hw-3

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Second Task

Theorem. $S\ K\ K = I$

Proof.

$$
\begin{aligned}
S\ K\ K = {} & \overbrace{(\lambda f\ g\ x.f\ x\ (g\ x))}^{S}\ \overbrace{(\lambda x\ y.x)}^{K}\ \overbrace{(\lambda x\ y.x)}^{K} \\
\to_\beta {} & [f := \lambda x\ y.x] \\
\to_\beta {} & (\lambda g\ x.(\lambda x\ y.x)\ x\ (g\ x))\ (\lambda x\ y.x) \\
\to_\alpha {} & (\lambda g\ x.(\lambda x'\ y.x')\ x\ (g\ x))\ (\lambda x\ y.x) \\
\to_\beta {} & [g := \lambda x\ y.x] \\
\to_\beta {} & \lambda x.(\lambda x'\ y.x')\ x\ ((\lambda x\ y.x)\ x) \\
\to_\alpha {} & \lambda x.(\lambda x'\ y.x')\ x\ ((\lambda x''\ y.x'')\ x) \\
\to_\beta {} & [x' := x] \\
\to_\beta {} & \lambda x.(\lambda y.x)\ ((\lambda x''\ y.x'')\ x) \\
\to_\beta {} & [y := (\lambda x''\ y.x'')\ x] \\
\to_\beta {} & \lambda x.x \\
= {} & I
\end{aligned}
$$

$\square$