Разкриване на неопределености

2 осн. неопределености $\left[\frac{0}{0}\right]$ $\left[\frac{\infty}{\infty}\right]$

4 теореми на Лопитал

Правила на Лопитал

Гайсн ученик на Бернули 1701 г. $\underline{I}$ ~~печатна~~ книга по матем.

$f(x) \to 0$, $g(x) \to 0$, $x \to x_0$

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = \lim_{x\to x_0} \frac{f''(x)}{g''(x)} = \dots = \lim_{x\to x_0} \frac{f^{(n)}(x)}{g^{(n)}(x)}$$

$\frac{0}{0}$ $\frac{0}{0}$

$f(x) \to \infty$, $g(x) \to \infty$, $x \to x_0$

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = \dots = \lim_{x\to x_0} \frac{f^{(n)}(x)}{g^{(n)}(x)}$$

$\frac{\infty}{\infty}$ $\frac{0}{0}$

$$[\infty\, 0] = \begin{cases} \left[\frac{1}{0}\, 0\right] \sim \left[\frac{0}{0}\right] \\ \left[\infty\, \frac{1}{\infty}\right] \sim \left[\frac{\infty}{\infty}\right] \end{cases} \qquad [\infty - \infty] = \left[\frac{1}{0} - \frac{1}{0}\right] \sim \left[\frac{0}{0}\right]$$

$$[1^{\infty}] \qquad \left(1+\frac{1}{x}\right)^{x} \xrightarrow[x\to\infty]{} e \qquad \ln \qquad [\infty \ln 1] \sim [\infty\, 0]$$

$$[0^{0}] \qquad [0 \ln 0] \sim [0\, \infty]$$

$$[\infty^{0}] \qquad [0 \ln \infty] \sim [0\, \infty]$$

$$\lim_{x\to-1} \frac{\ln(1+x)}{tg\frac{\pi x}{2}} = \lim_{x\to-1} \frac{\frac{1}{1+x}}{\frac{1}{\cos^2\frac{\pi x}{2}}\cdot\frac{\pi}{2}} = \frac{2}{\pi} \lim_{x\to-1} \frac{\cos^2\frac{\pi x}{2}}{1+x} = \cancel{\frac{2}{\pi}} \lim_{x\to-1} \frac{-2\cos\frac{\pi x}{2}\sin\frac{\pi x}{2}\cdot\cancel{\frac{\pi}{2}}}{1}$$

$$\frac{0}{0}$$

$$-\frac{\infty}{-\infty}$$

$$= -\lim_{x\to-1} \sin \pi x = 0$$

$$\lim_{x\to 1} \ln x \cdot \ln(x-1) = \lim_{x\to 1} \frac{\ln(x-1)}{\frac{1}{\ln x}} = \lim_{x\to 1} \frac{\frac{1}{x-1}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}} = \lim_{x\to 1} \frac{x \ln^2 x}{x-1} =$$

$0 \cdot \infty$ $\qquad \frac{\infty}{\infty}$ $\qquad \frac{0}{0}$

$$= -\lim_{x\to 1} \frac{\ln^2 x + 2\cancel{x} \ln x \cdot \frac{1}{\cancel{x}}}{1} = -\lim_{x\to 1} (\underbrace{\ln^2 x}_{\downarrow\, 0} + 2 \underbrace{\ln x}_{\downarrow\, 0}) = 0$$

$$\lim_{x\to 0} (\operatorname{cotg} x)^{\frac{1}{\ln x}} \qquad y = (\operatorname{cotg} x)^{\frac{1}{\ln x}} \;\Big|\ln \Rightarrow \ln y = \frac{\ln \operatorname{cotg} x}{\ln x}$$

$$\infty^0 \qquad \lim_{x\to 0} \ln y = \underline{\ln \lim_{x\to 0} y} = \lim_{x\to 0} \frac{\ln \operatorname{cotg} x}{\ln x} = \lim_{x\to 0} \frac{-\frac{1}{\operatorname{cotg} x}\cdot\frac{1}{\sin^2 x}}{\frac{1}{x}} =$$

$$= -2\lim_{x\to 0} \frac{x}{\sin 2x} = -\lim_{x\to 0} \frac{2x}{\sin 2x} \overset{\frac{\infty}{\infty}}{=} \underline{-1} \Rightarrow \lim_{x\to 0} y = e^{-1}$$

$$y=\left(\frac{2}{\pi}\operatorname{arctg} x\right)^{x} \Rightarrow \ln y = x \ln\left(\frac{2}{\pi}\operatorname{arctg} x\right)$$

$$\lim_{x\to\infty}\left(\frac{2}{\pi}\operatorname{arctg} x\right)^{x} \quad 1^{\infty} \qquad \lim_{x\to\infty}\ln y = \underline{\ln \lim_{x\to\infty} y} = \lim_{x\to\infty} x\ln\left(\frac{2}{\pi}\operatorname{arctg} x\right) = \quad \infty\, 0$$

$$= \lim_{x\to\infty}\frac{\ln\left(\frac{2}{\pi}\operatorname{arctg} x\right)}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\frac{1}{\cancel{\frac{2}{\pi}}\operatorname{arctg} x}\cdot\cancel{\frac{2}{\pi}}\frac{1}{1+x^2}}{-\frac{1}{x^2}} = -\lim_{x\to\infty}\frac{1x^2}{(1+x^2)\operatorname{arctg} x} =$$

$$\frac{0}{0}$$

$$= -1\cdot\frac{2}{\pi} = \underline{-\frac{2}{\pi}} \Rightarrow \lim_{x\to\infty} y = e^{-\frac{2}{\pi}}$$

$$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right) = \lim_{x\to 1}\frac{\ln x - x + 1}{(x-1)\ln x} = \lim_{x\to 1}\frac{\frac{1}{x}-1}{\ln x + \frac{x-1}{x}} = \lim_{x\to 1}\frac{1-x}{x\ln x + x - 1}$$

$$\infty - \infty \qquad \frac{0}{0} \qquad \frac{0}{0}$$

$$\lim_{x \to 1} \frac{1-x}{x\ln x + x - 1} = \lim_{x \to 1} \frac{-1}{\ln x + \cancel{\frac{x}{x}} + x - \cancel{1}} = -\lim_{x \to 1} \frac{1}{\ln x + x} = \frac{-1}{0+1} = -1$$

$\frac{0}{0}$

g. p. No 7

① $$\lim_{x \to \infty} \left[x\left(\operatorname{arctg}\frac{x+1}{x+2} - \operatorname{arctg}\frac{x}{x+2}\right)\right]$$

$\infty \cdot 0$

② $$\lim_{x \to 0} x^{\frac{1}{\ln(e^x - 1)}}$$

0^0