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CAP4613
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Assignment 8

a.
Given:
Inputs: $x_0 = 1,\ x_1 = 1,\ x_2 = 0$
Initial Weights:

$$\omega_{x0,h1} = 0.18,\ \omega_{x1,h1} = 0.32,\ \omega_{x2,h1} = 0.42$$
$$\omega_{x0,h2} = 0.51,\ \omega_{x1,h2} = 0.64,\ \omega_{x2,h2} = 0.12$$
$$\omega_{x0,h3} = 0.43,\ \omega_{x1,h3} = 0.72,\ \omega_{x2,h3} = 0.33$$
$$\omega_{h0,O1} = 0.53,\ \omega_{h1,O1} = 0.22,\ \omega_{h2,O1} = 0.19,\ \omega_{h3,O1} = 0.61$$
$$\omega_{h0,O2} = 0.61,\ \omega_{h1,O2} = 0.38,\ \omega_{h2,O2} = 0.21,\ \omega_{h3,O2} = 0.15$$

Calculating net input for hidden layer neurons:

1. h1:
   $$z_{h1} = \omega_{x0,h1} * x_0 + \omega_{x1,h1} * x_1 + \omega_{x2,h1} * x_2$$
   z_{h1} = 0.18 · 1+0.32 · 1+0.42 · 0 = 0.50
2. h2:
   $$z_{h2} = \omega_{x0,h2} * x_0 + \omega_{x1,h2} * x_1 + \omega_{x2,h2} * x_2$$
   z_{h2} = 0.51 · 1+0.64 · 1+0.12 · 0 = 1.15
3. h3:
   $$z_{h3} = \omega_{x0,h3} * x_0 + \omega_{x1,h3} * x_1 + \omega_{x2,h3} * x_2$$
   z_{h3} = 0.43 · 1+0.72 · 1+0.33 · 0 = 1.15

Applying sigmoid activation function for hidden layer neurons:
h1:

$$\alpha_{h1} = \sigma(z_{h1}) = \frac{1}{1+e^{-0.50}} = 0.062$$

h2:

$$\alpha_{h2} = \sigma(z_{h2}) = \frac{1}{1+e^{-1.15}} = 0.760$$

h3:

$$\alpha_{h3} = \sigma(z_{h3}) = \frac{1}{1+e^{-1.15}} = 0.760$$

Calculating net input for output layer neurons:

o1:

$$z_{o1} = \omega_{h0,o1} + \omega_{h1,o1} * a_{h1} + \omega_{h2,o1} * \alpha_{h2} + \omega_{h3,o1} * \alpha_{h3}$$

z_{o1} = 0.53 · 1+0.22 · 0.622+0.19 · 0.760+0.61 · 0.760 = 1.275

o2:

$$z_{o2} = \omega_{h0,o2} + \omega_{h1,o2} * a_{h1} + \omega_{h2,o2} * \alpha_{h2} + \omega_{h3,o2} * \alpha_{h3}$$

$$z_{o2} = 0.61 \cdot 1 + 0.38 \cdot 0.622 + 0.21 \cdot 0.760 + 0.15 \cdot 0.760 = 1.120$$

Applying sigmoid activation function for output layer neurons:

o1:

$$\alpha_{o1} = \sigma(zo1) = \frac{1}{1+e^{-1.275}} = 0.781$$

o2:

$$\alpha_{o2} = \sigma(zo2) = \frac{1}{1+e^{-1.120}} = 0.754$$

y_{h1}	y_{h2}	y_{h3}	y_1	y_2
0.622	0.760	0.760	0.781	0.754

b.

o1:

$\delta_{o1} = (a_{o1} - y_1) * a_{01} * (1 - a_{o1})$

δ_{o1} = (0.781−1) · 0.781 · (1−0.781) = - 0.037

o2:

$\delta_{o2} = (a_{o2} - y_1) * a_{o2} * (1 - a_{o2})$

δ_{o2} = (0.754−0) · 0.754 · (1−0.754) = 0.140

Computing the local gradients for the hidden neurons:

h1:

$\delta_{h1} = (\omega_{h1,o1} * \delta_{o1} + \omega_{h1,o2} * \delta_{o2}) * a_{h1} * (1 - a_{h1})$

δ_{h1} = (0.22 · −0.037+0.38 · 0.140) · 0.622 · (1−0.622) = 0.0106

h2:

$\delta_{h2} = (\omega_{h2,o1} * \delta_{o1} + \omega_{h2,o2} * \delta_{o2}) * a_{h2} * (1 - a_{h2})$

δ_{h2} = (−0.00703+0.0294) · 0.759 · 0.241 = 0.0041

h3:

$\delta_{h3} = (\omega_{h3,o1} * \delta_{o1} + \omega_{h3,o2} * \delta_{o2}) * a_{h3} * (1 - a_{h3})$

δ_{h3} = (0.61 · −0.037+0.15 · 0.140) · 0.759 · (1−0.759) = −0.0003

δ_{h1}	δ_{h2}	δ_{h3}	δ_{o1}	δ_{o2}
0.0106	0.0041	-0.0003	-0.037	0.140

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c.

Bias weights update rule: $\omega_{new} = \omega_{old} - \eta * \delta$

Bias weights changes and updates:

h1:

$\Delta\omega_{x0,h1} = - \eta * \delta_{h1}$

$\Delta\omega_{x0,h1} = -0.5 \cdot 0.0106 = -0.0053$

$\omega_{x0,h1}^{new} = \omega_{x0,h1} + \Delta\omega_{x0,h1} = 0.18 + (-0.0053) = 0.1747$

h2:

$\Delta\omega_{x0,h2} = - \eta * \delta_{h2}$

$\Delta\omega_{x0,h2} = -0.5 \cdot 0.0041 \approx -0.00205$

$\omega_{x0,h2}^{new} = \omega_{x0,h2} + \Delta\omega_{x0,h2} = 0.51 + (-0.00205) = 0.50795$

h3:

$\Delta\omega_{x0,h3} = - \eta * \delta_{h3}$

$\Delta\omega_{x0,h3} = -0.5 \cdot -0.0003 \approx 0.00015$

$\omega_{x0,h3}^{new} = \omega_{x0,h3} + \Delta\omega_{x0,h3} = 0.43 + 0.00015 = 0.50795$

o1:

$\Delta\omega_{h0,o1} = - \eta * \delta_{o1}$

$\Delta\omega_{h0,o1} = -0.5 \cdot -0.037 = 0.0185$

$\omega_{h0,o1}^{new} = \omega_{h0,o1} + \Delta\omega_{h0,o1} = 0.53 + 0.0185 = 0.5485$

o2:

$\Delta\omega_{h0,o2} = \quad - \eta * \delta_{o2}$

$\Delta\omega_{h0,o2} = -0.5 \cdot 0.140 = -0.07$

$\omega_{h0,o2}^{new} = \omega_{h0,o2} + \Delta\omega_{h0,o2} = 0.61 + (-0.07) = 0.54$

Change in weights

$\Delta\omega_{x0,h1}$	$\Delta\omega_{x0,h2}$	$\Delta\omega_{x0,h3}$	$\Delta\omega_{h0,o1}$	$\Delta\omega_{h0,o2}$
-0.0053	-0.00205	0.00015	0.0185	-0.07

Updated weights

$\omega_{x0,h1}^{new}$	$\omega_{x0,h2}^{new}$	$\omega_{x0,h3}^{new}$	$\omega_{h0,o1}^{new}$	$\omega_{h0,o2}^{new}$
0.1747	0.50795	0.43015	0.5485	0.54
