Lesson 8

Triangles with 3 Common Measures

Goals

- Describe, compare, and contrast (orally and in writing) triangles that share three common measures of angles or sides.
- Justify (orally and using other representations) whether triangles are identical copies or are "different" triangles.
- Recognize that examining which side lengths and angle measures are adjacent can help determine whether triangles are identical copies.

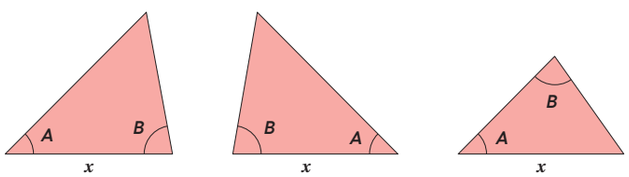
Learning Target

I understand that changing which sides and angles are next to each other can make different triangles.

Lesson Narrative

In this lesson, students examine sets of triangles in which all the triangles share 3 common measures of angles or sides. Students learn to recognize when triangles are "identical copies" that are oriented differently on the page, and when they are different triangles (meaning triangles that are not identical copies). This prepares them for trying to draw more than one triangle given 3 measures, in the next lesson.

For example, suppose a triangle has angles that measure A and B and a side length that measures x. Here are 3 triangles that have these measures:



This example shows 2 "different triangles" (triangles that are not identical copies). The first two triangles are identical copies, so they are the same, but the third is not, so it is different from the other two.

Access for Students with Diverse Abilities

- Representation (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 1)
- MLR3: Critique, Correct, Clarify (Activity 2)

Instructional Routines

- MLR3: Critique, Correct, Clarify

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down
10 min	15 min	10 min	10 min	5 min

Assessment

Lesson 8

Triangles with 3 Common Measures

Lesson Narrative (continued)

Students construct arguments and critique the reasoning of others as they decide whether triangles are identical copies or different triangles. In the example, the first two figures have angles A and B adjacent to side x. However, in the third figure angle B is no longer adjacent to side x. Here students can see that a good way to try to make a different triangle with the same 3 measures is to change which sides and angles are adjacent.

Students do not need to memorize how many different kinds of triangles are possible given different combinations of angles and sides, and they do not need to know criteria such as angle-side-angle for determining if two triangles are identical copies.

Student Learning Goal

Let's contrast triangles.

Warm-up

3 Sides; 3 Angles

10 min

Activity Narrative

The purpose of this *Warm-up* is to begin looking at the different triangles that can be drawn when three measures are specified. The first set of triangles in this activity all share the same 3 side lengths. The second set of triangles all share the same 3 angle measures. Students consider which set of conditions mean that the created triangles must be identical.

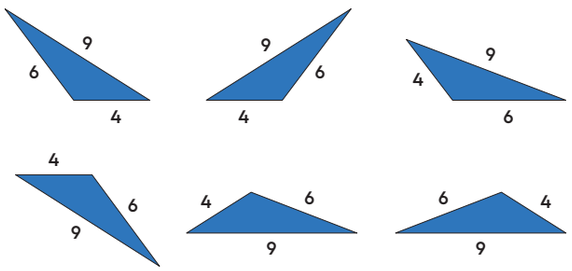
Launch

Provide access to geometry toolkits. Give students 1 minute of quiet think time, followed by a whole-class discussion.

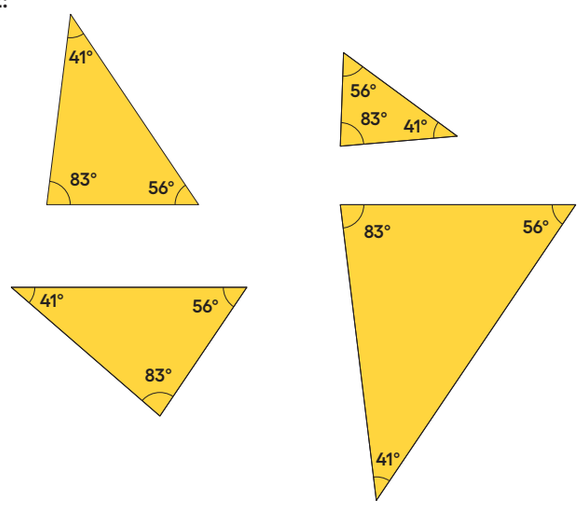
Student Task Statement

Examine each set of triangles. What do you notice? What is the same about the triangles in the set? What is different?

Set 1:



Set 2:



Set 1: All of the side lengths and angles are the same size. These triangles are identical copies. The triangles face different directions.

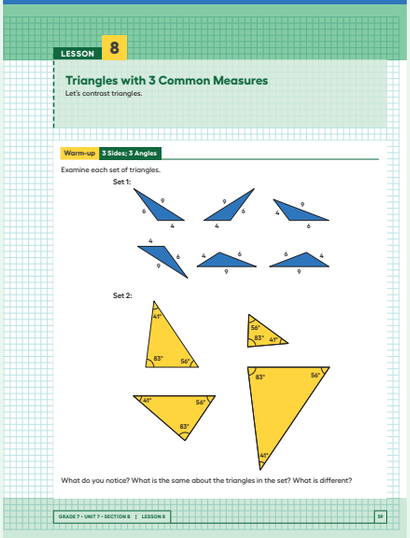
Set 2: These triangles all have the same angles, but different side lengths. They could be scaled copies that are oriented differently.

Building on Student Thinking

Some students may say that all the triangles in the second set are "the same shape." This statement can result from two very different misconceptions. Listen to the students' reasoning and explain as needed:

1. Just because they are all in the same category, "triangles," doesn't mean they are all the same shape. If we can take two shapes and position one exactly on top of the other, so all the sides and corners line up, then they are identical copies.
2. These triangles are scaled copies of each other, but that does not make them "the same" because their side lengths are still different. Only scaled copies made using a scale factor of 1 are identical copies.

Student Workbook


LESSON 8

Triangles with 3 Common Measures

Let's contrast triangles.

Warm-up 3 Sides; 3 Angles

Examine each set of triangles.

Set 1:

Set 2:

What do you notice? What is the same about the triangles in the set? What is different?

Access for Multilingual Learners (Activity 1, Student Task)

MLR2: Collect and Display.
Collect the language that students use to compare triangles and to describe the relative position of sides and angles. Display words and phrases, such as "side length," "angle measure," "between," "next to," and "turned." During the *Activity Synthesis*, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Activity Synthesis

Invite students to share things they notice—things that are the same and things that are different about the triangles. Record and display these ideas for all to see.

If these discussion points do not come up in students' explanations, make them explicit:

In the first set:

- All the triangles are identical copies, just in different orientations.
- They have the same 3 side lengths.
- They have the same 3 angle measures (can be checked with tracing paper or a protractor).

In the second set:

- The triangles are not identical copies.
  - Note: Students may recognize that these triangles are scaled copies of each other, since they have the same angle measures. However, this is the first time students have seen scaled copies in different orientations, and it is not essential to this lesson that students recognize that these triangles are scaled copies.
- They have the same 3 angle measures.
- They have different side lengths (can be checked with tracing paper or a ruler).

The goal is to make sure students understand that the second set has 3 different triangles (because they are different sizes) and that the first set really shows only 1 triangle in many different orientations. Tracing paper may be helpful to convince students of this.

Activity 1

2 Sides and 1 Angle

15 min

Activity Narrative

In this activity, students examine different orientations of triangles that all share 2 sides lengths and one angle measure. They recognize that some of these triangles are identical copies and others are different triangles (not identical copies).

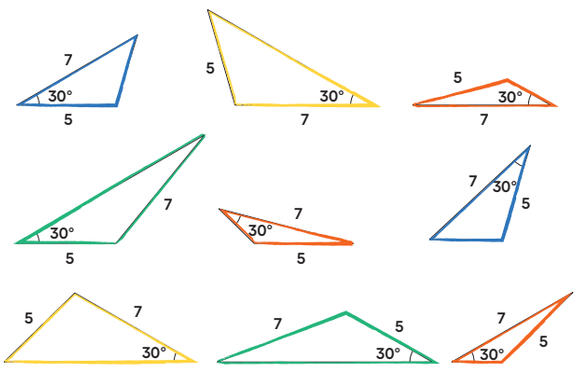
In the coming lessons, students are asked to draw their own triangles. On their own, students often have trouble thinking about triangles where the three given conditions are not necessarily intended to be placed adjacent to one another. For example, when given two sides and an angle, many students will immediately think of putting the given angle between the two sides, but struggle with visualizing putting the angle anywhere else. This task is important for helping students view this as a viable option.

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet work time followed by time to discuss their explanations with a partner. Follow with a whole-class discussion. Provide access to geometry toolkits.

Student Task Statement

Examine this set of triangles.



1. What is the same about the triangles in the set? What is different?

   All of the triangles have one side of length 5 and one side of length 7. And all of them have a 30-degree angle. The triangles are oriented differently, and the two sides and one angle are in a different order.

2. How many different triangles are there? Explain or show your reasoning.

   There are 4 different triangles.

   Sample explanation:

   - The triangles marked in blue have the common measurements in this order: 7 units, 30°, 5 units.
   - The triangles marked in green have the common measurements in this order: 30°, 5 units, 7 units.
   - The triangles marked in both yellow and red have the common measurements in this order: 30°, 7 units, 5 units, but the yellow triangles are larger and the red triangles are smaller.

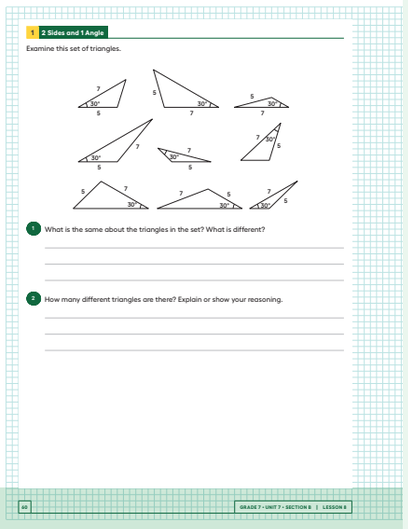
Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with a printed copy of the triangles for them to cut out and rearrange to determine the number of different triangles.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Building on Student Thinking

Some students may say that there are 9 different triangles, because they do not recognize that some of them are identical copies oriented differently. Prompt them to use tracing paper to compare the triangles.

Student Workbook



Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify. This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Activity Synthesis

Select students to share the similarities and differences between the triangles in the set.

Trace a few of the triangles from the set and show how you can turn, flip, or move some of them to line up while others cannot be lined up. Ask students what this means about all the triangles in the set.

they are not all identical to each other

Explain that,

- *"While there are certainly times when the position of a triangle is important ('I wouldn't want my roof upside down!'), for this unit in geometry, we will consider shapes to be the same if they are identical copies."*

To highlight the differences among the triangles, ask students:

- *"Is there only one possible triangle that could be created from the given conditions?"*

  No, there were 4.

- *"How would you explain what is different about these four triangles?"*

  Some have the 30° angle between the two sides of known length and others have the 30° angle next to the side of unknown length.

Explain to students that it seems that the order in which the conditions are included in the triangle (for example, is the angle between the two sides or not?) matters in creating different triangles. Emphasize that the three required pieces (2 sides and 1 angle) do not have to all be put next to one another. When they are asked to draw triangles with three or more conditions, they should consider the way in which the conditions are arranged in their drawing. For example, think about whether the given angle must go between the two sides or not.

Activity 2

2 Angles and 1 Side

10 min

Activity Narrative

This activity builds from previous work with determining unique triangles with certain conditions. In this activity, the conditions given are 2 angles and 1 side.

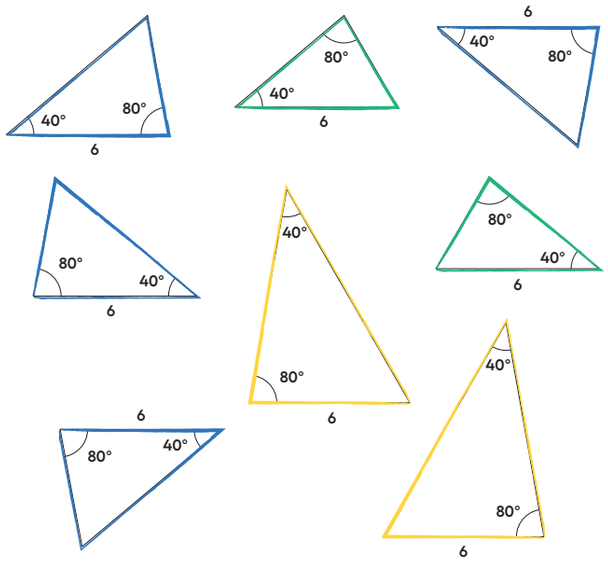
In this activity, students critique a statement or response that is intentionally unclear, incorrect, or incomplete and improve it by clarifying meaning, correcting errors, and adding details.

Launch

Keep students in the same groups. Tell students that this activity is similar to the previous one, and that they should pay close attention to what they find different here. Provide access to geometry toolkits. Give students 2–3 minutes of quiet work time, followed by time to discuss their explanations with a partner. Follow that with a whole-class discussion.

Student Task Statement

Examine this set of triangles.



1. What is the same about the triangles in the set? What is different?

   All of the triangles have one side that is 6 units long and two angles that have the measures 40 degrees and 80 degrees. The triangles are oriented differently, and the one side and two angles are in a different order.

2. How many different triangles are there? Explain or show your reasoning.

   There are 3 different triangles.

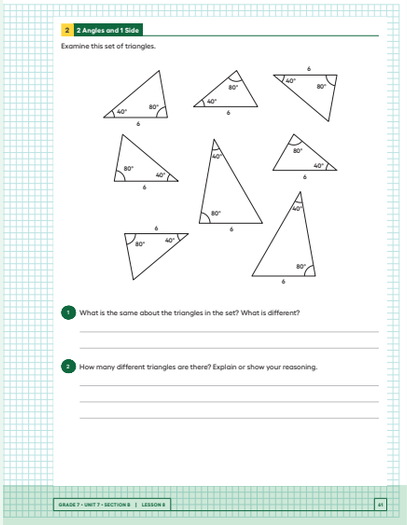
   Sample explanation:

   - The triangles marked in blue have the common measurements in this order: 40°, 6 units, 80°.
   - The triangles marked in yellow have the common measurements in this order: 40°, 80°, 6 units.
   - The triangles marked in green have the common measurements in this order: 80°, 40°, 6 units.

Building on Student Thinking

Some students may say there are only 2 different triangles in this set, because they do not notice the slight size difference between the smaller two groups of triangles. Prompt them to look at where the 80° angle is located in comparison to the 6-unit side.

Student Workbook


2 Angles and 1 Side

Examine this set of triangles.

1. What is the same about the triangles in the set? What is different?

2. How many different triangles are there? Explain or show your reasoning.

Activity Synthesis

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response to "How many different triangles are there? Explain or show your reasoning" by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

  "There are 2 different sets of triangles. One set is the triangles where the side that is 6 units is between the 2 angles. The other set is the triangles where the side that is 6 units is not between the 2 angles."

  Ask,

💬 ***"What parts of this response are unclear, incorrect, or incomplete?"***

  As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

If time permits, consider asking students to use a protractor to measure the unlabeled angle from each of the three different triangles. Discuss what they notice about the third angle.

It's the same size in every triangle.

Explain to students that here we see another example of different triangles that can be made using the same conditions (2 angles and 1 side) in different orders (side between the two angles, side next to the 40-degree angle, and side next to the 80-degree angle). Tell them that in upcoming lessons we will continue to investigate what they noticed here, with the addition of drawing the different triangles.

Lesson Synthesis

Here are some questions to help students articulate how they can construct triangles with given conditions:

- For what we have done today, what does it mean for two triangles to be "different?"

  They are not identical copies.

- If you have a drawing of two triangles, how can you tell if they are identical copies?

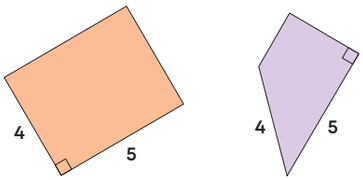
  If I trace one triangle and can move the tracing to perfectly line up with the other, then they are identical copies.

- When trying to draw different triangles with the same set of conditions, what are some things to try?

  Change the order of the conditions in the triangle.

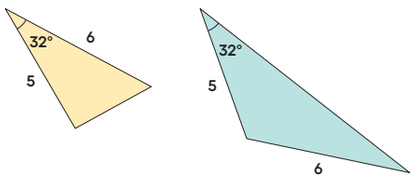
Lesson Summary

Both of these quadrilaterals have a right angle and side lengths 4 and 5:



However, in one case, the right angle is between the two given side lengths, and in the other, it is not.

If we create two triangles with three equal measures, but these measures are not next to each other in the same order, that usually means that the triangles are different. Here is an example:



Cool-down

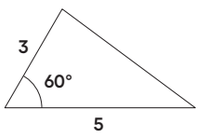
Comparing Andre's and Noah's Triangles

5 min

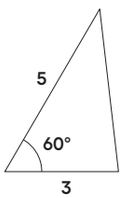
Student Task Statement

Andre and Noah each drew a triangle with side lengths of 5 cm and 3 cm and an angle that measures 60°, and then they showed each other their drawings.

Andre's triangle



Noah's triangle



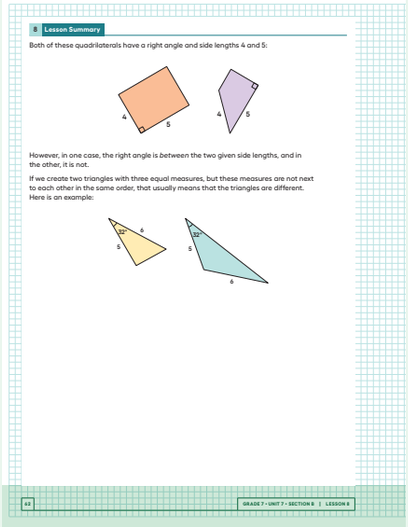
1. Did Andre and Noah draw different triangles? Explain your reasoning.

   These are both the same triangle. In both cases, the 60° angle is between the 3-cm and 5-cm sides. If you trace one triangle, flip it and turn it, it can line up exactly with the other triangle.

2. Explain what Andre and Noah would have to do to draw another triangle that is different from what either of them has already drawn.

   To draw a different triangle, they should try putting the 60° angle next to the side of unknown length, instead of between the two known sides.

Student Workbook


8 Lesson Summary

Both of these quadrilaterals have a right angle and side lengths 4 and 5:

However, in one case, the right angle is between the two given side lengths, and in the other, it is not.

If we create two triangles with three equal measures, but these measures are not next to each other in the same order, that usually means that the triangles are different. Here is an example:

Responding To Student Thinking

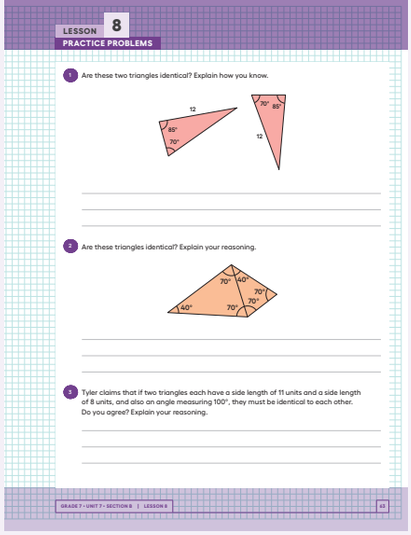
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 8

Practice Problems

7 Problems

Student Workbook


LESSON 8
PRACTICE PROBLEMS

1 Are these two triangles identical? Explain how you know.

12
85°
70°
70°
85°
12

2 Are these triangles identical? Explain your reasoning.

70°
40°
70°
70°
40°
70°

3 Tyler claims that if two triangles each have a side length of 11 units and a side length of 8 units, and also an angle measuring 100°, they must be identical to each other. Do you agree? Explain your reasoning.

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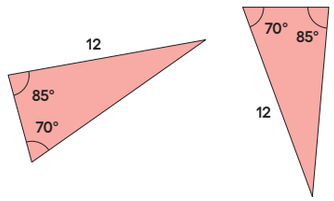
Problem 1

Are these two triangles identical? Explain how you know.

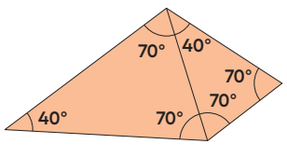


No, these two triangles are not identical.

They have two of the same angle measures and one side length is the same, but the sides and angles are arranged differently in the triangles. In the triangle on the left, the side marked 12 is adjacent to the 85° angle. In the triangle on the right, the side marked 12 is adjacent to the 70° angle.

Problem 2

Are these triangles identical? Explain your reasoning.

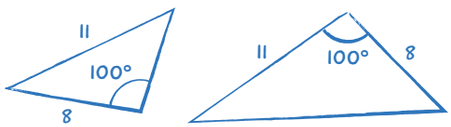


No, they are not identical.

Although they have the same angle measurements, two of the side lengths are different.

Problem 3

Tyler claims that if two triangles each have a side length of 11 units and a side length of 8 units, and also an angle measuring 100°, they must be identical to each other. Do you agree? Explain your reasoning.



No, it is possible to build two different triangles with these measurements.

Problem 4

from Unit 5, Lesson 8

The markings on the number line are equally spaced. Label the other markings on the number line.

Problem 5

from Unit 5, Lesson 9

A passenger on a ship dropped his camera into the ocean. If it is descending at a rate of -4.2 meters per second, how long until it hits the bottom of the ocean, which is at -1,875 meters?

It will take about 446 seconds, which is about 7 and a half minutes.

Problem 6

from Unit 4, Lesson 3

Apples cost $1.99 per pound.

a. How much do $3\frac{1}{4}$ pounds of apples cost?

$6.47 (this number is rounded to the nearest cent)

b. How much do x pounds of apples cost?

1.99x

c. Clare spent $5.17 on apples. How many pounds of apples did Clare buy?

About 2.6 pounds

$1.99x = 5.17$, so $x \approx 2.598$. Most grocery-store scales round to the nearest tenth.

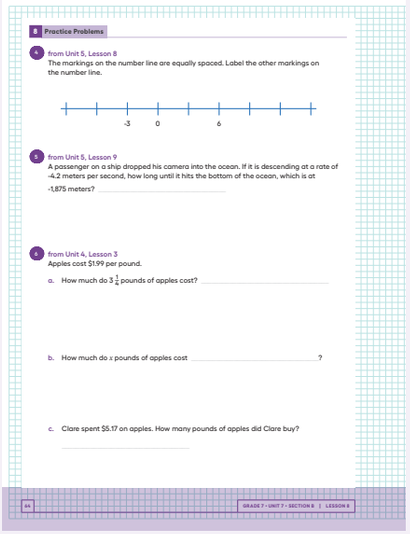
Problem 7

from Unit 3, Lesson 5

Diego has a glue stick with a diameter of 0.7 inches. He sets it down 3.5 inches away from the edge of the table, but it rolls onto the floor. How many rotations did the glue stick make before it fell off of the table?

3.5 ÷ 2.2 times (about 1.6 times)

Student Workbook


8 Practice Problems

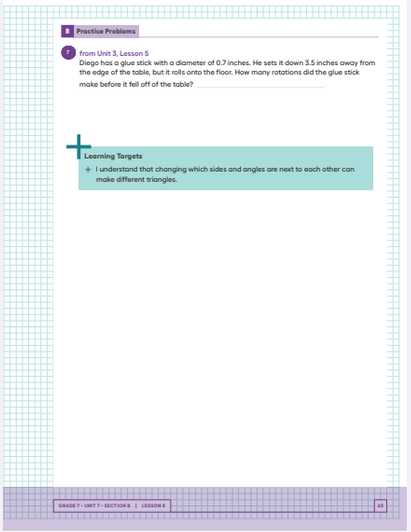
4 from Unit 5, Lesson 8
The markings on the number line are equally spaced. Label the other markings on the number line.

-3 0 6

5 from Unit 5, Lesson 9
A passenger on a ship dropped his camera into the ocean. If it is descending at a rate of -4.2 meters per second, how long until it hits the bottom of the ocean, which is at -1,875 meters? _____

6 from Unit 4, Lesson 3
Apples cost $1.99 per pound.
a. How much do $3\frac{1}{4}$ pounds of apples cost? _____
b. How much do x pounds of apples cost _____?
c. Clare spent $5.17 on apples. How many pounds of apples did Clare buy?
_____

Student Workbook


8 Practice Problems

7 from Unit 3, Lesson 5
Diego has a glue stick with a diameter of 0.7 inches. He sets it down 3.5 inches away from the edge of the table, but it rolls onto the floor. How many rotations did the glue stick make before it fell off of the table? _____

Learning Targets
+ I understand that changing which sides and angles are next to each other can make different triangles.