

# **Applications of Seismic Metamaterial Cloaking**

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## **Abstract**

Seismic protection is a vitally important field in modern civil engineering, with rapidly accelerating population growth in urban areas leading to densification on an unprecedented level. High concentrations of people incur more risk from earthquakes and seismic waves, but recent advancements in metamaterial cloaking can create safer, more habitable cities. The field of seismic metamaterials in the context of cloaking, or preventing seismic surface waves like Rayleigh or Love waves from reaching protected areas, can be divided into seismic soil metamaterials and above-surface resonators. To mitigate the devastating effects of earthquakes, such as soil liquefaction and building collapse, techniques to redirect surface waves are paramount to the study of seismic metamaterials. Implementations of resonant metawedges, acting as above-surface metamaterials, have been found to successfully convert surface waves of varying frequencies to bulk shear waves [1, 2]. Computer models have shown that seismic-mufflers, or soil metamaterials, effectively act as cloaking devices and lessen the ground motion caused by incident seismic waves [3]. Seismic metamaterials are found in all areas of civil engineering, earthquake protection, and even in nature, and their continued study is vital for the better understanding of building in the context of seismic disturbances. Seismic cloaking research will lead to better urban design and disaster prevention, a key investment for all of humanity.

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## Introduction

Seismic disturbances caused by earthquakes have long been a risk to those who live in danger zones, and they are important to all planners and geologists, as well as engineers and everyday citizens because of the massive implications of city-level structural failure. Each year, more than 10,000 people die due to earthquake related situations, mostly resulting from building collapse [4]. But how exactly do these disturbances occur? When the movement of tectonic plates causes stress to build up deep below the surface in the crust, fracture can eventually occur along a fault, or divide. This causes a slip to occur along the fault, which rapidly propagates as a crack. This rupturing causes waves to spread through the body of the earth, called bulk waves. When these bulk waves reach an interface with a different material, they are called surface waves, which can be split into Rayleigh and Love waves, depending on their plane of oscillation. Rayleigh waves move elliptically in a plane parallel to the direction of wave propagation, or up and down, similar to an ocean wave, while Love waves shake the ground horizontally on the surface plane. These surface waves have a high amplitude and can cause large amounts of damage to surface dwellings, like cities and towns [5]. Not only can surface waves endanger life and habitable buildings, they can completely destroy infrastructure important to civilization like oil and water pipelines. Previously, protection against earthquake damage has been considered through fortifying structural parts of buildings [6]. The research that delves into using seismic metamaterials shifts its focus to seismically isolating the base of buildings by attenuating or diverting the ground motion of damaging surface waves. The purpose of this study is to create a comprehensive literature review of current metamaterial research for seismic protection. To cloak cities, or effectively dampen or deflect incident seismic waves, seismic soil resonators and above-surface resonators count themselves among the many tools civil engineers and city planners use to protect structures. We will attempt to demystify the world of seismic metamaterial cloaking and the advances that are being taken to make our cities safer.

## Seismic Metamaterials

Despite the pressing need for effective seismic protection in urban communities and otherwise, seismological metamaterial research is still developing as a field, and thus, there is much to be learned about how to protect against earthquakes. That being said, the effects of seismic destruction have been known since the dawn of modern civilization; even Pliny the Elder of ancient Rome in the 1<sup>st</sup> century described methods of slip-techniques to protect building foundations [7]. Metamaterials, however, remained an elusive key to effective earthquake mitigation for millennia. Following the advent of research in auxetic electromagnetic materials in the 1970s, it wasn't until the past decade or so that these principles were applied to soil science and seismic wave mitigation at urban-level scales [8]. In fact, while the development of electromagnetic metamaterials such as stop band photonic crystals was occurring as early as 1987, we did not see phononic, or vibrational mode, applications on plates and surfaces until 2014 [9]. Just as photonic crystals can have a remarkable influence on incident electromagnetic waves in a range of frequencies, phononic crystals can perform similar duties with acoustic waves; the name seismic metamaterials actually refers to large scale phononic crystals used to control seismic waves [1]. We can see the complementarity of metamaterials and civil engineering in the literature available to us today. Phononic crystals have the ability to form bandgaps in their frequency response. These band gaps ensure that the incident wave is completely reflected by the phononic crystal surface at the unique frequency range it was designed for, and forbids the passage of elastic waves at those frequencies [6].

Metamaterials can broadly be defined as man-made materials that can achieve properties not found in nature [1], which are often expressed as repeating cellular structures. A rule of thumb is that the elementary cell size of the metamaterial should be at least three times smaller than the wavelength of the wave it is trying to influence so the metamaterial can effectively interact with the wave in ways that cannot be attained by conventional materials.

## Buried Metamaterials

Much work has been done in the exploration of various buried resonators, also known as seismic-soil metamaterials. Seismic-soil metamaterials are structured soils that are made up of either rigid additions or voids embedded into the geological medium [1]. The constructed network of boreholes in the geological medium make up the metamaterial and modify the constitutive properties of the medium. Strategic alterations to the geological medium through the construction of a large-scale metamaterial can be utilized in the seismic protection of vulnerable land by impeding and diverting seismic surface waves that would otherwise be destructive to that land. A majority of the studies have investigated cylindrical voids up until this point, with a couple of exceptions that will be discussed later.

Perhaps the simplest seismic soil metamaterial is the cylindrical void version, which was carried out in full-scale in Grenoble, France, in 2012. A lens consisting of a rectangular block of 23 holes (5m deep, 2m diameter) with triangular grid spacing of 7.07m was placed between a seismograph and an impact location, as seen in *Fig. 1*. This lens effectively creates artificial anisotropy in which the voids scatter incident waves, causing them to interfere with one another. The interference of seismic waves is very much related to similar concepts from acoustic theory, where similar sound waves can interfere constructively or destructively with each other. This interference attenuates the ground acceleration around and behind the lens, which decreases the horizontal ground motion in certain frequency ranges, thus effectively creating a large-scale metamaterial stop band [1]. In fact, one could theoretically create a seismic cloak that “detours waves around a protected area,” [1] leading to an enclosed space perfectly protected from seismic activity. There is a catch, however. This seismic cloaking, dependent as it is on wave interference, must also take into consideration wave frequency, which is determined by the source, or earthquake, thus capping the cloaking ability to a restricted range of frequencies. In this specific set up, seismic soil metamaterials, or the cylindrical voids, result in horizontal motion attenuation

between 1-2 Hz and 5-7 Hz (*Fig. 2*), precisely the fundamental frequency of 5-10 story buildings. This technique can help avoid resonance of the waves with the buildings behind the shield. These seismic soil metamaterial stop bands, calculated and constructed to protect the buildings behind the lens, are incredibly valuable for their adjustable ability to defend against surface waves.

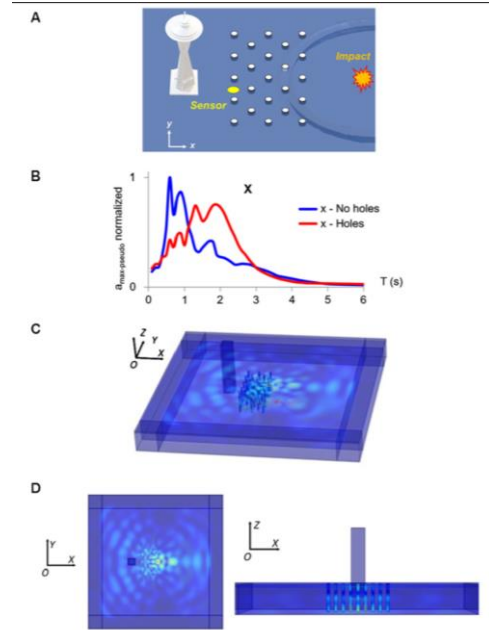


Figure 1 Experimental test for 23 holes in the soil for theoretical studies of the dynamic response  $x(t)$  of a building modeled as a SDOFO [1].

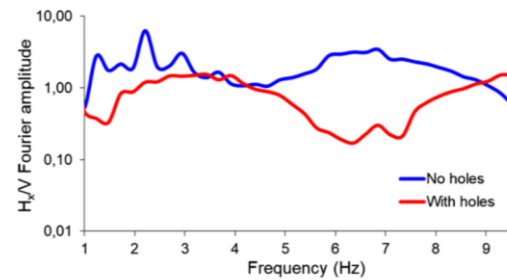


Figure 2 Experimental test in Grenoble, France. Ratio of the horizontal  $H_x$  to the vertical  $V$  components of ground motion versus frequency; soil without holes (blue solid line) and holey soil (red solid line) [1].

A significant solution to explore is the seismic-muffler. In its simplest form, a seismic-muffler consists of vertical open-air trenches embedded in a medium and was first explored by May and Bolt. These trenches act as a barrier

surrounding an area that would then be protected from seismic ground wave destruction. Through numerical modeling, May and Bolt concluded that in order to attenuate seismic waves amplitude past the trench wall, the trenches must be hundreds of meters deep [10]. Complications arise when implementing vertical trenches that are hundreds of meters deep and 1 meter across that render them impractical. Additionally, even with the muffler present, there is still a risk of seismic energy diffraction both underneath and around the muffler inlet. This diffraction leaves the intended protected land vulnerable to ground waves [3]. To address the shortcomings of the basic seismic-muffler associated with their construction, a V-shaped seismic metamaterial muffler can be considered using a network of angled boreholes. The transition to using an array of deep air-filled boreholes as opposed to deep air-filled trenches is due to the practicality of implementing boreholes when compared to trenches. Additionally, the angling of the boreholes implies that the boreholes do not need to be dug as deep for the same length of borehole which further lessens the construction concerns. The decision to angle the boreholes is also due to the additional protection it provides. In the examination of the V-shaped muffler the performance was measured through the transmission loss, defined as the summation of the loss through the inlet of the muffler and the loss through the muffler itself  $TL = TL_{con} + TL_{ent}$  [3]. Areas at risk for earthquakes usually comprise soft rock/dense soil and stiff soil. The shear and compressional velocities used in these simulations were 360-760 m/s and 180-360 m/s to be representative of these soft rock/dense soil and stiff soil conditions, respectively. The seismic velocities used in these simulations were taken from the National Institute of Building Sciences [3]. The analysis of various slope angles for the V-shaped seismic-muffler by Haupt et. al. was done through bench-scale experiments that supported 3D numerical simulations leading to the reasonable earth-scale modelling [3]. Seismic transmission loss was found to increase as the slope of the boreholes with respect to the horizontal decreased. The angled borehole orientation is advantageous because it not only

diverts the horizontally travelling surface waves, but also attenuates diffracted waves that propagate vertically to the ground surface by means of its sloped geometry [3]. The vertical seismic motion has marginal effects on the ground surface with the angled seismic-muffler in place. The sloped seismic-muffler studied here was able to reduce seismic wave amplitudes by 30-50 decibels [4]. Research also found that increasing the muffler outlet diameter at a constant muffler wall slope had marginal effects on the transmission loss and therefore the V-shaped seismic-muffler can be used for critical infrastructure of various sizes [3]. Increasing the muffler outlet diameter too greatly increases the risk for an earthquake to originate within the muffler itself, rendering it useless.

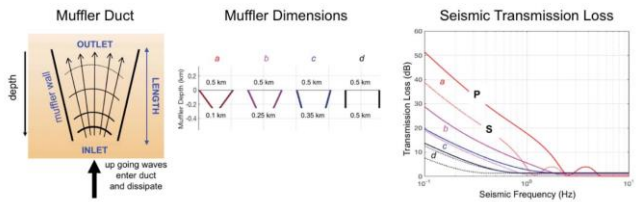


Figure 3 Effect of muffler inlet diameter and wall slope [3].

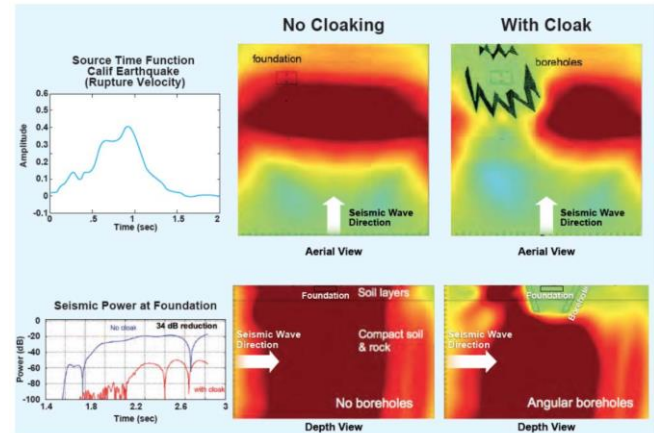
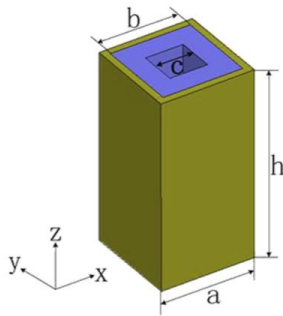


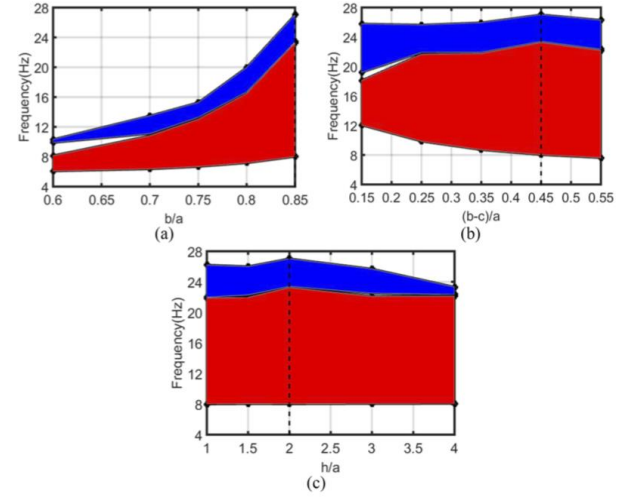
Figure 4 Finite Difference Model of the effects on seismic wave propagation from seismic cloaking [4].

Along with the placement and orientation of the boreholes, there are additional material and geometric parameters that have an effect on the constitutive properties of the medium. The finite element method (FEM) creates a mesh of finite elements connected by nodes [11]. In this practice, the Rayleigh waves were modelled with traction free conditions at the top and bottom surfaces of each RVE whereas surface waves were modelled

with the consideration of a semi-infinite medium [11]. To explore the effects of both the material properties and the geometry of each void or addition, the FEM was applied to various makeups. These designs included air-filled circular pipes, air-filled square pipes, air-filled rectangular pipes, and concrete-filled square pipes. Upon investigating and comparing the lower band edges and upper band edges the square pipes were found to have wider complete bandgaps (18.62 Hz) when compared to the bandgaps of the circular pipes (7.78 Hz) [11]. These complete bandgaps are frequency ranges in which “the seismic shield can provide a virtually unperturbed area, even if reflections at the free boundaries of the volume are included” [11]. *Fig. 6* quantifies various ratios of the geometric parameters in order to maximize the bandgap, making the metamaterial effective for a larger range of frequencies. Increasing the ratio between the outer diameter of the pile ( $b$ ) and the square lattice constant ( $a$ ) increases the bandgap of the material, leveling out at around  $b/a=0.7$  [11]. The ratio between the thickness of the square pipe ( $b-c$ ) and the square lattice constant ( $a$ ) maximizes the bandgap at  $(b-c)/a=0.45$  [11]. This demonstrates that an air-filled square pipe is more desirable than a concrete-filled square pipe.



*Figure 5 Geometric parameters of the hollow square pile [11].*



*Figure 6 The complete bandgaps bounded by the upper and lower frequency for a hollow square pile, dependent on geometric parameters [11].*

## Above Surface Metamaterials

While acoustic theory can be used to model the wave interference for seismic soil resonators, seismic waves can also be described using elastodynamic theory, at least for low magnitude earthquakes. Just as soil inclusions like air pillars or trenches can be used to cloak areas by redirecting and attenuating surface waves, above surface resonators can also be used to deflect surface waves back into the ground as bulk shear waves. Similar in operation to a tuned mass damper (TMD), which reduces the dynamic response of a main structure when the damper is excited by a particular frequency, above surface resonators like tall poles can perform tuned mass damping on the ground. When an above surface resonator reaches its specific structural frequency based on its geometrical and physical characteristics, it resonates out of phase with the ground to help deflect surface waves back into the ground. Interestingly, forests can act as above surface resonators, as trees with their varied heights act as ersatz tuned mass dampers. A forest is an example of a naturally occurring geophysical metamaterial that has the capability of mode converting dangerous surface Rayleigh waves to less destructive bulk shear waves [2]. The variation of tree heights performs as a type of band gap, as different heights correspond to different damping frequencies, as shown in *Fig. 7*. Additionally, as



above surface seismic metamaterials are locally resonant, they can be spaced randomly, which is why densely packed forests are ideal to redirect surface waves.

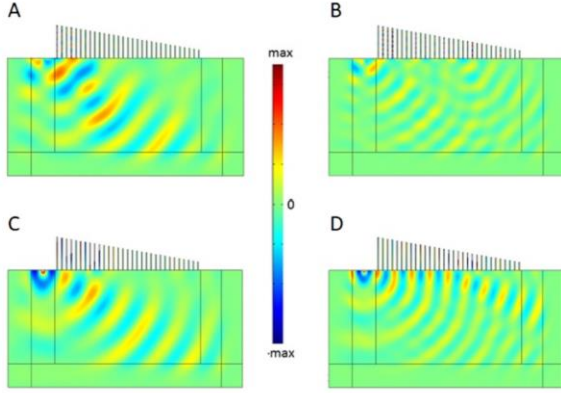


Figure 7 Two-dimensional in-plane elastodynamic simulation of a Rayleigh wave that propagates at a frequency 70 Hz (A, C) and 120 Hz (B, D) in a forest of trees of decreasing height (14 m to 4 m) and same diameter (0.3 m) [1].

A concept that has gained much traction in recent years is the design of the seismic metawedge, a metastructure capable of segregating and controlling the propagation of seismic waves through a graded array of vertical subwavelength resonators. They generally are sorted in order of ascending or descending height to form a wedge shape. The different heights of vertical resonators are tuned such that their first resonance frequency, given by:

$$f = \frac{\pi}{2h} \sqrt{\frac{E_r}{\rho_r}}$$

As the resonance frequency is inversely proportional to the height of the resonator, it's useful to have tall resonators to attenuate low frequency surface wave amplitudes, and shorter resonators to attenuate higher frequency waves. Keep in mind, as Rayleigh waves move up and down, the vibrational modes to observe for the vertical resonators should be longitudinal, not transverse, to match the excitation. In the case of the classic metawedge, incident waves slow down upon reaching the shortest resonators until reaching the resonator with the same fundamental frequency as the source. That resonator is located at what is referred to as the turning point. At the turning point, the surface wave diverges and

creates a band gap, which reflects the surface wave back, shown in Fig. 8. This reflected wave causes a seismic rainbow, which can increase wave amplitude in the shorter resonators, causing damage. In the case of the inverse metawedge, the incident surface wave approaches the tallest resonators, and upon reaching the turning point, instead of being reflected like in the case of the classic metawedge, the Rayleigh wave is mode converted into a bulk S-wave at an angle predicted by Snell's law:

$$\theta = \cos^{-1}\left(\frac{v_s}{v_r}\right)$$

With  $v_s$  representing the compressional wave speed in the resonator and  $v_r$  being the Rayleigh wave speed. The performance of the resonant metawedge is shown in Fig. 9, showing the ability of the metawedge to massively reduce the amplitude of surface waves upon exiting the metawedge, practically lowering the spectral ratio to 0 [2]. Such an attenuation proves that the metawedge is mightily effective and important in seismic consideration.

A study by Avarantos found that unit cells made of different materials resulted in different band gaps depending on their specific sound velocity. Their results showed that steel had a significant (20-30%) larger band gap than concrete, likely because waves travel faster in that medium [6]. This also can be extended to the example of forests as resonant metawedges, as sound travels faster in hardwood than it does in concrete, showing that a forest has a larger effective protected seismic frequency range than the so called "concrete jungle." The same study found that for their unit cell with lattice constant increased to 100m, the band gaps fell between 0.5 and 50 Hz, the frequency range of seismic waves. This lattice constant, while seemingly large, is reasonable for a city scale or multi-building foundation, and performs remarkably well, showing transmission drops inside the bandgap between 10 and 26 dB [6].

A notable detail of the experiment performed by Colombi et al. is the sizing of the metawedge. Their numerical simulations used a wedge ranging from 1-14m, which corresponds to a protected frequency range of 30-120 Hz, and the source



frequency was a Rayleigh wave at 50 Hz. The dominant frequencies of earthquakes generated by anthropic sources may well be within this range, but common seismic frequencies are generally lower, and range between 0.5 to 50 Hz. Nevertheless, the utility of the metawedge as a metamaterial for seismic control is an avenue with much reward for future research, and cannot be understated.

The resonant metawedge is a metamaterial capable of controlling the propagation of incident surface waves from both directions, whether by means of reflection or by mode conversion, and it represents a key tool to the design of safer cities. These metamaterials act as an ultra broadband shield for Rayleigh waves, and prove the feasibility and efficacy of large-scale mechanical metamaterials in a geophysical context.

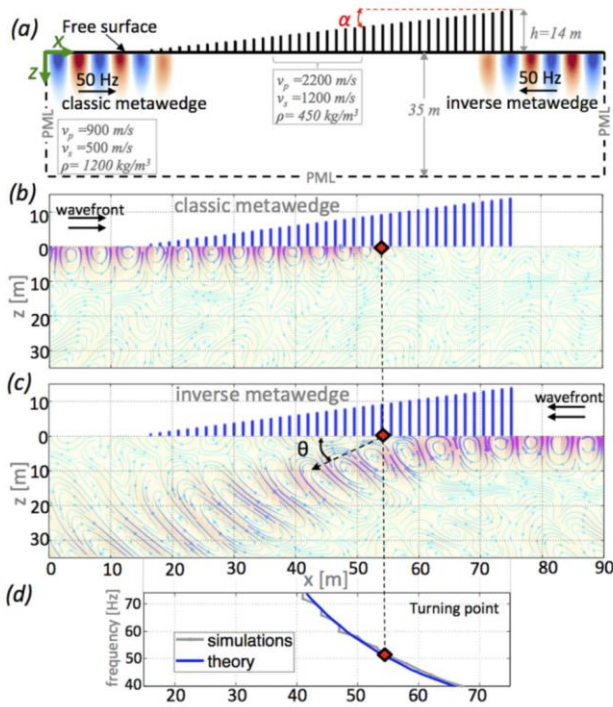


Figure 8 The metawedge with an angle of 13 degrees acting classically and inversely shown with the elastic streamlines where the colorscale and linewidth represent the magnitude of the displacement field [2].

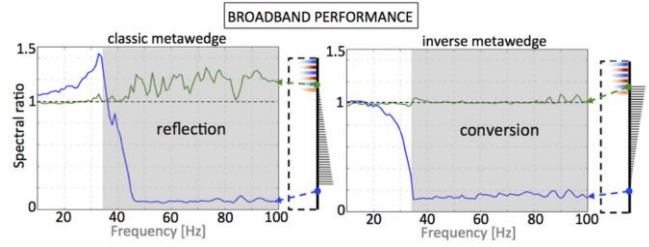


Figure 9 The efficiency of the metawedge shown via the spectral ratio calculated just before (green) and just after (blue) the wedge [2].

## Limitations of Seismic Metamaterials

Three significant limitations common to various designs of seismic cloaks that have been presented so far are the narrowband resonant nature of the structures, the lack of scalability of the designs, and the required directionality of the structure. The narrowband resonant nature of the structure limits the range of frequencies in which the cloak remains functional and effective [4]. Many of the designs presented depend on the size of each column or borehole to address a specific frequency. In order to account for the broadband nature of seismic surface waves there would need to be multiple resonant dampers each addressing a unique range of frequencies. Structures that require multiple resonant dampers to address a unique frequency band, like the resonant metawedge, work in bench-scale experiments but difficulties arise when attempting to scale up to an earth sized model. In some models, the resonators needed for lower frequency waves may exceed 100 meters in height! In this regard, scalability can present itself an issue as the size of structure necessary to address a broadband frequency goes beyond the realm of possibility when it comes to construction. When considering an above-surface metamaterial such as the resonant metawedge, the direction of implementation is crucial to its functionality. The metawedge is most effective if the source of the seismic wave is located such that the inverse metawedge is formed. Effects from an earthquake can be disastrous if the seismic wave approaches from the opposing side.

Upon delving into the experimental research, some apparent limitations come to light. The numerical simulations and experiments performed by Brule in 2014 showed the efficacy of seismic

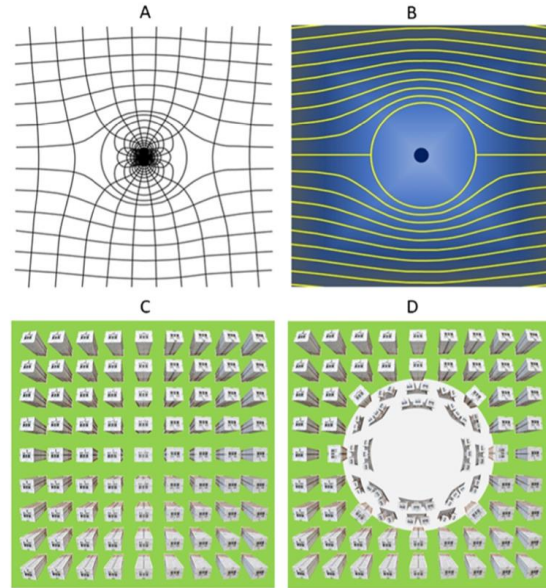
metamaterials for Rayleigh waves, but these experiments were performed on an aluminum plate for validation. Metal is geometrically homogeneous and isotropic, and thus simple to simulate, while soil is a complex medium, both heterogeneous and highly anisotropic [1]. Simulating the actual conditions of surface waves on soil would likely be numerically inefficient and intensive, but current experimental validation shows that such simplifications do not significantly alter results. We can trust, for the most part, the results of numerical simulations on the effectiveness of seismic metamaterials, as long as we take it with a grain of salt.

### Application of Seismic Metamaterials

One particularly interesting application of seismic metamaterials is in urban design. Cities remarkably resemble artificial forests, with buildings extending both above and below ground as seismic soil metamaterials and above surface resonators. Urban centers can theoretically act as meta-metamaterials, and effectively use the city itself to protect against seismic interference. By arranging the buildings to fit the spectral parameter curves of the Kirchhoff-Love biharmonic equation, one can theoretically create a seismic invisibility cloak that shields the city from earthquakes. By mapping a city to the spectral parameter, as shown in *Fig. 10* we can see the city center is void of seismic activity; this technique in transformed urbanism is called a metacity cloak [1]. The metacity helps avoid some of the limitations of stand-alone seismic metamaterials, like scaling and construction issues, as it is no longer inefficient to construct resonators on the order of hundreds of meters, as buildings themselves serve as these resonators. Furthermore, due to the wide variety of buildings in a typical city grid, the protected frequency range can be quite large.

Additionally, recent developments in seismic metamaterial theory have found that by altering the unit cell, lower and lower band gaps can be achieved. Mass-in-mass, spherical harmonic oscillators, and concrete layered with rubber are among the many designs for foundational unit cells that can achieve laboratory band gaps as low as 2.5

Hz [5]. Typical unit cells include a core of dense mass connected to an external element using elastic tendons, and has been numerically shown to provide low frequency band gaps without an excess requirement of unit cells. These cells, however, are notoriously complex to perform computations on, and have not been shown in practice, as the simpler unit cells of boreholes and metawedges make up the bulk of available research.



*Figure 10 Concept of Metacity cloak based upon transformation urbanism using a conformal mapping [1].*

As previously discussed, the angled seismic-muffler is a significant potential use of seismic metamaterials for practical uses. This muffler is important because it is practical in terms of construction feasibility and it is not directionally dependent. A notable use for this seismic metamaterial would be the protection of critical infrastructure such as nuclear power plants, oil refineries, or pipelines.

### Conclusion

Critical infrastructure such as oil and water pipelines or nuclear power plants left vulnerable to potential earthquakes could result in disastrous destruction. Combatting the devastating effects of earthquakes has long been carried out through strengthening buildings themselves such that they can withstand the extreme surface waves. More recent research has considered the application of

seismic metamaterials to redirect and attenuate the ground motion of surface waves, isolating building foundations. This study consists of a comprehensive literature review into key research on the implementation of metamaterials for seismic protection. Through the exploration of both buried metamaterials and above-ground metamaterials, the limitations and potential of this developing technology became more clear. Key limitations of this technology include the narrowband nature of each seismic metamaterial, the manufacturing challenges that arise when moving from bench-scale experiments to an earth-

sized models, and the directionality necessary when implementing certain metastructures. Effectively accomplishing a seismic cloak that attenuates and diverts incident seismic waves is possible given the computational power and the foresights for the aforementioned limitations. The research examined here is paving the way for the advent of seismic metamaterials for the protection of critical infrastructures against damaging earthquakes, and creating urban environments that are not only wonders of architecture, but also engineering ingenuity.

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