

Vector Spaces

Let V be a non Empty set of elements called as vectors and K, m be the scalars. If the set V satisfies following axioms then it is called as Vector space.

I) AXioms

a) clasure property

Let X, Y, Z EV, K&l be scalars

C .: X + Y E V

C, : KX E V

b) Addition axioms

A: Commutative x + Y = Y + X E V

Az: Associative (X+Y) + Z = X + (Y+Z)

As: Existence of identity

70 EV Such that X+0=X

OEV is called as additive identity

Ay: Existence of inverse

7 - X E V Such that X + (-x) = 0

c) Scalar Multiplication

M.: K(X+Y) = KX + KY E V

M2: (K+1)X = KX+1X E V

M3: (Kl) X = K(lX) E V
M4: Existence of multiplicative identity

(There must 7 1 E V Such that 1. X = X

Let V be a set of positive real no. with Addition and scalar multiplication define as x+ Y = XY and Cx = X° whose x, Y & V and C be a scalar.

To prove V is a Vector space we prove the following Ans 10 axioms

10 axioms x+y=xy and $cx=x^c$ (i) and c be a scalar

Let X, Y, Z & V be vectors K & l be the scalous 1) Closure axiom $C_1: X+Y-XY-XY-XX \in V$ $C_2: L.X=X^2 \in V$ as $X^2 \in \text{geal}(R)$ 2) Addition axiom A.: Commutative $X+Y=XY=YX=Y+X\in V$ A: Associative $1^R = 1^R = 1$ Au: Existence of inverse $X + 1 = X \cdot 1 = 1$: Yox is the additive inverse in V 3) Scalar Multiplication

M,: K(X+Y) = K(XY) = (XY) = XKYK = XK+YK $(x+1)x = x^{(x+1)} = x^{(x+1$ M_3 : $(Kl) X = X^{Kl} = (X^l)^K = K(X^l) = K(l)$ My: Multiplicative identity JJ J 1 E V- Such that 1. X = X 1 = X

1 is multiplicative identity. · V is a Vector space.



8.2] Examine wheather the set of all oreal numbers (x, y) with operation $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ (i) $K(x_1, y_1) = (K^2x_1, K^2y_1)$

Ans. (x, y,), (x_2, y_2) , $(x_3, y_3) \in V$ and PK and P be the scalars

C. : $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \in V$

C2: K(x, y.) = (K2x, K2y.) E V

2) Addition axiom A,: commutative

 $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2) = (x_2 + x_1, y_2 + y_1)$ $= (x_2, y_2) + (x_1, y_1)$

A: Associative

 $[(\alpha_1, y_1) + (\alpha_2, y_2)] + (\alpha_3, y_3) = (\alpha_1 + \alpha_2, y_1 + y_2) + (\alpha_3, y_3)$ $= (\alpha_1 + \alpha_2 + \alpha_3, y_1 + y_2 + y_3)$ $= (\alpha_1 + \alpha_2 + \alpha_3, y_1 + y_2 + y_3)$ $= (\alpha_1, y_1) + [(\alpha_2 + \alpha_3), y_1 + (y_2 + y_3)]$ $= (\alpha_1, y_1) + [(\alpha_2, y_2) + (\alpha_3, y_3)]$

As: Existence of identity $f(0,0) \in V$ such that $(\alpha, y,) + (0,0) = (\alpha, +0, y, +0) = (\alpha, y, y,)$

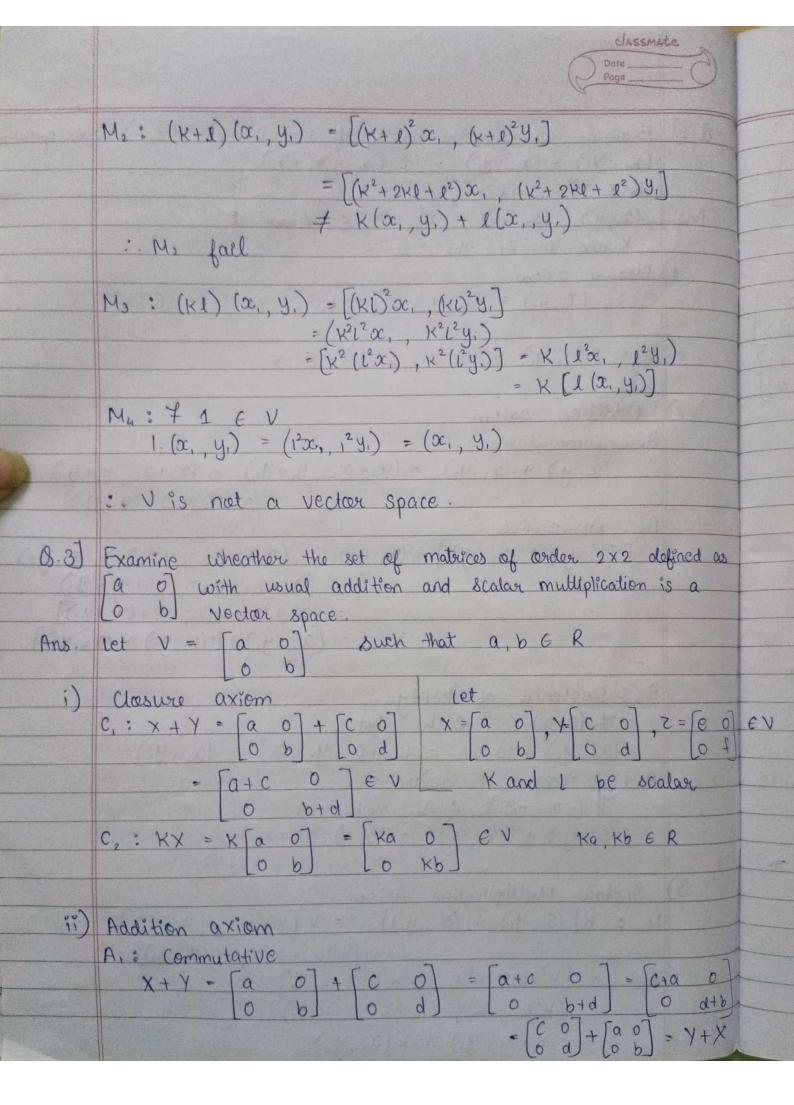
A4: Existence of inverse $f(-x, -y,) \in V$ such that (x, y,) + (-x, -y,) = (0, 0)

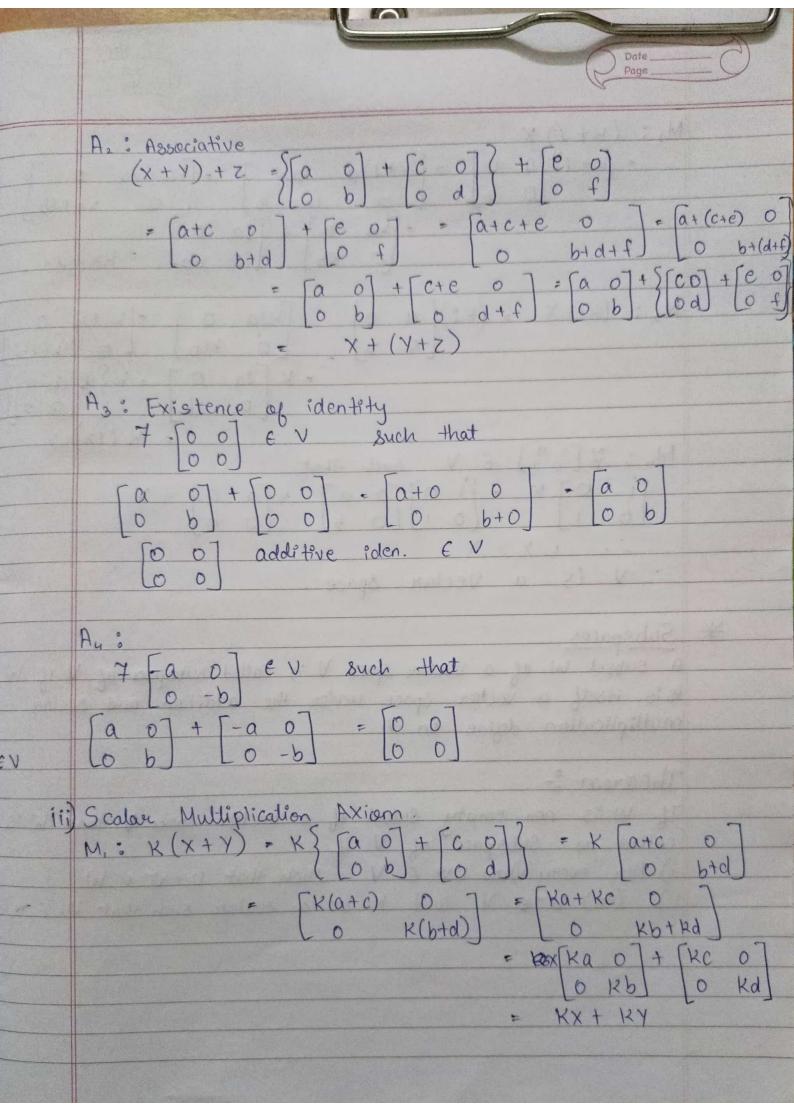
3) Scalar Multiplication Axiom

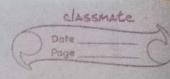
M.: $K[(x_1, y_1) + (o(x_1, y_2)] = K[(x_1 + x_2), y_1 + y_2)]$ $= [K^2(x_1 + x_2), K^2(y_1 + y_2)]$ $= [K^2(x_1 + x_2), K^2(y_1 + x_2)]$

= $(K^2\alpha_1, K^2y_1) + (K^2\alpha_2, K^2y_2)$

= $K(\alpha_1, y_1) + K(\alpha_2, y_2)$







M₂: $(K+l) \times$ - $(K+l) \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ = $(K+l) a & 0 \\ 0 & (K+l) b \end{bmatrix}$ · (Ka+la & 0)- (Ka & 0) + (la & 0)- $(K+l) \times = (Kl) \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ = $(K+la) \times = (Kla) \times = (Kla)$

* Subspaces.

A subset W of a vector space V is called subspace of V if W is itself a vector space under the addition and scalar multiplication define on V

Theorem :-

If W is non empty subset of a vector space V than W is called as Subspace of V if
i) From every (+) too UY & V such that U+V & W
ii) (+) U & V and K be a scalar such that Ku & W