monte carlo

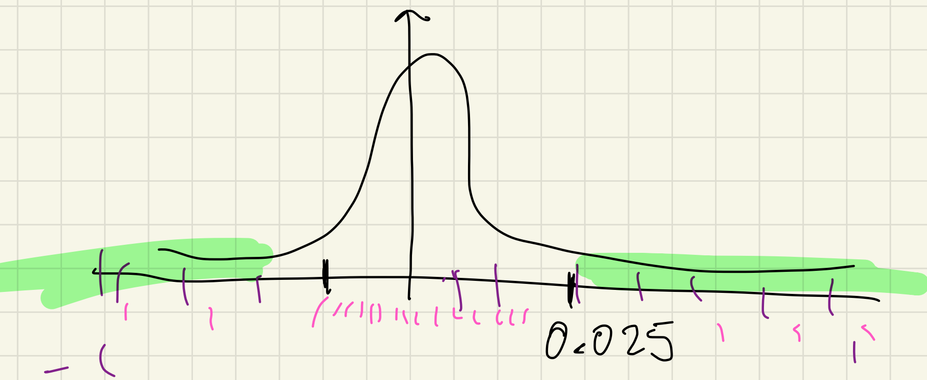
Nikki
Rider
F2020

$$\Phi_f = \int_a^b f(x)\,dx$$

$$\hat{\Phi}_f = \frac{b-a}{R} \sum_{r=1}^{R} f(X_r)$$

where

$X_r \in [a, b]$ & $\Delta X = \frac{b-a}{R}$



Rejection sampling

(a)

$N(\mu, \sigma)$

$$N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 0.68$$

(b)

$$c\,Q(x) > P(x)$$

proposal (prior)

target (posterior)

$cQ(x)$ $\quad c = \max(P/Q)$

$P(x)$

random # pairs (x, u)

$0 \le u \le 1$

keep if $u \le P(x)/c\,Q(x)$

area under Q $= (b-a)\max(P(x))$

- draw a random number x from $Q(x)$ for first candidate for our set of $\{X_r\}$
  - R_{tot} += 1
- draw a random number u uniformly between $[0, 1]$
- if $u \leq P(x)/cQ(x)$
  - keep current x as a support point $\{X_r\}$
  - R_{suc} += 1
- otherwise start over again!

functions at top

r *** gives R random variables distributed according to the corr. dist fcn

d *** dist fcn

np. random.rand( )

line 131 take out normed=false

(d) $Q(x) = \frac{1}{\pi(1+x^2)}$

$$|Q(x)\, dx| = |q(u)\, du|$$

assume positive

u is uniformly distributed so $q(u) = 1$

$$Q(x)\, dx = du$$

solve for expression $x(u)$

$$x = \tan\left(\pi(u+c)\right)$$

$$-\infty \leq x \leq \infty$$

$$-\frac{1}{2} \leq u+c \leq \frac{1}{2}$$

$$0 \leq u \leq 1$$

$$\longrightarrow \quad c = -1/2$$

plots for HW03

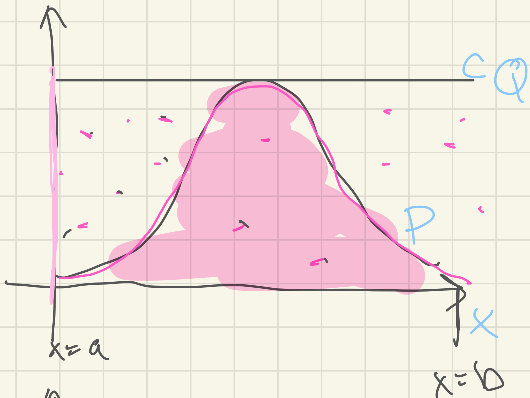
for an appropriate R value of your choosing, you should include histograms for:

3c
- exponential target P
  uniform proposal Q
- normal target
  uniform proposal

3e
- normal target
  cauchy proposal

(3b) $\frac{N_R}{N_{tot}} \propto \int_a^b P(x)\,dx$

$$\frac{N_R}{N_{tot}} = (\text{const}) \int_a^b P(x)\,dx$$



(x, u)

(3d)

$$Q(x) = \frac{1}{\pi(1+x^2)}$$

$$-\infty \leq x \leq \infty$$

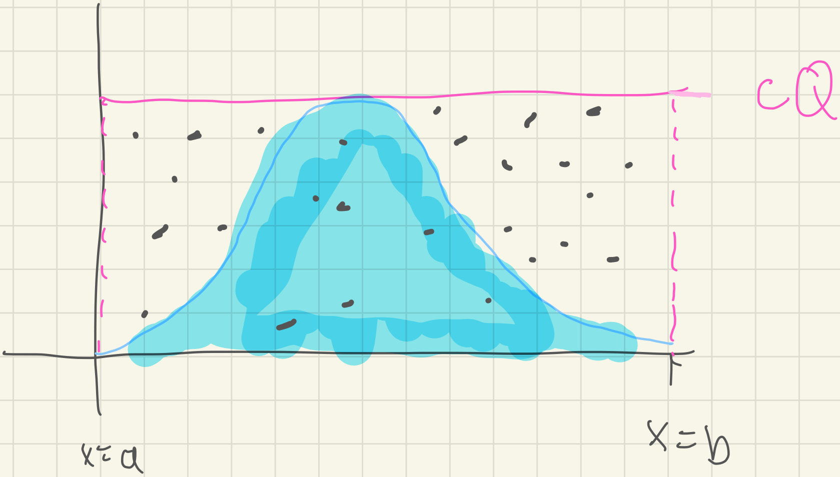
$$0 \leq u \leq 1 \quad \longrightarrow \quad q(u) = 1$$

$$Q(x)\,dx = q(u)\,du$$

$$\int \underline{Q(x)}\,dx = \int du$$

get $x(u)$

use variable limits to find integration constant



$$(x, cQu)$$

[0, 1]

$$\frac{N_R}{N_{tot}} = \frac{\text{blue area}}{\text{pink area}}$$

$$= \frac{\int_a^b P(x)\,dx}{(b-a)\,cQ}$$