$$I=\int\sqrt{x^2+2x+5}\,dx=\int\sqrt{(x+1)^2+2^2}\,d(x+1)=\int\sqrt{u^2+4}\,du=u\sqrt{u^2+4}-\int u\,d\sqrt{u^2+4}=$$

$$x^2+2x+5=x^2+2x\cdot1+1^2-1^2+5=(x+1)^2+2^2$$

$$u=x+1 \qquad d(u^2+4)^{\frac{1}{2}}=\frac{1}{\cancel{2}}(u^2+4)^{\frac{1}{2}-1}\cancel{2}u\,du$$

$$=u\sqrt{u^2+4}-\int\frac{u^2+4-4}{\sqrt{u^2+4}}\,du=u\sqrt{u^2+4}-\underbrace{\int\sqrt{u^2+4}\,du}_{I}+4\int\frac{du}{\sqrt{u^2+4}}$$

$$2I=u\sqrt{u^2+4}+\frac{4\cancel{2}}{\cancel{2}}\int\frac{d\left(\frac{u}{2}\right)}{\sqrt{\left(\frac{u}{2}\right)^2+1}}=u\sqrt{u^2+4}+4\ln\left|\frac{u}{2}+\sqrt{\left(\frac{u}{2}\right)^2+1}\right|$$

$$I=\frac{x+1}{2}\sqrt{(x+1)^2+4}+\frac{4}{2}\ln\left|\frac{x+1}{2}+\frac{1}{2}\sqrt{(x+1)^2+4}\right|+C$$

$$I=\frac{x+1}{2}\sqrt{x^2+2x+5}+\ln\left(\frac{x+1}{2}+\frac{1}{2}\sqrt{x^2+2x+5}\right)^2+C$$

$$\int \frac{x+3}{(x^2+2x+5)^2}\,dx = \int \frac{x+1+2}{\left[(x+1)^2+2^2\right]^2}\,d(x+1) = \int \frac{u\,du}{(u^2+4)^2} + 2\underbrace{\int \frac{du}{(u^2+4)^2}}_{I_2} =$$

$$x^2+2x+5 = x^2+2x\cdot 1+1^2-1^2+5 = (x+1)^2+2^2$$

$$u = x+1$$

$$= \frac{1}{2}\int (u^2+4)^{-2}\,d(u^2+4) + \frac{2}{4}\int \frac{u^2+4-u^2}{(u^2+4)^2}\,du = -\frac{1}{2}(u^2+4)^{-1} + \frac{1}{2}\underbrace{\int \frac{du}{u^2+4}}_{I_1} - \frac{1}{2}\int \frac{u\cdot u\,du}{(u^2+4)^2} =$$

$$\int v^{-2}\,dv = \frac{v^{-2+1}}{-2+1}$$

$$= -\frac{1}{2}\,\frac{1}{u^2+4} + \frac{1\cdot 2}{2\cdot 4}\int \frac{d\left(\frac{u}{2}\right)}{1+\left(\frac{u}{2}\right)^2} - \frac{1}{4}\int \frac{u\,d(u^2+4)}{(u^2+4)^2} =$$

$$= -\frac{1}{2}\,\frac{1}{u^2+4} + \frac{1}{4}\,\mathrm{arctg}\,\frac{u}{2} + \frac{1}{4}\int u\,d\left(\frac{1}{u^2+4}\right) = -\frac{1}{2}\,\frac{1}{u^2+4} + \frac{1}{4}\,\mathrm{arctg}\,\frac{u}{2} + \frac{1}{4}\left(\frac{u}{u^2+4} - \underbrace{\int \frac{du}{u^2+4}}_{I_1}\right)$$

$$= -\frac{1}{2}\,\frac{1}{u^2+4} + \frac{1}{4}\,\frac{u}{u^2+4} + \frac{1}{4}\,\mathrm{arctg}\,\frac{u}{2} - \frac{1\cdot 2}{4\cdot 4}\int \frac{d\left(\frac{u}{2}\right)}{1+\left(\frac{u}{2}\right)^2} = \frac{u-2}{4(u^2+4)} + \left(\frac{1}{4}-\frac{1}{8}\right)\mathrm{arctg}\,\frac{u}{2} + C$$

$$\frac{u-2}{4(u^2+4)}+\left(\frac{\overline{1}}{4}-\frac{1}{8}\right)\operatorname{arctg}\frac{u}{2}+C\,\overline{\overline{\overline{1}}}_1=\frac{x-1}{4(x^2+2x+5)}+\frac{1}{8}\operatorname{arctg}\frac{x+1}{2}+C$$

$u=x+1$

e) рационални ф-ции $\displaystyle\int\frac{P_m(x)}{Q_n(x)}dx=?$ $\displaystyle\int Ax^m dx=\frac{Ax^{m+1}}{m+1}+C$

$$\int\frac{B}{(x-a)^m}dx=B\int(x-a)^{-m}d(x-a)=B\frac{(x-a)^{-m+1}}{-m+1}+C=\frac{B}{1-m}\frac{1}{(x-a)^{m-1}}+C$$

$m\neq 1$

$$\int\frac{B}{x-a}dx=B\int\frac{d(x-a)}{x-a}=\ln|x-a|+C\ ;\quad \int\frac{Mx+N}{(x^2+px+q)^m}dx=\int\frac{Mx+N}{\left[\left(x+\frac{p}{2}\right)^2+\left(\sqrt{\frac{4q-p^2}{2}}\right)^2\right]^m}$$

$m=1$ $\qquad p^2-4q<0$

$$x^2+px+q=x^2+2x\frac{p}{2}+\frac{p^2}{4}-\frac{p^2}{4}+q=$$

$$=\left(x+\frac{p}{2}\right)^2+\left(\sqrt{\frac{4q-p^2}{2}}\right)^2$$

$u=x+\frac{p}{2}$

$\sqrt{\frac{4q-p^2}{2}}=D>0$ конст.

$$\int \frac{Mx+N}{\left[\left(x+\frac{p}{2}\right)^2+\left(\frac{\sqrt{4q-p^2}}{2}\right)^2\right]^m}dx = M\int \frac{\left(x+\frac{p}{2}-\frac{p}{2}\right)d\left(x+\frac{p}{2}\right)}{\left[\left(x+\frac{p}{2}\right)^2+D^2\right]^m} + N\int \frac{d\left(x+\frac{p}{2}\right)}{\left[\left(x+\frac{p}{2}\right)^2+D^2\right]^m} =$$

$$= M\int \frac{u\,du}{\left(u^2+D^2\right)^m} - M\frac{p}{2}\underbrace{\int \frac{du}{\left(u^2+D^2\right)^m}}_{I_m} + N\int \frac{du}{\left(u^2+D^2\right)^m} = \frac{M}{2}\int \left(u^2+D^2\right)^{-m} d\left(u^2+D^2\right) +$$

$$\left(N - M\frac{p}{2}\right)I_m = \frac{M}{2}\frac{\left(u^2+D^2\right)^{-m+1}}{-m+1} + \left(N-M\frac{p}{2}\right)I_m + C =$$

$$= \frac{M}{2(1-m)}\frac{1}{\left(x^2+px+q\right)^{m-1}} + \left(N-M\frac{p}{2}\right)\int \frac{dx}{\left(x^2+px+q\right)^m} + C$$

$$\int \frac{P_m(x)}{Q_n(x)}dx \qquad \deg P_m(x) = m$$
$$\deg Q_n(x) = n$$

$$I=\int\sqrt{a^2+x^2}\,dx = \cancel{x}\sqrt{a^2+x^2}-\int x\,d\sqrt{a^2+x^2} = x\sqrt{a^2+x^2}-\int\frac{x\cdot\cancel{2}x}{\cancel{2}\sqrt{a^2+x^2}}dx =$$

$a\neq 0$ $\quad\underline{x=a\,tg\,t}$

$t=arctg\frac{x}{a}$ $\qquad 1+tg^2x=1+\frac{\sin^2x}{\cos^2x}=\frac{1}{\cos^2x}$ $\qquad = x\sqrt{a^2+x^2}-\int\frac{x^2+a^2-a^2}{\sqrt{a^2+x^2}}dx=$

$$=x\sqrt{a^2+x^2}-\underbrace{\int\sqrt{a^2+x^2}\,dx}_{I}+a^2\int\frac{dx}{\sqrt{a^2+x^2}}\Longrightarrow 2I=x\sqrt{a^2+x^2}+\frac{a^2}{a}\int\frac{d\left(\frac{x}{a}\right)}{\sqrt{\left(\frac{x}{a}\right)^2+1}}$$

$$2I=x\sqrt{a^2+x^2}+a^2\ln\left|\frac{x}{a}+\sqrt{\left(\frac{x}{a}\right)^2+1}\right|$$

$$I=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|+C$$

$$dx=\frac{a}{\cos^2t}dt\qquad I=a\int\sqrt{a^2+a^2tg^2t}\,\frac{dt}{\cos^2t}=a^2\int\frac{\sqrt{1+tg^2t}}{\cos^2t}dt=$$

$$1+tg^2t=1+\frac{\sin^2t}{\cos^2t}=\frac{1}{\cos^2t}\qquad =a^2\int\frac{dt}{\cos^3t}=a^2\int\frac{1}{\cos t}\frac{1}{\cos^2t}dt$$

$d\,tg\,t$

$$I = a\int \sqrt{\frac{a^2+a^2 tg^2 t}{\cos^2 t}}\, dt = a^2\int \frac{\sqrt{1+tg^2 t}}{\cos^2 t}\, dt =$$

$$= \frac{1}{\cos^2 t} \qquad = a^2\int \frac{dt}{\cos^3 t} = a^2\int \frac{1}{\cos t}\left(\underbrace{\frac{1}{\cos^2 t}dt}_{d\,tgt}\right) = a^2\int \frac{d\,tgt}{\cos t} =$$

$$= a^2\left[\frac{tgt}{\cos t} - \int tgt\, d\left(\frac{1}{\cos t}\right)\right] = a^2\left(\frac{tgt}{\cos t} - \int \frac{tgt \sin t}{\cos^2 t}\, dt\right) = a^2\left[\frac{tgt}{\cos t} - \frac{1}{2}\frac{\sin t}{\cos^2 t} - \frac{1}{2}\ln tg\left(\frac{t}{2}+\frac{\pi}{4}\right)\right] + C$$

$$d\cos^{-1} t = +\cos^{-2} t \sin t\, dt$$

$$\int \frac{tgt \sin t}{\cos^2 t}\, dt = \int tgt \sin t\, d\,tgt = \frac{1}{2}\int \sin t\, d\,tg^2 t = \frac{1}{2}\Big(\sin t\, tg^2 t -$$

$$- \int tg^2 t \cos t\, dt\Big) = \frac{1}{2}\sin t \cdot tg^2 t - \frac{1}{2}\int \frac{\sin^2 t}{\cos^2 t}\cancel{\cos t}\, dt = \frac{1}{2}\sin t\cdot tg^2 t -$$

$$-\frac{1}{2}\ln tg\left(\frac{t}{2}+\frac{\pi}{4}\right)$$

$$d\,tg\left(\frac{t}{2}+\frac{\pi}{4}\right)$$

$$-\frac{1}{2}\int \frac{1-\cos^2 t}{\cos t}\, dt = \frac{1}{2}\sin t\cdot tg^2 t - \frac{1}{2}\int \frac{dt}{\sin\left(t+\frac{\pi}{2}\right)} + \frac{1}{2}\int \cos t\, dt =$$

$$= \frac{1}{2}\sin t\cdot tg^2 t - \frac{1}{2}\int \frac{d\left(\frac{t}{2}+\frac{\pi}{4}\right)}{\sin\left(\frac{t}{2}+\frac{\pi}{4}\right)\cos\left(\frac{t}{2}+\frac{\pi}{4}\right)} + \frac{1}{2}\sin t = \frac{1}{2}\sin t\left(1+tg^2 t\right) - \frac{1}{2}\int \frac{\frac{d\left(\frac{t}{2}+\frac{\pi}{4}\right)}{\cos^2\left(\frac{t}{2}+\frac{\pi}{4}\right)}}{tg\left(\frac{t}{2}+\frac{\pi}{4}\right)}$$

д. р. № 10

1 $\displaystyle\int \frac{x^2-x+1}{(x^2+x+1)^2}\,dx$

2 $\displaystyle\int \frac{dx}{\sqrt{2+3x-2x^2}}$