$$A = \begin{pmatrix} 5 & 12 & -6 \\ -3 & -10 & 6 \\ -3 & -12 & 8 \end{pmatrix} \qquad |A - \lambda E| = \begin{vmatrix} 5-\lambda & 12 & -6 \\ -3 & -10-\lambda & 6 \\ -3 & -12 & 8-\lambda \end{vmatrix}^{P_1+P_2} = 0$$

$$= \begin{vmatrix} 5-\lambda & 12 & -6 \\ 2-\lambda & 2-\lambda & 0 \\ -3 & -12 & 8-\lambda \end{vmatrix}^{-4C_1+C_2} = \begin{vmatrix} 5-\lambda & 4(\lambda-2) & -6 \\ 2-\lambda & -3(2-\lambda) & 0 \\ -3 & 0 & 8-\lambda \end{vmatrix} = -3(-1)^{3+1}\begin{vmatrix} 4(\lambda-2) & -6 \\ 3(\lambda-2) & 0 \end{vmatrix} +$$

$$-4(5-\lambda)+12 = 4\lambda-8$$

$$-4(2-\lambda)+(2-\lambda)$$

$$+(8-\lambda)(-1)^{3+3}\begin{vmatrix} 5-\lambda & 4(\lambda-2) \\ 2-\lambda & +3(\lambda-2) \end{vmatrix} = -3.18(\lambda-2)+$$

$$+(8-\lambda)\left[3(5-\lambda)(\lambda-2)+4(\lambda-2)^2\right] = -54(\lambda-2)+$$

$$+(8-\lambda)(\lambda-2)(15-3\lambda+4\lambda-8) = -54(\lambda-2)+(8-\lambda)(\lambda-2)(\lambda+7) =$$

$$=(\lambda-2)[\underbrace{\;}_{-54}8\lambda-\lambda^2+56-7\lambda] = (\lambda-2)(-\lambda^2+\lambda+2) = -(\lambda-2)(\lambda^2-\lambda-2) = 0$$

$$\frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} = 2;\ -1 \qquad (\lambda-2)^2(\lambda+1) = 0$$

$\lambda_1 = -1$, $\lambda_{2,3} = 2$ – хар. корени, които са и собств. стойн. на A

$$\lambda_1 = -1 \Longrightarrow A+E = \begin{pmatrix} 5+1 & 12 & -6 \\ -3 & -10+1 & 6 \\ -3 & -12 & 8+1 \end{pmatrix} = \begin{pmatrix} 6 & 12 & -6 \\ -3 & -9 & 6 \\ -3 & -12 & 9 \end{pmatrix} \overset{\frac{1}{6}p_1 \ \frac{1}{3}p_{2,3}}{\sim} \begin{pmatrix} 1 & 2 & -1 \\ -1 & -3 & 2 \\ -1 & -4 & 3 \end{pmatrix} \overset{p_1+p_{2,3}}{\sim}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{pmatrix} \overset{-2p_2+p_3}{\sim} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3=p \end{array} \\ \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$-x_2 = -p \Longrightarrow x_2 = p$

$x_1 = -2x_2 + p = -p$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -p \\ p \\ p \end{pmatrix} = p\underbrace{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{v}_1}$$

$r=2$ б.м.р., завис.

$n=3$ от $3-2=1$ парам.

$\lambda_{2,3} = 2 \Longrightarrow$

$$A-2E = \begin{pmatrix} 5-2 & 12 & -6 \\ -3 & -10-2 & 6 \\ -3 & -12 & 8-2 \end{pmatrix} = \begin{pmatrix} 3 & 12 & -6 \\ -3 & -12 & 6 \\ -3 & -12 & 6 \end{pmatrix} \sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2=q & x_3=s \end{array} \\ \begin{pmatrix} 1 & 4 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$r=1$

$n=3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \sim \begin{pmatrix} -4q+2s \\ q \\ s \end{pmatrix} = \begin{pmatrix} -4q \\ q \\ 0 \end{pmatrix} + \begin{pmatrix} 2s \\ 0 \\ s \end{pmatrix} = q\underbrace{\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}}_{\vec{v}_2} + s\underbrace{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}}_{\vec{v}_3}$$

$\vec{v}_1\ \vec{v}_2\ \vec{v}_3$ – ЛН в-ри

| собствен базис

$$= p\begin{pmatrix}-1\\1\\1\end{pmatrix} = \vec{v}_1$$

$r=2$ б.м.р., завис.

$n=3$ от $3-2=1$ па...

$$A-2E = \begin{pmatrix} 5-2 & 12 & -6 \\ -3 & -10-2 & 6 \\ -3 & -12 & 8-2 \end{pmatrix} = ($$

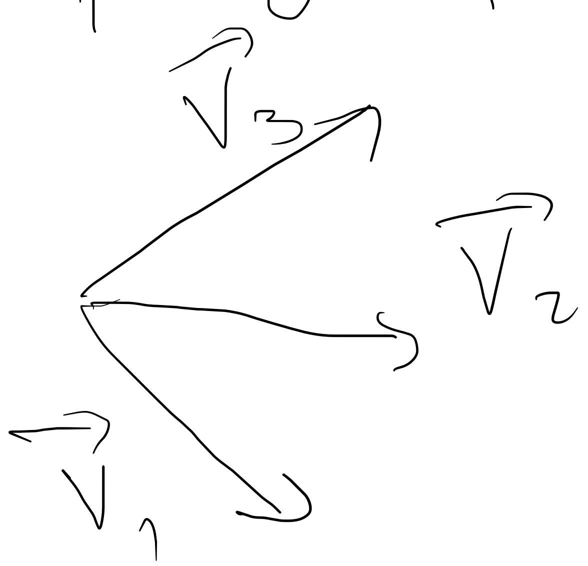
$$\alpha_1\vec{V}_1 + \alpha_2\vec{V}_2 + \alpha_3\vec{V}_3 = \vec{0}$$

$$\alpha_1\begin{pmatrix}-1\\1\\1\end{pmatrix} + \alpha_2\begin{pmatrix}-4\\1\\0\end{pmatrix} + \alpha_3\begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\ldots 2s) = \begin{pmatrix}-4q\\q\\0\end{pmatrix} + \begin{pmatrix}2s\\0\\s\end{pmatrix} = q\underbrace{\begin{pmatrix}-4\\1\\0\end{pmatrix}}_{\vec{V}_2} + s\underbrace{\begin{pmatrix}2\\0\\1\end{pmatrix}}_{\vec{V}_3}$$

$$\begin{cases} -\alpha_1 - 4\alpha_2 + 2\alpha_3 = 0 \\ \alpha_1 + \alpha_2 = 0 \\ \alpha_1 + \alpha_3 = 0 \end{cases}$$

$$\begin{vmatrix} -1 & -4 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -1-2+4 \neq 0 \Longrightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0 \Longrightarrow \vec{V}_1, \vec{V}_2, \vec{V}_3 \text{ л.н.}$$



$$S^{-1}AS = \overset{D}{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

$$A \sim D$$

$$S = [\vec{V}_1\ \vec{V}_2\ \vec{V}_3]$$

$$\alpha_1 \begin{pmatrix}-1\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}-4\\1\\0\end{pmatrix} + \alpha_3 \begin{pmatrix}2\\0\\1\end{pmatrix} = \qquad S = \begin{pmatrix}-1 & -4 & 2\\1 & 1 & 0\\1 & 0 & 1\end{pmatrix}$$

$$SE = \left(\begin{array}{ccc|ccc}-1 & -4 & 2 & 1 & 0 & 0\\1 & 1 & 0 & 0 & 1 & 0\\1 & 0 & 1 & 0 & 0 & 1\end{array}\right) \begin{array}{l}P_1 + P_{2,3}\end{array} \sim \left(\begin{array}{ccc|ccc}-1 & -4 & 2 & 1 & 0 & 0\\0 & -3 & 2 & 1 & 1 & 0\\0 & -4 & 3 & 1 & 0 & 1\end{array}\right) \begin{array}{l}-\frac{4}{3}P_2 + P_3\end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc}-1 & -4 & 2 & 1 & 0 & 0\\0 & -3 & 2 & 1 & 1 & 0\\0 & 0 & \frac{1}{\cancel{3}} & -\frac{1}{\cancel{3}} & -\frac{4}{\cancel{3}} & \cancel{1}\,3\end{array}\right) \begin{array}{l}3P_3\\-2P_3 + P_{1,2}\end{array} \sim \left(\begin{array}{ccc|ccc}-1 & -4 & 0 & 3 & 8 & -6\\0 & -3 & 0 & 3 & 9 & -6\\0 & 0 & 1 & -1 & -4 & 3\end{array}\right) \begin{array}{l}-\frac{4}{3}P_2 + P_1\end{array} \sim \left(\begin{array}{ccc|ccc}-1 & 0 & 0 & -1 & -4 & 2\\0 & 1 & 0 & -1 & -3 & 2\\0 & 0 & 1 & -1 & -4 & 3\end{array}\right)$$

$$-\frac{8}{3} + 3$$

$$-\frac{4}{3}3 + 3 \qquad -\frac{4}{3}9 + 8 \qquad +\frac{4}{3}6 - 6$$

$$\sim \left(\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 4 & -2\\0 & 1 & 0 & -1 & -3 & 2\\0 & 0 & 1 & \underbrace{-1 & -4 & 3}_{S^{-1}}\end{array}\right)$$

$$\underbrace{\begin{pmatrix} 1 & 4 & -2 \\ -1 & -3 & 2 \\ -1 & -4 & 3 \end{pmatrix}}_{S^{-1}} \qquad S^{-1} A S \overset{?}{=} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 4 & -2 \\ -1 & -3 & 2 \\ -1 & -4 & 3 \end{pmatrix}\begin{pmatrix} 5 & 12 & -6 \\ -3 & -10 & 6 \\ -3 & -12 & 8 \end{pmatrix}}\begin{pmatrix} -1 & -4 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5-12+6 & 12-40+24 & -6+24-16 \\ -5+9-6 & -12+30-24 & 6-18+16 \\ -5+12-9 & -12+40-36 & 6-24+24 \end{pmatrix}\begin{pmatrix} -1 & -4 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & -4 & 2 \\ -2 & -6 & 4 \\ -2 & -8 & 6 \end{pmatrix}\begin{pmatrix} -1 & -4 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-4+2 & 4-4 & -2+2 \\ 2-6+4 & 8-6 & -4+4 \\ 2-8+6 & 8-8 & -4+6 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Зад. $A = \begin{pmatrix} -7 & -4 \\ 18 & 11 \end{pmatrix}$ $\quad A^3 = ?$ $\quad S^{-1} A S = D$ (диаг. матр. от собств. стойн.

$A = S D S^{-1}$ $\qquad A^2 = A . A = S D \underbrace{S^{-1} S}_{E} D S^{-1} = S D^2 S^{-1}$

$A^k = S D^k S^{-1}$

$$D^k = \operatorname{diag}(\lambda_1^k, \lambda_2^k, \ldots, \lambda_n^k)$$

$$|A - \lambda E| = \begin{vmatrix} -7-\lambda & -4 \\ 18 & 11-\lambda \end{vmatrix} = -(\lambda+7)(11-\lambda) + 72 = 0$$

$$-(11\lambda + 77 - \lambda^2 - 7\lambda) + 72 = -(-\lambda^2 + 4\lambda + 77) + 72 = \lambda^2 - 4\lambda - 5 = 0$$

$$\lambda_{1,2} = 2 \pm \sqrt{4+5} = 2 \pm 3 = 5; -1 \qquad D = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\lambda_1 = 5 \Rightarrow A - 5E = \begin{pmatrix} -7-5 & -4 \\ 18 & 11-5 \end{pmatrix} = \qquad D^3 = \begin{pmatrix} 5^3 & 0 \\ 0 & (-1)^3 \end{pmatrix} = \begin{pmatrix} 125 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & -4 \\ 18 & 6 \end{pmatrix} \sim \begin{pmatrix} -3 & -1 \\ 3 & 1 \end{pmatrix} \sim \begin{array}{cc} x_1 & x_2 = p \end{array} \begin{pmatrix} -3 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{p}{3} \\ p \end{pmatrix} = \frac{p}{3} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\lambda_2 = -1 \Rightarrow A + E = \begin{pmatrix} -7+1 & -4 \\ 18 & 11+1 \end{pmatrix} = \begin{pmatrix} -6 & -4 \\ 18 & 12 \end{pmatrix} \sim \begin{pmatrix} -3 & -2 \\ 3 & 2 \end{pmatrix} \sim \begin{array}{cc} x_1 & x_2 = q \end{array} \begin{pmatrix} -3 & -2 \\ 0 & 0 \end{pmatrix} \quad \vec{v}_1$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3}q \\ q \end{pmatrix} = \frac{q}{3} \underbrace{\begin{pmatrix} -2 \\ 3 \end{pmatrix}}_{\vec{v}_2} \qquad S = [\vec{v}_1, \vec{v}_2] = \begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix}$$

$$S = [\vec{v}_1, \vec{v}_2] = \begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix} \qquad SE = \left(\begin{array}{cc|cc} -1 & -2 & 1 & 0 \\ 3 & 3 & 0 & 1 \end{array}\right) \overset{3p_1 + p_2}{\sim} \left(\begin{array}{cc|cc} -1 & -2 & 1 & 0 \\ 0 & -3 & 3 & 1 \end{array}\right) \overset{-\frac{1}{3}p_2}{\sim}$$

$$\sim \left(\begin{array}{cc|cc} -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & -\frac{1}{3} \end{array}\right) \overset{2p_2 + p_1}{\sim} \left(\begin{array}{cc|cc} +1 & 0 & +1 & +\frac{2}{3} \\ 0 & 1 & -1 & -\frac{1}{3} \end{array}\right)$$

$$\underbrace{\begin{matrix} +1 & +\frac{2}{3} \\ -1 & -\frac{1}{3} \end{matrix}}_{S^{-1}}$$

$$A^3 = \frac{1}{3} \underbrace{\begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix}}_{S} \begin{pmatrix} 125 & 0 \\ 0 & -1 \end{pmatrix} \underbrace{\begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix}}_{S^{-1}} = \frac{1}{3} \begin{pmatrix} -125 & 2 \\ 375 & -3 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix} =$$

$$= \frac{1}{3} \begin{pmatrix} -375 - 6 & -250 - 2 \\ 375 + 9 & 750 + 3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -381 & -252 \\ 384 & 753 \end{pmatrix} = \begin{pmatrix} 127 & -84 \\ 128 & 251 \end{pmatrix}$$

д.р. No 4

① За матр. $A=\begin{pmatrix} 1 & -3 & 3 \\ -2 & -4 & 6 \\ -2 & -6 & 8 \end{pmatrix}$ конструирайте матр. S, която диагонализира A и проверете, че $S^{-1}AS=diag(\lambda_1,\lambda_2,\lambda_3)$, λ_i, $i=1,2,3$ са собств. стойн. на A

② A, B: $B^2=A$. Казв., че $B=\sqrt{A}$

$$S^{-1}AS=diag(\lambda_1,\dots,\lambda_n)=D$$

$$\sqrt{A}=S\sqrt{D}S^{-1} \qquad \sqrt{D}=diag(\sqrt{\lambda_1},\sqrt{\lambda_2},\dots,\sqrt{\lambda_n})$$

$$A=\begin{pmatrix} 6 & -2 \\ -3 & 7 \end{pmatrix} \qquad \sqrt{A}=?$$