#### **Changing Elevation**

#### Goals

- Comprehend that the term "opposite" (in spoken and written language) refers to numbers with the same magnitude but different signs.
- Create and interpret equations and diagrams that represent adding signed numbers in the context of elevation.
- Generalize (orally) a method for determining the sum of two signed numbers.

#### **Learning Target**

I can add positive and negative numbers.

#### Student Learning Goal

Let's solve problems about adding signed numbers.

# Access for Students with Diverse Abilities

• Representation (Activity 1, Activity 3)

#### **Access for Multilingual Learners**

- MLR1: Stronger and Clearer Each Time (Activity 2)
- MLR2: Collect and Display (Activity 3)

#### **Required Materials**

#### **Materials to Gather**

• Receipt tape: Activity 3

#### **Required Preparation**

#### **Activity 1:**

For the digital version of the activity, acquire devices that can run the applet.

#### Activity 3:

Cut 1 strip of receipt tape for every 2 students. Each strip of receipt tape should be at least 4 feet long. You may want to prepare some strips that are even longer in case groups choose extra long objects.

#### **Lesson Narrative**

In this lesson, students build fluency with adding signed numbers. First, they examine changes in elevation. They write addition expressions and draw number line diagrams to help find the final elevation after each change. Then, they apply what they have learned to add signed numbers without a context.

Students generalize about the sum of two signed numbers based on the signs and relative magnitudes of the addends, including these observations:

- Adding two numbers with the same sign gives a sum that is farther away from zero than either addend.
- Adding two numbers with different signs gives a sum with the same sign as the addend with the greater magnitude.
- Adding a number and its opposite gives a sum of 0. (Opposites are numbers that are the same distance from zero but on opposite sides of the number line.)

#### **Lesson Timeline**

5 min

Warm-up

15 min

Activity 1

10 min

**Activity 2** 

10 min

**Activity 3** 

10 min

**Lesson Synthesis** 

**Assessment** 

5<sub>min</sub>

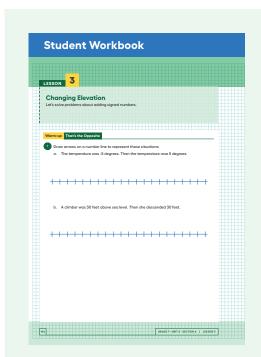
Cool-down

### **Changing Elevation**

#### **Lesson Narrative (continued)**

As students use observations about the number line and the meaning of opposites to develop these generalizations, they are making use of repeated reasoning.

The last activity is optional because it provides an opportunity for additional practice adding signed numbers using physical manipulatives.



# Student Workbook Minning That the Opposite When's the opposite? a. Burning 100 feet cost. b. Amping down 10 steps. c. Pouring 8 gallons into a fish tonk.

#### Warm-up

#### That's the Opposite



#### **Activity Narrative**

In this *Warm-up*, students think about opposite values and actions in different real-world situations. Students represent adding a number and its opposite on a number line. This will be useful when students think about opposite numbers and situations in following activities.

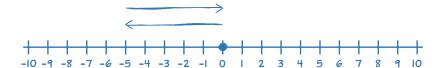
## Launch 22

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by 1 minute of partner discussion, then follow with whole-class discussion.

#### **Student Task Statement**

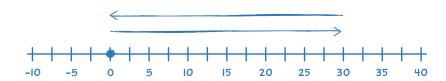
- 1. Draw arrows on a number line to represent these situations:
  - **a.** The temperature was -5 degrees. Then the temperature rose 5 degrees.

#### Sample response:



**b.** A climber was 30 feet above sea level. Then she descended 30 feet.

#### Sample response:



- 2. What's the opposite?
  - a. Running 150 feet east.

running 150 feet west

**b.** Jumping down 10 steps.

jumping up 10 steps

c. Pouring 8 gallons into a fish tank.

pouring 8 gallons out of a fish tank

**Lesson 3** Warm-up Activity 1 Activity 2 Activity 3 Lesson Synthesis Cool-down

#### **Activity Synthesis**

The goal of this discussion is for students to come up with many different concrete examples of opposites. Begin by inviting a few students to share their responses. Then ask students to describe additional pairs of situations and numbers that are opposites. For example, students might say that 5 hours later is the opposite of 5 hours earlier or that turning 90 degrees clockwise is the opposite of turning 90 degrees counterclockwise.

#### **Activity 1**

#### **Cliffs and Caves**

15 min

#### **Activity Narrative**

#### There is a digital version of this activity.

In this activity, students return to the context of height and depth to continue making sense of adding signed numbers. They examine a variety of situations, which include starting in the positives, starting in the negatives, moving away from 0, moving towards 0, crossing over 0, and ending exactly on 0. From this variety of situations, students see that the length of the arrows in the number line diagrams correspond to the magnitude of the addends. Using repeated reasoning, students see that when the addends have opposite signs, the longer arrow (the number with the larger magnitude) determines the sign of the sum.

In the digital version of the activity, students use an applet to represent a mountaineer climbing up or down a cliff. The applet allows students to set the starting elevation and the elevation change. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

## Launch 🙎

Explain that a mountaineer is someone who climbs mountains and a *spelunker* is someone who explores caves and caverns. Arrange students in groups of 2. Give students 3 minutes of quiet work time followed by partner discussion after the first question. Then ask students to work on the second question, and follow with whole-class discussion.

# Access for Students with Diverse Abilities (Activity 1, Launch)

# Representation: Internalize Comprehension.

Represent the same information through different modalities by using physical objects to represent abstract concepts. Create an interactive display that allows students to experience physically moving an object (mountaineer) up and down a number line. Highlight connections between the act of physically moving the object up and down with arrows on a number line diagram. Alternatively, provide students access to the digital version of this activity.

Supports accessibility for: Conceptual Processing, Visual-Spatial Processing Warm-up

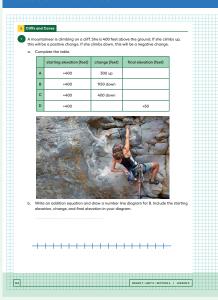
#### **Building on Student Thinking**

If some students struggle with working backwards to fill in the change when given the final elevation, consider asking:

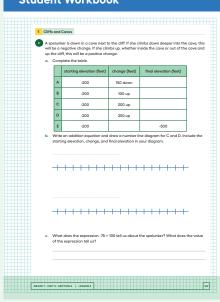
"Did the mountaineer go up or down in elevation?"

"How far did the mountaineer go up or down?"

#### Student Workbook



#### Student Workbook

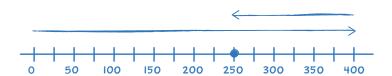


#### **Student Task Statement**

- **1.** A mountaineer is climbing on a cliff. She is 400 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.
  - **a.** Complete the table.

|   | starting<br>elevation<br>(feet) | change (feet) | final<br>elevation<br>(feet) |
|---|---------------------------------|---------------|------------------------------|
| Α | +400                            | 300 up        | +700                         |
| В | +400                            | 150 down      | +250                         |
| С | +400                            | 400 down      | 0                            |
| D | +400                            | 350 down      | +50                          |

**b.** Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.



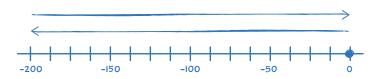
- **2.** A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.
  - a. Complete the table.

|   | starting<br>elevation<br>(feet) | change (feet) | final<br>elevation<br>(feet) |
|---|---------------------------------|---------------|------------------------------|
| Α | -200                            | 150 down      | -350                         |
| В | -200                            | 100 up        | -100                         |
| С | -200                            | 200 up        | 0                            |
| D | -200                            | 250 up        | +50                          |
| Е | -200                            | 300 down      | -500                         |

**b.** Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

Warm-up

C: -200 + 200 = 0



D: -200 + 250 = 50

A number line diagram is drawn with an arrow from 0 to -200, an arrow from -200 to 50, and a point at 50.

**c.** What does the expression -75 + 100 tell us about the spelunker? What does the value of the expression tell us?

The spelunker was at an elevation of -75 feet, then went 100 feet up. The spelunker is now at an elevation of 25 feet.

#### **Activity Synthesis**

The purpose of this activity is to emphasize how to tell whether the sum of two numbers will be positive, negative, or 0 based on the magnitude of the addends. Begin by displaying both of the tables from the *Task Statement*, and invite students to share their responses, recording them for all to see. Discuss the following:

"What addition equations can be written to describe the mountaineer's elevation in the first row of each table?"

Print: 400 + 300 = 700 and -200 + (-150) = -350 Digital: 200 + 75 = 275 and -20 + (-15) = -35

"What patterns do you notice?"

Adding two positive numbers results in another positive number, and adding two negative numbers results in another negative number.

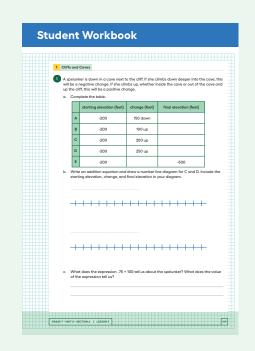
Invite students to share their equations for row B in the first table and rows C and D from the second table. Record their responses for all to see, and ask,

"What patterns do you notice?"

All these equations are adding a positive number and a negative number. The sum can be positive, negative, or 0.

"When adding two numbers with different signs, how can we tell what the sign of the sum will be?"

The sum will have the sign of the addend with the larger magnitude. If the magnitudes of both numbers are the same, the sum will be 0.



Warm-up

# Access for Multilingual Learners (Activity 2, Synthesis)

# MLR1: Stronger and Clearer Each Time.

Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response on how finding the sum of two numbers with the same sign is similar or different from finding the sum of two numbers with different signs. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.

Advances: Writing, Speaking, Listening

#### **Activity 2**

#### **Adding Rational Numbers**



#### **Activity Narrative**

In this activity, students find sums of signed numbers. No scaffolding is given, and students may use any strategy to find the sums.

Monitor for students who use different strategies to find the sums, such as:

- Drawing a diagram or number line.
- Reasoning about the magnitude and sign of the numbers.

## Launch 🙎

Arrange students in groups of 2. Give 2 minutes of quiet work time, followed by time for partner discussion. Then follow with whole-class discussion.

#### **Student Task Statement**

Find the sums.

0

11.5

**3.** 
$$\frac{1}{2}$$
 +  $\left(-\frac{3}{4}\right)$ 

- 1

**4.** What do you notice about finding the sum of two numbers with the same sign compared to finding the sum of two numbers with different signs?

Sample response: When two numbers have the same sign, their sum will be larger in magnitude and have the same sign as the two numbers. When two numbers have different signs, their sum will have the sign of the number with the larger magnitude.

#### **Are You Ready for More?**

Find the sum without a calculator.

0

#### **Activity Synthesis**

Invite selected students to share their solutions and reasoning to the first three problems. Help students connect the different reasoning strategies by asking the following questions:

"What do the different strategies have in common? How are they different?"
"Did anyone solve the problem the same way but would explain it differently?"

#### **Activity 3: Optional**

#### **School Supply Number Line**

10 min

#### **Activity Narrative**

In this activity, students use the structure of the number line and the relative positions of numbers to compare them. Instead of working with specific given numbers, they start with two different, arbitrary lengths. They use these lengths to label various locations on a number line. Students see that even though each group may have worked with different lengths for a and b, they still get the same answers to the final set of questions because everyone started with a > b. Students use a number line to support their reasoning about comparisons between signed numbers.

Warm-up

Identify a group that has a small difference between the lengths of their two objects and a group that has a large difference between the lengths of their two objects.

## Launch 🙎

Arrange students in groups of 2. Provide each group 1 blank strip of receipt tape. The length of tape has to be longer than four times the length of their longer object. Ensure that each group has access to 2 objects of appropriate length.

#### **Student Task Statement**

Your teacher will give you a long strip of paper.

- 1. Follow these instructions to create a number line.
  - Fold the paper in half along its length and along its width.
  - Unfold the paper, and draw a line along each crease.
  - Label the line in the middle of the paper 0. Label the right end of the paper + and the left end of the paper -.
  - Select two objects of different lengths, for example, a pen and a glue stick. The length of the longer object is a, and the length of the shorter object is b.
  - Use the objects to measure and label each of the following points on your number line.

a

**2***b* 

-b

b

a + b

a + -b

**2***a* 

-a

b + -a

#### Sample response:



# Access for Students with Diverse Abilities (Activity 3, Launch)

# Representation: Develop Language and Symbols.

Provide students with access to a reference sheet that displays the symbols <, >, and = and their respective meanings: "less than," "greater than," and "equal to."

Supports accessibility for: Language, Memory

# Access for Multilingual Learners (Activity 3, Student Task)

#### MLR2: Collect and Display.

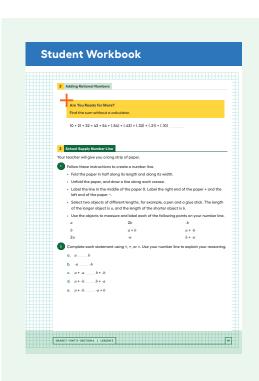
Collect the language that students use to describe how they are measuring and labeling lengths on their number line. Display words and phrases, such as "-b is located on the left side of the number line" and "-a is less than -b because ..." During the Activity Synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

#### **Building on Student Thinking**

Some students might forget which symbol means greater than or less than.

Some students may be confused between comparing the value of the expression and the magnitude of the expression. Explain that the number to the left on a number line has the lesser value, even if it may have the greater magnitude (farther away from zero).



- **2.** Complete each statement using <, >, or =. Use your number line to explain your reasoning.
  - $\mathbf{a}.a \geq b$

Activity 1

Sample reasoning: Both values are positive, and a is the value of the longer object.

**b.**-a <u>≤</u> -b

Sample reasoning: Both numbers are negative but the same distance away from 0 as a and b.

**c.** a + -a = b + -b

Sample reasoning: Both sides are 0.

**d.**  $a + -b \ge b + -a$ 

Sample reasoning: The expression on the left is greater than 0, and the one on the right is less than 0.

**e.** a + -b  $\geq$  -a + b

Sample reasoning: Since b + (-a) is the same as -a + b, the same reason applies as before.

#### **Activity Synthesis**

The goal of this discussion is for students to understand that even without knowing the actual numbers, knowing how the signs and magnitudes of two numbers compare is enough to determine whether their sum will be positive or negative. Begin by displaying the number lines from the previously selected groups for all to see (one with a small difference between a and b and one with a large difference).

Discuss the following questions:

"Which points are in the same relative position?"

For example, a + b is greater than 2b and less than 2a in both number lines.

"Which points are in different relative positions?"

For example, 2b may be greater than a on one diagram but less than a on the other.

Then invite students to indicate whether they used <, >, or = and explain their reasoning for each part of the last question. Point out the different positions on the number lines that students refer to during their explanations.

If time allows, extend the discussion by highlighting the fact that all the comparisons in the *Task Statement* had the same answer for every group. Ask students to invent another comparison that would have a different answer for some of the groups than others.

#### **Lesson Synthesis**

Share with students,

"Today we represented changes in elevation with number line diagrams and addition expressions. We saw some patterns that happen when adding signed numbers."

To review these patterns, consider asking:

"What happens when we add two numbers with the same sign?"

Both arrows point in the same direction, so we add their magnitudes and keep the same sign for the sum.

"What happens when we add two numbers with different signs?"

The arrows point in opposite directions, so we subtract their magnitudes and use the sign of the number with the larger magnitude for sign of the sum.

"What is the opposite of a number?"

the number that is the same distance from 0 on the number line but on the other side of 0

 $\bigcirc$  "What is the opposite of 5? Of -8? Of  $\frac{1}{3}$ ? Of -0.6?"

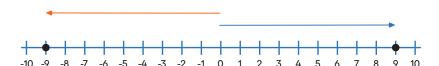
$$-5; 8; -\frac{1}{3}; 0.6$$

"What happens when we add a number and its opposite?"

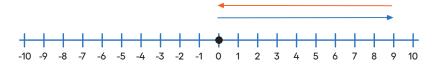
The two arrows are the same length and pointing in opposite directions, so the sum is zero.

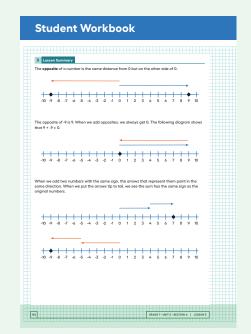
#### **Lesson Summary**

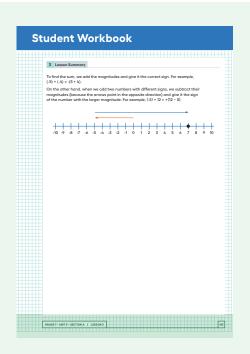
The **opposite** of a number is the same distance from 0 but on the other side of 0.



The opposite of -9 is 9. When we add opposites, we always get 0. The following diagram shows that 9 + -9 = 0.





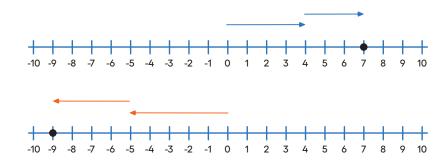


#### **Responding To Student Thinking**

#### Points to Emphasize

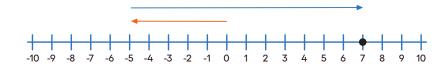
If students struggle with adding signed numbers, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about the addition expression that represents each transaction in this activity:

Grade 7, Unit 5, Lesson 4, Activity 1 Earning and Spending When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign as the original numbers.



To find the sum, we add the magnitudes and give it the correct sign. For example, (-5) + (-4) = -(5 + 4).

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and give it the sign of the number with the larger magnitude. For example, (-5) + 12 = +(12 - 5).



#### Cool-down

Add 'Em Up

5 mir

#### **Student Task Statement**

Find each sum.

**1.** 56 + (-56)

0

**2.** -240 + 370

130

3.-5.7 + (-4.2)

-9.9

#### **Practice Problems**

#### 5 Problems

#### Problem 1

What is the final elevation if

a. A bird starts at 20 m and changes 16 m?

$$36 \text{ m, because } 20 + 16 = 36$$

**b.** A butterfly starts at 20 m and changes -16 m?

$$4 \text{ m, because } 20 + (-16) = 4$$

c. A diver starts at 5 m and changes -16 m?

-II m, because 
$$5 + (-16) = -II$$

d. A whale starts at -9 m and changes 11 m?

$$2 \text{ m, because } -9 + 11 = 2$$

e. A fish starts at -9 meters and changes -11 meters?

$$-20$$
 m, because  $-9 + (-11) = -20$ 

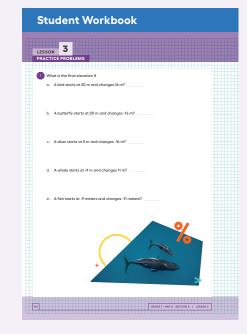
#### **Problem 2**

One of the particles in an atom is called an electron. It has a charge of -1. Another particle in an atom is a proton. It has a charge of +1.

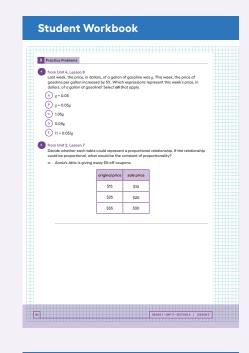
The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and -6 + 6 = 0.

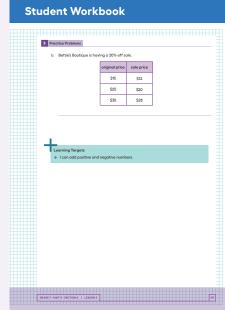
Find the overall charge for the rest of the elements on the list.

|        | charge from electrons | charge from protons | overall<br>charge |
|--------|-----------------------|---------------------|-------------------|
| carbon | -6                    | +6                  | 0                 |
| neon   | -10                   | +10                 | 0                 |
| oxide  | -10                   | +8                  | -2                |
| copper | -27                   | +29                 | +2                |
| tin    | -50                   | +50                 | 0                 |



| 3 Practice Proble               | ms  |                             |                      |
|---------------------------------|---|-----------------------------|----------------------|
|                                 | rticles in an atom is called o                              |                             |                      |
|                                 | a proton. It has a charge of<br>e electrons and the protons |                             |                      |
| because it has<br>of the elemen | s 6 electrons and 6 protons                                 | and -6 + 6 = 0. Find the ov | erall charge for the |
|                                 | charge from electrons                                       | charge from protons         | overall charge       |
| carbon                          | -6  | +6                          | 0                    |
| neon                            | -10   | +10                         |                      |
| oxide                           | -10   | +8                          |                      |
| copper                          | -27   | +29                         |                      |
| tin                             | -50   | +50                         |                      |
| b9.2 + 4.4                      |   |                             |                      |
| c81.4 + (-1:                    | 2)  |                             |                      |
|                                 |   |                             |                      |





#### **Problem 3**

Find the value of each expression.

**a.** 14.7 + 28.9 **43.6** 

**b.** -9.2 + 4.4 **-4.8** 

c. -81.4 + (-12) -93.4

**d.** 51.8 + (-0.8) 51

#### Problem 4

from Unit 4, Lesson 8

Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select **all** that apply.

**A.** g + 0.05

**B.** *g* + 0.05*g* 

**C.** 1.05g

**D.** 0.05g

**E.** (1 + 0.05)g

#### **Problem 5**

from Unit 2, Lesson 7

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. Annie's Attic is giving away \$5-off coupons.

| original price | sale price |
|----------------|------------|
| \$15           | \$10       |
| \$25           | \$20       |
| \$35           | \$30       |

#### Not proportional

The ratio between the original price and sale price is not always the same.

**b.** Bettie's Boutique is having a 20%-off sale.

| original price | sale price |
|----------------|------------|
| \$15           | \$12       |
| \$25           | \$20       |
| \$35           | \$28       |

#### Proportional

The constant of proportionality is 0.8 dollar of the sale price per dollar of the original price.