HYPOTHESIS TESTING

Analytics Primer

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

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Flip Number	Result	Probability
1	Heads	0.50

Hypothesis Testing Through Example

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- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25

Do you still think the coin is fair?

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
3	Heads	

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
3	Heads	0.125

Do you still think the coin is fair?

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
3	Heads	0.125
4	Heads	0.0625

Do you still think the coin is fair?

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
3	Heads	0.125
4	Heads	0.0625
5	Heads	0.03125

Do you still think the coin is fair?

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
3	Heads	0.125
4	Heads	0.0625
5	Heads	0.03125

- No longer believe coin is fair.

Hypothesis Testing Through Example

- I have a coin that you believe is fair to start. NULL Hypothesis
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results. Test Statistic

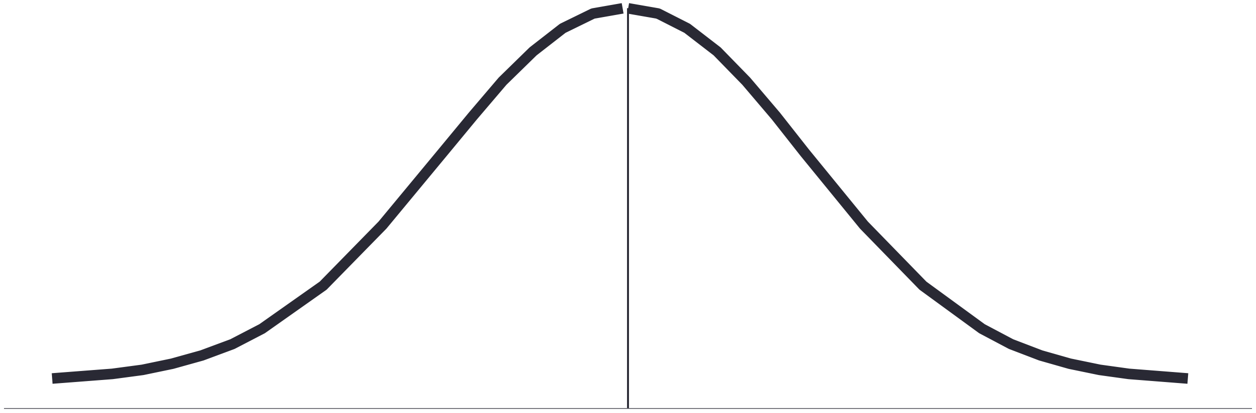
Flip Number	Result	P-Value
1	Heads	0.50
2	Heads	0.25
3	Heads	0.125
4	Heads	0.0625
5	Heads	0.03125

- No longer believe coin is fair.

Reject the NULL Hypothesis

Example with Means

- According to the CLT, sample means follow a Normal distribution as long as the sample size is big enough.



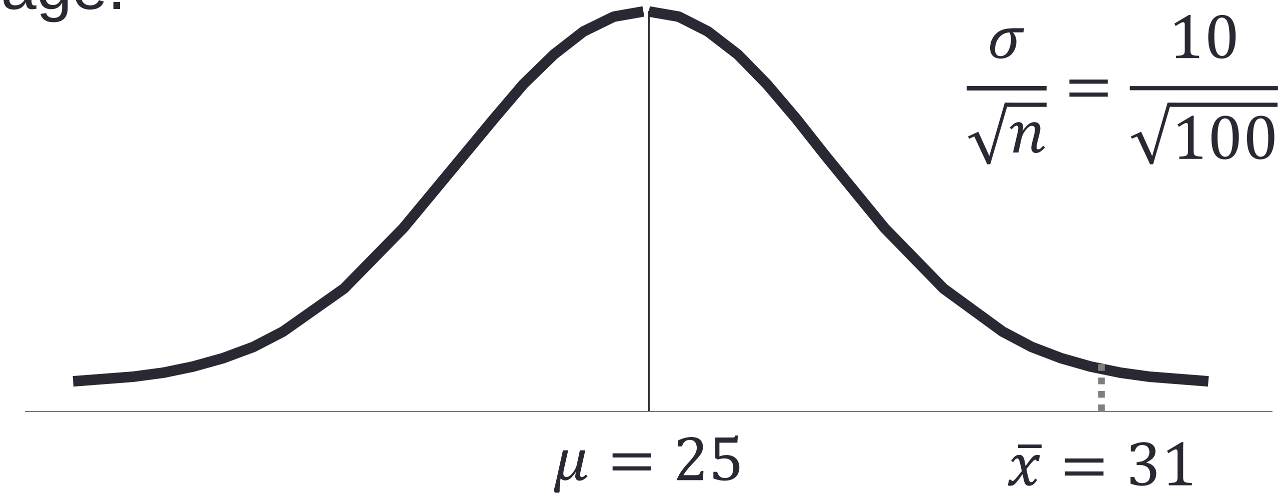
Example with Means

- You believe the average age of your customers is 25 years old with standard deviation of 10.

Example with Means

- You believe the average age of your customers is 25 years old with standard deviation of 10.
- You take a sample of 100 of your customers and collect their age.

$$\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{100}} = 1$$



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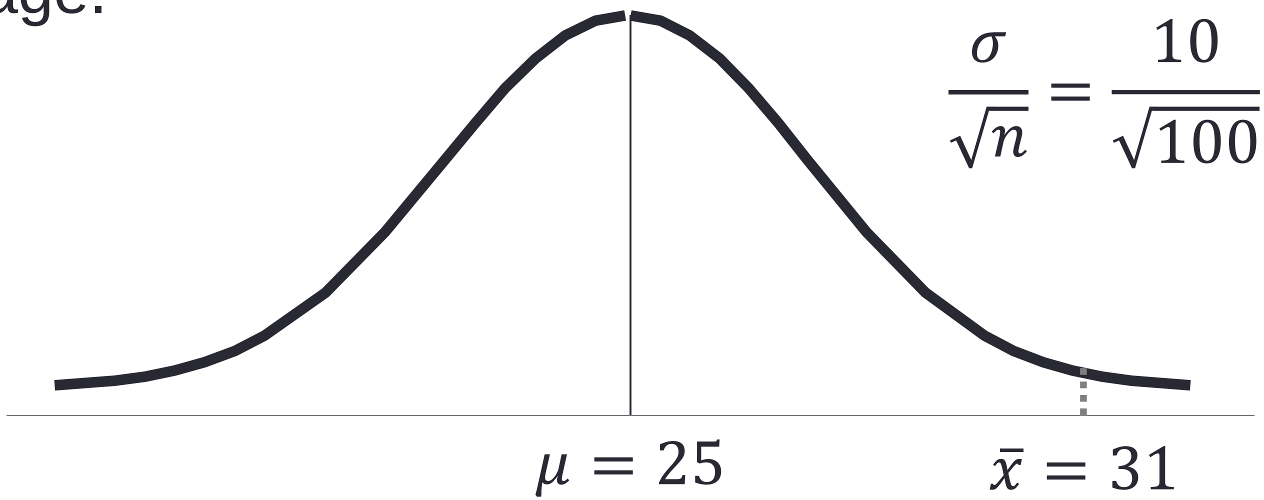

- What is the probability you see this under your original thought of 25 years old?

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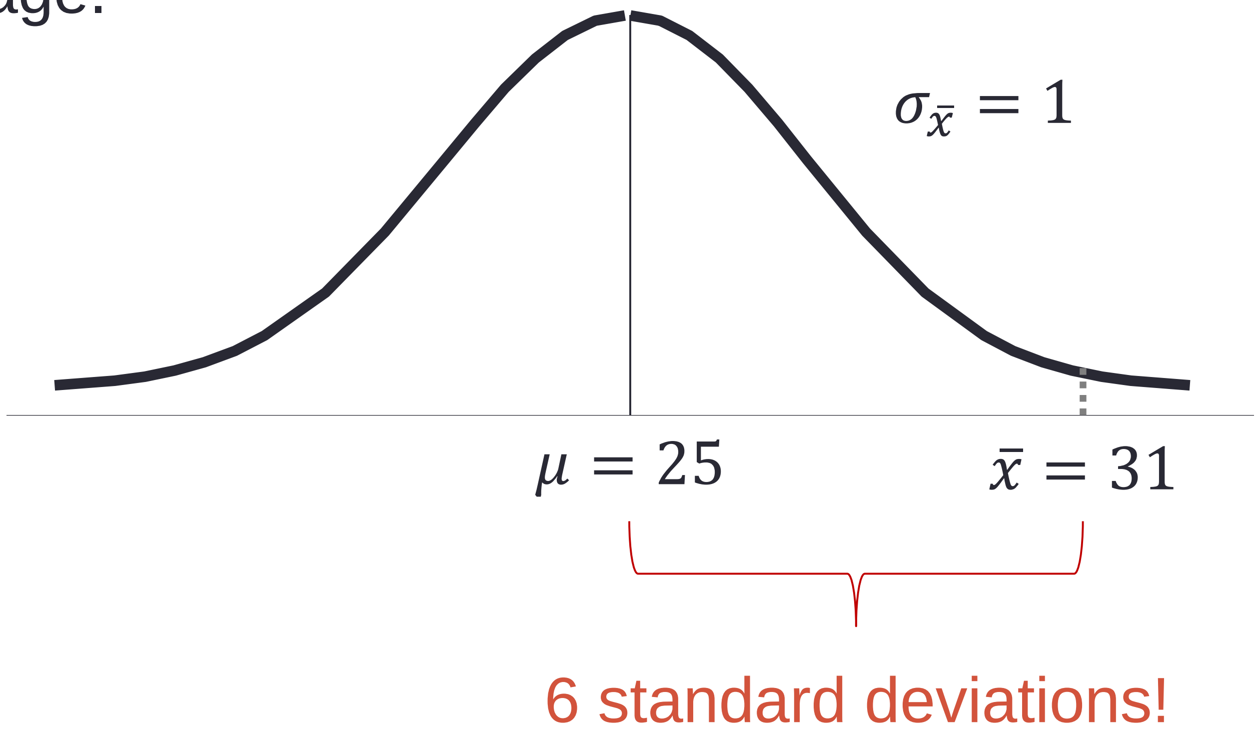

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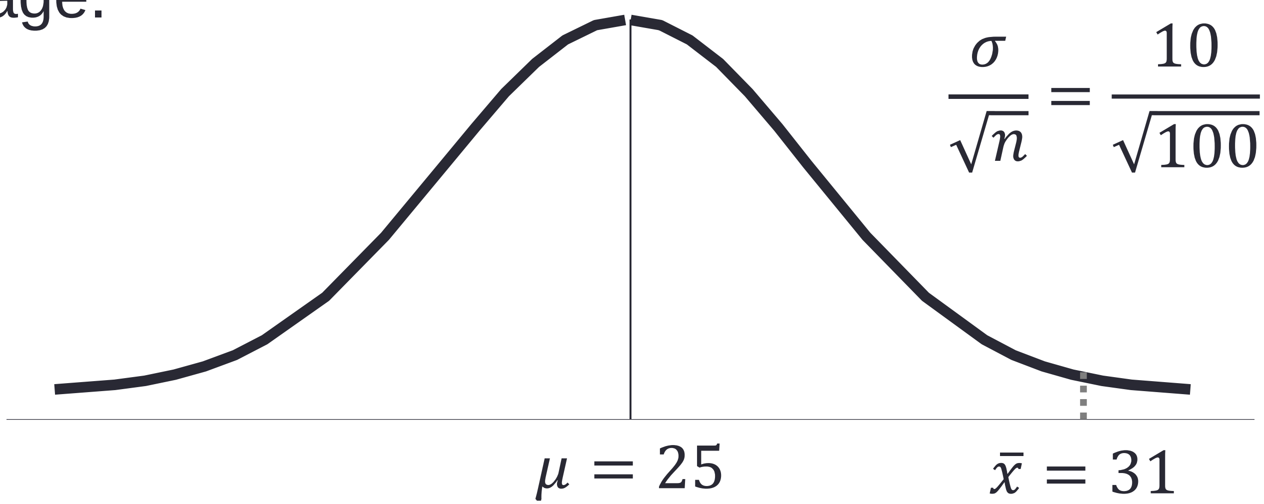


- What is the probability you see this under your original thought of 25 years old? < 0.0001!

Example with Means

- You believe the average age of your customers is 25 years old with standard deviation of 10.
- You take a sample of 100 of your customers and collect their age.
- What is the probability you see this under your original thought of 25 years old? < 0.0001!
- Do you still believe your original hypothesis?

Example with Means

- You believe the average age of your customers is 25 years old with standard deviation of 10. NULL Hypothesis
- You take a sample of 100 of your customers and collect their age. Test Statistic
- What is the probability you see this under your original thought of 25 years old? P-value
- Do you still believe your original hypothesis?

Decision on NULL Hypothesis

Hypothesis Test Process

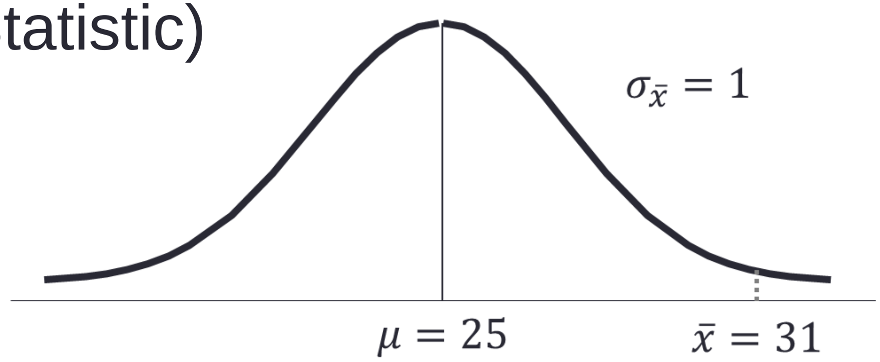
1. Develop your Hypothesis Statements (H_0 and H_a)
2. Collect Data (Test Statistic)
3. What is probability this happens? (P-value)
4. Decision About Null Hypothesis
5. Summarize

Hypothesis Test Process

1. Develop your Hypothesis Statements

$$H_0: \mu = 25 \qquad H_a: \mu \neq 25$$

2. Collect Data (Test Statistic)



3. What is probability this happens? (P-value)

$$0.00006$$

4. Decision About Null Hypothesis
5. Summarize

NULL AND ALTERNATIVE HYPOTHESIS

Hypothesis Testing

- **Hypothesis Testing** can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The **null hypothesis**, denoted by H_0, is a tentative assumption about a population parameter.
- The **alternative hypothesis**, denoted by H_a, is the opposite of what is stated in the null hypothesis.
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H_0 and H_a.

Developing Null and Alternative

- It is not always obvious how the null and alternative hypotheses should be formulated.
- The context of the situation is very important in determining how the hypotheses should be stated.
- In some cases it is easier to identify the alternative hypothesis first!
- Typically, the alternative is what we are trying to test and want to collect evidence for.

Null Hypothesis, H_0

- The **null hypothesis** is the status quo, or the initial claim about the data.
- For example, my customers have an average age of 25 years.

$$H_0: \mu = 25$$

Null Hypothesis, H_0

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- For example, my customers have an average age of 25 years.

$$H_0: \mu = 25$$

- The null hypothesis is about population parameters, NOT sample statistics.
- Parameters are unknown, statistics are known.

$$H_0: \mu = 25 \qquad \cancel{H_0: \bar{x} = 25}$$

Null Hypothesis, H_0

- The **null hypothesis** is the status quo, or the initial claim about the data.
- For example, my customers have an average age of 25 years.

$$H_0\colon \mu = 25$$

- This is the truth until you can prove otherwise – innocent until proven guilty.
- Always contains **one** of the following: $=,\ \geq,\ \leq$
- May reject or fail to reject.

Alternative Hypothesis, H_a

- The **alternative hypothesis** is the opposite of the null hypothesis.
- For example, the average age of my customers is not 25 years old.

$$H_a\colon \mu \neq 25$$

- This is typically what we are trying to prove.
- Always contains **one** of the following: $\neq$, $<$, $>$
- Never say we prove it!

Summary of Null vs. Alternative

- Equality piece of the hypothesis is contained in the null hypothesis.
- Hypotheses are about a population parameter like μ.
- General Forms:

$$\boxed{\begin{array}{l} H_0\!: \mu \le \mu_0 \\ H_a\!: \mu > \mu_0 \end{array}} \qquad \boxed{\begin{array}{l} H_0\!: \mu \ge \mu_0 \\ H_a\!: \mu < \mu_0 \end{array}} \qquad \boxed{\begin{array}{l} H_0\!: \mu = \mu_0 \\ H_a\!: \mu \ne \mu_0 \end{array}}$$

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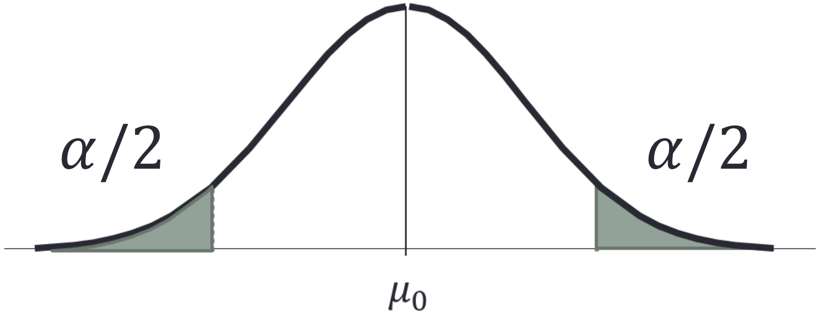
One-Sided Tests

Two-Sided Test

Rejection Region

Two-Sided

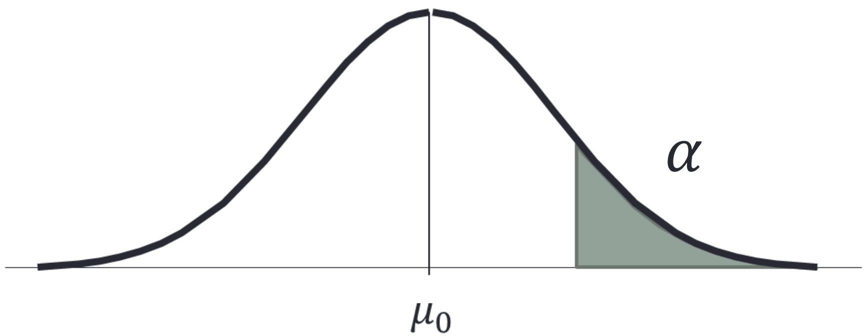
$$\boxed{\begin{aligned} H_0&: \mu = \mu_0 \\ H_a&: \mu \neq \mu_0 \end{aligned}}$$



Rejection Region

One-Sided

$$H_0: \mu \leq \mu_0$$
$$H_a: \mu > \mu_0$$



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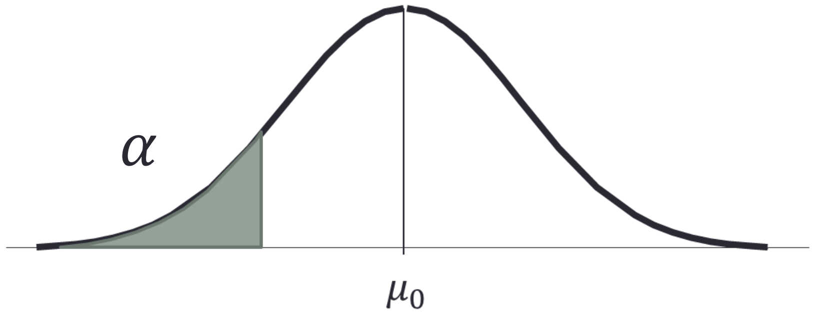

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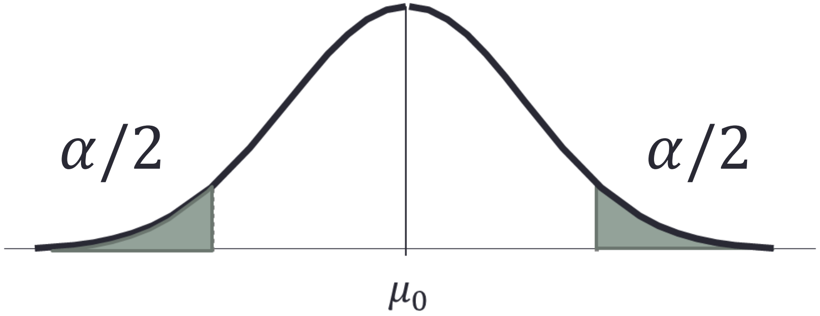

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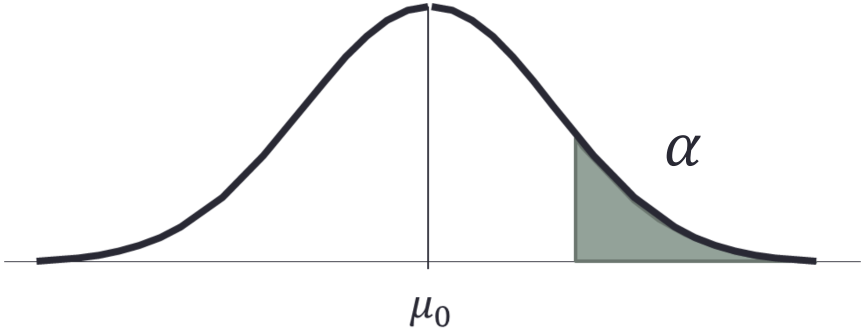

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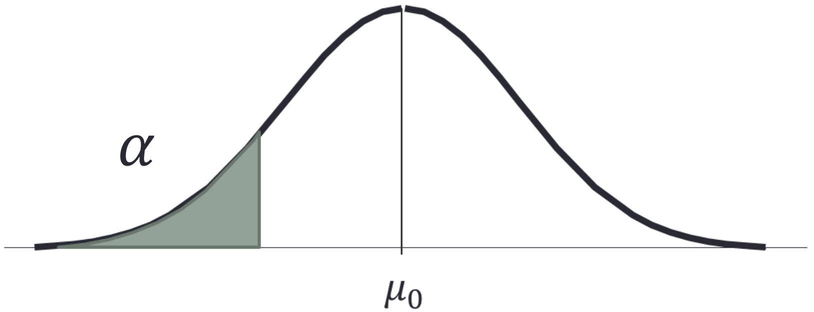

Summary of Null vs. Alternative

- Equality piece of the hypothesis is contained in the null hypothesis.
- Hypotheses are about a population parameter like p.
- General Forms:

$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$

One-Sided Tests

Two-Sided Test

TEST STATISTIC

Test Statistic

- The **test statistic** is a way to "standardize" the information from the sample.
- We will use probability to help us make our decision about the population parameter, but in order to calculate a probability, we need to "standardize the statistic" we obtained from the sample..
- Test statistics have a common form:

Mean is given in
Null Hypothesis!

$$\text{Test Statistic} = \frac{\text{Statistic} \; - \text{Mean}}{\text{Standard Error}}$$

Test Statistic

- The **test statistic** summarizes the amount of information provided in the sample.
- Imagine this like evidence in a court case.
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Sample Information

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Null Hypothesis Information

Test Statistic

- The **test statistic** summarizes the amount of information provided in the sample.
- Imagine this like evidence in a court case.
- Test statistics have a common form:

$$\text{Test Statistic} = \frac{\text{Statistic} - \text{Null Value}}{\text{Standard Error}}$$

Estimated Error from Sampling Distribution of Statistic

Test Statistic for Means

- The **test statistic** summarizes the amount of information provided in the sample.
- Sample means need the t-distribution because of the unknown values of the population standard deviation.

$$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

Test Statistic for Proportions

- The **test statistic** summarizes the amount of information provided in the sample.
- Sample proportions use the Normal distribution.

$$z = \frac{\hat{p} - p_0}{\left(\sqrt{\frac{p_0(1-p_0)}{n}}\right)}$$

P-VALUE & SIGNIFICANCE LEVEL APPROACH

P-values

- Once the test statistic has been determined, we can calculate the probability that we got the information we did from our sample, **assuming that the null hypothesis is true**.
- The **p-value** is the probability we got our sample, or a sample more extreme, under the null hypothesis.

Significance Level vs. P-value

- If the p-value is low, this implies that the sample we obtained from the population is **extremely rare** IF we assume that the null hypothesis is true.
- This leads us to question the validity of the null hypothesis – rejecting the null hypothesis if the p-value is low enough.
- How low is low enough?

Significance Level vs. P-value

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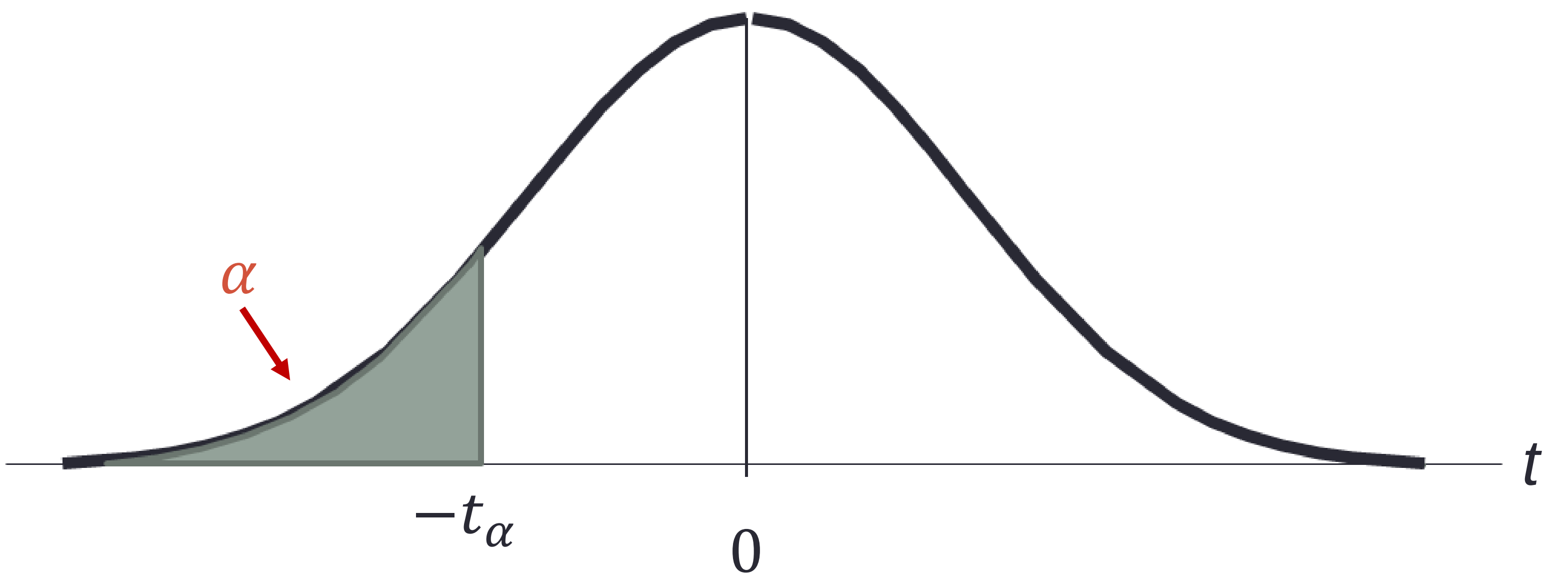
- How low is low enough?
  - Significance Level - α

Significance Level vs. P-value

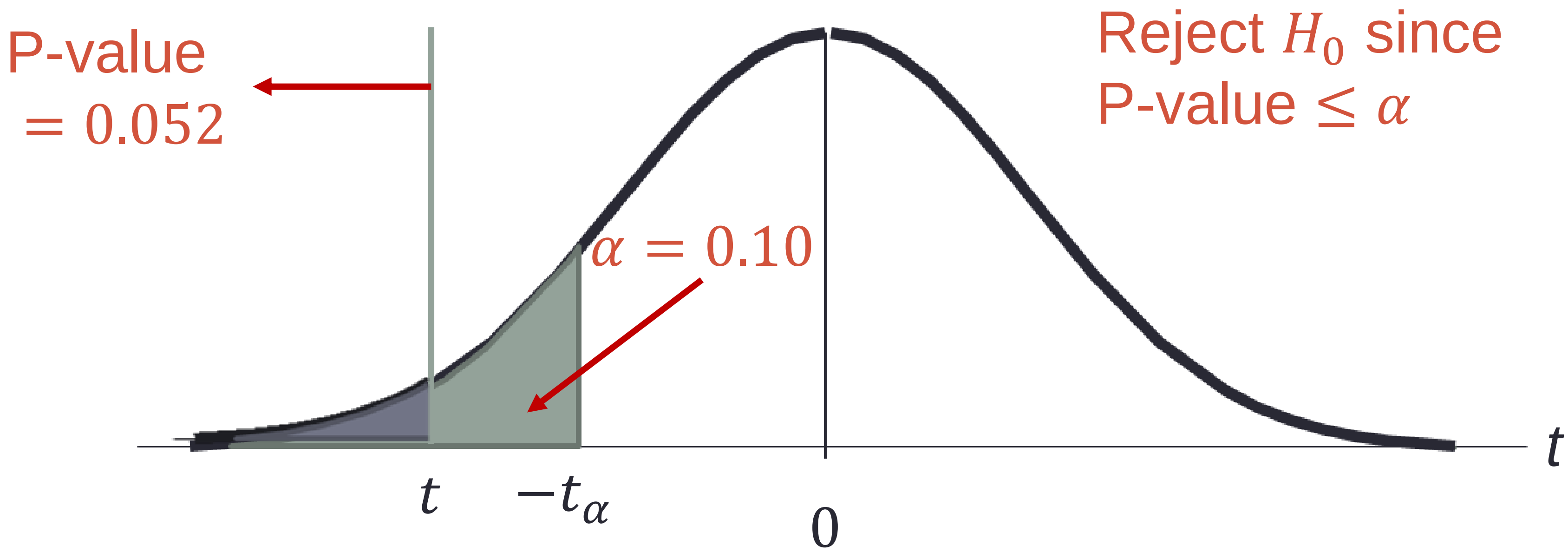
- How low is low enough?
  - Significance Level - α
- If the p-value is less than or equal to the level of significance (α), the value of the test statistic is in our rejection region.
- The rejection rule is the following:
  - Reject H_0 if p-value $\leq \alpha$

Lower-Tailed Test with P-value

$H_a: \mu < \mu_0$



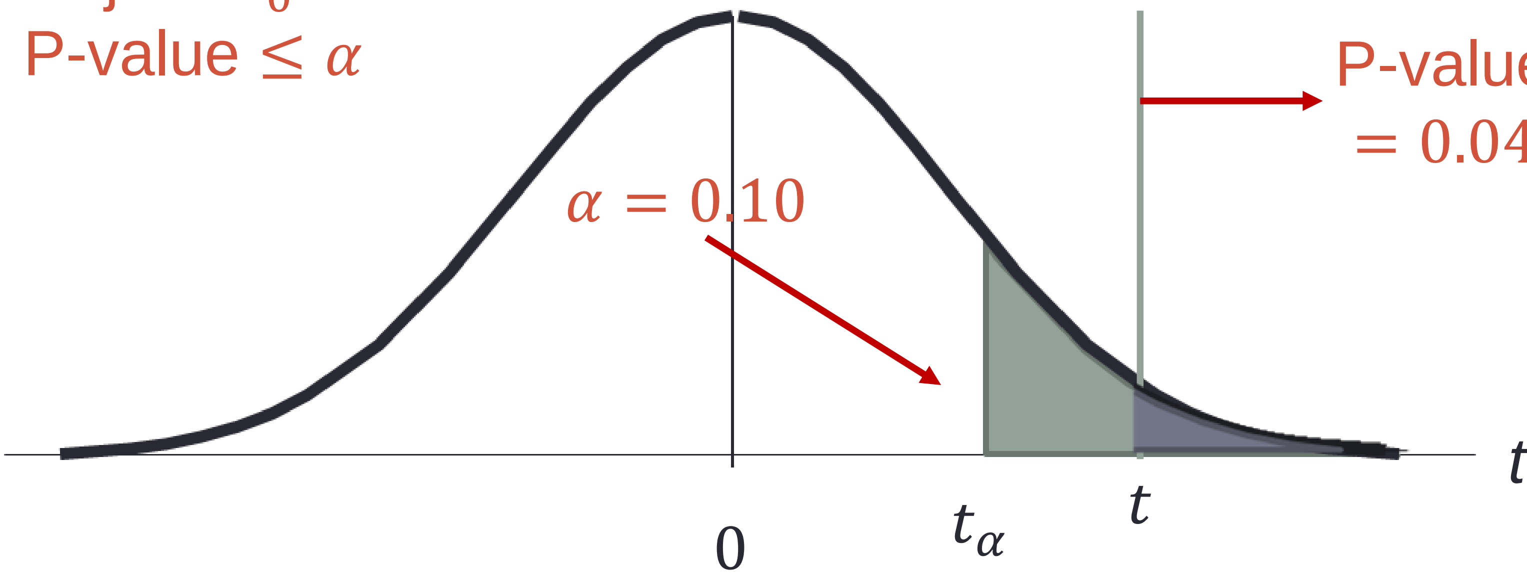
Lower-Tailed Test with P-value



Upper-Tailed Test with Critical Value

$H_a: \mu > \mu_0$

Reject H_0 since
P-value $\leq \alpha$



Errors in Hypothesis Test

- I have a coin that you believe is fair to start.
- To test if this coin is fair you ask me to flip the coin repeatedly and record the results.

Flip Number	Result	Probability
1	Heads	0.50
2	Heads	0.25
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- No longer believe coin is fair – but could it be?

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- No longer believe coin is fair – but could it be? **YES!**

Errors in Hypothesis Test

- Hypothesis tests depend on sample data.
- Therefore, hypothesis tests may be wrong!
- There are two types of errors in hypothesis testing – **Type I and Type II errors**.

Type I Error

- A **Type I error** is rejecting the null hypothesis when the null hypothesis was actually true.
- In other words, you have a false rejection.
- The probability of making a Type I error in a hypothesis test is called the **significance level**.
- Most hypothesis tests are referred to as **significance tests** because they only control the Type I error.

Type II Error

- A **Type II error** is “accepting” the null hypothesis when the null hypothesis was actually false.
- In other words, you have falsely accepted.
- The probability of NOT making a Type II error in a hypothesis test is called the **power**. Power of a test = probability of rejecting the null hypothesis when null hypothesis is false.
- Difficult to control the Type II error.
- Can only control for Type I or Type II at a time.

Type I vs Type II Errors

TRUTH

		H_0 True	H_0 False
CHOICE	Accept H_0	Correct	Type II
	Reject H_0	Type I	Correct

Significance Level, α

- Defines the unlikely values of the sample statistic **if the null hypothesis is true**.
- This area is typically called the **rejection region** of the sampling distribution.
- Selected before the hypothesis test is even run!
- Typical values are 0.01, 0.05, 0.10.

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THIS IS CHANGING!

Hypothesis Test Process

1. Develop your Hypothesis Statements (H_0 and H_a)
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3. What is probability this happens? (P-value)
4. Decision About Null Hypothesis
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TEST FOR MEANS

One Tailed Test Using P-Value Approach

Example for One-Tail Hypothesis Test

- You work for a business school as an analyst.
- The dean of the business school just went on record saying that students who just graduated **average** less than $3000 per month in salary.
- With a significance level of 0.05, conduct a hypothesis test on this claim.

Example for One-Tail Hypothesis Test

1. $H_0: \mu \geq \$3000$

   $H_a: \mu < \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7, n = 12$

$$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

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P-VALUE APPROACH!

Example for One-Tail Hypothesis Test

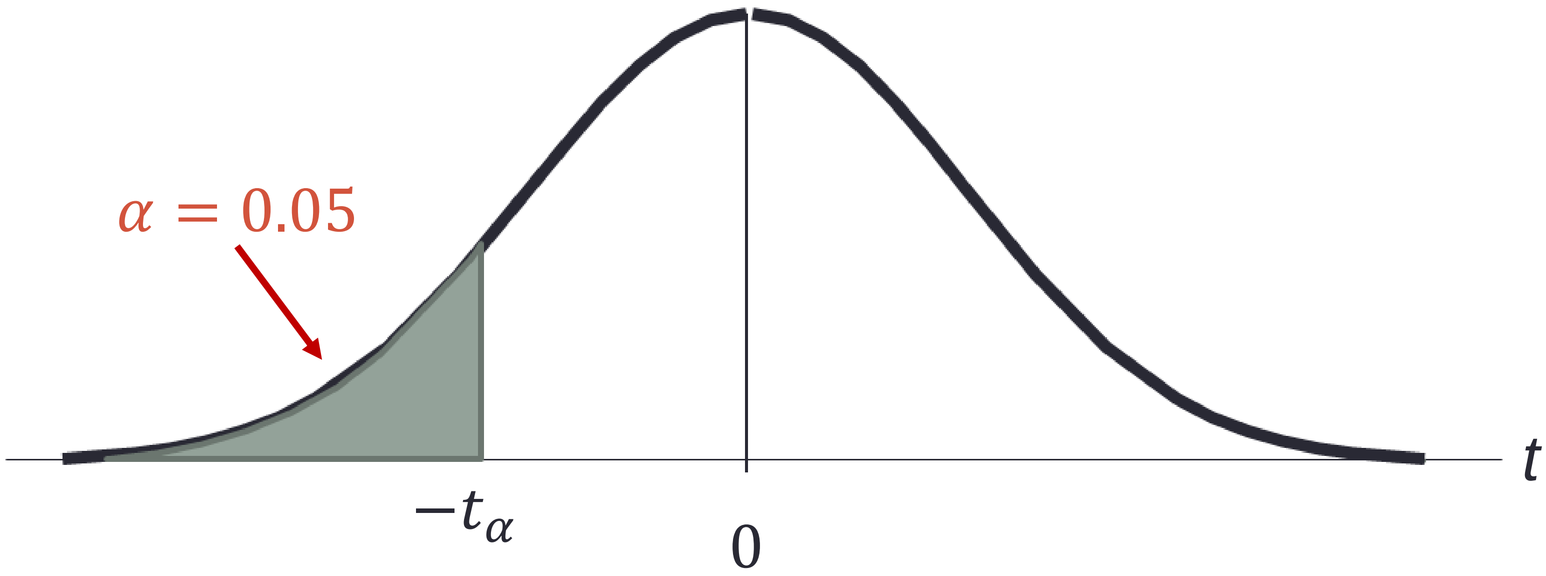
1. $H_0: \mu \geq \$3000$

   $H_a: \mu < \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7$

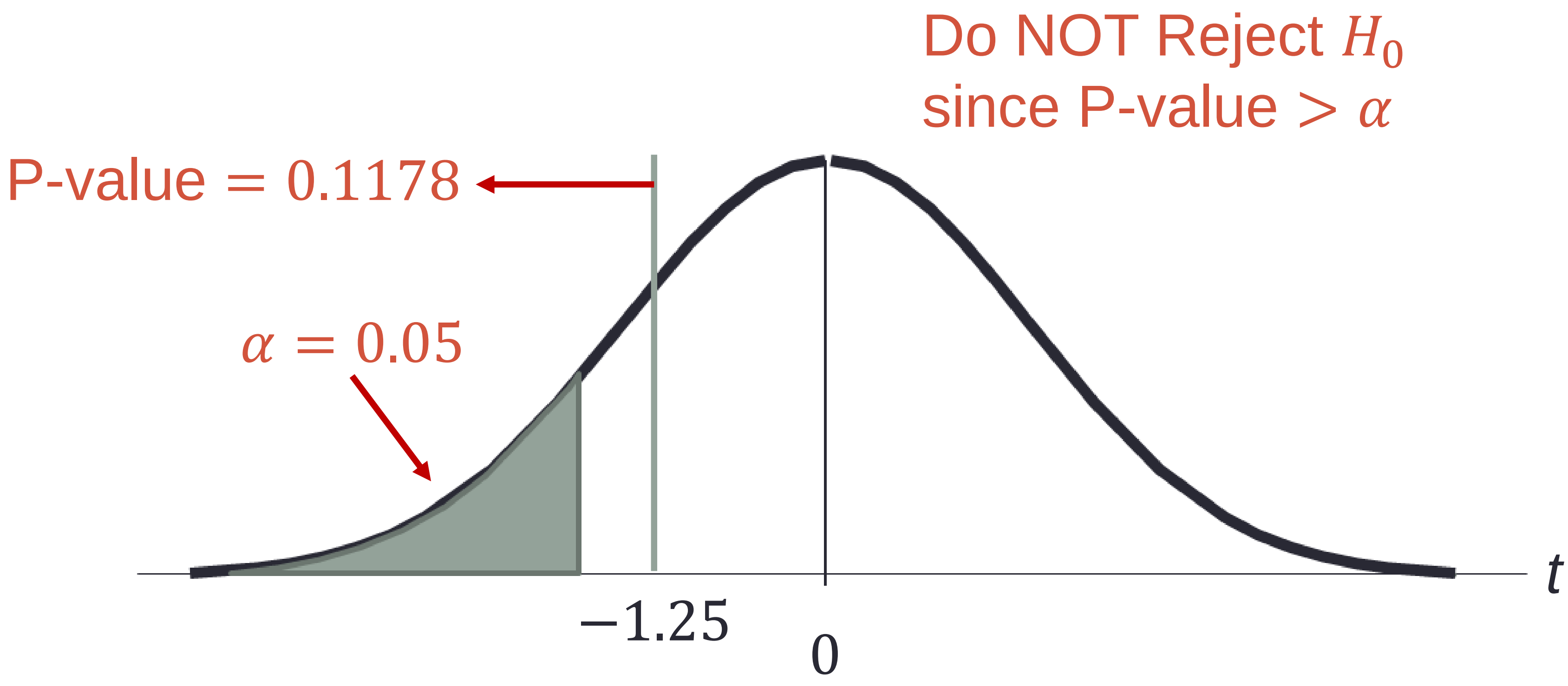
$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

3. P-Value = 0.1178

Lower-Tailed Test with P-value



Lower-Tailed Test with P-value



Example for One-Tail Hypothesis Test

1. $H_0: \mu \geq \$3000$

   $H_a: \mu < \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

3. Significance level $\alpha = 0.05$.; P-Value = 0.1178
4. Do NOT Reject H_0

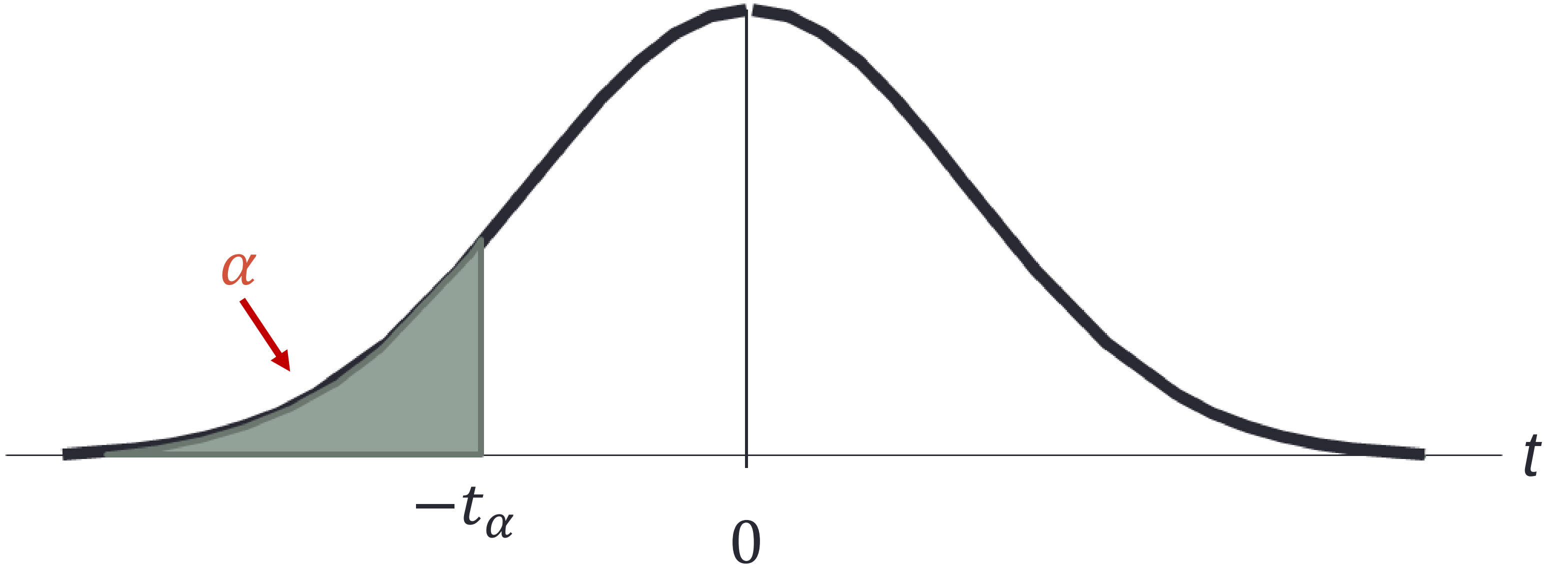
CRITICAL VALUE APPROACH

OPTIONAL SELF STUDY

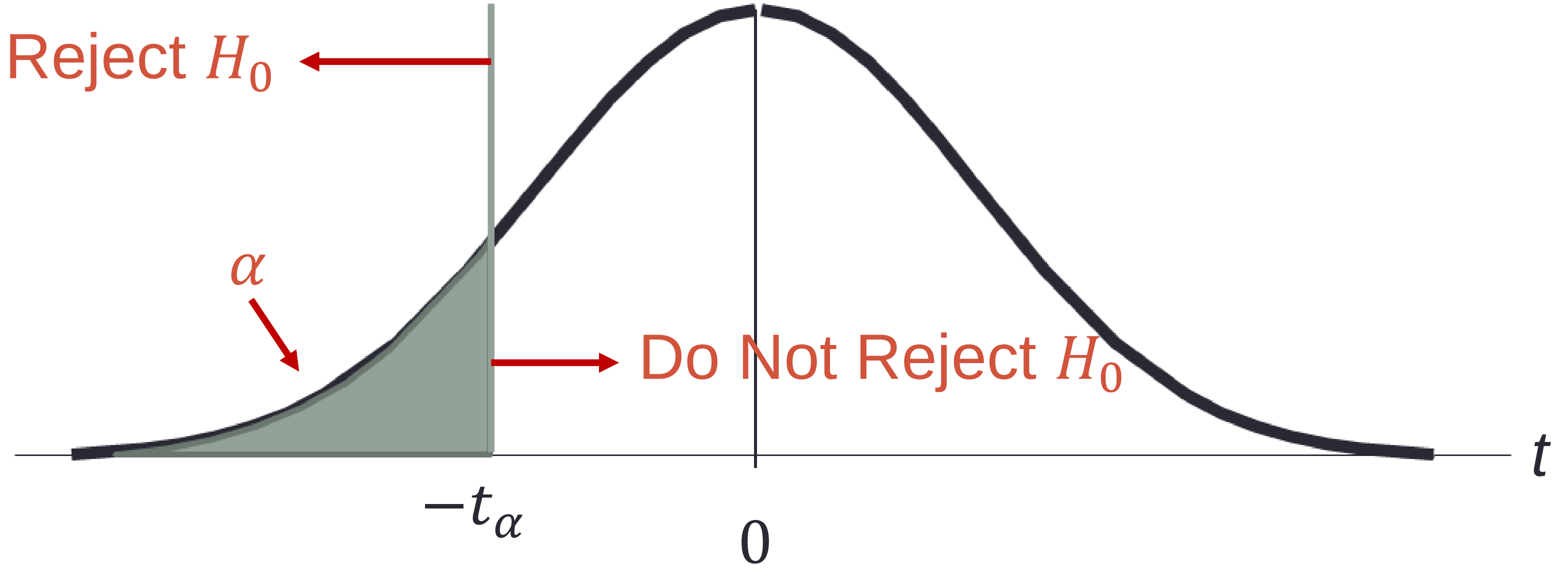
Critical Value Approach

- We can use the t-distribution with $n - 1$ degrees of freedom to find the t-value with an area of α in the lower (or upper) tail of the distribution.
- The value of the test statistic that established the boundary of the rejection region is called the **critical value** for the test.
- The rejection rule is the following:
  - Lower Tail: Reject H_0 if $t \leq -t_\alpha$
  - Upper Tail: Reject H_0 if $t \geq t_\alpha$

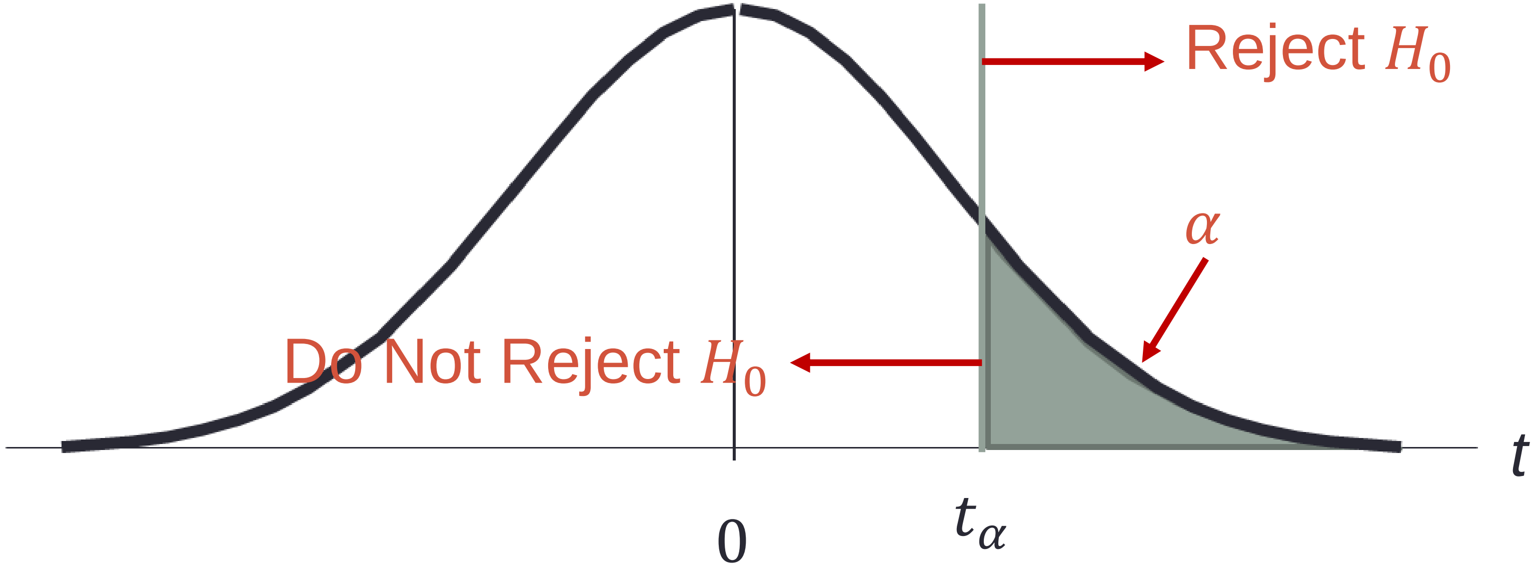
Lower-Tailed Test with Critical Value



Lower-Tailed Test with Critical Value



Upper-Tailed Test with Critical Value



How to Find t_α

One-Tail	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	.0005
Two-Tail	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
.	.	.	.	.	.	.	.	.	.	.
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
.	.	.	.	.	.	.	.	.	.	.

How to Find t_α

One-Tail	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	.0005
Two-Tail	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
.	.	.	.	.		.	.	.	.	.
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
.	.	.	.	.	.	.	.	.	.	.

Example for One-Tail Hypothesis Test

- You work for a business school as an analyst.
- The dean of the business school just went on record saying that students who just graduated **average** less than $3000 per month in salary.
- With a significance level of 0.05, conduct a hypothesis test on this claim (n=12).

How to Find t_α

One-Tail	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	.0005
Two-Tail	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
.	.	.	.	.	.	.	.	.	.	.
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
.	.	.	.	.	.	.	.	.	.	.

How to Find t_α

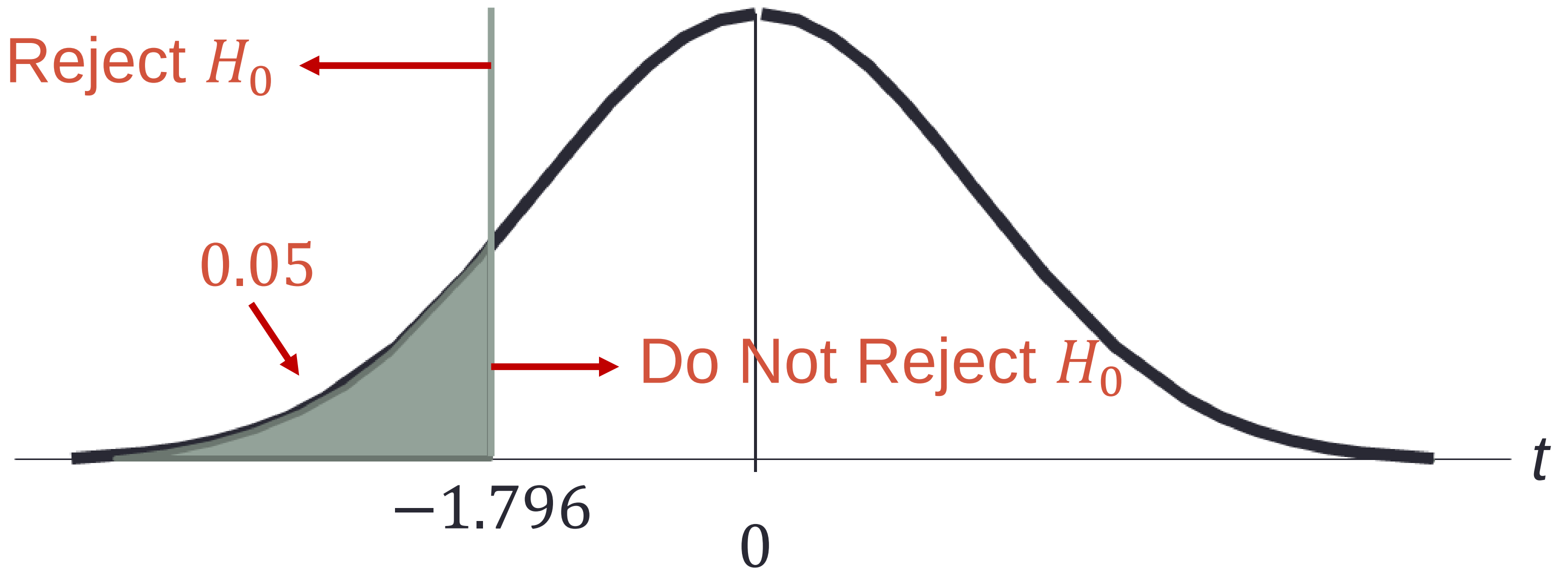
One-Tail	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	.0005
Two-Tail	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
.	.	.	.	.	.	.	.	.	.	.
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
.	.	.	.	.	.	.	.	.	.	.

Example for One-Tail Hypothesis Test

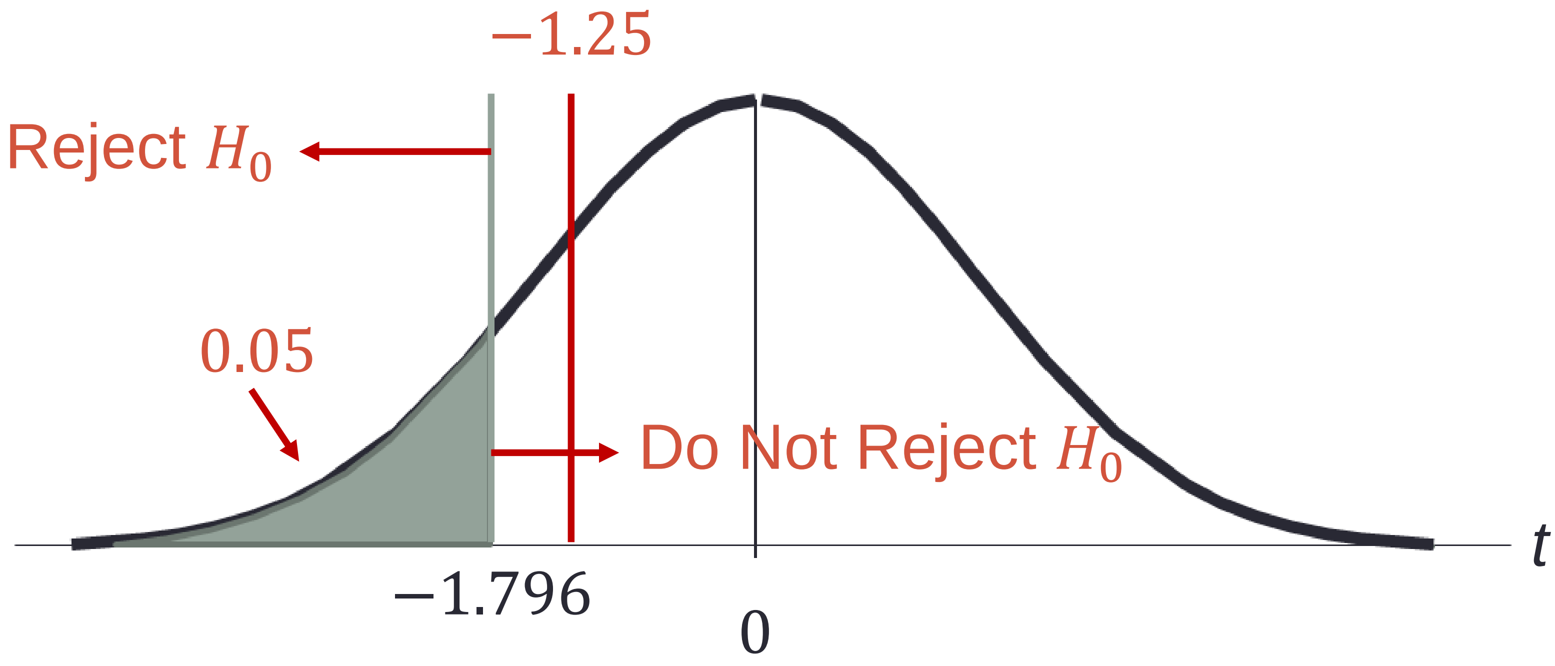
1. $H_0: \mu \geq \$3000$
   $H_a: \mu < \$3000$
2. Significance level $\alpha = 0.05$: Critical Value = -1.796
   2. So, Reject the null hypothesis if t$\leq$ -1.796
3. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

Lower-Tailed Test with Critical Value



Lower-Tailed Test with Critical Value



Example for One-Tail Hypothesis Test

1. $H_0: \mu \geq \$3000$

   $H_a: \mu < \$3000$
2. Significance level $\alpha = 0.05$. Critical Value = -1.796.
3. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

4. Do NOT Reject H_0

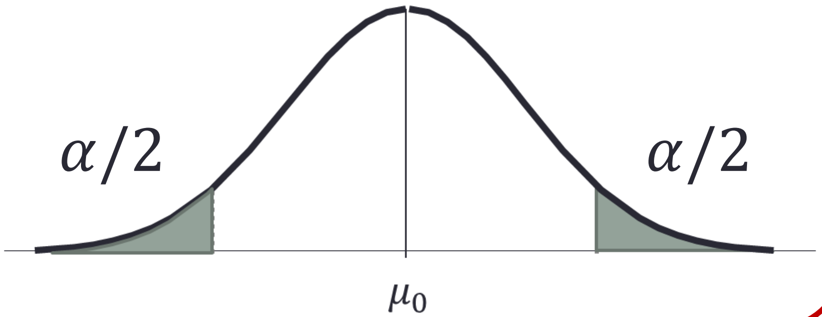
TEST FOR MEANS

Two Tailed Test Using P-Value Approach

Rejection Region

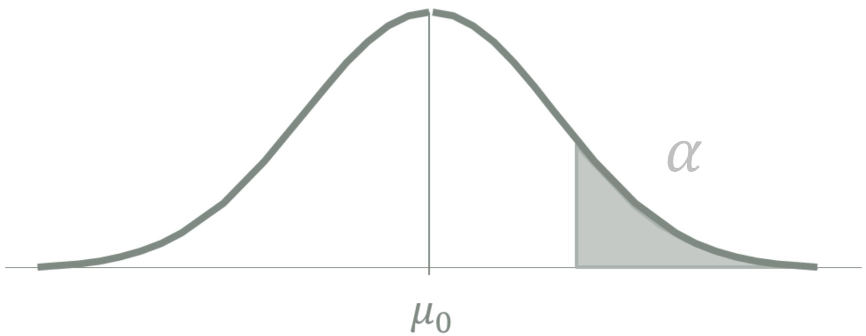
Two-Sided

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

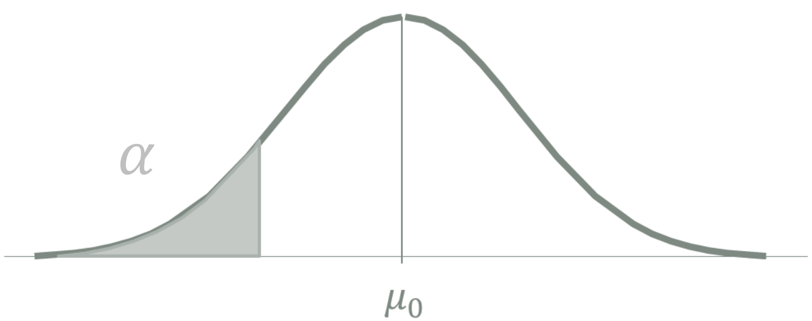


One-Sided

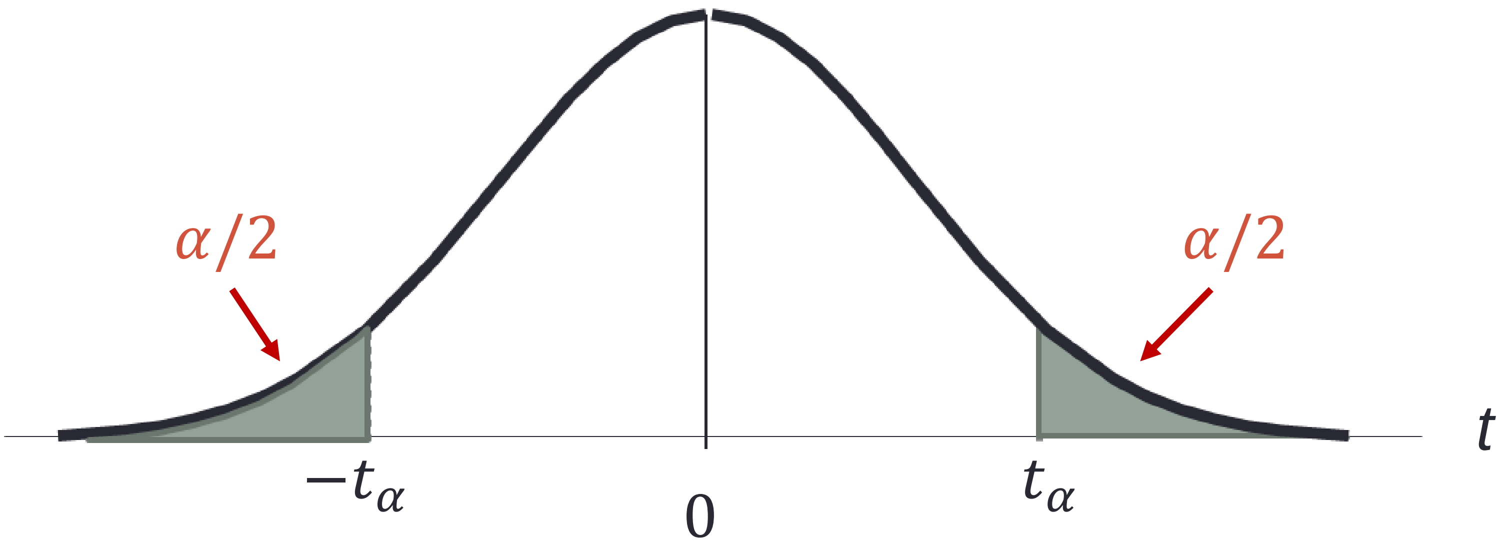
$$H_0: \mu \leq \mu_0$$
$$H_a: \mu > \mu_0$$



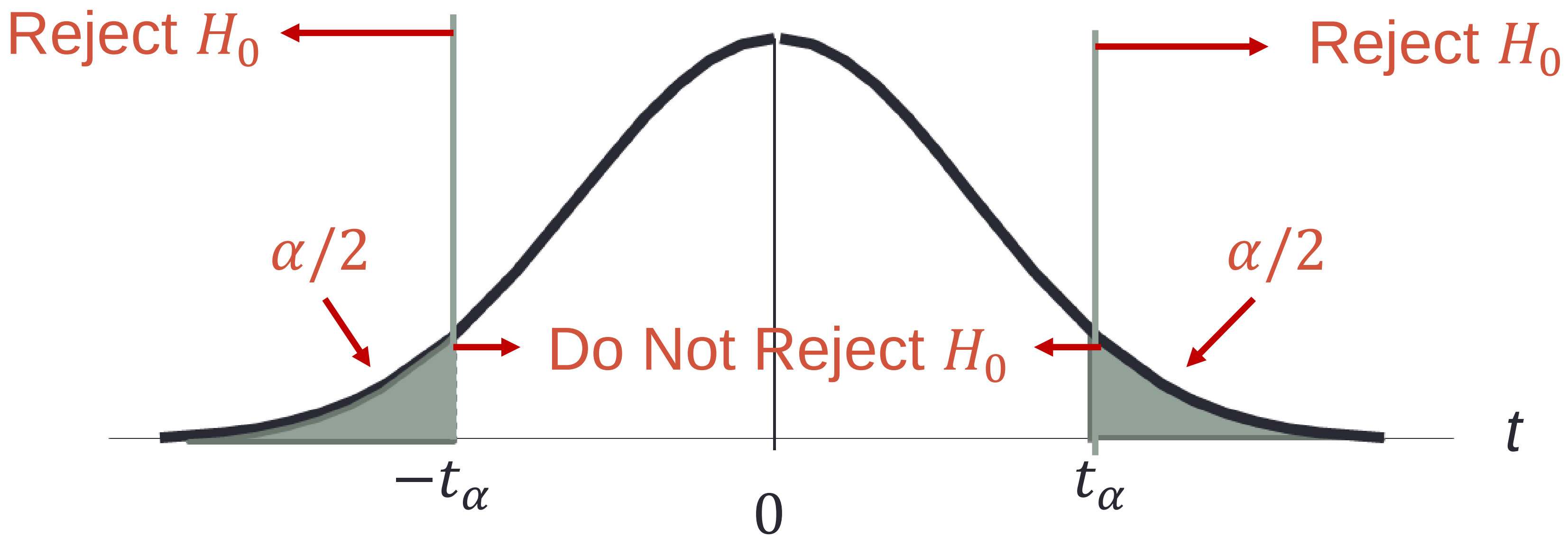
$$H_0: \mu \geq \mu_0$$
$$H_a: \mu < \mu_0$$



Two-Sided Test with Critical Value

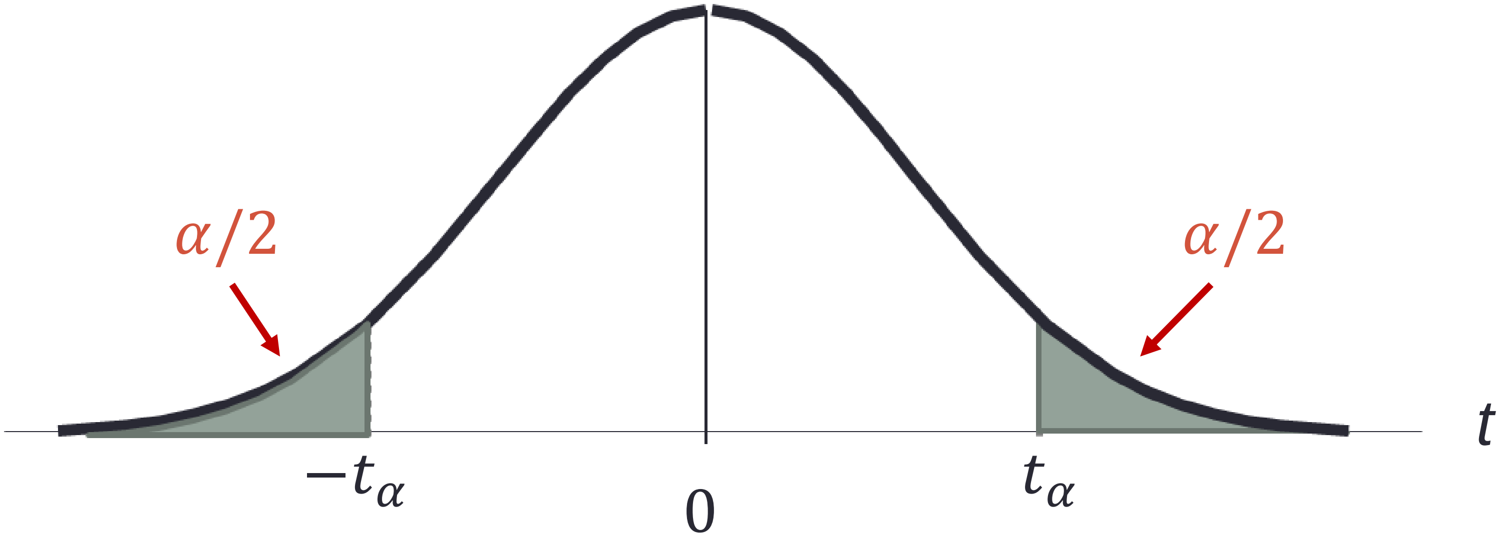


Two-Sided Test with Critical Value

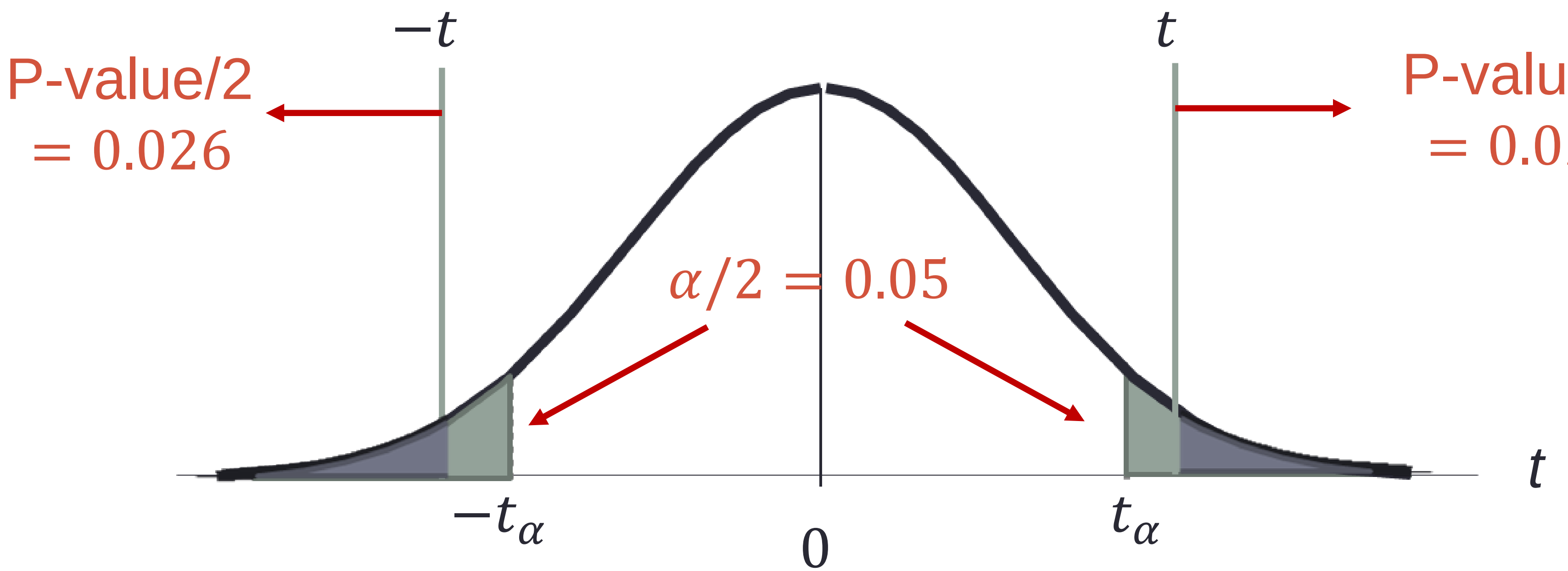


- The rejection rule is the following:
  - Reject H_0 if $t \leq -t_\alpha$ or $t \geq t_\alpha$

Two-Sided Test with P-value

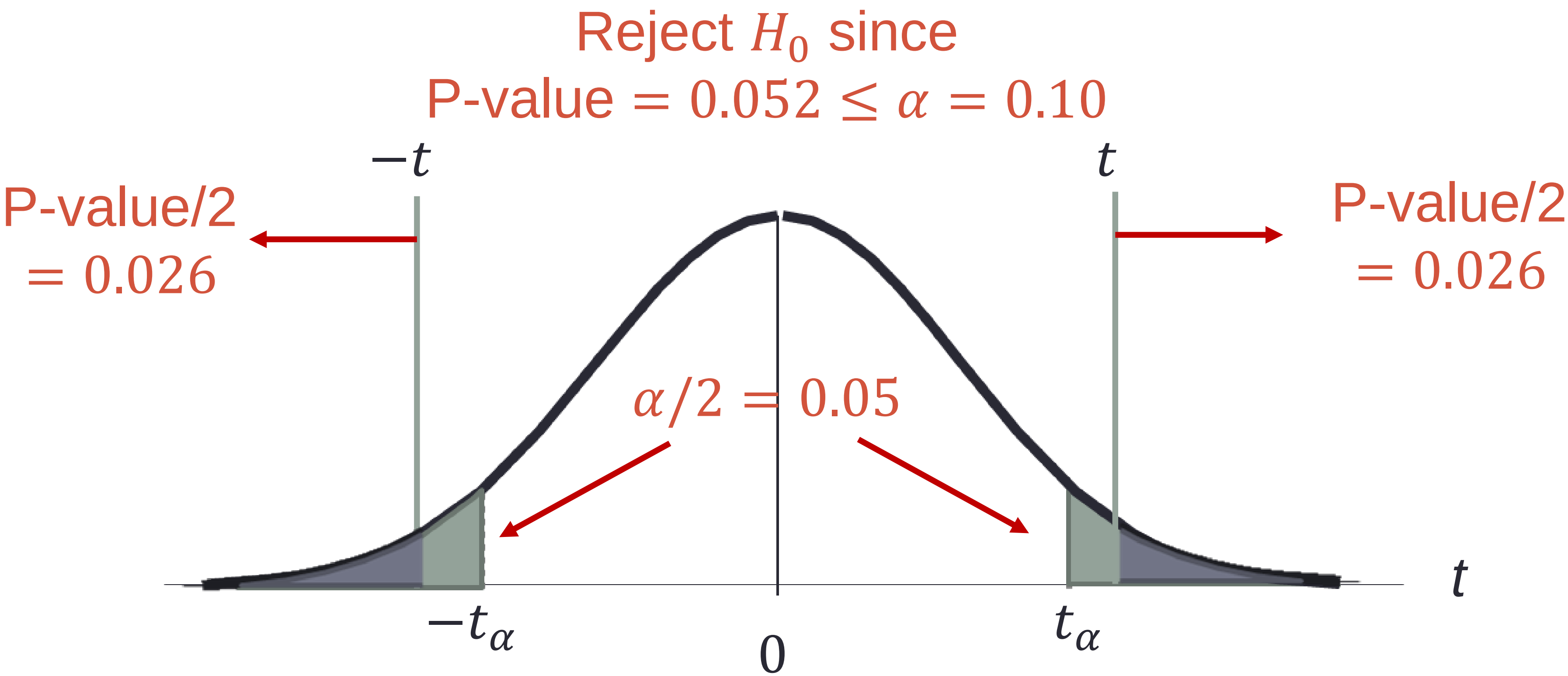


Two-Sided Test with P-value



- The rejection rule is the following:
  - Reject H_0 if p-value $\leq \alpha$

Two-Sided Test with P-value



- The rejection rule is the following:
  - Reject H_0 if p-value $\leq \alpha$

Example for Two-Tail Hypothesis Test

- You work for a business school as an analyst.
- The dean of the business school just went on record saying that students who just graduated do not **average** $3000 per month in salary.
- With a significance level of 0.05, conduct a hypothesis test on this claim.

Example for Two-Tail Hypothesis Test

1. $H_0: \mu = \$3000$

   $H_a: \mu \neq \$3000$

2. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

Example for Two-Tail Hypothesis Test

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   $H_a: \mu \neq \$3000$

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$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

P-VALUE APPROACH!

Example for Two-Tail Hypothesis Test

1. $H_0: \mu = \$3000$

   $H_a: \mu \neq \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

3. Significance level $\alpha = 0.05$.; P-Value = 0.2356

Example for Two-Tail Hypothesis Test

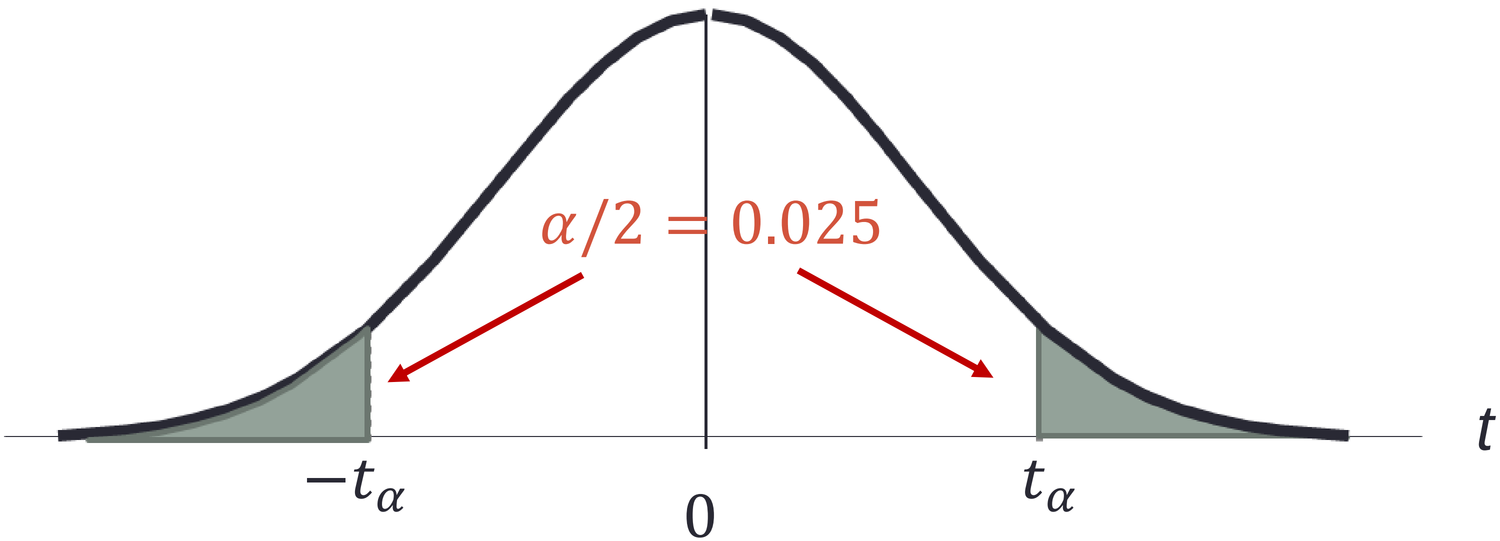
1. $H_0: \mu = \$3000$

   $H_a: \mu \neq \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7$

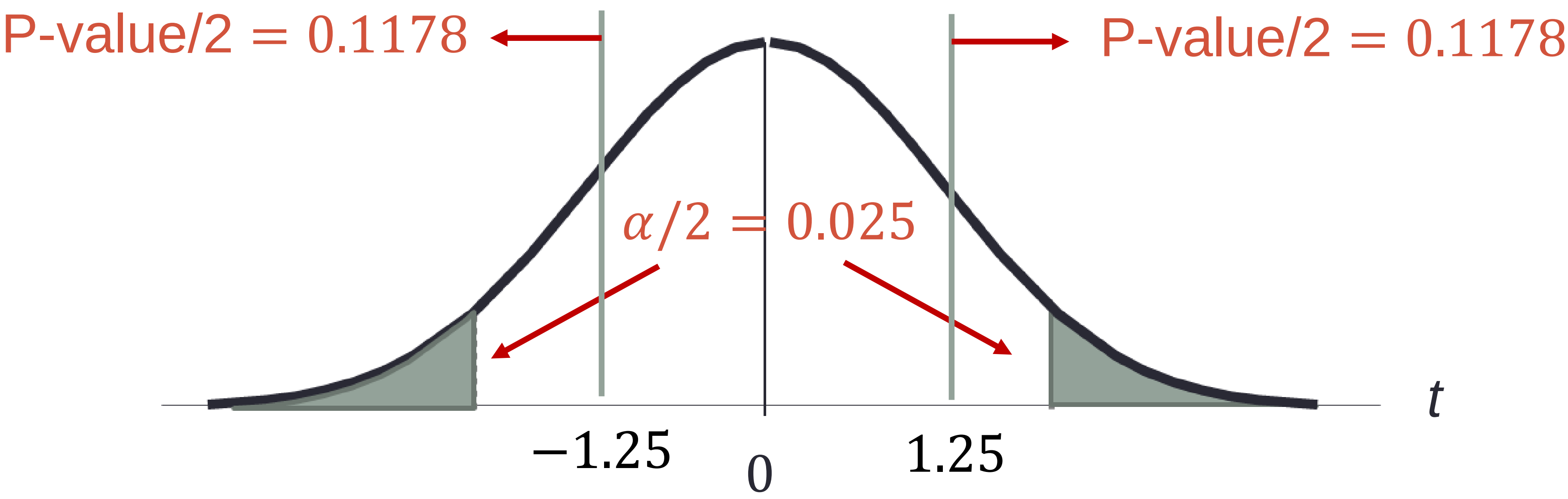
$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

3. Significance level $\alpha = 0.05$; P-Value = 0.2356 (Twice that of One-Sided P-value)

Two-Tailed Test with P-value



Two-Tailed Test with P-value



Do NOT Reject H_0

since P-value $= 0.2356 > \alpha = 0.05$

Example for Two-Tail Hypothesis Test

1. $H_0: \mu = \$3000$

   $H_a: \mu \neq \$3000$
2. Sample data: $\bar{x} = \$2940, s = 165.7$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{2940 - 3000}{\left(\frac{165.7}{\sqrt{12}}\right)} = -1.25$$

3. Significance level $\alpha = 0.05$; P-Value = 0.2356
4. Do NOT Reject H_0

TEST FOR PROPORTIONS

One Tailed Test Using P-Value Approach

Example for One-Tail Hypothesis Test

- You are interested in hair color and eye color across 2 different regions of the country.
- You want to know if less than 32% of people have blue eyes.
- You have a sample of 762 people.

Example for One-Tail Hypothesis Test

1. $H_0 : p \geq .32$

   $H_a : p < .32$

2. Sample data: $\hat{p} = 0.2913$

$$z = \frac{\hat{p} - p_0}{\left(\sqrt{\frac{p_0(1 - p_0)}{n}}\right)} = \frac{0.2913 - 0.32}{\left(\sqrt{\frac{0.32(1 - 0.32)}{762}}\right)} = -1.70$$

Example for One-Tail Hypothesis Test

1. $H_0 : p \geq .32$

   $H_a : p < .32$

2. Sample data: $\hat{p} = 0.2913$

$$z = \frac{\hat{p} - p_0}{\left(\sqrt{\frac{p_0(1 - p_0)}{n}}\right)} = \frac{0.2913 - 0.32}{\left(\sqrt{\frac{0.32(1 - 0.32)}{762}}\right)} = -1.70$$

P-VALUE APPROACH!

Example for One-Tail Hypothesis Test

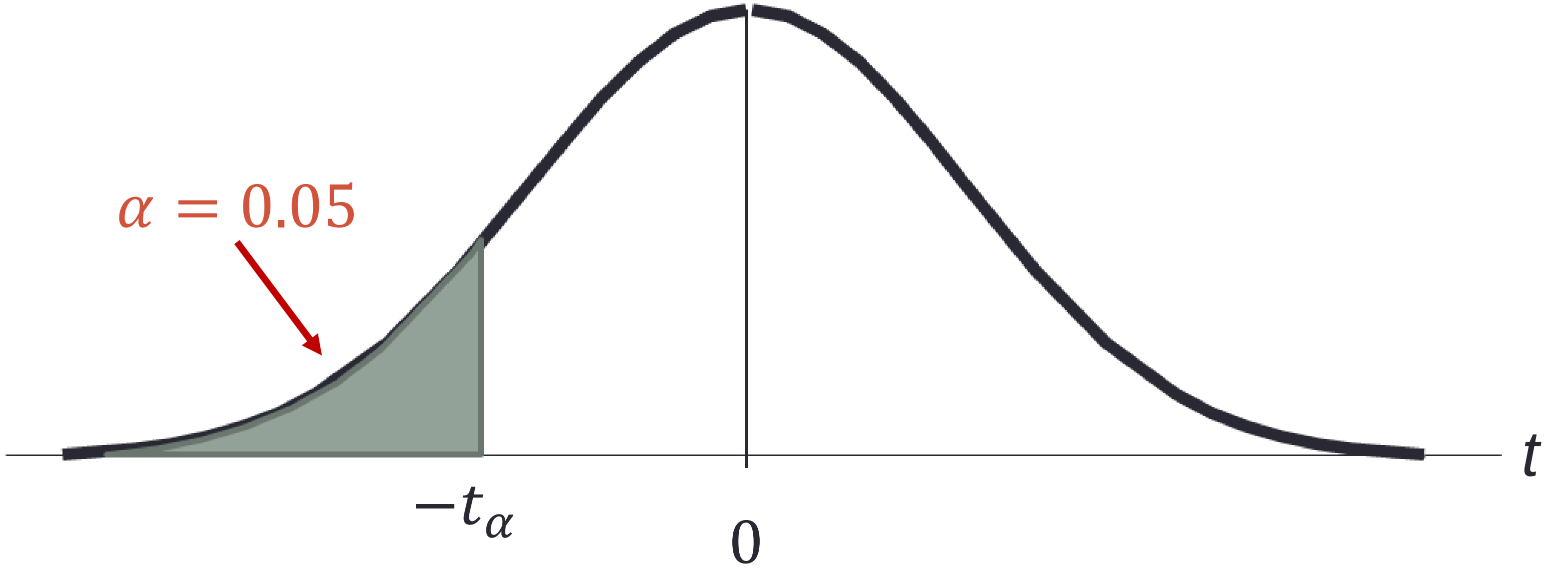
1. $H_0: p \geq .32$

   $H_a: p < .32$
2. Sample data: $\hat{p} = 0.2913$

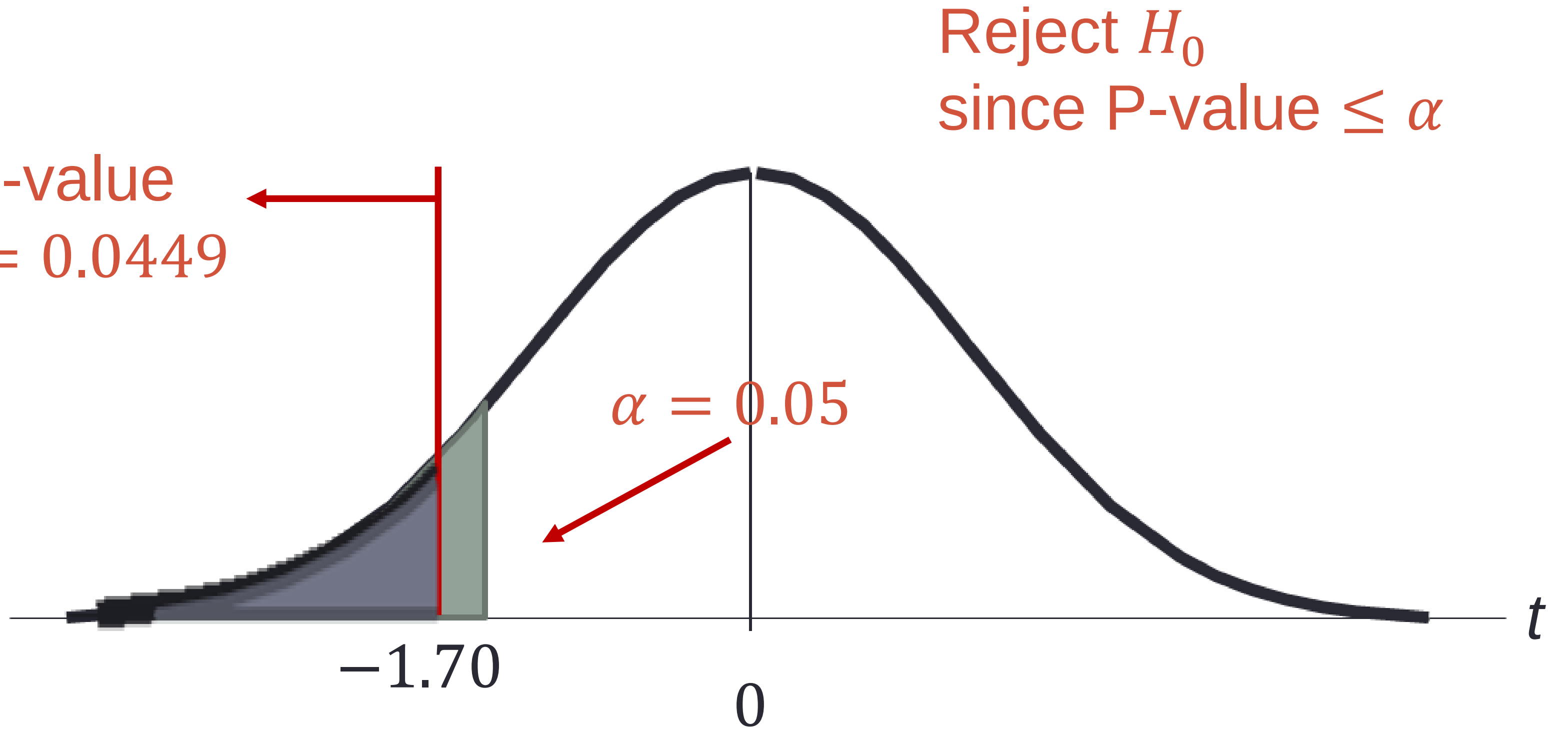
$$z = \frac{\hat{p} - p_0}{\left(\sqrt{\frac{p_0(1-p_0)}{n}}\right)} = \frac{0.2913 - 0.32}{\left(\sqrt{\frac{0.32(1-0.32)}{762}}\right)} = -1.70$$

3. Significance level $\alpha = 0.05$; P-value = 0.0449

Lower-Tailed Test with P-value



Lower-Tailed Test with P-value



Example for One-Tail Hypothesis Test

1. $H_0\!: p \geq .32$

   $H_a\!: p < .32$
2. Sample data: $\hat{p} = 0.2913$

$$z = \frac{\hat{p} - p_0}{\left(\sqrt{\frac{p_0(1 - p_0)}{n}}\right)} = \frac{0.2913 - 0.32}{\left(\sqrt{\frac{0.32(1 - 0.32)}{762}}\right)} = -1.70$$

3. Significance level $\alpha = 0.05$; P-value = 0.0449
4. Reject H_0

HYPOTHESIS TESTS VS. CONFIDENCE INTERVALS

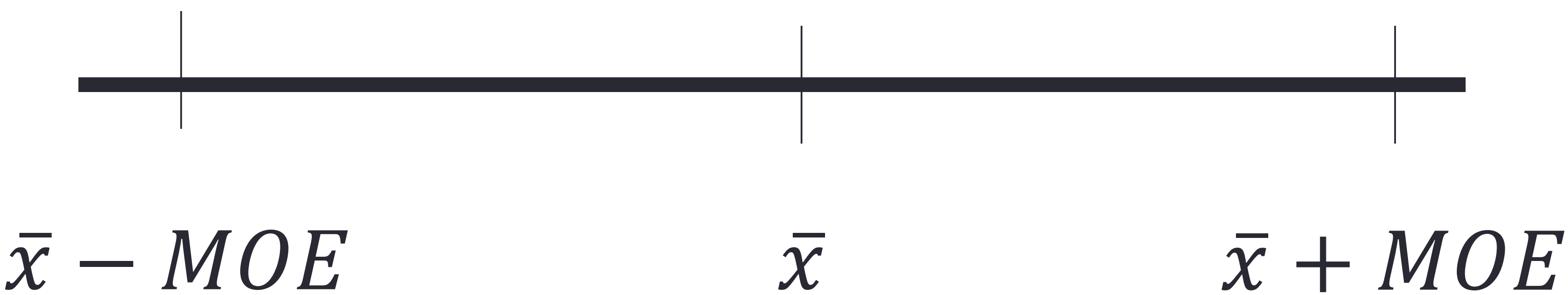
Same Test?

- Under certain conditions, hypothesis tests and confidence intervals are conducting the same test.
- It is best to understand this concept visually

How do Confidence Intervals and Hypothesis test relate to each other

- Only TWO-SIDED hypothesis tests can relate to confidence intervals!
- Need to have the same "error rate" (i.e. α needs to be the same!!)
- 95% Confidence Interval $\Longleftrightarrow$ Hypothesis Test with $\alpha = 0.05$
- 99% Confidence Interval $\Longleftrightarrow$ Hypothesis Test with $\alpha = 0.01$

Confidence Interval for μ



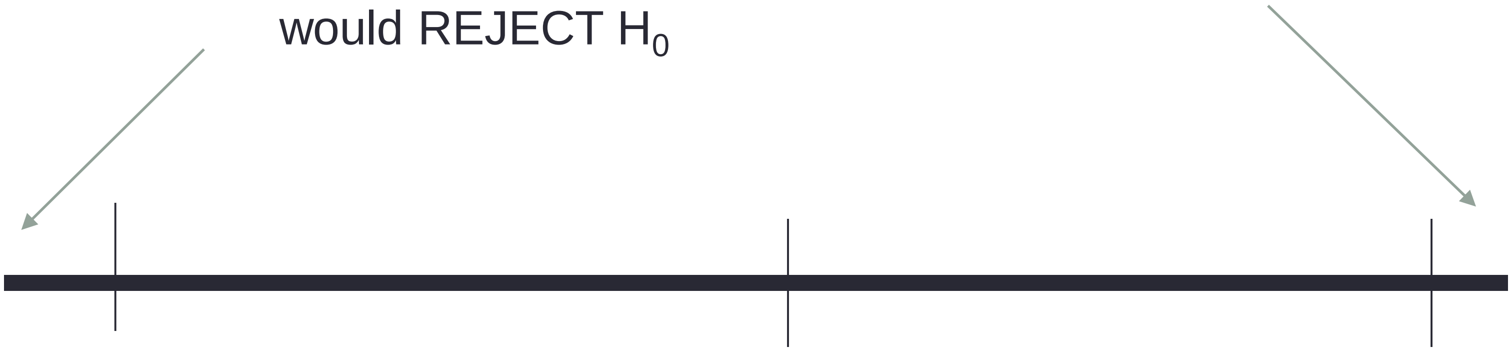
Confidence Interval for μ

If μ_0 falls INSIDE confidence interval, you would FAIL TO REJECT H_0

$\bar{x} - MOE$ $\qquad$ $\bar{x}$ $\qquad$ $\bar{x} + MOE$

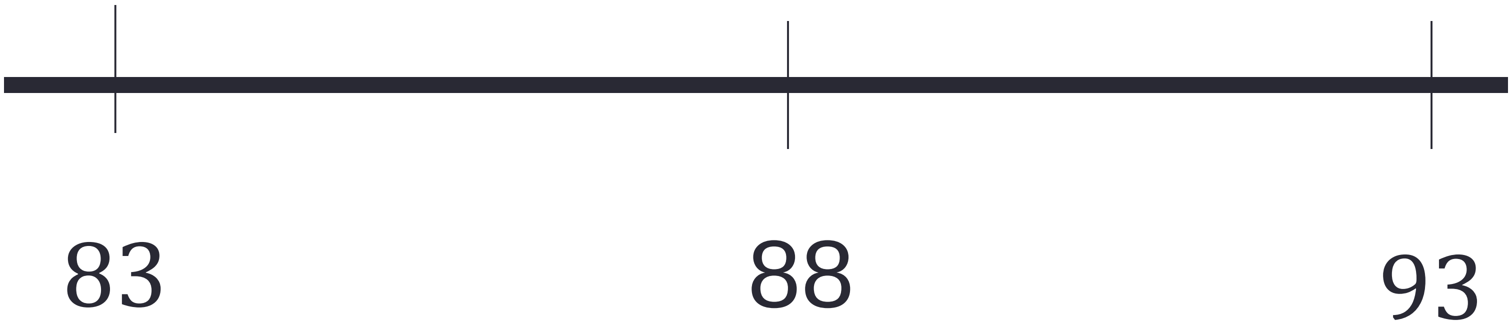
Confidence Interval for μ

If μ_0 falls OUTSIDE confidence interval, you would REJECT H_0



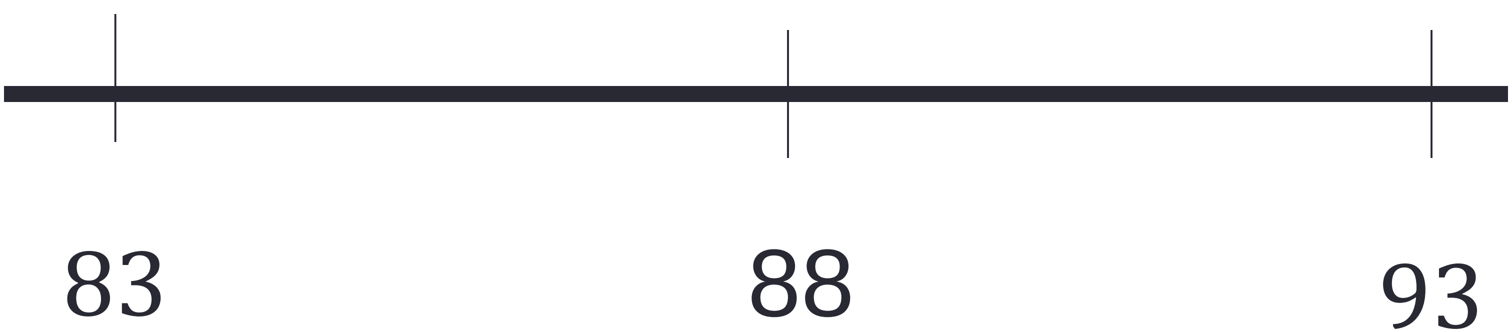
$\bar{x} - MOE$ $\qquad\qquad$ $\bar{x}$ $\qquad\qquad$ $\bar{x} + MOE$

Example: 95% Confidence Interval



Example: 95% Confidence Interval

H0: $\mu = 90$
HA: $\mu \neq 90$
$\alpha = 0.05$

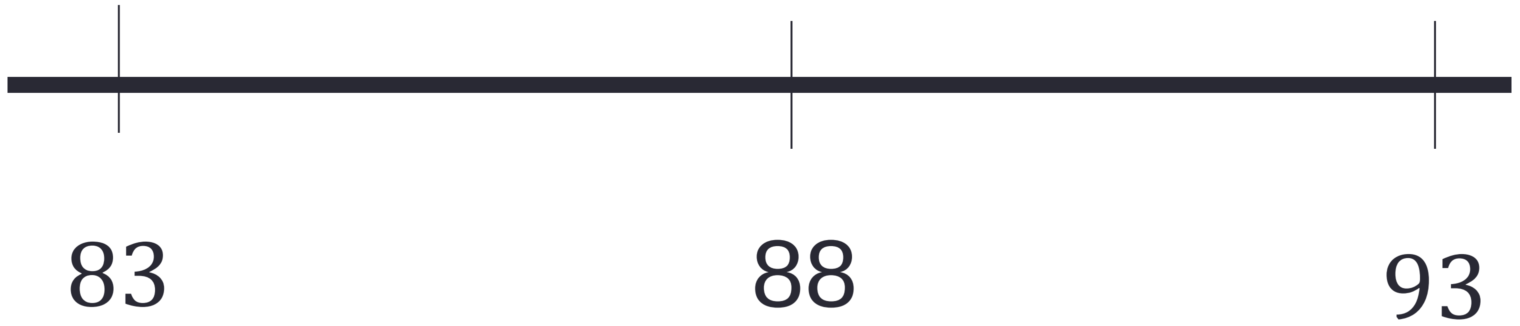


Example: 95% Confidence Interval

H0: $\mu = 90$
HA: $\mu \neq 90$
$\alpha = 0.05$

Example: 95% Confidence Interval

H0: $\mu = 80$
HA: $\mu \neq 80$
$\alpha = 0.05$



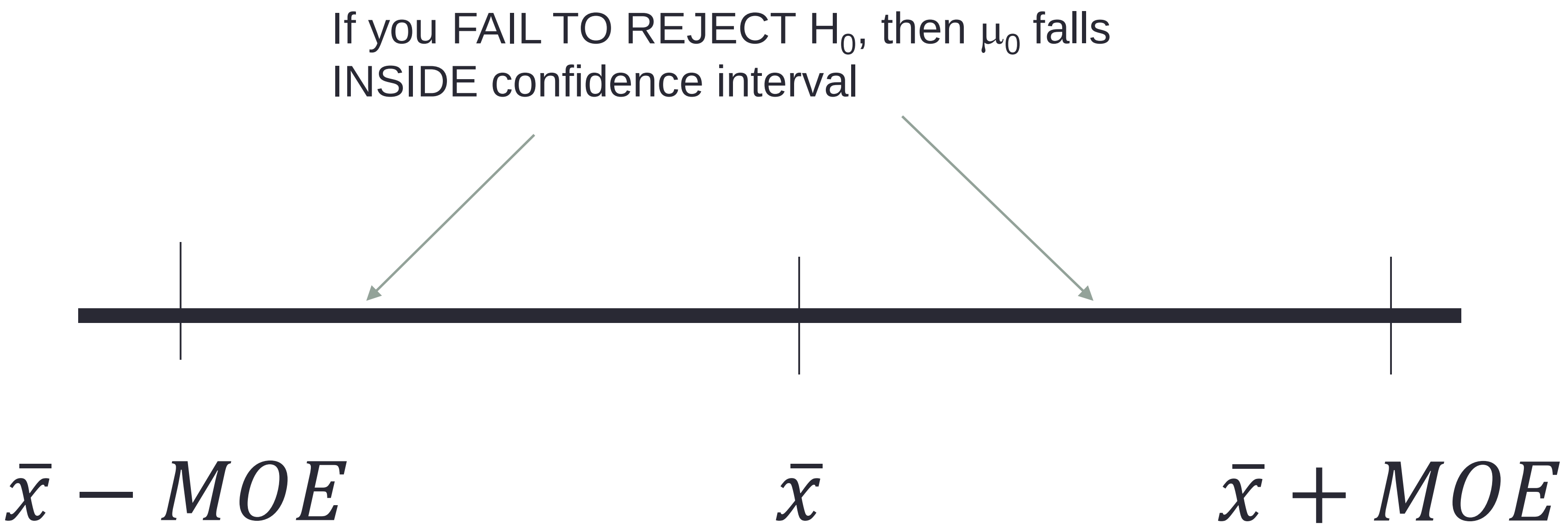
Example: 95% Confidence Interval

H0: $\mu = 80$
HA: $\mu \neq 80$
$\alpha = 0.05$

Hypothesis Test to Confidence Intervals

- Still need same conditions:
  - TWO-SIDED Hypothesis Test and “error” is the same!
- IF you reject the null hypothesis, then the value of μ_0 is OUTSIDE confidence interval.
- IF you fail to reject the null hypothesis then the value of μ_0 is INSIDE confidence interval.

Confidence Interval for μ



Confidence Interval for μ

If you REJECT H_0, then μ_0 falls OUTSIDE confidence interval

$\bar{x} - MOE$ $\qquad$ $\bar{x}$ $\qquad$ $\bar{x} + MOE$

Same Test?

- Under certain conditions, hypothesis tests and confidence intervals are conducting the same test.
- It is best to understand this concept visually through distributions.
- Conditions:
  1. The hypothesis test is two-sided
  2. $C = (1 - \alpha)$

Significance Level

Confidence Level