Lesson 14

Solving Problems with Rational Numbers

Goal

Apply operations with rational numbers to solve problems involving repeated gains or losses, and present (orally, in writing, and using other representations) the solution method.

Learning Targets

- I can represent situations with expressions that include rational numbers.
- I can solve problems using the four operations with rational numbers.

Student Learning Goal

Let's use all four operations with signed numbers to solve problems.

Lesson Narrative

In this lesson students put together what they have learned about rational number arithmetic and the interpretation of negative quantities, such as negative time or negative rates of change. They solve problems with rational numbers in the context of scoring margins between sports teams and negative charges on an electricity bill or account. Students may use strategies such as filling in tables and making numerical calculations to solve the problems as they reason quantitatively and abstractly about the meaning of negative quantities in the situations.

It is not necessary that students write and solve algebraic equations in this lesson, as they will move toward this strategy in later lessons. Instead, students may notice patterns in the operations and quantities they use when solving the problems.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR8: Discussion Supports (Activity 2, Launch)

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports
- Notice and Wonder
- Which Three Go Together?

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
5 min	15 min	15 min	10 min	5 min

Warm-up

Which Three Go Together: Equations

5 min

Activity Narrative

This *Warm-up* prompts students to compare four equations. It gives students a reason to use language precisely. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three images that go together and can explain why. Next, tell students to share their response with their group and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A. $\frac{1}{2}x = -50$

B. $x + 90 = -100$

C. $-60t = 30$

D. $-0.01 = -0.001x$

Sample responses:

A, B, and C go together because:

- Each equation does not have any decimals.
- Each equation has a variable on the left side of the equal sign.
- The solutions to these equations are all negative.

A, B, and D go together because:

- Each equation has an x-variable.
- The solutions to these equations are all integers.
- The number on the side that does not have a variable is negative.

A, C, and D go together because:

- Each equation has multiplication.
- Each equation does not have addition.

B, C, and D go together because:

- Each equation does not have any fractions.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations, and ensure the reasons given are correct.

Instructional Routines

Which Three Go Together?

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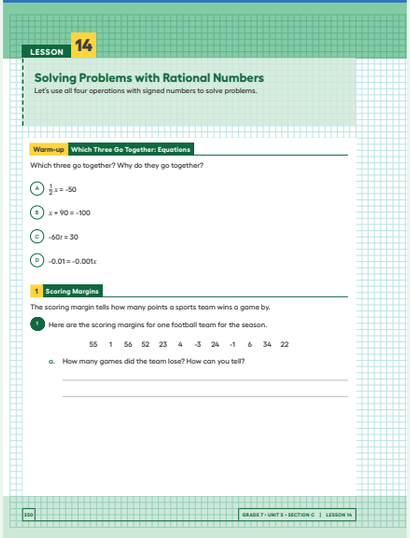
Student Workbook


LESSON 14

Solving Problems with Rational Numbers

Let's use all four operations with signed numbers to solve problems.

Warm-up Which Three Go Together: Equations

Which three go together? Why do they go together?

A. $\frac{1}{2}x = -50$

B. $x + 90 = -100$

C. $-60t = 30$

D. $-0.01 = -0.001x$

1 Scoring Margins

The scoring margin tells how many points a sports team wins a game by.

1. Here are the scoring margins for one football team for the season.

55 1 56 52 23 4 -3 24 -1 6 34 22

a. How many games did the team lose? How can you tell?

Instructional Routines

5 Practices

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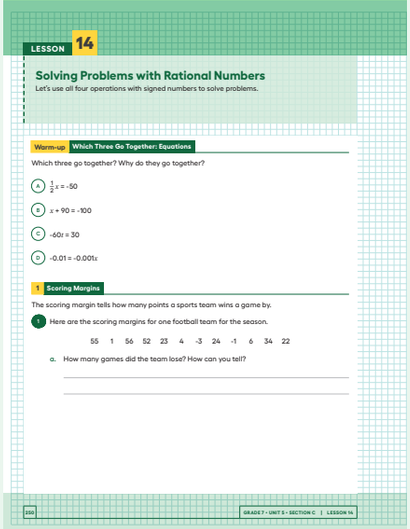
Access for Students with Diverse Abilities (Activity 2, Launch)

Engagement: Provide Access by Recruiting Interest.
Optimize meaning and value. Invite students to share sports teams that they are familiar with. Consider replacing the data in this activity with scoring margins for two rival teams that students are interested in. The first problem should result in a positive average scoring margin for the season, and the second problem should result in a negative average scoring margin.

Supports accessibility for: Conceptual Processing, Social-Emotional Functioning

Student Workbook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1 Scoring Margins

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1. Here are the scoring margins for one football team for the season.

55 1 56 52 23 4 -3 24 -1 6 34 22

a. How many games did the team lose? How can you tell?

During the discussion, prompt students to explain the meaning of any terminology they use, such as "positive," "negative," "addition," and "subtraction," and to clarify their reasoning as needed. Consider asking:

- *"How do you know … ?"*
- *"What do you mean by … ?"*
- *"Can you say that in another way?"*

Activity 1
Scoring Margins

15 min

Activity Narrative

In this activity, students use the context of the scoring margins for a football team to calculate the average of a set of signed numbers. A positive number represents the number of points the team won by, and negative numbers represent the number of points the team lost by. Students reason abstractly and quantitatively about the teams' scores over the season in order to make comparisons about the teams in the situation.

Monitor for groups who use these different strategies to find the sum of the numbers in the second problem. Here are some strategies students may use, from less efficient to more efficient:

- Add the numbers one at a time, in the order listed.
- Add all the positive numbers, then add all the negative numbers, and then add the two partial sums.
- Find combinations of positive and negative numbers that have opposite values, and understand that these will have a sum of 0. For example 3 + 3 is the opposite of -6.

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet work time followed by time for partner discussion. Follow with a whole-class discussion.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

The scoring margin tells how many points a sports team wins a game by.

1. Here are the scoring margins for one football team for the season.

55 1 56 52 23 4 -3 24 -1 6 34 22

a. How many games did the team lose? How can you tell?

2 games

Sample response: The scoring margins are negative.

b. What was their average scoring margin for the season?

22.75, because 273 ÷ 12 = 22.75

2. Here are the scoring margins for another football team for the season.

26 8 -29 3 -4 -32 -14 -14 -6 3 24 -22

What was this team's average scoring margin for the season?

-4.75, because -57 ÷ 12 = -4.75

3. Which team's average scoring margin was higher? How much higher?

The first team's scoring margin was higher by 27.5, because 22.75 – (-4.75) = 27.5.

Activity Synthesis

The purpose of this discussion is for students to share strategies they used when performing arithmetic operations with rational numbers.

Invite previously selected groups to share their strategies for the second problem. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- *"What is the same and what is different about the strategies?"*
- *"Which strategy do you think is the most efficient for adding a large collection of positive and negative numbers?"*
- *"Do any of the strategies help improve the accuracy of the calculation?"*
- *"Are there certain situations where one strategy might be more efficient than another strategy?"*
- *"Did you use the same or a different strategy when finding the sum of the first set of scoring margins? Why?"*

Activity 2

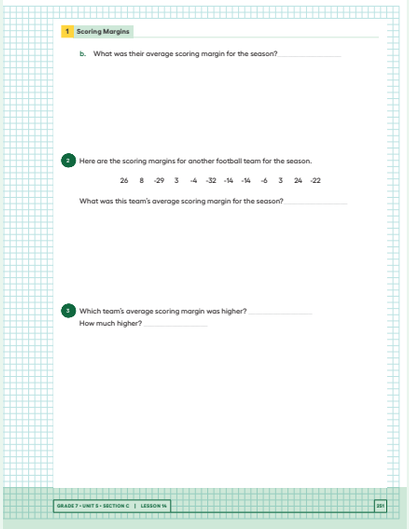
Solar Power

15 min

Activity Narrative

In this activity, students perform operations with rational numbers using the context of energy consumption. Students use what they have learned about signed numbers to interpret what positive and negative energy consumption means and determine the net consumption of electricity by a family that has installed solar panels. They determine the cost to purchase electricity per kWh and the cost to sell it back to the grid. They determine the effect an additional battery has on energy consumption and their net cost for electricity in a month.

Student Workbook


1 Scoring Margins

b. What was their average scoring margin for the season? _____

2 Here are the scoring margins for another football team for the season.

26 8 -29 3 -4 -32 -14 -14 -6 3 24 -22

What was this team's average scoring margin for the season? _____

3 Which team's average scoring margin was higher? _____

How much higher? _____

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Instructional Routines

Notice and Wonder

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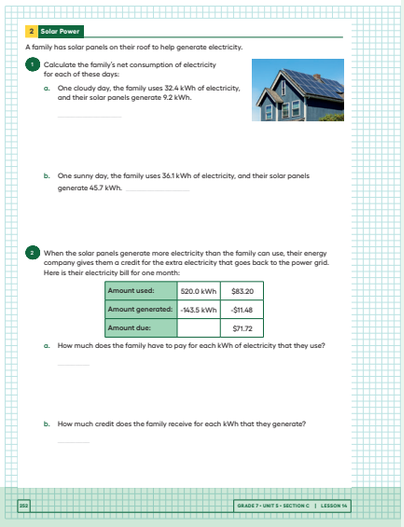
Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Student Workbook



2 Solar Power

A family has solar panels on their roof to help generate electricity.

1. Calculate the family's net consumption of electricity for each of these days:

   a. One cloudy day, the family uses 32.4 kWh of electricity, and their solar panels generate 9.2 kWh.

   b. One sunny day, the family uses 36.1 kWh of electricity, and their solar panels generate 45.7 kWh.

2. When the solar panels generate more electricity than the family can use, their energy company gives them a credit for the extra electricity that goes back to the power grid. Here is their electricity bill for one month:

Amount used:	520.0 kWh	$83.20
Amount generated:	-143.5 kWh	-$11.48
Amount due:		$71.72

   a. How much does the family have to pay for each kWh of electricity that they use?

   b. How much credit does the family receive for each kWh that they generate?

Launch

Tell students to close their books or devices (or to keep them closed). Display the image of the solar panels and the first table from the *Task Statement* for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image or table.

If these ideas do not come up during the conversation, ask students to discuss them:

- The image shows solar panels.
- The table is describing electricity usage and costs.
- What does kWh measure?

Explain that "kWh" stands for "kilowatt-hour," which is a unit of energy. The amount of electricity a customer consumes is typically measured by the kWh and called their "consumption." Solar panels can generate electricity. A customer's "net consumption" is the difference between the electricity used and the electricity generated.

Arrange students in groups of 2. Give students 4 minutes of quiet work time followed by partner discussion. Follow with a whole-class discussion.

Student Task Statement

A family has solar panels on their roof to help generate electricity.

1. Calculate the family's net consumption of electricity for each of these days:

   a. One cloudy day, the family uses 32.4 kWh of electricity, and their solar panels generate 9.2 kWh.

   23.2 kWh

   b. One sunny day, the family uses 36.1 kWh of electricity, and their solar panels generate 45.7 kWh.

   -9.6 kWh

2. When the solar panels generate more electricity than the family can use, their energy company gives them a credit for the extra electricity that goes back to the power grid. Here is their electricity bill for one month:

Amount used:	520.0 kWh	$83.20
Amount generated:	-143.5 kWh	-$11.48
Amount due:		$71.72

a. How much does the family have to pay for each kWh of electricity that they use?

$0.16

b. How much credit does the family receive for each kWh that they generate?

$0.08

3. The family adds a backup battery to their solar panel system. Here is their energy usage for a new month. Calculate their amount due to the energy company this month.

Amount used:	164.5 kWh	$26.32
Amount generated:	-203.0 kWh	-$16.24
Amount due:		$10.08

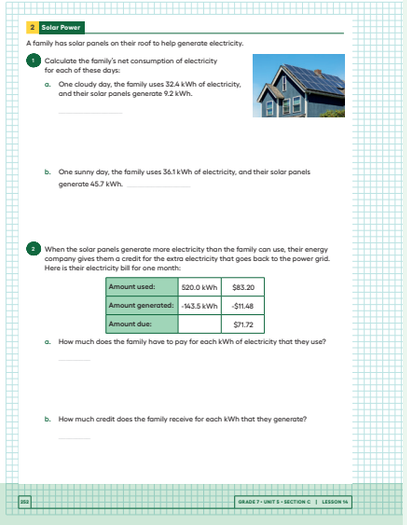
Are You Ready for More?

While most rooms in any building are all at the same level of air pressure, hospitals make use of "positive pressure rooms" and "negative pressure rooms." What do you think it means to have negative pressure in this setting? What could be some uses of these rooms?

Here the pressure of a room is being measured relative to the air pressure outside of the room, by taking the quantity (air pressure inside) – (air pressure outside).

So a positive pressure room is one where there air pressure inside the room is greater than the air pressure outside the room, and the reverse for negative pressure rooms. In positive pressure rooms, air does not naturally flow into the room, so it is a good place to keep patients who have a weakened immune system and are very susceptible to getting infected by airborne diseases. In negative pressure rooms, air does not naturally flow out of the room, so it is a good place to keep patients who are highly contagious.

Student Workbook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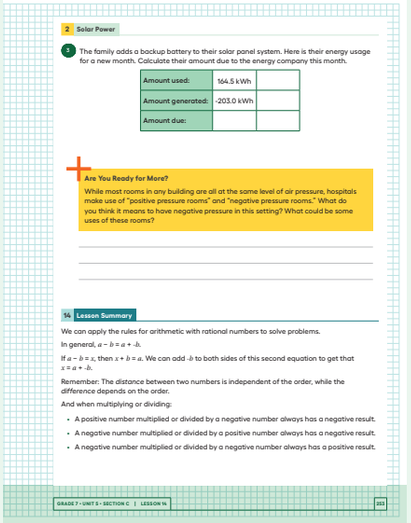
Student Workbook


2 Solar Power

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Amount used:	164.5 kWh	
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Amount due:		

Are You Ready for More?

While most rooms in any building are all at the same level of air pressure, hospitals make use of "positive pressure rooms" and "negative pressure rooms." What do you think it means to have negative pressure in this setting? What could be some uses of these rooms?

14 Lesson Summary

We can apply the rules for arithmetic with rational numbers to solve problems.

In general, $a - b = a + -b$.

If $a - b = x$, then $x + b = a$. We can add $-b$ to both sides of this second equation to get that $x = a + -b$.

Remember: The distance between two numbers is independent of the order, while the *difference* depends on the order.

And when multiplying or dividing:

- A positive number multiplied or divided by a negative number always has a negative result.
- A negative number multiplied or divided by a positive number always has a negative result.
- A negative number multiplied or divided by a negative number always has a positive result.

Access for Students with Diverse Abilities (Activity 2, Synthesis)

Engagement: Provide Access by Recruiting Interest.
Optimize meaning and value. Invite students to share their work on this problem and any related information that they research with a family member or other member of the community.

Supports accessibility for: Conceptual Processing, Organization

Activity Synthesis

The goal of this discussion is for students to share their responses and discuss the implications of electricity consumption in their community. Begin by inviting students to share their responses and reasoning to the last question. Consider discussing the following questions:

- *"How did you determine the cost of the electricity used?"*

  I multiplied the number of kWh used by the cost for one kWh.

- *"How did you determine the cost of the electricity generated?"*

  I multiplied the number of kWh generated by the cost for one kWh.

- *"What does a negative cost for the electricity generated mean in this situation?"*

  A negative cost to the consumer means it is like a credit on their bill.

- *"Compare the electricity used and generated in the two months. What do you notice?"*

  The amount used went down by a lot, while the amount generated went up.

If time allows, consider asking students to research average electricity consumption in the school's city or state. Discuss the main sources of energy in the area and the potential benefits or consequences of alternative sources of energy.

Lesson Synthesis

Share with students,

- *"Today we saw that signed numbers can be used to represent situations where amounts are changing in different ways."*

To review using positive and negative numbers to represent situations, consider asking:

- *"In the activity about the sports teams, what did a positive amount represent?"*

  having a higher score than another team

- *"What did a negative amount represent?"*

  having a lower score than another team

- *"In the activity about the price of electricity, what did a positive amount represent?"*

  money the customer owed the company

- *"What did a negative amount represent?"*

  money the company owed the customer

- *"How are the representations of these amounts similar and different?"*

  In both activities, the positive or negative number represented a difference, so it was comparing two things. In the sports activity, we were comparing how many points different teams were winning by, but in the electricity activity we were comparing how much electricity was being used or generated over the month.

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Cool-down

Charges and Checks

5 min

Student Task Statement

Lin's sister has a checking account. If the account balance ever falls below $0, the bank charges her a fee of $5.95 per day. Today, the balance in Lin's sister's account is -$2.67.

1. If she does not make any deposits or withdrawals, what will be the balance in her account after 2 days?

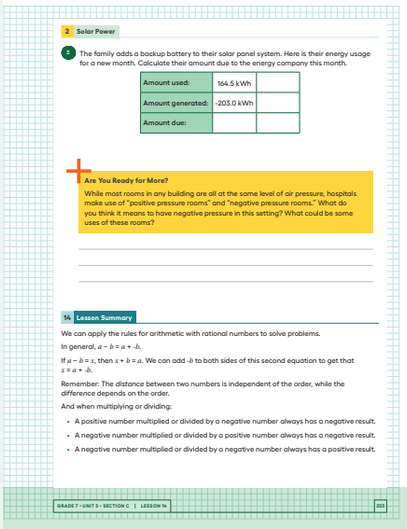
   -$14.57

2. In 14 days, Lin's sister will be paid $430 and will deposit it into her checking account. If there are no other transactions besides this deposit and the daily fee, will Lin continue to be charged $5.95 each day after this deposit is made? Explain or show your reasoning.

   No

   Sample reasoning: Even if the fee was $10 per day, that would total $140, which is much less than what she will deposit.

Student Workbook



2 Solar Power

3 The family adds a backup battery to their solar panel system. Here is their energy usage for a new month. Calculate their amount due to the energy company this month.

Amount used:	164.5 kWh	
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Amount due:		

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Responding To Student Thinking

Points to Emphasize

If most students struggle with solving problems that involve negatives, review this concept as opportunities arise over the next several lessons. For example, invite multiple students to share their thinking about what calculations are needed to complete the table in this activity:

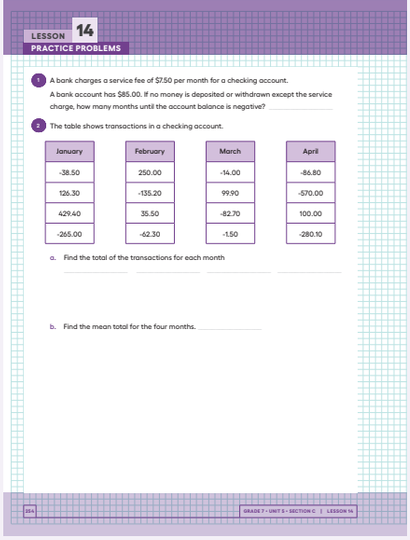
Grade 7, Unit 5, Lesson 17, Activity 1
Gains and Losses

Lesson 14

Practice Problems

5 Problems

Student Workbook



LESSON 14
PRACTICE PROBLEMS

1. A bank charges a service fee of $7.50 per month for a checking account. A bank account has $85.00. If no money is deposited or withdrawn except the service charge, how many months until the account balance is negative?
2. The table shows transactions in a checking account.

January	February	March	April
-38.50	250.00	-14.00	-86.80
126.30	-135.20	99.90	-570.00
429.40	35.50	-82.70	100.00
-265.00	-62.30	-1.50	-280.10

a. Find the total of the transactions for each month

b. Find the mean total for the four months.

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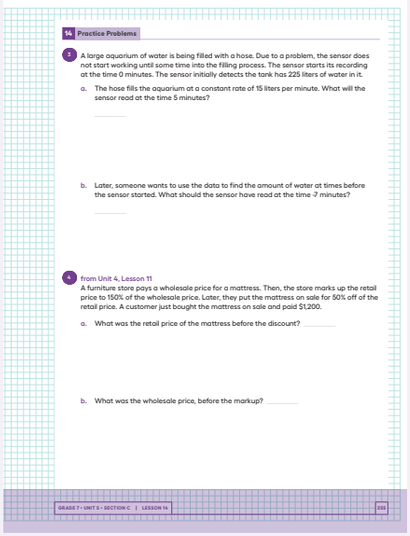
Student Workbook



14 Practice Problems

3. A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time 0 minutes. The sensor initially detects the tank has 225 liters of water in it.
   a. The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?
   b. Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time -7 minutes?
4. from Unit 4, Lesson 11
   A furniture store pays a wholesale price for a mattress. Then, the store marks up the retail price to 150% of the wholesale price. Later, they put the mattress on sale for 50% off of the retail price. A customer just bought the mattress on sale and paid $1,200.
   a. What was the retail price of the mattress before the discount?
   b. What was the wholesale price, before the markup?

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Problem 1

A bank charges a service fee of $7.50 per month for a checking account.

A bank account has $85.00. If no money is deposited or withdrawn except the service charge, how many months until the account balance is negative?

12, because $85 \div 7.50 = 11\frac{1}{3}$, which means it will take 12 months to have a negative balance in the account

Problem 2

The table shows transactions in a checking account.

January	February	March	April
-38.50	250.00	-14.00	-86.80
126.30	-135.20	99.90	-570.00
429.40	35.50	-82.70	100.00
-265.00	-62.30	-1.50	-280.10

a. Find the total of the transactions for each month.

252.20 88 1.70 -836.90

b. Find the mean total for the four months.

-123.75, because $[252.20 + 88 + 1.70 + (-836.9)] \div 4 = -123.75$

Problem 3

A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time 0 minutes. The sensor initially detects the tank has 225 liters of water in it.

a. The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?

300 liters, because $225 + 15 \cdot 5 = 300$

b. Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time -7 minutes?

120 liters, because $225 + 15 \cdot -7 = 120$

Problem 4

from Unit 4, Lesson 11

A furniture store pays a wholesale price for a mattress. Then, the store marks up the retail price to 150% of the wholesale price. Later, they put the mattress on sale for 50% off of the retail price. A customer just bought the mattress on sale and paid $1,200.

a. What was the retail price of the mattress before the discount?

$2,400, because 1,200 ÷ 0.5 = 2,400

b. What was the wholesale price, before the markup?

$1,600, because 2,400 ÷ 1.5 = 1,600

Problem 5

from Unit 4, Lesson 10

a. A restaurant bill is $21. You leave a 15% tip. How much do you pay including the tip?

$24.15

b. Which of the following represents the amount a customer pays including the tip of 15% if the bill was b dollars? Select **all** that apply.

A. $15b$

B. $b + 0.15b$

C. $1.15b$

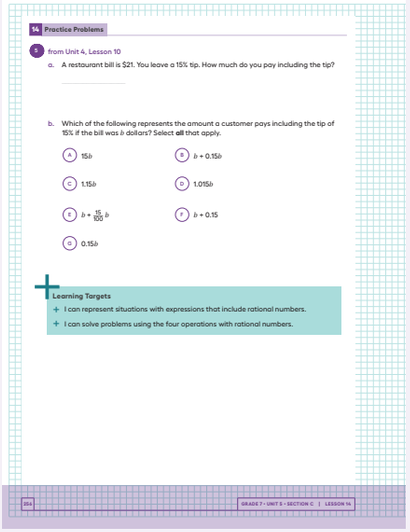
D. $1.015b$

E. $b + \frac{15}{100}b$

F. $b + 0.15$

G. $0.15b$

Student Workbook


14 Practice Problems

5 from Unit 4, Lesson 10

a. A restaurant bill is $21. You leave a 15% tip. How much do you pay including the tip?

b. Which of the following represents the amount a customer pays including the tip of 15% if the bill was b dollars? Select **all** that apply.

A $15b$ B $b + 0.15b$

C $1.15b$ D $1.015b$

E $b + \frac{15}{100}b$ F $b + 0.15$

G $0.15b$

Learning Targets

+ I can represent situations with expressions that include rational numbers.
+ I can solve problems using the four operations with rational numbers.