Interpreting Inequalities

Goals

- Critique (orally) a solution method for a problem involving an inequality.
- Identify the inequality that represents a situation, and justify (in writing) the choice.
- Present (orally, in writing, and using other representations) the solution method for a problem involving an inequality, and interpret the solution.

Learning Targets

- I can match an inequality to a situation it represents, solve it, and then explain what the solution means in the situation.
- If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation.

Access for Students with Diverse Abilities

- Representation (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR2 (Activity 1)

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Narrative

In this lesson, students identify which inequality, from several options, matches a given situation. They make sense of what each part of the inequality represents in the context. Then they write a question about the situation that could be answered by solving the inequality. Finally, students solve the inequality and create a visual display explaining their solution. As students create their displays and examine others' displays, they construct and critique arguments.

Student Learning Goal

Let's write inequalities.

Lesson Timeline

				Assessment
5 min	**15** min	**15** min	**10** min	**5** min
Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down

Warm-up
Notice and Wonder: Unequal

5 min

Activity Narrative

The purpose of this *Warm-up* is to re-introduce tape diagrams for inequalities, which may be useful when students interpret related quantities in word problems in a later activity. While students may notice and wonder many things about these images, these are the important discussion points:

- The whole tape is shorter than the length labeled 40.
- We don't know the length of $3x + 16$ but we do know it's shorter than 40.
- We could represent the diagram with the inequality $3x + 16 < 40$ (or equivalent).

This prompt gives students opportunities to see and make use of structure. They might apply the structure that is familiar from representing equations and recognize how the same structure can be applied to inequalities.

Launch 👥

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder.

Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?
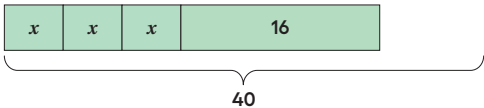
Students may notice:

- It's a tape diagram.
- It has three lengths each labeled *x* and one labeled 16.
- There's a longer length labeled 40.

Students may wonder:

- Why is the 40 longer than the tape?
- How long is the tape?

Access for Multilingual Learners (Activity 1, Task Statement)

MLR2: Collect and Display. Collect the language that students use to explain why an inequality represents a situation. Display words and phrases, such as "most," "least," "lose," "gain," "bottom," "top," "above," "below," "under," "over," "maximum," and "minimum." During the *Activity Synthesis*, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading

Student Workbook

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Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If representing the tape diagram with an inequality does not come up during the conversation, ask students to discuss this idea.

Activity 1
Club Activities Matching
15 min

Activity Narrative

In this activity, students analyze four situations and select the inequality that best represents each situation.

Students may notice and make use of structure when reasoning about how the relationships in the situations can be represented by a specific inequality.

Launch

Tell students that their job in this activity is to read four situations carefully and decide which inequality best represents each situation.

Give 5–10 minutes of quiet work time.

If needed, point out that none of the situations give the total. Students must infer what the total is based on the given inequality choices. For example, in the first situation every inequality has 3 numbers: $\frac{1}{2}$, 5, and 25. The problem states that $\frac{1}{2}$ and 5 describe the parts, so we can infer that 25 is the total.

Student Task Statement

Choose the inequality that best matches each given situation. Explain your reasoning.

1. The Garden Club is planting fruit trees in the school garden. There is one large tree that needs 5 pounds of fertilizer. The rest are newly planted trees that need $\frac{1}{2}$ pound of fertilizer each.

   A. $25x + 5 \leq \frac{1}{2}$

   B. $\frac{1}{2}x + 5 \leq 25$

   C. $\frac{1}{2}x + 25 \leq 5$

   D. $5x + \frac{1}{2} \leq 25$

   Sample reasoning: $\frac{1}{2}$ a pound for each of an unknown number of trees, plus 5 pounds of fertilizer, is less than or equal to 25, which is likely the maximum amount of fertilizer available.

2. The Hiking Club is on a hike down a cliff. They begin at an elevation of 12 feet and descend at the rate of 3 feet per minute.

A. $37x - 3 \geq 12$

B. $3x - 37 \geq 12$

C. $12 - 3x \geq -37$

D. $12x - 37 \geq -3$

Sample reasoning: They start at 12 feet and then lose 3 feet per minute. If x is the number of minutes they hike, then $-3x$ is the change in elevation. Their elevation must be above -37 feet; perhaps that elevation is the bottom of the cliff.

3. The Science Club is researching boiling points. They learn that at high altitudes, water boils at lower temperatures. At sea level, water boils at 212 °F. With each increase of 500 feet in elevation, the boiling point of water is lowered by about 1 °F.

A. $212 - \frac{1}{500}x < 195$

B. $\frac{1}{500}x - 195 < 212$

C. $195 - 212x < \frac{1}{500}$

D. $212 - 195x < \frac{1}{500}$

Sample reasoning: The boiling point is 212 °F at sea level, and decreases $\frac{1}{500}$ of a degree for every foot of elevation, x.
The solution will tell us for which elevations the temperature is below 195 degrees Fahrenheit.

Activity Synthesis

If needed, discuss which inequalities are correct. Otherwise, proceed to the next activity where students will continue working with these situations. Consider not validating which inequalities are correct at this time. When students get into groups for the next activity, they can compare their responses with the members of their groups and resolve any discrepancies at that time.

Activity 2

Club Activities Display

15 min

Activity Narrative

In this activity, students interpret parts of an inequality in context, term by term; for example, what quantity must $\frac{1}{2}x$ represent? Then they make sense of the entire inequality by thinking about what question would be answered by the solution to the inequality. Monitor for groups that create displays that communicate their mathematical thinking clearly, contain an error that would be instructive to discuss, or organize the information in a way that is useful for all to see. At this point, there is very little scaffolding for the solving of the inequality itself.

Student Workbook

Access for Students with Diverse Abilities (Activity 1, Synthesis)

Representation: Internalize Comprehension.
Use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight key words or phrases in the situation with its corresponding number or symbol in the matching inequality.

Supports accessibility for: Visual-Spatial Processing

Access for Students with Diverse Abilities (Activity 2, Task Statement)

Engagement: Develop Effort and Persistence.
Provide guides or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, create an exemplar display including all required components, highlighting different ways to communicate mathematical thinking clearly.

Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook

Launch 👥👤

Arrange students in groups of 2–3 and provide tools for making a visual display. Assign one situation to each group. Note that the level of difficulty increases for the situations, so this is an opportunity to differentiate by assigning more or less challenging situations to different groups.

Student Task Statement

Your teacher will assign your group *one* of the situations from the previous task. Create a visual display about your situation. In your display:

- Explain what the variable and each part of the inequality represent.
- Write a question that can be answered by the solution to the inequality.
- Show how you solved the inequality.
- Explain what the solution means in terms of the situation.

1. Sample responses:

- x represents the number of small trees that can be fertilized with the remaining fertilizer. $\frac{1}{2}x$ represents the number of pounds of fertilizer needed to grow x small trees. 5 is the number of pounds of fertilizer for the large tree. 25 is the total weight of fertilizer available in pounds.
- How many small trees can be planted with the available fertilizer?
- $\frac{1}{2}x + 5 \le 25$, $\frac{1}{2}x \le 20$, $x \le 40$
- Up to 40 small trees can be planted with the fertilizer available.

2. Sample responses:

- x is the number of minutes the students have been hiking. −3 represents the change in elevation in feet each minute. −3x is the total feet of elevation change after x minutes. 12 is the initial elevation. 12 − 3x represents the students' elevation after hiking for x minutes. −37 represents the elevation at the bottom of the cliff, which is 37 feet below sea level.
- How long can the students hike before getting to the bottom of the cliff, if the elevation at the bottom is 37 feet below sea level?
- $12 - 3x \ge -37$, $-3x \ge -49$, $x \le 16\frac{1}{3}$
- The students can hike for a time period of up to $16\frac{1}{3}$ minutes, at which point they come to the bottom of the cliff.

3. Sample responses:

- x represents the elevation in feet. The term $-\frac{1}{500}x$ represents the change in the boiling point of water in degrees Fahrenheit after an increase in elevation of x feet. 212 represents the boiling point of water, in degrees Fahrenheit, at sea level, which is an elevation of 0 feet. $212 - \frac{1}{500}x$ is the boiling point of water in degrees Fahrenheit at elevation x.
- At which elevations is the boiling point of water below 195 degrees Fahrenheit?
- $212 - \frac{1}{500}x < 195$, $-\frac{1}{500}x < -17$, $x > 8,500$
- At elevations greater than 8500 feet, the boiling point of water is less than 195 degrees Fahrenheit.

Are You Ready for More?

{3, 4, 5, 6} is a set of four consecutive integers whose sum is 18.

1. How many sets of three consecutive integers are there whose sum is between 51 and 60? Can you be sure you've found them all? Explain or show your reasoning.

   4 sets: {16, 17, 18}, {17, 18, 19}, {18, 19, 20}, {19, 20, 21}

   Sample reasoning: $x + (x + 1) + (x + 2) \geq 51$ and $x + (x + 1) + (x + 2) \leq 60$, $16 \leq x \leq 19$.

2. How many sets of four consecutive integers are there whose sum is between 59 and 82? Can you be sure you've found them all? Explain or show your reasoning.

   Both of these problems can be solved by intelligent guess-and-check, or other more conceptual strategies, and by using the first answer one finds to generate the others. If students use these strategies, help them clarify their reasoning: how do they know they have found all of the sets? Also encourage students to see if they can write inequalities in addition to (not instead of!) whatever strategies they use.

   5 sets: {14, 15, 16, 17}, {15, 16, 17, 18}, {16, 17, 18, 19}, {17, 18, 19, 20}, {18, 19, 20, 21}.

   Sample reasoning: $x + (x + 1) + (x + 2) + (x + 3) \geq 59$ and $x + (x + 1) + (x + 2) + (x + 3) \leq 82$, $13.25 \leq x \leq 18.5$.

Activity Synthesis

Select groups to share their visual displays. Encourage students to ask questions about the mathematical thinking or design approach that went into creating the display. Here are questions for discussion, if not already mentioned by students:

- How did you figure out what each term in the inequality represents?
- How did you decide on the direction of the inequality for the solutions?
- Did anyone with the same problem do one of the steps differently? Share what you did differently so we can learn from what happened.
- How do you know there are 25 pounds of fertilizer available?

Alternatively, have students do a Gallery Walk in which they leave written feedback on sticky notes for the other groups. Here is guidance for the kind of feedback students should aim to give each other:

- What is one thing this group did that would have made your project better if you had done it?
- What is one thing your group did that would have improved their project if they did it too?
- How did this group decide the direction of inequality for the solutions?
- Does their answer make sense in the situation?
- Is their math work clear and correct?
- If there was a mistake, what could they be more careful about in similar problems?

Student Workbook

Responding To Student Thinking

Points to Emphasize

If students struggle with interpreting the inequality or its solution, plan to review this concept when opportunities arise over the next several lessons. For example, make sure to invite multiple students to share their thinking about how the inequalities represent the situations in this activity:

Student Workbook

Lesson Synthesis

Share with students,

💬 *"Today we solved inequalities and explained what our solutions represented about the situation."*

Ask students to consider,

💬 *"What if someone asked for your help with how to solve inequalities? What would you tell them? How would you describe to someone how to solve any inequality?"*

Ask students to either record their answer or share their thoughts with their partner. Consider creating a persistent visual display that shows the procedure, using language the class develops, along with an example.

Lesson Summary

Many real-world problems can be represented and solved by using inequalities. Writing inequalities is very similar to writing equations to represent a situation. The expressions that make up the inequalities can be thought of in much the same way as the expressions that make up equations. For inequalities, we also have to think about how expressions compare to each other—which one has a greater value, which one has a lesser value, and can they also be equal?

For example, a school fundraiser has a minimum target of $500. Faculty have donated $100 and there are 12 student clubs that are participating with different activities. How much money would each club need to raise if the 12 clubs shared the responsibility of meeting the goal equally? If n is the amount of money that each club raises, then the solution to $100 + 12n = 500$ is the amount each club has to raise to meet the goal. It is more realistic, though, to use the inequality $100 + 12n \geq 500$, since the more money raised, the more successful the fundraiser. There are many solutions because there are many different amounts of money the clubs could raise that would get them above their minimum goal of $500.

Cool-down

Party Decorations

5 min

Student Task Statement

Andre is making paper cranes to decorate for a party. He plans to make one large paper crane for a centerpiece and several smaller paper cranes to put around the table. It takes Andre 10 minutes to make the centerpiece and 3 minutes to make each small crane. He will only have 30 minutes to make the paper cranes once he gets home.

1. Andre wrote the inequality $3x + 10 \leq 30$ to plan his time. Describe how this inequality represents the situation.

   Sample response: The variable x represents the number of small paper cranes Andre can make. $3x$ is the amount of time it takes to make x small cranes. 10 is the number of minutes it takes to make the centerpiece. 30 is Andre's time limit in minutes.

2. Solve Andre's inequality, and explain what the solution means.

   $x \leq 6\frac{2}{3}$ Sample response: Andre can make up to 6 small cranes.

Practice Problems   6 Problems

Problem 1

Priya looks at the inequality $12 - x > 5$ and says "I subtract a number from 12 and want a result that is bigger than 5. That means that the solutions should be values of x that are smaller than something."

Do you agree with Priya? Explain your reasoning and include solutions to the inequality in your explanation.

Yes, I agree with Priya

Sample reasoning: By subtracting different numbers from 12, we can see that 12 − 3 is greater than 12 − 8 because subtracting 3 is subtracting less. When $x = 7$, the inequality is not true anymore, but for anything less than 7, it is still true. The solution to the inequality is all values less than 7, or $x < 7$.

Problem 2

Each day, a store opens with 175 shirts out on display. On average, 6 shirts are sold per hour. The store manager likes to keep at least 150 shirts on display at all times. If there are fewer than 150 on display, more shirts are brought out from the storeroom. The manager writes the inequality $-6x + 175 \geq 150$ to describe the situation.

a. Explain what x represents in the inequality.

Sample response: the number of hours the store has been open

b. Solve the inequality.

Sample response: $-6x + 175 \geq 150$, $-6x \geq -25$, $x \leq 4\frac{1}{6}$

c. Explain what the solution means in the situation.

Sample response: At any time during the first $4\frac{1}{6}$ hours (or 4 hours and 10 minutes) after opening, there will be enough shirts on display.

Problem 3   from Unit 6, Lesson 13

You know x is a number less than 4. Select **all** the inequalities that *must* be true.

A. $x < 2$

B. $x + 6 < 10$

C. $5x < 20$

D. $x - 2 > 2$

E. $x < 8$

Problem 4   from Unit 6, Lesson 13

Here is an unbalanced hanger diagram.

a. If you know each circle weighs 6 grams, what do you know about the weight of each triangle? Explain your reasoning.

Sample response: Each triangle weighs more than 4 grams. The 3 triangles weigh more than 2 circles. The 2 circles together weigh 12 grams, so that means each triangle has to weigh more than 4 grams.

b. If you know each triangle weighs 3 grams, what do you know about the weight of each circle? Explain your reasoning.

Sample response: Each circle weighs less than 4.5 grams. The 3 triangles weigh 9 grams, and this is more than the weight of 2 circles. So each circle weighs less than 4.5 grams.

Lesson 16 Practice Problems

Problem 5

from Unit 6, Lesson 13

Match each sentence with the inequality that could represent the situation.

3 — **A.** Han got $2 from Clare, but still has less than $20. **1.** $x - 2 < 20$

1 — **B.** Mai spent $2 and has less than $20. **2.** $2x < 20$

2 — **C.** If Tyler had twice the amount of money he has, he would have less than $20. **3.** $x + 2 < 20$

4 — **D.** If Priya had half the money she has, she would have less than $20 **4.** $\frac{1}{2}x < 20$

Problem 6

from Unit 4, Lesson 12

At a skateboard shop:

a. The price tag on a shirt says $12.58. Sales tax is 7.5% of the price. How much will the shirt cost including tax?

$13.52

b. The shop buys a helmet for $19.00 and sells it for $31.50. What percentage was the markup?

65.8% or 66%

c. The shop pays workers $14.25 per hour plus 5.5% commission. If someone works 18 hours and sells $250 worth of merchandise, what is the total earning on their paycheck for this pay period? Explain or show your reasoning.

$270.25

Sample reasoning: $18 \cdot (14.25) + (0.055) \cdot 250 = 270.25$