Tax and Tip

Goals

- Comprehend "sales tax" and "tip" as two contexts that involve adding a percentage of the initial amount.
- Explain (orally) how to calculate the total cost including a tax or tip, given the "subtotal" and the percentage.
- Explain (orally) how to determine the percentage of the subtotal that a tax or tip is.

Learning Target

I understand and can solve problems about sales tax and tip.

Lesson Narrative

In this lesson, students are introduced to two common applications of percent increase: sales tax and tips. Students can use any representation they would like to make sense of the situations. The tables presented in this lesson provide an opportunity for students to be more efficient by using an equation of the form y = kx. By repeatedly calculating the tax for different prices and then generalizing the process, they are engaging in expressing regularity in repeated reasoning.

There is an optional activity in which the tax rate is fractional, and the given dollar amounts are rounded to the nearest cent. This provides an opportunity for students to practice finding the constant of proportionality when rounding error makes it look like the relationship is not exactly proportional.

Student Learning Goal

Let's learn about sales tax and tips.

Access for Students with Diverse Abilities

- Engagement (Activity 2)
- Action and Expression (Activity 3)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 3)
- MLR5: Co-Craft Questions (Activity 1)
- MLR8: Discussion Supports (Activity 2)

Instructional Routines

- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

 Four-function calculators: Activity 1, Activity 2, Activity 3

Required Preparation

Lesson:

It is recommended that students be provided access to four-function calculators so that they can focus on reasoning about how numbers are related to each other, representing those relationships, and deciding which operations are appropriate (rather than focusing on computation).

Lesson Timeline

5_{min}

Warm-up

20 min

Activity 1

10 min

Activity 2

15 min

Activity 3

10 min

Lesson Synthesis

Assessment

5 min

Cool-down

Warm-up

Notice and Wonder: The Price of Sunglasses



Activity Narrative

The purpose of this *Warm-up* is to introduce students to the meaning of sales tax, which will be useful when students calculate prices including tax in a later activity. While students may notice and wonder many things about this situation, the important discussion point is why the total price is more than the price tag.

This *Warm-up* prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved.

Launch 🙎

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

You are on vacation and want to buy a pair of sunglasses for \$10 or less. You find a pair with a price tag of \$10. The cashier says the total cost will be \$10.45.



What do you notice? What do you wonder?

Students may notice:

- The price of the sunglasses is \$10.
- The total cost is more than the price of the sunglasses.
- · Sales tax was added to the cost of the sunglasses.

Students may wonder:

- If the cashier made a mistake.
- Why the total cost is more than the price listed on the sunglasses.
- · What the tax is on the sunglasses.
- If something else was purchased to make the price more.

Instructional Routines

Notice and Wonder ilclass.com/r/10694948

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Instructional Routines

MLR5: Co-Craft Questions

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Access for Multilingual Learners (Activity 1)

MLR5: Co-Craft Questions

This activity uses the Co-Craft Questions math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

"Is there anything on this list that you are wondering about now?"

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If questioning why the total price is higher than the price tag does not come up during the conversation, ask students to discuss this idea. Ask students if they have ever heard of sales tax before, and if some have, ask them to share their understanding.

Explain that sales tax is a fee (an amount of money) paid to the government. The amount of tax is a percentage of the price of the item. Different states charge different sales tax percentages, and additionally some local governments, like for counties and cities, also charge a sales tax.

To start to help make sense of how sales tax works, ask questions like:

- "How much sales tax is being collected on the \$10 sunglasses?"
 - \$0.45 or 45 cents
- "45 cents is what percent of \$10?"
 4.5%
- "What is the sales tax rate for our local area?"
 Answer varies based on location.

Activity 1

Shopping in Two Different Cities



Activity Narrative

In this activity, students calculate total price including sales tax. They repeat the calculation for several items and with two different tax rates, giving them an opportunity to make use of repeated reasoning. Students also write expressions to represent the amount of the sales tax and the total price including tax for an item with any price, x. This builds on their previous work writing equations to represent situations involving percent increase.

Arrange students in groups of 2. Introduce the context of purchasing items and being charged sales tax. Use *Co-Craft Questions* to orient students to the context and elicit possible mathematical questions.

- Display only the problem stem and the first row of each table, without revealing the questions. Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.
- Invite several partners to share one question with the class and record responses. Ask the class to make comparisons among the shared questions and their own. Ask,
- "What do these questions have in common? How are they different?"
 Listen for and amplify language related to the learning goal, such as "sales tax," "tax rate," "percentage," and "price including tax."
 - Reveal the instruction "Complete the tables" and the remaining rows of each table. Give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences between the questions.

Give students 4–5 minutes of quiet work time followed by time for partner discussion. Then hold a whole-class discussion.

Student Task Statement

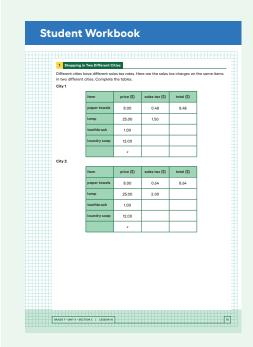
Different cities have different sales tax rates. Here are the sales tax charges on the same items in two different cities. Complete the tables.

City 1

item	price (\$)	sales tax (\$)	total (\$)
paper towels	8.00	0.48	8.48
lamp	25.00	1.50	26.50
toothbrush	1.00	0.06	1.06
laundry soap	12.00	0.72	12.72
	X	0.06 <i>x</i>	1.06x

City 2

item	price (\$)	sales tax (\$)	total (\$)
paper towels	8.00	0.64	8.64
lamp	25.00	2.00	27.00
toothbrush	1.00	0.08	1.08
laundry soap	12.00	0.96	12.96
	X	0.08 <i>x</i>	1.08 <i>x</i>



Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Differentiate the degree of difficulty or complexity. Begin with having students calculate the tax on an item which involves them rounding to the nearest cent, possibly by using the local tax rate.

Supports accessibility for: Conceptual Processing, Memory

Activity Synthesis

Invite students to share their strategies for calculating the sales tax and total price for the laundry soap in each city. Ask students to describe any patterns they see in the table, for example:

- The values in the total column can be found by adding the previous two columns.
- The sales tax is proportional to the price with a constant of proportionality of 0.06 or 0.08.
- The total is proportional to the price with a constant of proportionality of 1.06 or 1.08.

Next, invite students to share their expressions for the last row of each table. Make sure students see the connection between this row and their previous work on percent increase. Point out that sometimes we want to know just the amount of the tax, 0.06x, and sometimes we want to know the total, which is the price plus the sales tax, x + 0.06x = 1.06x.

If time permits, introduce the term "tax rate." Explain that when there is a certain tax that gets applied to a class of goods, it is called a tax rate. Tax rates are usually described in terms of percentages. Ask students:

"What is the tax rate in City 1?"

6%

"What is the tax rate in City 2?"

8%

Activity 2: Optional

Shopping in a Third City

10 min

Activity Narrative

In this activity, students encounter a situation in which rounding error makes it look like the relationship between the price of an item and the sales tax is not quite proportional. Students see that this is due to having a fractional percentage for the tax rate and the custom of rounding dollar amounts to the nearest cent. As students explain which tax rate matches the given amount of sales tax for each item, they attend to precision.

Launch

Keep students in the same groups.

Give students 3–4 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

Student Task Statement

Here is the sales tax on the same items in City 3.

item	price (\$)	sales tax (\$)
paper towels	8.00	0.58
lamp	25.00	1.83
toothbrush	1.00	0.07
laundry soap	12.00	

1. What is the tax rate in this city?

The tax rate is about 7.3%. (This rate can give all the dollar amounts in the table when rounded to the nearest cent. 7.31% would also work.)

- For the paper towels, $0.073 \cdot 8.00 = 0.584$, which rounds to \$0.58.
- For the lamp, $0.073 \cdot 25.00 = 1.825$, which rounds to \$1.83.
- For the toothbrush, $0.073 \cdot 1.00 = 0.073$, which rounds to \$0.07.
- **2.** For the sales tax on the laundry soap, Kiran says it should be \$0.84. Lin says it should be \$0.87. Do you agree with either of them? Explain your reasoning.

No, I don't agree with either of them.

Sample reasoning: Since the sales tax on the lamp was \$1.83, both Kiran and Lin's answers are too small.

- To get Kiran's answer of \$0.84, the tax rate would be just 7%, but then the tax on the lamp would have been only \$1.75.
- To get Lin's answer of \$0.87, the tax rate could be 7.25%, but then the tax on the lamp would still have been only \$1.81.
- The sales tax on the laundry soap should be \$0.88, because 0.073 · 12.00 = 0.876, which rounds to \$0.88.

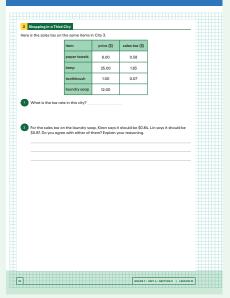
Building on Student Thinking

Some students may say that the relationship is not proportional. Remind them of the activity in a previous unit where they measured the length of the diagonal and the perimeter of several squares and determined that there was really a proportional relationship, even though measurement error made it look like there was not an exact constant of proportionality.

Some students may say that the tax rate is exactly 7%. Prompt them to calculate what the sales tax would have been for the paper towels and the lamp if the tax rate were exactly 7%.

Some students may use 7.25% as the tax rate since that is what comes from the first item (paper towels), but in this case they did not check this number against the tax on the other items provided. Prompt students to use the additional information they have to check their answer before proceeding to solve the row with laundry soap.

Student Workbook



Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.

Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking

Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 3)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 3, Student Task)

Action and Expression: Internalize Executive Functions.

To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time, and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may attempt to write an equation, but place numbers in the wrong place. Ask them what each piece of their equation means in this situation. In particular, monitor for students who struggle with understanding the second part the first question. Help these students understand by rephrasing the question as, "The total is what percent of the subtotal?" and helping them to see that the answer should be greater than 100% since the total is greater than the subtotal.

Students might need a way to keep track of all the information. Suggest using a table that keeps track of original price and percentage.

Activity Synthesis

The purpose of this discussion is to highlight how a fractional tax rate, along with the custom of rounding dollar amounts to the nearest cent, can explain the apparent inconsistencies in the table.

Consider asking:

(in the previous activity?" How did you determine the tax rates for the items in City 1 and City 2 from the previous activity?"

I divided the sales tax by the price.

- "How was determining the tax rate for City 3 different from Cities 1 and 2?"
 Since the tax rates were not the same for each item, I had to determine what tax rate might give each value listed.
- "Is there a proportional relationship between the price of the item and the amount of sales tax for City 3? How do you know?"

Yes, there is a proportional relationship. The constant of proportionality for each row is approximately 0.073.

Consider reminding students about when they measured the length of the diagonal and the perimeter of a square. They saw that measurement error made it look like the relationship was not quite a proportional relationship.

Activity 3

Dining at a Restaurant

15 min

Activity Narrative

In this activity, students practice finding the percentage, the original amount, or the new amount in the context of sales tax and tips. They apply the strategies they learned previously with percent increase, such as representing the percentage as a decimal and writing and solving an equation. As students apply what they learned about percent increase to represent situations involving tax and tip, they are making use of structure.

Launch

Keep students in the same groups. Explain that in some restaurants, people pay the server a *tip* in addition to paying for the meal. Tips usually range between 10% and 20% of the cost of the meal.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Give students 4–6 minutes of quiet work time, followed by time for partner discussion.

Then hold a whole-class discussion.

Student Task Statement

1. Jada has a meal in a restaurant. She adds up the prices listed on the menu for everything they ordered and gets a subtotal of \$42.00.



a. When the check comes, it says they also need to pay \$3.99 in sales tax. What percentage of the subtotal is the sales tax?

9.5% (3.99 ÷ 42 = 0.095)

b. After tax, the total is \$45.99. What percentage of the subtotal is the total?

 $109.5\% (45.99 \div 42 = 1.095)$

c. They actually pay \$52.99. The additional \$7 is a tip for the server. What percentage of the subtotal is the tip?

16.67% (7 ÷ 42 ≈ 16.67)

2. The tax rate at this restaurant is 9.5%.

Date:	Sep. 12th
Time:	6:04 PM
Server:	#27
Bread Stix	9.50
Ravioli Bites	10.50
Cheesecake	4.95
Subtotal	24.95
Sales Tax	
Total	
V*	w "

Another person's subtotal is \$24.95. How much will their sales tax be?

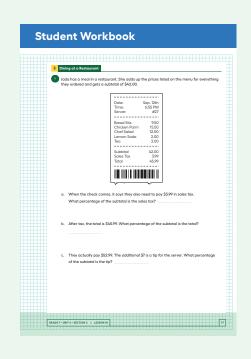
 $$2.37 (24.95 \cdot 0.095 = 2.37025)$



Some other person's sales tax is \$1.61. How much was their subtotal?

Any number in the range of \$16.90 to \$16.99 is reasonable.

Sample response: \$16.95 (1.61 ÷ 0.095 ≈ 16.95)





Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Are You Ready for More?

Elena's cousins went to a restaurant. The part of the entire cost of the meal that was tax and tip together was 25% of the cost of the food alone. What could the tax rate and tip rate be?

Sample response: 5% tax rate and 20% tip

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they solved each problem, including which values were given in the situation and which values they were trying to find. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases, such as "sales tax," "tip," "subtotal," and "total including tax." Ask students to suggest ways to update the display:

"Are there any new words or phrases that you would like to add? Is there any language you would like to revise or remove?"

If no student uses the strategy of writing and solving an equation, ask students:

- (How we might use equations to solve the last two problems?" Set up an equation that shows the percentage times the subtotal equals the new amount, such as $0.095 \cdot 24.95 = x$ or 0.095x = 1.61
- "How can we represent the tax rate 9.5% as a decimal?"
 0.095 for just the sales tax; 1.095 for the total including tax

the subtotal or the original amount—is known or unknown.

"How do you know whether to multiply or divide to solve the problem?"
It depends on whether the number next to the tax rate—in other words,

Lesson Synthesis

Share with students,

"Today we calculated sales tax and tips. These are two different examples of percent increase."

To review the meanings of these terms, consider asking students:

"What is sales tax?"

Sales tax is an extra amount of money, added to the price of an item, that is paid to the government.

"If the tax rate is 7%, how can you calculate the price of an item including sales tax?"

I multiply the price by 1.07.

○ "What is a tip?"

A tip is an extra amount of money, added to the price of a meal, that is given to the server at a restaurant.

"If you want to leave a 15% tip, how can you calculate the total amount to pay for a meal?"

I multiply the bill by 1.15.

Lesson Summary

Many places have *sales tax*. A sales tax is an amount of money that a government agency collects on the sale of certain items. For example, a state might charge a tax on all cars purchased in the state. Often, the tax rate is given as a percentage of the cost. For example, a state's tax rate on car sales might be 2%, which means that for every car sold in that state, the buyer has to pay a tax that is 2% of the sales price of the car.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone buys something, they have to add 0.075 times the amount on the price tag to determine the total cost of the item.

For example, if the price tag on a T-shirt in Arizona says \$11.50, then the sales tax is $(0.075) \cdot 11.5 = 0.8625$, which rounds to 86 cents. The customer pays 11.50 + 0.86, or \$12.36 for the shirt.

The total cost to the customer is the item price plus the sales tax. We can think of this as a percent increase. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A *tip* is an amount of money that a person gives someone who provides a service. It is customary in many restaurants to give a tip to the server that is between 10% and 20% of the cost of the meal. If a person plans to leave a 15% tip on a meal, then the total cost will be 115% of the cost of the meal.

Cool-down

A Restaurant in a Different City

This *Cool-down* assesses whether students understand how to calculate a tax rate based on a price before tax was added and the amount of tax added. Additionally, they must use proportional reasoning to determine the tax on another item with the same tax rate.

Student Task Statement

At a dinner, the meal costs \$22, and a sales tax of \$1.87 is added to the bill.

1. How much would the sales tax be on a \$66 meal?

 $$5.61(22 \cdot 3 = 66 \text{ and } 1.87 \cdot 3 = 5.61)$

2. What is the tax rate for meals in this city?

8.5% (1.87 ÷ 22 = 0.085)



Student Workbook 10 22.00 The soles too rate in New Mexico is 5.125%. Select all the equations the soles too, t, that one would pay in New Mexico for an item of cost d^2 . $d^2 = 5.125c$. $d^2 = 5.025c$. $d^2 = 0.05125c$. $d^2 = 0.05125c$. $d^2 = 0.05125c$.

Student Workbook a. If you pay \$30, what percentage is th

Problem 1

In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total price including tax for each item.

item	price before tax (\$)	sales tax (\$)	price including tax (\$)
pillow	8.00	0.58	8.58
blanket	22.00	1.60	23.60
trash can	14.50	1.05	15.55

Problem 2

The sales tax rate in New Mexico is 5.125%. Select **all** the equations that represent the sales tax, t, that one would pay in New Mexico for an item of cost c?

- **A.** t = 5.125c
- **B.** t = 0.5125c
- **C.** t = 0.05125c
- **D.** $t = c \div 0.05125$
- **E.** $t = \frac{5.125}{100} c$

Problem 3

Here are the prices of some items and the amount of sales tax charged on each in Nevada.

cost of item (\$)	sales tax (\$)
10	0.46
50	2.30
5	0.23

- a. What is the sales tax rate in Nevada? 4.6%
- **b.** Write an expression for the amount of sales tax charged, in dollars, on an item that costs c dollars. 0.046c (or equivalent)

Problem 4

Your total bill at a restaurant is \$24.

- a. If you pay \$30, what percentage is the tip? 25%
- b. If you pay \$27, what percentage is the tip? 12.5%

Problem 5

from Unit 4, Lesson 9

Find each amount:

- **a.** 3.8% of 25 0.95
- **b.** 0.2% of 50 0.1
- c. 180.5% of 99 178.695

Problem 6

from Unit 4, Lesson 8

On Monday, the high was 60 degrees Fahrenheit. On Tuesday, the high was 18% more. How much did the high increase from Monday to Tuesday?

10.8 degrees Fahrenheit

Problem 7

from Unit 3, Lesson 4

Complete the table. Explain or show your reasoning.

object	radius	circumference
ceiling fan	2.8 ft	17.6 ft
water bottle cap	13 mm	82 mm
bowl	9 cm	56.5 cm
drum	I2 in	75.4 in

Sample reasoning: The constant of proportionality is $2 \cdot \pi$. The given radii are multiplied by 6.28 to find the missing circumferences, and the given circumferences are divided by 6.28 to find the missing radii.

