Demand Estimation

Mixtape Session

Jeff Gortmaker and Ariel Pakes

Last Class

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$$\text{subject to} \quad s_{jt} = \sum_{i \in \mathcal{I}_t} w_{it} \cdot \frac{\exp[\delta_{jt} + \mu_{ijt}(\theta)]}{1 + \sum_{k \in \mathcal{J}_t} \exp[\delta_{kt} + \mu_{ikt}(\theta)]}$$

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- Let's go over your second coding exercise.

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  - $\rightarrow$ Results in unrealistically limited substitution between similar cereals.
- Also can't estimate a parameter in Π on log income alone.
  - $\rightarrow$ Market fixed effects are collinear with market-level income means.
  - $\rightarrow$ Unrealistic that overall cereal preference doesn't vary with income.

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- Typical example is consumer survey data.
  - → Internal surveys conducted by firms.
  - → Ad-hoc surveys conducted by academics.
  - → Marketing research datasets (e.g. NielsenIQ’s Consumer Panel).
  - → Regulatory agencies like the UK’s antitrust authority (Reynolds and Walters, 2008).

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- Let's incorporate answers to these questions into estimation.
  - $\rightarrow$ We'll set up a general framework and come back to these when we have notation to do so.

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

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  - $\rightarrow$ Ratios (e.g. mean income given mushy), correlations (e.g. between income and price), etc.
- The resulting "micro BLP" estimator is used a lot in industrial organization.

Micro BLP Popularity

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Petrin (2002)	Automobiles	United States	1981–1993
Berry, Levinsohn, and Pakes (2004)	Automobiles	United States	1993
Thomadsen (2005)	Fast Food	United States	1999
Goeree (2008)	Personal Computers	United States	1996–1998
Ciliberto and Kuminoff (2010)	Cigarettes	United States	1993–2002
Nakamura and Zerom (2010)	Coffee	United States	2000–2004
Beresteanu and Li (2011)	Automobiles	United States	1999–2006
Li (2012)	Automobiles	United States	1999–2006
Copeland (2014)	Automobiles	United States	1999–2008
Starc (2014)	Health Insurance	United States	2004–2008
Ching, Hayashi, and Wang (2015)	Nursing Homes	United States	1999
Li, Xiao, and Liu (2015)	Automobiles	China	2004–2009
Nurski and Verboven (2016)	Automobiles	Belgium	2010–2011
Barwick, Cao, and Li (2017)	Automobiles	China	2009–2011
Murry (2017)	Automobiles	United States	2007–2011
Wollmann (2018)	Commercial Vehicles	United States	1986–2012
Li (2018)	Automobiles	China	2008–2012
Li, Gordon, and Netzer (2018)	Digital Cameras	United States	2007–2010
Backus, Conlon, and Sinkinson (2021)	Cereal	United States	2007–2016
Grieco, Murry, and Yurukoglu (2021)	Automobiles	United States	1980–2018
Neilson (2021)	Primary Schools	Chile	2005–2016
Armitage and Pinter (2022)	Automobiles	United States	2009–2017
Döpper, MacKay, Miller, and Stiebale (2022)	Retail	United States	2006–2019
Durrmeyer (2022)	Automobiles	France	2003–2008
Weber (2022)	Trucks	United States	2010–2018
Bodéré (2023)	Preschools	United States	2010–2018
Montag (2023)	Laundry Machines	United States	2005–2015
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- We'll use the standardized framework for PyBLP from Conlon and Gortmaker (2023).

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    1. Micro statistics $f(\overline{v}) = [f_1(\overline{v}), \ldots, f_M(\overline{v})]'$.
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- Statistically, we need $f(\overline{v}) \to f(v(\theta_0))$ as the micro dataset expands.
    - $\to$ This gives what we'll call $m = 1, \ldots, M$ different "micro moments."
    - $\to$ A bit different from our "aggregate moments" $\mathbb{E}[\xi_{jt} \cdot z_{jt}] = 0$.

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- Different weights w_{dijt}, values v_{pijt}, and functions $f_m(\cdot)$ support most summary stats.

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- Lastly, you need to define your micro moment m.
  - $\rightarrow$ The identity function $f_m(\overline{v}_p) = \overline{v}_p$ just matches the mean surveyed income.
  - $\rightarrow$ You also need to specify the actual value of the micro statistic $\overline{v}_1$.

Model Analogues

$$f_m(\overline{v}_p) \to f_m(v_p(\theta_0))$$

- For each guess of θ, PyBLP will compute the model analogue $f_m(v_p(\theta))$.

Model Analogues

$$f_m\left(\frac{1}{N_d}\sum_{n\in\mathcal{N}_d} v_{pi_nj_nt_n}\right) \to f_m\left(\frac{\cdots v_{pijt}}{\cdots}\right)$$

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  4. Selected to be in the survey with known probability $w_{di_nj_nt_n}$.

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

Adding Micro Moments

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  - → Start by choosing a single micro moment that "targets" the parameter.
- What if you could estimate a parameter with either aggregate or micro variation?
  - → Could just choose the variation that seems more "credible." Often the micro moment.
  - → Can use both. Micro moments can reduce large SEs from limited aggregate variation.

Targeting Micro Moments

- Simplest case: 1 characteristic x_{jt} (e.g. price), 1 demographic y_{it} (e.g. income).

$$u_{ijt} = \beta_1 + \pi_1 y_{it} + (\beta_x + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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  - $\rightarrow$ Relationship between income and price targets how income shifts price sensitivity.
  - $\rightarrow$ Other common examples include "$\mathbb{E}[y_{it} \mid x_{jt} < \overline{x}]$" and "$\mathbb{E}[x_{jt} \mid y_{it} < \overline{y}]$."

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- Micro data is not directly informative about "linear parameters" β_1 or β_x.
  - $\rightarrow$ Mean utility $\delta_{jt} = \beta_1 + \beta_x x_{jt} + \xi_{jt}$ is already pinned down by market shares s_{jt}.

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- Direct measures of substitution are very informative about Σ.
  - $\rightarrow$ Recall the red bus/blue bus example that motivated adding preference heterogeneity.
  - $\rightarrow$ Each second choice is like observing a new market with the first choice removed.

Second Choice Moments

- Extend the simple example from before with unobserved preference heterogeneity:

$$u_{ijt} = \beta_1 + \sigma_1 \nu_{1it} + \pi_1 y_{it} + (\beta_x + \sigma_x \nu_{2it} + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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  - $\rightarrow$ In your exercise, you'll match the share "$\mathbb{P}(\text{mushy}_{jt} \text{ and } \text{mushy}_{kt} \mid j \neq 0)$."
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- Diversion ratios straightforward to interpret and collect.

Outside Substitution and Market Size

- Estimating a σ_1 is important if you're interested in inside-outside substitution.
  - $\rightarrow$ How many consumers will stop purchasing soda if all sodas are taxed?
  - $\rightarrow$ How much larger will a market grow if innovation reduces production costs?

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- On day 1, we discussed how choosing a market size is neither easy nor innocuous.
  - $\rightarrow$ Assuming a small market size M_t means assuming a small outside share s_{0t}.
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  - $\rightarrow$ Implies little substitution to the outside good in counterfactuals.
- Directly matching an outside diversion ratio will help discipline outside substitution.
  - $\rightarrow$ If $\hat{\sigma}_1$ is large, many people will dislike all inside goods and usually choose $j = 0$.
  - $\rightarrow$ This reduces the *effective* market size, helping to compensate for a too-large M_t.
  - $\rightarrow$ See Zhang (2023) for more on how σ_1 can help and other solutions.

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  - → Clarity: Matching a single statistic makes it clear where identification comes from.

- But adding more info can greatly increase the precision of our estimates.
  - → Ideally we'd observe a complete micro dataset $\{t_n, j_n, k_n, y_{i_n t_n}\}_{n \in \mathcal{N}_d}$.

Maximum Likelihood

- If we only had micro data, we may want to just work with its log likelihood: (technically, this likelihood is conditional on the aggregate data)

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- For a modern take on this "MLE" approach, see Grieco, Murry, Pinkse and Sagl (2023).
  - $\rightarrow$ Combine 1, 2, and the likelihood for aggregate market shares into a single objective.
  - $\rightarrow$ Their Julia package Grumps.jl efficiently handles the high-dimensional $\delta = \{\delta_{jt}\}_{j,t}$.

Optimal Micro Moments

- In micro BLP, optimal micro moments match the first-order conditions in MLE:

$$f^*(\overline{v}) = \frac{1}{N_d} \sum_{n \in \mathcal{N}_d} \frac{\partial \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta)}{\partial \theta}$$

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- These use all the information in a micro dataset (Conlon and Gortmaker, 2023).
  - $\rightarrow$ Intuition for statistical efficiency here is just that MLE is efficient.
- Can be a bit tricky to compute, but only a few lines of code with PyBLP.
  - $\rightarrow$ Like optimal IVs, can update along with the weighting matrix for a second GMM step.

Roadmap

Coding Exercise 3

- This is the end of the class, but try to do the third exercise on your own.
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- I'll post the remaining solutions today after questions.

Good luck with estimating your own demand systems!

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