A hybrid VND method for the split delivery vehicle routing problem

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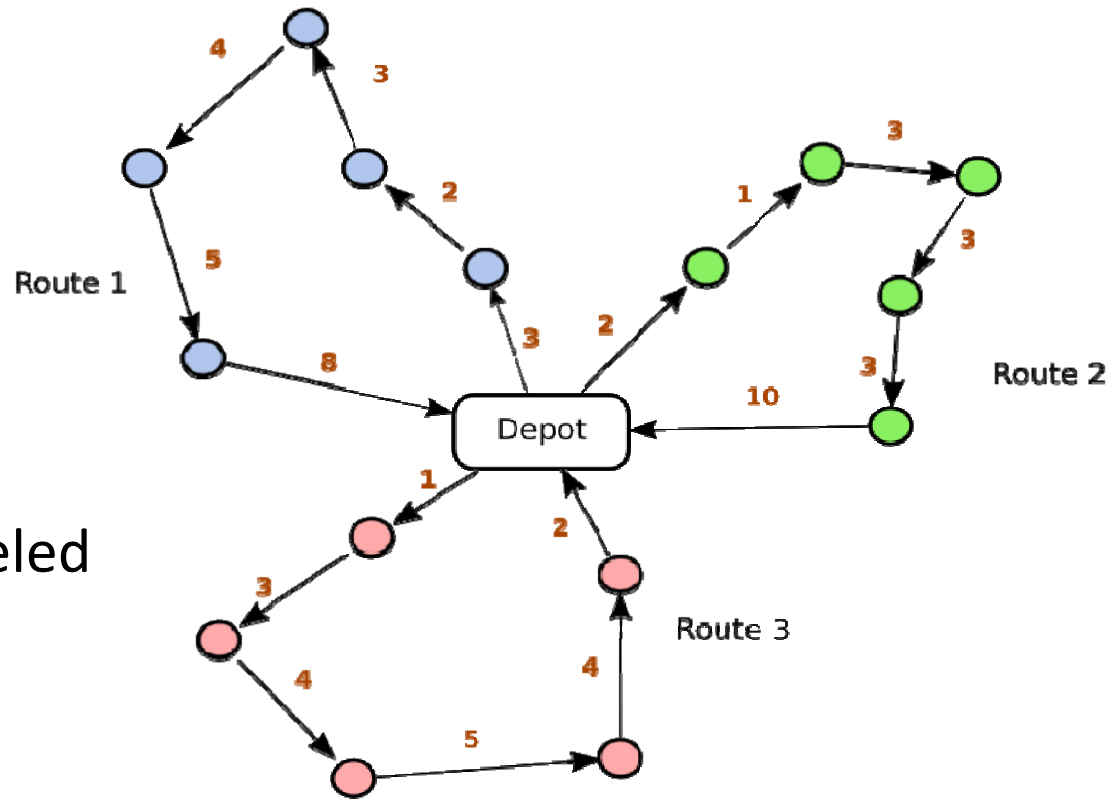
Overview

- SDVRP formulation
- Main idea of algorithm
- Coding solutions
- Variable neighborhood descent
- Tabu Search procedure
- Algorithm overview
- Computational results

SDVRP Formulation

- Vehicles with capacity
- customers with demand



- ***Objective:***
– minimize the total distance traveled
- ***Constraints:***
– serve all customers
– do not exceed the vehicle capacity
– customers can be serviced by more than one vehicle

Bounded Formulation (Frizzell & Giffin 1992)

d_{ij} cost of traveling between customer i and customer j

w_i demand of customer i

Q capacity of vehicle

Variables:

x_{ijk} 1 if the vehicle k travels directly from customer i to customer j,

0 otherwise

f_{ik} the fraction of demand of customer i delivered by vehicle k

y_{ik} surplus variables for subtour elimination

$$\min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{v} d_{ij} x_{ijk}$$

$$\sum_{k=1}^{v} \sum_{i=0}^{n} x_{ijk} \geq 1;$$

$$\sum_{i=0}^{n} x_{ijk} = \sum_{j=0}^{n} x_{ijk};$$

$$\sum_{k=1}^{v} f_{ik} = 1; \quad \sum_{i=1}^{n} w_i f_{ik} \leq Q; \quad f_{ik} \leq \sum_{j=0}^{n} x_{jik};$$

$$y_{ik} - y_{jk} + (n+1) x_{ijk} \leq n.$$

Main idea

Let $\pi: \{1, \dots, n\} \to \{1, \dots, n\}$ is a permutation of customers.

R^{π} is the set of all possible routes, obtained from the permutation π.

For each $r \in R^{\pi}$ vehicle can travel from customer i to customer j only if $y_i < y_j$

Theorem 1

Let the distance matrix (d_{ij}) satisfies the triangle inequality and let $R^* = (r^1, \dots, r^F)$ is the set of routes in optimal solution. Then there is a permutation π such that $r \in R^{\pi}$ for each $r \in R^*$

Theorem 2

If (d_{ij}) satisfies the triangle inequality, there always exists an optimal SDVRP solution such that $y_{ik_1} = y_{ik_2}$ for all i, k_1, k_2.

Theorem 3

SDVRP is NP-hard for given permutation π.

Algorithmic idea

1. Apply local search for permutations
2. For each permutation we apply heuristics to obtain set of routes

Encoding solutions

Permutation U without split demands

List of customer visits:	5 3 6 2 1 4
Amounts delivered:	9 5 7 4 8 6

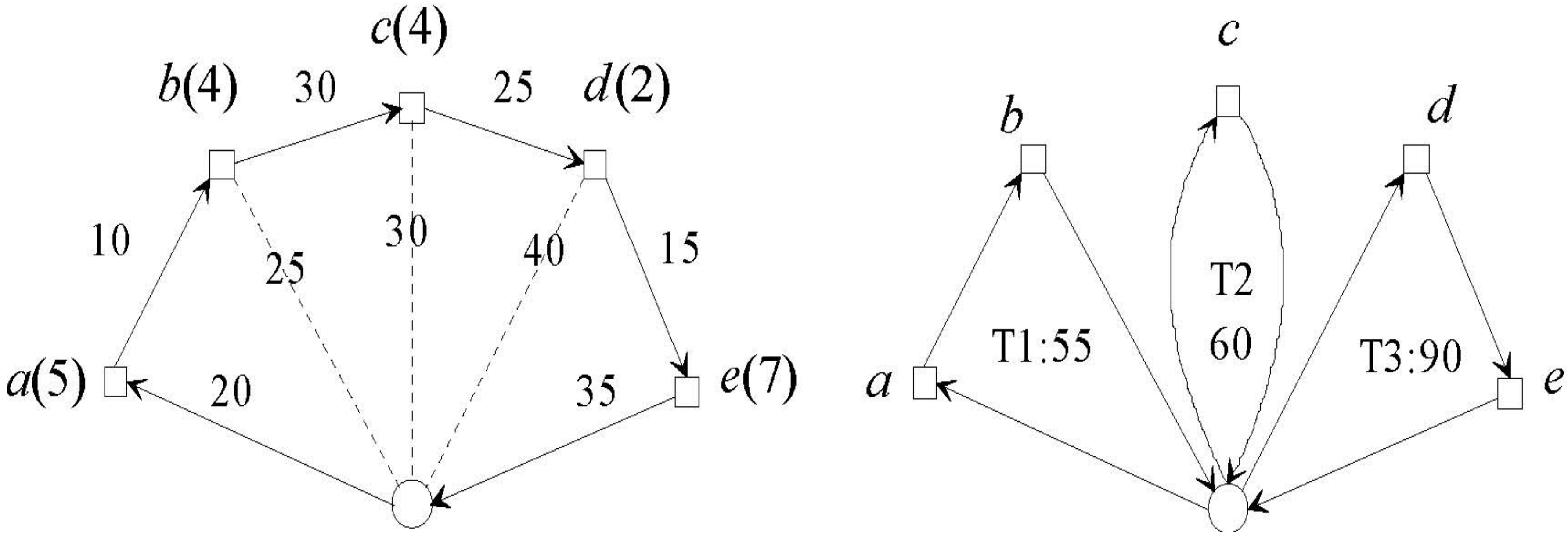
Permutation V with 3 split demands

List of customer visits:	5 3 3 6 6 2 1 1 4
Amounts delivered:	9 1 4 6 1 4 5 3 6

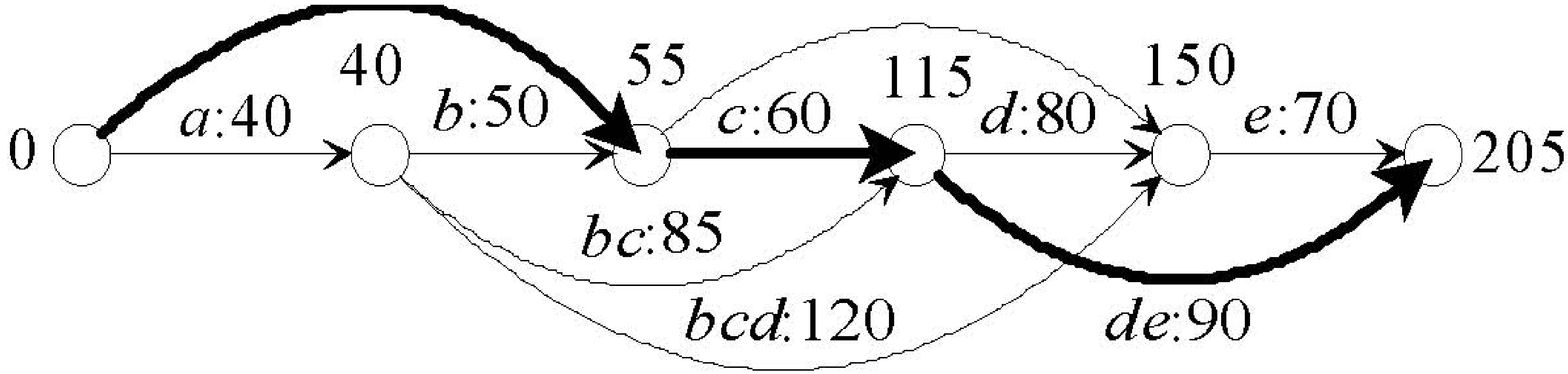
Permutation E with dummy customers

List of customer visits:	5 3 3 6 -- 6 2 1 -- 1 4
Amounts delivered:	9 1 4 6 5 1 4 5 7 3 6

Decoding procedures: Exact (Prins 2004)



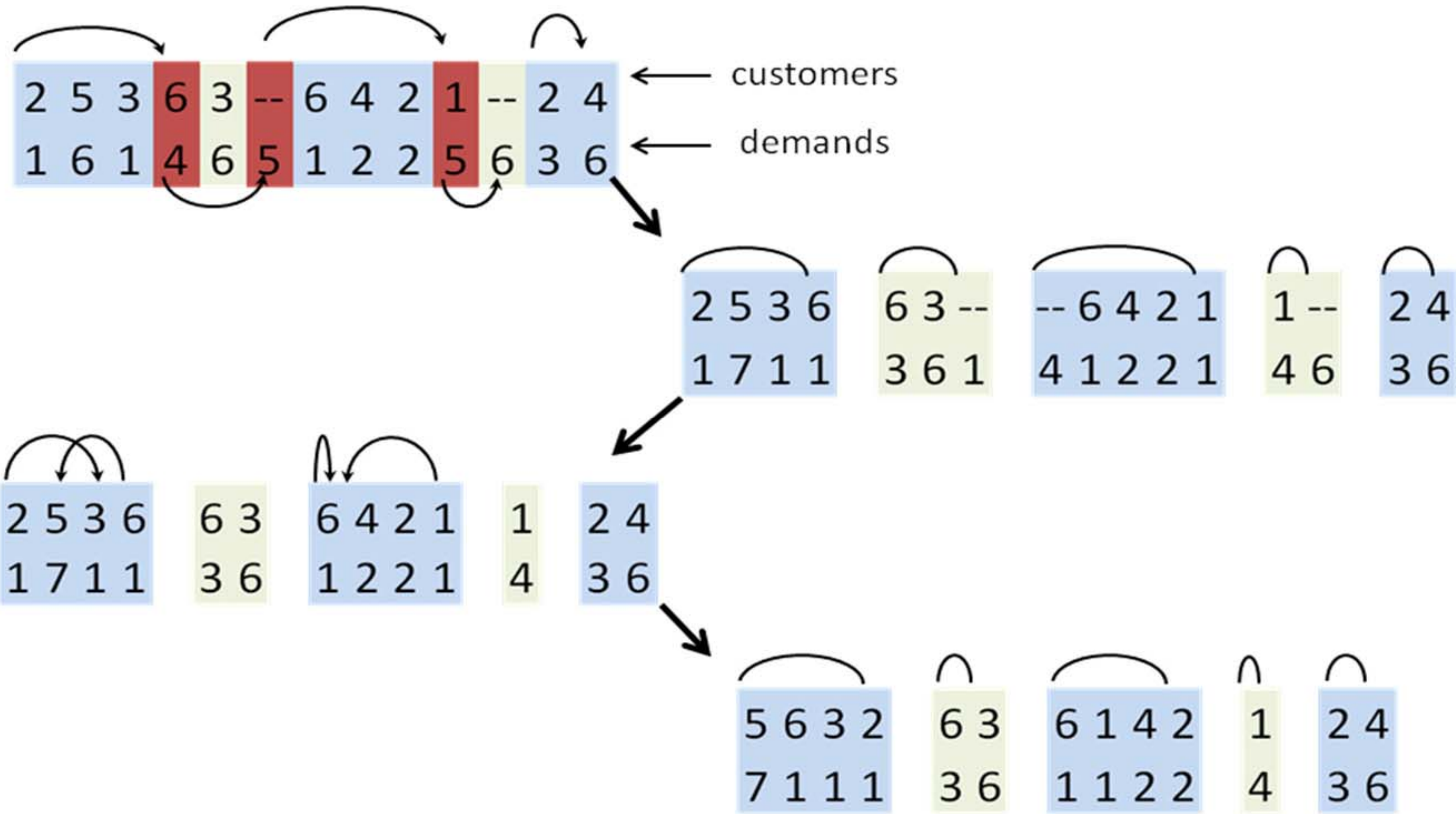
Permutation $S = (a, b, c, d, e)$ *and optimal splitting for VRP, cost 205*



Auxiliary graph of possible trips for $Q = 10$ and shortest path in boldface

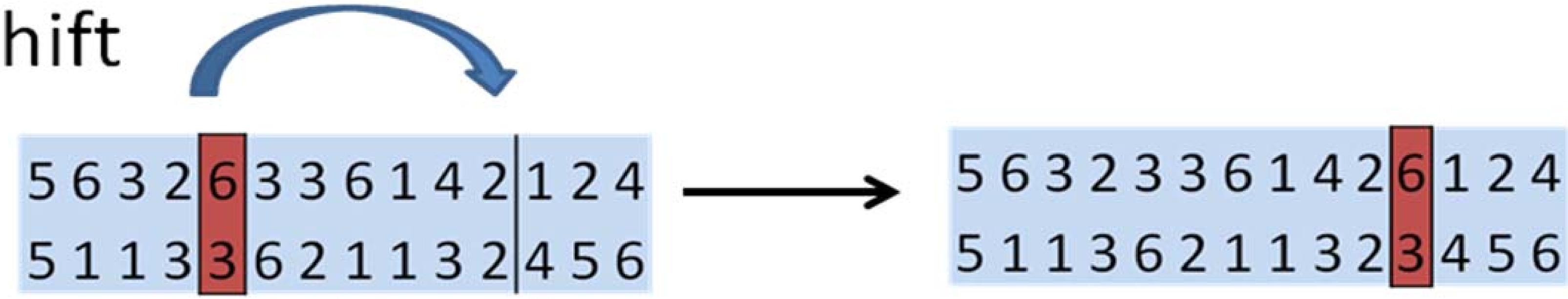
(Bellman's algorithm for directed acyclic graphs)

Decoding procedures: Greedy

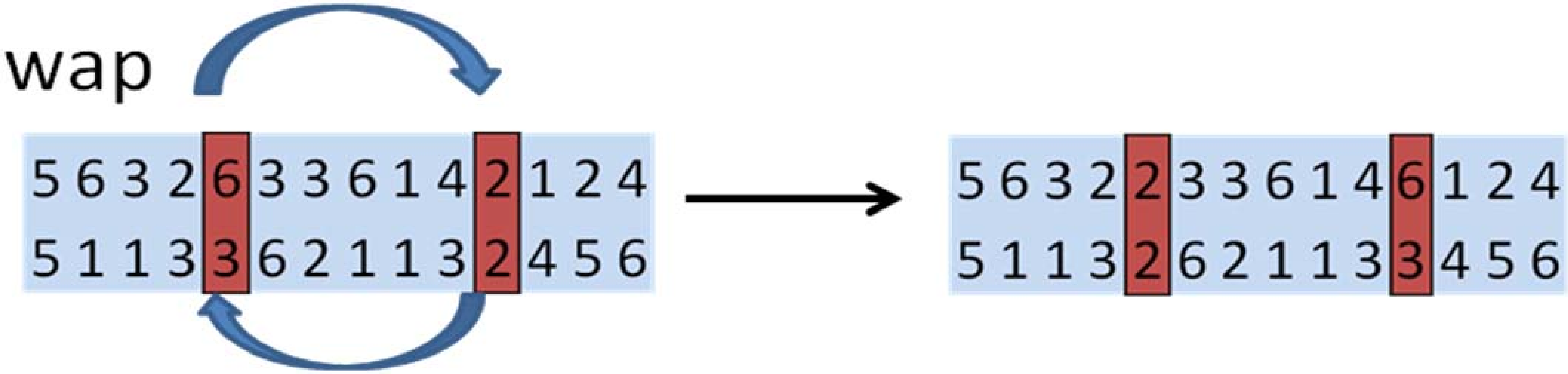


Local search moves

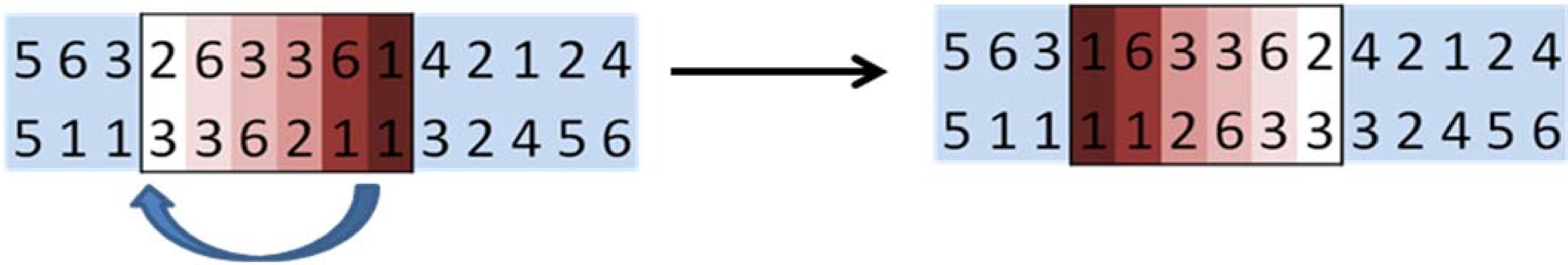
- Shift



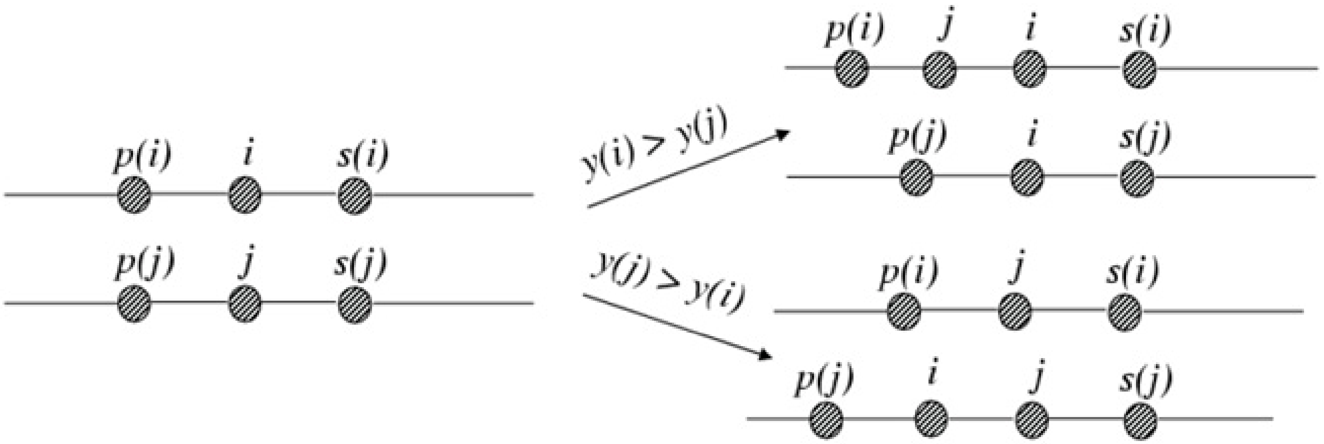
- Swap

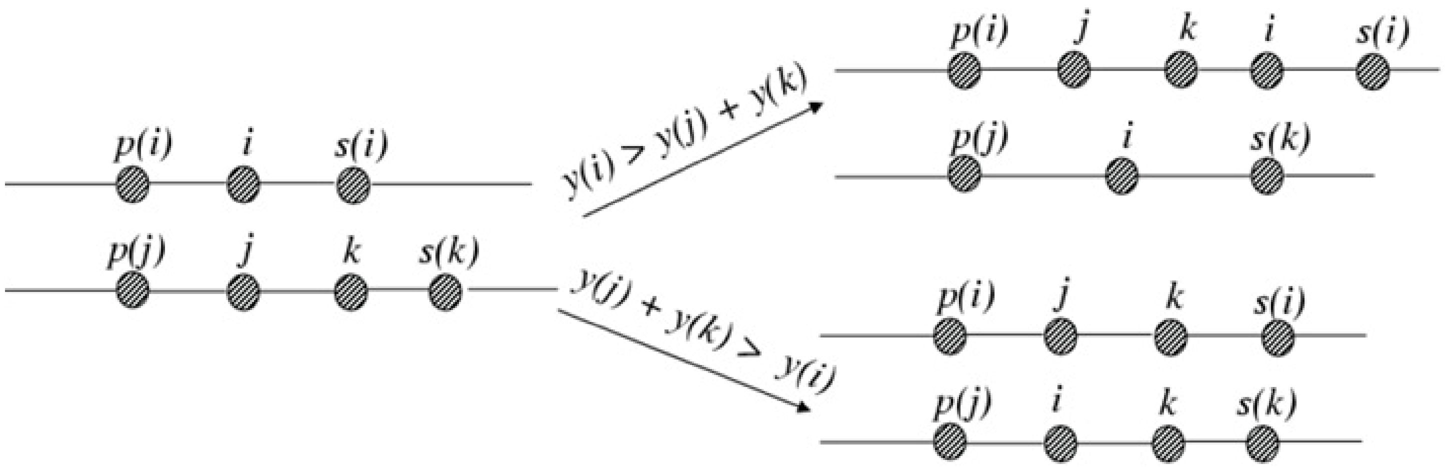


- 2-Opt

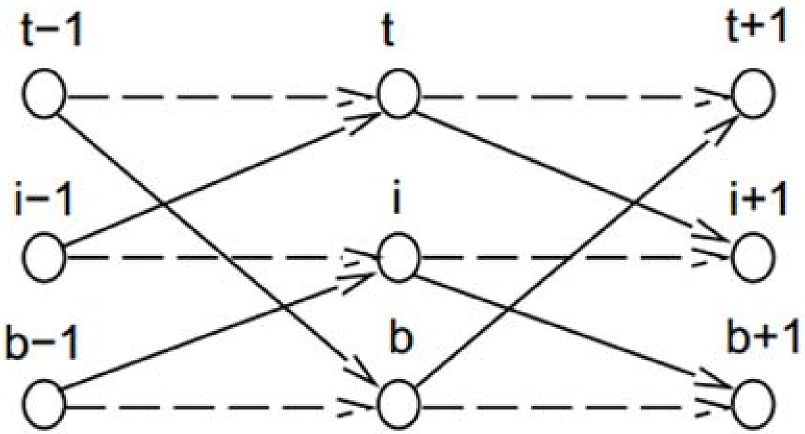


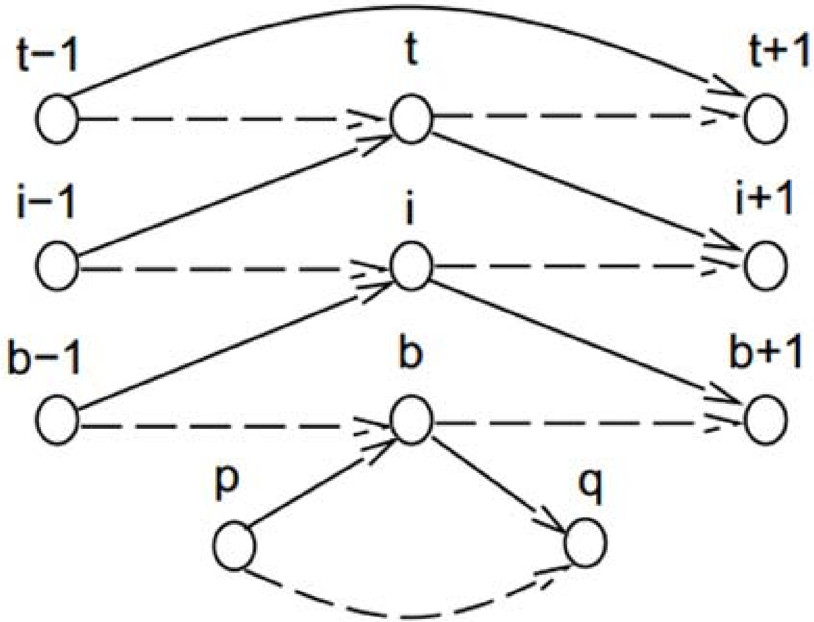
Split moves (Boudia 2007)





Ejection chains





Variable neighborhood descent

1. Choose initial solution
2. Apply a local search method to find local optimum for current neighborhood
3. Switch neighborhood
4. Repeat steps 2 and 3 until it is local optimum for each neighborhood

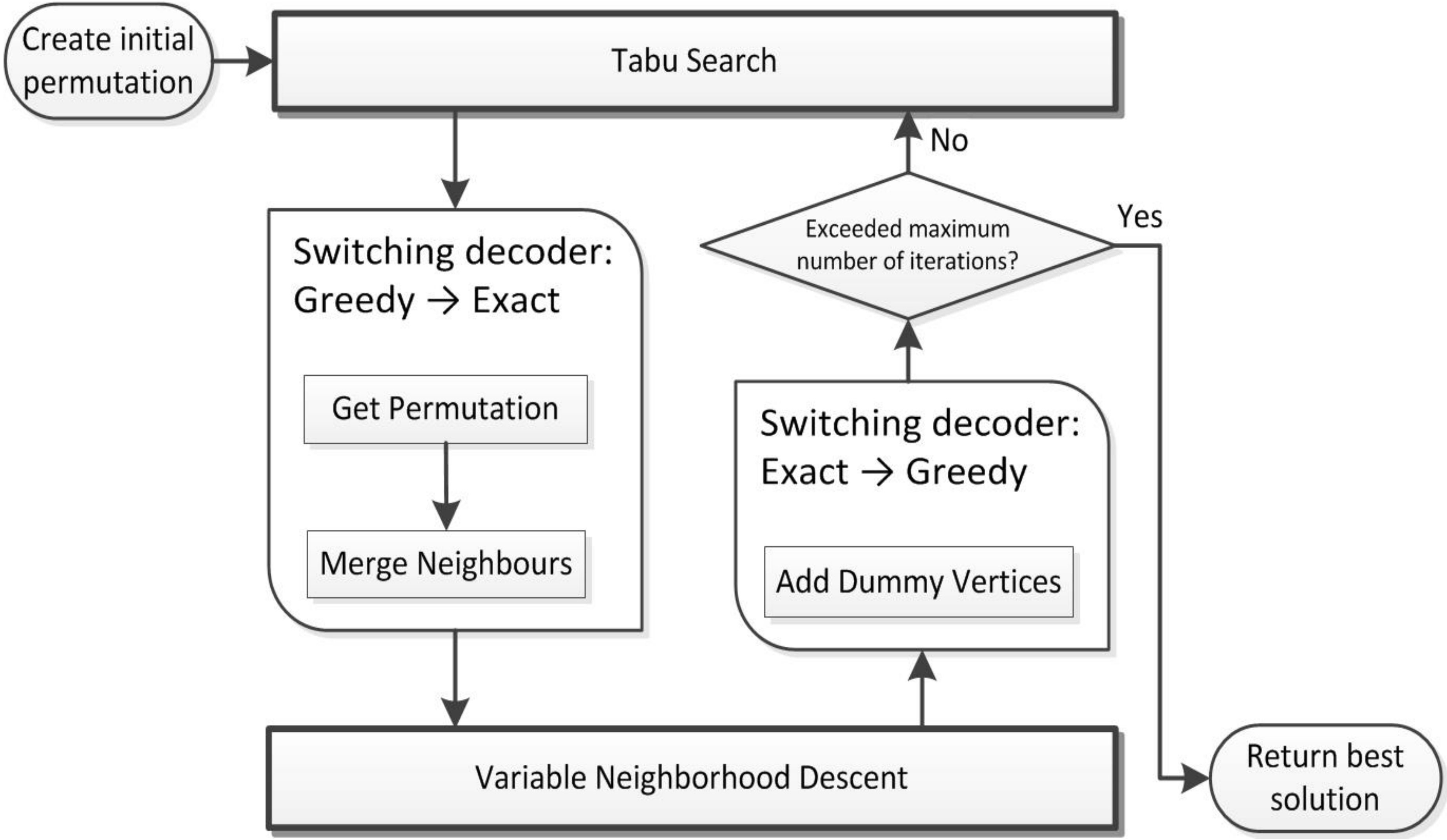
We use exact decoder.

Tabu search procedure

1. Choose an initial solution, tabu list is empty
2. Move to the best available neighboring solution
3. Update tabu list
4. Repeat steps 2 and 3 until the stopping condition is met

- We use greedy decoder.
- We use randomized 2-opt neighborhood.
- Stopping condition is maximum number of iterations.
- Diversification: we return the best solution.

Algorithm overview



		TSVBA		MA|MP		Hybrid			
File	Demand	Cost	Time	Cost	Time	Cost	Time	Saving	Best known
p1-50	[0.01-0.1]	466,74	20	460,79	12	461,87	82	-0,23	460,79
p2-75	[0.01-0.1]	614,09	136	600,06	19	600,32	170	-0,68	596,25
p3-100	[0.01-0.1]	741,60	1 944	726,81	37	745,62	317	-2,59	726,81
p4-150	[0.01-0.1]	891,10	2 641	875,61	100	885,93	951	-2,26	866,31
p5-199	[0.01-0.1]	1 069,24	11 216	1 018,71	356	1 048,63	1 337	-2,97	1 018,38
p6-120	[0.01-0.1]	990,59	2 736	976,57	73	1 003,75	450	-2,78	976,57
p7-100	[0.01-0.1]	658,99	462	649,73	35	637,46	243	-0,58	633,80
p1-50	[0.1-0.3]	753,98	23	751,41	10	766,94	36	-3,49	741,06
p2-75	[0.1-0.3]	1 085,70	97	1 074,46	34	1 084,41	69	-1,56	1 067,80
p3-100	[0.1-0.3]	1 416,35	161	1 392,85	78	1 403,99	100	-1,94	1 377,28
p4-150	[0.1-0.3]	1 929,91	755	1 878,71	148	1 904,92	226	-1,59	1 875,09
p5-199	[0.1-0.3]	2 408,16	1 544	2 340,14	347	2 366,39	399	-1,59	2 329,37
p6-120	[0.1-0.3]	2 755,74	464	2 720,38	144	**2 718,23**	146	**0,08**	2 720,38
p7-100	[0.1-0.3]	1 441,48	98	1 417,28	43	1 426,20	94	-0,74	1 415,78
p1-50	[0.1-0.5]	1 023,24	18	988,31	12	**988,05**	25	**0,03**	988,31
p2-75	[0.1-0.5]	1 458,59	68	1 413,80	37	1 436,98	63	-2,75	1 398,53
p3-100	[0.1-0.5]	1 886,70	145	1 845,30	28	1 859,68	122	-1,75	1 827,65
p4-150	[0.1-0.5]	2 647,17	470	2 561,65	225	2 581,26	259	-1,63	2 539,75
p5-199	[0.1-0.5]	3 296,69	1 217	3 191,25	475	3 216,43	535	-1,14	3 180,30
p6-120	[0.1-0.5]	4 010,80	341	3 934,39	163	**3 925,04**	147	**0,24**	3 934,39
p7-100	[0.1-0.5]	2 010,00	85	1 994,59	51	**1 989,40**	99	**0,26**	1 994,59

		TSVBA		MA|MP		Hybrid			
File	Demand	Cost	Time	Cost	Time	Cost	Time	Saving	Best known
p1-50	[0.1-0.9]	1 530,81	19	1 467,06	21	**1 461,64**	54	**0,37**	1 467,06
p2-75	[0.1-0.9]	2 164,74	62	2 102,58	46	2 112,65	123	-1,22	2 087,22
p3-100	[0.1-0.9]	2 874,86	125	2 780,95	84	2 789,66	132	-0,31	2 780,95
p4-150	[0.1-0.9]	4 151,90	452	4 045,87	245	4 050,19	534	-0,34	4 036,44
p5-199	[0.1-0.9]	5 066,24	109	4 941,22	726	**4 890,86**	608	**1,02**	4 941,22
p6-120	[0.1-0.9]	6 308,76	419	6 318,37	196	**6 227,87**	173	**1,43**	6 318,37
p7-100	[0.1-0.9]	3 157,48	98	3 113,72	52	**3 105,25**	143	**0,27**	3 113,72
p1-50	[0.3-0.7]	1 505,38	19	1 477,01	25	1 498,21	52	-1,44	1 477,01
p2-75	[0.3-0.7]	2 182,33	55	2 132,16	52	2 149,66	138	-0,82	2 132,16
p3-100	[0.3-0.7]	2 929,29	135	2 858,87	100	2 866,12	191	-0,25	2 858,87
p4-150	[0.3-0.7]	4 151,90	449	4 045,87	245	**4 037,89**	395	**0,20**	4 045,87
p5-199	[0.3-0.7]	5 281,50	119	5 155,36	750	**5 134,90**	745	**0,40**	5 155,36
p6-120	[0.3-0.7]	6 511,08	437	6 424,71	271	**6 349,54**	253	**0,78**	6 399,42
p7-100	[0.3-0.7]	3 200,62	65	3 155,69	91	**3 142,46**	153	**0,42**	3 155,69
p1-50	[0.7-0.9]	2 219,32	24	2 154,35	23	2 176,75	122	-1,04	2 154,35
p2-75	[0.7-0.9]	3 278,33	85	3 200,35	27	3 244,74	343	-1,39	3 200,35
p3-100	[0.7-0.9]	4 435,56	186	4 312,95	56	4 339,39	675	-0,61	4 312,95
p4-150	[0.7-0.9]	6 416,12	679	6 267,48	402	6 305,73	767	-0,61	6 267,48
p5-199	[0.7-0.9]	8 333,61	153	8 081,58	572	**8 075,79**	1 101	**0,07**	8 081,58
p6-120	[0.7-0.9]	10 186,06	30	10 063,47	298	10 031,42	523	-0,14	10 017,47
p7-100	[0.7-0.9]	4 996,88	153	4 919,48	180	4 934,64	296	-0,31	4 919,48

	TSVBA		EMIP + VRTP		Hybrid			
Instance	Cost	Time	Cost	Time	Cost	Time	Saving	Best known
c-SD01-008	228,28	0,00	228,28	0,70	228,28	0,37	0,00	228,28
c-SD02-016	708,28	0,02	714,40	54,40	714,40	2,58	-0,86	708,28
c-SD03-016	430,58	0,03	430,61	67,30	430,58	2,07	0,00	430,58
c-SD04-024	631,05	0,08	631,06	400,00	631,05	6,78	0,00	631,05
c-SD05-032	1 390,57	0,13	1 408,12	402,70	1 403,99	17,74	-0,97	1 390,57
c-SD06-032	831,24	0,14	831,21	408,30	831,24	17,58	0,00	831,21
c-SD07-040	3 640,00	0,09	3 714,40	403,20	3 640,00	30,76	0,00	3 640,00
c-SD08-048	5 068,28	0,14	5 200,00	404,10	5 108,28	54,87	-0,79	5 068,28
c-SD09-048	2 071,03	0,36	2 059,84	404,30	**2 048,67**	62,11	**0,54**	2 059,84
c-SD10-064	2 747,83	0,89	2 749,11	400,00	**2 716,96**	85,03	**1,12**	2 747,83
c-SD11-080	13 280,00	0,41	13 612,12	400,10	13 280,00	148,30	0,00	13 280,00
c-SD12-080	7 213,62	0,84	7 399,06	408,30	7 236,66	109,80	-0,32	7 213,62
c-SD13-096	10 110,58	1,20	10 367,06	404,50	10 110,58	191,14	0,00	10 110,58
c-SD14-120	10 802,87	2,31	11 023,00	5 021,70	**10 776,82**	237,35	**0,24**	10 802,87
c-SD15-144	15 153,45	3,20	15 271,77	5 042,30	**15 121,94**	289,28	**0,21**	15 153,45
c-SD16-144	3 446,43	7,59	3 449,05	5 014,70	**3 381,31**	258,50	**1,89**	3 446,43
c-SD17-160	26 493,56	7,27	26 665,76	5 023,60	26 584,90	421,31	-0,34	26 493,56
c-SD18-160	14 323,04	27,95	14 546,58	5 028,60	**14 288,57**	416,28	**0,24**	14 323,04
c-SD19-192	20 157,10	11,95	20 559,21	5 034,20	**20 109,05**	530,70	**0,24**	20 157,10
c-SD20-240	39 722,86	11,02	40 408,22	5 053,00	**39 697,18**	1 056,28	**0,06**	39 722,86
c-SD21-288	11 458,76	111,56	11 491,67	5 051,00	**11 292,96**	1 241,37	**1,45**	11 458,76

Conclusion

- We found some new properties of the problem
- We developed new hybrid heuristic based on idea of different decoders
- It improves 21 best-known solutions in the 70 test instances with number of customers up to 280