Maulana Azad National Institute of Technology, Bhopal
End-Term Examination (Mathematics 3) [MTH 231]
Semester-III
Date: 25/11/2024

Branch: CSE

Course: B. Tech.

Max. Marks: 50

Time: 180 Minutes

Scholar No.:

Name:

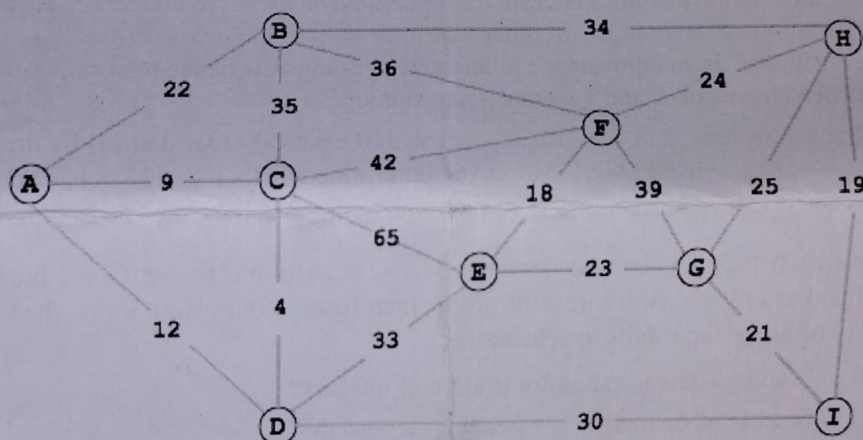**Please read the following instructions carefully.**

- There are five questions. Attempt all questions. Give a proper justification for your answers. Usual notations are used. Assume missing data, if any.

- **Notations:** $\mathbb{N}$: Set of natural numbers; $\mathbb{Z}$: Set of integers; $\mathbb{R}$: Set of real numbers; $\mathbb{C}$: Set of complex numbers; $P(A)$: Power set of A; |: divisibility symbol.

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1. (a) Let R be the relation of congruence modulo 3 defined on the set of integers $\mathbb{Z}$, such that for $a, b \in \mathbb{Z}$ and $aRb \Leftrightarrow a \equiv b \pmod 3$. In other words, $R = \{(a, b) : a, b \in \mathbb{Z}$ and 3 devides $(a - b)\}$. Check whether R is an equivalence relation. If R is an equivalence relation, find all the distinct equivalence classes of R and generate a partition of $\mathbb{Z}$. **[5 Marks]**

    (b) Given the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$, with the partial order defined by divisibility. Then, prove that $< A, | >$ is a POSET. Moreover, determine whether $< A, | >$ is a bounded lattice or not. If it is a bounded lattice, represent the same with the help of the Hasse diagram. **[5 Marks]**

2. (a) Determine whether the algebraic structure $(G, *)$ is a group if the set G and binary operation $*$ are defined as follows. (If it forms the group, then Justify your claim; if not, then identify which property of the group it fails to achieve.) **[4 Marks]**

    i. $G = \mathbb{Z}$ and $*$ is the usual multiplication of integers?

    ii. $G = \mathbb{R}$ and $*$ is defined by $a * b = 0$ for all a, b in $\mathbb{Z}$

    (b) Determine whether both $(\mathbb{R}^2, +)$ and $(\mathbb{C}, +)$ are both groups. If yes, check whether $f(x, y) = x + iy, \forall (x, y) \in \mathbb{R}^2$ is a isomorphism between $(\mathbb{R}^2, +)$ and $(\mathbb{C}, +)$. **[6 Marks]**

3. (a) Define the generating function and, using the generating functions, determine the sum $3 \times 2 \times 1 + 4 \times 3 \times 2 + 5 \times 4 \times 3 + \cdots + (r + 1)r(r - 1)$. **[5 Marks]**

    (b) Determine a_r for given $a_o = 1, a_1 = -2$ and $a_2 = 1$ satisfying the following recurrence relation, for $r > 2$, **[5 Marks]**

    $$a_r + 3a_{r-1} + 3a_{r-2} + a_{r-3} = 0$$

4. (a) State Bayes Theorem and Using the application of Bayes Theorem solve the problem:

    In 2023, there will be three candidates for the principal position- Mr. Chatterji, Mr. Ayangar, and Dr. Singh - whose chances of getting the appointment are in the proportion 4:2:3, respectively. The probability that Mr. Chatterji, if selected, would introduce co-education in the college is 0.3. The probabilities of Mr. Ayangar and Dr. Singh doing the same are respectively 0.5 and 0.8. **[5 Marks]**

i. What is the probability that there will be coeducation in the college in 2023?

ii. If there is coeducation in the college in 2023, what is the probability that Dr. Singh is the principal?

(b) A call center receives an average of 4 calls per hour. Assume the number of calls follows a Poisson distribution, then answer the following: **[5 Marks]**

    i. What is the probability that the center receives exactly 2 calls in an hour?

    ii. What is the probability that the center receives at least 3 calls in an hour?

    iii. If the center operates for 5 hours, what is the expected number of calls received during this time?

    iv. Calculate the variance and mode for the number of calls received in an hour.

5. (a) Use Prim's algorithm to compute the Minimum Spanning Tree (MST). Start from vertex A. Show all intermediate steps, including each iteration's priority queue updates and selected edges. Which MST algorithm is more suitable for dense graphs, and why? **[5 Marks]**

(b) Using an appropriate algorithm, compute the shortest path from the source vertex a to all other graph vertices. In addition, draw the graph and annotate the shortest paths from a to each vertex, including the total distance for each path (if reachable). **[5 Marks]**