Lesson 7

Connecting Representations of Functions

Goals

- Compare and contrast (orally) representations of functions, and describe (orally) the strengths and weaknesses of each type of representation.
- Interpret multiple representations of functions, including graphs, tables, and equations, and explain (orally) how to find information in each type of representation.

Learning Target

I can compare inputs and outputs of functions that are represented in different ways.

Access for Students with Diverse Abilities

- Representation (Activity 1, Activity 2)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR5: Co-Craft Questions (Activity 1)

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display

Required Materials

Materials to Gather

- Math Community Chart: Lesson

Required Preparation

Activity 2:
For the digital version of the activity, acquire devices that can run the applet.

Lesson Narrative

In this lesson, students compare two functions represented in different ways (graph and table, graph and equation, and, optionally, table and verbal description). In each case, students use the different representations to find outputs for different inputs. Even though they use different representations, students are looking for the same information about the contexts and need to interpret each representation appropriately.

In a graph, students identify the input on the horizontal axis, then find the corresponding coordinate point on the graph, which lets them read the associated output. In a table, they find the input value in the first row (or column) and read the output value in the second. For functions represented by equations, students substitute the input value into the expression on one side of the equation and compute the corresponding output value on the other. Students also look for inputs corresponding to a given output by trying to reverse these procedures.

Each representation has strengths and weaknesses. Comparing the different strengths of these representations in the *Lesson Synthesis* helps students make decisions about how to use these tools strategically in the future.

Lesson Timeline

Warm-up	Activity 1	Activity 2	Activity 3	Lesson Synthesis	Assessment: Cool-down
5 min	10 min	10 min	10 min	10 min	5 min

Lesson 7

Connecting Representations of Functions

Lesson Narrative (continued)

The last activity is optional. Consider using it if students would benefit from additional practice of comparing a function represented by words with one represented by a table.

Note that this lesson specifically avoids comparisons of linear functions to other linear functions in order to avoid students associating "function" with only linear relationships. In a later lesson, students revisit some of these ideas and compare linear functions.

Student Learning Goal

Let's connect tables, equations, graphs, and stories of functions.

Warm-up
Which Are the Same? Which Are Different?

5 min

Activity Narrative

The purpose of this activity is for students to identify connections between three different representations of functions: equation, graph, and table. Two of the functions displayed are the same but with different variable names. It is important for students to focus on comparing input-output pairs when deciding how two functions are the same or different.

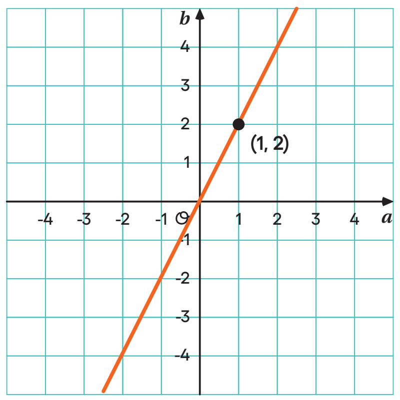
Launch

Give students 1–2 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Here are three different ways of representing functions.

$$y = 2x$$



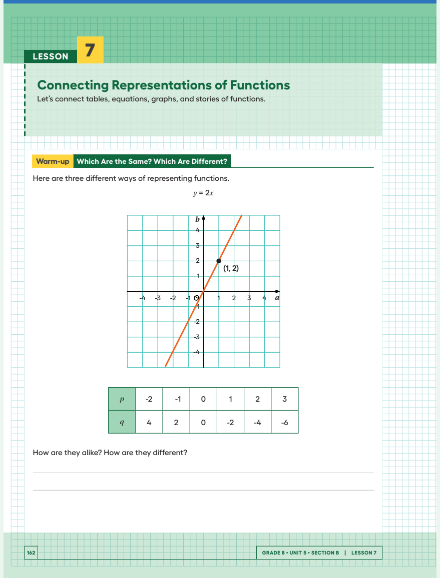
p	-2	-1	0	1	2	3
q	4	2	0	-2	-4	-6

How are they alike? How are they different?

Sample responses:

- The equations for the first two would have the same form but different variables.
- The graphs for the first two are identical except for the labels on the axes.
- A table of values for both the equation and graph would have the same ordered pairs, but the variables names would be different.
- The third one has opposite outputs for the same input as the first two. The graph would be a line reflected across the y-axis as compared with the first two.

Student Workbook



Access for Students with Diverse Abilities (Activity 1, Launch)

Representation: Internalize Comprehension.
Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, ask students to use the same color to highlight the temperatures for each city that occurred at the same time, such as blue for 4:00 p.m.

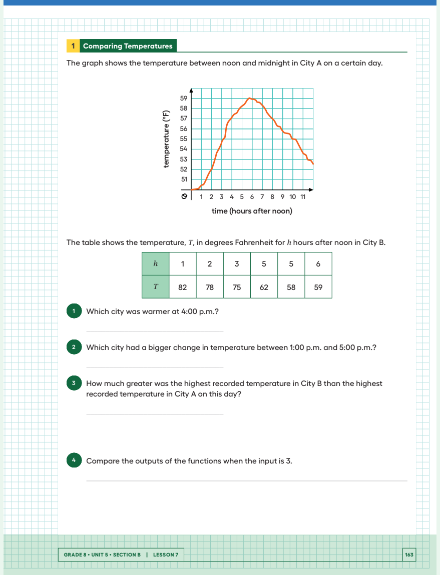
Supports accessibility for: Visual-spatial processing

Access for Multilingual Learners (Activity 1, Launch)

MLR5: Co-Craft Questions.
Keep student workbooks or devices closed. Display only the graph and the table, without revealing the questions. Give students 2–3 minutes to write a list of mathematical questions that could be asked about this situation, before comparing their questions with a partner. Invite each group to contribute one written question to a whole-class display. Ask the class to make comparisons among the shared questions and their own. Reveal the intended questions for this task, and invite additional connections.

Advances: Reading, Writing

Student Workbook


1 Comparing Temperatures

The graph shows the temperature between noon and midnight in City A on a certain day.

The table shows the temperature, T, in degrees Fahrenheit for h hours after noon in City B.

h	1	2	3	5	5	6
T	82	78	75	62	58	59

1. Which city was warmer at 4:00 p.m.?
2. Which city had a bigger change in temperature between 1:00 p.m. and 5:00 p.m.?
3. How much greater was the highest recorded temperature in City B than the highest recorded temperature in City A on this day?
4. Compare the outputs of the functions when the input is 3.

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Activity Synthesis

Ask students to share ways the representations are alike and different. Record and display the responses for all to see. To help students clarify their thinking, ask students to reference the equation, graph, or table when appropriate. If the relationship between the inputs and outputs in each representation does not arise, ask students what they notice about this relationship in each representation.

Activity 1
Comparing Temperatures

10 min

Activity Narrative

This is the first of three activities in which students make connections between different functions represented in different ways. In this activity, students are given a graph and a table of temperatures from two different cities and are asked to make sense of the representations in order to answer questions about the context.

Launch

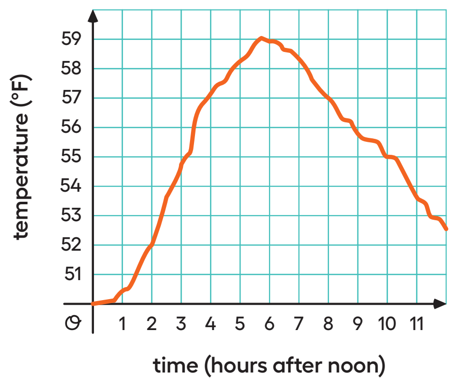
Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time and then time to share responses with their partner.

Follow with a whole-class discussion.

Student Task Statement

The graph shows the temperature between noon and midnight in City A on a certain day.



The table shows the temperature, T, in degrees Fahrenheit for h hours after noon in City B.

h	1	2	3	4	5	6
T	82	78	75	62	58	59

1. Which city was warmer at 4:00 p.m.?

   City B

   From the graph, the temperature in City A is 57 °F at 4:00 p.m., and from the table, the temperature in City B is 62 °F

2. Which city had a bigger change in temperature between 1:00 p.m. and 5:00 p.m.?

   City B

   From the graph, the temperature in City A increased about 7.5 °F, from just under 50.5 °F to just over 58 °F. From the table, the temperature in City B decreased 24 °F, from 82 °F down to 58 °F.

3. How much greater was the highest recorded temperature in City B than the highest recorded temperature in City A on this day?

   About 23 °F

   From the graph, the highest recorded temperature in City A is about 59 °F. From the table, the highest recorded temperature in City B is 82 °F. 82 − 59 = 23.

4. Compare the outputs of the functions when the input is 3.

   The first function gives the temperature in City A at 3:00 p.m., which is about 54.5 °F. The second function gives the temperature in City B at 3:00 p.m., which is 75 °F. City B is hotter than City A at that time by about 20.5 °F, since 75 − 54.5 = 20.5.

Activity Synthesis

Display the graph and table for all to see. Select groups to share how they used the two different representations to get their answers for each question. To further student thinking about the advantages and disadvantages of each representation, ask:

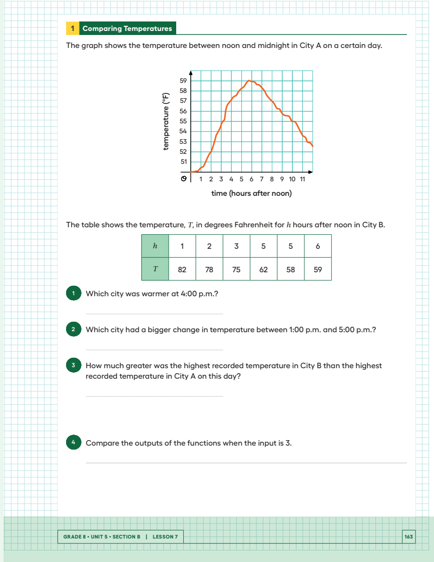
- *"Which representation do you think is better for identifying the highest recorded temperature in a city?"*

  The graph, since I just have to find the highest part. In the table I have to read all the values in order to find the highest temperature.

- *"Which representation do you think is quicker for figuring out the change in temperature between 1:00 p.m. and 5:00 p.m.?"*

  The table is quicker since the numbers are given, and I only have to subtract. In the graph, I have to figure out the temperature values for both times before I can subtract.

Student Workbook



1 Comparing Temperatures

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Instructional Routines

MLR2: Collect and Display

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Access for Multilingual Learners (Activity 2)

MLR2: Collect and Display
This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Access for Students with Diverse Abilities (Activity 2, Launch)

Representation: Develop Language and Symbols.
Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, show or provide students with a cube and a sphere. Discuss the relationship between the side length or radius and the volume of the object.
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Activity 2

Comparing Volumes

10 min

Activity Narrative

There is a digital version of this activity.

This is the second of three activities in which students make connections between different functions represented in different ways. In this activity, students are given an equation and a graph of the volumes of two different objects. Students then compare inputs and outputs of both functions and what those values mean in the context of the shapes.

In the digital version of the activity, students use an applet to compare the two functions. The applet allows students to have an interactive version of the graph to identify coordinates. Use the digital version if it would benefit students identifying points more precisely.

Launch

Arrange students in groups of 2.

Give students 3–5 minutes of quiet work time and then time to share their responses with their partner.

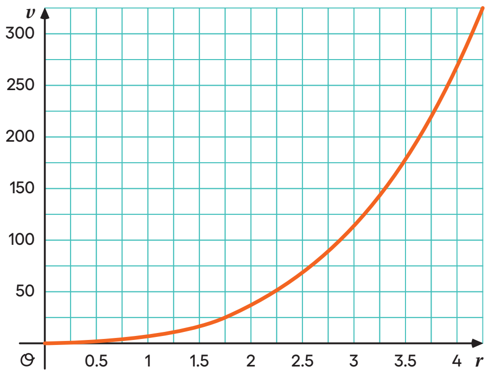
Follow with a whole-class discussion.

As students work, use *Collect and Display* to create a shared reference that captures students' developing mathematical language. Collect the language that students use to describe the connections across the two different representations. Display words and phrases, such as "If $s = 3$, then the volume of the cube is 27 cm^3 since the equation is $V = s^3$," "Going up from $r = 3$ on the axis, the graph is at a vertical height of about 110 cm^3," "If the input is 2, then on the point on the graph where $r = 2$, I can trace to the left until I reach the vertical axis, where the value of V is about 35 cm^3," and "If the input is 2, then in the equation, s is equal to 2, so $V = 2^3 = 8$."

Student Task Statement

The volume, V, of a cube with edge length s cm is given by the equation $V = s^3$.

The volume of a sphere is a function of its radius (in cm), and the graph of this relationship is shown here.



1. Is the volume of a cube with edge length $s = 3$ greater or less than the volume of a sphere with radius 3?

   Less

   The volume of a cube with edge length is 27 cm^3, since $27 = 3^3$. From the graph, the volume of a sphere of radius 3 cm^3 is over 100 cm^3.

2. If a sphere has the same volume as a cube with edge length 5, estimate the radius of the sphere.

   About 3.1 cm

   The volume of a cube with edge length 5 cm is 125 cm^3, since $125 = 5^3$. From the graph, the volume of a sphere with radius 3.1 cm is about 125 cm^3.

3. Compare the outputs of the two volume functions when the inputs are 2.

   The output of the cube volume function is 8 cm^3 when the input is 2 cm, since $8 = 2^3$. From the graph, the output of the sphere volume function when the input is 2 cm is about 35 cm^3.

Are You Ready for More?

Estimate the edge length of a cube that has the same volume as a sphere with radius 2.5 cm.

About 4 cm

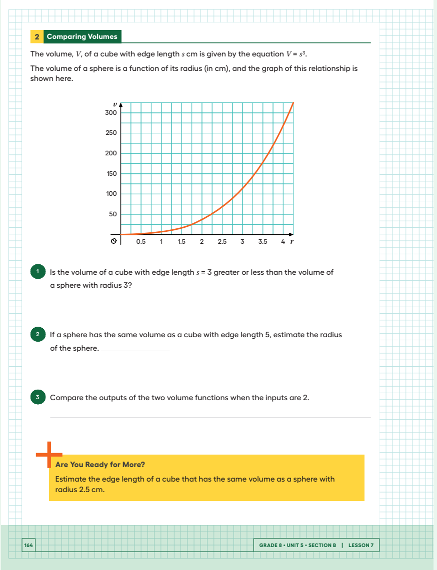
From the graph, the volume of a sphere of radius 2.5 cm is about 65 cm^3, whereas a cube of side length 4 cm has volume $4^3 = 64$.

Building on Student Thinking

Some students may struggle with the many parts of the second question. These two questions can help scaffold the question for students who need it:

> *"What information is given to you, and what can you do with it?"*
> *"What information is the focus of the question, and what would you like to know to be able to answer that question."*

Student Workbook


2 Comparing Volumes

The volume, V, of a cube with edge length s cm is given by the equation $V = s^3$.

The volume of a sphere is a function of its radius (in cm), and the graph of this relationship is shown here.

1 Is the volume of a cube with edge length $s = 3$ greater or less than the volume of a sphere with radius 3?

2 If a sphere has the same volume as a cube with edge length 5, estimate the radius of the sphere.

3 Compare the outputs of the two volume functions when the inputs are 2.

Are You Ready for More?

Estimate the edge length of a cube that has the same volume as a sphere with radius 2.5 cm.

Instructional Routines

MLR1: Stronger and Clearer Each Time

ilclass.com/r/10695479
Please log in to the site before using the QR code or URL.

Building on Student Thinking

If students do not notice that Elena's family's speed has different units than Andre's family, ask

"What do you notice so far about the information you have for the two families?"
"What needs to be true to compare two rates?"

Student Workbook


3 It's Not a Race

Elena's family is driving on the freeway at 55 miles per hour.

Andre's family is driving on the same freeway, but not at a constant speed. The table shows how far Andre's family has traveled in miles, d, every minute for 10 minutes.

t	1	2	3	4	5	6	7	8	9	10
d	0.9	1.9	3.0	4.1	5.1	6.2	6.8	7.4	8	9.1

1. How many miles per minute is 55 miles per hour?
2. Who has traveled farther after 5 minutes? After 10 minutes?
3. How long did it take Elena's family to travel as far as Andre's family had traveled after 8 minutes?
4. For both families, the distance in miles is a function of time in minutes. Compare the outputs of these functions when the input is 3.

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Activity Synthesis

The purpose of this discussion is for students to think about how they used the information from the different representations to answer questions about the context.

Display the equation and graph for all to see. Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they used the representations to answer the questions. Invite students to borrow language from the display as needed. As they respond, update the reference to include additional phrases. Consider asking the following questions to prompt students to expand on their answers:

"How did you use the given representations to find an answer? How did you use the equation? The graph?"

"For which problems was it nicer to use the equation? The graph? Explain your reasoning."

Activity 3: Optional
It's Not a Race

10 min

Activity Narrative

In this activity, students continue their work comparing properties of functions represented in different ways. Students are given a verbal description and a table to compare and decide whose family traveled farther over the same time intervals. The purpose of this activity is for students to continue building their skill interpreting and comparing functions.

Launch

Give students 3–5 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Elena's family is driving on the freeway at 55 miles per hour.

Andre's family is driving on the same freeway, but not at a constant speed. The table shows how far Andre's family has traveled in miles, d, every minute for 10 minutes.

t	1	2	3	4	5	6	7	8	9	10
d	0.9	1.9	3.0	4.1	5.1	6.2	6.8	7.4	8	9.1

1. How many miles per minute is 55 miles per hour?

   0.92 miles per minute

2. Who has traveled farther after 5 minutes? After 10 minutes?

Andre has traveled farther after 5 minutes. Elena has traveled 4.6 miles, because $5 \cdot 0.92 = 4.6$. From the table, we see that Andre has traveled 5.1 miles in that same time.

Elena has traveled farther after 10 minutes. Elena has traveled 9.2 miles, because $10 \cdot 0.92 = 9.2$. Andre has traveled 9.1 miles in that same time.

3. How long did it take Elena's family to travel as far as Andre's family had traveled after 8 minutes?

About 8.04 minutes

After 8 minutes, Andre has traveled 7.4 miles. To find the number of minutes, t, it takes Elena to travel 7.4 miles at 0.92 miles per minute, we solve the equation $0.92t = 7.4$ to find that t is approximately 8.04 minutes.

4. For both families, the distance in miles is a function of time in minutes. Compare the outputs of these functions when the input is 3.

The function for Andre's family gives an output of 3.0 miles for an input of 3 minutes. The function for Elena's family gives an output of 2.76 for an input of 3 minutes, since $3 \cdot 0.92 = 2.76$. Therefore, Andre's family has traveled a greater distance after 3 minutes than has Elena's family.

Activity Synthesis

The purpose of this discussion is for students to think about how they use a verbal description and table to answer questions related to the context. Ask students to share their solutions and how they used the equation and graph. Consider asking some of the following questions:

"How did you use the table to get information? How did you use the verbal description?"

"What did you prefer about using the description to solve the problem? What did you prefer about using the table to solve the problem?"

Lesson Synthesis

Conclude this lesson by inviting students to summarize some of the strengths and weaknesses of the representations students worked with during the lesson.

Arrange students in groups of 2–3.

Give students 3–4 minutes to prepare to answer these question, calling on 1–3 groups for each question:

"What is something you did today with all the different representations?"

We used each representation to find outputs for different inputs.

"What was your favorite representation? Why?"

I like equations because if I have an equation, I can make a graph or a table.

"What are some strengths and weaknesses of the different representations?"

Graphs require estimation but easily let us identify important features such as highest point or steepest section. Tables immediately let us find output values but only for limited input values. Equations let us precisely compute outputs for all inputs but only one at a time.

Student Workbook

3 It's Not a Race

Elena's family is driving on the freeway at 55 miles per hour.

Andre's family is driving on the same freeway, but not at a constant speed. The table shows how far Andre's family has traveled in miles, d, every minute for 10 minutes.

t	1	2	3	4	5	6	7	8	9	10
d	0.9	1.9	3.0	4.1	5.1	6.2	6.8	7.4	8	9.1

1. How many miles per minute is 55 miles per hour? ____

2. Who has traveled farther after 5 minutes? ____
After 10 minutes? ____

3. How long did it take Elena's family to travel as far as Andre's family had traveled after 8 minutes? ____

4. For both families, the distance in miles is a function of time in minutes. Compare the outputs of these functions when the input is 3.

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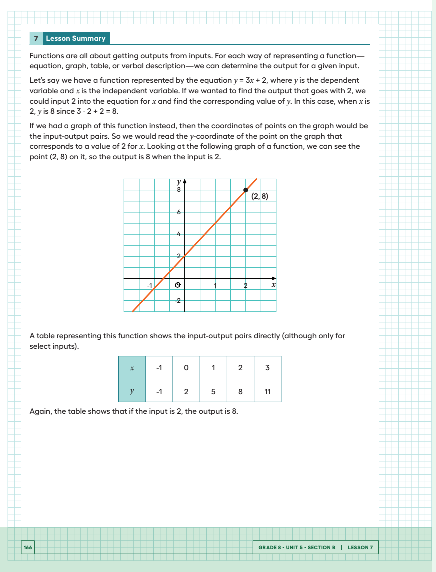
Student Workbook

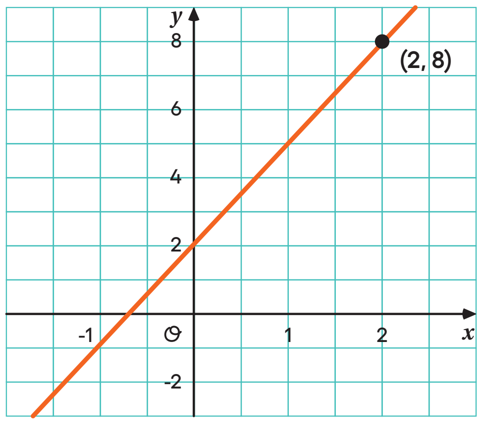


Lesson Summary

Functions are all about getting outputs from inputs. For each way of representing a function—equation, graph, table, or verbal description—we can determine the output for a given input.

Let's say we have a function represented by the equation $y = 3x + 2$, where y is the dependent variable and x is the independent variable. If we wanted to find the output that goes with 2, we could input 2 into the equation for x and find the corresponding value of y. In this case, when x is 2, y is 8 since $3 \cdot 2 + 2 = 8$.

If we had a graph of this function instead, then the coordinates of points on the graph would be the input-output pairs. So we would read the y-coordinate of the point on the graph that corresponds to a value of 2 for x. Looking at the following graph of a function, we can see the point (2, 8) on it, so the output is 8 when the input is 2.



A table representing this function shows the input-output pairs directly (although only for select inputs).

x	-1	0	1	2	3
y	-1	2	5	8	11

Again, the table shows that if the input is 2, the output is 8.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

"What norm(s) should stay the way they are?"

"What norm(s) do you think should be made more clear? How?"

"What norms are missing that you would add?"

"What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs,* identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Discuss the potential revisions over the next few lessons.

Responding To Student Thinking

Points to Emphasize

If most students struggle with comparing functions represented in different ways, focus on connections between representations as opportunities arise. For example, in the activity referred to here, invite 2–3 students to share how they answered the first question and made connections between Tank A's equations and Tank B's description.

Grade 8, Unit 5, Lesson 8, Activity 2 Is It Filling Up or Draining Out?

Cool-down

Comparing Different Areas

5 min

Student Task Statement

The table shows the area of a square for specific side lengths.

side length (inches)	0.5	1	2	3
area (square inches)	0.25	1	4	9

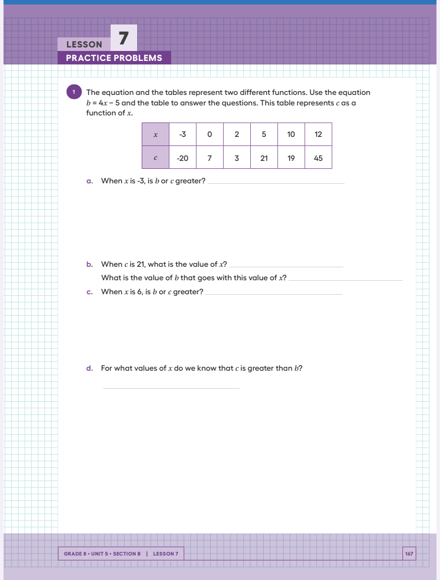
The area A of a circle with radius r is given by the equation $A = \pi \cdot r^2$.

Is the area of a square with side length 2 inches greater than or less than the area of a circle with radius 1.2 inches?

Less than

From the table, we see that the area of a square of side length 2 inches is 4 square inches, whereas from the equation, we find that the area of a circle with radius 1.2 inches is about 4.52 square inches.

Student Workbook


LESSON 7
PRACTICE PROBLEMS

1 The equation and the tables represent two different functions. Use the equation $b = 4x - 5$ and the table to answer the questions. This table represents c as a function of x.

x	-3	0	2	5	10	12
c	-20	7	3	21	19	45

a. When x is -3, is b or c greater?

b. When c is 21, what is the value of x? What is the value of b that goes with this value of x?

c. When x is 6, is b or c greater?

d. For what values of x do we know that c is greater than b?

Lesson 7

Practice Problems

4 Problems

Problem 1

The equation and the tables represent two different functions. Use the equation $b = 4x - 5$ and the table to answer the questions. This table represents c as a function of x.

x	-3	0	2	5	10	12
c	-20	7	3	21	19	45

a. When x is -3, is b or c greater?

b

b. When c is 21, what is the value of x? What is the value of b that goes with this value of x?

$x = 5$; $b = 15$

c. When x is 6, is b or c greater?

There is not enough information to answer this question since 6 is not in the table for x.

d. For what values of x do we know that c is greater than b?

0, 5, and 12

Problem 2

Elena and Lin are training for a race. Elena runs her mile at a constant speed of 7.5 miles per hour.

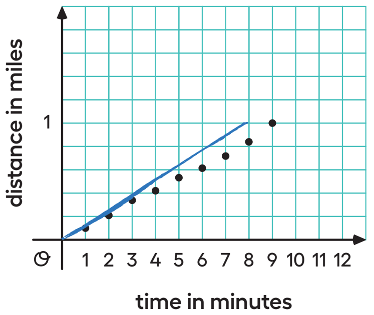
Lin's total distance is recorded every minute:

time (minutes)	1	2	3	4	5	6	7	8	9
distance (miles)	0.11	0.21	0.32	0.41	0.53	0.62	0.73	0.85	1

a. Who finished their mile first? Elena

Sample reasoning: It took her 8 minutes to complete her mile, but it took Lin 9 minutes.

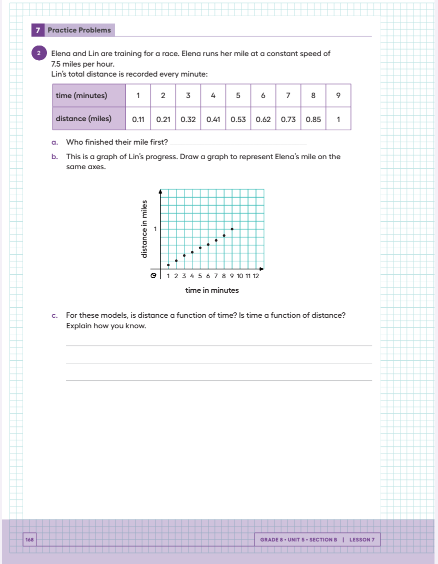
b. This is a graph of Lin's progress. Draw a graph to represent Elena's mile on the same axes.



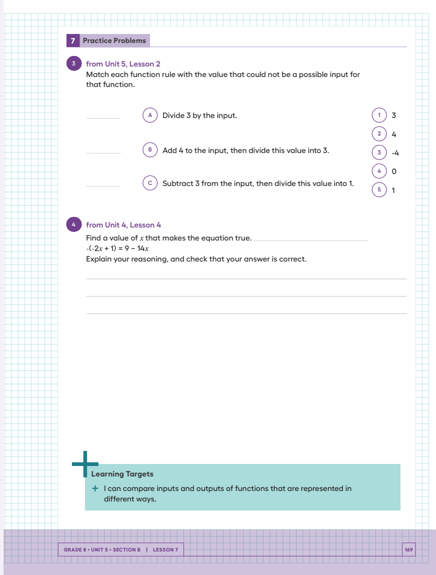
c. For these models, is distance a function of time? Is time a function of distance? Explain how you know.

Sample response: In both models, distance is a function of time, and time is also a function of distance. Given a time for either runner, the distance can be found, and vice versa.

Student Workbook



Student Workbook


7 Practice Problems

3 from Unit 5, Lesson 2

Match each function rule with the value that could not be a possible input for that function.

A Divide 3 by the input.

B Add 4 to the input, then divide this value into 3.

C Subtract 3 from the input, then divide this value into 1.

1 3

2 4

3 -4

4 0

5 1

4 from Unit 4, Lesson 4

Find a value of x that makes the equation true.

$-(-2x + 1) = 9 - 14x$

Explain your reasoning, and check that your answer is correct.

Learning Targets

+ I can compare inputs and outputs of functions that are represented in different ways.

Problem 3

from Unit 5, Lesson 2

Match each function rule with the value that could not be a possible input for that function.

4 **A.** Divide 3 by the input.

3 **B.** Add 4 to the input, then divide this value into 3.

1 **C.** Subtract 3 from the input, then divide this value into 1.

1. 3

2. 4

3. -4

4. 0

5. 1

Problem 4

from Unit 4, Lesson 4

Find a value of x that makes the equation true. Explain your reasoning, and check that your answer is correct.

$-(-2x + 1) = 9 - 14x$

$x = \frac{5}{8}$

Sample reasoning: This is the same as $2x - 1 = 9 - 14x$. If 1 is added to each side, that results in $2x = 10 - 14x$. If $14x$ is added to each side, then $16x = 10$. Both sides are then multiplied by $\frac{1}{16}$ to find $x = \frac{10}{16}$, or $\frac{5}{8}$. This is correct because $-\left(-2\left(\frac{5}{8}\right) + 1\right) = \frac{5}{4} - 1 = \frac{1}{4}$, and $9 - 14\left(\frac{5}{8}\right) = \frac{36}{4} - \frac{35}{4} = \frac{1}{4}$.