Lesson 1

Tiling the Plane

Goals

- Compare (orally) areas of the shapes that make up a geometric pattern.
- Comprehend that the word "area" (orally and in writing) refers to how much of the plane a shape covers.

Learning Target

I can explain the meaning of "area."

Lesson Narrative

In the first lesson of the course, students compare the amounts of a plane covered by different shapes and recall what they know about **area**. The investigations allow students to experience two important ideas that will be made explicit in the next lesson:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- The area of a **region** does not change when the region is decomposed and rearranged.

At the end of this lesson, students are asked to write their best definition of "area." It is important to let them formulate their definition in their own words. It is especially important to encourage Multilingual learners to use their own words and the words of their peers. In a future lesson, students will revisit the definition of "area" as "the number of square units that cover a region without gaps or overlaps."

While the mathematics that students explore in this lesson is not complicated and offers a low threshold for entry, it does prompt students to make sense of problems and persevere in solving them. The activities also enable the teacher to begin setting the expectations for mathematical discourse; that is for students to construct logical arguments and listen to the reasoning of others.

The lesson allows some time for the teacher to begin establishing classroom norms and routines.

Instructional Routines

- 5 Practices
- Which Three Go Together?

Required Materials

Materials to Gather

- Chart paper: Warm-up
- Sticky notes: Warm-up
- Geometry toolkits: Activity 1

Materials to Copy

- 6–12 Blank Math Community Chart (1 copy for every 30 students): Warm-up

Required Preparation

Warm-up:
Make a space for students to place their sticky notes at the end of the *Warm-up*. For example, hang a sheet of chart paper on a wall near the door.

Activity 1:
Assemble geometry toolkits. Toolkits include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Prepare pattern blocks (with triangles, rhombuses, and trapezoids), if available.

For the digital version of the activity, acquire devices that can run the applet.

Lesson:
Assemble geometry toolkits. Toolkits include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Lesson Timeline

Warm-up	Activity 1	Lesson Synthesis	Assessment: Cool-down
10 min	25 min	10 min	5 min

Lesson 1

Tiling the Plane

Lesson Narrative (continued)

A note about terminology:

In these materials, when we talk about a two-dimensional figure, such as a rectangle, triangle, or circle, we usually mean the boundary of the figure (such as the sides of a rectangle), not including the region inside. However, we also use shorthand language such as "the area of a rectangle" to mean the "the area of the region inside the rectangle." The term "shape" could refer to a figure with or without its interior. Although the terms "figure," "region," and "shape" are used without being defined precisely for students, help students understand that sometimes our focus is on the boundary (which in this unit will always be composed of black line segments), and sometimes it is on the region inside (which in this unit will be shown in color and referred to as "the shaded region").

Math Community

This is the first exercise that focuses on the work of building a mathematical community. Students have the opportunity to think about what a mathematical community is and to share their initial thoughts about what it looks like and sounds like to do math together in a community.

Student Learning Goal

Let's look at tiling patterns and think about area.

Warm-up

Which Three Go Together: Tilings

10 min

Activity Narrative

This is the first *Which Three Go Together* routine in the course. In this routine, students are presented with four items or representations and asked: "Which three go together? Why do they go together?"

Students are given time to identify a set of three items, explain their rationale, and refine their explanation to be more precise or find additional sets. The reasoning here prompts students to notice common mathematical attributes, look for structure, and attend to precision, which deepens their awareness of connections across representations.

This *Warm-up* prompts students to compare four geometric patterns. It gives students a reason to use language precisely and allows the teacher to hear how students use terminology in describing geometric characteristics. Comparing the patterns also urges students to think about shapes that cover the plane without gaps and overlaps, which supports future conversations about the meaning of "area."

Before students begin their work, consider establishing a small, discreet hand-signal that students can display when they have an answer that they can support with reasoning. This might include a thumbs-up or a certain number of fingers that indicates the number of responses that they have. Using a signal is a quick way to see if students have had enough time to think about the problem. A subtle signal keeps students from being distracted or rushed by seeing hands being raised around the class.

Students may choose to describe the patterns in terms of:

- Colors (blue, green, yellow, white, or no color).
- Size of shapes or other measurements.
- Geometric shapes (squares, pentagons, hexagons, or polygons).
- Relationships of shapes (whether each side of the polygons meets the side of another polygon, what polygon is attached to each side, whether there is a gap between polygons, and so on).

Launch

Arrange students in groups of 2–4. Display the four patterns for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three patterns that go together and can explain why. Next, tell students to share their response with their group and then work together to find as many sets of three as they can.

Instructional Routines

Which Three Go Together?

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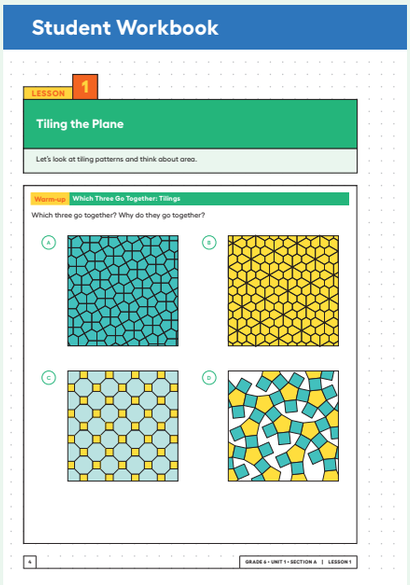
Student Workbook

LESSON 1

Tiling the Plane

Let's look at tiling patterns and think about area.

Warm-up Which Three Go Together: Tilings

Which three go together? Why do they go together?

A B C D

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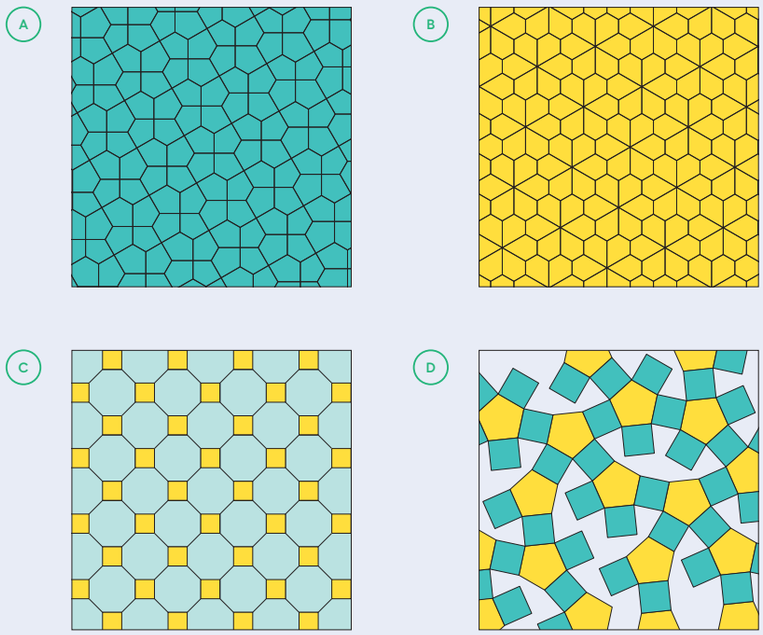
Student Task Statement

Which three go together? Why do they go together?



Sample responses:

A, B, and C go together because:

- All the shapes have color (or are shaded).
- All the colored shapes meet other colored shapes on all sides. There are no gaps between the shapes.
- They don't have white (or non-shaded) regions.
- The shapes have different side lengths.

A, B, and D go together because:

- They all have pentagons.

A, C, and D go together because:

- They have the color blue.
- They have shapes with one or more right angles.

B, C, and D go together because:

- They have the color yellow.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Because there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure that the reasons given are correct.

During the discussion, ask students to explain the meaning of any geometric terminology they use (names of polygons or angles, parts of polygons, "area") and to clarify their reasoning as needed. For example, a student may say that

Patterns A, B, and C each have shapes with different side lengths, but all the shapes in Pattern D have the same side lengths. Ask how they know that is the case, and whether this is true for the white (or non-filled) regions in Pattern D.

Explain to students that covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps is called "tiling" the plane. Patterns A, B, and C are examples of tiling. Tell students that they will explore more tilings in upcoming activities.

Math Community

Tell students that today is the start of planning the type of mathematical community they want to be a part of for this school year. The start of this work will take several weeks as the class gets to know one another, reflects on past classroom experiences, and shares their hopes for the year.

Display and read aloud the question "What do you think it should look like and sound like to do math together as a mathematical community?" Give students 2 minutes of quiet think time and then 1–2 minutes to share with a partner. Ask students to record their thoughts on sticky notes and then place the notes on the sheet of chart paper. Thank students for sharing their thoughts and tell them that the sticky notes will be collected into a class chart and used at the start of the next discussion.

After the lesson is complete, review the sticky notes to identify themes. Make a Math Community Chart to display in the classroom. See the blackline master Blank Math Community Chart for one way to set up this chart. Depending on resources and wall space, this may look like a chart paper hung on the wall, a regular sheet of paper to display using a document camera, or a digital version that can be projected. Add the identified themes from the students' sticky notes to the student section of the "Doing Math" column of the chart.

Activity 1

More Red, Green, or Blue?

25 min

Activity Narrative

There is a digital version of this activity.

This activity asks students to compare the amounts of the plane covered by two tiling patterns, with the aim of supporting two big ideas of the unit:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- A region can be decomposed and rearranged without changing its area. Students are likely to notice that in each pattern:
- The same 3 polygons (triangles, rhombuses, and trapezoids) are used as tiles.
- The shapes are arranged without gaps and overlaps, but their arrangements are different.
- A certain set of smaller tiles form a larger hexagon. Each hexagon has 3 trapezoids, 4 rhombuses, and 7 triangles.
- The entire tiling pattern is composed of these hexagons.

Expect students to begin their comparison by counting each shape, either in the entire pattern or in a portion that repeats. For students to effectively

Instructional Routines

5 Practices

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compare how much of the plane is covered by each shape, they need to be aware of the relationships between the shapes. For example, two green triangles can be placed on top of a blue rhombus so that they match up exactly. This means that two green triangles cover the same amount of the plane as one blue rhombus covers. (Students don't need to use the word "area" in their explanations. At this point, phrases such as "they match up" or "two triangles make one rhombus" suffice.)

Geometry toolkits are introduced here, giving students an opportunity to use tools strategically. Students could, for instance, use tracing paper to trace shapes, use a straightedge to extend lines, or use scissors to cut out shapes.

Monitor for the different ways in which students make comparisons. Here are some approaches students may take, from more common to less common:

- Compare the numbers of shapes in the entire pattern—56 green triangles, 32 blue rhombuses, and 24 red trapezoids. This doesn't yet tell us which shape covers more of the plane.
- Compare the numbers of shapes in a larger hexagon—7 green triangles, 4 rhombuses, and 3 trapezoids. There are fewer shapes to count, but these numbers alone also don't tell us which shape covers more of the plane.
- Compare the numbers of shapes and their relationships. Examples: One rhombus matches up exactly with 2 triangles. One trapezoid matches up exactly with 3 triangles. In a larger hexagon, there are 7 green triangles, 4 rhombuses that match up with 8 triangles, and 3 trapezoids that match up with 9 triangles. (Students may also relate the areas of shapes in the entire pattern this way. See student responses for additional examples.)

The routine of Anticipate, Monitor, Select, Sequence, Connect requires a balance of planning and flexibility. The anticipated approaches might not surface in every class, and there may be reason to change the order in which strategies are presented. While monitoring, keep in mind the learning goal and adjust the order to ensure that all students have access to the first idea presented (whether that be a common misconception or a different approach).

In the digital version of the activity, students use an applet to explore the shapes that comprise the pattern. The applet allows students to see the patterns on a triangular grid and to frame the repeating larger hexagons. It also allows students to isolate a larger hexagon and to move and rotate individual shapes within it. These features might help students in quantifying and relating the shapes.

Launch

Arrange students in groups of 2. Ask one partner to analyze Pattern A and the other to analyze Pattern B. Tell students that their job is to compare the amount of the plane covered by each shape in their pattern.

Before students begin their work, introduce them to the geometry toolkits. Encourage students to consider using one or more tools in the toolkits for help, if needed. If pattern blocks are available, they can be offered as well.

Give students 4–5 minutes of quiet think time. Then ask students to discuss their responses with their partner.

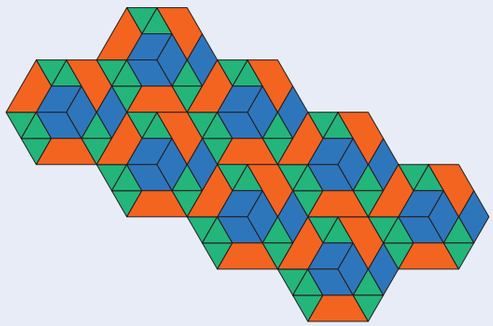
Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

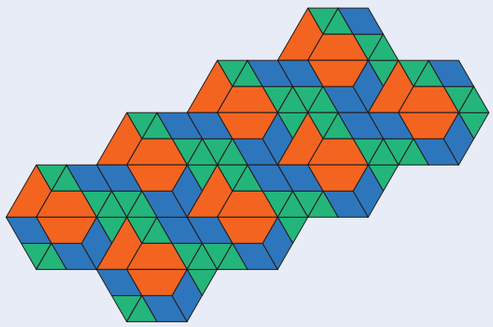
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B



In both Patterns A and B, more of the plane is covered by red trapezoids than by green triangles or blue rhombuses.

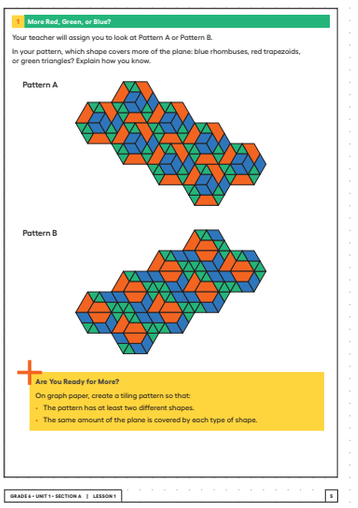
Sample reasoning:

- Patterns A and B are each made of 56 green triangles, 32 blue rhombuses, and 24 red trapezoids.
  - One blue rhombus covers the same amount of the plane as do 2 green triangles, so 32 blue rhombuses cover the same amount of the plane as do 64 green triangles. This is more than the amount covered by 56 green triangles. So, blue rhombuses cover more of the plane than green triangles cover.
  - One red trapezoid covers the same amount of the plane as do 3 green triangles, so 24 red trapezoids cover the same amount of the plane as do 72 green triangles. This is more than the amount covered by 64 green triangles. So, red trapezoids cover more of the plane than blue rhombuses cover.
- Each pattern is composed of 8 larger hexagons. In each of these hexagons there are 3 red trapezoids, 4 blue rhombuses, and 7 green triangles.
  - Comparing 3 trapezoids and 4 rhombuses: Two red trapezoids can be arranged into a small hexagon. Three rhombuses can also be arranged into the same small hexagon. This means that 2 trapezoids cover the same amount of the plane as do 3 rhombuses. The last red trapezoid covers more of the plane than does the last rhombus. So, red trapezoids cover more of the plane than blue rhombuses cover.

Building on Student Thinking

Students may say more of the area is covered by the color they see the most in each image, saying, for example, "It just looks like there is more red." Ask these students if there is a way to prove their observations. Students may only count the number of green triangles, red trapezoids, and blue rhombuses but not account for the area covered by each shape. If students suggest that the shape with the most pieces in the pattern covers the most amount of the plane, ask them to test their hypothesis. For example, ask, "Do 2 triangles cover more of the plane than 1 trapezoid covers?"
Students may not recall the terms "trapezoid," "rhombus," and "triangle." Consider reviewing the terms, although they do not need to know the formal definitions to work on the task.

Student Workbook


1 More Red, Green, or Blue?

Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A

Pattern B

Are You Ready for More?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

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- Comparing 3 trapezoids and 7 triangles: One trapezoid covers the same amount of the plane as do 3 triangles, so 2 trapezoids cover the same amount of the plane as do 6 triangles. The last trapezoid covers more of the plane than does the last triangle. So, red trapezoids cover more of the plane than green triangles cover.

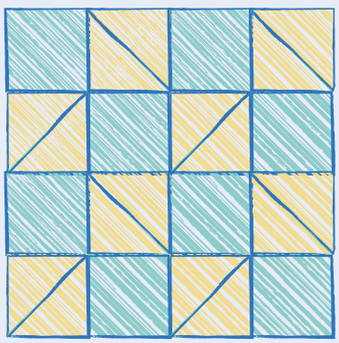
Are You Ready for More?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Answers vary.

Sample response:



Activity Synthesis

The purpose of this discussion is to help students connect their comparisons of shapes to the idea of area.

Invite previously selected students to share their answers and explanations. Sequence the discussion of strategies in the order listed in the *Activity Narrative*. Be sure to discuss the idea of comparing shapes by placing them on top of one another and seeing if or how they match. Consider demonstrating this idea using the applet.

Make it explicit that when students are asked, "Which shape covers more of the plane?", they are being asked to compare the areas covered by the different shapes.

Connect the discussion to the learning goals by asking questions, such as:

- *"How does the area of the trapezoid compare to the area of the triangle?"*

  The area of the trapezoid is three times the area of the triangle.

- *"How does the area of the rhombus compare to the area of the triangle?"*

  The area of the rhombus is twice the area of the triangle.

- *"Is it possible to compare the area that all of the rhombuses cover in Pattern A to the area that all the triangles cover in Pattern B? If yes, how? If no, why not?"*

  Yes, we can count the number of rhombuses in A and the number of triangles in B. Because 2 triangles have the same area as does 1 rhombus, divide the number of rhombuses in A by 2. Then compare the result to the number of triangles in B.

Lesson Synthesis

In this lesson, students started to reason about what it means for two shapes to have the same area. They also started to think about tools that can help them do mathematics. Ask students:

- *"Draw two shapes that you know do not have the same area. How can you tell?"*
- *"What are some of the tools in the geometry toolkit and what are they used for?"*

Tell students that they will continue to think about area and to learn to use tools strategically when doing mathematics.

Lesson Summary

In this lesson, we learned about *tiling* the plane, which means "covering a two-dimensional **region** with copies of the same shape or shapes such that there are no gaps or overlaps."

Then we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

In future lessons, we will continue with this reasoning, and we will continue learning how to use mathematical tools strategically to help us do mathematics.

Cool-down

What is Area?

5 min

The purpose of this *Cool-down* is to check how students are thinking about area after engaging in the activities. While the task prompts students to reflect on the work in this lesson, ideas about area from students' prior work in grades 3–5 may also emerge. Knowing the range of student thinking will help to inform the next day's lesson.

Student Task Statement

Think about your work today, and write your best definition of "area."

Sample responses:

- The amount of space inside a two-dimensional shape
- The measurement of the inside of a shape
- The number of square units inside a shape
- The amount of space a shape covers
- The amount of the plane a shape covers

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Student Workbook


1 Lesson Summary

In this lesson, we learned about tiling the plane, which means "covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps."

Then we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about area.

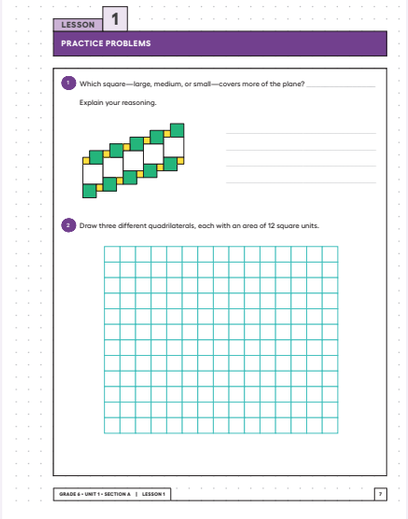
In future lessons, we will continue with this reasoning, and we will continue learning how to use mathematical tools strategically to help us do mathematics.

Lesson 1

Practice Problems

5 Problems

Student Workbook


LESSON 1

PRACTICE PROBLEMS

1 Which square—large, medium, or small—covers more of the plane? Explain your reasoning.

2 Draw three different quadrilaterals, each with an area of 12 square units.

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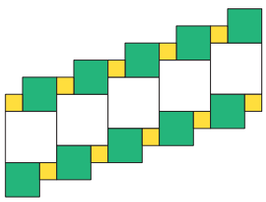
Problem 1

Which square—large, medium, or small—covers more of the plane? Explain your reasoning.



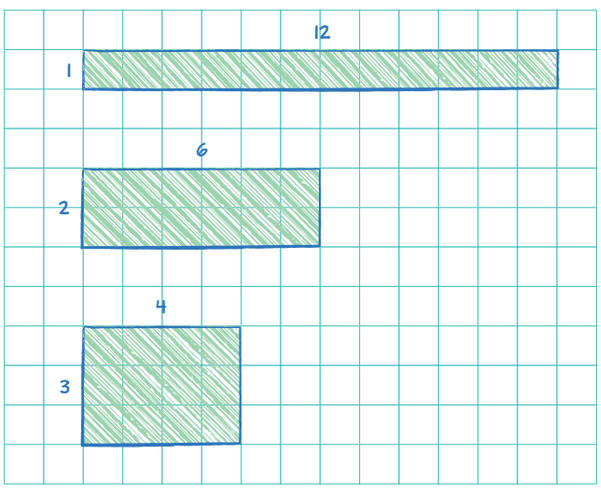
The large square covers more of the plane.

Sample reasoning: A large square can fit exactly 9 small squares. A medium square can fit exactly 4 small squares. There are 5 large squares, which cover the same amount of the plane as do 45 small squares. There are 10 medium squares, which cover the same amount of the plane as do 40 small squares. There are only 10 small squares.

Problem 2

Draw three different quadrilaterals, each with an area of 12 square units.

Sample response:

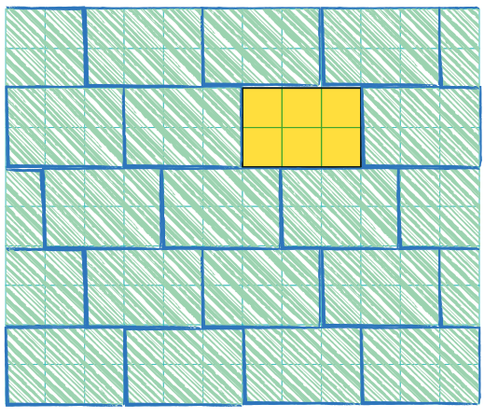


Problem 3

Use copies of the rectangle to show how a rectangle could:

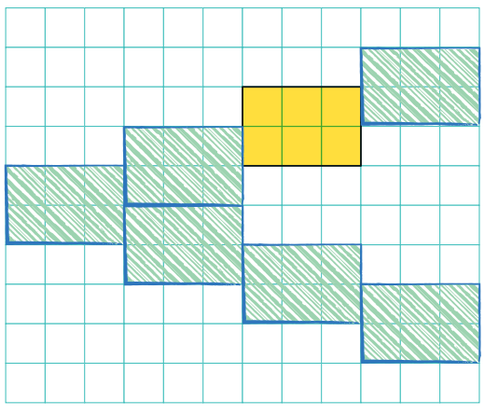
a. Tile the plane.

Sample response:

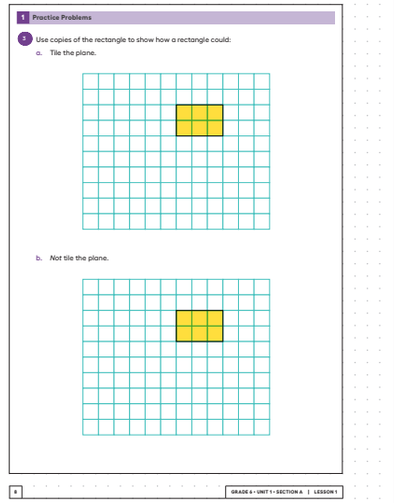


b. *Not* tile the plane.

Sample response:



Student Workbook


1 Practice Problems

3 Use copies of the rectangle to show how a rectangle could:

a. Tile the plane.

b. *Not* tile the plane.

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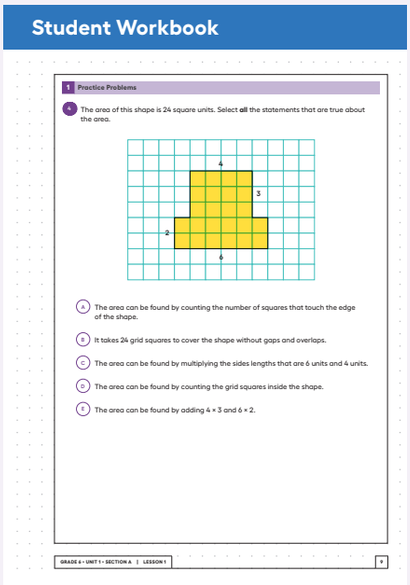
Student Workbook


1 Practice Problems

4 The area of this shape is 24 square units. Select **all** the statements that are true about the area.

A The area can be found by counting the number of squares that touch the edge of the shape.

B It takes 24 grid squares to cover the shape without gaps and overlaps.

C The area can be found by multiplying the sides lengths that are 6 units and 4 units.

D The area can be found by counting the grid squares inside the shape.

E The area can be found by adding 4 × 3 and 6 × 2.

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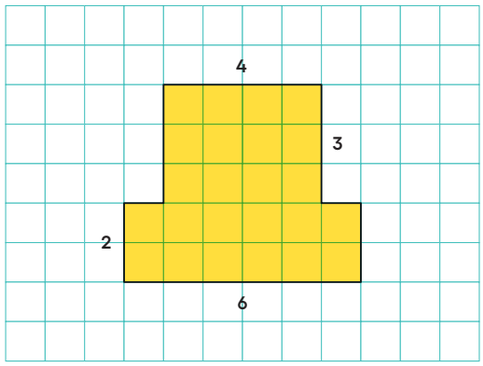
Problem 4

The area of this shape is 24 square units. Select **all** the statements that are true about the area.



A. The area can be found by counting the number of squares that touch the edge of the shape.

B. It takes 24 grid squares to cover the shape without gaps and overlaps.

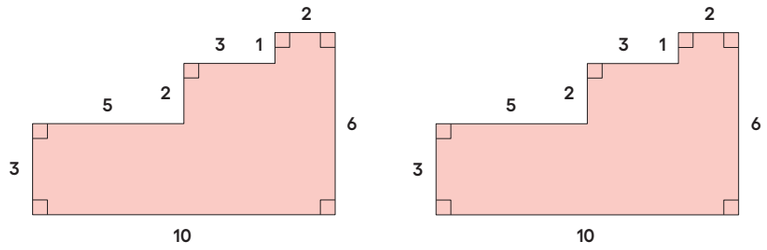
C. The area can be found by multiplying the sides lengths that are 6 units and 4 units.

D. The area can be found by counting the grid squares inside the shape.

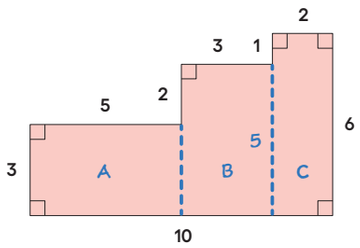
E. The area can be found by adding 4 × 3 and 6 × 2.

Problem 5

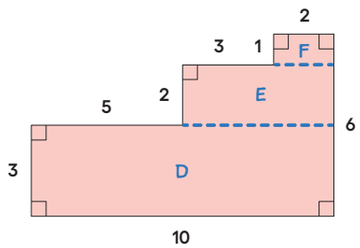
Here are two copies of the same figure. All angles are right angles. Show two different ways for finding the area of the shaded region.



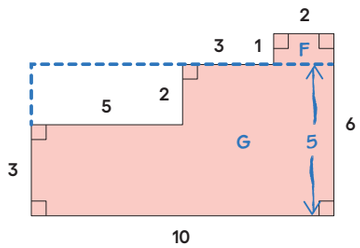
Sample reasoning:



Area of A is 15 square units. Area of B is 15 square units. Area of C is 12 square units. The area of the entire region is 15 + 15 + 12, or 42 square units.



Area of D is 30 square units. Area of E is 10 square units. Area of F is 2 square units. The area of the entire region is 30 + 10 + 2 or 42 square units.



Area of F is 2 square units. Area of G is the area of the 10-by-5 rectangle subtracted by the area of the 5-by-2 rectangle in the upper left.
(10 × 5) − (5 × 2) = 50 − 10 = 40, so the area of G is 40 square units. The total area is 40 + 2, or 42 square units.

Student Workbook