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Mid Term Examination (Mathematics - 3) [MTH 231]

Course: B. Tech.	Semester-III	Branch: CSE
Time: 90 Minutes	Date: 23/09/2024	Max. Marks: 20
Name:		Scholar No.:

There are six questions. Attempt all questions. **Give proper justification for your answers.** Usual notations are used. Assume missing data if any.
Notation: $\mathbb{R}$ represents set of real numbers, $\mathbb{R}^+$ represents set of positive real numbers, I is a set of all integers and Q^+ is a set of all positive rational numbers.

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1. Let S be a set of all integer divisors of 24, and a relation W is defined as $W = \{(x, y) : x, y \in S \text{ and } x \text{ divides } y\}$. Check whether the relation W is a POSET or equivalence relation. [3 Marks]

2. Determine whether relation $(\mathcal{P}(S), \subseteq)$ is lattice, where $\mathcal{P}(S)$ is the power set of $S = \{1, 3, 9\}$. Also check whether it is bounded lattice or not? Draw the Hasse diagram. [4 Marks]

3. Check whether the following algebraic structures are subgroups of $\mathbb{R}$ or not [4 Marks]

   (a) $(I \cup Q^+, +)$

   (b) $(I \cap Q^+, \cdot)$

4. Given $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$ are additive and multiplicative groups, respectively. Then, check whether f is an isomorphism between $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$, where f is defined as $f(x) = 7^x, \forall x \in \mathbb{R}$. [3 Marks]

5. Solve the recurrence relation $a_r - 3a_{r-1} + 2a_{r-2} = 2^r + 3$. [3 Marks]

6. Determine the numeric function corresponding to generating function

$$G(z) = \frac{3}{(1-z)^3} + \frac{7}{1 - 3z + 2z^2}.$$

[3 Marks]