(10)

$H_0: \xi \sim p_0(x) = 1\{(0;1)\}$

$H_1: \xi \sim p_1(x) = \dfrac{e^{1-x}}{e-1}\{(0;1)\}$

a) $n = 1$, α

$$l = \frac{L_1}{L_0} = \frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{(e-1)\cdot 1} \geq C$$

$$1 - x \geq \ln C(e-1)$$

$$x \leq \underbrace{1 - \ln C(e-1)}{} = A$$

$$G_{kp.}: x \leq A$$

$$P(x \leq A | H_0) = \alpha$$

$$\int_0^A \underbrace{p_0(x)}_{1} dx = \alpha; \quad A = \alpha; \quad G_{kp.}: x \leq \alpha.$$

$$\underline{\alpha_1 = \alpha.}$$

$$W = P(x \leq A | H_1) = \int_0^\alpha p_1(x) dx = \int_0^\alpha \frac{e^{1-x}}{e-1} dx =$$

$$= -\frac{e^{1-x}}{e-1}\Big|_0^\alpha = \underline{\frac{e}{e-1}\left(1 - e^{-\alpha}\right)}$$

$$\alpha_2 = 1 - W = \underline{1 - \frac{e}{e-1}\left(1 - e^{-\alpha}\right)}$$

б) $n=2,\ \alpha$

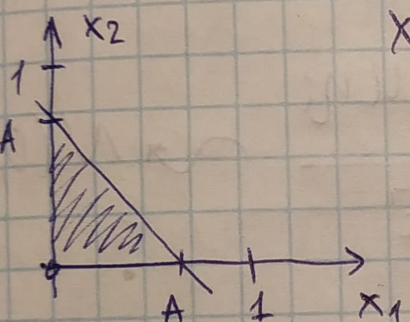
$$l = \frac{L_1}{L_0} = \frac{p_1(x_1)\cdot p_1(x_2)}{p_2(x_1)\cdot p_2(x_2)} = \frac{e^{1-x_1}\cdot e^{1-x_2}}{(e-1)^2\ 1\cdot 1} \geq c$$

$$e^{-x_1-x_2} \geq \frac{c(e-1)^2}{e^2}$$

$$-x_1-x_2 \geq \ln\left(\frac{c(e-1)^2}{e^2}\right)$$

$$G_{кр.}:\ x_1+x_2 \leq A.$$

$$P(x_1+x_2 \leq A|H_0) = \alpha$$



$$X_1+X_2 = A.$$

$$\iint\limits_{x_1+x_2\leq A} 1\cdot 1\cdot dx_1 dx_2 = \alpha$$

$$\frac{1}{2}A^2 = \alpha\ ;\quad A = \sqrt{2\alpha}$$

$$G_{кр.}:\ x_1+x_2 \leq \sqrt{2\alpha}$$

$$\underline{\alpha_1 = \sqrt{2\alpha}}$$

$$W = P(x_1+x_2 \leq A|H_1) = \iint\limits_{x_1+x_2\leq A} \frac{e^{1-x_1}e^{1-x_2}}{(e-1)^2}dx_1dx_2 =$$

$$= \frac{e^2}{(e-1)^2}\int_0^A dx_1 \int_0^{A-x_1} e^{-x_1}e^{-x_2}dx_2 = \frac{e^2}{(e-1)^2}\int_0^A e^{-x_1}\left(1-e^{-A+x_1}\right)dx_1 =$$

$$= \frac{e^2}{(e-1)^2}\left[1-e^{-A}-e^{-A}A\right],$$

$$\alpha_2 = 1 - w = 1 - \underbrace{\frac{e^2}{(e-1)^2}\left[1 - e^{-A} - e^{-A}A\right]}$$

c) оптимал. критерий: n, α.

$$l = \frac{L_1}{L_0} = \frac{\Pi p_1(x_i)}{\Pi p_0(x_i)} = \Pi \frac{p_1(x_i)}{p_0(x_i)} \geq c$$

Логарифмируем:

$$\sum_{i=1}^{n} \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i} \geq \ln c$$

$$\left\{ \text{ЦПТ} \quad \frac{\sum \eta_i - n M\eta_i}{\sqrt{n D\eta_i}} \rightsquigarrow N(0,1) \right\}$$

$$\eta_i = \ln \frac{e^{1-x_i}}{(e-1)\cdot 1} = \ln \frac{e}{e-1} - x_i$$

H_0: $$M\eta_i = M\left[\ln \frac{e}{e-1} - x_i\right] = \ln \frac{e}{e-1} - \frac{1}{2}$$

$$D\eta_i = D\left[\ln \frac{e}{e-1} - x_i\right] = Dx_i = \frac{1}{12}$$

$$P(\ln l \geq \ln c | H_0) = \alpha$$

$$P(\sum \eta_i \geq \ln c | H_0) =$$

$$= P\left(\frac{\sum \eta_i - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n\cdot 1/12}} \geq \frac{\ln C - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n\cdot 1/12}}\right)$$

$$\overset{\shortparallel}{\text{\ae}} \sim N(0,1) \qquad \overset{\shortparallel}{A}$$

$$\int_A^{\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,d\text{\ae} = \alpha \Rightarrow$$

$$A = u_{1-\alpha}$$

$$\ln \ell \geq \ln C$$

$$\sum \eta_i = n\ln\frac{e}{e-1} - \sum x_i$$

$$\frac{\ln C - n\ln\frac{e}{e-1} + n/2}{\sqrt{n/12}} = u_{1-\alpha}$$

$$\ln C = n\ln\frac{e}{e-1} - \frac{n}{2} + u_{1-\alpha}\sqrt{\frac{n}{12}}$$

$$G_{\text{кр.}}: \ \overline{\text{\ae}} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$$

$$W = P(\underset{\sum\eta_i}{\underset{\shortparallel}{\ln \ell}} \geq \ln C | H_1)$$

ЦПТ:
$$P\left(\frac{\sum\eta_i - n M\eta_i}{\sqrt{n D\eta_i}} \geq \frac{\ln C - n M\eta_i}{\sqrt{n D\eta_i}}\right) =$$

$$\overset{\shortparallel}{\text{\ae}} \sim N(0,1) \qquad \overset{\shortparallel}{B}$$

$$= \int_B^{+\infty} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx$$

$$\eta_i = \ln \frac{e^{1-x_i}}{e-1} = \ln \frac{e}{e-1} - x_i$$

$$H_1: \ M\eta_i = \ln\frac{e}{e-1} - Mx_i = \ln\frac{e}{e-1} - \underbrace{\int_0^1 x\,\frac{e^{1-x}}{e-1}\,dx}_{\frac{e-2}{e-1}} = \ln\frac{e}{e-1} + \frac{2-e}{e-1}$$

$$D\eta_i = \frac{e^2-3e+1}{(e-1)^2}$$

$$\alpha_2 = 1 - W$$

$$\alpha_1 = \alpha$$

$$W = \int_B^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx \longrightarrow \int_{-\infty}^{\infty} e^{-x^2/2}\frac{1}{\sqrt{2\pi}}\,dx = 1$$

$$B = \frac{n\left(\frac{e-2}{e-1} - \frac{1}{2}\right) + u_{1-\alpha}\cdot\sqrt{\frac{n}{12}}}{\sqrt{n\cdot\frac{e^2-3e+1}{(e-1)^2}}} = E + \underbrace{F}_{<0}\sqrt{n}$$

$n \to \infty$, $B \to -\infty$.

т.к. $W \xrightarrow[n\to\infty]{} 1$, то крит. сост-лен

d) $G_{кр}: x_{min} < c$ + исслед. на сост.

n, α.

$P(\vec{x}_n \in G_{кр.} | H_0) = \alpha$; $P(x_{min} < c | H_0) = \alpha$ найти

$H_0: \xi \sim R(0,1)$

$\xi \sim F(x) \Rightarrow x_{min} \sim 1-(1-F(x))^n$

$$F_0(x) = \begin{cases} 0, & x \le 0 \\ x, & x \in (0,1) \\ 1, & x \ge 1 \end{cases}$$

$$\alpha = P(x_{min} < c) = 1-(1-F(c))^n$$

$$\alpha = 1-(1-c)^n$$

$$c = 1-\sqrt[n]{1-\alpha}$$

$G_{кр.}: \ x_{min} < 1-\sqrt[n]{1-\alpha}$ – критерий.

$\alpha_1 = \alpha$

$$W = P(\vec{x}_n \in G_{кр.} | H_1) = P(x_{min} < 1-\sqrt[n]{1-\alpha} | H_1) =$$

$$= 1-\left(1-F_1\left(1-\sqrt[n]{1-\alpha}\right)\right)^n$$

$$H_1: \xi \sim p_1(x) = \begin{cases} \dfrac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

$$F_1(x) = \int_{-\infty}^{x} p_1(t)\,dt = \int_0^x e^{-t}dt \cdot \frac{e}{e-1} =$$

$$= \frac{e}{e-1}\left(1-e^{-x}\right).$$

$$W = 1-\left(1-\frac{e}{e-1}\left(1-e^{-1+\sqrt[n]{1-\alpha}}\right)\right)^n =$$

$$= 1-\left(1-\frac{e}{e-1}\left(1-e^{-1}\cdot e^{\left(1+\frac{\ln(1-\alpha)}{n}+o\left(\frac{1}{n}\right)\right)}\right)\right)^n =$$

$$= 1-\left(1-\frac{e}{e-1}\left(1-e^{-1}e\cdot\left(1+\frac{\ln(1-\alpha)}{n}+o\left(\frac{1}{n}\right)\right)\right)\right)^n =$$

$$= 1-\left(1+\frac{\ln(1-\alpha)}{(e-1)\cdot n}+o\left(\frac{1}{n}\right)\right)^n \longrightarrow$$

$$\longrightarrow 1-e^{\frac{\ln(1-\alpha)}{e-1}} = 1-(1-\alpha)^{1/e-1} = 0{,}029$$

не явл. сост.

$\alpha_2 = 1 - W$