1.

$$\frac{\partial \log p(x_{t+1}|x_{1:t})}{\partial \alpha} = \frac{1}{p(x_{t+1}|x_{1:t})} \sum_{r_t} p(x_{t+1}|r_t, x_{t-r_t:t}) p(r_t|x_{1:t}) \frac{\partial \log p(r_t|x_{1:t})}{\partial \alpha}$$

2.

$$\frac{\partial \log p(r_t|x_{1:t})}{\partial \alpha} = \frac{1}{p(r_t, x_{1:t})} - \frac{\partial \log p(x_{1:t})}{\partial \alpha}$$

where

$$\frac{\partial \log p(x_{1:t})}{\partial \alpha} = \frac{1}{p(r_t, x_{1:t})} \sum_{r_t} p(r_t, x_{1:t}) \frac{\partial \log p(r_t, x_{1:t})}{\partial \alpha}$$

3.

$$\frac{\partial \log p(r_t = r_{t-1}+1, x_{1:t})}{\partial \alpha} = \frac{\partial \log p(r_t = r_{t-1}+1|r_{t-1}, \alpha)}{\partial \alpha} + \frac{\partial \log p(r_{t-1}, x_{1:t-1})}{\partial \alpha}$$

$$\frac{\partial \log p(r_t = 0, x_{1:t})}{\partial \alpha} = A/B$$

where

$$A = \sum_{r_{t-1}} p(r_t = 0|r_{t-1}, \alpha) p_{prior}(x_t) p(r_{t-1}, x_{1:t-1}) \left[\frac{\partial \log p(r_t = 0|r_{t-1}, \alpha)}{\partial \alpha} + \frac{\partial \log p(r_{t-1}, x_{1:t-1})}{\partial \alpha} \right]$$

and

$$B = \sum_{r_{t-1}} p(r_t = 0|r_{t-1}, \alpha) p_{prior}(x_t) p(r_{t-1}, x_{1:t-1})$$