Lesson 6

Building Polygons (Part 1)

Goals

- Comprehend that two shapes are considered "identical copies" if they can be placed on top of each other and match up exactly.
- Generalize (orally) that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle.
- Use manipulatives to create a polygon with given side lengths, and describe (orally) the resulting shape.

Learning Targets

- I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle.
- I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.

Lesson Narrative

This lesson is the first in a series of lessons in which students create shapes with given conditions. In this lesson, students experiment with making polygons of various numbers and combinations of side lengths, using cardboard strips and metal fasteners.

The goal of the lesson is for students to make sense of the different shapes that are possible under given constraints about side lengths, including whether only one shape is possible or no shape is possible. In this lesson, students do not try to formulate general rules about what side lengths are possible; in the next lesson, they formulate such a rule for triangles.

Access for Students with Diverse Abilities

- Action and Expression (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR7: Compare and Connect (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2, Activity 3
- Metal paper fasteners: Activity 1, Activity 2, Activity 3
- Pre-printed slips, cut from copies of the blackline master: Activity 1, Activity 2, Activity 3

Materials to Copy

- What Can You Build? Cutouts (1 copy for every 2 students): Activity 1

Required Preparation

Lesson:

For the activities in this lesson and the next, you will need slips cut from copies of the "What Can You Build?" blackline master. Prepare 1 copy for every 2 students. These slips can be reused from one class to the next. To make the slips sturdier, it is recommended to copy them onto card stock. If card stock is not available, consider gluing each copy to light cardboard, such as a cereal box. Also if possible, copy each set of slips on a different color of paper, so that a stray strip can quickly be put back.

After the slips are cut, punch holes into the endpoints of each segment. A standard hole punch makes holes that are a little larger than needed for the metal paper fasteners, causing the cardboard strips to wiggle around. If possible, find a way to punch holes that are slightly smaller than the size of a standard hole punch.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	10 min	5 min	10 min	5 min

Lesson 6

Building Polygons (Part 1)

Student Learning Goal

Let's build shapes.

Put each set of strips in an envelope. Prepare to distribute at least 12 metal paper fasteners (i.e., brass brads) to each group.

Note: If using the digital version of every activity, the strips and fasteners will not be needed.

Instructional Routines

Notice and Wonder

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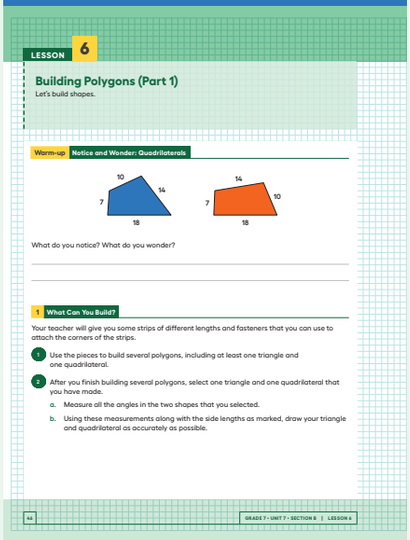
Building on Student Thinking

Some students may try evaluating each side of each equation. Encourage them to look for patterns or shortcuts that would help them answer each question without doing all the calculations.

Student Workbook


LESSON 6

Building Polygons (Part 1)

Let's build shapes.

Warm-up Notice and Wonder: Quadrilaterals

10 14 7 18 14 7 10 18

What do you notice? What do you wonder?

1 What Can You Build?

Your teacher will give you some strips of different lengths and fasteners that you can use to attach the corners of the strips.

1. Use the pieces to build several polygons, including at least one triangle and one quadrilateral.
2. After you finish building several polygons, select one triangle and one quadrilateral that you have made.
   a. Measure all the angles in the two shapes that you selected.
   b. Using these measurements along with the side lengths as marked, draw your triangle and quadrilateral as accurately as possible.

Warm-up

Notice and Wonder: Quadrilaterals

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to orient themselves to images of quadrilaterals with the same side lengths, which will be useful when students build quadrilaterals with fixed side lengths in a later activity. While students may notice and wonder many things about these images, similarities and differences between the figures are the important discussion points.

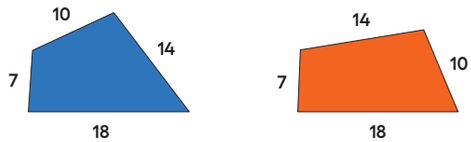
This *Warm-up* prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved.

Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner the things they notice and wonder.

Student Task Statement

What do you notice? What do you wonder?



Students may notice:

- Both of the shapes are quadrilaterals.
- The shapes have side lengths with the same values, but in a different order.
- One shape looks taller and thinner at the top, and the other looks wider and flatter.

Students may wonder:

- Do the shapes have the same perimeter?
- Do the shapes have the same area?
- Are the angle measures different in each shape?
- Are the angle measures the same but in a different order?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses, without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students,

💬 *"Is there anything on this list that you are wondering about now?"*

Encourage students to observe what is on display to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

Activity 1
What Can You Build?

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to explore a physical representation of polygons and make observations about triangles and quadrilaterals. This introductory activity serves to familiarize students with the tools and definitions they will use in future activities. Students may notice that some sets of 3 strips cannot make a triangle, but formalizing rules about what lengths can and cannot be used to form a triangle is *not* the goal of this lesson.

This is the place to notice that pairs of triangles with 3 matching lengths make identical triangles, whereas pairs of quadrilaterals with 4 matching lengths do not necessarily make identical quadrilaterals. Students will express this observation more formally in the next activity.

As students work, select at least one group's triangle and another group's quadrilateral to recreate for the whole-class discussion.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and students who benefit from extra processing time.

Launch

Arrange students in groups of 2. Remind students that a polygon is a closed shape with straight sides. If necessary, demonstrate how to use the fasteners to connect the strips.

Distribute one set of strips and fasteners to each group. Provide access to a geometry toolkit, including rulers and protractors. Give students 5–6 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Your teacher will give you some strips of different lengths and fasteners that you can use to attach the corners of the strips.

1. Use the pieces to build several polygons, including at least one triangle and one quadrilateral.
2. After you finish building several polygons, select one triangle and one quadrilateral that you have made.
   a. Measure all the angles in the two shapes that you selected.
   b. Using these measurements along with the side lengths as marked, draw your triangle and quadrilateral as accurately as possible.

Answers vary.

Access for Students with Diverse Abilities (Activity 1)

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a device that can run the digital applet. *Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization*

Building on Student Thinking

If students try to bend the strips to make shapes with curved sides, consider asking:

"Can polygons have curved sides?"

"Can you use a different length to make a polygon?"

Access for Multilingual Learners (Activity 1, Synthesis)

MLR8: Discussion Supports.
As each group shares the side lengths that they chose for their triangle, invite students to turn to a partner and restate what they heard, using precise mathematical language.
Advances: Listening, Speaking

Instructional Routines

MLR7: Compare and Connect

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Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Activity Synthesis

The purpose of this discussion is to establish what is meant when we say two shapes are identical copies. While students don't use the word *congruent* until grade 8, they should recognize that two shapes are identical only when they can match perfectly on top of each other by movements that don't change lengths or angles.

Invite a group to share the side lengths they chose for their triangle. Create a copy of the triangle with another set of strips and fasteners. Display it for all to see alongside the group's original triangle, but oriented differently. Ask students,

💬 *"Are the two shapes identical? How can you tell?"*

Yes, because you can put them on top of each other and they match up exactly.

Demonstrate turning or flipping the triangle, as needed, to place one copy on top of the other, and show that they match.

Repeat the demonstration with a group's quadrilateral. Create a copy that has the same side lengths as what they used, but different angles. Demonstrate the "floppiness" of the quadrilateral (that is, the angles can change even though the side lengths remain the same). Make sure students realize that the two quadrilaterals are not necessarily identical copies, even though they have the same side lengths.

Activity 2

Building Diego's and Jada's Shapes

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to reinforce that some conditions define a unique polygon while others do not. Students build polygons given only a description of the polygons' side lengths. They articulate that this is not enough information to guarantee that a pair of quadrilaterals are identical copies. On the other hand, triangles have a special property that three specific side lengths result in a unique triangle. Students should notice that their recreation of Jada's triangle is rigid—the side lengths and angles are all fixed.

Monitor for students who try putting the side lengths together in different orders to build different polygons, and invite them to share during the whole-class discussion.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and for students who benefit from extra processing time.

Launch

Arrange students in groups of 2. Ensure that each group has one set of strips and fasteners, as well as access to the geometry toolkit, including rulers and protractors. Encourage students to think about whether there are different shapes that would fulfill the given conditions. Give students 5–6 minutes of group work time followed by a whole-class discussion.

Student Task Statement

1. Diego built a quadrilateral using side lengths of 4 in, 5 in, 6 in, and 9 in.
   a. Build such a shape.
   b. Is your shape an identical copy of Diego's shape? Explain your reasoning.

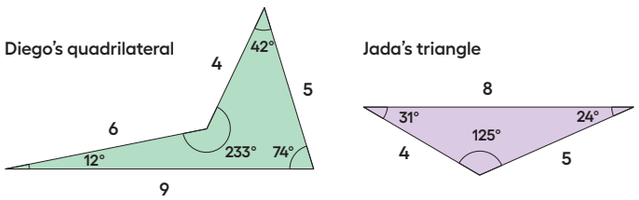
   No, my quadrilateral is probably not an identical copy of Diego's quadrilateral, because it is floppy. There are lots of different angles I can use to make a quadrilateral with these side lengths.
2. Jada built a triangle using side lengths of 4 in, 5 in, and 8 in.
   a. Build such a shape.
   b. Is your shape an identical copy of Jada's shape? Explain your reasoning.

   Yes, my triangle should be an identical copy of Jada's triangle, because it is not floppy. There is no way to change the angles to make a different triangle with these side lengths.

Activity Synthesis

Select previously identified students to share their constructions and explanations. Display each student's example for all to see.

If desired, reveal Diego and Jada's shapes, and display for all to see alongside students' work.



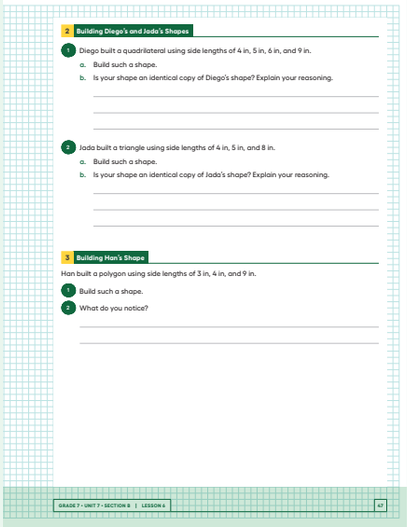
Ask students:

- *"Is this what you thought Jada and Diego's shapes looked like?"*
  "Which shape did you make an identical copy of?"
  Jada's triangle.
- *"Why did you not make an identical copy of Diego's shape?"*
  Because you can make quadrilaterals with the same side lengths but different angle measure.

Building on Student Thinking

Students may think that their triangle is different from Jada's because hers is "upside down." Ask the student to turn their triangle around and ask them if it is now a different triangle. While there is a good debate to be had if they continue to insist they are different, let the students know that, for this unit, we will consider shapes that have been turned or flipped or moved to be identical copies and thus "not different."

Student Workbook


2 Building Diego's and Jada's Shapes

1 Diego built a quadrilateral using side lengths of 4 in, 5 in, 6 in, and 9 in.
a. Build such a shape.
b. Is your shape an identical copy of Diego's shape? Explain your reasoning.

2 Jada built a triangle using side lengths of 4 in, 5 in, and 8 in.
a. Build such a shape.
b. Is your shape an identical copy of Jada's shape? Explain your reasoning.

3 Building Han's Shape

Han built a polygon using side lengths of 3 in, 4 in, and 9 in.
1 Build such a shape.
2 What do you notice?

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Access for Multilingual Learners (Activity 2, Synthesis)

MLR7: Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different representations. Display a student's quadrilateral that is different from Diego's and ask,

"How are these quadrilaterals the same? How are they different?"
"How does the order of the sides impact the quadrilateral?"
Advances: Representing, Conversing

Access for Students with Diverse Abilities (Activity 3, Launch)

Action and Expression: Provide Access for Physical Action.
Provide access to tools and assistive technologies such as a device that can run the digital applet.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Organization

Building on Student Thinking

Students may say that there is no way Han could have built this shape, because they are assuming it must be a triangle. Ask students if the question specifies that the shape is a triangle. If needed, remind students of the definition of *polygon* and prompt them to consider what they could do to finish building a closed shape with all straight sides.

Activity 3: Optional
Building Han's Shape

5 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is for students to see that sometimes it is impossible to build a polygon with certain conditions, but also that they need to think carefully about the information they are given before making assumptions.

In this case, students are given 3 side lengths that cannot form a triangle and are told to build a polygon. Students may assume that because they were given 3 side lengths, their shape is supposed to be a triangle., They will, however, succeed in building a polygon using the specified side lengths only if it has more than 3 sides. Formalizing rules about what lengths can and cannot be used to form a triangle is *not* the goal of this activity.

As students work on the task, monitor for students who realize that the shape cannot be a triangle and for students who realize it can be a polygon with more than 3 sides.

In the digital version of the activity, students use an applet to create polygons dynamically. The digital version may reduce barriers for students who need support with fine-motor skills and for students who benefit from extra processing time.

Launch

Arrange students in groups of 2. Ensure that each group has one set of strips and fasteners, as well as access to the geometry toolkit, including rulers and protractors. Give students 3–4 minutes of group work time, followed by a whole-class discussion.

Student Task Statement

Han built a polygon using side lengths of 3 in, 4 in, and 9 in.

1. Build such a shape.
2. What do you notice?

Sample response: I notice that this shape cannot be a triangle. I had to include a fourth side to make a quadrilateral.

Activity Synthesis

Select previously identified students to share their explanations. Make sure that students realize that this shape cannot be built as a triangle, but it is possible to build a polygon with more than 3 sides. Tell students

"As the unit progresses, you will be asked to create or draw shapes that include some conditions, but there may be some flexibility with the pieces that are not mentioned. Be aware of what must be included and what is not mentioned."

If time permits, consider asking some of the following questions:

- *"What length did you choose to use for your fourth side? Would another choice have worked?"*
- *"If another group used the same length for their fourth side as you did, does their polygon have to be an identical copy of yours? How do you know?"*
- *"Did any group choose to have more than 4 sides on their polygon? Does such a shape fulfill the given conditions?"*

Lesson Synthesis

The purpose of this discussion is for students to consider how given side lengths affect the conditions of polygons they create. Here are some questions to elicit student thinking:

- What kinds of shapes could you build with side lengths 4 inches, 4 inches, and 4 inches?

  triangle, square, another quadrilateral, pentagon, etc.

- What kinds of shapes could you **not** build with this set of side lengths and fasteners?

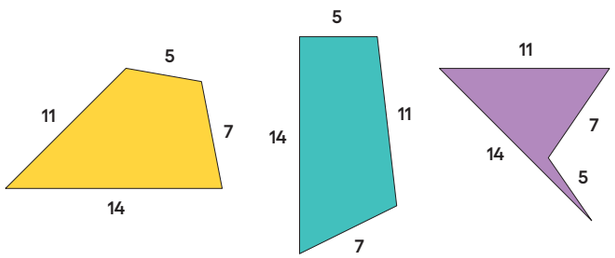
  circle, oval, a 9 inch square, etc.

- How is building a triangle with three given side lengths different from building a quadrilateral with four given side lengths?

  The triangle must be a specific one, but the quadrilateral might be a lot of different things by changing the angles.

Lesson Summary

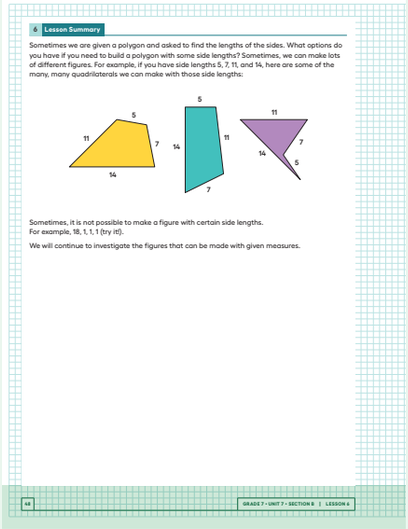
Sometimes we are given a polygon and asked to find the lengths of the sides. What options do you have if you need to build a polygon with some side lengths? Sometimes, we can make lots of different figures. For example, if you have side lengths 5, 7, 11, and 14, here are some of the many, many quadrilaterals we can make with those side lengths:



Sometimes, it is not possible to make a figure with certain side lengths. For example, 18, 1, 1, 1 (try it!).

We will continue to investigate the figures that can be made with given measures.

Student Workbook


6 Lesson Summary

Sometimes we are given a polygon and asked to find the lengths of the sides. What options do you have if you need to build a polygon with some side lengths? Sometimes, we can make lots of different figures. For example, if you have side lengths 5, 7, 11, and 14, here are some of the many, many quadrilaterals we can make with those side lengths:

Sometimes, it is not possible to make a figure with certain side lengths. For example, 18, 1, 1, 1 (try it!).

We will continue to investigate the figures that can be made with given measures.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

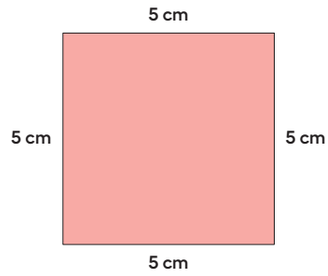
Cool-down

An Equilateral Quadrilateral

5 min

Student Task Statement

When asked to draw a quadrilateral with all four sides measuring 5 cm, Jada drew a square.



1. Does Jada's shape meet the requirements?

   Yes, Jada's shape has 4 sides, all measuring 5 cm.

2. Is there a different shape that would also meet the requirements? Explain your reasoning.

   A rhombus could be made with all four sides the same length, but without right angles.

Lesson 6

Practice Problems

6 Problems

Problem 1

A rectangle has side lengths of 6 units and 3 units. Can you make a different quadrilateral that also has two side lengths of 3 and two side lengths of 6? If so, describe it.

Yes, you could make a parallelogram or a kite using the side lengths 3, 3, 6, and 6.

Problem 2

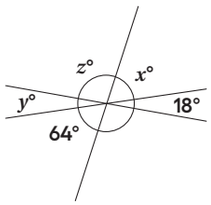
Come up with an example of three side lengths that cannot possibly make a triangle, and explain how you know.

Sample response: The lengths 1 foot, 1 inch, and 1 inch cannot possibly make a triangle, because if you attach the 1-inch lengths to either end of the 1-foot length, the 1-inch lengths are too short to connect at their other ends.

Problem 3

from Unit 7, Lesson 3

Find x, y, and z.



- $x = 64$
- $y = 18$
- $z = 98$

Problem 4

from Unit 7, Lesson 1

How many right angles need to be put together to make:

a. 360 degrees?

4

b. 180 degrees?

2

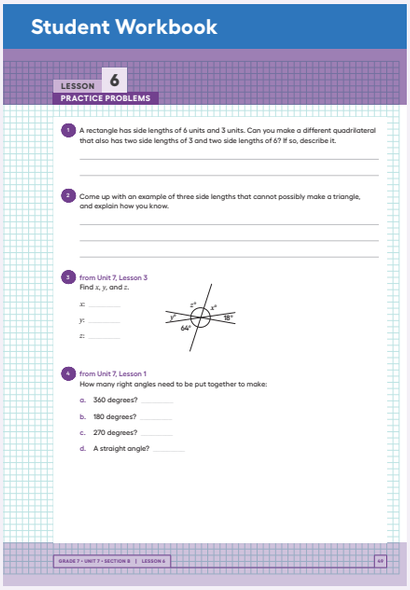
c. 270 degrees?

3

d. A straight angle?

2

Student Workbook


LESSON 6
PRACTICE PROBLEMS

1. A rectangle has side lengths of 6 units and 3 units. Can you make a different quadrilateral that also has two side lengths of 3 and two side lengths of 6? If so, describe it.

2. Come up with an example of three side lengths that cannot possibly make a triangle, and explain how you know.

3. from Unit 7, Lesson 3
Find x, y, and z.
x: _____
y: _____
z: _____

4. from Unit 7, Lesson 1
How many right angles need to be put together to make:
a. 360 degrees? _____
b. 180 degrees? _____
c. 270 degrees? _____
d. A straight angle? _____

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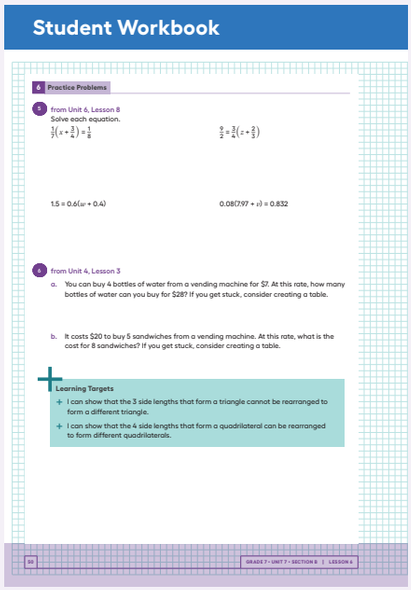
Student Workbook

6 Practice Problems

5 from Unit 6, Lesson 8
Solve each equation.
$\frac{1}{7}\left(x+\frac{3}{4}\right)=\frac{1}{8}$
$\frac{9}{2}=\frac{3}{4}\left(z+\frac{2}{3}\right)$
$1.5 = 0.6(w + 0.4)$
$0.08(7.97 + v) = 0.832$

6 from Unit 4, Lesson 3
a. You can buy 4 bottles of water from a vending machine for $7. At this rate, how many bottles of water can you buy for $28? If you get stuck, consider creating a table.
b. It costs $20 to buy 5 sandwiches from a vending machine. At this rate, what is the cost for 8 sandwiches? If you get stuck, consider creating a table.

Learning Targets
+ I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle.
+ I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.

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Problem 5

from Unit 6, Lesson 8

Solve each equation.

$\frac{1}{7}\left(x+\frac{3}{4}\right)=\frac{1}{8}$

$\frac{1}{8}$

$\frac{9}{2}=\frac{3}{4}\left(z+\frac{2}{3}\right)$

$\frac{16}{3}$

$1.5 = 0.6(w + 0.4)$

2.1

$0.08(7.97 + v) = 0.832$

2.43

Problem 6

from Unit 4, Lesson 3

a. You can buy 4 bottles of water from a vending machine for $7. At this rate, how many bottles of water can you buy for $28? If you get stuck, consider creating a table.

16

b. It costs $20 to buy 5 sandwiches from a vending machine. At this rate, what is the cost for 8 sandwiches? If you get stuck, consider creating a table.

$32