Решение транспортной задачи:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | Запасы | | *B*1 | *B*2 | *B*3 | *B*4 | | *A*1 | |  | | --- | | 1 | | |  | | --- | | 8 | | |  | | --- | | 2 | | |  | | --- | | 3 | | |  | | --- | | 30 | | | *A*2 | |  | | --- | | 4 | | |  | | --- | | 7 | | |  | | --- | | 5 | | |  | | --- | | 1 | | |  | | --- | | 50 | | | *A*3 | |  | | --- | | 5 | | |  | | --- | | 3 | | |  | | --- | | 4 | | |  | | --- | | 4 | | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | |  | | --- | | 15 | | |  | | --- | | 40 | | |  | | --- | | 30 | | |  | | --- | | 0 | | |

Число пунктов отправления *m=*3, а число пунктов назначения *n*=4. Следовательно опорный план задачи определяется числами, стоящими в *m+n*−1=3+4−1=6 заполненых клетках таблицы. Тарифы перевозок единицы груза из каждого пункта отправления во все пункты назначения задаются матрицей

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *C=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 1 | | |  | | --- | | 8 | | |  | | --- | | 2 | | |  | | --- | | 3 | | | |  | | --- | | 4 | | |  | | --- | | 7 | | |  | | --- | | 5 | | |  | | --- | | 1 | | | |  | | --- | | 5 | | |  | | --- | | 3 | | |  | | --- | | 4 | | |  | | --- | | 4 | | |  | |

Наличие груза у поставщиков равно:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *A*i= | |  | | --- | | 30 | | + | |  | | --- | | 50 | | + | |  | | --- | | 20 | | = | |  | | --- | | 100 | |  |

Общая потребность в грузе в пунктах назначения равна:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *B*i= | |  | | --- | | 15 | | + | |  | | --- | | 15 | | + | |  | | --- | | 40 | | + | |  | | --- | | 30 | | = | |  | | --- | | 100 | |  |

∑ *A*i=∑ *B*i. Модель транспортной задачи является закрытой. Следовательно она разрешима.

**Этап I. Нахождение первого опорного плана**

Найдем опорный план задачи *методом северно-западного* угла.

*A*1>*B*1. Следовательно в клетку (*A*1, *B*1) помещаем число *min*(*A*1, *B*1)=15. Потребности пункта *B*1 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*1 и будем считать запасы пункта *A*1 равными 30−15=15.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 15 | | |  | |  | | --- | | 15 | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

*A*1≤*B*2. Следовательно в клетку (*A*1, *B*2 ) помещаем число *min*(*A*1, *B*2 )=15. Запасы пункта *A*1 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*1 и будем считать потребности пункта *B*2 равными 15−15=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

*A*2>*B*2. Следовательно в клетку (*A*2, *B*2) помещаем число *min*(*A*2, *B*2)=0. Потребности пункта *B*2 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*2 и будем считать запасы пункта *A*2 равными 50−0=50.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  | |  | | --- | | 0 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

*A*2>*B*3. Следовательно в клетку (*A*2, *B*3) помещаем число *min*(*A*2, *B*3)=40. Потребности пункта *B*3 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*3 и будем считать запасы пункта *A*2 равными 50−40=10.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 10 | | |  |  |  | |  | | --- | | 0 | |  | |  | | --- | | 40 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

*A*2≤*B*4. Следовательно в клетку (*A*2, *B*4 ) помещаем число *min*(*A*2, *B*4 )=10. Запасы пункта *A*2 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*2 и будем считать потребности пункта *B*4 равными 30−10=20.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 0 | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 20 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

*A*3≤*B*4. Следовательно в клетку (*A*3, *B*4 ) помещаем число *min*(*A*3, *B*4 )=20. Запасы пункта *A*3 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*3 и будем считать потребности пункта *B*4 равными 20−20=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 0 | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 50 | |  |  | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 0 | | |  |  |  |  |  |  |  | |  | | --- | | 20 | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 20 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 100 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 15 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 40 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 30 | |  |  | | | |

**Этап II. Улучшение опорного плана**

Найдем оптимальный план транспортной задачи *методом потенциалов*.

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 15 | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 40 | | |  | | --- | | 10 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 15 | + | 8 | · | 15 | + | 7 | · | 0 | + | 5 | · | 40 | + | 1 | · | 10 | + | 4 | · | 20 | = | 425 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=1
* β2−α1=8
* β2−α2=7
* β3−α2=5
* β4−α2=1
* β4−α3=4

Полагая α1=0, находим β1=1 β2=8 α2=1 β3=6 β4=2 α3=-2 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α13=4, α14=-1, α21=-4, α31=-2, α32=7, α33=4.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | −4 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  |  | | --- | --- | | |  | | --- | | 7 | | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 7 находится в пересечении строки *A*3 и столбца *B*2. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | | − | |  | | --- | | 5 | |  | |  | | --- | | 1 | | + | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | −4 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | | + | |  | | --- | | 4 | |  | |  | | --- | | 4 | | − | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  |  | | --- | --- | | |  | | --- | | 7 | | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Наименьшее из чисел в минусовых клетках равно 0. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 0, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  |  |  |  | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  |  |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  | |  | | --- | | 0 | |  |  |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 15 | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 40 | | |  | | --- | | 10 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 15 | + | 8 | · | 15 | + | 5 | · | 40 | + | 1 | · | 10 | + | 3 | · | 0 | + | 4 | · | 20 | = | 425 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=1
* β2−α1=8
* β3−α2=5
* β4−α2=1
* β2−α3=3
* β4−α3=4

Полагая α1=0, находим β1=1 β2=8 α3=5 β4=9 α2=8 β3=13 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α13=11, α14=6, α21=-11, α22=-7, α31=-9, α33=4.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 11 | | |  | |  |  | | --- | --- | | |  | | --- | | 6 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | −11 | | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | |  | |  | | --- | | 0 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 11 находится в пересечении строки *A*1 и столбца *B*3. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | | − | |  | | --- | | 2 | | + | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 11 | | |  | |  |  | | --- | --- | | |  | | --- | | 6 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | | − | |  | | --- | | 1 | | + | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | −11 | | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  | | --- | | 40 | |  | |  | | --- | | 10 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | | + | |  | | --- | | 4 | |  | |  | | --- | | 4 | | − | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | −9 | | |  | |  | | --- | | 0 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 20 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Наименьшее из чисел в минусовых клетках равно 15. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 15, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  |  |  | |  | | --- | | 15 | |  |  | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  |  |  | |  | | --- | | 25 | |  | |  | | --- | | 25 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  | |  | | --- | | 15 | |  |  |  | |  | | --- | | 5 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 25 | | |  | | --- | | 25 | | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 5 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 15 | + | 2 | · | 15 | + | 5 | · | 25 | + | 1 | · | 25 | + | 3 | · | 15 | + | 4 | · | 5 | = | 260 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=1
* β3−α1=2
* β3−α2=5
* β4−α2=1
* β2−α3=3
* β4−α3=4

Полагая α1=0, находим β1=1 β3=2 α2=-3 β4=-2 α3=-6 β2=-3 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=-11, α14=-5, α21=0, α22=-7, α31=2, α33=4.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −11 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | 0 | | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  | | --- | | 25 | |  | |  | | --- | | 25 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | 2 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 5 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 4 находится в пересечении строки *A*3 и столбца *B*3. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −11 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | | − | |  | | --- | | 1 | | + | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | 0 | | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  | | --- | | 25 | |  | |  | | --- | | 25 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | | + | |  | | --- | | 4 | | − | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | 2 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | 4 | | |  | |  | | --- | | 5 | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Наименьшее из чисел в минусовых клетках равно 5. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 5, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  |  |  | |  | | --- | | 15 | |  |  | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  |  |  |  |  | |  | | --- | | 20 | |  | |  | | --- | | 30 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  |  |  | |  | | --- | | 15 | |  | |  | | --- | | 5 | |  |  | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  | | --- | | 30 | | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 5 | | |  | | --- | | 0 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 15 | + | 2 | · | 15 | + | 5 | · | 20 | + | 1 | · | 30 | + | 3 | · | 15 | + | 4 | · | 5 | = | 240 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 6 уравнений с 7 неизвестными:

* β1−α1=1
* β3−α1=2
* β3−α2=5
* β4−α2=1
* β2−α3=3
* β3−α3=4

Полагая α1=0, находим β1=1 β3=2 α2=-3 α3=-2 β4=-2 β2=1 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=-7, α14=-5, α21=0, α22=-3, α31=-2, α34=-4.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | *B*4 | | | *A*1 | |  | | --- | | 1 | |  | |  | | --- | | 8 | |  | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 30 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −7 | | |  | |  | | --- | | 15 | |  | |  |  | | --- | --- | | |  | | --- | | −5 | | | | *A*2 | |  | | --- | | 4 | |  | |  | | --- | | 7 | |  | |  | | --- | | 5 | |  | |  | | --- | | 1 | |  | |  | | --- | | 50 | | |  | |  |  | | --- | --- | | |  | | --- | | 0 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  | | --- | | 20 | |  | |  | | --- | | 30 | | | *A*3 | |  | | --- | | 5 | |  | |  | | --- | | 3 | |  | |  | | --- | | 4 | |  | |  | | --- | | 4 | |  | |  | | --- | | 20 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  | | --- | | 15 | |  | |  | | --- | | 5 | |  | |  |  | | --- | --- | | |  | | --- | | −4 | | | | Потребности | |  | | --- | | 15 | | | |  | | --- | | 15 | | | |  | | --- | | 40 | | | |  | | --- | | 30 | | | |  | | --- | | 100 | | |

Среди чисел *α*ij нет положительных. Следовательно данный опорный план является оптимальным.

**Решение:**

Оптимальный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | |  | | --- | | 15 | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 20 | | |  | | --- | | 30 | | | |  | | --- | | 0 | | |  | | --- | | 15 | | |  | | --- | | 5 | | |  | | --- | | 0 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 15 | + | 2 | · | 15 | + | 5 | · | 20 | + | 1 | · | 30 | + | 3 | · | 15 | + | 4 | · | 5 | = | 240 | |

Ответ: оптимальная стоимость перевозок 240