Lesson 4

Reasoning about Equations and Tape Diagrams (Part 1)

Goals

- Coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form $px + q = r$.
- Interpret (in writing) the solution to an equation in the context of the situation it represents.

Learning Targets

- I can draw a tape diagram to represent a situation where there is a known amount and several copies of an unknown amount and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

Lesson Narrative

The focus of this lesson is situations that lead to equations of the form $px + q = r$. Tape diagrams are used to help students understand why these situations can be represented with equations of this form, and to help them reason about solving equations of this form. Students also attend to the meaning of the equation's solution in the context. Note that we are not generalizing solution methods yet; just using diagrams as a tool to reason about solving equations.

Student Learning Goal

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Access for Students with Diverse Abilities

- Action and Expression (Warm-up)
- Engagement (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR8 (Warm-up)

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Required Preparation

Activity 1:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment
10 min	15 min	10 min	10 min	5 min
				Cool-down

Instructional Routines

Math Talk

ilclass.com/r/10694967

Please log in to the site before using the QR code or URL.

Instructional Routines

MLR8: Discussion Supports

ilclass.com/r/10695617

Please log in to the site before using the QR code or URL

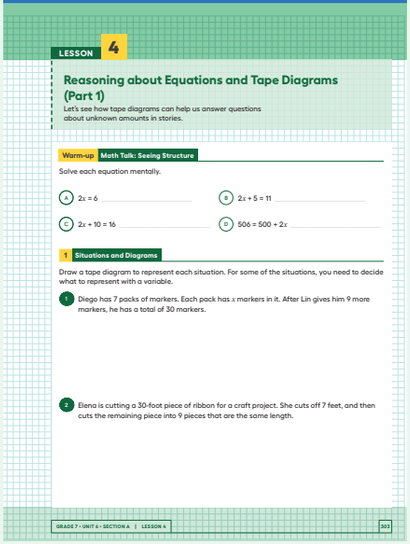
Access for Students with Diverse Abilities (Warm-up, Task Statement)

Action and Expression: Internalize Executive Functions.

To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Student Workbook



LESSON 4

Reasoning about Equations and Tape Diagrams (Part 1)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Math Talk: Seeing Structure

Solve each equation mentally.

A. $2x = 6$

B. $2x + 5 = 11$

C. $2x + 10 = 16$

D. $506 = 500 + 2x$

1 Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Diego has 7 packs of markers. Each pack has x markers in it. After Lin gives him 9 more markers, he has a total of 30 markers.

2. Elena is cutting a 30-foot piece of ribbon for a craft project. She cuts off 7 feet, and then cuts the remaining piece into 9 pieces that are the same length.

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Warm-up

Math Talk: Seeing Structure

10 min

Activity Narrative

This *Math Talk* focuses on analyzing a specific equation structure, $px + q = r$. It encourages students to recognize the importance of $r - q$, and to rely on the structure of expressions in this form to mentally solve problems. The strategies elicited here will be helpful later in the unit when students solve equations of this form.

To notice that each equation is a template where just two numbers vary, and therefore all the equations have the same solution, students need to look for and make use of structure.

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Student Task Statement

Solve each equation mentally.

A. $2x = 6$

$x = 3$

Sample reasoning: $\frac{6}{2} = 3$.

B. $2x + 5 = 11$

$x = 3$

Sample reasoning: The equation means 2 groups of something and 5 more equals 11. If I take the 5 more away, it means that 2 groups equal 6.

C. $2x + 10 = 16$

$x = 3$

Sample reasoning: When x is 1, $2x + 10 = 12$. When x is 2, $2x + 10 = 14$. When x is 3, $2x + 10 = 16$.

D. $506 = 500 + 2x$

$x = 3$

Sample reasoning: 506 equals 500 + 6, so $2x$ must equal 6.

Activity Synthesis

Ask students to identify what all the equations have in common. If it doesn't come up, point out that $2x$ must equal 6 in every equation. Therefore, the solution to each equation is $x = 3$.

To involve more students in the conversation, consider asking:

- *"Who can restate _____'s reasoning in a different way?"*
- *"Did anyone use the same strategy but would explain it differently?"*
- *"Did anyone solve the problem in a different way?"*
- *"Does anyone want to add on to _____'s strategy?"*
- *"Do you agree or disagree? Why?"*
- *"What connections to previous problems do you see?"*

Access for Multilingual Learners (Warm-up, Synthesis)

MLR8: Discussion Supports.

Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because ..." or "I noticed _____ so I ..." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Advances: Speaking, Representing

Activity 1

Situations and Diagrams

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students draw a tape diagram to represent each situation. The purpose is to show students that some situations can be represented by an equation of the form $px + q = r$ (or equivalent).

The last question is challenging to represent with a tape diagram, because part of the diagram would need to be divided into 30 equal pieces. This is intentional, and can be used to make the point that we are trying to develop ways to solve problems that are more efficient than drawing a diagram every time.

For each question, monitor for one student with a correct diagram. Encourage students to explain what any variables used to label the diagram represent in the situation.

Since students have not been shown how to represent situations like these with a tape diagram, they have to do a notable amount of sense-making to complete the task.

In the digital version of the activity, students use an applet to create their tape diagrams. The applet allows students to divide the tape into parts and label the parts and total.

The digital version may be helpful for students who struggle with drawing their own diagrams. However, even if the class is using the digital version, some students may prefer to draw the diagrams in their notebooks or on scratch paper.

Launch

Ensure students understand that the work of this task is to draw a tape diagram to represent each situation. There is no requirement to write an equation or solve a problem yet.

Arrange students in groups of 2.

Give 5–10 minutes to work individually or with their partner, followed by a whole-class discussion.

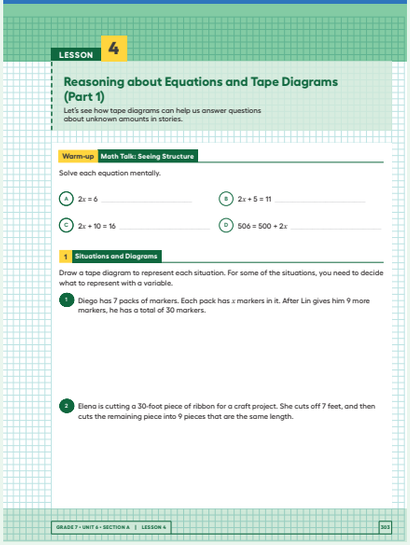
Access for Students with Diverse Abilities (Activity 1, Task Statement)

Engagement: Internalize Self Regulation.

Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to 2 out of the 3 situations. Require students to choose the last situation as it is different from the others.

Supports accessibility for: Organization, Attention

Student Workbook



LESSON 4

Reasoning about Equations and Tape Diagrams (Part 1)

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

Warm-up Math Talk: Seeing Structure

Solve each equation mentally.

A. $2x = 6$ B. $2x + 5 = 11$

C. $2x + 10 = 16$ D. $506 = 500 + 2x$

1 Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Diego has 7 packs of markers. Each pack has x markers in it. After Lin gives him 9 more markers, he has a total of 30 markers.

2. Elena is cutting a 30-foot piece of ribbon for a craft project. She cuts off 7 feet, and then cuts the remaining piece into 9 pieces that are the same length.

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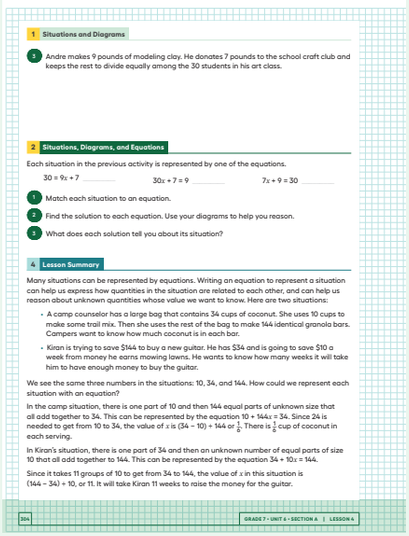
Student Workbook



1 Situations and Diagrams

3. Andre makes 9 pounds of modeling clay. He donates 7 pounds to the school craft club and keeps the rest to divide equally among the 30 students in his art class.

2 Situations, Diagrams, and Equations

Each situation in the previous activity is represented by one of the equations.

$30 = 9x + 7$ $30x + 7 = 9$ $7x + 9 = 30$

1. Match each situation to an equation.
2. Find the solution to each equation. Use your diagrams to help you reason.
3. What does each solution tell you about its situation?

4 Lesson Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are two situations:

- A camp counselor has a large bag that contains 34 cups of coconut. She uses 10 cups to make some trail mix. Then she uses the rest of the bag to make 144 identical granola bars. Campers want to know how much coconut is in each bar.
- Kiran is trying to save $144 to buy a new guitar. He has $34 and is going to save $10 a week from money he earns mowing lawns. He wants to know how many weeks it will take him to have enough money to buy the guitar.

We see the same three numbers in the situations: 10, 34, and 144. How could we represent each situation with an equation?

In the camp situation, there is one part of 10 and then 144 equal parts of unknown size that all add together to 34. This can be represented by the equation $10 + 144x = 34$. Since 24 is needed to get from 10 to 34, the value of x is $(34 - 10) \div 144$ or $\frac{1}{6}$. There is $\frac{1}{6}$ cup of coconut in each serving.

In Kiran's situation, there is one part of 34 and then an unknown number of equal parts of size 10 that all add together to 144. This can be represented by the equation $34 + 10x = 144$.

Since it takes 11 groups of 10 to get from 34 to 144, the value of x in this situation is $(144 - 34) \div 10$, or 11. It will take Kiran 11 weeks to raise the money for the guitar.

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Student Task Statement

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Diego has 7 packs of markers. Each pack has x markers in it. After Lin gives him 9 more markers, he has a total of 30 markers.
2. Elena is cutting a 30-foot piece of ribbon for a craft project. She cuts off 7 feet, and then cuts the remaining piece into 9 pieces that are the same length.
3. Andre makes 9 pounds of modeling clay. He donates 7 pounds to the school craft club and keeps the rest to divide equally among the 30 students in his art class.

Sample responses:

1. A tape diagram shows 7 same-size boxes, each marked x, one box marked 9, and a bracket showing the total is 30.
2. A tape diagram shows 9 same-size boxes, each marked x, one box marked 7, and a bracket showing the total is 30.
3. A tape diagram shows 30 same-size boxes, each marked x (or one marked 30x or equivalent), one box marked 7, and a bracket showing the total is 9.

Activity Synthesis

The purpose of this discussion is to understand how each situation is represented by a tape diagram. Select one student for each situation to present their correct diagram. Ensure that students explain the meaning of any variables used to label their diagram.

Possible questions for discussion:

- *"For the situations with no x, how did you decide what quantity to represent with a variable?"*

  Think about which amount is unknown but has a relationship to one or more other amounts in the story.

- *"How is the last situation different from the others?"*

  It's the only one where 30 is the coefficient of x rather than the total.

- *"Why was it tough to draw a diagram for the last question?"*

  You would have to divide part of the diagram into 30 equal pieces.

Activity 2
Situations, Diagrams, and Equations

10 min

Activity Narrative

This activity is a continuation of the previous one. Students match each situation from the previous activity with an equation, solve the equation by any method that makes sense to them, and interpret the meaning of the solution. Students are still using any method that makes sense to them to reason about a solution.

For each equation, monitor for a student using their diagram to reason about the solution and a student using the structure of the equation to reason about the solution.

Students reason abstractly and quantitatively when they attend to the meaning of the equation's solution in the context.

Launch

Keep students in the same groups.

Give 5 minutes to work individually or with a partner, followed by a whole-class discussion.

Student Task Statement

Each situation in the previous activity is represented by one of the equations.

- $30 = 9x + 7$
- $30x + 7 = 9$
- $7x + 9 = 30$

1. Match each situation to an equation.

- $9x + 7 = 30$: Situation 2 (ribbons)
- $30x + 7 = 9$: Situation 3 (modeling clay)
- $7x + 9 = 30$: Situation 1 (markers)

2. Find the solution to each equation. Use your diagrams to help you reason.

- $9x + 7 = 30$: $x = 2\frac{5}{9}$ or about 2.6
- $30x + 7 = 9$: $x = \frac{1}{15}$
- $7x + 9 = 30$: $x = 3$

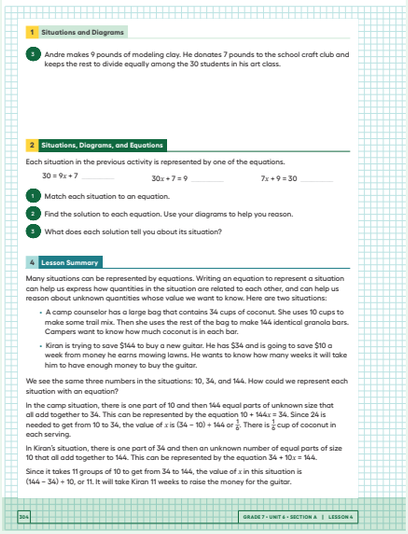
3. What does each solution tell you about its situation?

a. There are 3 markers in each pack.

b. Each of the 9 pieces is about 2.6 feet long.

c. Each student receives $\frac{1}{15}$ pound of modeling clay.

Student Workbook



1 Situations and Diagrams

3. Andre makes 9 pounds of modeling clay. He donates 7 pounds to the school craft club and keeps the rest to divide equally among the 30 students in his art class.

2 Situations, Diagrams, and Equations

Each situation in the previous activity is represented by one of the equations.

$30 = 9x + 7$ _____ $30x + 7 = 9$ _____ $7x + 9 = 30$ _____

1. Match each situation to an equation.
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4 Lesson Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are two situations:

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We see the same three numbers in the situations: 10, 34, and 144. How could we represent each situation with an equation?

In the camp situation, there is one part of 10 and then 144 equal parts of unknown size that all add together to 34. This can be represented by the equation $10 + 144x = 34$. Since 24 is needed to get from 10 to 34, the value of x is $(34 - 10) \div 144$ or $\frac{1}{6}$. There is $\frac{1}{6}$ cup of coconut in each serving.

In Kiran's situation, there is one part of 34 and then an unknown number of equal parts of size 10 that all add together to 144. This can be represented by the equation $34 + 10x = 144$.

Since it takes 11 groups of 10 to get from 34 to 144, the value of x in this situation is $(144 - 34) \div 10$, or 11. It will take Kiran 11 weeks to raise the money for the guitar.

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Develop Effort and Persistence.

Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation, such as: " I noticed _____ so I ...", "_____ corresponds to _____.", "It looks like _____ represents ...", or "How did you get ...?"

Supports accessibility for: Language, Social-Emotional Functioning

Activity Synthesis

The purpose of this discussion is to connect solving moves on the tape diagram with solving moves on the equation. Display the first situation next to its matching diagram and equation. Ask one student who reasoned with the diagram and one who reasoned only about the equation to explain their solutions. Point out or demonstrate connections between the two representations and solving moves. For example, "I subtracted the 9 extra markers and then divided the remaining 21 markers by 7," can be shown on a tape diagram and on a corresponding equation. It is not necessary to invoke the more abstract language of "doing the same thing to each side" of an equation yet.

Lesson Synthesis

Display one of the situations from the lesson and its corresponding equation. Ask students to explain:

- *"What does each number and letter in the equation represent in the situation?"*
- *"What is the reason for each operation (multiplication or addition) used in the equation?"*
- *"What is the solution to the equation? What does it mean to be a solution to an equation? What does the solution represent in the situation?"*

Lesson Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are two situations:

- A camp counselor has a large bag that contains 34 cups of coconut. She uses 10 cups to make some trail mix. Then she uses the rest of the bag to make 144 identical granola bars. Campers want to know how much coconut is in each bar.
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In Kiran's situation, there is one part of 34 and then an unknown number of equal parts of size 10 that all add together to 144. This can be represented by the equation $34 + 10x = 144$.

Since it takes 11 groups of 10 to get from 34 to 144, the value of x in this situation is $(144 - 34) \div 10$, or 11. It will take Kiran 11 weeks to raise the money for the guitar.

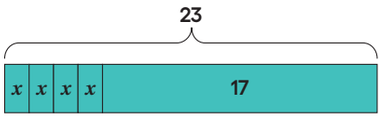
Cool-down

Finding Solutions

5 min

Student Task Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$4x + 17 = 23$

$x = 1\frac{1}{2}$

Sample explanation: The diagram and equation show that 4 groups plus 17 more equals a total of 23. If we take aways the 17 more, we have 4 groups that equal a total of 6, and $\frac{6}{4} = 1\frac{1}{2}$.

Responding To Student Thinking

More Chances

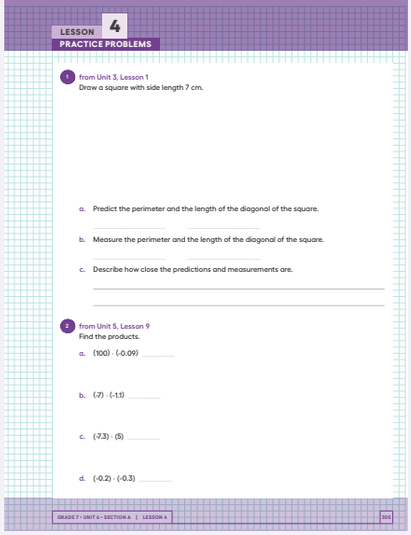
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 4

Practice Problems

5 Problems

Student Workbook


LESSON 4
PRACTICE PROBLEMS

1 from Unit 3, Lesson 1
Draw a square with side length 7 cm.

a. Predict the perimeter and the length of the diagonal of the square.

b. Measure the perimeter and the length of the diagonal of the square.

c. Describe how close the predictions and measurements are.

2 from Unit 5, Lesson 9
Find the products.

a. (100) · (-0.09)

b. (-7) · (-1.1)

c. (-7.3) · (5)

d. (-0.2) · (-0.3)

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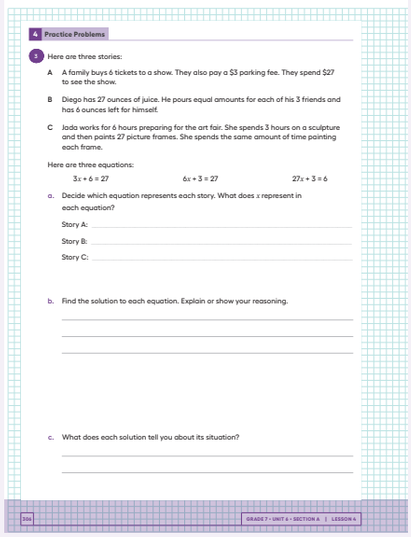
Student Workbook


4 Practice Problems

3 Here are three stories:

A A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 to see the show.

B Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.

C Jada works for 6 hours preparing for the art fair. She spends 3 hours on a sculpture and then paints 27 picture frames. She spends the same amount of time painting each frame.

Here are three equations:

$3x + 6 = 27$ $6x + 3 = 27$ $27x + 3 = 6$

a. Decide which equation represents each story. What does x represent in each equation?

Story A:

Story B:

Story C:

b. Find the solution to each equation. Explain or show your reasoning.

c. What does each solution tell you about its situation?

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Problem 1

from Unit 3, Lesson 1

Draw a square with side length 7 cm.

a. Predict the perimeter and the length of the diagonal of the square.

Perimeter: 28 cm

Length of diagonal: Approximately 9.9 cm.

b. Measure the perimeter and the length of the diagonal of the square.

Answers vary

c. Describe how close the predictions and measurements are.

Answers vary

Problem 2

from Unit 5, Lesson 9

Find the products.

a. (100) · (-0.09)

-9

b. (-7) · (-1.1)

7.7

c. (-7.3) · (5)

-36.5

d. (-0.2) · (-0.3)

0.06

Problem 3

Here are three stories:

A. A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 to see the show.

B. Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.

C. Jada works for 6 hours preparing for the art fair. She spends 3 hours on a sculpture and then paints 27 picture frames. She spends the same amount of time painting each frame.

Here are three equations:

$$3x + 6 = 27 \qquad 6x + 3 = 27 \qquad 27x + 3 = 6$$

a. Decide which equation represents each story. What does x represent in each equation?

Story A: Tickets to the show: $6x + 3 = 27$, x represents the cost of a ticket.

Story B: Diego's juice: $3x + 6 = 27$, x represents the number of ounces of juice he gave each friend.

Story C: $27x + 3 = 6$, x represents the number of hours spent on each picture frame.

b. Find the solution to each equation. Explain or show your reasoning.

Story A: $6x + 3 = 27$: $x = 4$

Story B: $3x + 6 = 27$: $x = 7$

Story C: $27x + 3 = 6$: $x = \frac{1}{9}$

Explanations vary.

c. What does each solution tell you about its situation?

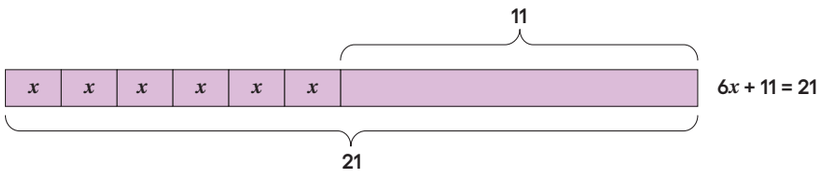
Story A: Each ticket to the show cost $4.

Story B: Diego gave each friend 7 ounces of juice.

Story C: Jada spent $\frac{1}{9}$ of an hour painting each picture frame.

Problem 4

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$x = \frac{10}{6}$ (or equivalent)

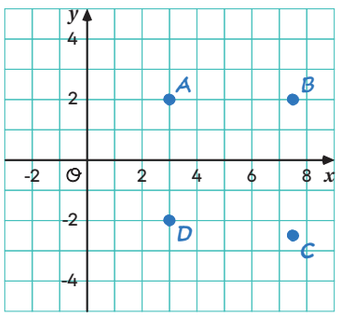
Sample reasoning: The total 21 is 11 plus 6 equal groups. So, the 6 equal groups equal a total of 10.

Problem 5

from Unit 5, Lesson 7

a. Plot these points on the coordinate plane:

$A = (3, 2)$, $B = (7.5, 2)$, $C = (7.5, -2.5)$, $D = (3, -2)$



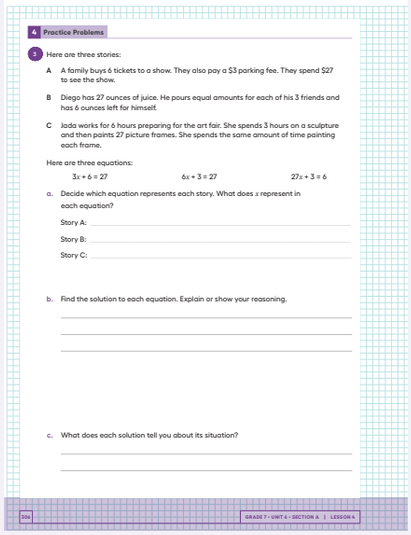
b. What is the vertical difference between D and A?

The vertical difference between D and A is -4 units.

c. Write an expression that represents the vertical distance between B and C.

An expression for the vertical distance between B and C is $|2 - (-2.5)|$.

Student Workbook



Student Workbook