Lesson 10

Representing Large Numbers on the Number Line

Goals

- Compare large numbers using powers of 10, and explain (orally) the solution method.
- Use number lines to represent (orally and in writing) large numbers as multiples of powers of 10.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals.
- I can write a large number as a multiple of a power of 10.

Lesson Narrative

In this lesson, students use number lines to visualize powers of 10, compare very large numbers, and make sense of orders of magnitude. They use the structure of a number line that is subdivided into 10 equal intervals to express large numbers as multiples of a power of 10.

In these materials, "multiple of a power of 10" does not necessarily mean an integer multiple of a power of 10. Students explore numbers of the form $b \cdot 10^n$, where b is some decimal number. When students are formally introduced to scientific notation, b is restricted to values between 1 and 10.

Student Learning Goal

Let's visualize large numbers on the number line using powers of 10.

Instructional Routines

- MLR5: Co-Craft Questions

Access for Multilingual Learners:

- MLR5: Co-Craft Questions (Activity 2)

Access for Students with Diverse Abilities

- Representation (Activity 1)

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	10 min	20 min	10 min	5 min

Warm-up
Labeling Tick Marks on a Number Line

5 min

Activity Narrative

This *Warm-up* prompts students to visualize and make sense of numbers expressed as a product of a single digit and a power of 10, in preparation for working with scientific notation.

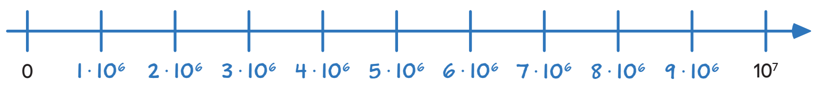
Expect student responses to include a variety of incorrect or partially-correct ideas. It is not important that all students understand the correct notation at this point, so it is not necessary to extend the time for this reason.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time and then 2 minutes to compare their number line with their partner's. Tell partners to try to come to an agreement on how to label the number line. Follow with a whole-class discussion.

Student Task Statement

Label the tick marks on the number line. Be prepared to explain your reasoning.



Activity Synthesis

The goal of this discussion is for students to see how to correctly label this number line. Begin by inviting selected students to explain how they labeled the number line. Record and display their responses on the number line for all to see. As students share, use their responses, correct or incorrect, to guide students to the understanding that the first tick mark is $1 \cdot 10^6$, the second is $2 \cdot 10^6$, and so on.

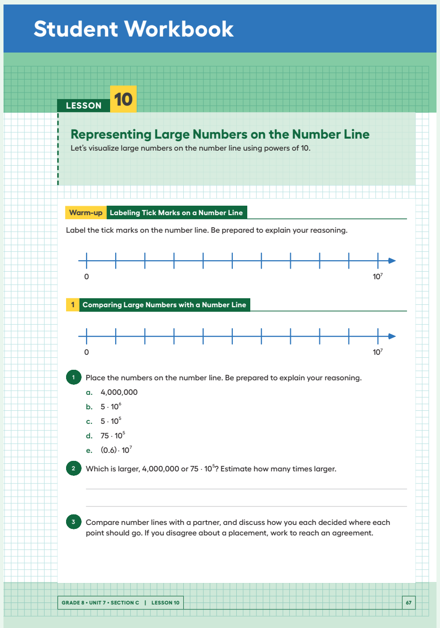
If not uncovered in students' explanations, ask the following questions to make sure students see how to label the number line correctly:

- *"How many equal parts is 10^7 being divided into?"*

  10

- *"If the number at the end of this number line were 20, how would we find the value of each tick mark?"*

  Divide 20 by 10



Student Workbook

LESSON 10

Representing Large Numbers on the Number Line

Let's visualize large numbers on the number line using powers of 10.

Warm-up Labeling Tick Marks on a Number Line

Label the tick marks on the number line. Be prepared to explain your reasoning.

0 10^7

1 Comparing Large Numbers with a Number Line

0 10^7

1. Place the numbers on the number line. Be prepared to explain your reasoning.
   a. 4,000,000
   b. $5 \cdot 10^6$
   c. $5 \cdot 10^5$
   d. $75 \cdot 10^5$
   e. $(0.6) \cdot 10^7$
2. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.
3. Compare number lines with a partner, and discuss how you each decided where each point should go. If you disagree about a placement, work to reach an agreement.

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Representation: Internalize Comprehension.
Begin with a physical demonstration of creating a number line from 0 to 70 and dividing the interval into 10 equal parts to support connections between new situations and prior understandings. Consider using the prompts:

"How many parts did we partition this number line into?"

"How should this number line be labeled and how do you know?"

"What do we know about the numbers as we move to the right on the number line?"

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

"Can we use the same reasoning with 10^7 at the end?"

Yes

"What is $10^7 \div 10$?"

10^6

"What does this number represent?"

The distance between two tick marks

"Can we write 10^6 as $1 \cdot 10^6$?"

Yes

"If the first tick mark is $1 \cdot 10^6$, then what is the second tick mark?"

$2 \cdot 10^6$

Activity 1
Comparing Large Numbers with a Number Line

10 min

Activity Narrative

There is a digital version of this activity.

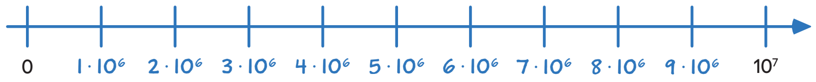
This activity encourages students to use the number line to make sense of powers of 10 and to think about how to rewrite expressions in the form $b \cdot 10^n$, where b is between 1 and 10 (as in the case of scientific notation). Students use the structure of the number line to compare numbers, and to extend their use to estimate relative sizes of other numbers when no number lines are given.

In the digital version of the activity, students use an applet to place points on a number line. The applet allows students to quickly check and revise their placement if necessary. The digital version may be helpful for students who rush to place the values on the number line and would benefit from immediate feedback.

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet time to work on the first two problems, followed by 1–2 minutes to discuss their work with their partner for the last problem. Follow with a brief whole-class discussion.

Student Task Statement



1. Place the numbers on the number line. Be prepared to explain your reasoning.
   a. 4,000,000

      4,000,000 is on the 4th tick mark because it's equal to $4 \cdot 10^6$.

   b. $5 \cdot 10^6$

      $5 \cdot 10^6$ is on the 5th tick mark.

   c. $5 \cdot 10^5$

      $5 \cdot 10^5$ is between 0 and the 1st tick mark because it is equal to $(0.5) \cdot 10^6$.

   d. $75 \cdot 10^5$

      $75 \cdot 10^5$ is between the 7th and 8th tick marks because it is equal to $(7.5) \cdot 10^6$.

   e. $(0.6) \cdot 10^7$

      $0.6 \cdot 10^7$ is on the 6th tick mark because it is equal to $6 \cdot 10^6$.

2. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.

   $75 \cdot 10^5$ (or $(7.5) \cdot 10^6$) is about twice as large as 4,000,000 (or $4 \cdot 10^6$), because 7.5 is roughly twice as large as 4.

3. Compare number lines with a partner, and discuss how you each decided where each point should go. If you disagree about a placement, work to reach an agreement.

   No answer required.

Activity Synthesis

The goal of this discussion is for students to see how rewriting numbers to have the same power of 10 makes it easier to compare their relative sizes. Here are some questions for discussion:

- *"How can 4,000,000, $75 \cdot 10^5$, and $(0.6) \cdot 10^7$ all be written as a multiple of 10^6?"*

  $4 \cdot 10^6$, $(7.5) \cdot 10^6$, $6 \cdot 10^6$

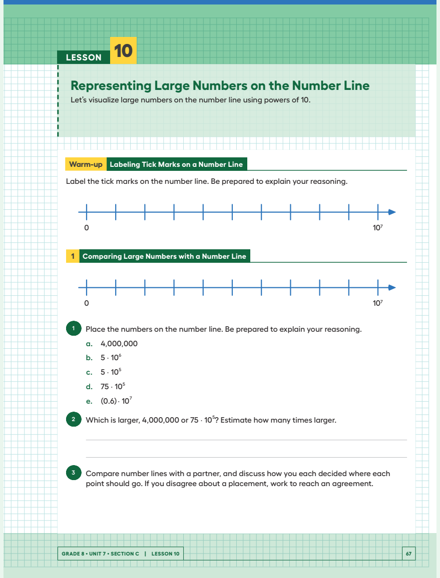
- *"Why are numbers easier to compare when they are all multiplied by the same power of 10?"*

  Since they are all multiplied by the same power of 10, we needed to compare only the first number of each expression.

- *"If the population of the United States is roughly $(3.5) \cdot 10^8$ people and the global population is roughly $8 \cdot 10^9$ people, about how many times larger is the global population than the U.S. population?"*

  $8 \cdot 10^9$ is roughly 20 times larger than $(3.5) \cdot 10^8$, because 8 is roughly twice as large as 3.5, and 10^9 is 10 times larger than 10^8.

Student Workbook


LESSON 10

Representing Large Numbers on the Number Line

Let's visualize large numbers on the number line using powers of 10.

Warm-up Labeling Tick Marks on a Number Line

Label the tick marks on the number line. Be prepared to explain your reasoning.

1 Comparing Large Numbers with a Number Line

1. Place the numbers on the number line. Be prepared to explain your reasoning.
   a. 4,000,000
   b. $5 \cdot 10^6$
   c. $5 \cdot 10^5$
   d. $75 \cdot 10^5$
   e. $(0.6) \cdot 10^7$
2. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.
3. Compare number lines with a partner, and discuss how you each decided where each point should go. If you disagree about a placement, work to reach an agreement.

Access for Multilingual Learners (Activity 2)

MLR5: Co-Craft Questions. This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Instructional Routines

MLR5: Co-Craft Questions

ilclass.com/r/10695544
Please log in to the site before using the QR code or URL.

Activity 2
The Speeds of Light

20 min

Activity Narrative

There is a digital version of this activity.

This activity promotes a deeper understanding of expressions written as the product of a single digit and a power of 10. Students investigate a table of the speed of light waves through different materials. To distinguish more easily between the different speeds of light through various materials, the interval between $2 \cdot 10^8$ and $3 \cdot 10^8$ is magnified on the number line. This illustrates the base-ten structure and allows for a more accurate placement of numbers with more decimal places.

Once a number line is labeled with powers of 10 and its structure is understood, numbers written as a multiple of the same power of 10 can be placed on the number line fairly straightforwardly.

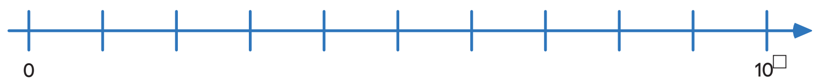
In the digital version of the activity, students use an applet to place points on a number line. The applet allows students to magnify an interval of the number line and easily change the placement of points. The digital version may be helpful for illustrating that the second number line is really a zoomed in version of an interval in the first number line.

Launch

Tell students to close their student workbooks or devices (or to keep them closed). Arrange students in groups of 2. Introduce the table only. Use *Co-Craft Questions* to orient students to the context and to elicit possible mathematical questions.

Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

Explain to students that as light moves through various materials, it slows down. The speed of light through empty space, with nothing in its way, is roughly 300,000,000 meters per second. The speed of light through olive oil is much slower at roughly 200,000,000 meters per second. Then display the number line for all to see.



Ask students to decide what power of 10 to use for the label of the rightmost tick mark on the number line so that the speed of light through space and through olive oil can both be plotted. Make sure students see that 10^9 is appropriate because for 200,000,000 (which is $2 \cdot 10^8$) to be plotted between 0 and the last tick mark, the last power of 10 needs to be greater than 10^8).

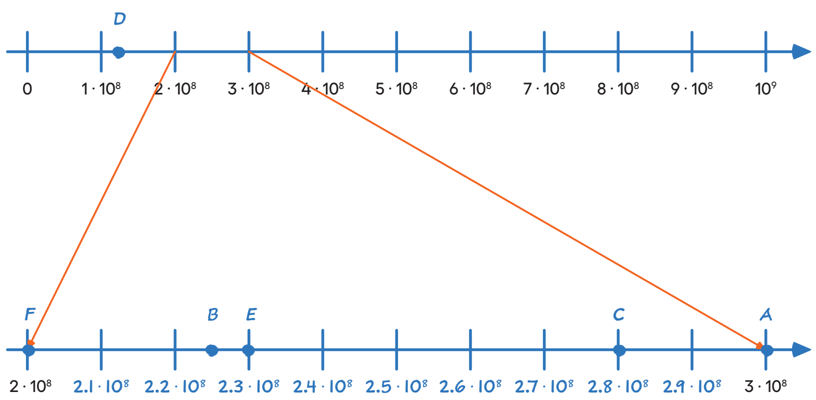
Give students 7–8 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The table shows how fast light waves can travel through different materials.

	material	speed (meters per second)
A	space	300,000,000
B	water	$(2.25) \cdot 10^8$
C	copper wire (electricity)	280,000,000
D	diamond	$124 \cdot 10^6$
E	ice	$(2.3) \cdot 10^8$
F	olive oil	200,000,000

Let's zoom in to highlight the values between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.



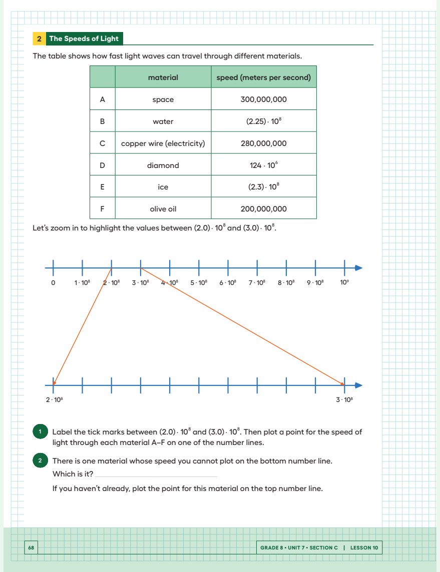
1. Label the tick marks between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$. Then plot a point for the speed of light through each material A–F on one of the number lines.
2. There is one material whose speed you cannot plot on the bottom number line.

   Which is it? Diamond If you haven't already, plot the point for this material on the top number line.
3. Which travels faster — light through diamond or light through ice? How can you tell from the given expressions for the speed of light? How can you tell from the number line?

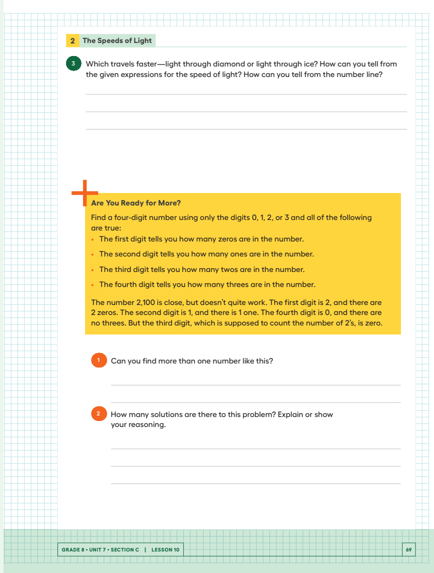
   Light travels faster through ice.

   Sample responses: If the expressions for speed are rewritten so they have the same power of 10, light travels through a diamond at $(1.24) \cdot 10^8$ meters per second and light travels through ice at $(2.3) \cdot 10^8$ meter per second and $2.3 > 1.24$. From the number line, the point for the speed of light through ice is farther to the right.

Student Workbook



Student Workbook



Are You Ready for More?

Find a four-digit number using only the digits 0, 1, 2, or 3 and all of the following are true:

- The first digit tells you how many zeros are in the number.
- The second digit tells you how many ones are in the number.
- The third digit tells you how many twos are in the number.
- The fourth digit tells you how many threes are in the number.

The number 2,100 is close, but doesn't quite work. The first digit is 2, and there are 2 zeros. The second digit is 1, and there is 1 one. The fourth digit is 0, and there are no threes. But the third digit, which is supposed to count the number of 2's, is zero.

1. Can you find more than one number like this?
2. How many solutions are there to this problem? Explain or show your reasoning.

The two possible solutions are 1,210 and 2,020. Explanations vary.

Sample explanation: Since this is a four-digit number and the digits of this number count how many occurrences of each digit there are, the sum of the digits must be four. There cannot be any 3's in the number, because that would mean some number needs to occur three times. But 3,000 doesn't work, nor do 2,322 or 1,131. Numbers of the form _333, 3_33, or 33_3 can't work, either, because the sum of the digits is more than four. Therefore, we are looking for combinations of the numbers 0, 1, and 2 that add up to 4, knowing that the last digit must be 0. At this point, there are not many choices left, and we can test them all.

Activity Synthesis

The goal of this discussion is to check that students understand how to rewrite very large numbers written using one power of 10 with a different power of 10.

Begin by asking students to explain how they were able to compare the speeds of light through the different materials even though they were each written in different ways. (Rewriting the numbers so they had the same power of 10 made it easier to compare them.) While students do not yet know the formal definition of scientific notation, this discussion should help illustrate how expressing values in this format allows for an easier comparison.

Invite students to share how they rewrote $124 \cdot 10^6$ using 10^8. If not brought up in students' explanations, consider asking the following series of questions:

- *"What can 10^6 be multiplied by to get 10^8?"*

  100 or 10^2

- *"If 10^6 is multiplied by 10^2 to get 10^8, the value of the expression will change. What can be done to keep the value of the expression the same?"*

  Divide the other factor by 10^2.

- *"What is the resulting equivalent expression?"*

  $1.24 \cdot 10^8$

Lesson Synthesis

The purpose of this discussion is for students to practice comparing large numbers by expressing them as multiples of the same power of 10. It is important for students to understand that:

- A "multiple of a power of 10" does not necessarily mean an integer multiple.
- Multiplying one factor of an expression by a power of 10 and then dividing another factor by the same power of 10 is equivalent to multiplying by 1, thus keeping the value of the expression the same.

Ask students:

"What are different ways to write 230,000,000 using powers of 10?"

Since the value of 230,000,000 will stay the same if it is multiplied by a power of 10 and then divided by the same power of 10, we can write: $230{,}000{,}000 = 23{,}000{,}000 \cdot 10 = 2{,}300{,}000 \cdot 10^2 = \ldots = 23 \cdot 10^7 = (2.3) \cdot 10^8$.

Then display this expression for all to see: $(5.2) \cdot 10^4$.

Give students a second expression, and ask them to work with a partner to determine which of the two expressions is larger. Complete as many comparisons as time allows.

- $53 \cdot 10^3$ Larger than $(5.2) \cdot 10^4$
- $425 \cdot 10^2$ Smaller than $(5.2) \cdot 10^4$
- $3.7 \cdot 10^7$ Larger than $(5.2) \cdot 10^4$
- $0.02 \cdot 10^6$ Smaller than $(5.2) \cdot 10^4$

Lesson Summary

Suppose we want to compare the number of pennies the U.S. Mint made in 2020, about 7,600,000,000, to the number of one dollar bills printed by the U.S. Bureau of Engraving and Printing in the same year (about 1.6 billion). There are many ways to do this.

We could write 1.6 billion as a decimal value, 1,600,000,000, and then we can tell that in 2020 there were more pennies made than one dollar bills printed.

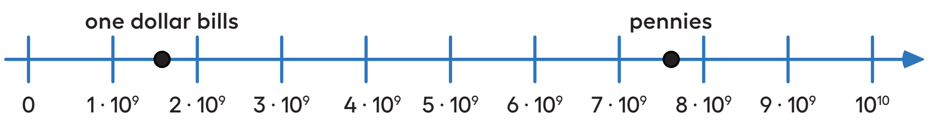
Or we could use powers of 10 to write these numbers:

$(7.6) \cdot 10^9$ for the number of pennies and

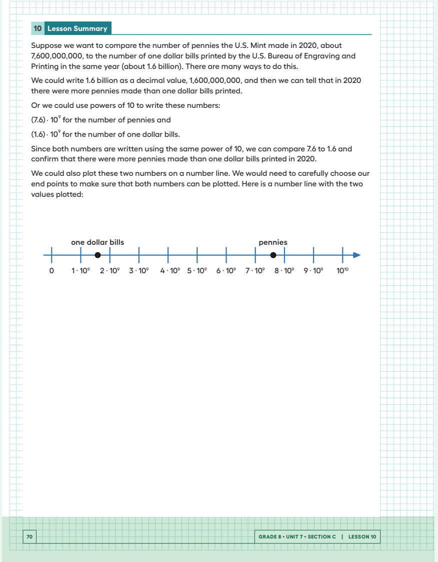
$(1.6) \cdot 10^9$ for the number of one dollar bills.

Since both numbers are written using the same power of 10, we can compare 7.6 to 1.6 and confirm that there were more pennies made than one dollar bills printed in 2020.

We could also plot these two numbers on a number line. We would need to carefully choose our end points to make sure that both numbers can be plotted. Here is a number line with the two values plotted:



Student Workbook



Responding To Student Thinking

Points to Emphasize
If most students struggle with representing large numbers written as a power of ten on a number line, revisit labeling number lines with powers of 10 in this lesson:
Unit 7, Lesson 12, Activity 2 That's a Tall Stack of Cash

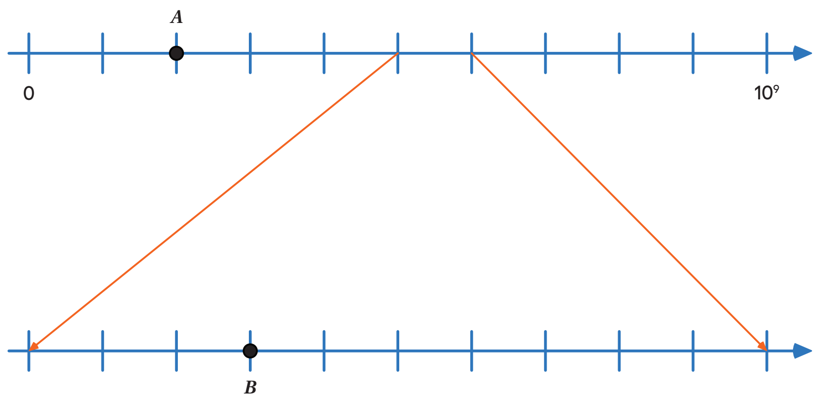
Cool-down

Describe the Point

5 min

Student Task Statement

The speed of light through ice can be written as a multiple of a power of 10, such as $(2.3) \cdot 10^8$ meters per second, or as a value, such as 230,000,000 meters per second. Use the number line to answer questions about points A and B.



1. Describe point A as:
   - **a.** A multiple of a power of 10 $2 \cdot 10^8$
   - **b.** A value 200,000,000
2. Describe point B as:
   - **a.** A multiple of a power of 10 $(5.3) \cdot 10^8$
   - **b.** A value 530,000,000
3. Plot a point C that is greater than A and less than B. Describe point C as:
   - **a.** A multiple of a power of 10
   - **b.** A value

   Sample response: any value between $2 \cdot 10^8$ and $(5.3) \cdot 10^8$.

Lesson **10**

Practice Problems

6 Problems

Problem 1

Write the number 437, 000 three different ways using powers of 10.

Sample responses: $4.37 \cdot 10^5$, $43.7 \cdot 10^4$, $437 \cdot 10^3$

Problem 2

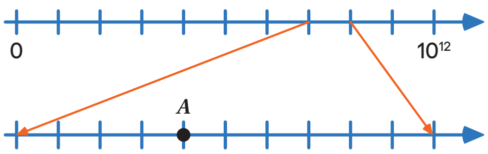
For each pair of numbers, circle the number with the larger value. Estimate about how many times larger.

a. $17 \cdot 10^8$ or $4 \cdot 10^8$ $17 \cdot 10^8$, about 4 times larger

b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$ $7.839 \cdot 10^6$, about 4 times larger

c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$ $8.5 \cdot 10^8$, about 2 times larger

Problem 3

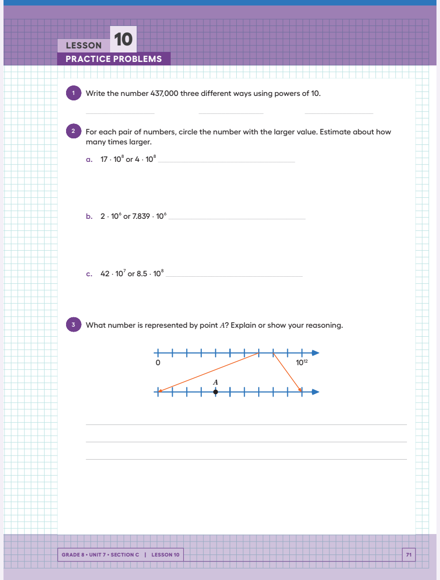


What number is represented by point A? Explain or show your reasoning.

$7.4 \cdot 10^{11}$

Sample reasoning: Point A lies between $7 \cdot 10^{11}$ and $8 \cdot 10^{11}$. It is $7.4 \cdot 10^{11}$ because it is four tick marks from $7.0 \cdot 10^{11}$.

Student Workbook


LESSON 10
PRACTICE PROBLEMS

1 Write the number 437,000 three different ways using powers of 10.

2 For each pair of numbers, circle the number with the larger value. Estimate about how many times larger.

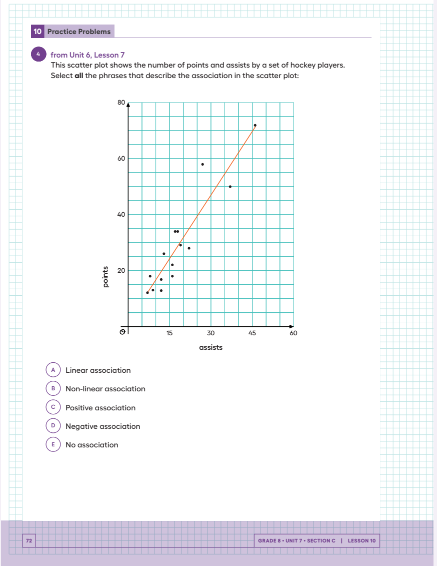
a. $17 \cdot 10^8$ or $4 \cdot 10^8$

b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$

c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$

3 What number is represented by point A? Explain or show your reasoning.

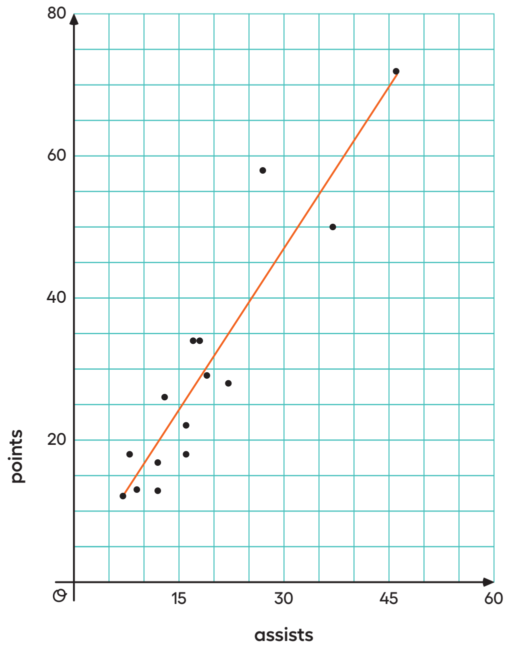
Student Workbook



Problem 4

from Unit 6, Lesson 7

This scatter plot shows the number of points and assists by a set of hockey players. Select **all** the phrases that describe the association in the scatter plot:



A. Linear association

B. Non-linear association

C. Positive association

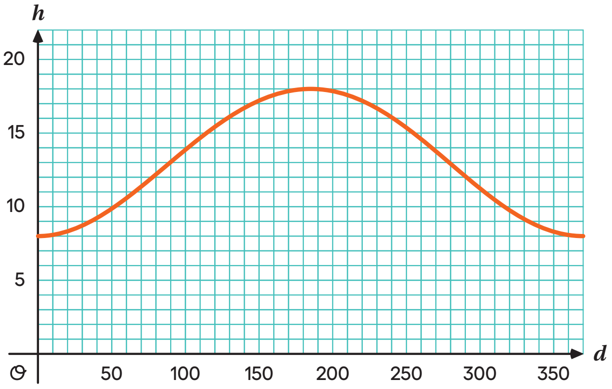
D. Negative association

E. No association

Problem 5

from Unit 5, Lesson 5

Here is a graph of the day of the year, d, and the predicted hours of sunlight, h, on that day.



a. Is hours of sunlight a function of days of the year? Explain how you know.

Yes, h is a function of d, because for every d there is one and only one value of h.

b. For what days of the year is the number of hours of sunlight increasing? For what days of the year is the number of hours of sunlight decreasing?

From day 0 to day 180, the hours of sunlight are increasing.
From day 180 to day 365, the hours of sunlight are decreasing.

c. Which day of the year has the greatest number of hours of sunlight?

The day with the greatest number of hours of sunlight is day 180.

Problem 6

from Unit 7, Lesson 8

Which expression has the same value as $5^3 \cdot 6^3$?

A. 11^6 **B.** 30^6 **C.** 30^3 **D.** 30^9

Student Workbook