SVKM's
D. J. Sanghvi College of Engineering

Program: B.Tech in AIML & AIDS **Academic Year: 2022** **Duration: 3 hours**

Date: 19.01.2023
Time: 09:00 am to 12:00 pm
Subject: Engineering Mathematics-III (Semester III) **Marks: 75**

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.
(1) This question paper contains two pages.
(2) All Questions are Compulsory.
(3) All questions carry equal marks.
(4) Answer to each new question is to be started on a fresh page.
(5) Figures in the brackets on the right indicate full marks.
(6) Assume suitable data wherever required, but justify it.
(7) Draw the neat labelled diagrams, wherever necessary.

Question No.	Question	Max. Marks
Q1 (a)	Show that V and W are subspaces of R^4: $V = \{(a,b,c,d): b - 2c + d = 0\}$ and $W = \{(a,b,c,d): a = d, b = 2c\}$. Also find a basis and the dimension of V and W. **OR** The vectors $(1,2,0,3), (4,0,5,8), (8,1,5,6)$ form a basis for three dimensional subspace V of R^4. Construct an orthonormal basis for V by using Gram-Schmidt process.	[07]
Q1 (b)	Show that R^n is a vector space with w.r.t usual vector addition and scalar multiplication defined as Addition: $u + v = (u_1 + v_1, \ u_2 + v_2, \dots, \ u_n + v_n)$ Scalar multiplication: $cu = (cu_1, cu_2, \dots, cu_n)$ For $u = (u_1, u_2, \dots, u_n)$, $v = (v_1, v_2, \dots, v_n)$ are elements of R^n.	[08]
Q2 (a)	Show that f: R^3→R is a linear transformation, where $f(x,y,z) = 3x + y - z$. What is the dimension of the Kernel space of f? Find a basis for the Kernel of f. **OR** If T:R^2→R^2 is defined by $T(x,y) = (2x - 3y, \ x + y)$, compute the matrix of T relative to the basis $\beta\{(1,2), \ (2,3)\}$.	[07]
Q2 (b)	Let T: R^5→R^5 be a linear mapping given by $T(a,b,c,d,e) = (b - d, d + e, b, 2d + e, b + e)$. Verify Rank Nullity Theorem. Is T invertible?	[08]

Q3 (a)	Use Caley Hamilton Theorem to compute A^{-1} and also A^9-$6A^8$+$10A^7$- A^6+A+I where A= $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ **OR** Determine if the following matrix is diagonalizable A= $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 0 \\ 1 & 1 & 5 \end{bmatrix}$. If diagonable, find an invertible matrix P such that $P^{-1}AP$ is diagonal , and use this to compute A^{17}.	[07]
Q3 (b)	Find the SVD of A, $U\Sigma V^T$, where $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$	[08]
Q4 (a)	Obtain half range sine series for $f(x) = \begin{cases} x & , \quad 0 < x < 1 \\ 2 - x & , \quad 1 < x < 2 \end{cases}$ hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$ **OR** Obtain the complex form of Fourier series for $f(x) = \cosh 3x + \sinh 3x$ in $(-3\ ,3)$.	[07]
Q4 (b)	Find a Fourier series to represent $f(x) = x^2$ in $(0\ ,\ 2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$	[08]
Q5 (a)	Solve the following LPP by Simplex Method / Big – M Method Maximize $z = 4x_1 - 2x_2 - x_3$ Subject to, $x_1 + x_2 + x_3 \le 3$, $2x_1 + 2x_2 + x_3 \le 4$ $x_1 - x_2 \le 0$ $x_1 ,\ x_2 ,\ x_3 \ge 0$ **OR** Use Dual – Simplex Method to solve the following LPP Maximize $z = -x_1 - 2x_2 - 3x_3$ Subject to, $2x_1 - x_2 - x_3 \ge 4$, $x_1 - x_2 + 2x_3 \le 8$ $x_1 , x_2 , x_3 \ge 0$	[07]
Q5 (b)	Using the method of Lagrange Multiplier solve the following N.L.P.P. Optimize $z = 12x_1 + 8x_2 + 6x_3 - {x_1}^2 - {x_2}^2 - {x_3}^2 - 23$ Subject to $x_1 + x_2 + x_3 = 10$ $x_1 , x_2 , x_3 \ge 0$	[08]