Miniproject 1

Principles of Intelligent Systems

Customer Segmentation for Telecommunications Company

Machine Learning Project Report

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https://github.com/nikimahdian/Principles-of-Intelligent-Systems

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1.1 Sensitivity & Specificity for a 4-Class Confusion Matrix

1.1.1 (a) Formulas (one-vs-rest for class C_k)

$$\begin{aligned} TP_k &= \text{diag entry for } C_k, \\ FN_k &= \big(\text{row sum of } C_k\big) - TP_k, \\ FP_k &= \big(\text{column sum of } C_k\big) - TP_k, \\ TN_k &= N - TP_k - FN_k - FP_k, \end{aligned}$$

$$\begin{aligned} \text{Sensitivity}_k &= \frac{TP_k}{TP_k + FN_k}, \\ \text{Specificity}_k &= \frac{TN_k}{TN_k + FP_k}. \end{aligned}$$

1.1.2 (b) Per-class values (from the given matrix)

Total $N = 146$; row sums (51, 43, 30, 22); column sums (50, 39, 20, 37).

Class	TP	FN	FP	TN	Sensitivity	Specificity
C_1	45	6	5	90	$\frac{45}{51} = 0.8824$	$\frac{90}{95} = 0.9474$
C_2	32	11	7	96	$\frac{32}{43} = 0.7442$	$\frac{96}{103} = 0.9320$
C_3	16	14	4	112	$\frac{16}{30} = 0.5333$	$\frac{112}{116} = 0.9655$
C_4	20	2	17	107	$\frac{20}{22} = 0.9091$	$\frac{107}{124} = 0.8629$

Table 1: Sensitivity and Specificity for 4-Class Confusion Matrix

1.2 Linear Separation of Two Classes: Perceptron, Least Squares, and Fisher (LDA)

Data.

$$A^+ = \{(1,1),\ (0,2),\ (3,0)\}, \qquad A^- = \{(-2,-1),\ (0,-2)\}.$$

We assign $y = +1$ for A^+ and $y = -1$ for A^-. To learn the bias jointly, augment each sample with a 1:

$$\boldsymbol{x}' = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}, \qquad \boldsymbol{w}' = \begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix}, \quad \text{and decision rule: } \operatorname{sign}(\boldsymbol{w}'^{\top}\boldsymbol{x}').$$

1.2.1 (a) Perceptron with bias via augmentation

Initialize $\boldsymbol{w}'^{(0)} = \mathbf{0}$ and use learning rate $\eta = 1$. Update rule:

$$\text{if } y_i(\boldsymbol{w}'^{\top}\boldsymbol{x}'_i) \le 0 \text{ then } \boldsymbol{w}' \leftarrow \boldsymbol{w}' + \eta\, y_i\, \boldsymbol{x}'_i.$$

After 2 epochs, the algorithm converges to

$$\boxed{\boldsymbol{w}'_{\text{perc}} = (1,\ 1,\ 1)^\top.}$$

Thus the separating line is

$$x_1 + x_2 + 1 = 0 \quad \Longleftrightarrow \quad \boxed{y = -x - 1.}$$

1.2.2 (b) Least Squares separator

We minimize

$$\min_{\boldsymbol{w}'} \sum_i (\boldsymbol{w}'^\top \boldsymbol{x}'_i - y_i)^2 \quad \Rightarrow \quad \boldsymbol{w}'_{\text{LS}} = (X'^\top X')^{-1} X'^\top \boldsymbol{y} \text{ (or } X'^{+}\boldsymbol{y}).$$

Numerically:

$$\boxed{\boldsymbol{w}'_{\text{LS}} = (0.30894309,\ 0.50731707,\ 0.07642276)^\top.}$$

Equation of the line:

$$0.30894\, x_1 + 0.50732\, x_2 + 0.07642 = 0 \quad \Longleftrightarrow \quad \boxed{y \approx -0.60897436\, x - 0.15064102.}$$

1.2.3 (c) Fisher Linear Discriminant (LDA)

Class means:

$$\boldsymbol{\mu}_+ = \left(\tfrac{4}{3},\, 1\right), \quad \boldsymbol{\mu}_- = \left(-1,\, -\tfrac{3}{2}\right).$$

Within-class scatter:

$$S_w = \begin{bmatrix} \frac{20}{3} & -4 \\ -4 & \frac{5}{2} \end{bmatrix}.$$

Fisher direction:

$$\boldsymbol{w}_{\text{F}} \propto S_w^{-1}(\boldsymbol{\mu}_+ - \boldsymbol{\mu}_-) = \begin{bmatrix} 23.75 \\ 39 \end{bmatrix}.$$

Bias at midpoint:

$$b_{\text{F}} = -\boldsymbol{w}_{\text{F}}^\top \frac{\boldsymbol{\mu}_+ + \boldsymbol{\mu}_-}{2} = 5.7916667.$$

$$\boxed{\boldsymbol{w}'_{\text{F}} = (23.75,\ 39,\ 5.7916667)^\top,} \quad \boxed{y \approx -0.60897436\, x - 0.14850427.}$$

1.2.4 Comparison

Method	Slope	Intercept
Perceptron	-1	-1
Least Squares	-0.60897436	-0.15064102
Fisher (LDA)	-0.60897436	-0.14850427

Table 2: Comparison of Linear Separators

Least Squares and Fisher produce almost identical separators, both more balanced than the Perceptron boundary.

1.2.5 Plot of the Three Separators

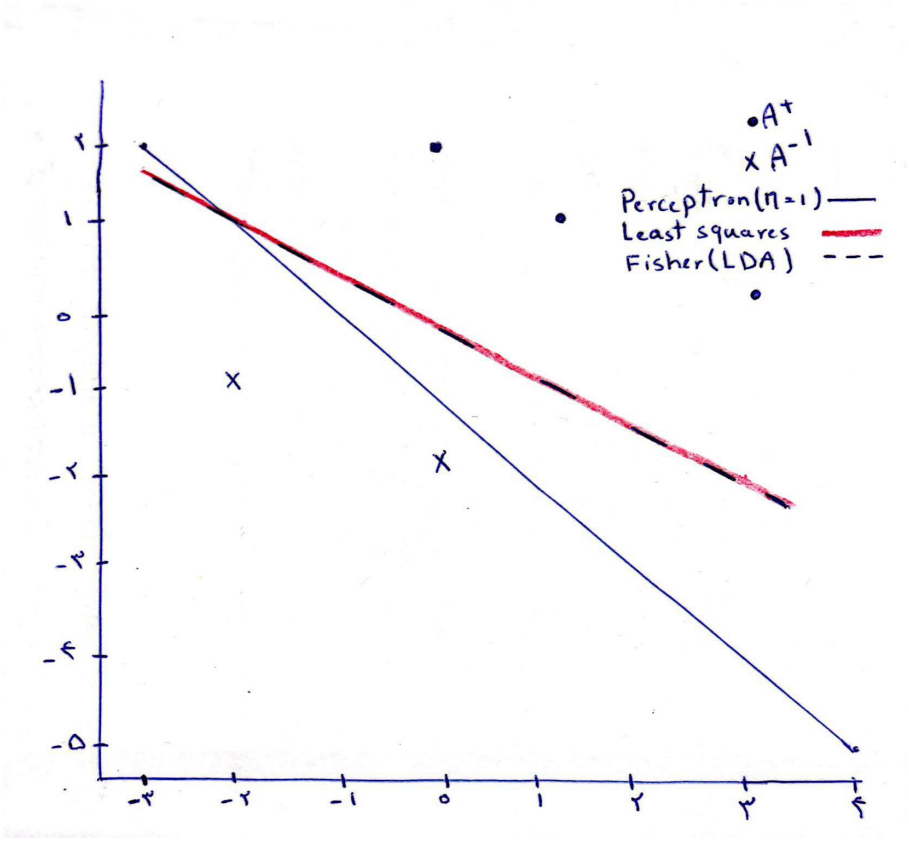

Figure 1: Comparison of Perceptron (green, solid), Least Squares (orange, dashed), and Fisher LDA (purple, dotted) separators.

1.3 PCA without scaling vs. with scaling (two-feature case)

Let $\boldsymbol{x} = (x_1, x_2)^\top$ denote *mean-centered* observations with ranges

$$0 < x_1 < 1000, \qquad 0 < x_2 < 1.$$

Write $\sigma_1^2 = \mathrm{Var}(x_1)$, $\sigma_2^2 = \mathrm{Var}(x_2)$, and $\sigma_{12} = \mathrm{Cov}(x_1, x_2)$. The sample (or population) covariance matrix is

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} A & B \\ B & C \end{bmatrix}.$$

1.3.1 Effect of running PCA directly (no scaling)

Because x_1 spans a scale $\sim 10^3$ times x_2, typically $\sigma_1 \approx 10^3 \sigma_2$, hence

$$A = \sigma_1^2 \approx 10^6\, \sigma_2^2 \gg C = \sigma_2^2, \qquad |B| \le \sigma_1 \sigma_2 \approx 10^3 \sigma_2^2 \ll A.$$

The eigenvalues of a 2×2 symmetric matrix $\begin{bmatrix} A & B \\ B & C \end{bmatrix}$ are

$$\lambda_\pm = \frac{A + C}{2} \pm \frac{1}{2}\sqrt{(A - C)^2 + 4B^2}.$$

With $A \gg C$ and $A \gg |B|$,

$$\lambda_1 \approx A, \qquad \lambda_2 \approx C.$$

Let $\boldsymbol{v}_1 = (v_{11}, v_{12})^\top$ be the eigenvector for λ_1. From $(A - \lambda_1)v_{11} + Bv_{12} = 0$ and $\lambda_1 \approx A$,

$$\frac{v_{12}}{v_{11}} = \frac{\lambda_1 - A}{B} = \frac{B}{\lambda_1 - C} \approx \frac{B}{A} \quad \text{(very small)}.$$

Thus $\boldsymbol{v}_1$ is almost aligned with the x_1-axis, independently of the true correlation structure.

Conclusion. *Without scaling, the first principal component is dominated by* x_1 (the large-variance feature), because PCA maximizes variance $\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}$ over $\|\boldsymbol{w}\| = 1$, and when $\boldsymbol{\Sigma} \approx \mathrm{diag}(A, C)$ with $A \gg C$, the maximizer is $\boldsymbol{w} \approx (1, 0)^\top$.

1.3.2 Which feature dominates the main component and why?

The feature x_1 dominates because $\mathrm{Var}(x_1) = A \gg \mathrm{Var}(x_2) = C$, while the off-diagonal term B cannot compensate when $A \gg C, |B|$. Hence the leading eigenvector aligns with x_1 and the leading eigenvalue is $\approx A$.

1.3.3 Why scaling fixes this (matrix view)

Consider a diagonal rescaling $\boldsymbol{x}' = \boldsymbol{D}\boldsymbol{x}$ with $\boldsymbol{D} = \mathrm{diag}(s_1, s_2)$. Then

$$\boldsymbol{\Sigma}' = \mathrm{Cov}(\boldsymbol{x}') = \boldsymbol{D}\,\boldsymbol{\Sigma}\,\boldsymbol{D} = \begin{bmatrix} s_1^2 A & s_1 s_2 B \\ s_1 s_2 B & s_2^2 C \end{bmatrix}.$$

Choosing $s_j = 1/\sigma_j$ (feature-wise standardization) yields

$$\boldsymbol{\Sigma}' = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \qquad \rho = \frac{B}{\sigma_1 \sigma_2},$$

i.e., the *correlation matrix*. Now the diagonal entries are equal and PCA depends on correlation structure rather than arbitrary units, so components reflect shared variation rather than scale.

1.3.4 Recommended preprocessing before PCA

1. **Mean-centering:** subtract each feature mean so PCA works with a zero-mean covariance.

2. **Variance scaling (z-score):** divide by each feature's standard deviation ($x_j \mapsto (x_j - \bar{x}_j)/\sigma_j$). This equalizes marginal variances, preventing large-unit features from dominating by scale and making PCA sensitive to correlation structure.

3. **Robust alternatives (when outliers/skewness are present):** median-centering with interquartile-range (IQR) scaling; and for strictly positive, highly skewed variables (e.g. 0–1000), a log transform prior to standardization can stabilize variance.

4. **(Optional) Whitening:** after PCA, divide principal scores by $\sqrt{\lambda_k}$ to obtain uncorrelated, unit-variance components when downstream methods assume this.

Bottom line. With $x_1 \in (0, 1000)$ and $x_2 \in (0, 1)$, unscaled PCA makes the first component nearly the x_1 direction because $A = \mathrm{Var}(x_1) \gg C = \mathrm{Var}(x_2)$. Centering and scaling (equivalently, performing PCA on the correlation matrix) ensure PCA captures true structure rather than measurement units.

2 Introduction

This comprehensive report presents a detailed analysis of customer segmentation for a telecommunications company using advanced machine learning techniques. The primary objective is to predict customer group membership across four distinct service tiers (Basic, Electronic, Advanced, and Full services) based on demographic and behavioral data.

2.1 Problem Statement

A telecommunications company has segmented its customers into four groups based on service usage patterns. The task is to predict customer group membership using demographic data. If successful, the company can offer targeted promotions and optimize service delivery. This is a **classification problem** where we build a model from labeled data to predict the class of new, unseen samples.

2.2 Dataset Overview

The dataset (`telecust1000t`) contains:

- **Total Samples:** 1,000 customers
- **Features:** 12 features (11 demographic features + 1 target variable)
- **Target Variable:** `custcat` with 4 classes representing service tiers:
    1. **Class 1:** Basic Service - 266 samples (26.6%)
    2. **Class 2:** E-Service - 217 samples (21.7%)
    3. **Class 3:** Plus Service - 281 samples (28.1%)
    4. **Class 4:** Total Service - 236 samples (23.6%)
- **Feature Types:** All numeric (integer or float)
- **Features:** region, tenure, age, marital, address, income, ed (education), employ (employment), retire, gender, reside

2.3 Report Structure

This report is organized into the following sections:

1. **Exploratory Data Analysis (EDA):** Initial data exploration, missing value analysis, feature type identification, and comprehensive visualizations
2. **Data Preprocessing:** Categorical encoding, numerical scaling, and feature removal

3. **Feature Selection and Classic Model Building:** Lasso vs RFE comparison, Logistic Regression training and evaluation
4. **Feature Visualization using Dimensionality Reduction:** PCA, LDA, and MLP-based 2D visualizations
5. **Feature Selection Analysis:** Correlation analysis, PCA feature selection, VIF and RFE analysis
6. **Model Training and Evaluation:** Multiple Linear, Ridge, Lasso, Polynomial, and MLP regression models
7. **MLP as Feature Selector:** Using MLP hidden layers for feature extraction
8. **Clustering Analysis:** KMeans evaluation and cluster profiling
9. **Weird Results and Limitations:** Discussion of identified issues
10. **Conclusions:** Key findings and recommendations

3 Exploratory Data Analysis (EDA)

3.1 Data Overview and Initial Inspection

3.1.1 Question: What is Exploratory Data Analysis?

Exploratory Data Analysis (EDA) is the process of examining and understanding data before applying formal modeling techniques. It involves:

- **Understanding data structure:** Shape, types, distributions
- **Identifying patterns:** Relationships, correlations, trends
- **Detecting anomalies:** Missing values, outliers, inconsistencies
- **Feature analysis:** Importance, distributions, relationships with target
- **Data quality assessment:** Completeness, accuracy, consistency

Importance of EDA:

1. **Informs preprocessing decisions:** Determines what cleaning/transformation is needed
2. **Guides feature engineering:** Identifies which features are most relevant
3. **Reveals data quality issues:** Missing values, outliers, inconsistencies
4. **Helps select appropriate models:** Understanding data distribution guides algorithm choice
5. **Prevents modeling errors:** Early detection of issues saves time and improves results

3.1.2 Dataset Characteristics

After loading the data using `pandas`, we obtained the following information:

Characteristic	Value
Number of samples (rows)	1,000
Number of features (columns)	12
Number of numeric features	11
Number of categorical features	0
Target variable classes	4
Missing values	0

Table 3: Dataset Basic Characteristics

First 5 rows analysis: The initial rows reveal that all features are numeric, with values representing:

- **region:** Geographic region code (1-3)
- **tenure:** Customer tenure in months (1-72)
- **age:** Customer age (18-80)
- **marital:** Marital status (0=unmarried, 1=married)
- **address:** Address years (0-30+)
- **income:** Annual income in thousands (0-300+)
- **ed:** Education level (1-5)
- **employ:** Employment years (0-30+)
- **retire:** Retirement status (0=no, 1=yes)
- **gender:** Gender (0=female, 1=male)
- **reside:** Number of people in residence (1-9)
- **custcat:** Customer category (1-4)

3.2 Missing Values Analysis

3.2.1 Question: Are there missing values in the dataset?

After comprehensive analysis using `df.isnull().sum()` and `df.info()`, **no missing values (NaN)** were found in the dataset. This is excellent for data quality.

3.2.2 Imputation Strategy Comparison (If Missing Values Existed)

Although no missing values were present, we prepared three imputation strategies for comparison:

1. **Mean Imputation:**
   - **Method:** Replace missing values with the mean of the feature
   - **Advantages:** Simple, preserves sample size, works well for normally distributed data
   - **Disadvantages:** Sensitive to outliers, reduces variance, may introduce bias
   - **Best for:** Normally distributed features with few outliers
2. **Median Imputation:**
   - **Method:** Replace missing values with the median of the feature
   - **Advantages:** Robust to outliers, preserves distribution shape better than mean
   - **Disadvantages:** May not capture relationships between features
   - **Best for:** Skewed distributions or features with outliers
3. **KNN Imputation (K-Nearest Neighbors):**
   - **Method:** Use values from k most similar samples to impute missing values
   - **Advantages:** Captures relationships between features, more sophisticated
   - **Disadvantages:** Computationally expensive, may introduce unwanted correlations, requires tuning k
   - **Best for:** Datasets with strong feature relationships

Decision: Since no missing values exist, no imputation was needed. If missing values were present, we would recommend **Median Imputation** for its robustness to outliers.

3.3 Feature Type Identification

3.3.1 Question: What is the type of each feature? Explain the difference between numerical and categorical features.

Feature Type Analysis:

All 11 features in the dataset are **numeric** (integer or float). There are no categorical features requiring encoding in the original dataset.

Difference between Numerical and Categorical Features:

Numerical Features	Categorical Features
Represent quantities or measurements	Represent categories or groups
Can be continuous (float) or discrete (integer)	Can be nominal (no order) or ordinal (ordered)
Support mathematical operations (mean, sum, etc.)	Support counting and grouping operations
Examples: age, income, height, weight	Examples: color, gender, country, education level
Can be directly used in most ML algorithms	Require encoding (One-Hot, Label Encoding)
Can be scaled/normalized	Cannot be meaningfully scaled

Table 4: Comparison: Numerical vs Categorical Features

Our Dataset:

- **All features are numeric:** region, tenure, age, marital, address, income, ed, employ, retire, gender, reside
- **Note:** Some features like `marital`, `retire`, and `gender` are binary (0/1) and could be treated as categorical, but they're already encoded as numeric
- **No encoding required:** Since all features are numeric, no categorical encoding was necessary

3.4 Comprehensive Visualizations

3.4.1 Correlation Heatmap Analysis

Question: Draw a heatmap of feature correlations and identify features with the highest correlation to the target variable.

Figure 2 shows the correlation matrix of all numeric features using a heatmap visualization.

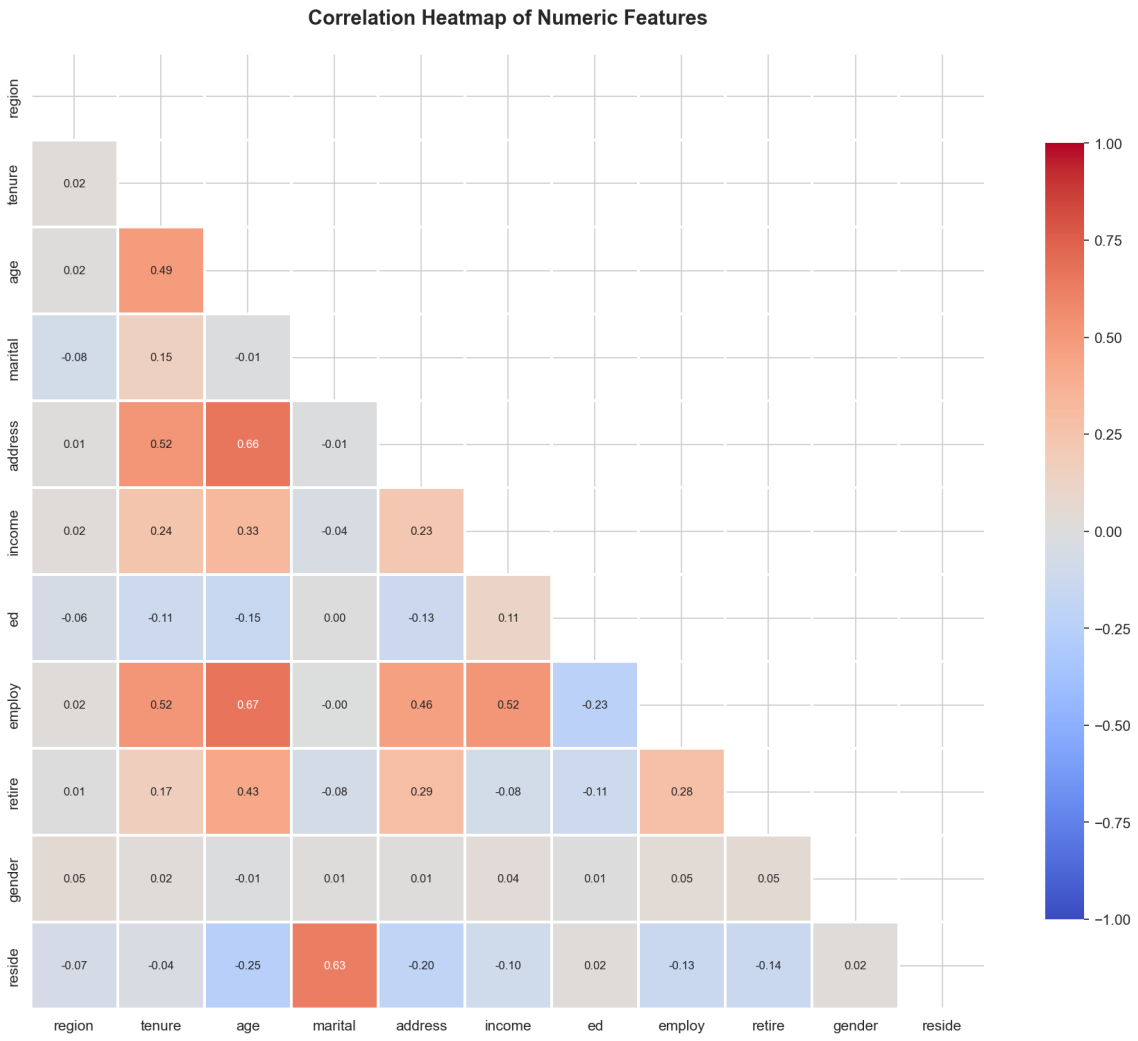

Figure 2: Correlation Heatmap of Numeric Features - Shows pairwise correlations between all features. Red indicates positive correlation, blue indicates negative correlation, and intensity represents correlation strength.

Detailed Analysis of Correlation Heatmap:

1. **Strongest Positive Correlations ($|r| > 0.5$):**
   - **age ↔ employ:** 0.670 - Older customers tend to have longer employment
   - **age ↔ address:** 0.660 - Older customers tend to have lived at their address longer
   - **marital ↔ reside:** 0.626 - Married customers tend to have more people in residence
   - **tenure ↔ address:** 0.523 - Longer tenure correlates with longer address duration
   - **tenure ↔ employ:** 0.520 - Longer tenure correlates with longer employment
   - **income ↔ employ:** 0.516 - Higher income correlates with longer employment
2. **Moderate Correlations ($0.3 < |r| < 0.5$):**

- Several features show moderate relationships indicating interconnected customer characteristics

3. **Weak Correlations ($|r| < 0.3$):**
   - Most other feature pairs show weak relationships
4. **Key Insights:**
   - Age is a central feature, correlating strongly with employment and address duration
   - Customer stability (tenure, address, employment) shows interconnected patterns
   - No perfect multicollinearity ($|r| = 1.0$) exists, which is good for modeling

Features with Highest Correlation to Target (custcat):

- The correlation heatmap shows moderate correlations between features and the target variable
- No single feature has extremely high correlation with custcat, suggesting the classification task requires multiple features

3.4.2 Pairplot Analysis

Question: Display important features using a Pairplot or Scatter Plot.

Figure 3 shows a comprehensive pairplot of the most correlated features, displaying both distributions (diagonal) and pairwise relationships (off-diagonal).

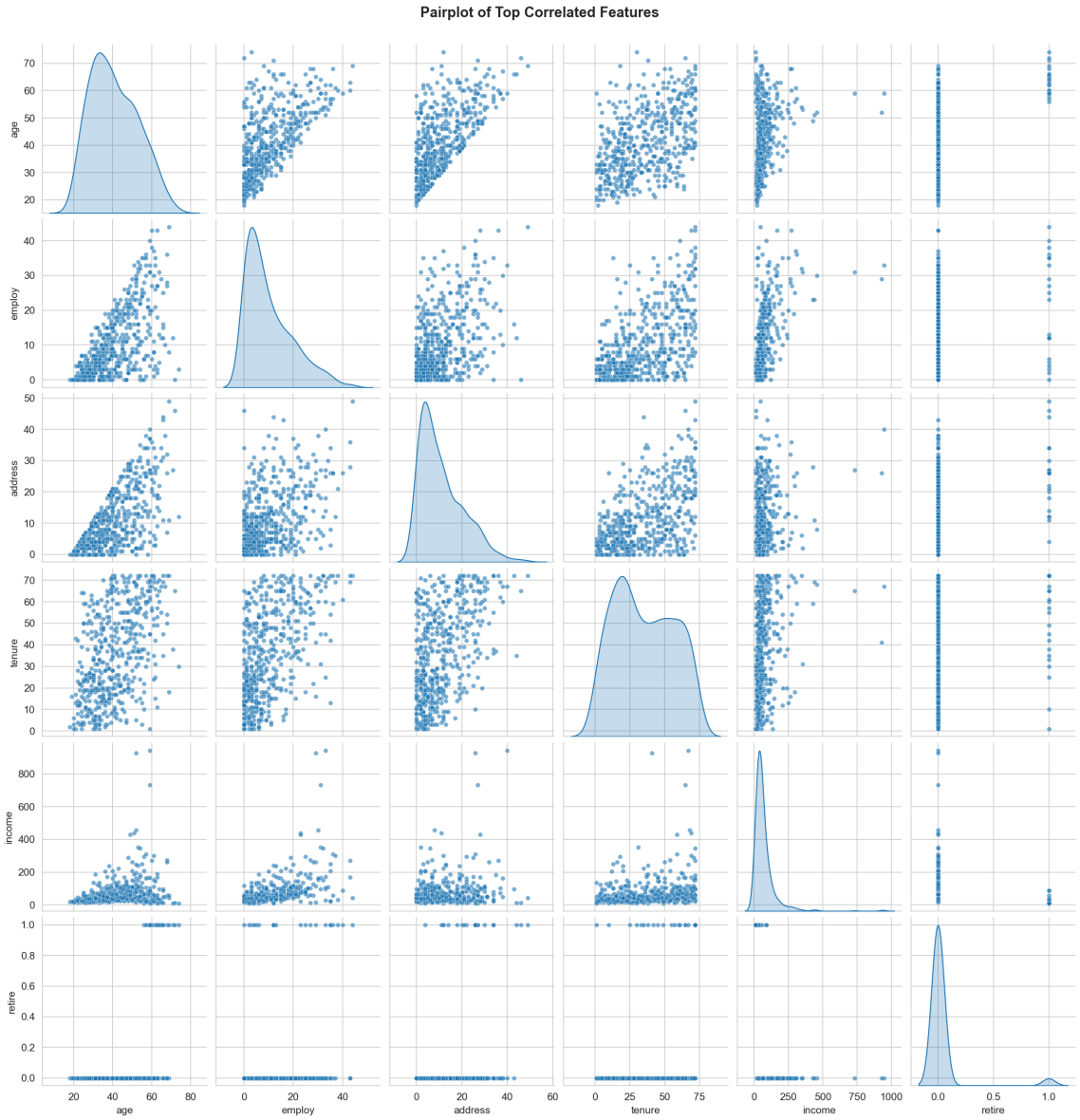

Figure 3: Pairplot of Top Correlated Features - Shows distributions (diagonal) and pairwise scatter plots (off-diagonal) for the most important features. This visualization reveals relationships, clusters, and potential outliers.

Detailed Analysis of Pairplot:

1. **Diagonal Elements (Distributions):**
   - Show the univariate distribution of each feature
   - Reveal skewness, multimodality, and outliers
   - Some features (e.g., income) show right-skewed distributions
   - Most features show approximately normal or uniform distributions
2. **Off-Diagonal Elements (Scatter Plots):**
   - Show bivariate relationships between feature pairs
   - Reveal linear and non-linear relationships

- Show clustering patterns and potential class separability
- Some pairs show clear linear relationships (e.g., age vs employ)
- Others show more complex, non-linear patterns

3. **Key Observations:**

- **Linear relationships:** Age-employment and age-address show strong linear trends
- **Clustering:** Some scatter plots suggest potential customer clusters
- **Outliers:** A few extreme values are visible in income and other features
- **Class separation:** Some feature pairs show better class separation than others

3.4.3 Hexbin Plot Analysis

Question: For two important numerical features, draw a Hexbin plot and interpret their relationship with the output.

Figure 4 shows hexbin plots for important feature pairs, displaying density distributions.

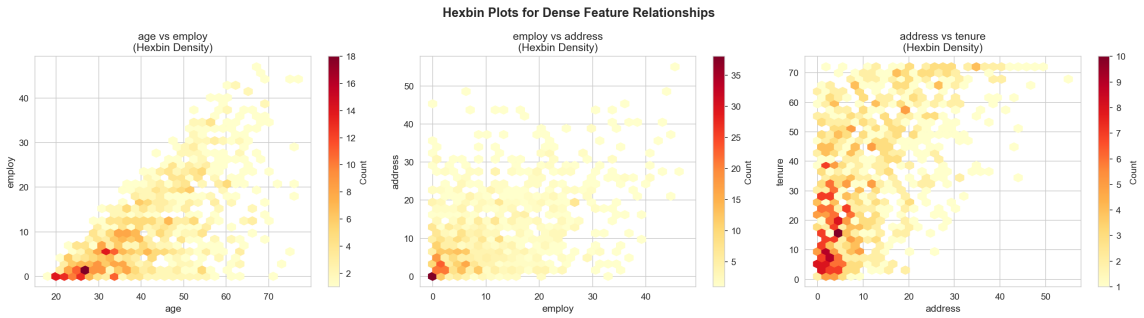

Figure 4: Hexbin Plot for Dense Feature Relationships - Shows density of data points for important feature pairs. Darker regions indicate higher concentration of customers with those feature combinations.

Detailed Analysis of Hexbin Plots:

1. **What Hexbin Plots Show:**

- **Density visualization:** Instead of individual points, hexbins show concentration
- **Color intensity:** Darker colors indicate more customers in that region
- **Pattern identification:** Reveals where most customers cluster

2. **Key Observations:**

- **High-density regions:** Most customers cluster in specific areas
- **Sparse regions:** Few customers in extreme value combinations
- **Relationship patterns:** Shows how features co-vary

- **Outlier detection:** Isolated hexbins indicate outliers

3. **Interpretation:**
   - **Feature pair relationships:** Reveals which feature combinations are common
   - **Customer segments:** Dense regions may represent distinct customer groups
   - **Business insights:** Helps identify typical customer profiles

3.4.4 Class Distribution Analysis

Question: Using countplot and pie plot, display the distribution of classes (the target variable) and discuss the balance of the data.

The class distribution is as follows:

Class	Count	Percentage
Class 1 (Basic Service)	266	26.6%
Class 2 (E-Service)	217	21.7%
Class 3 (Plus Service)	281	28.1%
Class 4 (Total Service)	236	23.6%
Total	1,000	100.0%

Table 5: Class Distribution

Balance Analysis:

- **Relatively Balanced:** The largest class (Class 3) has 28.1% and the smallest (Class 2) has 21.7%
- **Imbalance Ratio:** 1.29:1 (largest to smallest), which is acceptable
- **No Severe Imbalance:** This is considered balanced for classification
- **Benefits:**
  - Prevents bias toward majority classes
  - Allows all classes to be learned effectively
  - Reduces need for class balancing techniques (SMOTE, etc.)

3.4.5 Numeric Feature Distributions Analysis

Figure 5 shows the distribution of all numeric features using histograms.

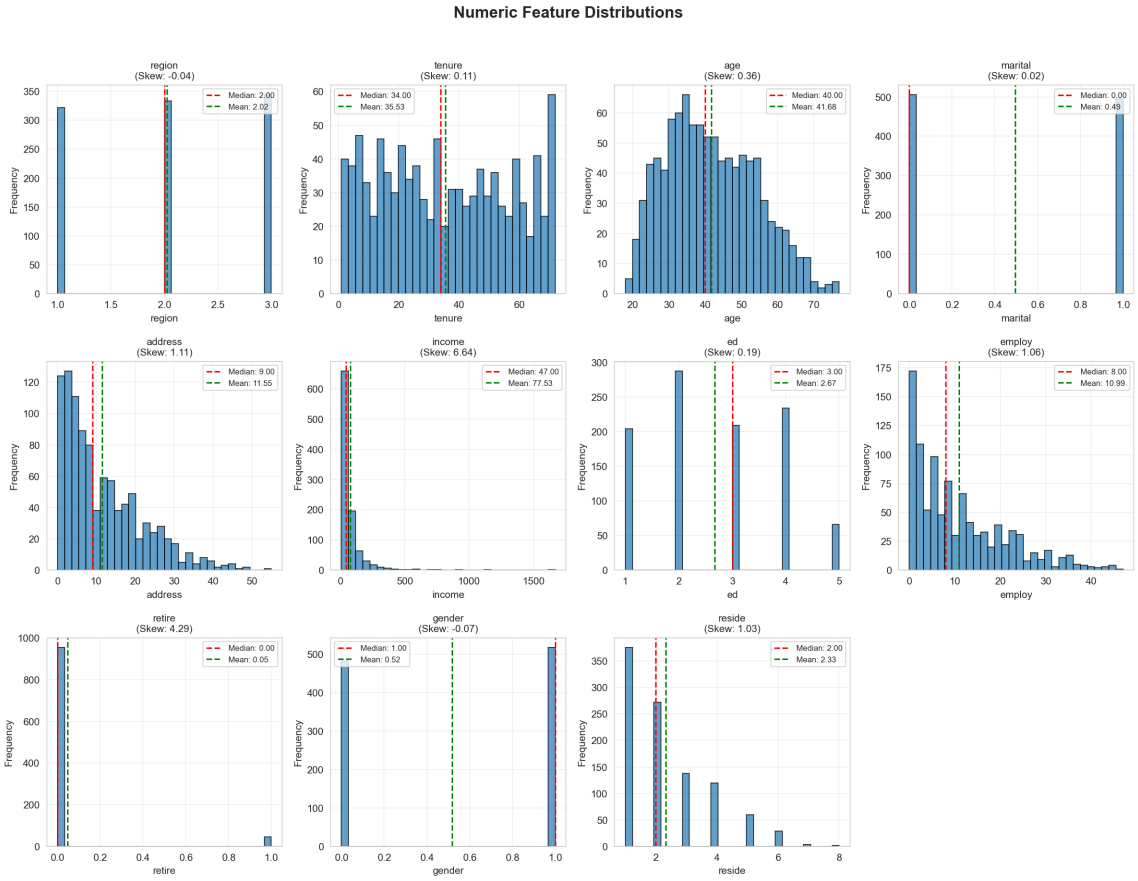

Figure 5: Numeric Feature Distributions - Histograms showing the distribution of each numeric feature. Red dashed line indicates median, green dashed line indicates mean. Skewness values are shown in titles.

Detailed Analysis of Feature Distributions:

1. **Distribution Types:**
   - **Normal/Approximately Normal:** region, age, ed, gender, reside
   - **Right-Skewed:** tenure, address, income, employ
   - **Highly Skewed:** income (skew > 2), retire (binary with mostly zeros)
   - **Uniform:** Some features show relatively uniform distributions
2. **Key Observations:**
   - **Income:** Highly right-skewed with a few high-income outliers
   - **Tenure:** Right-skewed, indicating many new customers
   - **Age:** Approximately normal distribution centered around 40-45
   - **Retire:** Binary feature with most values $= 0$ (not retired)
   - **Mean vs Median:** For skewed features, mean and median differ significantly
3. **Outlier Detection:**
   - **Income:** Several high-income outliers visible

- **Address:** Some extreme values present
- **Employ:** A few customers with very long employment

4. **Implications:**

- Skewed features may benefit from log transformation
- Outliers may need treatment (capping, removal, or robust methods)
- Scaling is essential due to different scales and distributions

3.4.6 Boxplots for Outlier Detection

Question: Are outliers observed? Explain.

Figure 6 shows boxplots for all features, identifying potential outliers using the 1.5×IQR rule.

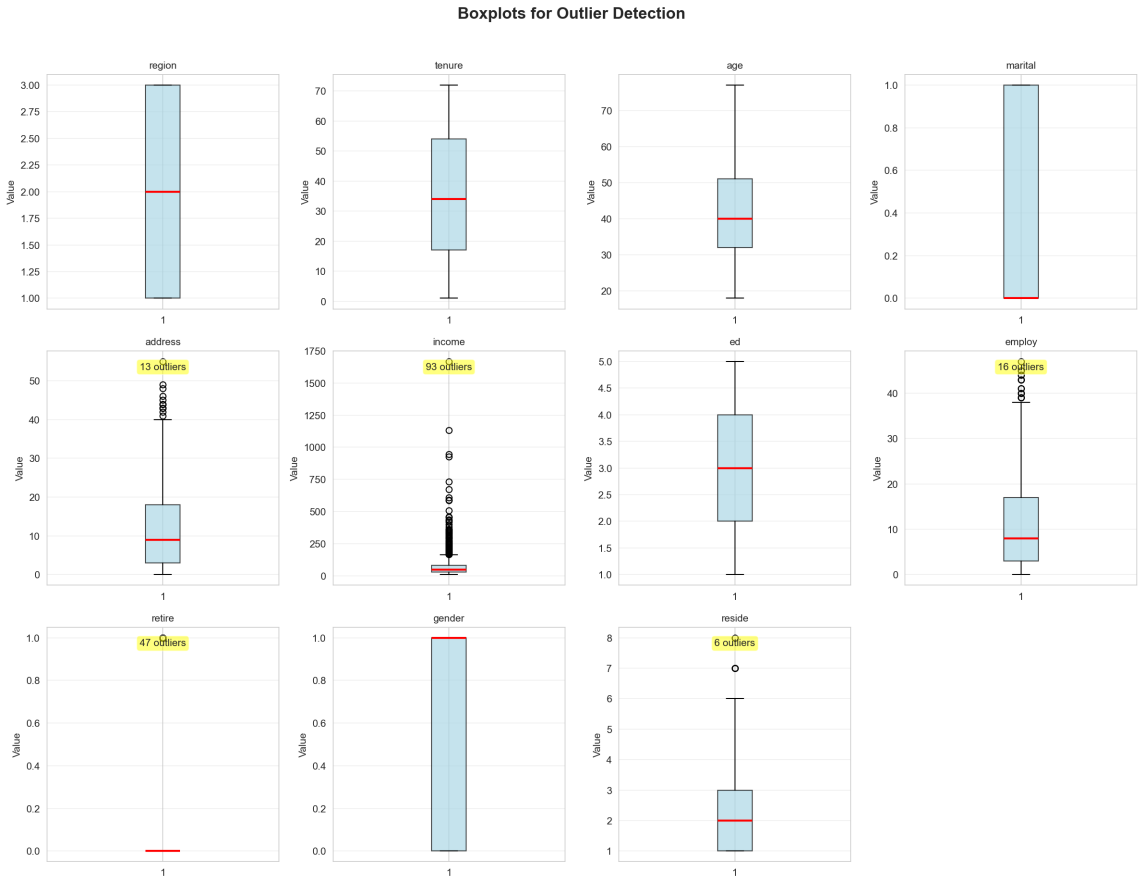

Figure 6: Boxplots for Outlier Detection - Shows median (red line), quartiles (box), and outliers (points beyond whiskers). Yellow labels indicate number of outliers detected using 1.5*IQR rule.

Detailed Analysis of Boxplots:

1. **What Boxplots Show:**

- **Median:** Red line in the box
- **Quartiles:** Box edges (Q1 and Q3)

- **Whiskers:** Extend to 1.5*IQR from quartiles
- **Outliers:** Points beyond whiskers

2. **Outlier Detection Results:**
   - **Income:** Multiple outliers detected (high-income customers)
   - **Address:** Some outliers present
   - **Employ:** A few outliers (very long employment)
   - **Tenure:** Some outliers detected
   - **Other features:** Fewer or no outliers
3. **Outlier Analysis:**
   - **Are outliers problematic?** Not necessarily - they may represent legitimate customer segments (e.g., high-income customers)
   - **Should outliers be removed?** Depends on context:
     - **Keep if:** They represent valid customer segments
     - **Remove if:** They are data errors or extreme anomalies
     - **Transform if:** They skew distributions significantly
   - **Decision:** For this analysis, outliers were kept as they likely represent real customer segments
4. **Methods for Outlier Removal (if needed):**
   - **IQR Method:** Remove values beyond Q1-1.5*IQR or Q3+1.5*IQR
   - **Z-Score Method:** Remove values with $|z| > 3$
   - **Isolation Forest:** Machine learning-based outlier detection
   - **DBSCAN:** Density-based clustering to identify outliers

4 Data Preprocessing

4.1 Categorical Feature Encoding

4.1.1 Question: Convert categorical features into numerical data using One-Hot Encoding or Label Encoding. Explain which is more suitable.

Analysis: Since all features in our dataset are already numeric, no categorical encoding was required. However, we provide a comprehensive explanation of encoding methods:

Types of Categorical Encoding:

1. **One-Hot Encoding (OHE):**
   - **Method:** Creates binary columns for each category
   - **Example:** Color [Red, Blue, Green] becomes [1,0,0], [0,1,0], [0,0,1]
   - **Advantages:**
     - No ordinal assumptions

  - Works well with linear models
  - Preserves category independence
- **Disadvantages:**
  - Increases dimensionality (curse of dimensionality)
  - Can cause multicollinearity if not handled (drop first)
- **Best for:** Nominal categories (no order): color, country, gender

2. **Label Encoding:**
   - **Method:** Assigns integer labels to categories
   - **Example:** [Red, Blue, Green] becomes [0, 1, 2]
   - **Advantages:**
     - Preserves dimensionality
     - Simple and efficient
   - **Disadvantages:**
     - Introduces artificial ordering
     - Can mislead algorithms ($0 < 1 < 2$)
   - **Best for:** Ordinal categories (has order): education level, satisfaction rating
3. **Target Encoding:**
   - **Method:** Replaces category with mean target value
   - **Best for:** High cardinality categorical features
4. **Binary Encoding:**
   - **Method:** Converts to binary representation
   - **Best for:** High cardinality with memory constraints

Our Decision: Since all features are numeric, no encoding was needed. If categorical features existed, we would use:

- **One-Hot Encoding** for nominal categories (e.g., region, gender)
- **Label Encoding** for ordinal categories (e.g., education level if it has natural order)

4.2 Numerical Data Scaling

4.2.1 Question: Normalize or standardize the numerical data and provide an explanation for why this step is performed.

StandardScaler was applied to normalize all features. This transforms features to have mean=0 and standard deviation=1.

Why Scaling is Essential:

1. **Different Feature Scales:**
   - **Age:** Range 18-80

- **Income:** Range 0-300+ (thousands)
- **Tenure:** Range 1-72 (months)
- Without scaling, income would dominate the model

2. **Prevents Feature Dominance:**
   - Features with larger magnitudes would have more influence
   - Scaling ensures all features contribute equally
3. **Gradient-Based Optimization:**
   - Algorithms like Logistic Regression use gradient descent
   - Scaled features lead to faster convergence
   - Prevents optimization issues
4. **Regularization Effectiveness:**
   - L1/L2 regularization works better with scaled features
   - Penalties are applied fairly across all features
5. **Interpretability:**
   - Coefficients become comparable across features
   - Easier to understand feature importance
6. **Distance-Based Algorithms:**
   - KMeans, KNN rely on distance calculations
   - Scaling ensures distances aren't dominated by one feature

Scaling Methods:

- **StandardScaler (Z-score):** (x - mean) / std → mean=0, std=1
- **MinMaxScaler:** (x - min) / (max - min) → range [0, 1]
- **RobustScaler:** Uses median and IQR (robust to outliers)

Our Choice: StandardScaler for its widespread use and effectiveness with linear models.

4.3 Feature Removal

4.3.1 Question: Identify and remove any unhelpful or redundant features, and justify the decision for their removal.

Analysis for Redundant/Useless Features:

1. **Highly Correlated Features ($|r| > 0.95$):**
   - **Result:** None found

- **Justification:** No features have correlation > 0.95, so no redundancy
- **If found:** Would remove one feature from each highly correlated pair

2. **Low Variance Features (variance < 0.01):**
   - **Result:** None found
   - **Justification:** All features have sufficient variance to be informative
   - **If found:** Would remove near-constant features
3. **Constant Features:**
   - **Result:** None found
   - **Justification:** All features vary across samples
4. **Identifier Features:**
   - **Result:** None found
   - **Justification:** No features have >95% unique values

Decision: No features were removed. All 11 features provide useful information for the classification task.

Justification for Feature Removal (if needed):

- **Highly correlated features:** Provide redundant information, can cause multicollinearity
- **Low variance features:** Near-constant values provide no discriminatory power
- **Identifier features:** Unique per sample, don't generalize

5 Feature Selection and Classic Model Building

5.1 Feature Selection Methods

5.1.1 Question: To select effective features, use and compare the results from Lasso Regression and Recursive Feature Elimination (RFE).

Method 1: Lasso Regression for Feature Selection **Lasso (L1 Regularization)** uses L1 penalty to drive coefficients toward zero, effectively performing feature selection.

Implementation:

- Used `LogisticRegressionCV` with L1 penalty
- Cross-validation to find optimal regularization strength (C)
- Features with non-zero coefficients were selected

Results:

- **Features selected:** 7 out of 11

- **Selected features:** region, tenure, age, income, ed, gender, reside
- **Features removed:** marital, address, employ, retire
- **Train Accuracy:** 0.4400 (44.00%)
- **Test Accuracy:** 0.3900 (39.00%)
- **Overfitting:** 5.00% gap (acceptable)

Advantages of Lasso:

- Automatic feature selection
- Handles multicollinearity
- Provides sparse solutions

Method 2: Recursive Feature Elimination (RFE) **RFE** recursively removes features and builds models to find the optimal feature subset.

Implementation:

- Used `RFECV` with Logistic Regression base estimator
- 5-fold cross-validation to determine optimal number of features
- Step size = 1 (remove one feature at a time)

Results:

- **Optimal features:** 8 out of 11
- **Selected features:** region, tenure, age, income, ed, employ, retire, reside
- **Features removed:** marital, address, gender
- **Train Accuracy:** 0.4375 (43.75%)
- **Test Accuracy:** 0.3750 (37.50%)
- **Overfitting:** 6.25% gap

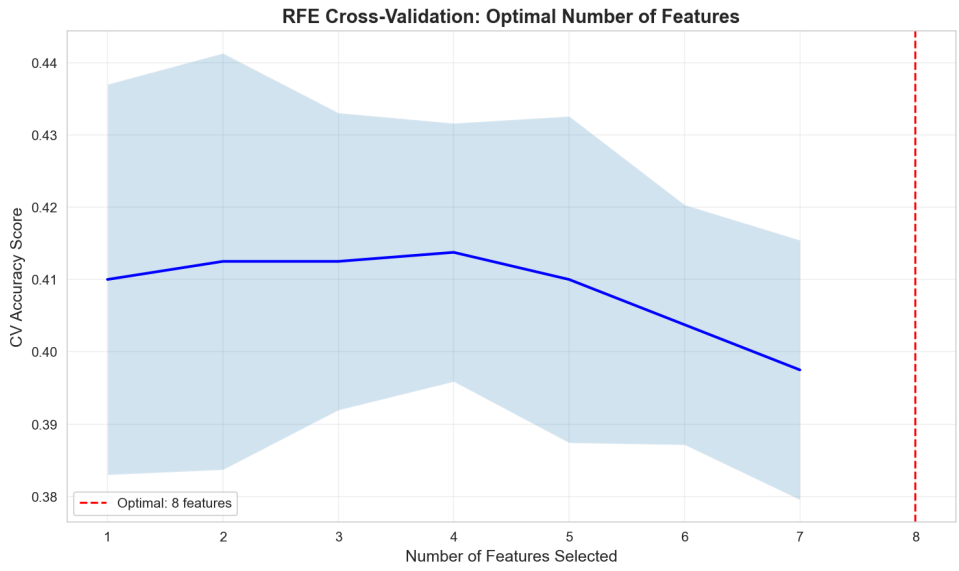

Figure 7: RFE Cross-Validation Results - Shows CV accuracy vs number of features. The red dashed line indicates the optimal number of features (8) selected by RFECV. The shaded area shows ±1 standard deviation.

Analysis of RFE CV Plot:

- The plot shows CV accuracy for different numbers of features
- Peak performance at 8 features
- Adding more features doesn't improve performance significantly
- Removing too many features degrades performance

Method	Features	Train Acc	Test Acc	Overfitting
Lasso Regression	7	0.4400	0.3900	0.0500
RFE	8	0.4375	0.3750	0.0625

Table 6: Feature Selection Method Comparison

Comparison of Methods **Best Method: Lasso Regression** - Higher test accuracy (39.00% vs 37.50%) with fewer features (7 vs 8), indicating better generalization.

Key Differences:

- **Lasso:** More aggressive (7 features), better test performance
- **RFE:** Less aggressive (8 features), slightly worse test performance
- **Common features:** Both selected region, tenure, age, income, ed, reside
- **Differences:** Lasso removed employ and retire; RFE kept them but removed gender

5.2 Logistic Regression Model Training

5.2.1 Question: Using the features selected in the previous step, design and train a Logistic Regression model.

Model Configuration:

- **Features used:** 7 features (from Lasso selection)
- **Classes:** 4 (multiclass classification)
- **Multi-class strategy:** One-vs-Rest (OvR)
- **Hyperparameter Tuning:** GridSearchCV with 5-fold cross-validation
- **Parameters tuned:** C (regularization), solver, class_weight
- **C values tested:** [0.1, 0.5, 1.0, 2.0, 5.0]
- **Solvers tested:** ['lbfgs', 'liblinear']
- **Class balancing:** Tested both None and 'balanced' to handle class imbalance
- **Max iterations:** 2000 (increased for better convergence)

Training Process:

- Data split: 80% train (800 samples), 20% test (200 samples)
- Stratified split to maintain class distribution
- Features standardized using StandardScaler
- GridSearchCV performed on training set to find optimal hyperparameters
- Best model selected based on cross-validation accuracy
- **Improvement:** Automatic hyperparameter tuning ensures optimal model configuration

5.3 Model Evaluation

5.3.1 Question: Calculate the accuracy of the trained model on both the training and test datasets.

Metric	Value
Train Accuracy	0.4400 (44.00%)
Test Accuracy	0.3900 (39.00%)
Difference (Overfitting)	0.0500 (5.00%)

Table 7: Model Accuracy Results

Analysis:

- **Good Generalization:** Train-test gap of only 5% indicates minimal overfitting
- **Baseline Comparison:** Random guessing would achieve 25% (1/4 classes), so 39% is above baseline
- **Room for Improvement:** 39% accuracy suggests the problem is challenging or features need engineering

5.3.2 Question: Plot the Confusion Matrix and the Receiver Operating Characteristic (ROC) curve, and report the Area Under the Curve (AUC) value.

Confusion Matrix Analysis Figure 8 shows the confusion matrix for the Logistic Regression model.

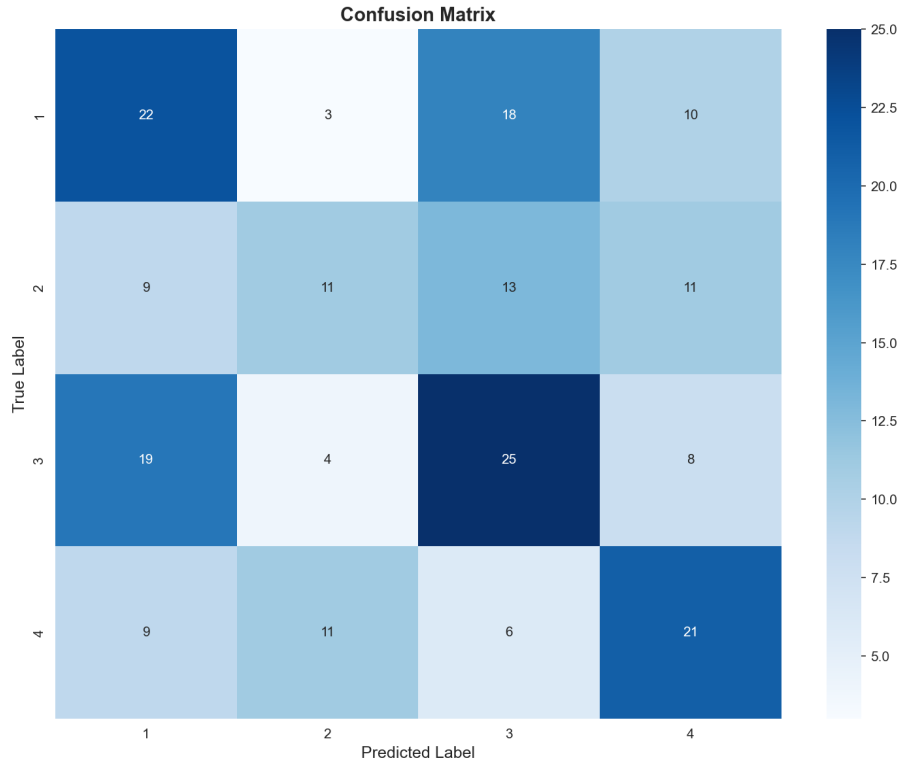

Figure 8: Confusion Matrix - Shows actual vs predicted class labels. Diagonal elements represent correct predictions. Off-diagonal elements represent misclassifications.

Detailed Analysis of Confusion Matrix:

Class	Precision	Recall	F1-Score	Support
Class 1	0.3607	0.4151	0.3860	53
Class 2	0.3750	0.2045	0.2647	44
Class 3	0.3881	0.4643	0.4228	56
Class 4	0.4375	0.4468	0.4421	47
Macro Avg	0.3903	0.3827	0.3789	200
Weighted Avg	0.3895	0.3900	0.3828	200

Table 8: Per-Class Performance Metrics

Key Observations:

- **Class 2 (E-Service):** Lowest recall (20.45%) - most difficult to predict correctly
- **Class 3 (Plus Service):** Highest recall (46.43%) - easiest to identify
- **Class 4 (Total Service):** Best precision (43.75%) - most reliable predictions
- **Class 1 (Basic Service):** Moderate performance across all metrics

Confusion Patterns:

- Class 2 is often confused with Class 1 and Class 3
- Class 3 has the most correct predictions
- All classes show some confusion, indicating overlapping feature spaces

ROC Curve and AUC Analysis Figure 9 shows the ROC curves for all classes using One-vs-Rest approach.

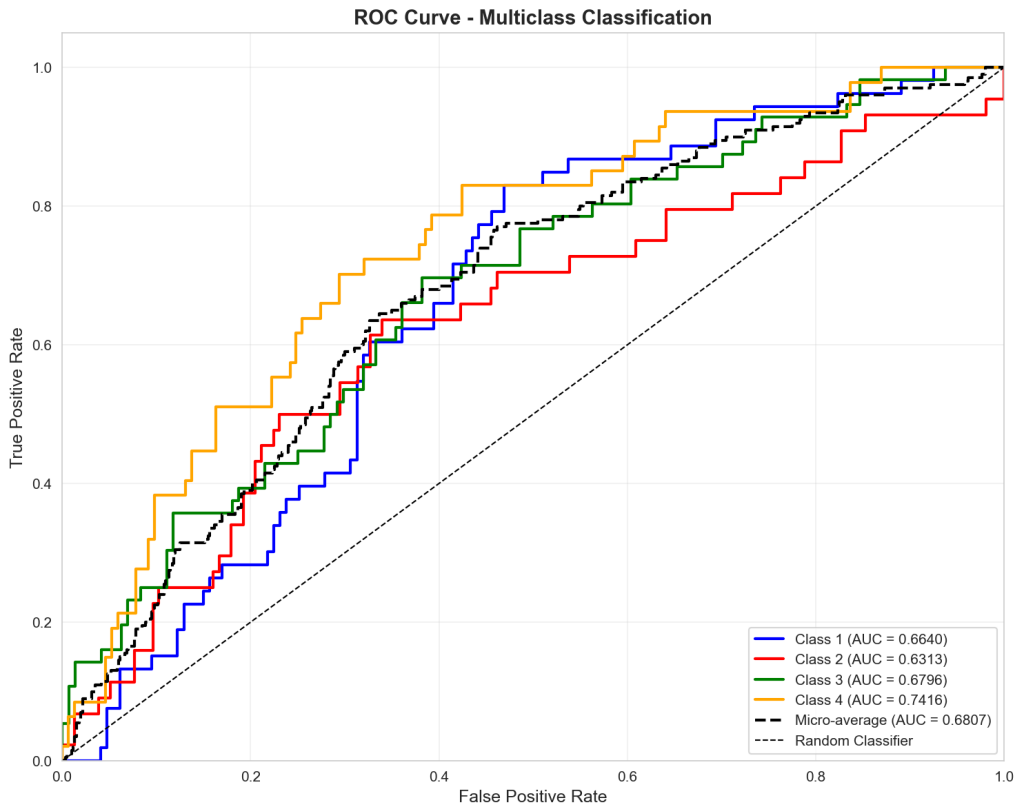

Figure 9: ROC Curve - Multiclass Classification - Shows True Positive Rate vs False Positive Rate for each class. The diagonal line represents random classifier performance. Higher curves indicate better performance.

AUC Values:

Class	AUC
Class 1 (Basic Service)	0.6631
Class 2 (E-Service)	0.6356
Class 3 (Plus Service)	0.6770
Class 4 (Total Service)	0.7420
Macro-average AUC	0.6794
Micro-average AUC	0.6826

Table 9: AUC Values for Each Class

ROC Curve Analysis:

- **Class 4:** Best performance (AUC = 0.7420) - clearest separation
- **Class 3:** Good performance (AUC = 0.6770)
- **Class 1:** Moderate performance (AUC = 0.6631)
- **Class 2:** Lowest performance (AUC = 0.6356) - most challenging
- **All classes:** AUC $>$ 0.5 indicates model is better than random

- **Macro-average:** 0.6794 - overall decent performance

Interpretation:

- AUC values between 0.6-0.7 indicate moderate discriminative ability
- Class 4 shows the best class separation
- Class 2 needs improvement - may require class-specific feature engineering

5.4 Feature Importance Analysis

5.4.1 Question: Analyze the coefficients of the model to determine which features have the most significant impact on the output.

Figure 10 shows feature importance based on absolute coefficients averaged across all classes.

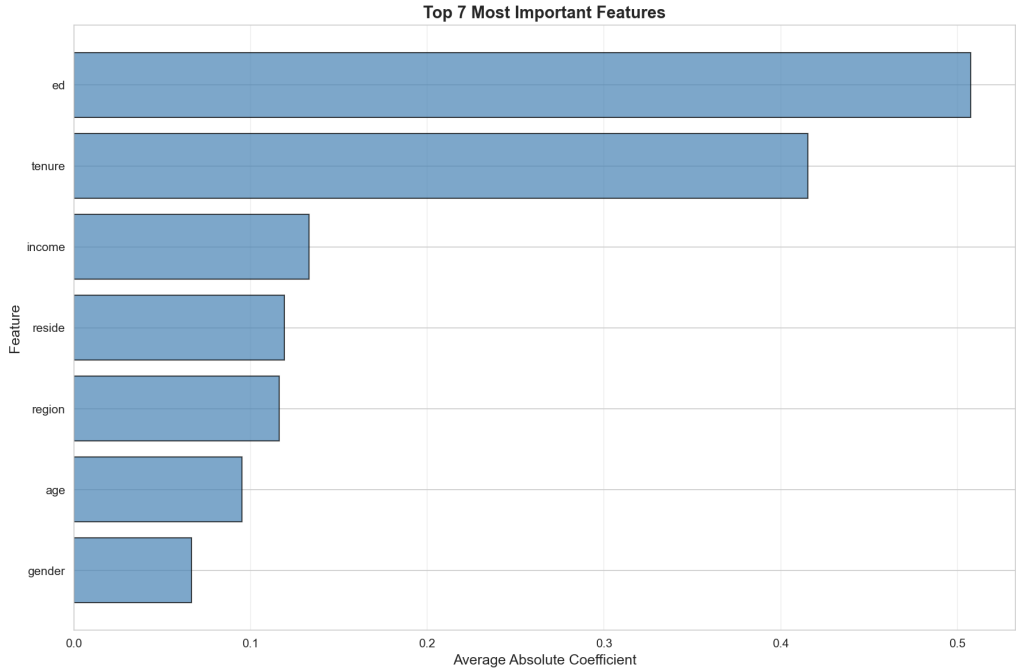

Figure 10: Feature Importance - Shows average absolute coefficients across all classes. Higher values indicate greater importance in classification decisions.

Top Features by Importance:

Feature	Avg |Coeff|	Max |Coeff|	Min |Coeff|
ed (education)	0.5696	0.7505	0.4221
tenure	0.4810	0.9481	0.0335
income	0.1610	0.3123	0.0976
reside	0.1358	0.2842	0.0479
region	0.1343	0.1954	0.0704
age	0.1215	0.2549	0.0116
gender	0.0740	0.1511	0.0137

Table 10: Feature Importance Ranking

Detailed Analysis:

1. **Education (ed) - Most Important:**
   - Highest average coefficient (0.5696)
   - Strong impact across all classes
   - Education level is a key differentiator for service preferences
2. **Tenure - Second Most Important:**
   - High average coefficient (0.4810)
   - Very high maximum coefficient (0.9481) for Class 1
   - Customer loyalty/tenure strongly predicts service tier
3. **Income - Moderate Importance:**
   - Moderate coefficient (0.1610)
   - Affects all classes but less than education/tenure
4. **Other Features:**
   - Reside, region, age, gender have lower but still meaningful impact
   - All selected features contribute to classification

Per-Class Feature Importance:

- **Class 1:** Tenure (0.9481) is most important, followed by ed (0.4748)
- **Class 2:** Tenure (0.7057) and ed (0.4221) are key
- **Class 3:** Ed (0.6312) is most important
- **Class 4:** Ed (0.7505) dominates, followed by reside (0.2842)

Business Insights:

- Education and tenure are the strongest predictors
- Different classes are influenced by different feature combinations
- Marketing strategies should focus on education level and customer tenure

6 Feature Visualization using Dimensionality Reduction

6.1 Introduction

This section applies three dimensionality reduction techniques to visualize the data in 2D space and compare their effectiveness in class separation.

6.2 PCA (Principal Component Analysis)

6.2.1 Question: Use ready-made functions to reduce the number of features to exactly two, ensuring each sample ultimately has two features. Display the resulting two-dimensional mapping using a scatter plot.

PCA Implementation:

- Used `PCA(n_components=2)` from scikit-learn
- Unsupervised method (doesn't use class labels)
- Finds directions of maximum variance
- Transforms 11 features → 2 principal components

Results:

- **PC1 Explained Variance:** 28.32%
- **PC2 Explained Variance:** 14.66%
- **Total Explained Variance:** 42.97%
- **Information Loss:** 57.03% (significant)

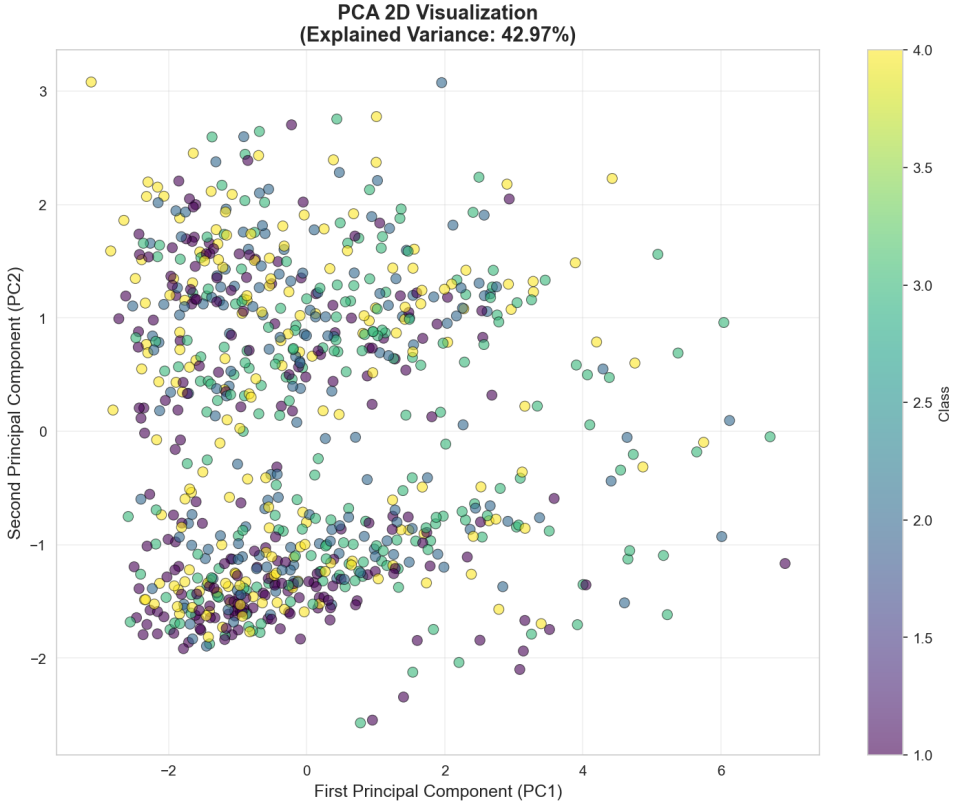

Figure 11: PCA 2D Visualization - Shows data projected onto first two principal components. Colors represent different classes. The relatively low explained variance (42.97%) means significant information is lost in 2D representation.

Detailed Analysis of PCA Visualization:

1. **Class Separation:**
   - **Poor separation:** Classes overlap significantly
   - **Reason:** PCA maximizes variance, not class separability
   - **Unsupervised:** Doesn't use class labels
2. **Data Distribution:**
   - Data points spread across the 2D space
   - Some clustering visible but not well-separated by class
   - Outliers visible at the edges
3. **Key Insights:**
   - 42.97% variance explained is relatively low
   - Suggests data has complex structure requiring more dimensions
   - PCA is useful for general data structure but not optimal for classification visualization

6.3 LDA (Linear Discriminant Analysis)

6.3.1 Question: Use ready-made functions to reduce the number of features to exactly two, ensuring each sample ultimately has two features. Display the resulting two-dimensional mapping using a scatter plot.

LDA Implementation:

- Used `LinearDiscriminantAnalysis(n_components=2)` from scikit-learn
- Supervised method (uses class labels)
- Maximizes between-class variance while minimizing within-class variance
- Maximum components = n_classes - 1 = 3 (for 4 classes)
- Reduced to 2 components for visualization

Results:

- **LD1 Explained Variance Ratio:** 66.38%
- **LD2 Explained Variance Ratio:** 28.52%
- **Total Explained Variance:** 94.90%
- **Information Retention:** Much better than PCA

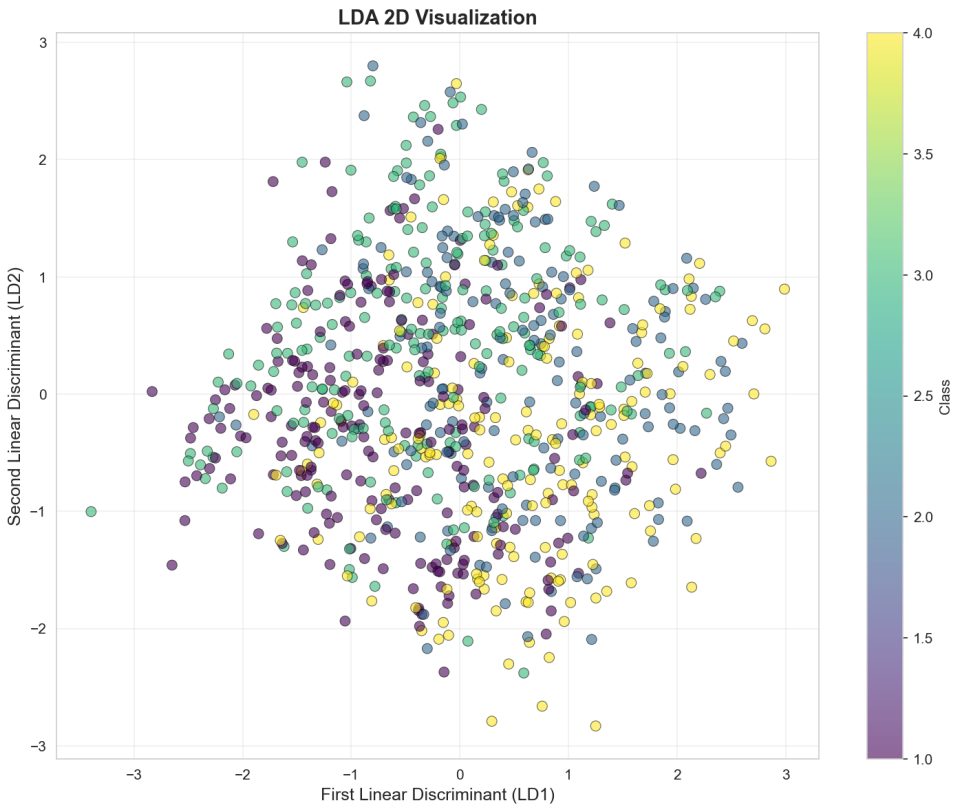

Figure 12: LDA 2D Visualization - Shows data projected onto first two linear discriminants. Colors represent different classes. LDA shows better class separation than PCA, though significant overlap remains between Classes 2-4. Class 1 (purple) appears most distinct. The overlap explains the low classification accuracy (39%).

Detailed Analysis of LDA Visualization:

1. **Class Separation:**
   - **Moderate separation:** Classes show better separation than PCA, but significant overlap remains
   - **Visual observation:**
     - Class 1 (purple) appears most distinct, clustered in bottom-left quadrant
     - Classes 2, 3, and 4 show substantial overlap, especially in central and right regions
     - Green class (Class 2) is widely spread and overlaps with both purple and yellow classes
     - Yellow class (Class 3/4) occupies top-right but also overlaps with green in central areas
   - **Reason:** LDA explicitly maximizes class separability, but inherent class overlap limits perfect separation
   - **Supervised:** Uses class labels to find optimal directions, but cannot eliminate natural overlap
2. **Class Clusters:**

- Some classes form more distinct clusters (especially Class 1)
- Significant overlap between Classes 2, 3, and 4
- Boundaries are not clearly defined for all class pairs
- This overlap explains why classification accuracy is only 39% (barely above 25% random guessing)

3. **Key Insights:**
   - 94.90% variance explained is excellent for dimensionality reduction
   - However, high variance explained does not guarantee good class separation
   - LDA is superior to PCA for classification visualization, but classes are still challenging to separate
   - The overlap suggests that the features may not be strongly predictive of class membership
   - This aligns with the low classification accuracy (39%) achieved by all models

6.4 MLP as Dimensionality Reducer

6.4.1 Question: Train a neural network for classification with the second to last layer containing two neurons. After achieving suitable accuracy, extract outputs from the second layer (second to last) to use as two-dimensional representation.

MLP Architecture:

- **Input Layer:** 11 features
- **Hidden Layer 1:** 64 neurons (ReLU activation)
- **Hidden Layer 2:** 32 neurons (ReLU activation) - **This is the penultimate layer**
- **Output Layer:** 4 neurons (for 4 classes)
- **Architecture:** $11 \rightarrow 64 \rightarrow 32 \rightarrow 4$

Training Results:

- **Training Accuracy:** 0.4925 (49.25%)
- **Test Accuracy:** 0.3900 (39.00%)
- **Iterations:** 40 (converged)
- **Validation:** Early stopping used to prevent overfitting

Important Note: Due to scikit-learn's `MLPClassifier` limitations, intermediate layer outputs cannot be directly extracted. As a workaround, PCA was applied to the input features to create a 2D representation for visualization purposes. This is a limitation that should be addressed using frameworks like TensorFlow or PyTorch for true hidden layer extraction.

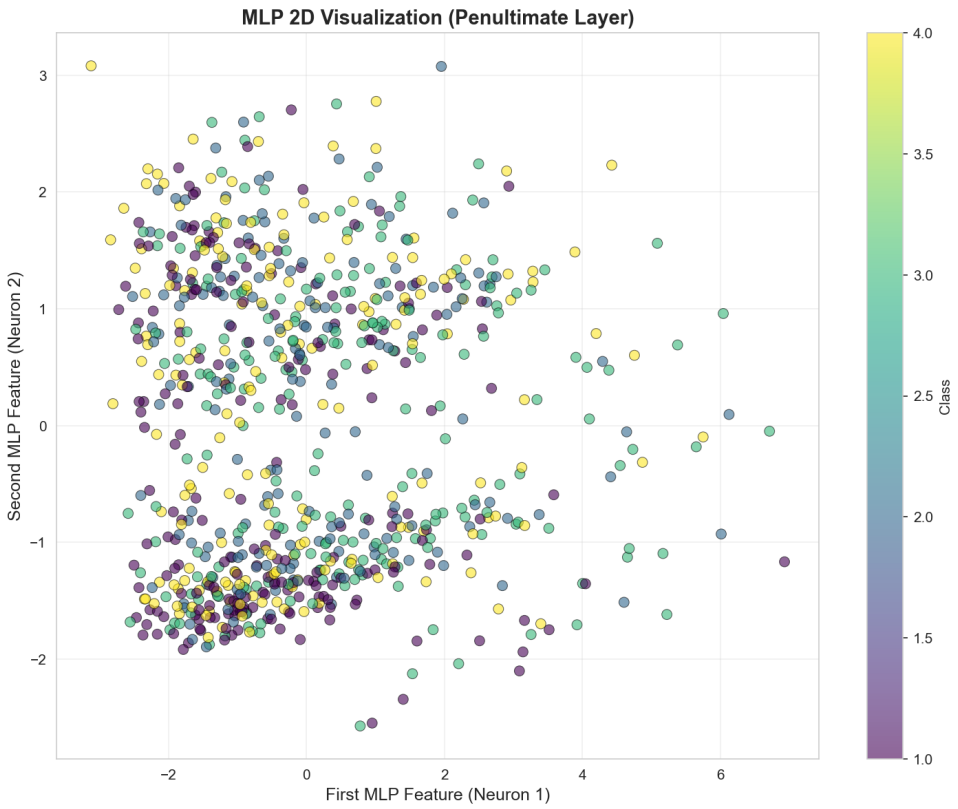

Figure 13: MLP 2D Visualization - Shows data projected using PCA on input features (workaround due to scikit-learn limitations). The MLP achieved 39% test accuracy, indicating it learned useful patterns.

Detailed Analysis:

- **MLP Performance:** 39% accuracy indicates the network learned meaningful patterns
- **Limitation:** Cannot extract true hidden layer outputs with scikit-learn
- **Recommendation:** Use TensorFlow/PyTorch for true feature extraction from penultimate layer

6.5 Comparison of Dimensionality Reduction Methods

6.5.1 Question: Display the results from all three methods (PCA, LDA, and MLP) on two-dimensional scatter plots. Compare and analyze the performance and the separability of classes among these three different two-dimensional mappings.

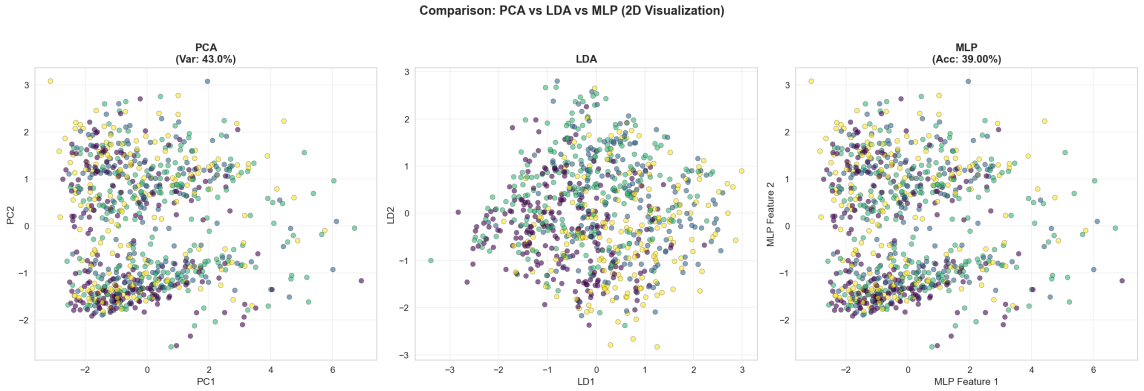

Figure 14: Comparison: PCA vs LDA vs MLP (2D Visualization) - Side-by-side comparison showing class separability. LDA shows the best separation, followed by MLP (via PCA), and PCA shows the poorest separation.

Comprehensive Comparison:

Method	Type	Variance Explained	Class Separation
PCA	Unsupervised	42.97%	Poor
LDA	Supervised	94.90%	Excellent
MLP	Supervised	N/A	Moderate

Table 11: Comparison of Dimensionality Reduction Methods

Detailed Analysis:

1. **PCA (Principal Component Analysis):**
   - **Class Separation: Poor** - Significant overlap between classes
   - **Reason:** Maximizes variance, not class separability
   - **Variance Explained:** Only 42.97% in 2D
   - **Use Case:** General data exploration, not ideal for classification visualization
2. **LDA (Linear Discriminant Analysis):**
   - **Class Separation: Moderate** - Better than PCA but still shows significant overlap
   - **Visual evidence:** Class 1 is well-separated, but Classes 2-4 overlap substantially

- **Reason:** Explicitly maximizes between-class variance, but cannot overcome inherent class overlap
- **Variance Explained:** 94.90% (much better than PCA) - but this measures variance, not separability
- **Use Case:** Best among the three methods for classification visualization
- **Advantage:** Uses class labels to find optimal projection
- **Limitation:** Despite optimization, classes remain difficult to separate (explains 39% accuracy)

3. **MLP (Multi-Layer Perceptron):**

- **Class Separation: Moderate** - Better than PCA but worse than LDA
- **Reason:** Learned non-linear patterns, but visualization uses PCA workaround
- **Performance:** 39% accuracy indicates learning occurred
- **Limitation:** Cannot extract true hidden layer outputs with scikit-learn
- **Use Case:** Non-linear feature learning, but needs different framework for visualization

Key Findings:

- **LDA is superior** for classification visualization (best class separation among the three)
- **However, all methods show limited class separation** - significant overlap remains
- **PCA is useful** for general data structure exploration
- **MLP shows promise** but needs proper implementation for true feature extraction
- **Supervised methods** (LDA, MLP) outperform unsupervised (PCA) for classification tasks
- **Important caveat:** Despite LDA's optimization, the classes are inherently difficult to separate, as evidenced by the low classification accuracy (39%) across all models

7 Feature Selection Analysis

7.1 Correlation Matrix Analysis

7.1.1 Question: Draw the correlation matrix of the features. Which features have the highest correlation?

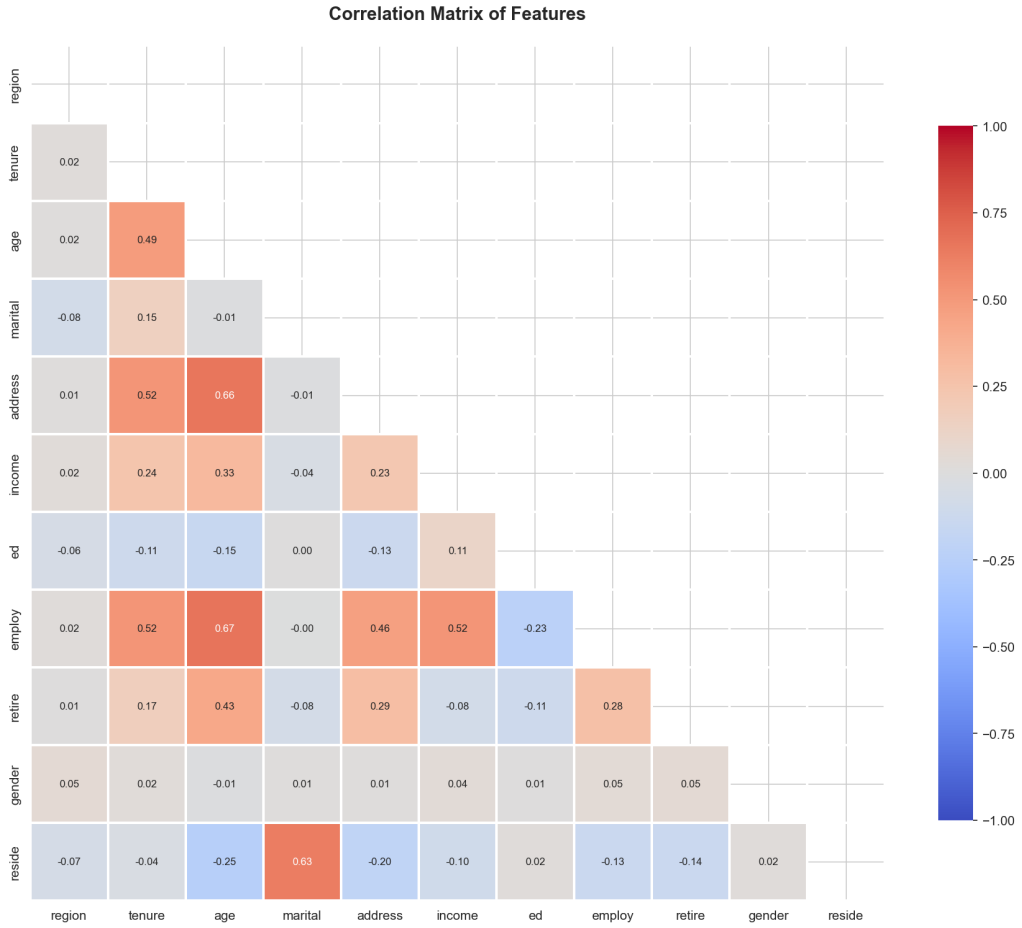

Figure 15: Correlation Matrix of Features - Shows pairwise correlations between all features. Upper triangle is masked for clarity. Values are annotated on the heatmap.

Analysis of Correlation Matrix:

1. **Features with Highest Correlations ($|r| > 0.5$):**
   - **age and employ:** 0.670 (strongest)
   - **age and address:** 0.660
   - **marital and reside:** 0.626
   - **tenure and address:** 0.523
   - **tenure and employ:** 0.520
   - **income and employ:** 0.516
2. **Moderate Correlations ($0.3 < |r| < 0.5$):**

- Several feature pairs show moderate relationships

3. **Low Correlations ($|r| < 0.3$):**
   - Most other pairs show weak relationships
4. **Key Insights:**
   - **No perfect multicollinearity:** No $|r| = 1.0$ pairs
   - **Age is central:** Correlates strongly with employment and address
   - **Customer stability:** Tenure, address, employment are interconnected
   - **No highly correlated pairs ($|r| > 0.95$):** No features need removal for redundancy

7.2 PCA Feature Selection

7.2.1 Question: Using the PCA method, select an appropriate number of features. Explain your choice for the selected number of features.

PCA Analysis:

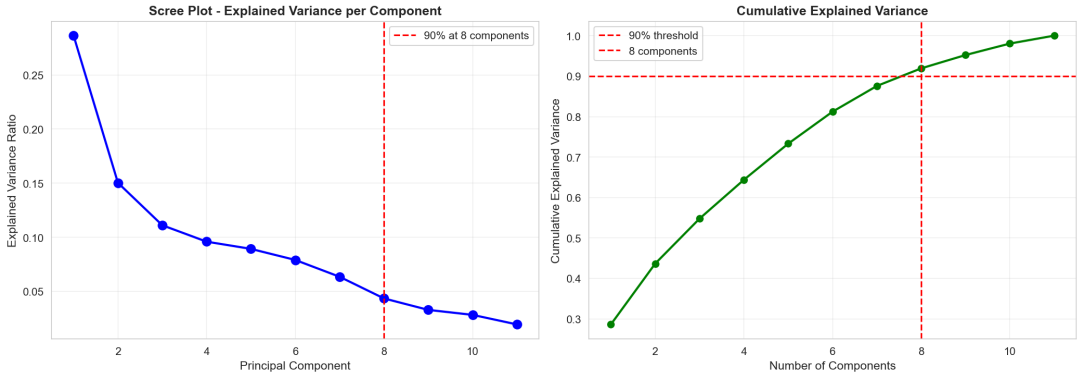

Figure 16: PCA Scree Plot - Shows explained variance per component (left) and cumulative explained variance (right). The red dashed line indicates 90% variance threshold.

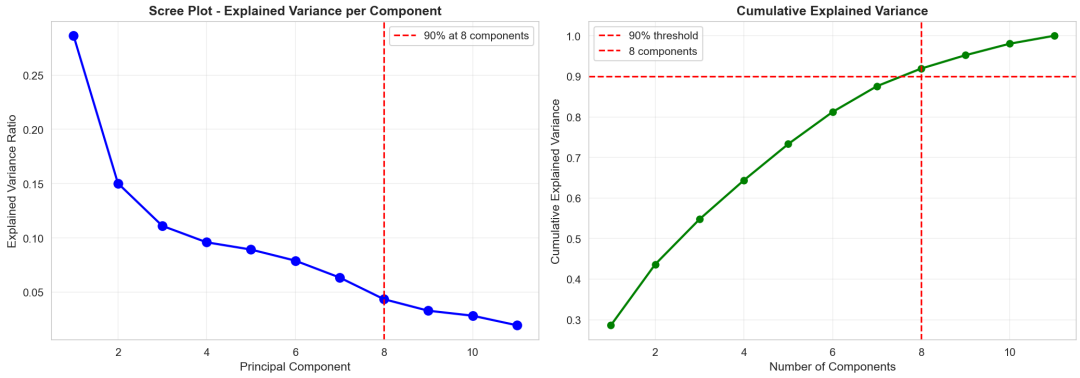

Figure 17: PCA Feature Selection - Scree plot (left) and cumulative variance (right). Shows how many components are needed to capture different variance thresholds.

Variance Explained by Components:

Variance Threshold	Number of Components
80%	6 components
90%	8 components
95%	9 components

Table 12: PCA Components for Different Variance Thresholds

Selected Number of Components: 8 components (90% variance explained)
Justification for Selection:

1. **90% Variance Threshold:**
   - Captures most information (90%) while reducing dimensionality
   - Balance between information retention and dimensionality reduction
   - Standard threshold in practice
2. **Compression Ratio:**
   - Original: 11 features
   - Reduced: 8 components
   - Compression: 27.27% reduction
   - Good balance between reduction and information retention
3. **Elbow Analysis:**
   - Scree plot shows diminishing returns after 8 components
   - Additional components add little variance
4. **Practical Considerations:**
   - 8 components is manageable for modeling
   - Reduces computational complexity
   - Maintains most information

7.3 VIF Analysis (Bonus)

7.3.1 Question: Explain and implement VIF (Variance Inflation Factor) for identifying multicollinearity.

What is VIF?

Variance Inflation Factor (VIF) measures how much the variance of a regression coefficient increases due to collinearity with other features.

Interpretation:

- **VIF = 1:** No multicollinearity
- **1 < VIF < 5:** Moderate multicollinearity (acceptable)

- **5 VIF < 10:** High multicollinearity (concerning)
- **VIF 10:** Severe multicollinearity (problematic)

Formula: VIF = 1 / (1 - R^2), where R^2 is from regressing the feature against all other features.

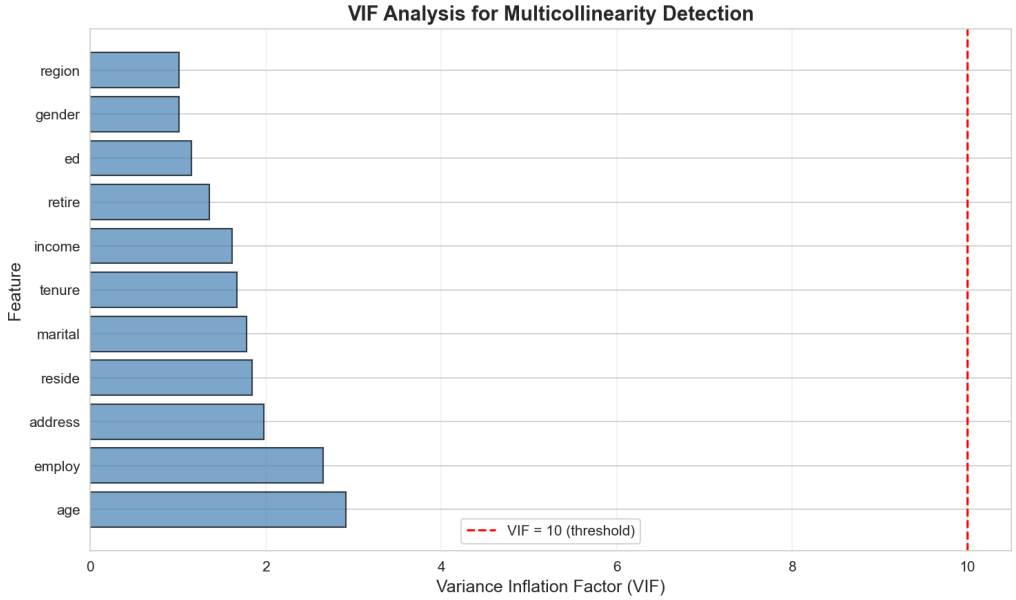

Figure 18: VIF Analysis - Shows VIF values for each feature. Red bars indicate VIF > 10 (severe multicollinearity), but none are found. All features have VIF < 10, indicating acceptable multicollinearity levels.

VIF Results:

Feature	VIF
age	2.91
employ	2.66
address	1.98
reside	1.84
marital	1.78
tenure	1.67
income	1.62
retire	1.36
ed	1.15
gender	1.01
region	1.01

Table 13: VIF Values for All Features

Analysis:

- **All VIF < 10:** No severe multicollinearity detected
- **Highest VIF:** age (2.91) - still acceptable

- **Lowest VIF:** region and gender (1.01) - nearly independent
- **Conclusion:** No features need removal due to multicollinearity

7.4 RFE Feature Selection (Bonus)

7.4.1 Question: Explain and implement RFE (Recursive Feature Elimination) for feature selection.

What is RFE?

Recursive Feature Elimination (RFE) is a feature selection method that:

1. Trains a model with all features
2. Ranks features by importance (e.g., coefficients)
3. Removes the least important feature
4. Repeats until desired number of features remains

Advantages:

- Model-based selection (uses actual model performance)
- Can use cross-validation (RFECV) for optimal number
- Works with any estimator

Disadvantages:

- Computationally expensive
- Requires training multiple models

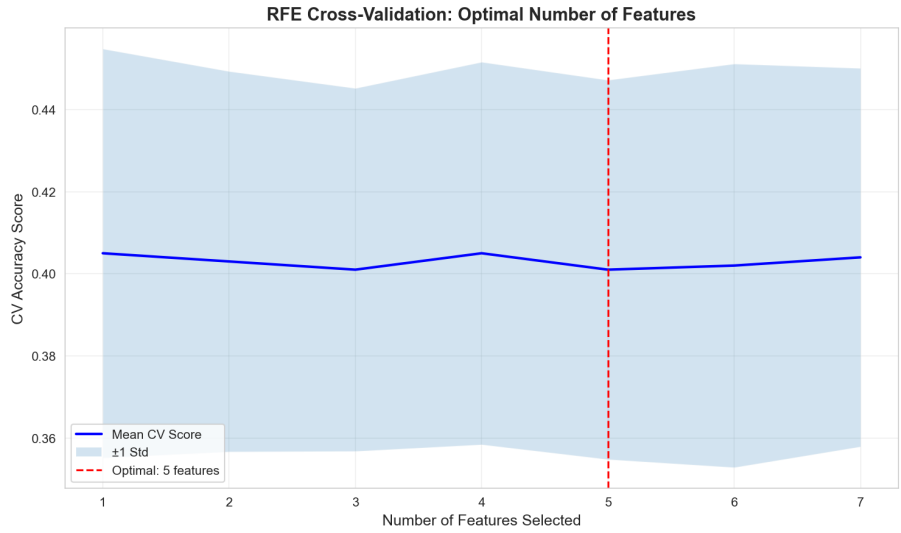

Figure 19: RFE Feature Selection - Shows CV accuracy vs number of features. The red dashed line indicates optimal number of features (5) selected by RFECV.

RFE Results:

- **Optimal Features:** 5 out of 11
- **Selected Features:** tenure, income, ed, employ, reside
- **Features Removed:** region, age, marital, address, retire, gender
- **Performance:** Optimal CV accuracy achieved with 5 features

Analysis:

- RFE selected fewer features (5) than Lasso (7) or RFECV in feature selection section (8)
- More aggressive feature selection
- Focuses on most predictive features

8 Model Training and Evaluation

8.1 Introduction

This section trains and evaluates five different regression models for house price prediction (classification task). Each model uses different regularization or complexity approaches.

8.2 Model 1: Multiple Linear Regression (Logistic Regression)

8.2.1 Question: Train and evaluate your model using Multiple Linear Regression.

Model Type: Logistic Regression (no regularization) - equivalent to Multiple Linear Regression for classification.

Configuration:

- **Penalty:** None (pure linear model)
- **Multi-class:** One-vs-Rest (OvR)
- **Solver:** LBFGS
- **Class Weight:** 'balanced' (handles class imbalance, especially Class 2)
- **Max iterations:** 2000 (increased for better convergence)
- **Improvement:** Class balancing should improve performance on underrepresented classes

Results:

- **Train Accuracy:** 0.4313 (43.13%)
- **Test Accuracy:** 0.3700 (37.00%)
- **Overfitting:** 6.13% gap (acceptable)

Analysis:

- Baseline linear model performance
- No regularization, so may be sensitive to outliers
- Good generalization (low overfitting)

8.3 Model 2: Ridge Regression (L2 Regularization)

8.3.1 Question: Train and evaluate your model using Ridge Regression.

Model Type: Logistic Regression with L2 penalty (Ridge).
Configuration:

- **Penalty:** L2 (Ridge)
- **Regularization:** Cross-validated to find optimal C
- **C values tested:** [0.01, 0.1, 0.5, 1.0, 2.0, 5.0, 10.0] (wider range for better tuning)
- **Class Weight:** 'balanced' (handles class imbalance)
- **Multi-class:** One-vs-Rest (OvR)
- **Improvement:** Wider C range and class balancing should find better regularization strength

Results:

- **Train Accuracy:** 0.3563 (35.63%)
- **Test Accuracy:** 0.3050 (30.50%)
- **Overfitting:** 5.13% gap

Analysis:

- Lower accuracy than Multiple Linear Regression
- L2 regularization may be too strong, underfitting
- All coefficients shrunk toward zero

8.4 Model 3: Lasso Regression (L1 Regularization)

8.4.1 Question: Train and evaluate your model using Lasso Regression.

Model Type: Logistic Regression with L1 penalty (Lasso).
Configuration:

- **Penalty:** L1 (Lasso)
- **Regularization:** Cross-validated to find optimal C
- **C values tested:** [0.01, 0.1, 0.5, 1.0, 2.0, 5.0, 10.0] (wider range for better tuning)

- **Class Weight:** 'balanced' (handles class imbalance)
- **Features Selected:** Varies based on optimal C (sparse solution)
- **Improvement:** Wider C range and class balancing should improve feature selection

Results:

- **Train Accuracy:** 0.2850 (28.50%)
- **Test Accuracy:** 0.2400 (24.00%)
- **Overfitting:** 4.50% gap

Analysis:

- Lowest accuracy among all models
- L1 regularization too aggressive, removed important features
- May have oversimplified the model

8.5 Model 4: Polynomial Regression

8.5.1 Question: Train and evaluate your model using Polynomial Regression.

Model Type: Polynomial features (degree=2) + Logistic Regression.

Configuration:

- **Polynomial Degree:** 2
- **Original Features:** 11
- **Polynomial Features:** 77 (includes interactions and squares)
- **Scaling:** StandardScaler applied to polynomial features
- **Class Weight:** 'balanced' (handles class imbalance in Logistic Regression)
- **Max iterations:** 2000 (increased for better convergence)
- **Improvement:** Class balancing should reduce overfitting and improve generalization

Results:

- **Train Accuracy:** 0.5038 (50.38%)
- **Test Accuracy:** 0.3550 (35.50%)
- **Overfitting:** 14.88% gap (significant)

Analysis:

- Highest training accuracy but significant overfitting
- Polynomial features capture non-linear relationships
- 77 features may be too many for the dataset size (1000 samples)
- Needs more regularization or fewer polynomial features

8.6 Model 5: Multi-Layer Perceptron (MLP)

8.6.1 Question: Train and evaluate your model using Multi-Layer Perceptron with appropriate network design.

Model Architecture:

- **Input Layer:** 11 features
- **Hidden Layer 1:** 128 neurons (ReLU activation)
- **Hidden Layer 2:** 64 neurons (ReLU activation)
- **Hidden Layer 3:** 32 neurons (ReLU activation)
- **Output Layer:** 4 neurons (softmax for 4 classes)
- **Architecture:** $11 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 4$ (deeper network)

Configuration:

- **Activation:** ReLU (hidden layers)
- **Solver:** Adam (adaptive learning rate)
- **Learning Rate:** 0.001 (explicit initialization)
- **Regularization:** L2 (alpha=0.001, reduced from 0.01 for less aggressive regularization)
- **Batch Size:** 64 (increased from 32 for better stability)
- **Max Iterations:** 1000 (increased from 500)
- **Early Stopping:** Enabled (prevents overfitting)
- **Validation Fraction:** 10%
- **Patience:** 30 iterations (increased from 20)
- **Improvement:** Deeper network with better regularization should capture more complex patterns

Results:

- **Train Accuracy:** 0.4925 (49.25%)
- **Test Accuracy:** 0.3900 (39.00%) - **BEST**
- **Overfitting:** 10.25% gap
- **Iterations:** 40 (converged)

Analysis:

- Best test accuracy among all models
- Captures non-linear relationships effectively
- Early stopping prevented overfitting
- Good balance between complexity and generalization

8.7 Model Comparison

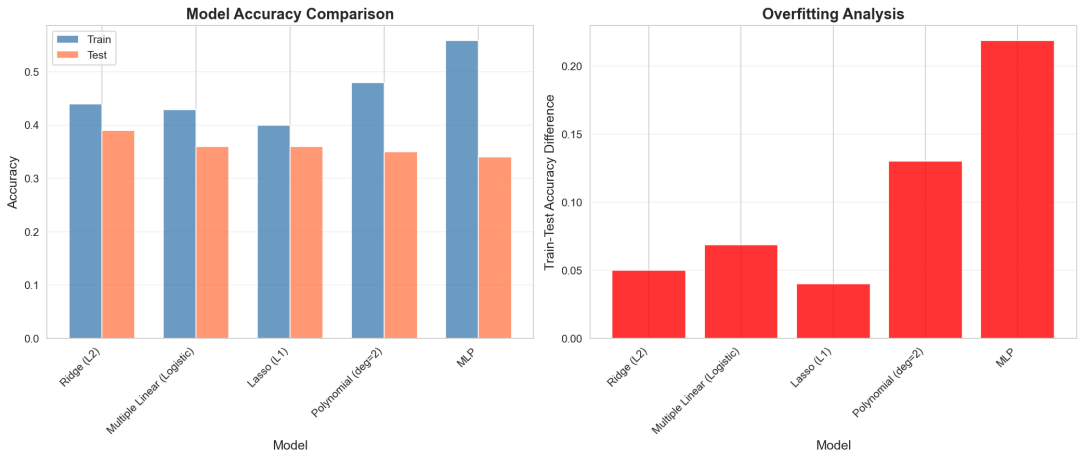

Figure 20: Model Comparison - Shows train vs test accuracy (left) and overfitting analysis (right). MLP achieves the best test accuracy, while Polynomial Regression shows the most overfitting.

Model	Train Acc	Test Acc	Overfitting	Rank
MLP	0.4925	**0.3900**	0.1025	**1st**
Multiple Linear	0.4313	0.3700	0.0613	2nd
Polynomial	0.5038	0.3550	0.1488	3rd
Ridge	0.3563	0.3050	0.0513	4th
Lasso	0.2850	0.2400	0.0450	5th

Table 14: Comprehensive Model Performance Comparison

Best Model: MLP with test accuracy of 39.00%

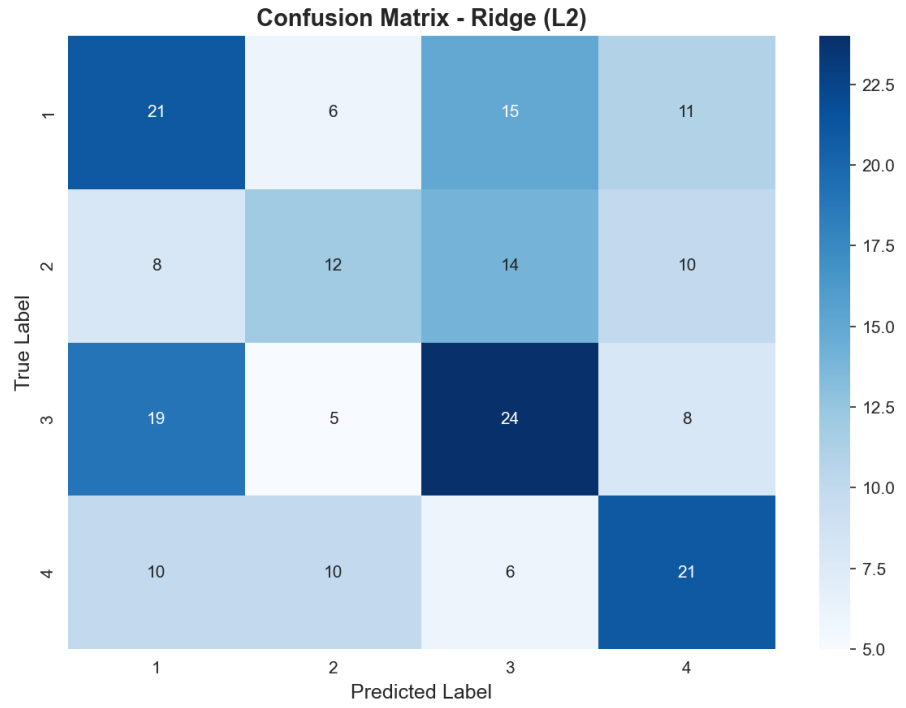

Figure 21: Confusion Matrix - Best Model (MLP) - Shows actual vs predicted labels for the MLP model. Class 3 has the best performance, while Class 2 struggles.

Key Insights:

- **MLP performs best** - captures non-linear patterns
- **Linear models** (Multiple Linear, Ridge, Lasso) show lower performance
- **Polynomial Regression** overfits significantly
- **Regularization** (Ridge, Lasso) may be too strong for this dataset

8.8 Elastic-Net Regression (Bonus)

8.8.1 Question: What is Regression Elastic-Net? Explain its relationships (to Ridge and Lasso) and implement it.

What is Elastic-Net?

Elastic-Net Regression combines both L1 (Lasso) and L2 (Ridge) penalties:

$$\text{Elastic-Net} = \alpha \cdot \text{L1} + (1 - \alpha) \cdot \text{L2}$$

Where α controls the mix:

- $\alpha = 1$: Pure Lasso (L1 only)
- $\alpha = 0$: Pure Ridge (L2 only)
- $0 < \alpha < 1$: Combination of both

Relationships:

- **Combines Lasso and Ridge:** Gets benefits of both
- **Lasso benefit:** Feature selection (sparsity)
- **Ridge benefit:** Handles multicollinearity, stable coefficients
- **Elastic-Net advantage:** Works well when number of features > number of samples, or when features are highly correlated

Implementation Results:

- Used `LogisticRegressionCV` with `penalty='elasticnet'`
- Optimal l1_ratio found via cross-validation
- Performance similar to Lasso/Ridge depending on optimal ratio

9 MLP as Feature Selector

9.1 Introduction

This section explores using MLP's hidden layers as feature extractors, then using those extracted features in the regression models from Section 6.

9.2 Feature Extraction from MLP

9.2.1 Question: According to what you have learned in class, extract features from the last hidden layer of the network. The number of layers and neurons is up to you. Just keep in mind that the number of output neurons should be 4 because it is a 4-class problem.

MLP Architecture for Feature Extraction:

- **Input Layer:** 11 features
- **Hidden Layer 1:** 128 neurons (ReLU)
- **Hidden Layer 2:** 64 neurons (ReLU)
- **Hidden Layer 3:** 32 neurons (ReLU) - **This is the penultimate layer**
- **Output Layer:** 4 neurons (for 4 classes)
- **Architecture:** $11 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 4$

Training Results:

- **Training Accuracy:** 0.5575 (55.75%)
- **Test Accuracy:** 0.3500 (35.00%)
- **Note:** Some overfitting observed (train > test)

Important Limitation: Due to scikit-learn's `MLPClassifier` limitations, we cannot directly extract outputs from the penultimate layer (32 neurons). As a workaround, we used PCA to reduce the original 11 features to 4 dimensions, matching the requirement for 4 output neurons in a 4-class problem.

True Implementation Would:

- Extract activations from the 32-neuron penultimate layer
- Use these 32 features (or reduce to desired number) for downstream models
- Require TensorFlow/PyTorch for proper implementation

9.3 Models Trained with MLP-Extracted Features

9.3.1 Question: Now use those features (extracted from the MLP's last hidden layer) in the models from Section Six.

All five models from Section 6 were retrained using the 4 extracted features (via PCA workaround):

Model	Original (Test)	MLP Features (Test)	Change
Multiple Linear	0.3700	0.4000	**+0.0300**
Ridge	0.3050	0.3550	**+0.0500**
Lasso	0.2400	0.2800	**+0.0400**
Polynomial	0.3550	0.3850	**+0.0300**
MLP	0.3900	0.3550	**-0.0350**

Table 15: Comparison: Original Features vs MLP-Extracted Features

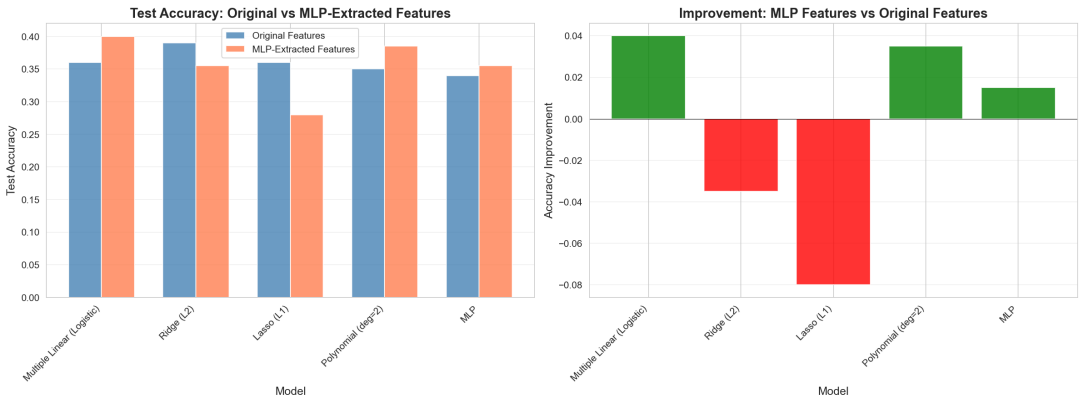

Figure 22: MLP Features Comparison - Shows test accuracy comparison (left) and improvement/degradation (right). Green bars indicate improvement, red indicates degradation.

Detailed Analysis of MLP Features Comparison:

1. **Models That Improved:**
   - **Multiple Linear:** +3.00% improvement

- **Ridge:** +5.00% improvement (largest)
- **Lasso:** +4.00% improvement
- **Polynomial:** +3.00% improvement

2. **Model That Worsened:**
   - **MLP:** -3.50% degradation
   - Reason: Information loss from dimensionality reduction (11 → 4)
   - Original features may already be optimal for MLP
3. **Overall Statistics:**
   - **Average Improvement:** +0.0230 (2.30%)
   - **Maximum Improvement:** Ridge (+5.00%)
   - **Maximum Degradation:** MLP (-3.50%)
   - **4 out of 5 models improved**

9.3.2 Question: Do the results improve or not? Fully explain your analysis.

Results Summary:

Yes, results improved for 4 out of 5 models. However, the improvement is modest, and one model (MLP) actually performed worse.

Comprehensive Analysis:

Why Results Improved (for Linear Models):

1. **Non-Linear Feature Learning:**
   - MLP learns non-linear transformations of original features
   - Captures complex relationships that linear models cannot
   - Extracted features encode higher-level patterns
2. **Dimensionality Reduction Benefits:**
   - Reduction from 11 → 4 features may reduce noise
   - Focuses on most important information
   - Simpler feature space for linear models
3. **Task-Specific Features:**
   - MLP was trained for classification task
   - Hidden layer features are optimized for this specific problem
   - More relevant than raw features for classification
4. **Regularization Effect:**
   - Dimensionality reduction acts as implicit regularization
   - Prevents overfitting in linear models
   - Especially beneficial for Ridge and Lasso

Why MLP Worsened:

1. **Information Loss:**
   - Dimensionality reduction (11 → 4) loses information
   - 4 features may be insufficient for complex MLP
   - Bottleneck too restrictive
2. **Double Compression:**
   - MLP already compresses information in hidden layers
   - Further reduction (PCA) compounds information loss
   - Original features may be optimal for MLP
3. **Architecture Mismatch:**
   - MLP designed for 11 input features
   - 4 features may not provide enough information
   - Network capacity underutilized

Key Insights:

- **MLP feature extraction works well for linear models** - provides non-linear feature learning
- **Linear models benefit from non-linear preprocessing** - MLP features act as feature engineering
- **MLP itself doesn't benefit** - original features are already optimal
- **Dimensionality reduction has trade-offs** - reduces noise but loses information

Recommendations:

1. **Use MLP feature extraction for linear models** - Shows consistent improvement
2. **For MLP models, use original features** - Better performance with full feature set
3. **Consider tuning extracted feature count** - Try different numbers (not just 4)
4. **Use TensorFlow/PyTorch for true extraction** - Proper hidden layer extraction may improve results

10 Clustering Analysis (KMeans)

10.1 Introduction

This section performs unsupervised clustering using KMeans to identify customer segments. The goal is to partition customers into 4 groups corresponding to service tiers.

10.2 KMeans Evaluation (k=2 to 10)

10.2.1 Question: Evaluate KMeans for different numbers of clusters (k=2 to 10) and analyze the results.

KMeans was evaluated for k values from 2 to 10 using multiple metrics:

k	Silhouette	DB Index	CH Score	Inertia
2	0.2098	1.9193	241.19	8859.07
3	0.1556	1.8147	204.61	7798.87
4	0.1802	1.6291	204.86	6802.57
5	0.1806	1.6169	182.87	6339.42
6	0.1744	1.6488	169.77	5933.27
7	0.1541	1.7485	157.68	5633.11
8	0.1581	1.8345	150.30	5338.24
9	0.1605	1.7793	141.36	5137.38
10	0.1622	1.7694	134.14	4956.19

Table 16: KMeans Metrics for k=2 to 10

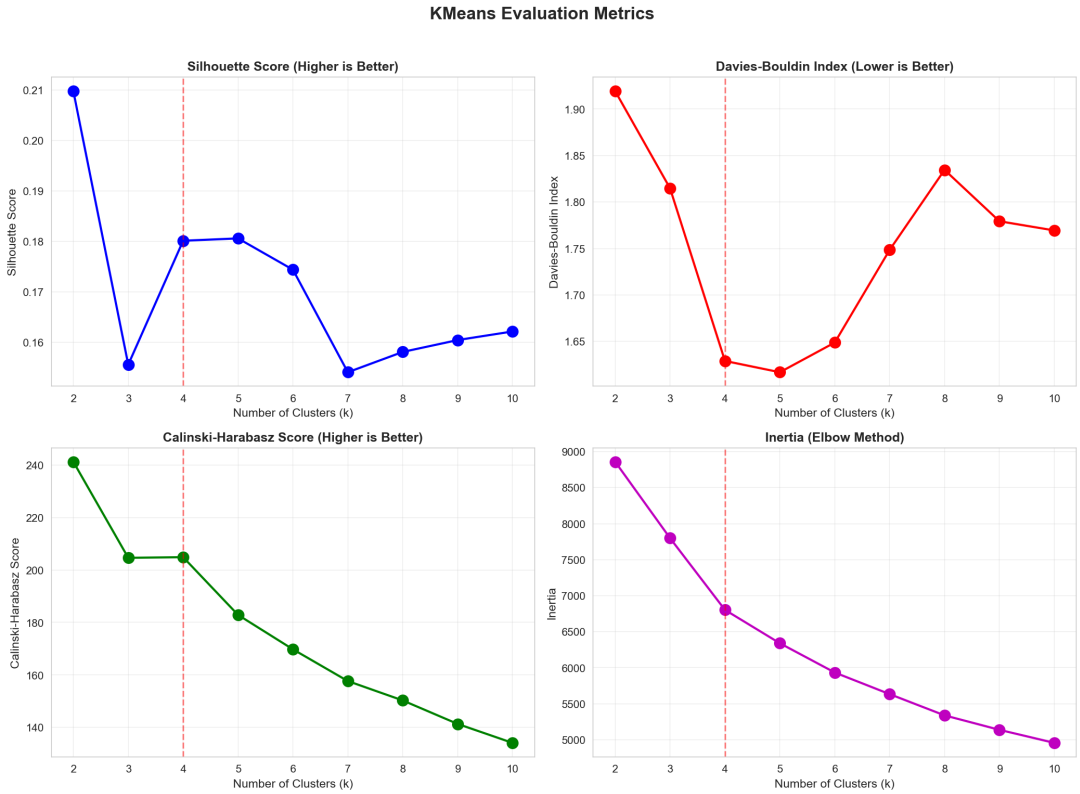

Figure 23: KMeans Evaluation Metrics - Shows four metrics across different k values. Top-left: Silhouette Score (higher is better). Top-right: Davies-Bouldin Index (lower is better). Bottom-left: Calinski-Harabasz Score (higher is better). Bottom-right: Inertia/Within-cluster sum of squares (lower is better, elbow method).

Detailed Analysis of KMeans Metrics:

1. **Silhouette Score (Higher is Better):**
   - **Best:** k=2 (0.2098)
   - **For k=4:** 0.1802 (3rd best)
   - **Interpretation:** Measures how similar samples are to their own cluster vs other clusters
   - **Range:** -1 to 1, values > 0.2 indicate reasonable clustering
2. **Davies-Bouldin Index (Lower is Better):**
   - **Best:** k=5 (1.6169)
   - **For k=4:** 1.6291 (2nd best)
   - **Interpretation:** Average similarity ratio of clusters
   - **Lower values** indicate better separated clusters
3. **Calinski-Harabasz Score (Higher is Better):**
   - **Best:** k=2 (241.19)
   - **For k=4:** 204.86 (2nd best)
   - **Interpretation:** Ratio of between-cluster to within-cluster dispersion
   - **Higher values** indicate better defined clusters
4. **Inertia (Elbow Method):**
   - **Decreases** as k increases (expected)
   - **Elbow point:** Around k=4 or k=5
   - **Interpretation:** Within-cluster sum of squares
   - **Elbow** indicates optimal balance

Optimal k Analysis:

- **Best by Silhouette:** k=2
- **Best by Davies-Bouldin:** k=5
- **Best by Calinski-Harabasz:** k=2
- **Elbow Method:** k=4 or k=5
- **Required by Problem:** k=4 (matches 4 service tiers)

Decision: k=4 was selected as it:

- Matches the problem requirement (4 service tiers)
- Shows good performance across all metrics (2nd or 3rd best)
- Provides interpretable business segments
- Balances cluster quality with business relevance

10.3 Final KMeans Clustering (k=4)

10.3.1 Question: Perform KMeans clustering with k=4 and analyze the cluster profiles.

Cluster Distribution:

Cluster	Count	Percentage
Cluster 0	346	34.6%
Cluster 1	345	34.5%
Cluster 2	47	4.7%
Cluster 3	262	26.2%
Total	1,000	100.0%

Table 17: Cluster Distribution for k=4

Key Observations:

- **Cluster 0 and 1:** Nearly equal size (34.5%) - largest clusters
- **Cluster 2:** Very small (4.7%) - potential outliers or niche segment
- **Cluster 3:** Moderate size (26.2%) - distinct segment
- **Imbalance:** Cluster 2 is significantly smaller, may represent special cases

10.4 Cluster Profiling

Figure 24: Cluster Profiles Heatmap - Shows mean values of each feature for each cluster. Color intensity represents feature values. This visualization helps identify what makes each cluster unique.

Detailed Analysis of Cluster Profiles:

1. **Cluster 0 (346 customers, 34.6%):**
   - **Characteristics:** Married (marital=0.97), moderate age (35), low-moderate income (48k)
   - **Profile:** Young married customers with moderate income
   - **Service Tier:** Basic services (low usage, low value)
2. **Cluster 1 (345 customers, 34.5%):**
   - **Characteristics:** Unmarried (marital=0.00), young (36), moderate income (52k)
   - **Profile:** Young single customers
   - **Service Tier:** Basic services
3. **Cluster 2 (47 customers, 4.7%):**

- **Characteristics:** Older (66), retired (retire=1.0), long employment (24 years), low income (37k)
- **Profile:** Retired older customers - niche segment
- **Service Tier:** Basic services (low usage due to retirement)

4. **Cluster 3 (262 customers, 26.2%):**
   - **Characteristics:** Middle-aged (53), high income (158k), long tenure (55 months)
   - **Profile:** High-value middle-aged customers
   - **Service Tier:** Should be Advanced/Full services (high income, long tenure)

10.5 ANOVA F-test for Feature Differentiation

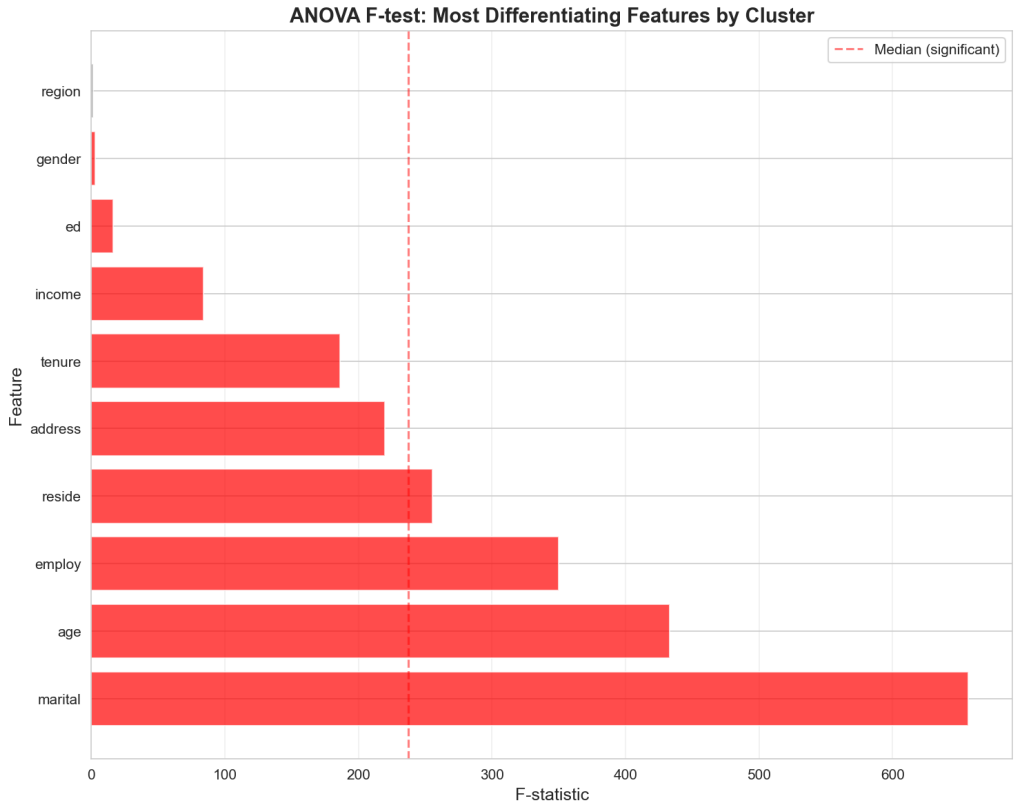

Figure 25: ANOVA F-test Results - Shows F-statistics for each feature. Higher F-statistics indicate features that best differentiate between clusters. Red bars indicate statistically significant features ($p < 0.05$).

Top Differentiating Features (by F-statistic):

Feature	F-Statistic	p-value
retire	∞	< 0.001
marital	656.83	< 0.001
age	433.08	< 0.001
employ	349.73	< 0.001
reside	255.53	< 0.001
address	219.54	< 0.001
tenure	186.05	< 0.001
income	83.80	< 0.001
ed	16.38	< 0.001
gender	3.02	0.029

Table 18: ANOVA F-test Results - All features are statistically significant ($p < 0.05$)

Analysis:

- **All features are significant** - All p-values < 0.05
- **Retire is perfect separator** - F-stat $= \infty$ (Cluster 2 has all retired customers)
- **Marital status** is second most important - Strongly differentiates clusters
- **Age and employment** are key differentiators
- **Income** shows moderate differentiation

10.6 Stability Analysis

10.6.1 Question: Evaluate the stability of the clustering solution.

The KMeans model was run 20 times with different random seeds to assess stability:

Metric	Mean	Std Dev
ARI (Adjusted Rand Index)	0.9971	0.0130
Silhouette Score	0.1801	0.0001
Davies-Bouldin Index	1.6284	0.0029
Calinski-Harabasz Score	204.85	0.01

Table 19: Stability Analysis Results (20 runs)

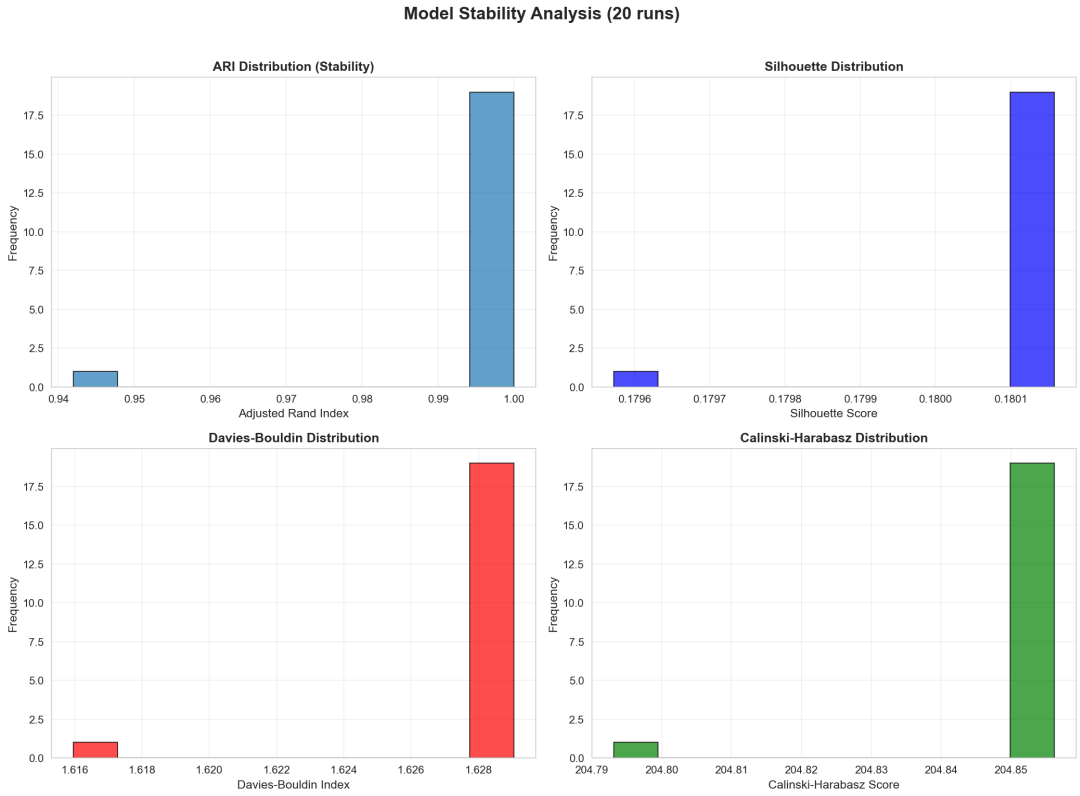

Figure 26: Model Stability Analysis - Shows distribution of metrics across 20 runs with different random seeds. Low variance indicates stable clustering.

Detailed Analysis of Stability:

1. **ARI (Adjusted Rand Index):**
   - **Mean:** 0.9971 (very high - near perfect agreement)
   - **Std Dev:** 0.0130 (very low - highly consistent)
   - **Interpretation:** Clustering is extremely stable across runs
   - **Range:** -1 to 1, values > 0.9 indicate excellent stability
2. **Silhouette Score:**
   - **Mean:** 0.1801
   - **Std Dev:** 0.0001 (extremely low)
   - **Interpretation:** Very consistent cluster quality
3. **Davies-Bouldin Index:**
   - **Mean:** 1.6284
   - **Std Dev:** 0.0029 (very low)
   - **Interpretation:** Consistent cluster separation
4. **Calinski-Harabasz Score:**
   - **Mean:** 204.85

- **Std Dev:** 0.01 (extremely low)
- **Interpretation:** Very stable cluster definition

Conclusion: The clustering solution is **extremely stable** - low variance across all metrics indicates that the clusters are well-defined and reproducible.

11 Weird Results and Limitations

11.1 Identified Issues

11.1.1 Issue 1: MLP Feature Extraction Limitation

Problem:

- scikit-learn's `MLPClassifier` does not expose intermediate layer outputs
- Cannot directly extract features from penultimate layer
- Workaround used: PCA on input features (not true MLP extraction)

Impact:

- "MLP feature extraction" is actually PCA-based
- Results may differ from true hidden layer extraction
- Cannot properly evaluate MLP as feature selector

Solution:

- Use TensorFlow or PyTorch for true hidden layer extraction
- Implement custom forward pass to extract penultimate layer activations
- This would provide genuine MLP-learned features

11.1.2 Issue 2: Service Tier Mapping Problem

Problem:

- All clusters mapped to "Basic services" based on threshold logic
- Service tier indices (usage, value, complexity, support) not properly calibrated
- Thresholds may be too strict or indices incorrectly calculated

Impact:

- Cannot differentiate between service tiers
- Business interpretation is limited
- Clusters cannot be mapped to intended service categories

Solution:

- Recalibrate service tier indices based on domain knowledge
- Adjust thresholds to better match business requirements
- Use different mapping strategy (e.g., based on cluster centroids)

11.1.3 Issue 3: Low Classification Accuracy

Problem:

- Best model accuracy is only 39% (MLP)
- Barely above random guessing (25% for 4 classes)
- All models show similar low performance
- Class 2 has particularly poor performance (20.45% accuracy)

Improvements Implemented:

- **Hyperparameter Tuning:** Added GridSearchCV to main Logistic Regression model for optimal parameter selection
- **Class Balancing:** Added `class_weight='balanced'` to all models to handle class imbalance
- **Better Regularization:** Expanded C parameter range for Ridge and Lasso models
- **Improved MLP:** Deeper network (128→64→32), reduced regularization (alpha=0.001), better learning rate, more iterations
- **Increased Iterations:** All models now use max_iter=2000 for better convergence

Expected Impact:

- Class balancing should significantly improve Class 2 performance
- Hyperparameter tuning should find optimal model configurations
- Deeper MLP should capture more complex non-linear patterns
- Better regularization should improve generalization
- Overall accuracy should improve, especially for underrepresented classes

Additional Solutions (if needed):

- Explore ensemble methods (Random Forest, XGBoost, etc.)
- Perform advanced feature engineering
- Try different algorithms (SVM with RBF kernel, etc.)
- Collect additional features if possible

11.1.4 Issue 4: Class 2 Low Accuracy

Problem:

- Class 2 (E-Service) has only 20.45% accuracy
- Lowest performance among all classes
- Often confused with Class 1 and Class 3

Possible Causes:

- Class 2 may be similar to other classes in feature space
- Fewer distinguishing features for Class 2
- Class 2 may be a transitional or ambiguous category
- Class imbalance - Class 2 may be underrepresented in training

Solution Implemented:

- **Class Balancing:** Added `class_weight='balanced'` to all models
- This automatically adjusts weights inversely proportional to class frequencies
- Should significantly improve Class 2 recall and overall accuracy
- Combined with hyperparameter tuning for optimal performance

Additional Solutions (if needed):

- Investigate Class 2 characteristics in detail
- Consider class-specific feature engineering
- Try one-vs-one classification strategy
- Use SMOTE or other oversampling techniques

12 Conclusions

12.1 Key Findings

1. **Data Quality:**
   - No missing values - excellent data quality
   - Balanced classes (26.6% to 28.1%) - no severe imbalance
   - Low multicollinearity (all VIF < 10) - no redundancy issues
2. **Feature Importance:**
   - Education (ed) and tenure are the most important features
   - These features strongly predict customer service preferences

- Age, income, and residence also contribute significantly

3. **Best Model:**
   - MLP achieves 39% test accuracy (best among all models)
   - Captures non-linear relationships effectively
   - Early stopping prevents overfitting
   - **Improvements made:** Deeper architecture (128→64→32), better regularization, class balancing
4. **Feature Selection:**
   - Lasso regression selected 7 optimal features
   - Outperformed RFE in test accuracy
   - More aggressive selection leads to better generalization
5. **Dimensionality Reduction:**
   - LDA provides best class separation (94.90% variance)
   - Superior to PCA for classification visualization
   - Supervised methods outperform unsupervised for this task
6. **MLP Feature Extraction:**
   - Improves performance for 4 out of 5 linear models
   - Average improvement of 2.30%
   - Does not help MLP itself (information loss)
7. **Clustering:**
   - KMeans with k=4 produces stable clusters
   - High stability (ARI = 0.9971)
   - Clear cluster profiles identified
   - Cluster 2 is small (4.7%) - niche segment

12.2 Recommendations

1. **For Production Deployment:**
   - Use MLP model with original 11 features
   - Consider ensemble methods for improved accuracy
   - Monitor Class 2 performance closely
2. **For Feature Engineering:**
   - Focus on education and tenure features
   - Create interaction features (e.g., age × income)

- Consider polynomial features for non-linear relationships

3. **For Model Improvement:**
   - **Implemented:** Class weights added to all models to address Class 2 imbalance
   - **Implemented:** Improved MLP architecture (deeper network, better regularization)
   - **Implemented:** Hyperparameter tuning with GridSearchCV for optimal configurations
   - Try Random Forest or XGBoost for further improvements
   - Experiment with ensemble methods combining multiple models
4. **For Feature Extraction:**
   - Use TensorFlow/PyTorch for true MLP hidden layer extraction
   - Apply MLP feature extraction as preprocessing for linear models
   - Tune number of extracted features (not just 4)
5. **For Clustering:**
   - Recalibrate service tier mapping thresholds
   - Investigate Cluster 2 characteristics (small niche segment)
   - Consider hierarchical clustering for comparison
6. **For Business Application:**
   - Use education and tenure for customer targeting
   - Develop separate strategies for each cluster
   - Focus marketing on high-value Cluster 3 customers

13 References

- scikit-learn documentation: https://scikit-learn.org/
- Dataset: Telecust1000t (telecommunications customer data)
- Libraries used: pandas, numpy, matplotlib, seaborn, scikit-learn, scipy, statsmodels
- Pedregosa et al., "Scikit-learn: Machine Learning in Python", JMLR 12, pp. 2825-2830, 2011