Low Probability = Rare/ unusual occurrences.

Event - A collection of outcomes of a procedure. Head.

Simple Event - A single outcome. {H}

Sample Space - All the simple events. Every possible outcome. {H, T}

Probability - The likelihood of an event occuring. P(A)

You can't flip a coin enough times to get "exact" probability.

3 Types: -

① Observed Probability - Probability that is estimated based on an observation.

$$P(A) = \frac{\#\text{ of times event occurred}}{\#\text{ of observation}}$$ = what actually happens.

② Classical probability - Probability based on the chance of an event occuring. (each simple chance must have an equal chance of occuring)

$$P(A) = \frac{\#\text{ of ways "A" could occur}}{\#\text{ of obs}}$$ = what could happen.

③ Subjective Probability : Educated guess, not usually used.
eg:- chances of getting hit by a meteor.
eg:- a bird will poop on your car today.

$P = 0 \rightarrow$ impossible event
$P = 1 \rightarrow$ certain event.

Complementary events = mutually exclusive events.

Complement of event A = $\bar{A}$

Die 5 $P(5) = 1/6$ $P(A) + P(\bar{A}) = 1$.

$P(\bar{5}) = 5/6$

Addition Rule:

Compound event: An event which joins two or more simple events.
ex:- Probability of rolling a 1 or 5 "OR". one, or the other or BOTH.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \text{ or } B) = P(A) + P(B) - P(\text{Both})$ in a single trial.

Ex:-

	Didn't do it	Did it
Guilty	False 11 Positive	True 72 Positive
Innocent	True 85 Negative	False 9 Negative

TRUE/ FALSE → whether you predicted is right/ wrong True/False

POSITIVE/ NEGATIVE → whether you actually did it (+ve, -ve)

How many people are "Guilty" or "Did it"?

Guilty = 11 + 72

Did it = 72 + 9

NOT mutually exclusive.

Added two times - goes above our population

Guilty "or" did it = 11 + 72 + 9 = 92

$$P\left(\text{Guilty or did it}\right) = 92/177$$

Confusion Matrix in Machine Learning:
https://lucdemortier.github.io/articles/16/PerformanceMetrics

Disjoint Events :- Events that are mutually exclusive (They cannot happen at the same time)

Multiplication Rule: prob. of 'A' happening and then 'B' happening. (not a single trial, successive trials)

$P(A \text{ and } B) = P(A) \times P(B)$

$P(A \text{ and then } B)$

	Didn't do it	Did it
Guilty	False 11 Positive	True 72 Positive
Innocent	True 85 Negative	False 9 Negative

Prob. of selecting "Guilty" and then "Not Guilty".

$P(\text{Guilty}) = \frac{83}{177}$.

and then

$P(\text{Not Guilty}) = \frac{94}{176}$

done without replacement. 177 becomes 176.

One event affected the other event

Conditional Probability: The prob. of an event occuring given that some other event has already occurred.

$P(B|A)$ = Prob. of event B occurring given that event 'A' has already occurred.

These are dependent events.

Independent events :- The occurrence of one event does not affect the other.

NOTE: If A and B are independent,

$P(B|A) = P(B)$ ⇐ It doesn't matter if 'A' is occurred, hence its only the prob. of B.

eg:- Roll a die

1. $P(2|3) = P(2) = 1/6$. {Because independent events}

2. Drawing Cards.

w/o replacement $P(9|9) = 4/51$ ⇐ Dependent with replacement $P(Q|Q) = P(Q) = 4/52$ ← Independent

$P(A \text{ and then } B) = P(A) \cdot P(B)$ ← works for independent events.

$= P(A) \cdot P(B|A)$ ← for dependent events.

need to make a separate section on Bayes theorem

* Bayes theorem - 3B1B video.

Prob. of "At least one" :- one or more,
$P(\text{At least one}) = 1 - P(\text{none})$

Fundamental rules of counting says that if an event 'A' can occur 'm' times and event 'B' can occur 'n' times, then together they occur $m \times n$ ways.

* $0! = 1$

Factorial (!) Arrang 5 fruits in order = 5! unique ways.

Permutations - Items have to be unique.

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

n = Total items
r = # of items to be arranged out of 'n'.

← order of items matter.

for non-distinct items,

$$\frac{n!}{n_1! \, n_2! \, n_3! \ldots}$$

n = Total items
n_1, n_2, are the counts of non-distinct events.

Combinations - order doesn't matter.

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Discrete Probabilities
↪ countable, finite.

Vocab:

Random Variable - A variable, x, that has a value for each outcome of a procedure that is determined by chance.

Probability Distributions - A table that gives the probabilities for each value of a random variable.

Discrete Random Variable - A variable with a countable or finite # of values.

Continous Random Variable - A variable with infinite number of possible values. (usually a measurement)

Histograms from PDF :-

$0 \le P(x) \le 1$

$\Sigma P(x) = 1$

Horizontal - Values of random variable.
Vertical - Probability.
Same as frequency distribution.

Mean, Variance and Std. dev of a PDF -

Mean :-

$$\mu = \frac{\Sigma (x \cdot f)}{N}$$

$$= \Sigma \left[x \cdot \frac{f}{N} \right] \qquad \frac{f}{N} = p(x)$$

$$\mu = \Sigma \left[x \cdot P(x) \right] \leftarrow \text{mean. Expected value. } E[x].$$

Variance :- $\sigma^2 = \Sigma \left[x^2 \cdot P(x) \right] - \mu^2$

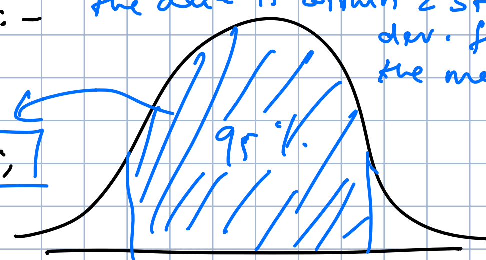
Std. dev -

Usual and Unusual :-

Values are unusual if they lie outside of :- $\mu + 2\sigma$, $\mu - 2\sigma$

If the prob. of an event is less than $\boxed{5\%}$, it is considered unusual.

According to the empirical rule, 95% of the data is within 2 std. dev. from the mean.



Ex :- Flip a coin 1000 times.

P(Exactly 501 heads) = 0.0252 ≤ 0.05. (unusual)

P(501 or more heads) = 0.487 ≥ 0.05 (usual)

Binomial Probability Distributions.

A prob. dist that has only Two outcomes. success & failure.

Rules :- Must be a fixed # of trials.
- trials must be independent.
- each trial has only two outcomes. success or failure.
- the prob. of success is the same.

n = # of trials.
$p \rightarrow$ The prob. of a successful outcome in a single trial
$q \rightarrow$ The prob. of failure...

$X \rightarrow$ The number of successes that occur in the trials.
$P(X) \rightarrow$ The prob. of getting "x" successes.

$$P(x) = \frac{n!}{(n-x)!\,x!} \leftarrow \text{similar to combination formula.}$$

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{(n-x)}$$
$$= {}^{n}C_{x}\, p^{x}\, (1-p)^{(n-x)}.$$

Mean, variance and std. dev of Binomial distribution,

1. Mean - The # of succes you expect to occur from your procedure.

$$\mu = n \cdot P(x)$$

2. variance - $\sigma^2 = n \cdot p \cdot q$.

3. std. dev - $\sigma = \sqrt{npq}$.