$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\operatorname{tg} z = \frac{\sin z}{\cos z} = -i\,\frac{e^{iz} - e^{-iz}}{e^{iz} + e^{iz}}$$

$$\operatorname{cotg} z = \frac{1}{\operatorname{tg} z}$$

$$\operatorname{sh} z = \frac{e^{z} - e^{-z}}{2}$$

$$\operatorname{ch} z = \frac{e^{z} + e^{-z}}{2}$$

$$\cos z = \operatorname{ch} iz$$

$$\sin z = -i \operatorname{sh} iz$$

$$\operatorname{tg} z = -i \operatorname{th} iz$$

$$\operatorname{cotg} z = i \operatorname{cth} iz$$

$y = \sin x$

$$\sin z = \sin(x+iy) =$$

$$= \sin x \cos iy + \cos x \sin iy$$

$$\cos iy = \frac{e^{i^2 y} + e^{-i^2 y}}{2} = \frac{e^{-y} + e^{y}}{2} =$$

$$= \operatorname{ch} y$$

$z \in \mathbb{C}$

$$\sin iy = \frac{e^{i^2 y} - e^{-i^2 y}}{2i} = -i\,\frac{e^{-y} - e^{y}}{2} =$$

$$= i\,\frac{e^{y} - e^{-y}}{2} = i \operatorname{sh} y$$

$$\sin z = \underbrace{\sin x \operatorname{ch} y}_{Re} + i\,\underbrace{\cos x \operatorname{sh} y}_{Im}$$

$\operatorname{ch} z = \cos iz$

$\operatorname{sh} z = -i \sin iz$

$\operatorname{th} z = -i \operatorname{tg} iz$

$\operatorname{cth} z = i \operatorname{cotg} iz$

$\operatorname{Arch} z = \ln\left(z + \sqrt{z^2 - 1}\right) \quad z \geq 1$

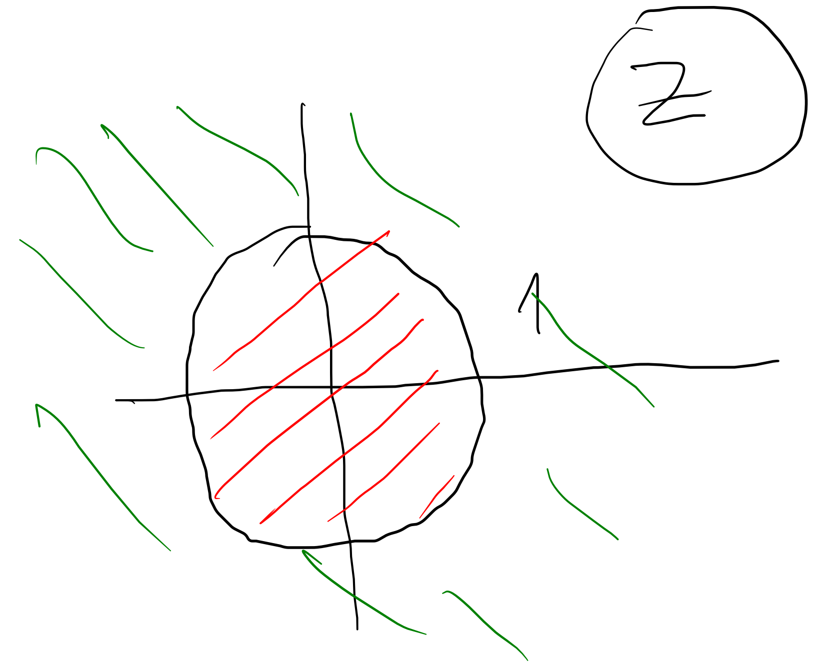
ареа-косинус хиперболичен

$\operatorname{Arsh} z = \ln\left(z + \sqrt{z^2 + 1}\right)$

$\operatorname{Arth} z = \ln \sqrt{\frac{1+z}{1-z}} \quad |z| < 1$

$\operatorname{Arcth} z = \ln \sqrt{\frac{z+1}{z-1}} \quad |z| > 1$

$|z| = \sqrt{x^2 + y^2} \quad x^2 + y^2 \lesseqgtr 1$



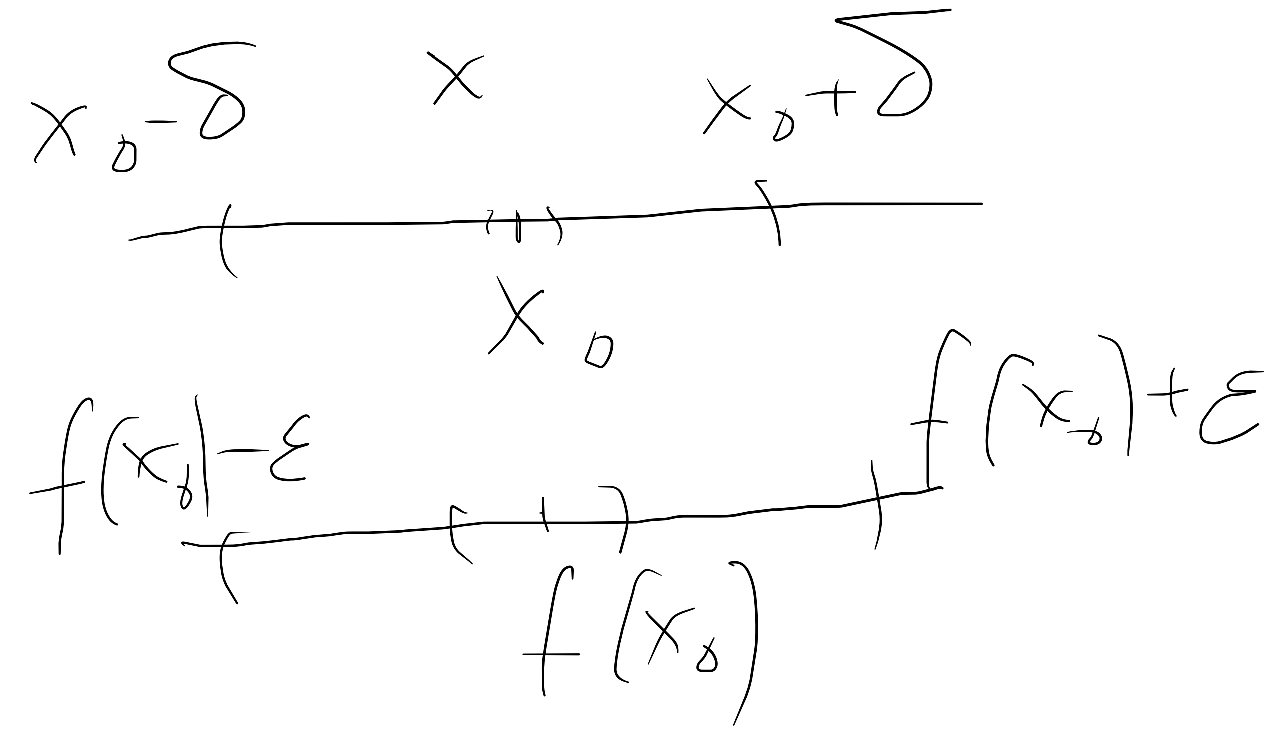
Непрекъснатост на ф-ция

Def (Коши) Казваме, че ф-ята $f(x)$ е непрекъсната в т. $x_0 \in D$, ако за $\forall \varepsilon > 0$ $\exists \delta(\varepsilon, x_0) > 0$ и такова, че когато $|x - x_0| < \delta$ да е изпълн. и условието $|f(x) - f(x_0)| < \varepsilon$.

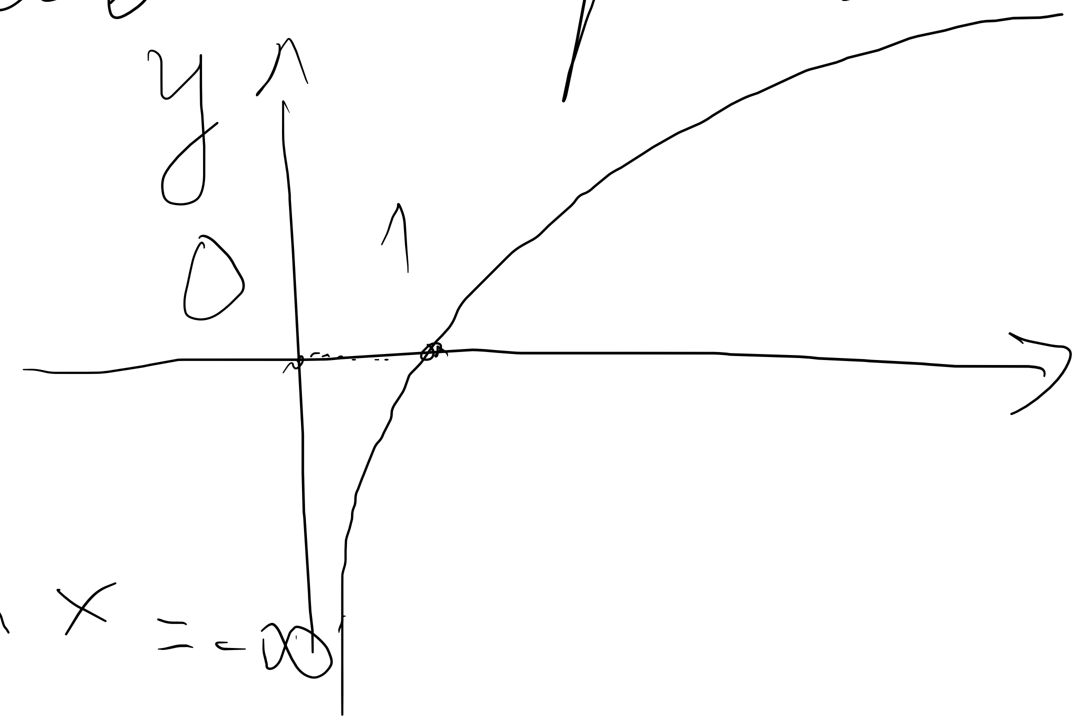
Def (Хайне) Казваме, че ф-цията $f(x)$ е непрекъсн. в т. $x_0 \in D$, ако за $\forall \{x_n\} \to x_0$ съответната $\{f(x_n)\} \to f(x_0)$.

th Двете дефиниции са еквивал.

поточкова непрекъсн.



$y = \ln x$



$\lim\limits_{x \to 0} \ln x = -\infty$

Def (Коши) Казв., че ф-цията $f(x)$ е равномерно непр. в $[a, b] \subset D$, ако за $\forall \varepsilon > 0$ $\exists\, \delta(\varepsilon) > 0$ и такова, че за $\forall\, x_1 \neq x_2 \in [a, b]$, за които $|x_1 - x_2| < \delta$ да е изпълн.

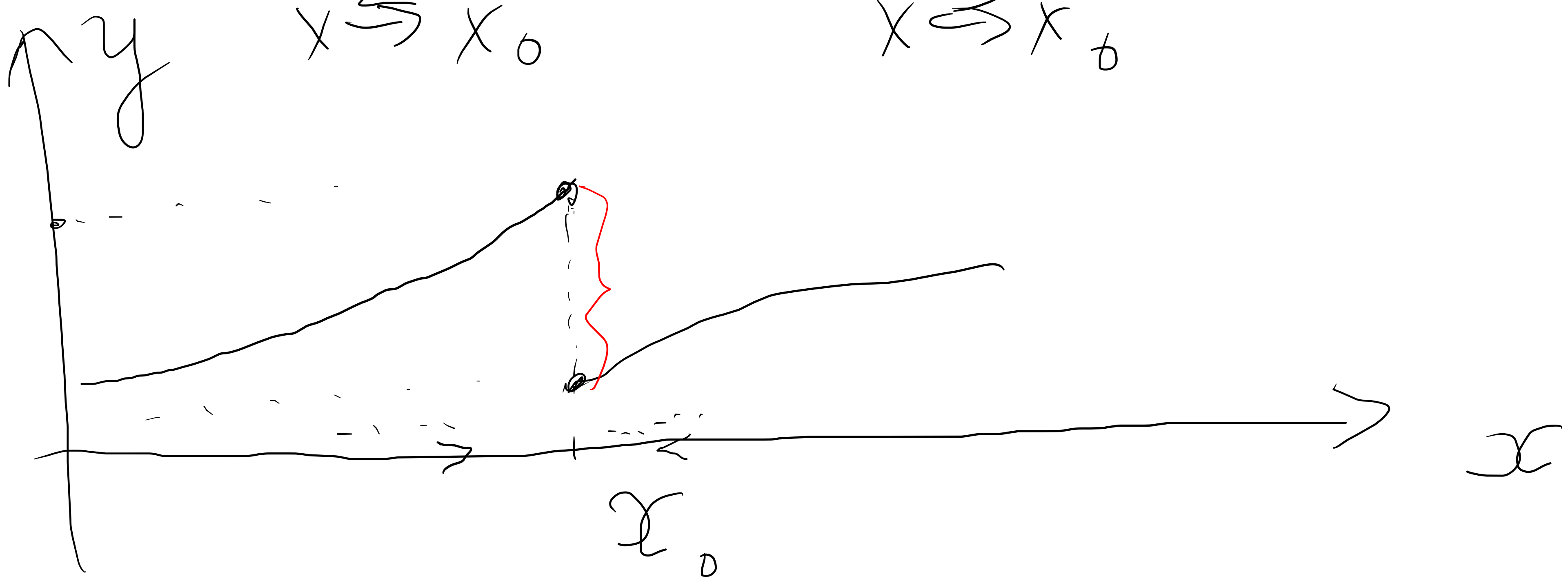
и условието $|f(x_1)-f(x_2)|<\varepsilon$ равномерна непрек.

точка на прекъсв. е точка, в която ф-цията не е непрекъсната.

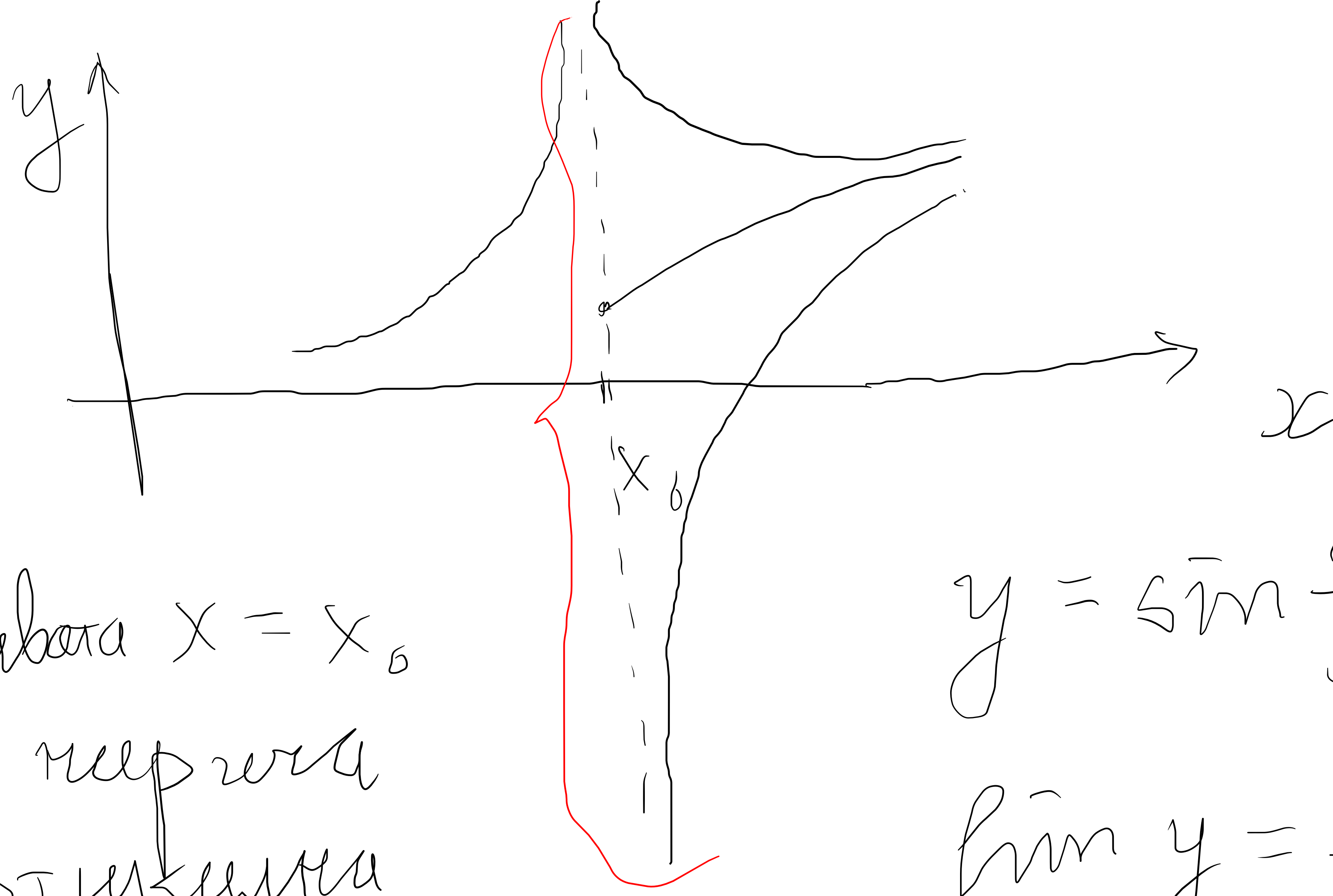
x_0 - точка на прекъсв. от I род, когато $\exists \lim\limits_{x \overset{<}{\to} x_0} f(x)$ и $\lim\limits_{x \overset{>}{\to} x_0} f(x)$, но $\lim\limits_{x \overset{<}{\to} x_0} f(x) \neq \lim\limits_{x \overset{>}{\to} x_0} f(x)$

лява гран

дясна гран.

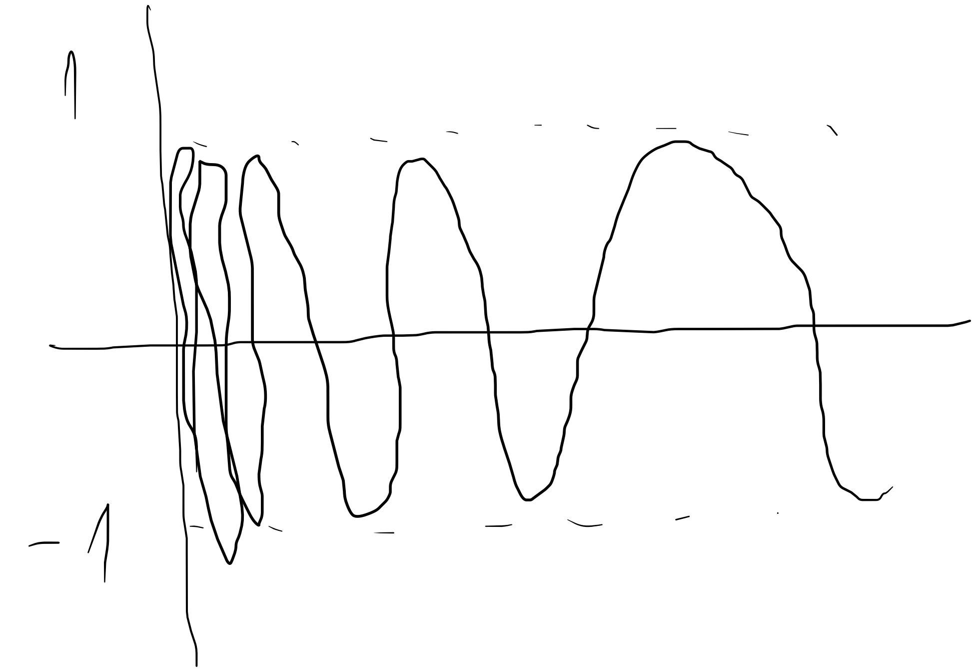


x_0 - точка на прекъсв. от II род, когато поне една от едностранните граници е несобствена $(\pm\infty)$ или $\nexists$



правата $x = x_0$ се нарича вертикална асимптота

$$y = \sin\frac{1}{x}$$



$$\lim_{x\to 0} y = \sin\infty$$

$\nexists$

Тригоном. ф-ции $\sin x$, $\cos x$, $tg\,x$, $cotg\,x$ нямат граници за $x\to\infty$

$$\lim_{x\to\infty}\sin x = \lim_{x_n\to\infty}\sin(x_0+2n\pi) =$$

$x_n = x_0 \pm 2n\pi$

$x_n \to \infty$

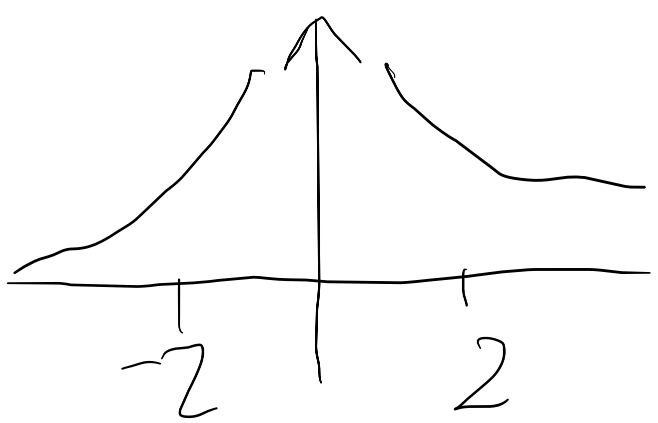
$|\sin x_0| \le 1$

$$= \lim_{x_n\to\infty}\left(\sin x_0 \underbrace{\cos 2n\pi}_{1} + \cos x_0 \underbrace{\sin 2n\pi}_{0}\right) = \lim_{x_n\to\infty}\sin x_0 = \sin x_0$$

пр. $y=\dfrac{1}{x^2-4}$

$x_{1,2}=\pm 2$ – точки на прекъсване

$y(-x)=y(x)$ четна

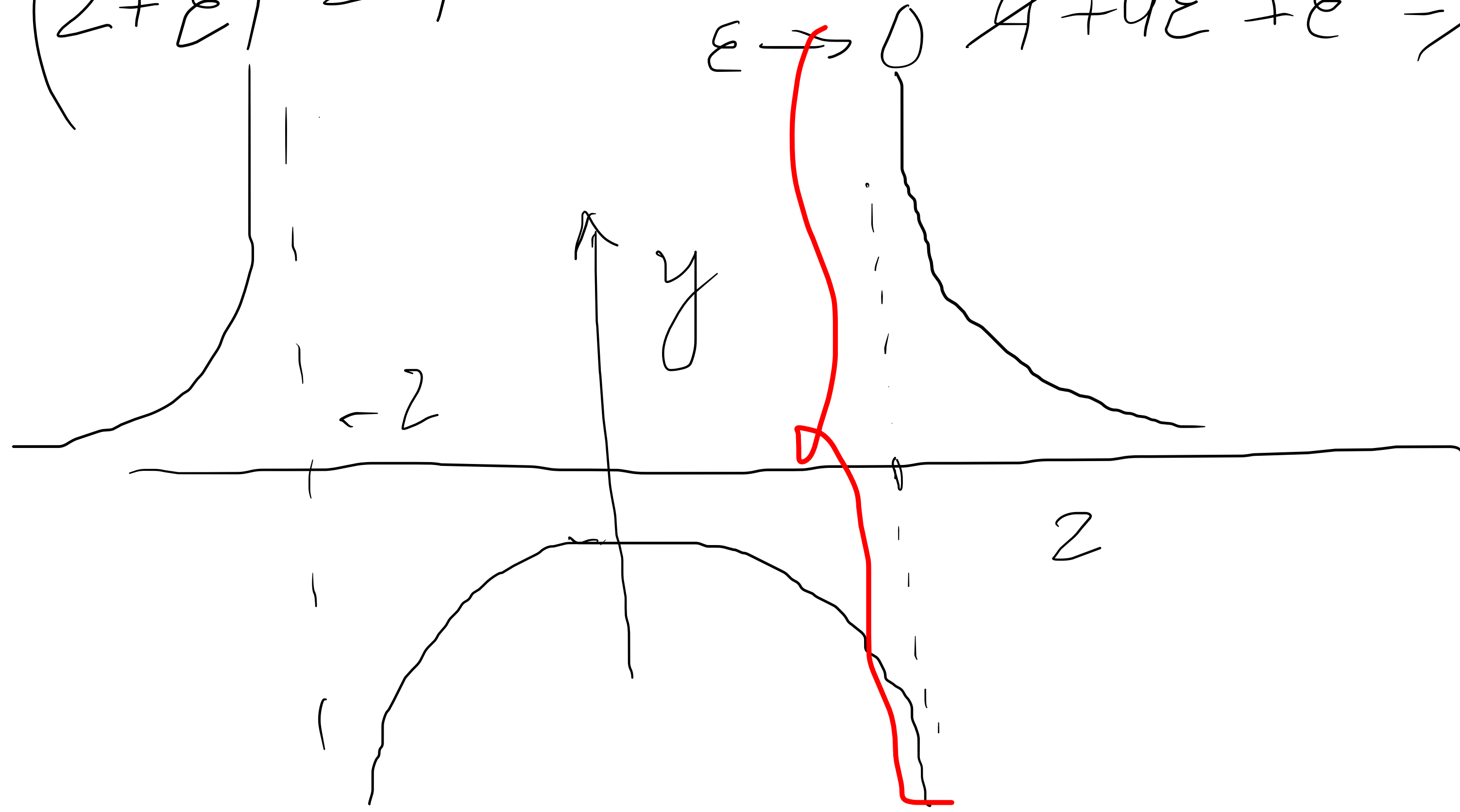


$$\lim_{x \overset{<}{\to} 2} y = \lim_{\substack{x=2-\varepsilon \\ \varepsilon>0,\ \varepsilon\to 0}} \frac{1}{(2-\varepsilon)^2-4} = \lim_{\varepsilon\to 0} \frac{1}{\cancel{4}-4\varepsilon+\varepsilon^2-\cancel{4}} =$$

$$= \lim_{\varepsilon\to 0} \frac{1}{\underbrace{\varepsilon}_{\downarrow\, 0}(\underbrace{\varepsilon-4}_{\downarrow\, -4})} = -\infty \Rightarrow x=2 \text{ е верт.}$$

асимпт. и т. $x=2$ е т. на прекъсв. от II род

$$\lim_{x \overset{>}{\to} 2} y = \lim_{\substack{x=2+\varepsilon \\ \varepsilon>0 \\ \varepsilon\to 0}} \frac{1}{(2+\varepsilon)^2-4} = \lim_{\varepsilon\to 0} \frac{1}{\cancel{4}+4\varepsilon+\varepsilon^2-\cancel{4}} = \lim_{\varepsilon\to 0} \frac{1}{\underbrace{\varepsilon}_{\downarrow\, 0}(\underbrace{\varepsilon+4}_{\downarrow\, 4})} = \infty$$



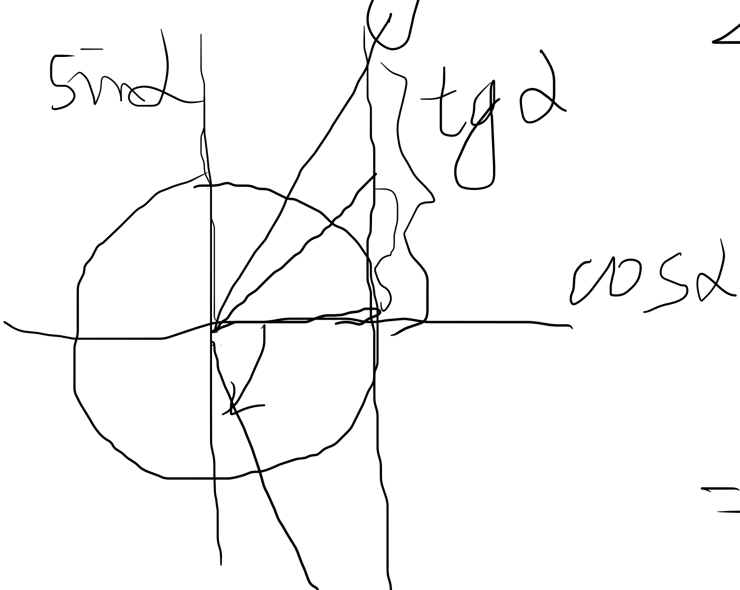
пр. $y=\operatorname{arctg}\frac{1}{1-x^2}$ $\qquad x=1$ $\qquad \lim\limits_{x\lessapprox 1} y=\lim \operatorname{arctg}\frac{1}{1-(1-\varepsilon)^2}=$

$y(-x)=y(x)$ $\qquad x=1-\varepsilon$, $\varepsilon>0$, $\varepsilon\to 0$

$x_{1,2}=\pm 1$ т. на прекъсв. $\qquad x=1+0$, $x=1-0$

$$=\lim_{\varepsilon\to 0}\operatorname{arctg}\frac{1}{\underbrace{\cancel{1}-(\cancel{1}-2\varepsilon+\varepsilon^2)}_{2\varepsilon-\varepsilon^2}}=\lim_{\varepsilon\to 0}\operatorname{arctg}\frac{1}{\underbrace{\varepsilon(2-\varepsilon)}_{\downarrow\ +0}}$$

$$=\operatorname{arctg}\infty=\frac{\pi}{2}$$

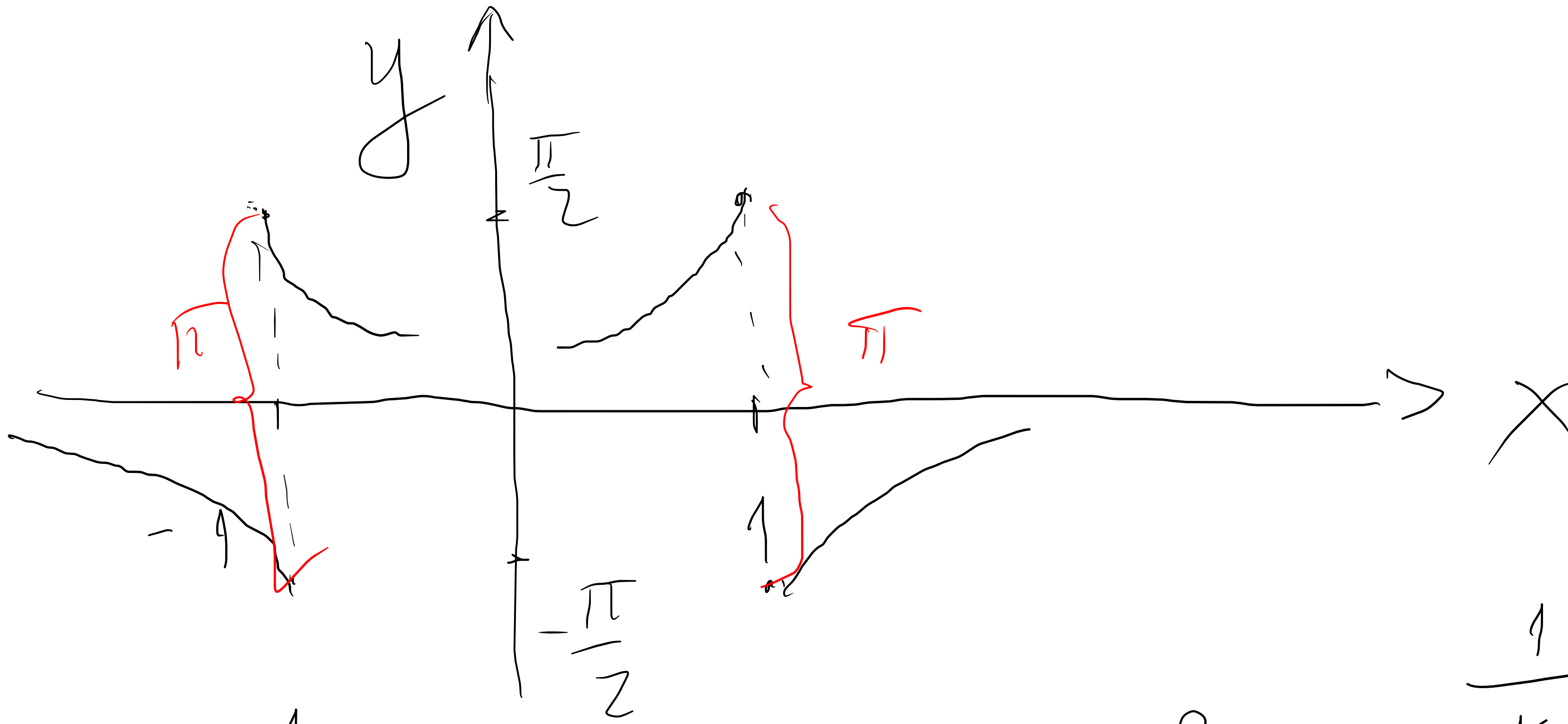


$$\lim_{x\gtrapprox 1} y=\lim_{\substack{x=1+\varepsilon\\ \varepsilon>0\\ \varepsilon\to 0}}\operatorname{arctg}\frac{1}{1-(1+\varepsilon)^2}=\lim_{\varepsilon\to 0}\operatorname{arctg}\frac{1}{\cancel{1}-(\cancel{1}+2\varepsilon+}$$

$$=\lim_{\varepsilon\to 0}\operatorname{arctg}\frac{1}{\underbrace{-\varepsilon(\varepsilon+2)}_{\downarrow\ -0}}=\operatorname{arctg}(-\infty)=-\frac{\pi}{2}\Rightarrow \text{т. } x=1 \text{ е}$$

т. прекъсв. от I род

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$



пр. $y = e^{\frac{1}{x-1}}$

$x = 1$ е т. на прек.

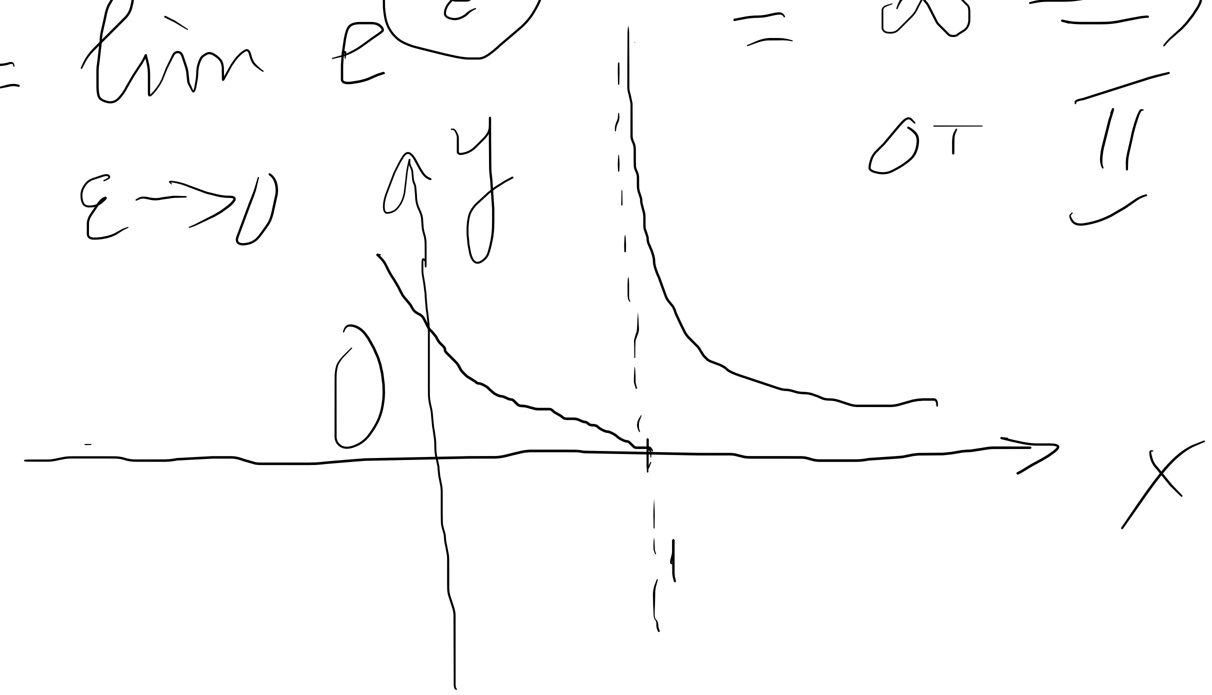
$\lim\limits_{x \to 1^-} y = \lim e^{\frac{1}{1-\varepsilon-1}} = \lim\limits_{\varepsilon \to 0} e^{-\frac{1}{\varepsilon}} = 0$

$x = 1-\varepsilon,\ \varepsilon > 0,\ \varepsilon \to 0$

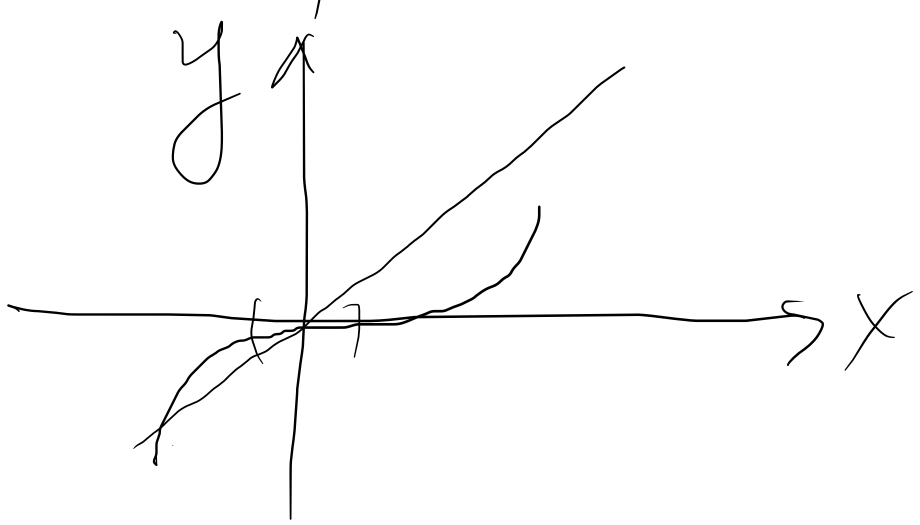
$\frac{1}{e^{\infty}} \qquad \frac{1}{e^{\frac{1}{\varepsilon}}}$

$\lim\limits_{x \to 1^+} = \lim e^{\frac{1}{1+\varepsilon-1}} = \lim\limits_{\varepsilon \to 0} e^{\frac{1}{\varepsilon}} = \infty \Rightarrow$ т. $x = 1$ е т. прекъс от II род $\Rightarrow$ правата $x = 1$ е вертик. асимптота

$x = 1+\varepsilon,\ \varepsilon > 0,\ \varepsilon \to 0$



пр. Док., че б.м. функц в околн. на т. $x=0$ са еквивал. а) $\alpha(x) = \arcsin x$; $\beta(x) = x$



$$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{t\to 0} \frac{t}{\sin t} = 1$$

$\sin t \sim t$
$t \to 0$

$$t = \arcsin x \quad | \sin$$

$$\sin t = \sin \arcsin x = x$$

$$x\to 0 \Rightarrow t \to 0$$

б) $\alpha(x) = \arcsin x + \sin^2 x$ $\qquad \lim\limits_{x\to 0} \dfrac{\arcsin x + \sin^2 x}{2x} =$

$\beta(x) = 2x$

$$= \frac{1}{2} \lim_{x\to 0} \frac{\arcsin x}{x} + \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \lim_{x\to 0} \frac{x}{2} = \frac{1}{2} \cdot 1 + 1 \cdot 0 = \frac{1}{2}$$

пр. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{\ln(1+tg^2 x)} = \lim\limits_{x\to 0} \dfrac{2\sin^2\frac{x}{2}}{\ln(1+tg^2 x)} = \lim\limits_{x\to 0} \dfrac{2\left(\frac{x}{2}\right)^2}{tg^2 x} =$

$\frac{0}{0}$ – неопред.

$= \frac{1}{2}\lim\limits_{x\to 0}\dfrac{\cancel{x^2}}{\cancel{x^2}} =$

$= \frac{1}{2}\cdot 1$

$1-\cos$ $\quad \sin^2\frac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$ $\quad 1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$

$\cos^2\frac{\alpha}{2} = \dfrac{1+\cos\alpha}{2}$

$\sin u \sim u$
$u\to 0$

$\underbrace{\ln(1+u)\sim u}_{\text{б.м.}}$

$tg\, u \sim u$

$\dfrac{x^{\cancel{2}}}{3\cancel{x}}$

пр. $\alpha(x) = x^2 + \ln(1+3x)$
$\underbrace{\beta(x)}_{\text{б.м.}} = 3x$ $\quad x\to 0$

$\lim\limits_{x\to 0}\dfrac{\alpha(x)}{\beta(x)} = \lim\limits_{x\to 0}\dfrac{x^2+\ln(1+3x)}{3x} = \frac{1}{3}\lim\limits_{x\to 0} x + \lim\limits_{x\to 0}\dfrac{\cancel{\ln(1+3x)}^{\,\sim 3x}}{3x}$

$\frac{0}{0}$

$= \frac{1}{3}\cdot 0 + 1 = 1$ $\quad \alpha(x)\sim\beta(x)$

пр. $\alpha(x) = \ln(1+x+x^2) + \arcsin 3x$ $\quad x \to 0$

$\beta(x) = 5x^3$ $\quad$ б. м.

$$\lim_{x\to 0} \frac{\alpha(x)}{\beta(x)} = \lim_{x\to 0} \frac{\ln(1+x+x^2)}{5x^3} + \lim_{x\to 0} \frac{\arcsin 3x}{5x^3} =$$

$\frac{0}{0}$

$$\ln(1+x+x^2) \underset{x\to 0}{\sim} x+x^2 \;;\; \arcsin 3x \underset{x\to 0}{\sim} 3x$$

$$= \lim_{x\to 0} \frac{x+x^2}{5x^3} + \lim_{x\to 0} \frac{3x}{5x^3} = \frac{1}{5}\lim_{x\to 0} \frac{1+x}{x^2} + \frac{3}{5}\lim_{x\to 0} \frac{1}{x^2} = \infty$$

∞

Производна на ф-ция $\frac{1}{x^2} + \frac{1}{x}$ ∞

$y = f(x)$ $\quad x + \Delta x$ – нарастване $\quad$ диференчно частно

$\Delta y = f(x+\Delta x)$ $\quad \frac{f(x+\Delta x) - f(x)}{\cancel{x} + \Delta x - \cancel{x}} = \frac{\Delta y}{\Delta x}$ $\quad \Delta x \to 0$

$\Delta \to d$ – диференциали

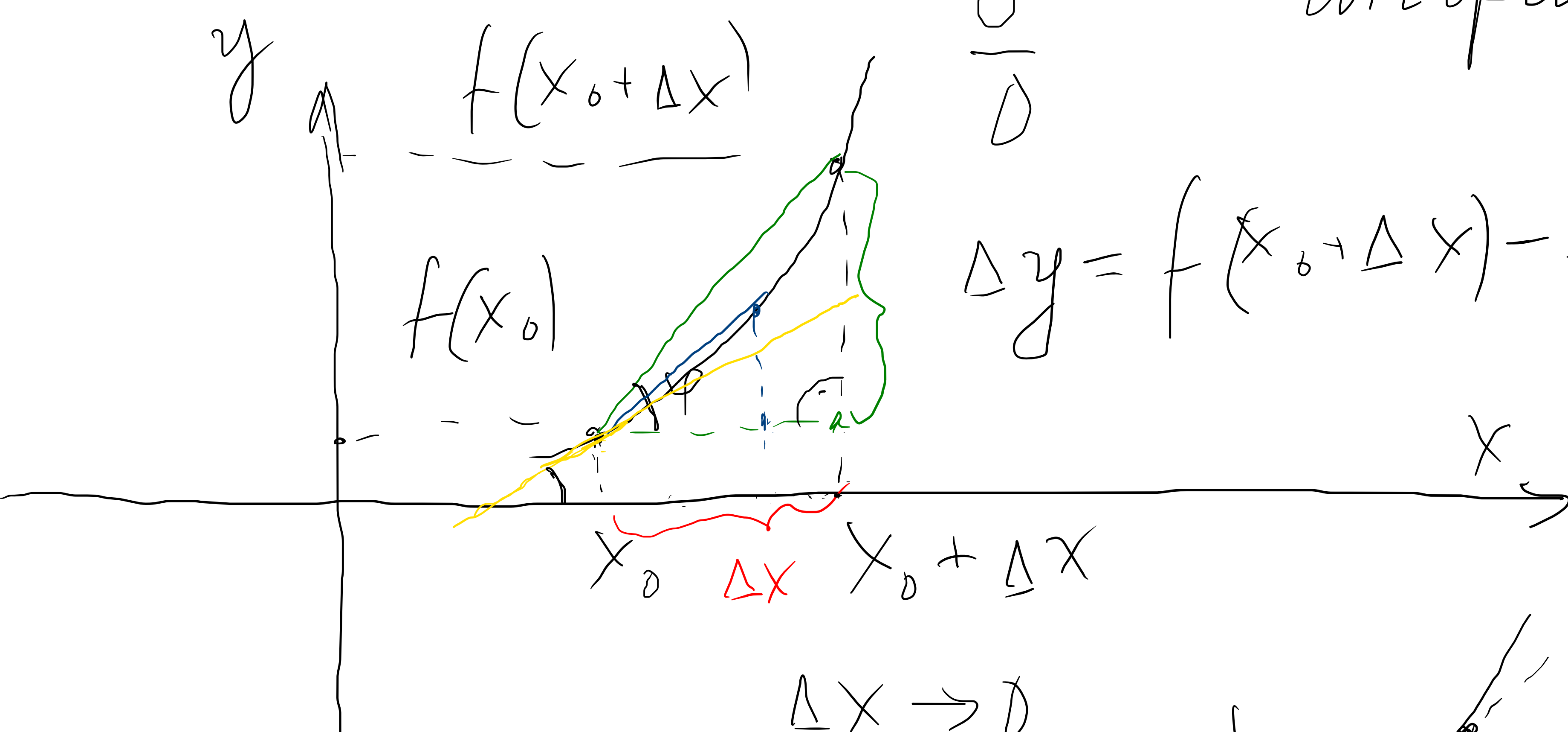
$$\frac{f(x+\Delta x)-f(x)}{\cancel{x}+\Delta x-\cancel{x}} = \frac{\Delta y}{\Delta x} \qquad \Delta x \to 0$$

Ако $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}\ \exists$, то

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x) = \frac{dy}{dx}$$

производна – отн. на безкрайно малки инфинитезимални

$\frac{0}{0}$

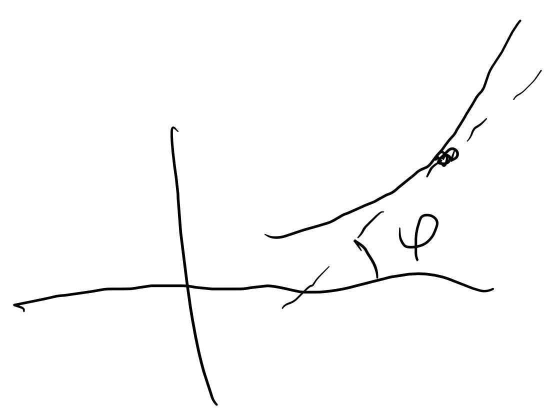


$$\Delta y = f(x_0+\Delta x) - f(x_0)$$

$$\frac{\Delta y}{\Delta x} = tg\,\varphi$$

$x_0 + \Delta x$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x_0) = tg\,\varphi$$



ъгълът φ е наклонът на допирателната (тангента) към полож. посока на абсцисната ос.