BT6270 Assignment 2

FitzHugh Nagumo Model of a Neuron

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Introduction:

The FitzHugh–Nagumo model is a simplified 2D version of the Hodgkin–Huxley model which models in a detailed manner activation and deactivation dynamics of a spiking neuron.
The FitzHugh-Nagumo Model is an example of a relaxation oscillator because, if the external stimulus I_{ext} exceeds a certain threshold value, the system will exhibit a characteristic excursion in phase space, before the variables v and w relax back to their rest values.This behaviour is typical for spike generations (a short, nonlinear elevation of membrane voltage v, diminished over time by a slower, linear recovery variable w)in a neuron after stimulation by an external input current.

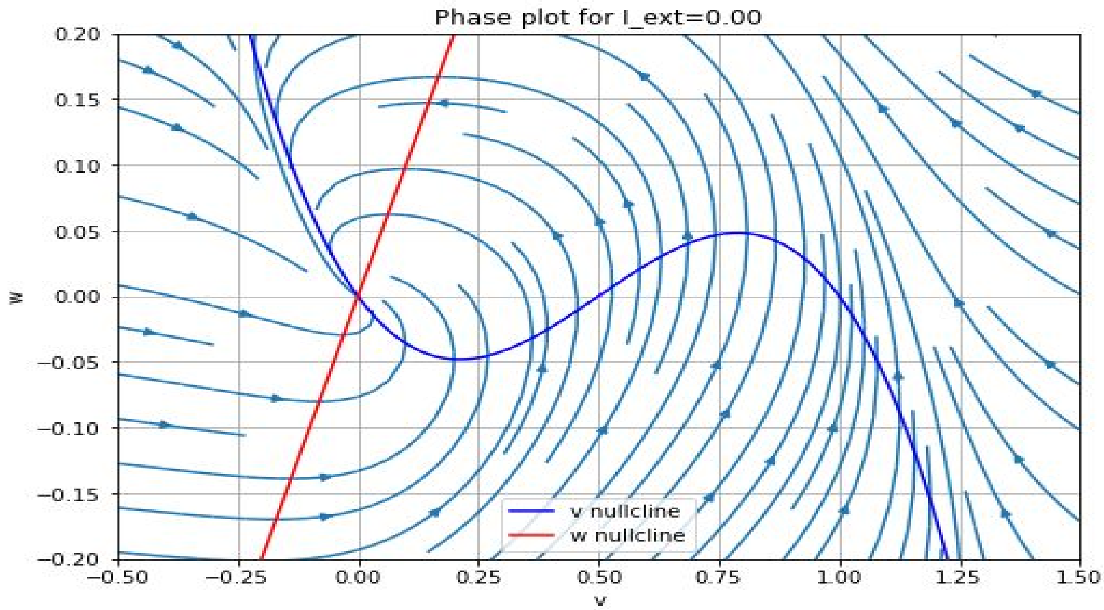
We have the following two equations determining the system:

$$dv / dt = f(v) - w + I_{ext} \qquad \text{where , } f(v) = v(a-v)(v-1)$$
$$dw / dt = bv - rw \qquad a=0.5\text{; choose } b,\ r=0.1$$

The simulation for the cases that are mentioned in the questions are done using python code and plots are attached as follow in this report:

Case 1: I_{ext}= 0

(a) Draw a Phase Plot Superimposed

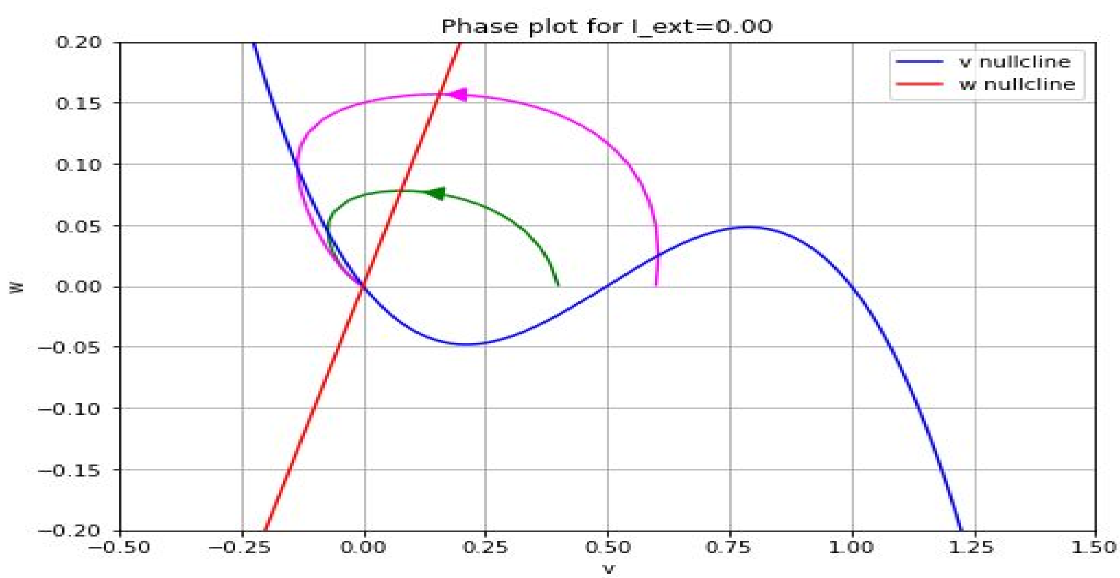


The above plot shows the phase trajectories as well as the null-clines.

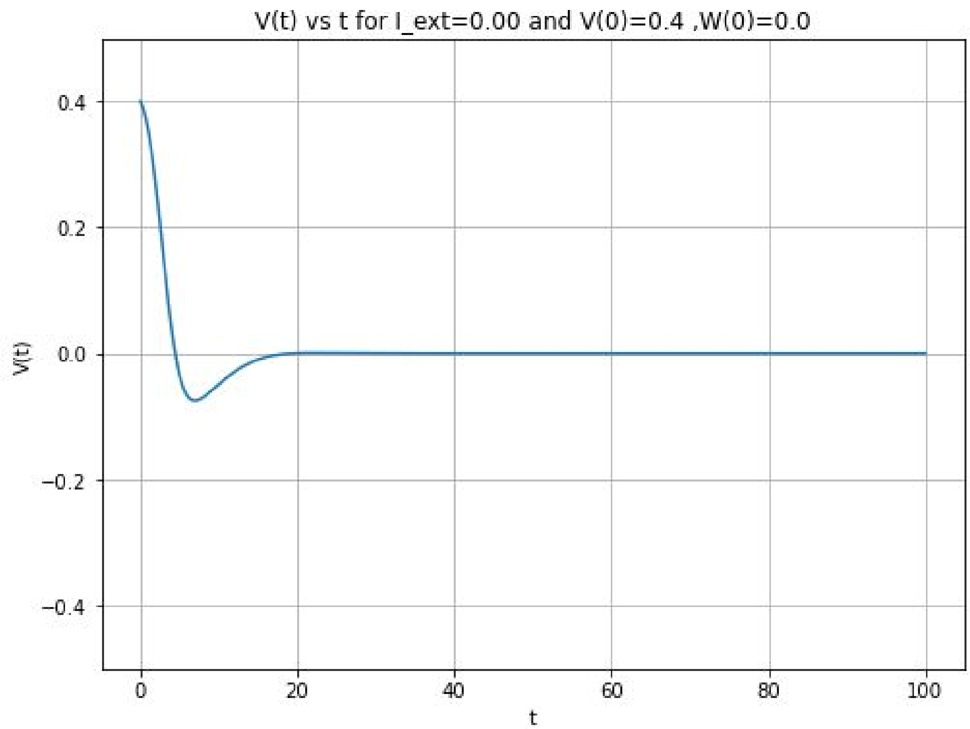
The plot below shows us the trajectories when:

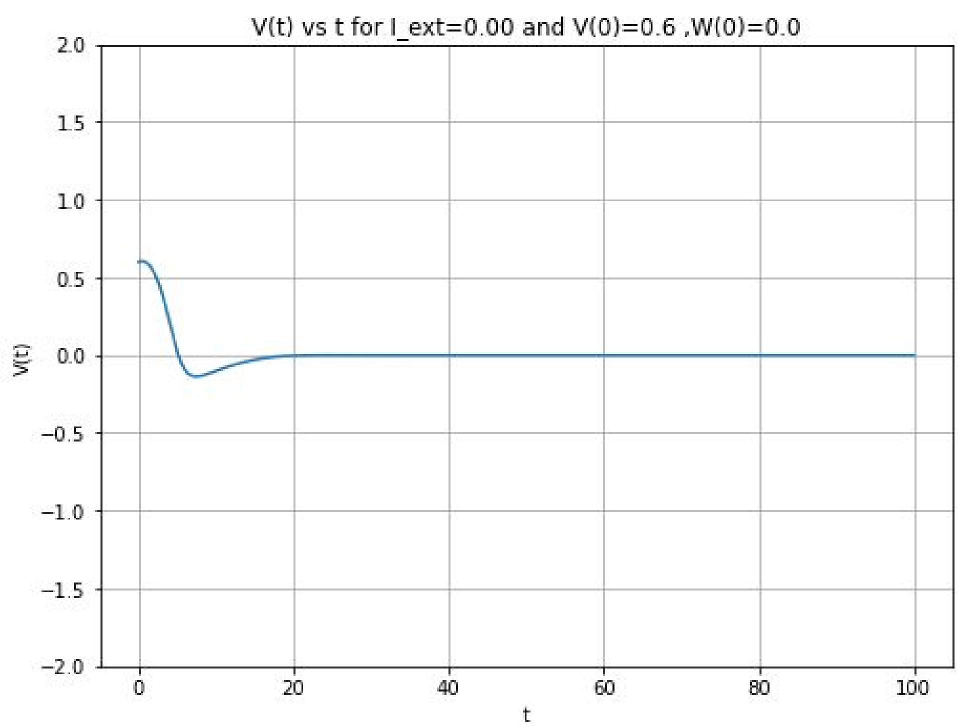
- v(0) =0.4: Green Trajectory (V(0) < a and W(0)= 0)
- v(0) =0.6: Pink Trajectory (V(0) > a and W(0)= 0)

Both of the points end at the fixed point (0, 0) which is stable.



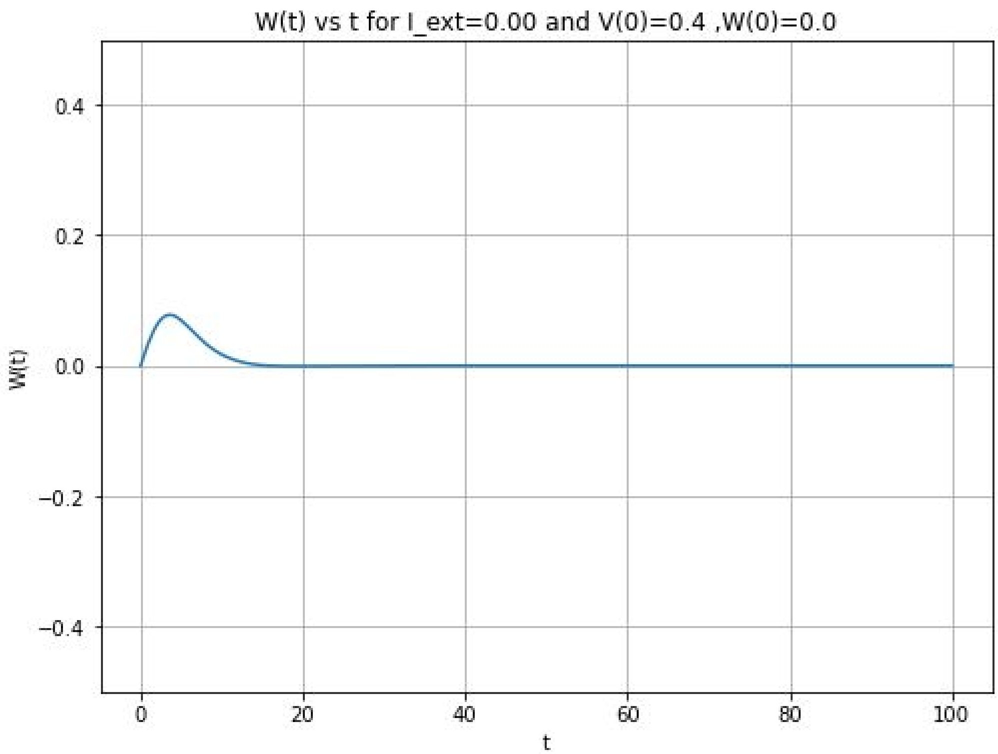
The plots below show the variation of V(t) with time.

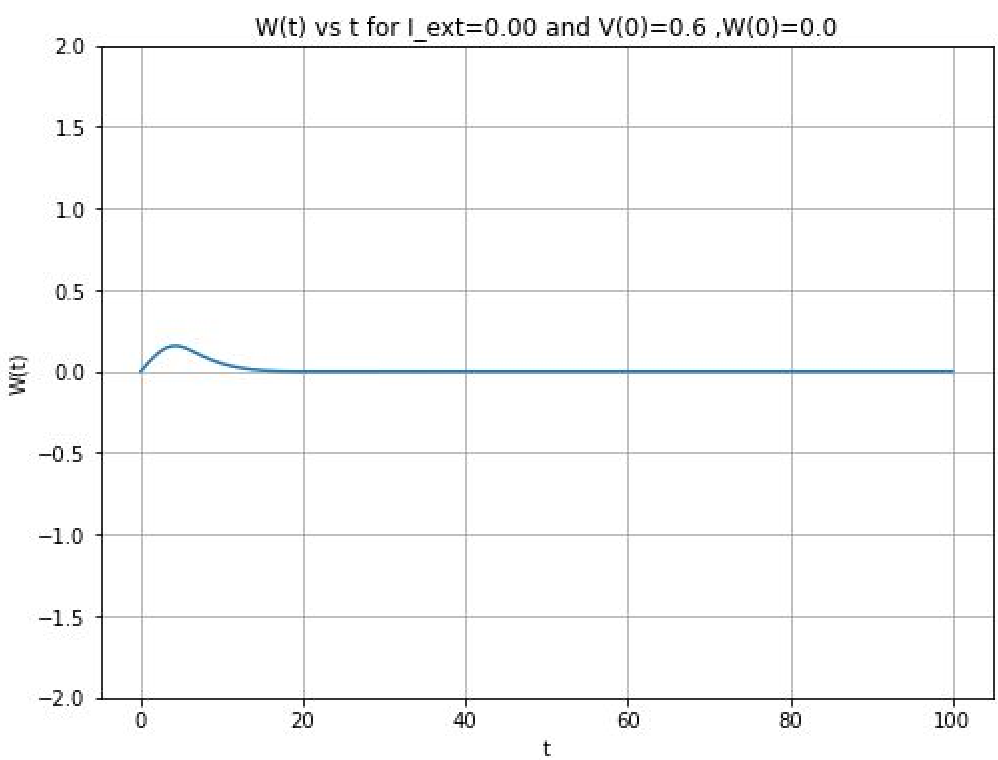




We can observe action potential.

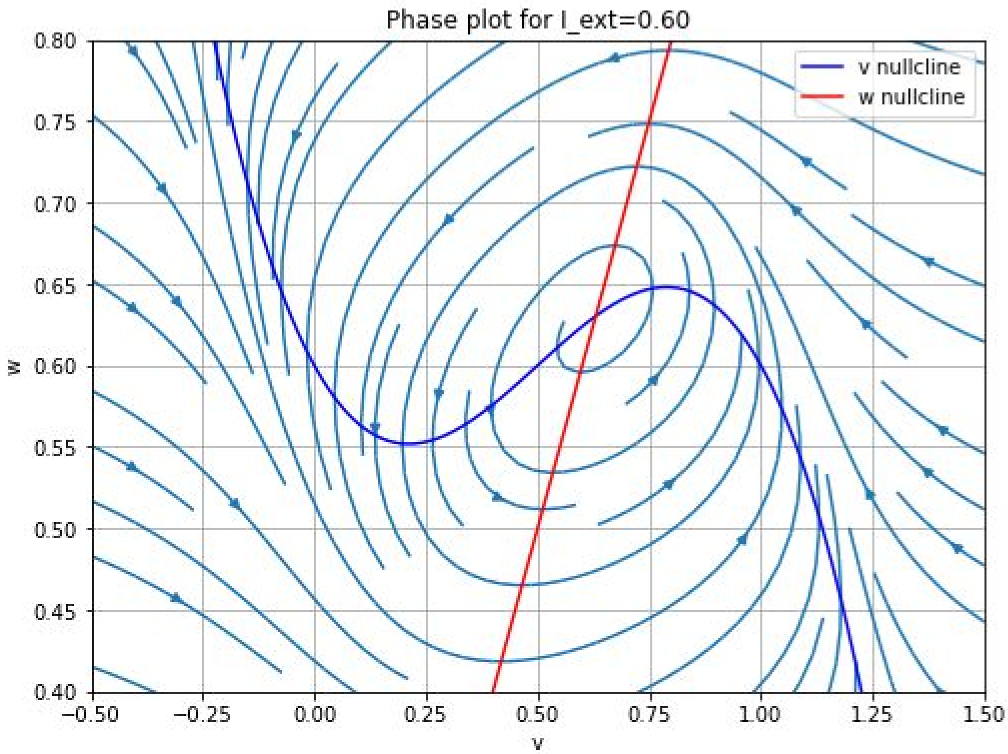
The plots below show the variation of W(t) with time.





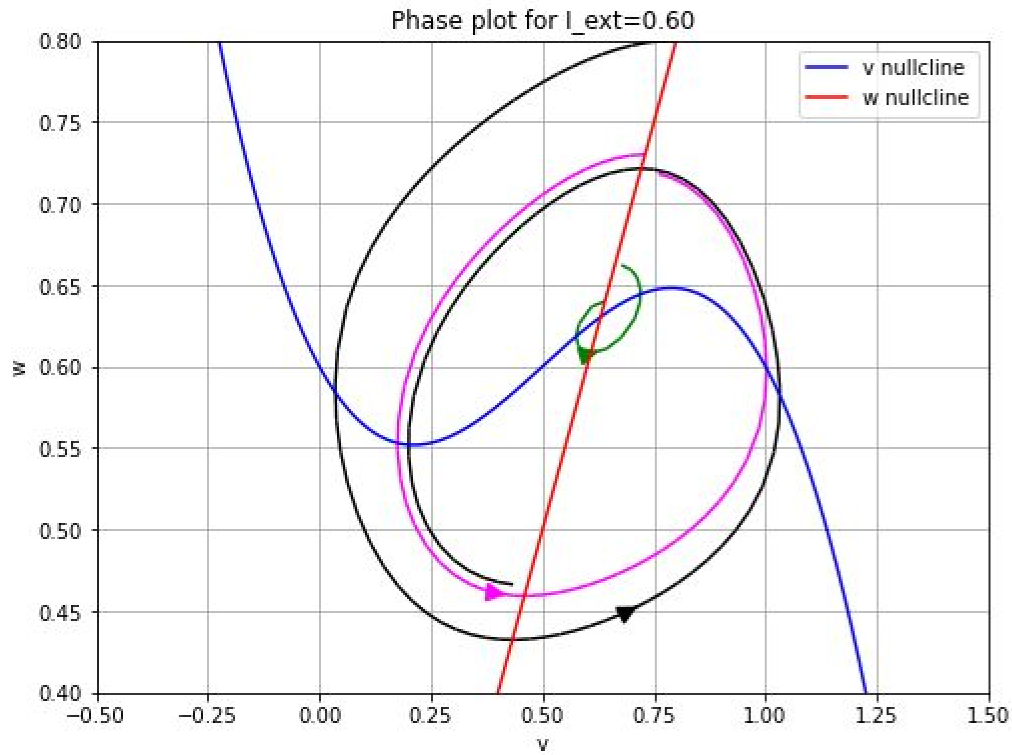
Case 2: I_{ext}=0.6

We take Iext value to be 0.6 and the phase plot for this value is plotted below.

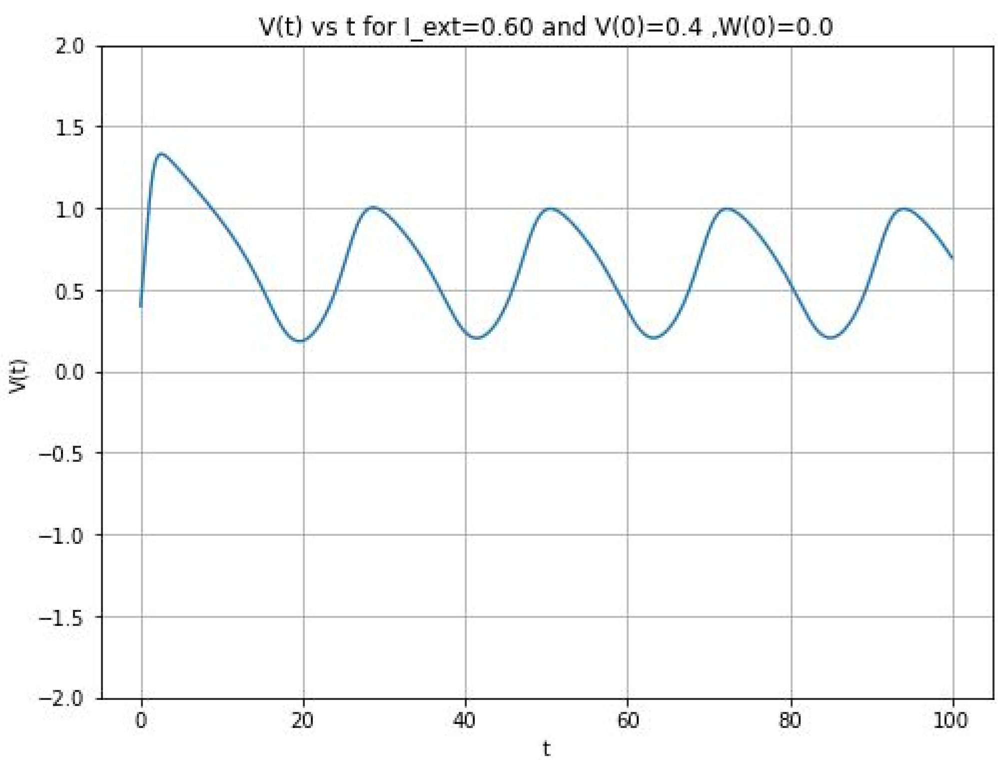


Also the fixed point is unstable i.e., for a small perturbation there is no return to the fixed point and it is shown in the trajectory on the phase plane in the plot below.

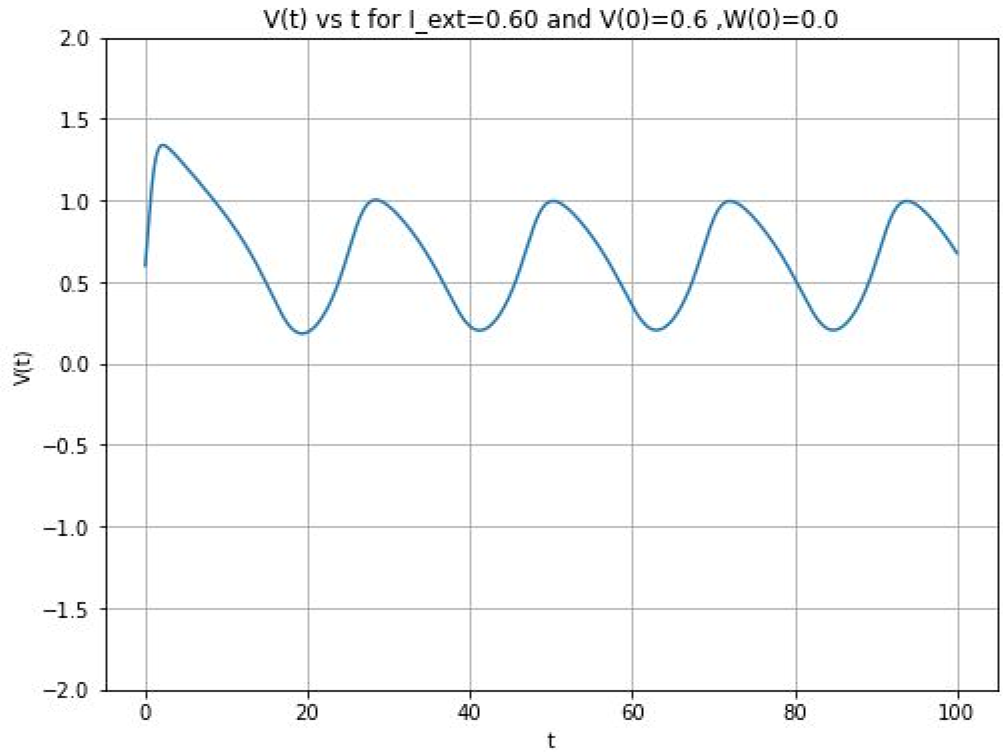
- Here in the plot the trajectory(green path) moves towards the limit cycle (pink trajectory).
- The black trajectory in the plot is the one where the point is from (0.8,0.8). This also moves towards the limit cycle proving that the limit cycle is stable.



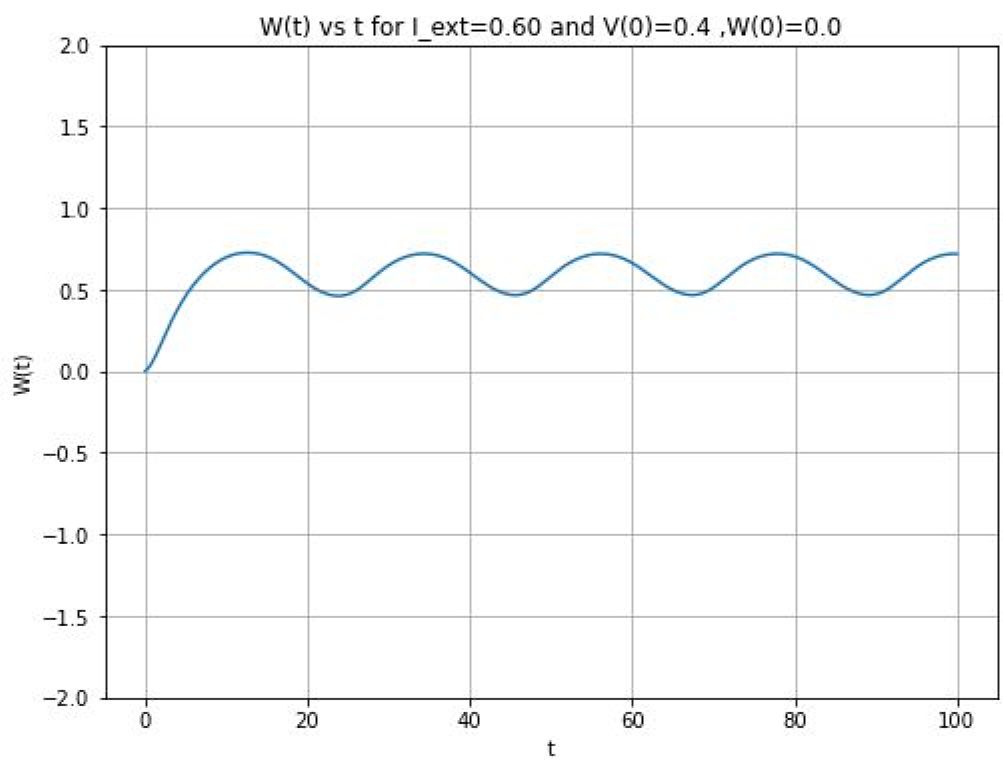
The plots below show the variation of V(t) with time.

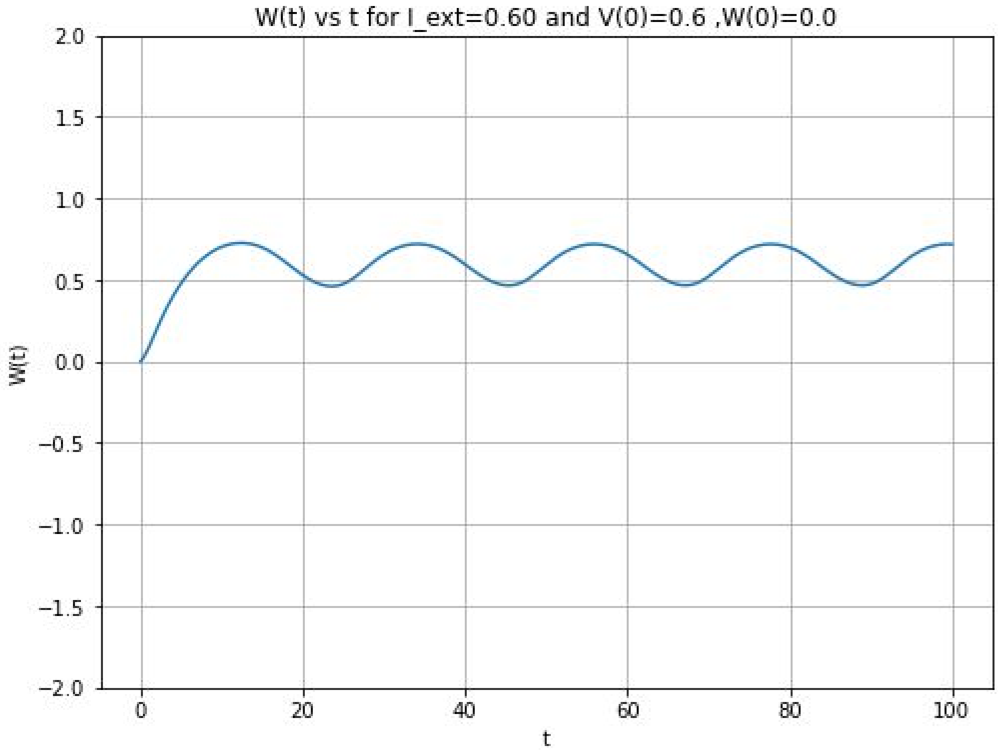


We can see we have continuous periodic firing.



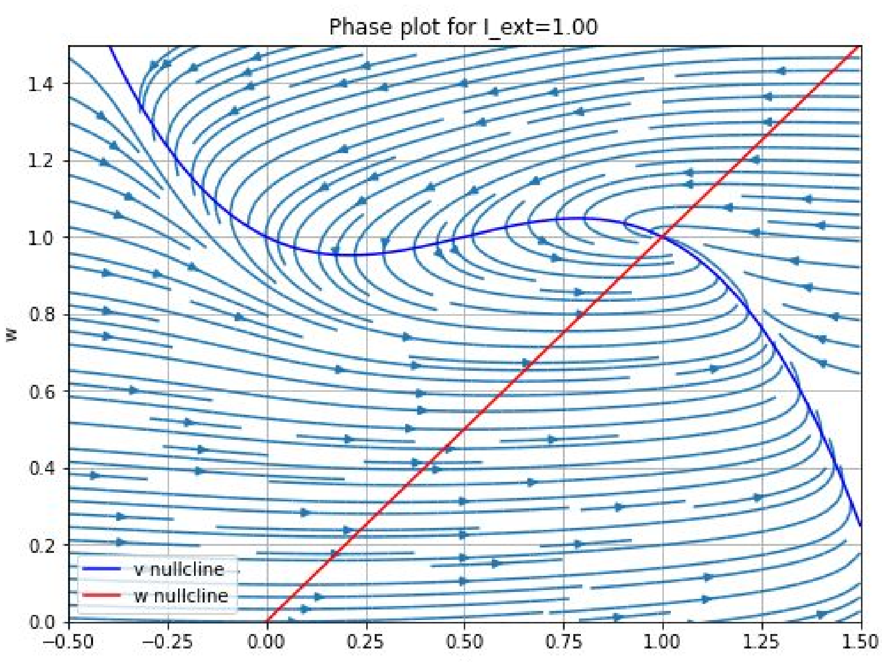
The plots below show the variation of W(t) with time.



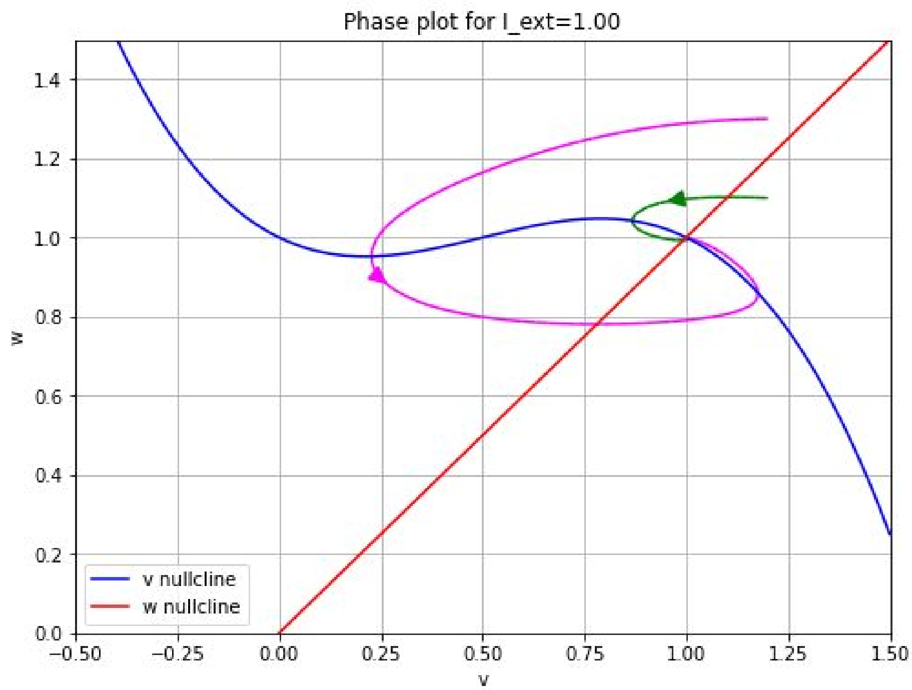


Case 3: I_{ext}=1

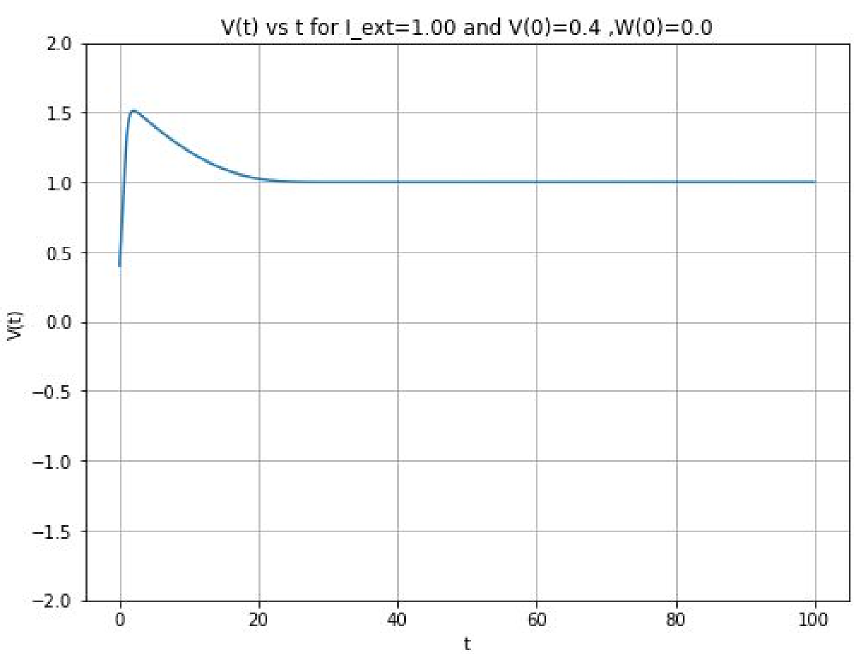
Here for (Iext > I2) we chose Iext value to be 1, and the phase plot for the same is below.

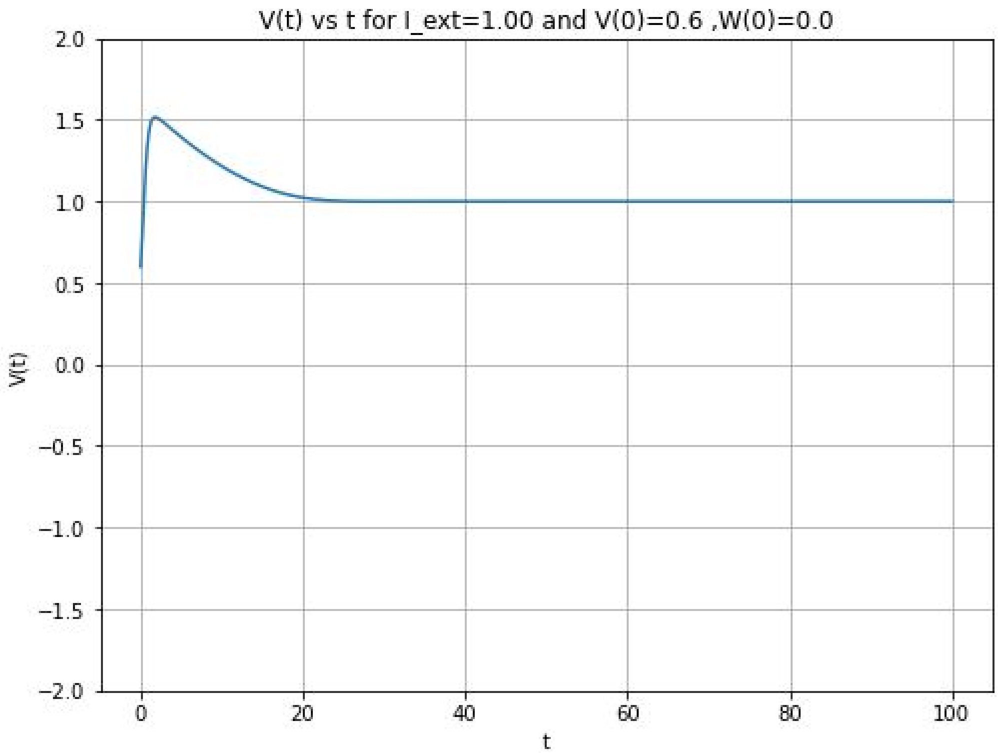


- Also the fixed point is stable ie., for a small perturbation there is a return to the fixed point and it is shown in the trajectory on the phase plane in the plot below. (Green and pink trajectories)

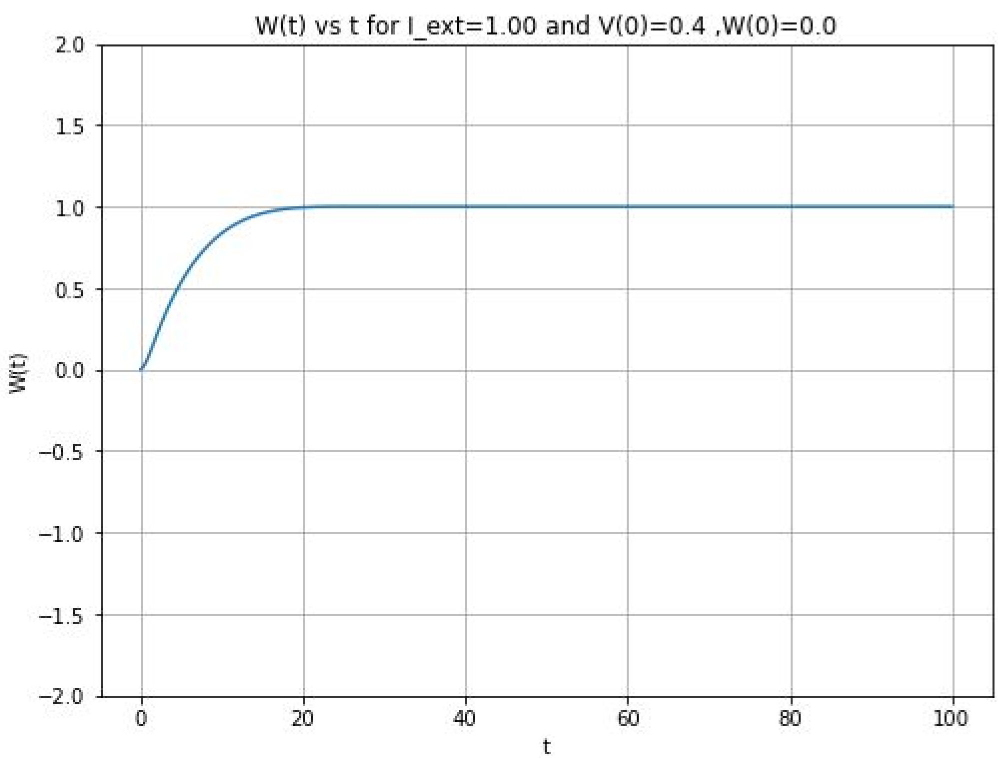


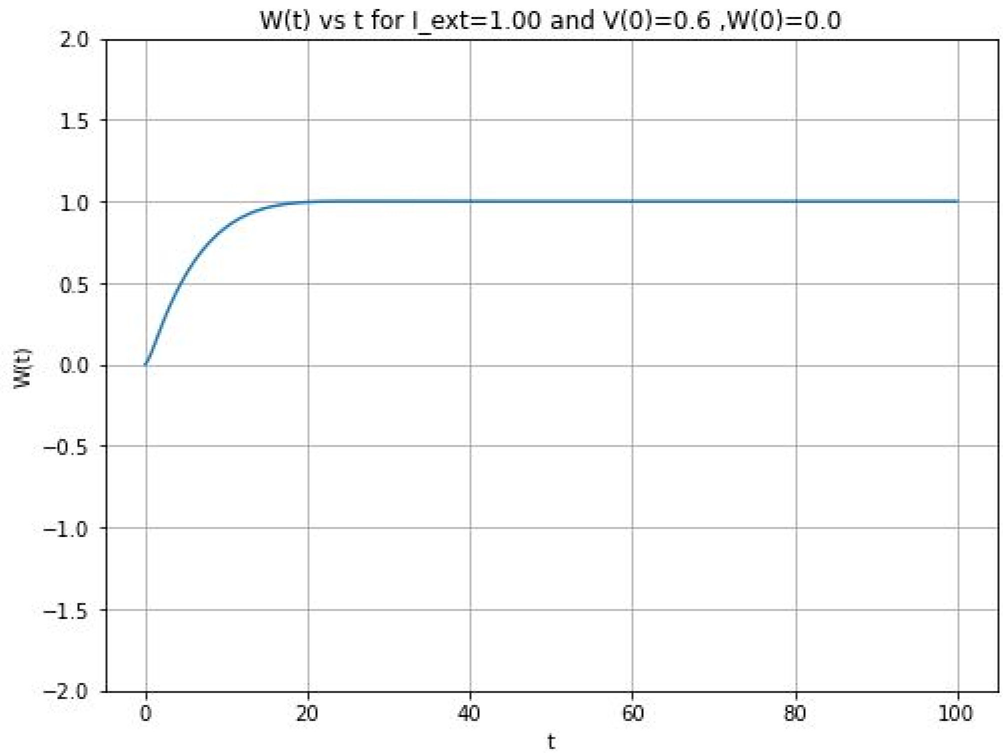
The plots below show the variation of V(t) with time.





The plots below show the variation of W(t) with time





CASE 4:

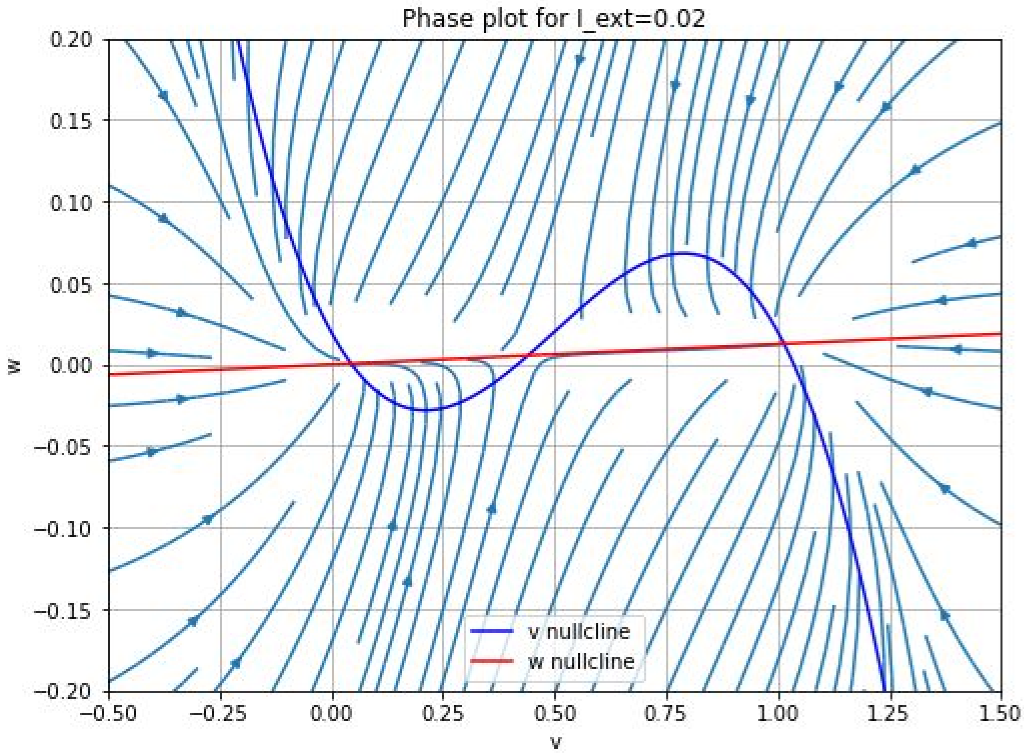
We set Iset value to be 0.02 .

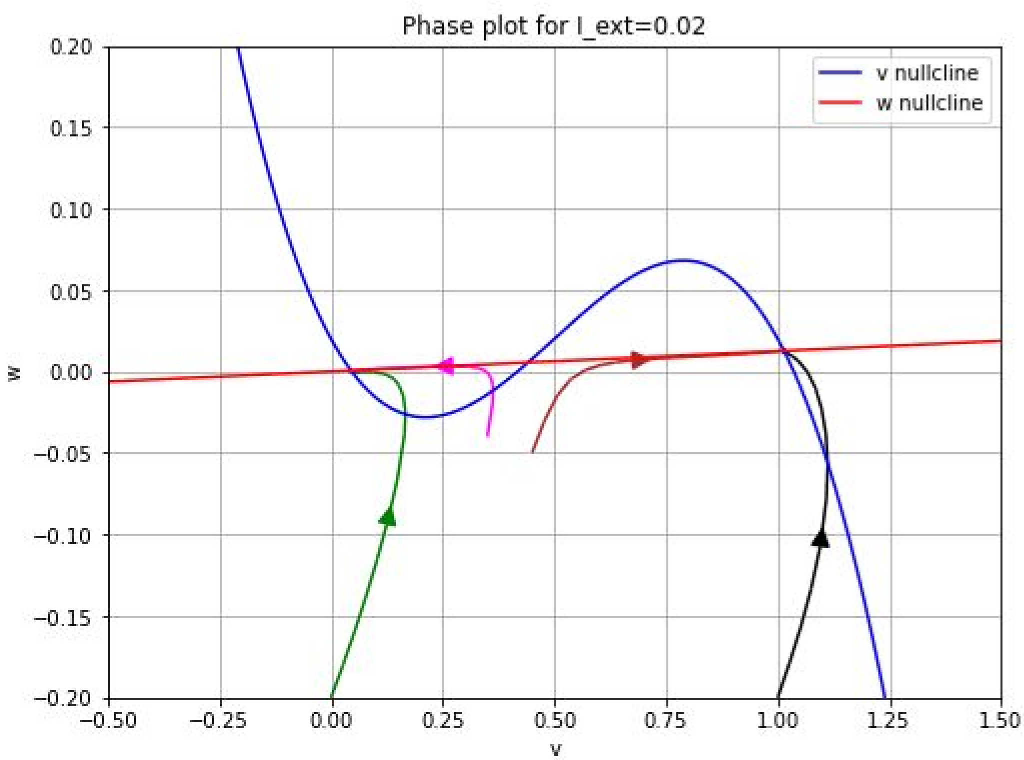
and we set the values :

- a=0.5.
- b=0.01
- r=0.8

such that the graph looks as the phase plot shown in the question.

The phase plot obtained is attached below.


Phase plot for I_ext=0.02
w
v
v nullcline
w nullcline

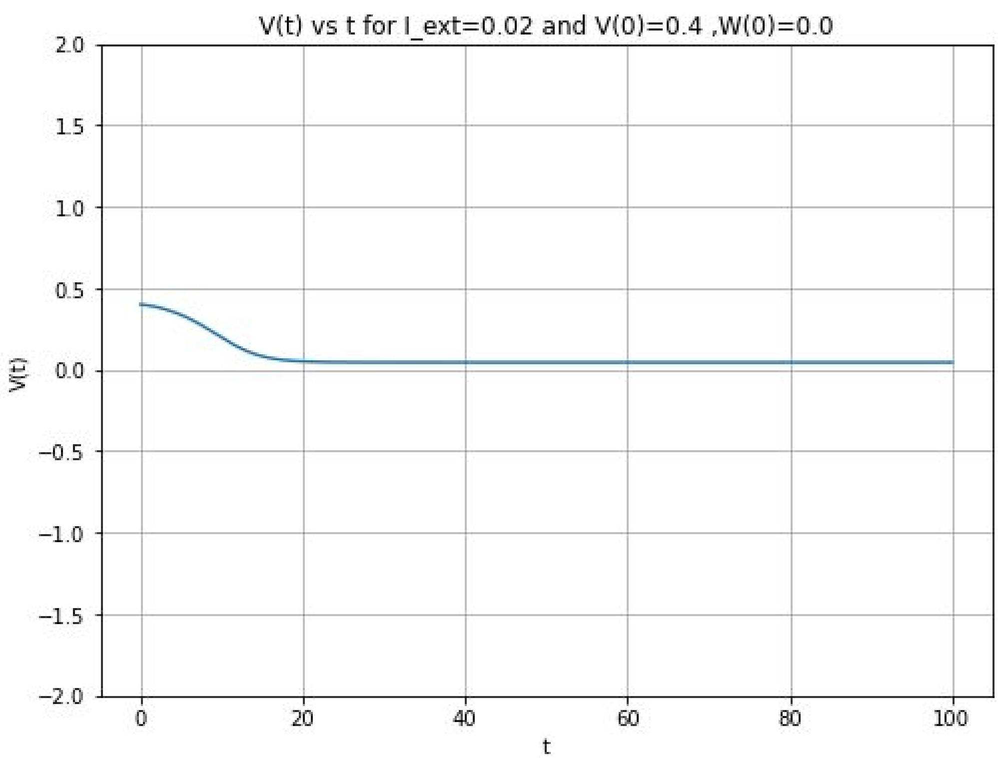

Phase plot for I_ext=0.02
w
v
v nullcline
w nullcline

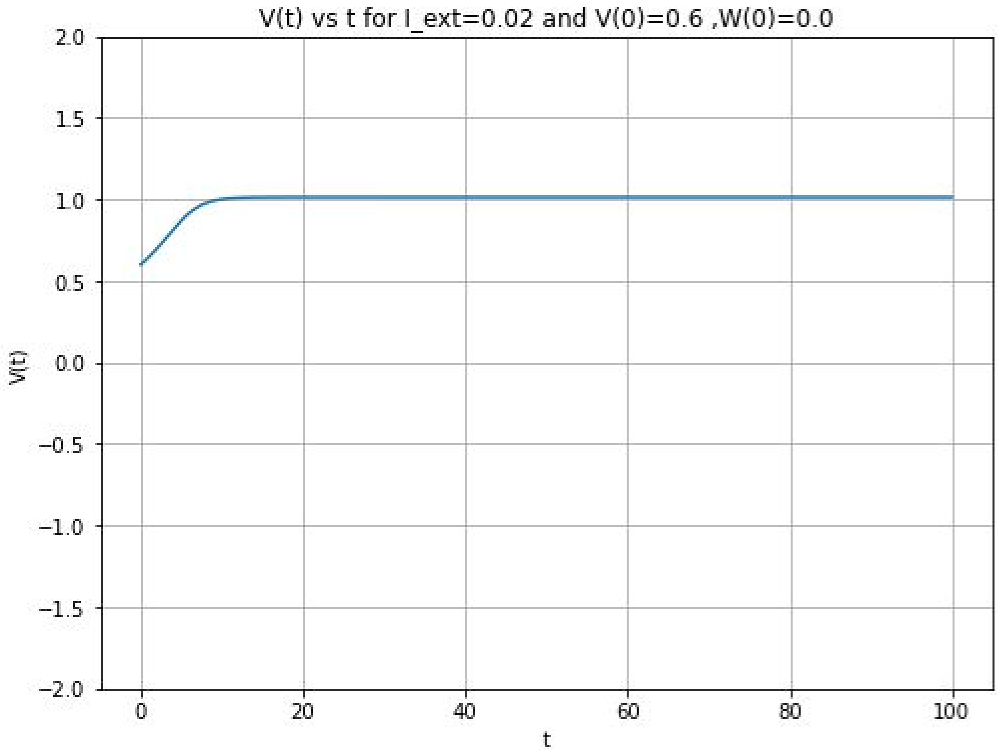
We can see that the red line (w nullcline) cuts the blue line (v nullcline) at three points, P1 , P2 , P3 (left to right).

From the plot we can observe that there are **two stable points** and **one unstable point**.

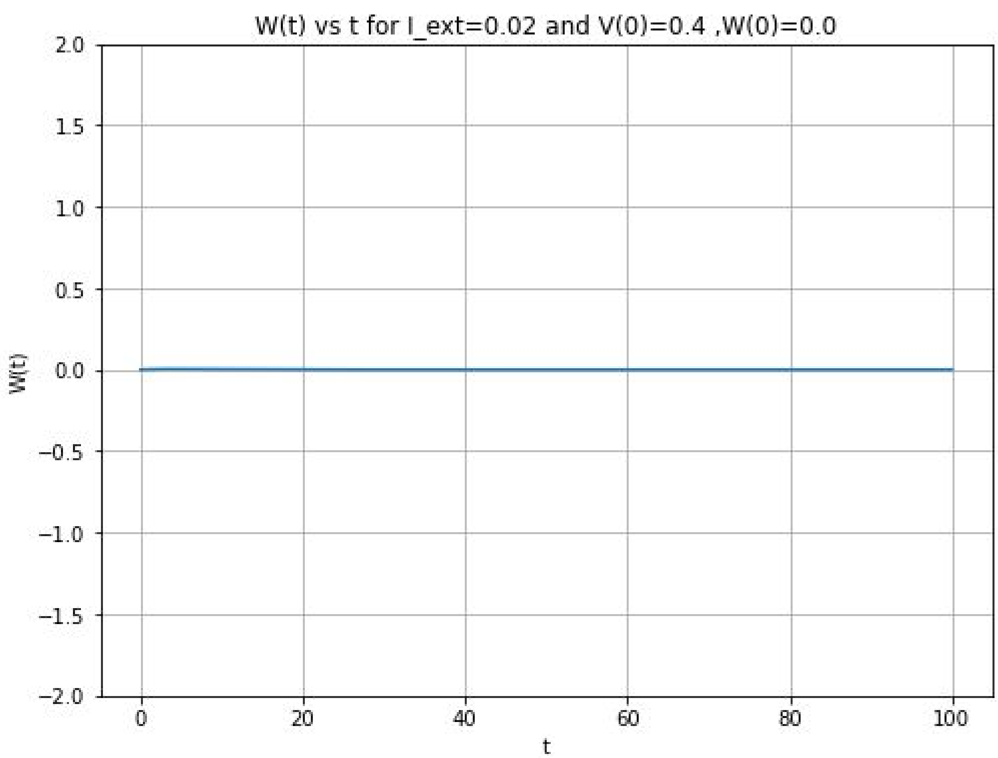
- We can see that **P1 is stable** as any small perturbation around it brings it back to that point(green trajectory).
- We can see that **P2 is unstable** as any small perturbation around it brings it to other two points P1 or P3(green trajectory).
- We can see that **P3 is stable** as any small perturbation around it brings it back to that point(green trajectory).

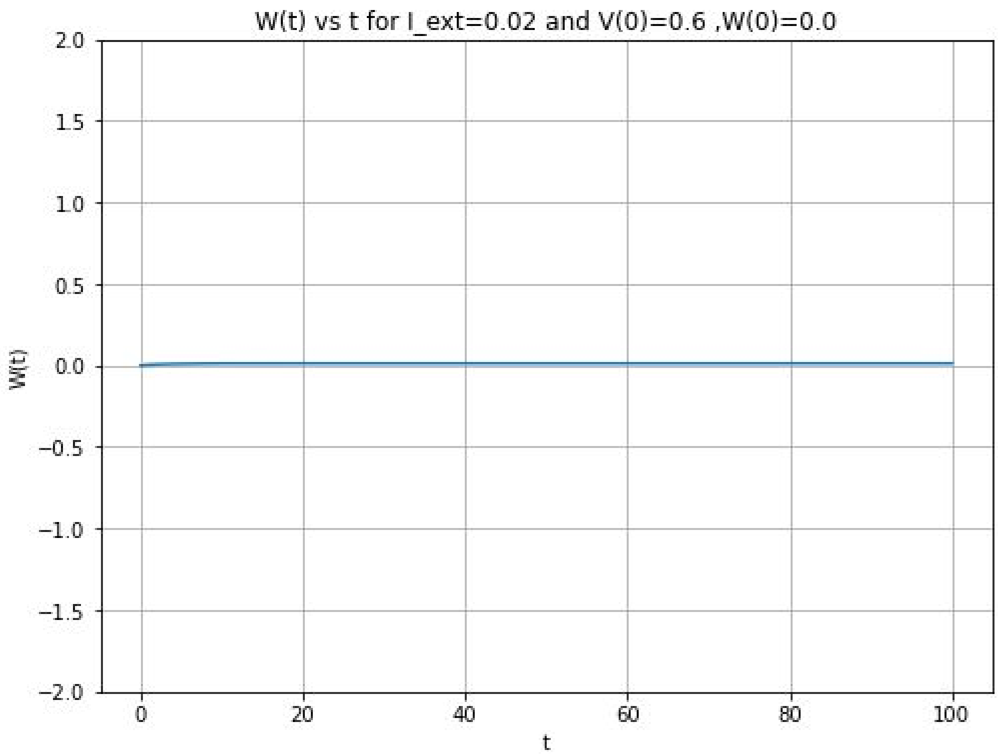
The plots below show the variation of V(t) with time.





The plots below show the variation of W(t) with time.




W(t) vs t for I_ext=0.02 and V(0)=0.6 ,W(0)=0.0
W(t)
t
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
20
40
60
80
100