Lesson 12

Congruent Polygons

Goals

- Critique arguments (orally) that two figures with congruent corresponding sides may be non-congruent figures.
- Justify (orally and in writing) that two polygons on a grid are congruent using the definition of "congruence" in terms of rigid transformations.

Learning Target

I can decide using rigid transformations whether or not two figures are congruent.

Student Learning Goal

Let's decide if two figures are congruent.

Lesson Narrative

In this lesson, students find rigid transformations that show two figures are congruent and make arguments for why two figures are not congruent. They learn that, for many shapes, simply having corresponding side lengths that are equal will not guarantee the figures are congruent.

In the previous lesson, students defined what it means for two shapes to be congruent and started to apply the definition to determine if a pair of shapes is congruent. In the first part of this lesson, students continue to determine whether or not pairs of shapes are congruent, but here they have the extra structure of a grid. With this extra structure, students try to communicate precisely when describing translations, reflections, and rotations. For example:

- Instead of "translate down and to the left," students can say, "translate 3 units down and 2 units to the left"
- Instead of "reflect the shape," students can say, "reflect the shape over vertical line ℓ."

In addition, students have to be careful how they name congruent polygons, making sure that corresponding vertices are listed in the proper order.

Access for Students with Diverse Abilities

- Engagement (Activity 1)
- Representation (Activity 3)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Take Turns

Required Materials

Materials to Gather

- Geometry toolkits: Warm-up, Activity 1, Activity 2
- Toothpicks, pencils, straws, or other objects: Activity 3

Activity 4:
If using this optional activity, have sets of objects ready for students to build quadrilaterals. Each pair of students requires 12 objects (such as toothpicks, pencils, or straws) to be used as sides of quadrilaterals: 8 objects of one length and 4 objects of a different length.

Lesson Timeline

5 min	15 min	15 min	10 min	10 min	Assessment: 5 min
Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Cool-down

Lesson 12

Congruent Polygons

Lesson Narrative (continued)

An optional part of the lesson begins to examine criteria to decide when two shapes are congruent. If two shapes are congruent, then their corresponding sides and angles are congruent. Is it true that having the same side lengths (or angles) is enough to determine whether or not two shapes are congruent? Students investigate this question for quadrilaterals of two types:

- 4 congruent side lengths
- 2 pairs of congruent side lengths where the pairs are of different length

Warm-up
Translated Images

5 min

Activity Narrative

The purpose of this activity is for students to connect rigid transformations with congruent figures. In this activity, students identify which figures are images of an original triangle under a translation. They may notice features of figures under a translation, such as parallel corresponding segments, or the orientation of the figure staying the same. This will be useful in upcoming activities as students describe a sequence of transformations from one figure to another.

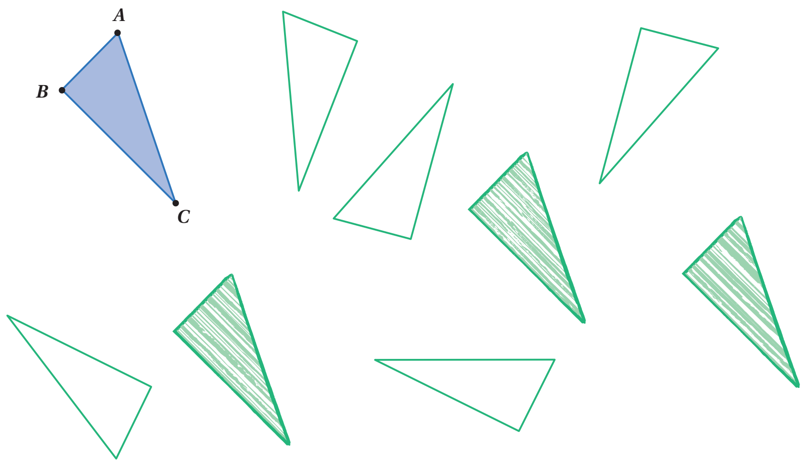
Launch

Provide access to geometry toolkits.

Allow for 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

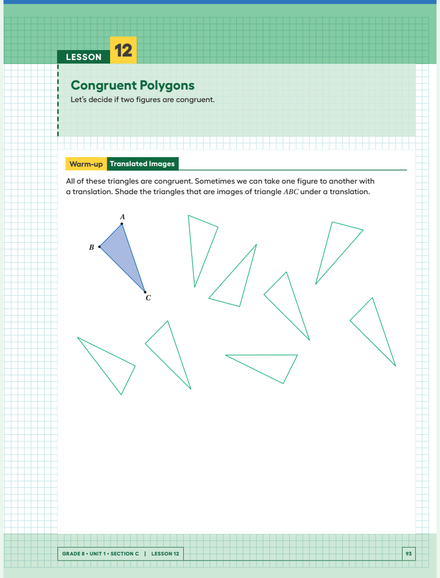
All of these triangles are congruent. Sometimes we can take one figure to another with a translation. Shade the triangles that are images of triangle ABC under a translation.



Building on Student Thinking

If any students assert that a triangle is a translation when it isn't really, ask them to use tracing paper to demonstrate how to translate the original triangle to land on it. Inevitably, they need to rotate or flip the paper. Remind them that a translation consists only of sliding the tracing paper around without turning it or flipping it.

Student Workbook



Access for Multilingual Learners (Activity 1)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Instructional Routines

MLR7: Compare and Connect

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Activity Synthesis

The purpose of this discussion is for students to articulate what features they can look for when they are identifying a translation. Ask students what they noticed about figures that were translations of triangle ABC.
If no students share these observations, suggest them now and ask students to discuss:

- The shape appears to be pointed in the same direction. A rotation may not have this property.
- The corresponding points go in the same order around the figure. A reflection does not have this property.
- Corresponding sides are parallel. This is a property of translating a line.

If time allows, choose a triangle that is not the image of triangle ABC under a translation, and ask students what rigid transformation would show that it is congruent to triangle ABC. If needed, demonstrate the rotation or reflection.

Activity 1
Congruent Pairs (Part 1)

15 min

Activity Narrative

In this activity, students apply the definition of congruence to figures on a square grid. Students may use the structure of the grid to describe rigid transformations, including reflecting over a specified line or identifying coordinates of corresponding points. Students may also wish to use tracing paper to execute the transformations.

Students are given several pairs of shapes on grids and asked to determine if the shapes are congruent. The congruent shapes are deliberately chosen so that more than one transformation will likely be required to show the congruence. In these cases, students will likely find different ways to show the congruence. Monitor for students who use different sequences of transformations to show congruence. For example, for the first pair of quadrilaterals, some different ways are:

- Translate $EFGH$ 1 unit to the right, and then rotate its image 180 degrees about $(0, 0)$.
- Reflect $ABCD$ over the x-axis, then reflect its image over the y-axis, and then translate this image 1 unit to the left.

For the pairs of shapes that are *not* congruent, students need to identify a feature of one shape not shared by the other in order to argue that it is not possible to move one shape on top of another with rigid motions. At this stage, arguments can be informal. Monitor for students who use different features to show figures are not congruent:

- The side lengths are different so it is not possible to make them match up.
- The angles are different so it is not possible to make them match up.
- The areas of the shapes are different.

Launch

Provide access to geometry toolkits.

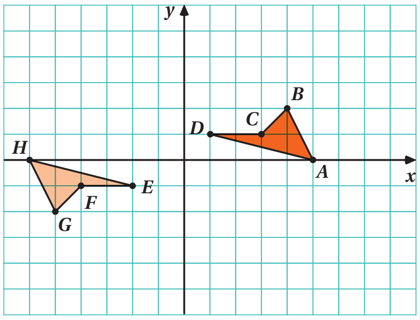
Allow for 5–10 minutes of quiet work time followed by a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

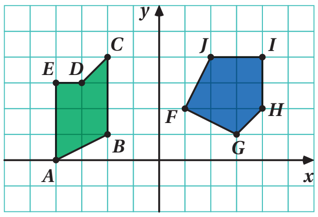
1.



These are congruent.

Sample reasoning: Rotate quadrilateral *ABCD* around *D* by 180 degrees, and then translate left 3 units and down 2 units. It matches up perfectly with *HGFE*.

2.



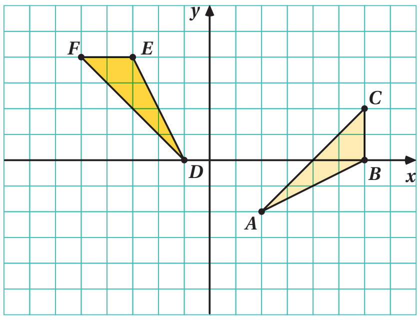
These are not congruent.

Sample reasoning 1: They are both pentagons, but *ABCDE* has a pair of opposite parallel sides while *FGHIJ* does not.

Sample reasoning 2: Angle *D* in *ABCDE* measures more than 180 degrees, while all angles in *FGHIJ* measure less than 180 degrees.

Sample reasoning 3: Side *DE* measures one unit in length, while all sides of *FGHIJ* measure more than 1 unit in length.

3.



These are congruent.

Sample reasoning: Rotate triangle *ABC* around $(0,0)$ counterclockwise by 90 degrees, then translate it down 2 units and left 3 units. It matches up with triangle *DEF* perfectly.

Access for Students with Diverse Abilities (Activity 1, Student Task)

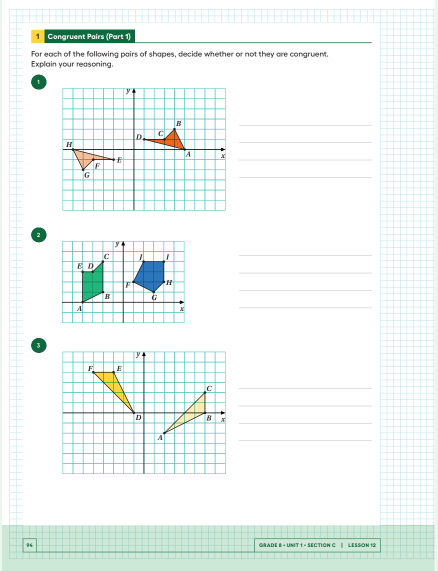
Engagement: Internalize Self Regulation.
Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to 2 out of 4 questions. Once students have successfully completed them, invite them to share with a partner prior to the whole class-discussion.

Supports accessibility for: Organization, Attention

Building on Student Thinking

Students may want to visually determine congruence each time or explain congruence by saying, "They look the same." Encourage those students to explain congruence in terms of translations, rotations, reflections, and side lengths. For students who focus on features of the shapes such as side lengths and angles, ask them how they could show the side lengths or angle measures are the same or different using the grid or tracing paper.

Student Workbook



Instructional Routines

Take Turns

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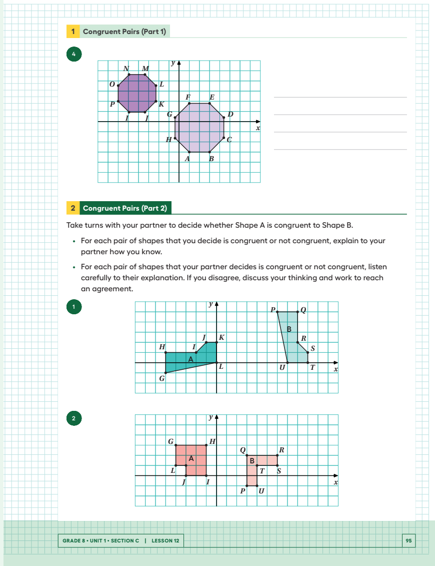
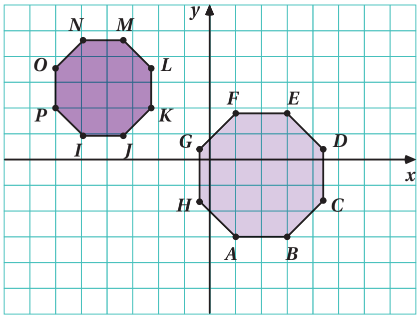
Student Workbook



4.



These are not congruent.

Sample reasoning: Both are regular octagons, but $ABCDEFGH$ is larger than $IJKLMNOP$. This can be seen by comparing the images or by looking at sides AB and IJ. Side AB is 2 units in length, while side IJ is less than 2 units in length.

Activity Synthesis

The goal of the discussion is for students to understand that when two shapes are congruent, there is a rigid transformation that matches one shape up perfectly with the other. Choosing the right sequence takes practice. Students should be encouraged to experiment using tools, such as tracing paper or technology when available. When two shapes are not congruent, there is no rigid transformation that matches one shape up perfectly with the other. It is not possible to perform every possible sequence of transformations in practice, so to show that one shape is *not* congruent to another, we identify a feature of one shape that is not shared by the other. For the shapes in this activity, students can focus on side lengths: For each pair of non-congruent shapes, one shape has a side length not shared by the other. Since rigid transformations do not change side lengths, this is enough to conclude that the two shapes are not congruent.

Display 2–3 approaches from previously selected students for all to see. Invite students to briefly describe their approaches to showing figures are congruent as well as not congruent. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

"What do the approaches have in common? How are they different?"

"Did anyone solve the problem the same way, but would explain it differently?"

"Why do the different approaches lead to the same outcome?"

Activity 2

Congruent Pairs (Part 2)

15 min

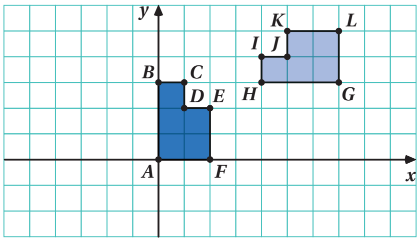
Activity Narrative

In this partner activity, students take turns claiming that two given polygons are or are not congruent and explaining their reasoning. The partner's job is to listen for understanding and challenge their partner if their reasoning is incorrect or incomplete. As students trade roles explaining their thinking and listening, they have opportunities to explain their reasoning and critique the reasoning of others.

This activity continues to investigate congruence of polygons on a grid. Unlike in the previous activity, the non-congruent pairs of polygons share the same side lengths.

Launch

Arrange students in groups of 2, and provide access to geometry toolkits. Display this image for all to see.



Demonstrate these steps of how to set up the activity after choosing a student as a partner:

- One partner claims whether the shapes are congruent or not.
- If the partner claims the shapes are congruent, they should describe a rigid transformation to show congruence, while the other partner checks the claim by performing the transformation.
- If the partner claims the shapes are not congruent, they should support this claim with an explanation to convince the other partner that they are not congruent.
- For each question, students switch roles.

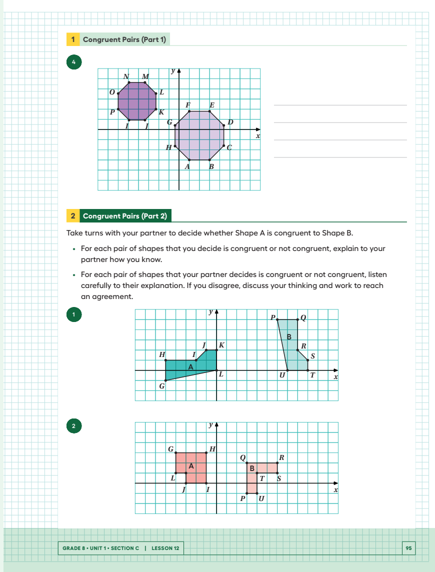
Students work through this same process with their own partners on the questions in the activity.

Building on Student Thinking

For Part 5, students may be correct in saying the shapes are not congruent but for the wrong reason. They may say one is a 3-by-3 square and the other is a 2-by-2 square, counting the diagonal side lengths as one unit. If so, have them compare lengths by marking them on the edge of a card, or measuring them with a ruler.

In discussing congruence for Part 3, students may say that quadrilateral $GHIJ$ is congruent to quadrilateral $PQRS$, but this is not correct. After a set of transformations is applied to quadrilateral $GHIJ$, it corresponds to quadrilateral $QRSP$. The vertices *must* be listed in this order to accurately communicate the correspondence between the two congruent quadrilaterals.

Student Workbook

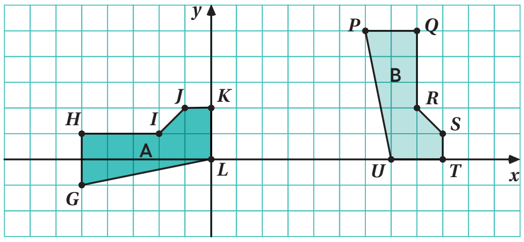


Student Task Statement

Take turns with your partner to decide whether Shape A is congruent to Shape B.

- For each pair of shapes that you decide is congruent or not congruent, explain to your partner how you know.
- For each pair of shapes that your partner decides is congruent or not congruent, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

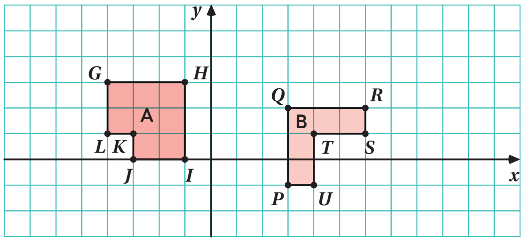
1.



Yes, they are congruent.

Sample reasoning: Rotate Hexagon A 90 degrees clockwise with center (0, 0), then translate it 7 units to the right. It matches up perfectly with Hexagon B.

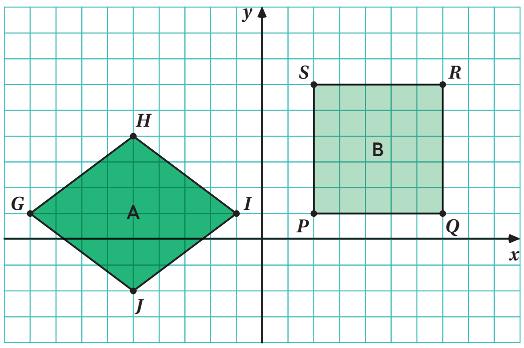
2.



No, they are not congruent.

Sample reasoning: Hexagon A has greater area (8 square units) than Hexagon B (6 square units). They are not congruent because translations, rotations, and reflections do not change the area of a figure.

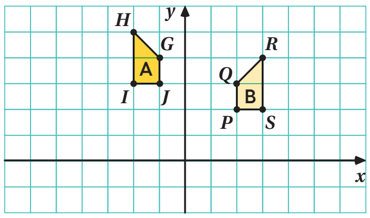
3.



No, they are not congruent.

Sample reasoning: Both shapes are quadrilaterals, and the side lengths all appear to be 5 units in length. But the angles are not the same. Quadrilateral B is a square with 4 right angles. Quadrilateral A is a rhombus. Angles G and I are acute while Angles H and J are obtuse. Since translations, rotations, and reflections do not change angle measures, there is no way to match up any of the angles of these quadrilaterals.

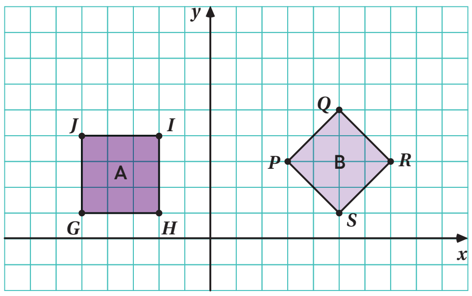
4.



Yes, they are congruent.

Sample reasoning: Reflect Quadrilateral A over the y-axis, and then translate one unit to the right and one unit down. It matches up perfectly with Quadrilateral B.

5.



No, they are not congruent.

Sample reasoning 1: Rotate Quadrilateral B about S by 45 degrees counterclockwise and then translate to the left by 7 units. Angle PSR matches up with angle GHI, but the sides of Quadrilateral B are a little shorter than those of Quadrilateral A, so the two shapes are not congruent.

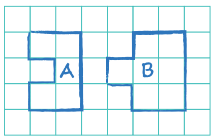
Sample reasoning 2: The area of square Quadrilateral A is 9 square units. The area of Quadrilateral B (which is also a square) is 8 square units because it contains 4 whole unit squares and then 8 half unit squares that make 4 more unit squares. Congruent shapes have the same area so these two shapes are not congruent.

Are You Ready for More?

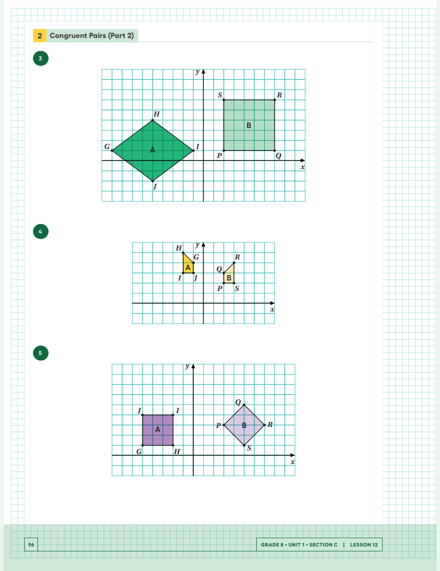
A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

1. Find a polygon with these properties.
2. Is there a second polygon, not congruent to the first, with these properties?

Here are two non-congruent shapes that meet the conditions.



Student Workbook



Instructional Routines

MLR8: Discussion Supports

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Access for Multilingual Learners (Activity 2)

MLR8: Discussion Supports

Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement "I told my partner to move the shape to the right" as "I told my partner to translate the shape 7 units to the right."

Advances: Listening, Representing

Activity Synthesis

To highlight student reasoning and language use, invite groups to respond to the following questions:

"For which congruent shapes was it easiest to explain your reasoning to your partner? Were some transformations harder to describe than others?"

"For the pairs of shapes that were not *congruent, how did you convince your partner?"*

"Did you use any measurements to help decide whether or not the pairs of shapes were congruent?"

Activity 3: Optional
Building Quadrilaterals

10 min

Activity Narrative

This activity is optional because it provides additional opportunity for students to reason about whether quadrilaterals are congruent based on features, such as side lengths and angle measures.

In this activity, students build quadrilaterals that contain congruent sides and investigate whether or not they form congruent quadrilaterals.

In addition to building an intuition for how side lengths and angle measures influence congruence, students also get an opportunity to revisit the taxonomy of quadrilaterals as they study which types of quadrilaterals they are able to build with specified side lengths.

Monitor for students who build both parallelograms and kites with the two pairs of sides of different lengths. Select them to share during the discussion.

Launch

There are two sets of building materials for this activity. Each set contains 4 side lengths. Set A contains 4 side lengths of the same size. Set B contains 2 side lengths of one size and 2 side lengths of another size.

Assign half of the class to work with Set A and the other half to work with Set B. Arrange students in groups of 2. Each group is given two of the same set of building materials. Students use the set of side lengths to build a quadrilateral at the same time, then compare their quadrilaterals with a partner to decide whether they are congruent.

Give students 5 minutes to work with their partner followed by a whole-class discussion.

Student Task Statement

Your teacher will give you a set of four objects.

1. Make a quadrilateral with your four objects and record what you have made.
2. Compare your quadrilateral with your partner's. Are they congruent? Explain how you know.
3. Repeat Steps 1 and 2, forming different quadrilaterals. If your first quadrilaterals were not congruent, can you build a pair that is? If your first quadrilaterals were congruent, can you build a pair that is not? Explain.

   There should be a variety of rhombuses and squares from Set A and parallelograms and kites from Set B. It is possible to build both congruent and non-congruent polygons from both sets of objects.

Activity Synthesis

The goal of this discussion is for students to connect angle measures and side lengths of polygons to the conditions for congruence.

To start the discussion, ask:

"Were the quadrilaterals that you and your partner built always congruent? How did you check?"

"Was it possible to build congruent quadrilaterals? What parts were important to be careful about when building them?"

Display this statement for all to see: "Two polygons are congruent if they have corresponding sides that are congruent *and* corresponding angles that are congruent." Ask students how this statement about congruent polygons connects with their understanding of rigid transformations.

Rigid transformations result in congruent figures, and rigid transformations keep the same angle measures and side lengths. This means that congruent figures must have the same angles and side lengths.

Invite previously selected students who created kits and parallelograms to share their quadrilaterals with the class. If no students bring it up, name the shapes with adjacent pairs of congruent sides as "kites" and the shapes with opposite pairs of congruent figures as "parallelograms." If time allows, invite students who created a square and a non-square rhombus to share their quadrilaterals, then name the shapes with all four sides congruent as "rhombuses."

Access for Students with Diverse Abilities (Activity 3, Student Task)

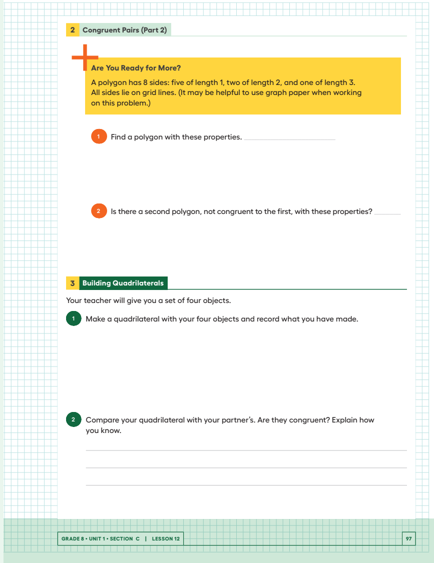
Representation: Develop Language and Symbols.
Provide students with access to charts with symbols and meanings. For example, display a chart of the taxonomy of quadrilaterals to provide access to precise language of the different types of quadrilaterals they are building.

Supports accessibility for: Language, Memory

Building on Student Thinking

Students may assume when building quadrilaterals with a set of objects of the same length that the resulting shapes are congruent. They may think that two shapes are congruent because they can physically manipulate them to make them congruent. Ask them to first build their quadrilateral and then compare it with their partner's. The goal is not to ensure that the two are congruent but to decide whether they *have to be* congruent.

Student Workbook


2 Congruent Pairs (Part 2)

Are You Ready for More?

A polygon has 8 sides: five of length 1, two of length 2, and one of length 3. All sides lie on grid lines. (It may be helpful to use graph paper when working on this problem.)

1. Find a polygon with these properties.
2. Is there a second polygon, not congruent to the first, with these properties?

3 Building Quadrilaterals

Your teacher will give you a set of four objects.

1. Make a quadrilateral with your four objects and record what you have made.
2. Compare your quadrilateral with your partner's. Are they congruent? Explain how you know.

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Lesson Synthesis

Display this question for all to see: “How can you determine when two shapes are congruent?” Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to “How can you determine when two shapes are congruent?” In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

“___ makes sense, but what do you mean when you say … ?”

“Can you describe that another way?”

“How do you know … ? What else do you know is true?”

Close the partner conversations and give students 3–5 minutes to revise their first draft.

Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

Here are some examples of a second draft:

- Two figures are congruent when there is a sequence of translations, rotations, and reflections that match one figure up perfectly with the other.
- We can use a grid to show that two shapes are congruent. For example, we can say how many grid units to move left for a translation or to reflect the shape over the x-axis.
- Two figures are not congruent if they have different side lengths, different angles, or different areas.
- Even if two figures have the same side lengths, they may not be congruent. With four sides of the same length, for example, we can make many different rhombuses that are not congruent to one another because the angles are different.

As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

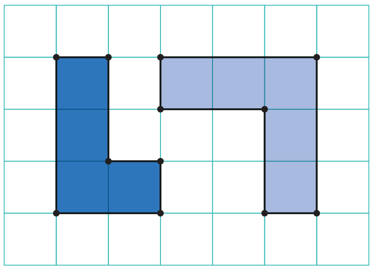
Lesson Summary

How do we know if two figures are congruent?

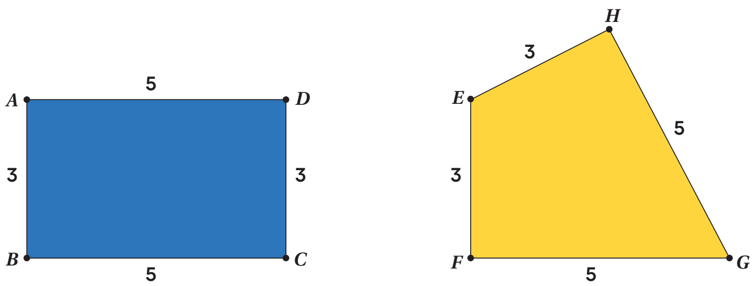
- If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- If we can describe a sequence of translations, rotations, and reflections that move one figure onto the other so they match up exactly, they are congruent.

How do we know that two figures are *not* congruent?

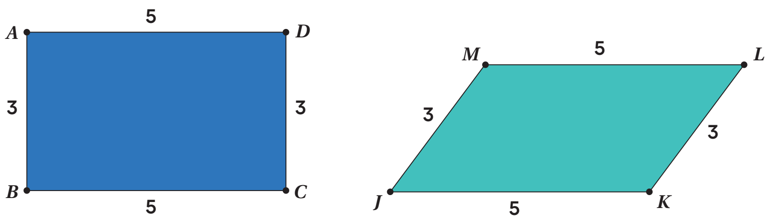
- If there is no correspondence between the figures where the parts have equal measure, that shows that the two figures are *not* congruent.
  - If two polygons have different sets of side lengths, they can't be congruent. For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.



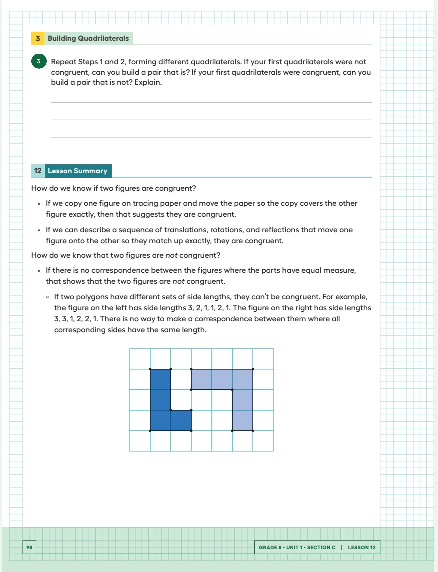
  - If two polygons have the same side lengths, but not in the same order, the polygons can't be congruent. For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order.



  - If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent. For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are right angles. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.



Student Workbook



Responding To Student Thinking

Points to Emphasize
If students struggle with identifying congruence and rigid transformations, focus on identifying congruence as opportunities arise in the next lesson. For example, in the activity referred to here, provide multiple opportunities for students to share their responses with tracing paper and to demonstrate their thinking or strategy for finding the sequence of transformations by connecting corresponding points and lengths.

Unit 1, Lesson 13, Activity 2 Congruent Ovals

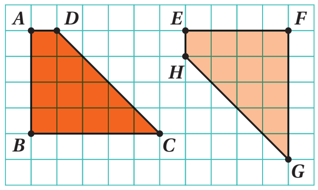
Cool-down

Moving to Congruence

5 min

Student Task Statement

Describe a sequence of reflections, rotations, and translations that shows that quadrilateral $ABCD$ is congruent to quadrilateral $EFGH$.



Sample response: Translate $ABCD$ down 1 and 5 to the right. Then reflect over line GH.

Lesson 12

Practice Problems

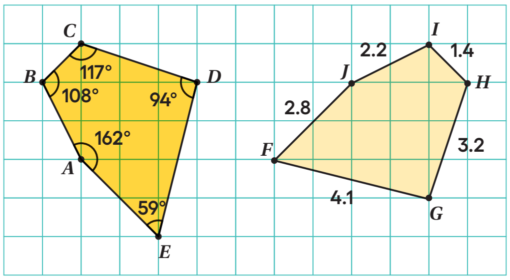
4 Problems

Problem 1

a. Show that the two pentagons in the following image are congruent.

Sample response: After performing a 90-degree clockwise rotation with center D, then translating 3 units down and 6 units to the right, $ABCDE$ matches up perfectly with $JIHGF$. The rotation and translation do not change side lengths or angle measures.

b. Find the side lengths of pentagon $ABCDE$ and the angle measures of pentagon $FGHIJ$.

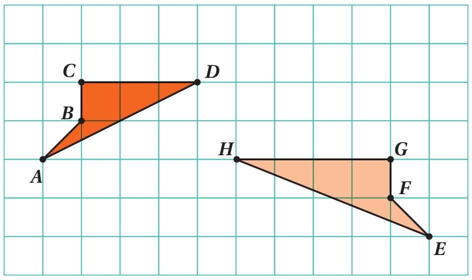


$AB = 2.2$, $BC = 1.4$, $CD = 3.2$, $DE = 4.1$, and $EA = 2.8$. Angle F: 59°, Angle G: 94°, Angle H: 117°, Angle I: 108°, and Angle J: 162°

Problem 2

For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.

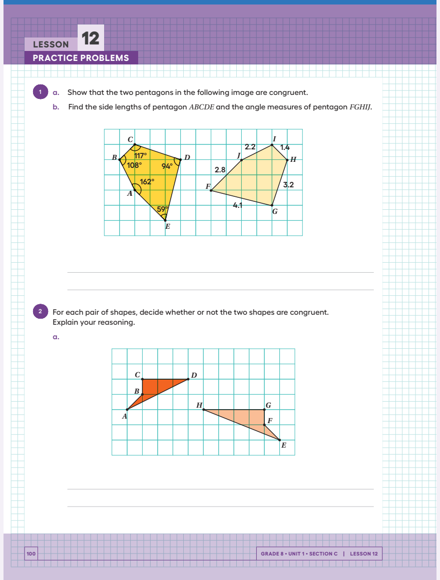
a.



Not congruent

Sample reasoning: Segment EH in polygon $EFGH$ is longer than any of the sides in polygon $ABCD$. A, B, and C can be matched up with vertices E, F, and G, but H does not match up with D.

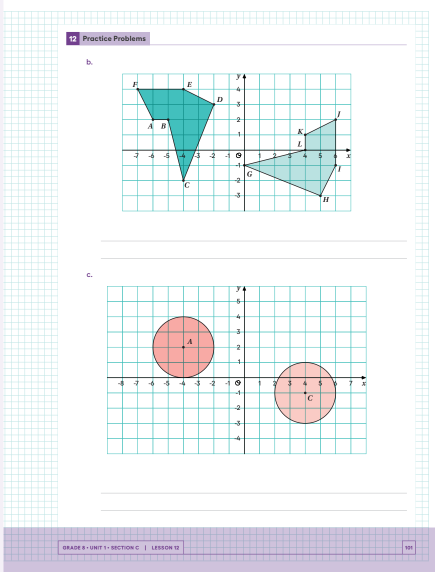
Student Workbook


LESSON 12
PRACTICE PROBLEMS

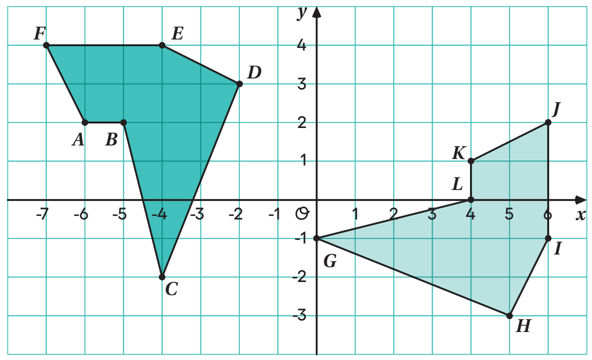
1 a. Show that the two pentagons in the following image are congruent.
b. Find the side lengths of pentagon $ABCDE$ and the angle measures of pentagon $FGHIJ$.

2 For each pair of shapes, decide whether or not the two shapes are congruent. Explain your reasoning.
a.

Student Workbook



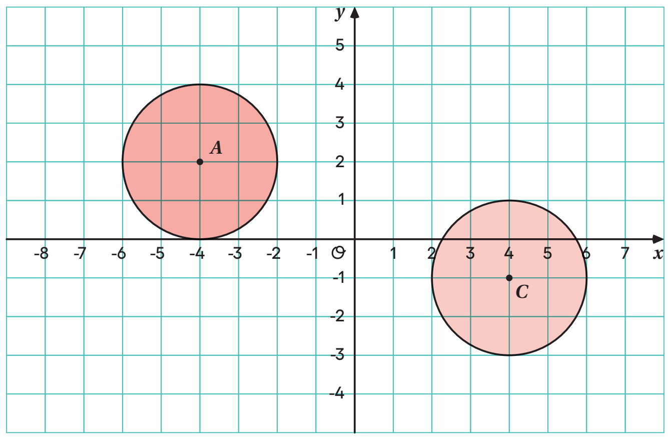
b.



congruent

Sample reasoning: If *ABCDEF* is rotated 90 degrees clockwise about *C* and then moved 4 units to the right and 1 unit up, it matches up perfectly with *GHIJKL*.

c.



congruent

Sample reasoning: If the circle on the top left is translated to the right by 8 units and down 3 units, it lands on top of the other circle.

Problem 3

from Unit 1, Lesson 8

a. Draw segment PQ.

Answers vary

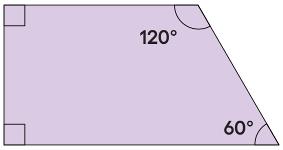
b. When PQ is rotated 180° around point R, the resulting segment is the same as PQ. Where could point R be located?

R must be the midpoint of PQ.

Problem 4

from Unit 1, Lesson 10

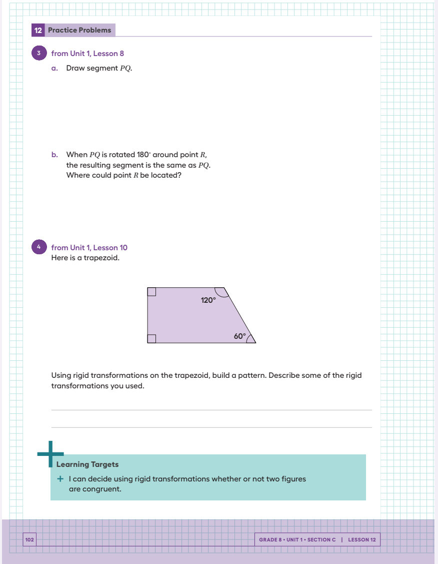
Here is a trapezoid.



Using rigid transformations on the trapezoid, build a pattern. Describe some of the rigid transformations you used.

Sample response: Clockwise rotations, centered at the vertex of the 60° angle, of 60°, 120°, 180°, 240°, and 300° make a "windmill" type figure with copies of the trapezoid.

Student Workbook


12 Practice Problems

3 from Unit 1, Lesson 8

a. Draw segment PQ.

b. When PQ is rotated 180° around point R, the resulting segment is the same as PQ. Where could point R be located?

4 from Unit 1, Lesson 10

Here is a trapezoid.

120°

60°

Using rigid transformations on the trapezoid, build a pattern. Describe some of the rigid transformations you used.

Learning Targets

+ I can decide using rigid transformations whether or not two figures are congruent.