Lesson 9

Side Length Quotients in Similar Triangles

Goals

- Calculate unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.
- Generalize (orally) that the quotients of pairs of side lengths in similar triangles are equal.

Learning Targets

- I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.
- I can find missing side lengths in a pair of similar triangles using quotients of side lengths.

Lesson Narrative

The purpose of this lesson is for students to see that the quotient of a pair of side lengths in one triangle will always be equal to the quotient of the corresponding side lengths in a similar triangle. While this fact is not limited to triangles, this lesson focuses on the particular case of triangles so that students are ready to investigate the concept of slope in upcoming lessons.

The lesson begins with students determining if two triangles are similar given only their side lengths and no drawing, prompting them to attempt finding a common scale factor between corresponding sides. Next, students calculate the quotients of pairs of side lengths in similar triangles and find that the quotients for corresponding pairs of sides are the same. Finally, students use this information to calculate missing side lengths in a set of three similar triangles.

Student Learning Goal

Let's find missing side lengths in triangles.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up

Required Preparation

Warm-up:
Provide access to geometry toolkits.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	15 min	15 min	10 min	5 min

Assessment: Cool-down

Warm-up

Two-three-four and Four-five-six

5 min

Activity Narrative

In this activity, students identify whether two triangles are similar or not. Since no drawing is given, students will need to recognize that there is no single scale factor that multiplies all of the side lengths in one triangle to get the side lengths in the other triangle.

Launch

Provide access to geometry toolkits.

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Triangle A has side lengths 2, 3, and 4. Triangle B has side lengths 4, 5, and 6. Is Triangle A similar to Triangle B? Be prepared to explain your reasoning.

No

Sample reasoning: The shortest side in Triangle A is doubled to get the shortest side in Triangle B, but the longest side in Triangle A is multiplied by 1.5 to get the longest side in Triangle B. Since the scale factor is not the same for all side lengths, the two triangles are not similar.

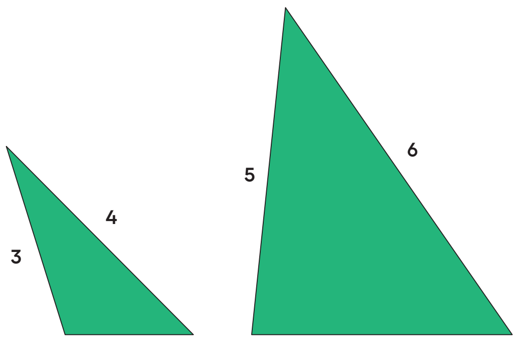
Activity Synthesis

The goal of this discussion is to make sure students understand that triangles cannot be similar if you cannot apply the same scale factor to each side of one triangle to get the corresponding sides of the other triangle.
Discuss with students:

"How can you tell if two figures are similar without drawing a diagram?"

Find a scale factor that takes the shortest side of one figure to the shortest side of the other figure. Test to see if the same scale factor works for all of the other sides.

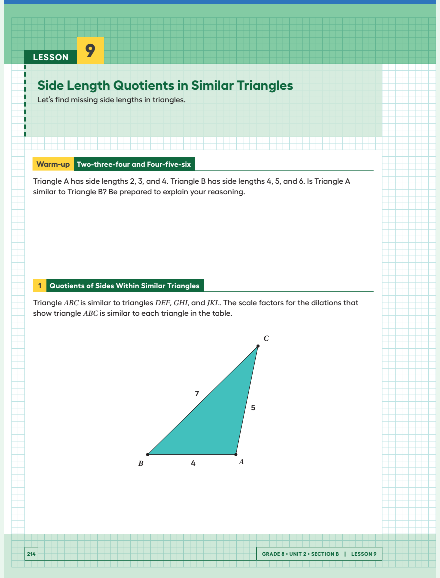
Display diagrams of the triangles for visual confirmation.



Building on Student Thinking

Some students may think that adding the same number to each side length will result in similar triangles. Draw a picture to help students see why this is not true.

Student Workbook


LESSON 9

Side Length Quotients in Similar Triangles

Let's find missing side lengths in triangles.

Warm-up Two-three-four and Four-five-six

Triangle A has side lengths 2, 3, and 4. Triangle B has side lengths 4, 5, and 6. Is Triangle A similar to Triangle B? Be prepared to explain your reasoning.

1 Quotients of Sides Within Similar Triangles

Triangle ABC is similar to triangles DEF, GHI, and JKL. The scale factors for the dilations that show triangle ABC is similar to each triangle in the table.

C
7
5
B
4
A

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Access for Students with Diverse Abilities (Activity 1, Launch)

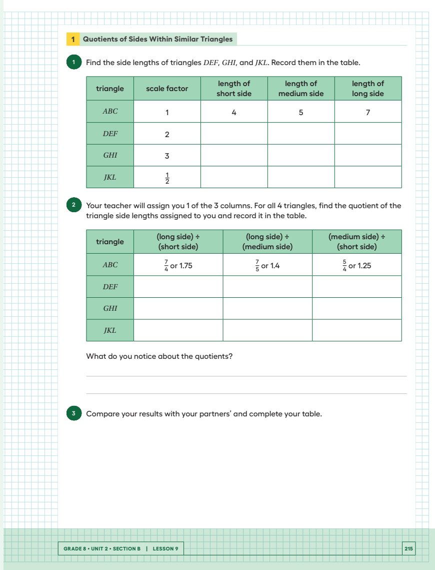
Engagement: Develop Effort and Persistence.
Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.

Supports accessibility for: Memory, Conceptual Processing

Building on Student Thinking

Some students may find quotients in fraction form. If necessary, prompt them to express their fractions in a way that helps them to recognize that the fractions are equivalent.

Student Workbook



1 Quotients of Sides Within Similar Triangles

1. Find the side lengths of triangles DEF, GHI, and JKL. Record them in the table.

triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

2. Your teacher will assign you 1 of the 3 columns. For all 4 triangles, find the quotient of the triangle side lengths assigned to you and record it in the table.

triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75	$\frac{7}{5}$ or 1.4	$\frac{5}{4}$ or 1.25
DEF			
GHI			
JKL			

What do you notice about the quotients?

3. Compare your results with your partners' and complete your table.

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Activity 1

Quotients of Sides Within Similar Triangles

10 min

Activity Narrative

While previous activities explored the ratio of side lengths between similar triangles, this activity explores ratios of side lengths within similar triangles. and how these compare for similar triangles. For example, if a and b are two side lengths of a triangle, then the corresponding side lengths of a similar triangle have lengths sa and sb for some positive scale factor s, and the ratios $a:b$ and $sa:sb$ are equivalent. By repeatedly dividing one side length of a triangle by another side length of the same triangle, students determine that the quotients of pairs of side lengths in similar triangles are equal.

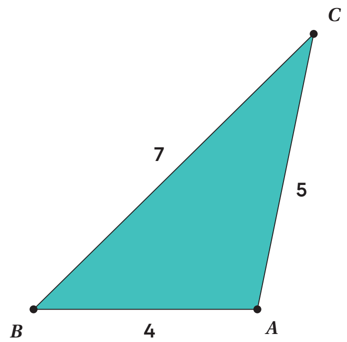
Launch

Arrange students in groups of 3. Assign each of the columns in the second table to 1 student in each group.

Give students 5 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

Triangle ABC is similar to triangles DEF, GHI, and JKL. The scale factors for the dilations that show triangle ABC is similar to each triangle in the table.



1. Find the side lengths of triangles DEF, GHI, and JKL. Record them in the table.

triangle	scale factor	length of short side	length of medium side	length of medium side
ABC	1	4	5	7
DEF	2	8	10	14
GHI	3	12	15	21
JKL	$\frac{1}{2}$	2	2.5	3.5

2. Your teacher will assign you 1 of the 3 columns. For all 4 triangles, find the quotient of the triangle side lengths assigned to you and record it in the table. What do you notice about the quotients?

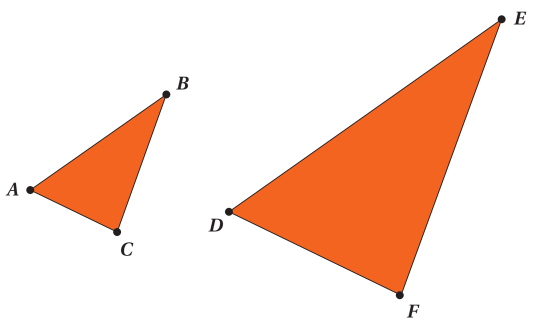
triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75	$\frac{7}{5}$ or 1.4	$\frac{5}{4}$ or 1.25
DEF	$\frac{14}{8}$ or 1.75	$\frac{14}{10}$ or 1.4	$\frac{10}{8}$ or 1.25
GHI	$\frac{21}{12}$ or 1.75	$\frac{21}{15}$ or 1.4	$\frac{15}{12}$ or 1.25
JKL	$\frac{3.5}{2}$ or 1.75	$\frac{3.5}{2.5}$ or 1.4	$\frac{2.5}{2}$ or 1.25

3. Compare your results with your partners' and complete your table.

No answer needed

Are You Ready for More?

Triangles ABC and DEF are similar. Explain why $\frac{AB}{BC} = \frac{DE}{EF}$.



There is a scale factor s such that $s \cdot AB = DE$ and $s \cdot BC = EF$. So $\frac{s \cdot AB}{s \cdot BC} = \frac{DE}{EF}$, and $\frac{AB}{BC} = \frac{DE}{EF}$.

Activity Synthesis

The goal of this discussion is to make sure students understand that quotients of corresponding side lengths in similar triangles are equivalent.

Discuss with students:

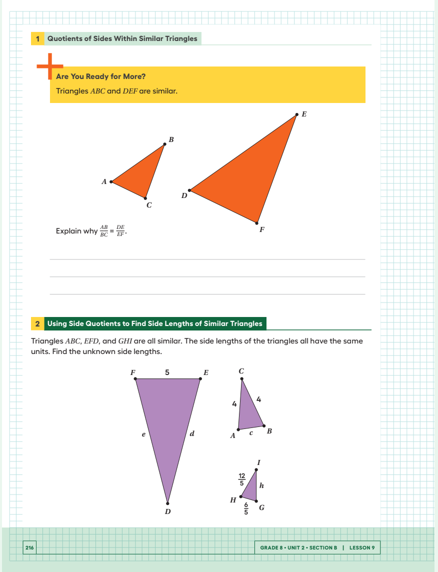
"For the triangles examined, what would the value of (medium side) ÷ (long side) be?"

$\frac{5}{7}$

"Do you think the value of (medium side) ÷ (long side) would be $\frac{5}{7}$ for any triangle similar to triangle ABC?"

Yes. Any triangle similar to ABC will have side lengths that are multiples of 4, 5, and 7. The medium side divided by the long side will always be a fraction equivalent to $\frac{5}{7}$.

Student Workbook



Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports. Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class. Display a sentence frame to support student-student rehearsals: "The value of the medium side divided by the long side would be _____ for any triangle similar to triangle ABC because ..."

Advances: Speaking

Activity 2

Using Side Quotients to Find Side Lengths of Similar Triangles

20 min

Activity Narrative

In this activity, students calculate side lengths of similar triangles. They need to think strategically about which side lengths to calculate first since there are many missing values. As they discover more side lengths, more paths for finding the remaining values open up.

Monitor for students who use these different strategies:

- Uses (external) scale factors to move from one triangle to another.
- Uses quotients of corresponding side lengths within a triangle (internal scale factors).

Launch

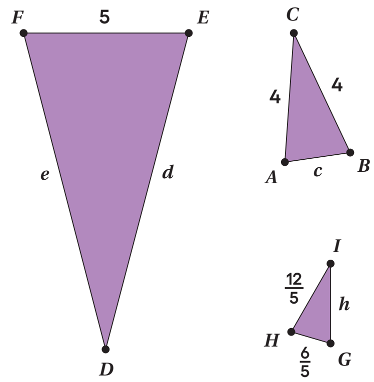
Arrange students in groups of 2.

Give students 5 minutes of quiet work time followed by a partner then whole-class discussion.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

Triangles ABC, EFD, and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths.



$c = 2$, $d = 10$, $e = 10$, $h = \frac{12}{5}$ (or equivalent)

Instructional Routines

MLR7: Compare and Connect

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2, Narrative)

MLR7: Compare and Connect.
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code corresponding sides of similar triangles and the ratios of their lengths.

Supports accessibility for: Visual-Spatial Processing

Building on Student Thinking

Some students may have trouble locating corresponding sides. Suggest they use tracing paper to rotate and or translate the triangles. Another technique is to color corresponding side lengths the same color. For example, they could color AB, EF, and GH all red.

Activity Synthesis

The goal of this discussion is to emphasize how multiple relationships can be used to find side lengths of similar triangles. Display 2–3 strategies from previously selected students for all to see. If time allows, invite students to briefly describe their strategies. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the different strategies have in common? How are they different?"

"Did anyone solve the problem the same way, but would explain it differently?"

"How does each method represent scale factor?"

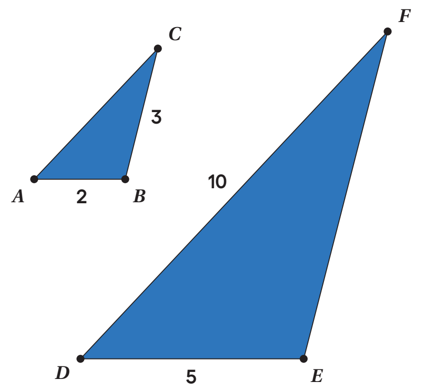
"Are there any benefits or drawbacks to one method compared to another?"

Both methods are efficient and the method to use is guided by what information is missing and the numbers involved in the calculations. Some key points to highlight are:

- Triangle ABC has 2 equal side lengths, so the other 2 triangles will as well. This insight is efficient for finding h.
- One side of triangle GHI is twice the length of another side, so this will be true for the other triangles as well. This insight is helpful for finding c, d, and e.

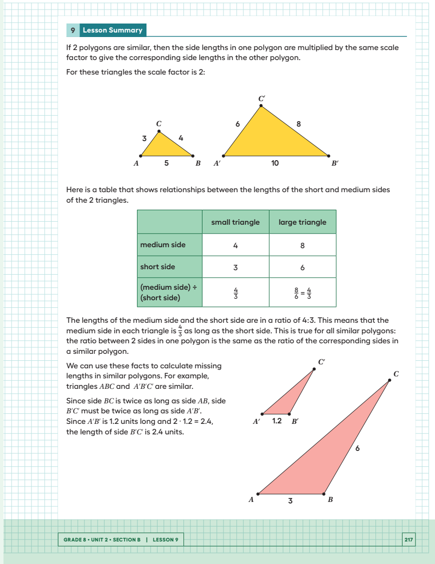
Lesson Synthesis

The purpose of this discussion is to compare the two different ways for finding the side lengths of similar triangles—using scale factors to to move from one triangle to another and using quotients of corresponding side lengths within the triangles. Display the image for all to see.



Tell students that these 2 triangles are similar, and since sides AB and DE are corresponding sides, the scale factor is $\frac{5}{2}$ or 2.5. Explain how there are 2 ways to find the length of AC. One way is to divide corresponding side DF by the scale factor $\frac{5}{2}$ giving a length of 4. Another way to do this is to notice that DF is twice the length of DE. This means that AC is twice the length of AB, which also gives 4.

Student Workbook

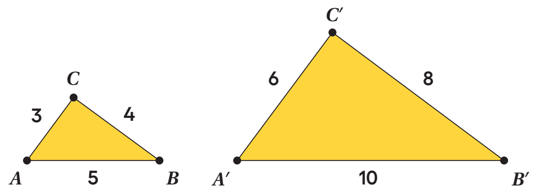


Sometimes both methods for calculating missing side lengths are equally effective. Invite students to use one of these strategies to calculate the length of EF. If time allows, ask several students to share their answers and reasoning. Side EF is $\frac{5}{2}$ the length of corresponding side BC, or 7.5. Side EF is also $\frac{3}{2}$ the length of DE, again 7.5.

Lesson Summary

If 2 polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon.

For these triangles the scale factor is 2:



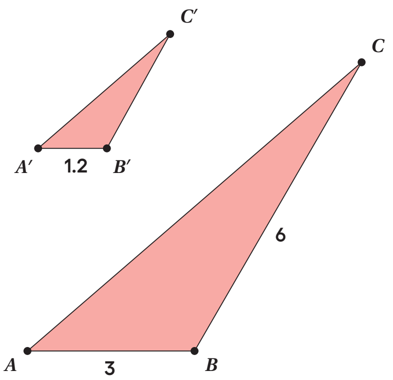
Here is a table that shows relationships between the lengths of the short and medium sides of the 2 triangles.

	small triangle	large triangle
medium side	4	8
short side	3	6
(medium side) ÷ (short side)	$\frac{4}{3}$	$\frac{8}{6} = \frac{4}{3}$

The lengths of the medium side and the short side are in a ratio of 4:3. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side. This is true for all similar polygons: the ratio between 2 sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

We can use these facts to calculate missing lengths in similar polygons. For example, triangles ABC and $A'B'C'$ are similar.

Since side BC is twice as long as side AB, side $B'C'$ must be twice as long as side $A'B'$. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.



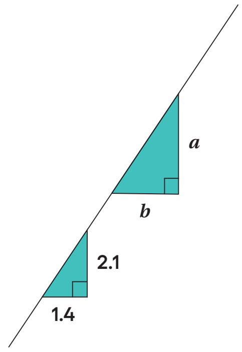
Cool-down

Similar Sides

5 min

Student Task Statement

The 2 triangles shown are similar. Find the value of $\frac{a}{b}$.



$\frac{3}{2}$ or 1.5 (or equivalent)

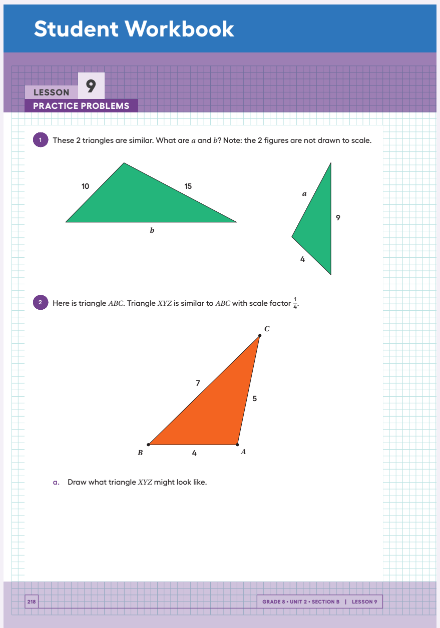
Responding To Student Thinking

Points to Emphasize

If students struggle with calculating quotients of side lengths in similar triangles, as opportunities arise over the next several lessons, revisit the idea that quotients of side lengths in similar figures are proportional.

For example, as students complete the table in the activity referred to here, invite multiple students to share their thinking about why the quotients of the vertical and horizontal side lengths are always equal for slope triangles whose longest sides lie on the same line.

Unit 2, Lesson 10, Activity 1 Similar Triangles on the Same Line

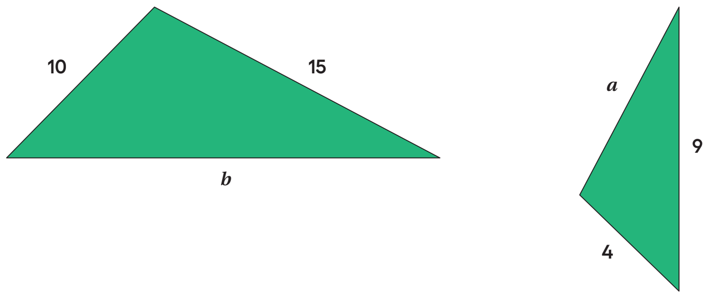

Student Workbook
LESSON 9
PRACTICE PROBLEMS
1 These 2 triangles are similar. What are a and b? Note: the 2 figures are not drawn to scale.
2 Here is triangle ABC. Triangle XYZ is similar to ABC with scale factor $\frac{1}{4}$.
a. Draw what triangle XYZ might look like.

Practice Problems

4 Problems

Problem 1

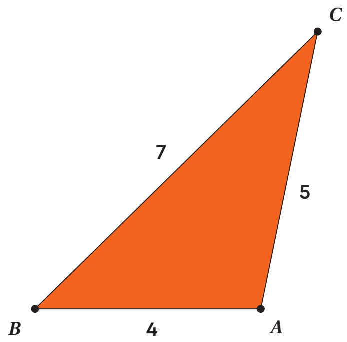
These 2 triangles are similar. What are a and b? Note: the 2 figures are not drawn to scale.



$a = 6$, $b = 22.5$

Problem 2

Here is triangle ABC. Triangle XYZ is similar to ABC with scale factor $\frac{1}{4}$.



a. Draw what triangle XYZ might look like.

Sample response: A similar triangle with lengths drawn approximately $\frac{1}{4}$ of the corresponding side lengths in triangle ABC.

b. How do the angle measures of triangle XYZ compare to triangle ABC? Explain how you know.

The angle measures are the same.

Sample reasoning: In similar polygons, corresponding angles are congruent.

c. What are the side lengths of triangle XYZ?

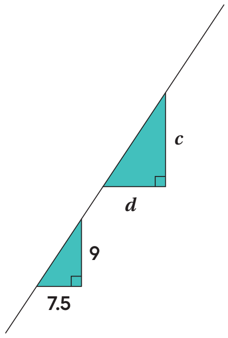
1 unit, $\frac{5}{4}$ unit, and $\frac{7}{4}$ unit

d. For triangle XYZ, calculate (long side) ÷ (medium side), and compare to triangle ABC.

The result is $\frac{7}{5}$ unit, the same as the corresponding result for triangle ABC.

Problem 3

The 2 triangles shown are similar. Find the value of $\frac{d}{c}$.

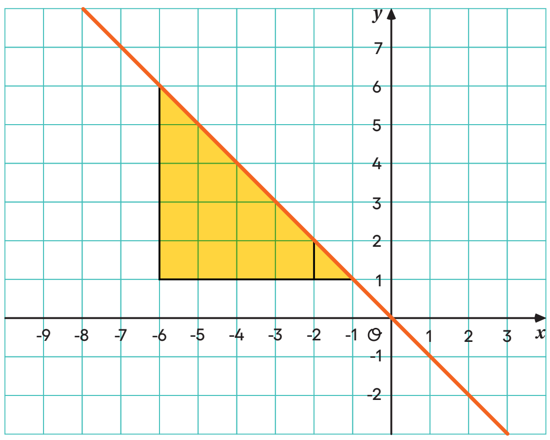


$\frac{5}{6}$ (or equivalent)

Problem 4

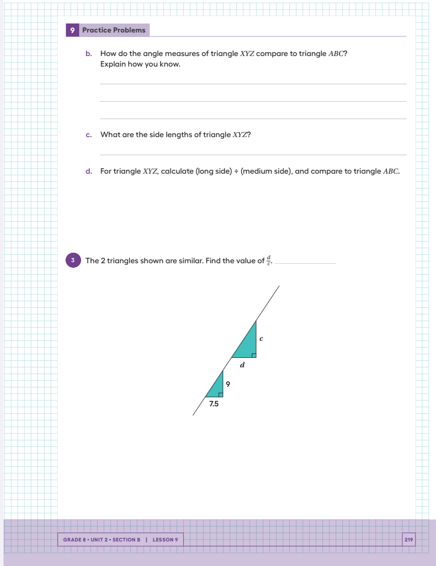
from Unit 2, Lesson 5

The diagram shows 2 nested triangles that share a vertex. Find a center and a scale factor for a dilation that would move the larger triangle to the smaller triangle.



Center: (-1, 1), scale factor: $\frac{1}{5}$ (or equivalent)

Student Workbook



Student Workbook