Lesson 9

Describing Large and Small Numbers Using Powers of 10

Goals

- Describe (orally and in writing) large and small numbers as multiples of powers of 10.
- Interpret a diagram for base-ten units, and explain (orally) how the small squares, long rectangles, and large squares relate to each other.

Learning Target

Given a very large or very small number, I can write an expression equal to it using a power of 10.

Instructional Routines

- MLR2: Collect and Display

Access for Multilingual Learners:

- MLR2: Collect and Display (Activity 2)
- MLR7: Compare and Connect (Activity 1)

Access for Students with Diverse Abilities:

- Engagement (Activity 1)
- Action and Expression (Activity 2)

Required Materials

Materials to Copy

- Using Powers of 10 to Describe Cards (1 copy for every 6 students): Activity 2

Lesson Narrative

In this lesson, students build on work done in previous grades with numbers in base ten. They use base-ten diagrams to represent different powers of 10 and review how multiplying and dividing by 10 affect the decimal representation of numbers. They use their understanding of base-ten structure as they express very large and very small numbers using exponents.

Students also practice communicating— by describing and writing—very large and small numbers, which requires attending to precision. This leads to a discussion about how powers of 10 can be used to more easily communicate such numbers. Students do not need to read or write numbers in formal scientific notation, as that will be introduced in a following lesson. The focus here is on introducing the idea that numbers can be written as a multiple of a power of 10 and to practice doing that flexibly.

Student Learning Goal

Let's find out how to use powers of 10 to write large or small numbers.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Cool-down (Assessment)
5 min	15 min	15 min	10 min	5 min

Warm-up
Thousand Million Billion Trillion

5 min

Activity Narrative

In this *Warm-up*, students connect thousand, million, billion, and trillion to their respective powers of ten—10^3, 10^6, 10^9, and 10^{12}. Understanding powers of 10 associated with these denominations will help students reason about quantities in real-world contexts, such as the number of cells in a human body (trillions) or the world population (billions).

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time and 1 minute to compare their responses with their partner. Given the limited time, it may not be possible for students to create examples for each of the values in the second question. Tell students to try to at least find 1 or 2 examples and then to find others as time allows. Follow with a whole-class discussion.

Student Task Statement

1. Match each expression with its corresponding value and word.

expression	value	word
10^{-3}	1,000,000,000,000	billion
10^6	$\frac{1}{100}$	milli-
10^9	1,000	million
10^{-2}	1,000,000,000	thousand
10^{12}	1,000,000	centi-
10^3	$\frac{1}{1{,}000}$	trillion

2. For each of the numbers, think of something in the world that is described by that number.

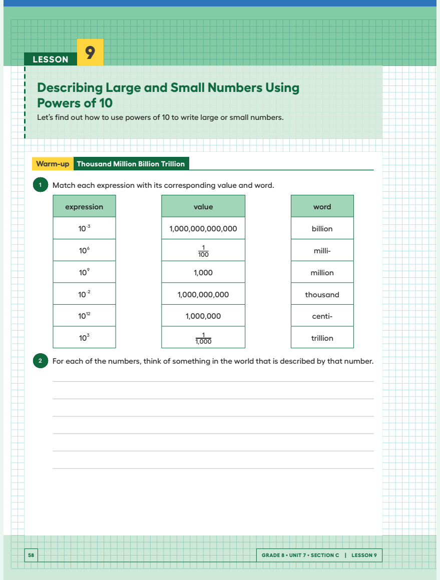
Answers vary.

Building on Student Thinking

If students confuse the prefix "milli-" with the word "million," consider explaining that:

- The word "million" literally means "a big thousand," and so both "million" and "mille" are related to the Latin "mille," meaning "thousand."
- While "milli-" is talking about a thousand parts (thousandths), "million" is talking about a thousand thousands.

Student Workbook



Activity Synthesis

The goal of this discussion is for students to get a sense of the comparative sizes of very large and very small numbers by connecting them to a concrete example. Begin by displaying the completed table for all to see and address any questions or disagreements. Then invite students to share their examples for the final question. After each student shares, ask the class whether they agree that the given example could be described by that value.

If necessary, consider sharing some of the following examples:

- Thousandth (10^{-3})
    - A milliliter is $\frac{1}{1,000}$ of a liter.
- Hundredth (10^{-2})
    - A centimeter is $\frac{1}{100}$ of a meter.
    - A penny is $\frac{1}{100}$ of a dollar.
    - A yard is $\frac{1}{100}$ of the distance between opposing end zones on an American football field.
- Thousand (10^{3})
    - Population of an endangered species
    - Gallons of fuel it would take to fill 50 cars
- Million (10^{6})
    - Population of the state of Delaware in 2022
    - Acres covered by the state of Rhode Island
    - Number of seconds in 12 days
- Billion (10^{9})
    - Population of India or China (each ~1.4 billion)
    - Number of seconds in 31 years
- Trillion (10^{12})
    - Number of red blood cells in one-half pint of blood
    - Number of stars in a giant galaxy
    - Number of seconds in 31,688 years

Activity 1
Base-Ten Representations Matching

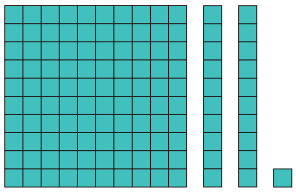
15 min

Activity Narrative

In this activity, students use their understanding of decimal place value and base-ten diagrams to practice working with the structure of scientific notation before it is formally introduced. They express numbers as sums of terms, with each term being a multiple of a power of 10. For example, 254 can be written as $2 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0$.

Launch

Display the diagram for all to see.



Ask students to explain their reasoning after each of the following questions:

- *"If each small square represents 1 tree, what does the whole diagram represent?"*

  121 trees

- *"If each small square represents 10 books, what does the whole diagram represent?"*

  1,210 books

- *"If each small square represents 1,000,000 stars, what does the whole diagram represent?"*

  121,000,000 stars

- *"If each small square represents 0.1 seconds, what does the whole diagram represent?"*

  12.1 seconds

- *"If each small square represents 10^3 people, what does the whole diagram represent?"*

  121,000 people

Give students 6 minutes of quiet work time followed by a whole-class discussion.

Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Provide Access by Recruiting Interest.
Provide choice. Invite students to decide which expression or diagram to start with when trying to find a match.

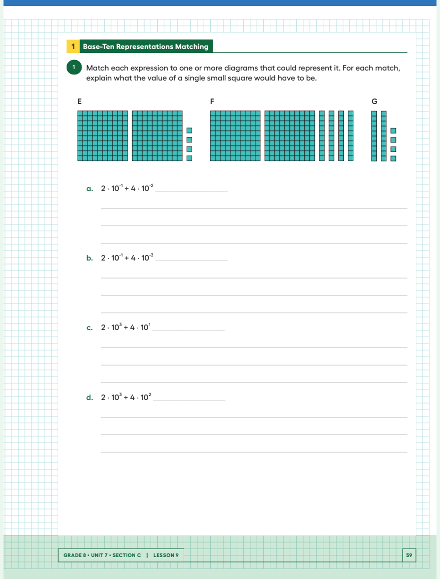
Supports accessibility for: Organization, Attention

Building on Student Thinking

If students struggle with writing the value of expressions involving negative powers of 10, consider asking:

"How would you expand this expression into factors using 10 or $\frac{1}{10}$?"
"What power of 10 would be 10 times larger than 10^3? Than 10^{-3}?"

Student Workbook


1 Base-Ten Representations Matching

1 Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.

a. $2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

b. $2 \cdot 10^{-1} + 4 \cdot 10^{-3}$

c. $2 \cdot 10^{3} + 4 \cdot 10^{1}$

d. $2 \cdot 10^{3} + 4 \cdot 10^{2}$

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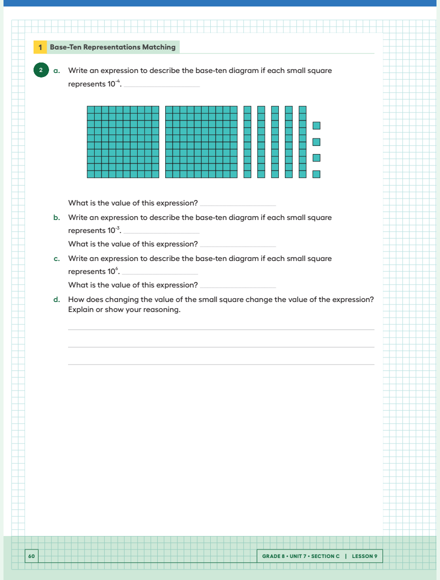
Student Workbook


1 Base-Ten Representations Matching

2 a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4}.

What is the value of this expression?

b. Write an expression to describe the base-ten diagram if each small square represents 10^{-3}.

What is the value of this expression?

c. Write an expression to describe the base-ten diagram if each small square represents 10^{6}.

What is the value of this expression?

d. How does changing the value of the small square change the value of the expression? Explain or show your reasoning.

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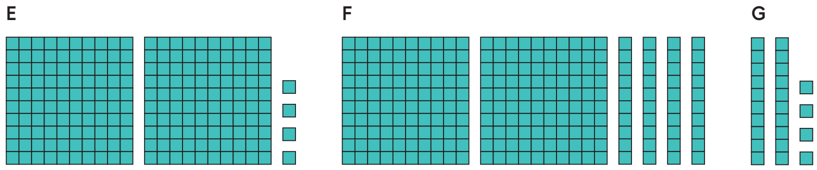
Student Task Statement

1. Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.



a. $2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

G if a small square is 10^{-2} or F if a small square is 10^{-3}

b. $2 \cdot 10^{-1} + 4 \cdot 10^{-3}$

E if a small square is 10^{-3}

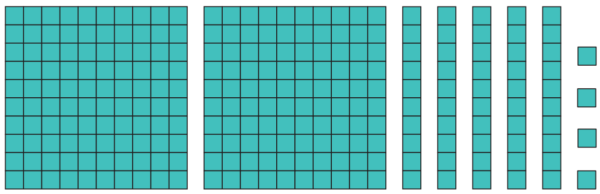
c. $2 \cdot 10^{3} + 4 \cdot 10^{1}$

E if a small square is 10^{1}

d. $2 \cdot 10^{3} + 4 \cdot 10^{2}$

G if a small square is 10^{2} or F if a small square is 10^{1}

2. a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4}.



What is the value of this expression?

$4 \cdot 10^{-4} + 5 \cdot 10^{-3} + 2 \cdot 10^{-2}$ which is 0.0254.

b. Write an expression to describe the base-ten diagram if each small square represents 10^{-3}. What is the value of this expression?

$4 \cdot 10^{-3} + 5 \cdot 10^{-2} + 2 \cdot 10^{-1}$ which is 0.254.

c. Write an expression to describe the base-ten diagram if each small square represents 10^{6}. What is the value of this expression?

$4 \cdot 10^{6} + 5 \cdot 10^{7} + 2 \cdot 10^{8}$ which is 254,000,000.

d. How does changing the value of the small square change the value of the expression? Explain or show your reasoning.

Sample response: Changing the value of the small square changes the power of 10. The long rectangle always has an exponent that is 1 higher than for the small square, and the large square always has an exponent that is 1 higher than for the long rectangle, or 2 higher than for the small square.

Activity Synthesis

The goal of this discussion is to make sure students see the connection between decimal place value and sums of terms that are multiples of powers of 10.

Begin by inviting students to share their responses to the first matching problem. If not brought up in students' responses, bring attention to the fact that for expressions a and d, both diagrams F and G could be used depending on the choice of the value of the small square. Discuss the following questions:

- *"How are the diagrams related to our base-ten numbers and place value system?"*

  In base-ten numbers, each place value is ten times larger than the one to its right; so every 1 unit of a place value can be composed of 10 units of the next place value to its right. The diagrams work the same way: Each shape representing a base-ten unit can be composed of 10 that represent another unit that is one tenth of its value.

- *"How are the diagrams related to numbers written using powers of 10?"*

  We can think of each place value as a power of ten. So a ten would be 10^1, a hundred would be 10^2, a tenth would be 10^{-1}, and so on.

- *"If each large square represents 10^2, what do 2 large squares and 4 long rectangles represent?"*

  A long rectangle is a tenth of the large square, so we know the long rectangle represents 10^1. This means that 2 large squares and 4 long rectangles represent $2 \cdot 10^2 + 4 \cdot 10^1$.

- *"Why is it possible for one base-ten diagram to represent many different numbers?"*

  Because the structure of our place value system—where every group of 10. of a base-ten unit composes 1 of the next higher unit—is consistent across all place values.

Activity 2

Using Powers of 10 to Describe Large and Small Numbers

15 min

Activity Narrative

This activity motivates students to find easier ways to communicate about very large and very small numbers by using powers of 10, in preparation for working with scientific notation. In describing quantities that have many digits, students must use language precisely.

Access for Multilingual Learners (Activity 1, Synthesis)

MLR7: Compare and Connect.
Lead a discussion comparing, contrasting, and connecting the different representations. Ask,

> *"How are the expression in part a and the expression in part b the same? How are they different?"*

and

> *"How are diagrams F and G the same? How are they different?".*

Advances: Representing, Conversing

Instructional Routines

MLR2: Collect and Display

ilclass.com/r/10690754
Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display.
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 2, Launch)

Action and Expression: Internalize Executive Functions.
Begin with a whole-class "think aloud" to demonstrate how students should work together.

Supports accessibility for: Memory; Conceptual Processing

Student Workbook


2 Using Powers of 10 to Describe Large and Small Numbers

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about ______ pencils are made each year.
2. The mass of a proton is ______ kilograms.
3. The population of Russia is about ______ people.
4. The diameter of a bacteria cell is about ______ meter.

Partner B's statements:

1. Light waves travel through space at a speed of ______ meters per second.
2. The population of India is about ______ people.
3. The wavelength of a gamma ray is ______ meters.
4. The tardigrade (water bear) is ______ meters long.

Are You Ready for More?

A "googol" is a name for a really big number: a 1 followed by 100 zeros.

1. If you square a googol, how many zeros will the answer have? ______ Show your reasoning.
2. If you raise a googol to the googol power, how many zeros will the answer have? ______ Show your reasoning.

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Launch

Arrange students in groups of 2. Distribute a pair of cards (one for Partner A and one for Partner B) from the blackline master to each group. Ask partners not to show their card to each other.

Tell students that one partner should read an incomplete statement in the materials and the other partner should read aloud the missing information on the card. The goal is for each partner to write down the missing quantity correctly. Partners should take turns reading and writing until all four statements for each person are completed.

Consider explaining (either up front or as needed during work time) that students who have the numbers can describe or name them in any way that they think conveys the quantities fully. Likewise, those writing the numbers can write in any way that captures the quantities accurately.

Give students 10 minutes to work. Leave a few minutes for a whole-class discussion.

Student Task Statement

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about _____ pencils are made each year.
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4. The diameter of a bacteria cell is about _____ meter.

Partner B's statements:

1. Light waves travel through space at a speed of _____ meters per second.
2. The population of India is about _____ people.
3. The wavelength of a gamma ray is _____ meters.
4. The tardigrade (water bear) is _____ meters long.

The blackline master shows the completed statements for each partner.

Are You Ready for More?

A "googol" is a name for a really big number: a 1 followed by 100 zeros.

Writing a googol as 10^{100} makes it easier to solve this problem.

1. If you square a googol, how many zeros will the answer have? Show your reasoning.

   200 zeros, because $(10^{100})^2 = 10^{200}$

2. If you raise a googol to the googol power, how many zeros will the answer have? Show your reasoning.

   10^{102} zeros, because $(10^{100})^{10^{100}} = 10^{100 \cdot 10^{100}} = 10^{10^2 \cdot 10^{100}} = 10^{10^{102}}$

Activity Synthesis

The purpose of the discussion is for students to hear different strategies for communicating very small and very large numbers. Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they described the very large numbers to their partner, or how the very large numbers were described to them. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional equivalent phrases. For example, 150,000,000,000 could be described as "one hundred fifty billion," as "15 followed by 10 zeros," as $150 \cdot 10^9$, or as $(1.5) \cdot 10^{11}$.

Display the first 2 rows of the table for all to see. If necessary, explain how 150,000,000,000 and $150 \cdot 10^9$ are equivalent. If time allows, ask students to come up with additional equivalent expressions for 150,000,000,000 that use powers of 10 ($15 \cdot 10^{10}$, $150,000 \cdot 10^6$).

quantities	using powers of 10
150,000,000,000 meters	$150 \cdot 10^9$ meters
300,000,000 meters per second	$300 \cdot 10^6$ meters per second
0.0000000000048 meters	$48 \cdot 10^{-13}$ meters
0.00000000000000000000000000167 kilogram	$167 \cdot 10^{-29}$ kilogram

Next, ask students to share how they described very small numbers to their partner, or how they were described to them. Display the remaining rows of the table for all to see. Tell students to consider the third row, and discuss the idea that multiplying by 10^{-1} means multiplying by $\frac{1}{10}$, and results in a number that is one place value smaller. That means that 0.0000000000048 is 48 multiplied by 0.1 or $\frac{1}{10}$ 13 times, or $48 \cdot (0.0000000000001)$. Another way to write this is $48 \cdot 10^{-13}$. If time allows, ask students to come up with additional equivalent expressions that use powers of 10 ($4.8 \cdot 10^{-12}$, $0.48 \cdot 10^{-11}$).

Lesson Synthesis

The purpose of this discussion is for students to see the structure of our place value system as a useful tool for describing large and small numbers in different ways. Here are some questions for discussion:

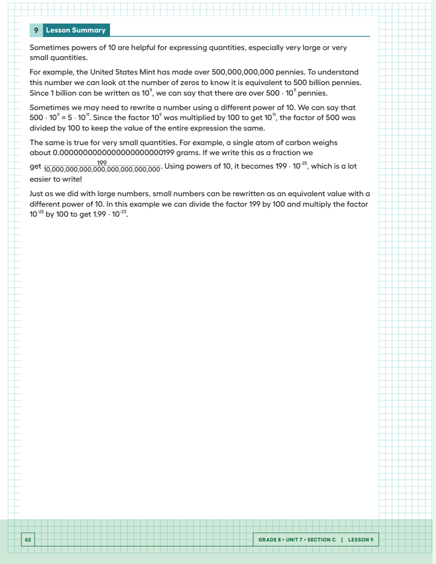
- *"How do base-ten diagrams help us make sense of (or explain) the exponents in powers of 10?"*

  When using diagrams, grouping 10 of the next smaller unit means multiplying by 10. When dealing with powers of 10, multiplying by 10 increases the exponent by 1. Likewise, decomposing a base-ten unit into 10 of the next smaller unit means multiplying by $\frac{1}{10}$, so the exponent in the power of 10 goes down by 1.

- *"How does using powers of 10 make it easier to communicate about very large or very small numbers?"*

  We can write in a smaller space. It's also faster to read and easier to understand the size of a number and to compare numbers. Using powers of 10 helps us avoid errors of missing zeros or extra zeros.

Student Workbook


9 Lesson Summary
Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities.
For example, the United States Mint has made over 500,000,000,000 pennies. To understand this number we can look at the number of zeros to know it is equivalent to 500 billion pennies. Since 1 billion can be written as 10^9, we can say that there are over $500 \cdot 10^9$ pennies.
Sometimes we may need to rewrite a number using a different power of 10. We can say that $500 \cdot 10^9 = 5 \cdot 10^{11}$. Since the factor 10^9 was multiplied by 100 to get 10^{11}, the factor of 500 was divided by 100 to keep the value of the entire expression the same.
The same is true for very small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams. If we write this as a fraction we get $\frac{199}{10,000,000,000,000,000,000,000,000}$. Using powers of 10, it becomes $199 \cdot 10^{-25}$, which is a lot easier to write!
Just as we did with large numbers, small numbers can be rewritten as an equivalent value with a different power of 10. In this example we can divide the factor 199 by 100 and multiply the factor 10^{-25} by 100 to get $1.99 \cdot 10^{-23}$.
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Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas here, so there is no need to slow down or add additional work to the next lessons.

"What are some different ways to describe a large number like 123 billion?"

$123 \cdot (1,000,000,000)$, or $123 \cdot 10^9$

"What are some different ways to describe a small number like 0.0000000789?"

789 ten-billionths, $789 \cdot \frac{1}{10,000,000,000}$, or $789 \cdot 10^{-10}$

Lesson Summary

Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities.

For example, the United States Mint has made over 500,000,000,000 pennies. To understand this number we can look at the number of zeros to know it is equivalent to 500 billion pennies. Since 1 billion can be written as 10^9, we can say that there are over $500 \cdot 10^9$ pennies.

Sometimes we may need to rewrite a number using a different power of 10. We can say that $500 \cdot 10^9 = 5 \cdot 10^{11}$. Since the factor 10^9 was multiplied by 100 to get 10^{11}, the factor of 500 was divided by 100 to keep the value of the entire expression the same.

The same is true for very small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams. If we write this as a fraction we get $\frac{199}{10,000,000,000,000,000,000,000,000}$. Using powers of 10, it becomes $199 \cdot 10^{-25}$, which is a lot easier to write!

Just as we did with large numbers, small numbers can be rewritten as an equivalent value with a different power of 10. In this example we can divide the factor 199 by 100 and multiply the factor 10^{-25} by 100 to get $1.99 \cdot 10^{-23}$.

Cool-down

Better with Powers of 10

5 min

Student Task Statement

1. Write 0.000000123 as a multiple of a power of 10. $(1.23) \cdot 10^{-7}$ (or equivalent)
2. Write 123,000,000 as a multiple of a power of 10. $(1.23) \cdot 10^{8}$ (or equivalent)

Lesson 9

Practice Problems

6 Problems

Problem 1

Match each number to its name.

a. 1,000,000
b. 0.01
c. 1,000,000,000
d. 0.000001
e. 0.001
f. 10,000

- One hundredth
- One thousandth
- One millionth
- Ten thousand
- One million
- One billion

Problem 2

Write each expression as a multiple of a power of 10:

a. 42,300 $423 \cdot 10^2$ (or equivalent)

b. 2,000 $2 \cdot 10^3$ (or equivalent)

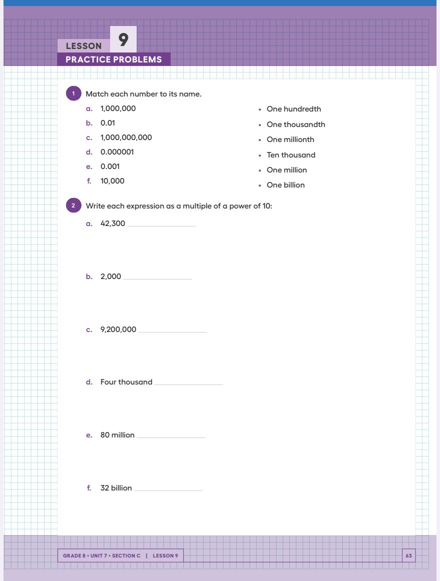
c. 9,200,000 $92 \cdot 10^5$ (or equivalent)

d. Four thousand $4 \cdot 10^3$ (or equivalent)

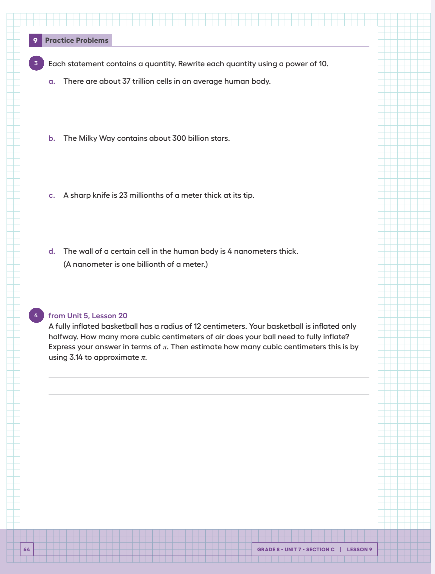
e. 80 million $8 \cdot 10^7$ (or equivalent)

f. 32 billion $32 \cdot 10^9$ (or equivalent)

Student Workbook



Student Workbook


9 Practice Problems

3 Each statement contains a quantity. Rewrite each quantity using a power of 10.

a. There are about 37 trillion cells in an average human body.

b. The Milky Way contains about 300 billion stars.

c. A sharp knife is 23 millionths of a meter thick at its tip.

d. The wall of a certain cell in the human body is 4 nanometers thick. (A nanometer is one billionth of a meter.)

4 from Unit 5, Lesson 20

A fully inflated basketball has a radius of 12 centimeters. Your basketball is inflated only halfway. How many more cubic centimeters of air does your ball need to fully inflate? Express your answer in terms of π. Then estimate how many cubic centimeters this is by using 3.14 to approximate π.

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Student Workbook


9 Practice Problems

5 from Unit 4, Lesson 5

Solve each of these equations. Explain or show your reasoning.

$2(3 - 2c) = 30$

$3x - 2 = 7 - 6x$

$31 = 5(b - 2)$

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Problem 3

Each statement contains a quantity. Rewrite each quantity using a power of 10.

a. There are about 37 trillion cells in an average human body.

$37 \cdot 10^{12}$ cells (or equivalent)

b. The Milky Way contains about 300 billion stars.

$300 \cdot 10^{9}$ stars (or equivalent)

c. A sharp knife is 23 millionths of a meter thick at its tip.

$23 \cdot 10^{-6}$ meter (or equivalent)

d. The wall of a certain cell in the human body is 4 nanometers thick. (A nanometer is one billionth of a meter.)

$4 \cdot 10^{-9}$ meter (or equivalent)

Problem 4

from Unit 5, Lesson 20

A fully inflated basketball has a radius of 12 centimeters. Your basketball is inflated only halfway. How many more cubic centimeters of air does your ball need to fully inflate? Express your answer in terms of π. Then estimate how many cubic centimeters this is by using 3.14 to approximate π.

$1{,}152\pi$ cubic centimeters

3,617.28 cubic centimeters

Problem 5

from Unit 4, Lesson 5

Solve each of these equations. Explain or show your reasoning.

$2(3 - 2c) = 30$

$c = -6$

Sample reasoning: Divide each side by 2, then subtract 3 from each side, then divide each side by -2.

$3x - 2 = 7 - 6x$

$x = 1$

Sample reasoning: Add 2 to each side, then add $6x$ to each side, then divide each side by 9.

$31 = 5(b - 2)$

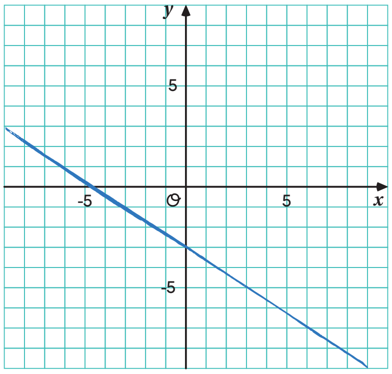
$b = \frac{41}{5}$

Sample reasoning: Distribute 5 on the right side, add 10 to each side, then divide each side by 5.

Problem 6

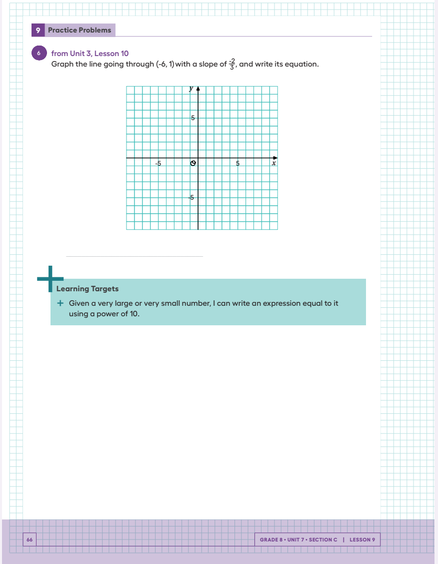
from Unit 3, Lesson 10

Graph the line going through (-6, 1) with a slope of $\frac{-2}{3}$, and write its equation.



$y = \frac{-2}{3}x - 3$ (or equivalent)

Student Workbook


Practice Problems

from Unit 3, Lesson 10

Graph the line going through (-6, 1) with a slope of $\frac{-2}{3}$, and write its equation.

Learning Targets

+ Given a very large or very small number, I can write an expression equal to it using a power of 10.