Lesson 2

Multiplying Powers of 10

Goal

Generalize a process for multiplying exponential expressions with the same base, and justify (orally and in writing) that $10^n \cdot 10^m = 10^{n+m}$.

Learning Target

I can explain and use a rule for multiplying powers of 10.

Lesson Narrative

In this lesson, students make use of repeated reasoning to discover the exponent rule $10^n \cdot 10^m = 10^{n+m}$.

Students begin by relating base-ten diagrams to exponents, specifically powers of 10. This allows them to consider the meaning of quantities and not just how to compute them.

Next, they expand expressions written as the product of two powers of 10 and notice patterns when asked to write the expression using a single power of 10. Students will extend this exponent rule to cases where the exponents are zero or negative in following lessons, but the focus here is on cases with positive exponents.

Student Learning Goal

Let's explore patterns with exponents when we multiply powers of 10.

Instructional Routines

- MLR2: Collect and Display

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)

Access for Students with Diverse Abilities

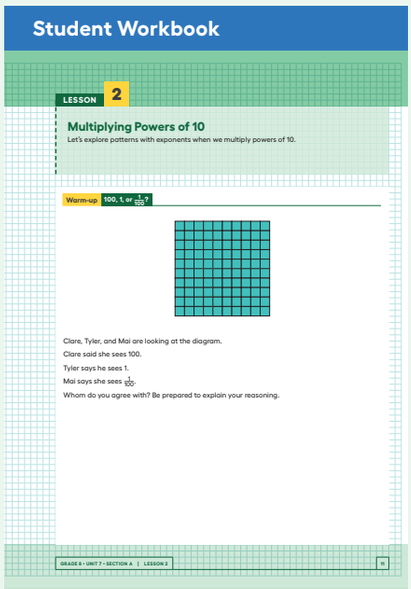
- Representation (Activity 1)

Required Preparation

Activity 2:
Create a visual display of the exponent rule $10^n \cdot 10^m = 10^{n+m}$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	15 min	15 min	10 min	5 min
				Cool-down


Student Workbook

LESSON 2

Multiplying Powers of 10

Let's explore patterns with exponents when we multiply powers of 10.

Warm-up 100, 1, or $\frac{1}{100}$?

Clare, Tyler, and Mai are looking at the diagram.
Clare said she sees 100.
Tyler says he sees 1.
Mai says she sees $\frac{1}{100}$.
Whom do you agree with? Be prepared to explain your reasoning.

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Warm-up

100, 1, or $\frac{1}{100}$?

5 min

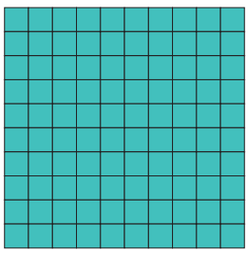
Activity Narrative

This *Warm-up* gives students a chance to think about the different numbers that a diagram might represent. Students will see related diagrams that represent different powers of 10 in a following activity.

Launch

Give students 1 minute of quiet think time followed by a whole class discussion.

Student Task Statement



Clare, Tyler, and Mai are looking at the diagram.

Clare said she sees 100.

Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Whom do you agree with? Be prepared to explain your reasoning.

Sample response: I agree with all of them. There are 100 small squares and 1 big square. Each small square is $\frac{1}{100}$ of the large square.

Activity Synthesis

The goal of this discussion is for students to share their reasoning. Survey the class to see who agrees with each person. Invite students to share their reasoning. Consider discussing the following questions:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone agree with _____ but would explain it differently?"*

Activity 1
Picture a Power of 10

15 min

Activity Narrative

The purpose of this activity is for students to develop a sense of visual scale between powers of 10. Students should understand that multiplying by 10 corresponds to increasing the exponent by 1.

This activity prompts students to make sense of quantities and their relationships. For example, even though the notation for 10^{100} does not appear to be much different than 10^{98}, it is 100 times larger. Small changes in the exponent can result in large changes in the value of the expression.

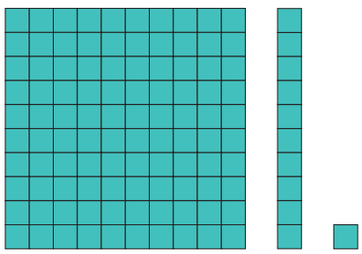
Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time followed by 3 minutes to share their responses with their partner. Conclude with a whole-class discussion.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use during the partner discussions about representations of different powers of 10. Display words and phrases such as "power of 10" and "multiply by 10" and expressions such as "10^2" and "$10 \cdot 10$." Record sketches or diagrams of the rectangle and squares from the *Task Statement*.

Student Task Statement

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could the large square be represented as a power of 10? Explain your reasoning.

   The large square can be represented by 10^2 because there are 100 small squares.

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

   The medium rectangle represents 10^3 because $10^2 \cdot 10 = 10^3$. The large square represents 10^4 because $10^2 \cdot 10^2 = 10^4$.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Provide students with additional blank copies of the base 10 diagram to record the different values given for each small square to develop a sense of visual scale between powers of 10.

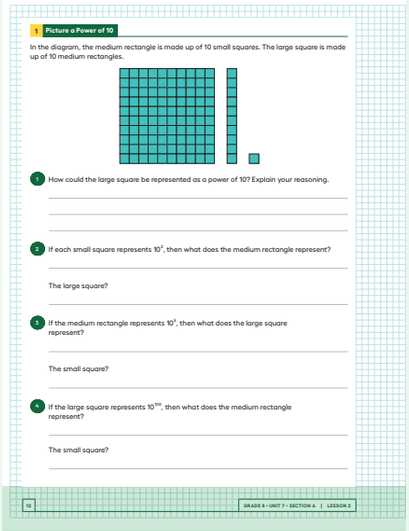
Supports accessibility for: Visual-Spatial Processing, Organization

Building on Student Thinking

If students multiply the exponent by 10 (for example thinking that $10^2 \cdot 10 = 10^{20}$), consider asking:

"How did you determine your exponents?"
"How do you write out $10^2 \cdot 10$ as a multiplication problem?"

Student Workbook



1 Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.

1. How could the large square be represented as a power of 10? Explain your reasoning.
2. If each small square represents 10^7, then what does the medium rectangle represent?

   The large square?
3. If the medium rectangle represents 10^5, then what does the large square represent?

   The small square?
4. If the large square represents 10^{100}, then what does the medium rectangle represent?

   The small square?

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3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?

The small square represents 10^4 because $10^5 \div 10 = 10^4$. The large square represents 10^6 because $10^5 \cdot 10 = 10^6$.

4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

The medium rectangle represents 10^{99} because $10^{100} \div 10 = 10^{99}$ and the small square represents 10^{98} because $10^{100} \div 10^2 = 10^{98}$.

Activity Synthesis

The purpose of this discussion is to make sure students understand that increasing the exponent of a power of 10 by 1 corresponds to multiplying by 10, and decreasing the exponent by 1 corresponds to dividing by 10.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their responses to each question. Record and display the responses for all to see, referring to the image from the *Task Statement* when necessary. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases.

If time allows, consider sharing this applet that illustrates animals measured in units that vary by powers of ten: **ilclass.com/l/688140**

Activity 2

Multiplying Powers of Ten

15 min

Activity Narrative

The goal of this activity is to help students flexibly transition between different notations for powers of 10 and to introduce the property of multiplication of values with the same base. Students observe patterns in the notations and generalize that $10^n \cdot 10^m = 10^{n+m}$ for the values of n and m that are positive integers.

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before inviting 1–2 students to share and explain their answers. When it is clear that students understand how to complete the table, explain that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 5–6 minutes to complete the remaining questions before a whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^7
$10^4 \cdot 10^4$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^3 \cdot 10^5$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^{18} \cdot 10^{23}$	skip	10^{41}

b. If you chose to skip one entry in the table, which entry did you skip? Why?

I chose to skip the expanded column of $10^{18} \cdot 10^{23}$ because there are too many factors that are 10 and they won't fit in the table.

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

$10^n \cdot 10^m = 10^{n+m}$ because multiplying n factors that are 10 with m factors that are 10 results in $n + m$ factors that are 10

b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

10^4 because $10^4 \cdot 10^0 = 10^{4+0}$. That means 10^0 must equal 1 for the rule to work.

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Sample response: There are 10^{20} bacteria because 10^7 people times 10^{13} bacteria per person is equal to 10^{20} total bacteria.

Are You Ready for More?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

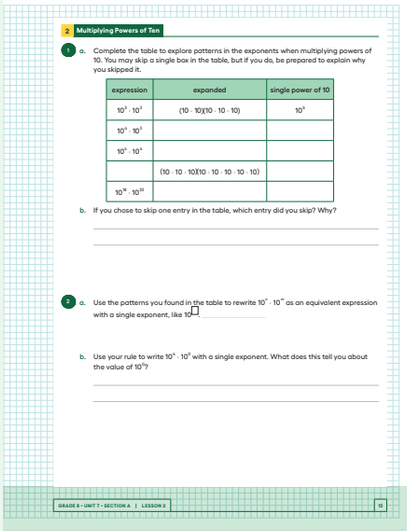
$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together? 10 ways

2. How about 10^7? 10^8? 14 ways, 22 ways

Student Workbook



2 Multiplying Powers of Ten

1 a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

2 a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

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Activity Synthesis

The goal of this discussion is to reinforce the exponent rule for multiplying exponential expressions with the same base. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ two factors that are ten · three factors that are ten = five factors that are ten

Continue to reinforce student understanding of this rule by writing out the expanded form of each expression and counting the total number of factors of 10 when discussing the following questions:

- *"What is $10^3 \cdot 10^1$ written as a single power of 10?"*

  $10^3 \cdot 10^1 = (10 \cdot 10 \cdot 10) \cdot (10) = 10^4$

- *"What is $10^5 \cdot 10^2$ written as a single power of 10?"*

  $10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7$

- *"What is $10^{20} \cdot 10^{17}$ written as a single power of 10?"*

  $10^{20} \cdot 10^{17} = 10^{20+17} = 10^{37}$

- *"What are some different ways to write 10^8 as the product of 2 powers of 10?"*

  $10^4 \cdot 10^4$ and $10^1 \cdot 10^7$

Lesson Synthesis

The purpose of this discussion is to address a common misconception when multiplying exponents with the same base. Display the following statement for all to see:

"$10^3 \cdot 10^4 = 10^{12}$ because 3 factors of 10 multiplied by 4 factors of 10 makes a total of 12 factors of 10."

Ask students if they agree or disagree with the statement and invite several students to share their reasoning. When the class is in agreement that the statement is incorrect, ask students to individually write an improved statement. Improved responses should include connections between the representations of a single expression, expanded form, and the single power of 10. For example, students may write:

$10^3 \cdot 10^4 = 10^7$ because $10^3 = 10 \cdot 10 \cdot 10$ and $10^4 = 10 \cdot 10 \cdot 10 \cdot 10$. There will be a total of 7 factors of 10 being multiplied together, which is written as 10^7.

Lesson Summary

In this lesson, we developed a rule for multiplying powers of 10: Multiplying powers of 10 corresponds to adding the exponents together.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ (two factors that are ten · three factors that are ten = five factors that are ten)

To see this, multiply 10^2 and 10^3. We know that 10^2 has two factors that are 10 and that 10^3 has three factors that are 10. That means that $10^2 \cdot 10^3$ has 5 factors that are 10.

This will work for other powers of 10, too.
For example, $10^{14} \cdot 10^{47} = 10^{(14+47)} = 10^{61}$.

Cool-down

That's a Lot of Office Space!

5 min

Student Task Statement

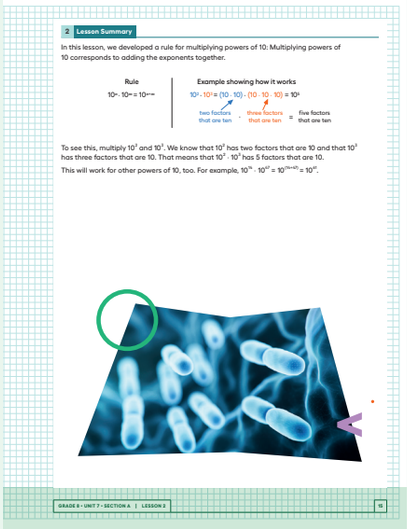
1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.

   10^{38}, because $10^{32} \cdot 10^6 = 10^{32+6} = 10^{38}$

2. A company leases out office space for 10^2 dollars per square foot. If this company owns approximately 10^6 square feet of office space in multiple locations worldwide, how much money could they make renting out all of their office space? Express your answer both as a power of 10 and as a dollar amount.

   10^8 or $100,000,000

Student Workbook


2 Lesson Summary

In this lesson, we developed a rule for multiplying powers of 10: Multiplying powers of 10 corresponds to adding the exponents together.

Rule: $10^n \cdot 10^m = 10^{n+m}$

Example showing how it works: $10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ (two factors that are ten · three factors that are ten = five factors that are ten)

To see this, multiply 10^2 and 10^3. We know that 10^2 has two factors that are 10 and that 10^3 has three factors that are 10. That means that $10^2 \cdot 10^3$ has 5 factors that are 10.
This will work for other powers of 10, too. For example, $10^{14} \cdot 10^{47} = 10^{(14+47)} = 10^{61}$.

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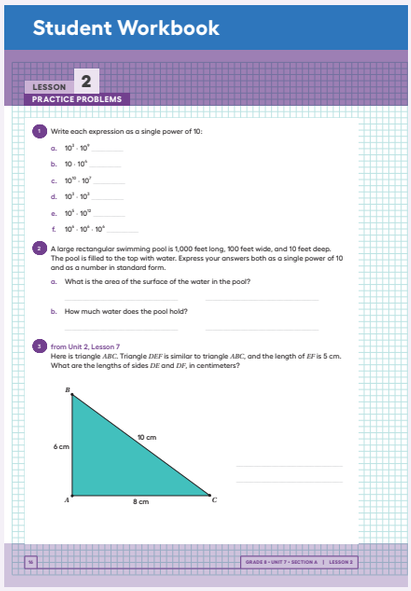
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Practice Problems

4 Problems


Student Workbook

LESSON 2
PRACTICE PROBLEMS

1. Write each expression as a single power of 10:
   a. $10^3 \cdot 10^9$ _____
   b. $10 \cdot 10^4$ _____
   c. $10^{10} \cdot 10^7$ _____
   d. $10^3 \cdot 10^3$ _____
   e. $10^5 \cdot 10^{12}$ _____
   f. $10^6 \cdot 10^6 \cdot 10^6$ _____
2. A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water. Express your answers both as a single power of 10 and as a number in standard form.
   a. What is the area of the surface of the water in the pool?
   b. How much water does the pool hold?
3. from Unit 2, Lesson 7
   Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?

Problem 1

Write each expression as a single power of 10:

a. $10^3 \cdot 10^9$ 10^{12}

b. $10 \cdot 10^4$ 10^5

c. $10^{10} \cdot 10^7$ 10^{17}

d. $10^3 \cdot 10^3$ 10^6

e. $10^5 \cdot 10^{12}$ 10^{17}

f. $10^6 \cdot 10^6 \cdot 10^6$ 10^{18}

Problem 2

A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water. Express your answers both as a single power of 10 and as a number in standard form.

a. What is the area of the surface of the water in the pool?

10^5 square feet, 100,000 square feet

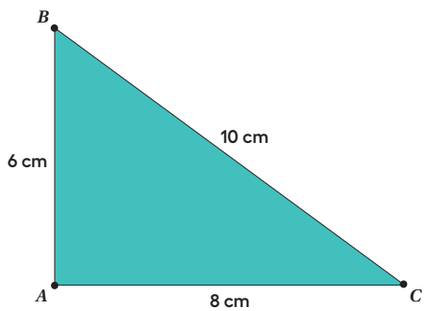
b. How much water does the pool hold?

10^6 cubic feet, 1,000,000 cubic feet

Problem 3

from Unit 2, Lesson 7

Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



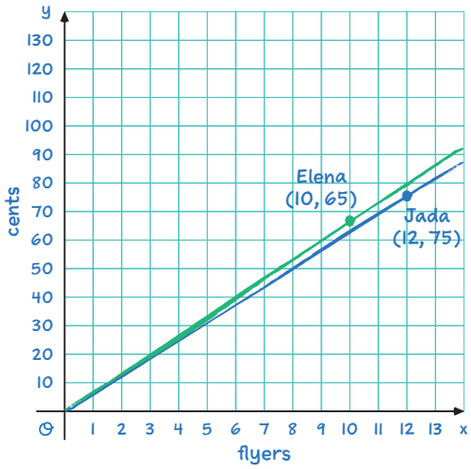
$DE = 3$ and $DF = 4$, because the scale factor is $\frac{1}{2}$, so each side length is half the corresponding side length of ABC

Problem 4

from Unit 3, Lesson 3

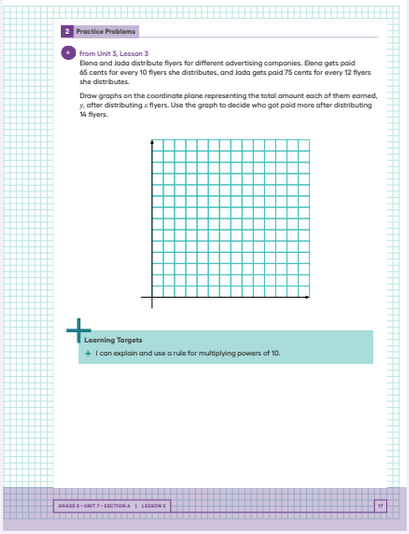
Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



Sample response: After distributing 14 flyers, the point on Elena's graph is higher than the point on Jada's graph.

Student Workbook


2 Practice Problems

4 from Unit 3, Lesson 3

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Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.

Learning Targets

+ I can explain and use a rule for multiplying powers of 10.