Lesson 4

Scaled Relationships

Goals

- Explain (orally and in writing) that corresponding angles in a figure and its scaled copies have the same measure.
- Identify (orally and in writing) corresponding distances or angles that can show that a figure is not a scaled copy of another.
- Recognize that corresponding distances in a figure and its scaled copy are related by the same scale factor as corresponding sides.

Learning Targets

- I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.
- When I see a figure and its scaled copy, I can explain what is true about corresponding angles.
- When I see a figure and its scaled copy, I can explain what is true about corresponding distances.

Lesson Narrative

In this lesson, students examine corresponding parts more closely, to make a case for whether a figure is or is not a scaled copy of another.

- Previously students saw that a scale factor relates the corresponding side lengths of a figure and a scaled copy. In this lesson, they notice that corresponding distances not connected by a segment are also related by the same scale factor as corresponding sides.
- Previously students used tracing paper to compare corresponding angles. In this lesson, they use protractors to test their observations about corresponding angles. They verify, in several sets of examples, that corresponding angles in a figure and its scaled copies are the same size.

Practice with the use of protractors helps develop a sense for measurement accuracy and for drawing conclusions from said measurements, such as determining whether or not two angles are the same.

Student Learning Goal

Let's find relationships between scaled copies.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR8: Discussion Supports (Activity 1, Activity 3)

Instructional Routines

- 5 Practices
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2, Activity 3

Required Preparation

Activity 3:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	20 min	15 min	10 min	5 min

Instructional Routines

Notice and Wonder

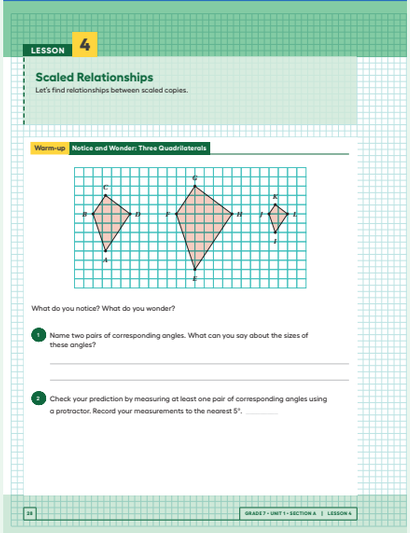
ilclass.com/r/10694948

Please log in to the site before using the QR code or URL.

Building on Student Thinking

Some students may read the wrong number on the protractor, moving down from the 180° mark instead of up from the 0° mark, or reading the measurement outside of one of the lines forming the angle instead of between the two lines. Clarify the angle being measured, how to line up the protractor, or how to read the markings correctly.

Student Workbook


LESSON 4

Scaled Relationships

Let's find relationships between scaled copies.

Warm-up Notice and Wonder: Three Quadrilaterals

What do you notice? What do you wonder?

1. Name two pairs of corresponding angles. What can you say about the sizes of these angles?

2. Check your prediction by measuring at least one pair of corresponding angles using a protractor. Record your measurements to the nearest 5°.

Warm-up

Notice and Wonder: Three Quadrilaterals

5 min

Activity Narrative

This is the first *Notice and Wonder* activity in the course. Students are shown an image and asked: "What do you notice? What do you wonder?"

Students are given time to write down what they notice and wonder about the image and then time to share their thoughts. Their responses are recorded for all to see. Often, the goal is to elicit observations and curiosities about a mathematical idea that students are about to explore. Pondering the two open questions allows students to build interest about and gain entry into an upcoming task.

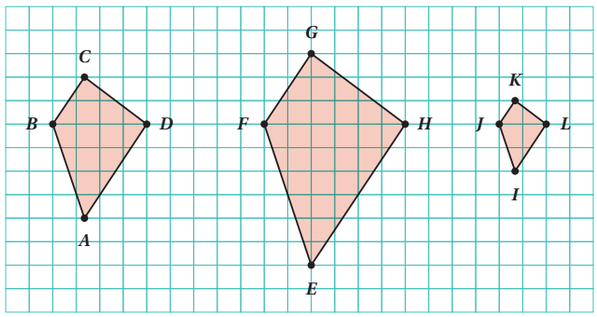
While students may notice and wonder many things about these figures, the main purpose of this *Warm-up* is for students to notice that the side lengths of the polygons in this task cannot be easily determined and to wonder how we can tell whether the figures are scaled copies. This will be useful when students examine corresponding distances that are not side lengths in a later activity.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement



What do you notice? What do you wonder?

Students may notice:

- There are 3 figures on a grid.
- Each figure has 4 sides.
- The figures are different sizes.
- The figures are not symmetric.
- All the sides are diagonal.
- Corresponding angles look like they might have the same measures.
- The figures look like they might be scaled copies.

Students may wonder:

- Why did someone draw this shape? Does it represent an actual object?
- How long are the side lengths?
- Why are the figures shown on a grid?
- Do corresponding angles have the same measure?
- Are these figures scaled copies? If so, what is the scale factor?

1. Name two pairs of corresponding angles. What can you say about the sizes of these angles?
2. Check your prediction by measuring at least one pair of corresponding angles using a protractor. Record your measurements to the nearest 5°.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If these ideas do not come up during the conversation, ask students to discuss:

- The fact that the side lengths are hard to determine given their orientation on the grid.
- The question of whether the figures are scaled copies and how that could be determined.

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Access for Multilingual Learners (Activity 1, Launch)

MLR8: Discussion Supports. Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.

Advances: Reading, Representing

Activity 1

Measuring the Three Quadrilaterals

10 min

Activity Narrative

In this activity, students investigate polygons on a grid to determine whether or not they are scaled copies. The sides of the polygons are diagonal, so the side lengths cannot be easily determined using the grid.

First, students measure the angles using a protractor to see if corresponding angles have the same measure. Next, they find other corresponding distances that are not side lengths and see that these distances are related by a consistent scale factor. Students learn that in scaled copies, any corresponding distances—not limited to lengths of sides or segments—are related by the same scale factor.

Students must take care when they identify corresponding parts. As students work, urge them to attend to the order in which points or segments are listed.

If students are not sure what to make of the values in the table, encourage them to consider the corresponding distances of two figures at a time. For example, ask:

- *"What do you notice about the corresponding vertical distances in $IJKL$ and $EFGH$? What about the corresponding horizontal distances in those two figures?"*

Launch

Ask students to name corresponding angles in these figures. Record their responses for all to see.

- Angles A, E, and I
- Angles B, F, and J
- Angles C, G, and K
- Angles D, H, and L

Tell students that they will test the angle measures of the figures, this time by using protractors instead of tracing paper. Provide access to protractors. Give students 2–3 minutes of quiet work time for the first two questions.

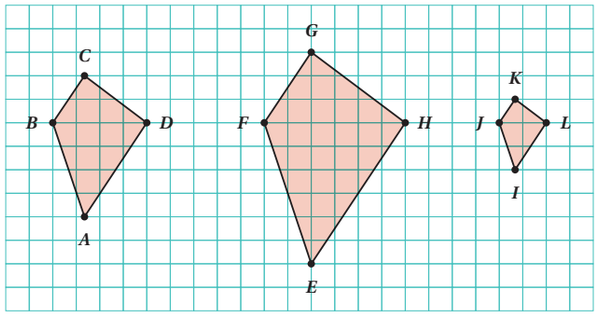
Some angles may be challenging to measure because of the size of the polygons. If students find the sides of a polygon not long enough to accommodate angle measurements, suggest that they extend the lines, or demonstrate how to do so (especially if using opaque protractors with holes in the middle rather than clear protractors without holes).

Select a few students to share their angle measurements. Discuss major discrepancies, if any. Students should be able to confirm that all corresponding angles in the polygons are equal.

Ask students if they can tell the lengths of segments GF or DC from the grid (without using rulers). Explain that they will explore another way to compare length measurements in scaled copies.

Give students 3–4 minutes of quiet work time for the remaining questions and 1 minute to discuss their responses with a partner before the whole-class discussion.

Student Task Statement



1. Measure at least one set of corresponding angles using a protractor. Record your measurements to the nearest 5°.

   At least one of these lists:

   a. Angles *A*, *E*, and *I* each measure about 50°.

   b. Angles *B*, *F*, and *J* each measure about 95°.

   c. Angles *C*, *G*, and *K* each measure about 90°.

   d. Angles *D*, *H*, and *L* each measure about 125°.

2. What do you notice about the angle measures?

   The corresponding angles have the same measure.

   Pause here so your teacher can review your work.

3. The side lengths of the polygons are hard to tell from the grid, but there are other corresponding distances that are easier to compare. Identify the distances in the other two polygons that correspond to DB and AC, and record them in the table.

quadrilateral	distance that corresponds to DB	distance that corresponds to AC
$ABCD$	$DB = 4$	$AC = 6$
$EFGH$	$HF = 6$	$EG = 9$
$IJKL$	$LJ = 2$	$IK = 3$

4. Look at the values in the table. What do you notice?

   These corresponding distances are related by the same scale factor even though they are not side lengths. For example:

   a. The scale factor from *ABCD* to *EFGH* is $1\frac{1}{2}$.

   b. The scale factor from *ABCD* to *IJKL* is $\frac{1}{2}$.

   c. The scale factor from *EFGH* to *ABCD* is $\frac{2}{3}$.

   d. The scale factor from *EFGH* to *IJKL* is $\frac{1}{3}$.

   e. The scale factor from *IJKL* to ABCD is 2.

   f. The scale factor from *IJKL* to *EFGH* is 3.

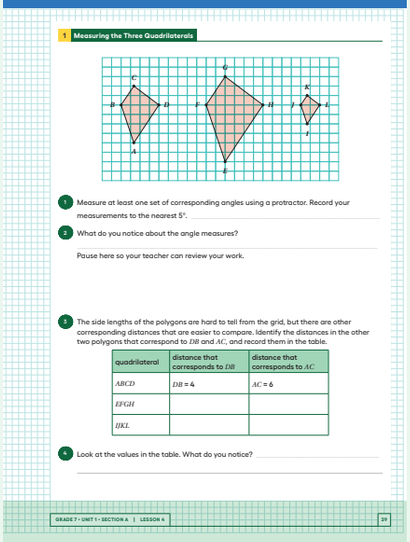
Building on Student Thinking

Students may list the corresponding vertices for distances in the wrong order. For example, instead of writing IK as the distance corresponding to AC, they may write KI. Remind students of the corresponding points by asking,

"Which vertex in IJKL corresponds to A? Which corresponds to C?"

and have them match the order of the vertices accordingly.

Student Workbook


1 Measuring the Three Quadrilaterals

1. Measure at least one set of corresponding angles using a protractor. Record your measurements to the nearest 5°.
2. What do you notice about the angle measures?

Pause here so your teacher can review your work.

3. The side lengths of the polygons are hard to tell from the grid, but there are other corresponding distances that are easier to compare. Identify the distances in the other two polygons that correspond to DB and AC, and record them in the table.

quadrilateral	distance that corresponds to DB	distance that corresponds to AC
$ABCD$	$DB = 4$	$AC = 6$
$EFGH$		
$IJKL$		

4. Look at the values in the table. What do you notice?

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Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color coding corresponding line segments.

Supports accessibility for: Visual-Spatial Processing

Instructional Routines

MLR1: Stronger and Clearer Each Time

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Access for Multilingual Learners (Activity 2, Narrative)

MLR1: Stronger and Clearer Each Time
This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing and speaking.

5. Are these three quadrilaterals scaled copies? Explain your reasoning.

Yes

Sample reasoning: They are scaled copies because the corresponding angles have equal measures and the corresponding distances are related by the same scale factor.

Activity Synthesis

Display the completed tables for all to see. To highlight how all distances in a scaled copy (not just the side lengths of the figure) are related by the same scale factor, ask questions such as:

- *"How do the distances in EFGH compare to the distances in ABCD? Do the corresponding distances share the same scale factor?"*

  The distances in *EFGH* are each $\frac{3}{2}$ times the corresponding distance in *ABCD*.

- *"How do distances in IJKL compare to the distances in EFGH? Do the corresponding distances share the same scale factor?"*

  The distances in *IJKL* are each $\frac{1}{3}$ times the corresponding distance in *EFGH*.

Activity 2

Scaled or Not Scaled?

20 min

Activity Narrative

The purpose of this activity is for students to see that figures may not be scaled copies, even though they have either corresponding angles with equal measures or corresponding distances multiplied by the same scale factor. This shows that to determine whether one figure is a scaled copy of another, we have to check both the corresponding angles and the corresponding distances.

As students work, monitor for convincing arguments about why one polygon is or is not a scaled copy of the other. Ask them to present their cases during the whole-class discussion. As they present and analyze different claims about the figures in the discussion, students construct arguments and critique reasoning.

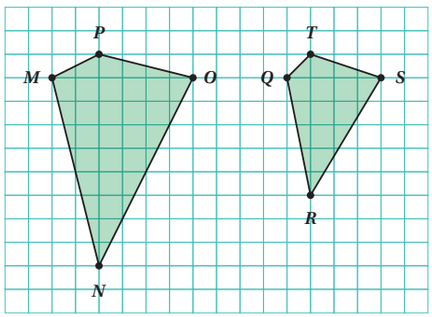
This is the first time Math Language Routine 1: *Stronger and Clearer Each Time* is suggested in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. It is not necessary that students finish this draft before moving to the structured partner meetings step. Students then meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, "What did you mean by … ?" and "Can you say that another way?" Finally, students write a second draft of their response, reflecting ideas from partners, and improvements on their initial ideas. Students should be encouraged to incorporate any good ideas and words they got from their partners to make their second draft stronger and clearer.

Launch

Keep students in the same groups. Provide access to geometry toolkits. Give students 6–7 minutes of quiet work time.

Student Task Statement

Here are two quadrilaterals.



1. Mai says that polygon $QRST$ is a scaled copy of polygon $MNOP$, but Noah disagrees. Do you agree with either of them? Explain or show your reasoning.

   I agree with Noah

   Sample reasoning: It is not a scaled copy because the corresponding angles are not equal. Mai may have noticed that the corresponding distances are multiplied by $\frac{3}{2}$ and thought this meant the polygons are similar.

2. Record the corresponding distances in the table. What do you notice?

quadrilateral	horizontal distance	vertical distance
$MNOP$	$MO = 6$	$NP = 9$
$QRST$	$QS = 4$	$RT = 6$

3. Measure at least three pairs of corresponding angles in $MNOP$ and $QRST$ using a protractor. Record your measurements to the nearest 5°. What do you notice?

   The corresponding angles are not all the same size. Rounded to the nearest 5°, the measures are:

MNOP	QRST
angle M measures 100°	angle Q measures 125°
angle N measures 40°	angle S measures 40°
angle O measures 75°	angle R measures 75°
angle P measures 140°	angle T measures 115°

Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. For example, present one question at a time. Check in with students to provide feedback and encouragement after each chunk.

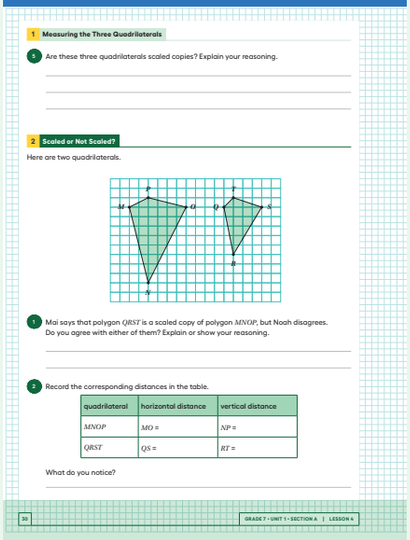
Supports accessibility for: Attention, Social-Emotional Functioning

Building on Student Thinking

Students may rely on the appearance of the figures rather than analyze given information to draw conclusions about scaling. Urge them to look for information about distances and angles (and to think about which tools could help them find such information) to support their argument.

Some students may struggle with comparing the corresponding angles in the first pair of figures. Remind students of the tools that are at their disposal, and that they could extend the sides of the polygons, if needed, to make it easier to measure the angles.

Student Workbook


1 Measuring the Three Quadrilaterals

3 Are these three quadrilaterals scaled copies? Explain your reasoning.

2 Scaled or Not Scaled?

Here are two quadrilaterals.

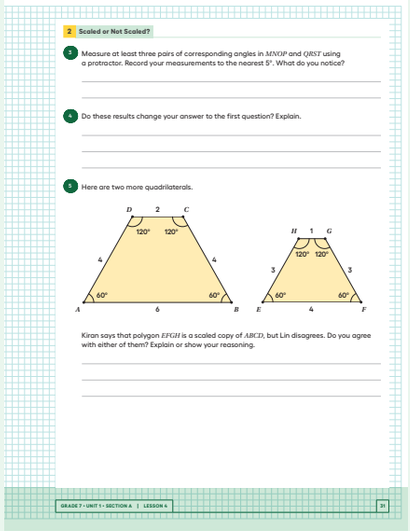
1 Mai says that polygon $QRST$ is a scaled copy of polygon $MNOP$, but Noah disagrees. Do you agree with either of them? Explain or show your reasoning.

2 Record the corresponding distances in the table.

quadrilateral	horizontal distance	vertical distance
$MNOP$	$MO =$	$NP =$
$QRST$	$QS =$	$RT =$

What do you notice?

Student Workbook



2 Scaled or Not Scaled?

3 Measure at least three pairs of corresponding angles in $MNOP$ and $QRST$ using a protractor. Record your measurements to the nearest 5°. What do you notice?

4 Do these results change your answer to the first question? Explain.

5 Here are two more quadrilaterals.

Kiran says that polygon $EFGH$ is a scaled copy of $ABCD$, but Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

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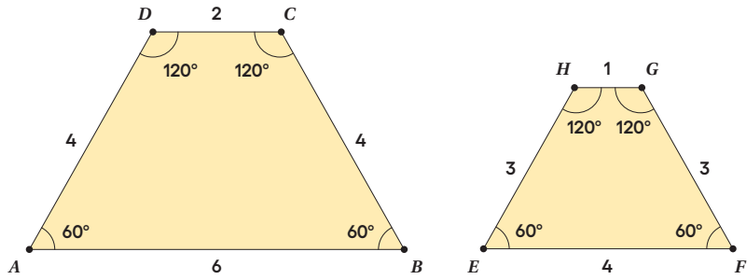
4. Do these results change your answer to the first question? Explain.

Since the corresponding angles are not equal, the polygons are definitely not scaled copies of one another.

5. Here are two more quadrilaterals.



Kiran says that polygon $EFGH$ is a scaled copy of $ABCD$, but Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

I agree with Lin. Sample reasoning: It is not a scaled copy because the corresponding distances are not multiplied by the same number (compared to *ABCD*, the top side in *EFGH* is half as long, while the bottom side is two-thirds as long). Kiran may have noticed that the corresponding angles are equal and thought this meant the polygons are similar. I noticed that the scale factors for the corresponding sides are not the same. *EF* and *EF* are related by a scale factor of $\frac{2}{3}$, but *DC* and *HG* are related by a scale factor of $\frac{1}{2}$.

Are You Ready for More?

All side lengths of Quadrilateral Y are 2, and all side lengths of Quadrilateral Z are 3. Does Quadrilateral Y have to be a scaled copy of Quadrilateral Z? Explain your reasoning.

No. Quadrilateral Y could be a square and Quadrilateral Z could be a rhombus that is not a square. Since the angles would be different, Quadrilateral Y would not be a scaled copy of Quadrilateral Z.

Activity Synthesis

The goal of this discussion is to make clear that angle measurements and distances are both important when deciding whether two polygons are scaled copies. To highlight the different arguments about whether one polygon is a scaled copy of another, consider debriefing with a role play. Ask two students to take on the roles of the characters Mai and Noah and make a brief argument about whether they believe one figure is a scaled copy of the other in the first question. After the cases are presented, ask students whether they agree with Mai or Noah.

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to the last question, about whether polygon $EFGH$ is a scaled copy of $ABCD$. In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "_____ makes sense, but what did you mean by … ?"
- "Can you say that another way?"
- "How do you know … ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer. If time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

Here is an example of a second draft:

> I agree with Lin. Even though the corresponding angles are equal, EFGH is not a scaled copy of ABCD. To get from AB to EF we multiply by $\frac{2}{3}$, but to get from CD to GH we multiply by $\frac{1}{2}$, so the corresponding side lengths don't all have the same scale factor.

If time allows, have students compare their first and second drafts.

After *Stronger and Clearer Each Time,* poll the class to find out whether they agree with Kiran or Lin.

Activity 3: Optional
Comparing Pictures of Birds

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use what they know about corresponding lengths and angles to show that one picture of a bird is not a scaled copy of the other. Unlike in previous tasks, minimal scaffolding is given here, so students need to decide what evidence is necessary to explain or show an absence of scaling. As students work, notice the different ways students use corresponding lengths and angles to think about scaling.

When students use corresponding measurements to show that one image is not a copy of the other, they practice looking for and making use of structure.

Monitor for students who use different approaches to show that one image is not a scaled copy of the other:

- Describe the copy as being stretched vertically or squished horizontally.
- Identify two pairs of corresponding distances in the images that have different scale factors.
- Identify corresponding angles in the two images that have different measures.

Plan to have students present in this order to support them in developing more precise ways of justifying that images are not scaled copies.

In the digital version of the activity, students use an applet to compare corresponding lengths and angles in two images. The applet allows students to add points and line segments and to measure distances and angles. This activity works best when each student has access to the applet because the level of precision is important. The digital version may help students measure quickly and accurately so they can focus more on the mathematical analysis.

Instructional Routines

5 Practices

ilclass.com/r/10690701

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

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Building on Student Thinking

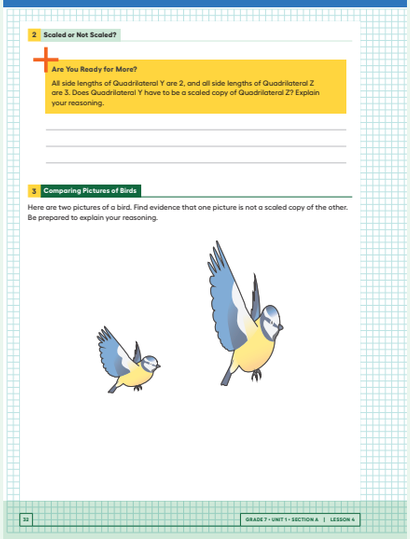
Students may draw two segments that do not share a point, or choose non-corresponding points and segments on the two figures. Refer students to earlier work involving polygons and point out pairs of distances that could be used for comparison and those that could not be. Remind them that we can compare only the corresponding parts, not just any two parts.

Access for Multilingual Learners (Activity 3, Synthesis)

MLR8: Discussion Supports.
At the appropriate time, give students 2–3 minutes to make sure that everyone in their group can explain why the copy of the bird image is not a scaled copy. Invite groups to rehearse what they will say when they share with the whole class.

Advances: Speaking, Conversing, Representing

Student Workbook


2 Scaled or Not Scaled?

Are You Ready for More?

All side lengths of Quadrilateral Y are 2, and all side lengths of Quadrilateral Z are 3. Does Quadrilateral Y have to be a scaled copy of Quadrilateral Z? Explain your reasoning.

3 Comparing Pictures of Birds

Here are two pictures of a bird. Find evidence that one picture is not a scaled copy of the other. Be prepared to explain your reasoning.

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Launch

Arrange students in groups of 3–4. Provide access to geometry toolkits.

Give students 2 minutes of quiet work time followed by 4–5 minutes to discuss in groups how to find evidence that one picture is not a scaled copy of the other. Select groups with different strategies, such as those described in the activity narrative, to share later.

If students' ideas deviate from drawing corresponding points and segments and comparing distances and angles, guide them with some prompts. For example:

- Pick a point that can be easily identified (such as the tip of one wing) in one picture. Ask for the corresponding point on the other.
- Ask if that pair of corresponding points could help them determine if one picture is scaled from the other. If not, ask what else might be needed.
- Add another point and a segment connecting the two points. Ask if or how the segment could help, and so on.

Student Task Statement

Here are two pictures of a bird. Find evidence that one picture is not a scaled copy of the other. Be prepared to explain your reasoning.

Answers vary. Sample reasoning:

- Corresponding angles do not match, so one picture cannot be a scaled copy of the other.
- The lengths of corresponding segments are not related by the same scale factor, so one picture cannot be a scaled copy of the other.

Activity Synthesis

The purpose of this discussion is to show various ways in which students can select evidence that shows the image is not a scaled copy. Invite previously selected groups to share their reasoning. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see. As students share their approaches, consider asking how they knew that certain pairs of points in the images were corresponding points. Students could identify a unique feature such as the eyes or the tip of the wing.

Connect the different responses to the learning goals by asking questions such as:

- *"How were corresponding points used in each approach?"*
- *"Which methods examined corresponding distances and which examined corresponding angles?"*
- *"How many pieces of evidence did it take for each approach to show that the image was not a scaled copy?"*

Make sure students understand that:

- Corresponding angles on an image and a scaled copy have the same measure.
- Corresponding distances on an image and a scaled copy are related by the same scale factor.
- To conclude that an image is not a scaled copy of another, it is sufficient to find one pair of corresponding angles with different measures or two pairs of corresponding distances that are related by different scale factors.

Lesson Synthesis

Share with students

- *"Today we saw that to determine whether a polygon is actually a scaled copy of another, we need to check corresponding lengths and angle measures."*

To help students generalize a process for determining whether a figure is a scaled copy of another, consider asking students:

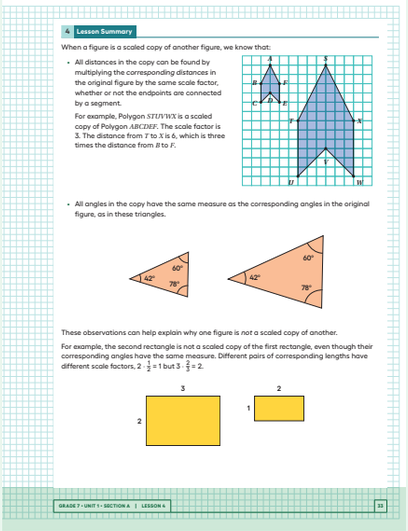
- *"Does a scale factor affect any other measurements other than segment lengths?"*

  The distances between any two points in the original figure, even those not connected by segments, are scaled by the same scale factor.

- *"How can we be sure that a figure is a scaled copy? What features do we check?"*

  If all the corresponding angles are the same size and all corresponding distances are scaled by the same factor, then we can conclude that it is a scaled copy of the other.

Student Workbook



4 Lesson Summary

When a figure is a scaled copy of another figure, we know that:

- All distances in the copy can be found by multiplying the corresponding distances in the original figure by the same scale factor, whether or not the endpoints are connected by a segment.

  For example, Polygon $STUVWX$ is a scaled copy of Polygon $ABCDEF$. The scale factor is 3. The distance from T to X is 6, which is three times the distance from B to F.

- All angles in the copy have the same measure as the corresponding angles in the original figure, as in these triangles.

60° 42° 78° 60° 42° 78°

These observations can help explain why one figure is *not* a scaled copy of another.

For example, the second rectangle is not a scaled copy of the first rectangle, even though their corresponding angles have the same measure. Different pairs of corresponding lengths have different scale factors: $2 \cdot \frac{1}{2} = 1$ but $3 \cdot \frac{1}{3} = 2$.

3 2 2 1

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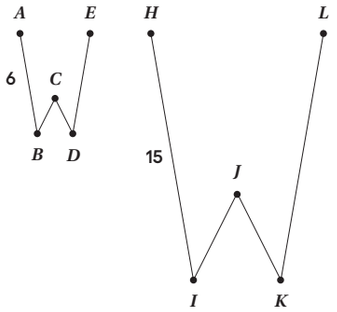
If desired, use this example to review these concepts. Figures $ABCDE$ and $HIJKL$ are scaled copies.



Display the image and consider asking:

- *"If the distance from A to E is 4 units, how long is the corresponding distance in the second figure?"*

  The distance from H to L is 10 units, because $15 \div 6 = 2.5$ and $4 \cdot 2.5 = 10$.

- *"If the distance from I to K is 5 units, how long is the corresponding distance in the first figure?"*

  The distance from B to D is 2, because $5 \div 2.5 = 2$.

- *"If angle CDE measures 37 degrees, how big is the corresponding angle in the second figure?"*

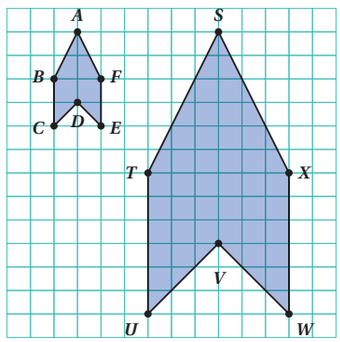
  Angle JKL measures 37 degrees, because corresponding angles have the same measure.

Lesson Summary

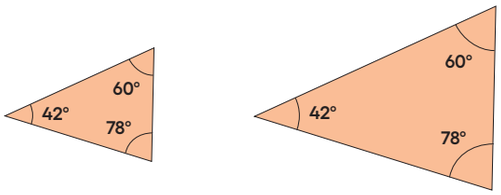
When a figure is a scaled copy of another figure, we know that:

- All distances in the copy can be found by multiplying the *corresponding distances* in the original figure by the same scale factor, whether or not the endpoints are connected by a segment.

  For example, Polygon $STUVWX$ is a scaled copy of Polygon $ABCDEF$. The scale factor is 3. The distance from T to X is 6, which is three times the distance from B to F.

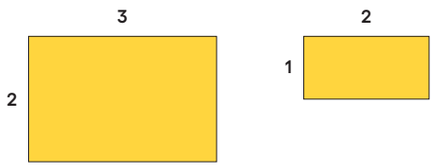


- All angles in the copy have the same measure as the corresponding angles in the original figure, as in these triangles.



These observations can help explain why one figure is *not* a scaled copy of another.

For example, the second rectangle is not a scaled copy of the first rectangle, even though their corresponding angles have the same measure. Different pairs of corresponding lengths have different scale factors, $2 \cdot \frac{1}{2} = 1$ but $3 \cdot \frac{2}{3} = 2$.



Responding To Student Thinking

Points to Emphasize
If students struggle with determining whether figures are scaled copies, focus on this when opportunities arise over the next several lessons. For example, highlight corresponding distances and corresponding angles in this activity:

Unit 1, Lesson 5, Activity 2 Scaling A Puzzle

Cool-down

Corresponding Polygons

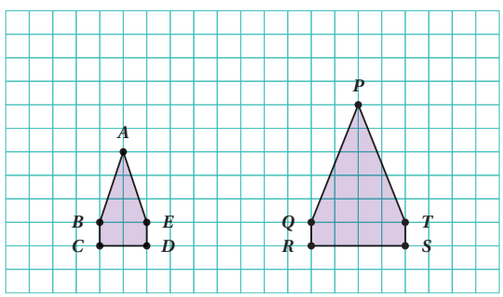
5 min

Launch

Provide access to geometry toolkits.

Student Task Statement

Here are two polygons on a grid.



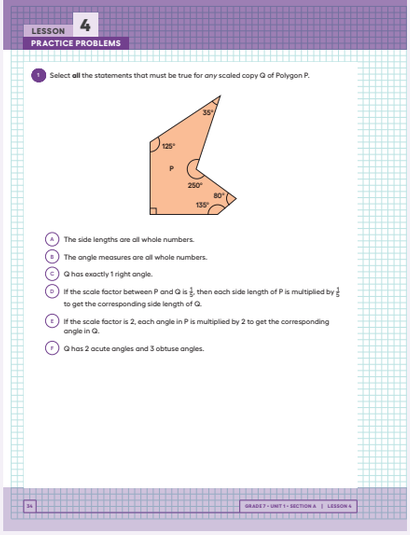
Is $PQRST$ a scaled copy of $ABCDE$? Explain your reasoning.

No. Sample reasoning: $PQRST$ is not a scaled copy of $ABCDE$ because we need to use different scale factors when comparing corresponding lengths (1 for corresponding segments BC and QR and 2 for corresponding segments CD and RS). Also, not all of their corresponding angles are the same size. Angle A and angle P are not the same size.

Practice Problems

4 Problems

Student Workbook

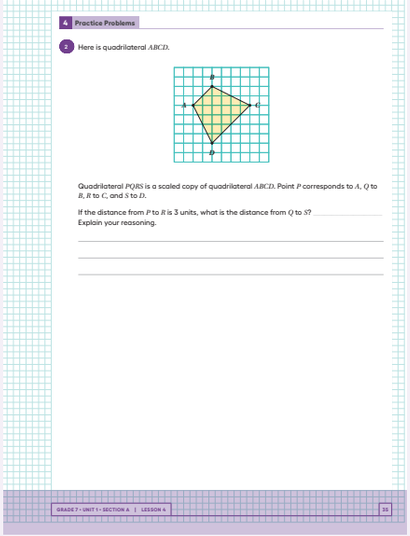

LESSON 4
PRACTICE PROBLEMS

1 Select **all** the statements that must be true for *any* scaled copy Q of Polygon P.

35°
125°
P
250°
80°
135°

A The side lengths are all whole numbers.
B The angle measures are all whole numbers.
C Q has exactly 1 right angle.
D If the scale factor between P and Q is $\frac{1}{5}$, then each side length of P is multiplied by $\frac{1}{5}$ to get the corresponding side length of Q.
E If the scale factor is 2, each angle in P is multiplied by 2 to get the corresponding angle in Q.
F Q has 2 acute angles and 3 obtuse angles.

Student Workbook


4 Practice Problems

2 Here is quadrilateral $ABCD$.

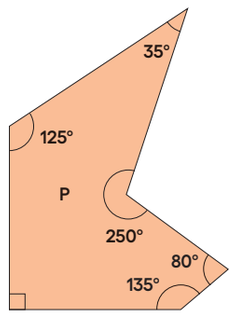
B
A
C
D

Quadrilateral $PQRS$ is a scaled copy of quadrilateral $ABCD$. Point P corresponds to A, Q to B, R to C, and S to D.

If the distance from P to R is 3 units, what is the distance from Q to S? _____
Explain your reasoning.

Problem 1

Select **all** the statements that must be true for *any* scaled copy Q of Polygon P.



A. The side lengths are all whole numbers.

B. The angle measures are all whole numbers.

C. Q has exactly 1 right angle.

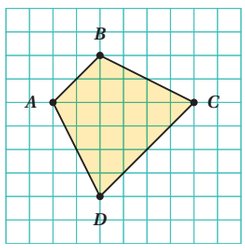
D. If the scale factor between P and Q is $\frac{1}{5}$, then each side length of P is multiplied by $\frac{1}{5}$ to get the corresponding side length of Q.

E. If the scale factor is 2, each angle in P is multiplied by 2 to get the corresponding angle in Q.

F. Q has 2 acute angles and 3 obtuse angles.

Problem 2

Here is quadrilateral $ABCD$.



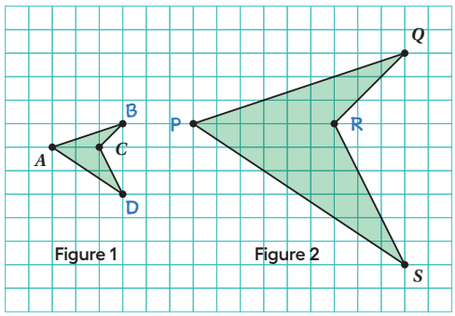
Quadrilateral $PQRS$ is a scaled copy of quadrilateral $ABCD$. Point P corresponds to A, Q to B, R to C, and S to D.

If the distance from P to R is 3 units, what is the distance from Q to S? Explain your reasoning.

The distance from Q to S is 3 units. Because the lengths of AC and BD are 6, and AC corresponds to PR, the scale factor must be $\frac{1}{2}$. Since QS corresponds to BD, QS must also be 3 units long.

Problem 3

Figure 2 is a scaled copy of Figure 1.



a. Identify the points in Figure 2 that correspond to the points A and C in Figure 1. Label them P and R. What is the distance between P and R?

6 units

b. Identify the points in Figure 1 that correspond to the points Q and S in Figure 2. Label them B and D. What is the distance between B and D?

3 units

c. What is the scale factor that takes Figure 1 to Figure 2?

3 because distances between points in Figure 2 are three times the corresponding distances in Figure 1

d. G and H are two points on Figure 1, but they are not shown. The distance between G and H is 1. What is the distance between the corresponding points on Figure 2?

3 units because the scale factor is 3

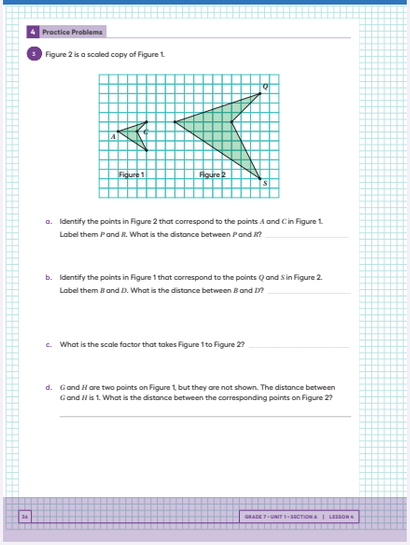
Problem 4

from an earlier course

To make 1 batch of lavender paint, the ratio of cups of pink paint to cups of blue paint is 6 to 5. Find two more ratios of cups of pink paint to cups of blue paint that are equivalent to this ratio.

Sample response: 12 cups of pink paint to 10 cups of blue paint and 18 cups of pink paint to 15 cups of blue paint (this is 2 batches and 3 batches, respectively, of this shade of lavender paint)

Student Workbook


4 Practice Problems

3 Figure 2 is a scaled copy of Figure 1.

Figure 1 Figure 2

a. Identify the points in Figure 2 that correspond to the points A and C in Figure 1. Label them P and R. What is the distance between P and R?

b. Identify the points in Figure 1 that correspond to the points Q and S in Figure 2. Label them B and D. What is the distance between B and D?

c. What is the scale factor that takes Figure 1 to Figure 2?

d. G and H are two points on Figure 1, but they are not shown. The distance between G and H is 1. What is the distance between the corresponding points on Figure 2?

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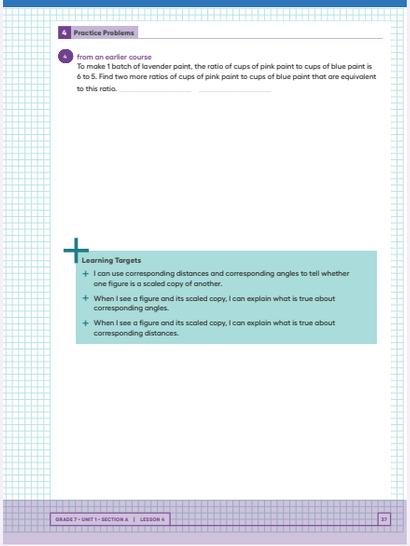
Student Workbook


4 Practice Problems

4 from an earlier course

To make 1 batch of lavender paint, the ratio of cups of pink paint to cups of blue paint is 6 to 5. Find two more ratios of cups of pink paint to cups of blue paint that are equivalent to this ratio.

Learning Targets

+ I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.
+ When I see a figure and its scaled copy, I can explain what is true about corresponding angles.
+ When I see a figure and its scaled copy, I can explain what is true about corresponding distances.

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