Lesson 4

More Balanced Moves

Goals

- Calculate a value that is a solution for a linear equation in one variable, and compare and contrast (orally) solution strategies with others.
- Critique (in writing) the reasoning of others in solving a linear equation in one variable.

Learning Target

I can make sense of multiple ways to solve an equation.

Lesson Narrative

In this lesson, students reinforce their understanding of a solution to an equation, that equivalent equations have the same solutions, and that performing certain moves can be used to write equivalent equations. A focus for this lesson is on solidifying the correct use of moves by analyzing common mistakes and checking solutions. As students suggest ways to improve incorrect solution paths, they must critique the arguments of the given solutions.

Student Learning Goal

Let's rewrite some more equations while keeping the same solutions.

Access for Students with Diverse Abilities

- Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR7: Compare and Connect (Warm-up)

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect

Lesson Timeline

Warm-up	Activity 1	Activity 2	Lesson Synthesis	Assessment: Cool-down
5 min	15 min	15 min	10 min	5 min

Instructional Routines

MLR7: Compare and Connect

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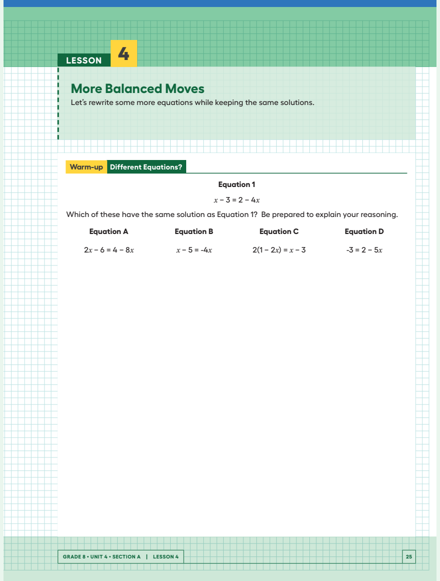
Access for Multilingual Learners (Warm-up)

MLR7: Compare and Connect
This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Student Workbook



Warm-up
Different Equations?

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to use the structure of equations to recognize when they are equivalent.

Monitor for students who use these different strategies:

- Solve each equation, then compare the solutions.
- Solve Equation 1, then substitute it into the other equations.
- Use valid moves correctly to make equations look alike.

Launch

Give students 2–3 minutes of quiet think time, and then facilitate a whole-class discussion.

Select work from students with different strategies, such as those described in the activity narrative, to share later.

Student Task Statement

Equation 1

$x - 3 = 2 - 4x$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

Equation A	**Equation B**	**Equation C**	**Equation D**
$2x - 6 = 4 - 8x$	$x - 5 = -4x$	$2(1 - 2x) = x - 3$	$-3 = 2 - 5x$

All of the other equations have the same solution as the first equation, $x = 1$.

Sample reasoning:

- Equation A: If you multiply each side of Equation 1 by 2, the result is Equation A. So if x makes Equation 1 true, then it makes Equation A true as well.
- Equation B: If you subtract 2 from each side of Equation 1, the result is Equation B. So if x makes Equation 1 true, then it makes Equation B true, too.
- Equation C: If you switch everything to the left of the equal sign and everything to the right of the equal sign on Equation C, and then rewrite the expression $2(1 - 2x)$ as $2 - 4x$ using the distributive property, the result is Equation 1. So if x makes Equation 1 true, then it makes Equation C true.
- Equation D: If you subtract x from each side of Equation 1, the result is Equation D. So if x makes Equation 1 true, then it also makes Equation D true.

Activity Synthesis

The goal of this discussion is for students to recognize that there are multiple ways to check that 2 equations are equivalent. In particular, students should recognize that equations do not need to be solved to determine that they have the same solution.

Display 2–3 strategies from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different strategies. Here are some questions for discussion:

- *"What do the strategies have in common? How are they different?"*

  "Which method of answering the question is the most efficient? After seeing all these ways to answer the question, which would you choose?"

  "What is an advantage of changing the equation to look like Equation 1? What is a disadvantage?"

  An advantage is that I could see quickly whether it would be the same as Equation 1, and I didn't have to keep going to actually figure out the value of x. A disadvantage would be that I never discovered what the value for x is that makes the equations true.

- *"How is writing equivalent equations similar to what we did in previous lessons with the balance hangers?"*

  In order to keep the hangers balanced, I had to make sure to do the same thing to each side of the hanger. In order to have each equation still be true, I have to make sure to do the same thing to each side of an equation.

If time allows, have students create another equation with the same solution as Equation 1 and trade with a partner. They should then explain to each other the step(s) necessary to make it look like Equation 1.

Activity 1

Step by Step by Step by Step

15 min

Activity Narrative

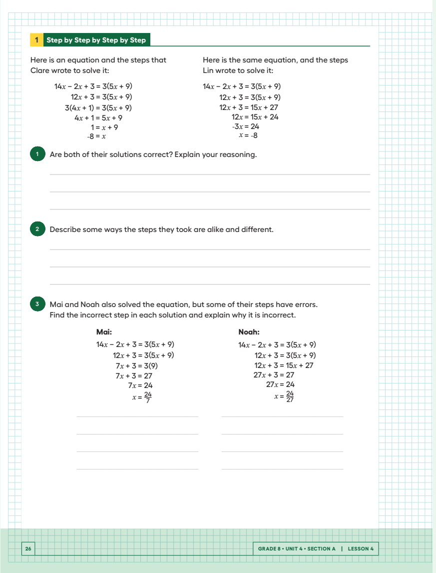
Before students work on solving complex equations on their own, in this activity they examine the work (both good and bad) of others. The purpose of this activity is to build student fluency solving equations by examining the solutions of others for both appropriate and inappropriate strategies.

Encourage students to use precise language when discussing the different steps made by the four students in the problem. For example, if a student says Clare used the distributive property to move from $12x + 3 = 3(5x + 9)$ to $3(4x + 1) = 3(5x + 9)$, ask them to be more specific about how Clare used the distributive property. Those specifics will help the whole class follow along. (Clare used the distributive property to re-write $12x + 3$ as $3(4x + 1)$.)

Access for Students with Diverse Abilities (Activity 1, Launch)

Engagement: Develop Effort and Persistence.
Chunk this task into more manageable parts. Demonstrate for students how to use an index card or a scrap piece of paper to cover and then unveil the steps one at a time. Invite students to make comparisons at each step. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Social-Emotional Functioning

Student Workbook



Launch

Arrange students in groups of 2.

Give students 4–5 minutes of quiet work time, and ask students to pause after the first two problems for a partner discussion.

Give 2–3 minutes for partners to work together on the final problem, and follow that with a whole-class discussion.

Student Task Statement

Here is an equation and the steps that Clare wrote to solve it:

$$14x - 2x + 3 = 3(5x + 9)$$
$$12x + 3 = 3(5x + 9)$$
$$3(4x + 1) = 3(5x + 9)$$
$$4x + 1 = 5x + 9$$
$$1 = x + 9$$
$$-8 = x$$

Here is the same equation, and the steps Lin wrote to solve it:

$$14x - 2x + 3 = 3(5x + 9)$$
$$12x + 3 = 3(5x + 9)$$
$$12x + 3 = 15x + 27$$
$$12x = 15x + 24$$
$$-3x = 24$$
$$x = -8$$

1. Are both of their solutions correct? Explain your reasoning.

   yes

   Sample reasoning: The solution of $x = -8$ is the only value of x that makes the equation true.

2. Describe some ways the steps they took are alike and different.

   Sample response: Both students combined like terms from line one to line two. Clare used the distributive property to re-write the left side as $3(4x + 1)$ moving from line two to line three, and Lin used the same property to distribute the 3 on the right side moving from line two to line three.

3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.

Mai:

$$14x - 2x + 3 = 3(5x + 9)$$
$$12x + 3 = 3(5x + 9)$$
$$7x + 3 = 3(9)$$
$$7x + 3 = 27$$
$$7x = 24$$
$$x = \frac{24}{7}$$

Noah:

$$14x - 2x + 3 = 3(5x + 9)$$
$$12x + 3 = 3(5x + 9)$$
$$12x + 3 = 15x + 2$$
$$27x + 3 = 27$$
$$27x = 24$$
$$x = \frac{24}{27}$$

Sample response: Mai made an error moving from line two to line three by subtracting $5x$ from each side of the equation before multiplying by 3 on the right hand side of the equation. Noah made an error moving from line three to line four by subtracting $15x$ from the right side of the equation but adding $15x$ to the left side of the equation. He should have subtracted it from the left side, too.

Activity Synthesis

The purpose of this discussion is recognize some common move errors that can result in equations that are not equivalent.

Begin the discussion by asking,

"How do you know when a solution to an equation is correct?"

> One way to know it is correct is by substituting the value of x into the original equation and seeing if it makes the equation true.

Display Clare's and Lin's solutions for all to see.

$$\begin{aligned}
14x - 2x + 3 &= 3(5x + 9)\\
12x + 3 &= 3(5x + 9)\\
3(4x + 1) &= 3(5x + 9)\\
4x + 1 &= 5x + 9\\
1 &= x + 9\\
-8 &= x
\end{aligned}$$

$$\begin{aligned}
14x - 2x + 3 &= 3(5x + 9)\\
12x + 3 &= 3(5x + 9)\\
12x + 3 &= 15x + 27\\
12x &= 15x + 24\\
-3x &= 24\\
x &= -8
\end{aligned}$$

Survey to see which solution they prefer. It is important to draw out that neither solution is better than the other, they are two ways of accomplishing the same task: solving for x. Invite groups to share ways in which the steps that Clare and Lin took are alike and different, and annotate the two solutions with students' observations. If none of the groups say it, point out that while the final steps may look different for Clare and Lin, their later steps worked to reduce the total number of terms until only an x-term and a number remained—one on each side of the equation.

Display Mai's and Noah's incorrect solutions for all to see.

$$\begin{aligned}
14x - 2x + 3 &= 3(5x + 9)\\
12x + 3 &= 3(5x + 9)\\
7x + 3 &= 3(9)\\
7x + 3 &= 27\\
7x &= 24\\
x &= \frac{24}{7}
\end{aligned}$$

$$\begin{aligned}
14x - 2x + 3 &= 3(5x + 9)\\
12x + 3 &= 3(5x + 9)\\
12x + 3 &= 15x + 27\\
27x + 3 &= 27\\
27x &= 24\\
x &= \frac{24}{27}
\end{aligned}$$

Invite groups to share an incorrect step that they found and what advice they would give to Mai and Noah for checking their work in the future.

Instructional Routines

MLR2: Collect and Display

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Please log in to the site before using the QR code or URL.

Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Student Workbook


2 Make Your Own Steps

Solve these equations for x.

Are You Ready for More?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second.

How many pencils does each cup contain?

GRADE 8 • UNIT 4 • SECTION A | LESSON 4

Activity 2

Make Your Own Steps

10 min

Activity Narrative

The purpose of this lesson is to increase fluency in solving equations. Students will solve equations individually and then compare differing, though accurate, solution paths in order to compare their work with others. This will help students recognize that while the final solution will be the same, there is more than one path to the correct answer that uses principles of balancing equations learned in previous lessons.

Launch

Arrange students in groups of 3–4. Give students quiet think time to complete the activity, and then tell groups to share how they solved the equations for x and to discuss the similarities and differences in their solution paths.

Use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe similarities and differences in their solution paths. Display words and phrases such as "multiply first," "divide first," "subtract x from each side," or "add x to each side."

Student Task Statement

Solve these equations for x.

1. $\frac{12+6x}{3}=\frac{5-9}{2}$

   $x=-3$

2. $x-4=\frac{1}{3}(6x-54)$

   $x=14$

3. $-(3x-12)=9x-4$

   $x=\frac{4}{3}$

Are You Ready for More?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

7, 8, and 9 pencils in the first, second, and third cups, respectively
$n+(n+1)+(n+2)=24$ so there are 7 pencils in the first cup, because $n=7$.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share their strategy for solving an equation. Invite students to borrow language from the display as needed, and update the reference to include additional phrases as they respond.

Students should take away from this activity the importance of using valid steps to solve an equation rather than following a specific solution path. Consider using some of the following prompts:

"How many different ways did your group members solve each problem?"

"When you compared solution paths, did you still come up with the same solution?"

Yes, even though we took different paths, we ended up with the same solutions.

"How can you make sure that the path you choose to solve an equation is a valid path?"

I can use the steps that we discovered earlier when we were balancing: adding the same value to each side, multiplying (or dividing) by the same value on each side, using the distribute property, and collecting like terms, whenever those steps are needed.

"What are some examples of steps that will not result in a valid solution?"

Performing an action to only one side of an equation and distributing incorrectly will give an incorrect solution.

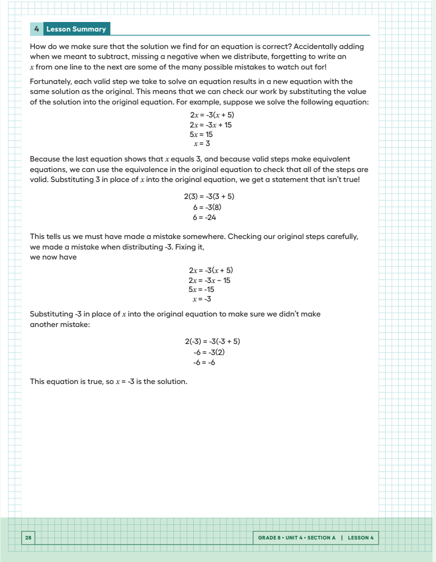
Lesson Synthesis

Display these prompts one at a time and, after each, ask students if the move described creates an equivalent equation:

- Subtract a number from each side (equivalent)
- Subtract $4x$ from each side (equivalent)
- Divide each side of the equation by 7 (equivalent)
- Add $5x$ to one side and 10 to the other (equivalent only if it is already known that $x = 2$)
- Add 4 to one side and add 5 to the other (not equivalent)

Ask students to write an equation and a solution to the equation that contains an error. Then, tell students to swap with a partner and try to find the error in their partner's solution.

Student Workbook


4 Lesson Summary

How do we make sure that the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an x from one line to the next are some of the many possible mistakes to watch out for!

Fortunately, each valid step we take to solve an equation results in a new equation with the same solution as the original. This means that we can check our work by substituting the value of the solution into the original equation. For example, suppose we solve the following equation:

$$\begin{aligned} 2x &= -3(x+5) \\ 2x &= -3x+15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Because the last equation shows that x equals 3, and because valid steps make equivalent equations, we can use the equivalence in the original equation to check that all of the steps are valid. Substituting 3 in place of x into the original equation, we get a statement that isn't true!

$$\begin{aligned} 2(3) &= -3(3+5) \\ 6 &= -3(8) \\ 6 &= -24 \end{aligned}$$

This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3. Fixing it, we now have

$$\begin{aligned} 2x &= -3(x+5) \\ 2x &= -3x-15 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

Substituting -3 in place of x into the original equation to make sure we didn't make another mistake:

$$\begin{aligned} 2(-3) &= -3(-3+5) \\ -6 &= -3(2) \\ -6 &= -6 \end{aligned}$$

This equation is true, so $x = -3$ is the solution.

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Lesson Summary

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This equation is true, so $x = -3$ is the solution.

Cool-down

Mis-Steps

5 min

Student Task Statement

Examine Lin's solution to $8(x - 3) + 7 = 2x(4 - 17)$.

$$
\begin{aligned}
8(x - 3) + 7 &= 2x(4 - 17) \quad && \text{4 − 17 = −13}\\
8(x - 3) + 7 &= 2x(13) &&\\
8x - 24 + 7 &= 26x &&\\
8x - 17 &= 26x &&\\
\text{subtract } 8x \quad -17 &= 34x && \text{should subtract, not add } 8x\\
-\tfrac{1}{2} &= x &&
\end{aligned}
$$

1. For each step, determine if the 2 equations are equivalent. If they are not, describe the error.
2. What is the correct solution to the original equation?

   Sample response: $x = \frac{1}{2}$ (or equivalent)

Responding To Student Thinking

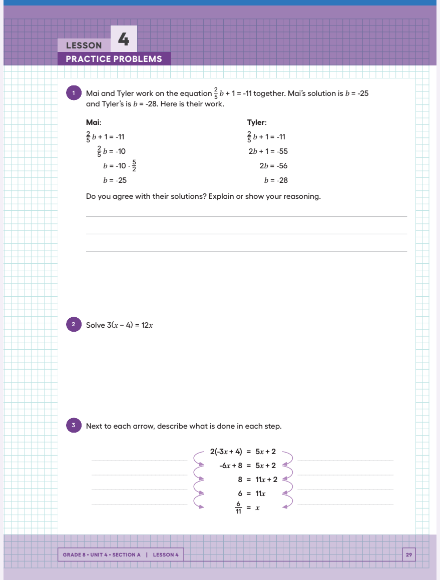
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 4

Practice Problems

6 problems

Student Workbook



Problem 1

Mai and Tyler work on the equation $\frac{2}{5}b + 1 = -11$ together. Mai's solution is $b = -25$ and Tyler's is $b = -28$. Here is their work. Do you agree with their solutions? Explain or show your reasoning.

Mai:	**Tyler:**
$\frac{2}{5}b + 1 = -11$	$\frac{2}{5}b + 1 = -11$
$\frac{2}{5}b = 10$	$2b + 1 = -55$
$b = -10 \cdot \frac{5}{2}$	$2b = -56$
$b = -25$	$b = -28$

I disagree with both solutions.

Sample reasoning: Mai added -1 on the left side and 1 on the right side of the equation. Tyler multiplied both sides of the equation by 5 but did not multiply the 1 by 5.

Problem 2

Solve $3(x - 4) = 12x$

$x = -\frac{4}{3}$

Sample reasoning: Distribute, subtract $3x$ from each side, and divide by 9. Or, first divide each side by 3, subtract x from each side, and then divide each side by 3.

Problem 3

Next to each arrow, describe what is done in each step.

	$2(-3x + 4) = 5x + 2$	
distributive property	$-6x + 8 = 5x + 2$	distributive property
add $6x$	$8 = 11x + 2$	add $6x$
subtract 2	$6 = 11x$	subtract 2
divide by 11	$\frac{6}{11} = x$	divide by 11

Problem 4

Andre solves an equation, but when he checks his answer he notices that his solution is incorrect. He knows he made an error, but he can't find it. Where is Andre's error and what is the solution to the equation?

$$\begin{aligned} -2(3x - 5) &= 4(x + 3) + 8 \\ -6x + 10 &= 4x + 12 + 8 \\ -6x + 10 &= 4x + 20 \\ 10 &= -2x + 20 \\ -10 &= -2x \\ 5 &= x \end{aligned}$$

Sample response: Andre's error occurs in the transition from the 3rd line to the 4th line. He adds $6x$ on the left side but subtracts $6x$ on the right side. The correct solution is $x = -1$.

Problem 5

from Unit 3, Lesson 13

Choose the equation that has solutions (5, 7) and (8, 13).

A. $3x - y = 8$

B. $y = x + 2$

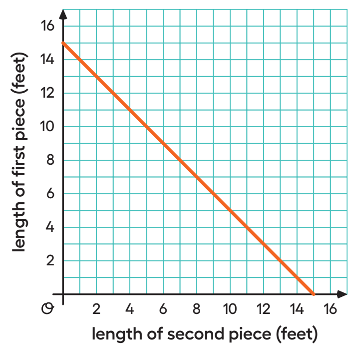
C. $y - x = 5$

D. $y = 2x - 3$

Problem 6

from Unit 3, Lesson 9

A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length (in feet) of the second piece, x, for each length of the first piece, y.



a. How long is the ribbon? Explain how you know.

15 feet

Sample reasoning: When the second piece is 0 feet long, the first is 15 feet long, so that is the length of the ribbon.

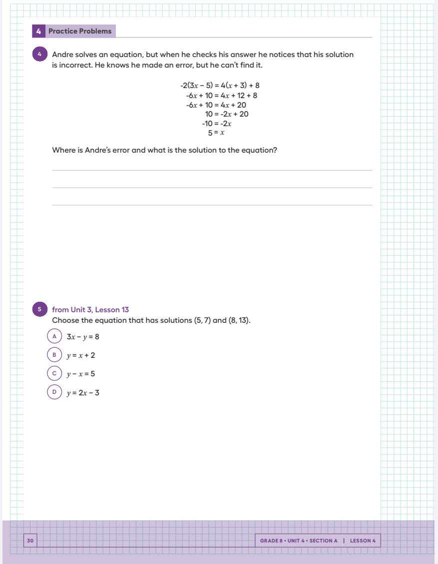
b. What is the slope of the line?

-1

c. Explain what the slope of the line represents and why it fits the story.

Sample response: The slope shows that for every 1 foot increase in the length of the second piece, the first piece must get 1 foot shorter. This makes sense because the total length of ribbon is constant.

Student Workbook



Student Workbook