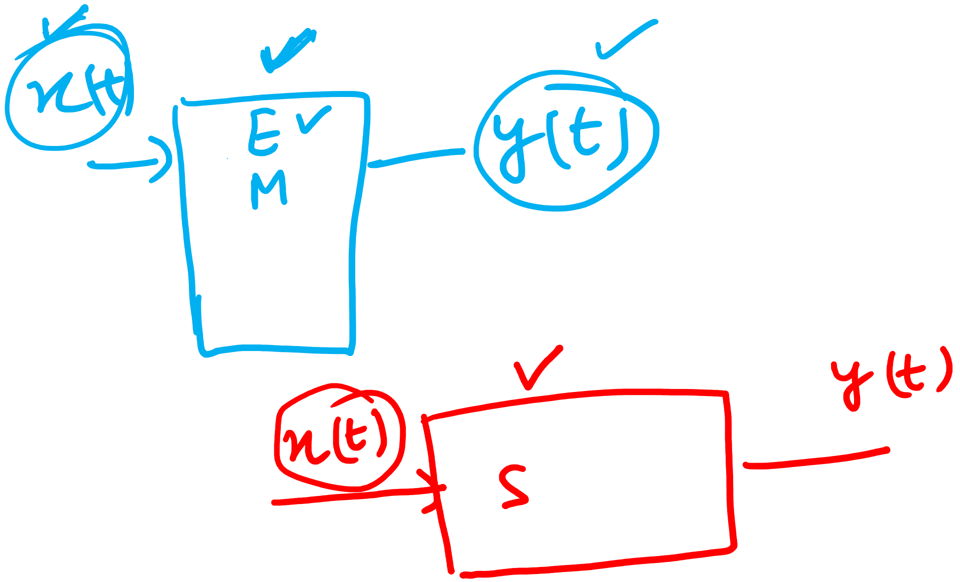
INTRODUCTION
Part-2

SIGNAL AND SYSTEM

ICICC03

Classification of systems :-

- Mathematical model of physical process/ discrete that describe the output signal for any given input signal.



Instantaneous (memoryless) and dynamic (with memory) systems

$x(t) \rightarrow y(t)$

$y(t) = k\,x(t)$

$V(t) = R\,i(t)$

$y(n) = k\,x(n)$

$y(t_0)$

$x(t_0)$

- The output at any instant *t* depends only on its input at that instant. In resistive networks, for example, any output of the network at some instant *t* depends only on the input at the instant *t*.
- Otherwise, the system is said to be *dynamic* (or a system with memory). A system whose response at *t* is completely determined by the input signals over the past *T* seconds [interval from (*t* – *T*) to *t*] is a *finite-memory system* with a memory of *T* seconds.
- Eg: Networks containing inductive and capacitive elements generally have infinite memory because the response of such networks at any instant *t* is determined by their inputs over the entire past (– ∞, *t*). This is true for the *RC* circuit.

$$V(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

Causal and noncausal systems

$y(t) \propto x(t), x(t-t_0)$

- The value of the output at the present instant depends only on the past and present values of input
- The input *x*(*t*), not on its future values. A system that violates the condition of causality is called a *noncausal* (or *anticipative*) system.

* $y(t)$ depends only on PAST values of $x(t)$ Not the future val

$$\boxed{y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau}$$

$\tau \leq t$

* $y(t) \propto$ future of input signal

$$\boxed{y(n) = \frac{1}{2N+1} \sum_{k=-N}^{N} x(n-k)},$$

$y(n) = x(n-N), x(n-N+1) \ldots x(n+N)$

$$y(t) = x(t-2) + x(t+2) \qquad (1.46)$$

For the input $x(t)$ illustrated in Fig. 1.30a, the output $y(t)$, as computed from Eq. (1.46) (shown in Fig. 1.30b), starts even before the input is applied. Equation (1.46) shows that $y(t)$, the output at t, is given by the sum of the input values 2 seconds before and 2 seconds after t (at $t - 2$ and $t + 2$, respectively). But if we are operating the system in real time at t, we do not know what the value of the input will be 2 seconds later. Thus it is impossible to implement this system in real time. For this reason, noncausal systems are unrealizable in *real time*.

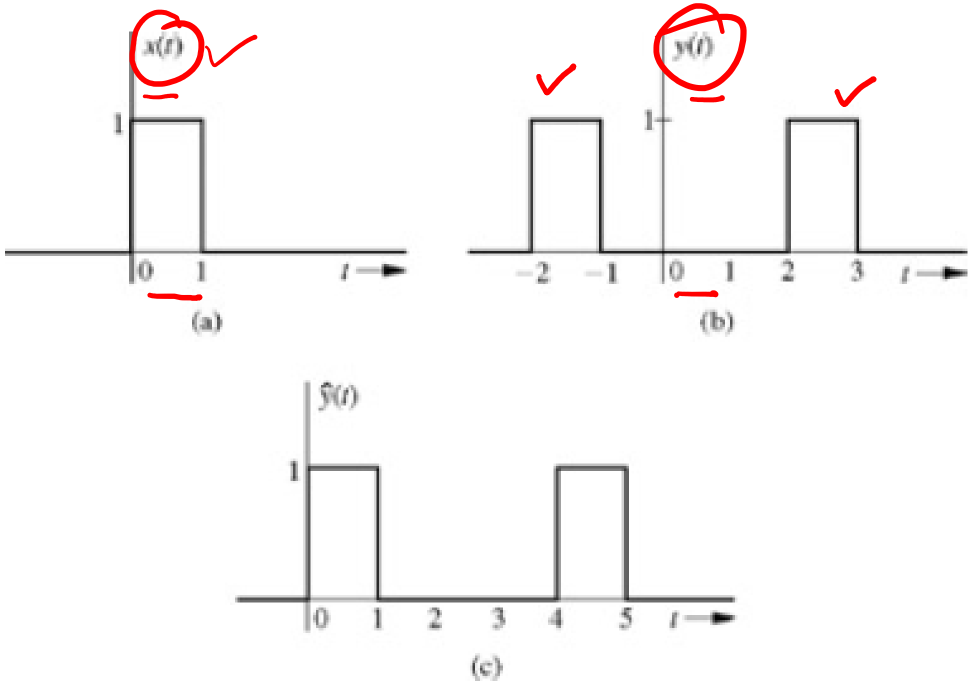


Figure 1.30: A noncausal system and its realization by a delayed causal system.

① ADDITIVITY ✓ +
② HomoGenty ✓

1.7-1 Linear and Nonlinear Systems ✓

THE CONCEPT OF LINEARITY

A system whose output is proportional to its input is an *example* of a linear system. But linearity implies more than this; it also implies the *additivity property*: that is, if several inputs are acting on a system, then the total effect on the system due to all these inputs can be determined by considering one input at a time while assuming all the other inputs to be zero. The total effect is then the sum of all the component effects. This property may be expressed as follows: for a linear system, if an input x_1 acting alone has an effect y_1, and if another input x_2, also acting alone, has an effect y_2, then, with both inputs acting on the system, the total effect will be $y_1 + y_2$. Thus, if

$$x_1 \longrightarrow y_1 \quad \text{and} \quad x_2 \longrightarrow y_2 \tag{1.37}$$

e.g → $y(t) = \frac{d\,x(t)}{dt}$

Linear

then for all x_1 and x_2

$$x_1 + x_2 \longrightarrow y_1 + y_2 \tag{1.38}$$

In addition, a linear system must satisfy the *homogeneity* or scaling property, which states that for arbitrary real or imaginary number k, if an input is increased k-fold, the effect also increases k-fold. Thus, if

$x \longrightarrow y$

then for all real or imaginary k

$$kx \longrightarrow ky \qquad k_1 x_1 \rightarrow k_1 y_1, \quad k_2 x_2 \rightarrow k_2 y_2 \tag{1.39}$$

e.g → $y(t) = 2x^2(t)$ ✓

Non linear

Thus, linearity implies two properties: homogeneity (scaling) and additivity.[†] Both these properties can be combined into one property (*superposition*), which is expressed as follows: If

$$x_1 \longrightarrow y_1 \quad \text{and} \quad x_2 \longrightarrow y_2$$

then for all values of constants k_1 and k_2,

$$\boxed{k_1 x_1 + k_2 x_2 \longrightarrow k_1 y_1 + k_2 y_2} \tag{1.40}$$

This is true for all x_1 and x_2.

$x(t) \longrightarrow y(t)$

$x(t-\tau) = y(t-\tau)$

Time-Invariant and Time-Varying Systems

- Systems whose parameters do not change with time are *time-invariant* (also *constant-parameter*) systems. For such a system, if the input is delayed by T seconds, the output is the same as before but delayed by T (assuming initial conditions are also delayed by T).

RC Day 1

RC Day 2

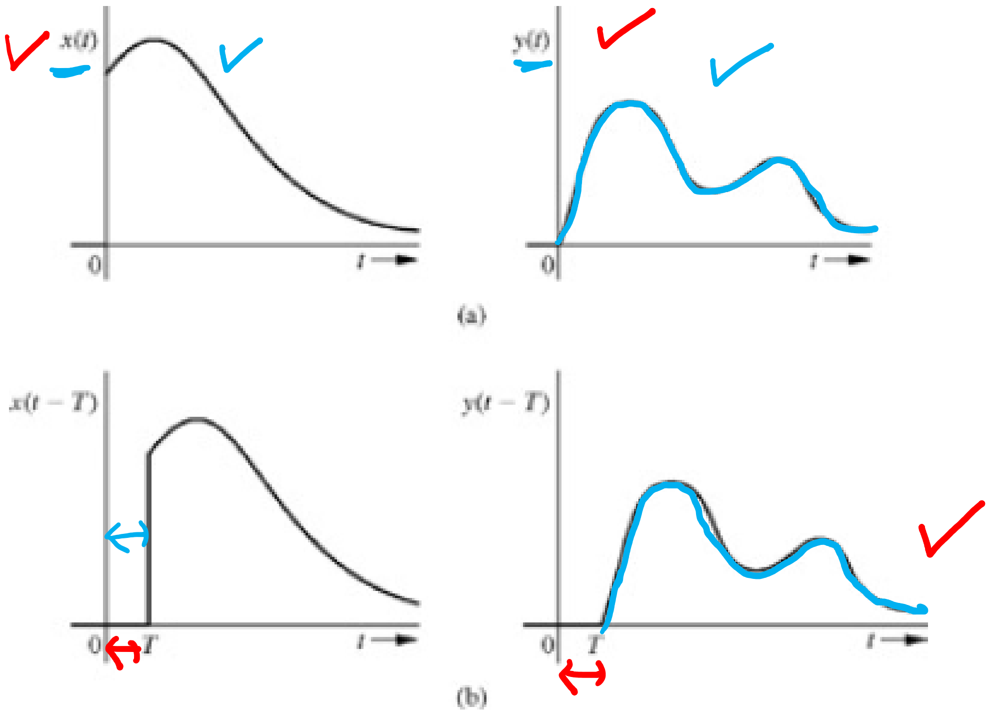


Figure 1.28: Time-invariance property.

Linear Time Invariant System LTI

Linear + Time Invariant →

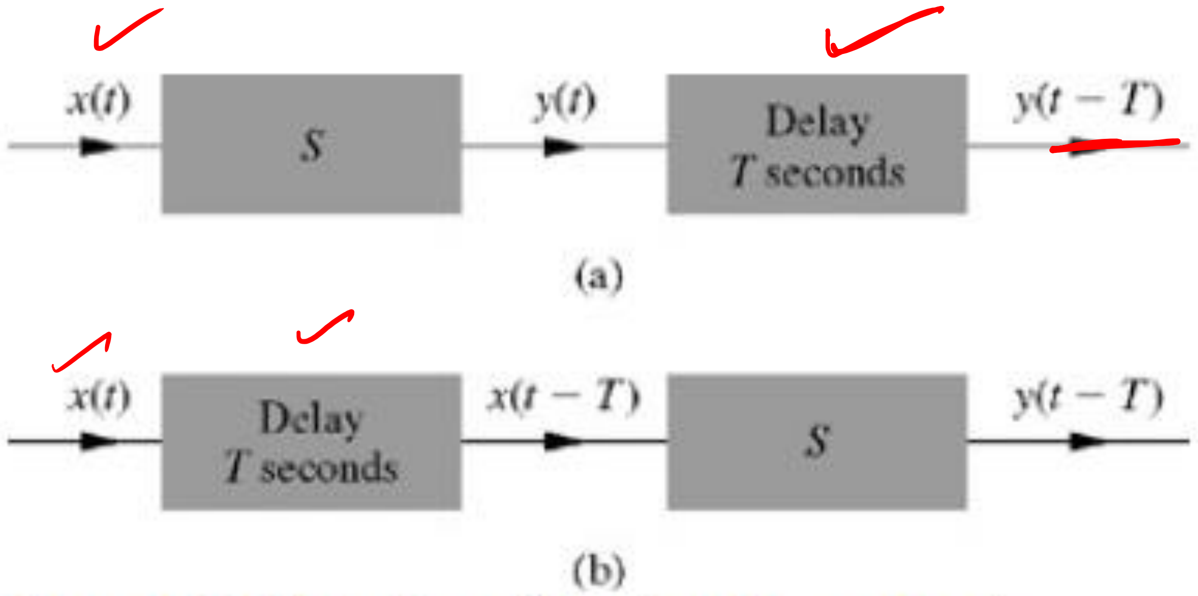


Figure 1.29: Illustration of time-invariance property.

Stable and unstable systems

Stability of system is very imp

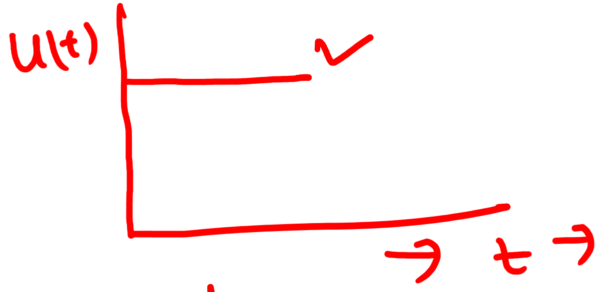
BIBO → Bounded input x_1 x_2
Bounded output

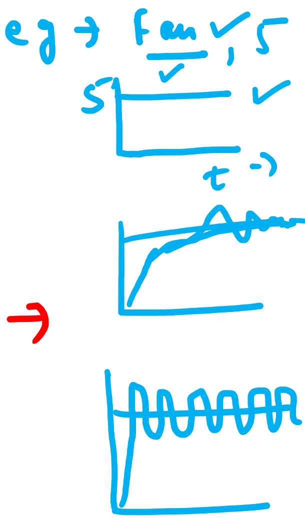
I/P

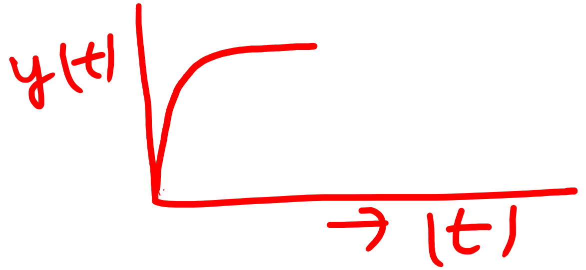
$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau$$

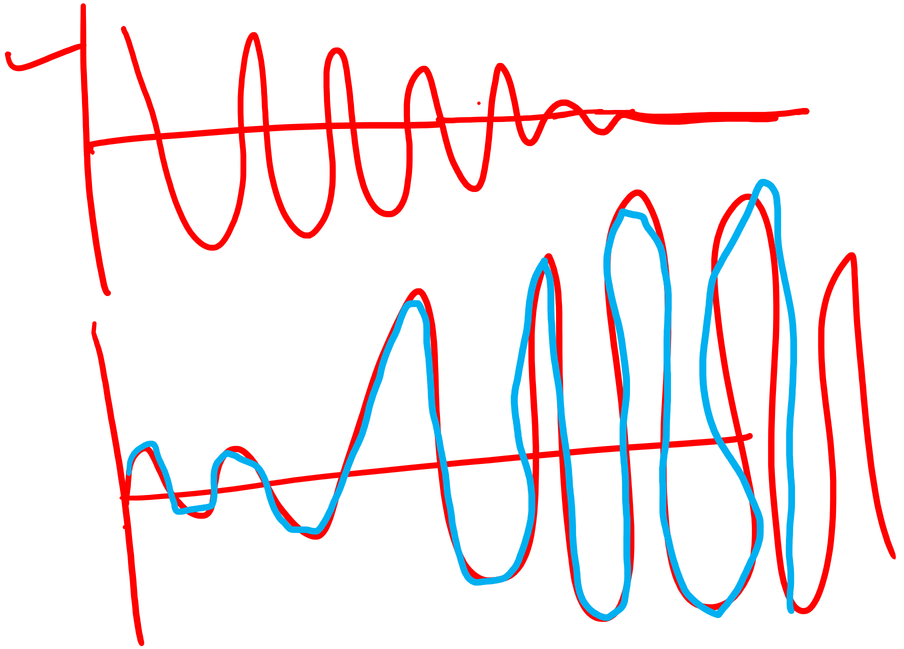
$$|x(\tau)| \le C_1$$

$$|y(t)| \equiv \left| \int_{t-T}^{t} x(\tau)\, d\tau \right| \le C_2$$









$x(t) \rightarrow x(t-t_0)$

Example 1.14

Consider the continuous-time system defined by

$$\boxed{y(t) = \sin[x(t)].} \tag{1.114}$$

To check that this system is time invariant, we must determine whether the time-invariance property holds for *any* input and *any* time shift t_0. Thus, let $x_1(t)$ be an arbitrary input to this system, and let

$$y_1(t) = \sin[x_1(t)] \tag{1.115}$$

be the corresponding output. Then consider a second input obtained by shifting $x_1(t)$ in time:

$$x_2(t) = x_1(t - t_0). \tag{1.116}$$

The output corresponding to this input is

$$y_2(t) = \sin[x_2(t)] = \sin[x_1(t - t_0)]. \tag{1.117}$$

Similarly, from eq. (1.115),

$$y_1(t - t_0) = \sin[x_1(t - t_0)]. \tag{1.118}$$

Comparing eqs. (1.117) and (1.118), we see that $y_2(t) = y_1(t - t_0)$, and therefore, this system is time invariant.

Consider the system

$$\boxed{y(t) = x(2t).} \qquad (1.120)$$

This system represents a time scaling. That is, $y(t)$ is a time-compressed (by a factor of 2) version of $x(t)$. Intuitively, then, any time shift in the input will also be compressed by a factor of 2, and it is for this reason that the system is not time invariant. To demonstrate this by counterexample, consider the input $x_1(t)$ shown in Figure 1.47(a) and the resulting output $y_1(t)$ depicted in Figure 1.47(b). If we then shift the input by 2—i.e., consider $x_2(t) = x_1(t-2)$, as shown in Figure 1.47(c)—we obtain the resulting output

$y(t) = x(2t)$

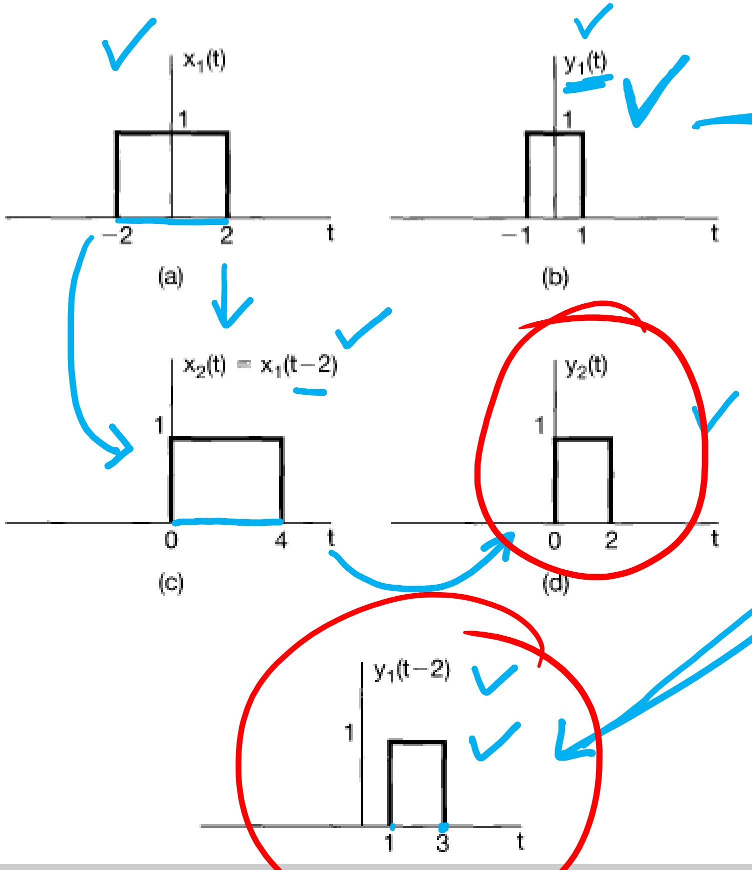


Figure 1.47 (a) The input $x_1(t)$ to the system in Example 1.16; (b) the output $y_1(t)$ corresponding to $x_1(t)$; (c) the shifted input $x_2(t) = x_1(t-2)$; (d) the output $y_2(t)$ corresponding to $x_2(t)$; (e) the shifted signal $y_1(t-2)$. Note that $y_2(t) \neq y_1(t-2)$, showing that the system is not time invariant.

$y_2(t) = y(t-1)$ ✓

$y_2(t) = x_2(2t)$ shown in Figure 1.47(d). Comparing Figures 1.47(d) and (e), we see that $y_2(t) \neq y_1(t-2)$, so that the system is not time invariant. (In fact, $\boxed{y_2(t) = y_1(t-1),}$ so that the output time shift is only half as big as it should be for time invariance, due to the time compression imparted by the system.)

Example 1.17

Consider a system S whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = tx(t)$$

To determine whether or not S is linear, we consider two arbitrary inputs $x_1(t)$ and $x_2(t)$.

Additivity

$$\begin{cases} x_1(t) \rightarrow y_1(t) = tx_1(t) \\ x_2(t) \rightarrow y_2(t) = tx_2(t) \end{cases}$$

$y_1 + y_2$
$= t\,(x_1(t) + x_2(t))$

Let $x_3(t)$ be a linear combination of $x_1(t)$ and $x_2(t)$. That is,

$$x_3(t) = ax_1(t) + bx_2(t)$$

where a and b are arbitrary scalars. If $x_3(t)$ is the input to S, then the corresponding output may be expressed as

$$\begin{aligned} y_3(t) &= tx_3(t) \\ &= t(ax_1(t) + bx_2(t)) \\ &= atx_1(t) + btx_2(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

$\rightarrow$ Homogenity

We conclude that the system S is linear.

Let us apply the linearity-checking procedure of the previous example to another system S whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = x^2(t)$$

Defining $x_1(t)$, $x_2(t)$, and $x_3(t)$ as in the previous example, we have

$$x_1(t) \rightarrow y_1(t) = x_1^2(t)$$
$$x_2(t) \rightarrow y_2(t) = x_2^2(t)$$

$$\Rightarrow y_1 + y_2 = x_1^2(t) + x_2^2(t)$$

and

$$(x_1 + x_2) \Rightarrow y_1 + y_2 = (x_1 + x_2)^2$$
$$= x_1^2 + x_2^2 + 2x_1x_2$$

$$\begin{aligned} x_3(t) \rightarrow y_3(t) &= x_3^2(t) \\ &= (ax_1(t) + bx_2(t))^2 \\ &= a^2x_1^2(t) + b^2x_2^2(t) + 2abx_1(t)x_2(t) \\ &= a^2y_1(t) + b^2y_2(t) + 2abx_1(t)x_2(t) \end{aligned}$$

Clearly, we can specify $x_1(t)$, $x_2(t)$, a, and b such that $y_3(t)$ is not the same as $ay_1(t) + by_2(t)$. For example, if $x_1(t) = 1$, $x_2(t) = 0$, $a = 2$, and $b = 0$, then $y_3(t) = (2x_1(t))^2 = 4$, but $2y_1(t) = 2(x_1(t))^2 = 2$. We conclude that the system S is not linear.

Is it a linear system ?

$x_1 \longrightarrow y_1 = 2x_1(n)+3$

$x_2 \longrightarrow y_2 =$

$$y[n] = 2x[n] + 3.$$

Example 1.20

Consider the system

$$y[n] = 2x[n] + 3. \tag{1.132}$$

This system is not linear, as can be verified in several ways. For example, the system violates the additivity property: If $x_1[n] = 2$ and $x_2[n] = 3$, then

$$x_1[n] \rightarrow y_1[n] = 2x_1[n] + 3 = 7, \tag{1.133}$$

$$x_2[n] \rightarrow y_2[n] = 2x_2[n] + 3 = 9. \tag{1.134}$$

However, the response to $x_3[n] = x_1[n] + x_2[n]$ is

$$y_3[n] = 2[x_1[n] + x_2[n]] + 3 = 13, \tag{1.135}$$

which does not equal $y_1[n] + y_2[n] = 16$. Alternatively, since $y[n] = 3$ if $x[n] = 0$, we see that the system violates the "zero-in/zero-out" property of linear systems given in eq. (1.125).

It may seem surprising that the system in the above example is nonlinear, since eq. (1.132) is a linear equation. On the other hand, as depicted in Figure 1.48, the output of this system can be represented as the sum of the output of a linear system and another signal equal to the *zero-input response* of the system. For the system in eq. (1.132), the linear system is

$$x[n] \rightarrow 2x[n],$$

and the zero-input response is

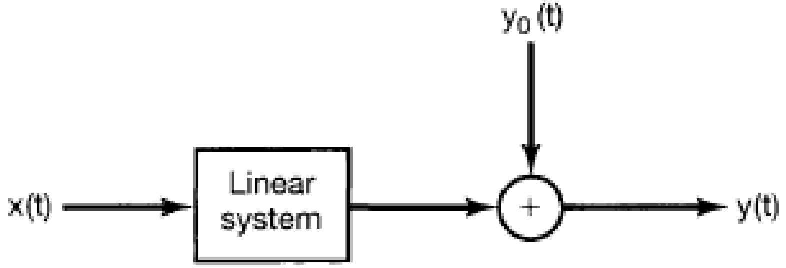


Figure 1.48 Structure of an incrementally linear system. Here, $y_0[n]$ is the zero-input response of the system.

There are, in fact, large classes of systems in both continuous and discrete time that can be represented as in Figure 1.48—i.e., for which the overall system output consists of the superposition of the response of a linear system with a zero-input response. As shown in Problem 1.47, such systems correspond to the class of *incrementally linear systems*—i.e., systems in continuous or discrete time that respond linearly to *changes* in the input. In other words, the *difference* between the responses to any two inputs to an incrementally linear system is a linear (i.e., additive and homogeneous) function of the *difference* between the two inputs. For example, if $x_1[n]$ and $x_2[n]$ are two inputs to the system specified by eq. (1.132), and if $y_1[n]$ and $y_2[n]$ are the corresponding outputs, then

$$y_1[n] - y_2[n] = 2x_1[n] + 3 - \{2x_2[n] + 3\} = 2\{x_1[n] - x_2[n]\}. \qquad (1.136)$$