Изследване на функции

деф. обл.; интервали на симетрия; асимптоти, интервали на монотонност, лок. екстремуми, изпъкналост, инфлексни точки, табл. с характерни точки, построяване на графики

$$y = \frac{x^3+4}{x^2} \qquad y(-x) = \frac{-x^3+4}{x^2} \neq \begin{matrix} y(x) \\ -y(x) \end{matrix} \qquad D.O = \{x : x \neq 0\}$$

н.т.н.м.

x	$-\infty$			0		2		∞
y	$-\infty$	$\nearrow\nearrow\nearrow\nearrow$	∞		∞ $\searrow$	min	$\nearrow\nearrow\nearrow$	∞
y'	+ + + + + + +		+		− − −	0	+ + +	+
y''	(+) (+) (+) (+)		(+)		(+)	(+)	(+)	(+)

$$\lim_{x\to\pm\infty} \frac{x^3+4}{x^2} = \lim_{x\to\pm\infty} \frac{1+\frac{4}{x^3}}{\underbrace{\frac{1}{x}}_{\to\pm 0}} = \pm\infty$$

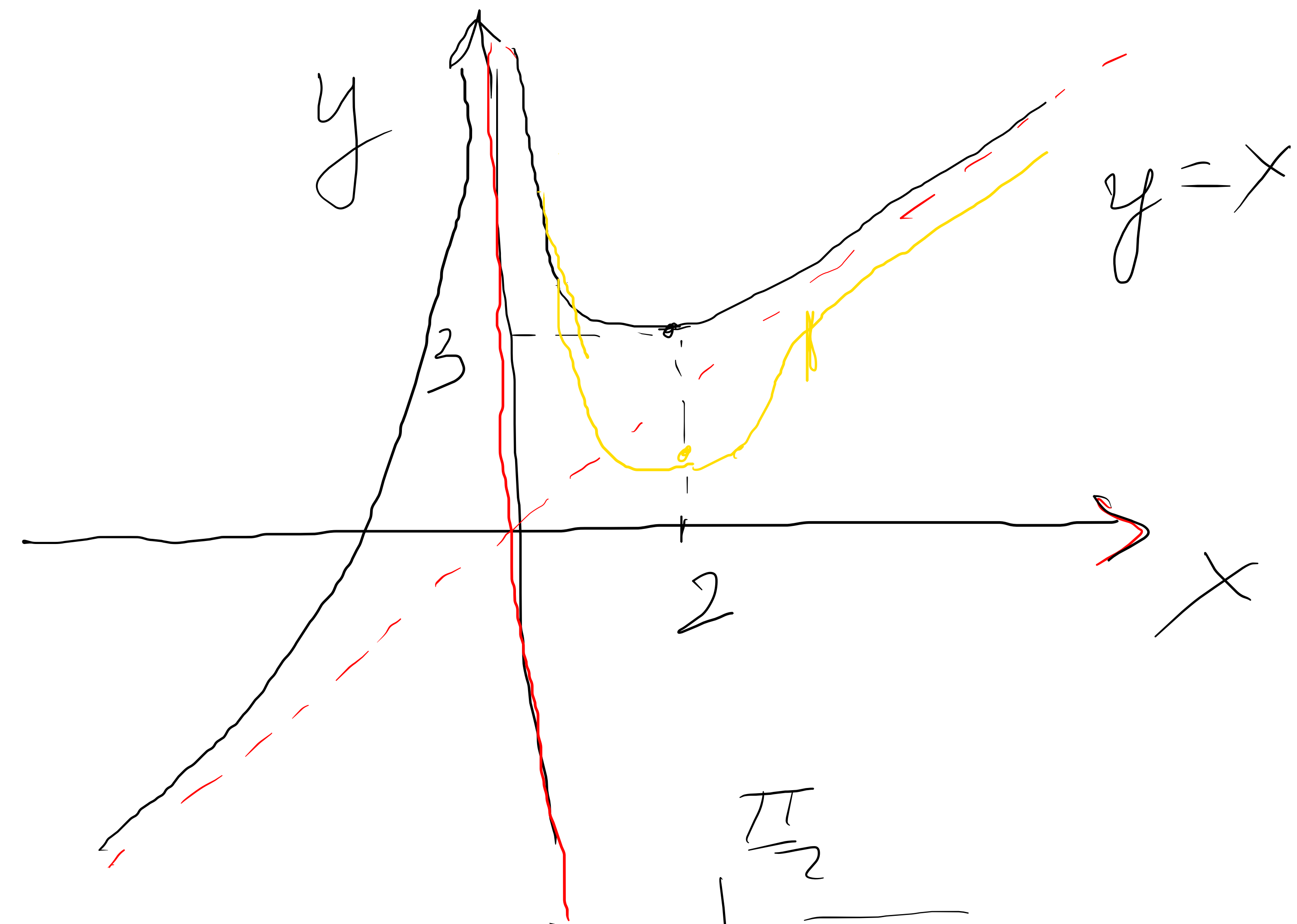
$$\lim_{x\xrightarrow{<}0} \frac{x^3+4}{x^2} = \lim_{\varepsilon\to 0} \frac{(0-\varepsilon)^3+4}{(0-\varepsilon)^2} = \lim_{\varepsilon\to 0} \frac{-\varepsilon^3+4}{\varepsilon^2} = \frac{4}{0} = +\infty$$

$x = 0-\varepsilon,\ \varepsilon > 0$

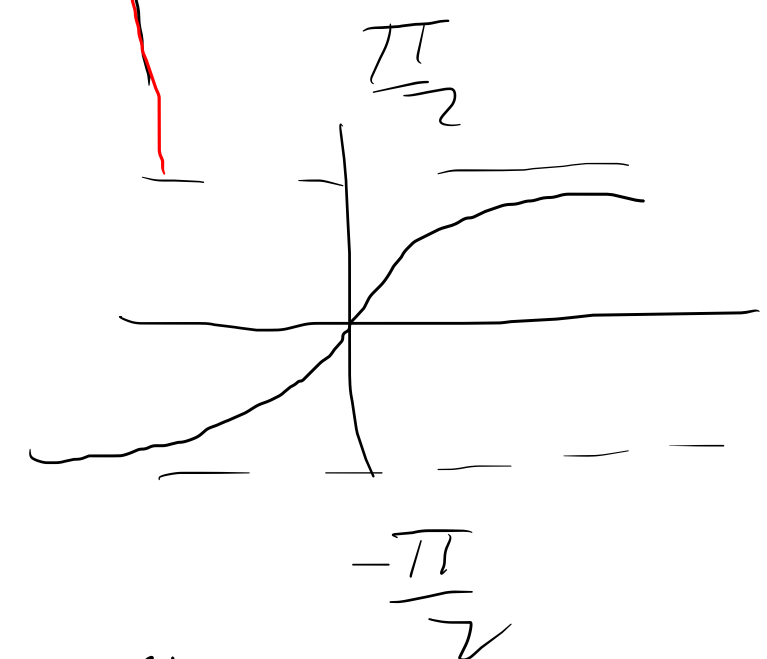
$$\lim_{x\xrightarrow{>}0} \frac{x^3+4}{x^2} = \lim_{\varepsilon\to 0} \frac{\varepsilon^3+4}{\varepsilon^2} = \infty$$

$x = 0+\varepsilon$

x	$-\infty$		0		2		∞
y	$-\infty$	$\nearrow\nearrow\nearrow\nearrow$	$\infty \mid \infty$	$\searrow$	min	$\nearrow\nearrow\nearrow$	∞
y'		$+ + + + + + + +$	$+ \mid - - -$		0	$+ + + +$	
y''		$\cup^{+}$ $\cup^{+}$ $\cup^{+}$ $\cup^{+}$ $\cup^{+}$		$\cup^{+}$ $\cup^{+}$		$\cup^{+}$ $\cup^{+}$	



$$y_{min} = y(2) = \frac{2^3+4}{2^2} = \frac{12}{4} = 3$$



$$y = x - 2\operatorname{arctg} x$$

$$y(-x) = -x - 2\underbrace{\operatorname{arctg}(-x)}_{-\operatorname{arctg} x} = -(x - 2\operatorname{arctg} x) = -y(x)$$ – нечетна

$$D' = \{x : x \geq 0\}$$

$$\lim_{x \to 0}(x - 2\operatorname{arctg} x) = 0 - 2\operatorname{arctg} 0 = 0 - 2 \cdot 0 = 0$$

$$\lim_{x \to \infty}(x - 2\operatorname{arctg} x) = \infty - 2\frac{\pi}{2} = \infty$$

x	0		1		∞
y	0	$\searrow$	min	$\nearrow$	∞
y'	$-\ -\ -$		0	$+\ +\ +\ +$	
y''	0 $\cup$	$\cup$	$\cup$	$\cup$	$\cup$

$$k = \lim_{x\to\infty} \frac{x - 2\operatorname{arctg} x}{x} = 1 - 2\lim_{x\to\infty}\frac{\operatorname{arctg} x}{x}$$

$$= 1 - 2\cdot 0 = 1 \qquad \frac{\frac{\pi}{2}}{\infty} \to 0$$

$$n = \lim_{x\to\infty}\left(\cancel{x} - 2\operatorname{arctg} x - \cancel{x}\right) = -2\lim_{x\to\infty}\operatorname{arctg} x = -2\frac{\pi}{2} = -\pi$$

$\Rightarrow$ $y = x - \pi$ – дясна асимптота

$$y' = 1 - \frac{2}{1+x^2} = \frac{x^2+1-2}{1+x^2} = \frac{x^2-1}{x^2+1} \qquad y' = 0 \Leftrightarrow x^2 - 1 = 0 \Rightarrow x_{1,2} = \pm 1$$

$$y' > 0 \qquad \frac{x^2-1}{\underset{\vee 0}{x^2+1}} > 0 \Rightarrow x^2 - 1 > 0$$

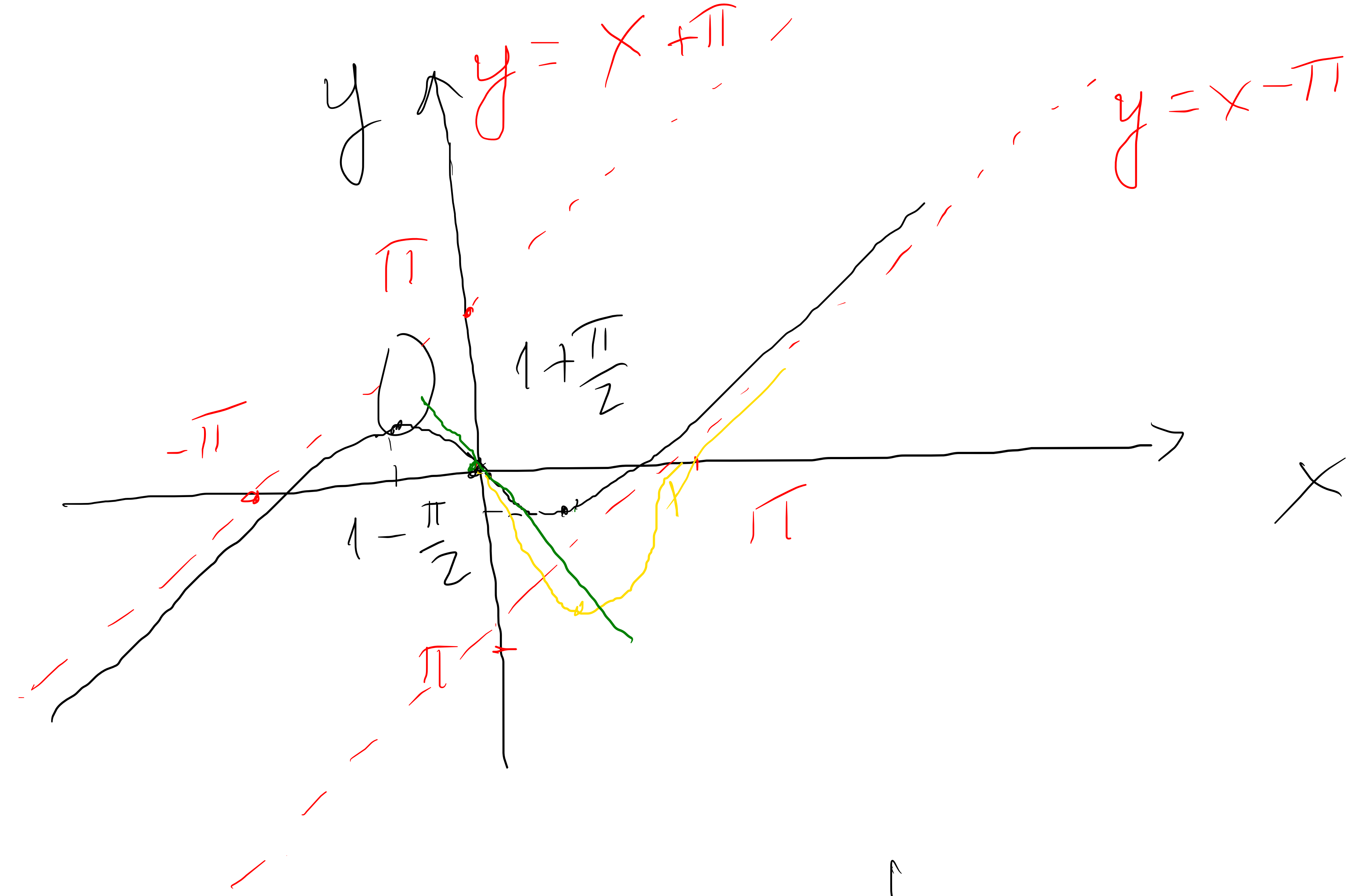
$+\ +\ +$ (-1) $-\ -$ (0) $-\ -$ (1) $+\ +\ +\ +$

$$y_{min} = y(1) = 1 - 2\operatorname{arctg} 1 = 1 - 2\frac{\pi}{4} = 1 - \frac{\pi}{2}$$

y''

$$y'' = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{2x(\cancel{x^2}+1-\cancel{x^2}+1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

x	0	1	∞
y	$0 \searrow$	min	$\nearrow \infty$
y'	$- - -$	0	$+ + + + +$
y''	$0 \oplus \oplus$	$\oplus$	$\oplus \oplus$

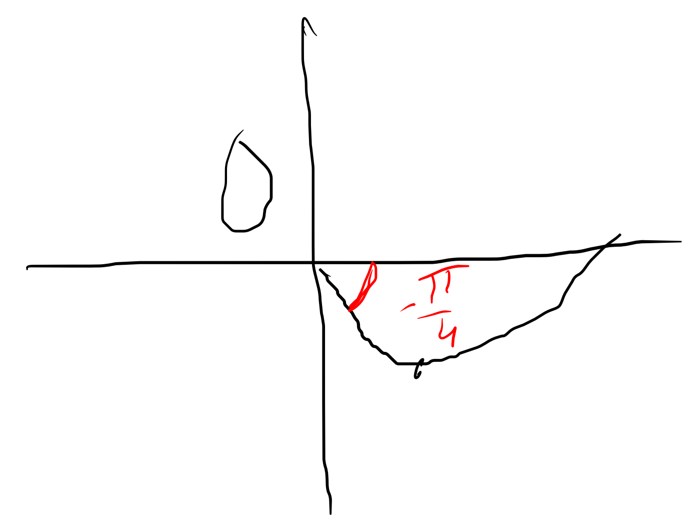


$y = x - \pi$

$y_{min} = y(1) = 1 - \frac{\pi}{2}$

$$\lim_{x \to 0} y' = \lim_{x \to 0} \frac{x^2 - 1}{x^2 + 1} = -1$$

$-\frac{\pi}{4}$



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1) $y = \operatorname{arcctg} \frac{1}{x^2 - 1}$ 2) $y = \ln \operatorname{tg}\left(\frac{\pi}{4} - \frac{x}{2}\right)$ 3) $y = (x+1)e^{\frac{1}{x}}$

$$y'' = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{2x(\cancel{x^2}+1-\cancel{x^2}+1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

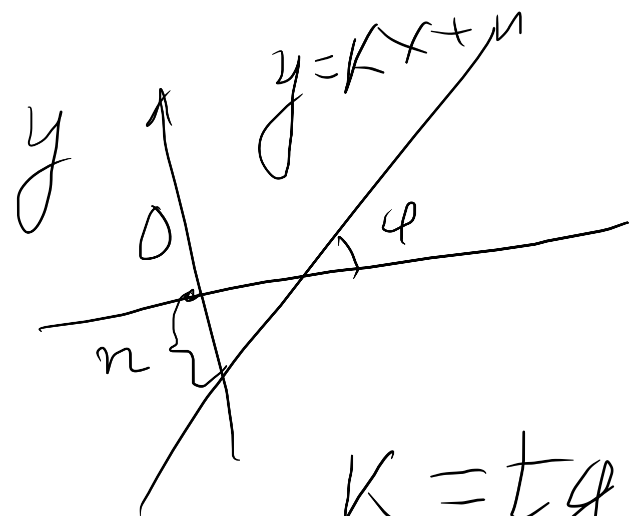
$$y'' = 0 \iff 4x = 0$$

$x = 0$ е инфл. т.

$$y'' > 0 \qquad \frac{4x}{(x^2+1)^2} > 0 \implies x > 0$$

$$(x^2+1)^2 > 0$$

$x=0$ е вертикална асимптота $\quad y = kx + n$



$K = tg\,\varphi$

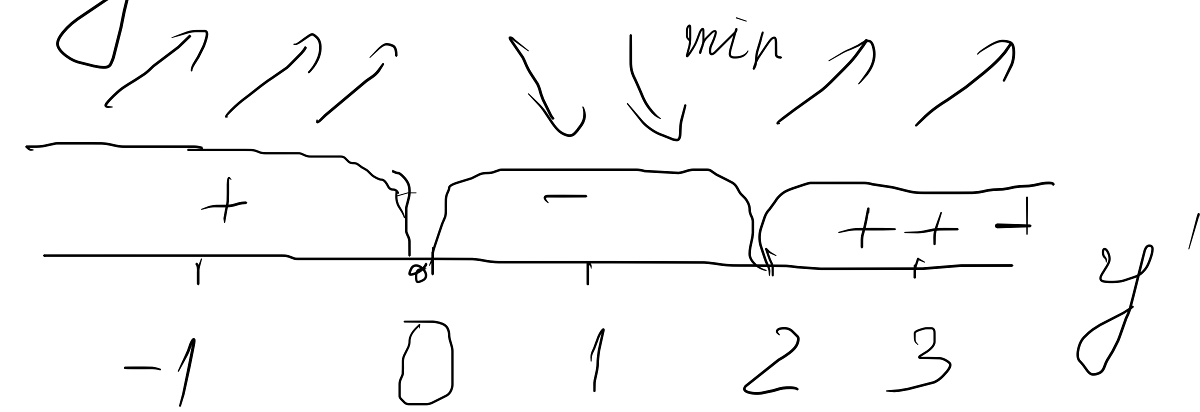
$$k = \lim_{x\to\pm\infty} \frac{x^3+4}{x^3} = 1$$

$$n = \lim_{x\to\pm\infty}\left(\frac{x^3+4}{x^2} - x\right) = \lim_{x\to\pm\infty} \frac{4}{x^2} = 0 \Rightarrow y = x$$ – наклонени асимптоти

$$y' = \frac{3x^2\cdot x^2 - 2x(x^3+4)}{x^4} = \frac{3x^{\cancel{4}3} - 2x^{\cancel{4}3} - 8\cancel{x}}{x^{\cancel{4}3}} = \frac{x^3-8}{x^3} \qquad y' = 0$$

$$\Leftrightarrow x^3 - 8 = 0 \Rightarrow x = 2$$ – стац. т. $\qquad y' > 0$

$$\frac{x^3-8}{x^3} > 0 \qquad \frac{(x-2)(x^2+2x+4)}{x^3} > 0$$



$$\frac{-\ +}{-} \qquad \frac{-\ +}{+} \qquad \frac{+\ +}{+}$$

$$y'' = \frac{3x^2x^3 - 3x^2(x^3-8)}{x^6}$$

$$y'' = \frac{3x^2 x^3 - 3x^2(x^3-8)}{x^6} = \frac{\cancel{3x^5} - \cancel{3x^5} + 24x^2}{x^6} = \frac{24}{x^4} > 0 \Rightarrow$$ изпъкн

(отдолу)