1 Face Recognition using PCA and LDA

Classifying images is a vital subset of computer vision. But an image these days can contain so much information that the processing time can end up becoming exponential. To combat this issue, techniques like PCA and LDA are utilized to reduce the dimension of the images and still make reasonable classifications.

1.1 **PCA**

The idea behind Principal Component Analysis or PCA consists of calculating the eigenvectors of the covariance matrix C and retaining eigenvectors corresponding to the P largest eigenvalues. This constitutes the orthogonal PCA feature set. The results of the accuracy using PCA for face detection for P eigenvectors is illustrated in figure 1

1.1.1 The Methodology

- 1. Vectorize each image such that the dimension of the image is converted from $m \times n$ to $mn \times 1$
- 2. Normalize the vectorized image x_i

$$\hat{x_i} = \frac{x_i}{\|x_i\|} \tag{1}$$

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3. Compute the global mean vector \vec{m} for all N images in the train dataset

$$\vec{m} = \frac{1}{N} \sum_{i=1}^{N} \hat{x_i}$$
 (2)

4. Subtract the global mean \vec{m} from each image from the training dataset

$$X = X - train - \vec{m} \tag{3}$$

5. Since finding the eigendecomposition of XX^T is computationally complex so instead we find the eigendecomposition of X^TX to get the eigenvectors \vec{u} first

- 6. We then get the eigenvectors \vec{w} of XX^T by multiplying X with \vec{u} and the normalizing it
- 7. Since the rank of XX^T is N, at most N eigenvectors can be picked. So when choosing P eigenvectors, P < N
- 8. We then project X onto the eigenspace by multiplying X with w
- 9. We utilize K-Nearest Neighbors classifier, with K=1 in the lower dimensional space to classify faces

1.2 LDA

The goal of Linear Discriminant Analysis or LDA is to find the directions in the underlying vector space that are maximally discriminating between the classes. A vector direction is maximally discriminating between the classes if it simultaneously maximizes the between-class scatter and minimizes the within-class scatter along that direction. The results of the accuracy using LDA for face detection for P eigenvectors is illustrated in figure 1

1.2.1 The Methodology

- 1. Vectorize each image such that the dimension of the image is converted from $m \times n$ to $mn \times 1$
- 2. Normalize the vectorized image x_i

$$\hat{x_i} = \frac{x_i}{\|x_i\|} \tag{4}$$

3. Compute the global mean vector $\vec{m_G}$ for all N images in the train dataset

$$\vec{m_G} = \frac{1}{N} \sum_{i=1}^{N} \hat{x_i} \tag{5}$$

4. Compute the class mean vector for class using the images in that particular class

$$\vec{m_C} = \frac{1}{C} \sum_{i=1}^{N} \hat{x} \tag{6}$$

5. The eigenvectors \vec{w} are computed such that it maximizes the ratio, called the Fisher Discriminant, of between-class scatter S_B to within-class scatter S_W will be the largest. This is accomplished by conducting a eigendecomposition of $S_W^{-1}S_B$ to get \vec{w} . Since S_W can be singular sometimes, its inverse may not exist. As a result the Yu and Yang algorithm is used to combat this.

- 6. We preserve P eigenvectors from \vec{w}
- 7. We then project the training samples onto the the eigenspace by

trainFeature =
$$\vec{w}(X$$
-train - $\vec{m_G}$) (7)

8. We utilize K-Nearest Neighbors classifier, with K=1 in the lower dimensional space to classify faces

2 Face Recognition using Autoencoders

An autoencoder is a type of neural network that is usually used for dimensionality reduction. It is comprised of two neural networks: an encoder and a decoder. The encoder takes in an high-dimensional sample and outputs a P-dimensional vector just like the PCA and LDA techniques. The decoder then takes the P-dimensional vector and outputs a sample in the original high-dimensional sample space. During training, the autoencoder refines itself by attempting to recreate the input sample from the corresponding P-dimensional vector representation.

The results and comparisions of the accuracies for facial recognition detected by PCA, LDA, and the autoencoder are illustrated below in figure 1

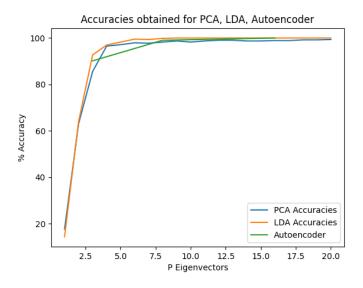


Figure 1: Accuracies for P Eigenvectors using PCA, LDA, and Autoencoder

2.1 Observations

From figure 1 we can see that all three techniques have come close or have achieved a full 100% accuracy. The PCA performed better for lower dimensions, then the LDA, and finally the au-

to encoder. At higher dimensions but less than N, all three techniques have a chieved full 100% accuracy.

3 Car Detection using AdaBoost

AdaBoost stands for Adaptive Boosting because it arranges a set of weak classifiers in a sequence in which each weak classifier is the best choice for a classifier at that point for rectifying the errors made by the previous classifier. A weak classifier is defined as having the ability to at least detect 50% of the images correctly.

3.1 The Methodology

- 1. Get the integral representation of each image is taken (taking the cumulative sum of each pixel in the image)
- 2. A feature matrix is created to convert the training dataset to create training feature vectors
- 3. For each cascade stage t out of a total T stages, a new AdaBoost classifier is added where
 - I. the weights are initialized first. Let p be the number of positive classes and n be the number of negative classes

$$w_i = \begin{cases} \frac{1}{2p} & \text{if } x = 1\\ \frac{1}{2n} & \text{if } x = 0 \end{cases}$$
 (8)

- II. for each n weak classifier in a total of N weak classifiers:
 - i) The weights are normalized $w_i = \frac{w_{ji}}{\sum_i w_{ji}}$
 - ii) Iterate over each feature of the images and sort them and their corresponding labels and weights based on the feature element values
 - iii) Compute the classification error of the feature elements that would be used as a threshold for both polarities

Error if polarity 1:
$$e_1 = S^+ + T^- - S^-$$
 (9)

Error if polarity -1:
$$e_{-1} = S^- + T^+ - S^+$$
 (10)

Where S^+ is the sum of weights of images belonging to the positive class whose feature value is less than the threshold, S^- is the sum of weights of images belonging to the negative class whose feature value is less than the threshold, T^+ is the sum of weights of images belonging to the positive class, and T is the sum of weights of images belonging to the negative class.

The classification error is calculated by taking the $min(e_1, e_{-1})$.

- iv) The best weak classifier is the weak classifier with the lowest classification error
- v) Update the weights with the error ϵ of the best weak classifier as shown below

$$\beta = \frac{\epsilon}{1 - \epsilon} \tag{11}$$

$$\alpha = \ln(\frac{1}{\beta}) \tag{12}$$

where β represents the confidence and the α is the trust factor

$$w_i = w_i \beta^{1 - \epsilon_i} \tag{13}$$

III. We combine all the weak classifiers to form one strong classifier

The AdaBoost implementation was inspired by Fangda Li's implementation.

3.2 Results

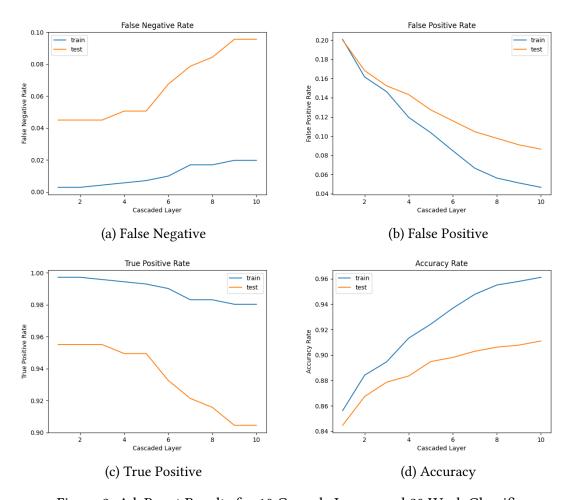


Figure 2: AdaBoost Results for 10 Cascade Layers and 20 Weak Classifiers

3.3 Observations

- As the cascade layer level increased, the false positive rate and true positive rate decreased
- The false negative rate on the other hand increased as the number of cascade layers increased
- The accuracy of detecting the cars increased with every cascade layer and this demonstrates how each weak classifier adds on to and rectifies the previous layers errors to make the detection even more accurate

4 Source Code

```
1 # Name: Nikita Ravi
2 # Class: ECE 66100
3 # Homework #10
4 # Deadline: 12/09/2022
6 # Import Modules
7 import cv2
8 import numpy as np
9 import os
import matplotlib.pyplot as plt
11 from autoencoder import evaluateautoencoder
def getimages(path):
   filenames = [image for image in os.listdir(path) if os.path.isfile(os.path.
     join(path, image))]
   images = [cv2.imread(os.path.join(path, image)) for image in os.listdir(path
15
     ) if os.path.isfile(os.path.join(path, image))]
16
   return images, filenames
18
 def displayimage(image, points=False):
   def clickevent(event, x, y, flags, params):
20
      # This function was inspired by https://www.geeksforgeeks.org/displaying-
     the-coordinates-of-the-points-clicked-on-the-image-using-python-opency/
     if(event == cv2.EVENTLBUTTONDOWN):
       print(x, y)
23
        font = cv2.FONTHERSHEYSIMPLEX
        cv2.circle(image, (x, y), 1, (0,0,255), thickness=-1)
        cv2.putText(image, str(x) + ', ' +
26
              str(y), (x,y), font,
              1, (0, 0, 255), 2)
28
        cv2.imshow('window', image)
29
30
   cv2.imshow("window", image)
31
   if(points):
32
      cv2.setMouseCallback('window', clickevent)
33
   cv2.waitKev(0)
34
   cv2.destroyAllWindows()
35
   quit()
36
37
 def calculatedistance(trainFeatureVectors, testFeatueVector):
   dist = np.linalg.norm(trainFeatureVectors - testFeatueVector, axis=0)
   return dist
40
42 class KNearestNeighbors:
   def init (self, k):
     self.k = k
44
45
   def fit(self, Xtrain, ytrain):
47
      self.Xtrain = Xtrain
     self.ytrain = ytrain
```

```
def predict(self, Xtest):
50
      Xtest = Xtest.T
51
      predictedclasses = np.zeros(len(Xtest))
52
      for idx, X in enumerate(Xtest):
53
        dist = calculatedistance(self.Xtrain, X.reshape(-1, 1))
54
        sorteddistidx = np.argsort(dist)
55
        predictedclass = self.ytrain[sorteddistidx[:self.k]]
        frequency = np.bincount(predictedclass)
        label = np.argmax(frequency)
59
        predictedclasses[idx] = label
61
      return predicted classes
63
64 class PCA:
    def init (self, Xtrain, ytrain, Xtest, ytest, P, k):
65
      self.k = k
      self.Xtrain = Xtrain
67
      self.ytrain = ytrain
68
      self.Xtest = Xtest
69
      self.ytest = ytest
70
      self.P = P
    def train(self):
73
      self.globalmean = np.mean(self.Xtrain, axis=1).reshape(-1, 1)
74
      X = self.Xtrain - self.globalmean
75
76
       , , u = np.linalg.svd(X.T @ X)
      w = X @ u # True eigenvectors of covariance
78
      w /= np.linalg.norm(w, axis=0)
79
80
      self.wp = w[:, :self.P] # Preserving P eigenvectors
      trainFeature = self.wp.T @ X # Project onto eigenspace
82
83
      self.knn = KNearestNeighbors(self.k)
84
      self.knn.fit(trainFeature, self.ytrain)
85
86
    def test(self):
87
      X = self.Xtest - self.globalmean
88
      testFeature = self.wp.T @ X # Project onto eigenspace
89
      predictedclasses = self.knn.predict(testFeature)
90
91
      acc = np.sum((predictedclasses - self.ytest) == 0) / np.float(self.
92
     ytest.size) * 100
      return acc
93
94
  class LDA:
    def init (self, Xtrain, ytrain, Xtest, ytest, P, NUMCLASSES, k):
96
      self.Xtrain = Xtrain
97
      self.ytrain = ytrain
98
      self.Xtest = Xtest
      self.ytest = ytest
100
      self.P = P
101
```

```
self.C = NUMCLASSES
      self.k = k
103
104
    def train(self):
105
      self.P = self.C - 1 if self.P ; self.C - 1 else self.P
106
      self.globalmean = np.mean(self.Xtrain, axis=1)
107
      self.classmeans = np.zeros((self.Xtrain.shape[0], self.C))
109
      for idx in range(self.C):
        self.classmeans[:, idx] = np.mean(self.Xtrain[:, self.ytrain == idx +
      1, axis=1)
      ,, ,, ,,
113
      It can be shown that when Sw is isotropic (all classes have identical
     variances in all of the same principal directions), the
      LDA eigenvectors are the eigenvectors of the Sb matrix. These correspond
     to the space spanned by the -C- - 1 mean difference
      mi - m.
116
      w = self.classmeans - self.globalmean.reshape(-1, 1) # Equation 26
118
      self.wp = w[:, :self.P] # Preserve P eigenvectors
119
      trainFeature = self.wp.T @ (self.Xtrain - self.globalmean.reshape(-1, 1)
     ) # Project onto eigenspace
      self.knn = KNearestNeighbors(self.k)
      self.knn.fit(trainFeature, self.ytrain)
124
    def test(self):
      X = self.Xtest - self.globalmean.reshape(-1, 1)
      testFeature = self.wp.T @ X
128
      predictedclasses = self.knn.predict(testFeature)
      acc = np.sum((predictedclasses - self.ytest) == 0) / np.float(self.
     vtest.size) * 100
      return acc
  def vectorizefaceimages(X):
    Xmean = np.mean(X)
134
    Xstd = np.std(X)
135
    Xnormalized = (X - Xmean) / Xstd
136
    return Xnormalized
138
139
  def createfacedatasets(IMAGESIZE, N, images, filenames):
    X = np.zeros((IMAGESIZE, N), dtype=np.float32)
141
    y = np.zeros(N, dtype=np.int64)
143
    for idx, image in enumerate(images):
      filename = filenames[idx]
145
      gray = cv2.cvtColor(image, cv2.COLORBGR2GRAY) if len(image.shape) == 3
     else image
      X[:, idx] = gray.flatten()
148
      y[idx] = int(filename.split(" ")[0])
```

```
X = vectorizefaceimages(X) # Normalize the images so the mean is at zero
     and standard deviation at 1
    return X, v
153
  def plotaccuracies(PCALDAaccuracies, autoencoderaccuracies):
154
    pca, lda = PCALDAaccuracies
156
    plt.plot(range(1, 20 + 1), pca, label="PCA Accuracies")
    plt.plot(range(1, 20 + 1), lda, label="LDA Accuracies")
158
    plt.plot([3, 8, 16], autoencoderaccuracies, label="Autoencoder") # From
159
     autoencoder script results
160
    plt.title("Accuracies obtained for PCA, LDA, Autoencoder")
    plt.xlabel("P Eigenvectors")
162
    plt.ylabel("% Accuracy")
164
    plt.legend()
    plt.show()
166
167
  def task1():
168
    # Constants
    TRAINDIR = "/Users/nikitaravi/Documents/Academics/Year 4/Semester 2/ECE
170
     66100/hw10/FaceRecognition/train"
    TESTDIR = "/Users/nikitaravi/Documents/Academics/Year 4/Semester 2/ECE
171
     66100/hw10/FaceRecognition/test"
    N = 630 # Number of images in both training and testing dataset
    IMAGESIZE = 128 * 128 # 128 x 128 resolution
    NUMCLASSES = 30 # Number of classes
    MAXP = 20 # Maximum number of eigenvalues we want
    NUMNEIGHBORS = 1 # 1-NN Classification
176
    trainImages, trainFileNames = getimages(TRAINDIR)
    testImages, testFileNames = getimages(TESTDIR)
179
180
    Xtrain, ytrain = createfacedatasets(IMAGESIZE, N, trainImages,
181
     trainFileNames) # Xtrain shape: (128x128, 630), ytrain shape: (630, 1)
    Xtest, ytest = createfacedatasets(IMAGESIZE, N, testImages,
182
     testFileNames) # Xtest shape: (128x128, 630), ytest shape: (630, 1)
    accuraciespcalda = []
183
184
    for mode in ["pca", "lda"]:
185
      print("======="+mode.upper()+"========")
186
      accuraciesforeachP = []
      for P in range(1, MAXP+1):
188
        accuracy = None
        if(mode == "pca"):
190
          pca = PCA(Xtrain, ytrain, Xtest, ytest, P, k=NUMNEIGHBORS)
          pca.train()
          accuracy = pca.test()
194
        elif(mode == "lda"):
          lda = LDA(Xtrain, ytrain, Xtest, ytest, P, NUMCLASSES, k=
     NUMNEIGHBORS)
```

```
lda.train()
          accuracy = lda.test()
198
        accuraciesforeachP.append(accuracy)
200
        print(f"Accuracy at -P eigenvectors: -accuracy ")
201
      accuraciespcalda.append(accuraciesforeachP)
202
203
    return accuraciespcalda
204
205
  def task2():
    accuraciesforp = []
207
    for p in [3, 8, 16]:
208
      Xtrain, ytrain, Xtest, ytest = evaluateautoencoder(p, training=False)
209
      knn = KNearestNeighbors(k=1)
      knn.fit(Xtrain.T, ytrain)
      predictedclasses = knn.predict(Xtest.T)
213
      accuracy = np.sum((predictedclasses - ytest) == 0) / np.float(ytest.
     size) * 100
      print(f"Accuracy at -p eigenvectors: -accuracy ")
      accuracies forp.append(accuracy)
    return accuraciesforp
218
219
  def createfeaturematrix(IMAGESIZE, height, width):
    # Haar Kernel Feature Extraction inspired by Fangda Li's code
    numfeatures = 47232 # Initially tried with 60000 but index ended with 47232
    featurematrix = np.zeros((numfeatures, IMAGESIZE), dtype=int)
223
    index = 0 # Keep track of the index of featurematric when appending to
     count number of features
    offset = 2 # offset of pixels from image borders
    # Horizontal haar filter
    widthstep, heightstep = 2, 1
228
    for i in range(1, height, heightstep): # row multiplier
229
      for j in range(1, width, widthstep): # height multiplier
230
        for y in range(offset, height - heightstep * i + 1 - offset): # Go
     through the image
          for x in range(offset, width - widthstep * j + 1 - offset):
            featurematrix[index, y * height + x] = 1.0
            featurematrix[index, y * height + x + widthstep * j//2] = -2.0
234
            featurematrix[index, y * height + x + widthstep * j] = 1.0
            featurematrix[index, (y + heightstep * i) * height + x] = -1.0
236
            featurematrix[index, (y + heightstep * i) * height + x +
     widthstep * j//2] = 2.0
            featurematrix[index, (y + heightstep * i) * height + x +
     widthstep * j] = -1.0
            index += 1
240
    # Vertical haar filter
241
    widthstep, heightstep = 1, 2
242
    for i in range(1, height, heightstep): # row multiplier
      for j in range(1, width, widthstep): # height multiplier
244
        for y in range(offset, height - heightstep * i + 1 - offset): # Go
```

```
through the image
          for x in range(offset, width - widthstep * j + 1 - offset):
246
            featurematrix[index, y * height + x] = -1.0
247
            featurematrix[index, y * height + x + widthstep * j] = 1.0
248
            featurematrix[index, (y + heightstep * i//2) * height + x] = 2.0
249
            featurematrix[index, (y + heightstep * i//2) * height + x +
     widthstep * j] = -2.0
            featurematrix[index, (y + heightstep * i) * height + x//2] = -1.0
251
            featurematrix[index, (y + heightstep * i) * height + x +
     widthstep * j] = 1.0
            index += 1
254
    print(f"The number of features are: -index ")
    return featurematrix
  class CascadedAdaBoost:
    # This class was inspired by Fangda Li's code
259
    def init (self, NUMCASCADES, NUMWEAKCLASSIFIERS, IMAGESIZE, height,
     width):
      self.NUMCASCADES = NUMCASCADES
261
      self.NUMWEAKCLASSIFIERS = NUMWEAKCLASSIFIERS
262
      self.IMAGESIZE = IMAGESIZE
      self.height = height
264
      self.width = width
265
      self.cascadedadaboost = []
      self.featurematrix = createfeaturematrix(self.IMAGESIZE, self.height,
267
     self.width)
      self.numtrainpositives = None
268
      self.numtrainnegatives = None
      self.Xtrain = None
      self.ytrain = None
271
    def fit(self, Xtrain, ytrain):
      self.Xtrain = Xtrain
274
      self.ytrain = ytrain
      self.numtrain = self.ytrain.size
276
277
    def train(self):
278
      self.trainFeatureVectors = self.featurematrix @ self.Xtrain
279
      self.numtrainpositives = int(np.sum(self.ytrain))
280
      self.numtrainnegatives = len(self.ytrain) - self.numtrainpositives
281
282
      self.positivetrainfeaturevector = self.trainFeatureVectors[:, self.
283
     ytrain == 1
      self.negativetrainfeaturevector = self.trainFeatureVectors[:, self.
284
     ytrain == 0
285
      currentpositivesfeaturevectors = self.positivetrainfeaturevector
      currentnegativesfeaturevectors = self.negativetrainfeaturevector
287
      falsepositivetrain = []
289
      falsenegativetrain = []
      truepositivetrain = []
291
      accuracytrain = []
```

```
falsepositivetest = []
294
      falsenegativetest = []
      truepositivetest = []
296
      accuracytest = []
29
      for idx in range(self.NUMCASCADES):
        print(f"Training AdaBoost with Cascade Number -idx+1")
300
        currentadaboostclassifier = self.addnewadaboostclassifier()
301
        currentadaboostclassifier.settrainingfeaturevectors(
     currentpositivesfeaturevectors, currentnegativesfeaturevectors)
303
        for weak in range(self.NUMWEAKCLASSIFIERS):
304
          print(f"Adding weak classifier number: -weak + 1")
          currentadaboostclassifier.addweakclassifier()
306
        self.FalsePositiveIdx, FalsePositive, FalseNegative, TruePositive,
308
     Accuracy = self.classifytrainingdata()
        falsepositivetrain.append(FalsePositive)
309
        falsenegativetrain.append(FalseNegative)
310
        truepositivetrain.append(TruePositive)
311
        accuracytrain.append(Accuracy)
313
        currentnegativesfeaturevectors = self.negativetrainfeaturevector
314
     [:, self.FalsePositiveIdx - self.numtrainpositives]
        FalsePositive, FalseNegative, TruePositive, Accuracy = self.
      classifytestingdata()
        falsepositivetest.append(FalsePositive)
316
        falsenegativetest.append(FalseNegative)
        truepositivetest.append(TruePositive)
318
        accuracytest.append(Accuracy)
319
      return falsepositivetrain, falsepositivetest, falsenegativetrain,
     falsenegativetest, truepositivetrain, truepositivetest,
     accuracytrain, accuracytest
322
    def addnewadaboostclassifier(self):
      adaboost = Adaboost()
324
      adaboost.setfeaturematrix(self.featurematrix)
      self.cascadedadaboost.append(adaboost)
327
      return adaboost
328
329
    def classifytrainingdata(self):
330
      tempfeaturevector = self.trainFeatureVectors
331
      posidx = np.arange(self.numtrain)
333
      for classifier in self.cascadedadaboost:
        predicted = classifier.classifyfeaturevectors(tempfeaturevector)
        tempfeaturevector = tempfeaturevector[:, predicted == 1]
        posidx = posidx[predicted == 1]
337
      # Sort TruePositives, FalsePositives, FalseNegatives
339
      FalsePositiveIdx = posidx[np.take(self.ytrain, posidx) == 0]
```

```
numtruepositives = np.sum(np.take(self.ytrain, posidx))
      TruePositive = numtruepositives / self.numtrainpositives
342
      FalsePositive = (posidx.size - numtruepositives) / self.
343
     numtrainnegatives
      FalseNegative = 1 - TruePositive
344
      w = self.numtrainpositives / (self.numtrainpositives + self.
     numtrainnegatives)
      Accuracy = TruePositive * w + (1 - FalsePositive) * (1 - w)
346
347
      print("Training FP = %.4f, FN = %.4f, TP = %.4f, Acc = %.4f" % (
     FalsePositive, FalseNegative, TruePositive, Accuracy))
      return FalsePositiveIdx, FalsePositive, FalseNegative, TruePositive,
     Accuracy
    def settesting(self, Xtest, ytest):
351
      self.Xtest = Xtest
352
      self.ytest = ytest
353
    def classifytestingdata(self):
355
      self.testFeatureVectors = self.featurematrix @ self.Xtest
356
      tempfeaturevector = self.testFeatureVectors
357
      self.numtestpositives = int(np.sum(self.ytest))
359
      self.numtestnegatives = len(self.ytest) - self.numtestpositives
360
      numtest = self.ytest.size
362
      posidx = np.arange(numtest)
363
364
      for classifier in self.cascadedadaboost:
        predicted = classifier.classifyfeaturevectors(tempfeaturevector)
        tempfeaturevector = tempfeaturevector[:, predicted == 1]
        posidx = posidx[predicted == 1]
368
      # Sort TruePositives, FalsePositives, FalseNegatives
370
      FalsePositiveIdx = posidx[np.take(self.ytest, posidx) == 0]
371
      numtruepositives = np.sum(np.take(self.ytest, posidx))
372
      TruePositive = numtruepositives / self.numtestpositives
      FalsePositive = (posidx.size - numtruepositives) / self.
374
     numtestnegatives
      FalseNegative = 1 - TruePositive
375
      w = self.numtestpositives / (self.numtestpositives + self.
376
     numtestnegatives)
      Accuracy = TruePositive * w + (1 - FalsePositive) * (1 - w)
377
      print("Testing FP = %.4f, FN = %.4f, TP = %.4f, Acc = %.4f" % (
379
     FalsePositive, FalseNegative, TruePositive, Accuracy))
      return FalsePositive, FalseNegative, TruePositive, Accuracy
380
  class Adaboost:
382
    # This class was inspired by Fangda Li's code
    def init (self):
384
      self.weakclassifierindex = np.array([], dtype=int)
      self.weakclassifierpolarities = np.array([])
386
      self.weakclassifierthreshold = np.array([])
```

```
self.weakclassifierweights = np.array([])
      self.weakclassifierresults = np.array([])
389
      self.weakclassifierweightedresults = None
      self.threshold = 1.0
391
      self.ytrain = None
392
      self.trainsortedidx = None
303
      self.trainfeaturevector = None
      self.numpositive = None
      self.numnegative = None
396
      self.weightsforsample = None
398
    def setfeaturematrix(self, featurematrix):
      self.featurematrix = featurematrix
400
    def settrainingfeaturevectors(self, positivefeaturevector,
402
     negativefeaturevector):
      self.NUMFEATURES = positivefeaturevector.shape[0]
403
      self.numpositive = positivefeaturevector.shape[1] # shape: numfeaturs
     =47232 x numimages
      self.numnegative = negativefeaturevector.shape[1]
405
      self.trainfeaturevector = np.hstack((positivefeaturevector,
406
     negativefeaturevector))
      self.ytrain = np.hstack((np.ones(self.numpositive), np.zeros(self.
407
     numnegative)))
      self.trainsortedidx = np.argsort(self.trainfeaturevector, axis=1)
408
      print(f"Number of positive training data: -self.numpositive / negative
410
     training data: -self.numnegative ")
    def addweakclassifier(self):
412
      # Initialize the weights for the first weak classifier
413
      if(not self.weakclassifierindex.size):
414
        self.weightsforsample = np.zeros(self.vtrain.size, dtype=float)
        self.weightsforsample[self.ytrain == 1] = 1 / (2 * self.numpositive)
416
        self.weightsforsample[self.ytrain == 0] = 1 / (2 * self.numnegative)
418
      # Normalize the weights
420
        self.weightsforsample /= np.sum(self.weightsforsample)
421
      # Get the best weak classifier that minimizes the error with the current
423
     weights
      bestfeatureindex, bestfeaturepolarity, bestfeaturethreshold,
424
     bestfeatureerror, bestfeatureresults = self.getbestweakclassifier
     ()
425
      self.weakclassifierindex = np.append(self.weakclassifierindex,
426
     bestfeatureindex)
      self.weakclassifierpolarities = np.append(self.
427
     weakclassifierpolarities, bestfeaturepolarity)
      self.weakclassifierthreshold = np.append(self.weakclassifierthreshold,
428
      bestfeaturethreshold)
429
      # Confidence
```

```
beta = bestfeatureerror / (1 - bestfeatureerror)
432
      # Trust Factor
      alpha = np.log(1 / np.abs(beta))
434
      self.weakclassifierweights = np.append(self.weakclassifierweights,
435
      e = np.abs(bestfeatureresults - self.ytrain)
437
      # Update the weights
      self.weightsforsample = self.weightsforsample * beta ** (1 - e)
440
      # Adjust the threshold
      if(not len(self.weakclassifierresults)):
442
        self.weakclassifierresults = bestfeatureresults.reshape(-1,1)
444
        self.weakclassifierresults = np.hstack((self.weakclassifierresults,
     bestfeatureresults.reshape(-1,1)))
      self.weakclassifierweightedresults = np.dot(self.
447
     weakclassifierresults, self.weakclassifierweights)
      self.threshold = min(self.weakclassifierweightedresults[self.ytrain ==
448
      1])
449
    def getbestweakclassifier(self):
450
      featureerrors = np.zeros(self.NUMFEATURES)
      featurethreshold = np.zeros(self.NUMFEATURES)
452
      featurepolarity = np.zeros(self.NUMFEATURES)
453
      featuresortedidx = np.zeros(self.NUMFEATURES, dtype=int)
454
      Tpos = np.sum(self.weightsforsample[self.ytrain == 1])
456
      Tneg = np.sum(self.weightsforsample[self.ytrain == 0])
457
      for f in range(self.NUMFEATURES):
458
        sortedweights = self.weightsforsample[self.trainsortedidx[f, :]]
        sortedlabels = self.ytrain[self.trainsortedidx[f, :]]
460
        Spos = np.cumsum(sortedlabels * sortedweights)
        Sneg = np.cumsum(sortedweights) - Spos
464
        Epos = Spos + Tneg - Sneg
        Eneg = Sneg + Tpos - Spos
        polarities = np.zeros(self.numpositive + self.numnegative)
468
        polarities[Epos ¿ Eneg] = -1
469
        polarities[Epos ;= Eneg] = 1
        # print(Epos ¿ Eneg)
471
        errors = np.minimum(Epos, Eneg)
473
        sortedidx = np.argmin(errors)
475
        minerrorsampleidx = self.trainsortedidx[f, sortedidx]
        minerror = np.min(errors)
477
        threshold = self.trainfeaturevector[f, minerrorsampleidx]
479
        polarities = polarities[sortedidx]
```

```
featureerrors[f] = minerror
482
        featurethreshold[f] = threshold
        featurepolarity[f] = polarities
484
        featuresortedidx[f] = sortedidx
485
      bestfeatureindex = np.argmin(featureerrors)
487
      bestfeaturethreshold = featurethreshold[bestfeatureindex]
      bestfeatureerror = featureerrors[bestfeatureindex]
489
      bestfeaturepolarity = featurepolarity[bestfeatureindex]
491
      bestfeatureresults = np.zeros(self.numpositive + self.numnegative)
      bestsortedindex = featuresortedidx[bestfeatureindex]
493
      if(bestfeaturepolarity == 1):
        bestfeatureresults[self.trainsortedidx[bestfeatureindex,
495
     bestsortedindex: | ] = 1
496
        bestfeatureresults[self.trainsortedidx[bestfeatureindex, :
     bestsortedindex]] = 1
498
      return bestfeatureindex, bestfeaturepolarity, bestfeaturethreshold,
     bestfeatureerror, bestfeatureresults
500
    def classifyfeaturevectors(self, trainfeaturevector):
501
      weakclassifiers = trainfeaturevector[self.weakclassifierindex, :]
502
503
      polarityvector = self.weakclassifierpolarities.reshape(-1, 1)
504
      thresholdvector = self.weakclassifierthreshold.reshape(-1, 1)
505
      # Predictions made by weak classifier
507
      weakclassifierpredictions = weakclassifiers * polarityvector ¿
     thresholdvector * polarityvector
      weakclassifierpredictions[weakclassifierpredictions == True] = 1
      weakclassifierpredictions[weakclassifierpredictions == False] = 0
511
      # Apply weak classifier weights
512
      strongclassifierresult = self.weakclassifierweights @
     weakclassifierpredictions
514
      # Apply strong classifier threshold
515
      finalpredictions = np.zeros(strongclassifierresult.size)
      finalpredictions[strongclassifierresult ¿= self.threshold] = 1
517
      return finalpredictions
518
  def getintegralimage(image):
520
    # Return integral representation of the image
    # https://towardsdatascience.com/understanding-face-detection-with-the-viola
522
     -jones-object-detection-framework-c55cc2a9da14
    # Cumulative sum of above and to the left of the current pixel
523
    return np.cumsum(np.cumsum(image, axis=0), axis=1)
524
525
  def createcardatasets(IMAGESIZE, positiveimages, negativeimages):
    N = len(positiveimages) + len(negativeimages)
527
    positiveflatteneddata = np.zeros((IMAGESIZE, len(positiveimages)))
```

```
negativeflatteneddata = np.zeros((IMAGESIZE, len(negativeimages)))
530
    for idx, image in enumerate(positiveimages):
531
      image = cv2.cvtColor(image, cv2.COLORRGBA2GRAY) if len(image.shape) == 3
532
     else image
      image = getintegralimage(image)
      positiveflatteneddata[:, idx] = image.flatten()
534
    positivelabels = np.ones(len(positiveimages))
536
    for idx, image in enumerate(negativeimages):
538
      image = cv2.cvtColor(image, cv2.COLORRGBA2GRAY) if len(image.shape) == 3
539
     else image
      image = getintegralimage(image)
      negativeflatteneddata[:, idx] = image.flatten()
541
    negativelabels = np.zeros(len(negativeimages))
543
    X = np.hstack((positiveflatteneddata, negativeflatteneddata))
545
    y = np.hstack((positivelabels, negativelabels))
546
    return X, y
547
  def plottask3results(train, test, title):
549
    plt.plot(range(1, len(train) + 1), train, label="train")
550
    plt.plot(range(1, len(test) + 1), test, label="test")
551
552
    plt.title(title)
553
    plt.xlabel("Cascaded Layer")
554
    plt.ylabel(title)
556
    plt.legend()
557
    plt.show()
558
560 def task3():
    # Constants
    TRAINDIR = "/Users/nikitaravi/Documents/Academics/Year 4/Semester 2/ECE
562
     66100/hw10/CarDetection/train"
    TESTDIR = "/Users/nikitaravi/Documents/Academics/Year 4/Semester 2/ECE
563
     66100/hw10/CarDetection/test"
    IMAGESIZE = 40 * 20 # 40x20 image
564
    HEIGHT, WIDTH = 20, 40
565
    NUMCASCADES = 10
566
    NUMWEAKCLASSIFIERS = 20
567
    trainpositiveimages, trainpositivefilenames = getimages(os.path.join(
569
     TRAINDIR, "positive"))
    trainnegativeimages, trainnegativefilenames = getimages(os.path.join(
570
     TRAINDIR, "negative"))
    testpositiveimages, testpositivefilenames = getimages(os.path.join(
571
     TESTDIR, "positive"))
    testnegativeimages, testnegativefilenames = getimages(os.path.join(
572
     TESTDIR, "negative"))
573
    Xtrain, ytrain = createcardatasets(IMAGESIZE, trainpositiveimages,
```

```
trainnegativeimages) # Xtrain: (800, 2468), ytrain: (2468,)
    Xtest, ytest = createcardatasets(IMAGESIZE, testpositiveimages,
575
     testnegativeimages) # Xtest: (800, 618), ytest: (618,)
576
    classifier = CascadedAdaBoost(NUMCASCADES, NUMWEAKCLASSIFIERS, IMAGESIZE
577
     , HEIGHT, WIDTH)
    classifier.fit(Xtrain, ytrain)
578
    classifier.settesting(Xtest, ytest)
579
    falsepositivetrain, falsepositivetest, falsenegativetrain,
580
     falsenegativetest, truepositivetrain, truepositivetest,
     accuracytrain, accuracytest = classifier.train()
581
    plottask3results(falsepositivetrain, falsepositivetest, "False
582
     Positive Rate")
    plottask3results(falsenegativetrain, falsenegativetest, "False
583
     Negative Rate")
    plottask3results(truepositivetrain, truepositivetest, "True Positive
584
     Rate")
    plottask3results(accuracytrain, accuracytest, "Accuracy Rate")
585
586
  if name == " main ":
587
    print("=====TASK 1=====")
    pcaldaaccuracies = task1()
589
590
    print("=====TASK 2====")
591
    autoencoderaccuracies = task2()
592
    plotaccuracies(pcaldaaccuracies, autoencoderaccuracies)
593
594
    print("=====TASK 3=====")
595
    task3()
```

Listing 1: The Source Code