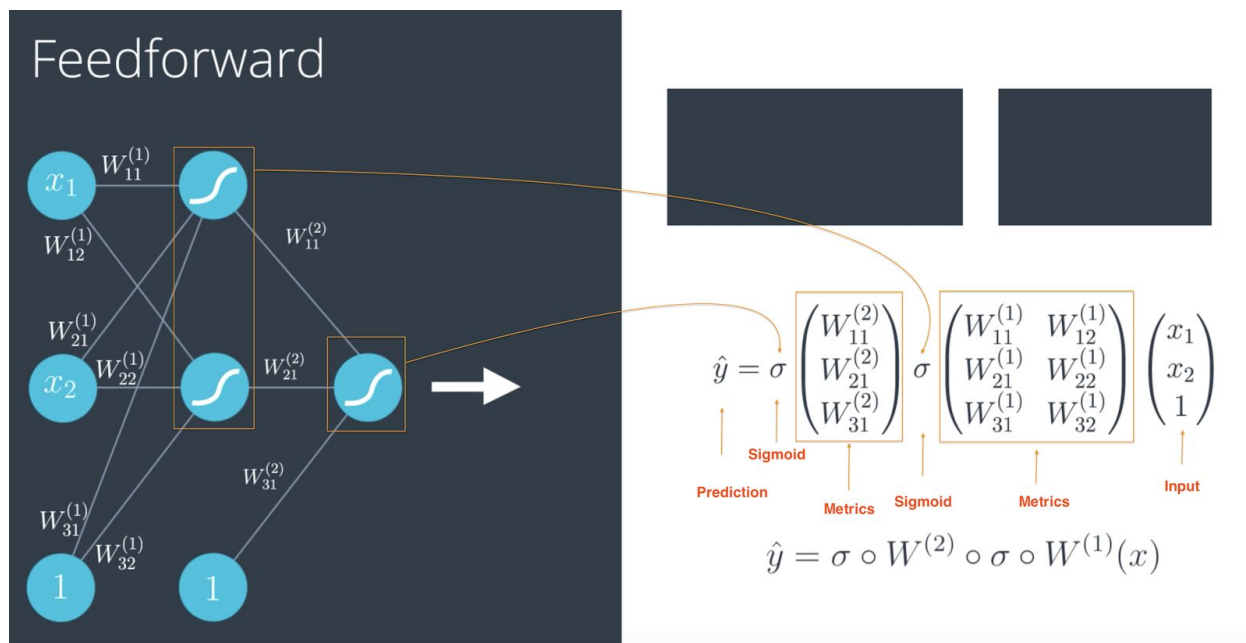


**Note:** This is the summary note from Udacity Introduction to Deep Learning with PyTorch

## Feedforward

- is the process neural networks use to turn the input into an output.
- First step of backpropagation (next lesson)
- **CNNs** and **RNNs** are just some special cases of Feedforward networks.
  - <https://towardsdatascience.com/feed-forward-neural-networks-c503faa46620>



- What is the error function for multi-layer perceptron ?

- Error Function:

$$E(W) = -\frac{1}{m} \sum_{i=1}^m y_i \ln(\hat{y}_i) + (1 - y_i) \ln(1 - \hat{y}_i)$$

- For Binary Error Function:

$$\text{We have } \hat{y} = \sigma(Wx + b)$$

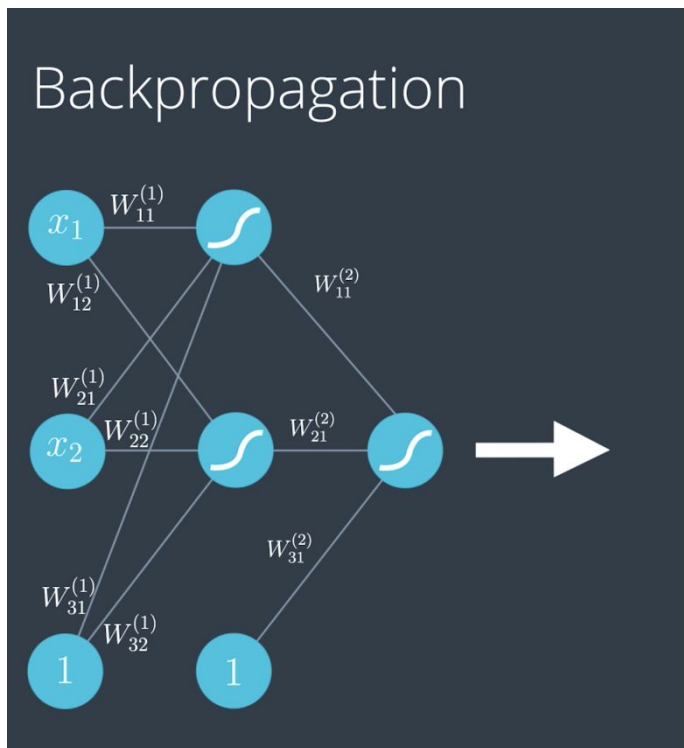
- For MLP:

$$\text{We have } \hat{y} = \sigma \circ W^{(n)} \circ \sigma \circ W^{(n-1)} \dots \circ \sigma \circ W^{(1)}(x)$$

## Backpropagation

- It's a method of training the neural network
- Consists of:
  - Doing a feedforward operation.
  - Comparing the output of the model with the desired output.
  - Calculating the error.
  - Running the feedforward operation backwards (backpropagation) to spread the error to each of the weights.
  - Use this to update the weights, and get a better model.
  - Continue this until we have a model that is good.
- The method is handled well by [Keras](#), a high-level neural networks API, written in Python and capable of running on top of [TensorFlow](#), [CNTK](#), or [Theano](#)
- Good article :  
<https://towardsdatascience.com/everything-you-need-to-know-about-neural-networks-and-backpropagation-machine-learning-made-easy-e5285bc2be3a>

- Backpropagation explained in Math
  - First it's good to understand what is composition function:
    - <https://www.onlinemathlearning.com/composite-functions.html>
    - $f(g(x))$  is the composite function that is formed when  $g(x)$  is substituted for  $x$  in  $f(x)$
    - $f(g(x))$  is read as "f of g of x"
    - $f(g(x))$  can also be written as  $(f \circ g)(x)$  or  $fg(x)$
    - In the composition  $(f \circ g)(x)$ , the domain of  $f$  becomes  $g(x)$
  - Formulas (prediction, gradient, gradient descent)



First layer

Second layer

$$\hat{y} = \sigma W^{(2)} \circ \sigma \circ W^{(1)}(x)$$

Metrics

$$W^{(1)} = \begin{pmatrix} W_{11}^{(1)} & W_{12}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} \\ W_{31}^{(1)} & W_{32}^{(1)} \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{pmatrix}$$

Partial Derivatives, just a long vector

Gradient

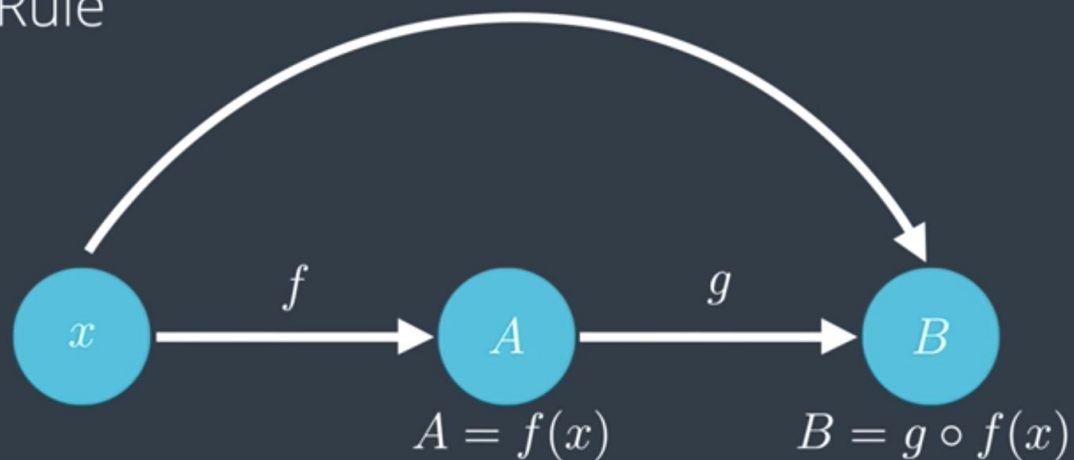
$$\nabla E = \begin{pmatrix} \frac{\partial E}{\partial W_{11}^{(1)}} & \frac{\partial E}{\partial W_{12}^{(1)}} & \frac{\partial E}{\partial W_{11}^{(2)}} \\ \frac{\partial E}{\partial W_{21}^{(1)}} & \frac{\partial E}{\partial W_{22}^{(1)}} & \frac{\partial E}{\partial W_{21}^{(2)}} \\ \frac{\partial E}{\partial W_{31}^{(1)}} & \frac{\partial E}{\partial W_{32}^{(1)}} & \frac{\partial E}{\partial W_{31}^{(2)}} \end{pmatrix}$$

Gradient Descent

$$W_{ij}^{(k)} \leftarrow W_{ij}^{(k)} - \alpha \frac{\partial E}{\partial W_{ij}^{(k)}}$$

Learning Rate

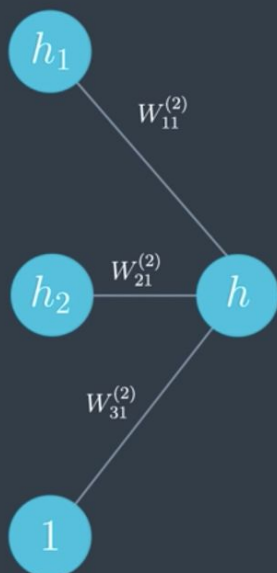
## Chain Rule



Partial derivative of  $B$  with respect to  $x$

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial A} \frac{\partial A}{\partial x}$$

## Backpropagation



$$h = W_{11}^{(2)}\sigma(h_1) + W_{21}^{(2)}\sigma(h_2) + W_{31}^{(2)}$$

$$\frac{\partial h}{\partial h_1} = W_{11}^{(2)} \sigma(h_1) [1 - \sigma(h_1)]$$

Derivate of sigmoid function

$$\begin{aligned} \sigma'(x) &= \frac{\partial}{\partial x} \frac{1}{1+e^{-x}} \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \\ &= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} \\ &= \sigma(x)(1 - \sigma(x)) \end{aligned}$$

## **Mini Summary**

- Feedforward is the process neural networks use to turn the input into an output.
- Backpropagation is a method of training the neural network.