Axioms of Semantic Geometry

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A Semantic Geometry of Meaning

1. Axioms

Axiom 0.1 (Directional Meaning). Semantic meaning is invariant under positive scalar multiplication. For any non-zero vector $x \in \mathbb{R}^d \setminus \{0\}$,

$$[x] := \{\alpha x \mid \alpha > 0\}$$

represents a single semantic meaning.

Axiom 0.2 (Spherical Normalization). All semantic representations admit a lossless projection to the unit sphere.

$$\pi : \mathbb{R}^d \setminus \{0\} \to \mathbb{S}^{d-1}, \quad \pi(x) = \frac{x}{\|x\|}$$

Axiom 1.1 (Angular Metricity). Semantic similarity depends only on angular separation.

$$d(u, v) := \arccos(u \cdot v), \quad u, v \in \mathbb{S}^{d-1}$$

Axiom 1.2 (Locality of Meaning). Every coherent semantic concept occupies a geodesically bounded region.

$$\forall C \subset \mathbb{S}^{d-1},\ \exists \mu \in \mathbb{S}^{d-1},\ \exists \theta \in (0, \pi) \text{ such that } C \subseteq \{x \mid d(x, \mu) \leq \theta\}$$

Axiom 2.1 (Witness Principle). Primitive semantic data are witnesses.

$$w \in \mathbb{S}^{d-1}$$

Axiom 2.2 (Facts as Constraints). Facts are regions of admissible meanings.

$$F \subset \mathbb{S}^{d-1}, \quad w \text{ supports } F \iff w \in F$$

Axiom 2.3 (Fact Strengthening). Adding consistent witnesses can only restrict facts.

$$W_1 \subset W_2 \implies F(W_2) \subseteq F(W_1)$$

Axiom 2.4 (Fact Non-Emptiness).

$$F \neq \varnothing \implies \exists w \in F$$

Axiom 2.5 (Witness Non-Universality).

$$\forall w,\ F(w) \neq \{w\}$$

Axiom 3.1 (Semantic Axes). There exists a latent set of semantic axes

$$\mathcal{A} \subset \mathbb{S}^{d-1}$$

such that semantic variation corresponds to angular displacement along these axes.

Axiom 3.2 (Antipodal Negation). Negation is represented by antipodal direction.

$$\forall a \in \mathcal{A}, \quad \neg a = -a$$

Axiom 3.3 (Exclusion vs. Negation). Orthogonality represents independence, not negation.

$$a \cdot b = 0 \implies a \perp b$$

Axiom 3.4 (Modifier Projection). Modifiers act as projections.

$$M : \mathbb{S}^{d-1} \to \text{span}(U) \cap \mathbb{S}^{d-1}$$

Axiom 4.1 (No Linear Aggregation). There exists no linear operator

$$A : (\mathbb{S}^{d-1})^n \to \mathbb{S}^{d-1}$$

that preserves semantic consistency under contradiction.

Axiom 4.2 (Conflict as Dispersion). Semantic conflict is measurable as angular variance.

Axiom 4.3 (Coherent Region Requirement). Valid interpretations must lie within a common open neighborhood.

$$\exists U \subset \mathbb{S}^{d-1} \text{ open} : \quad W \subset U,\ \mu \in U$$

Axiom 5.1 (Local Admissibility). Reasoning is valid only within admissible regions.

Axiom 5.2 (No Global Coherence). Global semantic coherence cannot be assumed.

Axiom 5.3 (Impossibility).

$$\bigcap_i C_i = \varnothing \implies \text{impossible}$$

Axiom 6.1 (Geometry Precedes Logic). Logical inference operates only after geometric admissibility.

Axiom 6.2 (Local Soundness). Logic is sound only within a single coherent region.

Axiom 6.3 (No Global Closure). No globally complete semantic theory exists.

Axiom 7 (Hallucination). A hallucination is a meaning unsupported by any coherent witness region.

$$\forall U \subset \mathbb{S}^{d-1},\ r \notin \mathrm{Conv}(W_U)$$

2. Derived Theorems

Theorem 1 (Non-Existence of a Global Semantic Mean)**.** *There exists no function*

$$M : (\mathbb{S}^{d-1})^n \to \mathbb{S}^{d-1}$$

such that M yields a semantically valid interpretation for all finite witness sets.

Sketch. Assume such an M exists. By Axiom 0.1, semantic meaning is invariant under scaling. By Axiom 0.2, all meanings lie on $\mathbb{S}^{d-1}$. Consider witnesses $w_1 = a$ and $w_2 = -a$ for some semantic axis a (Axiom 3.1). Then w_1 and w_2 are contradictory (Axiom 3.2). Any deterministic M must return some $\mu \in \mathbb{S}^{d-1}$. However, μ cannot lie in a coherent region containing both w_1 and w_2 (Axiom 4.3). Therefore $M(W)$ violates the hallucination criterion (Axiom 7), yielding a contradiction. □

Theorem 2 (Geometric Detectability of Contradiction)**.** *Semantic contradiction is detectable via antipodal separation on $\mathbb{S}^{d-1}$.*

Sketch. By Axiom 3.1, semantic properties are organized along latent axes. By Axiom 3.2, negation corresponds to antipodal reversal along such an axis. Thus, for witnesses w_1, w_2 such that $w_1 \approx a$ and $w_2 \approx -a$, we have $d(w_1, w_2) \approx \pi$. By Axiom 4.2, angular dispersion detects conflict. Hence contradiction is geometrically observable. □

Theorem 3 (Invalidity of Centroid Aggregation)**.** *Centroid aggregation is semantically invalid in general.*

Sketch. Centroid aggregation computes

$$\mu = \frac{\sum_i w_i}{\|\sum_i w_i\|}.$$

This operation is linear prior to normalization. By Axiom 4.1, no linear operator preserves semantic consistency. Thus, for some witness set W, the centroid cannot lie in any coherent region containing W (Axiom 4.3). □

Theorem 4 (Centroid-Induced Hallucination)**.** *There exist witness sets W for which centroid aggregation produces a hallucinated interpretation.*

Sketch. Let $W = \{a, -a\}$ for some semantic axis a. Then $\sum_i w_i = 0$, and normalization yields an arbitrary μ. No coherent region contains μ and both witnesses (Axiom 4.3). By Axiom 7, μ constitutes a hallucination. □

Theorem 5 (Unavoidability of Semantic Ambiguity)**.** *There exist queries admitting multiple semantically valid interpretations.*

Sketch. By Axiom 1.2, semantic regions are local and bounded. Distinct regions may overlap without inclusion. For queries whose witness sets lie in such overlaps, multiple admissible interpretations satisfy Axiom 4.3. □

Theorem 6 (Irreducibility of Ambiguity)**.** *Semantic ambiguity cannot be eliminated without loss of information.*

Sketch. Resolving ambiguity requires collapsing overlapping regions. This merges distinct witness-supported directions, destroying witness separation. By Axiom 2.1, witnesses must be preserved. Therefore ambiguity removal implies semantic loss. □

Theorem 7 (Existence of Impossible Queries)**.** *There exist semantic queries with no admissible interpretation.*

Sketch. Each semantic constraint defines a subset of $\mathbb{S}^{d-1}$ (Axiom 2.2). Some families of constraints have empty intersection. By Axiom 5.3, such queries are semantically impossible. □

Theorem 8 (Necessity of Refusal)**.** *For impossible queries, refusal is the only semantically valid response.*

Sketch. By Theorem 7, no admissible μ exists. Any non-refusal output selects some μ. By Axiom 7, this constitutes hallucination. Thus refusal is required. □

Theorem 9 (Locality of Sound Reasoning)**.** *All semantically sound reasoning is local to a coherent region.*

Sketch. By Axiom 1.2, coherence is local. By Axiom 4.3, valid interpretations must lie within a coherent region. By Axiom 6.1, logic operates only after geometric admissibility. Hence reasoning outside a region is unsound. □

Theorem 10 (Non-Existence of a Global Semantic Ordering)**.** *There exists no partial or total order $\preceq$ on semantic concepts that is globally sound.*

Sketch. By Axiom 1.2, semantic concepts correspond to bounded, overlapping regions on $\mathbb{S}^{d-1}$. For any two overlapping but non-identical regions C_1, C_2, neither inclusion $C_1 \subseteq C_2$ nor $C_2 \subseteq C_1$ holds. Thus antisymmetry fails, and no globally valid order can exist. □

Theorem 11 (Semantic Drift Under Iteration)**.** *Repeated semantic synthesis without new witnesses leads to drift away from admissible regions.*

Sketch. By Axiom 2.1, semantic grounding requires witnesses. Iterated synthesis reuses previous outputs as inputs. By Theorem 3, aggregation introduces unsupported directions. Thus each iteration introduces error. By induction, distance from the original admissible region increases monotonically. □

Theorem 12 (Explanatory Non-Correction)**.** *No amount of post-hoc explanation can render an invalid semantic interpretation valid.*

Sketch. By Axiom 6.1, logic operates only after geometric admissibility. Explanations are logical structures applied to interpretations. If an interpretation violates Axiom 4.3, no logical transformation can relocate it into a coherent region. □

Theorem 13 (Orthogonality of Confidence and Truth)**.** *Semantic truth and confidence are independent quantities.*

Sketch. Truth is defined via region membership (Axiom 2.2, Axiom 4.3). Confidence is not a directional property and is invariant under scaling (Axiom 0.1). Therefore confidence cannot determine admissibility. □

Theorem 14 (Locality of Semantic Negation)**.** *Semantic negation is local to a semantic axis and does not distribute globally.*

Sketch. By Axiom 3.2, negation corresponds to antipodal reversal along a specific axis. Linguistic negation may apply to arbitrarily complex expressions. Therefore symbolic negation does not correspond to uniform geometric inversion. □

Theorem 15 (Non-Transitivity of Property Inheritance)**.** *Semantic property inheritance is non-transitive in general.*

Sketch. Inheritance requires region inclusion. By Axiom 1.2, regions overlap without global containment. Thus even if $A \subseteq B$ locally and $B \subseteq C$ locally, $A \subseteq C$ need not hold globally. □

Theorem 16 (Path Coherence Requirement for Multi-Hop Reasoning)**.** *Multi-hop reasoning is sound only if all intermediate steps remain within a single coherent region.*

Sketch. Each inference step induces a movement in semantic space. By Axiom 4.3, semantic validity requires remaining within a coherent region. If any step exits the region, Axiom 6.1 prohibits further reasoning. □

Theorem 17 (Informational Value of Disagreement)**.** *Witness disagreement increases semantic information.*

Sketch. By Axiom 4.2, angular dispersion is measurable. Dispersion reveals structure along latent semantic axes (Axiom 3.1). Therefore disagreement constrains admissible regions and increases information content. □

Theorem 18 (Irreversibility of Semantic Compression)**.** *Semantic compression induces irreversible information loss.*

Sketch. Compression collapses multiple witnesses into fewer representations. This requires aggregation. By Theorem 3, aggregation loses polarity. Lost directional information cannot be reconstructed. □

Theorem 19 (Context Window Limitation)**.** *Finite context windows impose hard limits on semantic coherence.*

Sketch. Only witnesses within the window participate in region determination. External constraints are excluded. Thus the admissible region is artificially truncated, regardless of ground truth. □

Theorem 20 (Non-Closure of Truth Under Conjunction)**.** *Truth is not closed under conjunction.*

Sketch. Truth corresponds to region membership. Two regions may each be nonempty while their intersection is empty. Thus jointly asserting two true statements may be invalid. □

Theorem 21 (Stability of Refusal)**.** *Refusal is a stable semantic fixed point.*

Sketch. Refusal introduces no semantic direction. Thus it cannot violate Axiom 7. Any alternative output under impossibility violates admissibility. □

Theorem 22 (Non-Propositionality of Truth)**.** *Truth is not an intrinsic property of sentences.*

Sketch. Sentences map to witnesses (Axiom 2.1). Truth applies to regions, not points (Axiom 2.2). Thus sentences alone cannot carry truth values. □

Theorem 23 (Non-Monotonicity of Knowledge)**.** *Knowledge is non-monotonic under evidence accumulation.*

Sketch. By Axiom 2.3, adding witnesses may shrink admissible regions. Previously valid interpretations may be excluded. Thus conclusions may be retracted. □

Theorem 24 (Impossibility of a Global World Model)**.** *A single globally coherent semantic world model cannot exist.*

Sketch. By Axiom 5.2, global coherence is forbidden. Disjoint coherent regions necessarily exist. Any global model collapses contradictions, violating Axiom 7. □

Theorem 25 (Hallucination in Always-Answering Systems)**.** *Any system that always produces an answer must hallucinate.*

Sketch. By Theorem 7, impossible queries exist. Any non-refusal output selects a semantic direction. By Axiom 7, such a direction is hallucinated. □

3. Stage 2: Semantic Energy

The semantic energy of a meaning μ given a set of witnesses W is defined as:

$$E(\mu \mid W) = \sum_{w \in W} (1 - \mu \cdot w)$$

The total energy is given by:

$$E = \alpha E_{\text{fit}} + \beta E_{\text{logic}} + \gamma E_{\text{time}} + \delta E_{\text{sheaf}}$$

Curvature.

$$K(\mu) = \operatorname{tr}\left(\nabla^2_{\text{tan}} E(\mu)\right)$$

Low curvature indicates admissibility, high curvature indicates contradiction, and divergent curvature indicates impossibility.