Root finding : bisection

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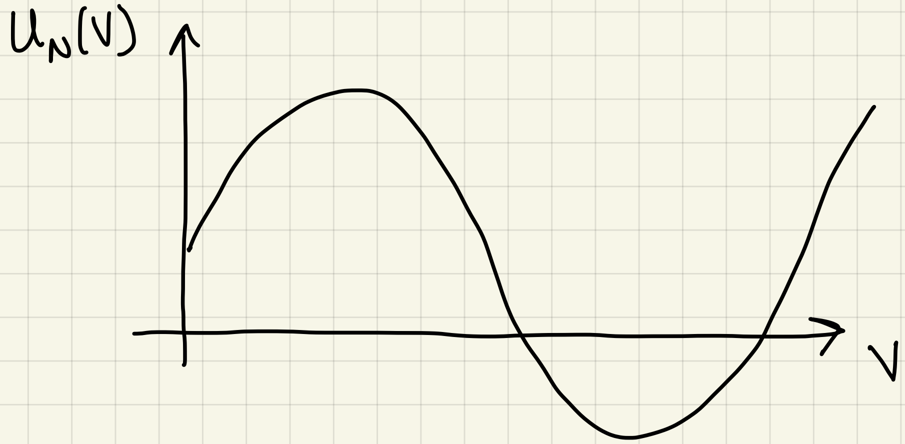
we want to find the roots of the function $U_N(V)$, which means we want to find the value of V that makes $U_N(V) = 0$ true.

for example:



we want to choose initial low & high values of V to begin "bracketing" the root

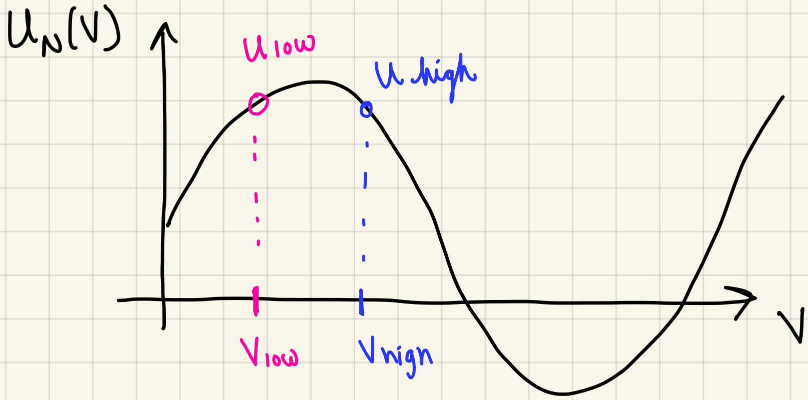
choose some V_0, let's say $V_0 = 0.5$

let $V_{low} = V_0$

and $V_{high} = 1.1 * V_0$

evaluate the corresponding $U_N(V)$ for each

case 1

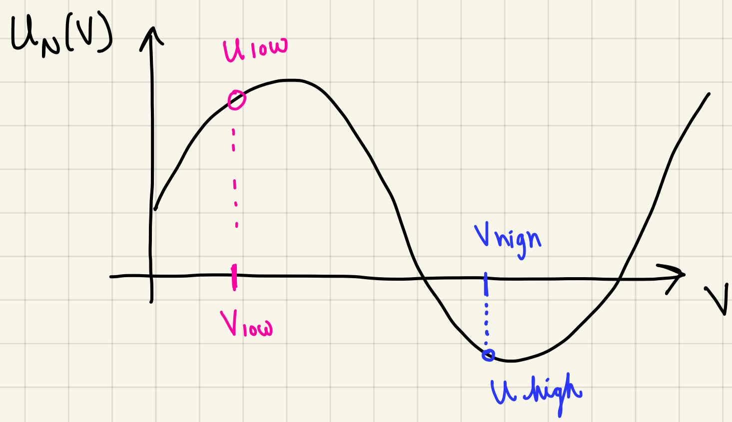


it might be the case that U low & U high are on the same side (both above or both below) the V - axis

in which case, U low * U high > 0

& the root is <u>not</u> bracketed

case 2



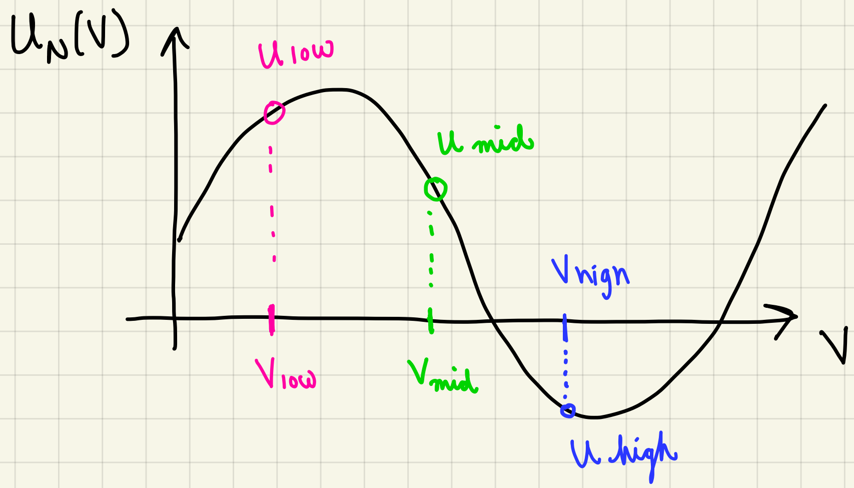
it might be the case that U low & U high are <u>not</u> on the same side (one above & one below) the V - axis

in which case, U low * U high < 0

& the root <u>is</u> bracketed

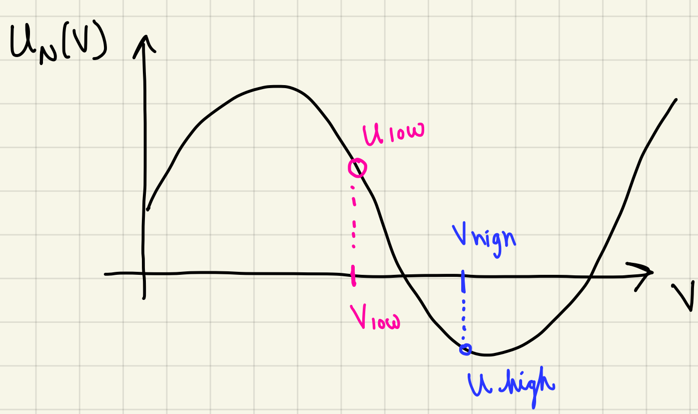
once it is bracketed,

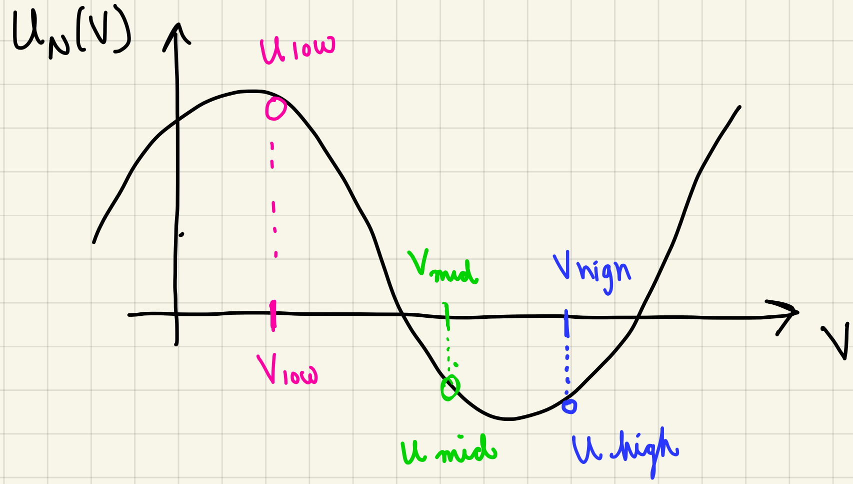
we want to close in on the root until we have reached the root up to some tolerance level. say, tol = 10^{-4}



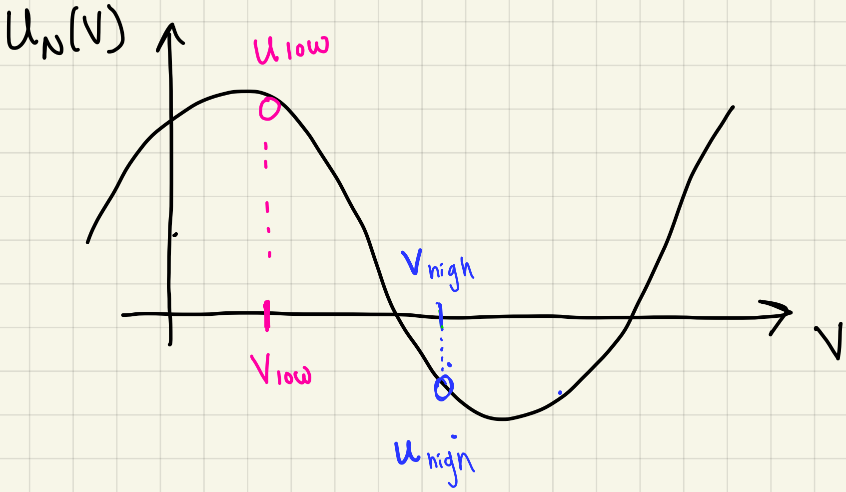
check if U mid & U low are on the same side of the V- axis

if they are reassign U low → U mid





U_N(V)
U low
V mid
V high
V
V low
U mid
U high



U_N(V)
U low
V high
V
V low
U high

if U_{mid} & U_{low} were on opposite sides of the V- axis

instead reassign $U_{high} \rightarrow U_{mid}$

continue until $\dfrac{|V_{high} - V_{low}|}{(V_{high} + V_{low})} < tol$

note: the denominator in this test is used to sort of "normalize" this difference.
you can also just use $|V_{high} - V_{low}|$