Lesson 19

Expanding and Factoring

Goals

- Apply the distributive property to expand or factor an expression that includes negative coefficients, and explain (orally and using other representations) the reasoning.
- Comprehend the terms "expand" and "factor" (in spoken and written language) in relation to the distributive property.

Learning Targets

- I can organize my work when I use the distributive property.
- I can use the distributive property to rewrite expressions with positive and negative numbers.
- I understand that "factoring" and "expanding" are words used to describe using the distributive property to write equivalent expressions.

Lesson Narrative

In this lesson, students practice using the distributive property to write equivalent expressions when there are rational coefficients. Students learn the terms **factor** and **expand** to describe applying the distributive property in opposite directions, that is, changing a sum of products to a product of a factor and a sum, or vice versa. Some of the expressions that students factor have a variable as the common factor instead of a constant, such as $17a - 13a = a(17 - 13)$. This shows how the distributive property is the basis for combining like terms. As students apply the distributive property to factor or expand expressions, they make use of structure.

Student Learning Goal

Let's use the distributive property to write expressions in different ways.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

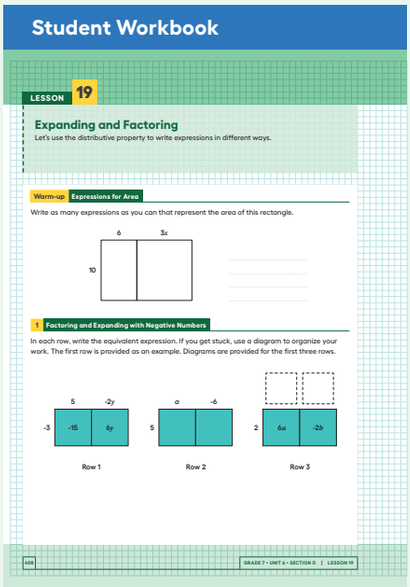
- Critique, Correct, Clarify (Activity 1)
- MLR8 (Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR8: Discussion Supports

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
5 min	20 min	10 min	10 min	5 min
				Cool-down


Student Workbook

LESSON 19

Expanding and Factoring

Let's use the distributive property to write expressions in different ways.

Warm-up Expressions for Area

Write as many expressions as you can that represent the area of this rectangle.

1 Factoring and Expanding with Negative Numbers

In each row, write the equivalent expression. If you get stuck, use a diagram to organize your work. The first row is provided as an example. Diagrams are provided for the first three rows.

Row 1 Row 2 Row 3

Warm-up

Expressions for Area

5 min

Activity Narrative

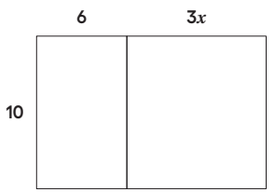
The purpose of this *Warm-up* is to reinforce the meaning of an area diagram, writing the area in two distinct ways to illustrate the distributive property. In this lesson, students will use the distributive property for increasingly complex expressions, and the area diagram is a useful support.

Launch

Give students 2 minutes to write as many expressions as they can think of.

Student Task Statement

Write as many expressions as you can that represent the area of this rectangle.



Sample responses:

- $(6 + 3x) \cdot 10$
- $60 + 30x$
- $30(x + 2)$
- $10(3x + 6)$
- $5(6x + 12)$

Activity Synthesis

The purpose of this discussion is to recall different ways to think about the area of the entire rectangle in order to write equivalent expressions. Ask a student to share an expression and record it for all to see. Ask if students agree that the expression represents the area of the rectangle, and if not, how they would amend it. Then ask for additional expressions and record them. Ensure at least one expression involves the height by the total width, such as $10(6 + 3x)$, and another expression involves the sum of the areas of the two smaller rectangles, such as $60 + 30x$.

Activity 1
Factoring and Expanding with Negative Numbers

20 min

Activity Narrative

In this activity, students practice rewriting expressions using the distributive property. Each expression involves subtraction or negative numbers.

Before completing the task, students are given an example student statement that is intentionally unclear, incorrect, or incomplete. Students critique the statement and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Arrange students in groups of 2. Instruct them to take turns writing an equivalent expression for each row. One partner writes the equivalent expression and explains their reasoning, while the other listens. If the partner disagrees, they work to resolve the discrepancy before moving to the next row.

Draw students' attention to the organizers that appear above the table, and tell them that these correspond to the first three rows in the table. Let students know that they are encouraged to draw more organizers like this for other rows, as needed.

Use *Critique, Correct, Clarify* to give students an opportunity to improve sample reasoning for why an expression is equivalent by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

💬 *"The expression -15 + 6y is equivalent to -3(5 – 2y) because you times by -3."*

Ask,

💬 *"What parts of this response are unclear, incorrect, or incomplete?"*

As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Display and review these criteria:
- Using "distributed" or "distributive property"
- Using "factor," "expand," or "terms."
- Rewriting subtraction as adding the opposite
- Rearranging terms that are being added
- Labeled an area diagram

- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners

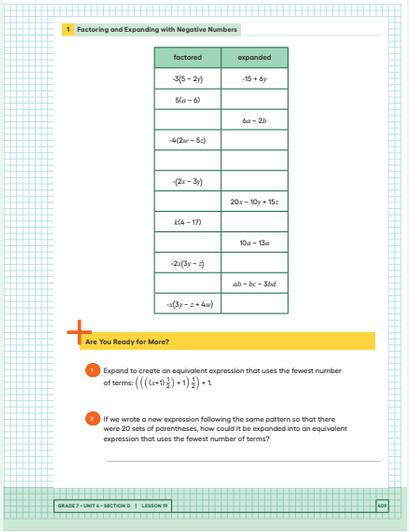
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.v

Access for Students with Diverse Abilities (Activity 1, Task Statement)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, color code each term of the factored and expanded expressions, in both the area diagram and the first row of the table.

Supports accessibility for: Visual-Spatial Processing

Student Workbook


1 Factoring and Expanding with Negative Numbers

factored	expanded
$-3(5-2y)$	$-15+6y$
$5(a-6)$	
	$6a-2b$
$-4(2w-5z)$	
$-(2x-3y)$	
	$20x-10y+15z$
$k(4-17)$	
	$10a-13a$
$-2x(3y-z)$	
	$ab-bc-3bd$
$-x(3y-z+4w)$	

Are You Ready for More?

1. Expand to create an equivalent expression that uses the fewest number of terms: $(((x+1)\frac{1}{2})+1)\frac{1}{2})+1$.

2. If we wrote a new expression following the same pattern so that there were 20 sets of parentheses, how could it be expanded into an equivalent expression that uses the fewest number of terms?

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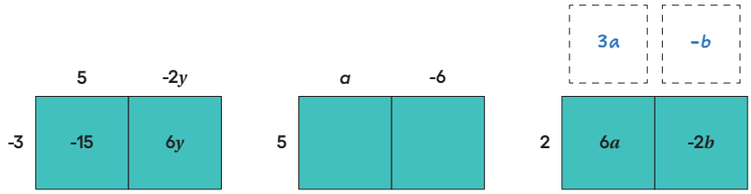
Building on Student Thinking

If students are unsure how to proceed, consider asking them to draw an organizer to represent the terms in the expression or to think about how subtraction can be rewritten as adding the opposite.

Student Task Statement

In each row, write the equivalent expression. If you get stuck, use a diagram to organize your work. The first row is provided as an example. Diagrams are provided for the first three rows.



factored	expanded
$-3(5-2y)$	$-15+6y$
$5(a-6)$	$5a-30$
$2(3a-b)$	$6a-2b$
$-4(2w-5z)$	$-8w+20z$
$-(2x-3y)$	$-2x+3y$
$5(4x-2y+3z)$	$20x-10y+15z$
$k(4-17)$	$4k-17k$
$a(10-13)$	$10a-13a$
$-2x(3y-z)$	$-6xy+2xz$
$b(a-c-3d)$	$ab-bc-3bd$
$-x(3y-z+4w)$	$-3xy+xz-4xw$

Expressions equivalent to these are also acceptable. For example, instead of $a(10-13)$, one could write $(10-13)\cdot a$ or $-3a$.

Are You Ready for More?

1. Expand to create an equivalent expression that uses the fewest number of terms: $\left(\left(\left((x+1)\frac{1}{2}\right)+1\right)\frac{1}{2}\right)+1$.

   $\frac{1}{4}(x+7)$ (or equivalent)

2. If we wrote a new expression following the same pattern so that there were 20 sets of parentheses, how could it be expanded into an equivalent expression that uses the fewest number of terms?

   $\frac{1}{2^{10}}(x+2^{11}-1)$ (or equivalent)

Activity Synthesis

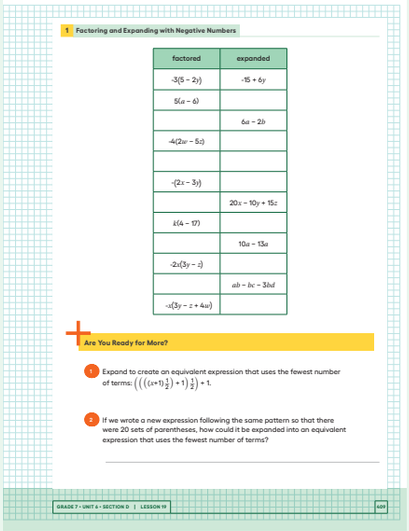
Much of the discussion will take place in small groups. The purpose of the discussion is to use diagrams and carefully rewrite expressions to understand why expressions are equivalent.

Display the correct equivalent expressions and work to resolve any discrepancies. Expanding the term $-(2x-3y)$ may require particular care. One way to interpret it is to rewrite as $-1 \cdot (2x-3y)$. If any confusion about handling subtraction arises, encourage students to employ the strategy of rewriting subtraction as adding the opposite.

To wrap up the activity, ask:

- *"Which rows did you and your partner disagree about? How did you resolve the disagreement?"*
- *"Which rows are you the most unsure about?"*
- *"Why are all the expressions in this column called 'factored expressions'? Why are these called 'expanded expressions'?"*
- *"Describe a process or procedure for taking a factored expression and writing its corresponding expanded expression."*
- *"Describe a process or procedure for taking an expanded expression and writing its corresponding factored expression."*

Student Workbook



1 Factoring and Expanding with Negative Numbers

factored	expanded
$-3(5-2y)$	$-15+6y$
$5(a-b)$	
	$6a-2b$
$-4(2w-5z)$	
$-(2x-3y)$	
	$20x-10y+15z$
$k(4-17)$	
	$10a-13a$
$-2x(3y-z)$	
	$ab-bc-3bd$
$-x(3y-z+4w)$	

Are You Ready for More?

1. Expand to create an equivalent expression that uses the fewest number of terms: $\left(\left(\left((x+1)\frac{1}{2}\right)+1\right)\frac{1}{2}\right)+1$.

2. If we wrote a new expression following the same pattern so that there were 20 sets of parentheses, how could it be expanded into an equivalent expression that uses the fewest number of terms?

Instructional Routines

MLR8: Discussion Supports

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Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy, such as "I see a common factor is _____, so ..." "I noticed _____, so I ..." "What do you notice about ... ?" or "I agree/disagree because ..."

Supports accessibility for: Language, Social-Emotional Functioning

Access for Multilingual Learners (Activity 2, Synthesis)

MLR8: Discussion Supports.
Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____, so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Activity 2
Seeing Structure

10 min

Activity Narrative

In this activity, students factor more complex expressions. This activity is an opportunity to notice and make use of structure in order to apply the distributive property in more sophisticated ways.

Launch

Display the expression 18 – 45 + 27, and ask students to calculate as quickly as they can. Invite students to explain their strategies. If no student brings it up, ask if the three numbers have anything in common. (They are all multiples of 9.) One way to quickly compute would be to notice that 18 – 45 + 27 can be written as $2 \cdot 9 - 5 \cdot 9 + 3 \cdot 9$ or $(2 - 5 + 3) \cdot 9$ which can be quickly calculated as $0 \cdot 9$ or 0. Tell students that noticing common factors in expressions can help us write them with fewer terms, which can make the expressions easier to use.

If needed, remind students that the instruction "Factor each expression" means to apply the distributive property to rewrite each sum as a product: $ab + ac = a(b + c)$. The result is an equivalent expression with fewer terms.

Keep students in the same groups.

Give them 5 minutes of quiet work time and time to share their expressions with their partner, followed by a whole-class discussion.

Student Task Statement

Factor each expression. Be prepared to explain your reasoning.

1. $3 \cdot 15 + 4 \cdot 15 - 5 \cdot 15$

$(3 + 4 - 5) \cdot 15$

2. $3x + 4x - 5x$

$(3 + 4 - 5)x$

3. $3(x - 2) + 4(x - 2) - 5(x - 2)$

$2(x - 2)$

4. $3\left(\frac{5}{2}x + 6\frac{1}{2}\right) + 4\left(\frac{5}{2}x + 6\frac{1}{2}\right) - 5\left(\frac{5}{2}x + 6\frac{1}{2}\right)$

$2\left(\frac{5}{2}x + 6\frac{1}{2}\right)$

Accept all equivalent forms for each answer.

Activity Synthesis

The purpose of this discussion is to highlight the use of the distributive property. For each expression, invite a student to share their process for rewriting it with fewer terms.

Lesson Synthesis

Share with students,

- *"Today we learned to factor and expand expressions by using the distributive property."*

To review this new vocabulary, consider asking students:

- *"What does it mean to expand an expression?"*

  Multiply the factor outside the parentheses by each term inside the parentheses.

- *"Give an example of expanding an expression."*

  $5(3x - 4) = 15x - 20$

- *"What does it mean to factor an expression?"*

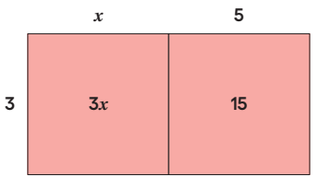
  Divide each term in the expression by a common factor and write that factor outside a set of parentheses.

- *"Give an example of factoring an expression."*

  $6x + 8 = 2(3x + 4)$

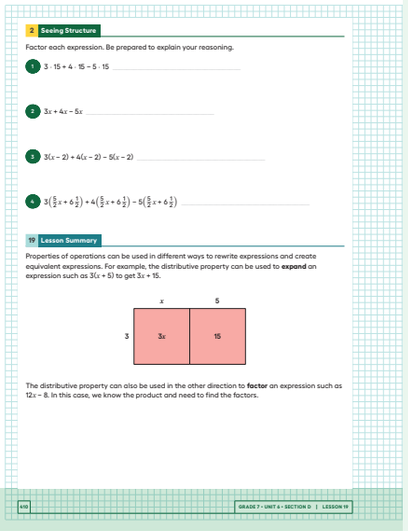
Lesson Summary

Properties of operations can be used in different ways to rewrite expressions and create equivalent expressions. For example, the distributive property can be used to **expand** an expression such as $3(x + 5)$ to get $3x + 15$.



The distributive property can also be used in the other direction to **factor** an expression such as $12x - 8$. In this case, we know the product and need to find the factors.

Student Workbook


2 Seeing Structure

Factor each expression. Be prepared to explain your reasoning.

1. $3 \cdot 15 + 4 \cdot 15 - 5 \cdot 15$
2. $3x + 4x - 5x$
3. $3(x - 2) + 4(x - 2) - 5(x - 2)$
4. $3\left(\frac{5}{2}x + 6\frac{1}{2}\right) + 4\left(\frac{5}{2}x + 6\frac{1}{2}\right) - 5\left(\frac{5}{2}x + 6\frac{1}{2}\right)$

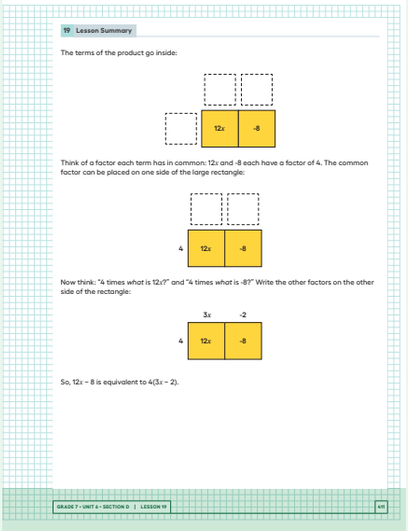
19 Lesson Summary

Properties of operations can be used in different ways to rewrite expressions and create equivalent expressions. For example, the distributive property can be used to **expand** an expression such as $3(x + 5)$ to get $3x + 15$.

x 5
3 3x 15

The distributive property can also be used in the other direction to **factor** an expression such as $12x - 8$. In this case, we know the product and need to find the factors.

Student Workbook


19 Lesson Summary

The terms of the product go inside:

12x -8

Think of a factor each term has in common: 12x and -8 each have a factor of 4. The common factor can be placed on one side of the large rectangle:

4 12x -8

Now think: "4 times what is 12x?" and "4 times what is -8?" Write the other factors on the other side of the rectangle:

3x -2

4 12x -8

So, 12x − 8 is equivalent to 4(3x − 2).

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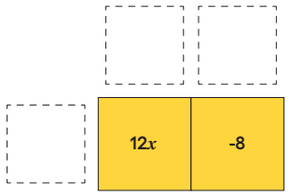
Responding To Student Thinking

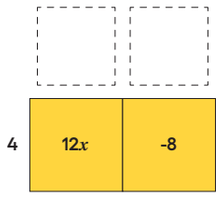
Points to Emphasize

If students struggle with applying the distributive property to factor and expand expressions, plan to review this concept when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about the parentheses in this activity: Grade 7, Unit 6, Lesson 21, Activity 1 Seeing It Differently

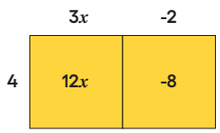
The terms of the product go inside:



Think of a factor each term has in common: $12x$ and -8 each have a factor of 4. The common factor can be placed on one side of the large rectangle:



Now think: "4 times *what* is 12x?" and "4 times *what* is –8?" Write the other factors on the other side of the rectangle:



So, $12x - 8$ is equivalent to $4(3x - 2)$.

Cool-down

Equivalent Expressions

5 min

Student Task Statement

1. Expand to write an equivalent expression: $-\frac{1}{2}(-2x + 4y)$

Sample response: $x - 2y$

2. Factor to write an equivalent expression: $26a - 10$

If you get stuck, use a diagram to organize your work.

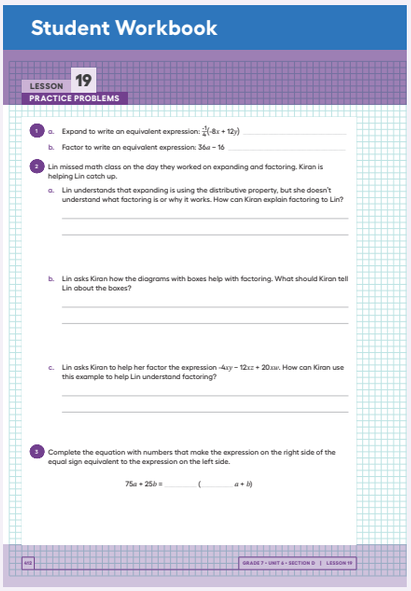
Sample response: $2(13a - 5)$

Expressions equivalent to these are also acceptable, such as $(13a - 5) \cdot 2$.

Lesson 19

Practice Problems

6 Problems



Problem 1

a. Expand to write an equivalent expression: $\frac{-1}{4}(-8x + 12y)$

Sample response: $2x - 3y$

b. Factor to write an equivalent expression: $36a - 16$

Sample response: $4(9a - 4)$ (or $2(18a - 8)$)

Problem 2

Lin missed math class on the day they worked on expanding and factoring. Kiran is helping Lin catch up.

a. Lin understands that expanding is using the distributive property, but she doesn't understand what factoring is or why it works. How can Kiran explain factoring to Lin?

Sample response: Factoring is the distributive property in the other direction. Instead of expanding a product to a sum of terms, factoring takes a sum of terms and makes it into a product by looking for common factors in the terms that can be written outside the parentheses.

b. Lin asks Kiran how the diagrams with boxes help with factoring. What should Kiran tell Lin about the boxes?

Sample response: The expression in each box is the product of the term to the left of the big rectangle and the term above each box, just as the area of a rectangle is length times width. Together, the boxes form a long rectangle, so it is still true that the term to the left of the big rectangle times the sum of the terms above each box equals the sum of all the terms in the boxes. If you want to factor an expression, look for a common factor in each box, and place it to the left of the rectangle. To decide what to write above each box, think, "What times that common factor equals what is in the box?"

c. Lin asks Kiran to help her factor the expression $-4xy - 12xz + 20xw$. How can Kiran use this example to help Lin understand factoring?

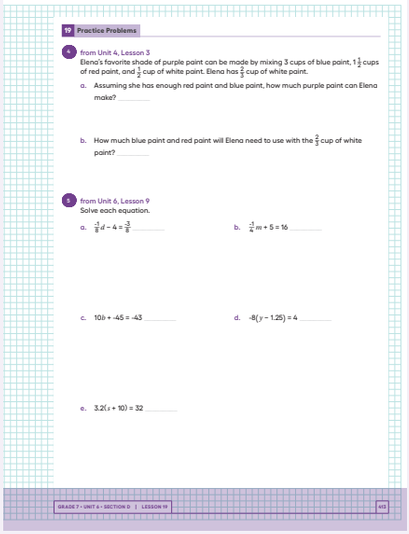
Sample response: First, find a common factor, like $4x$. Write "$4x(\ldots)$." We are going to decide what needs to go in the parentheses to make an expression equivalent to $-4xy - 12xz + 20xw$. To get $-4xy$, we need to multiply by $-y$. Using similar reasoning, we can fill in the rest: $4x(-y - 3z + 5w)$.

Problem 3

Complete the equation with numbers that make the expression on the right side of the equal sign equivalent to the expression on the left side.

$75a + 25b =$ 25$(3a + b)$

Student Workbook



19 Practice Problems

4 from Unit 4, Lesson 3
Elena's favorite shade of purple paint can be made by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ cup of white paint. Elena has $\frac{2}{3}$ cup of white paint.

a. Assuming she has enough red paint and blue paint, how much purple paint can Elena make? _____

b. How much blue paint and red paint will Elena need to use with the $\frac{2}{3}$ cup of white paint? _____

5 from Unit 6, Lesson 9
Solve each equation.

a. $\frac{-1}{8}d - 4 = \frac{-3}{8}$ _____

b. $\frac{-1}{4}m + 5 = 16$ _____

c. $10b + -45 = -43$ _____

d. $-8(y - 1.25) = 4$ _____

e. $3.2(s + 10) = 32$ _____

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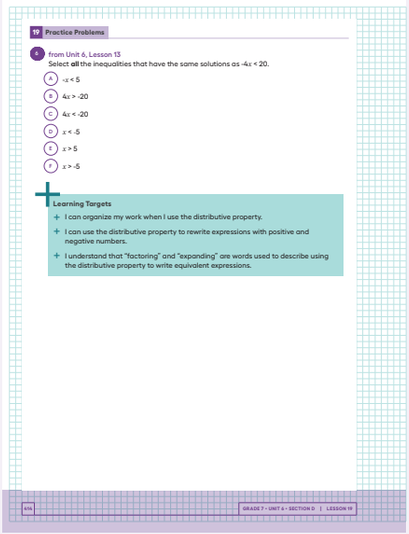
Student Workbook



19 Practice Problems

6 from Unit 6, Lesson 13
Select **all** the inequalities that have the same solutions as $-4x < 20$.

A. $-x < 5$
B. $4x > -20$
C. $4x < -20$
D. $x < -5$
E. $x > 5$
F. $x > -5$

Learning Targets
- I can organize my work when I use the distributive property.
- I can use the distributive property to rewrite expressions with positive and negative numbers.
- I understand that "factoring" and "expanding" are words used to describe using the distributive property to write equivalent expressions.

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Problem 4

from Unit 4, Lesson 3

Elena's favorite shade of purple paint can be made by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ cup of white paint. Elena has $\frac{2}{3}$ cup of white paint.

a. Assuming she has enough red paint and blue paint, how much purple paint can Elena make?

$\frac{20}{3}$ cups

Sample reasoning: One batch of purple paint makes 5 cups. Elena can make $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$ batches, so that's $\frac{20}{3}$ cups.

b. How much blue paint and red paint will Elena need to use with the $\frac{2}{3}$ cup of white paint?

4 cups of blue paint and 2 cups of red paint

Problem 5

from Unit 6, Lesson 9

Solve each equation.

a. $\frac{-1}{8}d - 4 = \frac{-3}{8}$

$d = -29$

b. $\frac{-1}{4}m + 5 = 16$

$m = -44$

c. $10b + -45 = -43$

$b = \frac{1}{5}$ (or equivalent)

d. $-8(y - 1.25) = 4$

$y = 0.75$ (or equivalent)

e. $3.2(s + 10) = 32$

$s = 0$

Problem 6

from Unit 6, Lesson 13

Select **all** the inequalities that have the same solutions as $-4x < 20$.

A. $-x < 5$

B. $4x > -20$

C. $4x < -20$

D. $x < -5$

E. $x > 5$

F. $x > -5$