Lesson 11

Stained-Glass Windows (Optional)

Goals

- Apply circumference and area of circles to calculate the cost of a stained-glass window, and explain (orally and in writing) the solution method.
- Design a stained-glass window that could be built for a given dollar amount, and present (orally, in writing, and through other representations) a justification that it costs less than the limit.
- Make simplifying assumptions to solve problems about a stained-glass window.

Learning Target

I can apply my understanding of area and circumference of circles to solve more complicated problems.

Lesson Narrative

This culminating lesson is optional. In this lesson students work on several tasks that combine circumference and area ideas and computations. First, students are given a design for a stained-glass window and the prices of the different components. They decide if it would be possible to produce the window for a certain amount of money. Students must make some assumptions about the shapes in the design and about how the different materials are sold.

The following two activities are optional, so teachers can choose what best fits the needs of their students. The first optional activity asks how scaling the window will affect the cost, bringing in ideas from a previous unit. Since measurements of both length and area are involved, the total cost does not simply increase by the scale factor nor by the square of the scale factor. In the second optional activity, students invent their own design for a stained-glass window that could be produced given a cost constraint. The series of tasks provides many opportunities to engage in different aspects of mathematical modeling.

Student Learning Goal

Let's use circumference and area to design stained-glass windows.

Lesson Timeline

Activity 1	Activity 2	Activity 3
25 min	15 min	20 min

Access for Students with Diverse Abilities

- Action and Expression (Activity 1)
- Engagement (Activity 2)

Access for Multilingual Learners

- MLR3: Critique, Correct, Clarify (Activity 2)
- MLR6: Three Reads (Activity 1)
- MLR8: Discussion Supports (Activity 3)

Instructional Routines

- MLR3: Critique, Correct, Clarify
- MLR6: Three Reads
- Notice and Wonder

Required Materials

Materials to Gather

- Blank paper: Activity 2
- Compasses: Activity 2
- Geometry toolkits: Activity 2

Required Preparation

Lesson:
Four-function calculators are optional but recommended to take the focus off computation.

Instructional Routines

MLR6: Three Reads

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Access for Multilingual Learners (Activity 1)

MLR6: Three Reads.
This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Instructional Routines

Notice and Wonder

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Access for Students with Diverse Abilities (Activity 1, Student Task)

Action and Expression: Internalize Executive Functions.
To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, begin by providing students with just one of the six rectangular pieces of the window to analyze and make sense of the shapes.

Supports accessibility for: Organization, Attention

Activity 1

Cost of a Stained-Glass Window

25 min

Activity Narrative

In this activity students apply what they have learned about circles to solve a multi-step problem. Students find the area and perimeter of geometric figures whose boundaries are line segments and fractions of circles and use that information to calculate the cost of a project. The shape of the regions in the stained-glass window are left unspecified on purpose to give students an opportunity to engage in an important step of the mathematical modeling cycle—making simplifying assumptions. Assuming that the curves in the design are arcs of a circle is reasonable and expedient.

Another opportunity for mathematical modeling in this activity is to discuss if it is reasonable that a person has to pay only for the glass used in the final window and not for possible scraps of glass left over from cutting out the shapes. In reality, if they had to buy the glass at a store, the glass would likely come in square or rectangular sheets, and they would need to buy more than they were going to use. If these issues come up, encourage students to keep note of the decisions they are making and to recognize that different choices would lead to different results.

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the diagram, without revealing the question.

- For the first read, read the problem aloud and then ask,

💬 *"What is this situation about?"*

designing a stained-glass window

Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured (the length and width of the window, the cost per square foot of colored glass, the cost per square foot of clear glass, the cost of the frame).
- After the third read, reveal the question:

💬 *"Do they have enough money to cover the cost of making the window?"*

and ask,

💬 *"What are some ways we might get started on this?"*

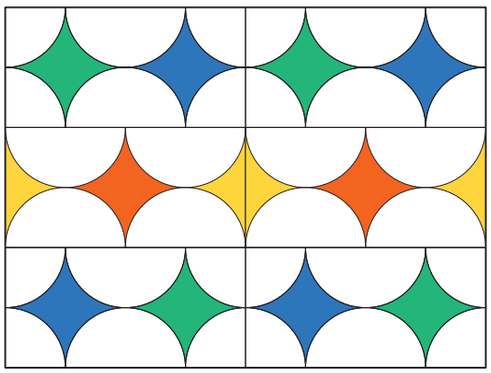
- Invite students to name some possible starting points, referencing quantities from the second read (decompose the design into parts of circles, determine the radius of the circular pieces).

Arrange students in groups of 2. Give students 8–10 minutes of partner work time followed by small-group and whole-class discussions.

As students work, prompt them to recognize any assumptions that they are making and to make them explicit.

Student Task Statement

The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 feet tall and 4 feet wide. Here is their design.



- They have raised $100 for the project.
- The colored glass costs $5 per square foot.
- The clear glass costs $2 per square foot.
- The material they need to join the pieces of glass together costs 10 cents per foot.
- The frame around the window costs $4 per foot.

Do they have enough money to cover the cost of making the window?

Yes, they need about $93.

Sample reasoning:

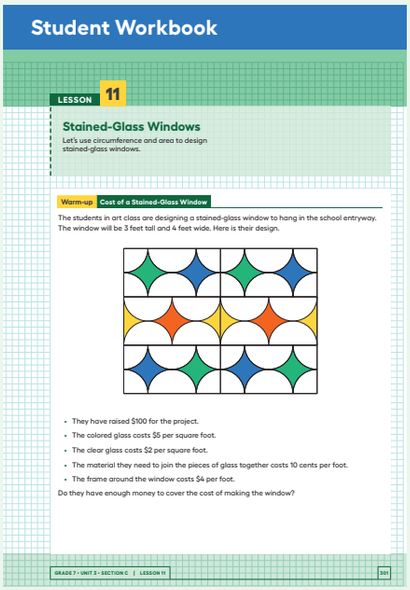
Assume that the students have to pay only for the glass they use and not for the scraps they cut away.

First, we need to find the area of the clear glass and the area of the colored glass. The entire window is 3 ft by 4 ft and has an area of 12 ft^2. There are 6 smaller rectangles. Each of these rectangles has a total of 2 full circles of clear glass, because $\frac{4}{2} = 2$, and $\frac{2}{2} + \frac{4}{4} = 2$. In the entire window, there are 12 complete circles of clear glass. Each circle has a diameter of 1 ft, a radius of $\frac{1}{2}$ ft, and an area of $\frac{1}{4}\pi$ ft^2. The area of the clear glass is $12 \cdot \frac{1}{4}\pi$, or approximately 9.42 ft^2. That means the area of colored glass is approximately $12 - 9.42$, or 2.58 ft^2.

Next, we need to find the total length of the seams between the pieces of glass and the total length of the frame around the window. The 12 circles each have a circumference of 1π ft, which makes 12π ft or about 37.68 ft of curved seams. There are also 11 ft of straight seams, because $4 + 4 + 3 = 11$. All together there are about 48.68 ft of seams. Finally, there is 14 ft of frame all the way around the window.

Next, we can calculate how much each material will cost. The clear glass will cost $9.42 \cdot 2$, or $18.84. The colored glass will cost $2.58 \cdot 5$, or $12.90. The seams will cost $48.68 \cdot 0.10$, or $4.87. The frame will cost $14 \cdot 4$, or $56. The total cost of all the materials is about $93, because $18.84 + 12.90 + 4.87 + 56.00 = 92.61$.
If these assumptions are accurate, they have just enough money to buy the materials, but if they need to pay for the scraps they cut off or if accidentally they break pieces as they go, they don't have a lot of extra money.

Student Workbook



LESSON 11

Stained-Glass Windows

Let's use circumference and area to design stained-glass windows.

Warm-up Cost of a Stained-Glass Window

The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 feet tall and 4 feet wide. Here is their design.

- They have raised $100 for the project.
- The colored glass costs $5 per square foot.
- The clear glass costs $2 per square foot.
- The material they need to join the pieces of glass together costs 10 cents per foot.
- The frame around the window costs $4 per foot.

Do they have enough money to cover the cost of making the window?

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Instructional Routines

MLR3: Critique, Correct, Clarify

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Access for Multilingual Learners (Activity 2)

MLR3: Critique, Correct, Clarify
This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Building on Student Thinking

Because there are multiple steps in solving this problem, some students may benefit from having their calculations checked along the way so that one early error does not impact the final result.

Some students may struggle to find the diameter or radius lengths. Encourage these students to cut one individual panel, separate the clear glass from the colored glass, and rearrange the figures to see how to determine the length of the diameter and radius.

Activity Synthesis

The purpose of this discussion is to highlight assumptions that students made while solving the problem.

As groups complete the activity, combine groups of 2 to make groups of 4. If possible, combine groups who solved the problem in different ways. Display the following questions for all to see and ask the groups to discuss:

- *"Did you get the same answer? Why or why not?"*
- *"Did you use the same strategy? What was the same or different in your work?"*
- *"Did you make any assumptions as you worked on the problem?"*

For the whole-class discussion, invite groups to share the similarities and differences they noticed between their small groups. As students share, revoice comparison statements and assumptions that students state. Ask for details and examples as needed to help clarify students' reasoning.

After each group shares, consider asking the class to indicate if they had any of the same conversations in their own group so as to not have repetitive explanations. Every group does not need to share if the same conversation was had.

If not mentioned in students' explanations, highlight the assumptions that the shapes are parts of circles and that the total cost takes into account only the exact area and lengths shown in the figure.

Activity 2: Optional

A Bigger Window

15 min

Activity Narrative

In this activity, students calculate the cost of a larger version of the stained-glass window from the previous activity. The window is scaled by a factor of 3. Students recognize that the lengths of the frame and seams will increase by a factor of 3, while the area of the glass will increase by a factor of 3^2.

If students observe that the material for the seams and the frame has width and that the scale factor would need to be applied to this measurement, ask them if they can make a simplifying assumption. The width of the seams is never specified or taken into account in the calculations in the previous activity so it is appropriate to continue to put this to the side, as part of the modeling process.

Launch

Keep students in the same groups. Give students 2–3 minutes of quiet work time followed by partner work time.

As students work, monitor for how students make use of the given scale factor. Select students who solve the problem in different ways to share during the whole-class discussion. If there is a student who quickly assumes they could just multiply their cost from the previous activity by 3, but later realizes why they could not do that, select them to share their reasoning.

Student Task Statement

A local community member sees the school's stained-glass window and really likes the design. They ask the students to create a larger copy of the window using a scale factor of 3.

Would $450 be enough to buy the materials for the larger window? Explain or show your reasoning.

No, $450 is not enough money. They would need about $468.27. The lengths of the seams and the frame are one-dimensional, so they scale by 3. The areas of the clear glass and the colored glass are two-dimensional, so they scale by 9. So, $18.84 \cdot 9 + 12.90 \cdot 9 + 4.87 \cdot 3 + 56.00 \cdot 3 = 468.27$

Building on Student Thinking

Some students might think that they can multiply the original cost by 3. Encourage them to compute the lengths and areas of the new window, or remind them that while the side lengths in scaled copies increase by the scale factor, the area increases by the square of the scale factor.

Activity Synthesis

The purpose of this discussion is for students to reiterate that when all the lengths in a figure are scaled by a scale factor, the area changes by (scale factor)^2.

Ask selected students to share their reasoning. If there are students who still think that $450 is enough money, ask them to share their reasoning. Discuss why you cannot just multiply the price of the original design by 3 to find the price of the scaled stained-glass window.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response by correcting errors, clarifying meaning, and adding details.

- Display this first draft:

> *"I learned that to scale something, you multiply it by the scale factor. Since they want a window that is 3 times as big, I multiplied the cost of the small window by 3. Because 93 · 3 = 279, then $450 dollars is more than enough."*

Ask,

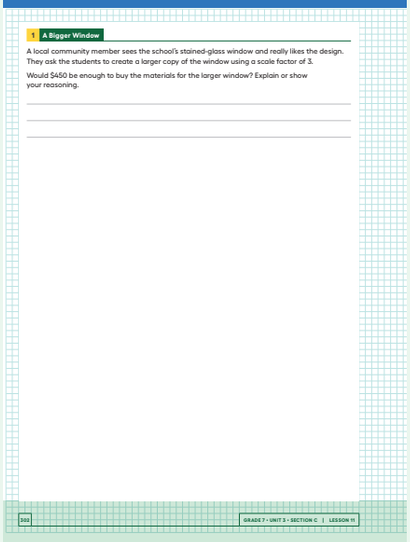
> *"What parts of this response are unclear, incorrect, or incomplete?"*

Access for Students with Diverse Abilities (Activity 2, Student Task)

Engagement: Develop Effort and Persistence.
Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "Let's try …", "We are trying to …", "We already know …", and "We will need to know …"

Supports accessibility for: Language, Social-Emotional Functioning

Student Workbook


1 A Bigger Window

A local community member sees the school's stained-glass window and really likes the design. They ask the students to create a larger copy of the window using a scale factor of 3.

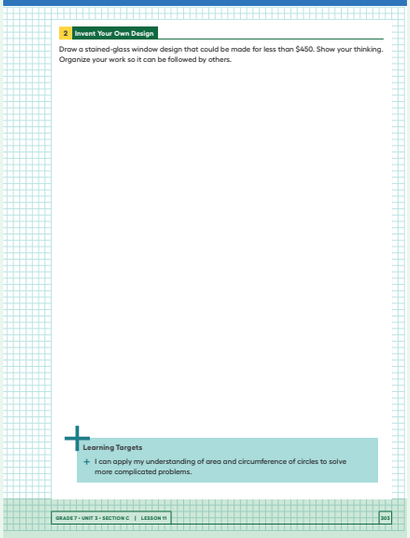
Would $450 be enough to buy the materials for the larger window? Explain or show your reasoning.

Access for Multilingual Learners (Activity 2, Student Task)

MLR8: Discussion Supports. At the appropriate time, give groups 2–3 minutes to plan what they will say if they are chosen to present to the class. "Practice what you will say when you share your design with the class. Talk about what is important to say, and decide who will share each part." During group presentations, invite the student who is not speaking to follow along and point to the corresponding parts of the display.

Advances: Speaking, Conversing, Representing

Student Workbook


2 Invent Your Own Design

Draw a stained-glass window design that could be made for less than $450. Show your thinking. Organize your work so it can be followed by others.

Learning Targets

+ I can apply my understanding of area and circumference of circles to solve more complicated problems.

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As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.

- Give students 2–4 minutes to work with a partner to revise the first draft.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft even more clear and more convincing.

Activity 3: Optional
Invent Your Own Design

20 min

Activity Narrative

In this activity students create their own stained-glass design for a given amount of money. The activity is purposefully left open to allow students to either tweak the previous design or create something completely new. As students apply what they have learned about perimeter and area, choose tools and strategies, and come up with a design that fits the given constraints, they are modeling with mathematics.

Launch

Keep students in the same groups. Encourage students to include whole or partial circles in their designs.

Student Task Statement

Draw a stained-glass window design that could be made for less than $450. Show your thinking. Organize your work so it can be followed by others.

Answers vary.

Building on Student Thinking

Some students may think that they need to create a new design and, therefore, struggle to get started. Point these students to the designs in previous activities and ask how they could modify these designs to meet the cost requirement.

Activity Synthesis

Display students' designs for all to see and ask students to explain how they knew their design met the cost requirement. Allow other students to ask questions of the student who is sharing their design. Consider displaying sentence frames to support students in asking questions of the presenters, such as:

- *"Can you say more about … ?"*
- *"Why did you … ?"*
- *"How do you know … ?"*