Chance Experiments

Goals

- Comprehend and use the terms "impossible," "unlikely," "equally likely as not," "likely," and "certain" (in spoken and written language) to describe the likelihood of an event.
- Interpret percentages, fractions, and decimals that represent the likelihood of events.
- Order a given set of events from least likely to most likely, and justify (orally) the reasoning.

Learning Targets

- I can describe the likelihood of events using the words "impossible," "unlikely," "equally likely as not," "likely," and "certain."
- I can tell which event is more likely when the chances of different events are expressed as fractions, decimals, or percentages.

Access for Students with Diverse Abilities

• Engagement (Activity 1)

Access for Multilingual Learners

- MLR2: Collect and Display (Activity 2)
- MLR8: Discussion Supports (Activity 1)

Instructional Routines

Card Sort

Required Materials

Materials to Gather

• Number cubes: Activity 1, Activity 2

Materials to Copy

 Likelihood Cards (1 copy for every 2 students): Activity 3

Lesson Narrative

In this lesson students investigate chance experiments. A **chance experiment** is something that can be repeated with results that are based on chance. The result of a chance experiment is called an *outcome*. They use language like "impossible," "unlikely," "equally likely as not," "likely," and "certain" to describe the likelihood of a chance event. An **event** is a set of one or more outcomes in a chance experiment. Students make sense of situations and sort them into these categories based on their intuition about how often they expect an event to occur. In some cases, a value is assigned to the likelihood of an event using a fraction, decimal, or percentage chance. By comparing informal categories early and numerical quantities later, students are attending to precision when sorting the scenarios. Later, students will connect this language to more precise numerical values on their own.

Student Learning Goal

Let's investigate chance.

Lesson Timeline

5 min

Warm-up

10 min

Activity 1

10 min

Activity 2

10 min

Activity 3

10 min

Lesson Synthesis

Assessment

5_{min}

Cool-down

Warm-up

Which Is More Likely?



Activity Narrative

The purpose of this *Warm-up* is to engage students' intuition about likelihood of events. The following activities in this lesson continue to develop more formal ways of thinking about likelihood leading to the definition of probability in the next lesson.

Launch 🞎

Arrange students in groups of 2.

Give students 2 minutes of quiet work time and time to share their response with a partner.

Follow with a whole-class discussion.

Student Task Statement

Which is more likely to happen? Explain your reasoning.

- **A.** When reaching into a dark closet and pulling out one shoe from a pile of 20 pairs of shoes, a left shoe is pulled out.
- **B.** When listening to a 5-song playlist in shuffle mode, the first song on the playlist plays first.

Sample reasoning: It is more likely that a left shoe is pulled out than the first song on the playlist being played first in shuffle mode. Since the shoes come in pairs, it is equally likely that a left or right shoe would be pulled out, so half of the time we would expect to get a right shoe. For the playlist, there are 5 different songs that could play first, and only I of them is the first song on the list.

Activity Synthesis

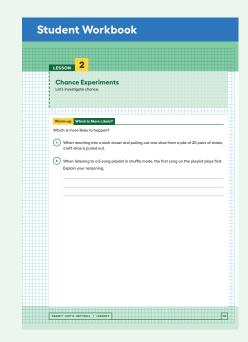
The purpose of the discussion is to help students recognize their own intuition about the likelihood of an event even when prior outcomes are not available.

Ask partnerships to share responses with the class, and ask at least one student that chose each option for their reasoning.

It may be helpful to reiterate that the likelihood of these actions only give a sense of what might be expected to happen. Even though the first situation is more likely to happen, it is possible someone might pull out a right shoe and have the first song on the list play first, but it might be a little more surprising than if someone pulled out a left shoe and a different song played first.

Building on Student Thinking

Students may think that it is required to pull out a specific shoe rather than any left shoe. Ask students to visualize the problem and determine how many left shoes are in the closet.



Activity 1

How Likely Is It?



Activity Narrative

As preparation for talking about probability, students are asked to engage their intuition about the concept by loosely grouping scenarios into categories based on their likelihood. Some of the categories are meant to be loosely interpreted, while others such as "certain" and "impossible" have more precise meanings. Students should be able to construct arguments for their classification.

Launch 🙎

Arrange students in groups of 2.

Tell students that a "standard number cube" is a cube that has the numbers 1 through 6 printed on it so that each face shows a different number. This item will be referred to throughout the unit.

Describe a **chance experiment** as something that can be done for which the result is not known. The result of a chance experiment is called an **outcome**. For example, opening a 100-page book to a random page is a chance experiment because the page one opens to is not known. The outcome might be that it opened to page 42.

Define an **event** as a result of a chance experiment that someone is interested in. This could be one or more outcomes. For example, one might open a 100-page book to a random page and be interested in the event that they open to an even-numbered page.

It may help students to understand the categories of likelihood with an example of opening a book to a random page:

- Impossible: opening a 100-page book to page -300
- Unlikely: opening a 100-page book to page 45
- Equally likely as not: opening a 100-page book to a page numbered less than 51
- Likely: opening a 100-page book to a page numbered greater than 10
- Certain: opening a 100-page book to a page numbered less than 1,000

Allow students 5–7 minutes quiet work time followed by partner and whole-class discussion.

Student Task Statement

1. For each **chance experiment**, label the **event** with one of these options: impossible, unlikely, equally likely as not, likely, certain.

a. A raffle prize is given to the person holding 1 ticket drawn from the 100 given out.

Event: One of the 2 tickets you have is chosen.

unlikely

b. You time how long you wait before someone takes your order at a fast-food restaurant.

Event: It takes less than 10 minutes.

likelu

c. You roll a standard number cube.

Event: You get an even number on top.

equally likely as not

d. A random 4-year-old child is selected and their height is measured.

Event: The chosen child is over 6 feet tall.

impossible (unlikely is acceptable)

e. You write down the weather on July 1.

Event: It snowed on that day.

impossible (or possibly unlikely depending on the location of your school)

f. You toss a ball into water.

Event: The ball gets wet.

certain

g. You spin the spinner here and find out which part the arrow points to.

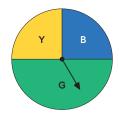
Event: It points to green.

equally likely as not

h. You spin the spinner here and find out which part the arrow points to.

Event: It points to red.

impossible



Access for Students with Diverse Abilities (Activity 1, Student Task)

Engagement: Provide Access by Recruiting Interest.

Leverage choice around perceived challenge. Invite students to select 5–6 of the situations to complete.

Supports accessibility for: Organization, Social-Emotional Functioning

Access for Multilingual Learners (Activity 1, Student Task)

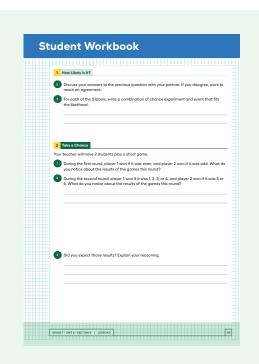
MLR8: Discussion Supports.

Invite students to begin partner interactions by repeating the question, "Is the event impossible, unlikely, equally likely as not, likely, or certain?" This gives both students an opportunity to produce language.

Advances: Conversing

Student Workbook





2. Discuss your answers to the previous question with your partner. If you disagree, work to reach an agreement.

No response needed.

3. For each of the 5 labels, write a combination of chance experiment and event that fits the likelihood.

Sample response:

- Impossible: You choose a rectangle with side lengths selected at random.

 The chosen rectangle has 5 different side lengths.
- Unlikely: A peacock is chosen at random.
 It is albino.
- Equally likely as not: A coin is flipped. It lands with tails showing.
- Likely: A box of crayons is selected from a store. A red crayon is in the box.
- Certain: A triangle is selected.
 It has 3 sides.

Activity Synthesis

The purpose of this discussion is for students to see that the loose categories can be understood in a more formal way. Some students may begin to attach numbers to the likelihood of the events and that is a good way to begin the transition to thinking about probability.

There may be some discussion about the category "equally likely as not." Are events in this category required to be at exactly 50%, or would an event with a 55% or 48% likelihood be placed in this category as well? At this stage, the categories are meant to be loose, so it is not necessary that everyone agrees on what goes in each category.

Discuss questions such as:

- When an event is close to the edge between the categories
 - When an event is close to the edge between the categories, it can be hard to label it well.
- "Which categories are the most strict about what can go in them?"
 - "Certain" should represent scenarios that must happen with 100% certainty. "Impossible" should represent scenarios that cannot happen.
- "What does it mean for an event to be certain?"

that it must happen

"What does it mean for an event to be likely?"

between about 50% and 100% chance

Take a Chance



Activity Narrative

In this lesson, students begin to move towards a more quantitative understanding of likelihood by observing a game that has two rounds with different requirements for winning in each round. The game is also played multiple times to help students understand that the actual number of times an outcome occurs may differ from expectations based on likelihood at first, but should narrow towards the expectation in the long-run. By repeating the process many times, students recognize a structure beginning to form with the results. Activities in later lessons will more formally show this structure forming from the repeated processes.



Arrange students in groups of 2.

Following the demonstration game, allow 5 minutes for partners to play the game and answer the questions followed by a whole-class discussion.

Select 2 students to play this game of chance that consists of 2 rounds. Give 1 standard number cube to each group.

Round 1: Player 1 wins with evens and player 2 wins with odds. Roll the number cube. Record whether evens or odds wins the round. After each time, switch the roles. Demonstrate this round twice.

Round 2: In round 2, assign player 1 the numbers 1, 2, 3, or 4 to win, while player 2wins with 5 or 6. Roll the number cube. Record whether 1, 2, 3, or 4 wins the round, or 5 or 6 wins the round. After each time, switch the roles. Demonstrate this round twice.

Have groups play this game of chance by playing the first round 10 times, then playing the second round 10 times. Display the results of each round for all to see. For example:

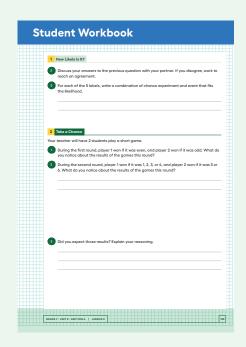
	round 1 winner	round 2 winner
game 1	player 2	player 1
game 2	player 1	player 1
game 3	player 2	player 2
game 4	player 1	player 1

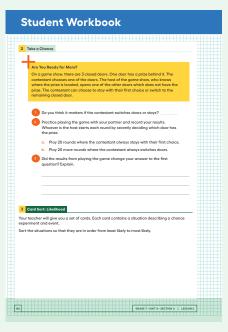
Access for Multilingual Learners (Activity 2, Student Task)

MLR2: Collect and Display.

Collect the language students use to describe the likelihood of either player winning each round of the game. Display words and phrases such as "more ways to win," "greater percentage," or "4 out of 6 outcomes." During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.

Advances: Conversing, Reading





Student Task Statement

Your teacher will have 2 students play a short game.

- 1. During the first round, player 1 won if it was even, and player 2 won if it was odd. What do you notice about the results of the games this round?

 Sample response: I notice that there was not really an advantage for either
 - Sample response: I notice that there was not really an advantage for either player in this round.
- **2.** During the second round, player 1 won if it was 1, 2, 3, or 4, and player 2 won if it was 5 or 6. What do you notice about the results of the games this round?
 - Sample response: I notice that player I won more often than player 2.
- **3.** Did you expect those results? Explain your reasoning.
 - Sample response: I expected player I to usually win.
 - Sample reasoning: Player I had 4 ways to win, while the other player only had 2.

Are You Ready for More?

On a game show, there are 3 closed doors. One door has a prize behind it. The contestant chooses one of the doors. The host of the game show, who knows where the prize is located, opens one of the *other* doors which does not have the prize. The contestant can choose to stay with their first choice or switch to the remaining closed door.

- Do you think it matters if the contestant switches doors or stays?
 Sample response: It appears like it doesn't matter: there are two doors left, one has the prize, and one doesn't.
- Practice playing the game with your partner and record your results.Whoever is the host starts each round by secretly deciding which door has the prize.
 - a. Play 20 rounds where the contestant always stays with their first choice.
 - **b.** Play 20 more rounds where the contestant always switches doors.

No response needed.

- **3.** Did the results from playing the game change your answer to the first question? Explain.
 - Sample response: Switching should win more often than not switching, but other results are possible by chance.

Activity Synthesis

The purpose of this discussion is to begin moving students towards quantifying the likelihood of events. Invite students to share their answers and reasoning to the last question.

Some questions for discussion:

"What was the chance experiment in this game?"

rolling the number cube

"What number would you assign to the likelihood of evens winning in the first round? What about the likelihood of 1–4 winning in the second round?"

For the first round of the game, there is a 50% or $\frac{1}{2}$ chance for evens to win. For the second round of the game, I-4 wins about 67% or $\frac{4}{6}$ or $\frac{2}{3}$ of the time.

"What percentage or fraction would you assign to waiting for less than 10 minutes before your order is taken at a fast-food restaurant?"

Values between 80% and 100% are expected. It is not necessary to agree on a particular value at this point.

"How could we get more evidence to support these answers?"

Collect data from fast-food restaurants to find the fraction of customers who order their food within IO minutes.

Activity 3

Card Sort: Likelihood

10 min

Activity Narrative

Students sort different descriptions of chance experiments and events during this activity. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections.

Launch 22

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

Give students 1 minute to place all the cards face up and start thinking about possible ways to sort the cards into categories.

- Pause the class and select 1–3 students to share the categories they identified.
- Discuss as many different categories as time allows.

Attend to the language that students use to describe their categories and situations, giving them opportunities to describe their situation more precisely. Highlight the use of terms like "percentage," "fraction," "decimal," "more likely," and "less likely." After a brief discussion, invite students to continue with the activity.

Instructional Routines

Card Sort

ilclass.com/r/10783726

Please log in to the site before using the QR code or URL.



Building on Student Thinking

Students may have trouble understanding the "rock paper scissors" context. Tell these students that a player randomly chooses one of the three items to play in each round. If students still struggle, tell them that each of the three items are expected to be played with equal likelihood.



Student Task Statement

Your teacher will give you a set of cards. Each card contains a situation describing a chance experiment and event.

Sort the situations so that they are in order from least likely to most likely.

numbered balls, left-handed, letter T, rain, rock, toothpaste, cards, flies, medical test, pattern blocks

Activity Synthesis

The purpose of the discussion is for students to talk about the methods they used to sort the cards and compare likelihood of different situations.

Some questions for discussion:

- "How are the numerical values of the likelihoods written?"
 Some are written as percentages, some as fractions, and some as decimal values.
- (in the situations when there is a mix of percentages, fractions, and decimals?"

We convert the percentages to fractions of the form $\frac{P}{100}$ and fractions to decimals by dividing, then order the decimals.

"Some of the cards do not have a percentage, fraction, or decimal. How can you determine where those cards would go in the order?"

We think of them as fractions with the number of ways the event might happen in the numerator and the number of ways the chance experiment might result as the denominator.

Lesson Synthesis

Ask students,

☐ "What is a chance experiment?"

In groups of 2, have partners come up with a chance experiment and examples of events that are:

- · Impossible.
- · Unlikely.
- Equally likely as not.
- · Likely.
- · Certain.

Ask partners to share responses with the class.

Lesson Summary

A **chance experiment** is something that can be done over and over again, and what ends up happening is unknown before doing the experiment. For example, flipping a coin is a chance experiment because we don't know if the result will be a head or a tail. An **outcome** of a chance experiment is one of the things that can happen when the experiment is done. For example, when a coin is flipped, one possible outcome is that it will be a head. An **event** is a set of one or more outcomes.

We can describe events using these words and phrases:

- Impossible
- Unlikely
- · Equally likely as not
- Likely
- Certain

For example, if we flip a coin:

- It is impossible that the coin will turn into a bottle of ketchup.
- It is unlikely that the coin will land on its edge.
- It is equally likely as not that we will get a tail.
- It is likely that you will get a head or a tail.
- It is certain that the coin will land somewhere.

The *probability* of an event is a measure of the likelihood that an event will occur. We will learn more about probabilities in the lessons to come.



Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Cool-down

According To



Student Task Statement

Here are some situations:

- According to market research, a business has a 75% chance of making money in the first three years.
- According to lab testing, $\frac{5}{6}$ of a certain kind of experimental light bulb will work after three years.
- According to experts, the likelihood of a car needing major repairs in the first three years is 0.7.
- 1. Write these situations in order of likelihood from least to greatest after three years: The business makes money, the light bulb still works, and the car needs major repairs.

The car needs major repairs, the business makes money, the light bulb still works.

2. Select one of the situations. Write a chance experiment and event that has the same likelihood as the situation you selected.

Sample responses:

- · flipping two coins and at least one not landing on heads
- rolling a standard number cube and it landing with any number other than I face up
- selecting a number greater than 3 when selecting a number between I and IO randomly

Practice Problems

6 Problems

Problem 1

The likelihood that Han makes a free throw in basketball is 60%. The likelihood that he makes a 3-point shot is 0.345. Which event is more likely, Han making a free throw or making a 3-point shot? Explain your reasoning.

It is more likely that Han makes a free throw.

Sample reasoning: Since 0.345 is less than 0.5, making a 3-pointer is an unlikely event. Since 60% is greater than 50%, making a free throw is a more likely event.

Problem 2

Different events have these likelihoods. Sort them from least to greatest:

60% 8 out of 10 0.37 20%

20%, 0.37, 60%, 8 out of 10, $\frac{5}{6}$

Problem 3

There are 25 prime numbers between 1 and 100. There are 46 prime numbers between 1 and 200. Which situation is more likely? Explain your reasoning.

- A computer produces a random number between 1 and 100 that is prime.
- A computer produces a random number between 1 and 200 that is prime.

A computer produces a random number between I and IOO that is prime. Sample reasoning: There is a 25% chance of getting a prime number from the first IOO numbers and only a 23% chance from the first 200 numbers.

Problem 4

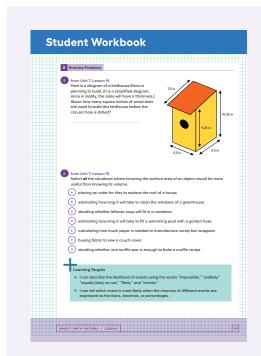
from Unit 4, Lesson 2

It takes $4\frac{3}{8}$ cups of cheese, $\frac{7}{8}$ cups of olives, and $2\frac{5}{8}$ cups of sausage to make a signature pizza. How much of each ingredient is needed to make 10 pizzas? Explain or show your reasoning.

number of pizzas	cups of cheese	cups of olives	cups of sausage
I	<u>35</u> 8	7/8	<u>21</u> 8
10	$\frac{350}{8}$ (or equivalent)	$\frac{70}{8}$ (or equivalent)	$\frac{210}{8}$ (or equivalent)

With decimals, the answers are 43.75 cups of cheese, 8.75 cups of olives, and 26.25 cups of sausage.

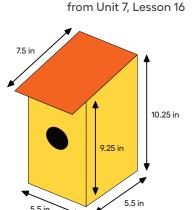
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PRA	CTICE PROBLEMS	-
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0	Different events have these likelihoods. Sort them from least to greatest:	
_	60% 8 out of 10 0.37 20% 5	
	•	
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	reasoning.	



Problem 5

Here is a diagram of a birdhouse Elena is planning to build. (It is a simplified diagram, since in reality, the sides will have a thickness.) About how many square inches of wood does she need to build this birdhouse before the circular hole is drilled?

286 square inches



Problem 6

from Unit 7, Lesson 15

Select **all** the situations where knowing the surface area of an object would be more useful than knowing its volume.

- **A.** placing an order for tiles to replace the roof of a house
- **B.** estimating how long it will take to clean the windows of a greenhouse
- C. deciding whether leftover soup will fit in a container
- **D.** estimating how long it will take to fill a swimming pool with a garden hose
- **E.** calculating how much paper is needed to manufacture candy bar wrappers
- **F.** buying fabric to sew a couch cover
- G. deciding whether one muffin pan is enough to bake a muffin recipe

LESSON 2 • PRACTICE PROBLEMS