

Demand Estimation

MIXTAPE SESSION

Jeff Gortmaker and Ariel Pakes



Who Am I?

- A fifth-year Economics PhD candidate at Harvard University.

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- A fifth-year Economics PhD candidate at Harvard University.
- Making BLP-style estimation more accessible to researchers.
 - Best practices papers ([Conlon and Gortmaker, 2020, 2023](#)).
 - Open-source Python package ([PyBLP](#)).
 - This course!

This Course

- Three days, 6pm-9pm.
 1. Today: BLP model, pure logit, price endogeneity.
 2. Wednesday: Mixed logit, identification, numerical best practices.
 3. Friday: Micro BLP, consumer survey data, other extensions.

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- Ask questions in the Discord chat!
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- Three coding exercises, one after each day.
 - Try these on your own or with your classmates' help. Use Discord rooms!
 - I'll do the first two exercises live at the start of days 2 and 3. We'll post solutions.

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- None of these are required for the course, but I recommend taking a look afterwards.

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- BLP can be used to better understand all sorts of decisions.
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 - Product purchases, hospital visits, school choice, voting behavior, etc.
- Typically used for **counterfactual analysis** of something that hasn't happened.
 - Need a structural model when we can't just estimate a treatment effect.
- Running example: **What if we halved an important product's price?**
 - Practitioners: Increased sales vs. cannibalization?
 - Regulators: Revenue loss from eliminating a tax?
 - Academics: Welfare consequences?

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

Model Overview

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 - Original [BLP \(1995\)](#) also modeled firm price-setting. We'll focus on demand.

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 - Different demographics and preferences.
- Individuals are faced with [choices](#) denoted by $j \in \mathcal{J}_t$.
 - Products, hospitals, candidates, etc.
 - Outside option $j = 0$: no purchase, no treatment, no vote, etc.

Utility Maximization

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt}$$

- Individuals choose an alternative to maximize (indirect) utility u_{ijt} .
 - We will specify a function for u_{ijt} and use revealed preference to estimate it.

Utility Maximization

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 3. Idiosyncratic heterogeneity ε_{ijt} : Superimposed noise that accommodates estimation.
- We will parameterize δ_{jt} and μ_{ijt} and make a convenient assumption about ε_{ijt} .

Aggregate Market Shares

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- Assume a convenient distribution for ε_{ijt} : iid type I extreme value.

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$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt} \implies s_{ijt} = \mathbb{P}_{\varepsilon_{it}} \left(u_{ijt} \geq u_{ikt} \text{ for all } k \in \mathcal{J}_t \cup \{0\} \right)$$

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 - Possible to eliminate ε_{ijt} but computation gets difficult (Berry and Pakes, 2007).

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- We'll match these to observed quantities $q_{jt} = s_{jt} \cdot M_t$ in our data.

Choosing a Market Size

- In our data, we observe quantities $q_{jt} = s_{jt} \cdot M_t$.
 - Need to divide by some market size M_t to get our model's market shares s_{jt} .
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- But typically, the choice of market size is **neither easy nor innocuous**.
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- You should try different assumptions and see how they change your results.
 - In general, the bigger the market size, the more substitution to the outside good.
 - We'll learn how to discipline these assumptions with data on day 3.

Identification and Normalizations

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt}$$

- We will estimate our utility function with **revealed preference**.
 - Holding μ_{ijt} fixed, a higher quantity $q_{jt} > q_{kt}$ implies a higher mean utility $\delta_{jt} > \delta_{kt}$.

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- Utility is invariant to positive affine transformations. Need two normalizations.

$$u_{ijt} > u_{ikt} \quad \overset{b>0}{\iff} \quad a + b \cdot u_{ijt} > a + b \cdot u_{ikt}$$

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- Now that our model can in theory be identified, how do we estimate it?

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

Pure Logit Model

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt} = \delta_{jt} + \cancel{\mu_{ijt}}^0 + \varepsilon_{ijt}$$

- Start with the simplest case: no heterogeneous utility. We'll add μ_{ijt} back on day 2.

Pure Logit Model

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- Market shares simplify. No aggregation over individual types.
 - The 1 in the denominator is from our level normalization $u_{i0t} = \varepsilon_{i0t}$, i.e. $\delta_{0t} = 0$.
- We can recover mean utilities from observed market shares (Berry, 1994).
 - If we specify a function for δ_{jt} , we'll have a linear regression!

Pure Logit Estimating Equation

$$\log \frac{s_{jt}}{s_{0t}} = \delta_{jt}$$

- Running example: What if we halved an important product's price?
 - In your exercise, products j are breakfast cereals; markets t are city-quarters.
 - If we estimate the model, we can change p_{jt} and estimate how consumers react.

Pure Logit Estimating Equation

$$\log \frac{s_{jt}}{s_{0t}} = \delta_{jt} = \alpha p_{jt} + x'_{jt} \beta + \xi_{jt}$$

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 - If we estimate the model, we can change p_{jt} and estimate how consumers react.
- Specify δ_{jt} as a function of price p_{jt} and other product characteristics x_{jt} .
 - In your exercise, p_{jt} is per serving; x_{jt} includes a constant, a “mushy” dummy, etc.

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 - In your exercise, p_{jt} is per serving; x_{jt} includes a constant, a “mushy” dummy, etc.
- Interpret the regression error ξ_{jt} as unobserved product quality not in our data.
 - Unobserved characteristics, advertising, average taste variation, “demand shocks,” etc.

Interpreting Parameters

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 - Instead, report own-price elasticities, or a quantity-weighted average/median.
 - You can derive elasticities by differentiating the multinomial logit expression for s_{jt} .

$$\eta_{jjt} = \frac{\partial \log q_{jt}}{\partial \log p_{jt}} = \frac{\partial q_{jt}}{\partial p_{jt}} \frac{p_{jt}}{q_{jt}} = \frac{\partial s_{jt}}{\partial p_{jt}} \frac{p_{jt}}{s_{jt}} = \alpha \cdot p_{jt} \cdot (1 - s_{jt})$$

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- If x_{jt} is a “mushy” cereal dummy, β is “utils” from mushyness. Again, not helpful.
 - Instead, report β/α , the dollar willingness to pay for mushyness.

Roadmap

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Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

Endogeneity Concerns

$$\delta_{jt} = \alpha p_{jt} + x'_{jt}\beta + \xi_{jt}$$

- In your coding exercise, you'll run an OLS regression of δ_{jt} on p_{jt} and x_{jt} .

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 - Firms know more than us about demand when setting prices.
 - Often, $\mathbb{C}(p_{jt}, \xi_{jt}) > 0$, so $\hat{\alpha} < 0$ is biased towards zero. \mathbb{C} means covariance.

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- Today we'll focus on handling just price endogeneity for simplicity.

Fixed Effects

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- Adding product and market fixed effects to x_{jt} can eliminate a lot of bias.
 - E.g. if p_{jt} is correlated with fixed effects ξ_j and/or ξ_t in $\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}$.
 - But do need multiple observations per product and market to add ξ_j and ξ_t .

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 - Aside: Related to dynamic panel approach. Let $\xi_{jt} = \rho\xi_{jt-1} + \Delta\xi_{jt}$, estimate ρ .

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 - Aside: Related to dynamic panel approach. Let $\xi_{jt} = \rho\xi_{jt-1} + \Delta\xi_{jt}$, estimate ρ .
- Modern grocery scanner datasets have many thousands of products/markets.
 - Dummies take too much memory, so we “absorb” them, i.e. iteratively de-mean.
 - Stata: [Reghdfe](#). R: [Fixest](#). Python: [PyFixest](#). Coding exercise: [PyBLP](#) via [PyHDFE](#).

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- Adding product and market fixed effects to x_{jt} can eliminate a lot of bias.
 - E.g. if p_{jt} is correlated with fixed effects ξ_j and/or ξ_t in $\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}$.
 - But do need multiple observations per product and market to add ξ_j and ξ_t .
 - Aside: Related to dynamic panel approach. Let $\xi_{jt} = \rho\xi_{jt-1} + \Delta\xi_{jt}$, estimate ρ .
- Modern grocery scanner datasets have many thousands of products/markets.
 - Dummies take too much memory, so we “absorb” them, i.e. iteratively de-mean.
 - Stata: [Reghdfe](#). R: [Fixest](#). Python: [PyFixest](#). Coding exercise: [PyBLP](#) via [PyHDFE](#).
- Helpful but insufficient: ξ_{jt} typically varies by product *and* market, e.g. $\mathbb{C}(p_{jt}, \Delta\xi_{jt}) > 0$.

Instrumental Variables

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- With or without fixed effects, a carefully-chosen IV can be a good solution.
→ Relevance: $\mathbb{C}(p_{jt}, z_{jt}) \neq 0$. Exclusion: $\mathbb{C}(\xi_{jt}, z_{jt}) = 0$.

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 - Does the sign of the coefficient on z_{jt} make sense?
 - Is the instrument strong, or should you worry about weak instruments?
- Many places to look. I'll discuss the most common ones.

Typical Instruments for Price

$$\delta_{jt} = \alpha p_{jt} + x'_{jt}\beta + \xi_{jt}$$

- Typically, prices are marginal costs plus a markup term.
 - We want valid instruments that shift costs and/or markups.

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- **Cost-shifters**: Measures of input prices, tariffs, etc.
 - Consumers should only care about them through their effect on prices.

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- Typically, prices are marginal costs plus a markup term.
- **Cost-shifters**: Measures of input prices, tariffs, etc.
- **Hausman**: Current price of the same product averaged across *other* locations.
 - Need costs to be correlated across locations, but not unobserved quality.

Typical Instruments for Price

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- Typically, prices are marginal costs plus a markup term.
- **Cost-shifters**: Measures of input prices, tariffs, etc.
- **Hausman**: Current price of the same product averaged across *other* locations.
- **Waldfoegel**: Average consumer characteristics in *nearby* locations.
 - Helpful that retailers tend to do “uniform pricing” (**DellaVigna and Gentzkow, 2019**).
 - With uniform pricing, your neighbors’ demographics will affect your prices.

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- **BLP**: Average characteristics x_{kt} of *competing* products $k \neq j$.
 - Characteristics of competing products affect markups.
 - We'll come back to these later, since they can also serve a different purpose.

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- **BLP**: Average characteristics x_{kt} of *competing* products $k \neq j$.
- I recommend starting with just one. A straightforward cost-shifter if you have it.

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

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- Try to do the first exercise before day 2's class, when I'll do it live.
 1. Getting set up with Python and PyBLP.
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 3. Running the price cut counterfactual.

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 - Do the substitution patterns you estimate seem reasonable?

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- When doing the exercise, think critically about the pure logit model's limitations.
 - Do the substitution patterns you estimate seem reasonable?
- If you have time, try the supplemental exercises.
 - Statistical inference.
 - Modeling the supply side.
 - Checking your code by simulating data.

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