

# COMP1215 - Combinatorics

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11 November 2019

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## 1 Inclusion and Exclusion

1.  $|A \cup B| = |A| + |B| - |A \cap B|$

## 2 Pigeonhole principle

If  $|A| > |B|$  (size of  $A$  > size of  $B$ ), then every function from  $A$  to  $B$  maps at least 2 distinct elements of  $A$  to the same element of  $B$ .

In other words:

For  $f : A \rightarrow B \exists x_1, x_2 \in A, x_1 \neq x_2$  such that  $f(x_1) = f(x_2)$

**Example 2.1.**

$$A = \{x \mid x \in \mathbb{Z} \text{ and } |x| \leq 5\}$$

$$B = \{x \mid x \in \mathbb{N} \text{ and } x \leq 5\}$$

As you can see,  $|A| < |B|$ , so all valid functions between  $A$  and  $B$  must map at least one pair of  $x$  values to the same  $y$  value.

$$f : A \rightarrow B$$

$$f(x) = x^2$$

$$f(-5) = 25$$

$$f(5) = 25$$