

Liquid Haskell

Haskell as a Theorem Prover

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Software bugs are everywhere



On a more serious note, software bugs can lead to deaths, which is what happened in May 2015, when Airbus crashed in Spain due to a bug in the software control engine.

4 deaths, Spain, software control

Airbus A400M crashed due to a software bug.
— May 2015

Software bugs are everywhere



Heartbleed is another bug that got publicity last year, partially because of its pretty logo.

that due to buffer overflow attackers could get access to encrypted information.

Information protected by the SSL protocol is stored next to unprotected information. Due to buffer overflow attackers can steal encrypted information, like user passwords.

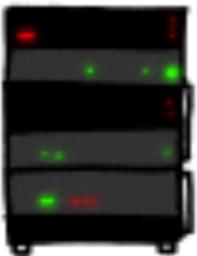
The Heartbleed Bug.
Buffer overflow in OpenSSL. 2015

HOW THE HEARTBLEED BUG WORKS:

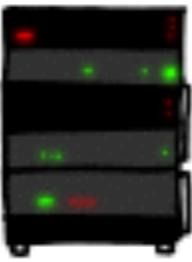
SERVER, ARE YOU STILL THERE?
IF SO, REPLY "POTATO" (6 LETTERS).



User Eric wants pages about "boats". User Erica requests secure connection using key "4538538374224". User Meg wants these 6 letters: POTATO. User Ada wants pages about "irl games". Unlocking secure records with master key 5130985733435. Macie (chrome user) sends this message: "H



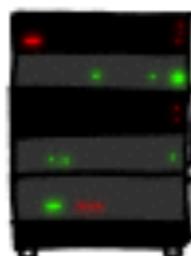
POTATO



SERVER, ARE YOU STILL THERE?
IF SO, REPLY "BIRD" (4 LETTERS).



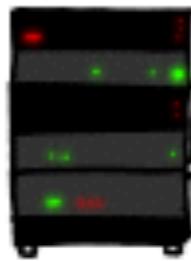
User Olivia from London wants pages about "new bees in car why". Note: Files for IP 375.381.283.17 are in /tmp/files-3843. User Meg wants these 4 letters: BIRD. There are currently 345 connections open. User Brendan uploaded the file selfie.jpg (contents: 834ba962e2ceb9ff89bd3bfff84)



HMM...



BIRD



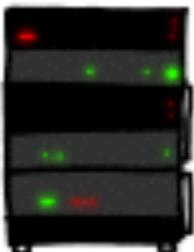
SERVER, ARE YOU STILL THERE?

a connection. Jake requested pictures of deer. User Meg wants these 500 letters: HAT. Lucas

SERVER, ARE YOU STILL THERE?
IF SO, REPLY "HAT" (500 LETTERS).

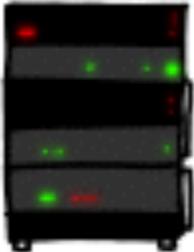


a connection. Jake requested pictures of deer.
User Meg wants these 500 letters: HAT. Lucas
requests the "missed connections" page. Eve
(administrator) wants to set server's master
key to "14835038534". Isabel wants pages about
"snakes but not too long". User Karen wants to
change account password to "CoHoBaSt". User



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Make bugs difficult to express

Using Modern Programming Languages

F#, Ocaml, Scala, Haskell

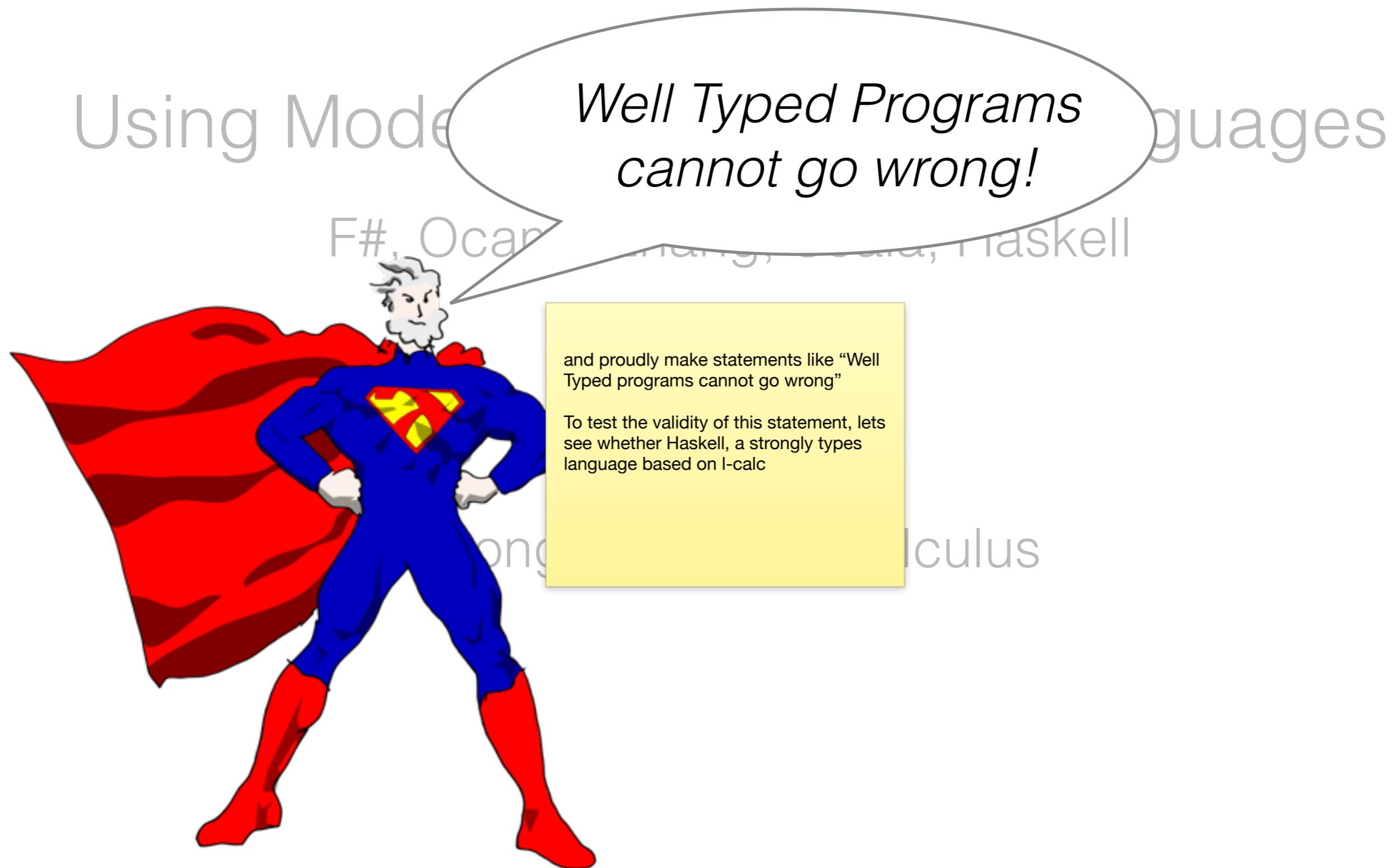
Because of
Strong Types + λ -Calculus

The goal of research in programming languages is not only to track these bugs, but rather make such kind of software bugs difficult to express.

So, these days we use fancy FPL like that make heavy use

Towards this goal, PL researchers have built fancy functional programming languages such as ... that make heavy use of the well studied theories of strong typing and lambda calculus

Make bugs difficult to express





can be used to encode Heartbleed





can be used to encode Heartbleed



I am importing data text, a library fir
unicode mania
I am creating a text niki
and will use a take function to index
pack .

Note that take comes from a library
marked as unsafe, so safety relies only
on type system and not on runtime
checks.

```
λ> :m +Data.Text Data.Text.Unsafe
```

```
λ> let pack = "hat"
```

```
λ> :t takeWord16
```

```
takeWord16 :: Text -> Int -> Text
```



VS.



If I ask for the True first characters
lambda man is smart enough to tell me
that True is an invalid argument for take

```
λ> :m +Data.Text Data.Text.Unsafe
```

```
λ> let pack = "hat"
```

```
λ> takeWord16 pack True
```

Type Error: Cannot match **Bool** vs **Int**



VS.



```
λ> :m +Data.Text Data.Text.U
```

```
λ> let pack = "hat"
```

```
λ> takeWord16 pack 500
```

“hat\58456\2594\SOH\NUL...

But, If I ask for the 8 first carachets I encoded HD take will return the 4 first carachets and 4 more characters that are stored in the memory next to Niki

My goal is to use types and statically catch this error



VS.



Valid Values for takeWord16?

the type of takes states that all ints are valid indexes

`takeWord16 :: t:Text -> i:Int -> Text`

All Ints

`..., -2, -1, 0, 1, 2, 3, ...`

Valid Values for takeWord16?

what i want is to split ints and specify
tats the valid arguments for take are
only ints that are less than the size of t

`takeWord16 :: t:Text -> i:Int -> Text`

Valid Ints

`0, 1 ... , len t`

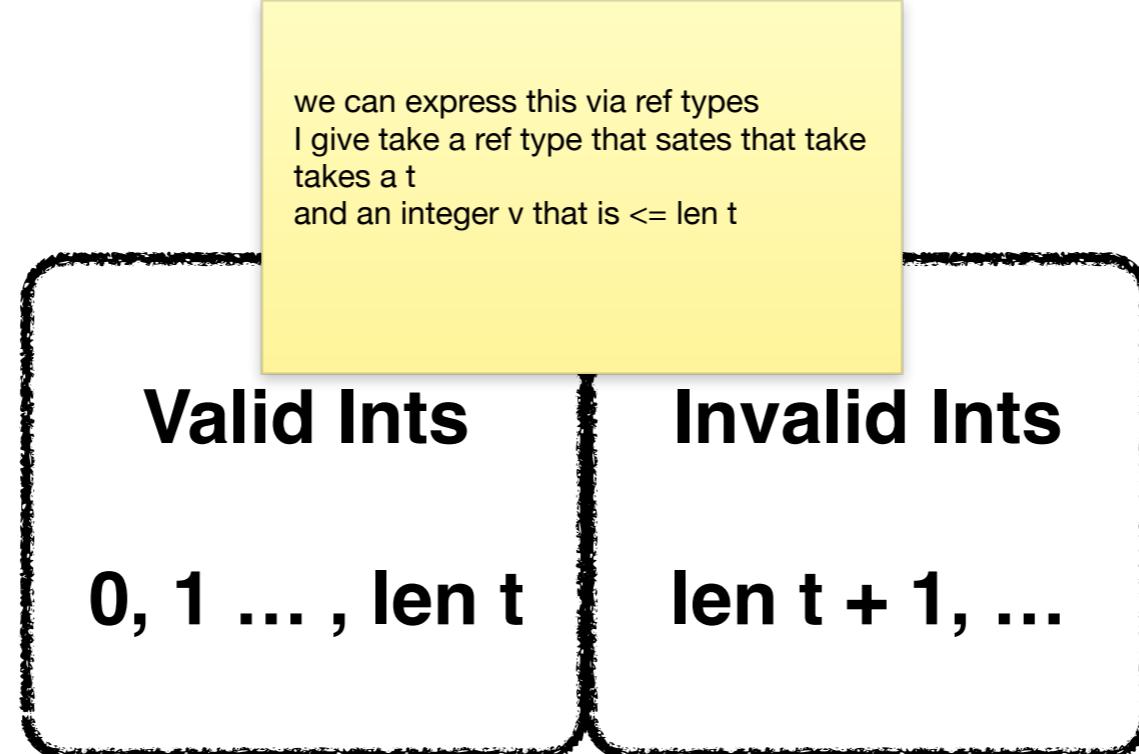
Invalid Ints

`len t + 1, ...`

Refinement Types

```
take :: t:Text -> {v:Int | v <= len t} -> Text
```

we can express this via ref types
I give take a ref type that states that take
takes a t
and an integer v that is $\leq \text{len } t$



Refinement Types

```
take :: t:Text -> {v:Int | v <= len t} -> Text
```

```
λ> :m +Data.Text Data.Text.
```

```
λ> let pack = "hat"
```

```
λ> take pack 500
```

```
Refinement Type Error
```

going back to the code, with the refined type for take, asking 8 elemns of Niki will return a refinement type error,

My goal is to allow this reasoning by embedding refinements into haskell's type system

Goal: Refinement Types for Haskell

Goal: Refinement Types for Haskell

specifically during my PhD I developed a sound and expressive refinement type system for Haskell programs and what is important is that I achieved both of these under the requirement of decidable and predictable verification

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

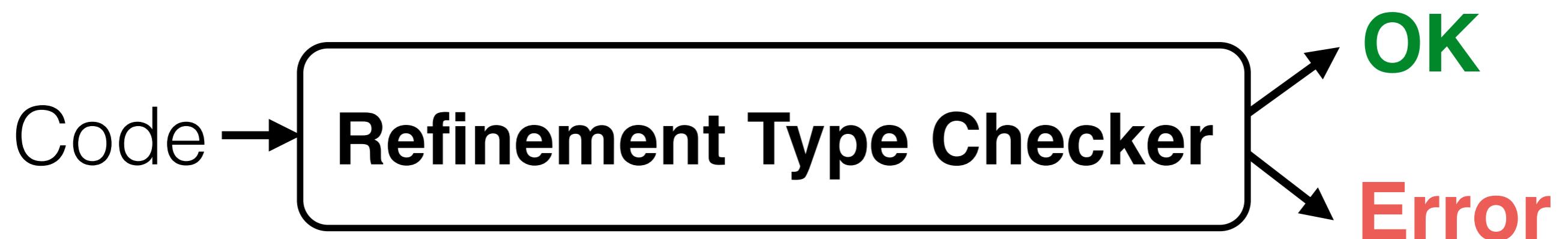
With automatic & predictable verification.

Lets start by building a refinement type checkers that is sound unders

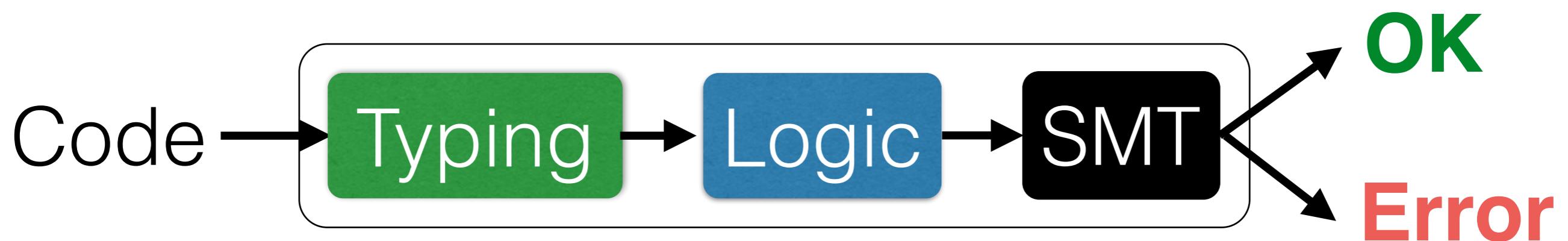
1. Soundness

Under Lazy Evaluation

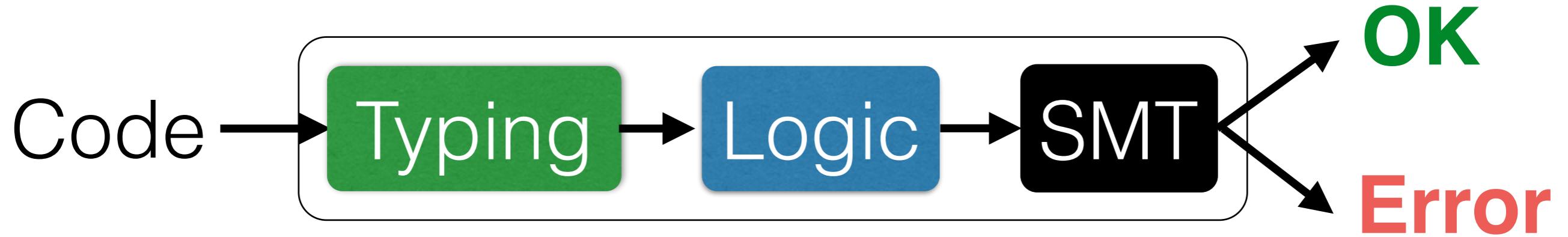
Refinement Typing 101



Refinement Typing 101



1. Source Code to **Type constraints**
2. **Type Constraints** to **Verification Condition (VC)**
3. Check **VC validity** with **SMT Solver**



```
take :: t:Text -> {v | v <= len t} -> Text
heartbleed = let x = "hat"
            in take x 500
```

Code → Typing

```
take :: t:Text -> {v | v <= len t} -> Text
heartbleed = let x = "hat"
             in take x 500
```

x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}

Code → Typing

```
take :: t:Text -> {v | v <= len t} -> Text
heartbleed = let x = "hat"
            in take x 500
```

x: {v | len v = 3} |- {v | v = 500} <: {v | v <= len x}

Code → Typing

```
take :: t:Text -> {v | v <= len t} -> Text
heartbleed = let x = "hat"
            in take x 500
```

$x : \{v \mid \text{len } v = 3\} \dashv \{v \mid v = 500\} \lhd \{v \mid v \leq \text{len } x\}$

Code → Typing

```
take :: t:Text -> {v | v <= len t} -> Text
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$x : \{v \mid \text{len } v = 3\} \dashv \{v \mid v = 500\} \lhd \{v \mid v \leq \text{len } x\}$

Code → Typing

```
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```

$x : \{v \mid \text{len } v = 3\} \dashv \{v \mid v = 500\} \lhd \{v \mid v \leq \text{len } x\}$



Encode Subtyping as Logical VC

If VC valid then Subtyping holds

Clearly state that on the left I have my environment



Encode Subtyping as Logical VC

$$x : \{v \mid \text{len } v = 3\} \dashv \vdash \{v \mid v = 500\} \lhd \{v \mid v \leq \text{len } x\}$$

$x : \{v \mid p\}$

$\{v \mid p\} \lhd \{v \mid q\}$





to encode an environment binding we use flanagan;s meaning that says

x: {v | p}

Means*: if x reduces to a value then $p[x/v]$

Encoded as: “ x has a value” $\Rightarrow p[x/v]$

* Flanagan “Hybrid Type Checking” (POPL ’06)



$$\{\textcolor{blue}{v \mid p}\} \lhd \{\textcolor{blue}{v \mid q}\}$$

Means: if $y : \{\textcolor{blue}{v \mid p}\}$ then $y : \{\textcolor{blue}{v \mid q}\}$

Encoded as: $p \Rightarrow q$

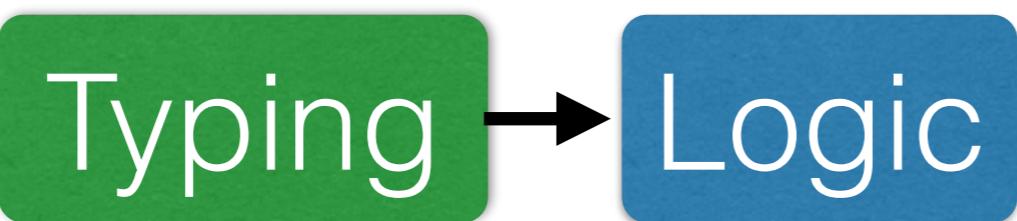


Encode Subtyping ...

```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
```

... as Logical VC

```
(“x has a value” => len x = 3)  
=> (v = 500) => (v <= len x)
```



Encode Subtyping ...

$x : \{v \mid \text{len } v = 3\}$ \vdash $\{v \mid v = 500\} <: \{v \mid v \leq \text{len } x\}$

... as Logical VC

(“ x has a value” \Rightarrow $\text{len } x = 3$)
 $\Rightarrow (v = 500) \Rightarrow (v \leq \text{len } x)$

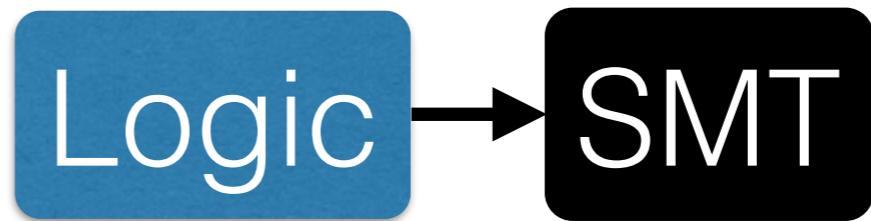


Encode Subtyping ...

$x : \{v \mid \text{len } v = 3\} \dashv \vdash \{v \mid v = 500\} <: \{v \mid v \leq \text{len } x\}$

... as Logical VC

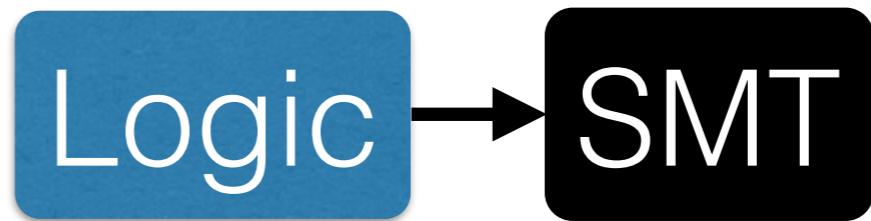
(“ x has a value” $\Rightarrow \text{len } x = 3$)
 $\Rightarrow (v = 500) \Rightarrow (v \leq \text{len } x)$



(“`x` has a value” \Rightarrow `len x = 3`)
 $\Rightarrow (\text{v} = 500) \Rightarrow (\text{v} \leq \text{len x})$

How to encode “`x` has a value” ?

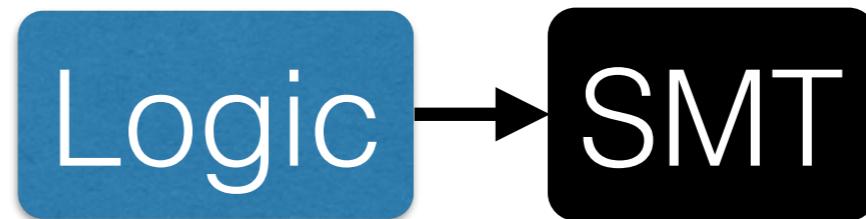
(In a decidable manner)



(“`x` has a value” $\Rightarrow \text{len } x = 3$)
 $\Rightarrow (v = 500) \Rightarrow (v \leq \text{len } x)$

Assumption: Binders Are Values!

i.e. `x` is guaranteed to be a value



len x = 3
=> (v = 500) => (v <= len x)

Assumption: Binders Are Values!

i.e. x is guaranteed to be a value



```
take :: t:Text -> {v | v <= len t} -> Text
heartbleed = let x = "hat"
            in take x 500
```

Checker soundly reports **Error**

Assumption: Binders must be values

Not true under Haskell's lazy semantics

Ignoring “has a value” is unsound!

Under Haskell’s lazy semantics

Ignoring “has a value” is unsound!

```
spin :: Int -> Int  
spin x = spin x
```

Ignoring “has a value” is unsound!

```
spin :: Int -> {v:Int | false}  
spin x = spin x
```

OK

As `spin` does not return any value

Ignoring “has a value” is unsound!

```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int | false}
```

```
heartbleed = let x = "hat"
              y = spin 0
            in take x 500
```

OK? or Error?

Ignoring “has a value” is unsound!

```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int | false}

heartbleed = let x = "hat"
            y = spin 0
            in take x 500
```

OK under **CBV** evaluation

Ignoring “has a value” is unsound!

```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int | false}

heartbleed = let x = "hat"
            y = spin 0
            in take x 500
```

Error under Lazy evaluation

Ignoring “has a value” is unsound!

Reports **Erroneous** code as **OK**

Ignoring “has a value” is unsound!

How to encode “has a value” ?

How to encode “has a value” ?

Most expressions provably reduce to a value

How to encode “has a value” ?

Most expressions provably reduce to a value

If x reduces to a value,

Then encode “x has a value” by **true**

Solution: Stratified Types

$x : \{v : \text{Int} \mid p\}$

Must reduce to a Value

$x : \{v : \text{Int}^{\uparrow} \mid p\}$

May-not reduce to a Value

Stratified Types to Logic

$x : \{v : \text{Int} \mid p\}$ encoded as $p[x/v]$

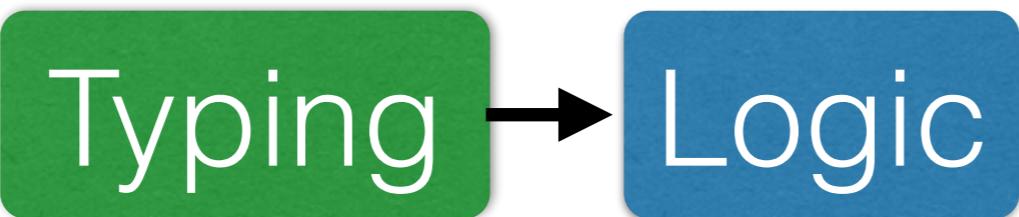
$x : \{v : \text{Int}^{\uparrow} \mid p\}$ encoded as true

Code → Typing

```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int↑ | false}
```

```
heartbleed = let x = "hat"
              y = spin 0
            in   take x 500
```

x:{v | len v=3}
y:{v:Int[↑] | false} |- {v | v=500} <: {v | v <= len x}



```
x:{v|len v=3}
y:{v:Int↑|false} |- {v|v=500} <: {v|v<=len x}
```

len x = 3

true $\Rightarrow v = 500 \Rightarrow v \leq \text{len } x$



len x=3

true

$\Rightarrow v = 500 \Rightarrow v \leq \text{len } x$



```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int↑ | false}

heartbleed = let x = pack "hat"
            y = spin 0
            in take x 500
```

How to enforce stratification?

How to enforce stratification?

$x : \{v : \text{Int} \mid p\}$

Must have a Value

Terminating expressions must have a value

Solution

Check termination with Refinement Types!

We saw how regain soundness under
lazy evaluation using type stratification

1. Soundness Under Lazy Evaluation

Goal: Refinement Types for ~~Tool~~~1~11

What is important is that this technique allows for decidable and predictable verification

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

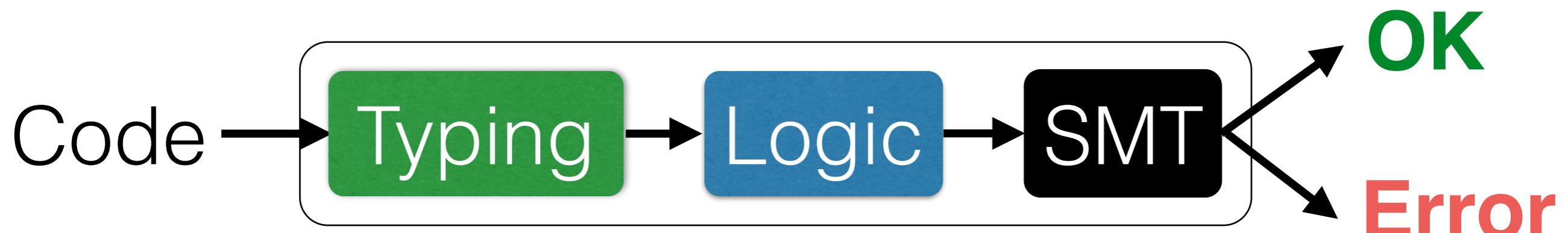
With automatic & predictable verification.

What is important is that this technique allows for decidable and predictable verification

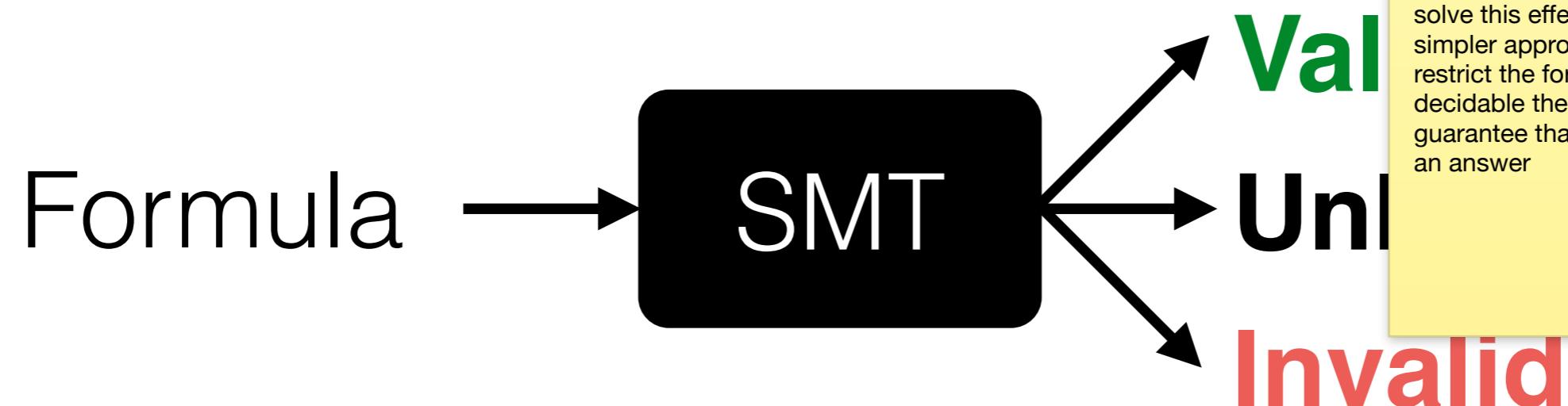
With automatic & predictable verification.

With automatic & predictable error detection.

Recall the structure of the type checker
that extracts the VC from types and
checks the verification of VC using an
SMT solver



With automatic & predictable verification.



An SMT solver is a black box that given a formula it can

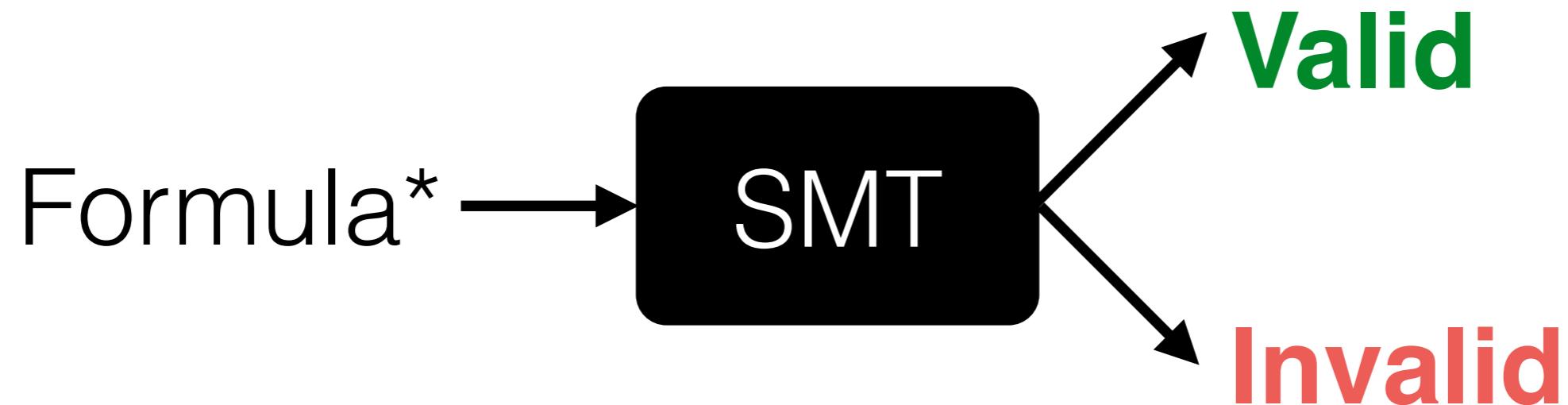
1. return valid if the formula is always true,
2. return unsat if the formula is always false, or
3. timeout with unknown

Timeouts lead to unpredictable behaviours. what Rustan calls the butterfly event: minor modification at one part of your programs can change the verification of another part

Rustan carefully selects triggers to solve this effect, while we chose a simpler approach that says that if we restrict the formula to efficient and decidable theories, then we have the guarantee that the SMT will return with an answer

With automatic & predictable verification.

How do we prove it's testable?



*From Decidable Logic



The VC comes via conjunctions and implications from the refinements of types, thus

Encode Subtyping ...

$$x_1 : \{v \mid p_1\}, \dots, x_n : \{v \mid p_n\} \vdash \{v \mid q_1\} <: \{v \mid q_2\}$$

... as Logical Verification Condition

$$p_1 \wedge \dots \wedge p_n \Rightarrow q_1 \Rightarrow q_{12}$$

For predictability, p from decidable theories

$$\{v:a \mid p\}$$

... as Logical Verification Condition

$$p_1 \wedge \dots \wedge p_n \Rightarrow q_1 \Rightarrow q_{12}$$

where dec theories are theories for which the SMT has dec & efficient procedures, like ...

For predictability, p from decidable theories

$$\{v:a \mid p\}$$

Boolean Logic

(QF) Linear Arithmetic

Uninterpreted Functions ...

For automatic & predictable verification.

where dec theories are theories for which the SMT has dec & efficient procedures, like ...

For predictability, p from decidable theories

{v:a | p }

Boolean Logic

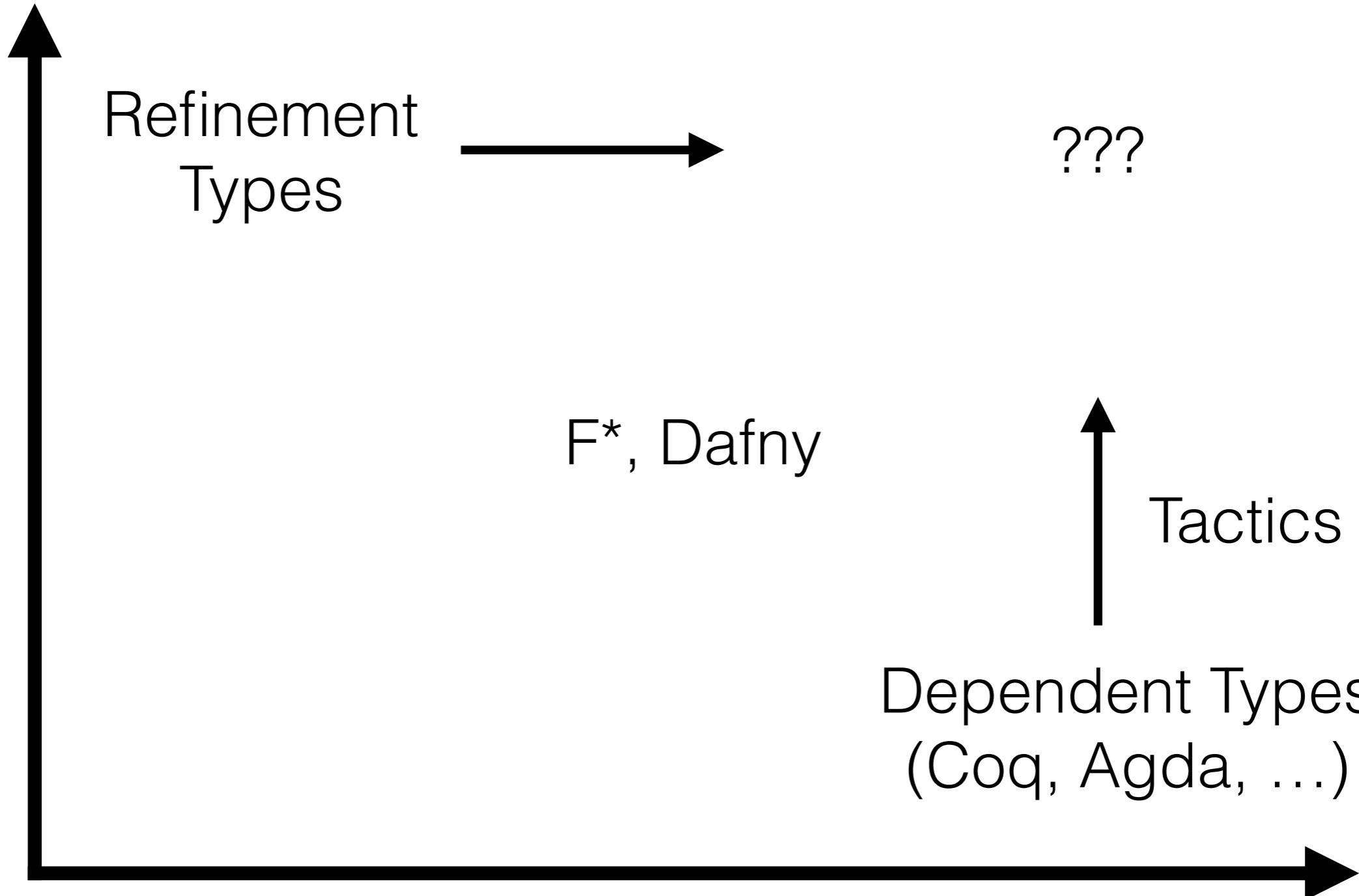
(QF) Linear Arithmetic

Uninterpreted Functions ...

For automatic & predictable verification.

What about expressiveness?

Automation



Expressiveness

Goal: Refinement Types for Haskell

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

With automatic & predictable verification.

Goal: Modular & Decidable Specification

As a first example, consider a function maximum that takes as input a list of ints and returns the maximum element of the list

```
maximum      :: [Int] -> Int
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

Client Verification

```
p = maximum [1, 2, 3]
assert (p > 0)

n = maximum [-1, -2, -3]
assert (n < 0)
```

Goal: Modular & Decidable Specifications

maximum [x] = x

maximum (x:xs) = let y = maximum xs in
if x < y then y else x

Specification?

Goal: Modular & Decidable Specifications

maximum
maximum

[x]
(x:xs)

= x
Let y = maximum xs in
if x < y then y else x

Modular
First-Order Specification

maximum : [{v:Int | 0 < v}] -> {v:Int | 0 < v}

Goal: Modular & Decidable Specifications

maximum
maximum

[x]
(x : xs)

=
let

y = maximum xs in
if $x < y$ then y else x

Higher-Order Specification
ensures \exists x in xs. r = x

Decidable

Observation:

maximum returns one of the input elements

Observation:

maximum returns one of the input elements

```
maximum :: [{v:Int | 0 < v}] -> {v:Int | 0 < v}
```

if all input elements are **positive**

then the result is **positive**

Observation:

maximum returns one of the input elements

```
maximum :: [{v:Int | v<0}] -> {v:Int | v<0}
```

if all input elements are **negative**
then the result is **negative**

Observation:

`maximum` returns one of the input elements

```
maximum :: [{v:Int | p v}] -> {v:Int | p v}
```

if all input elements satisfy `p`

then the result satisfies `p`

Observation:

maximum returns one of the input elements

maximum:: $\forall p. [\{v: \text{Int} \mid p v\}] \rightarrow \{v: \text{Int} \mid p v\}$

for all p

if all input elements satisfy p

then the result satisfies p

Observation:

maximum returns one of the input elements

maximum:: $\forall p. [\{v:\text{Int} \mid p v\}] \rightarrow \{v:\text{Int} \mid p v\}$

In SMT p is “Uninterpreted Function”

$$\forall \bar{x}, \bar{y}. \bar{x} = \bar{y} \Rightarrow (p \bar{x}) = (p \bar{y})$$

Observation:

maximum returns one of the input elements

maximum:: $\forall p. [\{v: \text{Int} \mid p v\}] \rightarrow \{v: \text{Int} \mid p v\}$

Is Verification Decidable?

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
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```

Assume

Prove

Is Verification Decidable?

maximum ::= $\forall p. [\{v: \text{Int} \mid p v\}] \rightarrow \{v: \text{Int} \mid p v\}$

maximum $[x]$ = x

maximum $(x: xs)$ = let y = maximum xs in
if $x < y$ then y else x

Assume

Prove

$x: \{x: \text{Int} \mid p x\}$

$xs: [\{v: \text{Int} \mid p v\}]$

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

Assume

$x : \{x : \text{Int} \mid p x\}$
 $xs : [\{v : \text{Int} \mid p v\}]$
 $y : \{y : \text{Int} \mid p y\}$

Prove

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

Assume

$x: \{x: \text{Int} \mid p x\}$

$xs: [\{v: \text{Int} \mid p v\}]$

$y: \{y: \text{Int} \mid p y\}$

Prove

$y: \{x: \text{Int} \mid p y\}$

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

Assume

$x: \{x: \text{Int} \mid p x\}$
 $xs: [\{v: \text{Int} \mid p v\}]$
 $y: \{y: \text{Int} \mid p y\}$

Prove

$y: \{x: \text{Int} \mid p y\}$
 $x: \{x: \text{Int} \mid p x\}$

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else xOK
```

Are SMT Valid ? Yes!

$$\begin{aligned} p x \wedge p y &\Rightarrow p y \\ p x \wedge p y &\Rightarrow p x \end{aligned}$$

Is Verification Decidable?

```
maximum ::= ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]      = x
maximum (x:xs) = let y = maximum xs in
                  if x < y then y else x
```

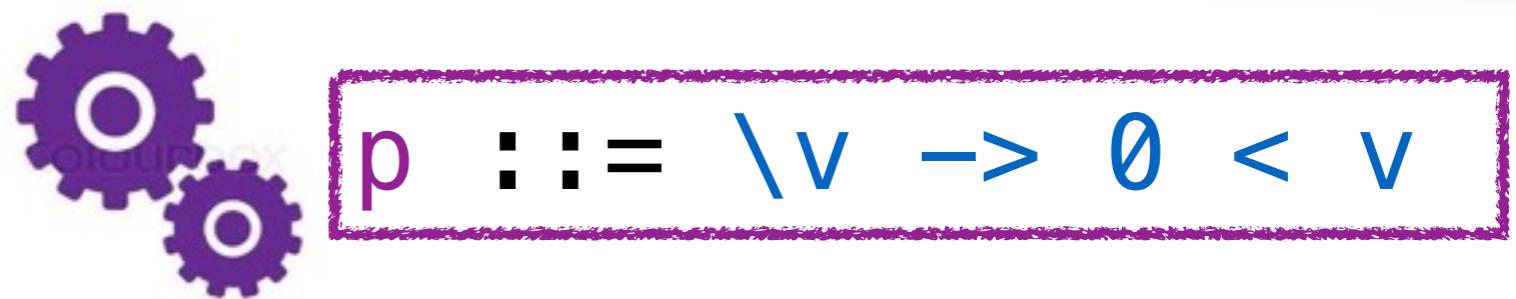
Verification is **Decidable** ...

... but is Specification **Modular**?

add the “negative example”

Client Verification

maximum:: $\forall p. [\{v:\text{Int} \mid p v\}] \rightarrow \{v:\text{Int} \mid p v\}$



maxPos:: $[\{v:\text{Int} \mid 0 < v\}] \rightarrow \{v:\text{Int} \mid 0 < v\}$

maxPos xs = maximum xs

OK

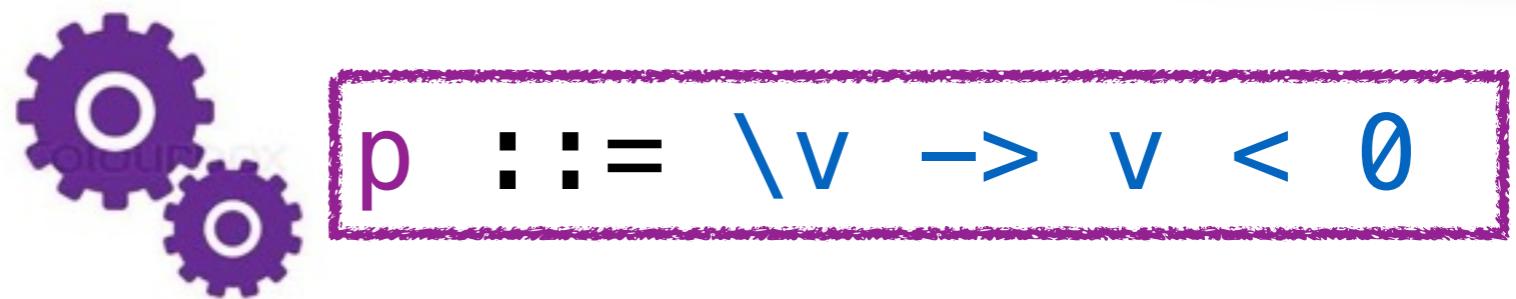
Automatic Instantiation via Liquid Types*

* Rondon, Kawaguchi, Jhala. “Liquid Types” (PLDI ’08)

add the “negative example”

Client Verification

maximum:: $\forall p. [\{v: \text{Int} \mid p v\}] \rightarrow \{v: \text{Int} \mid p v\}$



maxNeg:: $[\{v: \text{Int} \mid v < 0\}] \rightarrow \{v: \text{Int} \mid v < 0\}$

maxNeg xs = maximum xs

OK

Modular & Decidable Specifications

Step 1: Abstract Specification Refinements @Lib

Higher Order VS Modular

Step 2: Installation Requirements @Clt

Applications

Higher-Order Functions

Higher-Order Functions

Examples

compose, foldr, filter, ...

Higher-Order Functions

Examples

compose, **foldr**, filter, ...

`foldr f b []` = b

`foldr f b (x:xs)` = f x (foldr f b xs)

Specification

`foldr f b [] = b`

`foldr f b (x:xs) = f xs x (foldr f b xs)`

`inv xs b` := list `xs` satisfies `inv` at value `b`

Goal:

Specify, by induction, that the result satisfies `inv`

Specify, by induction, that the result satisfies `inv`

Inductive Hypothesis:

```
f: (xs: [a] -> x:a -> {b:b | inv xs b}
      -> {r:b | inv (x:xs) r})
```

Specify, by induction, that the result satisfies `inv`

Inductive Hypothesis:

$$\begin{aligned} f: (\text{xs}: [\text{a}] \rightarrow \text{x}: \text{a} \rightarrow \{\text{b}: \text{b} \mid \text{inv } \text{xs } \text{b}\} \\ \quad \rightarrow \{\text{r}: \text{b} \mid \text{inv } (\text{x}: \text{xs}) \text{ r}\}) \end{aligned}$$

Base Case:

$$b: \{\text{b} \mid \text{inv } [] \text{ b}\}$$

Specify, by induction, that the result satisfies `inv`

Inductive Hypothesis:

$$\begin{aligned} f: (\text{xs}: [\text{a}]) \rightarrow & \quad x: \text{a} \rightarrow \{\text{b}: \text{b} \mid \text{inv } \text{xs } \text{b}\} \\ & \rightarrow \{\text{r}: \text{b} \mid \text{inv } (\text{x}: \text{xs}) \text{ r}\} \end{aligned}$$

Base Case:

$$b: \{\text{b} \mid \text{inv } [] \text{ b}\}$$

Conclusion:

$$\{\text{v}: \text{b} \mid \text{inv } \text{xs } \text{v}\}$$

Specify, by induction that the result satisfies `inv`

$$\begin{aligned}\text{foldr } f \ b \ [] &= b \\ \text{foldr } f \ b \ (x:xs) &= f \ xs \ x \ (\text{foldr } f \ b \ xs)\end{aligned}$$

```
foldr :: forall inv.  
         -> f:(xs:[a] -> x:a -> {b:b|inv xs b})  
              -> {r:b|inv (x:xs) r})  
         -> b:{b|inv [] b}  
         -> xs:[a]  
         -> {v:b|inv xs v}
```

Hack: `xs` as extra argument on `f`

Specify, by induction that the result satisfies `inv`

$$\begin{aligned}\text{foldr } f \ b \ [] &= b \\ \text{foldr } f \ b \ (x:xs) &= f \ x \ (\text{foldr } f \ b \ xs)\end{aligned}$$

$$\begin{aligned}\text{foldr} &:: \text{forall } \text{inv}. \\ \rightarrow &f: (xs: [a] \rightarrow x:a \rightarrow \{b:b \mid \text{inv } xs \ b\}) \\ &\quad \rightarrow \{r:b \mid \text{inv } (x:xs) \ r\}) \\ \rightarrow &b: \{b \mid \text{inv } [] \ b\} \\ \rightarrow &xs: [a] \\ \rightarrow &\{v:b \mid \text{inv } xs \ v\}\end{aligned}$$

Problem: type mismatch

Solution: helper abstract refinement

Solution: helper abstract refinement

```
f: (xs:[a] -> x:a -> {b:b|inv xs b})  
    -> {r:b|inv (x:xs) r})
```

Simplifies to

```
f: (x:a -> b:b -> {r:b|stp x b r})
```

where

forall x xs b r.

$$\begin{aligned} \text{inv } xs \ b &\Rightarrow \text{stp } x \ b \ r \\ &\Rightarrow \text{inv } (x:xs) \ r \end{aligned}$$

Solution: helper abstract refinement

```
f: (xs:[a] -> x:a -> {b:b|inv xs b})  
    -> {r:b|inv (x:xs) r})
```

Simplifies to

```
f: (x:a -> b:b -> {r:b|stp x b r})
```

where

```
bound Ind inv stp = \x xs b r ->  
    inv xs b => stp x b r  
    => inv (x:xs) r
```

Specification

```
foldr :: (Ind inv stp)
        => f:(x:a -> b:b -> {r:b|stp x b r})
        -> {b:b|inv [] b}
        -> xs:[a]
        -> {v:b|inv xs v}
```

```
bound Ind inv stp = \x xs b r ->
    inv xs b -> stp x b r
    -> inv (x:xs) r
```

Higher-Order Functions

Examples

compose, **foldr**, filter, ...

Applications

Higher-Order Functions

Recursive Invariants

Floyd-Hoare Logic (in ST)

Capability-Safe IO Monad

2. Expressiveness

Modular Higher-Order Specifications

2. Expressiveness

Refinement Reflection

Specifications of functions

```
fib :: {i:Int | 0<=i} -> {v:Int | 0<=v}
fib i
| i <= 1      = i
| otherwise   = fib (i-1) + fib (i-2)
```

Specifications of functions

```
fib :: {i:Int | 0<=i} -> {v:Int | i<=v}
fib i
| i <= 1      = i
| otherwise   = fib (i-1) + fib (i-2)
```

Specifications of functions

```
fib :: {i:Int | 0<=i} -> {v:Int | i<=v}
fib i
| i <= 1      = i
| otherwise    = fib (i-1) + fib (i-2)
```

How to express **theorems** about functions?

\forall i. $0 \leq i \Rightarrow \text{fib } i \leq \text{fib } (i+1)$

How to express **theorems** about functions?

Step 1: Definition

In SMT **fib** is “Uninterpreted Function”

```
\forall i j. i = j => fib i = fib j
```

How to express **theorems** about functions?

```
fibCon :: i:Nat -> j:Nat  
        -> {v:() | i = j => fib i = fib j}  
fibCon _ _ = ()                                OK
```

```
\forall i j. i = j => fib i = fib j
```

How to express **theorems** about functions?

```
fibCon :: i:Nat -> j:Nat  
        -> {i = j => fib i = fib j}  
fibCon _ _ = ()                                OK
```

\forall i j. i = j => fib i = fib j

How to express **theorems** about functions?

```
fibOne :: {fib 1 = 1}  
fibOne = ()
```

Error

How to connect logic fib with target fib?

How to connect logic fib with target fib?

~~$\text{fib} :: i:\text{Nat} \rightarrow \text{Nat}$~~

~~$\text{fib } i$~~

~~$| \quad 1 \leq 1$~~

~~$| \quad \text{otherwise} = \text{fib } (i-1) + \text{fib } (i-2)$~~

Not decidable

~~$\forall i.$~~

~~$\text{if } i \leq 1 \text{ then } \text{fib } i = i$~~

~~$\text{else } \text{fib } i = \text{fib } (i-1) + \text{fib } (i-2)$~~

SMT Axiom

How to connect logic fib with target fib?

```
fib :: i:Nat -> Nat
fib i
| i <= 1      = i
| otherwise    = fib (i-1) + fib (i-2)
```

Refinement Reflection

```
fib :: i:Nat -> {v:Nat | v = fib i &&
if i <= 1 then fib i = i
else fib i = fib (i-1) + fib (i-2)
}
```

Refinement Reflection

Step 1: Definition

Step 2: Reflection

```
fib :: i:Nat -> {v:Nat | v = fib i &&
if i <= 1 then fib i = i
else fib i = fib (i-1) + fib (i-2)
}
```

Refinement Reflection

Step 1: Definition

Step 2: Reflection

```
fib :: i:Nat -> {v:Nat | v = fib i &&
if i <= 1 then fib i = i
else fib i = fib (i-1) + fib (i-2)
}
```

Step 3: Application

```
fib 1 :: {v:Nat | v = fib 1 && fib 1 = 1}
```

Refinement Reflection

```
fibOne :: {fib 1 = 1}  
fibOne = let _ = fib 1  
        in ()
```

OK

```
fib 1 :: {v:Nat | v = fib 1 && fib 1 = 1}
```

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
fibTwo = let _ = fib 2
         _ = fib 1
         _ = fib 0
       in ()
```

OK

Problem: Proof is Unreadable!

Solution: Proof Combinators!

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
fibTwo
  =  fib 2
  ==. fib 1 + fib 0
*** QED
```

OK

Problem: Proof is Unreadable!

Solution: Proof Combinators!

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
```

```
fibTwo
```

```
= fib 2
```

```
==. fib 1 + fib 0
```

```
==. 1 + 1
```

```
==. 2
```

```
*** QED
```

OK

Can we reuse existing proofs?

Refinement Reflection

```
fibThree :: {fib 3 = 3}

fibThree
= fib 3
==. fib 2 + fib 1
==. 2 + 1           ? fibTwo      OK
==. 3
*** QED
```

Combining Proofs

Refinement Reflection

Prove **theorems** about functions.

Step 1: Definition

Step 2: Reflection

Step 3: Application

Refinement Reflection

Turns Haskell into a theorem prover.

Refinement Reflection

Turns Haskell into a theorem prover.

Express **theorems** via refinement types.

Express **proofs** via functions.

Check that functions prove theorems.

Refinement Reflection

```
fibUp :: i:Nat -> {fib i ≤ fib (i+1)}
fibUp i
| i == 0
= fib 0 <. fib 1
*** QED
| i == 1
= fib 1 <=. fib 1 + fib 0 <= fib 2
*** QED
| otherwise
= fib i
==. fib (i-1) + fib (i-2)
<=. fib i      + fib (i-2) ? fibUp (i-1)
<=. fib i      + fib (i-1) ? fibUp (i-2)
<=. fib (i+1)
*** QED
```

Refinement Reflection

Higher Order Theorems

```
fMono :: f:(Nat -> Int)
      -> fUp:(z:Nat -> {f z <= f (z+1)})
      -> x:Nat
      -> y:{Nat | x < y}
      -> {f x <= f y}
```

```
fibMono :: x:Nat -> y:{Nat | x < y}
          -> {fib x <= fib y}
fibMono = fMono fib fibUp
```

Applications

Arithmetic Properties

(fib, ackermann)

Applications

Arithmetic Properties

(fib, ackermann)

Class Laws

(monoid, functor, monad)

Parallelization Equivalence

(of monoid morphisms)

Parallelization Equivalence

MapReduce: Apply $f \ x$ in parallel

Parallelization Equivalence

MapReduce: Apply $f \ x$ in parallel

1. Chunk input x in i chunks (`chunk i x`)
2. Apply f to each chunk in parallel (`map f`)
3. Reduce all results op (`foldr op (f [])`)

Parallelization Equivalence

MapReduce: Apply $f \ x$ in parallel

1. Chunk input x in i chunks ($\text{chunk } i \ x$)
2. Apply f to each chunk in parallel ($\text{map } f$)
3. Reduce all results op ($\text{foldr } op \ (f \ [])$)

```
mapReduce i f op xs  
= foldr op (f []) (map f (chunk i xs))
```

Example: Parallel Sums

```
sum :: [Int] -> Int
sum []      = 0
sum (x:xs) = x + sum xs
```

```
psum :: Int -> xs:[Int] -> Int
psum i xs = mapReduce i sum (+) xs
```

Question: is parallel sum equivalent to sum?

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int]->\{sum xs = psum i xs}
sumEq i xs
  =
  psum i xs
  ==.
  sum xs
    ? mapReduceEq i sum (+) rightId distr xs
*** QED
```

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int] -> {sum xs = psum i xs}
sumEq i xs
  =
  psum i xs
==.
  sum xs
    ? mapReduceEq i sum (+) rightId distr xs
*** QED
```

```
rightId :: xs:[Int] -> ys:[Int]
          -> {sum xs + sum [] = sum xs}
```

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int]->\{sum xs = psum i xs}
sumEq i xs
  =
  psum i xs
  ==.
  sum xs
    ?
    mapReduceEq i sum (+) rightId distr xs
*** QED
```

```
rightId :: xs:[Int] -> ys:[Int]
          -> \{sum xs + sum [] = sum xs\}
```

```
distr   :: xs:[Int] -> ys:[Int]
          -> \{sum (xs++ys) = sum xs + sum ys\}
```

Parallelization Equivalence

```
mapReduceEq
  :: i:Int
  -> f:([a] -> b)
  -> op:(b -> b -> b)
  -> rId:(xs:[a] -> {f xs `op` f [] = f xs})
  -> dis:(xs:[a] -> ys:[a]
           -> {f (xs++ys) = f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce i f op xs }
```

Parallelization Equivalence

mapReduceEq

```
:: i:Int  
-> f:( [a] -> b)  
-> op:(b -> b -> b)  
-> rId:(xs:[a] -> {f xs `op` f [] = f xs})  
-> dis:(xs:[a] -> ys:[a]  
       -> {f (xs++ys) = f xs `op` f ys})  
-> xs:[a]  
-> { f x = mapReduce i f op xs }
```

Generalization:

Let b be a monoid with an assoc operator

Parallelization Equivalence

```
mapReduceEq :: (Monoid b)
  => n:Int
  -> f:([a] -> b)
  -> rId:(xs:[a] -> {f xs `op` f [] = f xs})
  -> dis:(xs:[a] -> ys:[a]
           -> {f (xs++ys) = f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Generalization:

Let b be a **verified** monoid with right identity

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
=> n:Int
-> f:([a] -> b)
-> dis:(xs:[a] -> ys:[a]
      -> {f (xs++ys) = f xs `op` f ys})
-> xs:[a]
-> { f x = mapReduce n f xs }
```

Generalization:

Let b be a **verified** monoid with right identity

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
  => n:Int
  -> f:([a] -> b)
=> dis:(xs:[a] -> ys:[a])
=> {f(xs++ys) = f xs `op` f ys}
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Generalization:

Let f be a morphism among a and $[b]$

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
=> n:Int
-> f:Morphism [a] b
-> xs:[a]
-> { f x = mapReduce n f xs }
```

Applications

Arithmetic Properties

(fib, ackermann)

Class Laws

(monoid, functor, monad)

Parallelization Equivalence

(of monoid morphisms)

Parallelization of String Matching

2. Expressiveness

Refinement Reflection

Goal: Refinement Types for Haskell

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

With decidable & predictable verification.

Implementation

LiquidHaskell

LiquidHaskell

Liquid Types *

Decidable Type Inference

* Rondon, Kawaguchi, Jhala. “Liquid Types” (PLDI ’08)

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

Soundness under Laziness

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

Desugar to Ghost Variables

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

+

Bounded Refinement Types [ICFP 15]

Desugar to Ghost Functions

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

+

Bounded Refinement Types [ICFP 15]

+

Refinement Reflection [Submitted 16]

Liquid Haskell is a theorem prover

LiquidHaskell

Termination, Totality, Functional Correctness

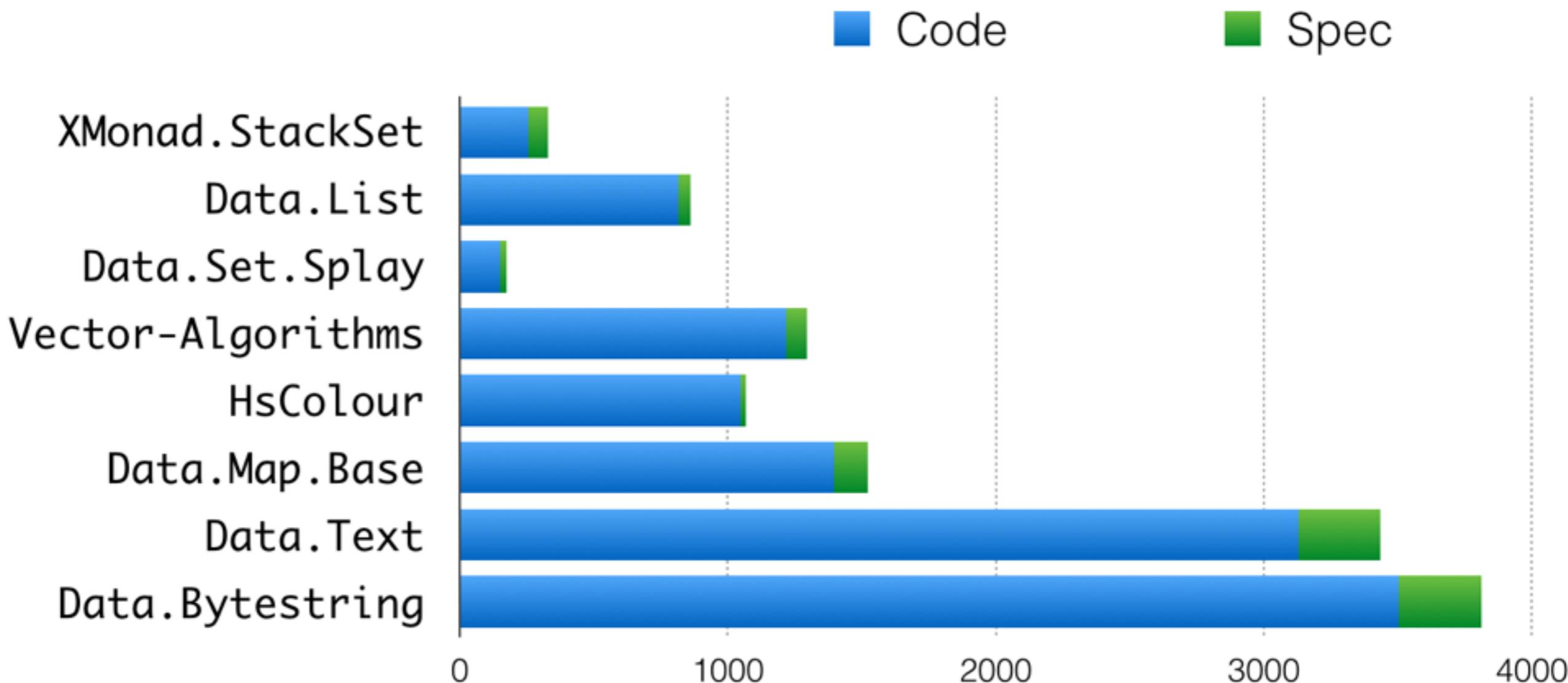
Data.List: list sortedness, safe indexing

Data.Map: binary-search tree preservation

Text, ByteString: low-level memory safety

XMonad: uniqueness of windows

LiquidHaskell



specs: 1/10 LOC

Modest Annotations

time: 0.5s/10 LOC

LiquidHaskell

as a theorem prover

Modest Annotations BUT Verbose Proofs

LiquidHaskell

as a theorem prover

Case study: String matcher parallelization

200LoC “runtime” code

100LoC specs on “runtime” code

1300LoC proof terms

200LoC theorem specs

proofs = 8 x code

Verbose Proofs

Refinement Types for Haskell

1. Soundness

Under Lazy Evaluation

Thanks!

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

3. Implementation

LiquidHaskell

With decidable & predictable verification.

END

My research goal is to integrate verification into code development chain

Envision:

Integrate Verification into Code Development

LiquidHaskell

The crew

Alexander Bakst

Ranjit Jhala

Simon Peyton-Jones

Eric Seidel

Dimitrios Vytiniotis

...

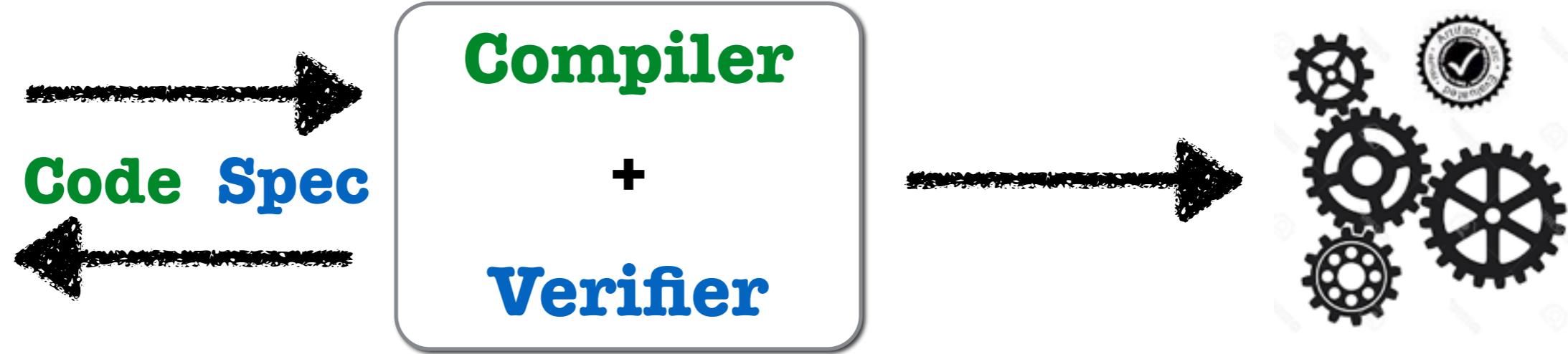
My research goal is to integrate verification into code development chain

Envision:

Integrate Verification into Code Development

Goal: Verification into Code Development

What I envision is that in this process the programmer also provides some code specifications that are automatically checked by a verifier ensuring that the final application satisfies the specifications



To achieve this, we need to persuade
the programmer to write specifications

Goal: Verification into Code D

User should **naturally** write Specifications ...



Code Spec



and the verifier should provide useful feedback

Goal: Verification into Code D

User should **naturally** write Specifications ...



... and get **useful** feedback.

User should **naturally** write Specifications ...

Natural Integration

Expressiveness

I believe that refinement types are in the right direction of naturally integrating specs into the language, as they simply refine the existing type system with logical predicates.

So the user does not need to learn a new language to express code of specification.

A second requirement for natural specification, is that spec language should be arbitrary expressive and this is a requirement that LH fails to satisfy

Expressiveness

LiquidHaskell:
Functions cannot appear in spec:

```
fib_increasing :: x:Nat -> y:Nat  
                  -> { x < y => fib x < fib y }  
fib_increasing x y = AutoProof
```

Right now LH does not allow arbitrary functions to appear inside the specifications which crucially restricts expressiveness.

As an example if I want to prove that fib is increasing I need to refer to the fib function inside the spec and say...

This is my current work, How to allow arbitrary functions appear inside specifications, while retaining decidability and predictability,

Expressiveness

LiquidHaskell:
Functions cannot appear in spec:

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fib_increasing :: x:Nat -> y:Nat  
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As an example if I want to prove that fib is increasing I need to refer to the fib function inside the spec and say...

This is my current work, How to allow arbitrary functions appear inside specifications, while retaining decidability and predictability,

Expressiveness

Once I achieve that we could express interesting higher order properties like that the monadic list implementation satisfies the left identity list monad, just by providing a functions that satisfies the specification that forall m:pa[if I bind m to return I get m

LiquidHaskell:
Functions cannot appear in specifications.

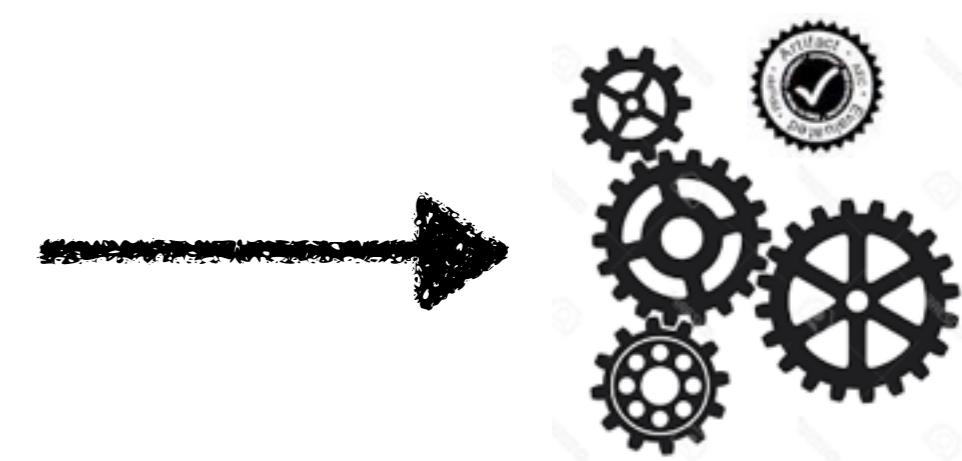
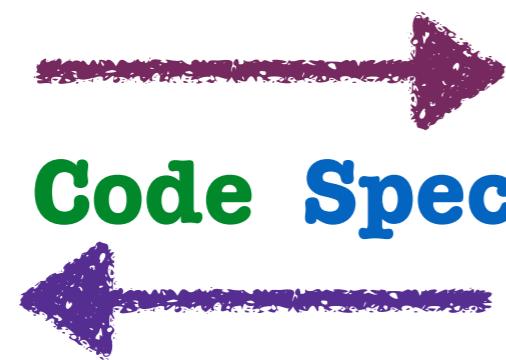
```
left_id    :: m:[a] -> { m >>= return == m }
left_id m = AutoProof
```

Then I am going to describe what I see
as useful feedback by an example

Goal: Verification into Code D

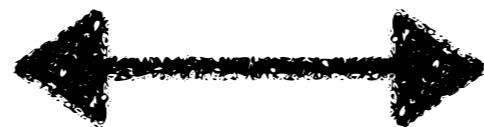
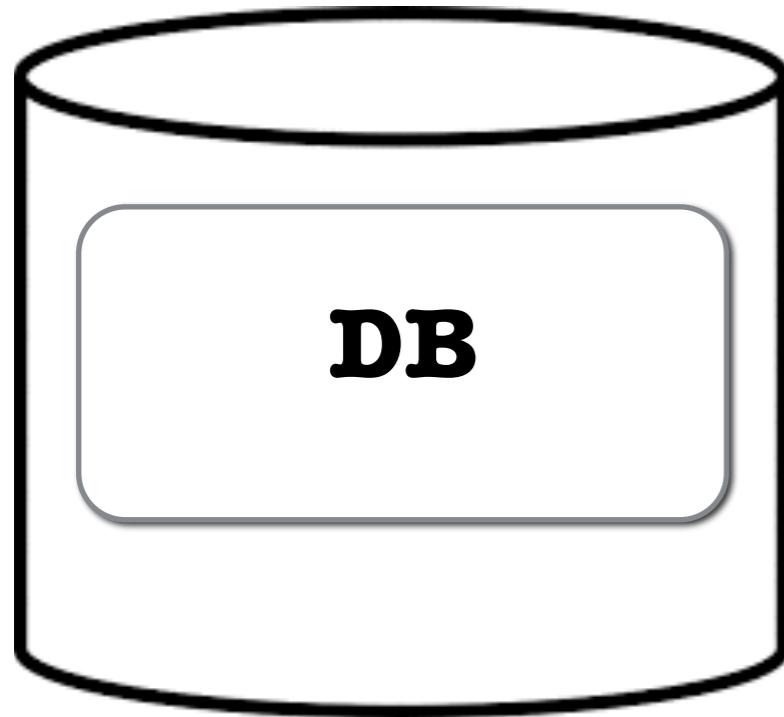
Natural Integration

Expressiveness



... and get **useful** feedback.

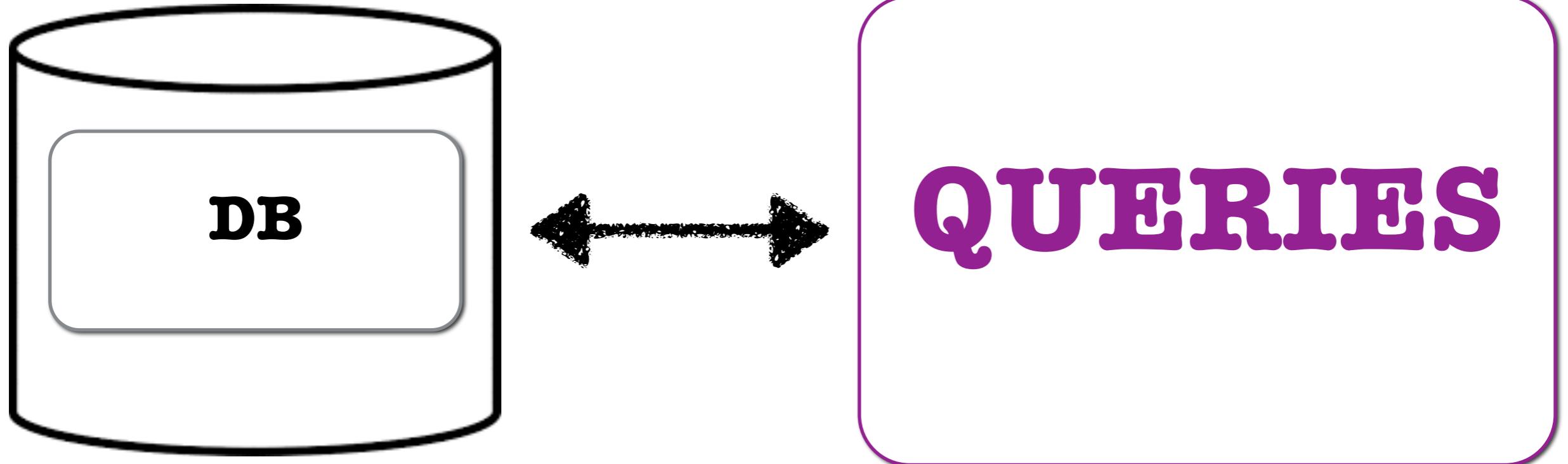
... and get useful feedback



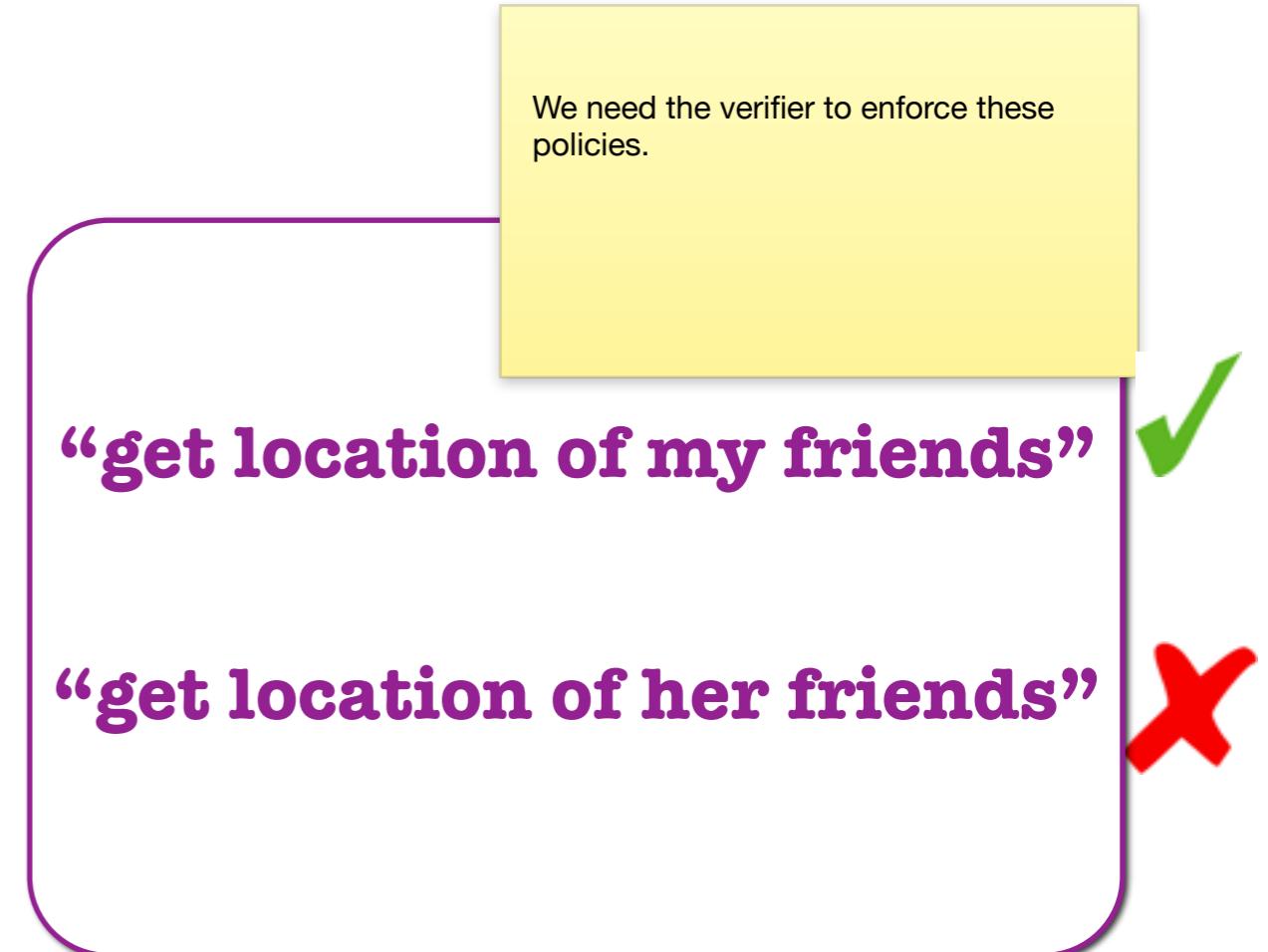
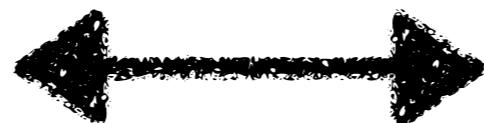
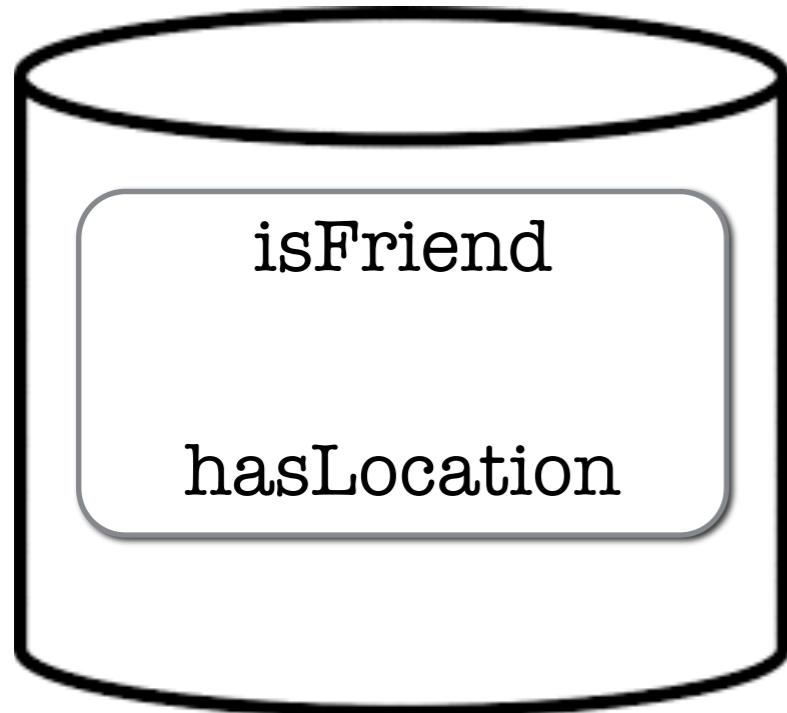
QUERIES

Consider a db and an API that makes queries. it is not important what the query language is, but to be concrete I am going to use HAXI

... and get useful feedback.



... and get useful feedback.



P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

... and get useful feedback.

```
main :: IO ()  
main = do
```

- me and her are two random users

```
let (myid, herid) = (42, 24)
```

- initialize the environment with my credentials

```
env <- initEnv $ UserState { me = myid }
```

- fetch the locations of MY friends

```
locs <- runHaxl env (getFriendsLocations myid)
```

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

Haxl*: A Haskell library that accesses remote data

*Simon Marlow et al. There is no Fork. (ICFP ’14)

... and get useful feedback.

```
main :: IO ()
```

```
main = do
```

- me and her are two random users

```
let (myid, herid) =
```

- initialize the environment

```
env <- initEnv $
```

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

If my query is SAFE,

Verification should succeed
automatically

- fetch the locations of MY friends

```
locs <- runHaxl env (getFriendsLocations myid)
```



Automation

... and get useful feedback.

```
main :: IO ()
```

```
main = do
```

– me and her are two random users

```
let (myid, herid) =
```

– initialize the environment

```
env <- initEnv $
```

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

If my query is UNSAFE,

Verification should fail immediately

th my credentials

```
me = myid }
```

– fetch the locations of MY friends

```
locs <- runHaxl env (getFriendsLocations herid)
```



Speed

Automation

... and get useful feedback.

```
main :: IO ()
```

```
main = do
```

- me and her are two random users

```
let (myid, herid) = (42, 24)
```

- initialize the environment with my credentials

```
env <- initEnv $ UserState { me = myid }
```

- fetch the locations of MY friends

```
locs <- runHaxl env (getFriendsLocations herid) ✗
```

Speed

Automation

Error Diagnosis

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

Error: Violation of P2

... and get useful feedback.

```
main :: IO ()
```

```
main = do
```

- me and her are two random users

```
let (myid, herid) = (42, 24)
```

- initialize the environment with my credentials

```
env <- initEnv $ UserState { me = myid }
```

- fetch the locations of MY friends

```
locs <- runHaxl env (getFriendsLocations !)
```

Speed

Automation

Error Diagnosis

Program Synthesis

P1: “Anyone can look at my friend list.”

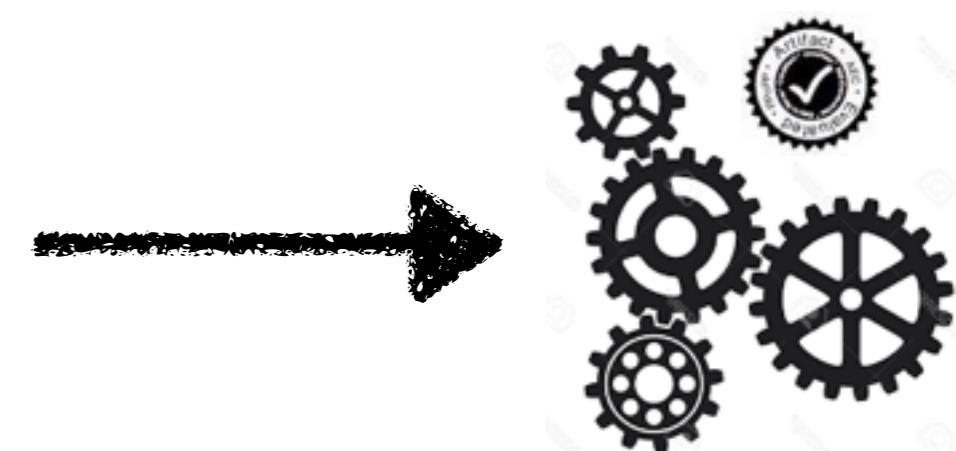
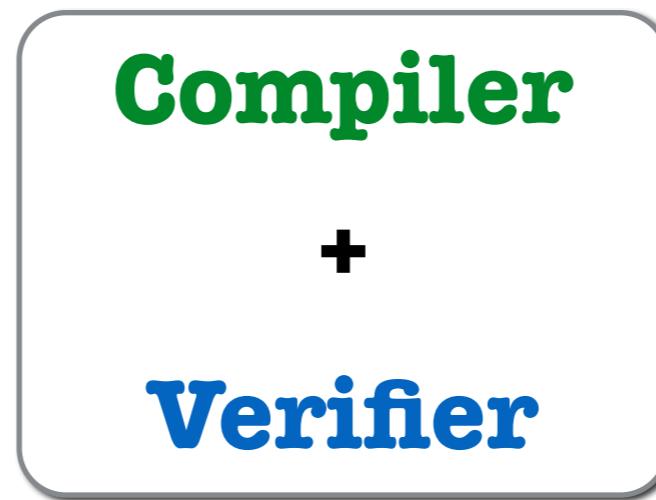
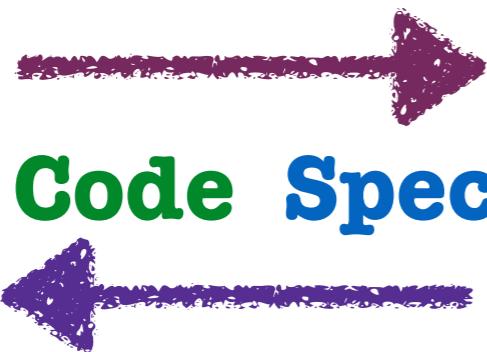
P2: “Only my friends can get my location.”

myid	p1, p2
herid	p1, p2

Goal: Verification into Code Development

Natural Integration

Expressiveness



Speed

Automation

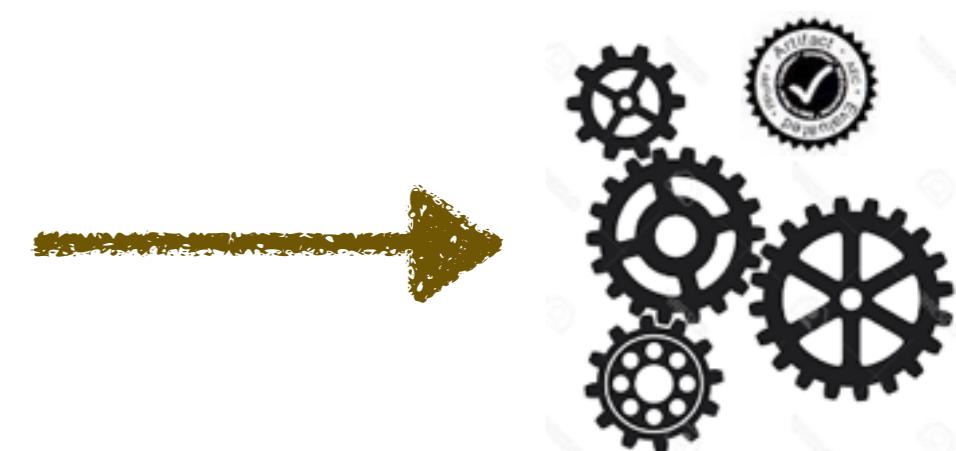
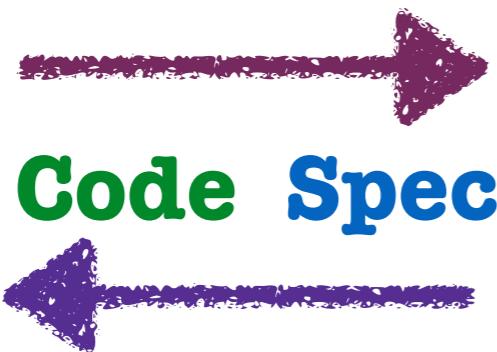
Error Diagnosis

Program Synthesis

Goal: Verification into Code Development

Natural Integration
Expressiveness

Data Privacy
Web Security



Speed
Automation
Error Diagnosis
Program Synthesis

Thanks!

END

The Haxl Monad

For this specific example, HAxl provides a run function that says that if you give me an env that stores my credentials and a computation that decries a query, I will give you back the result of the query.

```
runHaxl :: env:Env      - the environment
          -> haxl:Haxl a - the computation
          -> IO a         - fetched data
```

Refine the Haxl Monad

```
runHaxl :: (Valid Env (Haxl a))  
          => env:Env  
          -> {haxl:Haxl a | valid env haxl}  
          -> IO a
```

The Haxl Monad

Haxl is concurrency monad that performs requests of data `a`

```
newtype Haxl a = Haxl
{ unHaxl :: Env           – environment reader
  -> IORef (RequestStore) – cached fetches reader
  -> IO (Result a) }      – data fetched
```

```
runHaxl :: env:Env       – the environment
          -> haxl:Haxl a – the computation
          -> IO a         – fetched data
```



LiquidHaskell

Alexander Bakst

Ranjit Jhala

Simon Peyton-Jones

Eric Seidel

Michael Smith

Christopher Tetrault

Dimitrios Vytiniotis

...

Usable Program Verification

Expressiveness

Abstract Refinement Types

N. Vazou, P. M. Rondon, and R. Jhala. ESOP ‘13

Bounded Refinement Types

N. Vazou, A. Bakst, and R. Jhala. ICFP ‘15

Error Reporting & Diagnosis

Type Targeted Testing

E. Seidel, **N. Vazou**, and R. Jhala. ESOP ‘15

Prototype: LiquidHaskell

Refinement Types for Haskell

N. Vazou, E. Seidel, R. Jhala, D. Vytiniotis, and S. Peyton-Jones. ICFP ‘15

Experience with Refinement Types in the Real World

N. Vazou, E. Seidel, R. Jhala. Haskell ‘15