Analog and Digital Control

5th Group of Theoretical Exercises

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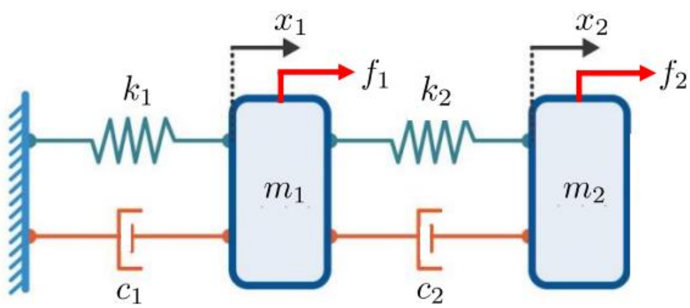
Άσκηση 1 :

Assuming the following mechanical physical system, which moves **only** on the horizontal axis, **without** any friction:



Inputs: $f_1,\ f_2$

Outputs: $x_1,\ x_2,\ f_{k_1}, f_{c_1}\ f_{k_2}, f_{c_2}$

Find the State Space representation of the system.

Tip: Use the Newton's second law to extract the differential equations of motion.

Για μάζα m_1 έχουμε ότι:

Απο $2^{ο}$ νόμο του Newton:

$$\Sigma F = m_1 \cdot a = m_1 \cdot \ddot{x}_1 \Rightarrow$$
$$-k_1 x_1 - c_1 \cdot \dot{x}_1 + k_2 (x_2 - x_1) + c_2 (\dot{x}_2 - \dot{x}_1) + f_1 = m_1 \cdot \ddot{x}_1 \Rightarrow$$
$$m_1 \ddot{x}_1 + \dot{x}_1 (c_1 + c_2) + x_1 (k_1 + k_2) = k_2 x_2 + c_2 \dot{x}_2 + f_1 \quad (1)$$

Για μάζα m_2 έχουμε ότι:

Απο $2^{ο}$ νόμο του Newton:

$$\Sigma F = m_2 \cdot \ddot{x}_2 \Rightarrow$$

$$-k_2 (x_2 - x_1) - c_2 (\dot{x}_2 - \dot{x}_1) + F_2 = m_2 \ddot{x}_2 \Rightarrow$$
$$m_2 \cdot \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = F_2 + c_2 \dot{x}_1 + k_2 x_1 \quad (2)$$

Έχουμε 4 μεταβλητές κατάστασης $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$

Τώρα μπορούμε να εκφράσουμε τις (1) και (2) ως προς z:

(1) $$m_1 \dot{z}_3 + z_3 (c_1 + c_2) + z_1 (k_1 + k_2) = k_2 z_2 + c_2 z_4 + f_1 \Rightarrow$$
$$\dot{z}_3 = \frac{k_2}{m_1} z_2 + \frac{c_2 z_4}{m_1} - \frac{z_1 (k_1 + k_2)}{m_1} - z_3 \frac{(c_1 + c_2)}{m_1} + \frac{1}{m_1} \cdot f_1 \quad (1)$$

(2) $$m_2 \dot{z}_4 + c_2 z_4 + k_2 z_2 = f_2 + c_2 \dot{x}_1 + k_2 x_1 \Rightarrow$$
$$\dot{z}_4 = \frac{c_2}{m_2} z_3 + \frac{k_2}{m_2} z_1 - \frac{c_2}{m_2} z_4 - \frac{k_2}{m_2} z_2 + \frac{1}{m_2} f_2 \quad (2)$$

Αρα:

$$Z=\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{k_1+k_2}{m_2} & \frac{k_2}{m_2} & -\frac{c_1+c_2}{m_2} & \frac{c_2}{m_2}\\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2}\end{bmatrix}\cdot\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{1}{m_1} & 0\\ 0 & \frac{1}{m_2}\end{bmatrix}\cdot\begin{bmatrix}f_1\\ f_2\end{bmatrix}$$

4×1 $\quad 4\times 4$ $\quad 4\times 1$ $\quad 4\times 2$ $\quad 2\times 1$

και:

$$y=\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\\ y_5\\ y_6\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ k_1 & 0 & 0 & 0\\ 0 & 0 & c_1 & 0\\ k_2 & -k_2 & 0 & 0\\ 0 & 0 & -c_2 & c_2\end{bmatrix}\cdot\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}f_1\\ f_2\end{bmatrix}$$

1×6 $\quad 6\times 4$ $\quad 4\times 1$ $\quad 6\times 2$ $\quad 2\times 1$

Άσκηση 2 :

Evaluate the inverse Laplace transform of the following quantity:

$$F(s)=\frac{1}{s\cdot(s+1)^2\cdot(s+2)^2\cdot(s^2+2\,s+5)}$$

$$F(s) = \frac{1}{s\cdot(s+1)^2\cdot(s+2)^2\cdot(s^2+2s+5)}$$

$$p_1 = 0\ ,\ p_{2,3} = -1\ ,\ p_{4,5} = -2\ ,\ p_{5,6} = -1 \pm 2i$$

$$F(s) = \frac{k_1}{s} + \frac{A_1}{(s+1)^1} + \frac{A_2}{(s+1)^2} + \frac{r_1}{(s-p_5)} + \frac{r_2}{(s-p_6)} + \frac{B_1}{(s+2)^1} + \frac{B_2}{(s+2)^2}$$

$$k_1 = s\cdot F(s)\Big|_{s=0} = \frac{1}{1^2+2^2+5} = \frac{1}{10} = 0.1$$

$$A_1 = \frac{1}{1!}\frac{d}{ds}\left[(s+1)^2\cdot F(s)\right]\Big|_{s=-1} = \frac{d}{ds}\left[\frac{1}{s(s+2)^2(s^2+2s+5)}\right]\Big|_{s=-1} =$$

$$-\frac{5s^3+14s^2+23s+10}{(s^4+4s^3+9s^2+10s)^2(s+2)}\Big|_{s=-1} = 1$$

$$A_2 = \frac{1}{0!}\cdot(s+1)^2F(s)\Big|_{s=-1} = \frac{1}{s\cdot(s+2)^2(s^2+2s+5)}\Big|_{s=-1} = -\frac{1}{4} = -0.25$$

$$B_1 = \frac{1}{1!}\frac{d}{ds}\left[(s+2)^2F(s)\right]\Big|_{s=-2} = \frac{d}{ds}\left[\frac{1}{s(s+1)^2(s^2+2s+5)}\right]\Big|_{s=-2} =$$

$$-\frac{3s^3+11s^2+19s+5}{(s^4+3s^3+7s^2+5s)^2(s+1)}\Big|_{s=-2} = 0.29$$

$$B_2 = \frac{1}{0!}(s+2)^2F(s)\Big|_{s=-2} = \frac{1}{s(s+1)^2(s^2+2s+5)}\Big|_{s=-2} = \frac{1}{-10} = -0.1$$

$$c_1 = (s-p_1)\cdot F(s)\Big|_{s=p_1} = \frac{1}{s\cdot(s+1)^2(s+2)^2}\Big|_{s=p_1} = \frac{1}{10+20i} = 0.02-0.04i$$

Apa:

$$f(t) = 0.1+e^{-t}+(-0.25)te^{-t}+0.29e^{-2t}+(-0.1)te^{-2t}+2\cdot 0.02e^{-t}\cos(2t)-$$
$$(2\cdot 0.04)e^{-t}\sin(2t)$$