Date........................

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(1) $f(x_1, x_2, x_3) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$

inputs:- $-1 \rightarrow$ False
$1 \rightarrow$ True

$u_i = sgn(x) = 1$, if $x > 0$
$sgn(x) = -1$, if $x < 0$
$sgn(0) = 0$

					f_1	f_2	$f = f_1 + f_2$
X_1	X_2	X_3	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_1 X_2 X_3$	$X_1 \bar{X}_2$	$\bar{X}_1 X_2 X_3 + X_1 \bar{X}_2$
1	1	1	-1	-1	-1	-1	-1
1	1	-1	-1	-1	-1	-1	-1
1	-1	1	-1	1	-1	+1	1
-1	1	1	1	-1	1	-1	1
-1	-1	1	1	1	-1	-1	-1
1	-1	-1	-1	1	-1	1	1
-1	1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	1	-1	-1	-1

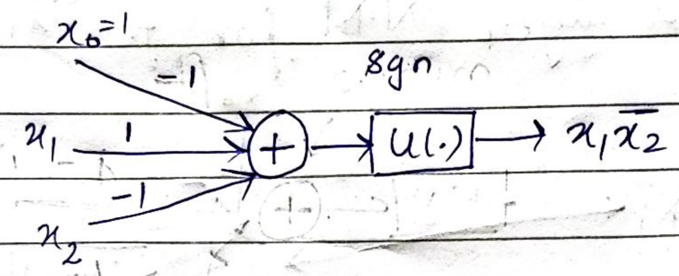
For $f = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 = f_1 + f_2$, Let the weights be, w_1'' & w_2''.
w_0'' is the bias

$\therefore u(w_0'' - w_1'' - w_2'') = -1 \Rightarrow w_0'' - w_1'' - w_2'' < 0$
$u(w_0'' - w_1'' + w_2'') = 1 \Rightarrow w_0'' - w_1'' + w_2'' > 0$
$u(w_0'' + w_1'' - w_2'') = 1 \Rightarrow w_0'' + w_1'' - w_2'' > 0$

by solving the above equations, we get $w_0'' = 1$, $w_1'' = 1$ & $w_2'' = 1$ for the final output layer.

for $f_2 = x_1\overline{x_2}$, Let w_0', w_1' & w_2' be the bias and weights.

$u(w_0' + w_1' + w_2') = -1$
$u(w_0' + w_1' - w_2') = +1$
$u(w_0' - w_1' - w_2') = -1$

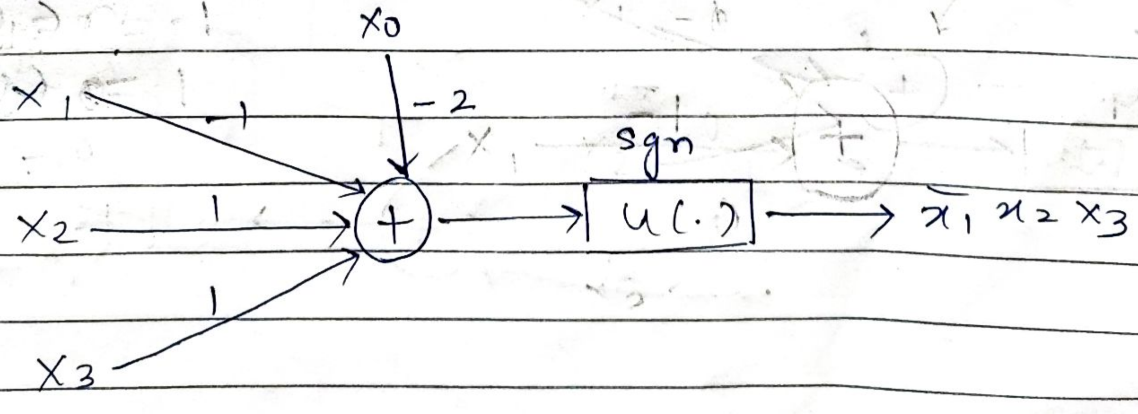


Solving the above equations gives us the values of $w_0' = -1$, $w_1' = 1$, $w_2' = -1$

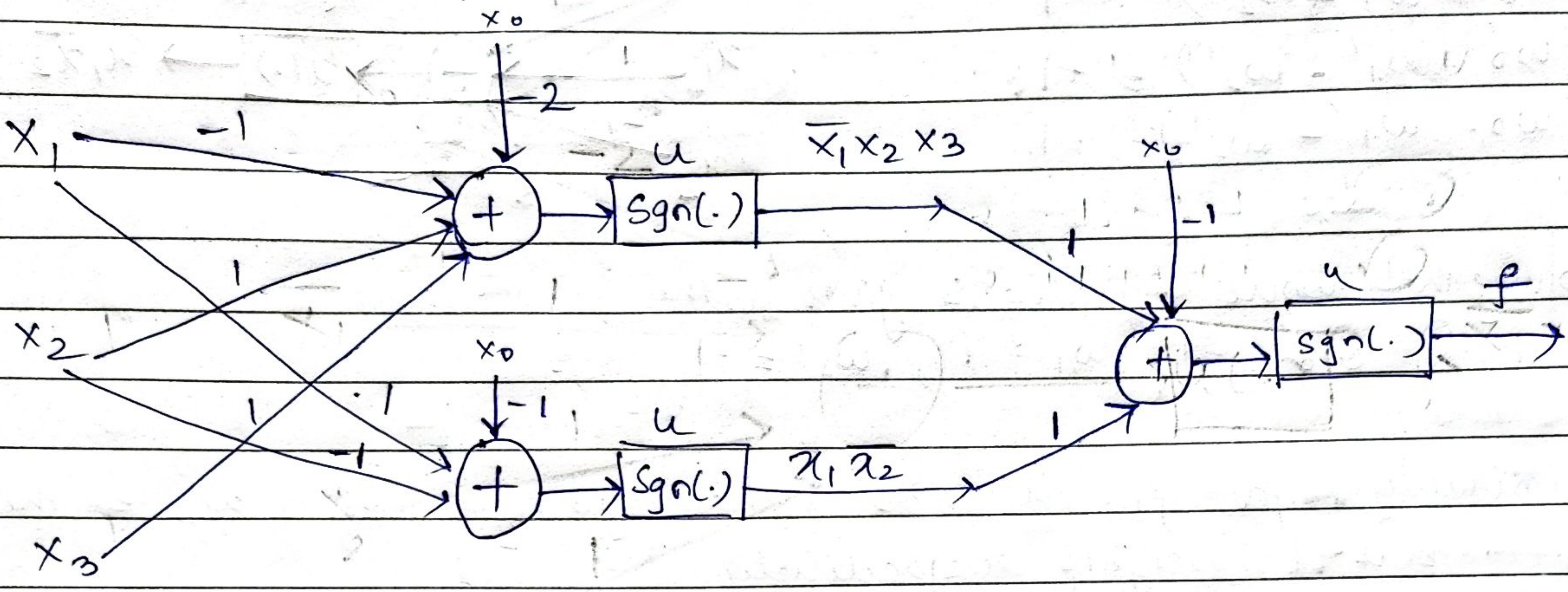
Similarly, for $f_1 = \overline{x_1}x_2x_3$, Let w_0, w_1, w_2 & w_3 be the ~~weio~~ bias & weights respectively

$u(w_0 + w_1 + w_2 + w_3) = -1 \Rightarrow w_0 + w_1 + w_2 + w_3 < 0$
$u(w_0 + w_1 + w_2 - w_3) = -1 \Rightarrow w_0 + w_1 + w_2 - w_3 < 0$
$u(w_0 + w_1 - w_2 + w_3) = -1 \Rightarrow w_0 + w_1 - w_2 + w_3 < 0$
$u(w_0 - w_1 + w_2 + w_3) = 1 \Rightarrow w_0 - w_1 + w_2 + 3w > 0$
$u(w_0 - w_1 - w_2 + w_3) = -1 \Rightarrow w_0 - w_1 - w_2 + w_3 < 0$
$u(w_0 + w_1 - w_2 - w_3) = -1 \Rightarrow w_0 + w_1 - w_2 - w_3 < 0$
$u(w_0 - w_1 + w_2 - w_3) = -1 \Rightarrow w_0 - w_1 + w_2 - w_3 < 0$
$u(w_0 - w_1 - w_2 - w_3) = -1 \Rightarrow w_0 - w_1 - w_2 - w_3 < 0$

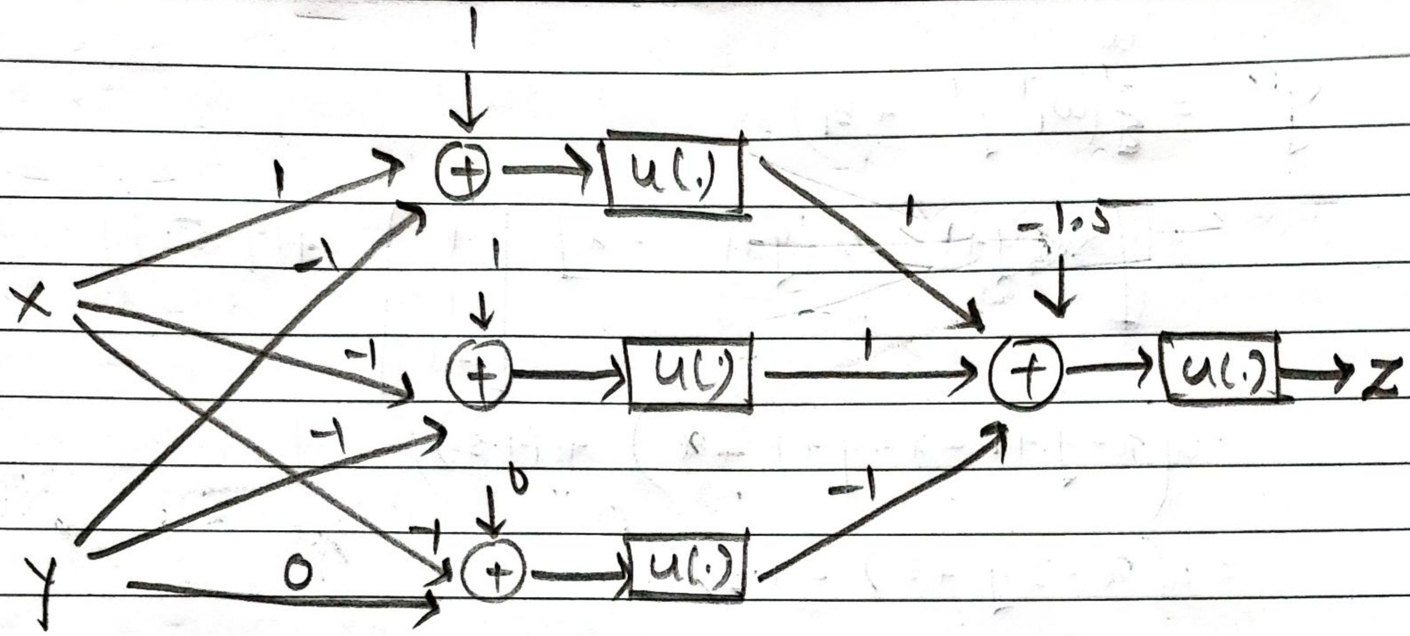
Solving the above inequalities gives us the weights to be $w_0 = -2$, $w_1 = -1$, $w_2 = 1$, $w_3 = 1$



2 layer NN for $f = \overline{x_1} x_2 x_3 + x_1 \overline{x_2}$



Q2



Activation $f^n \to u$

$$u(x) = 1 \quad, \text{if } x \geq 0$$
$$u(x) = 0 \quad, \text{if } x < 0$$

$x = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad w_0 = \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ $\quad b_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad w_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $\quad b_1 = -1.5$

$$y' = u\left(w_0^T x + b_0\right)$$

$$= u\left(\begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right)$$

$$= u\left(\begin{bmatrix} 1 & -1 \\ -1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right)$$

$$= u\left(\begin{bmatrix} x-y \\ -x-y \\ -x \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x-y+1 \\ -x-y+1 \\ -x \end{bmatrix}\right)$$

$$y'' = u\left(w_1^T y' + b_1\right)$$

$$= u\left(\begin{bmatrix}1\\1\\-1\end{bmatrix}^T u\left(\begin{bmatrix}x-y+1\\-x-y+1\\-x\end{bmatrix}\right) - 1.5\right)$$

$$= u\left(\begin{bmatrix}1 & 1 & -1\end{bmatrix} \cdot u\left(\begin{bmatrix}x-y+1\\-x-y+1\\-x\end{bmatrix}\right) - 1.5\right)$$

$$= u(x-y+1) + u(-x-y+1) - u(-x) - 1.5 \geq 0$$