Осн. граници $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$; $\lim_{t\to 0}(1+t)^{\frac{1}{t}} = e$; $\lim_{x\to\infty}\frac{1}{x}=0$

$t = \frac{1}{x}$

$\lim_{x\to 0}\frac{\sin x}{x} = 1$; $\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1$; $\lim_{x\to 0}\frac{e^x-1}{x} = 1$; $\lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$

няколко техники

$x\to\pm\infty$ $\frac{P(x)}{Q(x)} = \frac{\cancel{x^n}}{\cancel{x^n}}\frac{\dots}{\dots}$ $1+\left(\frac{1}{x}\right)^k$

субституция ; разкриване на неопред. при $x\to a$

$\frac{0}{0}$ - неопред $\frac{\cancel{x-a}}{\cancel{x-a}}\frac{\dots}{\dots e}$ унищожаване на ирационалности

$x\to a$

$\lim_{x\to 0}(1-4x)^{\frac{x-1}{x}} = \lim_{x\to 0}\left\{\left[1+(-4x)\right]^{\frac{1}{-4x}}\right\}^{\lim_{x\to 0}-4\frac{\cancel{x}(x-1)}{\cancel{x}}} = e^4$

1^∞ $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\lim_{x\to 0}\frac{\sin x}{x}} = e^1 = e$

$$\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}} = \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

$\sin x \sim x$

$x \to 0$

; $$\lim_{x\to 0}\frac{\ln(\cos x)}{x} = \quad \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{\ln\left(1-2\sin^2\frac{x}{2}\right)}{x} = \lim_{x\to 0}\frac{\ln\left(1+\left(-2\sin^2\frac{x}{2}\right)\right)}{-2\sin^2\frac{x}{2}} \lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{x} =$$

$$= 1\cdot(-2)\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \lim_{x\to 0}\frac{1}{4}x = -2\cdot 1\cdot 0 = 0$$

$$-2\sin^2\frac{x}{2} \sim -2\left(\frac{x}{2}\right)^2 = -\frac{x^2}{2}$$

$$= \lim_{x\to 0}\frac{\ln\left(1+\frac{x^2}{2}\right)^1}{\frac{x^2}{2}} \lim \frac{1}{2}\sqrt{x} = 1\cdot 0 = 0;$$

$$\lim_{x\to 0}\frac{a^x-1}{x} = \lim_{x\to 0}\frac{e^{x\ln a}-1}{x\ln a}\ln a$$

$a>0$ $\quad a^x = e^{x\ln a}$

$$= 1\cdot \ln a = \ln a \qquad a = e$$

$$\lim_{x\to 0} \frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} = \frac{1}{2} \lim_{x\to 0} \left(\frac{1}{x} \ln \frac{1+x}{1-x} \right) = \frac{1}{2} \lim_{x\to 0} \frac{\ln(1+x) - \ln(1-x)}{x}$$

$$\frac{0}{0}$$

$$= \frac{1}{2} \lim_{x\to 0} \frac{\ln(1+x)}{x} + \frac{1}{2} \lim_{x\to 0} \frac{\ln(1-x)}{-x} = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$$

$$\ln \sqrt{\frac{1+x}{1-x}} \sim x \quad ; \quad \lim_{x\to 0} \frac{\sin 5x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 5x}{5x} \lim_{x\to 0} \frac{1}{\frac{\sin 3x}{3x}} \cdot$$

$$x \to 0 \qquad\qquad \frac{0}{0}$$

$$\cdot \lim_{x\to 0} \frac{5\cancel{x}}{3\cancel{x}} = 1 \cdot 1 \cdot \frac{5}{3} = \frac{5}{3} \quad ; \quad \lim_{x\to 1} \frac{\cos \frac{\pi x}{2}}{1-\sqrt{x}} = \lim_{x\to 1} \frac{\sin \frac{\pi}{2}(1-x)}{\frac{\pi}{2}(1-x)} \frac{\frac{\pi}{2}(1-x)}{1-\sqrt{x}} =$$

$$\cos \frac{\pi x}{2} = \sin \frac{\pi}{2}(1-x) = \qquad \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{\sin \frac{\pi}{2}(1-x)}{\frac{\pi}{2}(1-x)} \frac{\pi}{2} (1+\sqrt{x}) =$$

$$= \sin\left(\frac{\pi}{2} - \frac{\pi x}{2}\right) = \underbrace{\sin \frac{\pi}{2}}_{1} \cos \frac{\pi x}{2} - \underbrace{\cos \frac{\pi}{2}}_{0} \sin \frac{\pi x}{2}$$

$$= 1 \cdot \frac{\pi}{2} \cdot 2 = \pi$$

$$\lim_{x\to\infty}\frac{1x}{\sqrt[3]{1x^3+10}} = \lim_{x\to\infty}\frac{\cancel{x}}{\cancel{x}}\frac{1}{\sqrt[3]{1+\frac{10}{x^3}}} = \underline{1}$$

$$\lim_{x\to a}\frac{x^2-(a+1)x+a}{x^3-a^3} = \lim_{x\to a}\frac{\cancel{(x-a)}(x-1)}{\cancel{(x-a)}(x^2+ax+a^2)} = \frac{a-1}{a^2+a^2+a^2} = \frac{a-1}{3a^2}$$

$$\cancel{a^2}-a(\cancel{a+1})+\cancel{a}$$

$$\frac{a+1\pm\sqrt{(a+1)^2-4a}}{2} = \frac{a+1\pm\sqrt{(a-1)^2}}{2} =$$

$$\frac{0}{0} \quad \frac{x-a}{x-a}$$

$$= \begin{cases} \frac{a+\cancel{1}+a-\cancel{1}}{2} = a \\ \frac{\cancel{a}+1-\cancel{a}+1}{2} = 1 \end{cases}$$

$$x^{\frac{1}{2}},\ x^{\frac{1}{3}}$$

$$\lim_{x\to 64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4} = \lim_{t\to 2}\frac{t^{\frac{6}{2}}-8}{t^{\frac{6}{3}}-4} = \lim_{t\to 2}\frac{t^3-8}{t^2-4} =$$

$$x = t^{\text{НОК (знам.)}} = t^6$$

$$x\to 64 \Rightarrow t\to 2$$

$$\frac{0}{0}$$

$$= \lim_{t\to 2}\frac{\cancel{(t-2)}(t^2+2t+4)}{\cancel{(t-2)}(t+2)} = \frac{2^2+4+4}{2+2} = \frac{12}{4} = 3$$

$$\lim_{x\to 0}\frac{\cos x-\sqrt[3]{\cos x}}{\sin^2 x}=\lim_{t\to 1}\frac{t-\sqrt[3]{t}}{1-t^2}=\lim_{t\to 1}\frac{t^3-t}{(1-t^2)\left(t^2+t\sqrt[3]{t}+\sqrt[3]{t^2}\right)}$$

$\frac{0}{0}$ $\cos x = t$ $1-t$

$x\to 0 \Rightarrow t\to 1$

$$=\lim_{t\to 1}\frac{t\cancel{(t-1)}(t+1)}{\cancel{(t-1)}(t+1)\left(t^2+t\sqrt[3]{t}+\sqrt[3]{t^2}\right)}=\frac{-1\cancel{(1+1)}}{\cancel{(1+1)}(1+1+1)}=-\frac{1}{3}$$

$$\lim_{x\to\infty}\left(x+\sqrt[3]{1-x^3}\right)=\lim_{x\to\infty}\frac{\cancel{x^3}+1-\cancel{x^3}}{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{(1-x^3)^2}}=0$$

$\infty-\infty$

$\infty+\infty+\infty$

$x^{\frac{2}{3}}, x^{\frac{1}{3}}, x^{\frac{3}{3}}$

$$\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=$$

субстит. $x=t^3$

$x\to 1 \Rightarrow t\to 1$

$=\frac{1}{3^2}=\frac{1}{9}$

$\frac{0}{0}$

$$=\lim_{t\to 1}\frac{t^{\frac{6}{3}}-2t^{\frac{3}{3}}+1}{(t^3-1)^2}=\lim_{t\to 1}\frac{t^2-2t+1}{(t^3-1)^2}=\lim_{t\to 1}\frac{\cancel{(t-1)^2}}{\cancel{(t-1)^2}(t^2+t+1)^2}$$

g.p No 6

1) $$\lim_{x\to\infty} \frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[3]{x^2+x}-x}$$

2) $$\lim_{x\to 1} (1-x)\,\mathrm{tg}\,\frac{\pi}{2}x$$