PDEs: advection

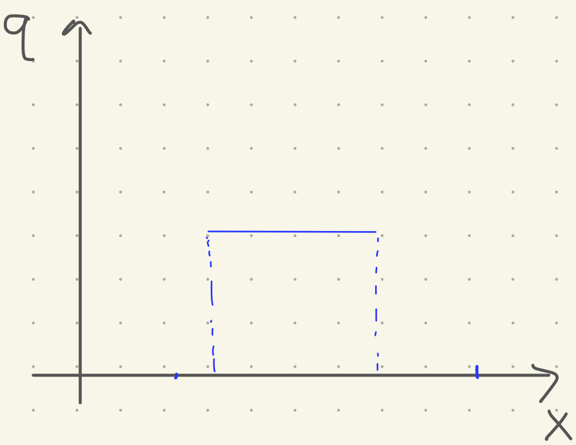
advection equation

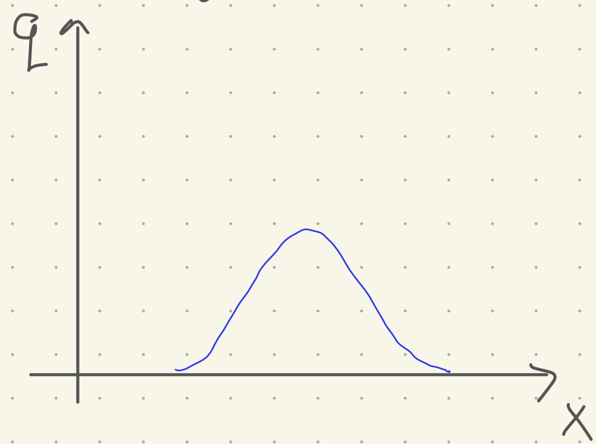
$$\frac{\partial q}{\partial t} = -c \frac{\partial q}{\partial x}$$

this equation describes a quantity q moving at a velocity c.

we have 2 initial conditions for q in our advection code:

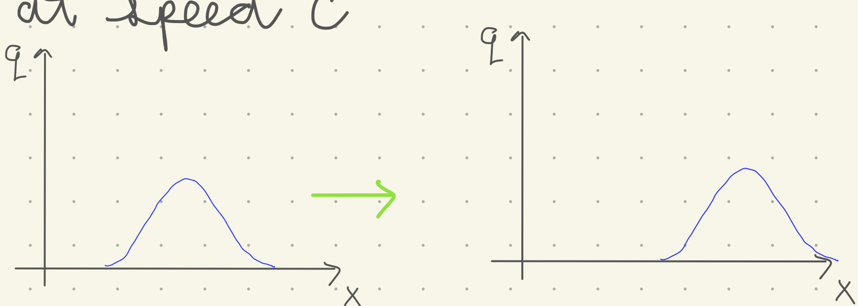
tophat & gaussian





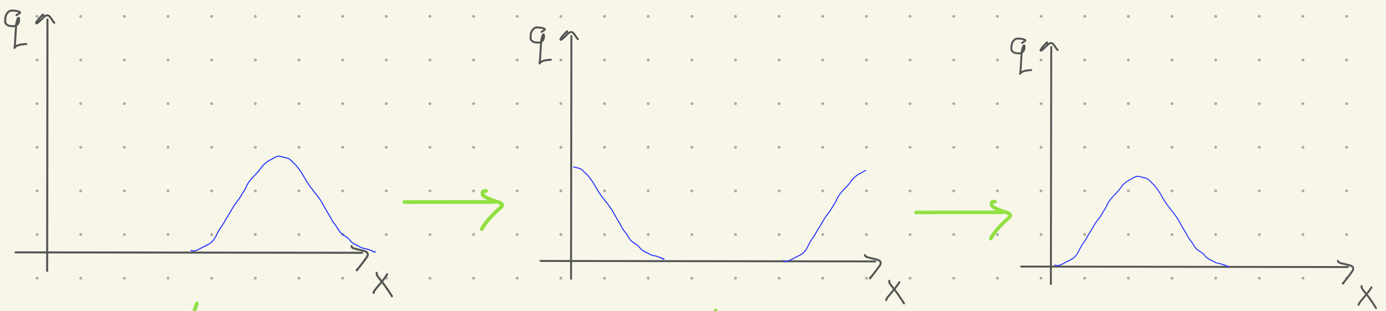
you can think of q as a blob of paint, so q represents paint density

in time the paint should advect (move) to the right at speed c



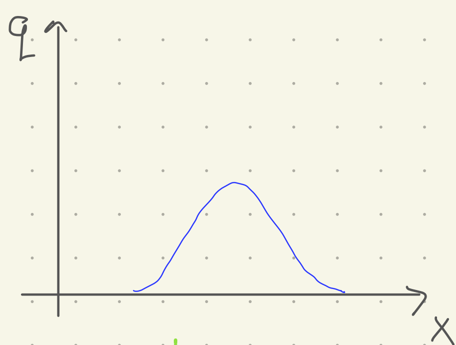
t_0 t_1

the code uses periodic boundary conditions, so once the blob leaves our domain it should appear on the other side



t_2 $\quad$ t_3 $\quad$ t_4

until one period $t = T$ is complete and the final result is back to the initial condition.



$t = T$

you will implement & explore stability for 4 methods of solving the advection eq.

this is very similar to the last HW, but the RHS of your time update will be different.

we used T or y but now we use q

- set up your (J+2, N) grid
- use initial condition FINC to fill 0^{th} column
- start time loop, where you set the boundary condition & time update

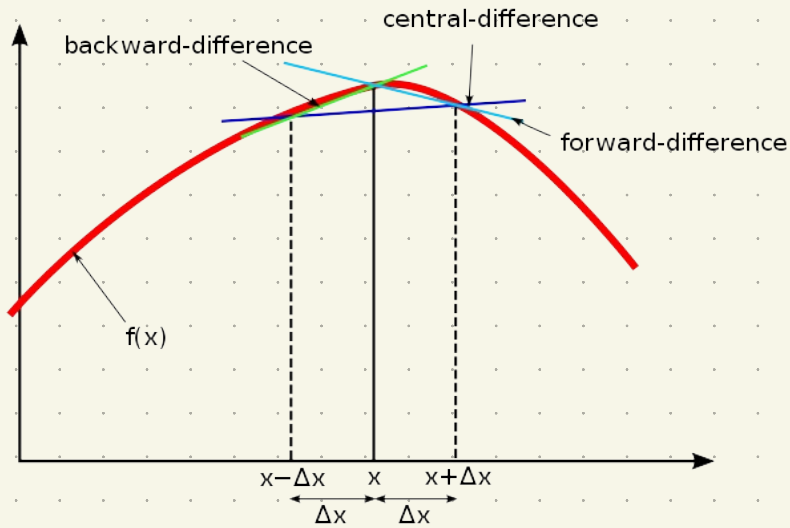
assume fourier solution to explore stability

$$q_j^n \equiv (\xi(k))^n e^{ikj\Delta x}$$

plug-in to method equation

& set $|\xi(k)| < 1$ for stability

first order vs second order finite difference



2ΔX

"central difference" is 2nd order accurate because it uses 2 values centered around q_j to determine the slope $\partial q / \partial x$

1st order:
(forward)

$$\left(\frac{\partial q}{\partial x}\right)_j \approx \frac{q_{j+1} - q_j}{\Delta x}$$

2nd order:
(centered)

$$\left(\frac{\partial q}{\partial x}\right)_j \approx \frac{q_{j+1} - q_{j-1}}{2\Delta x}$$

FTCS = "forward time, centered space"
uses 1st order for $\frac{\partial q}{\partial t}$ and 2nd order for $\frac{\partial q}{\partial x}$

for (9c) show

$$q_j^{n+1} = \frac{1}{2}(q_{j-1}^n + \cancel{q_{j+1}^n}) - \frac{\alpha}{2}(\cancel{q_{j+1}^n} - q_{j-1}^n).$$

if $\alpha = 1$

$= q^n_{j-1}$

is $\longrightarrow$

$$\frac{\partial q}{\partial t} = -c\frac{\partial q}{\partial x} + \frac{(\Delta x)^2}{2\Delta t}\frac{\partial^2 q}{\partial x^2},$$

by using $\dfrac{\partial q}{\partial t} = \dfrac{q_j^{n+1} - q_j^n}{\Delta t}$

& $\dfrac{\partial q}{\partial X} = \dfrac{q_{j+1}^n - q_{j-1}^n}{2\Delta X}$

& (from last HW)

$$\frac{\partial^2 q}{\partial X^2} = \frac{q_{j+1}^n - 2q_j^n + q_{j-1}^n}{(\Delta X)^2}$$

note for leapfrog method:

set up col 0 & 1 before loop