Liquid Haskell
Haskell as a Theorem Prover

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Software bugs are everywhere

On a more serious note, software bugs can lead to deaths, which is what happened in May 2015, when Airbus crashed in Spain due to a bug in the software control engine.

4 deaths, Spain, software control

Airbus A400M crashed due to a software bug.
— May 2015

Software bugs are everywhere

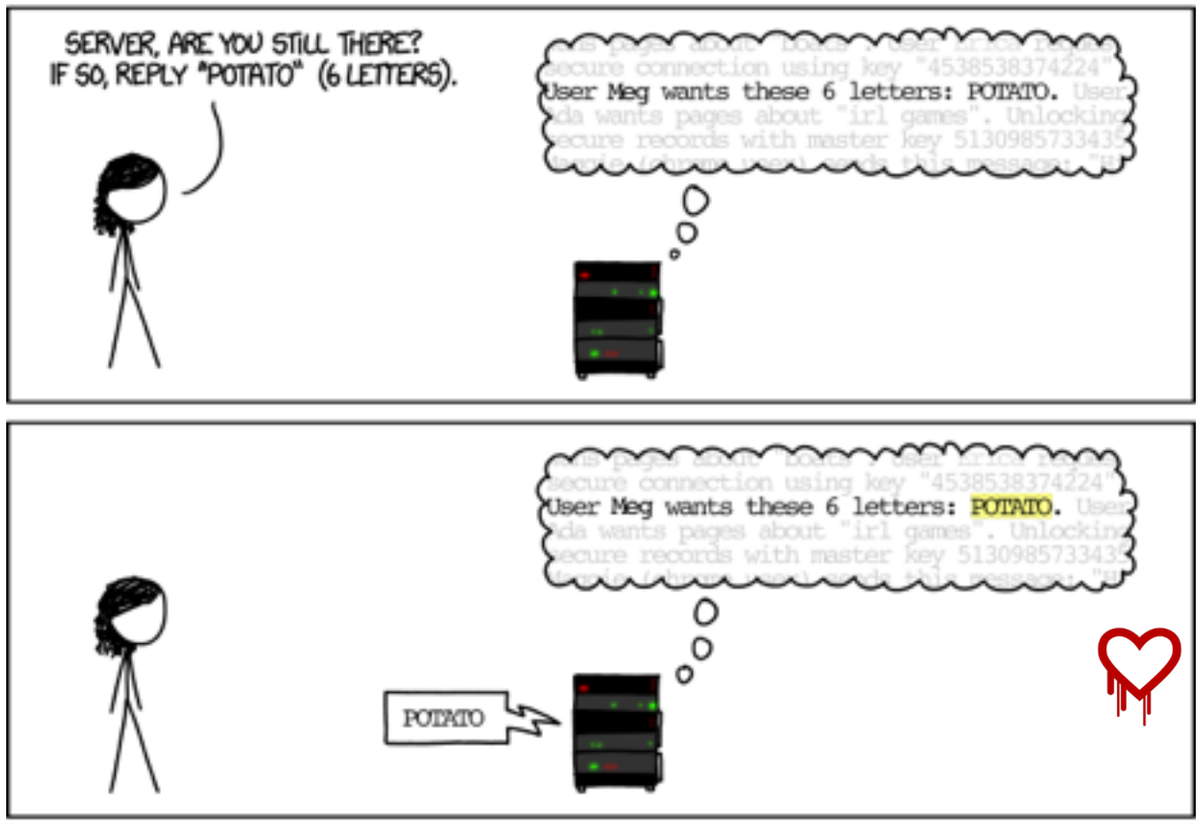
Heart bleed is another bug that got publicity last year, partially because of its pretty logo.

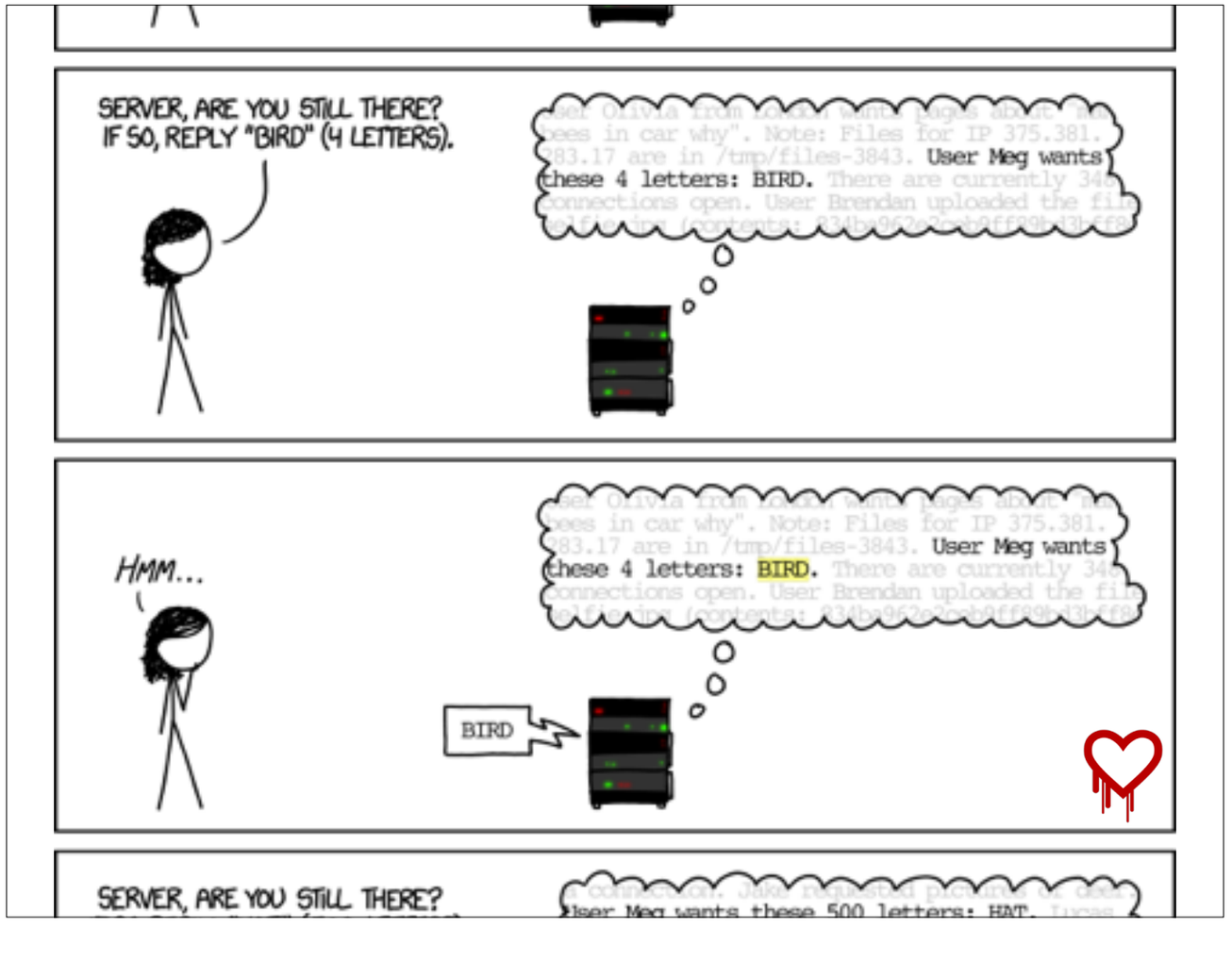
that due to buff over attackers could get access to encrypted informaiton.

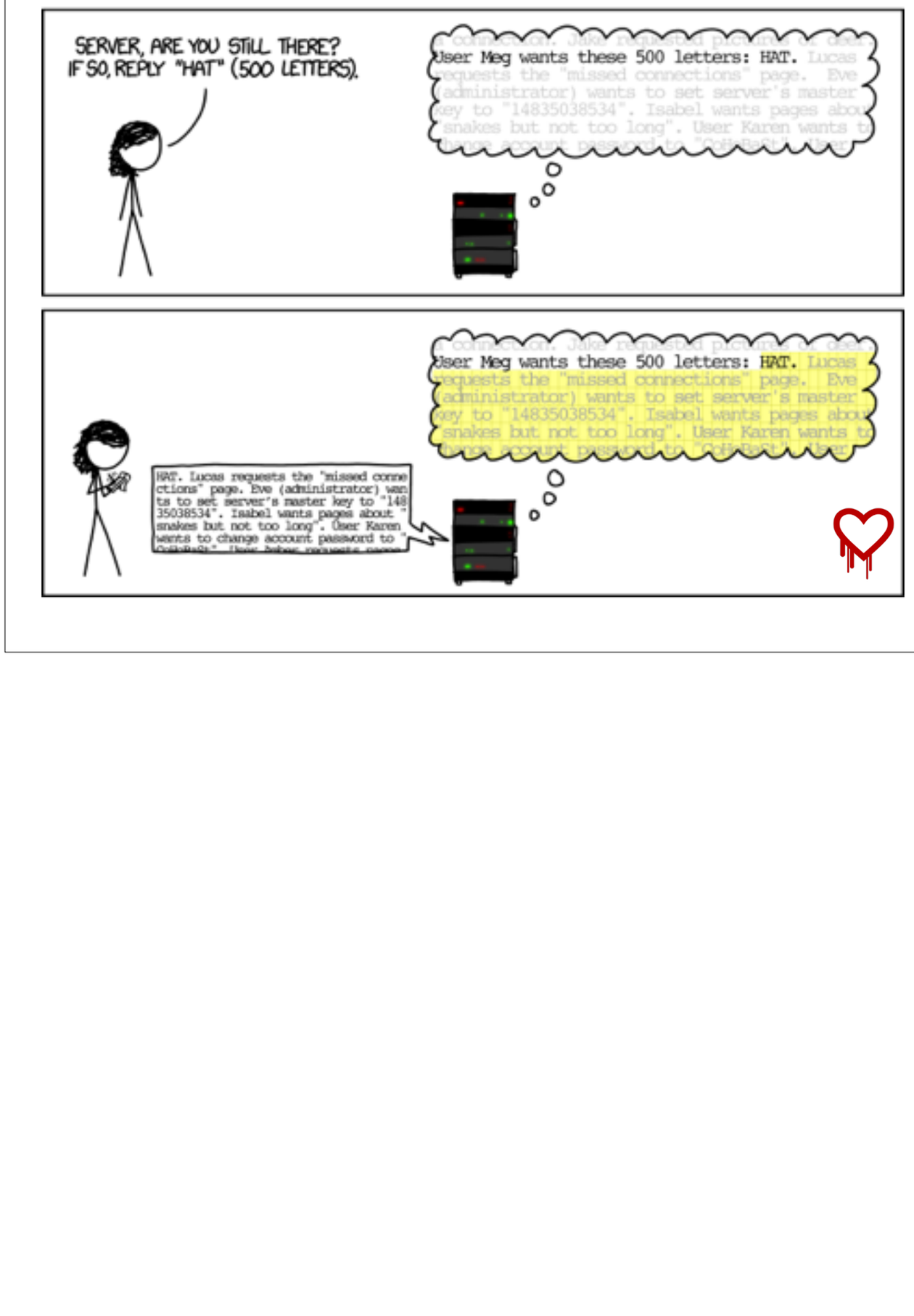
Information protected by the SSL protocol is stored next to unprotected information. Due to buffer overflow attackers can steal encrypted information, like user passwords.

The Heartbleed Bug.
Buffer overflow in OpenSSL. 2015

HOW THE HEARTBLEED BUG WORKS:




SERVER, ARE YOU STILL THERE? IF SO, REPLY "BIRD" (4 LETTERS).
these 4 letters: BIRD.
HMM...
these 4 letters: BIRD.
BIRD
SERVER, ARE YOU STILL THERE?


SERVER, ARE YOU STILL THERE? IF SO, REPLY "HAT" (500 LETTERS).
User Meg wants these 500 letters: HAT.
User Meg wants these 500 letters: HAT.
HAT. Lucas requests the "missed connections" page. Eve (administrator) wants to set server's master key to "14835038534". Isabel wants pages about "snakes but not too long". User Karen wants to change account password to "

Make bugs difficult to express

Using Modern Programming Languages

F#, Ocaml, Scala, Haskell

Because of

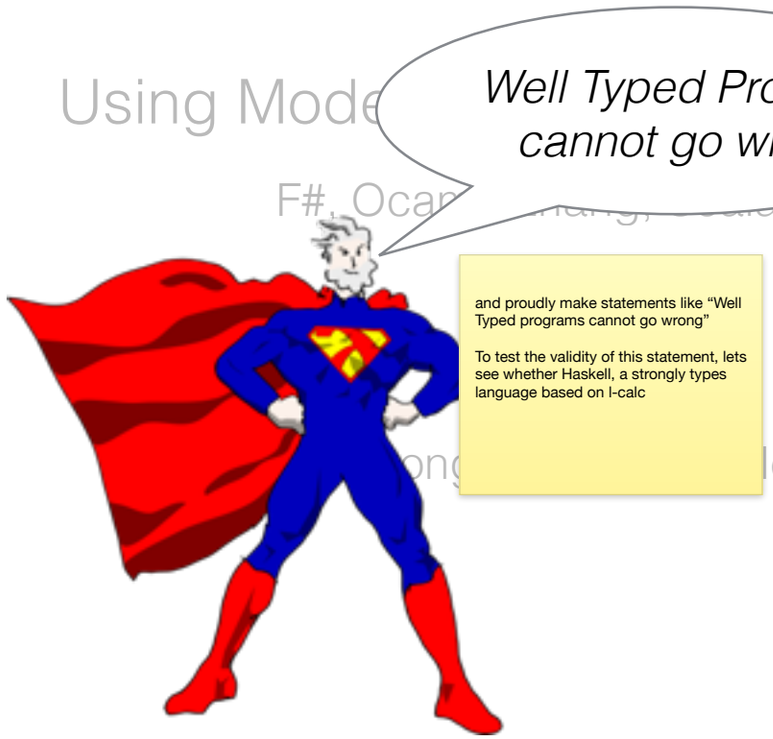
Strong Types + λ-Calculus

The goal of research in programming languages is not only to track these bug, but rether make such kind of software bugs difficult to express.

So, these days we use fancy FPL like that make heavy use

Towards this goal, PL researches have build fancy functional programming languages such as … that make heavy use of the well studied theories of strong typing and lambda calculus

Make bugs difficult to express



and proudly make statements like “Well Typed programs cannot go wrong”

To test the validity of this statement, lets see whether Haskell, a strongly types language based on l-calc

can be used to encode Heartbleed

can be used to encode Heartbleed

```
λ> :m +Data.Text Data.Text.Uns
λ> let pack = "hat"

λ> :t takeWord16
   takeWord16 :: Text -> Int -> Text
```

I am importing data text, a library fir unicode mania
I am creating a text niki
and will use a take function to index pack .

Note that take comes from a library marked as unsafe, so safety relies only on type system and not on runtime checks.

VS.

```
λ> :m +Data.Text Data.Text.Uns
λ> let pack = "hat"

λ> takeWord16 pack True
   Type Error: Cannot match Bool vs Int
```

If I ask for the True first characters lambda man is smart enough to tell me that True is an invalid argument for take

VS.

```
λ>  :m +Data.Text Data.Text.U
λ> let pack = "hat"

λ> takeWord16 pack 500
   “hat\58456\2594\SOH\NUL…
```

But, If I ask for the 8 first carachets I encoded HD
take will return the 4 first carachets and 4 more characters that are stored in the memory next to Niki

My goal is to use types and statically catch this error

VS.

Valid Values for `takeWord16`?

the type of takes states that all ints are valid indexes

```
takeWord16 :: t:Text -> i:Int -> Text
```

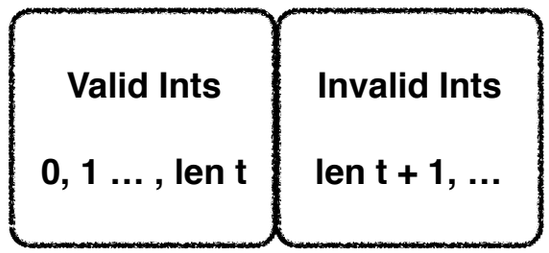
All Ints

…, -2, -1, 0, 1, 2, 3, …

Valid Values for `takeWord16`?

what i want is to split ints and specify tats the valid arguments for take are only ints that are less than the size of t

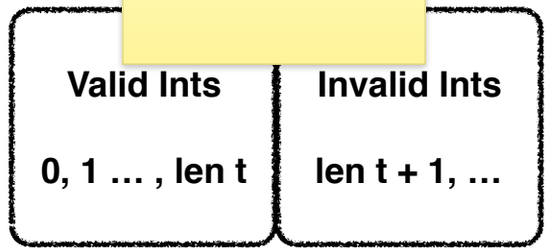
```
takeWord16 :: t:Text -> i:Int -> Text
```



Refinement Types

```
take :: t:Text -> {v:Int | v <= len t} -> Text
```

we can express this via ref types
I give take a ref type that sates that take takes a t
and an integer v that is <= len t



Refinement Types

```
take :: t:Text -> {v:Int | v <= len t} -> Text
```

```
λ> :m +Data.Text Data.Text.
λ> let pack = "hat"

λ> take pack 500
   Refinement Type Error
```

going back to the code, with the refined type for take, asking 8 elemns of Niki will return a refinement type error,

My goal is to allow this reasoning by embedding refinements into haskell's type system

Goal: Refinement Types for Haskell

Goal: Refinement Types for I

specifically during my PhD I developed a sound and expressive refinement type system for Haskell programs
and what is important is that I achieved both of these under the requirement of decidable and predictable verification

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

With automatic & predictable verification.

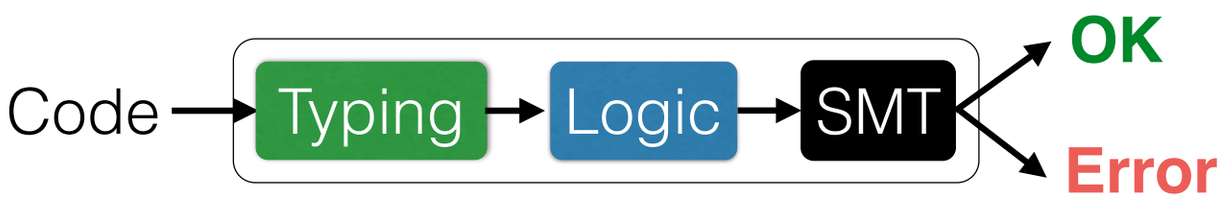
Lets start by building a refinement type checkers that is sound unders

1. Soundness

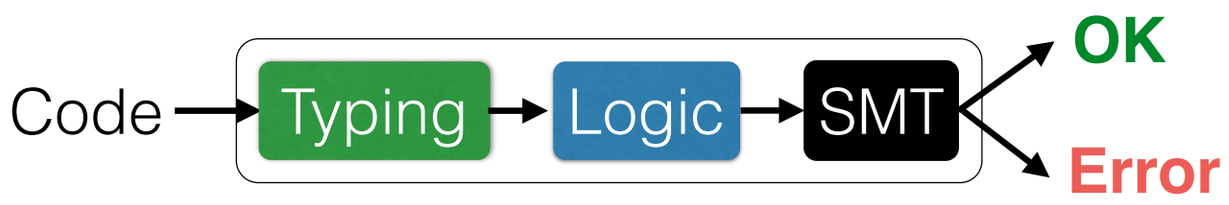
Under Lazy Evaluation

Refinement Typing 101

Refinement Typing 101



1. Source Code to **Type constraints**
2. **Type Constraints** to **Verification Condition (VC)**
3. Check **VC validity** with **SMT Solver**




```
take :: t:Text -> {v | v <= len t} -> Text

heartbleed = let x = "hat"
             in  take x 500
```

Code → Typing

```
take :: t:Text->{v|v <= len t}->Text
heartbleed = let x = "hat"
             in  take x 500
```

```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
```

Code → Typing

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heartbleed = let x = "hat"
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heartbleed = let x = "hat"
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```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
```

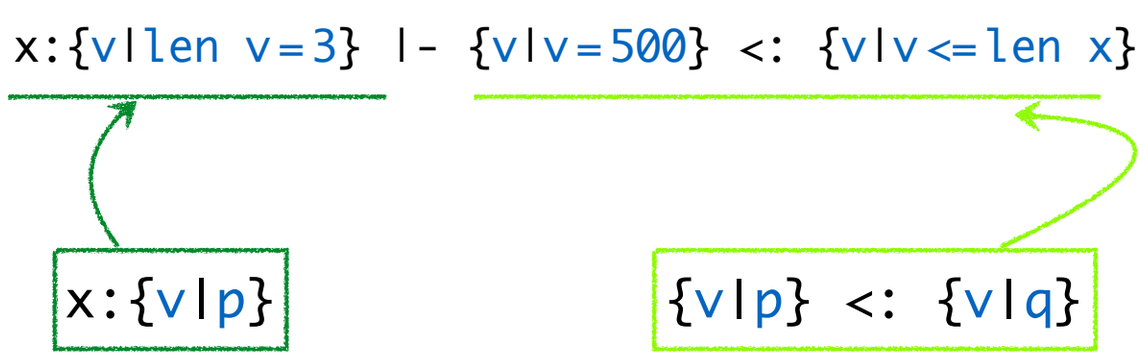
Encode Subtyping as Logical VC

If VC valid then Subtyping holds

Typing → Logic

Clearly state that on the left I have my environment

Encode Subtyping as Logical VC



to encode an environment binding we use flanagan;s meaning that says

x : { v | p }

Means*: *If* x reduces to a value *then* p[x/v]

Encoded as: “x has a value” => p[x/v]

* Flanagan “Hybrid Type Checking” (POPL ’06)


```
{v | p} <: {v | q}
```

Means: *if* `y:{v | p}` *then* `y:{v | q}`

Encoded as: `p => q`

Encode Subtyping …

```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
```

… as Logical VC

```
("x has a value" => len x = 3)
                 => (v = 500) => (v <= len x)
```

Encode Subtyping …

```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
```

… as Logical VC

```
("x has a value" => len x = 3)
                 => (v = 500) => (v <= len x)
```

Typing → Logic

Encode Subtyping …

```
x:{v|len v=3} |- {v|v=500} <: {v|v<=len x}
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… as Logical VC

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("x has a value" => len x = 3)
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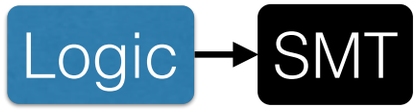
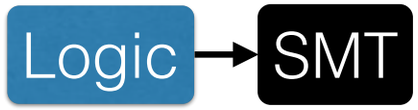



```
("x has a value" => len x = 3)
                 => (v = 500) => (v <= len x)
```

How to encode "x has a value" ?

(In a decidable manner)




```
("x has a value" => len x = 3)
                 => (v = 500) => (v <= len x)
```

Assumption: Binders Are Values!

i.e. x is guaranteed to be a value


```
len x = 3
=> (v = 500) => (v <= len x)
```

Assumption: Binders Are Values!

i.e. `x` is guaranteed to be a value


```
take :: t:Text->{v|v <= len t}->Text
heartbleed = let x = "hat"
             in  take x 500
```

Checker soundly reports **Error**

Assumption: Binders must be values

Not true under Haskell's lazy semantics

Ignoring “has a value” is unsound!

Under Haskell’s lazy semantics

Ignoring “has a value” is unsound!

```
spin :: Int -> Int
spin x = spin x
```

Ignoring “has a value” is unsound!

```
spin :: Int -> {v:Int|false}
spin x = spin x
```

OK

As `spin` does not return any value

Ignoring “has a value” is unsound!

```
take :: t:Text->{v|v <= len t}->Text
spin :: Int -> {v:Int | false}

heartbleed = let x = "hat"
                 y = spin 0
             in  take x 500
```

OK? or **Error?**

Ignoring “has a value” is unsound!

```
take :: t:Text->{v|v <= len t}->Text
spin :: Int -> {v:Int | false}

heartbleed = let x = "hat"
                 y = spin 0
             in  take x 500
```

OK under **CBV** evaluation

Ignoring “has a value” is unsound!

```
take :: t:Text->{v|v <= len t}->Text
spin :: Int -> {v:Int | false}

heartbleed = let x = "hat"
                 y = spin 0
             in  take x 500
```

Error under **Lazy** evaluation

Ignoring “has a value” is unsound!

Reports **Erroneous** code as **OK**

Ignoring “has a value” is unsound!

How to encode “has a value” ?

How to encode “has a value” ?

Most expressions provably reduce to a value

How to encode “has a value” ?

Most expressions provably reduce to a value

If x reduces to a value,
Then encode “x has a value” by `true`

Solution: Stratified Types

`x:{v:Int |p}`

Must reduce to a Value

`x:{v:Int`$^{\uparrow}$`|p}`

May-not reduce to a Value

Stratified Types to Logic

`x:{v:Int |p}` encoded as `p[x/v]`

`x:{v:Int`$^{\uparrow}$`|p}` encoded as `true`

Code → Typing

```
take :: t:Text->{v|v <= len t}->Text
spin :: Int -> {v:Int↑|false}

heartbleed = let x = "hat"
                 y = spin 0
             in  take x 500
```

```
x:{v|len v=3}
y:{v:Int↑|false}|-{v|v=500} <: {v|v<=len x}
```

Typing → Logic

```
x:{v|len v=3}
y:{v:Int↑|false}|-{v|v=500} <: {v|v<=len x}
```

```
len x=3
true                  => v=500 => v<=len x
```


```
len x = 3
true                => v = 500 => v <= len x
```


```
take :: t:Text -> {v | v <= len t} -> Text
spin :: Int -> {v:Int↑ | false}

heartbleed = let x = pack "hat"
                 y = spin 0
             in  take x 500
```

How to enforce stratification?

How to enforce stratification?

`x:{v:Int|p}`

Must have a Value

Terminating expressions must have a value

Solution

Check termination with Refinement Types!

We saw how regain soundness under lazy evaluation using type stratification

1. Soundness

Under Lazy Evaluation

Goal: Refinement Types for Haskell

What is important is that this technique allows for decidable and predictable verification

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

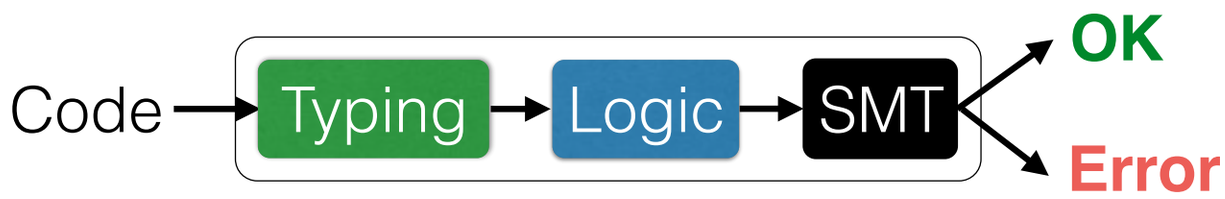
With automatic & predictable verification.

What is important is that this technique allows for decidable and predictable verification

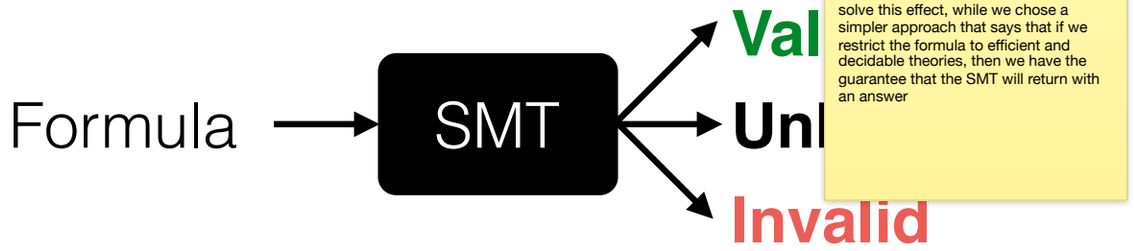
With automatic & predictable verification.

With automatic & predictable on.

Recall the structure of the type checker that extracts the VC from types and checks the verification of VC using an SMT solver



With automatic & predictable verification.



An SMT solver is a back box that given a formula it can

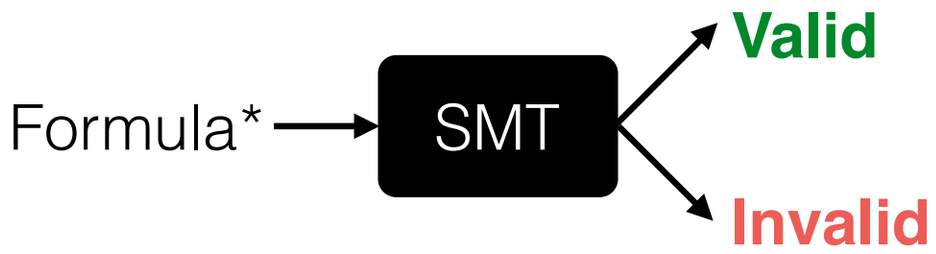
1. return valid is the formula is always true,
2. return invalid with a counter example that can be false, or
3. timeout with unknown

Timeouts lead to unpredictable behaviours. what Rustan calls the butterfly event: minor modification at one part of your programs can change the verification of another part

Rustan carefully selects triggers to solve this effect, while we chose a simpler approach that says that if we restrict the formula to efficient and decidable theories, then we have the guarantee that the SMT will return with an answer

With automatic & predictable verification.

How to learn vs. test code?



*From Decidable Logic

The VC comes via conjunctions and implications from the refinements of types, thus

Encode Subtyping …

$$x_1:\{v \mid p_1\}, \ldots, x_n:\{v \mid p_n\} \vdash \{v \mid q_1\} <: \{v \mid q_2\}$$

… as Logical Verification Condition

$$p_1 \wedge \ldots \wedge p_n \Rightarrow q_1 \Rightarrow q_{12}$$

For predictability, p from decidable theories

$$\{v : a \mid p\}$$

… as Logical Verification Condition

$$p_1 \wedge \ldots \wedge p_n \Rightarrow q_1 \Rightarrow q_{12}$$

For predictability, p from decidable theories

where dec theories are theories for which the SMT has dec & efficient procedures, like …

$$\{v : a \mid p\}$$

Boolean Logic

(QF) Linear Arithmetic

Uninterpreted Functions …

For automatic & predictable verification.

where dec theories are theories for which the SMT has dec & efficient procedures, like …

For predictability, p from decidable theories

$$\{v : a \mid p\}$$

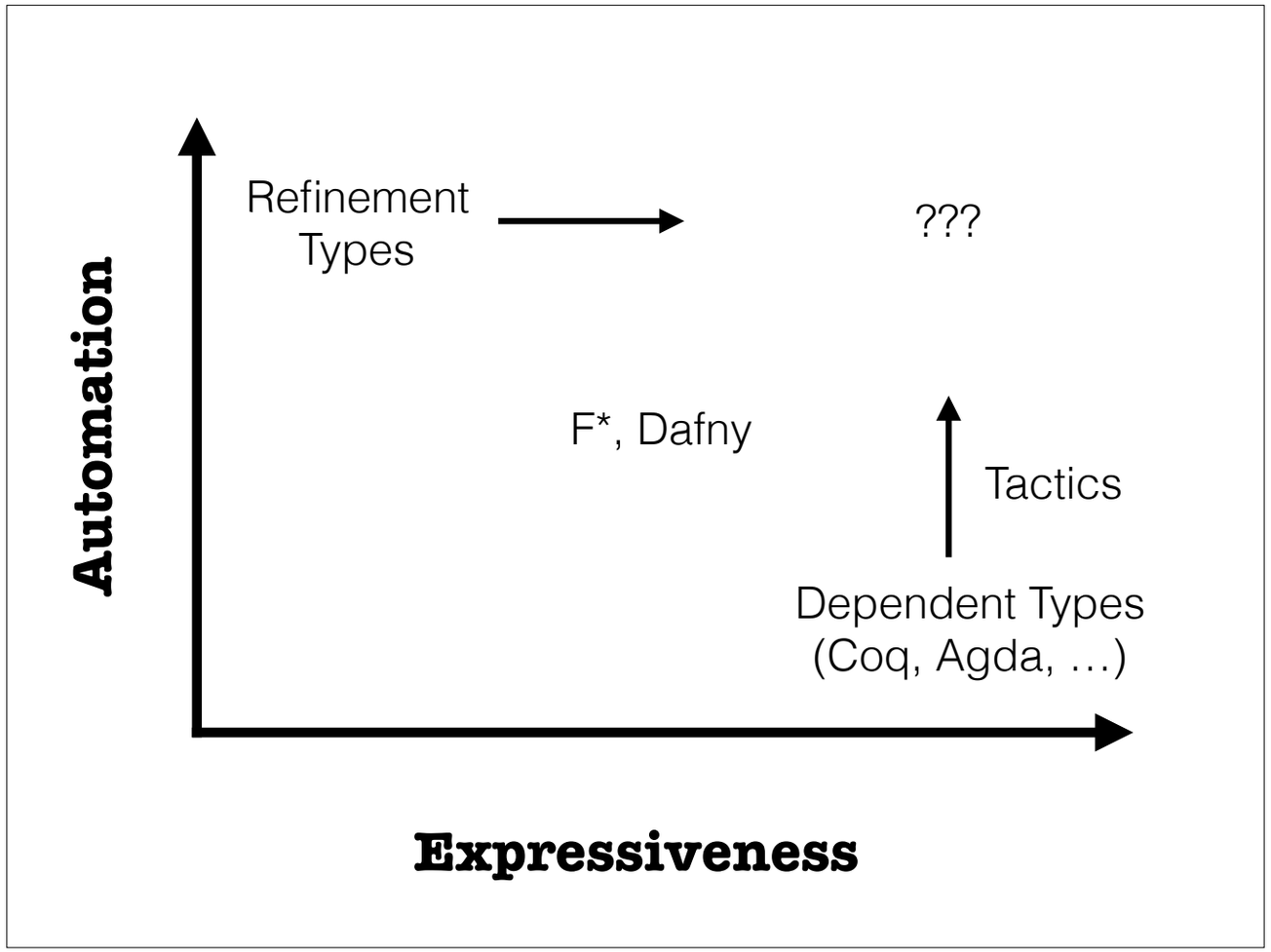
Boolean Logic

(QF) Linear Arithmetic

Uninterpreted Functions …

For automatic & predictable verification.

What about expressiveness?


Automation
Refinement Types
???
F*, Dafny
Tactics
Dependent Types
(Coq, Agda, …)
Expressiveness

Goal: Refinement Types for Haskell

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

With automatic & predictable verification.

Goal: Modular & Decidable Sp s

As a first example, consider a function maximum that
takes as input a list of ints and
returns the maximum element of the list

```
maximum          :: [Int] -> Int
maximum [x]      = x
maximum (x:xs)   = let y = maximum xs in
                   if x < y then y else x
```

Client Verification

```
p  = maximum [1, 2, 3]
assert (p > 0)
n  = maximum [-1, -2, -3]
assert (n < 0)
```

Goal: Modular & Decidable Specifications

```
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                 if x < y then y else x
```

Specification?

Goal: Modular & Decidable Specifications

```
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                 if x < y then y else x
```

Not Modular

First-Order Specification

```
maximum :: [{v:Int | 0<v}] -> {v:Int | 0<v}
```

Goal: Modular & Decidable Specifications

```
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                 if x < y then y else x
```

Not Decidable

Higher-Order Specification

```
ensures \exists x in xs. r = x
```

Observation:

`maximum` returns one of the input elements

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`maximum` returns one of the input elements

```
maximum :: [{v:Int| 0<v }] -> {v:Int| 0<v }
```

if all input elements are positive

then the result is positive

Observation:

`maximum` returns one of the input elements

```
maximum :: [{v:Int| v<0}] -> {v:Int| v<0}
```

if all input elements are negative

then the result is negative

Observation:

`maximum` returns one of the input elements

```
maximum :: [{v:Int | p v }] -> {v:Int | p v }
```

if all input elements satisfy `p`

then the result satisfies `p`

Observation:

`maximum` returns one of the input elements

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
```

for all p

if all input elements satisfy p

then the result satisfies p

Observation:

`maximum` returns one of the input elements

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
```

In SMT `p` is "Uninterpreted Function"

$$\forall \overline{x},\ \overline{y}.\ \overline{x} = \overline{y} \Rightarrow (p\ \overline{x}) = (p\ \overline{y})$$

Observation:

`maximum` returns one of the input elements

```
maximum:: ∀p. [{v:Int| p v}] -> {v:Int| p v}
```

Is Verification Decidable?

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Assume

Prove

Is Verification Decidable?

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maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
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```

Assume

```
x:{x:Int | p x}
xs:[{v:Int | p v}]
```

Prove

Is Verification Decidable?

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Assume

```
x:{x:Int | p x}
xs:[{v:Int | p v}]
y:{y:Int | p y}
```

Prove

Is Verification Decidable?

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maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
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maximum (x:xs) = let y = maximum xs in
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```

Assume

```
x:{x:Int | p x}
xs:[{v:Int | p v}]
y:{y:Int | p y}
```

Prove

```
y:{x:Int | p y}
```

Is Verification Decidable?

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                 if x < y then y else x
```

Assume

```
x:{x:Int | p x}
xs:[{v:Int | p v}]
y:{y:Int | p y}
```

Prove

```
y:{x:Int | p y}
x:{x:Int | p x}
```

Is Verification Decidable?

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
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```
OK

Are SMT Valid ? Yes!

$$p\ x \wedge p\ y \Rightarrow p\ y$$
$$p\ x \wedge p\ y \Rightarrow p\ x$$

Is Verification Decidable?

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
maximum [x]    = x
maximum (x:xs) = let y = maximum xs in
                 if x < y then y else x
```

Verification is **Decidable** ...

... but is Specification **Modular**?

add the “negative example”

Client Verification

```
maximum:: ∀p. [{v:Int | p v}] -> {v:Int | p v}
```

```
p ::= \v -> 0 < v
```

```
maxPos:: [{v:Int | 0<v}] -> {v:Int | 0<v}
maxPos xs = maximum xs
```

OK

Automatic Instantiation via Liquid Types*

* Rondon, Kawaguchi, Jhala. “Liquid Types” (PLDI ’08)

add the “negative example”

Client Verification

```
maximum:: ∀p.[{v:Int|p v}]->{v:Int|p v}
```

```
p ::= \v -> v < 0
```

```
maxNeg:: [{v:Int|v<0}]->{v:Int|v<0}
maxNeg xs = maximum xs
```

OK

Modular & Decidable Specifications

Step 1: Abstra finements @Lib

Step 2: Instantiate Refinements @Clt

Higher Order VS Modular

Applications

Higher-Order Functions

Higher-Order Functions

Examples

compose, foldr, filter, ...

Higher-Order Functions

Examples

compose, **foldr**, filter, ...

```haskell
foldr f b []     = b
foldr f b (x:xs) = f xs x (foldr f b xs)
```

Specification

```
foldr f b []     = b
foldr f b (x:xs) = f xs x (foldr f b xs)
```

`inv xs b` := list `xs` satisfies `inv` at value `b`

Goal:

Specify, by induction, that the result satisfies `inv`

Specify, by induction, that the result satisfies inv

Inductive Hypothesis:

```
f:(xs:[a] -> x:a -> {b:b|inv xs b}
          -> {r:b|inv (x:xs) r})
```

Specify, by induction, that the result satisfies inv

Inductive Hypothesis:

```
f:(xs:[a] -> x:a -> {b:b|inv xs b}
          -> {r:b|inv (x:xs) r})
```

Base Case:

```
b:{b|inv [] b}
```

Specify, by induction, that the result satisfies inv

Inductive Hypothesis:

```
f:(xs:[a] -> x:a -> {b:b|inv xs b}
          -> {r:b|inv (x:xs) r})
```

Base Case:

```
b:{b|inv [] b}
```

Conclusion:

```
{v:b|inv xs v}
```

Specify, by induction that the result satisfies inv

```
foldr f b []     = b
foldr f b (x:xs) = f xs x (foldr f b xs)
```

```
foldr :: forall inv.
      -> f:(xs:[a] -> x:a -> {b:b|inv xs b}
                   -> {r:b|inv (x:xs) r})
      -> b:{b|inv [] b}
      -> xs:[a]
      ->{v:b|inv xs v}
```

Hack: xs as extra argument on f

Specify, by induction that the result satisfies inv

```
foldr f b []     = b
foldr f b (x:xs) = f x (foldr f b xs)
```

```
foldr :: forall inv.
      -> f:(xs:[a] -> x:a -> {b:b|inv xs b}
                   -> {r:b|inv (x:xs) r})
      -> b:{b|inv [] b}
      -> xs:[a]
      ->{v:b|inv xs v}
```

Problem: type mismatch

Solution: helper abstract refinement

Solution: helper abstract refinement

```
f:(xs:[a] -> x:a -> {b:b|inv xs b}
          -> {r:b|inv (x:xs) r})
```

Simplifies to

```
f:(x:a -> b:b -> {r:b|stp x b r})
```

where

```
forall x xs b r.
    inv xs b ⇒ stp x b r
             ⇒ inv (x:xs) r
```

Solution: helper abstract refinement

```
f:(xs:[a] -> x:a -> {b:b|inv xs b}
          -> {r:b|inv (x:xs) r})
```

Simplifies to

```
f:(x:a -> b:b -> {r:b|stp x b r})
```

where

```
bound Ind inv stp = \x xs b r ->
        inv xs b ⇒ stp x b r
                 ⇒ inv (x:xs) r
```

Specification

```
foldr :: (Ind inv stp)
      => f:(x:a -> b:b -> {r:b|stp x b r})
      -> {b:b|inv [] b}
      -> xs:[a]
      -> {v:b|inv xs v}
```

```
bound Ind inv stp = \x xs b r ->
        inv xs b ⇒ stp x b r
                 ⇒ inv (x:xs) r
```

Higher-Order Functions

Examples

compose, **foldr**, filter, ...

Applications

Higher-Order Functions

Recursive Invariants

Floyd-Hoare Logic (in ST)

Capability-Safe IO Monad

2. Expressiveness

Modular Higher-Order Specifications

2. Expressiveness

Refinement Reflection

Specifications of functions

```
fib :: {i:Int|0<=i} -> {v:Int|0<=v}
fib i
  | i <= 1    = i
  | otherwise = fib (i-1) + fib (i-2)
```

Specifications of functions

```
fib :: {i:Int|0<=i} -> {v:Int|i<=v}
fib i
  | i <= 1    = i
  | otherwise = fib (i-1) + fib (i-2)
```

Specifications of functions

```
fib :: {i:Int|0<=i} -> {v:Int|i<=v}
fib i
  | i <= 1    = i
  | otherwise = fib (i-1) + fib (i-2)
```

How to express **theorems** about functions?

```
\forall i. 0 <= i => fib i <= fib (i+1)
```

How to express **theorems** about functions?

Step 1: Definition

In SMT `fib` is “Uninterpreted Function”

```
\forall i j. i = j => fib i = fib j
```

How to express **theorems** about functions?

```
fibCon :: i:Nat -> j:Nat
       -> {v:()| i = j => fib i = fib j}
fibCon _ _ = ()
```

OK

```
\forall i j. i = j => fib i = fib j
```

How to express **theorems** about functions?

```
fibCon :: i:Nat -> j:Nat
       -> {i = j => fib i = fib j}
fibCon _ _ = ()
```

OK

```
\forall i j. i = j => fib i = fib j
```

How to express **theorems** about functions?

```
fibOne :: {fib 1 = 1}
fibOne = ()
```

Error

How to connect logic fib with target fib?

How to connect logic fib with target fib?

```
fib :: i:Nat -> Nat
fib i
  | i <= 1    = i
  | otherwise = fib (i-1) + fib (i-2)
```

SMT Axiom

```
\forall i.
    if i <= 1 then fib i = i
    else fib i = fib (i-1) + fib (i-2)
```

Not Decidable

How to connect logic fib with target fib?

```
fib :: i:Nat -> Nat
fib i
  | i <= 1    = i
  | otherwise = fib (i-1) + fib (i-2)
```

Refinement Reflection

```
fib :: i:Nat -> {v:Nat| v = fib i &&
   if i <= 1 then fib i = i
   else fib i = fib (i-1) + fib (i-2)
    }
```

Refinement Reflection

Step 1: Definition

Step 2: Reflection

```
fib :: i:Nat -> {v:Nat| v = fib i &&
   if i <= 1 then fib i = i
   else fib i = fib (i-1) + fib (i-2)
    }
```

Refinement Reflection

Step 1: Definition

Step 2: Reflection

```
fib :: i:Nat -> {v:Nat| v = fib i &&
   if i <= 1 then fib i = i
   else fib i = fib (i-1) + fib (i-2)
    }
```

Step 3: Application

```
fib 1 :: {v:Nat| v = fib 1 && fib 1 = 1}
```

Refinement Reflection

```
fibOne :: {fib 1 = 1}
fibOne = let _ = fib 1
         in ()                                    OK
```

```
fib 1 :: {v:Nat| v = fib 1 && fib 1 = 1}
```

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
fibTwo = let _ = fib 2
             _ = fib 1
             _ = fib 0
         in ()                                  OK
```

Problem: Proof is Unreadable!

Solution: Proof Combinators!

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
fibTwo
 =    fib 2
 ==. fib 1 + fib 0
 *** QED                                    OK
```

Problem: Proof is Unreadable!

Solution: Proof Combinators!

Refinement Reflection

```
fibTwo :: {fib 2 = 2}
fibTwo
 =    fib 2
 ==. fib 1 + fib 0
 ==. 1 + 1                                    OK
 ==. 2
 *** QED
```

Can we reuse existing proofs?

Refinement Reflection

```
fibThree :: {fib 3 = 3}
fibThree
 =    fib 3
 ==. fib 2 + fib 1
 ==. 2 + 1            ? fibTwo        OK
 ==. 3
 *** QED
```

Combining Proofs

Refinement Reflection

Prove **theorems** about functions.

Step 1: Definition

Step 2: Reflection

Step 3: Application

Refinement Reflection

Turns Haskell into a theorem prover.

Refinement Reflection

Turns Haskell into a theorem prover.

Express **theorems** via refinement types.

Express **proofs** via functions.

Check that functions prove theorems.

Refinement Reflection

```
fibUp :: i:Nat -> {fib i ≤ fib (i+1)}
fibUp i
 | i == 0
 =   fib 0 <. fib 1
 *** QED
 | i == 1
 =   fib 1 <=. fib 1 + fib 0 <=. fib 2
 *** QED
 | otherwise
 =   fib i
 ==. fib (i-1) + fib (i-2)
 <=. fib i     + fib (i-2) ? fibUp (i-1)
 <=. fib i     + fib (i-1) ? fibUp (i-2)
 <=. fib (i+1)
 *** QED
```

Refinement Reflection

Higher Order Theorems

```
fMono :: f:(Nat -> Int)
      -> fUp:(z:Nat -> {f z <= f (z+1)})
      -> x:Nat
      -> y:{Nat| x < y}
      -> {f x <= f y}
```

```
fibMono :: x:Nat -> y:{Nat| x < y}
        -> {fib x <= fib y}
fibMono = fMono fib fibUp
```

Applications

Arithmetic Properties

(fib, ackermann)

Applications

Arithmetic Properties

(fib, ackermann)

Class Laws

(monoid, functor, monad)

Parallelization Equivalence

(of monoid morphisms)

Parallelization Equivalence

MapReduce: Apply `f` `x` in parallel

Parallelization Equivalence

MapReduce: Apply `f x` in parallel

1. Chunk input `x` in `i` chunks (`chunk i x`)
2. Apply `f` to each chunk in parallel (`map f`)
3. Reduce all results `op (foldr op (f []))`

Parallelization Equivalence

MapReduce: Apply `f x` in parallel

1. Chunk input `x` in `i` chunks (`chunk i x`)
2. Apply `f` to each chunk in parallel (`map f`)
3. Reduce all results `op (foldr op (f []))`

```
mapReduce i f op xs
 = foldr op (f []) (map f (chunk i xs)
```

Example: Parallel Sums

```
sum :: [Int] -> Int
sum []     = 0
sum (x:xs) = x + sum xs
```

```
psum :: Int -> xs:[Int] -> Int
psum i xs = mapReduce i sum (+) xs
```

Question: is parallel sum equivalent to sum?

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int]->{sum xs = psum i xs}
sumEq i xs
  =   psum i xs
  ==. sum xs
      ? mapReduceEq i sum (+) rightId distr xs
  *** QED
```

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int]->{sum xs = psum i xs}
sumEq i xs
  =    psum i xs
  ==.  sum xs
       ? mapReduceEq i sum (+) rightId distr xs
  *** QED
```

```
rightId :: xs:[Int] -> ys:[Int]
        -> {sum xs + sum [] = sum xs}
```

Question: is parallel sum equivalent to sum?

```
sumEq :: i:Int->xs:[Int]->{sum xs = psum i xs}
sumEq i xs
  =.   psum i xs
  ==.  sum xs
       ? mapReduceEq i sum (+) rightId distr xs
  *** QED
```

```
rightId :: xs:[Int] -> ys:[Int]
        -> {sum xs + sum [] = sum xs}
```

```
distr   :: xs:[Int] -> ys:[Int]
        -> {sum (xs++ys) = sum xs + sum ys}
```

Parallelization Equivalence

```
mapReduceEq
  :: i:Int
  -> f:([a] -> b)
  -> op:(b -> b -> b)
  -> rId:(xs:[a] -> {f xs `op` f [] = f xs})
  -> dis:(xs:[a] -> ys:[a]
        -> {f (xs++ys) =  f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce i f op xs }
```

Parallelization Equivalence

```
mapReduceEq
  :: i:Int
  -> f:([a] -> b)
  -> op:(b -> b -> b)
  -> rId:(xs:[a] -> {f xs `op` f [] = f xs})
  -> dis:(xs:[a] -> ys:[a]
        -> {f (xs++ys) =  f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce i f op xs }
```

Generalization:

Let b be a monoid with an assoc operator

Parallelization Equivalence

```
mapReduceEq :: (Monoid b)
  => n:Int
  -> f:([a] -> b)
  -> rId:(xs:[a]  -> {f xs `op` f [] = f xs})
  -> dis:(xs:[a] -> ys:[a]
        -> {f (xs++ys) =  f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Generalization:

Let b be a **verified** monoid with right identity

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
  => n:Int
  -> f:([a] -> b)
  -> dis:(xs:[a] -> ys:[a]
        -> {f (xs++ys) =  f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Generalization:

Let b be a **verified** monoid with right identity

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
  => n:Int
  -> f:([a] -> b)
  -> dis:(xs:[a] -> ys:[a]
          -> {f (xs++ys) = f xs `op` f ys})
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Generalization:

Let f be a morphism among a and [b]

Parallelization Equivalence

```
mapReduceEq :: (VerifiedMonoid b)
  => n:Int
  -> f:Morphism [a] b
  -> xs:[a]
  -> { f x = mapReduce n f xs }
```

Applications

Arithmetic Properties

(fib, ackermann)

Class Laws

(monoid, functor, monad)

Parallelization Equivalence

(of monoid morphisms)

Parallelization of String Matching

2. Expressiveness

Refinement Reflection

Goal: Refinement Types for Haskell

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

With decidable & predictable verification.

Implementation

LiquidHaskell

LiquidHaskell

Liquid Types *

Decidable Type Inference

* Rondon, Kawaguchi, Jhala. “Liquid Types” (PLDI ’08)

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

Soundness under Laziness

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

Desugar to Ghost Variables

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

+

Bounded Refinement Types [ICFP 15]

Desugar to Ghost Functions

LiquidHaskell

Liquid Types

+

Termination Checker [ICFP 14]

+

Abstract Refinement Types [ESOP 13]

+

Bounded Refinement Types [ICFP 15]

+

Refinement Reflection [Submitted 16]

Liquid Haskell is a theorem prover

LiquidHaskell

Termination, Totality, Functional Correctness

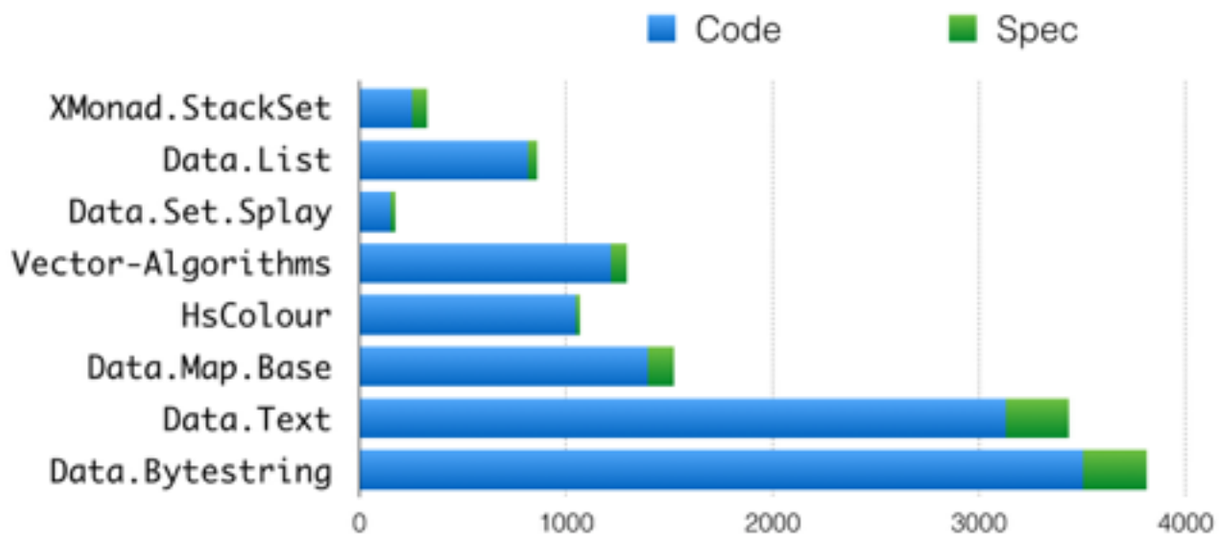
`Data.List`: list sortedness, safe indexing

`Data.Map`: binary-search tree preservation

`Text, Bytestring`: low-level memory safety

`XMonad`: uniqueness of windows

LiquidHaskell



specs: 1/10 LOC

time: 0.5s/10 LOC

Modest Annotations

LiquidHaskell

as a theorem prover

Modest Annotations BUT Verbose Proofs

LiquidHaskell

as a theorem prover

Case study: String matcher parallelization

200LoC	“runtime” code
100LoC	specs on “runtime” code
1300LoC	proof terms
200LoC	theorem specs

proofs = 8 x code

Verbose Proofs

Refinement Types for Haskell

Thanks!

1. Soundness

Under Lazy Evaluation

2. Expressiveness

Modular Higher-Order Specifications

Refinement Reflection

3. Implementation

LiquidHaskell

With decidable & predictable verification.

END

My research goal is to integrate verification into code development chain

Envision:

Integrate Verification into Code Development

LiquidHaskell

The crew

Alexander Bakst

Ranjit Jhala

Simon Peyton-Jones

Eric Seidel

Dimitrios Vytiniotis

...

My research goal is to integrate verification into code development chain

Envision:

Integrate Verification into Code Development

Goal: Verification into Code Development

What I envision is that in this process the programmer also provides some extra specifications that are automatically checked via a prover, ensuring that the final application satisfies the specifications

Goal: Verification into Code D nt

To acchieve this, we need to persuade the programmer to write specifications

User should **naturally** write Specifications ...

Goal: Verification into Code D nt

and the verifier should provide useful feedback

User should **naturally** write Specifications ...

... and get **useful** feedback.

User should **naturally** write Specifications ...

Natural Integrati

Expressiveness

I believe that refinement types are in the right direction of naturally integrating specs into the language, as they simply refine the existing type system with logical predicates.

So the user does not need to learn a new language to express code of specification.

A second requirement for natural specification, is that spec language should be arbitrary expressive and this is a requirement that LH fails to satisfy

Expressiveness

LiquidHaskell:
Functions cannot appear in spec

Right now LH does not allow arbitrary functions to appear inside the specifications which crucially restricts expressiveness.

As an example if I want to prove that fib is increasing I need to refer to the fib function inside the spec and say…

This is my current work, How to allow arbitrary functions appear inside specifications, while retaining decidability and predictability,

```
fib_increasing :: x:Nat -> y:Nat
                -> { x < y => fib x < fib y }
fib_increasing x y = AutoProof
```

Expressiveness

LiquidHaskell:
Functions cannot appear in spec

Right now LH does not allow arbitrary functions to appear inside the specifications which crucially restricts expressiveness.

As an example if I want to prove that fib is increasing I need to refer to the fib function inside the spec and say…

This is my current work, How to allow arbitrary functions appear inside specifications, while retaining decidability and predictability,

```
fib_increasing :: x:Nat -> y:Nat
               -> { x < y => fib x < fib y }
fib_increasing x y = AutoProof
```

Expressiveness

Once I achieve that we could express interesting higher order properties like that the monadic list implementation satisfies the left identity list monad, just by providing a functions that satisfies the specification that forall m:pa[if I bind m to return I get m

LiquidHaskell:
Functions cannot appear in specifications.

```
left_id   :: m:[a] -> { m >>= return == m }
left_id m = AutoProof
```

Goal: Verification into Code D nt

Then I am going to describe wha I see as useful feedback by an example

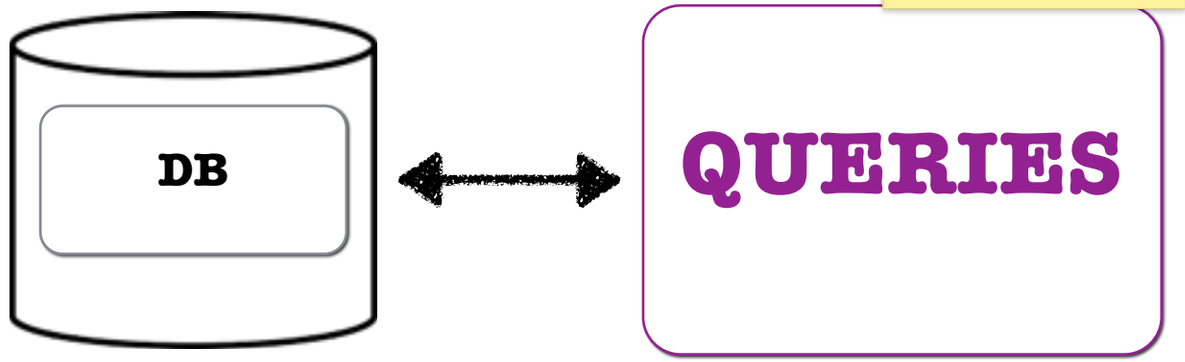
Natural Integration

Expressiveness

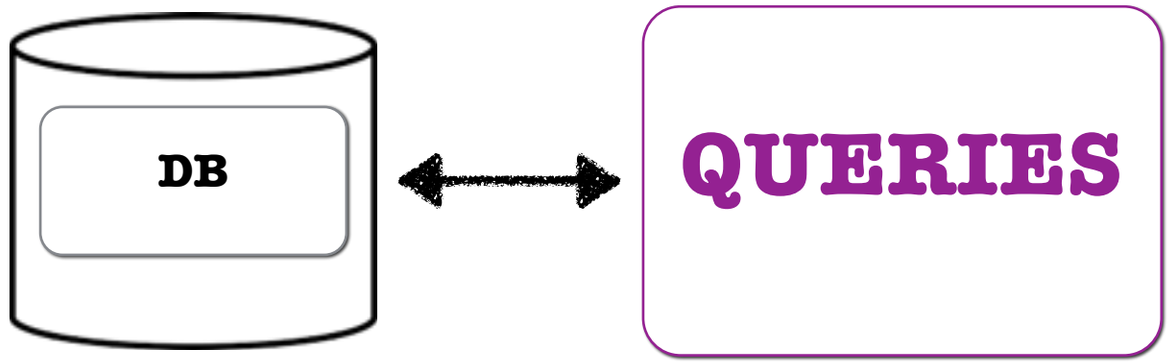
... and get **useful** feedback.

... and get **useful** fee

Consider a db and an API that makes queries. it is not important what the query language is, but to be concrete I am going to use HAXl

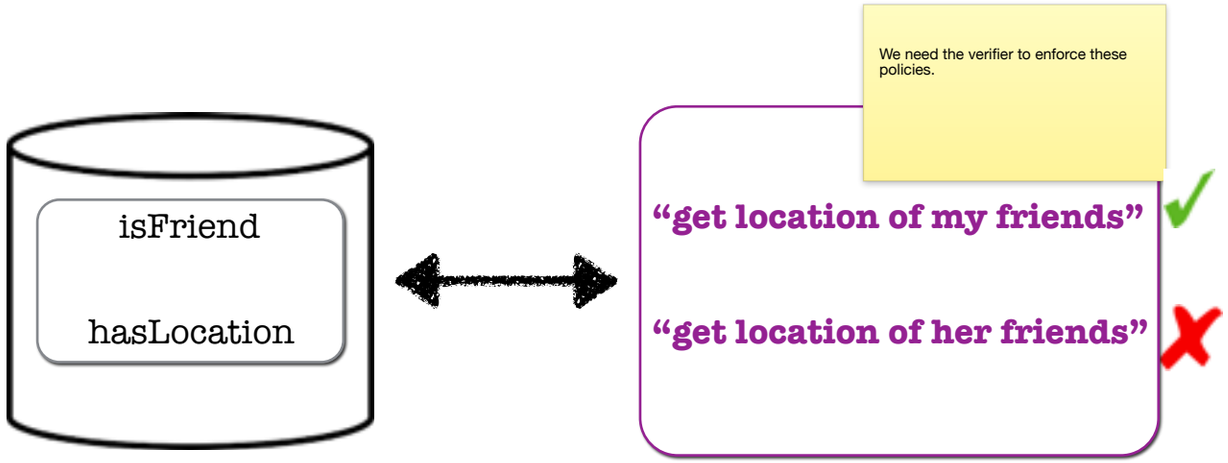


... and get **useful** feedback.



POLICIES

… and get **useful** feedback.



P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

… and get **useful** feedback.

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

```
main :: IO ()
main = do
 -- me and her are two random users
 let (myid, herid) = (42, 24)
 -- initialize the environment with my credentials
 env  <- initEnv $ UserState { me = myid }
 -- fetch the locations of MY friends
 locs <- runHaxl env (getFiendsLocations myid)
```

Haxl*: A Haskell library that accesses remote data

*Simon Marlow et al. There is no Fork. (ICFP ’14)

… and get **useful** feedback.

P1: "Anyone can look at my friend list."

P2: "Only my friends can get my location."

```
main :: IO ()
main = do
 – me and her are two random users
 let (myid, herid)
 – initialize the e                th my credentials
 env  <- initEnv $                 e = myid }
 – fetch the locations of MY friends
 locs <- runHaxl env (getFiendsLocations myid)
```

If my query is SAFE,

Verification should succeed automatically

✓

Automation

… and get **useful** feedback.

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

```
main :: IO ()
main = do
 – me and her are two random users
 let (myid, herid)

 – initialize the e                th my credentials
 env  <- initEnv $                 e = myid }

 – fetch the locations of MY friends
 locs <- runHaxl env (getFiendsLocations herid)
```

If my query is UNSAFE,

Verification should fail immediately

Speed

Automation

… and get **useful** feedback.

P1: “Anyone can look at my friend list.”

P2: “Only my friends can get my location.”

```
main :: IO ()
main = do
 – me and her are two random users
 let (myid, herid) = (42, 24)
 – initialize the environment with my credentials
 env  <- initEnv $ UserState { me = myid }
 – fetch the locations of MY friends
 locs <- runHaxl env (getFiendsLocations herid)
```

Speed

Automation

Error Diagnosis

```
Error: Violation of P2
```

… and get **useful** feedback.

P1: "Anyone can look at my friend list."

P2: "Only my friends can get my location."

```
main :: IO ()
main = do
 -- me and her are two random users
 let (myid, herid) = (42, 24)
 -- initialize the environment with my credentials
 env  <- initEnv $ UserState { me = myid }
 -- fetch the locations of MY friends
 locs <- runHaxl env (getFiendsLocations 1)
```

```
myid   p1, p2
herid  p1, p2
```

Speed

Automation

Error Diagnosis

Program Synthesis

Goal: Verification into Code Development

Natural Integration

Expressiveness

Speed

Automation

Error Diagnosis

Program Synthesis

Goal: Verification into Code Development

Natural Integration
Expressiveness

Data Privacy
Web Security

Speed
Automation
Error Diagnosis
Program Synthesis

Thanks!

END

The Haxl Monad

For this specific example, HAxl provides a run function that says that if you give me an env that stores my cresdntional sand a computation that decries a quesry, I will give you back the res of the query.

```
runHaxl :: env:Env      - the environment
        -> haxl:Haxl a  - the computation
        -> IO a         - fetched data
```

Refine the Haxl Monad

```
runHaxl :: (Valid Env (Haxl a))
        => env:Env
        -> {haxl:Haxl a | valid env haxl}
        -> IO a
```

The Haxl Monad

Haxl is concurrency monad that performs requests of data `a`

```
newtype Haxl a = Haxl
{ unHaxl :: Env                      - environment reader
         -> IORef (RequestStore)     - cached fetches reader
         -> IO (Result a) }          - data fetched
```

```
runHaxl :: env:Env      - the environment
        -> haxl:Haxl a  - the computation
        -> IO a         - fetched data
```

LiquidHaskell

Alexander Bakst

Ranjit Jhala

Simon Peyton-Jones

Eric Seidel

Michael Smith

Christopher Tetrault

Dimitrios Vytiniotis

...

Usable Program Verification

Expressiveness

Abstract Refinement Types

N. Vazou, P. M. Rondon, and R. Jhala. ESOP '13

Bounded Refinement Types

N. Vazou, A. Bakst, and R. Jhala. ICFP '15

Error Reporting & Diagnosis

Type Targeted Testing

E. Seidel, **N. Vazou**, and R. Jhala. ESOP '15

Prototype: LiquidHaskell

Refinement Types for Haskell

N. Vazou, E. Seidel, R. Jhala, D. Vytiniotis, and S. Peyton-Jones. ICFP '15

Experience with Refinement Types in the Real World

N. Vazou, E. Seidel, R. Jhala. Haskell '15