rational solutions.

Student Learning Goal

Let's solve equations that include negative values.

Lesson Narrative

In this lesson, students think about how to solve equations involving rational numbers. Students have previously solved equations of the form px=q and x+p=q with positive rational numbers. In this lesson, students build on that work by working with equations with negative solutions or with negative values of p or q. Students use the structure of equations involving negative coefficients and solutions in order to solve problems. They also reason about situations that can be represented using these types of equations in order to solve problems in context.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up, Activity 2)
- Engagement (Activity 1, Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 3)

Instructional Routines

- · Card Sort
- · Math Talk
- MLR7: Compare and Connect
- Take Turns

Required Materials

Materials to Copy

 Matching Inverses Cards (1 copy for every 2 students): Activity 3

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

Activity 3:

Copy each set of cards on a different color of paper so they can easily be sorted for the next class.

Lesson Timeline

5_{min}

Warm-up

10 min

Activity 1

20 min

Activity 2

10 min

Activity 3

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Warm-up

Math Talk: Opposites and Reciprocals



Activity Narrative

This Math Talk focuses on reminding students about opposites and reciprocals. It encourages students to think about how to get a sum of 0 or product of 1. The strategies elicited here will be helpful later in the lesson when students use the additive and multiplicative inverses to solve equations.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- · Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the Activity Synthesis to involve more students in the conversation before moving to the next problem. Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A.7 \cdot *b* = 1

 $b = \frac{1}{7}$. Sample reasoning: One seventh of 7 is I.

B. $c \cdot d = 1$

Answers vary

Sample reasoning: c and d can be any two numbers that are reciprocals of each other.

C. 11 + f = 0

f = -II. Sample reasoning: Adding the opposite value will give a sum of 0.

D.g + h = 0

Answers vary. Sample reasoning: g and h can be any two numbers that are opposites of each other.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate ______'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to ______'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

Instructional Routines

Math Talk

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Access for Students with Diverse Abilities (Warm-up, Launch)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

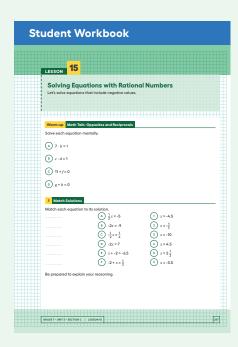
Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, _ because ..." or "I noticed _so I ..." Some students

may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing



Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.

Circulate to listen for and collect the language that students use as they solve the equations. On a visible display, record words and phrases, such as "Two negatives make a positive" or "The opposite of multiplication is division." Invite students to borrow language from the display as needed and update it throughout the lesson. For example, a student may rephrase "Two negatives make a positive" as "The product of two negative numbers is a positive number," or "The opposite of multiplication is division" as "The inverse operation of multiplication is division."

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Develop Effort and Persistence.

Differentiate the degree of difficulty or complexity. Begin with more accessible values, for example, x + 7 = 3 or 5x = -30.

Supports accessibility for: Conceptual Processing, Memory If not brought up in students' explanations, make these ideas explicit:

- The sum of a number and its opposite is 0.
- The product of a number and its reciprocal is 1.
- If you want to find a number that you can add to something and get 0 as a sum, use its opposite.
- If you want to find a number that you can multiply something by and get 1 as a product, use its reciprocal.

Activity 1

Match Solutions

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students connect their understanding of a solution to an equation as a value that makes the equation true with their understanding of operations involving negative numbers. Monitor for students who:

- Take an arithmetic approach by using substitution to check whether a given value makes the equation true.
- Take an algebraic approach by using inverse operations to solve for x.

In the digital version of the activity, students use an applet to match equations with their solution. The applet allows students to quickly check their answers. The digital version may be helpful for providing students with immediate feedback.

Launch

Give students 5 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Match each equation to its solution.

A.
$$\frac{1}{2}x = -5$$

B.
$$-2x = -9$$

2.
$$x = -\frac{1}{2}$$
 3. $x = -10$

C.-
$$\frac{1}{2}x = \frac{1}{4}$$

E.
$$x + -2 = -6.5$$

5.
$$x = 2\frac{1}{2}$$

F. -2 +
$$x = \frac{1}{2}$$

Be prepared to explain your reasoning.

Lesson 15 Warm-up **Activity 1 Activity 2** Activity 3 Lesson Synthesis Cool-down

Activity Synthesis

The goal of this discussion is for students to see different ways to match an equation with a given solution. Select an equation for which there is a student who took an arithmetic approach and a student who took an algebraic approach. An example of an arithmetic approach: "I know that -3.5 is the solution to -2x = 7 because I know that $-2 \cdot (-3.5) = 7$. An example of an algebraic approach: "If -2x = 7, then I know that $x = 7 \div -2$.)

Invite students to share their reasoning for why a solution is correct. Record their work, side by side, for all to see. Tell students that either approach is valid, but in the future there may be more complicated equations for which one approach might be more efficient than the other. If no students used "variable" as they described their approaches, invite them to identify the variables in some of the equations or share the definition of "variable" with the class.

Activity 2

Trip to the Mountains

20 min

Activity Narrative

In this activity, students reason quantitatively and abstractly as they interpret equations that represent situations. Students see that the structure of equations can be used to reason about a path to a solution even when negative values are included or when a variable represents a negative number. For example, students to see that equations of the form x + p = q can be solved by adding the opposite of p to the equation, regardless of whether p is positive or negative. Students also see that equations of the form px = q can be solved by multiplying the equation by the reciprocal of p.

Launch

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Give students 5–6 minutes of quiet work time, and follow with a wholeclass discussion.

Student Task Statement

The Hiking Club is on a trip to hike up a mountain.

- **1.** The members increased their elevation 290 feet during their hike this morning. Now they are at an elevation of 450 feet.
 - **a.** Explain how to find their elevation before the hike.

Sample response: Subtract the change of 290 from the current elevation of 450.

b. Han says the equation e + 290 = 450 describes the situation. What does the variable e represent?

Sample response: e represents the starting elevation.

Instructional Routines

MLR7: Compare and Connect

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Action and Expression: Internalize Executive Functions.

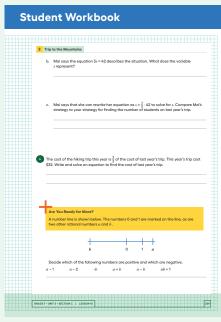
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time, and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may be misled by words to add or multiply (or subtract or divide) by the wrong numbers. For example, the word "increased" in the first situation may lead students to simply add the numbers they see, while the words "three times as many" in the last situation may lead students to multiply the numbers in the problem. Encourage students to make sense of the situations by acting them out or using visual diagrams. This will help them understand the actions and relationships in the stories.





c. Han says that he can rewrite his equation as e = 450 + -290 to solve for e. Compare Han's strategy to your strategy for finding the beginning elevation.

Sample response: Han used a variable to represent the unknown quantity, wrote an equation to describe the situation, and then solved by adding the opposite.

2. The temperature fell 4 degrees in the last hour. Now it is 21 degrees. Write and solve an equation to find the temperature it was 1 hour ago.

25 degrees, because t - 4 = 21, t - 4 + 4 = 21 + 4, t = 25

- **3.** There are 3 times as many students participating in the hiking trip this year than last year. There are 42 students on the trip this year.
 - **a.** Explain how to find the number of students that came on the hiking trip last year.

Sample response: Divide this year's number by 3, or multiply this year's number by $\frac{1}{3}$.

b. Mai says the equation 3s = 42 describes the situation. What does the variable s represent?

Sample response: s represents the number of students on the trip last year.

c. Mai says that she can rewrite her equation as $s = \frac{1}{3} \cdot 42$ to solve for s. Compare Mai's strategy to your strategy for finding the number of students on last year's trip.

Sample response: Mai used a variable to represent the unknown quantity, wrote an equation to describe the situation, and then solved by multiplying by the reciprocal.

4. The cost of the hiking trip this year is $\frac{2}{3}$ of the cost of last year's trip. This year's trip cost \$32. Write and solve an equation to find the cost of last year's trip.

48, because $\frac{2}{3}c = 32$, $c = 32 \cdot \frac{3}{2}$, c = 48

Are You Ready for More?

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other rational numbers a and b.



Decide which of the following numbers are positive and which are negative.

a - 1

a - I is positive because a is bigger than I.

a - 2

a-2 is negative because the distance between I and a is less than the distance between 0 and I. So a is less than 2.

-b

-b is positive because b is a negative number.

Lesson 15 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

a + b

a+b is negative because the distance from b to zero is greater than the distance from 0 to a.

a - b

a - b is positive because b is a negative value.

ab + 1

ab+1 is negative because both a and b have magnitudes greater than I, so ab will have magnitude greater than I. Also, ab will be negative as a is positive and b is negative. So ab < -1.

Activity Synthesis

The purpose of this discussion is for students to share strategies for determining which operation to use when solving problems. Tell students,

"We learned four things about the hiking trip in this activity: The students were climbing, the temperature was falling, there were more students this year than last, and the cost of the trip was less this year than last."

Then discuss the following questions:

"How did you know which operation described the rise in elevation, fall in temperature, rise in number of students, and fall in the cost?"

This conversation can highlight the problem with relying on "key words." For example, when students see "times as many," they might simply multiply the numbers they see in the problem. Encourage students to make sense of the situations by acting them out or drawing diagrams.

"How did you decide which strategy to use to solve for the unknown quantity?"

"What are some ways to know that a situation involves negative values?"

Activity 3: Optional

Card Sort: Matching Inverses

10 min

Activity Narrative

In this partner activity, students take turns matching numbers and their additive or multiplicative inverse. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Launch 🙎

Tell students that the cards contain numbers and that they will take turns matching the cards that are additive inverses. If necessary, remind students of the meaning of additive inverse and multiplicative inverse. If x + y = 0, then x and y are additive inverses. If $x \cdot 4 = 1$, then x and y are multiplicative inverses.

Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect.

Lead a discussion comparing, contrasting, and connecting the different equations students wrote to describe the temperature and the cost of the hiking trip in the 2nd and 4th problems. Ask, "How are equations for each situation the same?" "How are they different?" "How does the decrease in temperature (or the cost of the hiking trip) show up in each representation?" and "Are there any benefits or drawbacks to one representation compared to another?"

Advances: Representing, Conversing

Instructional Routines

Card Sort

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Instructional Routines

Take Turns

ilclass.com/r/10573524

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Access for Multilingual Learners (Activity 3, Launch)

MLR8: Discussion Supports.

Display sentence frames to support partner discussion. Examples: "I noticed _, so I matched...." Encourage students to challenge each other when they disagree.

Advances: Listening, Conversing

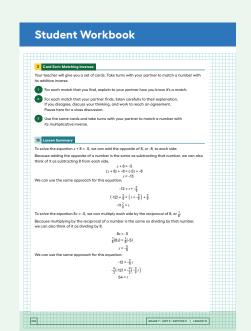
Lesson 15 Activity 1 Activity 2 **Activity 3** Lesson Synthesis Warm-up Cool-down

Access for Students with Diverse Abilities (Activity 3, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory



Explain how to set up and do the activity. If time allows, demonstrate the steps with a student as a partner. Consider demonstrating productive ways to agree or disagree, for example, by explaining mathematical thinking or asking clarifying questions.

Arrange students in groups of 2. Give each group a set of cards cut from the blackline master, and instruct students to match numbers that are additive inverses. Pause for a brief whole-class discussion before directing students to use the same set of cards and matching numbers that are multiplicative inverses.

Student Task Statement

Your teacher will give you a set of cards. Take turns with your partner to match a number with its additive inverse.

- 1. For each match that you find, explain to your partner how you know it's a
- 2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

Pause here for a class discussion.

3. Use the same cards and take turns with your partner to match a number with its multiplicative inverse.

Additive Inverses:

$$\frac{2}{1} \text{ and } -2; \frac{5}{10} \text{ and } -0.5; 3 \text{ and } -3;$$

$$\frac{1}{3} \text{ and } -\frac{1}{3}; 4 \text{ and } -4; 0.25 \text{ and } -\frac{1}{4};$$

$$\frac{10}{2} \text{ and } -5; 0.2 \text{ and } -0.2; 10 \text{ and } -\frac{10}{1};$$

$$0.1 \text{ and } -\frac{1}{10}; 25 \text{ and } -25;$$

$$\frac{4}{100} \text{ and } -\frac{4}{100}$$

$$2 \text{ and } \frac{5}{10}; -2 \text{ and } -0.5; 3 \text{ and } \frac{1}{3};$$

$$and -\frac{1}{3}; 4 \text{ and } 0.25; -4 \text{ and } -\frac{1}{4};$$

$$\frac{10}{2} \text{ and } 0.2; -5 \text{ and } -0.2; 10 \text{ and } -\frac{1}{10}; 25 \text{ and } \frac{4}{100};$$

$$-\frac{10}{100} \text{ and } -\frac{4}{100}$$

$$-25 \text{ and } -\frac{4}{100}$$

Multiplicative Inverses:

$$\frac{2}{1} \text{ and } -2; \frac{5}{10} \text{ and } -0.5; 3 \text{ and } -3; \\ \frac{1}{3} \text{ and } -\frac{1}{3}; 4 \text{ and } -4; 0.25 \text{ and } -\frac{1}{4}; \\ \frac{10}{2} \text{ and } -5; 0.2 \text{ and } -0.2; 10 \text{ and } -\frac{10}{1}; \\ 0.1 \text{ and } -\frac{1}{10}; 25 \text{ and } -25; \\ \frac{4}{100} \text{ and } -\frac{4}{100}; \\ \frac{2}{100} \text{ and } -\frac{4$$

Activity Synthesis

The purpose of this discussion is both to make connections and to contrast between a number's additive and multiplicative inverse. A key idea to emphasize is that multiplicative inverses require that the numbers have the same sign in order for the product to be positive, while additive inverses must have opposite signs in order for their sum to be 0.

Select 2–3 groups to share one of their sets of cards and how they matched a number with its additive inverse. Then repeat for 2–3 groups to share how they matched a number with its multiplicative inverse. Discuss as many different sets of cards as the time allows.

Here are some additional questions for discussion:

"Which matches were tricky? Explain why."

"Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?"

"How did matching for additive inverses differ from matching for multiplicative inverses?"

Lesson Synthesis

Share with students,

"Today we represented situations with equations and used inverses as a strategy to solve them."

To help students articulate the process of solving by using additive and multiplicative inverses (opposites and reciprocals), consider asking:

 \bigcirc "How can we solve an equation like x + (-9.2) = 7.5?"

We can add the opposite of -9.2 to 7.5.

 \bigcirc "How can we solve an equation like $x \cdot (-9.2) = 7.5$?"

We can multiply 7.5 by the reciprocal of -9.2.

 \bigcirc "Suppose we know that 60 is $\frac{4}{5}$ of a number. What is the difference between writing the equation $\frac{4}{5}x = 60$ and writing the equation $x = 60 \cdot \frac{5}{4}$?"

The first equation describes the situation, while the second shows a way to rewrite the equation to solve for the unknown.

Lesson Summary

To solve the equation x + 8 = -5, we can add the opposite of 8, or -8, to each side:

Because adding the opposite of a number is the same as subtracting that number, we can also think of it as subtracting 8 from each side.

$$x + 8 = -5$$

 $(x + 8) + -8 = (-5) + -8$
 $x = -13$

We can use the same approach for this equation:

-12 =
$$t + -\frac{2}{9}$$

(-12) $+\frac{2}{9}$ = $\left(t + -\frac{2}{9}\right) + \frac{2}{9}$
-11 $\frac{7}{9}$ = t

To solve the equation 8x = -5, we can multiply each side by the reciprocal of 8, or $\frac{1}{8}$:

Because multiplying by the reciprocal of a number is the same as dividing by that number, we can also think of it as dividing by 8.

$$8x = -5$$

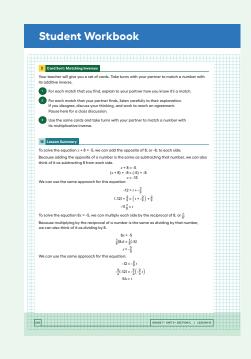
$$\frac{1}{8}(8x) = \frac{1}{8}(-5)$$

$$x = -\frac{5}{9}$$

We can use the same approach for this equation:

-12 =
$$-\frac{2}{9}t$$

- $\frac{9}{2}$ (-12) = $-\frac{9}{2}(-\frac{2}{9}t)$
54 = t



Lesson 15 Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis **Cool-down**

Responding To Student Thinking

Points to Emphasize

If most students struggle with solving equations that involve negatives, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about finding the solutions to the equations in this activity:

Grade 7, Unit 5, Lesson 16, Activity 1 Warmer or Colder Than Before?

Cool-down

Hiking Trip



Student Task Statement

The Hiking Club is taking another trip. The hike leader has a watch that shows that they have gained 296 feet in altitude from their starting position. Their altitude is now 285 feet. The equation x + 296 = 285 can be used to represent the situation.

1. Solve for *x*.

x = -11

2. What does x mean in the situation?

Sample response: Since x represents starting elevation, the Hiking Club started at an altitude of -II feet, or II feet below 0.

Practice Problems

6 Problems

Problem 1

Solve.

a.
$$\frac{2}{5}t = 6$$

$$t = 15$$

c.
$$\frac{1}{2} + p = -3$$

$$p = -3\frac{1}{2}$$

e.
$$-12 = -3y$$

b.
$$-4.5 = a - 8$$

$$a = 3.5$$

d.
$$12 = x \cdot 3$$

$$x = 4$$

Problem 2

Match each equation to a step that will help solve the equation.

A matches 3

A.
$$5x = 0.4$$

B matches I

B.
$$\frac{x}{5} = 8$$

C matches 2

C.
$$3 = \frac{-x}{5}$$

C.
$$3 = \frac{-x}{5}$$
 3. Multiply each side by $\frac{1}{5}$.

D matches 4

D.
$$7 = -5x$$

4. Multiply each side by
$$\frac{-1}{5}$$
.

Problem 3

from Unit 5, Lesson 13

Evaluate each expression if x is $\frac{2}{5}$, y is -4, and z is -0.2.

a.
$$x + y$$

$$-3\frac{3}{5}$$
 (or equivalent)

b.
$$2x - z$$

c.
$$x + y + z$$

d.
$$y \cdot x$$

$$\frac{-8}{5}$$
 (or equivalent)

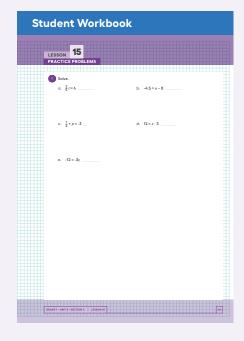
Problem 4

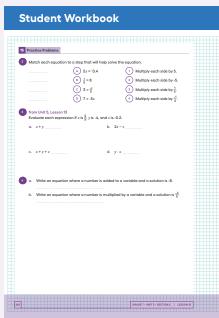
a. Write an equation where a number is added to a variable and a solution

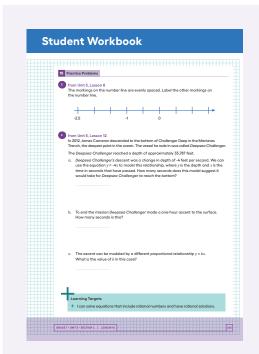
Sample response: x + 2 = -6

b. Write an equation where a number is multiplied by a variable and a solution is $\frac{-4}{5}$.

Sample response: -5x = 4







Problem 5

from Unit 5, Lesson 8

The markings on the number line are evenly spaced. Label the other markings on the number line.



Problem 6

from Unit 5, Lesson 12

In 2012, James Cameron descended to the bottom of Challenger Deep in the Mariana Trench, the deepest point in the ocean. The vessel he rode in was called *Deepsea Challenger*.

The Deepsea Challenger reached a depth of approximately 35,787 feet.

a. Deepsea Challenger's descent was a change in depth of -4 feet per second. We can use the equation y = -4x to model this relationship, where y is the depth and x is the time in seconds that have passed. How many seconds does this model suggest it would take for Deepsea Challenger to reach the bottom?

$$8,946.75$$
 seconds, because $-35,787 \div -4 = 8,946.75$

b. To end the mission *Deepsea Challenger* made a one-hour ascent to the surface. How many seconds is this?

3,600 seconds, because
$$60 \cdot 60 = 3,600$$

c. The ascent can be modeled by a different proportional relationship y = kx. What is the value of k in this case?

$$k = \frac{35,787}{3,600}$$

It took 3,600 seconds to go 35,787 feet up. This means the proportional relationship is $y = \frac{35,787}{3,600} x$.