Artificial Intelligence 2
Assignment #3
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Problem 1. Separable Convolution

I. Show that convolution with a 2D Gaussian kernel is a spatially separable convolution, i.e. there are two 1D kernels if applied to the image row-wise and column-wise in sequence, it is equivalent to convolving that image with the 2D Gaussian kernel.

$$G_\sigma(x,y) = \frac{1}{\sigma^2 2\pi} exp(-\frac{x^2+y^2}{2\sigma^2}) \qquad \text{- 2D Gaussian Kernel}$$

We also know that 1D Gaussian kernel is:

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{x^2}{2\sigma^2}) \qquad \text{-1D Gaussian Kernel}$$

To prove Gaussian filter is spatially separable, we need to prove that $G_\sigma(x,y) = G_\sigma(x) \times G_\sigma(y)$

$$\frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{x^2}{2\sigma^2}) \times \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{y^2}{2\sigma^2}) = \frac{1}{\sigma^2 2\pi} exp(-\frac{x^2}{2\sigma^2}) exp(-\frac{y^2}{2\sigma^2})$$

We know that 'exp' stands for exponential and is the same way as writing e^x

Therefore,

$$G_\sigma(x) \times G_\sigma(y) = \frac{1}{\sigma^2 2\pi} e^{(-\frac{x^2}{2\sigma^2})} e^{(-\frac{y^2}{2\sigma^2})} = \frac{1}{\sigma^2 2\pi} e^{(-\frac{x^2}{2\sigma^2})+(-\frac{y^2}{2\sigma^2})} = \frac{1}{\sigma^2 2\pi} e^{(-\frac{x^2+y^2}{2\sigma^2})}$$

So,

$$G_\sigma(x,y) = G_\sigma(x) \times G_\sigma(y) = \frac{1}{\sigma^2 2\pi} exp(-\frac{x^2+y^2}{2\sigma^2})$$

II. Is Sobel kernel spatially separable?

Sobel kernel is used to detect edges and can easily be separated spatially.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

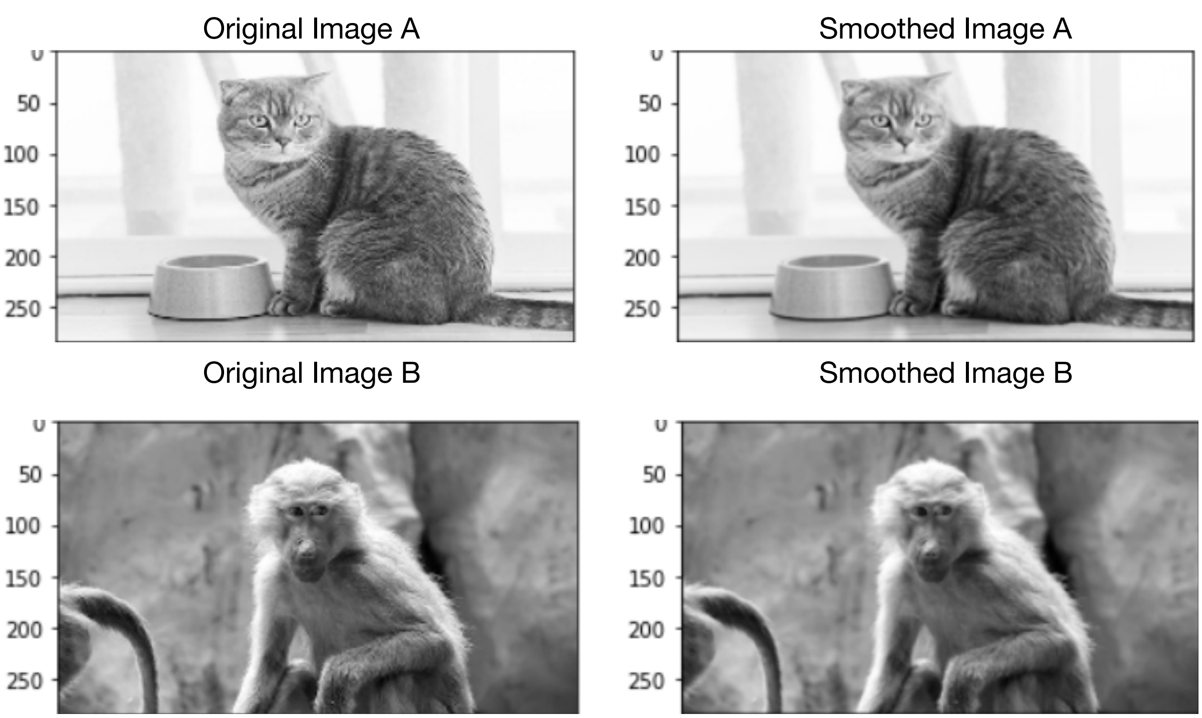
III.Why separable convolutions are preferred?

Because when using convolutions with for example a 2D kernel with size 3, instead of 9 multiplications, we do two convolutions with 3 multiplications (6 in total) to achieve same effect. This means we require less computations when running the network! Therefore, complexity goes down and we are able to run network faster. In other words, we prefer separable convolutions since they are more efficient and less computationally intensive.

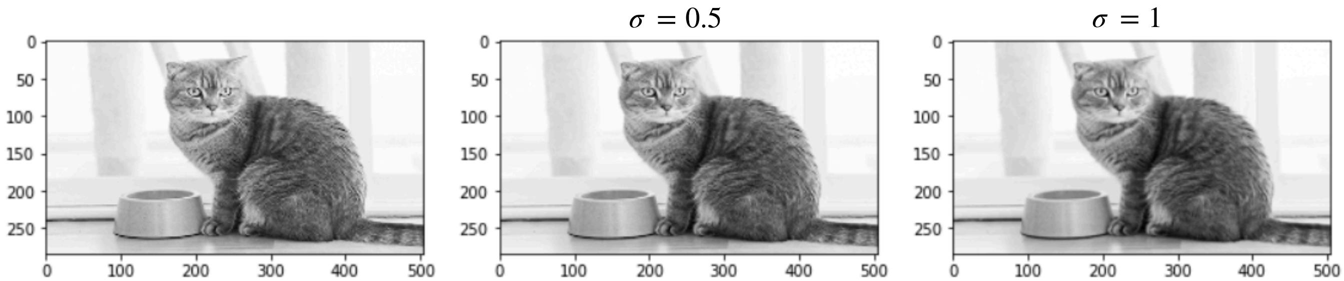
Problem 2. Hybrid Images

Gaussian Blur:

Gaussian Filter is a low pass filter. I will first apply this filter with $\sigma = 2$ and kernel size =3 to my images and plot the effect.

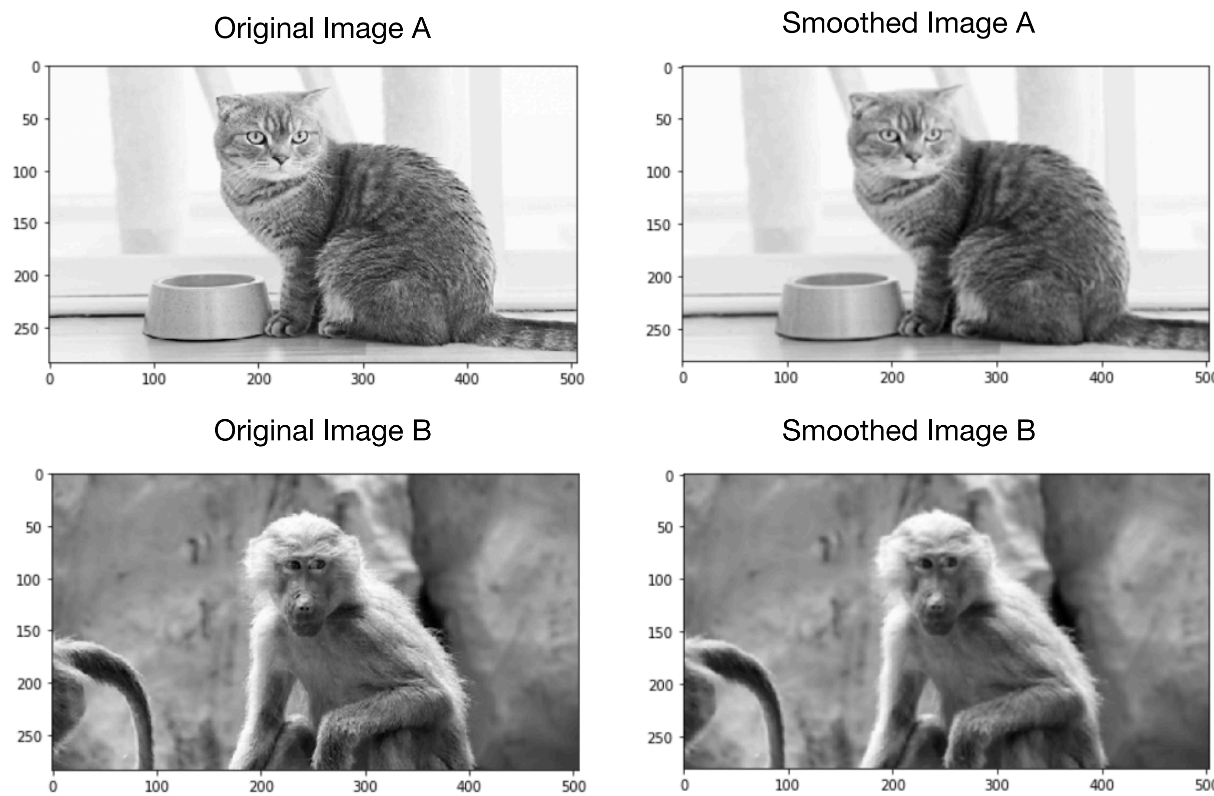


As we increase σ and kernel size, the output will be smoother.



Boxing Blur:

Box blur is also known as box linear filter. Box blurs are frequently used to approximate Gaussian blur. Here is the result of applying box blur with kernel size = 3 on both images.



Hybrid Image:

Gaussian filter:

The hybrid image when we want to see the cat up close is shown in the first row with different sigma value, and the hybrid for seeing the monkey up close is presented in the second row.

$\sigma = 1$ $\sigma = 5$

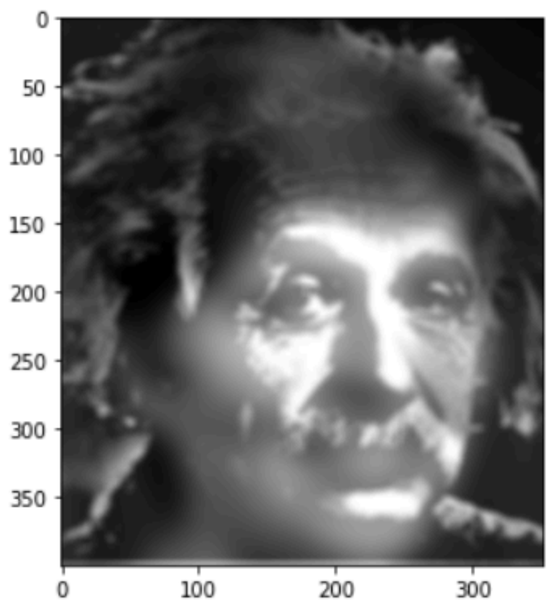
As we increase sigma, we pass higher frequencies in our low pass filter. This means the image corresponding to low pass frequency (far image) can be seen more clearly in the front and the other image corresponding to higher frequency (1-low pass) will have less frequency therefore is less visible in the front.

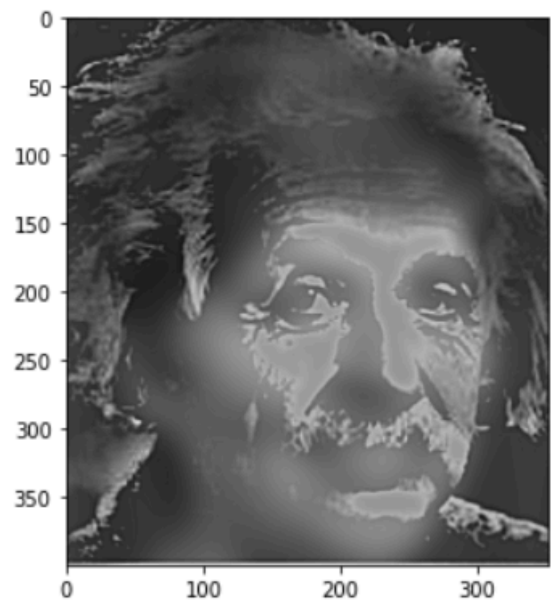
Box Blur:

Here is the result using this filter:

I think when using this filter it is really difficult to find the image that should be seen from a far distance, especially when the cat is seen in the front. As a result it is best to pick Gaussian filter and then tune parameter sigma to produce the desired result.

Problem 3. De-hybridizing





By applying a low pass filter on the hybrid image we can extract low frequencies that correspond to the far image. Then we will subtract them from hybrid image to get the high frequencies related to einstein image.