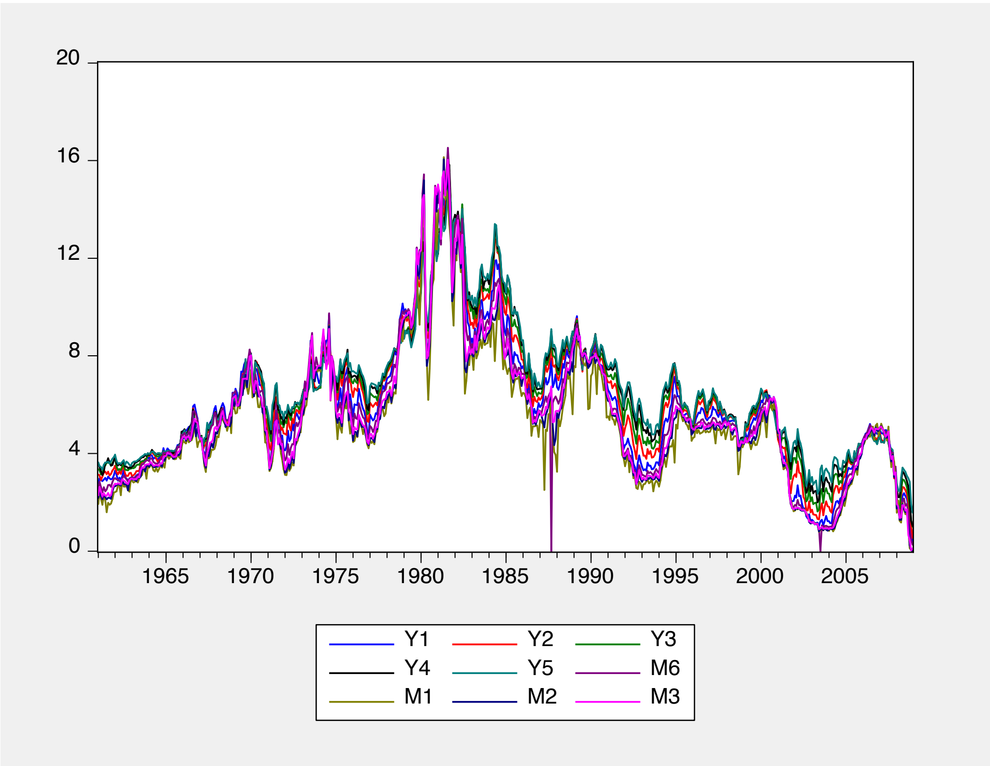
Nicole Maria Formenti
Università degli studi di Milano – DSE

TIME SERIES ECONOMETRICS – PROJECT

The aim of this project is to inspect a set of real data (US dollar LIBOR interbank rates, observed in the period 1961-2008) in order to understand whether the Pure Expectation Hypothesis holds and can be used to predict the markets' expectations of the future rates by using the term structure of today rates.
The PEH assumes that investors are interested in maximizing their gain. Supposing they can choose between 2 alternatives, which are investing on a long-term maturity or on the roll-over of short-term contracts, they will choose the most profitable one, causing the rates to adjust accordingly so that the expected return of both strategies will be the same. Information about the expectations of future rates can be found in the "spread", which is the difference between two interest rates.
According to empirical analysis, we know that rates are non-stationary and not integrated of order 0, but they're usually integrated of order 1. Moreover, they tend to move together, so they're defined as cointegrated. This assumption can be confirmed by the fact that there exists a linear combination between 2 cointegrated rates of order 1, which is the so called "spread", that is an integrated of order 0. This particular combination must have some characteristics that will be analysed afterwards

We start our analysis by inspecting the graph of the different rates.

1. <u>PRELIMINARY ANALYSIS OF THE GRAPH</u>



From the graph we can exclude that the time series are integrated of order 0, because there isn't any specific value to which the series tend to converge and adjust to. On the other hand, we can see that they tend to move together, so we can assume in the first place that they're cointegrated. Furthermore, they seem to have an intercept and no trend.

2. TEST FOR UNIT ROOTS: ORDER OF INTEGRATION

In order to check whether the time series are integrated of order 1 we test for unit roots on both levels the first differences, by including an intercept and no trend.

Null Hypothesis: M1 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.959225	0.0395
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: M2 has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.125743	0.2347
Test critical values:	1% level	-3.441533	
	5% level	-2.866365	
	10% level	-2.569399	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: M3 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.262989	0.1846
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: M6 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.430617	0.1337
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Y1 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.199465	0.2068
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Y2 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.987172	0.2926
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Y3 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.825867	0.3679
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Y4 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.442928	0.5619
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: Y5 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.410224	0.5782
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

From the test we obtain that for all of the series but m1, the null hypothesis that there is a unit root is accepted for every level of confidence. Regarding m1, the test rejects the null hypothesis at 5% level of confidence, but since it’s an approximation and from the graph we see the series have a similar pattern to the others, we decide to include it into the cointegration relation.

By taking the unit root test on the 1st difference (which results are not reported here), the null hypothesis is rejected, so we have no unit roots.
In conclusion all of the rates are proven to be integrated of order 1.

3. RIGHT NUMBER OF LAGS

In order to test for cointegration and estimate the VECM, we need to check for the right number of lags. First, we estimate a VAR model and check for lag length using the Schwarz criterium, which indicates 1 lag as the best solution.

VAR Lag Order Selection Criteria
Endogenous variables: M1 M2 M3 M6 Y1 Y2 Y3 Y4 Y5
Exogenous variables: C
Date: 12/09/19 Time: 15:42
Sample: 1961M01 2008M12
Included observations: 571

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-746.4429	NA	1.14e-10	2.646035	2.714558	2.672768
1	1646.676	4702.416	3.47e-14	-5.452455	-4.767228*	-5.185121
2	1805.066	306.2399	2.64e-14	-5.723525	-4.421592	-5.215590
3	1983.146	338.6937	1.88e-14	-6.063558	-4.144920	-5.315022*
4	2071.258	164.8058	1.84e-14	-6.088470	-3.553127	-5.099333
5	2190.134	218.5991*	1.61e-14*	-6.221137*	-3.069089	-4.991399

4. TEST FOR COINTEGRATION ON ALL THE VARIABLES

According to the theory, any set of n rates has a cointegrating rank of (n-1), which means that any set of (n-1) linearly independent spread vectors form a basis for the cointegrating space, which is the space spanned by the cointegrating vectors.
To test cointegration we use the Johansen cointegration test, selecting 0 lag since the test is made on first differences, intercept and no trend.

Date: 12/09/19 Time: 15:45
Sample (adjusted): 1961M02 2008M12
Included observations: 575 after adjustments
Trend assumption: Linear deterministic trend
Series: M1 M2 M3 M6 Y1 Y2 Y3 Y4 Y5
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.650258	2041.795	197.3709	0.0000
At most 1 *	0.485251	1437.724	159.5297	0.0000
At most 2 *	0.449127	1055.881	125.6154	0.0000
At most 3 *	0.367086	713.0363	95.75366	0.0000
At most 4 *	0.303865	450.0191	69.81889	0.0000
At most 5 *	0.232293	241.7470	47.85613	0.0000
At most 6 *	0.106857	89.74761	29.79707	0.0000
At most 7 *	0.038288	24.76788	15.49471	0.0015
At most 8	0.004026	2.319553	3.841466	0.1278

Trace test indicates 8 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

From the test we can see there are at most 8 cointegrating vectors, since the hypothesis that there are less cointegrated vectors are rejected with 0,05 confidence. The 8 rates are cointegrated with 7 cointegrating vectors, according to the theory

5. <u>ESTIMATION OF VECM</u>

To further investigate the possibility of 2 spreads being integrated, we can derive a useful representation:

$$i_{n,t} - i_{1,t} = T_n + \Delta_{i,t+1} + \varepsilon_{1,t}$$

Where $i_{n,t}$, $i_{1,t}$, $\Delta_{i,t+1}$, $\varepsilon_{1,t}$ are $I(0)$, and thus $i_{n,t} - i_{1,t}$ is $I(1)$

Let's start by analysing the terms on the right side. Since we assumed that $i_{1,t}$ is an integrated of order 1, then its first differences $\Delta\ i_{1,t+l}$ have to be an integrated of order 0. The error term $\varepsilon_{1,t}$, which is assumed to be a white noise, is also an integrated of order 0. T_n is the constant or intercept which represents the eventual term premia or term liquidity, which expresses the preference for a certain kind of maturities over others.
As a consequence, since the sum of integrated of order 0 is still an integrated of order 0, we need the terms on the left side to be integrated of order 0. We saw before that both rates are integrated of order 1, that means that their difference (the spread) is the linear combination that gives us an integrated of order 0. Our cointegrating vector is (1, -1)'.

As an example, we check the spread between a short-term rate, M2 and a long term rate, Y1, by testing for unit roots. The test confirms that there is no unit root, rejecting null hypothesis, so our spread is integrated of order 0.

Null Hypothesis: SPREAD has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.296151	0.0000
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Since we've confirmed that the rates are integrated of order 1 and cointegrated, we can give them a Vector Error Correction Model representation:

$$\Delta_{i,n} = (i_{n,t-1} - \beta i_{1,t-1} + T_n + \varepsilon_{1,t} + \beta\Delta_{i,t})$$

The ECM is the cointegrating term, which expresses how the short-term behaviour is adjusted to correct the deviation from the long run behaviour. In our data the ECM is represented by the spread, which adjusts so that the rates of the maturities will have the same long run behaviour. The coefficient β is equal to 1, because of the previous equation.

The purpose of investigating the cointegrating relation between a short term and a long-term rates is to check whether the PEH holds, in which case the spread between short and long-term rates should be a good predictor of the future short rates dynamics. According to empirical evidence, the long-term rates tend to drive the term structure and short-term rates adjust to their movements. We see that long-term Granger causes short term rates, that's because short-term rates moves according to expectations on the longer term ones, which also contain information about the future short term rates.

We find the correct lags for M2 and Y1 by estimating a VAR model between them. We obtain 2 lags according to the Schwarz criterium

VAR Lag Order Selection Criteria
Endogenous variables: M2 Y1
Exogenous variables: C
Date: 12/09/19 Time: 15:49
Sample: 1961M01 2008M12
Included observations: 571

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1814.100	NA	1.984502	6.361122	6.376349	6.367063
1	-555.3175	2504.339	0.024486	1.966086	2.011768	1.983908
2	-538.2174	33.90075	0.023388	1.920201	1.996337*	1.949905
3	-528.6285	18.94253	0.022934	1.900625	2.007216	1.942211
4	-519.2747	18.41283*	0.022508*	1.881873*	2.018918	1.935340*
5	-516.6049	5.236782	0.022613	1.886532	2.054032	1.951880

We proceed by estimating the unrestricted VECM between M2 and Y1

Vector Error Correction Estimates
Date: 12/10/19 Time: 15:23
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
Y1(-1)	1.000000	
M2(-1)	-1.019727 (0.02219) [-45.9611]	
C	-0.427749	
Error Correction:	**D(Y1)**	**D(M2)**
CointEq1	-0.050868 (0.04145) [-1.22720]	0.162782 (0.04299) [3.78667]
D(Y1(-1))	0.180326 (0.07229) [2.49452]	0.264925 (0.07381) [3.58905]
D(M2(-1))	-0.054325 (0.06903) [-0.78699]	-0.043535 (0.07049) [-0.61763]
C	-0.004081 (0.02107) [-0.19369]	-0.003634 (0.02151) [-0.16893]

We see that M2(-1) is near to -1, according to the assumption of PEH, which means that M2 and Y1 have a 1-1 relation. We also notice that the value of the variable D(Y1) in the ECM is near 0, which means that Y1 doesn't adjust toward the long run equilibrium and the rate of M2 is the only one who makes all the adjustments. We can then exclude Y1 from the long rate dynamics since the ECM doesn't lead it. We also see that among the independent variables (on the rows), M2 has little impact in the equations of D(Y1) and D(M2) compared to the impact of Y1.

We then test a restricted VECM by setting B(1,1)=1 (normalized condition), B2= -1 and A(1,1)=0.

Vector Error Correction Estimates
Date: 12/10/19 Time: 15:25
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1, B(1,2)=-1, A(1,1)=0
Convergence achieved after 1 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):
Chi-square(2) 2.748677
Probability 0.253007

Cointegrating Eq:	CointEq1	
Y1(-1)	1.000000	
M2(-1)	-1.000000	
C	-0.535184	
Error Correction:	**D(Y1)**	**D(M2)**
CointEq1	0.000000 (0.00000) [NA]	0.205535 (0.02506) [8.20061]
D(Y1(-1))	0.185563 (0.07227) [2.56763]	0.270558 (0.07394) [3.65898]
D(M2(-1))	-0.058501 (0.06886) [-0.84952]	-0.050323 (0.07046) [-0.71423]
C	-0.004071 (0.02105) [-0.19339]	-0.003657 (0.02154) [-0.16976]

The p-value of the restricted VECM is 0,25 > 0,05, which means the null hypothesis is not rejected and the restriction is a valid representation. We have proved that the spread has a cointegrating vector (1, -1)' and that we can exclude Y1 from the long-term dynamics.

6. GRANGER CAUSALITY

We test for Granger Causality to see whether a rate drives the other.

VEC Granger Causality/Block Exogeneity Wald Tests
Date: 12/10/19 Time: 15:56
Sample: 1961M01 2008M12
Included observations: 574

Dependent variable: D(Y1)

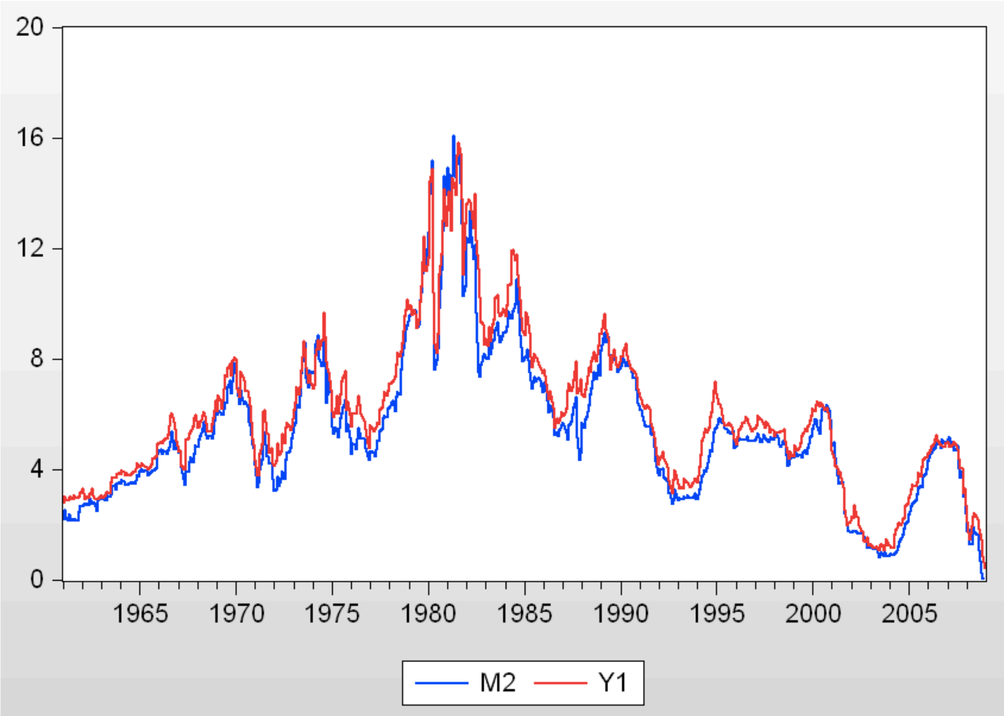
Excluded	Chi-sq	df	Prob.
D(M2)	0.619350	1	0.4313
All	0.619350	1	0.4313

Dependent variable: D(M2)

Excluded	Chi-sq	df	Prob.
D(Y1)	12.88127	1	0.0003
All	12.88127	1	0.0003

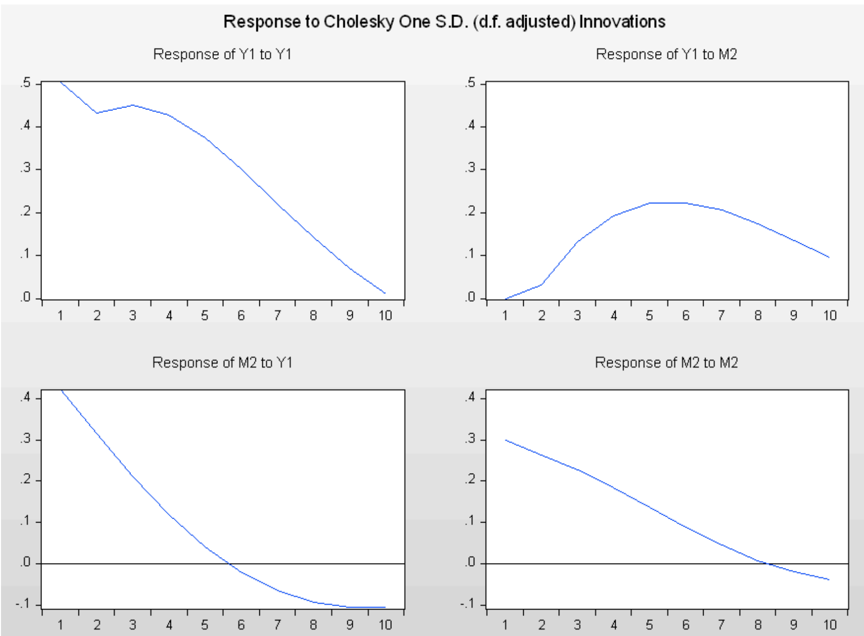
From the test we can conclude that Y1 Granger-causes M2, because we can't exclude the hypothesis that there are lagged value of Y1 in the equation of x1, while the opposite is rejected because we can exclude there are lagged value of M2 in Y1.

Granger- causality doesn't mean that in reality the relation between 2 variables operates in that direction, but only that the variable that Granger-causes the other moves before. That's because of the expectation on future short-term rates deriving from current long-term rates.
As a matter of fact, from the graph of M2 and Y1 we see that Y1 drives M2 because its changings usually precede the changings in M2.



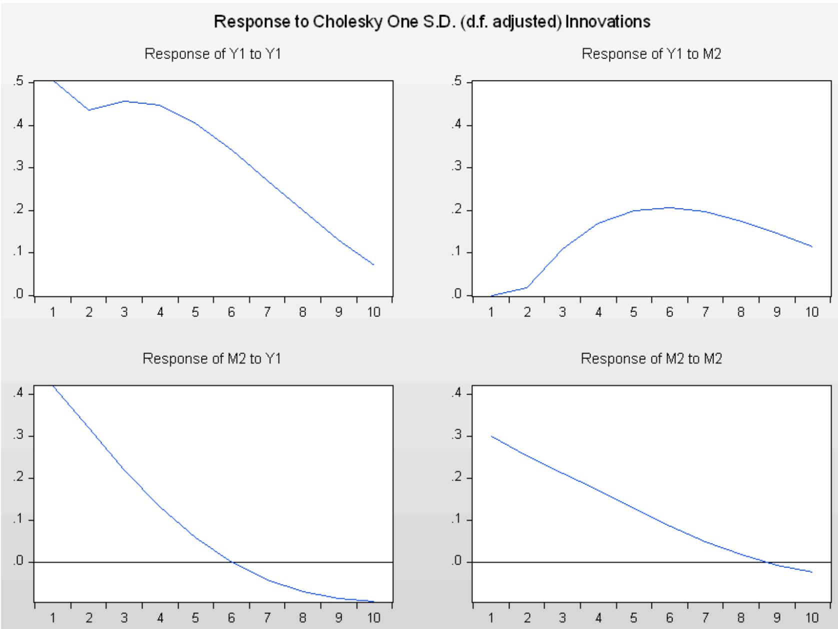
7. <u>IRF BETWEEN VARIABLES</u>

We plot the IRF both for VECM without restriction and with restriction, we expect them to be quite similar since the restriction model hasn't been rejected.
We order the variables with Y1 first and M2 afterwards, that's because we've seen from the Granger-causality test and the restricted model that M2 seems to follow Y1.
First, we plot the IRF for the VECM without restrictions



The IRF shows that we have a short-term response to Y1 both of Y1 and M2, which gradually decreases in the long term so it's not a permanent effect. M2 still has an effect on M2, even though it's smaller compared to Y1 but it decreases less sharply, and it has some impact on Y1. From these graphs we can conclude that Y1 seems to have some impact in the changes of M2 in the short-term, while M2 has less impact on Y1 and more in the long term.
This is in accordance with the PEH, for which the long-short rates spread is quite useful to predict the future rates of short-term rates.
An assumption for the response on the longer term of Y1 to M2, may be the fact that Y1 retains some information about short term rates, so it changes in response to them. On the other hand, short term rates changes based on the expectations of long-term rates.

Then we plot the IRF graph for the restricted VECM



The IRF graphs in the 2 cases are quite similar, confirming that the restricted VECM is a good representation.

8. FORECASTING

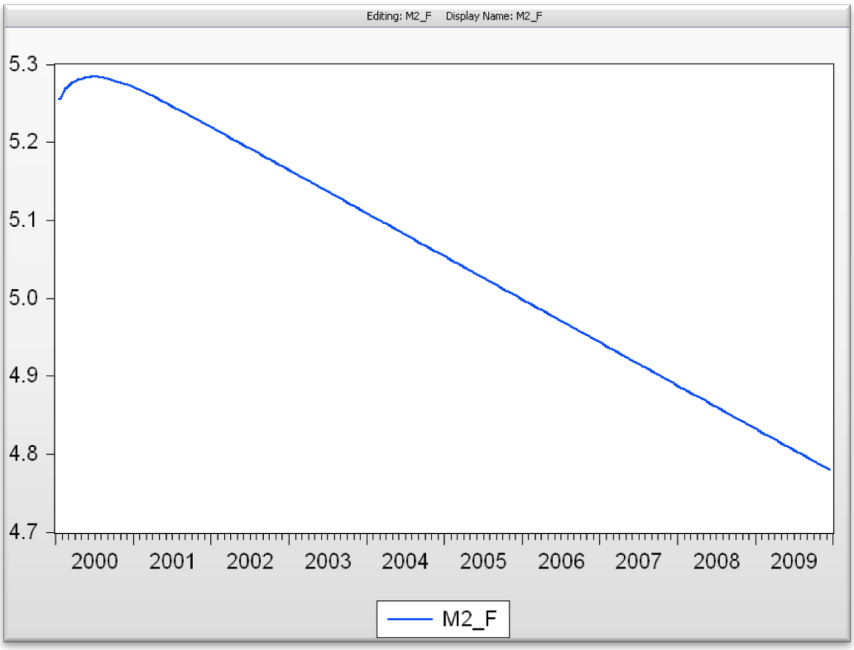
Since we have a valid VECM we can use it in order to forecast M2 and Y1. We make a forecasting from 2000 to 2008

Forecast Evaluation
Date: 12/10/19 Time: 20:40
Sample: 2000M01 2009M12
Included observations: 120

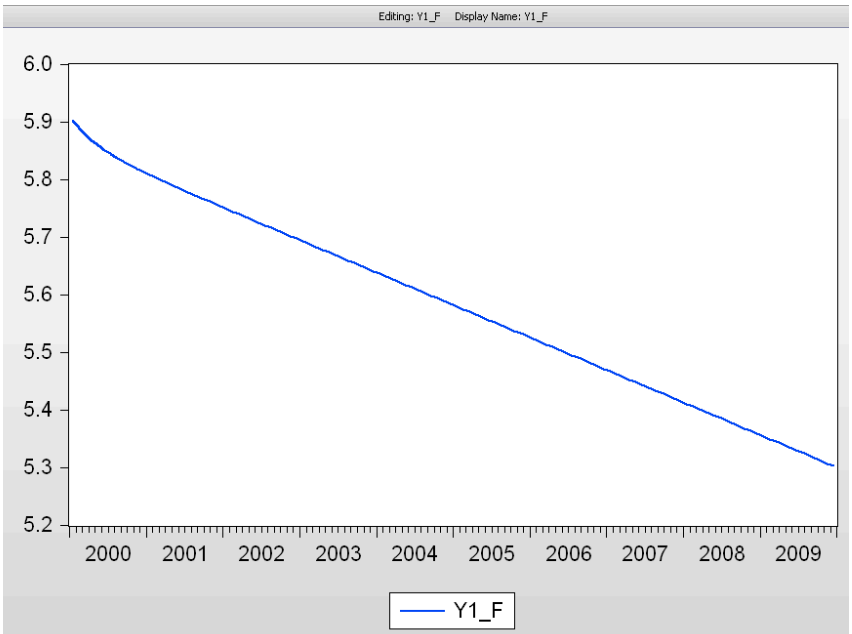
Variable	Inc. obs.	RMSE	MAE	MAPE	Theil
M2	108	2.757118	2.268156	44.66786	0.323794
Y1	108	2.887194	2.439654	43.51576	0.311578

RMSE: Root Mean Square Error
MAE: Mean Absolute Error
MAPE: Mean Absolute Percentage Error
Theil: Theil inequality coefficient

Forecasting for M2:



Forecasting for Y1:



From the statistics we can see that the root mean square error of the forecast of M2 (2.76) is slightly lower than the one of the forecast of Y1 (2.89), which means the forecast of M2 is better than the forecast of Y1.

CONCLUSIONS: this project shows that our set of 9 interbank rates, both short and long term, have proven to be cointegrated with a cointegrating rank of 8, thus moving together in the long run. This means that, according to the theory, their difference called "spread" is an integrated of order

0, and it is exactly the linear combination that transforms 2 series integrated of order 1 in an integrated of order 0. As a consequence, we've been able to estimate a restricted VECM between a long and a short-term rates which accepts the hypothesis that the cointegrating vector takes exactly value (1, -1). Moreover, we had excluded the variable Y1 from the long rates dynamics, which is in accordance with the PEH theory that assumes that the short rates adjust according to the expectations on the long rates, so it's the former which changes to maintain the long run equilibrium with the latter. Further evidence was given by the Granger Causality test that confirmed Y1 to Granger cause M2 and not the opposite, which means that Y1 temporally changes before M2. Finally, we've explored the IRF graphs and seen that Y1 has quite an impact on both, mainly in the short-run, whereas M2 still has a smaller impact.
In conclusion, we can assume the PEH holds for this specific case, even if not strongly, so the spread between long-term and short-term rates can be considered helpful for predicting future short term rates.