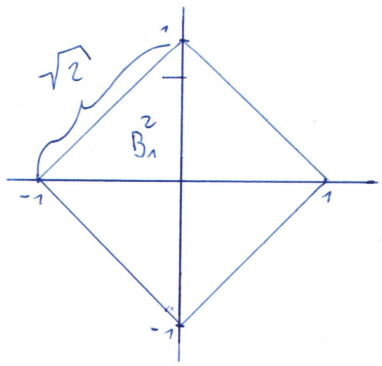
E2)

a)



$vol(B_1^2) = (\sqrt{2})^2 = 2$, since B_1^2 forms a square with side length $\sqrt{2}$.

$vol(B_1^3) = \frac{(\sqrt{2})^3}{3}\sqrt{2}$, since B_1^3 forms a Octahedron with side length $\sqrt{2}$.

b) For $B_2^\ell = \{x \in \mathbb{R}^\ell \mid \|x\|_2 \leq 1\}$ holds $\lim_{\ell \to \infty} vol(B_2^\ell) = 0$.

Let $x \in B_1^\ell = \{x \in \mathbb{R}^\ell \mid \|x\|_1 \leq 1\}$ it follows

$\|x\|_1 \leq 1$

$\Rightarrow \sum_i |x_i| \leq 1$

$|x_i| \leq 1 \Rightarrow \sum_i x_i^2 \leq 1$

$\Rightarrow \sqrt{\sum_i x_i^2} \leq 1$

$\Rightarrow \|x\|_2 \leq 1$

$\Rightarrow x \in B_2^\ell$

$\Rightarrow B_1^\ell \subseteq B_2^\ell$

therefore

$Vol(B_1^\ell) \leq vol(B_2^\ell)$

$\Rightarrow \lim_{\ell \to \infty} vol(B_1^\ell) = 0$