

Q.1)

$$T(n) = 3T(n/2) + n^2$$

$$T(n) = aT(n/b) + f(n)$$

$$a \geq 1, b \geq 1$$

On comparing,

$$a=3, b=2, f(n)=n^2$$

$$c = \log_b a = \log_2 3 = 1.584$$

$$n^c = n^{1.584} < n^2$$

$$\therefore f(n) > n^c$$

$$\therefore T(n) = O(n^2)$$

Q.3)

$$T(n) = T(n/2) + 2^n$$

$$a=1$$

$$b=2$$

$$f(n) = 2^n$$

$$c = \log_b a = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$f(n) > n^c$$

$$T(n) = O(2^n)$$

Q.5)

$$T(n) = 16T(n/4) + n$$

$$a=16, b=4, f(n)=n$$

$$c = \log_4 16 = 2$$

$$n^c = n^2$$

$$f(n) < n^c$$

$$\therefore T(n) = O(n^2)$$

Q.7)

$$T(n) = 2T(n/2) + n/\log n$$

$$a=2, b=2, f(n) = n/\log n$$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$\frac{n}{\log n} < n$$

$$f(n) < n^c$$

$$\therefore T(n) = O(n)$$

Q.2)

$$T(n) = 4T(n/2) + n^2$$

$$a \geq 1, b \geq 1$$

$$a=4, b=2, f(n)=n^2$$

$$c = \log_2 4 = 2$$

$$n^c = n^2 = f(n) = n^2$$

$$T(n) = O(n^2 \log_2 n)$$

Q.4)

$$T(n) = 2^n T(n/2) + n^n$$

$$a=2^n, b=2, f(n)=n^n$$

$$c = \log_b a = \log_2 2^n = n$$

$$n^c = n^n$$

$$f(n) = n^c$$

$$T(n) = O(n^2 \log_2 n)$$

Q.6)

$$T(n) = 2T(n/2) + n \log n$$

$$a=2, b=2, f(n)=n \log n$$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$n \log n > n$$

$$f(n) > n^c$$

$$T(n) = O(n \log n)$$

Q.6)

$$T(n) = 2T(n/4) + n^{0.51}$$

$$a=2, b=4, f(n)=n^{0.51}$$

$$c = \log_4 2 = 0.5$$

$$n^c = n^{0.5}$$

$$n^{0.51} < n^{0.51}$$

$$f(n) > n^c$$

$$\therefore T(n) = O(n^{0.51})$$

Q.9) $T(n) = 0.5T(n/2) + 1/n$
 $a = 0.5, b = 2$
 $a \geq 1$ but here a is 0.5 ,
 so we cannot apply
 Master's Theorem.

Q.11) $T(n) = 4T(n/2) + \log n$
 $a = 4, b = 2, f(n) = \log n$
 $c = \log_2 4 = 2$
 $n^c = n^2$
 $f(n) < n^c$
 $\therefore T(n) = \Theta(n^2)$

Q.13) $T(n) = 3T(n/2) + n$
 $a = 3, b = 2, f(n) = n$
 $c = \log_2 3 = 1.5849$
 $n^c = n^{1.5849}$
 $f(n) < n^c$
 $\therefore T(n) = \Theta(n^{1.5849})$

Q.15) $T(n) = 4T(n/2) + n$
 $a = 4, b = 2$
 $c = \log_2 4 = 2$
 $n^c = n^2$
 $f(n) < n^c$
 $\therefore T(n) = \Theta(n^2)$

Q.17) $T(n) = 3T(n/3) + n/2$
 $a = 3, b = 3$
 $c = \log_3 3 = 1$
 $f(n) = n/2$
 $n^c = n$
 $f(n) < n^c$
 $\therefore T(n) = \Theta(n)$

Q.10) $T(n) = 16T(n/4) + n!$
 $a = 16, b = 4, f(n) = n!$
 $c = \log_4 16 = 2$
 $n^c = n^2$
 As $n! > n^2$
 $\therefore T(n) = \Theta(n!)$

Q.12) $T(n) = \sqrt{n}T(n/2) + \log n$
 $a = \sqrt{n}, b = 2$
 $c = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$
 $\frac{1}{2} \log_2 n < \log(n)$
 $\therefore f(n) > n^c$
 $\therefore T(n) = \Theta(\log n)$

Q.14) $T(n) = 3T(n/3) + \sqrt{n}$
 $a = 3, b = 3$
 $c = \log_3 3 = 1$
 $n^c = n^1 = n$
 $f(n) < n^c$
 $\therefore T(n) = \Theta(n)$

Q.16) $T(n) = 3T(n/4) + n \log n$
 $a = 3, b = 4, f(n) = n \log n$
 $c = \log_4 3 = 0.792$
 $n^c = n^{0.792}$
 $f(n) \approx n \log n > n^c$
 $\therefore T(n) = \Theta(n \log n)$

Q.18) $T(n) = 16T(n/3) + n^2 \log n$
 $a = 16, b = 3, f(n) = n^2 \log n$
 $c = \log_3 16 = 1.6309$
 $n^c = n^{1.6309}$
 $f(n) > n^c$
 $\therefore T(n) = \Theta(n^2 \log n)$

Q.19) $T(n) = 4T(n/2) + n/\log n$
 $a = 4, b = 2, f(n) = \frac{n}{\log n}$

$$c = \log_b a = 2$$

$$n^c = n^2$$

$$\frac{n}{\log n} < n^2$$

$$\therefore T(n) = O(n^2)$$

Q.21) $T(n) = 7T(n/3) + n^2$
 $a = 7, b = 3, f(n) = n^2$

$$c = \log_3 7 = 1.7712$$

$$n^c = n^{1.7712}$$

$$f(n) > n^c$$

$$\therefore T(n) = O(n^2)$$

Q.20) $T(n) = 64T(n/8) - n^2 \log n$
 $a = 64, b = 8$

$$c = 2$$

$$n^c = n^2$$

$$f(n) > n^c$$

$$\therefore T(n) = O(n^2 \log n)$$

Q.22) $T(n) = T(n/2) + n(2 - \cos n)$
 $a = 1, b = 2, f(n) = n(2 - \cos n)$

$$c = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$f(n) > n^c$$

$$\therefore T(n) = O(n(2 - \cos n))$$