Analog and digital control

4^{th} Group of theoretical Exercises

Νικήτας Μενούνος ΤΛ20412

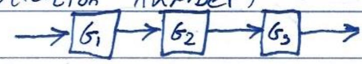
Κοσμάς Παπαζαχαρίας ΤΛ20441

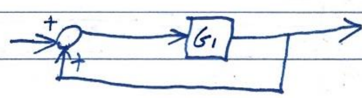
Παναγιώτης Κουζής ΤΛ20411

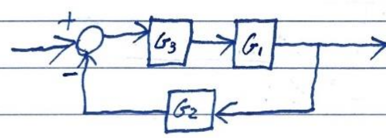
1) Assuming that:

$$G_1(s) = \frac{k+1}{s+1} \qquad G_2(s) = \frac{s+2}{s+3+k} \qquad G_3(s) = \frac{s^2+4s+3+k}{s}$$

simplify the following block diagrams into one single-transfer function. (Asume that k is the last digit of your Student registration number)

a) 

b) 

c) 

a) $k=1$

$$G_{ol} = (G_1 \cdot G_2) \cdot G_3 = \left(\left(\frac{2}{s+1}\right)\left(\frac{s+2}{s+4}\right)\right) \frac{s^2+4s+4}{s} = \frac{2s+4}{(s+1)(s+4)} \cdot \frac{s^2+4s+4}{s} =$$

$$\frac{(2s+4)(s^2+4s+4)}{(s+1)\cdot(s+4)\cdot s} = \frac{2s^3+8s^2+8s+4s^2+16s+16}{(s^2+s)\cdot(s+4)} =$$

$$\frac{2s^3+12s^2+24s+16}{s^3+5s^2+4s}$$

b) $K=1$

$$G_{o\lambda}=\frac{G_1}{1+G_1\cdot 1}=\frac{2/s+1}{1+\left(\frac{2}{s+1}\right)\cdot 1}=\frac{\frac{2}{s+1}}{\frac{s+3}{s+1}}=\frac{2\cancel{(s+1)}}{\cancel{(s+1)}(s+3)}=\frac{2}{s+3}$$

c) $k=1$

$$G_{o\lambda}=\frac{G_1\cdot G_3}{1+(G_1\cdot G_3)\cdot G_2\cdot -1}=\frac{G_1\cdot G_3}{1-(G_1\cdot G_3)\cdot G_2}$$

$$G_A=G_1\cdot G_3=\frac{2}{s+1}\cdot\frac{s^2+4s+4}{s}\Rightarrow\frac{2s^2+8s+8}{s^2+s}$$

Apa $$G_{o\lambda}=\frac{G_A}{1-G_A\cdot G_2}=\frac{\frac{2s^2+8s+8}{s^2+s}}{1-\frac{2s^2+8s+8}{s^2+s}\cdot\frac{s+2}{s+4}}=\frac{\frac{2s^2+8s+8}{s^2+s}}{1-\frac{2s^3+8s^2+8s+4s^2+16s+16}{s^3+4s^2+s^2+4s}}=$$

$$\frac{\frac{2s^2+8s+8}{s^2+s}}{1-\frac{2s^3+12s^2+24s+16}{s^3+4s^2+s^2+4s}}=\frac{\frac{2s^2+8s+8}{s^2+s}}{\frac{(s^3+6s^2+4s)-(2s^3+12s^2+24s+16)}{s^3+6s^2+4s}}=$$

$$\frac{\frac{2s^2+8s+8}{s^2+s}}{\frac{-(s^3+6s^2+20s+16)}{s^3+6s^2+4s}}=\frac{2s^3+16s^2+40s+32}{3s^3+17s^2+28s+16}$$

Ακολουθεί ο κώδικας σε matlab για την επαλήθευση των παραπάνω :

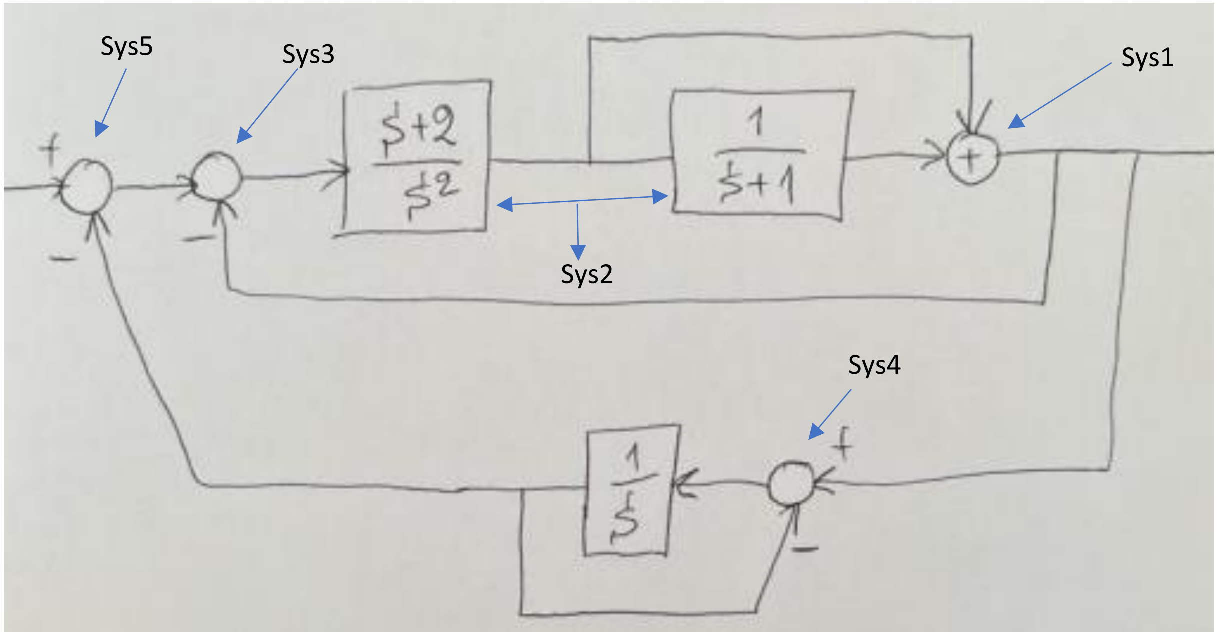
```
k =1 ;

sys1 = tf([k+1],[1 1]);
sys2 = tf([1 2],[1 3+k]);
sys3 = tf([1 4 3+k],[1 0]);

sys4 = series(sys1,sys2);
a = series(sys4,sys3)

b = feedback(sys1,1)

sys5 = series(sys1,sys3);
c = feedback(sys5,sys2,-1)
```

Ο κώδικας παράγει τα εξής αποτελέσματα :

```
a =

  2 s^3 + 12 s^2 + 24 s + 16
  --------------------------
      s^3 + 5 s^2 + 4 s

Continuous-time transfer function.


b =

    2
  -----
  s + 3

Continuous-time transfer function.


c =

  2 s^3 + 16 s^2 + 40 s + 32
  --------------------------
  3 s^3 + 17 s^2 + 28 s + 16

Continuous-time transfer function.
```

Άσκηση 2η

Using Matlab, simplify the following block diagram to one single system-transfer function :



Απλοποιούμε το παραπάνω block diagram σε μια ολική συνάρτηση μεταφοράς (sys5), με τον ακόλουθο κώδικα :

```
sys1 = feedback(tf([1],[1 1]),1)
sys2 = series(tf([1 2],[1 0 0]),sys1)
sys3 = feedback(sys2,1,-1)
sys4 = feedback(tf([1],[1 0]),1,-1)
sys5 = feedback(sys3 , sys4,-1)
```

Παράγοντας τα παρακάτω αποτελέσματα :

```
sys1 =

    1
  -----
  s + 2

Continuous-time transfer function.


sys2 =

     s + 2
  -----------
  s^3 + 2 s^2

Continuous-time transfer function.


sys3 =

         s + 2
  -------------------
  s^3 + 2 s^2 + s + 2

Continuous-time transfer function.


sys4 =

    1
  -----
  s + 1

Continuous-time transfer function.


sys5 =

          s^2 + 3 s + 2
  -----------------------------
  s^4 + 3 s^3 + 3 s^2 + 4 s + 4

Continuous-time transfer function.
```