Lesson 15

More Nets, More Surface Area

Goals

- Draw and assemble a net for the prism or pyramid shown in a given drawing.
- Interpret (using words and other representations) two-dimensional representations of prisms and pyramids.
- Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.

Learning Targets

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson Narrative

This lesson further develops students' ability to visualize the relationship between nets and polyhedra and their capacity to reason about surface area.

Previously, students identified polygons that make up the faces of a polyhedron and arranged them into a net. They also assembled given nets into polyhedra. In this lesson, students reason about the nets of polyhedra with less scaffolding. They practice mentally unfolding three-dimensional shapes, drawing two-dimensional nets, and using nets to calculate surface area.

As students coordinate edge lengths and arrangements of polygons in drawings of three-dimensional figures to those in the corresponding nets, they practice reasoning concretely and abstractly.

In an optional activity, students practice visualizing prisms that could be assembled from given nets (shown without a grid) and then compare and contrast their surface areas and volumes.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Representation (Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Warm-up)

Instructional Routines

- Math Talk
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Demonstration nets with and without flaps: Activity 1
- Geometry toolkits: Activity 1
- Glue or glue sticks: Activity 1
- Scissors: Activity 1
- Tape: Activity 1

Required Preparation

Activity 1:
Every student workbook contains a copy of the cards for this activity. Each group of 9 students will need one copy of the cards, so that each student gets one drawing of a polyhedron. Consider assignments of polyhedra in advance.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	30 min	15 min	10 min	5 min

Lesson 15

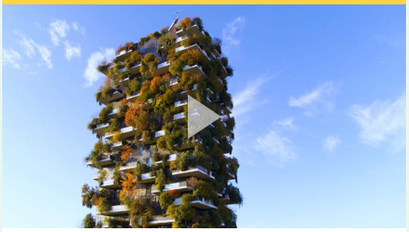
More Nets, More Surface Area

Student Learning Goal

Let's draw nets and find the surface area of polyhedra.

Inspire Math

Greening the City video



Go Online

Before the lesson, show this video to reinforce the real-world connection.

ilclass.com/l/458362

Please log in to the site before using the QR code or URL.

Instructional Routines (Warm-up, Student Task)

Math Talk

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Access for Students with Diverse Abilities

Action and Expression: Internalize Executive Functions.
To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Warm-up

Math Talk: Adjusting a Factor

5 min

Activity Narrative

This is the first *Math Talk* activity in the course. See the launch for extended instructions for facilitating this activity successfully.

This *Math Talk* focuses on multiplication of two whole numbers. It encourages students to observe the impact of adjusting a factor and to rely on the structure of base-ten numbers and the properties of operations to find products.

Each expression is designed to elicit slightly different reasoning. In explaining their strategies, students need to be precise in their word choice and use of language. While many ways of reasoning may emerge, it may not be feasible to discuss every strategy. Consider gathering only 2–3 different strategies per expression. As students explain their strategies, ask them how the factors impacted their approach.

Launch

This is the first time students do the *Math Talk* instructional routine in this course, so it is important to explain how it works before starting.

Explain that a *Math Talk* has four problems, revealed one at a time. For each problem, students have a minute to quietly think and are to give a signal when they have an answer and a strategy. The teacher then selects students to share different strategies (likely 2 or 3, given limited time), and might ask questions such as "Who thought about it in a different way?" The teacher then records the responses for all to see, and might ask clarification questions about the strategies before revealing the next problem.

Consider establishing a small, discreet hand signal that students can display when they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Find the value of each product mentally.

A. 6 · 15

90

Sample reasoning: (6 • 10) + (6 • 5) = 90

B. 12 · 15

180

Sample reasoning: Since the 6 from the first question doubled to 12, and the 15 stayed the same, the product doubles to 180. This is because there are twice as many groups of 15 as in the first question.

C. 6 · 45

270

Sample reasoning: Since the 6 is the same as the in the first question, and the 15 tripled to 45, the product triples to 270. This is because the number of groups stayed the same, but the amount in each group got three times as large.

D. 13 · 45

585

Sample reasoning: Since the 45 is the same as the previous question, we can double the 6 and the product to get 540. We need one more group of 45, and 540 + 45 = 585

Activity Synthesis

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Activity 1

Building Prisms and Pyramids

30 min

Activity Narrative

Previously, students used a given net of a polyhedron to find its surface area. Here they use a given polyhedron to draw a net and then calculate its surface area.

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Instructional Routines

MLR7: Compare and Connect

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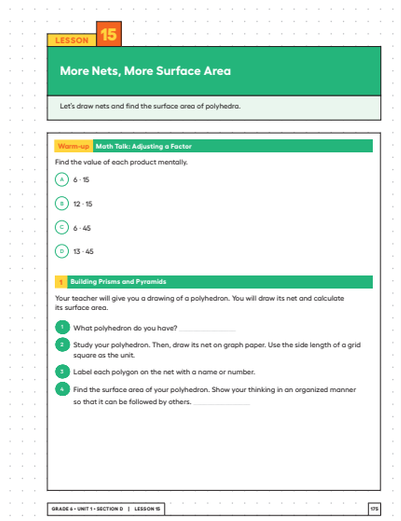
Student Workbook


LESSON 15

More Nets, More Surface Area

Let's draw nets and find the surface area of polyhedra.

Warm-up Math Talk: Adjusting a Factor

Find the value of each product mentally.

A. 6 · 15

B. 12 · 15

C. 6 · 45

D. 13 · 45

1 Building Prisms and Pyramids

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have? _____
2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.
3. Label each polygon on the net with a name or number.
4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others. _____

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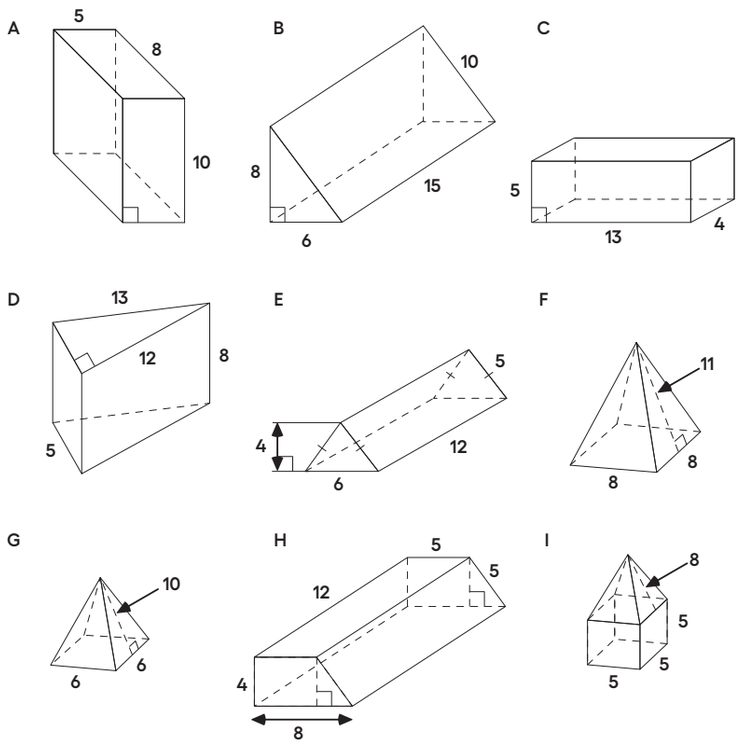
Use the provided polyhedra to differentiate the work for students with varying degrees of visualization skills. Rectangular prisms (A and C), triangular prisms (B and D), and square pyramids (F and G) can be managed by most students. Triangular Prism E requires a little more interpretive work (in that the measurements of some sides may not be immediately apparent to students). Trapezoidal Prism H and Polyhedron I (a composite of a cube and a square pyramid) require additional interpretation and reasoning. The work here offers opportunities for students to make sense of problems and persevere in solving them.

As students work, remind them of the organizational strategies discussed in previous lessons, such as labeling polygons, showing measurements on the net, and so on.



Launch

Arrange students in groups of 2–3. Assign each student in the group a different polyhedron from the student workbook and give them access to their geometry toolkits. Students need graph paper and a straightedge from their toolkits.

Explain to students that they will draw a net, find its surface area, and have their work reviewed by a peer. Give students 4–5 minutes of quiet time to draw their net on graph paper and then 2–3 minutes to share their net with their group and get feedback. When the group is sure that each net makes sense and all polygons of each polyhedron are accounted for, students can proceed and use the net to help calculate surface area.

If time permits, prompt students to cut and assemble their net into a polyhedron. Demonstrate how to add flaps to their net to accommodate gluing or taping. There should be as many flaps as there are edges in the polyhedron. (Remind students that this is different from the number of edges in the polygons of the net.)

Student Task Statement

Your teacher will assign you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?

   Sample responses:

   - A and C are rectangular prisms.
   - B, D and E are triangular prisms.
   - F and G are square pyramids.
   - H is a trapezoidal prism.
   - I is a composite of a cube and a square pyramid.

2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.

   Sample responses:

   - A and C should have 6 rectangles.
   - B, D, and E should have 5 polygons: 2 right triangles and 3 rectangles.
   - F and G should have 5 polygons: 1 square and 4 triangles.
   - H should have 6 polygons: 2 trapezoids and 4 rectangles.
   - I should have 9 polygons: 5 squares and 4 triangles.

3. Label each polygon on the net with a name or number.

   Answers vary.

4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

   Sample responses:

   - A: 340 square units. $2(5 \cdot 8) + 2(5 \cdot 10) + 2(8 \cdot 10) = 340$
   - B: 408 square units. $2\left(\frac{1}{2} \cdot 6 \cdot 8\right) + (6 \cdot 15) + (8 \cdot 15) + (10 \cdot 15) = 408$
   - C: 274 square units. $2(13 \cdot 4) + 2(13 \cdot 5) + 2(4 \cdot 5) = 274$
   - D: 300 square units. $2\left(\frac{1}{2} \cdot 5 \cdot 12\right) + (5 \cdot 8) + (12 \cdot 8) + (13 \cdot 8) = 300$
   - E: 216 square units. $2\left(\frac{1}{2} \cdot 6 \cdot 4\right) + (6 \cdot 12) + 2(5 \cdot 12) = 216$
   - F: 240 square units. $4\left(\frac{1}{2} \cdot 8 \cdot 11\right) + (8 \cdot 8) = 240$
   - G: 156 square units. $4\left(\frac{1}{2} \cdot 6 \cdot 10\right) + (6 \cdot 6) = 156$
   - H: 316 square units. The trapezoidal base can be decomposed into a 5-by-4 rectangle and a right triangle with a base of 3 units and a height of 4. $2(5 \cdot 4) + 2\left(\frac{1}{2} \cdot 3 \cdot 4\right) + (8 \cdot 12) + 2(5 \cdot 12) + (4 \cdot 12) = 316$
   - I: 205 square units. $5(5 \cdot 5) + 4\left(\frac{1}{2} \cdot 5 \cdot 8\right) = 205$

   Answers vary depending on the polyhedron received.

Building on Student Thinking

Students may know what polygons make up the net of a polyhedron but arrange them incorrectly on the net (for instance, allowing the faces to overlap instead of meeting at shared edges, orienting the faces incorrectly, or placing them in the wrong places). Suggest that students label some faces of the polyhedron drawing and transfer the adjacencies they see to the net. If needed, demonstrate the reasoning, for instance: "Face 1 and Face 5 both share the edge that is 7 units long, so I can draw them as two attached rectangles sharing a side that is 7 units long."

It may not occur to students to draw each face of the polyhedron to scale. Remind them to use the grid squares on their graph paper as units of measurement.

If a net is inaccurate, this becomes more evident when it is being folded. This may help students see which parts need to be adjusted and decide the best locations for the flaps. Reassure students that a few drafts of a net may be necessary before all the details are worked out, and encourage them to persevere.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect.
Invite students to prepare a visual display that shows their net drawings and surface area calculations. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students:

- "What did the nets and calculations have in common? How were they different?"
- "What kinds of additional details or language helped you understand the displays?"

Advances: Representing, Conversing

Access for Students with Diverse Abilities (Activity 2, Student Task)

Representation: Internalize Comprehension.
Provide students with a graphic organizer such as a two-column table to record measurements and calculations of surface area and volume. A table with space for students to record their calculations for volume and surface area for Boxes A, B, and C will help them keep their thinking organized.

Supports accessibility for: Visual-Spatial Processing, Organization

Activity Synthesis

Ask students who finish their calculation to find another person in the class who has the same polyhedron and discuss the following questions (displayed for all to see):

- *"Do your calculations match? Should they?"*
- *"Do your nets result in the same polyhedra? Should they?"*
- *"Do your models match the picture you were given? Why or why not?"*

If time is limited, consider having the answer key posted somewhere in the classroom so students can quickly check their surface area calculations.

Reconvene briefly for a whole-class discussion. Invite students to reflect on the process of drawing a net and finding surface area based on a picture of a polyhedron. Ask questions such as:

- *"How did you know that your net shows all the faces of your polyhedron?"*
- *"How did you know where to put each polygon or how to arrange all polygons so that, if folded, they can be assembled into the polyhedron in the drawing?"*
- *"How did the net help you find surface area?"*

Activity 2: Optional
Comparing Boxes

15 min

Activity Narrative

In this activity, students compare the surface areas and volumes of three rectangular prisms given nets that are not on a grid. To do this, they need to be able to visualize the three-dimensional forms that the two-dimensional nets would take when folded.

In grade 5, students had learned to distinguish area and volume as measuring different attributes. This activity clarifies and reinforces that distinction.

Launch

Keep students in the same groups of 2–3. Tell students that this activity involves working with both volume and surface area. To refresh students' understanding of volume from grade 5, ask students:

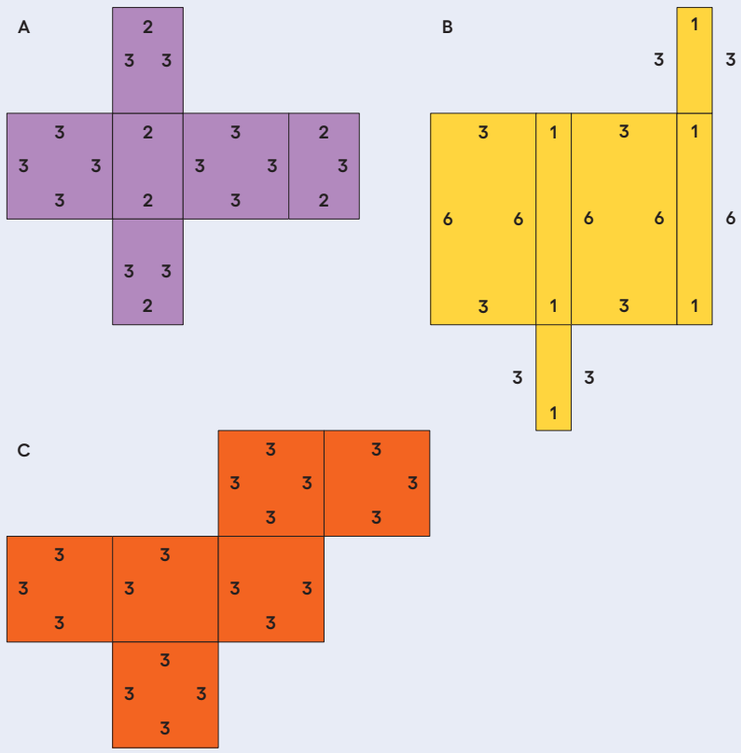
- *"When we find the volume of a prism, what are we measuring?"*
- *"How is volume different from surface area?"*
- *"How might we find the volume of a rectangular prism?"*

Reiterate that volume measures the number of unit cubes that can be packed into a three-dimensional shape and that we can find the number of unit cubes in a rectangular prism by multiplying the side lengths (length, width, and height) of a prism.

Give students 1–2 minutes to read the task statement and questions. Ask them to think about how they might go about answering each question and to be prepared to share their ideas. Give students a minute to discuss their ideas with their group. Then, ask groups to collaborate: Each member should perform the calculations for one prism (A, B, or C). Give students 5–7 minutes of quiet time to find the surface area and volume for their prism and then additional time to compare their results and answer the questions.

Student Task Statement

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.

   Box A uses the least cardboard.

   Sample reasoning: The surface area of A is 42 square centimeters. A: 4(2 • 3) + 2(3 • 3) = 42. The surface area of B and C is 54 square centimeters. B: 2(3 • 6) + 2(3 • 1) + 2(6 • 1) = 54. C: 6(3 • 3) = 54. Boxes B and C require the same amount of cardboard, both more cardboard than A.

2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

   Box C fits the most 1-centimeter cubes.

   Sample reasoning: The volume of A and B is 18 cubic centimeters. A: 3 • 2 • 3 = 18. B: 6 • 1 • 3 = 18. The volume of C is 27 cubic centimeter. C: 3 • 3 • 3 = 27. A and B fit the same number of cubes, but fewer than C.

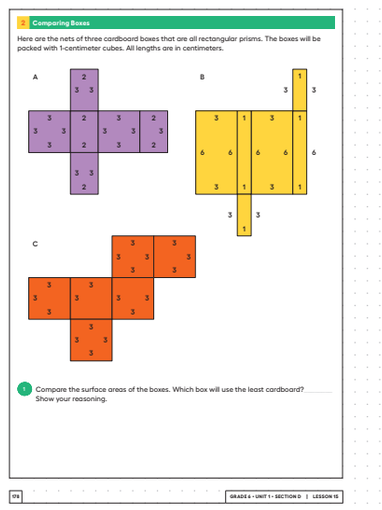
Building on Student Thinking

Students should have little trouble finding areas of rectangles but may have trouble keeping track of pairs of measurements to multiply and end up making calculation errors. Suggest that they label each polygon in the net and the corresponding written work and double-check their calculations to minimize such errors.

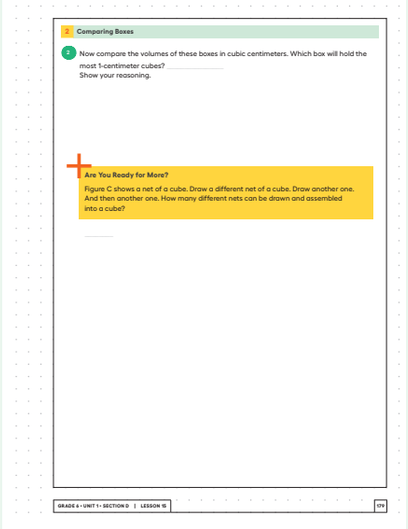
If students struggle to find the volume of their prism using information on a net, suggest that they sketch the prism that can be assembled from the net and label the edges of the prism.

Students may need a reminder that area is measured in square units and that volume is measured in cubic units.

Student Workbook



Student Workbook


2 Comparing Boxes

2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? _____ Show your reasoning.

Are You Ready for More?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

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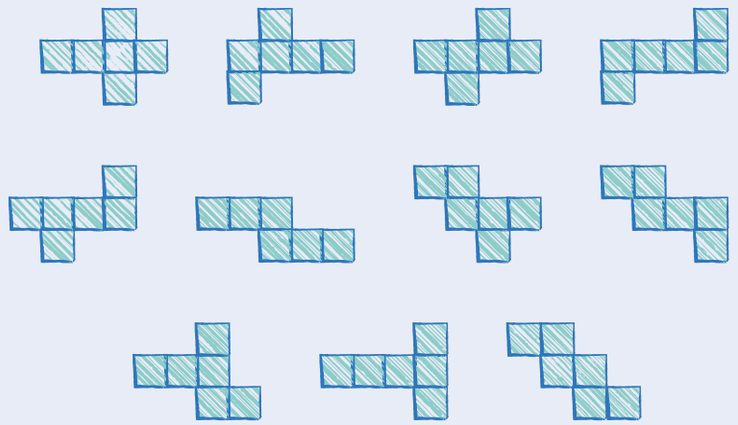
Are You Ready for More?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

There are 11 different nets for a cube. Any other net would be congruent to one of these.



Activity Synthesis

Ask students who worked on the same prism if they agree or disagree about the calculation. Record and display the results for all to see.

Invite students to share a few quick observations about the relationship between the surface areas and volumes for these three prisms, or between the amounts of material needed to build the boxes and the number of cubes that they can contain. Discuss questions such as:

- *"If these prisms are boxes, which prism—B or C—would take more material to build? Which can fit more unit cubes?"*

  Prisms B and C would likely take the same amount of material to build because their surface areas are the same. Prism C has a greater volume than does Prism B, so it can fit more unit cubes.

- *"Which prism—A or B—would take more material to build? Which can fit more unit cubes?"*

  A and B can fit the same number of unit cubes but, B would require more material to build.

- *"If two prisms have the same surface area, would they also have the same volume? How do you know?"*

  No, Prisms A, B, and C are examples of how two figures with the same volume may not have the same surface area, and vice versa.

Students will gain more insights into these ideas as they explore squares, cubes, and exponents in upcoming lessons. If students could benefit from additional work on distinguishing area and volume as different measures, do the optional lesson "Distinguishing Between Surface Area and Volume."

Lesson Synthesis

To highlight some key points from the lesson, display a picture of a prism or a pyramid and a drawing of its net. Discuss these questions:

"Can you find the surface area of a simple prism or pyramid from a picture, if all the necessary measurements are given?"

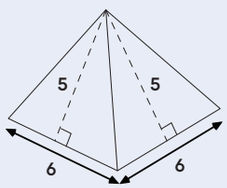
"Can you find the surface area from a net, if all the measurements are given?"

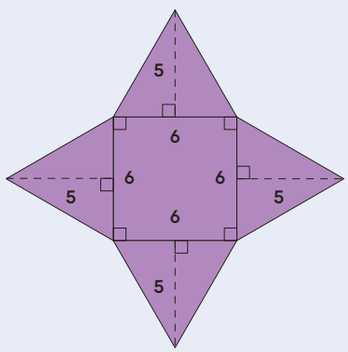
"Which might be more helpful for calculating surface area—a picture of a polyhedron or a net?"

If the polyhedron is simple—such as a cube or a square pyramid—and does not involve hidden faces with different measurements or require a lot of visualizing, either a picture or a net can work. Otherwise, a net may be more helpful because we can see all of the faces at once and can find the area of each polygon more easily. A net may also help us keep track of our calculations and notice missing or extra areas.

Lesson Summary

A net can help us find the surface area of a polyhedron that has different polygons for its faces. We can find the areas of all polygons in the net and add them.



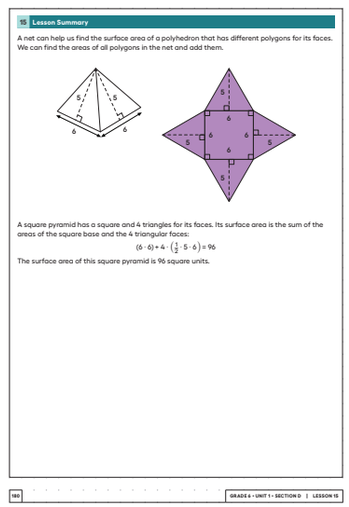


A square pyramid has a square and 4 triangles for its faces. Its surface area is the sum of the areas of the square base and the 4 triangular faces:

$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

The surface area of this square pyramid is 96 square units.

Student Workbook


15 Lesson Summary

A net can help us find the surface area of a polyhedron that has different polygons for its faces. We can find the areas of all polygons in the net and add them.

A square pyramid has a square and 4 triangles for its faces. Its surface area is the sum of the areas of the square base and the 4 triangular faces:

$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$

The surface area of this square pyramid is 96 square units.

Responding To Student Thinking

Points to Emphasize
If students struggle with relating polyhedra and their nets, discuss the connections between different representations of a polyhedron when opportunities arise in the next lesson. For example, review this practice problem, and discuss how students know the side lengths of the polygons on each net:

Unit 1, Lesson 15, Practice Problem 4

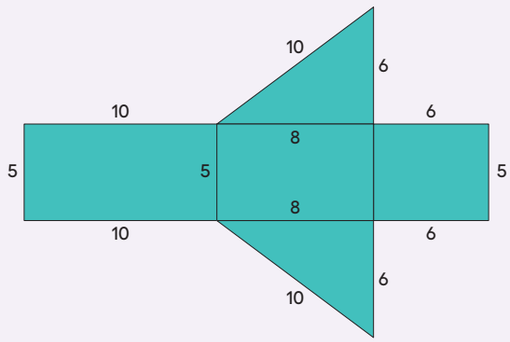
Cool-down

5 min

Surface Area of a Triangular Prism

Student Task Statement

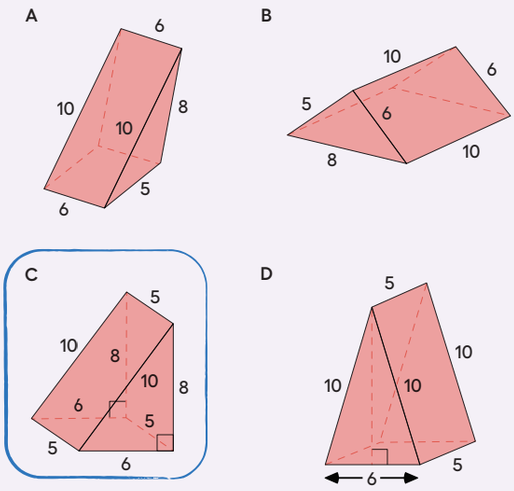
1. In this net, the two triangles are right triangles. All quadrilaterals are rectangles. What is its surface area in square units? Show your reasoning.



168 square units

Sample reasoning: There are two triangular faces with area of 24 square units each. $\frac{1}{2} \cdot 6 \cdot 8 = 24$. There is a rectangular face with area of 50 square units. $10 \cdot 5 = 50$. There is one rectangular face with area of 40 square units. $5 \cdot 8 = 40$. There is one rectangular face with area $5 \cdot 6 = 30$ square units. $2 \cdot 24 + 50 + 40 + 30 = 168$

2. If the net is assembled, which of the following polyhedra would it make?



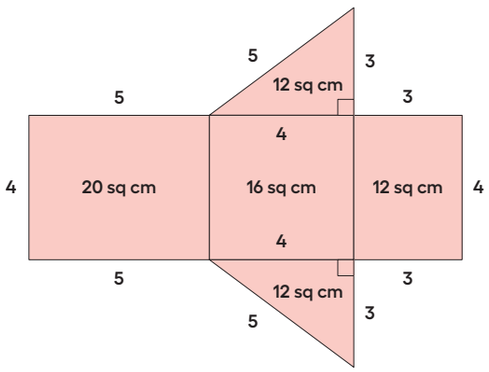
Lesson 15

Practice Problems

5 Problems

Problem 1

Jada drew a net for a polyhedron and calculated its surface area.



a. What polyhedron can be assembled from this net?

Triangular prism

b. Jada made some mistakes in her area calculation. What were the mistakes?

She calculated the areas of the two triangular faces incorrectly. The right triangles have a base of 4 cm and a height of 3 cm, so the area of each should be $\frac{1}{2} \cdot 4 \cdot 3$, or 6 sq cm. Jada wrote "12 sq cm" for the area of each triangle.

c. Find the surface area of the polyhedron. Show your reasoning.

60 sq cm

Sample reasoning: The triangular faces should be 6 sq cm each, so the surface area is 20 + 16 + 12 + 6 + 6, or 60

Problem 2

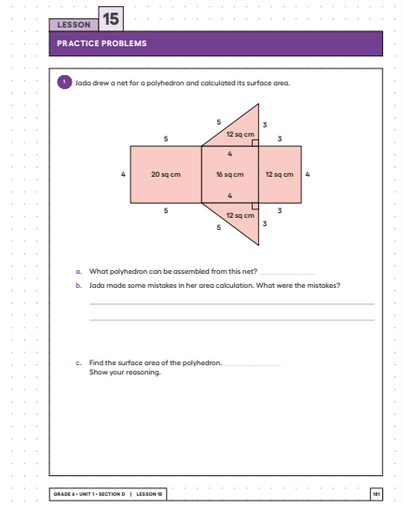
A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.

272 square inches

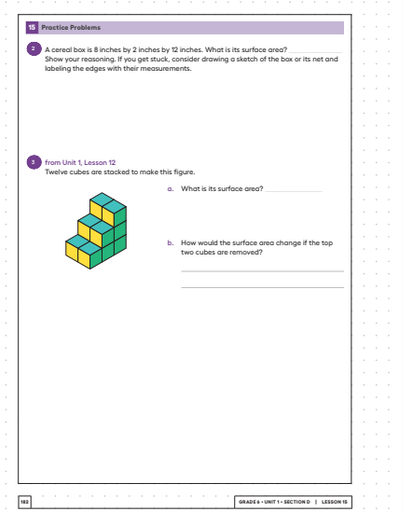
Sample reasoning:

- The top and bottom faces are 2 inches by 8 inches each, so their combined area is $2(2 \cdot 8)$ or 32 square inches.
- The front and back faces are 8 inches by 12 inches each, so their combined area is $2(8 \cdot 12)$ or 192 square inches.
- The side faces are 2 inches by 12 inches each, so their combined area is $2(2 \cdot 12)$ or 48 square inches.
- The surface area is 32 + 192 + 48 or 272 square inches.

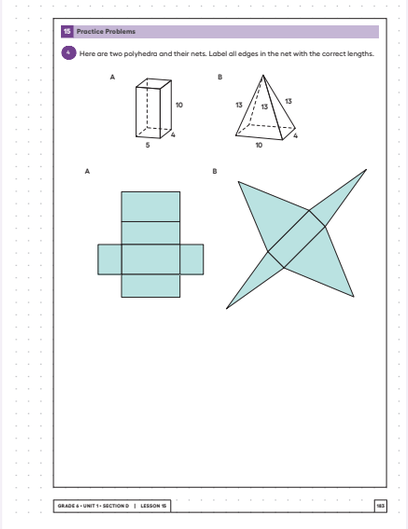
Student Workbook


LESSON 15
PRACTICE PROBLEMS
1 Jada drew a net for a polyhedron and calculated its surface area.
a. What polyhedron can be assembled from this net?
b. Jada made some mistakes in her area calculation. What were the mistakes?
c. Find the surface area of the polyhedron. Show your reasoning.

Student Workbook


15 Practice Problems
2 A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.
3 from Unit 1, Lesson 12
Twelve cubes are stacked to make this figure.
a. What is its surface area?
b. How would the surface area change if the top two cubes are removed?

Student Workbook


15 Practice Problems

A Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.

A B

10 5 4

13 13 13 10 4

A B

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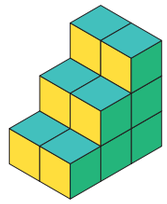
Problem 3

from Unit 1, Lesson 12

Twelve cubes are stacked to make this figure.



a. What is its surface area?

36 square units

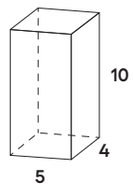
b. How would the surface area change if the top two cubes are removed?

The surface area would decrease by 6 square units.

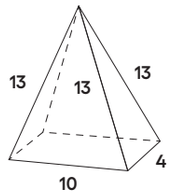
Problem 4

Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.

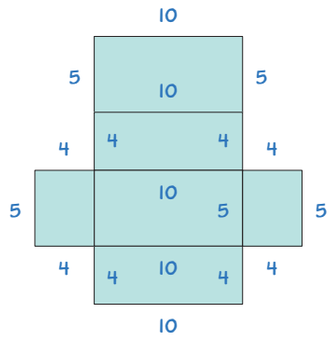
A



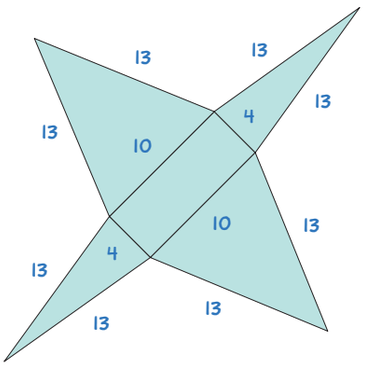
B



A



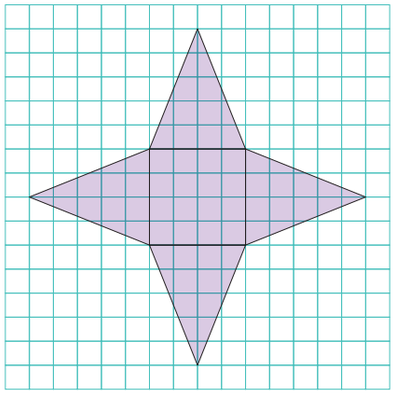
B



Problem 5

from Unit 1, Lesson 14

a. What three-dimensional figure can be assembled from the net?



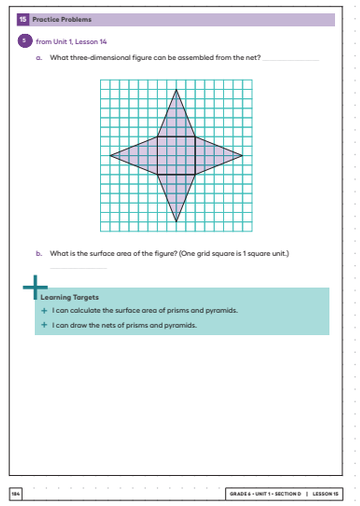
Square pyramid

b. What is the surface area of the figure? (One grid square is 1 square unit.)

56 square units

Sample reasoning: The area of the base is 16 square units. Each triangular face has a base of 4 units and a height of 5 units. This means each triangular face has an area of 10 square units. The total surface area is 56 square units, because 16 + 10 + 10 + 10 + 10 = 56

Student Workbook


15 Practice Problems

5 from Unit 1, Lesson 14

a. What three-dimensional figure can be assembled from the net?

b. What is the surface area of the figure? (One grid square is 1 square unit.)

Learning Targets

+ I can calculate the surface area of prisms and pyramids.
+ I can draw the nets of prisms and pyramids.

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