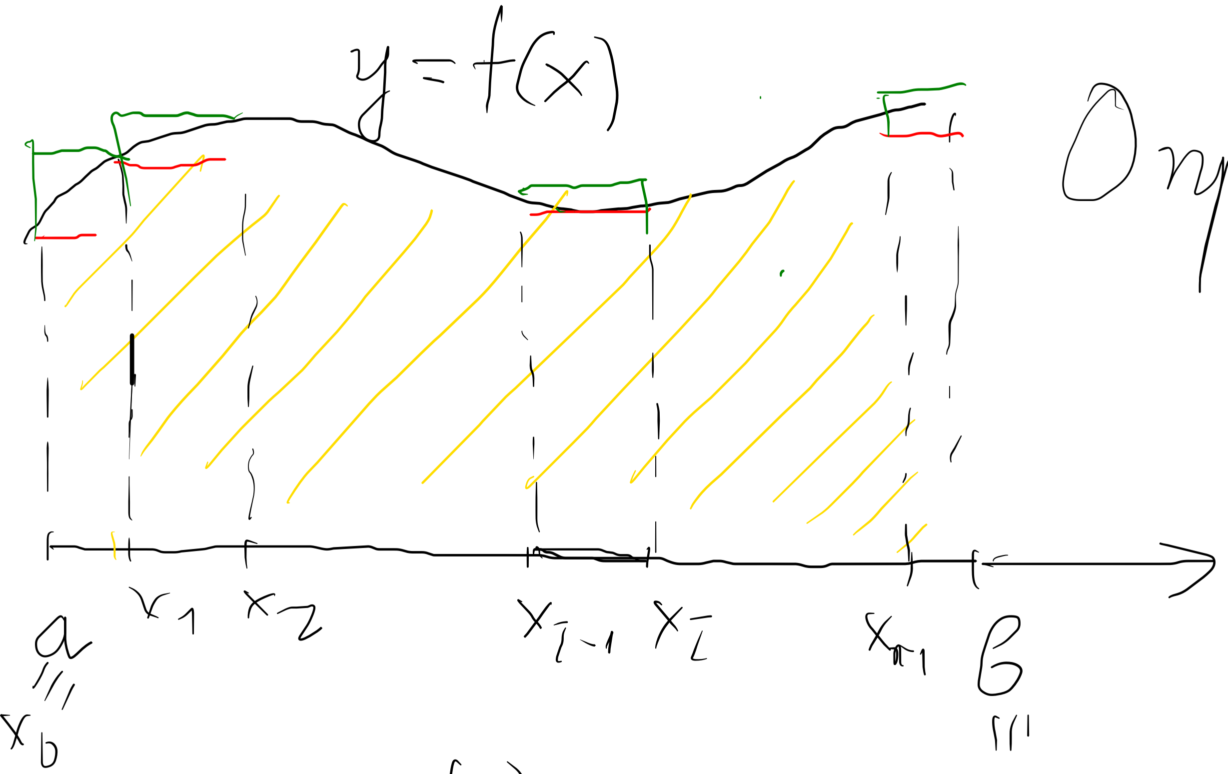
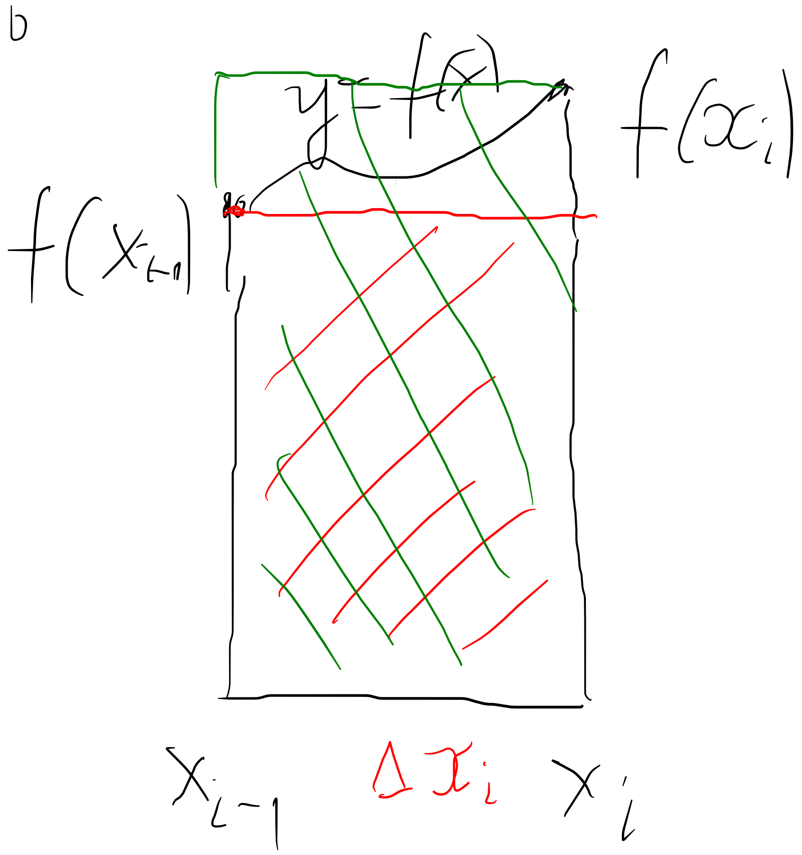


Определен интеграл (Риманов)

$$\Delta x_i = x_i - x_{i-1}$$

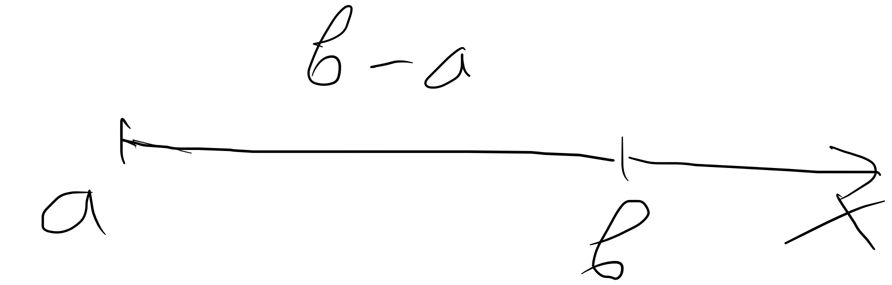
$$\underline{f}_i = \min_{x_{i-1} \le x \le x_i} f(x) \qquad \underline{\sigma} = \sum_{i=0}^{n} \underline{f}_i \Delta x_i$$ долна сума на Дарбу

$$\overline{f}_i = \max_{x_{i-1} \le x \le x_i} f(x) \qquad \overline{\sigma} = \sum_{i=0}^{n} \overline{f}_i \Delta x_i$$ горна сума на Дарбу



$$\lim_{\Delta x_i \to 0} \underline{\sigma} = \lim_{\Delta x_i \to 0} \overline{\sigma} = \int_a^b f(x)\,dx$$

Свойства



M = const $\quad \int_a^b M\,dx = M(b-a) \qquad M = 1 \quad \int_a^b 1\,dx = (b-a) =$

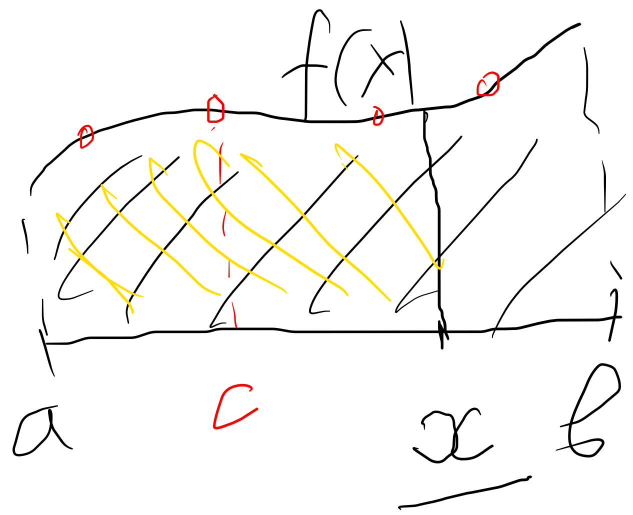
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \; ; \quad \int_a^b [A f_1(x) + B f_2(x)]\,dx = A \int_a^b f_1(x)\,dx + B \int_a^b f_2(x)\,dx$$

$f(x) \le \varphi(x)$

$a \le x \le b$

$$\int_a^b f(x)dx \le \int_a^b \varphi(x)dx; \quad \left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$$

$$\int_a^b f(x)dx \qquad \int_a^c f(x) + \int_c^b f(x)dx$$



$$\int_a^b f(x)dx = \int_a^b f_1(x)dx$$

интегралът като ф-ция на горната
граница

$$\int_a^b f(x)dx \qquad F(x) = \int_a^x f(u)du$$

$a \le x \le b$

$f \in C$, то $F \in C^1$ и

$F'(x) = f(x)$ за $a \le x \le b$

$$\int f(x)dx = \int_a^x f(u)du + C$$

Формула на Нютон-Лайбниц

$$\int_a^b f(x)\,dx = F(b)-F(a) = F(x)\Big|_a^b$$

$$F'(x) = f(x)$$

смяна на пром.

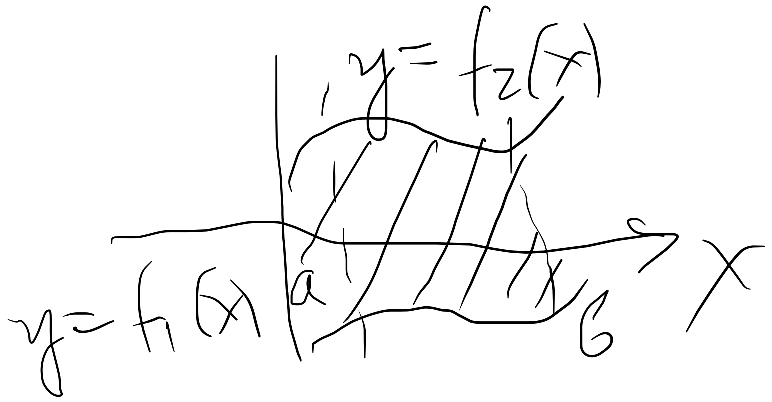
$$\int_a^b f(x)\,dx = \int_\alpha^\beta f(\varphi(t))\,\varphi'(t)\,dt$$

$x = \varphi(t)$, $\varphi \in C^1$ $\qquad \alpha = \varphi^{-1}(a)$

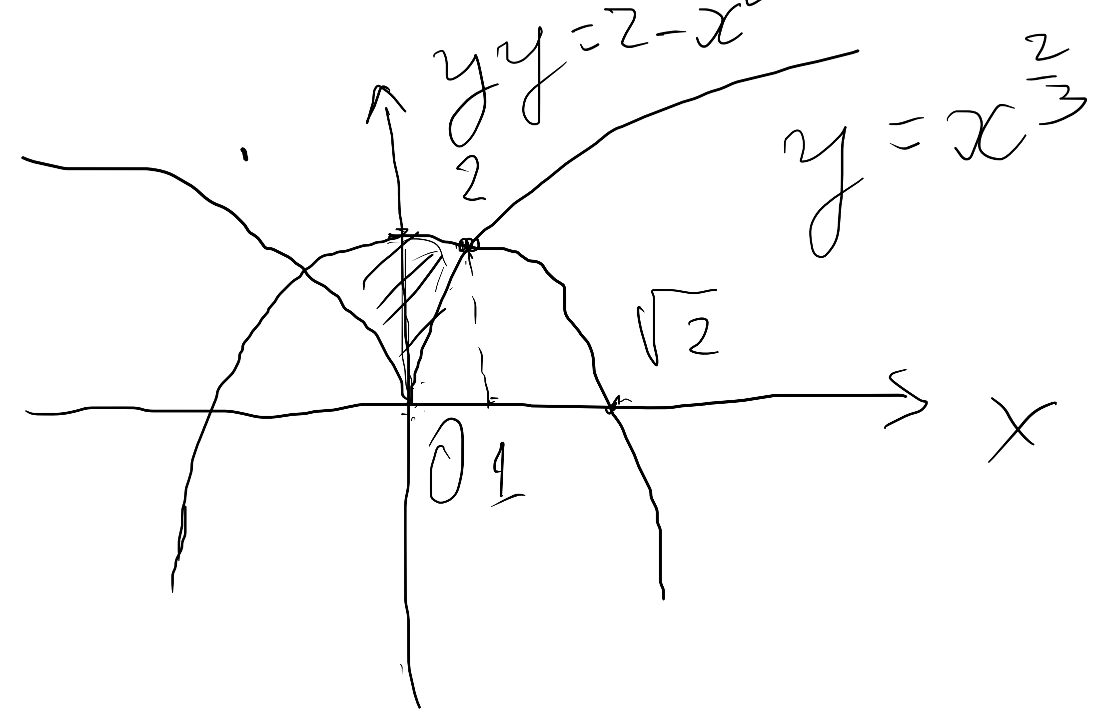
$dx = \varphi'(t)\,dt$ $\qquad \beta = \varphi^{-1}(b)$

формула за интегр. по части

$$\int_a^b u(x)\,dv(x) = u(x)v(x)\Big|_a^b - \int_a^b v(x)\,du(x)$$



$f_1(x) \le f_2(x)$ за $a \le x \le b$ $\quad S = \int_a^b [f_2(x) - f_1(x)]\,dx$



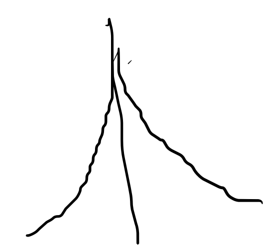
$$\begin{cases} y = 2 - x^2 \\ y = x^{\frac{2}{3}} \end{cases}$$

$$2 - x^2 = x^{\frac{2}{3}} \quad \Big|\,^3$$

$$(2 - x^2)^3 = x^2$$

$y = 2 - x^2$ Ним. лицето на обл., огран. от този криви

$y^3 = x^2$



$y = x^{\frac{2}{3}}$

$y' = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3} \frac{1}{\sqrt[3]{x}}$ $\quad x = 0 \quad y' \to \infty$ $\quad$ usp

$y(-x) = \sqrt[3]{(-x)^2} = \sqrt{x^2}$ $\qquad y'' = -\frac{2}{9} x^{-\frac{4}{3}} < 0$

$-\frac{2}{9} \frac{1}{\sqrt[3]{x^4}}$

$y' > 0$ за $x > 0$

$y' < 0$ за $x < 0$

$$(2-x^2)^3 = x^2 \qquad 8 - 12x^2 + 6x^4 - x^6 - x^2 = 0$$

$$-x^6 + 6x^4 - 13x^2 + 8 = 0 \qquad x^2 = u$$

$$u^3 - 6u^2 + 13u - 8 = 0$$

$$\pm 1, 2, 4, 8 \qquad x^2 = 1 \Longrightarrow$$

$$1 - 6 + 13 - 8 = 0 \qquad x_{1,2} = \pm 1$$

$$-1 - 6 - 13 - 8 \neq 0$$

$$S = 2\int_0^1 \left(\overbrace{2 - x^2}^{f_2} - \overbrace{x^{\frac{2}{3}}}^{f_1}\right) dx = 2\left(2x - \frac{x^3}{3} - \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}\right)\Bigg|_0^{1} =$$

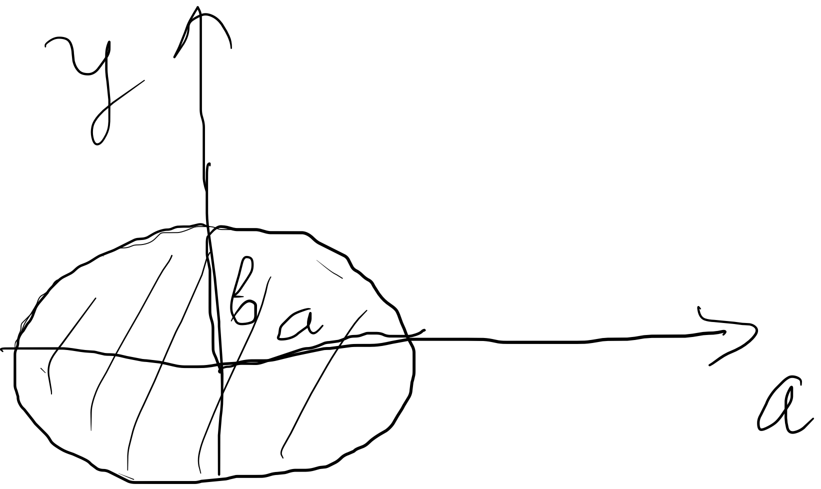
$$= 2\left(\overset{15}{2} - \overset{5}{\frac{1}{3}} - \overset{3}{\frac{3}{5}}\right) = 2\left(\frac{30-5-9}{15}\right) = \frac{32}{15}$$

$$4\int_0^1 1\,dx - 2\int_0^1 x^2\,dx - 2\int_0^1 x^{\frac{2}{3}}\,dx = 4x\Big|_0^1 - 2\frac{x^3}{3}\Big|_0^1 - 2\frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1}\Big|_0^1$$

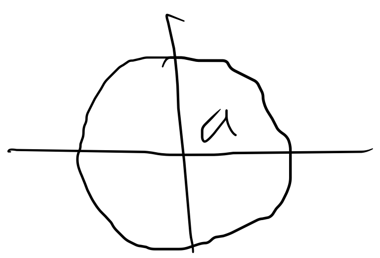
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$$

елипса

$$a=b \qquad x^2+y^2=a^2$$



$$y=\pm b\sqrt{1-\left(\frac{x}{a}\right)^2}$$



$$S_{кр}=\pi a^2$$

$$S_{ел}=4\int_0^a b\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx=4b\int_0^a\sqrt{1-\left(\frac{x}{a}\right)^2}\,dx=$$

$x=a\sin t \qquad dx=a\cos t\,dt$

тригон. суба.

x	0	a
t	0	$\frac{\pi}{2}$

$$=4b\int_0^{\frac{\pi}{2}}\sqrt{1-\left(\frac{\cancel{a}\sin t}{\cancel{a}}\right)^2}\,a\cos t\,dt=4ba\int_0^{\frac{\pi}{2}}|\cos t|\cos t\,dt=4ab\int_0^{\frac{\pi}{2}}\cos^2 t\,dt$$

$$4ab\int_0^{\frac{\pi}{2}}\cos^2 t\,dt = 4ab\int_0^{\frac{\pi}{2}}\frac{1+\cos 2t}{2}\,dt =$$

$$= 2ab\left(\int_0^{\frac{\pi}{2}}dt + \frac{1}{2}\int_0^{\frac{\pi}{2}}\cos 2t\,d2t\right) = 2ab\left(t + \frac{\sin 2t}{2}\right)\Bigg|_0^{\frac{\pi}{2}} = 2ab\left(\frac{\pi}{2} + \frac{0}{2} - 0 - 0\right) =$$

$$= \pi ab \qquad a = b$$

$$\pi a^2$$

$$\int_0^{\frac{\pi}{3}}\frac{\sin x - \cos x}{2\sin x + \cos x}\,dx =$$

$$\underline{tg\frac{x}{2} = t}$$

x	0	$\frac{\pi}{3}$
t	0	$\sqrt{3}$

$$= \int_0^{\sqrt{3}}\frac{\frac{t-1}{\cancel{\sqrt{1+t^2}}}\,\frac{dt}{1+t^2}}{\frac{2t+1}{\cancel{\sqrt{1+t^2}}}} = \int_0^{\sqrt{3}}\frac{t-1}{(2t+1)(1+t^2)}\,dt$$

$$R(-\sin x, -\cos x) = R(\sin x, \cos x)$$

$$tg\,x = t$$

$$= \int_0^{\sqrt{3}} \frac{t-1}{(2t+1)(1+t^2)}\, dt =$$

$$\frac{t-1}{(2t+1)(1+t^2)} = \frac{A}{2t+1} + \frac{Bt+C}{t^2+1}$$

$$t-1 = A(t^2+1) + (Bt+C)(2t+1)$$

$$t = -\frac{1}{2}: \quad -\frac{3}{2} = \frac{5}{4}A \Rightarrow A = -\frac{6}{5}$$

$$t = i: \quad i-1 = (Bi+C)(2i+1) = -2B + 2Ci + Bi + C$$

$$\begin{cases} -1 = -2B + C \\ 1 = 2C + B \end{cases}$$

$$\begin{cases} -2B + C = -1 \\ B + 2C = 1 \quad | \cdot 2 \end{cases} \Rightarrow 3C = 1 \Rightarrow C = \frac{1}{3}$$

$$B = 1 - 2C = 1 - \frac{2}{3} = \frac{1}{3}$$

$$= -\frac{6}{5}\int_0^{\sqrt{3}} \frac{dt}{2t+1} + \frac{1}{3}\int_0^{\sqrt{3}} \frac{t+1}{t^2+1}\, dt =$$

$$= -\frac{3}{5}\ln|2t+1|\Big|_0^{\sqrt{3}} + \frac{1}{6}\int_0^{\sqrt{3}} \frac{d(t^2+1)}{t^2+1} + \frac{1}{3}\operatorname{arctg} t\Big|_0^{\sqrt{3}}$$

$$= -\frac{3}{5}\Big(\ln(2\sqrt{3}+1) - \underbrace{\ln 1}_{0}\Big) + \frac{1}{6}\ln|t^2+1|\Big|_0^{\sqrt{3}} + \frac{1}{3}\Big(\operatorname{arctg}\sqrt{3} - \underbrace{\operatorname{arctg} 0}_{0}\Big) =$$

$$= -\frac{3}{5}\ln(2\sqrt{3}+1) + \frac{1}{6}\ln 4 + \frac{1}{3}\cdot\frac{\pi}{3} = -\frac{3}{5}\ln(2\sqrt{3}+1) + \ln 2^{\frac{2}{6}} + \frac{\pi}{9}$$

$$\ln 2^{\frac{1}{3}} = \ln\sqrt[3]{2}$$

g.p. No 11

1 $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin x - \cos x}{2\sin x + \cos x}\,dx$

c $\quad tg\,\frac{x}{2} = t$

2 $\displaystyle\int_0^1 \frac{dx}{x^4 + x^2 + 1}$

$$\frac{1}{(x^2+1)^2 - x^2 = (x^2 - x + 1)(x^2 + x + 1)}$$

3 $\displaystyle\int_1^4 \frac{dx}{(1 + \sqrt[4]{x})^2 \sqrt{x}}$

$$\frac{ax+b}{cx+d} = \frac{1x+0}{0x+1}$$