①

نیکی کدوریان ۴۰۱۲۳۱۵۳

$G(s) = \frac{k e^{-s\tau}}{s}$ $\quad G(j\omega) = k\frac{e^{-j\omega\tau}}{j\omega}$ $\quad -\tau\omega - ۹۰ = -۱۸۰$ $\quad \omega = \frac{۹۰}{\tau} = \frac{\pi}{۲\tau}$

$|G(j\omega)| = \left|\frac{k}{\omega}\right| \rightarrow \left|G\left(j\frac{\pi}{۲\tau}\right)\right| = \left|\frac{k}{\pi}\times ۲\tau\right| = \frac{۲k\tau}{\pi}$

$G_m = ۲۰\log \frac{۱}{|G(j\omega_p)|} = -۲۰\log\frac{۲k\tau}{\pi}$

$|G(j\omega)| = ۱ = ۰\,dB \Rightarrow k = \omega$ $\quad pm = \angle G(j\omega_g) - ۱۸۰$ $\quad pm = -k\tau + \frac{\pi}{۲}$

$G_m > ۰ \Rightarrow -۲۰\log\frac{۲k\tau}{\pi} > ۰ \Rightarrow ۰ < \frac{۲k\tau}{\pi} < ۱$ $\quad ۰ < k < \frac{\pi}{۲\tau}$

$P_m > ۰ \Rightarrow -k\tau + \frac{\pi}{۲} > ۰ \Rightarrow k < \frac{\pi}{۲\tau}$

$\Rightarrow ۰ < k < \frac{\pi}{۲\tau}$

②

$G(s) = \frac{k(s+۲)}{s^۲} \longrightarrow \frac{k(j\omega+۲)}{j^۲\omega^۲} = \frac{k(j\omega+۲)}{-\omega^۲}$

$|G(j\omega)| = \frac{k\sqrt{\omega^۲+۴}}{\omega^۲}$ $\quad |G(j\omega)| = ۱$ $\quad k\sqrt{\omega^۲+۴} = \omega^۲$

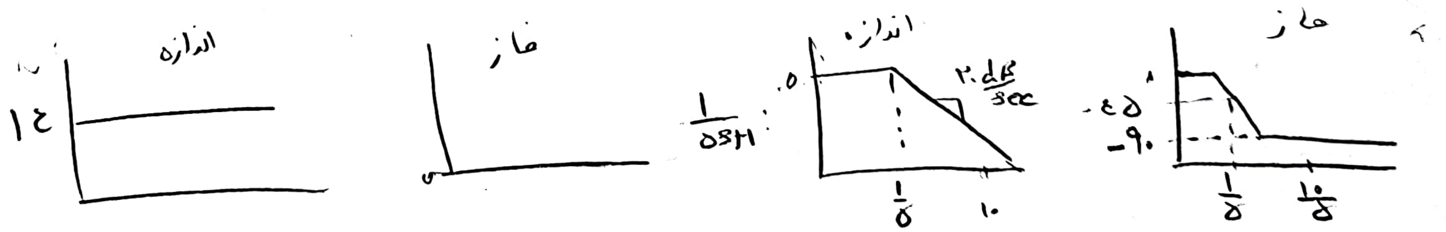
$P_m = \angle G(j\omega_c) - ۱۸۰$ $\quad \tan^{-1}\left(\frac{\omega}{۲}\right) - ۰ - ۱۸۰ = ۴۵^\circ$ $\quad \tan^{-1}\left(\frac{\omega_c}{۲}\right) = ۲۲۵$

$\omega_c = ۲$

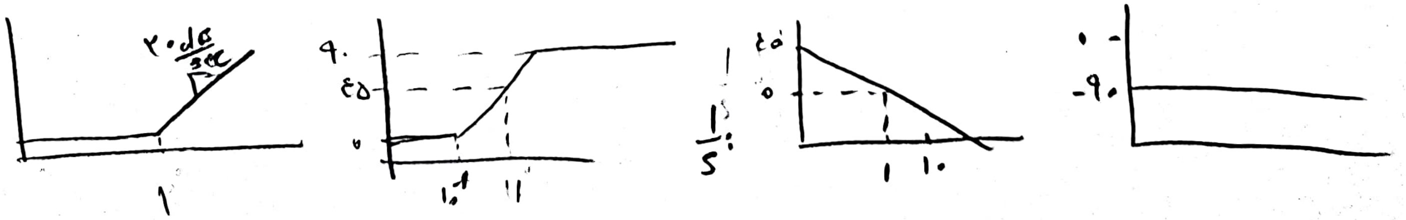
$k\sqrt{۸} = ۴ \quad k = \sqrt{۲}$

③

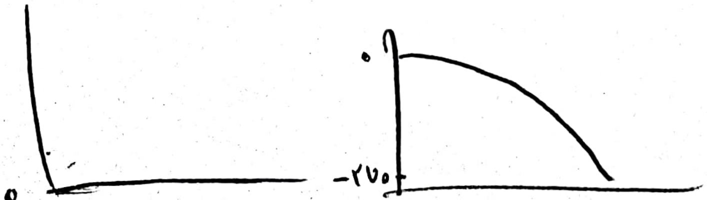
۵:



$s+۱$:



$e^{-۲s}$

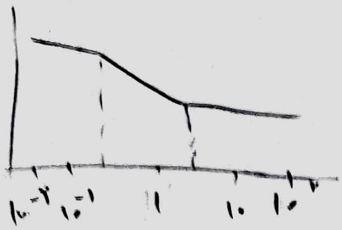


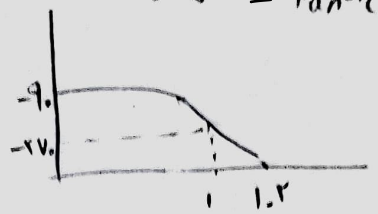
$$G = \frac{5}{5s+1} \times e^{-2s} \times \frac{s+1}{s}$$

$$G(j\omega) = \frac{5(j\omega+1)\,e^{-2j\omega}}{j\omega(5j\omega+1)}$$

$$\angle G(j\omega) = \tan^{-1}(\omega) - 2\omega - \left(90 + \tan^{-1}(5\omega)\right) = -180$$

$$\omega \simeq 0.421$$





$$|G(j\omega)| = \left| \frac{5\times(0.421j+1)\,e^{-0.842j}}{0.421(2.1j+1)\,j} \right| = \frac{5\sqrt{1+0.421^2}}{0.421\sqrt{1+2.1^2}} \simeq 5.54$$

$$G_m = -20\log 5.54 = -14.87\ \text{dB} < 0$$

$$|G(j\omega)| = 1 \qquad \frac{5\sqrt{\omega^2+1}}{\omega(25\omega^2+1)} = 1 \qquad \omega_c \simeq 1.3$$

$$P_M = \underbrace{\tan^{-1}(\omega) - \tan^{-1}(5\omega) - 2\omega - 90}_{\angle G} - 180 \Rightarrow \omega = 1.3$$

$$\Rightarrow P_M \simeq -447.79$$

$$\Rightarrow = -87.79^\circ$$

$$P_M < 0,\ G_m < 0 \Rightarrow$$ سیستم ناپایدار

(ج)

$S = j\omega$

$$G(j\omega) = \frac{4a^2}{(j\omega+a)^2} = \frac{4a^2}{a^2-\omega^2+2j\omega a} \qquad |G(j\omega)| = \frac{4a^2}{\sqrt{4a^2\omega^2+(a^2-\omega^2)^2}} = \frac{4a^2}{\omega^2+a^2}$$

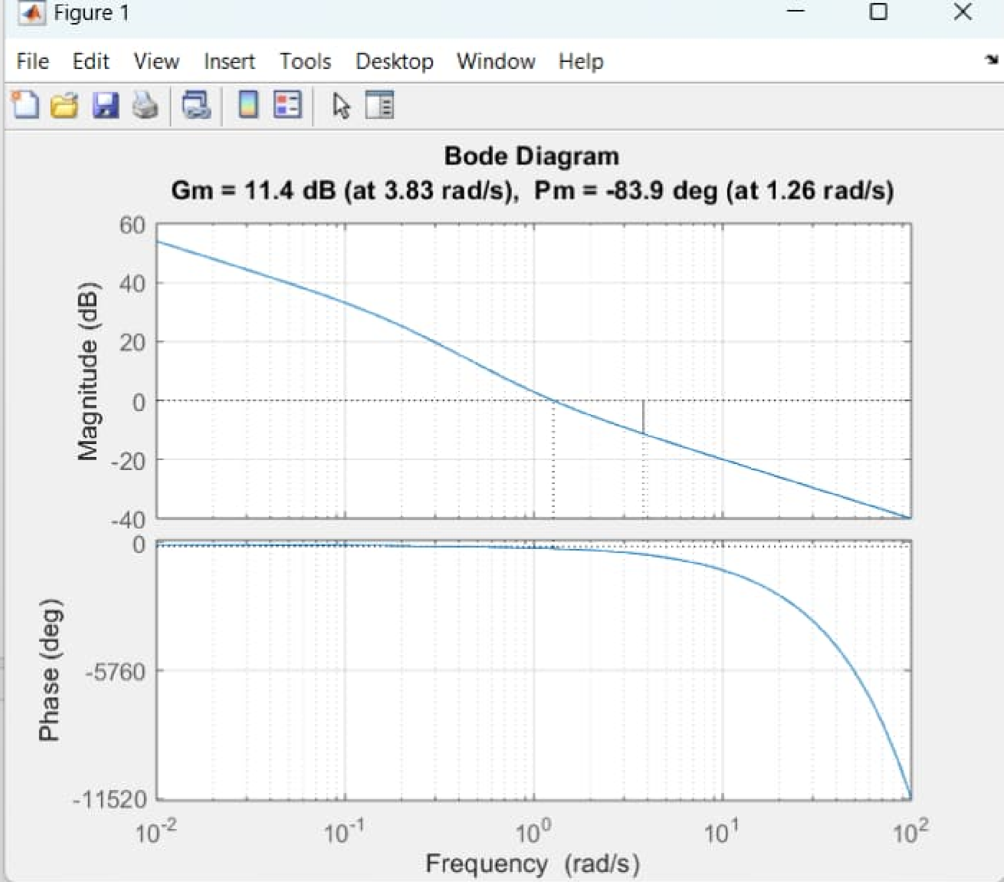
$$\angle G(j\omega) = 0 - 2\tan^{-1}\left(\frac{\omega}{a}\right) = -180 \qquad \tan^{-1}\left(\frac{\omega}{a}\right) = 90 \qquad \frac{\omega}{a} = \infty \Rightarrow G_m$$ ندارد

$$|G(j\omega)| = 1 \qquad 4a^2 = \omega^2 + a^2 \qquad \omega = \sqrt{3}\,a \qquad P_M = 0 - 2\tan^{-1}\left(\frac{\omega}{a}\right) - 180 = -300$$

$$\Rightarrow P_M = 90^\circ$$

سیستم ناپایدار ←

```
clc;
clear;
T = 2;
num = [5 5];
den = [5 1 0];
G = tf(num, den, 'InputDelay', T);
bode(G);
margin(G);
grid on;
```



Window