Lesson 9

Applying Area of Circles

Goals

- Calculate the area of a shape that includes circular or semi-circular parts, and explain (orally and in writing) the solution method.
- Comprehend and generate expressions in terms of π to express exact measurements related to a circle.

Learning Targets

- I can calculate the area of more complicated shapes that include fractions of circles.
- I can write exact answers in terms of π.

Lesson Narrative

In this lesson, students apply the formula $A = \pi r^2$ to solve problems involving the area of shapes made up of parts of circles and other shapes such as rectangles. Students are also introduced to the idea of expressing exact answers in terms of π.

The last activity is optional because it revisits a context that was explored in a previous optional activity, and it provides an additional opportunity for students to apply $A = \pi r^2$ to solve a problem.

Student Learning Goal

Let's find the areas of shapes made up of circles.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR8: Discussion Supports (Warm-up, Activity 2)

Instructional Routines

- 5 Practices
- Math Talk

Required Preparation

Lesson:
It is recommended that four-function calculators be made available to take the focus off of computation.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	25 min	15 min	10 min	5 min

Instructional Routines

Math Talk

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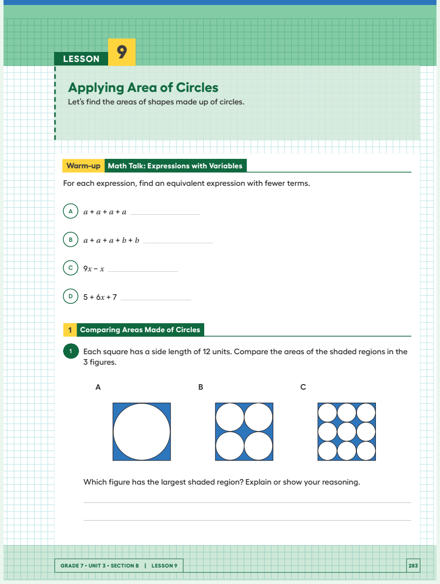
Access for Students with Diverse Abilities (Warm-up, Student Task)

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook


LESSON 9

Applying Area of Circles

Let's find the areas of shapes made up of circles.

Warm-up Math Talk: Expressions with Variables

For each expression, find an equivalent expression with fewer terms.

A $a+a+a+a$

B $a+a+a+b+b$

C $9x-x$

D $5+6x+7$

1 Comparing Areas Made of Circles

1 Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures.

A B C

Which figure has the largest shaded region? Explain or show your reasoning.

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Warm-up

Math Talk: Expressions with Variables

5 min

Activity Narrative

This *Math Talk* focuses on expressions with variables. It encourages students to think about equivalence and to rely on properties of operations to mentally solve problems. The understanding elicited here will be helpful later in the lesson when students express answers in terms of π.

To generate an equivalent expression with fewer terms, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

For each expression, find an equivalent expression with fewer terms.

A. $a + a + a + a$

$4a$ or equivalent

Sample reasoning: Adding a four times gives the same results as multiplying a by 4.

B. $a + a + a + b + b$

$3a + 2b$ or equivalent

Sample reasoning: Adding b twice gives the same result as multiplying b by 2. Also, $4a$ and $2b$ are not like terms, so they can't be combined.

C. $9x - x$

$8x$

Sample reasoning: By the distributive property, $9x - 1x = (9 - 1)x$.

D. $5 + 6x + 7$

$6x + 12$ or equivalent

Sample reasoning: By the commutative and associative properties, $5 + 6x + 7 = 6x + (5 + 7)$. Also, $6x$ and 12 are not like terms, so they can't be combined.

Activity Synthesis

To involve more students in the conversation, consider asking:

"Who can restate _____'s reasoning in a different way?"

"Did anyone use the same strategy but would explain it differently?"

"Did anyone solve the problem in a different way?"

"Does anyone want to add on to _____'s strategy?"

"Do you agree or disagree? Why?"

"What connections to previous problems do you see?"

The key takeaway is that applying properties of operations allows us to write equivalent expressions with fewer terms. This is often called "combining like terms."

Activity 1

Comparing Areas Made of Circles

25 min

Activity Narrative

In this activity, students find the areas of regions involving different-sized circles and compare the strategies used. In the first set of figures, students subtract to find the area around the outside of a circle. In the second set of figures, students divide to find the area of fractions of a circle. During the whole-class discussion, students consider leaving expressions in terms of pi as a means of increasing precision in describing circles.

Monitor for students who use different strategies for comparing the area of Figures A, B, and C, such as:

- Decomposing Figures B and C into smaller square sections, subtracting the area of each size of circle from its surrounding square, and then multiplying the result as needed (by 4 for Figure B and by 9 for Figure C).
- Multiplying the area of the circles as needed and then subtracting the result from the area of the full-size square.
- Recognizing that the areas have to be equal because Figure B is composed of 4 scaled copies of Figure A, each with $\frac{1}{4}$ as much area, and Figure C is composed of 9 scaled copies of Figure A, each with $\frac{1}{9}$ as much area.

Plan to have students present in this order to support moving them from less efficient to more efficient strategies. However, if the last strategy does not come up in students' work, it is not necessary for the teacher to bring it up during the discussion.

As students look for ways to be more efficient when comparing the areas of the regions, they make use of structure.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

5 Practices

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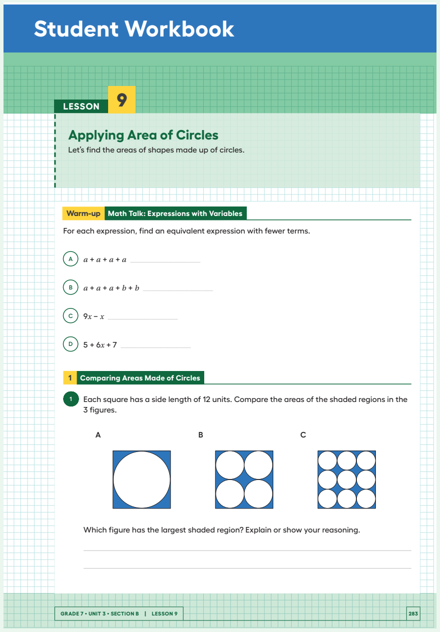
Student Workbook

LESSON 9

Applying Area of Circles

Let's find the areas of shapes made up of circles.

Warm-up Math Talk: Expressions with Variables

For each expression, find an equivalent expression with fewer terms.

A $a+a+a+a$

B $a+a+a+b+b$

C $9x-x$

D $5+6x+7$

1 Comparing Areas Made of Circles

1 Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures.

A B C

Which figure has the largest shaded region? Explain or show your reasoning.

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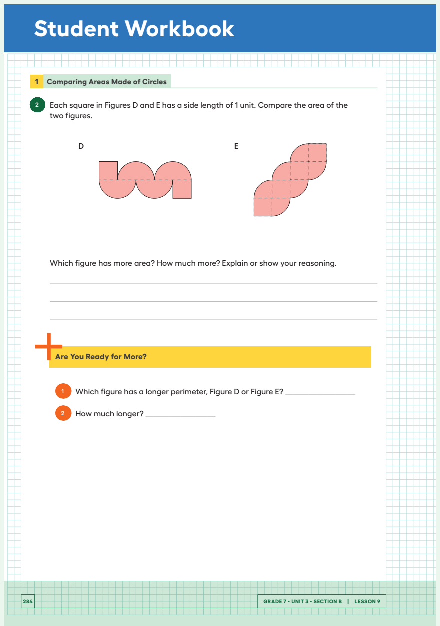
Student Workbook

1 Comparing Areas Made of Circles

2 Each square in Figures D and E has a side length of 1 unit. Compare the area of the two figures.

D E

Which figure has more area? How much more? Explain or show your reasoning.

Are You Ready for More?

1 Which figure has a longer perimeter, Figure D or Figure E?

2 How much longer?

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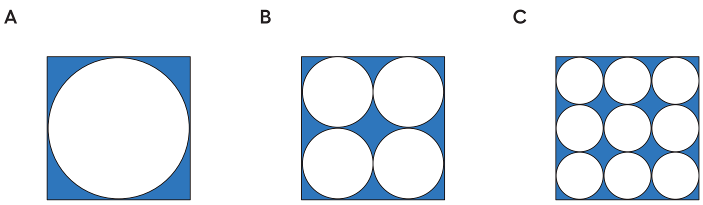
Launch

Arrange students in groups of 2. Display the image in the first question and ask students to make a prediction before calculating. Give 30 seconds of quiet think time before sharing with their partner. Give students partner work time followed by a whole-class discussion.

Select students who used each strategy described in the activity narrative to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

1. Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures. Which figure has the largest shaded region? Explain or show your reasoning.



The areas of all 3 shaded regions are equal: about 30.96 square units.

Sample reasoning:

- The area of the entire square is 144 square units, because $12 \cdot 12 = 144$.
- The area of the 1 large circle is approximately 113.04 square units, because $3.14 \cdot 6^2 = 113.04$.
- The area of the 4 medium circles is also 113.04 square units, because $3.14 \cdot 3^2 = 28.26$ and $28.26 \cdot 4 = 113.04$.
- The area of the 9 small circles is also 113.04 square units, because $3.14 \cdot 2^2 = 12.56$ and $12.56 \cdot 9 = 113.04$.
- The area of the shaded region in each figure is 30.96 square units, because $144 - 113.04 = 30.96$.

2. Each square in Figures D and E has a side length of 1 unit. Compare the area of the two figures. Which figure has more area? How much more? Explain or show your reasoning.

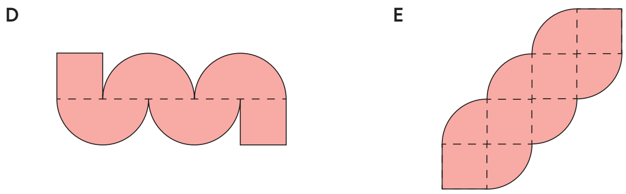


Figure E's area is about 0.43 square units larger than Figure D's.

- Figure D consists of 2 squares and 4 half circles, giving it an area of about 8.28 square units, because $4 \cdot 1.57 + 2 = 8.28$.
- Figure E consists of 4 squares and 6 quarter circles, giving it an area of about 8.71 square units, because $6 \cdot 0.785 + 4 = 8.71$.
- The difference in areas is 0.43, because $8.71 - 8.28 = 0.43$.

Building on Student Thinking

In the first question, students may not know how to find the radius of the circles. Suggest having them cut off the shaded regions and rearrange them to show that the length of each side fits half way across the circle (marking the radius).

In the second question, students might benefit from cutting and rearranging the figures. Some students might assume, based on previous activities, that the areas of both figures are equal. However, Figure D has more pieces that are parts of a circle, and Figure E has more units that are a full square. Ask students whether the fourth of the circle has the same area as the square.

Are You Ready for More?

Which figure has a longer perimeter, Figure D or Figure E? How much longer?

Figure D's perimeter is $\pi + 2$ units longer than Figure E's because $(4\pi + 6) - (3\pi + 4) = 1\pi + 2$.

Activity Synthesis

There are two main goals for this discussion: for students to notice ways to be more efficient when comparing the areas of the regions and to be introduced to expressing answers in terms of pi. These two goals are related because keeping the answers in terms of pi is an additional method that can help make the comparisons more efficient.

Display Figures A, B, and C for all to see. Invite previously selected students to share how they determined the area of the figures. Sequence the discussion of the strategies in the order listed in the activity narrative. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

"Why do the different approaches lead to the same outcome?"

"How does subtraction show up in each method?"

"Are there any benefits or drawbacks to one method compared to another?"

If not brought up by students, ask how we could have determined that the area of the shaded regions were equal without calculating the answer of 30.96. For example, determining that the unshaded area in each figure is 113.04 square units is enough to conclude that the shaded areas have to be equal, without completing the subtraction.

Next, focus the discussion on leaving answers in terms of π for each figure. Explain to students that in Figure A, the radius of the circle is 6, so the area of the circular region is $\pi \cdot 6^2$. Instead of multiplying by an approximation of π, we can express this answer as 36π. This is called answering "*in terms of π*." Consider writing "36π" inside the large circle of Figure A.

Ask students to answer these questions in terms of π:

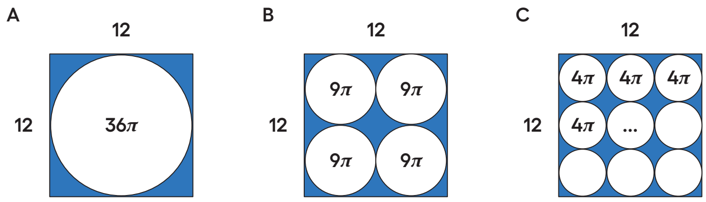
"What is the area of one of the circular regions in Figure B?"

9π

"What is the area of one of the circular regions in Figure C?"

4π

Consider writing "9π" and "4π" inside some of the circles in Figures B and C.



Ask students:

"What is the combined area of all four circles in Figure B?"

$4 \cdot 9\pi$, or 36π

"What is the combined area of all nine circles in Figure C?"

$9 \cdot 4\pi$, or 36π

"What is the area of the shaded region in each figure?"

$144 - 36\pi$

"Can the expression $144 - 36\pi$ be shortened by combining like terms?"

no

If time permits, ask students to express their answers for the area of Figures D and E in terms of π. Record and display their answers of $2 + 2/\pi$ and $4 + 1.5\pi$ for all to see. Ask students to discuss how they can tell Figure E's area is larger than Figure D's area when they are both written in terms of π. (because $2 > 0.5\pi$)

Activity 2: Optional
The Running Track Revisited

15 min

Activity Narrative

In this activity, students calculate the area of a running track. Students may have investigated the perimeter of this track in a previous lesson. Here, they apply the strategies of dividing to find the area of a fraction of a circle as well as subtracting to find the area of the region around a circle. As students decide how to decompose the running track into measurable pieces and how to use the given information to calculate areas, they reason abstractly and quantitatively.

Monitor for students who use different strategies for determining the area, such as:

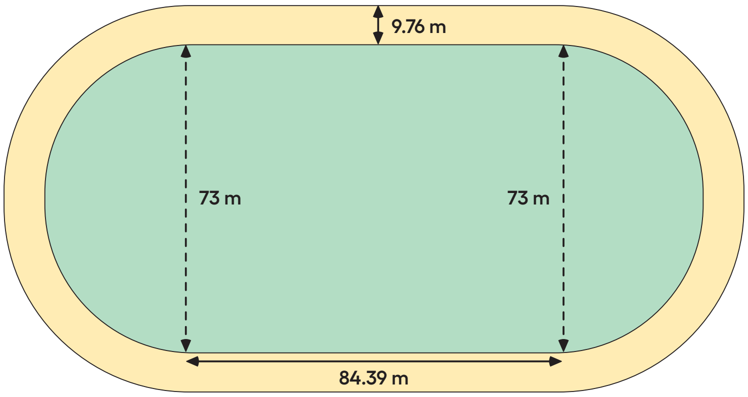
- Decomposing the track into two straight sections and two curved sections, finding the areas, and adding all four pieces together.
- Decomposing half of the track into one straight section and one curved section, finding and adding those areas, then multiplying the result by 2.
- Finding the area of the larger shape (the track and the field inside) and then subtracting the area of the field inside.

Launch

Arrange students in groups of 2. Give students 5–6 minutes of partner work time followed by small-group and whole-class discussions.

Student Task Statement

The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide, together with a half-circle at each end. The running lanes are 9.76 m wide all the way around.



What is the area of the running track that goes around the field? Explain or show your reasoning.

The area of the entire running track is $807.7376\pi + 823.6464$, or about 4,183.6 m². The area of the straight top and bottom of the running track are each 823.6464 m² because $84.39 \cdot 9.76 = 823.6464$. The left and right sides of the running track are half circles with a smaller half circle missing from the inside. The area of the inside circle is about 4,183.265 m² because $36.5^2 \cdot 3.14 = 4{,}183.265$. The area of the outside circle is about 6,719.561064 m² because $36.5 + 9.76 = 46.26$ and $46.26^2 \cdot 3.14 = 6{,}719.561064$. The area of each curved side of the running track is about 1,268.148032 m² because $(6{,}719.561064 - 4{,}183.265) \div 2 = 1{,}268.148032$. The area of the entire running track is about 4,183.6 m² because $2 \cdot 823.6464 + 2 \cdot 1{,}268.148032 = 4{,}183.588864$.

Building on Student Thinking

Some students may think they can calculate the area of the running track by multiplying half of the perimeter times the radius, as if the shape were just a circle. Prompt them to see that they need to break the overall shape into rectangular and circular pieces.

Activity Synthesis

Pair the groups of 2 to create groups of 4. Have students compare answers and explain their reasoning until they reach an agreement.

Then, ask the class how finding the area of the track in this activity was similar or different from solving the two area problems in the previous activity. Make sure to highlight these points:

- Like the first problem in the previous activity, finding the area of the track can be done by finding the area of the larger shape (the track and the field inside) and then taking away the area of the field inside.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.
At the appropriate time, give students 2–3 minutes to make sure that both partners can explain their process. Invite groups to rehearse what they will say when they share with the larger group.

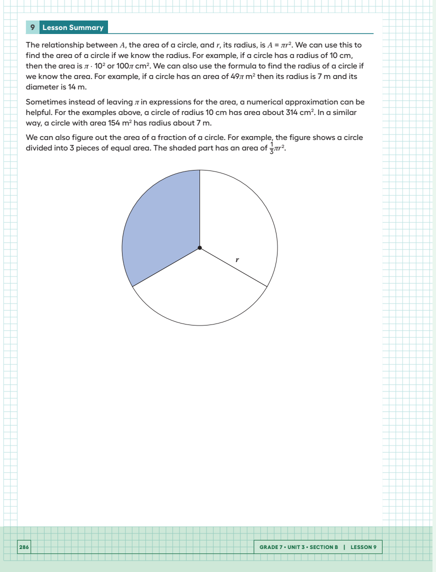
Advances: Speaking, Conversing, Representing

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Representation: Internalize Comprehension.
Provide a blank graphic organizer for students to organize the information provided in the problem and to structure their problem-solving strategy. The graphic organizer could include the prompts: "What do I need to find out?", "What do I need to do?", "How I solved the problem.", and "How I know my answer is correct."

Supports accessibility for: Organization, Attention

Student Workbook



- Like the second problem in the previous activity, there are semicircles whose area can be found by composing them to make a full circle or by taking half the area of the corresponding full circle.

Lesson Synthesis

Share with students:

"Today we applied the formula for finding the area of a circle to solve multi-step problems. We also practiced expressing answers in terms of π."

To give students extra practice expressing answers in terms of π, consider asking:

"What is the area, in terms of π, of a circle with a radius of 10?"

100π, because $10^2 = 100$

"What is the area, in terms of π, of a circle with a diameter of 10?"

25π because $10 \div 2 = 5$ and $5^2 = 25$

"What is the area, in terms of π, of a half-circle with a diameter of 10?"

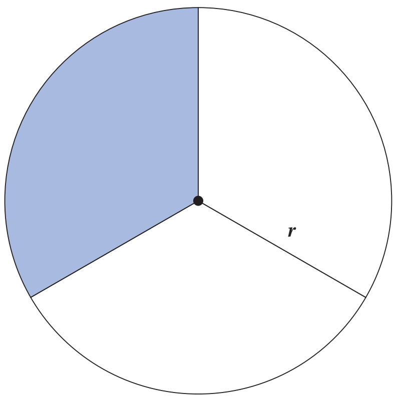
12.5π, because $25 \div 2 = 12.5$

Lesson Summary

The relationship between A, the area of a circle, and r, its radius, is $A = \pi r^2$. We can use this to find the area of a circle if we know the radius. For example, if a circle has a radius of 10 cm, then the area is $\pi \cdot 10^2$, or 100π cm^2. We can also use the formula to find the radius of a circle if we know the area. For example, if a circle has an area of 49π m^2 then its radius is 7 m and its diameter is 14 m.

Sometimes instead of leaving π in expressions for the area, a numerical approximation can be helpful. For the examples above, a circle of radius 10 cm has an area of about 314 cm^2. In a similar way, a circle with an area of 154 m^2 has a radius of about 7 m.

We can also figure out the area of a fraction of a circle. For example, the figure shows a circle divided into 3 pieces of equal area. The shaded part has an area of $\frac{1}{3}\pi r^2$.



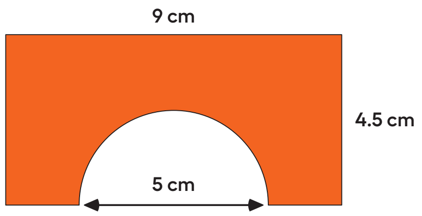
Cool-down

Area of an Arch

5 min

Student Task Statement

Here is a picture that shows one side of a child's wooden block with a semicircle cut out at the bottom.

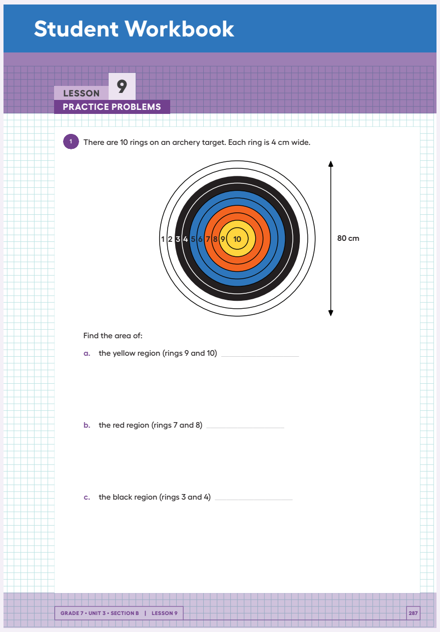


Find the area of the side. Explain or show your reasoning.

The area of the side of the block is about 30.68 cm². The area of the rectangle is $9 \cdot 4.5$, or 40.5 cm². The area of a circle with a diameter of 5 cm is 6.25π cm². The front face of the wooden block is a rectangle missing half of circle with diameter 5 cm, so its area in cm² is $40.5 - 3.125\pi$ or about 30.68.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.


Student Workbook

LESSON 9

PRACTICE PROBLEMS

1 There are 10 rings on an archery target. Each ring is 4 cm wide.

80 cm

Find the area of:

a. the yellow region (rings 9 and 10)

b. the red region (rings 7 and 8)

c. the black region (rings 3 and 4)

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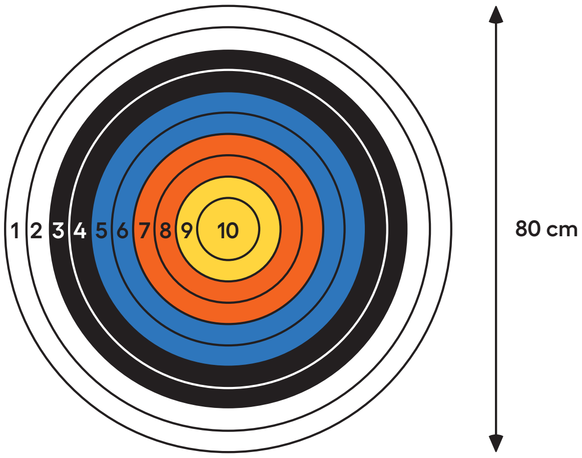
Practice Problems

6 Problems

Problem 1

There are 10 rings on an archery target. Each ring is 4 cm wide.



Find the area of:

a. the yellow region (rings 9 and 10)

64π cm^2, because $8^2 = 64$

b. the red region (rings 7 and 8)

194π cm^2, because $16^2 = 256$ and $256 - 64 = 192$

c. the black region (rings 3 and 4)

448π cm^2, because $32^2 - 24^2 = 448$

Problem 2

A circle with a 12-inch diameter is folded in half and then folded in half again. What is the area of the resulting shape?

9π in^2, or about 28 in^2, because $\frac{1}{4} \cdot 6^2\pi = 9\pi$

Problem 3

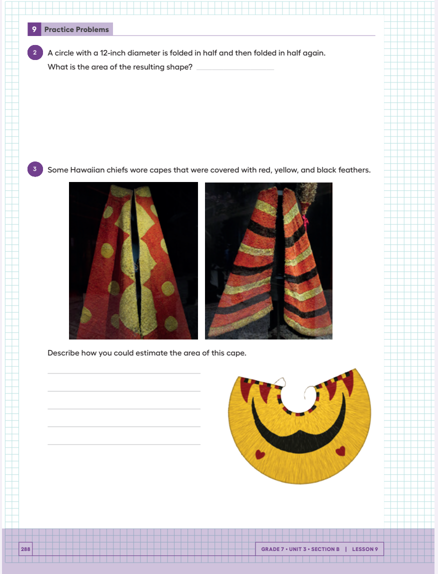
Some Hawaiian chiefs wore capes that were covered with red, yellow, and black feathers.

Describe how you could estimate the area of this cape.

Sample responses:

- Measure the radius of the large, outer circle. Square that value and multiply it by pi. Then multiply that answer by $\frac{3}{4}$ because the cape covers about $\frac{3}{4}$ of the full circle.
- Measure the diameters of the large, outer circle and the small, inner circle. Divide those each by 2 to find the radii. Use the formul $A = \pi^2$ to calculate the areas of each circle. Subtract the small circle's area from the large circle's area. Then multiply that answer by 0.8 because about 20% of each circle is missing.

Student Workbook

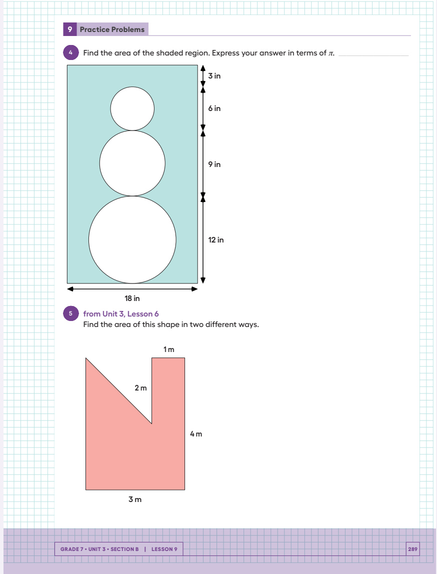

9 Practice Problems

2 A circle with a 12-inch diameter is folded in half and then folded in half again. What is the area of the resulting shape?

3 Some Hawaiian chiefs wore capes that were covered with red, yellow, and black feathers.

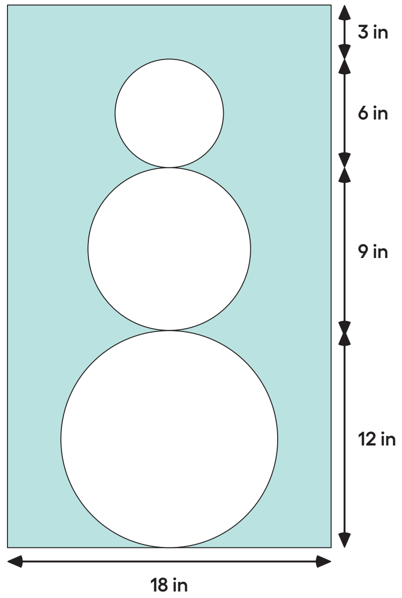
Describe how you could estimate the area of this cape.

Student Workbook



Problem 4

Find the area of the shaded region. Express your answer in terms of π.



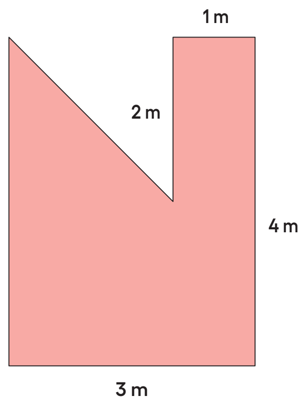
$540 - 65.25\pi$ in^2

Sample reasoning: Find the area of the rectangle by multiplying $18 \cdot 30 = 540$. Find the radii of the circles, square them, and add them together. $6^2 + 4.5^2 + 3^2 = 65.25$. Multiply 65.25 by π to get the total area of the circles. Subtract 65.25π from 540 to find the area of the shaded region.

Problem 5

from Unit 3, Lesson 6

Find the area of this shape in two different ways.



10 m^2

Sample responses:

It is a rectangle of area 12 m^2 with a triangle of area 2 m^2 missing.

It is a rectangle of area 6 m^2 plus a rectangle of area 2 m^2 plus a triangle of area 2 m^2.

Problem 6

from Unit 2, Lesson 5

Elena and Jada both read at a constant rate, but Elena reads more slowly. For every 4 pages that Elena can read, Jada can read 5.

a. Complete the table.

pages read by Elena	pages read by Jada
4	5
1	$\frac{5}{4}$ or 1.25
9	$\frac{45}{4}$ or 11.25
e	$\frac{5}{4}e$ or $1.25e$
12	15
$\frac{4}{5}j$ or $0.8j$	j

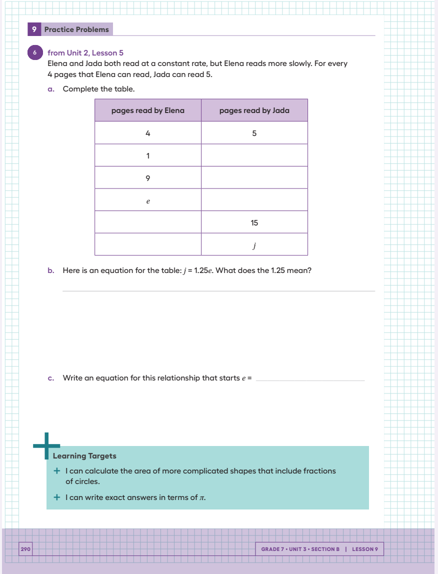
b. Here is an equation for the table: $j = 1.25e$. What does the 1.25 mean?

For every one page that Elena reads, Jada reads 1.25 pages.

c. Write an equation for this relationship that starts $e =$ ...

$e = \frac{4}{5}j$ or $e = 0.8j$

Student Workbook



9 Practice Problems

6 from Unit 2, Lesson 5

Elena and Jada both read at a constant rate, but Elena reads more slowly. For every 4 pages that Elena can read, Jada can read 5.

a. Complete the table.

pages read by Elena	pages read by Jada
4	5
1	
9	
e	
	15
	j

b. Here is an equation for the table: $j = 1.25e$. What does the 1.25 mean?

c. Write an equation for this relationship that starts $e =$

Learning Targets

+ I can calculate the area of more complicated shapes that include fractions of circles.
+ I can write exact answers in terms of π.

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