Lesson 10

Introducing Graphs of Proportional Relationships

Goals

- Compare and contrast (orally) graphs of relationships.
- Generalize (orally and in writing) that a proportional relationship can be represented in the coordinate plane by a line that includes the "origin" or by a collection of points that lie on such a line.
- Justify (orally) that a table and a graph represent the same relationship.

Learning Target

I know that the graph of a proportional relationship lies on a line through (0, 0).

Lesson Narrative

This lesson introduces an important way of representing a proportional relationship: its graph. Students plot points on the graph from tables, and, by the end of the lesson, start to see that the graph of a proportional relationship always lies on a line that passes through the **origin**, (0, 0). (They will be able to explain why this is true in grade 8.) They match tables and graphs of given situations and articulate their reasons for each match.

Student Learning Goal

Let's see how graphs of proportional relationships differ from graphs of other relationships.

Access for Students with Diverse Abilities

- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Warm-up, Activity 1)

Instructional Routines

- Card Sort
- MLR2: Collect and Display
- Take Turns

Required Materials

Materials to Gather

- Rulers: Activity 1
- Math Community Chart: Activity 2
- Pre-printed slips, cut from copies of the blackline master: Activity 2

Materials to Copy

- Matching Tables and Graphs Cards (1 copy for every 2 students): Activity 2

Warm-up:
For the digital version of the activity, acquire devices that can run the applet.

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Activity 2:
Printing one copy for each group of 2 students and cutting them up ahead of time. Prepare a few copies of an answer key and place them in envelopes for students to access to check their work when they finish.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	25 min	10 min	5 min

Warm-up

Notice These Points

5 min

Activity Narrative

There is a digital version of this activity.

This *Warm-up* prepares students for graphing proportional relationships in the **coordinate plane**. They practice graphing coordinate points and notice that all points lie on a straight line.

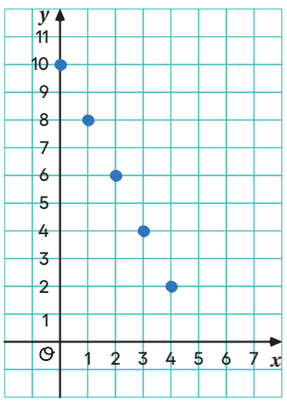
In the digital version of the activity, students use an applet to plot points on the coordinate plane. The applet allows students to add, remove, adjust, and label points. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. Plot the points (0, 10),(1, 8),(2, 6),(3, 4),(4, 2).



2. What do you notice about the graph?

Sample responses:

- The points line up so that they could all be connected with a single line.
- The line goes down when reading left to right.
- Every time the x-coordinate goes up 1, the y-coordinate goes down 2.

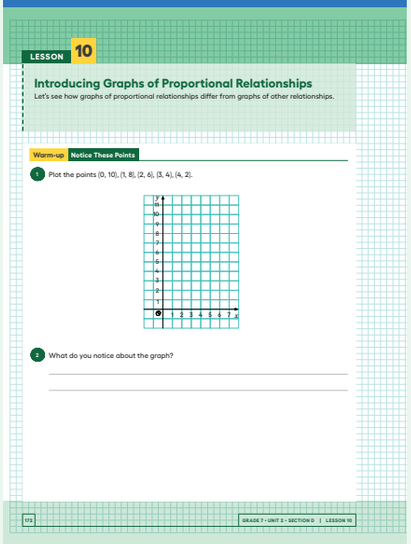
Activity Synthesis

The goal of this discussion is to review how to graph ordered pairs, (x, y), on the coordinate plane. Invite students to share their observations about the graph. Ask if other students agree. If some students do not agree that the points lie on a straight line, ask which points break the pattern and give students a chance to self-correct their work.

Building on Student Thinking

Some students may reverse the x- and y-coordinates when plotting points, such as plotting (10, 0) and (8, 1) instead of (0, 10) and (1, 8). Direct their attention to the x and y labels in the image and clarify that an ordered pair is written (x, y).

Student Workbook


LESSON 10

Introducing Graphs of Proportional Relationships

Let's see how graphs of proportional relationships differ from graphs of other relationships.

Warm-up Notice These Points

1. Plot the points (0, 10), (1, 8), (2, 6), (3, 4), (4, 2).

2. What do you notice about the graph?

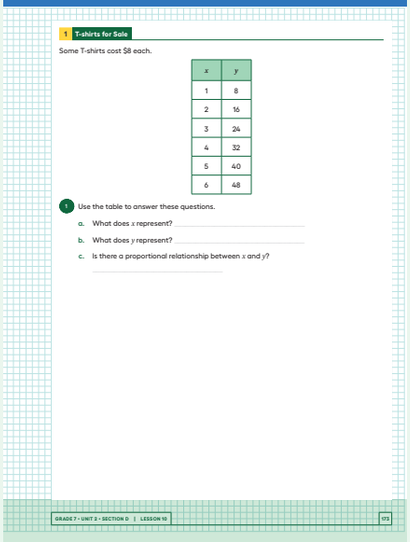
Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "The points are straight" as "The points line up on the coordinate plane so that they could all be connected with a single line."

Advances: Representing

Student Workbook



1 T-shirts for Sale

Some T-shirts cost $8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
   a. What does x represent?
   b. What does y represent?
   c. Is there a proportional relationship between x and y?

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Activity 1

T-shirts for Sale

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students create a graph to represent the proportional relationship given in a table. The goal is for students to notice that the points lie on a straight line that goes through the origin. The class discussion also prompts students to consider whether it makes sense to connect the points with a line.

In the digital version of the activity, students use an applet to plot points from a table on the coordinate plane. The applet allows students to add, remove, adjust, and label points. The digital version may help students graph quickly and accurately so they can focus more on the mathematical analysis.

Launch

Arrange students in groups of 2. Provide access to rulers. Give students 5 minutes of quiet work time followed by partner and whole-class discussion.

If students are unsure how to plot points from the table, consider rewriting the values in the first row of the table as an ordered pair, (1, 8), and demonstrating how to plot this point.

Student Task Statement

Some T-shirts cost $8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.

a. What does x represent?

x is the number of T-shirts

b. What does y represent?

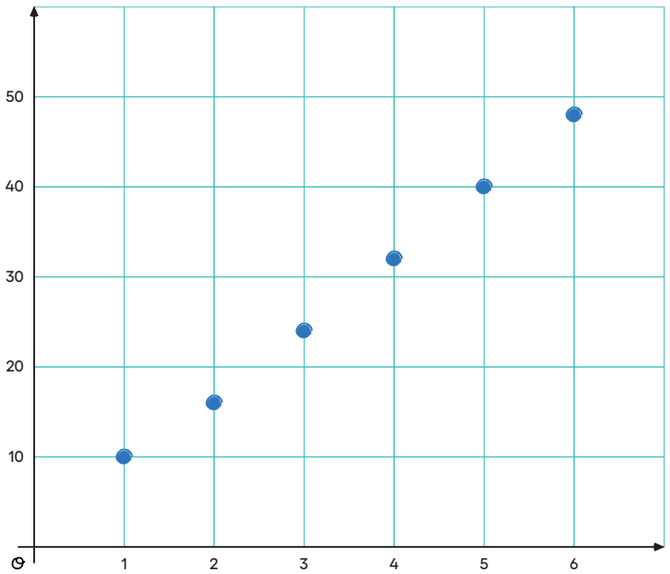
y is the total cost of those T-shirts

c. Is there a proportional relationship between x and y?

x is proportional to y

Students may identify 8 as the constant of proportionality.

2. Plot the pairs in the table on the **coordinate plane**.



3. What do you notice about the graph?

Students may notice that the points lie on a line.

Activity Synthesis

The goal of this discussion is to highlight the fact that the graph of a proportional relationship makes a straight line through the origin. Display a graph with the points plotted correctly for all to see. Invite students to share how to label the axes. (The x-axis represents "number of T-shirts" and the y-axis represents "cost in dollars".)

Ask students to share their observations about the plotted points. If not mentioned by students, highlight that the points lie on a straight line and that the line goes through (0, 0).

Direct students' attention to considering the meaning of other points that are also on this line but were not in the table. Ask,

- *"Could we buy 0 shirts? 7 T-shirts? 10 T-shirts? Can we buy half of a T-shirt?"*

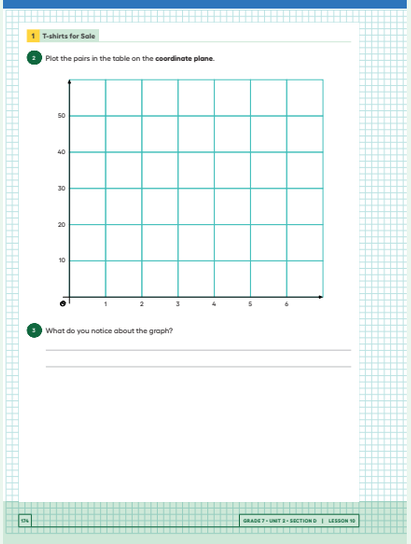
Note that the graph consists of discrete points because only whole numbers of T-shirts make sense in this context. However, people often connect discrete points with a line to make the relationship more clear, even when the in-between values don't make sense.

Ask the students,

- *"Suppose instead of price per shirt, this graph displayed the cost of cherries that are $8 per pound. Given that context, how should we change the graph?"*

Weights need not have integer values, so the graph is not restricted to discrete points. If you haven't done so already, draw the ray starting at (0, 0) that passes through the points.

Student Workbook


1 T-shirts for Sale

2 Plot the pairs in the table on the **coordinate plane**.

3 What do you notice about the graph?

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.

During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say ..." Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Instructional Routines

Card Sort

ilclass.com/r/10783726

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Instructional Routines

Take Turns

ilclass.com/r/10573524

Please log in to the site before using the QR code or URL.

Activity 2

Card Sort: Tables and Graphs

25 min

Activity Narrative

In this activity, students sort different graphs and tables that represent situations they have worked with during previous activities. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

Monitor for different ways groups choose to categorize the representations, but especially for categories that distinguish between proportional and nonproportional relationships. The purpose of this activity is to illustrate the idea that the graph of a proportional relationship is a line through the origin, though students will not have the tools for a formal explanation until grade 8.

Launch

Math Community

Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Arrange students in groups of 2 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 2 minutes to sort the cards into categories of their choosing.

- Pause for a class discussion, even if some groups haven't finished their sorting.
- Select groups to share their categories and how they started sorting their cards. If possible, select groups to share their categories in this order:
  - Separating the tables from the graphs
  - Matching pairs of a table and a graph that represent the same relationship
  - Separating the proportional relationships from the nonproportional relationships
- Discuss as many different types of categories as time allows.

Attend to the language that students use to describe their categories, giving them opportunities to describe the cards more precisely. Highlight the use of terms like "table," "graph," "proportional," "nonproportional," "straight line," and "corresponds."

After a brief discussion, invite students to complete the remaining questions.

Student Task Statement

Your teacher will give you a set of cards that show representations of relationships.

1. Sort the cards into categories of your choosing. Be prepared to describe your categories.
   Pause for a whole-class discussion.
2. Take turns with your partner to match a table with a graph.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

1H	2B	3G	4D	5A
6E	7F	8I	9C	10J

3. Which of the relationships are proportional?

2B	4D	7F	8I	9C

4. What do you notice about the graphs of proportional relationships? Do you think this will hold true for all graphs of proportional relationships?

   Sample responses: All points on a graph of a proportional relationship lie on a line. All such lines pass through (0, 0). The constant of proportionality can be seen in the graphs as the y-coordinate when x-coordinate is 1.

Are You Ready for More?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?

   The temperature graph could have negative coordinates because temperatures can be negative.

2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through (0, 0). What would the graph look like if k were a negative number?

   The line would still be through the origin, but it would slant downward from left to right.

Activity Synthesis

The goal of this discussion is to generalize that for all proportional relationships, the points lie on a straight line through the origin. Direct students' attention to the reference created using *Collect and Display.* Ask students to share how they determined which relationships were proportional. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.

Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual Processing, Organization, Memory

Building on Student Thinking

If students struggle to get started making any matches, ask questions like "How would we expect this row in the table to look on the graph?" or "See this point on the graph? What corresponds to it in the table?"

A common misunderstanding is to assume that if the points lie on a line, then the graph represents a proportional relationship. Ask questions about the table to assist students in realizing the error.

Student Workbook


2 Card Sort: Tables and Graphs

Your teacher will give you a set of cards that show representations of relationships.

1. Sort the cards into categories of your choosing. Be prepared to describe your categories. Pause for a whole-class discussion.
2. Take turns with your partner to match a table with a graph.
   a. For each match that you find, explain to your partner how you know it's a match.
   b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
3. Which of the relationships are proportional?
4. What do you notice about the graphs of proportional relationships? Do you think this will hold true for all graphs of proportional relationships?

Are you ready for more?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?
2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through (0, 0). What would the graph look like if k were a negative number?

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Ensure that students are in agreement that Cards B, C, D, F, and I are the graphs of proportional relationships. Then, invite students to share what they noticed about the characteristics of graphs of proportional relationships. Some observations might conclude:

- Points whose coordinates satisfy the relationship lie on a line.
- The line passes through the point (0, 0).

This would be a good place to either introduce the term **origin** to refer to the point (0, 0) or to remind students of it, if they have encountered it before.

If time permits, discuss which proportional relationships would warrant "connecting the dots." In other words, which proportional relationships are best represented with dots and which are best represented with an unbroken line? (It makes sense to draw an unbroken line for 7F and 8I. The rest should use dots that are not connected.) Students should realize that even when the graph of a proportional relationship is represented by unconnected points, they lie on a line through the origin.

Math Community

Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- *"I noticed our norm "_____" in action today, and it really helped me/my group because _____."*

Lesson Synthesis

Share with students,

- *"Today we examined graphs of relationships. Some were proportional and some were not."*

To help students generalize about graphs of proportional relationships, consider asking students:

- *"What characteristics were shared by all the graphs of proportional relationships that we saw?"*

  The points were arranged in a straight line. The point (0, 0) lines up with the other points.

- *"What characteristics might you see on a graph that would let you know that the relationship is not proportional?"*

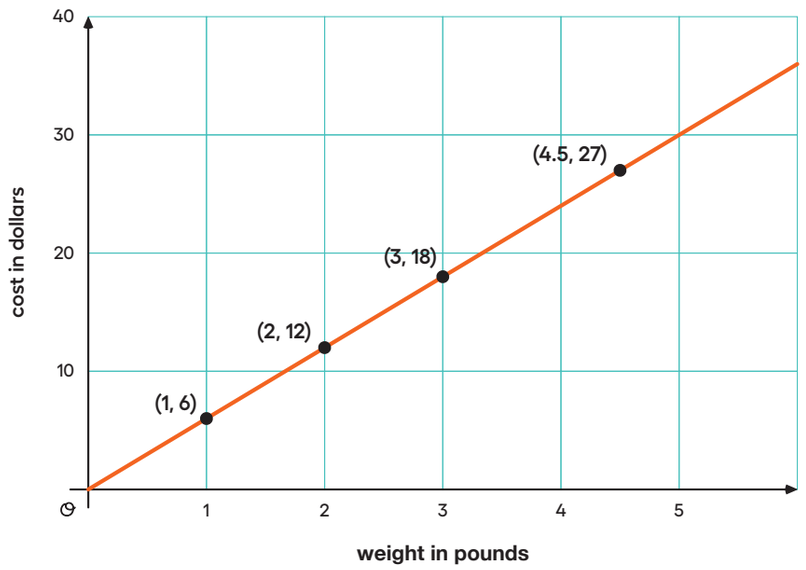
  The points are not arranged in a straight line. The point (0, 0) does not line up with the other points.

If desired, use the following quick activity to review these concepts. Tell students,

- *"Sketch a quick graph of a relationship that is or isn't proportional and trade sketches with your partner. Then, explain to your partner why their graph does or does not show a proportional relationship."*

Lesson Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, "Blueberries cost $6 per pound."



Different points on the graph tell us, for example, that 2 pounds of blueberries cost $12, and 4.5 pounds of blueberries cost $27.

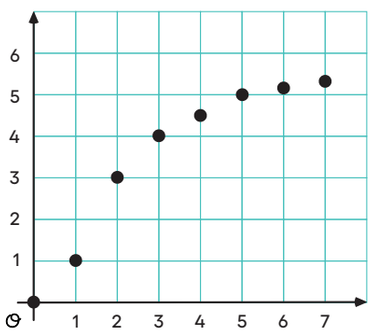
Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

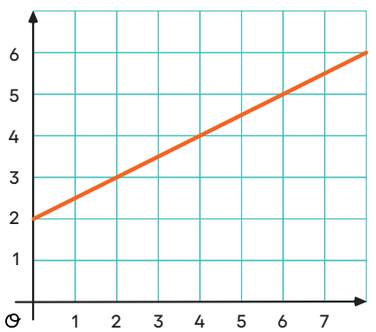
If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

Graphs that represent proportional relationships all have a few things in common:

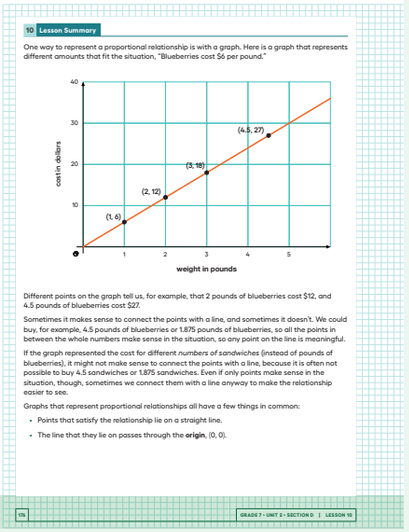
- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, (0, 0).

Here are some graphs that do *not* represent proportional relationships:





Student Workbook


10 Lesson Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, "Blueberries cost $6 per pound."

cost in dollars

weight in pounds

(1, 6) (2, 12) (3, 18) (4.5, 27)

Different points on the graph tell us, for example, that 2 pounds of blueberries cost $12, and 4.5 pounds of blueberries cost $27.

Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

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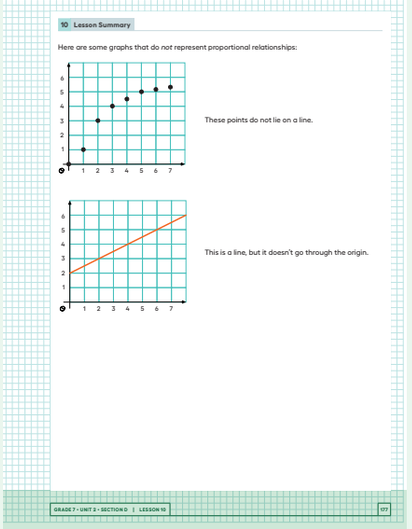
Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, (0, 0).

Student Workbook


10 Lesson Summary

Here are some graphs that do *not* represent proportional relationships:

These points do not lie on a line.

This is a line, but it doesn't go through the origin.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

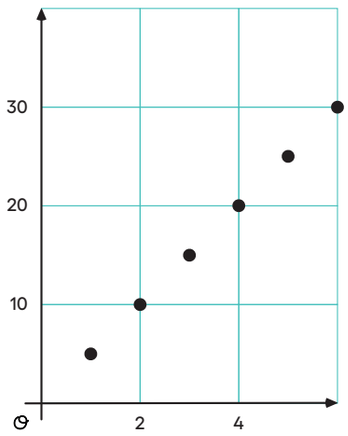
Which Are Not Proportional

5 min

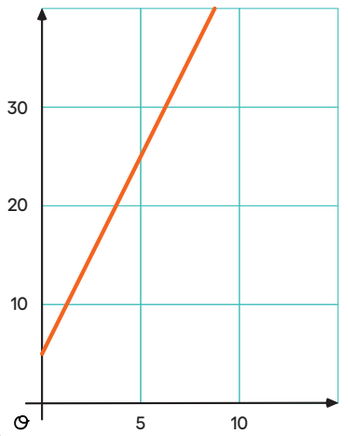
Student Task Statement

Which graphs cannot represent a proportional relationship? Select **all** that apply. Explain how you know.

A



B



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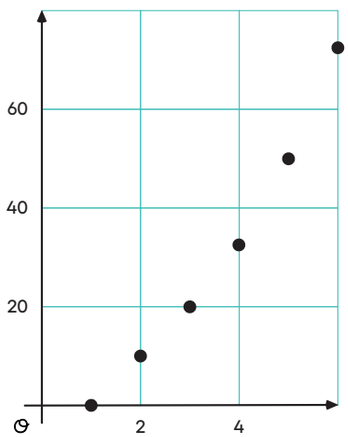
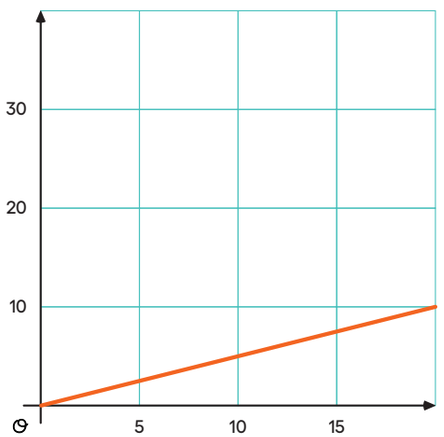


D



Sample reasoning: Since graph B does not go through the origin, it cannot be a proportional relationship. Since the points in graph C cannot be connected by a single, straight line, it cannot be a proportional relationship.

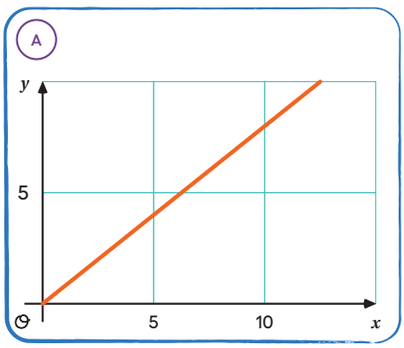
Lesson 10

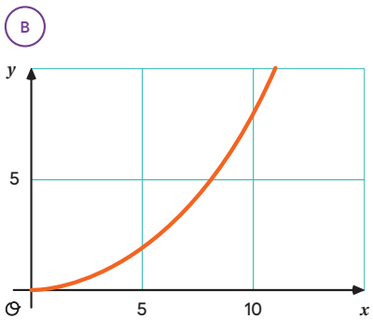
Practice Problems

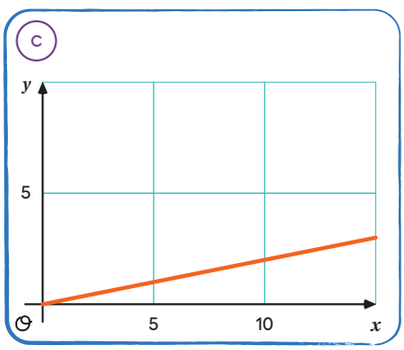
4 Problems

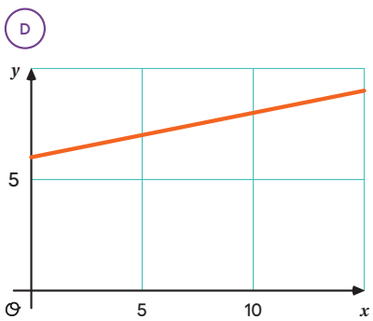
Problem 1

Which graphs could represent a proportional relationship?

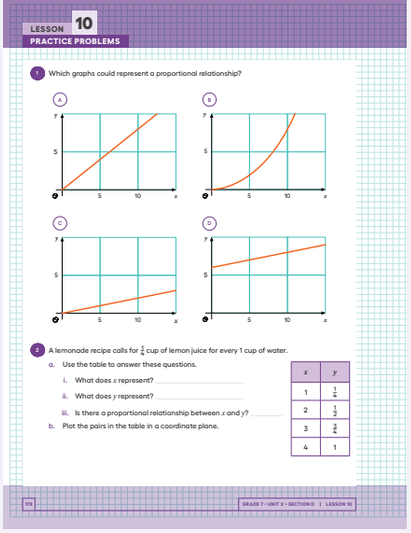








Student Workbook


LESSON 10
PRACTICE PROBLEMS

1 Which graphs could represent a proportional relationship?

A B C D

2 A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every 1 cup of water.

a. Use the table to answer these questions.

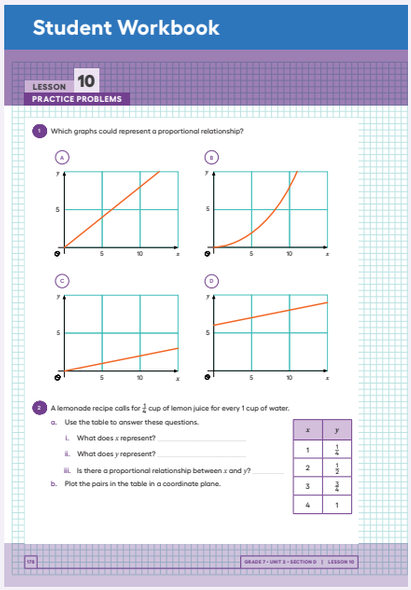
i. What does x represent? _____

ii. What does y represent? _____

iii. Is there a proportional relationship between x and y? _____

b. Plot the pairs in the table in a coordinate plane.

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1


Student Workbook

LESSON 10
PRACTICE PROBLEMS

1. Which graphs could represent a proportional relationship?

A

B

C

D

2. A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every 1 cup of water.

a. Use the table to answer these questions.

i. What does x represent? _____

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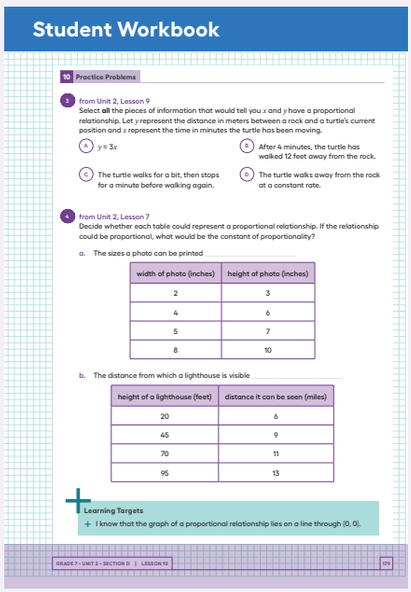
b. Plot the pairs in the table in a coordinate plane.

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1


Student Workbook

10 Practice Problems

3. from Unit 2, Lesson 9

Select **all** the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

A. $y = 3x$

B. After 4 minutes, the turtle has walked 12 feet away from the rock.

C. The turtle walks for a bit, then stops for a minute before walking again.

D. The turtle walks away from the rock at a constant rate.

4. from Unit 2, Lesson 7

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes a photo can be printed

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

b. The distance from which a lighthouse is visible

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Learning Targets

+ I know that the graph of a proportional relationship lies on a line through (0, 0).

Problem 2

A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every 1 cup of water.

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

a. Use the table to answer these questions.

i. What does x represent?

x represents the cups of water

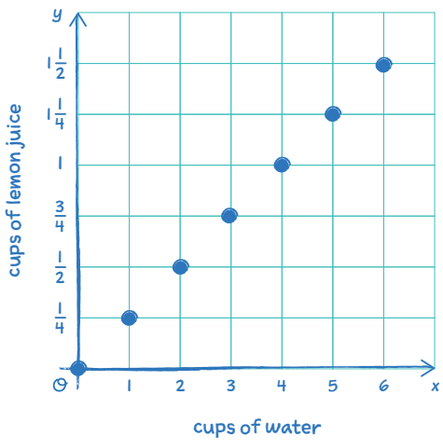
ii. What does y represent?

y represents the cups of lemon juice

iii. Is there a proportional relationship between x and y?

Yes

b. Plot the pairs in the table in a coordinate plane.



Problem 3

from Unit 2, Lesson 9

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Problem 4

from Unit 2, Lesson 7

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes a photo can be printed

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

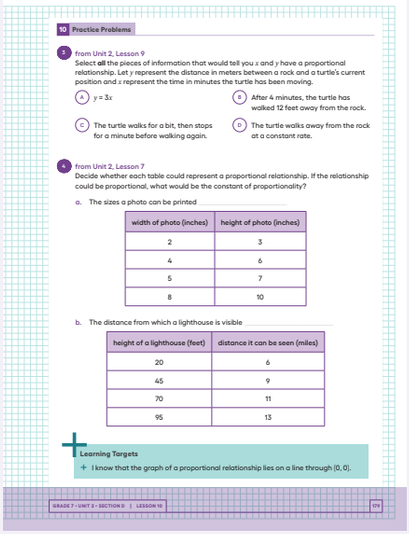
Not proportional, since the ratios of width to height are not all equivalent

b. The distance from which a lighthouse is visible

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Not proportional, since the ratios of height to distance are not all equivalent

Student Workbook



10 Practice Problems

3 from Unit 2, Lesson 9
Select **all** the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

A. $y = 3x$
B. After 4 minutes, the turtle has walked 12 feet away from the rock.
C. The turtle walks for a bit, then stops for a minute before walking again.
D. The turtle walks away from the rock at a constant rate.

4 from Unit 2, Lesson 7
Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes a photo can be printed

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

b. The distance from which a lighthouse is visible

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Learning Targets
+ I know that the graph of a proportional relationship lies on a line through (0, 0).

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