# Refinement Types for Ruby

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Abstract. Refinement types are a popular way to specify and reason about key program properties. In this paper, we introduce RTR, a new system that adds refinement types to Ruby. RTR is built on top of RDL, a Ruby type checker that provides basic type information for the verification process. RTR works by encoding its verification problems into Rosette, a solver-aided host language. RTR handles mixins through assume-guarantee reasoning, and uses just-in-time verification for metaprogramming. We formalize RTR by showing a translation from a core, Ruby-like language with refinement types into Rosette. We apply RTR to check a range of functional correctness properties on six Ruby programs. We find RTR can successfully verify key methods in these programs, taking only a few minutes to perform verification.

Keywords: ruby, rossette, refinement types, dynamic languages

## 1 Introduction

Refinement types combine types with logical predicates to encode program invariants [32, 43]. For example, the following refinement type specification:

**type**: incr\_sec , '(Integer  $x \{ 0 \le x < 60 \}$ )  $\rightarrow$  Integer  $r \{ 0 \le r < 60 \}$ ' describes a method incr\_sec that increments a second. With this specification, incr\_sec can only be called with integers that are valid seconds (between 0 and 59), and the method will always return valid seconds.

Refinement types were introduced to reason about simple invariants, like safe array indexing [43], but since then they have been successfully used to verify sophisticated properties including termination [39], program equivalence [9], and correctness of cryptographic protocols [28], in various languages (e.g., ML [18], Racket [21], and TypeScript [40]).

In this paper, we explore refinement types for Ruby, a popular, object-oriented, dynamic scripting language. Our starting place is RDL [17, 30], a Ruby type system recently developed by one of the authors and his collaborators. We introduce RTR, a tool that adds refinement types to RDL and verifies them via a translation into Rosette [38], a solver-aided host language. Since Rosette is not object-oriented, RTR encodes Ruby objects as Rosette structs that store object fields and an integer identifying the object's class. At method calls, RTR uses RDL's type information to statically overestimate the possible callees. When methods with refinement types are called, RTR can either translate the callee

directly, or treat it modularly by asserting the method preconditions and assuming the postcondition, using purity annotations to determine which fields (if any) the method may mutate. (§ 2).

In addition to standard object-oriented features, Ruby includes dynamic language features that increase flexibility and expressiveness. In practice, this introduces two key challenges in refinement type verification: *mixins*, which are Ruby code modules that extend other classes without direct inheritance, and *mertaprogramming*, in which code is generated on-the-fly during runtime and used later during execution. The latter feature is particularly common in Ruby on Rails (just "Rails" from now on), a popular Ruby web development framework.

To meet these challenges, RTR uses two key ideas. First, RTR incporates assume-guarantee checking [20] to reason about mixins. RTR verifies definitions of methods in mixins by assuming refinement type specifications for all undefined, external methods. Then, by dynamically intercepting the call that includes a mixin in a class, RTR verifies the appropriate class methods satisfy the assumed refinement types ( $\S$  3.1). Second, RTR uses just-in-time verification to reason about metaprogramming, following RDL's just-in-time type checking [30]. In this approach, (refinement) types are maintained at run-time, and methods are checked against their types after metaprogramming code has executed but before the methods have been called. ( $\S$  3.2).

We formalized RTR by showing how to translate  $\lambda^{RB}$ , a core Ruby-like language with refinement types, into  $\lambda^{I}$ , a core verification-oriented language. We then discuss how to map the latter into Rosette, which simply requires encoding  $\lambda^{I}$ 's primitive object construct into Rosette structs and translating some control-flow constructs such as return. (§ 4).

We evaluated RTR by using it to check a range of functional correctness properties on six Ruby and Rails applications. In total, we verified 31 methods, comprising 271 lines of Ruby, by encoding them as 1,061 lines of Rosette. We needed 73 annotations. Verification took a total median time of 437 seconds. Thus, we believe that RTR is a promising first step toward verification for Ruby.

## 2 Overview

We start with an overview of RTR, which extends RDL [30], a Ruby type checker, with refinement types. In RTR, program invariants are specified with refinement types (§ 2.1) and checked by translation to Rosette (§ 2.2). We translate Ruby objects to Rosette structs (§ 2.3) and method calls to function calls (§ 2.4).

## 2.1 Refinement Type Specifications

Refinement types in RTR are Ruby types extended with logical predicates. For example, we can use RDL's **type** method to link a method with its specification:

```
type '(Integer x \{ 0 \le x < 60 \}) \rightarrow Integer r \{ 0 \le r < 60 \}' def <code>incr_sec</code> (x) if (x==59) then 0 else x+1 end; end
```

This type indicates the argument and result of incr\_sec are integers in the range from 0 to 59. In general, refinements (in curly braces) may be arbitrary Ruby expressions that are treated as booleans, and they should be *pure*, *i.e.*, have no side effects, since effectful predicates make verification either complicated or imprecise [41]. As in RDL, the type annotation, which is a string, is parsed and stored in a global table which maintains the program's type environment.

#### 2.2 Verification using Rosette

To check if methods satisfy their specifications, RTR relies on Rosette [38], a solver-aided host language built on top of Racket. Among other features, Rosette can perform verification by using symbolic execution to generate logical constraints, which are discharged using Z3 [24]. Concretely, RTR generates Rosette verification queries that hold if and only if the Ruby specifications are satisfied.

For example, to check the safety of incr\_sec's specification, RTR defines the Rosette function incr\_sec using its Ruby definition and then verifies that, assuming incr\_sec's precondition, its postcondition always holds:

```
(define ( incr_sec x) ( if (x==59) then 0 else x + 1)) (define-symbolic x_in integer?) (verify #:assume (assert 0 \le x < 60) #:guarantee (assert (let ([r ( incr_sec x)]) 0 \le r < 60)))
```

Here we define an integer symbolic constant  $x_i$ n, standing for an unknown, arbitrary integer argument. Rosette can define symbolic constants of what it considers to be its solvable types: integers, booleans, bitvectors, reals, and uninterpreted functions. Finally, we use Rosette's **verify** function with assumptions and assertions to encode pre- and postconditions, respectively. Rosette searches for a binding to  $x_i$ n such that the assertion fails. If Rosette determines that no such binding exists, then the assertion is verified.

#### 2.3 Encoding and Reasoning about Objects

We encode Ruby objects in Rosette with a struct type we name **object** (similarly to prior work [19, 34]). The struct **object** contains an integer classid identifying the object's class, an integer **objectid** identifying the object itself, and a field for each instance variable of all objects encountered in the source Ruby program.

For example, consider a Ruby class **Time** with three instance variables **@sec**, **@min**, and **@hour**, representing the time components, and a method <code>is\_valid</code> that checks (in-bounds) validity for all the three time components.

```
class Time
  attr_accessor :sec, :min, :hour

def initialize (s, m, h)
  @sec = s; @min = m; @hour = h
end
```

```
type '() \rightarrow bool' def is_valid 0 \leq @\sec < 60 \ \land 0 \leq @min < 60 \ \land 0 \leq @hour < 24 end end
```

RTR observes three fields in this program, and thus it defines:

Here the **object** type includes five fields to represent the class ID, object ID and the three time components. Note that since the fields of the **object** struct are statically defined, our encoding cannot handle dynamically generated instance variables, which we leave as future work.

Given the **object** struct, we translate Ruby field reads and writes as getting or setting, respectively, the **object** fields in Rosette. For example, suppose we add a method mix to the **Time** class and specify it is only called with and returns valid times:

```
 \begin{array}{ll} \text{type}: \text{mix, '(Time t1 } \{ \ t1. \ \text{is\_valid } \}, & \textbf{Time t2 } \{ \ t2. \ \text{is\_valid } \}, \\ & \textbf{Time t3 } \{ \ t3. \ \text{is\_valid } \}) & \rightarrow \textbf{Time r } \{ \ r. \ \text{is\_valid } \}' \\ \text{def mix(t1,t2,t3)} & \text{@sec} = t1.\text{sec}; \ \text{@min} = t2.\text{min; @hour} = t3.\text{hour; } \textbf{self} \\ \textbf{end} \end{array}
```

Verification fails at the basic type checking stage, since the types of field getters and setters (e.g., sec and sec=) are unknown. We use **type** to specify such types:

```
\begin{array}{lll} \mbox{type}:\mbox{sec}\,, & \mbox{`()} & \rightarrow \mbox{Integer}\ \mbox{i} & \{\ \mbox{i} == \mbox{@sec}\,\}\mbox{'} \\ \mbox{type}:\mbox{sec}=, & \mbox{`(Integer}\ \mbox{i}) & \rightarrow \mbox{Integer}\ \mbox{out}\ \{\ \mbox{i} == \mbox{@sec}\,\}\mbox{'} \\ \end{array}
```

We note that these annotations can be generated automatically using our approach to metaprogramming (3.2). With these annotations, verification of mix succeeds. The verifier translates Ruby's mix to a homonymous Rosette function

```
(define (mix self t1 t2 t3)
  (set-object-@sec! self (sec t1))
  (set-object-@min! self (min t2))
  (set-object-@hour! self (hour t3))
  self)
```

where (**set-object**-x! y w) writes w to the x field of y and the field selectors **sec**, min, and hour are uninterpreted functions. Note that **self**, the receiver of a method call that is implicit in Ruby, turns into an explicit additional argument in the Rosette definition.

## 2.4 Method Calls

To translate a Ruby method call e.m(e1, ..., en), RTR needs to know the callee, which depends on the runtime type of the receiver e. RTR uses RDL's type information to overapproximate the set of possible receivers. For example, if e

has type A in RDL, then RTR translates the above as a call to A.m. If the e has a union type, RTR emits Rosette code that branches on the potential types of the receiver using **object** class IDs, and dispatches the appropriate method in each branch. This is similar to class hierarchy analysis [16], which also uses types to determine the set of possible method receivers and construct a call graph. Because type checking is necessary to method call translation, we require the programmer to provide type annotations for *all* methods and instance variables to be translated to Rosette.

Once the method being called is determined, we translate the call into Rosette. As an example, consider a method to\_sec that converts **Time** to seconds, after it calls the method incr\_sec from § 2.1.

```
type '(Time t { t. is_valid }) \rightarrow Integer r { 0 \le r < 90060 }' def to_sec(t) incr_sec(t.sec) + 60 * t.min + 3600 * t.hour end
```

The specification of to\_sec requires the result to be less than 90060, which must be true when the hour, min and increased sec are valid.

RTR's translation of to\_sec can simply call directly into incr\_sec's translation. This is morally equivalent to inlining incr\_sec's code. However, this is not always possible or desirable. A method's code may not be available because the method comes from a library, is external to the environment (§ 3.1), or hasn't even been defined yet (§ 3.2). The method may also contain constructs which are difficult to verify properties of, like diverging loops.

Instead, we can approximate the method call using the programmer provided method specification. To precisely reason with only a method's specification, we follow Dafny [22] and treat pure and impure methods differently.

Pure methods Pure methods have no side effects (including mutation) and return the same result for the same inputs (satisfying the congruence axiom). Thus pure methods can be represented using Rosette's uninterpreted functions. The method incr\_sec is indeed a pure function, so we can label it as such:

```
type : incr_sec , '(Integer \times \{0 \le x < 60\}) \rightarrowInteger r \{0 \le r < 60\}', :pure
```

With the above **pure** label the translation of to\_sec treats incr\_sec as an uninterpreted function. Furthermore, it asserts that the precondition  $0 \le x < 60$  holds, and assumes that the postcondition  $0 \le r < 60$  holds, which is enough information to prove to\_sec safe.

*Impure methods* Most Ruby methods have effects, usually mutating the fields of their inputs, and thus cannot be encoded as pure. As an example, consider incr\_min, an impure method that, given a **Time** object, increases the second and possibly the minute fields.

```
type '(Time t {t. is_valid \land t.min < 60}) 

\rightarrow Time r {r. is_valid }', protects: {t\Rightarrow@hour} 

def incr_min(t) 

if t.sec < 59 then t.sec = incr_sec(t.sec)
```

```
else t.min += 1; t.sec = 0 end return t end
```

A translated call to <code>incr\_min</code> generates a fresh symbolic value as the method's output, assumes the method's postcondition on that value, and then <code>havocs</code> (set to fresh symbolic values) all the fields of arguments to <code>incr\_min</code>, since they could be modified. To increase precision, we provide a <code>protects</code> label that lists fields of inputs that remain unchanged. In this example, <code>@hour</code> is listed as protected, and thus RTR does not havoc it. Currently, <code>pure</code> and <code>protects</code> labels are trusted; we leave checking them to future work.

#### 3 Just-In-Time Verification

Next, we see how labeled refinement types as method specifications are used to verify challenging Ruby code with dynamic bindings via mixins ( $\S$  3.1) and metaprogramming ( $\S$  3.2). In both cases verification happens just-in-time: verification of a method cannot happen statically, since the method's environment is not yet defined, but happens *before* the method is executed, thus ensuring that no violations occur at runtime.

#### 3.1 Mixins

Ruby implements mixins via its *module* system. A Ruby module is a collection of method definitions which at *runtime* can be added to any class that *includes* the module. Interestingly, because modules are meant to be a dynamic extension to other classes, they may refer to other methods which don't exist within their environment. Such incomplete environments pose a challenge for verification.

Consider the following method div\_by\_val, which is defined within the module **Arithmetic**, an adjustment of the Ruby Money library described in § 5.

#### module Arithmetic

The module method div\_by\_val divides its input x by value. RTR's built-in specification for / automatically requires that for the division to succeed, value cannot be 0. The above lack of division-by-zero example is elementary for standard refinement types, yet new light is shed in the dynamic contexts of Ruby.

The definition of value does not exist anywhere in the current environment. Rather, value is expected to be defined wherever the **Arithmetic** module is included. To proceed with verification, the programmer must provide an annotation for value which will be used by our translation (rules T-Pure1 of Figure 3) to approximate the method. The annotation

```
type :value, '() \rightarrow Float v { 0 < v }', :pure
```

encodes value as a positive uninterpreted symbol (*i.e.*, zero argument function) allowing verification of div\_by\_val to succeed. Note that if value were not pure, and thus labeled with protects, then verification of the postcondition of div\_by\_val would be impossible, since it relies on congruence of value.

RTR can verify external method annotations, like value's, following an assumeguarantee paradigm [20]. For instance, the **Arithmetic** module can be included in a class **Money** which defines value:

```
class Money
include Arithmetic
...
def value
if @val > 0 then return @val else return 0.01 end
end
end
```

We dynamically intercept the call to **include** to apply type annotations for methods not defined in a module. Since **Arithmetic** is included in **Money**, the annotation for value will be verified upon definition of the value method. Thus, we *assume* value's annotation to be true while we are verifying div\_by\_val, then we *quarantee* the annotation is indeed valid once value is defined.

#### 3.2 Metaprogramming

Metaprogramming in Ruby allows for editing and generating code at runtime. New methods can be defined on the fly, imposing a challenge for verification since properties rely on dynamically generated code. To achieve verification in the presence of metaprogramming, we employ just-in-time checking [30], in which, in addition to code, method annotations can also be generated dynamically.

We illustrate the just-in-time approach using an example from the Rails app Boxroom, an app for managing and sharing files in a web browser. The app defines the class **UserFile**, a Rails *model* that corresponds to a table in the database.

```
class UserFile < ActiveRecord::Base
  belongs_to : folder
  ...
  type '(Folder target) → Bool b { folder == target }'
  def move(target)
    self . folder = target; save!
  end
end</pre>
```

Every **UserFile** is associated with a folder (another model), as declared by the method call to belongs\_to. The move method updates the associated folder of a **UserFile** and saves the result to the database. We annotate move to specify that the new folder of the **UserFile** should be the same as the input target folder.

However, neither the method folder = nor folder are statically defined. Rather, the method call to belongs\_to results in the dynamic generation of these methods. We instrument the belongs\_to method so that it further generates type annotations for the generated methods, with the below code.

#### module ActiveRecord::Associations::ClassMethods

```
\begin{array}{ll} \textbf{pre}(: belongs\_to) \ do \ | * args | \\ name = args \ [0]. \ to\_s \\ cname = name.camelize \\ \textbf{type} \ '\#\{name\}' \ , \ '() \ \rightarrow \#\{cname\} \ c', : \textbf{pure} \\ \textbf{type} \ '\#\{name\}=', \ '(\#\{cname\} \ i) \ \rightarrow \#\{cname\} \ o \ \{\#\{name\}==i\}' \\ true \\ \textbf{end} \\ \textbf{end} \end{array}
```

We use **pre**, an RDL method which defines a precondition contract, to define a code block (*i.e.*, an anonymous function) which will be executed on each call to belongs\_to. First, the block sets variables name and cname to be the string version of the first argument passed to belongs\_to and its camelized representation, respectively. In our example belongs\_to : folder call, name and cname will be set to 'folder' and 'Folder', respectively. Then, we define the type annotations for the methods referred to by name and name=. Finally, we trivially return true so that the contract will always pass. In our example, the following two specifications will be generated:

```
type 'folder', '() \rightarrow Folder c', :pure type 'folder=', '(Folder i) \rightarrow Folder o {folder == i}'
```

These annotations will be generated when belongs\_to is invoked with the : folder argument, which happens exactly after the **UserFile** class is loaded. Thus, verification of the initial move method succeeds. Even though folder = is not labeled as pure (so it does not satisfy congruence), its postcondition exactly captures the verification requirement of move that folder == target.

In sum, with just-in-time specification we can verify methods in the face of metaprogramming. We use dynamically provided information to generate expressive annotations at runtime for methods that do not even exist statically.

#### 4 From Ruby to Rosette

In this section we formally describe our verifier and the translation from Ruby to Rosette programs. We start (§ 4.1) by defining  $\lambda^{RB}$  as the subset of Ruby that is important in our translation extended with refinement type specifications. We achieve the translation to Rosette by first translating  $\lambda^{RB}$  to an intermediate language  $\lambda^{I}$  (§ 4.2). Then (§ 4.3), we discuss how  $\lambda^{I}$  maps to a Rosette program. Finally (§ 4.5), we use this translation to construct a verifier for Ruby programs.

```
Constants
                            c := \mathtt{nil} \mid \mathtt{true} \mid \mathtt{false} \mid 0, 1, -1, \dots
                            e := c \mid x \mid x := e \mid \text{if } e \text{ then } e \text{ else } e \mid e ; e
   Expressions
                                | self | f | f := e | e.m(\overline{e}) | A.new | return(e)
                            t ::= \{x : A|e\}
Refined Types
                           P ::= \cdot \mid d, P \mid a, P
        Program
      Definition
                            d::=\operatorname{def}\ A.m(\overline{t})::t;l=e
    Annotation
                            a ::= A.m :: (\bar{t}) \to t ; l
                                                                                   with l \neq \texttt{exact}
             Labels
                            l ::= \mathtt{exact} \mid \mathtt{pure} \mid \mathtt{protects}[\overline{x.f}]
               x \in \text{var ids}, f \in \text{field ids}, m \in \text{meth ids}, A \in \text{class ids}
```

Fig. 1. Syntax of the Ruby Subset  $\lambda^{RB}$ .

Fig. 2. Syntax of the Intermediate Language  $\lambda^{I}$ .

## 4.1 Core Ruby $\lambda^{RB}$ & Intermediate Representation $\lambda^{I}$

 $\lambda^{RB}$  Figure 1 defines  $\lambda^{RB}$ , a core Ruby-like language with refinement types. Constants consist of nil, booleans, and integers. Expressions include constants, variables, assignment, conditionals, sequences, and the reserved variable self which refers to a method's receiver. Also included are references to an instance variable f, instance variable assignment, method calls, constructor calls A.new which create a new instance of class A, and return statements.

Refined types  $\{x:A|e\}$  refine the basic type A with the predicate e. The basic type A is used to represent both user defined and built-in classes including nil, booleans, integers, floats, etc. The refinement e is a pure, boolean valued expression that may refer to the refinement variable x of type A. In the interest of greater simplicity for the translation, we require that self does not appear in refinements e; however, extending the translation to handle this is natural, and our implementation allows for it. Sometimes we simplify the trivially refined type  $\{x:A|\text{true}\}$  to just A.

A program is a sequence of method definitions and type annotations over methods. A method definition  $\operatorname{def} A.m(\{x_1:A_1|e_1\},\ldots,\{x_n:A_n|e_n\})::t;l=e$  defines the method A.m with arguments  $x_1,\ldots,x_n$  and body e. The type

specification of the definition is a dependent function type: each argument binder  $x_i$  can appear inside the arguments' refinements types  $e_j$  for all  $1 \leq j \leq n$ , and can also appear in the refinement of the result type t. A method type annotation  $A.m:(\bar{t})\to t$ ; l binds the method named A.m with the dependent function type  $(\bar{t})\to t$ .  $\lambda^{RB}$  includes both method annotations and method definitions because annotations are used when a method's code is not available, e.g., in the cases of library methods, mixins, or metaprogramming.

A label l can appear in both method definitions and annotations to direct the method's translation into Rosette (as described in § 2.4). The label exact states that a called method will be exactly translated by using the translation of the body of the method. Since method type annotations do not have a body, they cannot be assigned the exact label. The pure label indicates that a method is pure and thus can be translated using an uninterpreted function. Finally, the protects [x.f] label is used when a method is impure. This means that the method may modify its inputs. The list of fields of method arguments given by  $\overline{x.f}$  captures all the argument fields which the method does not modify, information which we can use when translating the method call. Thus, our default assumption (when the list given to protects is empty) is imprecise but sound because it assumes all argument fields are modified.

 $\lambda^I$  Figure 2 defines  $\lambda^I$ , a core verification-oriented language that easily translates to Rosette (§ 4.3).  $\lambda^I$  maps objects to functional structures and methods to function calls. Concretely,  $\lambda^{RB}$  objects map to a special object struct type and  $\lambda^I$  provides primitives for creating, altering, and referencing instances of this type. Values in  $\lambda^I$  consist of constants c (defined identically as in  $\lambda^{RB}$ ) and object  $(i_1, i_2, [f_1 \ w_1] \dots [f_n \ w_n])$ , an instantiation of an object struct with class ID  $i_1$ , object ID  $i_2$ , and where each field  $f_i$  of the object is bound to value  $w_i$ . Expressions consist of let bindings (let  $([x_i \ u_i])$  in u) where each  $x_i$  may appear free in  $u_j$  if i < j, function calls, assert, assume, and return statements. They also include havoc(x.f), which mutates x's field f to a fresh symbolic value. Finally, they include field assignment x.f := u and field reads x.f, which respectively set and get the field f of the object x.

A program is a series of definitions and verification queries. A definition is a function definition, or a symbolic definition  $\operatorname{define-sym}(x, A)$ , where if A is a Rosette solvable type like boolean or integer, x gets bound to a new symbolic value of that type. Otherwise, x gets bound to a new object with symbolic fields defined depending on the type of A. Finally, a verification query  $\operatorname{verify}(\overline{u} \Rightarrow u)$  checks the validity of u assuming  $\overline{u}$ .

## 4.2 From $\lambda^{RB}$ to $\lambda^{I}$

Figure 3 defines the translation function  $e \rightsquigarrow u$  that maps expressions (and programs) from  $\lambda^{RB}$  to  $\lambda^{I}$ .

 $e \leadsto u$ 

Program Translation

 $P \leadsto Q$ 

Fig. 3. Translation from  $\lambda^{RB}$  to  $\lambda^{I}$ . For simplicity rules T-Pure1 and T-IMPURE1 assume single argument methods.

Global States The translation uses sets  $\mathcal{M}$ ,  $\mathcal{U}$ , and  $\mathcal{F}$ , to ensure all the methods, uninterpreted functions, and fields are well-defined in the generated  $\lambda^I$  term:

$$\mathcal{M} ::= A_1.m_1, \dots, A_n.m_n \quad \mathcal{U} ::= A_1.m_1, \dots, A_n.m_n \quad \mathcal{F} ::= f_1, \dots, f_n$$

In the translation rules, we use the standard set operations  $x \in \mathcal{X}$  and  $|\mathcal{X}|$  to check membership and the size of the set  $\mathcal{X}$ . Thus, the translation relation is actually defined over these sets:  $\mathcal{M}, \mathcal{U}, \mathcal{F} \vdash e \leadsto u$ . Since the rules do not modify these environments, in Figure 3 we simplify the rules to  $e \leadsto u$ . Note that the treatment of these sets is bidirectional: the translation rules assume that these sets are properly guessed, but in an algorithmic definition the rules are also used to construct the sets.

Expressions The rules T-Const and T-Var are identity while the rules T-If, T-Seq, T-Ret, and T-VarAssn are trivially inductively defined. The rule T-Self translates self into the special variable named self in  $\lambda^I$ . The self variable is always in scope, since each  $\lambda^{RB}$  method translates to a  $\lambda^I$  function with an explicit first argument named self. The rules T-Inst and T-InstAssn translate a reference from and an assignment to the instance variable f, to a respective reading from and writing to the field f of the variable self. Moreover, both the rules assume the field f to be in global field state  $\mathcal{F}$ . The rule T-New translates from a constructor call f and f and object instance. The classId(f function in the premise of this rule returns the class ID of f and f the freshID(f predicate ensures the new object instance has a fresh object ID. Each field of the new object, f is initially bound to nil.

Method Calls To translate the  $\lambda^{RB}$  method call  $e_F.m(\overline{e})$  we first use the function  $\mathsf{typeOf}(e_F)$  to  $\mathsf{type}\ e_F$  via RDL type checking [30]. If  $e_F$  is of type A we split cases of the method call translation based on the value of  $\mathsf{labelOf}(A.m)$ , the label specified in the annotation of A.m (as informally described in § 2.4).

The rule T-EXACT is used when the label is exact. The receiver  $e_F$  is translated to  $u_F$  which becomes the first (i.e., the self) argument of the function call to  $A_{-}m$ . Finally, A.m is assumed to be in the global method name set  $\mathcal{M}$  since it belongs to the transitive closure of the translation.

We note that for the sake of clarity, in the T-Purel and T-Impurel rules, we assume that the method A.m takes just one argument; the rules can be extended in a natural way to account for more arguments. The rule T-Purel is used when the label is pure. In this case, the call is translated as an invocation to the uninterpreted function A.m, so A.m should be in the global set of uninterpreted functions  $\mathcal{U}$ . Finally, the specification  $\operatorname{specOf}(A.m)$  of the method is enforced. Let  $(\{x:A_x|e_x\}) \to \{r:A_r|e_r\}$  be the specification. We assume that the binders in the specification are  $\alpha$ -renamed so that the binders x and r are fresh. We use the fresh  $\lambda^I$  binders x and r to bind the argument and the result respectively to ensure, via A-normal form conversion [33], that they will be evaluated exactly once, even though x and r they may appear many times in the refinements. To enforce the specification, we assert the method's precondition  $e_x$  and assume the postcondition  $e_r$ .

If a method is labeled with  $\mathtt{protects}[p]$  then the rule T-IMPURE1 is applied. Since the method is not pure, it cannot be encoded as an uninterpreted function. Instead, we locally define a new symbolic object as the return value, and we havout the fields of all arguments (including self) which are not in the  $\mathtt{protects}$  label, thereby assigning these fields to new symbolic values. Since we do not translate the called method at all, no global state assumptions are made.

Programs Finally, we use the translation relation to translate programs from  $\lambda^{RB}$  to  $\lambda^I$ , i.e.,  $P \leadsto Q$ . The rule T-ANN translates type annotations, by ignoring the annotations. The rule T-DEF translates a method definition for A.m to the function definition  $A_{-}m$  that takes the additional first argument self. The rule also considers the declared type of A.m and instantiates a symbolic value for every input argument. Finally, all refinements from the inputs and output of the method type are translated and the derived verification query is made.

## 4.3 From $\lambda^I$ to Rosette

We write  $Q \to R$  to encode the translation from the  $\lambda^I$  program Q to the Rosette program R. This translation is straightforward, since  $\lambda^I$  consists of Rosette extended with some macros to encode Ruby-verification specific operators, like define-sym and return. In fact, in the implementation of the translation (§ 5) we used Racket's macro expansion system to achieve this final transformation.

Handling objects  $\lambda^I$  contains multiple constructs for defining and altering objects, which are expanded in Rosette to perform the associated operations over object structs. object $(i_c, i_o, \overline{[fw]})$  binds x to a new object with class ID  $i_c$ , object ID  $i_o$  and where each field  $f_i$  of x is bound to  $u_i$ . define-sym(x, A) creates a new symbolic value bound to x. If A is one of Rosette's solvable types, x is bound to a symbolic value of type A. Otherwise, a new object is created with fields instantiated with symbolic objects of the appropriate type. Finally, havoc(x.f) is expanded to mutate all the f field of x to a new symbolic object of the appropriate type.

Control Flow Macro expansion is used to translate both return and assume in Rosette. In order to encode return, every function definition in  $\lambda^I$  is expanded to keep track of a local variable ret, which is initialized to a special undefined value, and is returned at the end of the function. Every statement return(e) is transformed to update the value of ret to e. Then, every expression u in a function is expanded to unless-done(u). unless-done is a function that checks the value of ret. If ret is undefined (i.e., nothing has been returned yet) then we proceed with executing u. Otherwise (i.e., something has been returned) u is not executed.

We used the encoding of return to encode more operators. For example, assume is encoded in Rosette as a macro that returns a special fail value when assumptions do not hold. The verification query then needs to be updated with the condition that fail is not returned. Similarly, in the implementation, we used macro expansion to encode and propagate exceptions.

#### 4.4 Primitive Types

The core language of  $\lambda^{RB}$  provides constructs for functions, assignments, control flow, etc., but does not provide the theories required to encode interesting verification properties, that for example reason about booleans and numbers. On the other hand, Rosette is a verification oriented language with special support for common theories over built-in datatypes, including booleans, numeric types, and vectors. To bridge this gap, we encode certain Ruby expressions, such as constants c in  $\lambda^{RB}$ , into Rosette's respective built-in datatypes.

Equality and Booleans To precisely reason about equality, we encode Ruby's == method over arbitrary objects using Rosette's equality operator equal? to check equality of objects' IDs. We encode Ruby's booleans and operations over them as Rosette's respective boolean data and operations.

Integers and Floats By default, we encode Ruby's infinite-precision Integer and Float objects as Rosette's built-in infinite-precision integer and real datatypes, respectively. The infinite-precision encoding is efficient and precise, but it may result in undecidable queries involving non-linear arithmetic or loops, for example. To reason in such cases we provide the configuration option of encoding Ruby's integers as Rosette's built-in finite sized bitvectors that come equipped with arithmetic operators.

Arrays Finally, we provide a special encoding for Ruby's arrays, since arrays in Ruby are commonly used both for storing arbitrarily large random-access data, and also for representing mixed-type tuples, stacks, queues, etc.. We encode Ruby's arrays as a Rosette struct composed of a fixed-size vector and an integer that represents the current size of the Ruby array. Because we used fixed-size vectors, we can only perform bounded verification over array operations. We chose to explicitly preserve the vector's size as a Rosette integer to get more efficient and precise reasoning when we index or loop over vectors.

## 4.5 Verification of $\lambda^{RB}$

We define a verification algorithm RTR<sup> $\lambda$ </sup> that, given a  $\lambda^{RB}$  program P checks if it is safe, i.e., all the definitions satisfy the specifications. The pseudo-code for this algorithm is shown below.

```
\begin{array}{l} \operatorname{def} \, \mathsf{RTR}^\lambda(P) \\ (\mathcal{F}, \,\, \mathcal{U}, \,\, \mathcal{M}) \, := \mathsf{guess}(P) \\ \operatorname{for} \,\, (f \in \mathcal{F}) \colon \, \mathsf{add} \,\, \mathsf{field} \,\, f \,\, \mathsf{to} \,\, \mathsf{object} \,\, \mathsf{struct} \\ \operatorname{for} \,\, (u \in \mathcal{U}) \colon \,\, \mathsf{define} \,\, \mathsf{uninterpreted} \,\, \mathsf{function} \,\, u \\ P \leadsto Q \twoheadrightarrow R \\ \operatorname{return} \,\, \mathsf{if} \,\, (\mathit{valid}(\mathsf{R})) \,\, \mathsf{then} \,\, \mathsf{SAFE} \,\, \mathsf{else} \,\, \mathsf{UNSAFE} \\ \mathsf{end} \end{array}
```

First, we guess the proper translation environments. In practice (as discussed in § 4.2) we use the translation of P to generate the minimum environments for which translation of P succeeds. We define an object struct in Rosette containing one field for each member of  $\mathcal{F}$ , and we define an uninterpreted function for each method in  $\mathcal{U}$ . Next, we translate P to a  $\lambda^I$  program Q via  $P \leadsto Q$  (§ 4.2) and Q to a the Rosette program R, via  $Q \twoheadrightarrow R$  (§ 4.3). Finally, we run the Rosette program R. The initial program P is safe iff the Rosette program R is valid, i.e., all the verify queries are valid.

We conclude this section with a discussion of the RTR $^{\lambda}$  verifier.

 $RTR^{\lambda}$  is Partial There exist expressions of  $\lambda^{RB}$  that fail to translate into a  $\lambda^{I}$  expression. The translation requires at each method call  $e_{F}.m(\overline{e})$  that the receiver has a class type A (i.e., typeOf( $e_{F}$ ) = A). There are three cases when this requirement fails 1)  $e_{F}$  has a union type, 2)  $e_{F}$  is ni1, 3) type checking fails and so  $e_{F}$  has no type. In our implementation (§ 5), we extend the translation to the first two cases. Handling for 1) is outlined in § 2.4. In case 2), we treat the receiver as self, matching the Ruby semantics. Case 3) can be caused by either a type error in the program or a lack of typing information for the type checker. In both cases translation cannot proceed.

 $RTR^{\lambda}$  may Diverge The translation to Rosette always terminates. All translation rules are inductively defined: they only recurse on syntactically smaller expressions or programs. Also, since the input program is finite, the minimum global environments required for translation are also finite. Finally, all the helper functions (including the type checking typeOf(·)) do terminate.

Yet, verification may diverge, as the execution of the Rosette program may diverge. Specifications can encode arbitrary expressions, thus it is possible to encode undecidable verification queries. Consider the following contrived Rosette program in which we attempt to verify an assertion over a recursive method:

```
(define (rec x) (rec x))
(define-symbolic b boolean?)
(verify (rec b))
```

Rosette attempts to symbolically evaluate this program, and thus diverges.

 $RTR^{\lambda}$  is Incomplete Verification is incomplete and its precision relies on the precision of the specifications. For instance, if a pure method A.m is marked as impure, the verifier will not prove the congruence property  $\forall x: A.m(x) = A.m(x)$ .

 $RTR^{\lambda}$  is Sound If the verifier decides that the input program is safe, then all definitions satisfy their specifications, assuming that (1) all the refinements are pure boolean expressions and (2) all the labels are sounds. The assumption (1) is required since verification under diverging (let alone effectful) specifications is difficult [41]. The assumption (2) is required since our translation encodes pure methods as uninterpreted functions while for the impure methods it only havoes the unprotected arguments. Checking these assumptions is left as future work.

#### 5 Evaluation

We implemented the Ruby refinement type checker RTR by extending RDL [30] with refinement types. Table 1 summarizes the evaluation of RTR.

Benchmarks To evaluate our technique, we used six popular Ruby libraries.

- Money [6] performs currency conversions over monetary quantities,
- BusinessTime [3] performs time calculations in business hours and days,
- Unitwise [7] performs various unit conversions,
- Geokit [4] performs calculations over locations on Earth,
- Boxroom [2] is a Rails app for sharing files in a web browser, and
- Matrix [5] is a Ruby standard library for matrix operations.

For verification we forked the original Ruby libraries and provided manually written method specifications in the form of refinement types. The forked repositories are publicly available [8].

We chose these libraries because they combine Ruby-specific features challenging for verification, like metaprogramming and mixins, with arithmetic heavy operations. In all libraries we verify both 1) functional correctness of arithmetic operations, e.g., no division-by-zero, the absolute value of a number should not be negative, and 2) data specific arithmetic invariants, e.g., integers representing months should always be in the range from 1 to 12, a data value added to an aggregate should always fall between maintained  ${\tt Qmin}$  and  ${\tt Qmax}$  fields. In the  ${\tt Matrix}$  library, we also verify code with heavy array operations, since matrices are implemented as an array of arrays. Concretely, we verify of a matrix multiplication method that, if we multiply a matrix with r rows by a matrix with c columns, the result will be a matrix of size  $r \times c$ .

Table 1. Evaluation of RTR. Methods is the number of methods verified. Ruby LoC is the sum total and range of the LoC per method of the verified methods and Rosette LoC is the sum total and range of the LoC per method of the Rosette translation. Verification Time is the median and semi-interquartile range of the total verification time in seconds for 11 runs. Spec is the number of total type specifications required for verification. Experiments were conducted on a 2014 Macbook Pro with a 3 GHz Intel Core i7 processor and 16 GB memory.

App	Methods	Rub	y LoC	Rosette LoC	Verification Time		Spec
					Time(s)	SIQR	
Money	7	43	[5, 11]	197 [24, 40]	23.24	0.29	10
BusinessTime	10	76	[5, 12]	337 [26, 58]	34.00	0.16	24
Unitwise	6	24	[4,4]	153 [22, 27]	19.51	0.21	6
Geokit	6	59	[5, 26]	246 [26, 68]	22.63	0.07	21
Boxroom	1	12	[12, 12]	34 [34, 34]	3.28	0.04	3
Matrix	1	57	[57, 57]	94 [94, 94]	334.35	3.99	9
Total	31	271	[4, 57]	1061 [22, 94]	437.01	4.76	73

Quantitative Evaluation Table 1 summarizes our evaluation in numbers. For each application we list the number of verified **Methods**. RTR acts at a method-level

granularity, that is, the programmer can specify exactly which methods should be verified, and when they should be verified (e.g., immediately when it is defined, prior to executing the method when it is called, or at a customized time). Yet, verified methods co-exist with (and can even call) the non-verified, modularly used methods. We only verified methods with interesting arithmetic properties.

The Ruby LoC and Rosette LoC columns give the sizes of the verified Ruby programs and translated Rosette programs, respectively. These metrics show the sum total lines of code (LoC) over all verified Ruby programs and resulting Rosette programs, followed by the range of LoC over these programs. For example, for the Money app, the total Ruby LoC verified, over all 7 methods, was 43, with the smallest method consisting of 5 LoC and the largest consisting of 11. For each method verified, RTR generates a separate Rosette program. We give the sizes of these programs and their sum in the Rosette LoC column. Unsurprisingly, the LoC of the Rosette generated program increases with the size of the source Ruby program.

We present the median (Time(s)) and semi-interquartile range (SIQR) of the verification time required to verify all methods for an application over 11 runs. For each application, the SIQR fell under 2% of the verification time, indicating relatively little variance in the time taken to verify all methods over 11 runs. Notably, the time taken to verify the Matrix multiplaction method was significantly higher than methods verified in other libraries. In general, RTR takes longer to verify assertions involving heavy array operations. This is magnified by the Matrix library, which operates over two-dimensional arrays.

Finally, Table 1 lists the number of type **specifications** required to verify an application's methods. These are comprised of method type annotations, including the annotations for the verified methods themselves, and variable type annotations for instance variables. Not listed, but nevertheless crucial to verification, are the number of type annotations from Ruby's standard and core libraries that are used; RDL includes an extensive list of annotations for Ruby library methods, so we did not need to write these manually for verification.

Notably, there is a high level of variation in the number of annotations required for each application. For example, in Unitwise we verified 6 methods and only required 6 annotations (one for each verified method), while in Geokit we verified 6 methods and required 21 annotations. The large variation results from the differing natures of the methods being verified. For each instance variable used in a method, and for each additional (non-standard/core library) method called, the programmer must provide an additional annotation. These requirements can account for large variation in number of annotations needed.

#### 5.1 Case Study

Next we illustrate the RTR verification process by presenting the exact steps required to specify and check the properties of a method from an existing Ruby library. For this process, we chose to verify the << method of the Aggregate library [1], a Ruby library for aggregating and performing statistical computations over some numeric data. The method << takes one input, data, and adds

it to the aggregate by updating (1) the minimum @min and maximum @max of the aggregate, (2) the count @count, sum @sum, and sum of squares @sum2 of the aggregate, and finally (3) the correct bucket in @buckets.

```
\begin{array}{l} \mbox{def} << \mbox{data} \\ \mbox{if} \ 0 == \mbox{@count} \\ \mbox{@min} = \mbox{data} \ ; \mbox{@max} = \mbox{data} \\ \mbox{else} \\ \mbox{@max} = \mbox{data} \ \mbox{if} \ \mbox{data} > \mbox{@max} \ ; \mbox{@min} = \mbox{data} \ \mbox{if} \ \mbox{data} < \mbox{@min} \\ \mbox{end} \\ \mbox{@count} \ += 1 \ ; \mbox{@sum} \ += \mbox{data} \ ; \mbox{@sum2} \ += \mbox{(data)} \\ \mbox{@buckets[to\_index(data)]} \ \ += 1 \ \mbox{unless outlier} \ ? \mbox{(data)} \\ \mbox{end} \end{array}
```

We specify functional correctness of the method << by providing a refinement type specification that declares that after the method is executed, the input data will fall between @min and @max.

```
type :<<, \label{eq:continuous} \begin{tabular}{ll} \begin{tabul
```

The type specification is stored in a global table and includes a user specified verification *alias*, in this case :bind. To verify the specification all we need to do is load the library and call the verifier with the method's verification alias:

```
rdl_do_verify: bind
```

Verification proceeds in three steps

- first use RDL to type check the basic types of the method,
- then translate the method to Rosette (using the translation of § 4), and
- finally run the Rosette program to check the validity of the specification.

At the current state, verification will fail in the first step, due to a lack of type information. Type checking will fail with the error

```
error: no type for instance variable '@count'
```

To fix this error, the user needs to provide the correct types for the instance variables using the below type annotations.

```
var_type :@count, 'Integer'
var_type :@min, :@max, :@sum, :@sum2, 'Float'
var_type :@buckets, 'Array<Integer>'
```

The << method also calls two methods which are *not* from Ruby's standard and core libraries: to\_index, which takes a numeric input and determines the index of the bucket the input falls in, and outlier?, which determines if the given data is an outlier based on provided specifications from the programmer. These methods provide challenging obstacles to verification. For example, the to\_index method makes use of non-linear arithmetic in the form of logarithms, and loops over arrays. Yet, neither of the calls to\_index or outlier? should affect verification of the specification of <<. So, it suffices to provide type annotations with a pure label, indicating we want to use uninterpreted functions to represent them:

```
type : outlier ?, '(Float i) \rightarrow Bool b', :pure type :to_index, '(Float i) \rightarrow Integer out', :pure
```

Given these annotations, the verifier has enough information to prove the post-condition on <<, and it will return the message to the user:

Aggregate instance method << is safe.

When verification fails, an unsafe message is provided, combined with a counterexample consisting of bindings to symbolic values which causes the postcondition to fail. For instance, if the programmer *incorrectly* specified that data is less than the Qmin, *i.e.*,

```
type :<<, '(Integer data) → Integer { data < @min }'
```

Then RTR would return the following message:

```
Aggregate instance method << is unsafe. Counterexample: (model [real_data 0][real_0min 0] ...)
```

This gives a binding to symbolic values in the translated Rosette program which would cause the specification to fail. We only show the bindings relevant to the specification here: when real\_data and real\_@min, the symbolic values corresponding to data and @min respectively, are both 0, the specification fails.

#### 6 Related Work

Verification for Ruby on Rails. Several prior systems can verify properties of Rails apps. Space [26] detects security bugs in Rails apps by using symbolic execution to generate a model of data exposures in the app and reporting a bug if the model does not match common access control patterns. Bocić and Bultan proposed symbolic model extraction [14], which extracts models from Rails apps at runtime, to handle metaprogramming. The generated models are then used to verify data integrity and access control properties. Rubicon [25] allows programmers to write specifications using a domain-specific language that looks similar to Rails tests, but with the ability to quantify over objects, and then checks such specifications with bounded verification. Rubyx [15] likewise allows programmers to write their own specifications over Rails apps and uses symbolic execution to verify these specifications.

In contrast to RTR, all of these tools are specific to Rails and do not apply to general Ruby programs, and the first two systems do not allow programmers to specify their own properties to be verified.

Rosette. Rosette has been used to help establish the security and reliability of several real-world software systems. Pernsteiner et al. [27] use Rosette to build a verifier to study the safety of the software on a radiotherapy machine. Bagpipe [42] builds a verifier using Rosette to analyze the routing protocols used by Internet Service Providers (ISPs). These results show that Rosette can be applied in a variety of domains.

Types For Dynamic Languages. There have been a number of efforts to bring type systems to dynamic languages including Python [10, 12], Racket [36, 37], and JavaScript [11, 23, 35], among others. However, these systems do not support refinement types.

Some systems have been developed to introduce refinement types to scripting and dynamic languages. Refined TypeScript (RSC) [40] introduces refinement types to TypeScript [13, 29], a superset of JavaScript that includes optional static typing. RSC uses the framework of Liquid Types [31] to achieve refinement inference. Refinement types have been introduced [21] to Typed Racket as well. As far as we are aware, these systems do not support mixins or metaprogramming.

General Purpose Verification Dafny [22] is an object-oriented language with built-in constructs for high-level specification and verification. While it does not explicitly include refinement types, the ability to specify a method's type and pre- and postconditions effectively achieves the same level of expressiveness. Dafny also performs modular verification by using a method's pre- and postconditions and labels (distinguishes between function and method definitions) indicating its purity or arguments mutated, an approach we largely emulate here. However, unlike Dafny, RTR leaves this modular treatment of methods as an option for the programmer. Furthermore, unlike RTR, Dafny does not include features such as mixins and metaprogramming.

#### 7 Conclusion and Future Work

We formalized and implemented RTR, a refinement type checker for Ruby programs using assume-guarantee reasoning and the just-in-time checking technique of RDL. Verification at runtime naturally adjusts standard refinement types to handle Ruby's dynamic features, such as metaprogramming and mixins. To evaluate our technique we used RTR to verify numeric properties on six commonly used Ruby and Ruby on Rails applications, simply by adding refinement type specifications to the existing method definitions.

Our work opens new directions for further Ruby verification. Currently the verifier trusts annotation labels that claim purity and immutability. Verification of the labels per se would provide better insight into Ruby methods. Furthermore, (refinement) type inference is a feature missing from our system, crucial for usability. Our plan is to investigate how standard inference techniques, like Hindley-Milner and liquid typing [31], adjust to just-in-time typing. We will also explore how Rosette's constructs for synthesis may be used toward the goal of refinement type inference. We also plan to extend the expressiveness of the system by adding support for loop invariants and dynamically defined instance variables, among other Ruby constructs. Finally, as Ruby is commonly used in the Ruby on Rails framework, we plan to extend RTR with modeling for webspecific constructs such as access control protocols and database operations in order to further support verification in the domain of web applications.

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