metropolis – hastings

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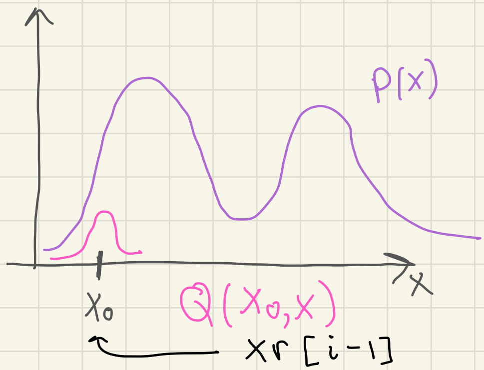
F2020

MH sampling method

in HW03 we used a single proposal distribution over the whole domain to draw samples $\{X_r\}$

now we will use a local distribution to draw our test samples.



an example of a local distribution is a Gaussian centered at our previous saved support point of width δ.

$$Q(x_0, x) = N(x, \delta) = \frac{1}{\delta} \frac{1}{\sqrt{2\pi}} e^{-(x-x_0)^2/2\delta^2}$$

we can draw a random x to test from this distribution using the Box-Müller method.

→ see BoxMuller1958.pdf for equations for 2 random x's drawn from a Gaussian centered at zero with width $\delta = 1$.

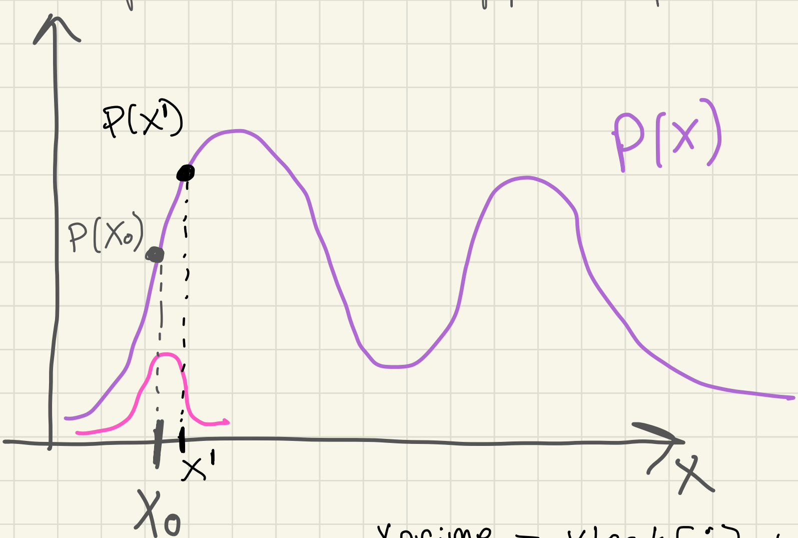
* edit the function rnorm(R) to return R of the random x's *

but we want these drawn from a gaussian centered at our previous saved support point of width δ.

so we will need to multiply these random x's by δ & add the value we want to center on

$$xtest = rnorm(R) * \delta \qquad xprime = xtest + x_0$$

now, how do we determine if we want to keep a trial support point?



$$xprime = xtest[i] + x0$$

again, we want more support points where P(x) is higher.

first acceptance condition:

$a \equiv P(x')/P(x_0)$

if $\underbrace{P(x') > P(x_0)}_{a > 1}$, equivalent to $a > 1$ where $a \equiv \frac{P(x')}{P(x_0)}$

then keep x' as one of our $\{x_r\}$ (easier to use "a" in code)

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xr[i] = xprime
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this saves more support points where $P(x)$ is higher, but will get stuck on a peak

so,

if the above condition is not met, let's still give a chance to save x

second acceptance condition:

get a random number $u \in [0, 1]$

if $u \leq a \longrightarrow$ accept x'

* if the second condition is still not met, save a copy of the old x (what i call x_0) to $\{x_r\}$ *

* this is an important difference from last week's HW.
this time we always save something to $\{X_r\}$, either the new X OR a copy of the previous.

a "for" loop will be needed then instead of a "while" loop like HW3.