Експоненциална функция

$a_n = \left(1 + \frac{1}{n}\right)^n \quad \lim_{n\to\infty} a_n = e \approx 2,7172\ldots$

Неперово число, експонента

$y = e^x \quad y = a^x, a > 0$ показателна ф-ция

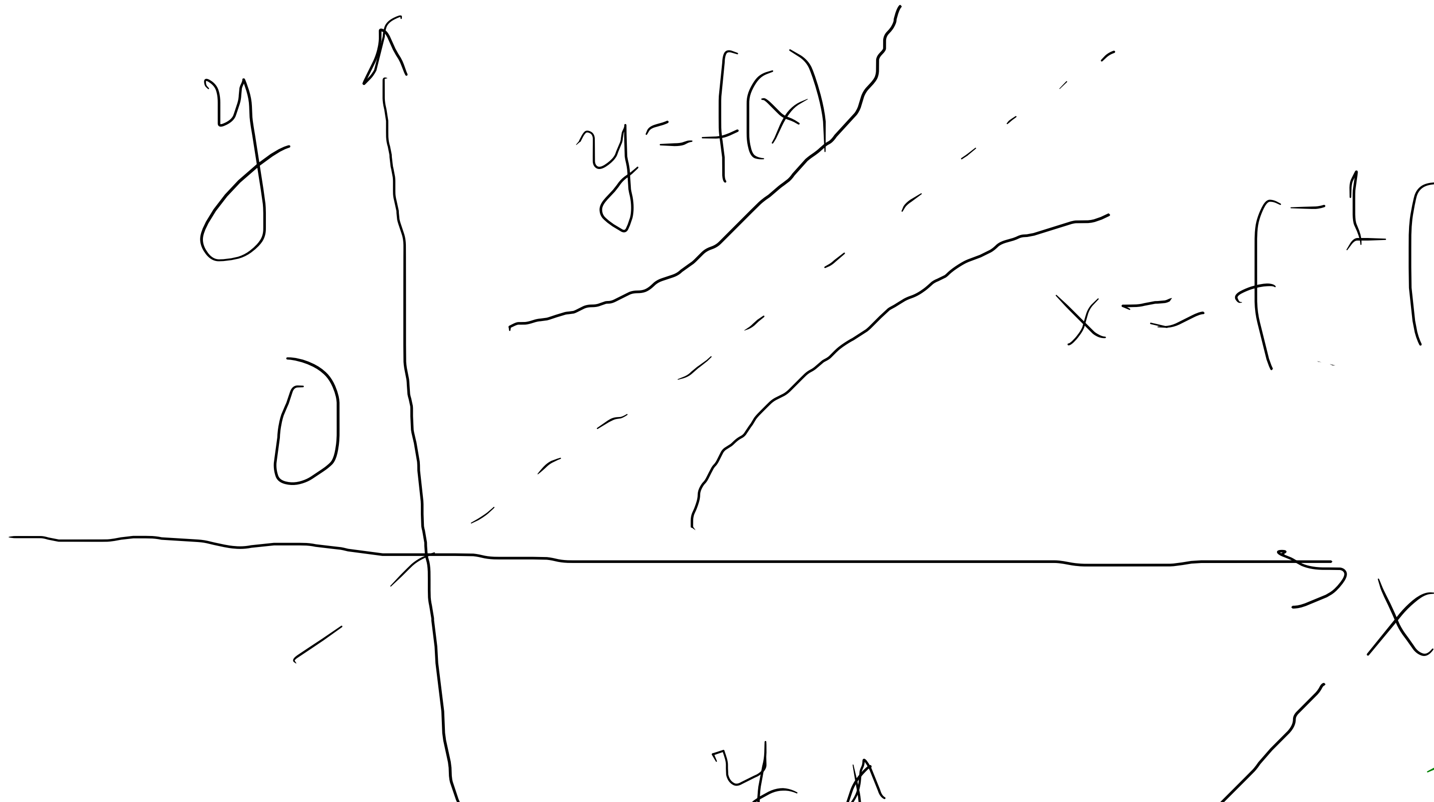
$y = f(x) \quad x = f^{-1}(y)$

Th Функцията $y = f(x)$ е обратима $\Leftrightarrow$ когато е монотонна.

Def Казв., че $f(x)$ е монотонно растяща, ако за $\forall$ x_1 и x_2: $x_1 \geq x_2$ да е изпълн. и усл. $f(x_1) \geq f(x_2)$

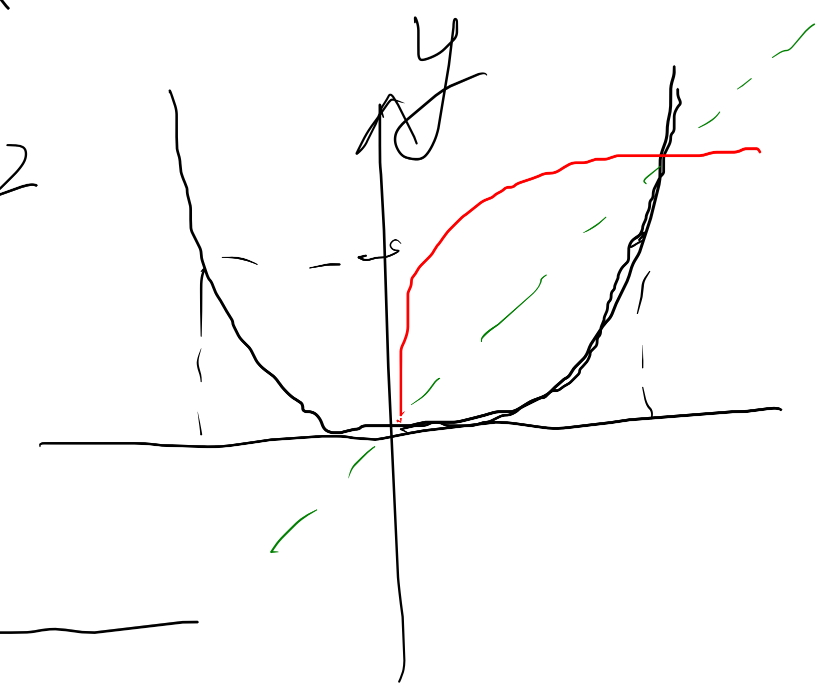
Ако за $x_1 \geq x_2$ — $\|$ — $f(x_1) \leq f(x_2)$

монотонно нарастваща ф-ция



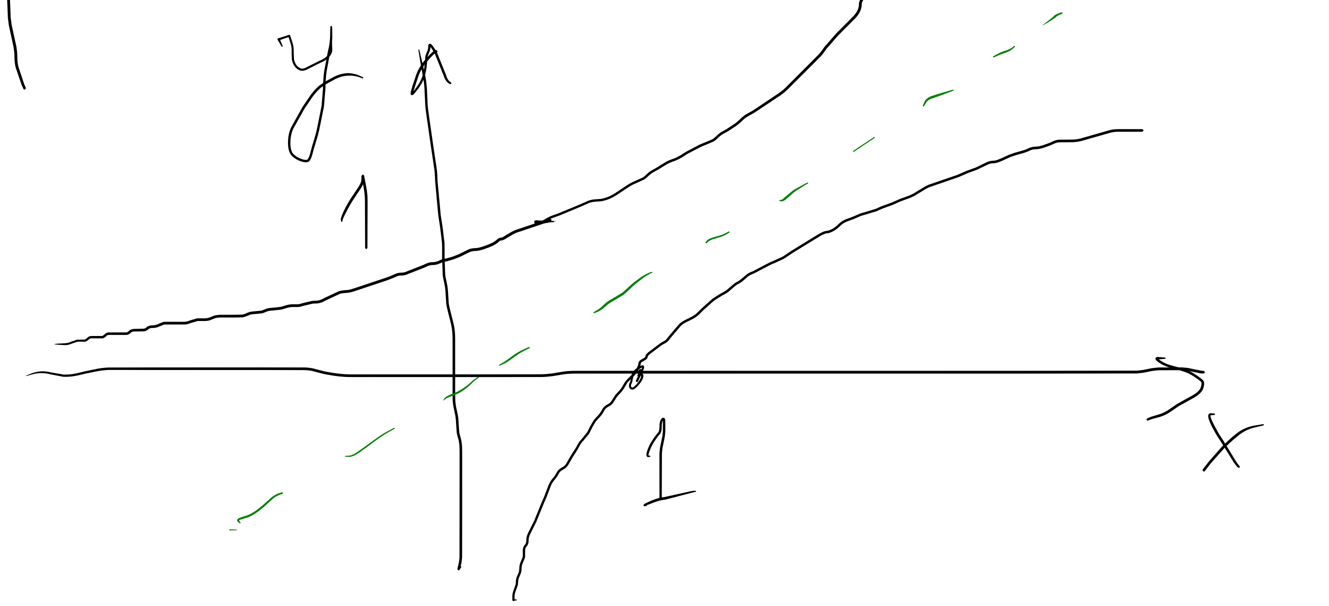
$y = x^2$

$x \geq 0$



$x = \sqrt{y}$

$y = e^x$



$x = \ln y$

$\log_a b$

$1 \neq a > 0, \; b > 0$

$\log_{10} b = \lg b$

$\log_e b = \ln b$

$\underbrace{y = a^x = e^{x \ln a}}$ $\ln$

$a > 0$

$\ln y = x \ln a \underbrace{\ln e}_{1}$

$= \ln a^x$

$\Rightarrow \ln y = \ln a^x \Rightarrow y = a^x$

Тъждества на Ойлер

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$y = \sin x$

$y = \cos x$

$T = 2\pi$

$y = tg\, x$

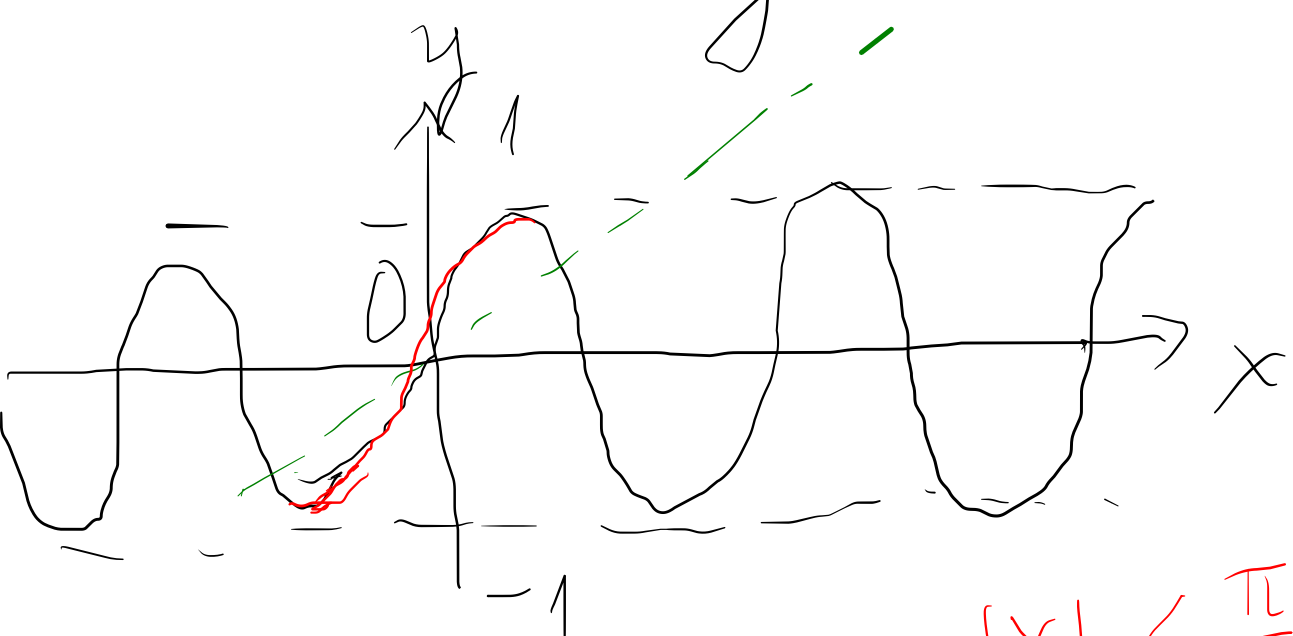
$y = cotg\, x$

$T = \pi$

$$tg\, x = \frac{\sin x}{\cos x} = \frac{e^{ix} - e^{-ix}}{2i} \cdot \frac{2}{e^{ix} + e^{-ix}} = -i\, \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

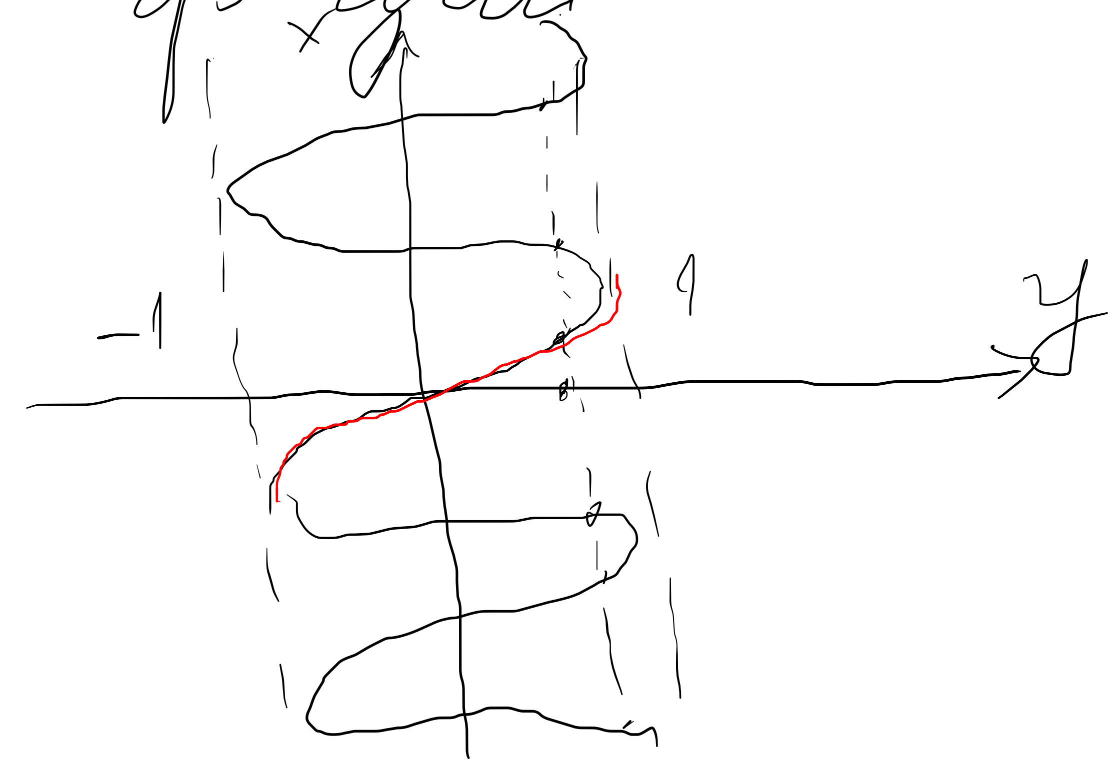
$$cotg\, x = \frac{1}{tg\, x} = i\, \frac{e^{ix} + e^{-ix}}{e^{ix} - e^{-ix}}$$

тригонометрични ф-ции



$|x| \leq \frac{\pi}{2}$

$y = \sin x$ е ф-ция



$y = \sin x \qquad |x| \le \frac{\pi}{2}$

$x = \arcsin y \qquad |y| \le 1$

$f \circ f^{-1}(x) = x$

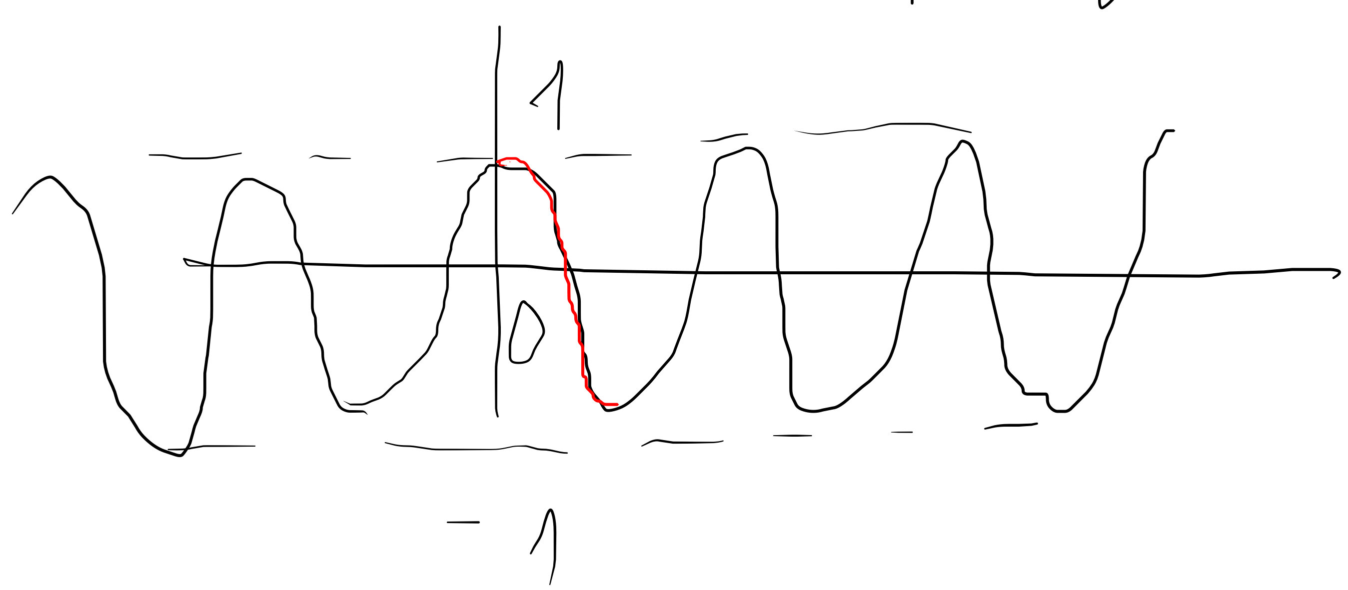
$\sin \arcsin u = u$

$\arcsin \sin u = u$

обратни тригонометрични ф-ции (кръгови ф-ции)

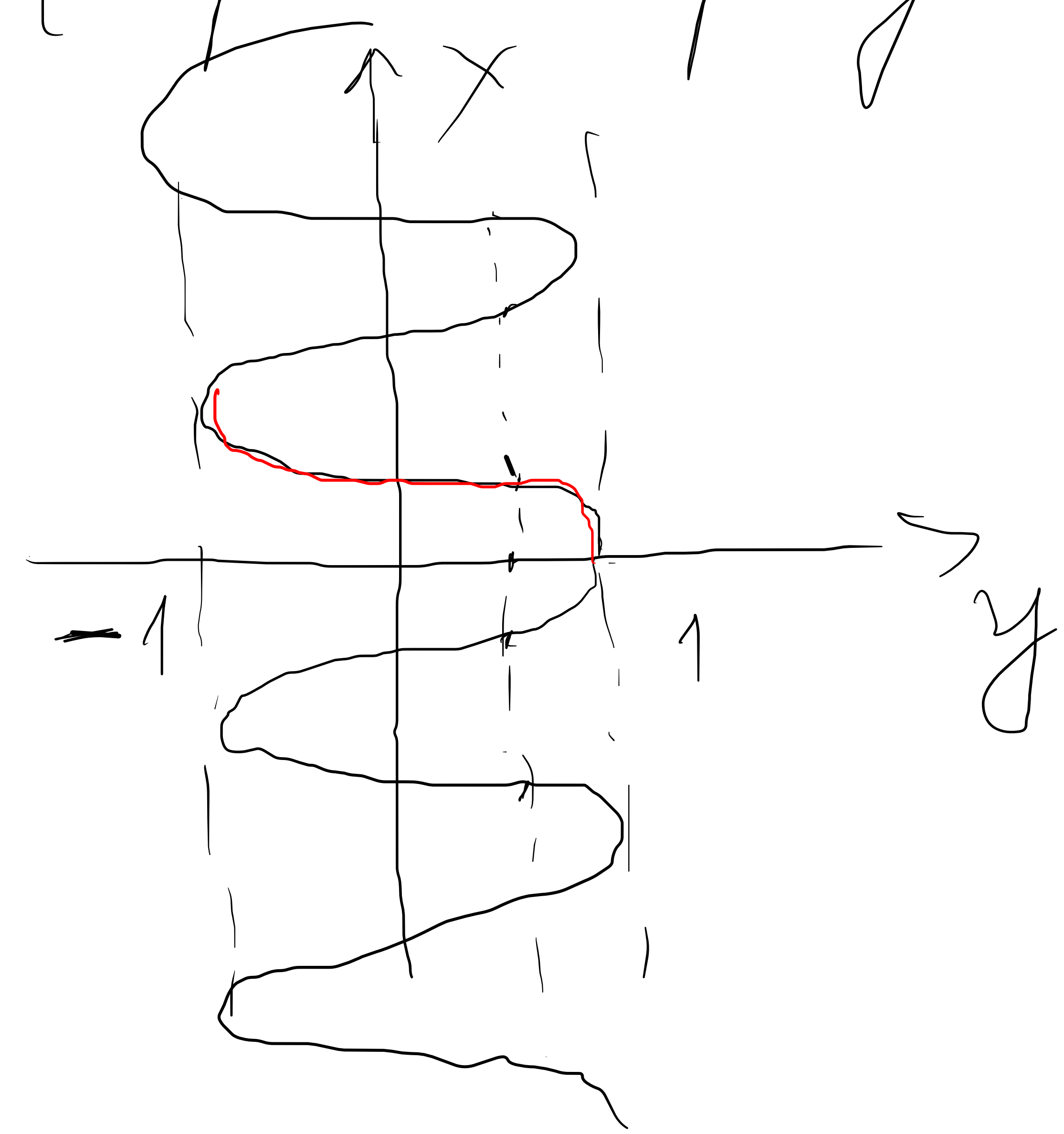
$y = \cos x$

$0 \le x \le \pi$



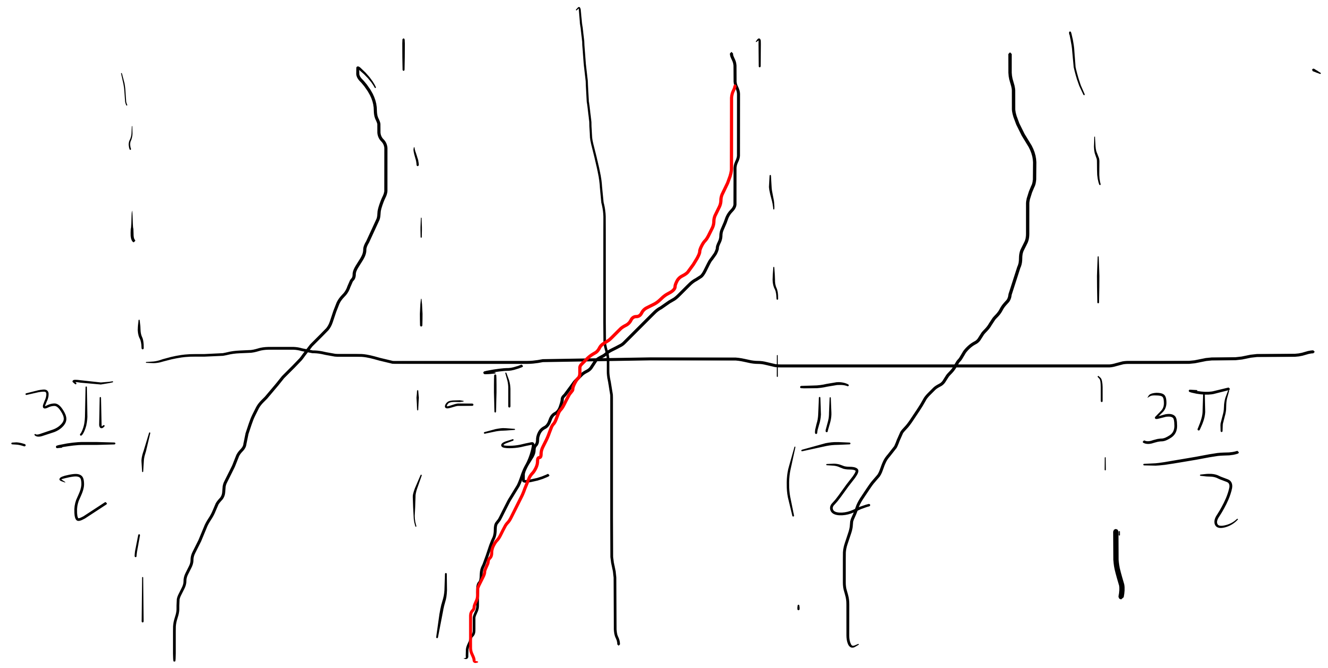
$x = \arccos y$

$|y| \le 1$



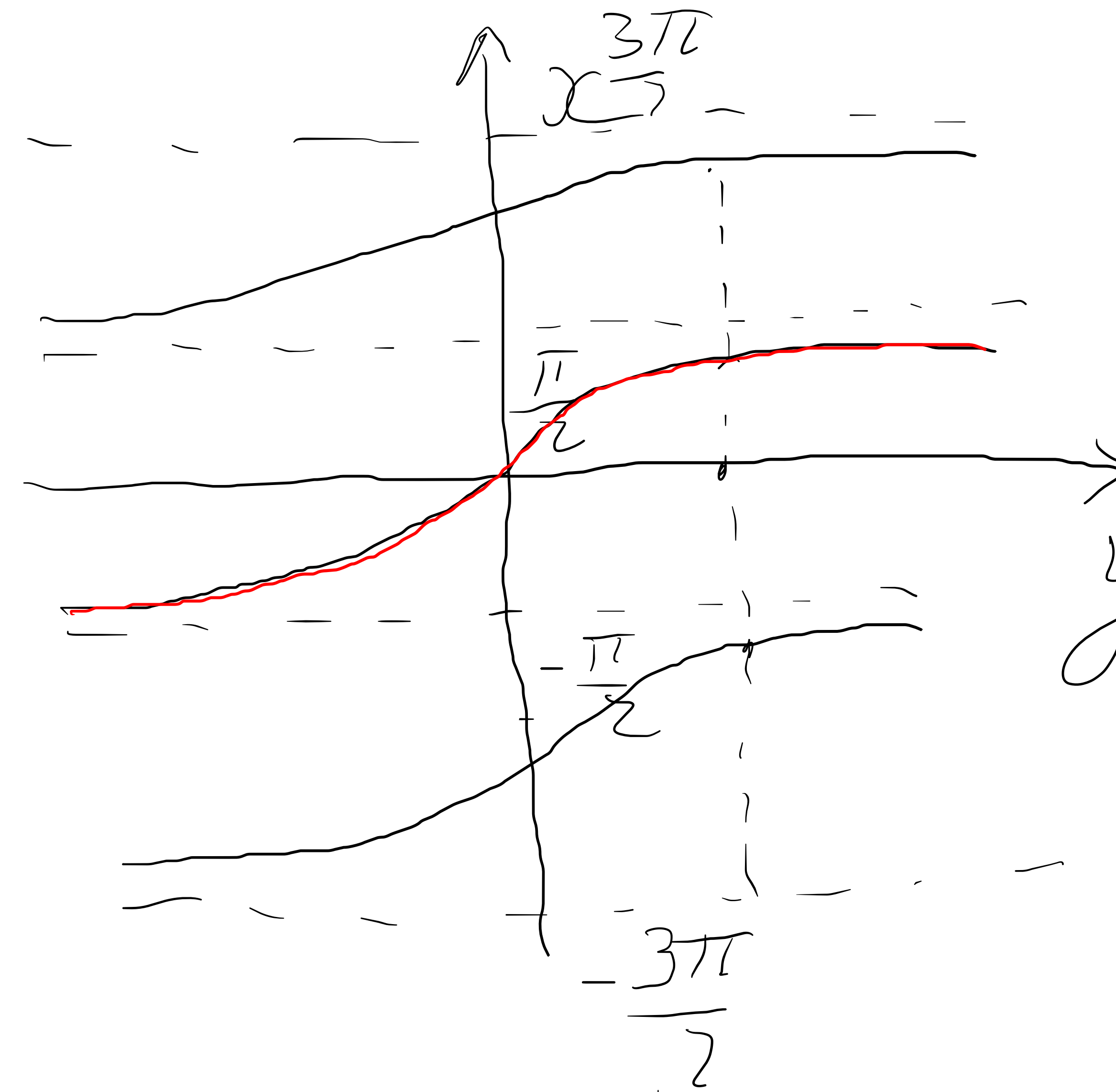
$y = \operatorname{tg} x$

$|x| < \frac{\pi}{2}$



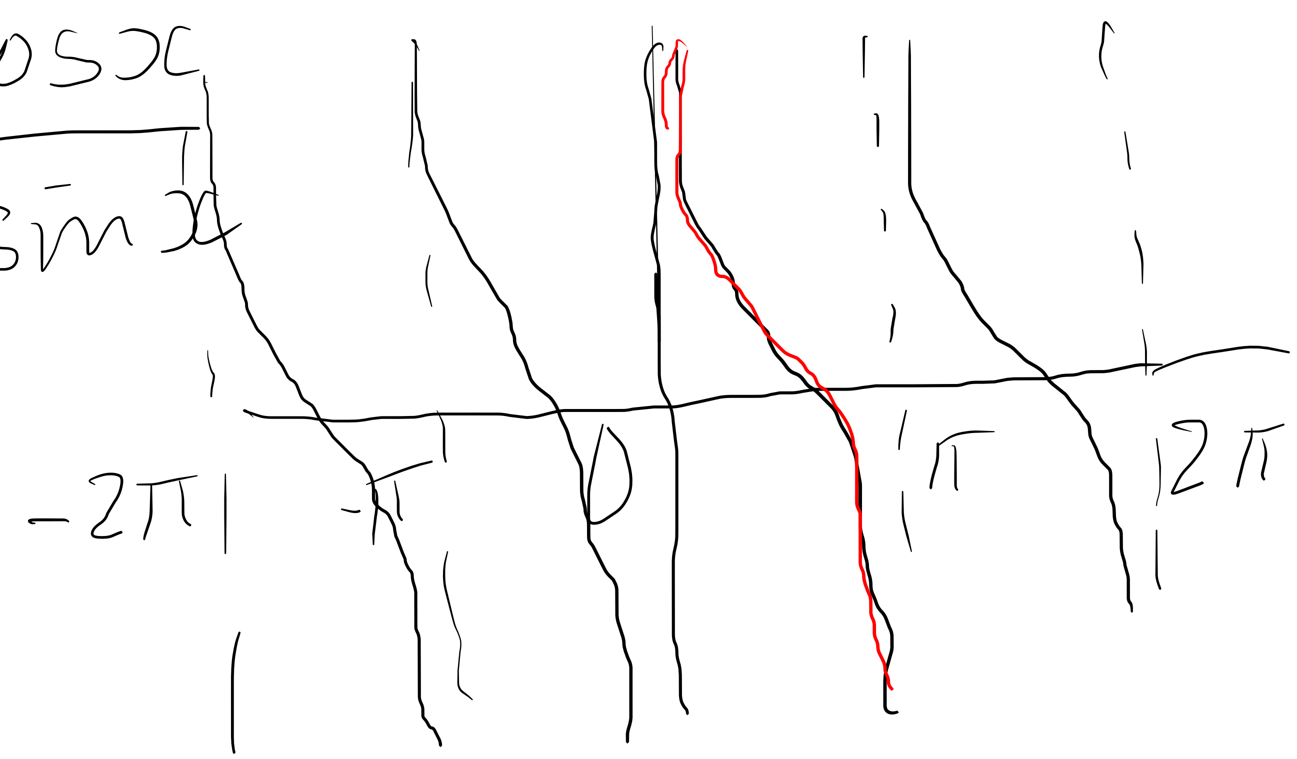
$x = \operatorname{arctg} y$

$y \in \mathbb{R}^1$



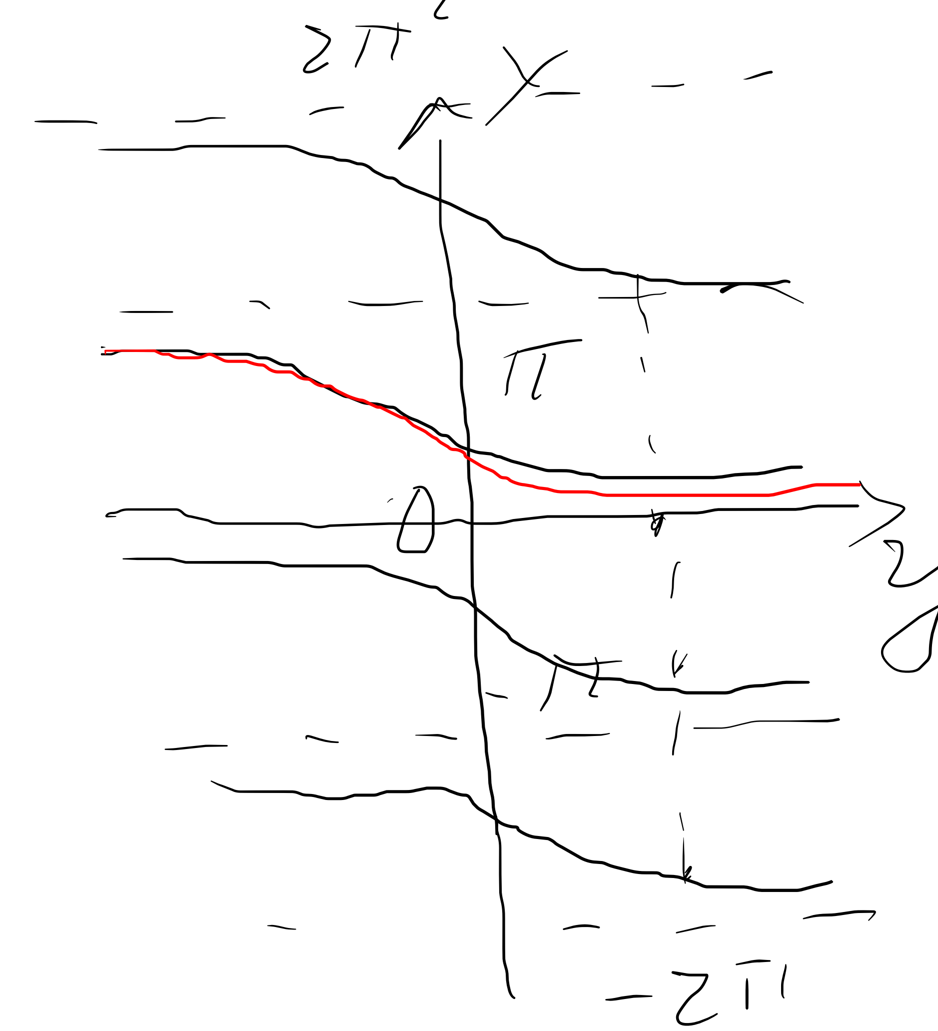
$y = \operatorname{cotg} x = \frac{1}{\operatorname{tg} x} = \frac{\cos x}{\sin x}$

$0 < x < \pi$



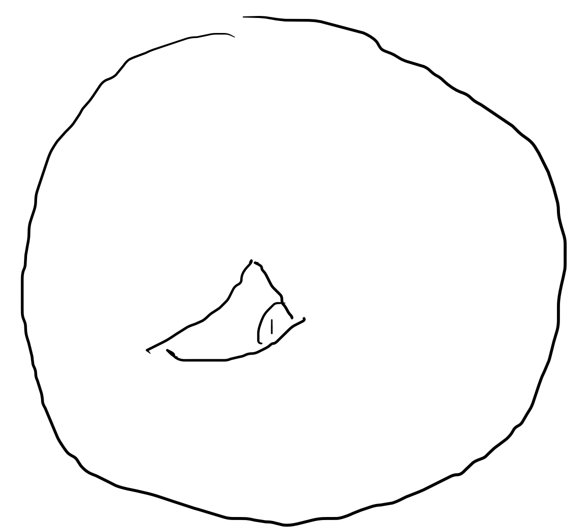
$x = \operatorname{arccotg} y$

$y \in \mathbb{R}^1$



Тъждества на Ойлер

$$ch\, x = \frac{e^{x} + e^{-x}}{2}$$

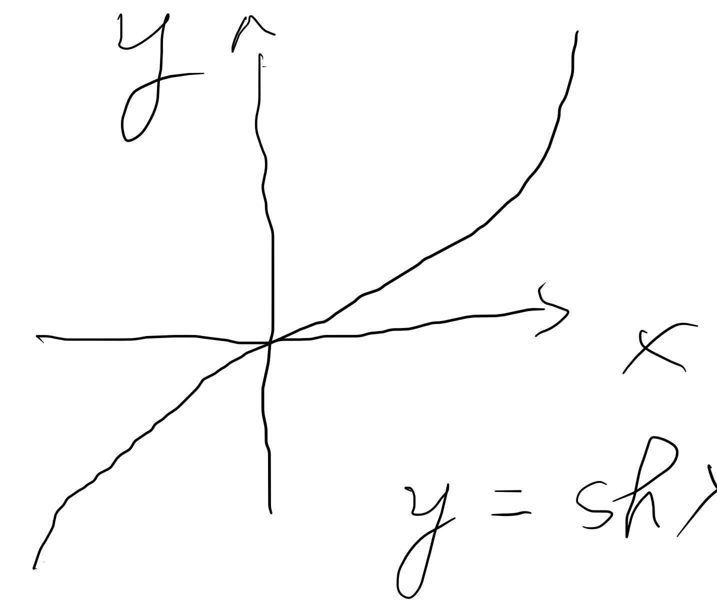


$$\cos^2\alpha + \sin^2\alpha = 1$$

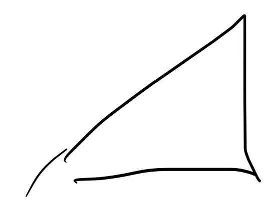
$$ch^2\alpha - sh^2\alpha = 1$$

$$sh\, x = \frac{e^{x} - e^{-x}}{2}$$

$$y = sh\, x$$



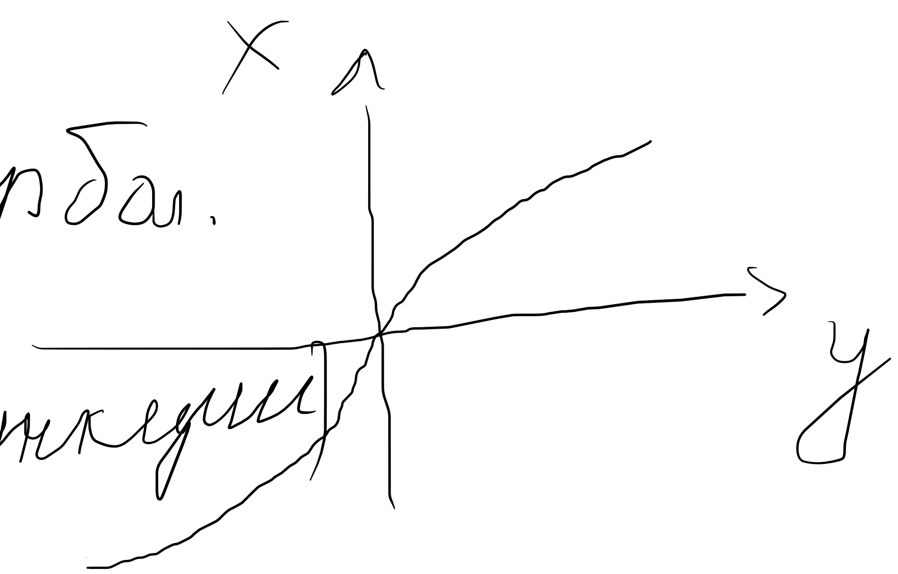
$$th\, x = \frac{sh\, x}{ch\, x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$



$$x \in \mathbb{R}^1$$

$$cth = \frac{1}{th\, x} = \frac{ch\, x}{sh\, x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

обратни хипербол. ф-ции (ареа-функции)



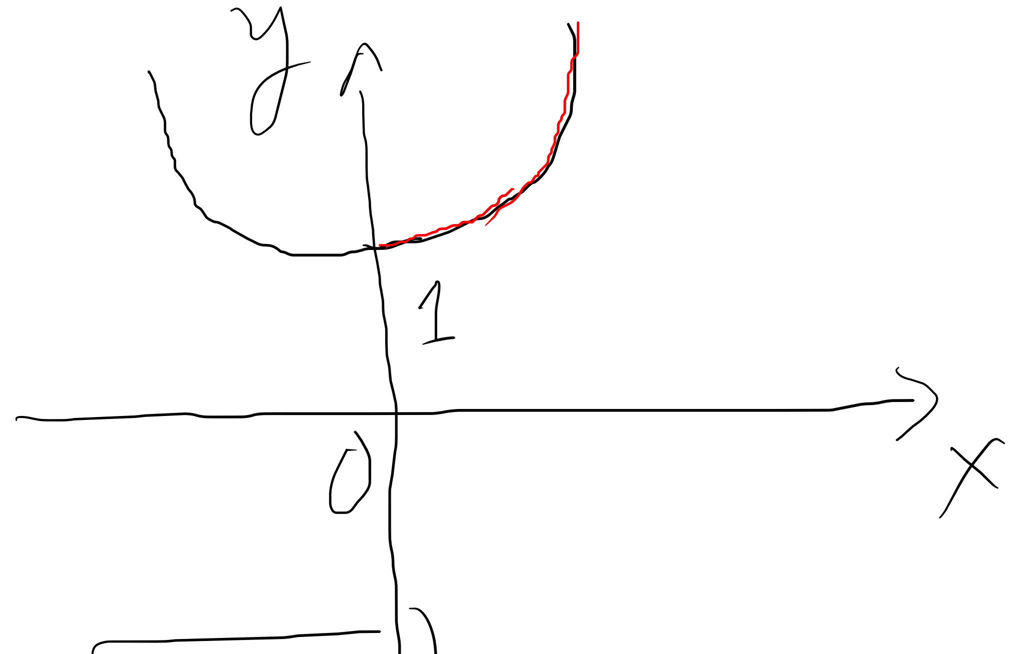
хиперболични функции arg

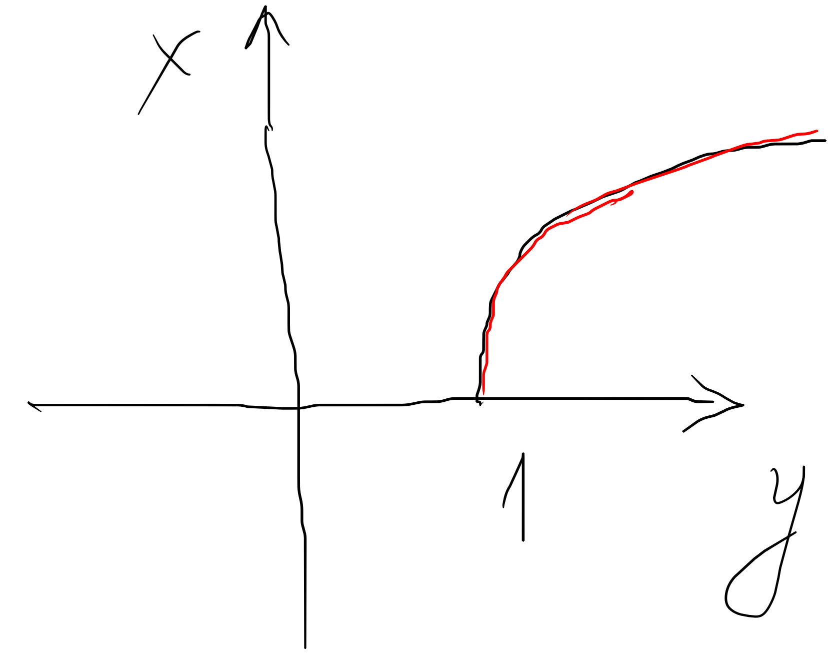
сферична геометрия

$x = Ar\, sh\, y \quad \forall\, y \in \mathbb{R}$

$Ar\, sh\, y = \ln\left(x + \sqrt{x^2+1}\right)$

$$y = \operatorname{ch} x = \frac{e^{x}+e^{-x}}{2}$$



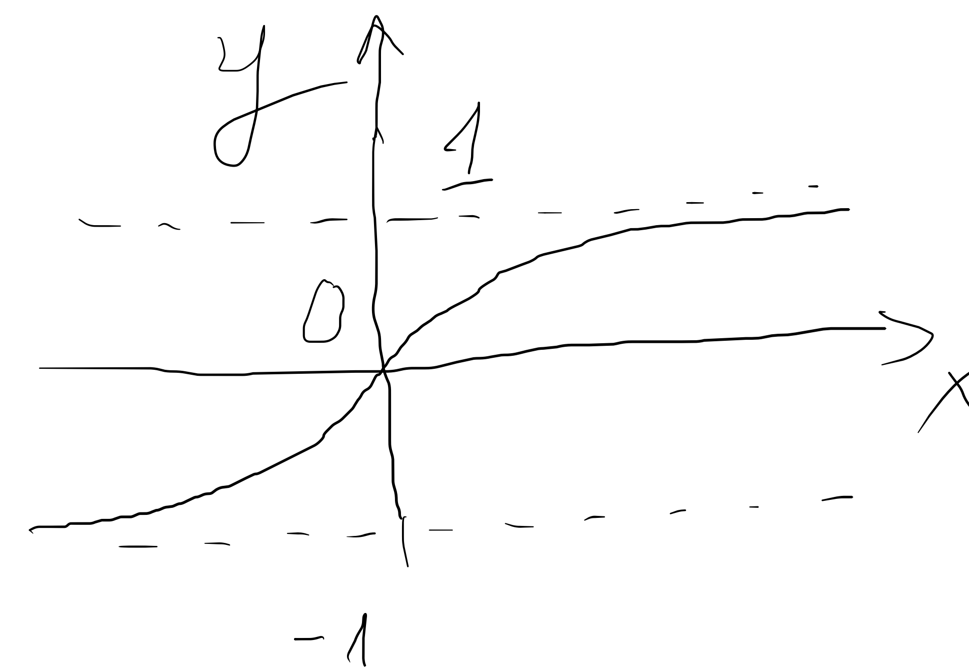


$$x \geq 0$$

$$x = \underbrace{\operatorname{Arch} y = \ln\left(y + \sqrt{y^2 - 1}\right)}_{y \geq 1}$$

$$y = \operatorname{th} x = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} = \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\,\frac{1-e^{-2x}}{1+e^{-2x}} \to 1$$

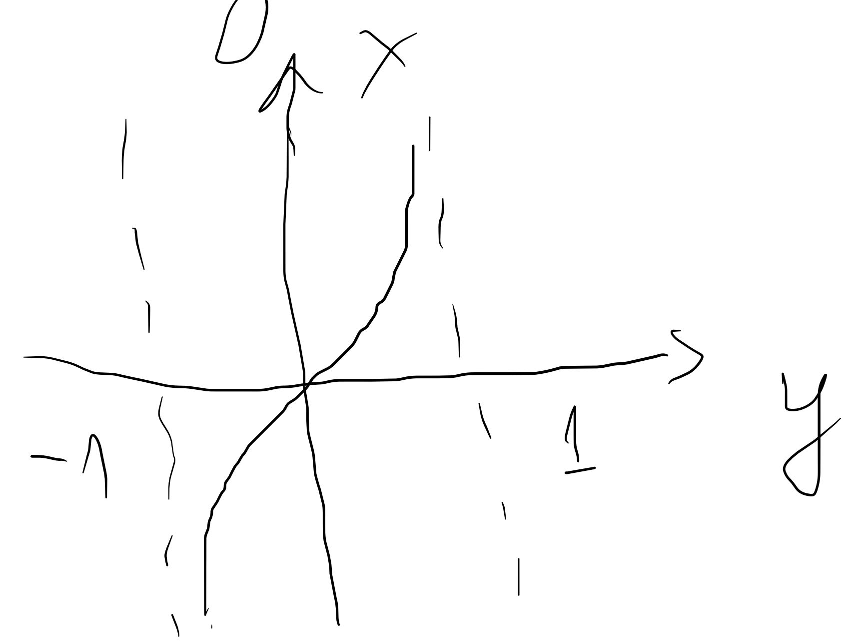
$$x \to \infty$$



$$\frac{\cancel{e^{-x}}}{\cancel{e^{-x}}}\,\frac{e^{2x}-1}{e^{2x}+1} \to -1$$

$$x \to -\infty$$

$$x \in \mathbb{R}^1$$



$$x = \operatorname{Arth} y$$

$$|y| < 1$$

$$\operatorname{Arth} y = \ln\sqrt{\frac{1+y}{1-y}}, \quad |y|<1$$

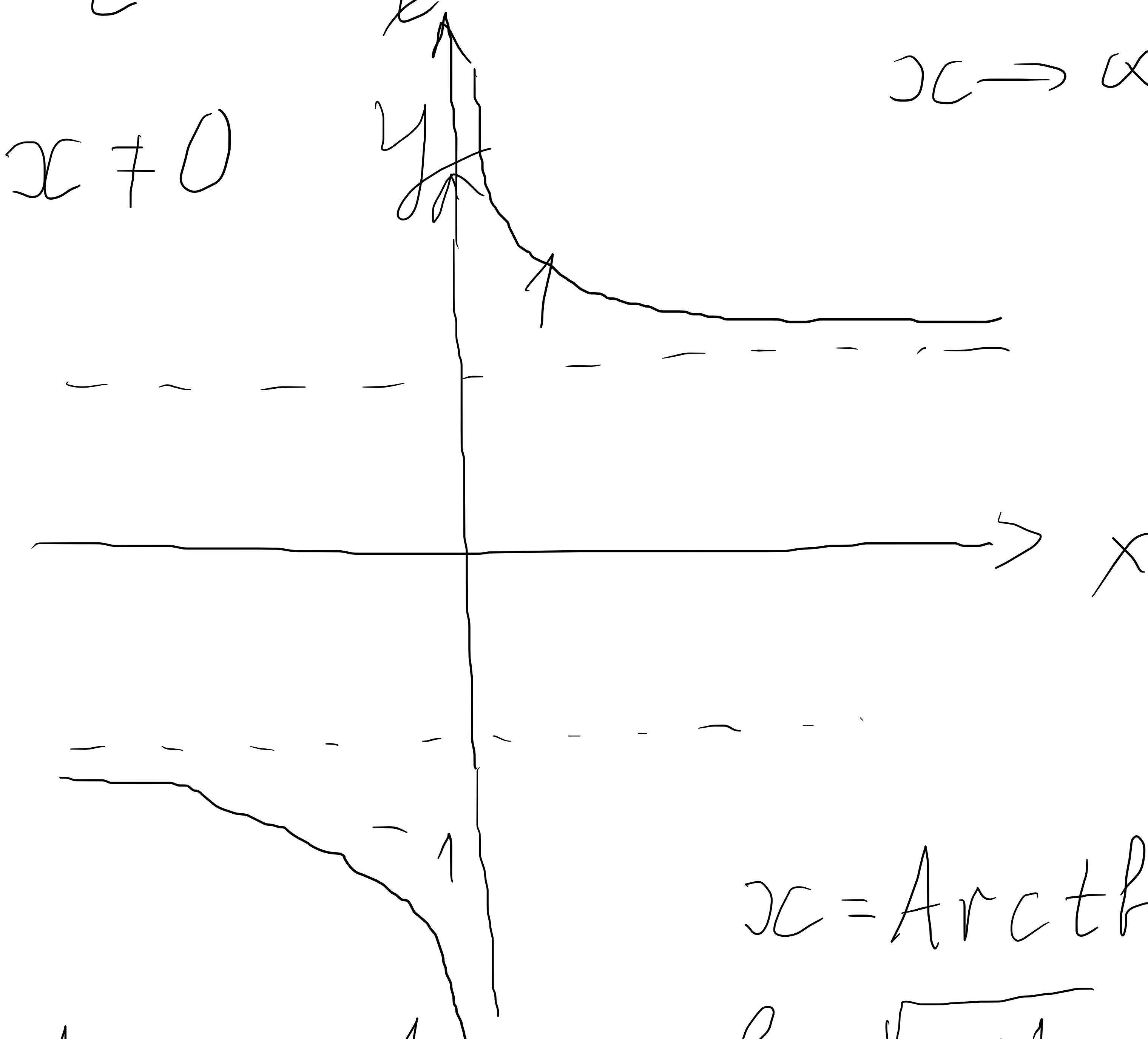
$$y = \operatorname{cth} x = \frac{\operatorname{ch} x}{\operatorname{sh} x} = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}} = \frac{\cancel{e^{x}}}{\cancel{e^{x}}}\,\frac{1+e^{-2x}}{1-e^{-2x}} \to 1$$

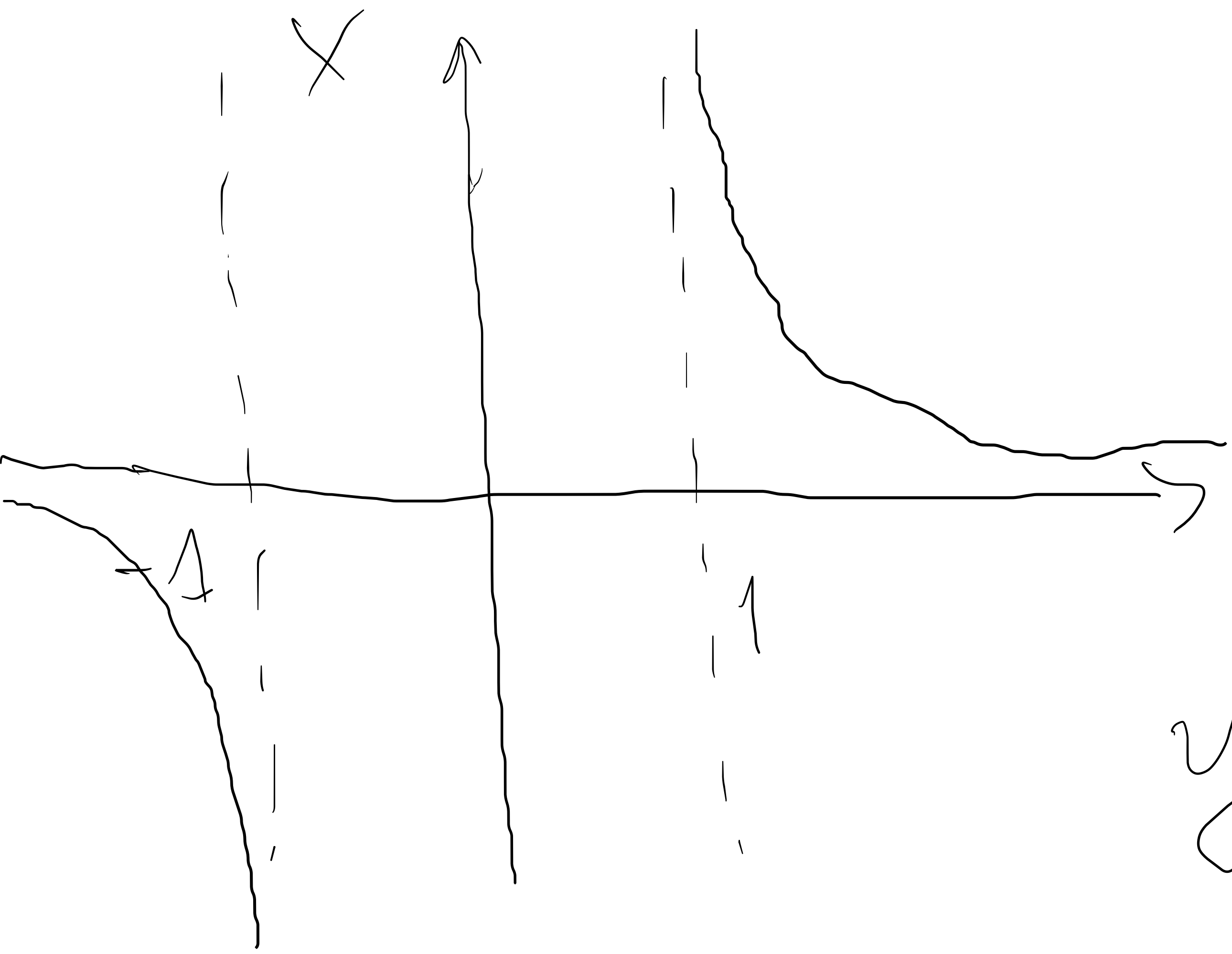
$x \to \infty$

$$\frac{\cancel{e^{-x}}}{\cancel{e^{-x}}}\,\frac{e^{2x}+1}{e^{2x}-1} \to -1$$

$x \to -\infty$

$\forall x \neq 0$





y

$y<-1 \cup y>1$

$$x = \operatorname{Arcth} y = \ln\sqrt{\frac{y+1}{y-1}}, \quad |y|>1$$

Граници на функции

Def (Коши) Казваме, че ф-ята $f(x)$ има граница A в т. x_0, ако за $\forall\, \varepsilon > 0\ \exists\, \delta(\varepsilon) > 0$ и такова, че когато $|x - x_0| < \delta$ да е изпълнено и условието $|f(x) - A| < \varepsilon$.

Def (Хайне) Казваме, че ф-ята $f(x)$ има гран. A в т. x_0, ако за $\forall\, \{x_n\} \to x_0$ съотв. функционални редици $\{f(x_n)\} \to A$.

Th Двете дефиниции са еквивалентни

$|x - x_0| < \delta$

$-\delta < x - x_0 < \delta$

$x_0 - \delta < x < x_0 + \delta$

$(x_0 - \delta, x_0 + \delta)$

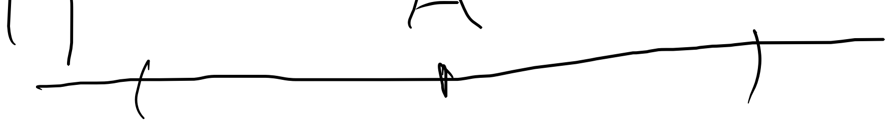
$\{x_n\} \to x_0$

$|f(x) - A| < \varepsilon$

$-\varepsilon < f(x) - A < \varepsilon$

$A - \varepsilon < f(x) < A + \varepsilon$

$\{f(x_n)\} \to A$



$\lim\limits_{x \to x_0} f(x) = A$

пр.: $y = \ln x$

$x \notin D.O.$

$x \to 0$

$\ln x \to -\infty$

$f(x)$ и $\lim\limits_{x \to x_0} f(x) = 0$

Казв., че ф-цията $f(x)$ е безкрайно малка в околност на т. x_0.

$f(x)$ $\lim\limits_{x \to x_0} f(x) = \pm\infty$ Казв., че ф-цията $f(x)$ е безкрайно голяма в околн. на т. x_0.

$f(x)$ и $g(x)$ б.м. в околн. на т. x_0

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \begin{cases} \pm\infty, & g(x) \text{ е б.м. от по-висок ред спрямо б.м.ф. } f(x) \\ 0, & f(x) \text{ е б.м. от по-висок ред с/но б.м. } g(x) \\ 0 \neq |A| < \infty & f(x) \text{ и } g(x) \text{ са б.м. от един и същ ред} \\ A = 1 & \text{б.м. } f(x) \text{ и } g(x) \text{ са еквиваленти в околн. на т. } x_0 \end{cases}$$

$\frac{0}{0}$

неопред.

$f(x) \underset{x \to x_0}{\sim} g(x)$

f и g са б.т. ф-ции

$\lim\limits_{x \to x_0} \frac{f(x)}{g(x)} = 1$ б.т. $f \underset{x \to x_0}{\sim} g$

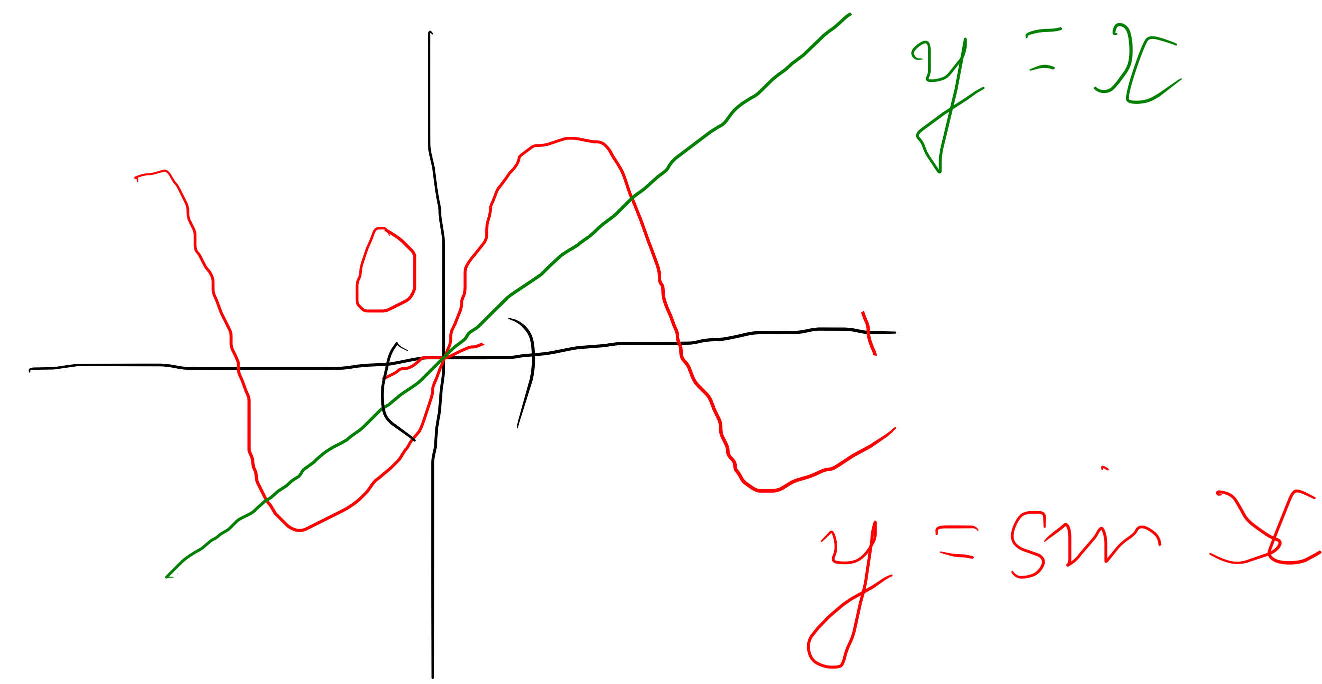
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

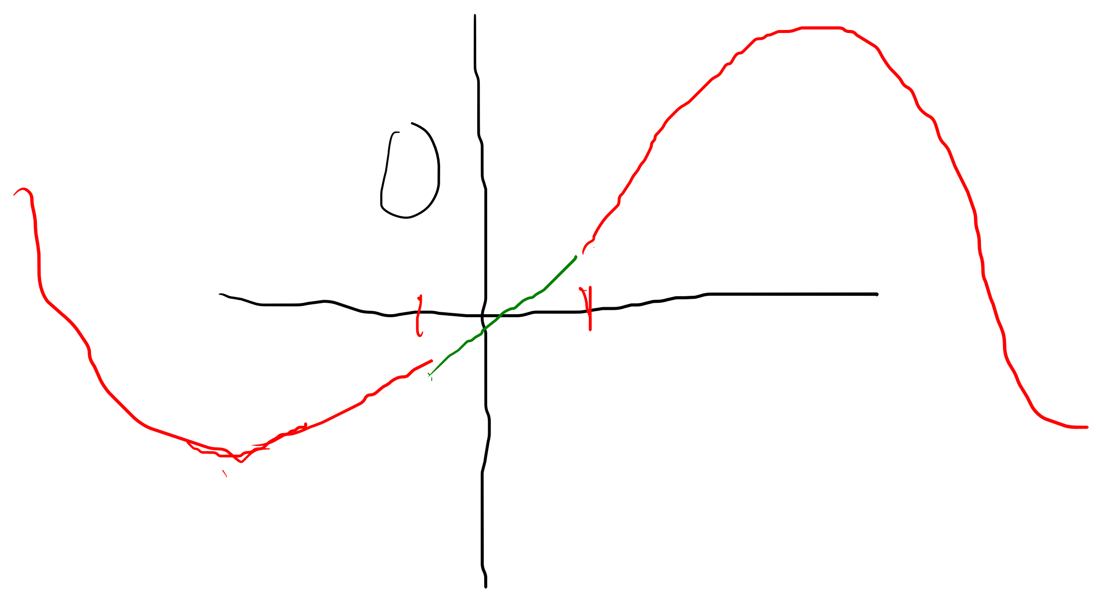
$\sin x \sim x$

$\sin x \to 0$ б.м. $x \to 0$

$x \to 0$ б.м.

$x \to 0$





$x \ll 1$

$\sin x \approx x$

$$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = 1$$

$\operatorname{tg} x \sim x$

$x \to 0$