Analog and Digital Control

3rd Group of Theoretical Exercises

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Άσκηση 1 :

Evaluate (do **NOT** use Matlab) all the characteristic quantities, the characteristic response times and the overshoot of the following systems (make all the necessary calculations on a piece of paper, then verify your results using Matlab):

$$G(s) = \frac{3 \cdot s + k + 1}{s^2 + 2 \cdot s + k + 12}$$

Evaluate all the quantities by using both the mathematical formulas (the system is of order 2) and the graphical approach. Is there an agreement between the results derived by both the mathematical formulas and the graphical approach? When using the graphical approach, sketch the system's unit step response in the time domain (read the next tip).

$$G(s) = \frac{1}{(s + 10) \cdot (s^2 + s + k + 2)}$$

Tip: Note that the system is of order 3 in the second case. Thus, (for the characteristic times and the overshoot) you need to evaluate the system's output (response) to a unit step input (in the time domain) via the inverse Laplace transform (using the partial fraction expansion method). Recall also the definition of the transfer function.

a) $k=2$

$$G(s)=\frac{3s+3}{s^2+2s+14}, \quad p_{1,2}=\frac{-B\pm i\sqrt{52}}{2}=-1\pm 3.6i$$

zeros: $z_1 \Rightarrow 3s+3=0 \Rightarrow 3s=-3 \Rightarrow s=-1=z_1$

system gain: $\frac{3}{1}\cdot\frac{s+1}{s^2+2s+14} \ : \ 3$

Time constants: $\tau_{1,2} \triangleq \frac{1}{|real(p_{1,2})|}=\frac{1}{|-1|}=1$ sec

Dc gain: $K_{ss}=\frac{3}{14}=0.214$

Natural Frequencies: $\omega_{n_{1,2}}=|p_1,p_2|=\sqrt{1^2+3.6^2}=3.736$ rad/sec

Damping ratio: $\zeta=\frac{|real(p)|}{\omega_n}=\cos(B)=\frac{|-1|}{3.736}=0.268$

Damping Frequency: $\omega_d=\omega_n\cdot\sqrt{1-\zeta^2} \Rightarrow \omega_{d_{1,2}}=3.736\cdot\sqrt{1-0.268^2}=3.6$ rad/sec

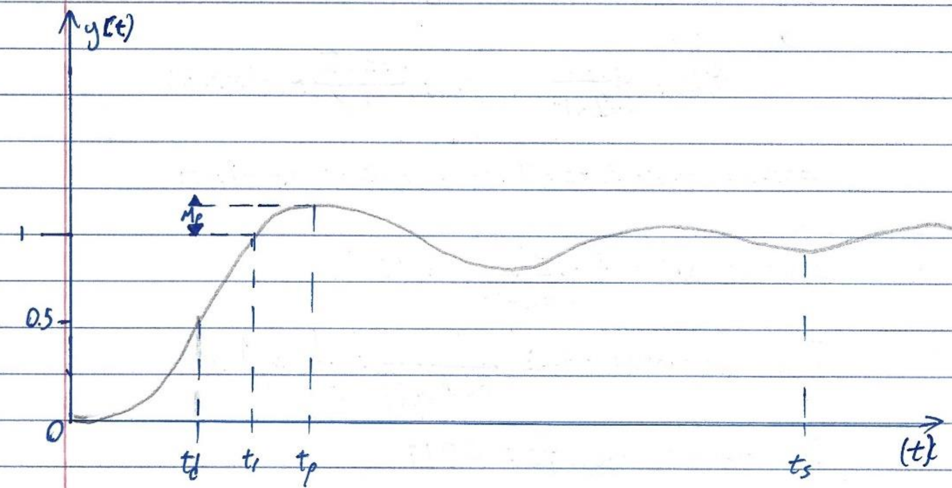
Delay time: $t_d=\frac{1+0.7\cdot\zeta}{\omega_n}=\frac{1+0.7\cdot 0.268}{3.736}=0.318$ sec

Rise time: $t_r=\frac{\pi-\beta}{\omega_d}$, $\beta=acos(\zeta)[rad]=1.3$ rad, Apa $t_r=0.517$ sec

Peak time $t_p=\frac{\pi}{\omega_d}=\frac{\pi}{3.6}=0.8726$

Settling time: $t_s=\frac{4}{\zeta\omega_n}=\frac{4}{0.268\cdot 3.736}=3.995$

Maximum Overshoot: $M_p=e^{-\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\pi}=e^{-\left(\frac{0.268}{0.963}\right)\pi}=0.417$



b)

$k=2$

$$G(s) = \frac{1}{(s+10)(s^2+s+4)} = \frac{1}{s^3+11s^2+14s+40}$$

$p_1 = -0.5+1.93i$

$p_2 = -0.5-1.93i$

$p_3 = -10$

zeros: Δεν υπάρχει

System gain: $\frac{1}{1}\frac{1}{s^3+11s^2+14s+40} \Rightarrow 1$

time constant: $\tau_{1,2} = \frac{1}{|real(p_{1,2})|} \Rightarrow \tau_{1,2} = \frac{1}{0.5} = 2$

$\tau_3 = \frac{1}{|real(p_3)|} = \frac{1}{10} = 0.1$

Dc gain: $K_{ss} = \frac{1}{40} = 0.025$

Natural frequency: $\omega_{n_{1,2,3}} = |p_{1,2}, p_3| \Rightarrow |p_{1,2}| = \sqrt{0.5^2 + 1.93^2} = 2$

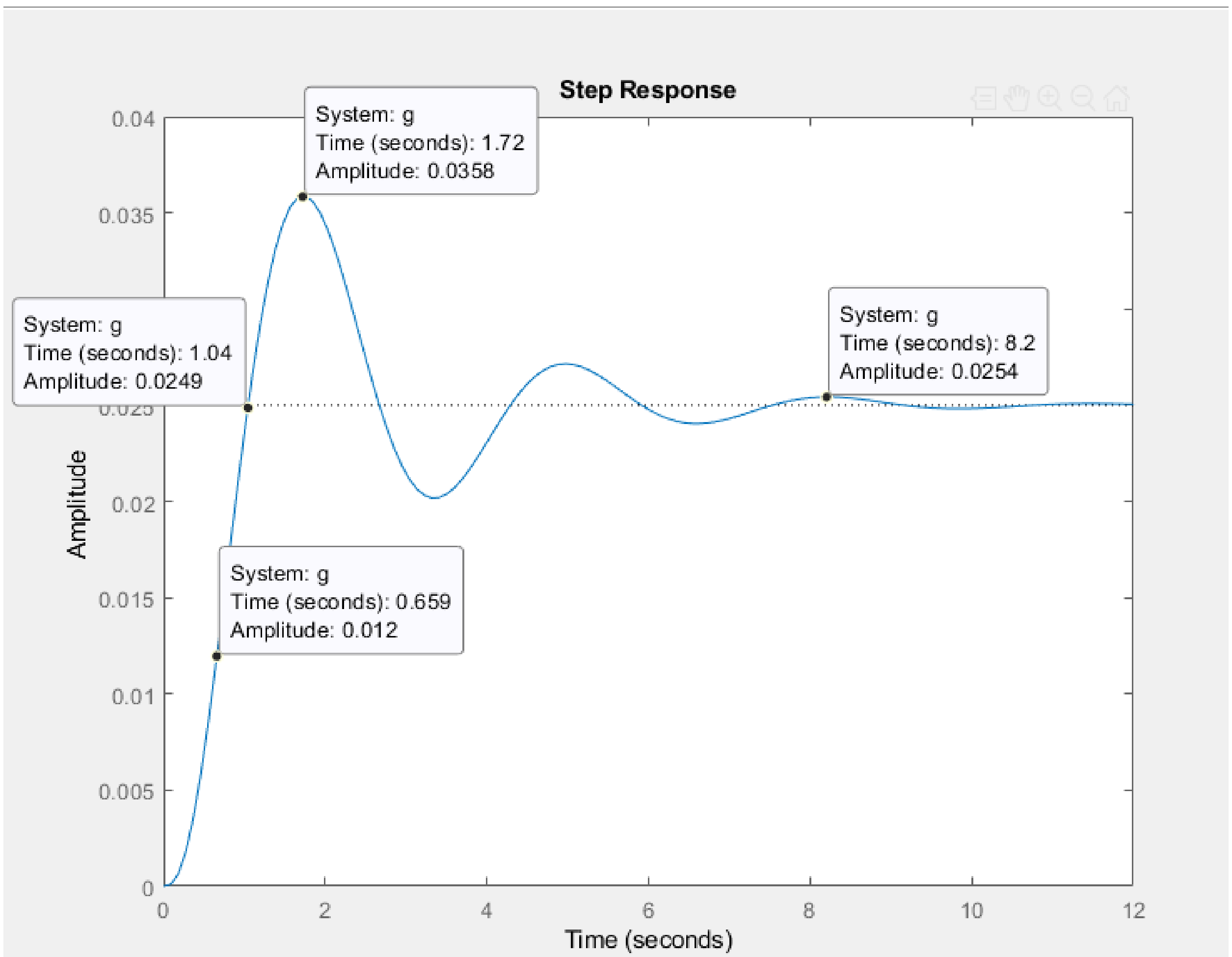
$|p_3| = \sqrt{10^2 + 0^2} = 10$

Damping ratio: $\zeta = \frac{|real(p)|}{\omega_n} = \cos(\beta) \Rightarrow \zeta_{1,2} = \frac{|-0.5|}{2} = 0.25$

$\zeta_3 = \frac{|-10|}{10} = 1$

Damping frequency: $\omega_d = \omega_n \cdot \sqrt{1-\zeta^2} \Rightarrow \omega_{d_{1,2}} = 2 \cdot \sqrt{1-0.25^2} = 1.936$

$\omega_{d_3} = 10 \cdot \sqrt{1-1^2} = 0$



Από την γραφική παράσταση παρατηρούμε ότι maximum overshoot = 0,0358-0,0249 = 0,0109

Άσκηση 2 :

Sketch "by hand" (and **NOT** using Matlab) the FRF (bode diagram) of the following system:

$$G(s) = \frac{1}{s+k+1}$$

Note (1): The Bode diagram is a double plot (magnitude in dB and phase in degrees, both versus the frequency ω. The common horizontal axis (frequency) should be in a logarithmic scale and in rad/secs. [0.01 < ω < 100] rad/sec.

Note (2): Recall that the FRF is evaluated by substituting (in the transfer function) "s" with "jω" and then evaluating the magnitude (in dB) and the angle=argument=phase (in degrees) of the complex number.

$$G(s) = \frac{1}{s+2+1} = \frac{1}{s+3} \Rightarrow G(j\omega) = \frac{1}{j\omega+3}$$

Πλάτος: $A_g = 20\log|G(j\omega)| = 20\log\left|\frac{1}{j\omega+3}\right| =$

$20(\log(1) - \log|j\omega+3|) = -20\log(\sqrt{\omega^2+3^2})$

Φάση: $\varphi = Arg\left(\frac{1}{j\omega+3}\right) = Arg(1) - Arg(j\omega+3) = -\tan^{-1}\left(\frac{\omega}{3}\right)$

Για $\omega = 0.01 \Rightarrow A_g = -20\log(\sqrt{\omega^2+3^2}) = -9.54$

$\angle G = -Arg(j\omega+3) = -0.19$

Για $\omega = 0.1 \Rightarrow A_g = -20\log(\sqrt{\omega^2+3^2}) = -9.5$

$\angle G = -\tan^{-1}\left(\frac{\omega}{3}\right) = 1.9$

Για $\omega = 1 \Rightarrow A_G = -20\log(\sqrt{\omega^2+3^2}) = -10$

$\angle G = -\tan^{-1}\left(\frac{\omega}{3}\right) = -18.4$

Για $w=10 \Rightarrow A_g = -20.3$, $\angle G = -73.3$

Για $w=100 \Rightarrow A_g = -40$, $\angle G = -88.2$