# **Increasing and Decreasing**

#### Goals

- Coordinate statements about "percent increase" or "percent decrease" with comparisons to the original amount, e.g., a 20% increase means the new value is 120% of the original value.
- Draw and label a tape diagram to represent a situation that involves adding or subtracting a percentage of the initial value.
- Explain (orally and in writing) how to calculate the new amount given the original amount and a percentage of increase or decrease.

# **Learning Targets**

- I can draw a tape diagram that represents a percent increase or decrease.
- When I know a starting amount and the percent increase or decrease, I can find the new amount.

# **Access for Students with Diverse Abilities**

• Representation (Activity 1)

#### **Access for Multilingual Learners**

- MLR7: Compare and Connect (Activity 1)
- MLR8: Discussion Supports (Activity 2)

#### **Instructional Routines**

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

#### **Lesson Narrative**

In this lesson, students make sense of situations involving **percent increase** and **percent decrease**, where a change is expressed as a percentage of the original amount. They use tape diagrams to represent the situations, and they calculate the new amount. This builds on students' previous work with situations where an increase or decrease is expressed as a fraction or decimal part of the original amount.

Students make sense of problems involving percent increase as they consider various strategies and representations to persevere in solving. As students use tape diagrams to make sense of the relationships between the quantities, they are making use of structure.

#### **Student Learning Goal**

Let's use percentages to describe increases and decreases.

#### **Lesson Timeline**

10 min

Warm-up

10 min

Activity 1

10 min

**Activity 2** 

10 min

**Lesson Synthesis** 

#### **Assessment**

5 min

Cool-down

#### Warm-up

## **Improving Their Game**



#### **Activity Narrative**

The purpose of this Warm-up is to encourage students to recognize that in some situations it is helpful to think about an increase in terms of what fraction or percentage it is of the original value. This will be useful when students interpret statements about percent increase in a later activity. While students may notice and wonder many things about this table, the important discussion points are that each team's increase is the same when viewed additively and that the increases are very different when viewed multiplicatively. For example, an increase of 8 points from 100 to 108 is not as significant as an increase from 4 points to 12 points, because the first increase is only 8% of the original value, while the second is 200% of the original value.

As students share things they notice, listen for language students use to discuss the significance of the different increases. For example, students may say that the baseball team tripled their score, which would be like the basketball team going from 100 to 300 points in the next game. When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see. They might first propose less formal or imprecise language and then restate their observation with more precise language in order to communicate more clearly.

# Launch

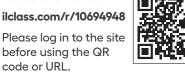
Arrange students in groups of 2. Display the table for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the table. If no students wonder which team improved the most, direct them to the second question, and give them 1 minute to work with a partner.

#### **Instructional Routines**

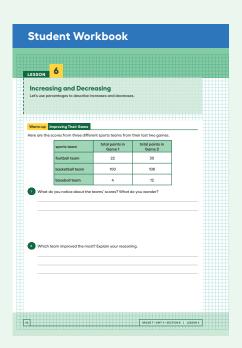
# **Notice and Wonder** ilclass.com/r/10694948





#### **Building on Student Thinking**

Students may say that the football team improved the least because the 8 points could have been scored from only 1 touchdown in football, but it would have to be 3 or 4 baskets in basketball and 8 separate runs in baseball. Prompt students to look at the significance of the 8 additional points in the context of each team's score in the Game 1 rather than the mechanics of scoring in each sport.



## **Student Task Statement**

Here are the scores from three different sports teams from their last two games.

sports team	total points in Game 1	total points in Game 2
football team	22	30
basketball team	100	108
baseball team	4	12

- 1. What do you notice about the teams' scores? What do you wonder? Students may notice:
  - · Each team improved by 8 points.
  - The basketball team had the most points.
  - The baseball team's score in Game 2 was 3 times as much as in Game I, a three-fold increase.

#### Students may wonder:

- What did each team do to improve their score?
- How did the baseball team improve so much?
- Which team improved the most?
- 2. Which team improved the most? Explain your reasoning.

Sample response: The baseball team improved the most because their 8 point increase tripled their previous score, while the other teams' scores were multiplied by smaller factors: about I.I for the basketball team and about I.4 for the football team.

## **Activity Synthesis**

Poll students on which team they think improved the most. First, ask a student who thinks they all improved by the same amount to share their reasoning. (Each team increased its score by 8 points.) Then, ask a few students who said the baseball team improved the most to share their reasoning.

There is no need to introduce the phrase "percent increase" at this time. The goal is to plant the idea that it sometimes makes sense to describe a change relative to a starting amount instead of just looking at absolute change. In the course of discussion, it may be natural to say things like:

☐ "The basketball team improved their score by 8% of their Game 1 score."

"The football team improved by about  $\frac{1}{3}$  of their Game 1 score."

"The baseball team tripled their Game 1 score."

# **Activity 1**

## More Shampoo and a Discounted Shirt



#### **Activity Narrative**

In this activity, students are given a percent increase or percent decrease and the original amount. They use this information to calculate the new amount. As students interpret the situations and choose a strategy for finding the new amount, they are making sense of problems and persevering in solving them.

Monitor for students who:

- Draw a double number line.
- · Create a table of equivalent ratios.
- Calculate the amount of the increase or decrease and then add or subtract it from the original amount.
- Find the percentage that the new amount is of the original amount and multiply the original amount by that.

The key focus is how students make sense of the multi-step problem and choose what steps are needed to calculate the new amount.

# Launch 🙎

Arrange students in groups of 2.

Give students 5 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

## **Student Task Statement**

**1.** A shampoo bottle says that now it contains 20% more. Originally, it came with 18.5 fluid ounces of shampoo. How much shampoo does the bottle come with now?



#### 22.2 fluid ounces of shampoo

#### Sample reasoning:

- The shampoo bottle gained 3.7 fluid ounces, because 20% of 18.5 fluid ounces is  $0.2 \cdot 18.5$ , or 3.7, fluid ounces. That means the shampoo bottle now has 22.2 fluid ounces, because 18.5 + 3.7 = 22.2.
- The shampoo bottle now has 120% as many fluid ounces of shampoo as it originally had, because 100 + 20 = 120. Now it has 22.2 fluid ounces of shampoo, because  $18.5 \cdot 1.2 = 22.2$ .

# Access for Multilingual Learners (Activity 1)

#### **MLR7: Compare and Connect**

This activity uses the Compare and Connect math language routine to advance representing and conversing as students use mathematically precise language in discussion.

#### **Instructional Routines**

# MLR7: Compare and Connect

#### ilclass.com/r/10695592





#### **Building on Student Thinking**

Have students use the double number line diagram if they need help figuring out the value that is 20% more.

#### **Student Workbook**



# Access for Students with Diverse Abilities (Activity 1, Synthesis)

# Representation: Develop Language and Symbols.

Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of percent increase and percent decrease.

Supports accessibility for: Conceptual Processing, Language **2.** The price of a shirt is \$18.50, but you have a coupon that lowers the price by 20%. What is the price of the shirt after using the coupon?



#### \$14.80

#### Sample reasoning:

- The price drops by \$3.70, because 20% of \$18.50 is 0.2 · 18.50, or \$3.70.
   That means the shirt will cost \$14.80, because 18.50 3.70 = 14.80.
- The sale price is 80% of the original price, because 100 20 = 80. The
  price of the shirt after using the coupon will be \$14.80 because 18.50.
  0.80 = 14.8.

#### **Are You Ready for More?**

Shirts are on sale for 20% off. You buy two of them. As you pay, the cashier says, "20% off of each shirt means 40% off of the total price."

Do you agree or disagree with this statement? Explain your reasoning.

#### disagree

Sample reasoning: It's still 20% off the total price. If we represent the discount from 20% off Shirt A as 0.2a and the discount from 20% off Shirt B as 0.2b, then the amount off the total price would be 0.2a + 0.2b or, by the distributive property, 0.2(a + b).

#### **Activity Synthesis**

The goal of this discussion is to highlight how students made sense of the percent increase and decrease situations and what steps they chose to calculate the new amount. Display 2–3 approaches from previously selected students for all to see. If time allows, invite students to briefly describe their approach. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the approaches have in common? How are they different?"
"How does the '20% more' or '20% less' show up in each method?"
"Did anyone solve the problem the same way, but would explain it differently?"

The key takeaway is that "20% more" means the new amount is 120% of the original amount, while "20% less" means the new amount is 80% of the original amount.

Tell students that the change on the cereal box is an example of a **percent increase**. The discount on the shirt is an example of a **percent decrease**. It may be mentioned that in both cases, 100% always corresponds to the original amount before the change; however, future activities will address this concept in depth.

# **Activity 2**

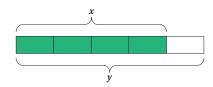
## **Using Tape Diagrams**



# **Activity Narrative**

In this activity, students match verbal descriptions to tape diagrams and see how the percentage of the increase or decrease determines the percentage that the new amount is of the original amount. For example, a 15% increase corresponds to 115% of the original amount, while a 30% decrease corresponds to 70% of the original amount. Then students create their own tape diagrams to represent two more situations. As students use the tape diagrams to make sense of the increase or decrease, they are making use of structure.

#### Launch



Display this image and the four expressions for all to see.

- 1.  $x + \frac{1}{4}x$
- **2.** y = 1.25x
- **3.** 125%
- 4. An increase of 25%

Ask students to explain how each of these expressions is related to the diagram. Then tell students they are going to match situations to tape diagrams.

Give students 3–4 minutes of quiet work time followed by time for partner discussion.

Then hold a whole-class discussion.

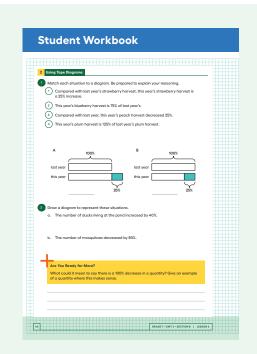
#### **Instructional Routines**

MLR8: Discussion Supports

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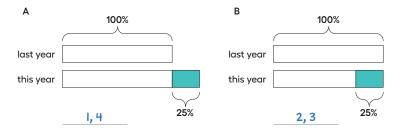
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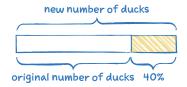
# **Student Task Statement**

- 1. Match each situation to a diagram. Be prepared to explain your reasoning.
  - **1.** Compared with last year's strawberry harvest, this year's strawberry harvest is a 25% increase.
  - 2. This year's blueberry harvest is 75% of last year's.
  - **3.** Compared with last year, this year's peach harvest decreased 25%.
  - **4.** This year's plum harvest is 125% of last year's plum harvest.



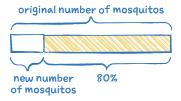
- 2. Draw a diagram to represent these situations.
  - a. The number of ducks living at the pond increased by 40%.

#### Sample response:



**b.** The number of mosquitoes decreased by 80%.

# Sample response:



## **Are You Ready for More?**

What could it mean to say there is a 100% decrease in a quantity? Give an example of a quantity where this makes sense.

Sample response: It means the quantity is now zero. For example, a person walks I mile one day and 0 miles the next day.

#### **Activity Synthesis**

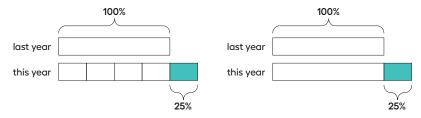
The purpose of this discussion is for students to explain the connections between the tape diagrams and the verbal descriptions, including where the 75% and 125% come from.

First, invite students to share how they reasoned about the fruit harvest situations and the given diagrams. For each diagram, consider asking,

 $\bigcirc$  "Why do both of these verbal descriptions match the same diagram?"

If possible, record the relevant reasoning on or near the image.

Display these two images for all to see.



Ask students,

"What is the same about these two diagrams? What is different?"

They both represent a 25% increase from last year to this year. The first diagram is partitioned into more pieces than the second diagram.

Explain that the tape diagrams that we have worked with in the past were divided into pieces that were all the same size. Here, we see that tape diagrams can be divided into pieces of different sizes, as long as the size of each piece is clearly indicated.

Next, invite students to share the diagrams they drew for the animal population situations. Consider asking:

"The new number of ducks is what percentage of the old number of ducks? How do you know?"

"The new number of mosquitos is what percentage of the old number of mosquitos? How do you know?"

#### **Lesson Synthesis**

Share with students,

"Today we looked at situations where a quantity increased or decreased by a percentage of the starting amount. We calculated the new amount after the increase or decrease."

To help students connect this topic with their previous learning, consider asking:

"What is another way to describe a 25% increase?"

adding on  $\frac{1}{4}$  of the original amount; ending up with 125% of the original amount

"What is another way to describe a 25% decrease?"

subtracting  $\frac{1}{4}$  of the original amount; ending up with 75% of the original amount

"When a quantity is increased or decreased, what percentage describes the original or starting value?"

100%

# Access for Students with Diverse Abilities (Activity 2, Synthesis)

# Representation: Internalize Comprehension.

Use color coding and annotations to highlight connections between representations in a problem. For example, annotate the section of Diagram B that represents 75% to emphasize that this section represents this year's blueberry harvest.

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# Access for Multilingual Learners (Activity 2, Synthesis)

#### MLR8: Discussion Supports.

For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

Advances: Listening, Speaking

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# **Responding To Student Thinking**

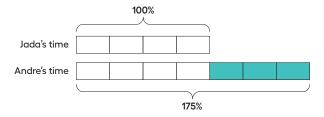
#### **More Chances**

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

# Student Workbook | Student Workbook | Student | Student

## **Lesson Summary**

Imagine that it takes Andre  $\frac{3}{4}$  more than the time it takes Jada to get to school. Then we know that Andre's time is  $1\frac{3}{4}$ , or 1.75, times Jada's time. We can also describe this in terms of percentages:



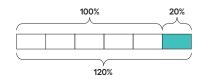
We say that Andre's time is 75% more than Jada's time. We can also see that Andre's time is 175% of Jada's time. In general, the terms **percent increase** and **percent decrease** describe an increase or decrease in a quantity as a percentage of the starting amount.

For example, if there were 500 grams of cereal in the original package, then "20% more" means that 20% of 500 grams has been added to the initial amount,  $500 + (0.2) \cdot 500 = 600$ , so there are 600 grams of cereal in the new package.



We can see that the new amount is 120% of the initial amount because

$$500 + (0.2) \cdot 500 = (1 + 0.2)500.$$



#### Cool-down

## **Fish Population**

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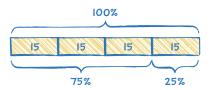
#### **Student Task Statement**

The number of fish in a lake decreased by 25% between last year and this year. Last year there were 60 fish in the lake. What is the population this year? If you get stuck, consider drawing a diagram.

There are 45 fish in the lake this year.

#### Sample reasoning:

- The number of fish decreased by 15, because 0.25 60 = 15. That means there are 45 fish left, because 60 15 = 45.
- There are only 75% as many fish this year, because 100 25 = 75. We can multiply  $0.75 \cdot 60 = 45$ .
- · Here is a tape diagram that shows there are 45 fish left:

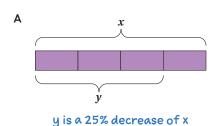


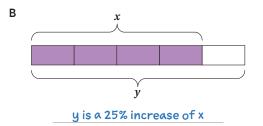
## **Practice Problems**

5 Problems

#### **Problem 1**

For each diagram, decide if y is an increase or a decrease relative to x. Then determine the percent increase or decrease.





# Problem 2

Draw diagrams to represent the following situations.

- **a.** The amount of flour that a bakery used this month was a 50% increase relative to last month.
- **b.** The amount of milk that a bakery used this month was a 75% decrease relative to last month.

Answers vary.

#### **Problem 3**

Write each percent increase or decrease as a percentage of the initial amount. The first one is done for you.

**a.** This year, there was 40% more snow than last year.

The amount of snow this year is 140% of the amount of snow last year.

**b.** This year, there were 25% fewer sunny days than last year.

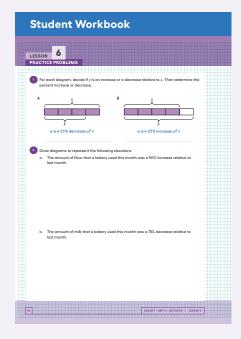
The number of sunny days this year is 75% of the number of sunny days last year.

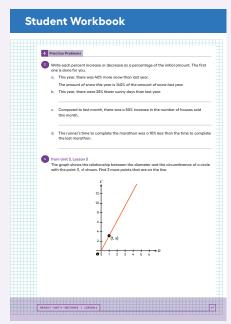
**c.** Compared to last month, there was a 50% increase in the number of houses sold this month.

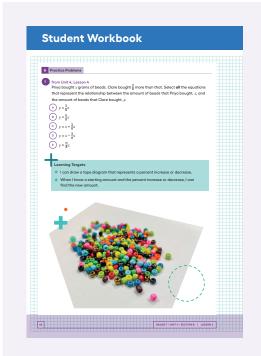
The number of houses sold this month is 150% of the number of houses sold last month.

**d.** The runner's time to complete the marathon was a 10% less than the time to complete the last marathon.

The runner's time to complete the marathon is 90% of the time to complete the last marathon.



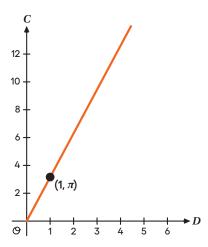




# Problem 4

from Unit 3, Lesson 3

The graph shows the relationship between the diameter and the circumference of a circle with the point (1,  $\pi$ ) shown. Find 3 more points that are on the line.



Sample response: (0,0),  $(2,2\pi)$ , (3,9.4)

# **Problem 5**

from Unit 4, Lesson 4

Priya bought x grams of beads. Clare bought  $\frac{3}{8}$  more than that. Select **all** the equations that represent the relationship between the amount of beads that Priya bought, x, and the amount of beads that Clare bought, y.

**A.** 
$$y = \frac{3}{8}x$$

**B.** 
$$y = \frac{5}{8}y$$

**C.** 
$$y = x + \frac{3}{8}x$$

**D.** 
$$y = x - \frac{3}{8}x$$

**E.** 
$$y = \frac{11}{8}x$$