Lesson 1

Writing Equivalent Equations

Goals

- Describe moves (orally and in writing) that can be used to create equivalent equations.
- Write equivalent equations using described moves.

Learning Target

I can describe moves that change one equation into an equivalent equation.

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1, Activity 2)

Instructional Routines

- MLR2: Collect and Display

Lesson Narrative

Students explore equivalent equations by rewriting expressions in multiple ways, then describing moves that result in equivalent equations. The focus of the lesson is on building a list of moves that result in equivalent equations and playing with ways to rewrite equations without a particular goal in mind. Students should be able to construct arguments for why their moves result in equivalent equations.

Student Learning Goal

Let's write equations in new ways.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Let Me Count the Ways

5 min

Activity Narrative

In this activity, students create equivalent expressions to given expressions. Students are encouraged to be creative in their answers, but should ensure that the new expressions are equivalent to the original. This work prepares students for writing equivalent equations in later activities.

Launch

Arrange students in groups of 2.

Display for all to see: $100, 100 \cdot 1, 98 + 1 + 1, \frac{200}{3-1}$. Ask students what they notice. Move to the task when students notice that all the expressions are equivalent to 100.

For the task, encourage students to be creative, but check with their partner that their expressions are still equivalent to the original. Tell students to try to give examples that they think nobody else in the class has written.

Student Task Statement

Write as many equivalent expressions for each as you have time.

A. 10

Sample responses: $9 + 1, 10 + 0, 12 - 2, \frac{30}{2+1}, 4 \cdot 4 - 3 \cdot 2$

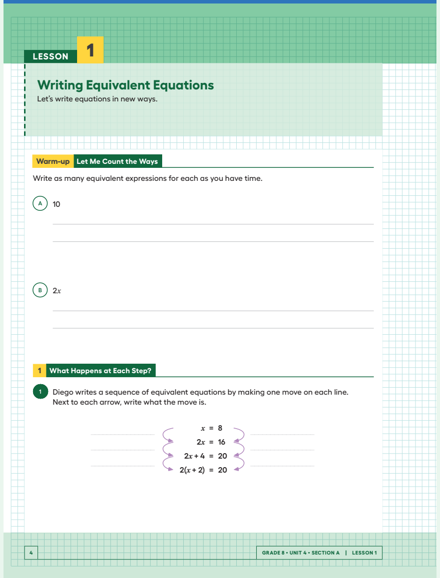
B. $2x$

Sample responses: $x \cdot 2, 10x - 8x, x + x, 3 + 2x - 3, \frac{40}{20}x$

Activity Synthesis

Invite 3–5 students to share examples that they have written for each expression. After each example, ask if other students can explain how it is equivalent to the original.

Student Workbook


LESSON 1

Writing Equivalent Equations

Let's write equations in new ways.

Warm-up Let Me Count the Ways

Write as many equivalent expressions for each as you have time.

(A) 10

(B) $2x$

1 What Happens at Each Step?

1. Diego writes a sequence of equivalent equations by making one move on each line. Next to each arrow, write what the move is.

$$\begin{aligned} x &= 8 \\ 2x &= 16 \\ 2x + 4 &= 20 \\ 2(x+2) &= 20 \end{aligned}$$

4 GRADE 8 • UNIT 4 • SECTION A | LESSON 1

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Activity 1

What Happens at Each Step?

15 min

Activity Narrative

In this activity, students first concentrate on identifying and informally recording moves used to create equivalent equations. They are introduced to recording the moves by using arrows that connect each equation to the next and labeling the arrows that show the moves. A distinction is made between moves that change how an expression on one side of the equation looks (distributive property, combine like terms) and moves that change each side of the equation (adding the same value to each side, multiplying the same value to each side). In the second problem, students practice making their own sequence of moves, starting from a simple equation, and recording their moves using the labeled arrows.

Students have solved these types of equations in previous grades. In this activity, students are encouraged to be more playful and to write equivalent equations that are not necessarily moving toward a goal of finding a solution.

Launch

Arrange students in groups of 2.

Display this equation for all to see: $2x = 10$. Remind students that, in the *Warm-up,* they were able to rewrite the expression on each side of this equation in many different ways. Ask students,

"What other types of moves are possible when rewriting an equation in another form?"

Doing operations like adding, subtracting, multiplying, or dividing each side of the equation by the same value.

Tell students that equivalent equations are equations that have the same solution. For example, $2x = 6$ and $2x + 1 = 7$ are equivalent equations because $x = 3$ is a solution for both of them.

Ask students,

"What is another way you could write an equation equivalent to $2x = 10$ using just 1 of these types of moves?"

After a brief quiet think time, select a student, and record the student's response for all to see below the original equation. Draw and label arrows connecting the equations explaining how the equation changed. For example,

$$\text{multiply by } \tfrac{1}{2} \qquad \begin{aligned} 2x &= 10 \\ x &= 5 \end{aligned} \qquad \text{multiply by } \tfrac{1}{2}$$

Ask students to think of another equation that is equivalent to the new equation. After a brief quiet think time, select a student to share their answer and description of the move. Record the response for all to see with arrows connecting the equations describing the move.

Tell students that it is important to explain their reasoning for writing equivalent equations and that we will begin by using the arrows.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe the moves. Display words and phrases such as "to each side," "distributive property," and "do the same thing on both sides."

Student Task Statement

1. Diego writes a sequence of equivalent equations by making one move on each line. Next to each arrow, write what the move is.

	$x = 8$	
multiply by 2	$2x = 16$	multiply by 2
add 4	$2x + 4 = 20$	add 4
distributive property	$2(x + 2) = 20$	distributive property

2. Write your own sequence of equivalent equations starting from $x = 12$. Use at least 3 different moves and write the moves next to each arrow.

Sample response:

	$x = 12$	
add 3	$x + 3 = 15$	add 3
multiply by 4	$4(x + 3) = 60$	multiply by 4
subtract 5	$4(x + 3) - 5 = 55$	subtract 5

Activity Synthesis

The goal of this discussion is to begin creating a shared language and way of writing descriptions of moves for writing equivalent equations.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their labels for the arrows in the first question. Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond.

Highlight the different ways in which students describe the moves going to the final row. Make sure students understand that when one side of the equation is changing how it looks—such as when using the distributive property or combining like terms—there may not be an equivalent move on the other side of the equation.

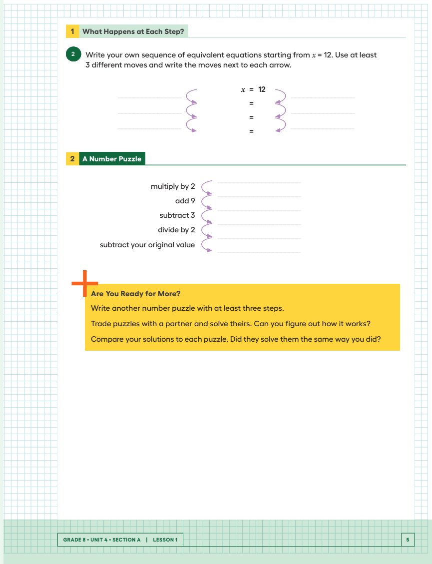
Establish a good convention to use with the class. In these materials, we are leaving the arrows blank to indicate that nothing has changed on that side of the equation. This can look unfinished, though. Encourage students to write "copy" or "no change" or use a similar phrase or indicator on the side of the equation opposite one of the moves that changes the look of an expression.

Invite 2–3 groups to share their sequence of equivalent equations for the last question. Display their equations for all to see. Ask other groups to provide the reasoning to label the arrows for each equivalent expression.

Building on Student Thinking

Students may write "factor" as the reasoning for the last step of the first question. This is okay, but remind students that the distributive property is also a valid description of what has happened and can be used to rewrite $2x + 4$ as $2(x + 2)$ or the other way around.

Student Workbook


1 What Happens at Each Step?

2 Write your own sequence of equivalent equations starting from $x = 12$. Use at least 3 different moves and write the moves next to each arrow.

$x = 12$

2 A Number Puzzle

multiply by 2
add 9
subtract 3
divide by 2
subtract your original value

Are You Ready for More?

Write another number puzzle with at least three steps.

Trade puzzles with a partner and solve theirs. Can you figure out how it works?

Compare your solutions to each puzzle. Did they solve them the same way you did?

GRADE 8 • UNIT 4 • SECTION A | LESSON 1

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Activity 2
A Number Puzzle

15 min

Activity Narrative

In this activity, students select a number and follow a series of instructions to change their number into a surprising result. Students then follow the same series of instructions for a variable to reveal how the result was not surprising at all, but a natural consequence of the instructions. This helps illustrate valid moves that can be done to an equation.

Monitor for students who combine like terms as they go as well as those who do not.

Launch

Begin the activity by telling students not to communicate with one another until instructed to do so. This includes talking or showing each other what is on their papers.

Tell students to select a number and write it on the first line. Then follow the instructions to write the correct number on the next line. Continue filling in the blanks until all have been filled.

After all students have had a chance to complete all of the instructions, tell students to compare the number on the last blank with one another.

Ask students,

"What do you notice about the final answers?"

Everyone got 3, regardless of their starting number.

Demonstrate how to begin the next part activity. Write "$5 = x$" on the first line, then draw an arrow on the right side of the equation and label it "multiply by 2," then write "$10 = 2x$" on the next line.

$$\begin{tikzcd} 5 \arrow[d, bend right, "\text{multiply by 2}"'] & = & x \arrow[d, bend left, "\text{multiply by 2}"] \\ 10 & = & 2x \end{tikzcd}$$

To see why this works, tell students to write "$= x$" on the right side of their paper next to their original number. They should continue writing equivalent equations using all of the instructions and blanks for the equation. For the last move, they should subtract x (their original number) on the right side to show what happens for whatever number might be chosen.

Select students who combine like terms as they follow the instructions, as well as those who do not. Ask them to share later.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier activity. Ask students to suggest ways to update the display:

"Are there any new words or phrases that you would like to add?"

"Is there any language you would like to revise or remove?"

Encourage students to use the display as a reference.

Student Task Statement

multiply by 2	$= 2x$
add 9	$= 2x + 9$
subtract 3	$= 2x + 9 - 3$ or $= 2x + 6$
divide by 2	$= \frac{2x + 9 - 3}{2}$ or $= x + 3$
subtract your original value	$= \frac{2x + 9 - 3}{2} - x$ or $= 3$

Are You Ready for More?

Write another number puzzle with at least three steps.

Trade puzzles with a partner and solve theirs. Can you figure out how it works?

Compare your solutions to each puzzle. Did they solve them the same way you did?

Sample response:

1. Think of a number.
2. Add 6.
3. Double it.
4. Add the original number.
5. Divide by 3.
6. Subtract 4.

The result is the original number again. This works because $\frac{2(x + 6) + x}{3} - 4 = x$.

Activity Synthesis

Invite selected students to share their equations involving x. If all students combine like terms, ask them what each equation would look like if they did not. For example, on the third line, write $= 2x + 9 - 3$ as well as $= 2x + 6$. Ask students,

"What are the benefits and drawbacks of writing it each way?"

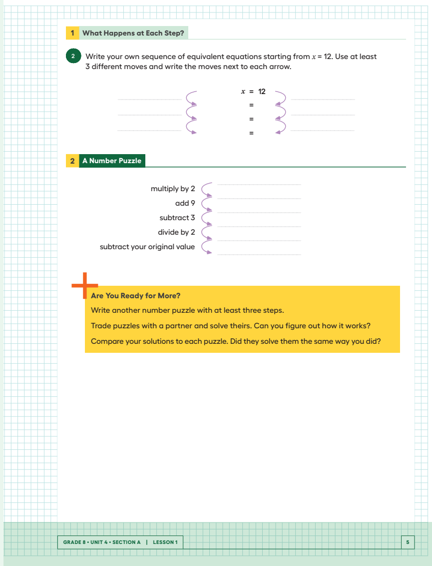
The first set of expressions show all of the instructions and how they affect x, but there are more terms so an instruction like "divide by 2" requires more work. The second set of expressions hides the instructions, but is easier to understand how to get the number in the blank.

Building on Student Thinking

If students divide only 1 term by 2 instead of the whole expression, consider asking:

"How can you check that each of your equations are equivalent?"
"How could you use substituting a value or x to help?"

Student Workbook



Tell students that all of these equations are equivalent because these are valid moves done correctly. Ask students what moves they have seen that create equivalent equations. Direct students' attention to the reference created using *Collect and Display*. Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond. Update the display to create a semi-permanent display of these valid moves to remain available in the classroom until the end of the unit.

- Use the distributive property. ($2(x + 6)$ is equivalent to $2x + 12$)
- Combine like terms. ($2x + 1 - x + 5$ is equivalent to $x + 6$)
- Add the same value on each side.
- Subtract the same value on each side.
- Multiply each side by the same non-zero value.
- Divide each side by the same non-zero value.

Lesson Synthesis

Display the partially completed set of equivalent equations.

$$\begin{array}{rcll} x & = & 5 & \\ & = & 7 & \text{add 2} \\ & = & 21 & \text{multiply by 3} \end{array}$$

Ask students,

- *"For these 3 equations to be equivalent, what should be written next to each arrow on the left?"*

  add 2 and multiply by 3

- *"What should the expressions on the left side of the equations be if the equations are equivalent?"*

  $x + 2$ and $3(x + 2)$ or $3x + 6$

Then display this pair of equations.

$$\begin{array}{rcl} 2(x+4) & = & 10 \\ 2x + 8 & = & 10 \end{array}$$

Ask students,

- *"What should we write next to each arrow?"*

  The left arrow should be labeled "distributive property." The right arrow should be labeled according to the class convention established in this lesson.

- *"Are these equations equivalent? Explain your reasoning."*

  Yes, because correctly applying the distributive property to one side of an equation is a valid move.

Lesson Summary

Equivalent equations are equations that we know are true for the same values of any variables. One way to create equivalent equations is to correctly use valid moves.

Valid moves include:

- Using the distributive property. ($2(x + 6)$ is equivalent to $2x + 12$)
- Combining like terms. ($2x + 1 - x + 5$ is equivalent to $x + 6$)
- Adding the same value to each side.
- Subtracting the same value from each side.
- Multiplying each side by the same non-zero value.
- Dividing each side by the same non-zero value.

For example, all of these equations are equivalent:

$$
\begin{array}{rrcll}
 & 2(3x+1)-5 & = & 15 & \\
\text{add } 5 & 2(3x+1) & = & 20 & \text{add } 5 \\
\text{distributive property} & 6x+2 & = & 20 & \\
\text{subtract } 2 & 6x & = & 18 & \text{subtract } 2 \\
\text{multiply by } \frac{1}{6} & x & = & 3 & \text{multiply by } \frac{1}{6}
\end{array}
$$

For these equations, the valid moves are used correctly, so all of the equations are equivalent. The last equation shows that 3 is the value for x that makes the equation true. Because all of the equations are equivalent, 3 is the value for x that makes each of these equations true.

Cool-down

Explain the Reasoning

5 min

Student Task Statement

$$
\begin{array}{rrcll}
 & 2x+3 & = & 7 & \\
\text{subtract 3 (or add -3)} & 2x & = & 4 & \text{subtract 3 (or add -3)} \\
\text{divide by 2 (or multiply by } \frac{1}{2}\text{)} & x & = & 2 & \text{divide by 2 (or multiply by } \frac{1}{2}\text{)}
\end{array}
$$

1. Label all 4 arrows to describe what happens in each move.
2. Are the equations equivalent? Explain your reasoning.

   yes

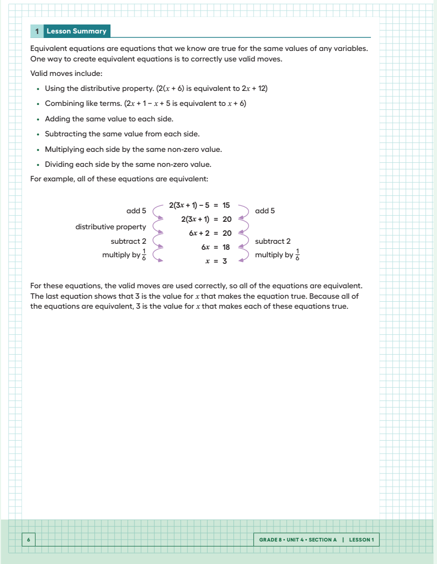
   Sample reasoning: As long as the same operations are done correctly to each side, the equations remain equivalent.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

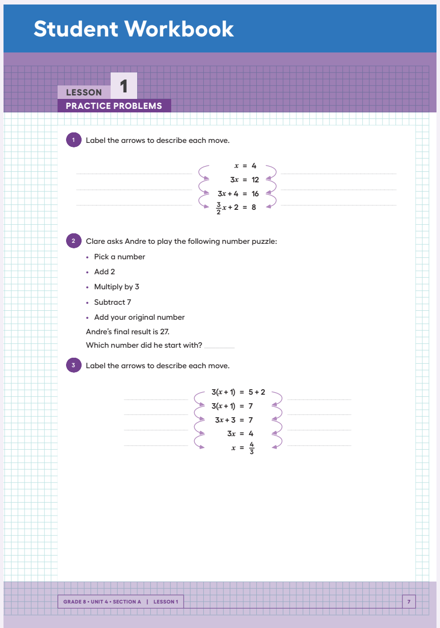
Student Workbook



Lesson 1

Practice Problems

7 Problems



Problem 1

Label the arrows to describe each move.

	$x = 4$	
multiply by 3	$3x = 12$	multiply by 3
add 4	$3x + 4 = 16$	add 4
divide by 2 (or multiply by $\frac{1}{2}$)	$\frac{3}{2}x + 2 = 8$	divide by 2 (or multiply by $\frac{1}{2}$)

Problem 2

Clare asks Andre to play the following number puzzle:

- Pick a number
- Add 2
- Multiply by 3
- Subtract 7
- Add your original number

Andre's final result is 27.
Which number did he start with? 7

Note: $3(x + 2) - 7 + x$ simplifies to $4x - 1$. If the final result is 27, then write $4x - 1 = 27$, which has the solution $x = 7$.

Problem 3

Label the arrows to describe each move.

	$3(x + 1) = 5 + 2$	
combine like terms	$3(x + 1) = 7$	combine like terms
distributive property	$3x + 3 = 7$	distributive property
subtract 3 (or add -3)	$3x = 4$	subtract 3 (or add -3)
multiply by $\frac{1}{3}$ (or divide by 3)	$x = \frac{4}{3}$	multiply by $\frac{1}{3}$ (or divide by 3)

Problem 4 from Unit 3, Lesson 13

Select **all** of the given points in the coordinate plane that lie on the graph of the linear equation $4x - y = 3$.

A. (-1, -7)

B. (0, 3)

C. $\left(\frac{3}{4}, 0\right)$

D. (1, 1)

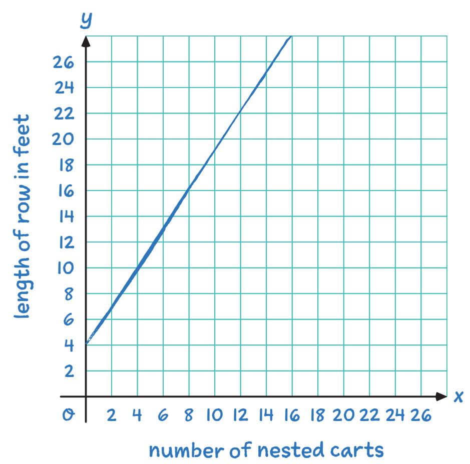
E. (2, 5)

F. (4, -1)

Problem 5 from Unit 3, Lesson 5

A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 feet long, followed by the nested carts (so 0 nested carts means there's just the starting cart). The store measured a row of 13 nested carts to be 23.5 feet long, and a row of 18 nested carts to be 31 feet long.

a. Create a graph of the situation.



b. How much does each nested cart add to the length of the row? Explain your reasoning.

1.5 feet

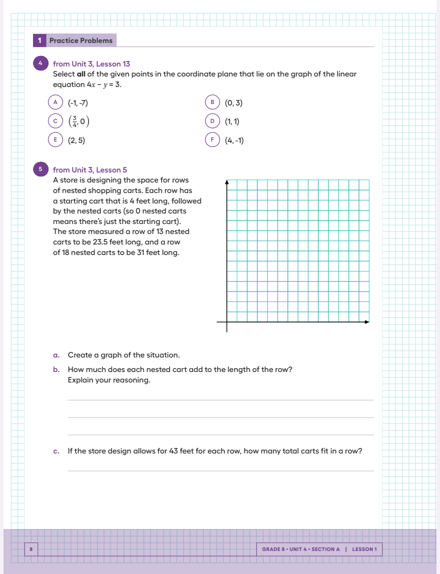
Sample reasoning: The slope, which can be found with the calculation $\frac{31 - 23.5}{18 - 13}$ tells the rate of change, or amount that each nested cart adds.

c. If the store design allows for 43 feet for each row, how many total carts fit in a row?

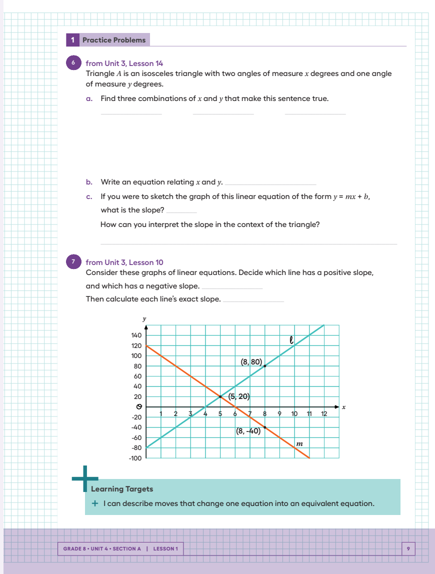
26 nested carts, or 27 carts total

Sample reasoning: Subtract 4 feet from 43 feet for the starting cart and then divide by 1.5 to find the number of nested carts that will fit. Use a table and repeatedly add 1.5. There are 12 more feet from 31 to 43, so 12 ÷ 1.5, or 8 more carts, can be added to 18.

Student Workbook



Student Workbook



Problem 6

from Unit 3, Lesson 14

Triangle A is an isosceles triangle with two angles of measure x degrees and one angle of measure y degrees.

a. Find three combinations of x and y that make this sentence true.

The 3 angles must sum to 180.

Sample responses: $x = y = 60$, or $x = 30$ and $y = 120$ or $x = 45$ and $y = 90$, that is, the three angles must sum to 180.

b. Write an equation relating x and y.

$2x + y = 180$ (or equivalent)

c. If you were to sketch the graph of this linear equation of the form $y = mx + b$, what is the slope? $\underline{-2}$

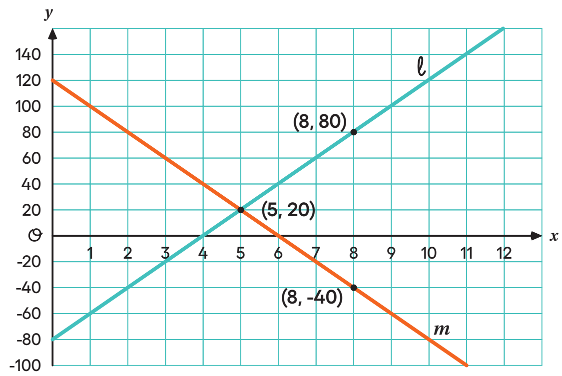
How can you interpret the slope in the context of the triangle?

Sample response: In the context of the triangle, for every 1 degree increase of x, y decreases by 2 degrees.

Problem 7

from Unit 3, Lesson 10

Consider these graphs of linear equations. Decide which line has a positive slope, and which has a negative slope. Then calculate each line's exact slope.



Line ℓ: positive, line m: negative.

The slope of ℓ is $\frac{80-20}{8-5} = \frac{60}{3} = 20$.

The slope of m is $\frac{-40-20}{8-5} = \frac{-60}{3} = -20$.