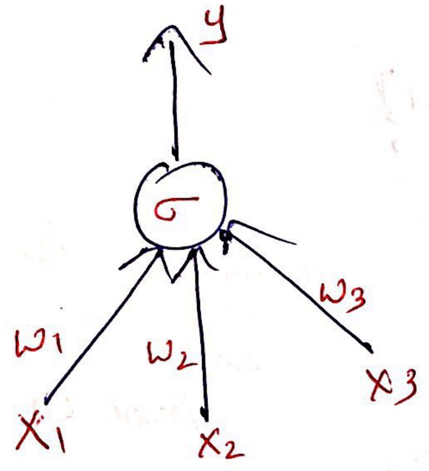
Lecture - 2

Part 1

Biological Neuron

10/5/19
3:00 pm.



Layer 1 → Detect Edge & Corners

Layer 2 → form feature Groups

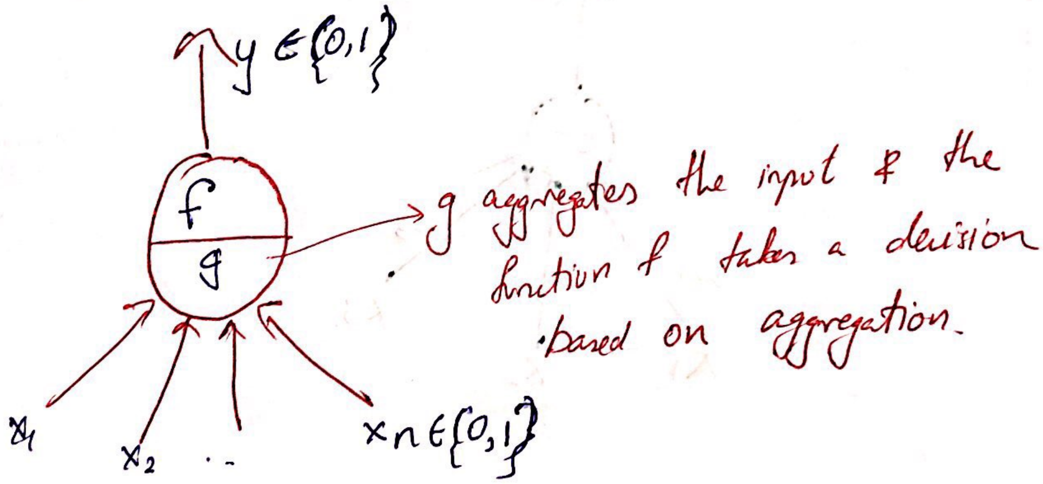
Layer 3 → detect high level features

Scanned with CamScanner

lecture - 2
Episode - 2 {2.2}

McColloch Pitts Neuron

Highly simplified computation model

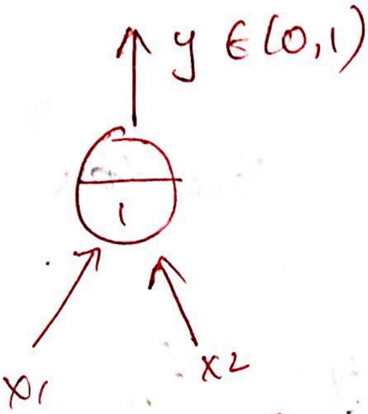


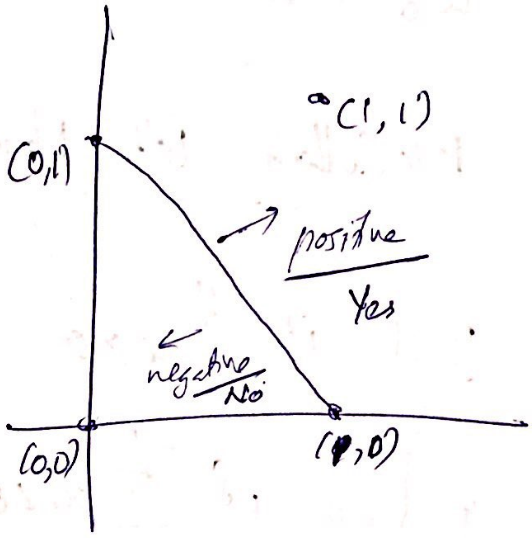
$$g(x_1, x_2 \ldots x_n) = g(x) = \sum_{i=1}^{n} x_i$$

$$y = \begin{cases} f(g(x)) = 1, & \text{if } g(x) \geq \theta \\ = 0, & \text{if } g(x) < \theta \end{cases}$$

Thresholding Parameter

Thresholding Logic.





$x_1 + x = \sum_{i=1}^{2} x_i \geq 1$

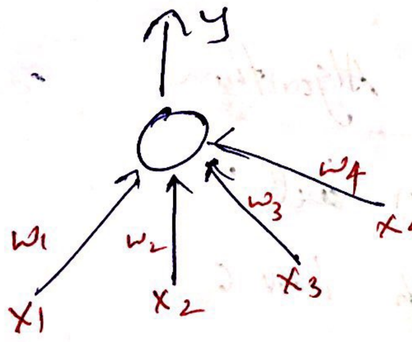
$x_1 + x_2 \geq 1$ (line)

Lecture-2 {2.3}
Episode-3

Perceptrons

→ Non-Boolean (say, real) inputs?

1955, Frank Rosenblatt



{no longer boolean values}

$y = 1 \quad \text{if} \quad \sum_{i=1}^{n} w_i * x_i \geq \theta$

$= 0 \quad \text{if} \quad \sum_{i=1}^{n} w_i * x_i < \theta$

w_0 - bias.

McCulloch Pitts Neuron	Perceptron
$y = \begin{cases} 1 & \text{if } \sum_{i=0}^{n} x_i \geq \theta \\ 0 & \text{if } \sum_{i=0}^{n} x_i < \theta \end{cases}$	$y = 1$ if $\sum_{i=0}^{n} w_i x_i \geq \theta$ $= 0$ if $\sum_{i=0}^{n} w_i x_i < \theta$

Lecture 2
Chapter 4

Error & Error Surfaces
To minimise error.

Lecture 2
Chapter - 5

Perceptron Learning Algorithm

$P \leftarrow$ inputs with label 1

$N \leftarrow$ inputs with label 0

$W = [w_1, w_2, \ldots w_n]$ randomly;

Scanned with CamScanner

while !convergence do

Pick random $x \in P \cup N$;

if $x \in P$ and $\sum_{i=0}^{n} w_i * x_i < 0$ (error)

$w = w + x$

end

if $x \in N$ and $\sum_{i=0}^{n} w_i * x_i \geq 0$ then (true).

$W = W - X$

end

end

convergence

(no more errors)

Two vector

$W = [w_0, w_1 \ldots w_n]$

$X = [x_0 \ldots x_n]$

$$W \cdot X = W^T X = \sum_{i=0}^{n} w_i * x_i$$

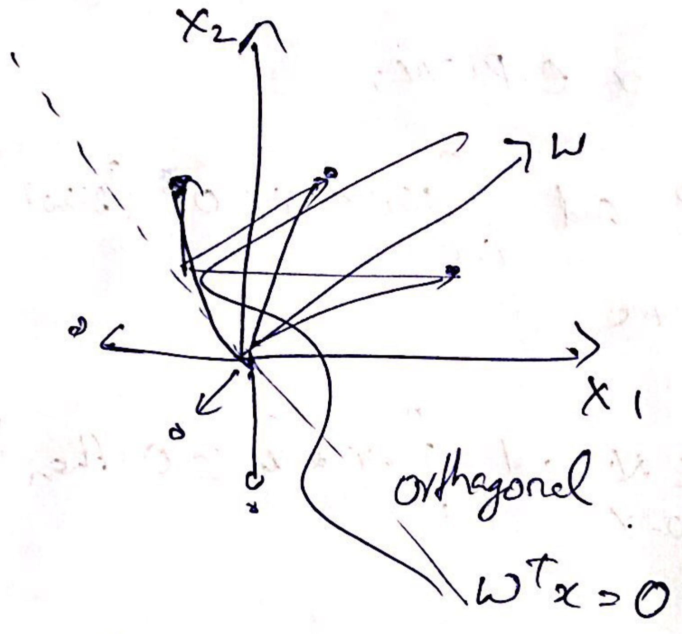
$$y = 1 \quad \text{if } W^T x \geq 0$$

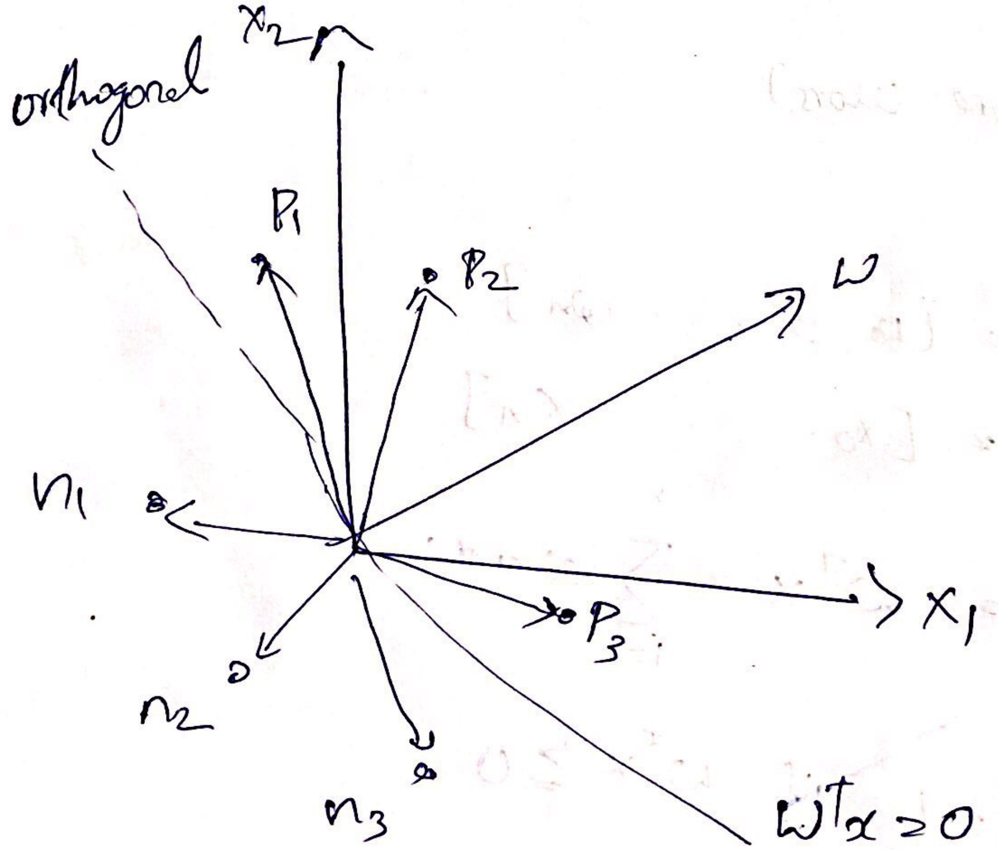
$$0 \quad \text{if } W^T x < 0$$

$[W^T x = 0]$ line divides into.

(α) Angle b/w w & x should be $90°$

$$\cos(\alpha) = \frac{W^T x}{||w|| \, |x||} = 0$$





~~(α) Angle of~~ P

(Positive points)

(α) Angle of $P_1, P_2, P_3 < 90$

(α') Angle of $n_1, n_2, n_3 > 90$

(negative points)

$x \in P$ and $w \cdot x < 0$ then

$w = w + x$

How this works !!

$w_{new} = w + x.$

$$\cos(\alpha_{new}) \propto w_{new}^T x$$
$$\propto (w + x)^T x$$
$$\propto w^T x + x^T x$$
$$\propto \cos\alpha + x^T x$$

$$\cos(\alpha_{new}) > \cos\alpha$$

$x \in N$ and $w \cdot x \geq 0$. then

$w = w - x$

$$\cos(\alpha_{new}) \propto w_{new}^T x$$
$$\propto (w - x)^T x$$
$$\propto w^T x - x^T x$$

$$\cos(\alpha_{new}) < \cos\alpha.$$

Lecture-6

Proof of Convergence of Perceptron Learning Algorithm

P & N are finite & linearly separable, the perceptron learning algorithm updates the weights vector w_t, a finite number of times,

---

- $x \in N$ then $-x \in P$

  $(\because w^T x < 0 \Rightarrow w^T(-x) \geq 0)$

- $P' = P \cup N^-$, for every element $p \in P'$, $w^T p \geq 0$

$P \leftarrow$ inputs label 1:

$N \leftarrow$ inputs label 0:

$N^- \leftarrow$ negations of N

$P' \leftarrow P \cup N^-$

Initialise w randomly

Scanned with CamScanner

while ! convergence do

random $p \in P'$:

$$p \leftarrow \frac{p}{||p||}$$

if $w.p < 0$ then

$w = w + p$:

end

end

$w^* \rightarrow$ solution vector

$(2^2)^n$

$(2^2)^3$

Lecture-7 2.7

Linearly Separable Boolean Functions

2.8

Representation Power of Network of Perceptrons