

# CS & IT ENGINEERING

Database Management System

FD's & Normalization

**DPP – 04**

DISCUSSION NOTES



**By- Vijay Agarwal sir**



# TOPICS TO BE COVERED

01 Question

02 Discussion



Q.1



Consider the following two sets of functional dependencies

[MCQ]

$X = \{P \rightarrow Q, Q \rightarrow R, R \rightarrow P, P \rightarrow R, R \rightarrow Q, Q \rightarrow P\}$

$Y = \{P \rightarrow Q, Q \rightarrow R, R \rightarrow P\}$

Which of the following is true?

A.  $X \subset Y$

B.  $Y \subset X$

☒ C.  $X \equiv Y$

D.  $X \neq Y$

X Cover Y

$\checkmark P \rightarrow Q$   $[P]^+ = [PQR\dots]$   
 $\checkmark Q \rightarrow R$   $[Q]^+ = [QRP\dots]$   
 $\checkmark R \rightarrow P$   $[R]^+ = [RPQ\dots]$

True

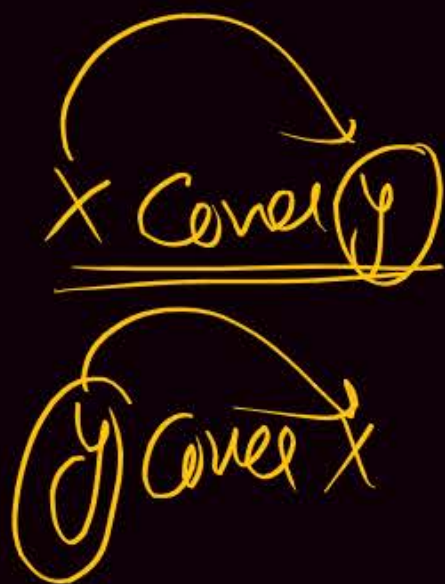
Y Cover X

$\checkmark P \rightarrow Q$   $[P]^+ = [PQR]$   
 $\checkmark Q \rightarrow R$   $[Q]^+ = [QRP]$   
 $\checkmark R \rightarrow P$   $[R]^+ = [RPQ]$   
 $\checkmark P \rightarrow R$   
 $\checkmark R \rightarrow Q$   
 $\checkmark Q \rightarrow P$   
True

Ans [C]

$F \equiv G$

<u>X Cover y</u>	True	False	True	False
y Cover x	<u>False</u>	<u>True</u>	<u>True</u>	<u>False</u>
	$x > y$	$y > x$	$x \equiv y$	$x \neq y$





Q.2

Consider a relation with schema  $R(P, Q, R, S, T)$  and FD set  $(PQ \rightarrow R, R \rightarrow S, S \rightarrow P)$ . How many super keys in relation R contains? 7



$$P.A = [\check{T}, \check{Q}, \check{P}, \check{S}, \check{R}]$$

[NAT]

$R(P, Q, R, S, T) [PQ \rightarrow R, R \rightarrow S, S \rightarrow P]$

$$[PQ]^+ = [PQ, RS]$$

$$[PQT]^+ = [PQ, RST]$$

$PQT$  is c.k. — (1)

$$\begin{array}{c} R \rightarrow S \\ [RQT]^+ = [PQ, RST] \end{array}$$

$RQT$  is c.k. — (2)

X Attribute  $\rightarrow$  [Prime Attribute]

$$\begin{array}{c} S \rightarrow P \\ [SAT]^+ = [SAT, PR] \end{array}$$

$SAT$  is c.k. — (2)

$PQT$   
 $QST$   
 $QTR$

$R(P, Q, R, S, T)$

$$PQT \Rightarrow PQT, PQTR, PQTS, PQRST$$

$R(P, Q, R, S, T)$

$$QST \Rightarrow QST, QSTP, QSTR, PQRST$$

$R(P, Q, R, S, T)$

$$QTR \Rightarrow QTR, QTRP, QTRS, PQRST$$

$PQT, QST, QTR, PQTR, PQTS, QSTR, PQRST$



PQT, QST, QTR

~~R(PQST)~~

$$\rightarrow \underline{PQT} \underline{RS} \Rightarrow \underline{2^2} = \underline{4}$$

$$\underline{QST} (R) \Rightarrow 2^1 = \underline{2}$$

$$\underline{QTR} = QTR = 1$$

7 Ans

Example

R(ABCDE)

C.K A, C, DE

$$\underline{A} (\underline{BCDE}) \Rightarrow 2^4 = 16$$

$$\underline{C} (\underline{BDE}) \Rightarrow 2^3 = 8$$

$$\underline{DE} (\underline{B}) \Rightarrow 2^1 = 2$$

26



Q.3



Consider a relation  $R(P, Q, R, S, T)$  with the set of functional dependencies  $\{P \rightarrow QR, RS \rightarrow T, Q \rightarrow S, \text{ and } T \rightarrow P\}$ . How many super keys are possible in  $R$ ? 27.  $PA = [\checkmark P, \checkmark T, \checkmark R, S, Q]$  [NAT]

$R(PQRST)$   $[P \rightarrow QR, RS \rightarrow T, \underline{Q \rightarrow S}, T \rightarrow P]$

$(P)^+ = [PQRST]$

$P$  is Candidate key

$T \rightarrow P$

$(T)^+ = [PQRST]$

$T$  is candidate key

$RS \rightarrow T$

$(RS)^+ = [PQRST]$

$\rightarrow (R)^+ = [R]$

$(S)^+ = [S]$

$RS$  is C.K. — (3)

$Q \rightarrow S$

$(QR)^+ = [PQRST]$

$(Q)^+ = [QS]$

$QR$  is C.K. — (4)

Uck

P
T
<u>RS</u>
RQ

$n$ : # Attributes

$$\text{Total \# Super key} = 2^n - 1 \Rightarrow 2^5 - 1 = 31 \text{ Super key}$$

Out 31 Super key

$R, S, Q, QS$  Not C.K.

$$31 - 4 = 27 \text{ Super key}$$



R(PQRST)

$\begin{matrix} \text{ck} \\ \text{P} \\ \text{T} \\ \text{RS} \\ \text{RQ} \end{matrix} \quad \begin{matrix} \cancel{\text{P}}\text{QRST} \\ \cancel{\text{T}}\text{QRS} \\ \text{RSQ} \\ \text{RQ} \end{matrix} \quad \begin{matrix} \Rightarrow \\ = \\ = \\ = \end{matrix} \quad \begin{matrix} 2^4 \\ 2^3 \\ 2^1 \\ 1 \end{matrix}$

$\left. \begin{matrix} \text{P, PQ, QR, RS, PT, PQR, PQS, PQT,} \\ \text{PRS, PRT, \dots, \underline{PQRST}} \end{matrix} \right\} (16)$

$16^1$   
 $8 \Rightarrow \text{T, TQ, TR, TS, TQR, TRS, TQS, \underline{TRS}} \Bigg\} 8$   
 $2 \rightarrow \text{RS, RSQ} \Bigg\} (2) \xrightarrow{\text{TRS}}$   
 $1 \quad \text{RQ} \Bigg\} (1)$

27 Super key Avg



Q.4

Consider the relation schema  $R(\underline{P}, \underline{Q}, \underline{R}, \underline{S}, \underline{T}, \underline{U}, \underline{V}, \underline{W}, \underline{X}, \underline{Y})$  and the set of functional dependencies on R are:



[MCQ]

$F = \{PQ \rightarrow R, Q \rightarrow TU, PS \rightarrow VW, V \rightarrow X, W \rightarrow Y\}$ . Which of the following can be the candidate key for R?

A.

PQT

$F: [PQ \rightarrow R, Q \rightarrow TU, PS \rightarrow VW, V \rightarrow X, W \rightarrow Y]$

B.

PQS

$[PQ]^+ = [PQRTU]$

C.

PQSR

$[PQS]^+ = [PQRSTUUVWX Y]$

$[PQT]^+ = [PQRTU]$

D.

PQSVW

PQS is C.K

~(D)

Ans(B)

Q.5

Let a relation R have attributes {P, Q, R, S, T} and "PQR" is the candidate key, then how many super keys are possible 4?



[NAT]

$R(PQRST) \Rightarrow R(\underline{PQR} ST) \Rightarrow 2^2 = 4 \text{ super key}$

PQR, PQRS, PQR T, PQRST

Ans (4)



Q.6

[MCQ]



Consider the following FD sets:

$$S_1 = \{P \rightarrow R, PR \rightarrow S, T \rightarrow PS, T \rightarrow U\}$$

$$S_2 = \{P \rightarrow S, QR \rightarrow PS, R \rightarrow Q, T \rightarrow P, T \rightarrow S, T \rightarrow U\}$$

$$S_3 = \{P \rightarrow S, R \rightarrow P, R \rightarrow Q, T \rightarrow PU\}$$

Which of the following sets is equivalent?

~~A.~~

$$S_1 \equiv S_2$$

$S_1$  Cover  $S_2$

$\checkmark P \rightarrow S$   $(P)^+ = [PRS]$

$\times QR \rightarrow PS$   $(QR)^+ = [QR]$

$R \rightarrow Q$

$T \rightarrow P$

$T \rightarrow S$

$T \rightarrow U$

False

$S_1 \neq S_2$

Not Check  
 $S_2$  Cover  $S_1$

B.

$$S_2 \equiv S_3$$

$S_1$  Cover  $S_3$

$\checkmark P \rightarrow S$   $(P)^+ = [PRS]$

$\times R \rightarrow P$   $(R)^+ = [R]$

$R \rightarrow Q$

$T \rightarrow PU$

False

$S_1 \neq S_3$

~~C.~~

$$S_1 \equiv S_3$$

~~D.~~

$$S_1 \equiv S_2 \equiv S_3$$

Ans (B)



Q.6

[MCQ]



Consider the following FD sets:

$$S_1 = \{P \rightarrow R, PR \rightarrow S, T \rightarrow PS, T \rightarrow U\}$$

$$S_2 = \{P \rightarrow S, QR \rightarrow PS, R \rightarrow Q, T \rightarrow P, \underline{T \rightarrow S}, \underline{T \rightarrow U}\}$$

$$S_3 = \{\underline{P \rightarrow S}, R \rightarrow P, R \rightarrow Q, \underline{T \rightarrow PU}\}$$

Which of the following sets is equivalent?

S<sub>3</sub> Cover S<sub>2</sub>

A.

$$S_1 \equiv S_2$$

S<sub>2</sub> Cover S<sub>3</sub>

✓  $P \rightarrow S$   $(P)^+ = [PS]$

✓  $R \rightarrow P$   $(R)^+ = [RQPS]$

✓  $R \rightarrow Q$   
✓  $T \rightarrow PU$   $(T)^+ = [TPSU]$

B.

$$S_2 \equiv S_3$$

C.

$$S_1 \equiv S_3$$

D.

$$S_1 \equiv S_2 \equiv S_3$$

True

$$S_2 \equiv S_3$$

✓  $P \rightarrow S$

✓  $QR \rightarrow PS$

✓  $R \rightarrow Q$

✓  $T \rightarrow P$

✓  $T \rightarrow S$

✓  $T \rightarrow U$

True

$$(P)^+ = [PS]$$

$$(QR)^+ = [QRPS]$$

$$(R)^+ = [RQPS]$$

$$(T)^+ = [TPUS]$$



# Q.7



Consider a relation  $R = \{P, Q, R, S, T, U, V, W\}$  with the functional dependency sets  $S = \{PR \rightarrow V, S \rightarrow TV, QR \rightarrow S, RV \rightarrow QS, PRS \rightarrow Q, RT \rightarrow PV\}$

The minimum numbers of single functional dependency in the minimal cover of  $F$  is 6?

[NAT]

Step 1:  $\checkmark PR \rightarrow V$ ,  $S \rightarrow T$ ,  $S \rightarrow V$ ,  $\checkmark QR \rightarrow \underline{S}$ ,  $\checkmark RV \rightarrow Q$ ,  $\checkmark RV \rightarrow S$ ,  ~~$PRS \rightarrow Q$~~ ,  $\checkmark RT \rightarrow P$ ,  $\checkmark RT \rightarrow V$

Step 2: L.H.S Extra  
 $\checkmark \underline{PR \rightarrow V}$

$(P)^+ = [P]$   $(R)^+ = [R]$

$\checkmark \underline{QR \rightarrow S}$   $(Q)^+ = [Q]$   $(R)^+ = [R]$

$\checkmark \underline{RV \rightarrow Q}$   $(R)^+ = [R]$   $(V)^+ = [V]$

~~$PRS \rightarrow Q$~~   
 $PR \rightarrow Q$

$\checkmark RT \rightarrow P$

$[PR]^+ = [PRVQ\underline{S}]$   $S$  is extraneous

$(R)^+ = [R]$   
 $(T)^+ = [T]$

$S$  is extra  
 $PR \rightarrow Q$



Q.7



Consider a relation  $R = \{P, Q, R, S, T, U, V, W\}$  with the functional dependency sets  $S = \{PR \rightarrow V, S \rightarrow TV, QR \rightarrow S, RV \rightarrow QS, PRS \rightarrow Q, RT \rightarrow PV\}$

The minimum numbers of single functional dependency in the minimal cover of  $F$  is 6

Redundant FD

Step 3

①  ~~$PR \rightarrow V$~~  ②  $S \rightarrow T, S \rightarrow V$  ③  $QR \rightarrow S$  ④  $RV \rightarrow Q, RV \rightarrow S$  ⑤  $PR \rightarrow Q$  ⑥  $RT \rightarrow P, RT \rightarrow V$  ⑦  $PR \rightarrow Q$  ⑧  $RT \rightarrow P, RT \rightarrow V$  ⑨  ~~$RT \rightarrow V$~~

$$\textcircled{1} [PR]^+ = [PRQSVT]$$

$$\textcircled{5} [RV]^+ = [RVSTPQ]$$

$$\textcircled{8} [RT]^+ = [RTVS]$$

$$\textcircled{2} [S]^+ = [SV]$$

$$\textcircled{3} [S]^+ = [ST]$$

$$\textcircled{4} [QR]^+ = [QR]$$

$$\textcircled{6} [RV]^+ = [RV]$$

$$\textcircled{7} [PR]^+ = [PR]$$

$$\textcircled{9} [RT]^+ = [RTPQSV]$$

Ans (6)



## Minimal Cover

Step 1: Split the FD such that R.H.s contain single Attribute

Step 2: We find Redundant (Extra) Attribute on L.H.s

AB  $\rightarrow$  C

A is extra if  $(B)^+ = ( \dots \underline{A} )$

B is extra if  $(A)^+ = ( \dots \underline{B} )$

Step 3: Find Redundant FD (Extra FD)

F:  $(A \rightarrow B, \underline{B \rightarrow C}, \underline{A \rightarrow C})$

A  $\rightarrow$  C is R.F.D

$\{A \rightarrow B, B \rightarrow C\}$

$(A)^+ = (AB C) =$



# Q.8



Consider a relation  $R(P, Q, R, S, T)$  with the following functional dependencies:  $PQR \rightarrow ST$  and  $S \rightarrow PQ$ , then the number of super keys in  $R$  is 10?

P.A. [R, P, Q, S], [NAT]

$R(P, Q, R, S, T)$  {  $PQR \rightarrow ST$ ,  $S \rightarrow PQ$  }

Sol<sup>n</sup>  $(PQR)^+ = \{P, Q, R, S, T\}$

$PQ, QR, PR$  ✗

$S \rightarrow PQ$   
 $(SR)^+ = \{S, R, P, Q, T\}$

Total # Super key  
Ans (10)

$PQR$  is Candidate key ①  $SR$  is c.k ②

2ck  $PQR$ ,  $SR$

$2^{5-3} + 2^{5-2} - 2^{5-4}$

$2^2 + 2^3 - 2^1$

$4 + 8 - 2 =$

10 Ans

