$$y = \ln(1+x) \qquad y^{(n)} = ? \qquad d^n y = y^{(n)}(x)\,dx^n$$

$$y' = \frac{1}{1+x} = (1+x)^{-1} \qquad \left[(1+x)^{-1}\right]' = -(1+x)^{-1-1} = -(1+x)^{-2}$$

$$y'' = -\frac{1}{(1+x)^2} = -(1+x)^{-2}$$

$$y^{(k+1)} = \left(y^{(k)}\right)' =$$

$$y''' = 2(1+x)^{-3} = \frac{2}{(1+x)^3}$$

$$= (-1)^{k+1}(k-1)!\,(-k)(1+x)^{-k-1} =$$

$$y^{IV} = -2\cdot 3\,(1+x)^{-4}$$

$$= (-1)^{k+2}\,k!\,(1+x)^{-(k+1)}$$

$\vdots$

$$y^{(k)} = (-1)^{k+1}(k-1)!\,(1+x)^{-k} \Longrightarrow y^{(n)} = \frac{(-1)^{n+1}(n-1)!}{(1+x)^n}$$

$k > 4$

$$d^n y = \frac{(-1)^{n+1}(n-1)!}{(1+x)^n}\,dx^n$$

$$[u(x)v(x)]^{(n)} = \sum_{i=0}^{n} \binom{n}{i} u^{(n-i)} v^{(i)}$$ – формула на Нютон - Лайбниц

$$y = \frac{1+x}{\sqrt{x}}$$

$v = 1+x$; $v' = 1$; $v^{(k)} = 0$, $k \geq 2$

$u = x^{-\frac{1}{2}}$; $u' = -\frac{1}{2} x^{-\frac{3}{2}}$; $u'' = \frac{1.3}{2^2} x^{-\frac{5}{2}}$;

$u''' = -\frac{1.3.5}{2^3} x^{-\frac{7}{2}}$, ... $\boxed{u^{(k)} = (-1)^k \frac{1.3 \ldots (2k-1)}{2^k} x^{-\frac{2k+1}{2}}}$

$1.3 \ldots 2k-1 = (2k-1)!!$

$2.4.6 \ldots 2k = (2k)!!$

$v^{(0)} \stackrel{def}{=} v$

$$\frac{1+x}{\sqrt{x}} = \binom{n}{0} u^{(n)} v + \binom{n}{1} u^{(n-1)} v' + 0$$

$$= (-1)^n \frac{(2n-1)!!}{2^n} x^{-\frac{2n+1}{2}} (1+x) + n \frac{(-1)^{n-1} (2n-3)!!}{2^{n-1}} x^{-\frac{2n-1}{2}}$$

$$(-1)^n \frac{(2n-1)!!}{2^n} x^{-\frac{2n+1}{2}} (1+x) + \frac{n(-1)^{n-1}(2n-3)!!}{2^{n-1}} x^{-\frac{2n-1}{2}} =$$

$$-\frac{2n+1}{2}+1 = -\frac{2n-1+2}{2} = \frac{-2n+1}{2} = -\frac{2n-1}{2}$$

$$= x^{-\frac{2n-1}{2}}\left[(-1)^n \frac{(2n-1)!!}{2^n} + \frac{n(-1)^{n-1}(2n-3)!!}{2^{n-1}}\right] + x^{-\frac{2n+1}{2}}\left[(-1)^n \frac{(2n-1)!!}{2^n}\right]$$

$$f(x) = (1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \dots + \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!}x^n + R_n$$

$$\left(1+\frac{x}{a}\right)^{\frac{1}{2}} = 1 + \frac{\frac{1}{2}}{1!}\frac{x}{a} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{a}\right)^2 + R_2 =$$

$$\alpha = \frac{1}{2}$$

$$x \to \frac{x}{a}$$

$$= 1 + \frac{1}{2}\left(\frac{x}{a}\right) - \frac{1}{8}\left(\frac{x}{a}\right)^2 + R_2$$

$$\left|\frac{x}{a}\right| \ll 1$$

$$\sqrt{1+\frac{x}{a}} \simeq 1 + \frac{1}{2}\left(\frac{x}{a}\right)$$

$$\left(1-\frac{x}{a}\right)^{-\frac{1}{2}} = 1 + \frac{-\frac{1}{2}}{1!}\left(-\frac{x}{a}\right) + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{x}{a}\right)^2 + R_2 =$$

$$\parallel$$

$$\frac{1}{\sqrt{1-\frac{x}{a}}} \qquad \alpha = -\frac{1}{2} \qquad x \to -\frac{x}{a} \qquad = \boxed{1 + \frac{x}{2a}} + \frac{3}{8}\left(\frac{x}{a}\right)^2 + R_2$$

$$\left|\frac{x}{a}\right| \ll 1 \qquad \frac{\frac{1}{2}\,\frac{3}{2}}{2!}$$

$$\frac{1}{\sqrt{1-\frac{x}{a}}} \approx 1 + \frac{x}{a}$$

$$\sqrt{\frac{a+x}{a-x}} \approx \left(1+\frac{x}{a}\right)^2 = 1 + \left(\frac{x}{a}\right)^2 + 2\frac{x}{a}$$

$$a = 1 \qquad f(x) = e^{\frac{x}{a}}; \quad f' = \frac{1}{a}e^{\frac{x}{a}}; \quad f'' = \frac{1}{a^2}e^{\frac{x}{a}}; \quad \dots \quad f^{(n)} = \frac{1}{a^n}e^{\frac{x}{a}}$$

$$f(0) = 1; \quad f'(0) = \frac{1}{a}; \quad f''(0) = \frac{1}{a^2}; \quad \dots \quad f^{(n)}(0) = \frac{1}{a^n}$$

$$e^{\frac{x}{a}} = 1 + \frac{1}{1!\,a}x + \frac{1}{2!\,a^2}x^2 = 1 + \frac{x}{a} + \frac{1}{2}\left(\frac{x}{a}\right)^2 + \frac{1}{6}\left(\frac{x}{a}\right)^3 + \dots$$

$$f(x) = \operatorname{arctg} x \qquad x \in [0,1]$$

$$f(0) = \operatorname{arctg} 0 = 0$$

$$f' = \frac{1}{1+x^2} \qquad f'(0) = 1 \qquad f^{IV}(0) = 0$$

$$f'' = -\frac{2x}{(1+x^2)^2} \qquad f''(0) = 0$$

$$f'''(0) = \frac{2(-1)}{1} = -2$$

$$f''' = -2\,\frac{\left[(1+x^2)^{\cancel{2}} - x\,2\,\cancel{(1+x^2)}\cdot 2x\right]}{(1+x^2)^{\cancel{4}\,3}} = -2\,\frac{1+x^2-4x^2}{(1+x^2)^3} = 2\,\frac{3x^2-1}{(1+x^2)^3}$$

$$f^{IV} = 2\,\frac{6x(1+x^2)^{\cancel{3}} - (3x^2-1)\,3\,\cancel{(1+x^2)^2}\,2x}{(1+x^2)^{\cancel{6}\,4}} = 2\,\frac{6x+6x^3-18x^3+6x}{(1+x^2)^4} =$$

$$= 2\,\frac{-12x^3+12x}{(1+x^2)^4} =$$

$$= -24\,\frac{x\,\cancel{(1+x^2)}}{(1+x^2)^{\cancel{4}\,3}} = -24\,\frac{x}{(1+x^2)^3}$$

$$-24\frac{x}{(1+x^2)^3}$$

$$f^{V} = \frac{-24\left((1+x^2)^{\cancel{3}} - x\,3\cancel{(1+x^2)}^2\cdot 2x\right)}{(1+x^2)^{\cancel{6}\,4}} =$$

$$= -24\,\frac{1+x^2-6x^2}{(1+x^2)^4} = 24\,\frac{5x^2-1}{(1+x^2)^4} \qquad f^{V}(0) = -24$$

$$\underset{\underset{f(x)}{\parallel}}{\operatorname{arctg} x} = \operatorname{arctg} 0 + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{IV}(0)}{4!}x^4 +$$

$$+ R_4 = 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 - \frac{2}{3!}x^3 + \frac{0}{4!}x^4 +$$

$$4! = 2\cdot 3\cdot 4$$

$$+ \underbrace{\frac{x^5}{5!}\,24\,\frac{5\xi^2-1}{(1+\xi^2)^4}}_{R_4} = \qquad 0<\xi<1 \qquad 0\le x\le 1$$

$$+\frac{x^5}{5}-\frac{x^7}{7}$$

$$= \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5}\,\frac{5\xi^2-1}{(1+\xi^2)^4}$$

$$= \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} \frac{5\xi^2 - 1}{(1+\xi^2)^4}$$

$$\left| \frac{x^5}{5} \frac{5\xi^2-1}{(1+\xi^2)^4} \right| \leq \frac{1}{5} \frac{5\xi^2-1}{(1+\xi^2)^4}$$

$$= \left| \frac{\xi^2}{(1+\xi^2)^4} - \frac{1}{5(1+\xi^2)^4} \right| \leq \frac{\xi^2}{(1+\xi^2)^4} + \frac{1}{5(1+\xi^2)^4}$$

$$\frac{1}{5(1+\xi^2)^4} \leq \frac{1}{5}$$

За кое $n = ?$

$\varepsilon = 10^{-2}, 10^{-3}$

д. р. No 8

① $y = \frac{1+x}{1-x}$. Нам. $y^{(n)} = ?$ и $d^n y = ?$ по 2 начина

② $f(x) = e^x$ и напишете до 3-та степен разлож. в околн. на т. $x = 0$ (Мак Лорен) и на т $x = -1$ (Тейлор) (по степените на полинома $x+1$).