Lesson 1

Lots of Flags

Goals

- Compare (orally and in writing) the dimensions and scale factors of multiple scaled copies of the same figure.
- Explain (orally) how to estimate or calculate the percentage of a rectangular area that is covered by another region.
- Generate the dimensions for a scaled copy of an original figure that has fractional side lengths.

Learning Targets

- I can find dimensions on scaled copies of a rectangle.
- I remember how to compute percentages.

Lesson Narrative

In this lesson, students apply proportional reasoning to situations involving fractional values. First, students revisit scaled copies. They calculate possible dimensions for a scaled version of the United States flag. Then students review percentages. They determine the percentage of the flag's area that is red, white, and blue for a variety of different flags from around the world.

As students compare multiple flags, they make use of repeated reasoning. As students explain their reasoning about the scaled figures or the percentages, they construct arguments and critique the reasoning of others.

The last activity is optional because it provides an opportunity for additional practice by combining the main ideas of the previous two activities.

Student Learning Goal

Let's explore the U.S. flag.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)
- Engagement (Activity 3)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)
- MLR:8 Discussion Supports (Activity 2)

Instructional Routines

- Take Turns
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	15 min	10 min	5 min

Warm-up

Scaled or Not?

5 min

Activity Narrative

In this *Warm-up* students reason about figures on a grid and determine which are scaled copies. This reminds students of proportional reasoning in a geometric context, which was done earlier in the course.

Launch

Arrange students in groups of 2.

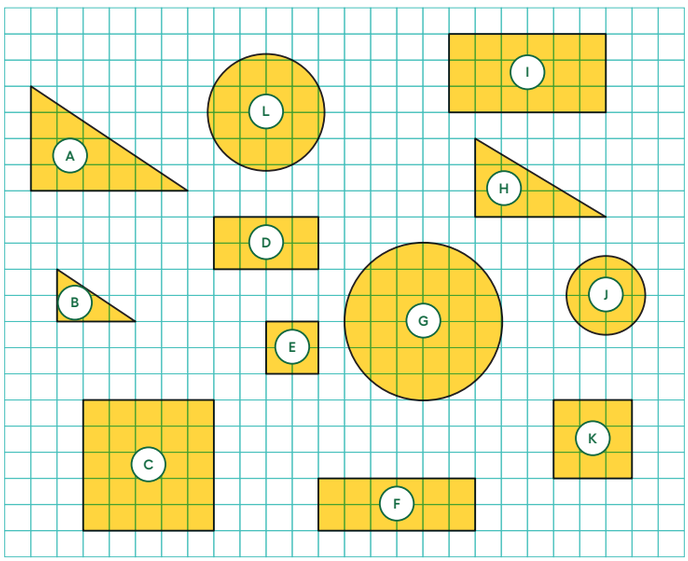
Give students 1 minute of quiet work time and another 1–2 minutes to share their solutions with their partner.

Ask students to make sure they have the same objects identified in the first question. If one partner is missing a set of scaled objects, they should add them to their list during their partner discussion.

As students discuss their answers with their partner, select students to share their answers to the second question during the whole-class discussion. Select students so that different sets of objects and their scale factors are represented in the discussion.

Student Task Statement

1. Which of the geometric objects are scaled versions of each other?



- A and B
- C, E, and K
- D and I
- G, J, and L

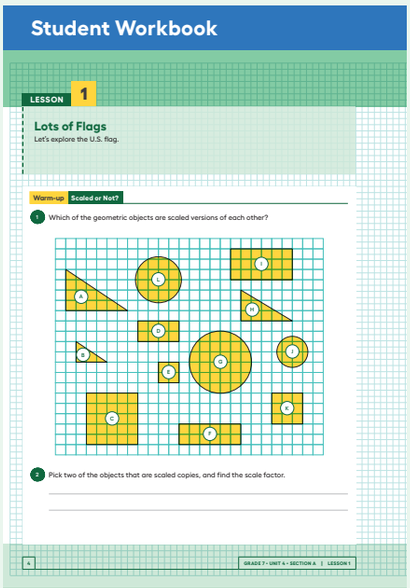
2. Pick two of the objects that are scaled copies, and find the scale factor.

Sample response: A is 2 times the size of B. The height of A is 4, and the length of its base is 6. The height of B is 2, and the length of its base is 3.

Building on Student Thinking

Students might think H is a scaled version of A or B. Suggest that they consider possible scale factors to get from, for example, A to H.

Student Workbook


LESSON 1

Lots of Flags

Let's explore the U.S. flag.

Warm-up Scaled or Not?

1. Which of the geometric objects are scaled versions of each other?

2. Pick two of the objects that are scaled copies, and find the scale factor.

Instructional Routines

MLR7: Compare and Connect

ilclass.com/r/10695592

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.

Activate or supply background knowledge. Provide a document including strategies of multiplying and dividing mixed numbers for students to use as a reference.

Supports accessibility for: Memory, Organization

Activity Synthesis

Invite previously selected students to share their answers to the second question. Ask the rest of the class whether they agree or disagree with the responses.

If time permits, ask students,

- *"Were there any figures that you initially believed were scaled versions of one another but later decided that they weren't? How did you know?"*

Activity 1

Flags Are Many Sizes

15 min

Activity Narrative

In this activity, students extend their recent work with scale drawings to scale figures with fractional side lengths. As they explore connections between the dimensions of the different sizes of flags and the ratios of the side lengths, students make use of repeated reasoning. This prepares students for upcoming work with ratios and rates involving fractions.

Monitor for groups who choose to work on each of the different options for flag uses: classroom flag, postage stamp, space shuttle, shoulder patch, or an original idea.

A note about flag dimensions: The official United States government flag has sides with ratio 1 : 1.9, that is the width of the flag is 1.9 times its height. However, there are many commercially sold flags that use different ratios. This activity is working with the official ratio. If there is a flag displayed in the classroom, it would be interesting to check if it uses the official ratio or one of the other common commercial ratios, such as 2 : 3 or 5 : 8 or 6 : 10.

Launch

Ask students where they have seen the United States flag displayed. Encourage them to list examples of flags of a variety of different sizes.

Tell students,

- *"The United States flag is displayed in many different sizes and for different purposes. One standard size is 19 feet by 10 feet. What wouldbe a possible use for a flag of this size?"*

Arrange students in groups of 2–3.

Give students 5–6 minutes of group work time.

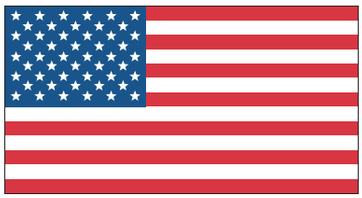
Select students with different sizes of flags to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially students who haven't shared recently.

Student Task Statement

One standard size for the United States flag is 19 feet by 10 feet. On a flag of this size, the union (the blue rectangle in the top-left corner) is about $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet.

There are many places that display flags of different sizes.

- Many classrooms display a U.S. flag.
- Flags are often displayed on stamps.
- There was a flag on the space shuttle.
- Astronauts on the Apollo missions had a flag on a shoulder patch.



1. Choose one of the four options, or invent another example of your own. Decide on a size that would be appropriate for this flag. Then, find the size of the union.

   Sample responses:

   - Classroom dimensions are 3.8 feet by 2 feet (scale factor $\frac{1}{5}$), and union dimensions are 1.525 feet by 1.075 feet.
   - Stamp dimensions are 0.079 foot by 0.042 foot (scale factor $\frac{1}{240}$), and union dimensions 0.032 foot by 0.022 foot.
   - Shuttle dimensions are 9.5 feet by 5 feet (scale factor $\frac{1}{2}$), and union dimensions 3.813 feet by 2.688 feet.
   - Patch dimensions are 0.32 foot by 0.167 foot (scale factor $\frac{1}{60}$), and union dimensions 0.127 foot by 0.090 foot.

2. Share your answer with another group that used a different option. What do your dimensions have in common?

   Sample response: The height of the union is approximately $\frac{2}{3}$ or $\frac{3}{4}$ of the width of the union.

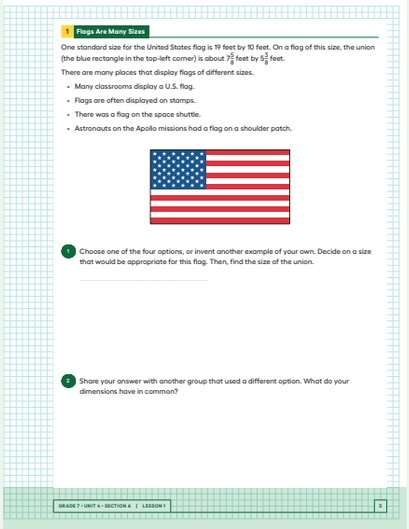
Activity Synthesis

The goal of this discussion is to highlight equivalent ratios between the flags' measurements. Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches.

Here are some questions for discussion:

- *"What do the approaches have in common? How are they different?"*
- *"Is this flag a scaled copy of the original 19-by-10-foot flag? How do we know?"*
- *"How does the scale factor show up in each representation?"*

Student Workbook


1 Flags Are Many Sizes

One standard size for the United States flag is 19 feet by 10 feet. On a flag of this size, the union (the blue rectangle in the top-left corner) is about $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet.

There are many places that display flags of different sizes.

- Many classrooms display a U.S. flag.
- Flags are often displayed on stamps.
- There was a flag on the space shuttle.
- Astronauts on the Apollo missions had a flag on a shoulder patch.

1 Choose one of the four options, or invent another example of your own. Decide on a size that would be appropriate for this flag. Then, find the size of the union.

2 Share your answer with another group that used a different option. What do your dimensions have in common?

GRADE 7 • UNIT 4 • SECTION A | LESSON 1

Instructional Routines

Take Turns

ilclass.com/r/10573524

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Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports.

Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed _____, so I matched ..." When students disagree, encourage them to challenge each other using these sentence frames: "I agree because ..." and "I disagree because ..." This will help students clarify their reasoning about the percentages of each color.

Advances: Speaking, Conversing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Access for Perception.

Add labels to the display to help students with color blindness to be able to distinguish between the red and blue regions of each flag.

Supports accessibility for: Visual-Spatial Processing

The key takeaway is that the ratios between the lengths in the scaled copy are equivalent to the ratios between lengths in the original figure. Even when lengths have fractional values, they can still be scaled by the same scale factor.

Activity 2

What Percentage Is Each Color?

15 min

Activity Narrative

In this partner activity, students take turns matching images of flags to verbal descriptions of what percentage of the flag's total area is red, white, and blue. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

This activity reminds students of previous work they did with percentages. It prepares students for later in this unit when they will calculate the percentage that one number is of another number.

Launch

Arrange students in groups of 2. Display the task for all to see. Tell students that for each flag image, there is a corresponding description. If time allows, choose a student to be your partner and demonstrate how to set up and do the activity.

- One partner picks an image of a flag.
- They identify a description of the percentages of each color and explain why they think it is equivalent.
- The other partner listens and makes sure they agree with the match and the reasoning.
- If they don't agree, the partners discuss until they come to an agreement.
- For the next flag image, the students swap roles.

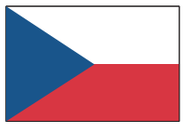
Student Task Statement

Here are some flags that use the colors red, white, and blue.

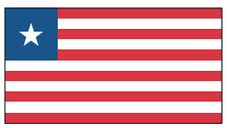
Chile

Puerto Rico

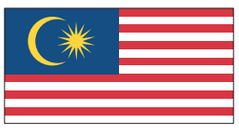
Czech Republic



Liberia



Malaysia



Samoa

1. Take turns with your partner to match each flag with a description. The description gives the **percentage** of the flag's area that is each color.

- For each match that you find, explain to your partner how you know it's a match.
- For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

a. 36% red, 33% white, 24% blue Malaysia

b. 37% red, 37% white, 26% blue Czech Republic

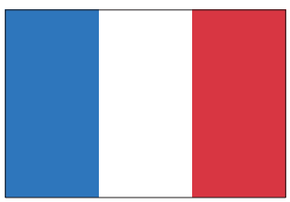
c. 46% red, 27% white, 27% blue Puerto Rico

d. 48% red, 42% white, 10% blue Liberia

e. 50% red, 35% white, 15% blue Chile

f. 75% red, 1% white, 24% blue Samoa

2. Here is an image of the flag of France. The three color bands are each the same width.



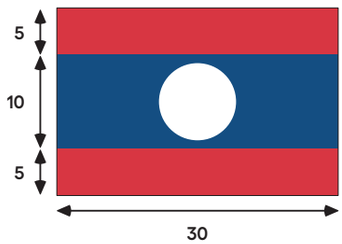
What percentage of this flag is:

a. red? $33\frac{1}{3}\%$

b. white? $33\frac{1}{3}\%$

c. blue? $33\frac{1}{3}\%$

3. Here is an image of the flag of Laos. The white circle has a diameter of 8 units.



What percentage of this flag is red? 50%

Student Workbook

2 What Percentage is Each Color?

Here are some flags that use the colors red, white, and blue.

Chile Puerto Rico Czech Republic

Liberia Malaysia Samoa

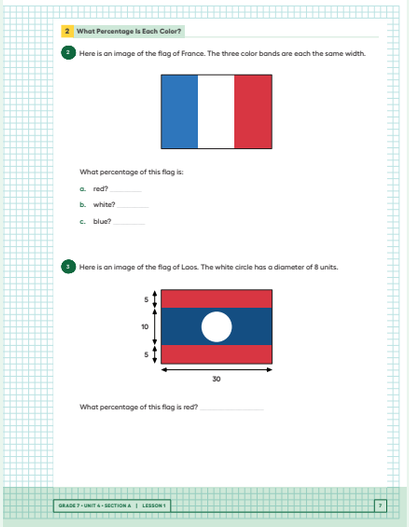
1 Take turns with your partner to match each flag with a description. The description gives the **percentage** of the flag's area that is each color.

- For each match that you find, explain to your partner how you know it's a match.
- For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking, and work to reach an agreement.

a. 36% red, 33% white, 24% blue _____
b. 37% red, 37% white, 26% blue _____
c. 46% red, 27% white, 27% blue _____
d. 48% red, 42% white, 10% blue _____
e. 50% red, 35% white, 15% blue _____
f. 75% red, 1% white, 24% blue _____

A GRADE 7 • UNIT 4 • SECTION A | LESSON 1

Student Workbook



2 What Percentage is Each Color?

2 Here is an image of the flag of France. The three color bands are each the same width.

What percentage of this flag is:

a. red? _____
b. white? _____
c. blue? _____

3 Here is an image of the flag of Laos. The white circle has a diameter of 8 units.

5
10
5
30

What percentage of this flag is red? _____

GRADE 7 • UNIT 4 • SECTION A | LESSON 1 7

Activity Synthesis

The purpose of this discussion is to remind students of strategies for determining the percentage that one number is of another number. Much of the discussion takes place between partners. First, invite students to share how they matched the flags with the descriptions of the percentages.

If not mentioned in students' explanations, highlight the use of benchmark percentages, such as:

- One-half of Chile's flag is red, which is 50%.
- Three-fourths of Samoa's flag is red, which is 75%.
- On the Czech Republic's flag, the red area is equal to the white area.

If not mentioned in students' explanations, ask students what the percentages of each color add up to. Key takeaways include:

- For the flags that are only red, white, and blue, the three percentages add up to 100%.
- On the Malaysia flag, there is also some yellow, so the red, white, and blue regions do not add up to 100%.

Next, invite students to share their reasoning for the flags from France and Laos. To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____' strategy?"*
- *"Do you agree or disagree? Why?"*

The key takeaways are:

- Percentages do not have to be whole numbers. For the three equally sized regions of the French flag to add up to 100%, they each need to be $33\frac{1}{3}$%.
- We can find the percentage that one number is of another number by dividing. On Laos's flag, the red area is 300 square units, while the whole flag is 600 sq. units. 300 ÷ 600 = 0.5, so the red area is 50% of the flag.

Activity 3: Optional

What Percentage Is the Union?

15 min

Activity Narrative

In this activity students calculate percentages of areas on the United States flag.

Knowing the side lengths of the flag and of the union allows students to compute the area of the flag and of the union. They can then compute what percentage of the flag is taken up by the union. Finding out what percentage of the flag is red requires additional reasoning. Students can either compute the area of the red stripes or they can see what fraction of the non-union part of the flag is red.

This is a good opportunity for students to estimate their answers and get a visual idea of the size of different percentages.

Launch

Keep students in the same groups. Tell students that they will return to examining the United States flag, this time looking at its area.

Give students 4–5 minutes of partner work time.

Student Task Statement

On a U.S. flag that is 19 feet by 10 feet, the union is about $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet. For each question, first estimate the answer, then compute the actual percentage.

1. What percentage of the flag is taken up by the union?

   approximately 20%

   The area of the union is about 41 square feet, and the area of the flag is 190 square feet, so the percentage is 41 ÷ 190 or about 0.216.

2. What percentage of the flag is red? Be prepared to share your reasoning.

   approximately 40%

   The total red area is 78.875 square feet, and the area of the flag is 190 square feet, so the percentage is 78.875 ÷ 190, or about 0.415.

Are You Ready for More?

The largest U.S. flag in the world is 225 feet by 505 feet.

1. Is the ratio of the length to the width equivalent to 1 : 1.9, the ratio for official government flags?

   no

2. If a square yard of the flag weighs about 3.8 ounces, how much does the entire flag weigh in pounds?

   about 3,000 pounds

Activity Synthesis

The purpose of this discussion is to emphasize the use of proportional reasoning when working with percentages. Ask students to share how their estimated percentages compared with their calculated percentages. Consider asking questions such as:

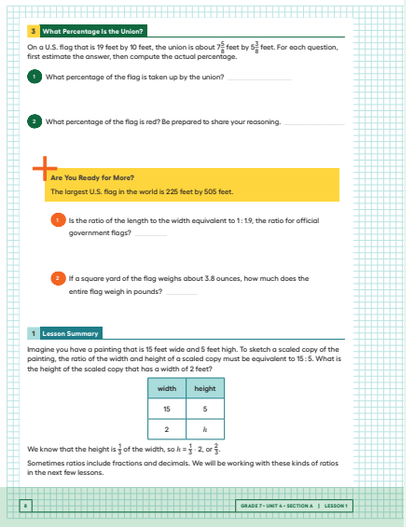
- *"How did you use the image of the flag to guide your estimate?"*
- *"Where did you use rounding in your estimations or calculations?"*

Access for Students with Diverse Abilities (Activity 3, Synthesis)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as "First, I _____ because ..." "I noticed _____, so I ..." "Why did you ... ?" "I agree/disagree because ..." *Supports accessibility for: Language, Social-Emotional Functioning*

Student Workbook



3 What Percentage is the Union?

On a U.S. flag that is 19 feet by 10 feet, the union is about $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet. For each question, first estimate the answer, then compute the actual percentage.

1. What percentage of the flag is taken up by the union?

2. What percentage of the flag is red? Be prepared to share your reasoning.

Are You Ready for More?

The largest U.S. flag in the world is 225 feet by 505 feet.

1. Is the ratio of the length to the width equivalent to 1 : 1.9, the ratio for official government flags?

2. If a square yard of the flag weighs about 3.8 ounces, how much does the entire flag weigh in pounds?

1 Lesson Summary

Imagine you have a painting that is 15 feet wide and 5 feet high. To sketch a scaled copy of the painting, the ratio of the width and height of a scaled copy must be equivalent to 15 : 5. What is the height of the scaled copy that has a width of 2 feet?

width	height
15	5
2	h

We know that the height is $\frac{1}{3}$ of the width, so $h = \frac{1}{3} \cdot 2$, or $\frac{2}{3}$.

Sometimes ratios include fractions and decimals. We will be working with these kinds of ratios in the next few lessons.

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Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson Synthesis

Share with students,

- *"Today we used what we know about scale factors and percentages to answer questions related to the United States flag and other flags."*

To review these concepts, consider asking students:

- *"How can you tell whether two objects are scaled versions of one another?"*

  Check whether corresponding lengths are related by the same scale factor.

- *"What properties stay the same when an object is scaled up or down?"*

  corresponding angles; the ratios between corresponding lengths

- *"How can you find the percentage of the total area that a particular region is?"*

  Divide the area of the region by the total area, or compare the region's area to known benchmarks, like 25% or 50%.

Lesson Summary

Imagine you have a painting that is 15 feet wide and 5 feet high. To sketch a scaled copy of the painting, the ratio of the width and height of a scaled copy must be equivalent to 15 : 5. What is the height of the scaled copy that has a width of 2 feet?

width	height
15	5
2	h

We know that the height is $\frac{1}{3}$ of the width, so $h = \frac{1}{3} \cdot 2$, or $\frac{2}{3}$.

Sometimes ratios include fractions and decimals. We will be working with these kinds of ratios in the next few lessons.

Cool-down

Colorado State Flag

5 min

Student Task Statement

The side lengths of the state flag of Colorado are in the ratio 2:3. If a flag is 12 feet long, what is its height?

8 feet

Lesson 1

Practice Problems

7 Problems

Problem 1

A rectangle has a height to width ratio of 3 : 4.5. Give two examples of dimensions for rectangles that could be scaled versions of this rectangle.

Sample response: a rectangle measuring 6 units by 9 units and a rectangle measuring 9 units by 13.5 units

Problem 2

One rectangle measures 2 units by 7 units. A second rectangle measures 11 units by 37 units. Are these two figures scaled versions of each other? If so, find the scale factor. If not, briefly explain why.

No, these two figures are not scaled versions of each other. The 2-unit side is scaled by a factor of 5.5 to correspond to the 11-unit side, but 7 multiplied by 5.5 is 38.5, not 37.

Problem 3

In Russia people make matryoshka dolls, which are hollow, wooden dolls that fit inside each other.

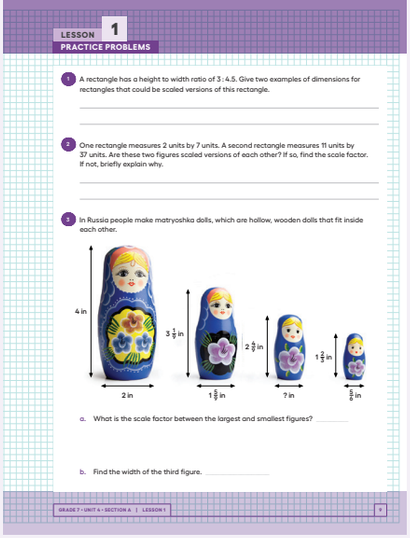
a. What is the scale factor between the largest and smallest figures?

$\frac{5}{12}$ or $\frac{12}{5}$

b. Find the width of the third figure.

$1\frac{2}{9}$ inches

Student Workbook


LESSON 1
PRACTICE PROBLEMS

1 A rectangle has a height to width ratio of 3 : 4.5. Give two examples of dimensions for rectangles that could be scaled versions of this rectangle.

2 One rectangle measures 2 units by 7 units. A second rectangle measures 11 units by 37 units. Are these two figures scaled versions of each other? If so, find the scale factor. If not, briefly explain why.

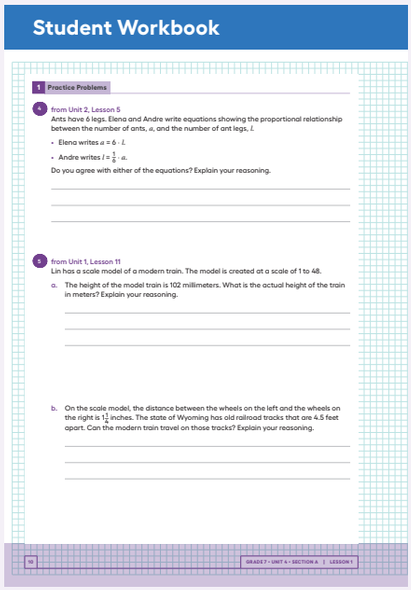
3 In Russia people make matryoshka dolls, which are hollow, wooden dolls that fit inside each other.

4 in, 2 in, 3 1/9 in, 1 5/9 in, 2 4/9 in, ? in, 1 2/3 in, 5/6 in

a. What is the scale factor between the largest and smallest figures?

b. Find the width of the third figure.

GRADE 7 • UNIT 4 • SECTION A | LESSON 1


Student Workbook

1 Practice Problems

4 from Unit 2, Lesson 5
Ants have 6 legs. Elena and Andre write equations showing the proportional relationship between the number of ants, a, and the number of ant legs, l.
- Elena writes $a = 6 \cdot l$.
- Andre writes $l = \frac{1}{6} \cdot a$.

Do you agree with either of the equations? Explain your reasoning.

5 from Unit 1, Lesson 11
Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.
a. The height of the model train is 102 millimeters. What is the actual height of the train in meters? Explain your reasoning.

b. On the scale model, the distance between the wheels on the left and the wheels on the right is $1\frac{1}{4}$ inches. The state of Wyoming has old railroad tracks that are 4.5 feet apart. Can the modern train travel on those tracks? Explain your reasoning.

10 GRADE 7 • UNIT 4 • SECTION A | LESSON 1

Problem 4

from Unit 2, Lesson 5

Ants have 6 legs. Elena and Andre write equations showing the proportional relationship between the number of ants, a, and the number of ant legs, l.

- Elena writes $a = 6 \cdot l$.
- Andre writes $l = \frac{1}{6} \cdot a$.

Do you agree with either of the equations? Explain your reasoning.

Neither of them are correct. Although 6 and $\frac{1}{6}$ are the correct constants of proportionality, they are being multiplied by the wrong variables. For example, using Elena's equation, 1 leg is equal to 6 ants.

Problem 5

from Unit 1, Lesson 11

Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.

a. The height of the model train is 102 millimeters. What is the actual height of the train in meters? Explain your reasoning.

4.896 meters

Sample reasonings:

- The actual height is 48 times the scaled height: $102 \cdot 48 = 4,896$. And 4,896 millimeters is 4.896 meters.
- 102 millimeters is 0.102 meters. The actual train height is 48 times 0.102 meters. $0.102 \cdot 48 = 4.896$.

b. On the scale model, the distance between the wheels on the left and the wheels on the right is $1\frac{1}{4}$ inches. The state of Wyoming has old railroad tracks that are 4.5 feet apart. Can the modern train travel on those tracks? Explain your reasoning.

no

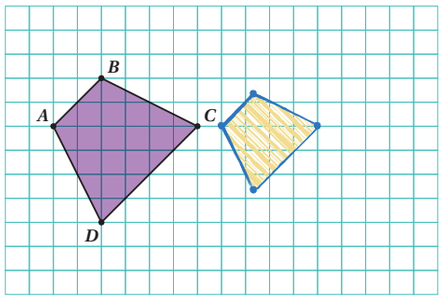
Sample reasoning: The modern train needs tracks that are 60 inches apart, because $1\frac{1}{4} \cdot 48 = 60$. The old tracks are only 54 inches apart, so they are not wide enough.

Problem 6

from Unit 1, Lesson 4

On the grid, draw a scaled copy of quadrilateral $ABCD$ with a scale factor $\frac{2}{3}$.

Sample response:



Problem 7

from Unit 1, Lesson 5

Solve each equation mentally.

a. $\frac{5}{2} \cdot x = 1$

$x = \frac{2}{5}$

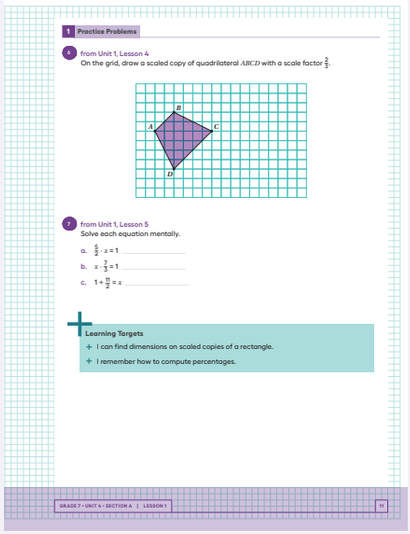
b. $x \cdot \frac{7}{3} = 1$

$x = \frac{3}{7}$

c. $1 \div \frac{11}{2} = x$

$x = \frac{2}{11}$

Student Workbook


1 Practice Problems

6 from Unit 1, Lesson 4
On the grid, draw a scaled copy of quadrilateral $ABCD$ with a scale factor $\frac{2}{3}$.

7 from Unit 1, Lesson 5
Solve each equation mentally.
a. $\frac{5}{2} \cdot x = 1$
b. $x \cdot \frac{7}{3} = 1$
c. $1 \div \frac{11}{2} = x$

Learning Targets
+ I can find dimensions on scaled copies of a rectangle.
+ I remember how to compute percentages.